Theorem 2.2.1 *Let $G = (V, E)$ be a graph with n vertices and e edges. Then G contains a bipartite subgraph with at least $e/2$ edges.*

Proof. Let $T \subseteq V$ be a random subset given by $\Pr[x \in T] = 1/2$, these choices being mutually independent. Set $B = V - T$. Call an edge $\{x, y\}$ crossing if exactly one of x, y is in T. Let X be the number of crossing edges. We decompose

$$X = \sum_{\{x,y\} \in E} X_{xy} ,$$

where X_{xy} is the indicator random variable for $\{x, y\}$ being crossing. Then

$$\mathrm{E}[X_{xy}] = \frac{1}{2}$$

as two fair coin flips have probability $1/2$ of being different. Then

$$\mathrm{E}[X] = \sum_{\{x,y\} \in E} \mathrm{E}[X_{xy}] = \frac{e}{2} .$$