# پاسخنامه تئوری

تابستان 87 - دوره ی 21 - امتحان اول

1) تست دوم :

$$\begin{cases} m v_0 = (m+M)v \rightarrow \boxed{v = \dfrac{m v_0}{m+M}} \\ -v_0^2 = -2\left(\mu g + \dfrac{\mu m g}{M}\right)x \rightarrow \boxed{x = \dfrac{M v_0^2}{2\mu g (m+M)}} \end{cases}$$

$$, \quad -v_0 = -\left(\mu g + \frac{\mu m g}{M}\right)t \rightarrow \boxed{t = \frac{M v_0}{\mu g (m+M)}}$$

تست اول :

$$2\left(\mu g + \frac{\mu m g}{M}\right)L = -v_{0\,min}^2 \rightarrow \boxed{v_{0\,min} = \sqrt{\frac{2\mu g L (m+M)}{M}}}$$

$$L = -\frac{\mu g (M+m)}{2M}t^2 + v_0 t \rightarrow \left(\frac{\mu g (m+M)}{2M}\right)t^2 - v_0 t + L = 0 \rightarrow t = \frac{v_0 \pm \sqrt{v_0^2 - \dfrac{2\mu g L (m+M)}{M}}}{\dfrac{\mu g (m+M)}{M}} \rightarrow$$

$$\boxed{t = \frac{M}{\mu g (m+M)}\left(v_0 - \sqrt{v_0^2 - \frac{2\mu g L (m+M)}{M}}\right)}$$

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$$x_2 = \dot{x}_{1\,(t-T)}T + x_{1\,(t-T)} \rightarrow x_{2(t)} = a\omega\sqrt{\frac{2h}{g}}\cos(\omega(t-T)) + a\sin(\omega(t-T)) \rightarrow$$

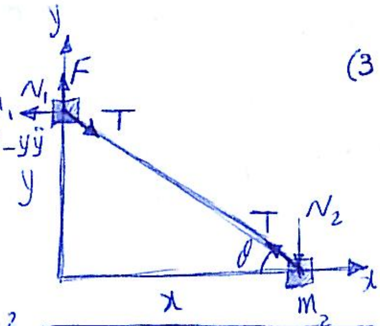
$$T = \sqrt{\frac{2h}{g}} \quad , \quad \dot{x}_{1(t)} = a\omega\cos(\omega t)$$

$$\boxed{x_{2(t)} = a\omega\sqrt{\frac{2h}{g}}\cos\left(\omega\left(t - \sqrt{\frac{2h}{g}}\right)\right) + a\sin\left(\omega\left(t-\sqrt{\frac{2h}{g}}\right)\right)}$$

یا

$$\boxed{x_{2(t)} = a\sqrt{1 + \frac{2\omega^2 h}{g}}\,\sin\left(\omega\left(t - \sqrt{\frac{2h}{g}}\right) + \tan^{-1}\left(\omega\sqrt{\frac{2h}{g}}\right)\right)}$$

(3




$$\begin{cases} F - T\sin\theta = m_1 \ddot{y} \\ -T\cos\theta = m_2 \ddot{x} \end{cases} \rightarrow \tan\theta = \frac{m_1\ddot{y} - F}{m_2 \ddot{x}}$$

$$\dot{x}\cos\theta = -\dot{y}\sin\theta \rightarrow x\dot{x} = -y\dot{y} \rightarrow \dot{x}^2 + x\ddot{x} = -y\ddot{y} - \dot{y}^2 \rightarrow L\cos\theta\,\ddot{x} + v_0^2 = -v_0^2\cot^2\theta - y\ddot{y}$$

$$\rightarrow \ddot{x} = -\ddot{y}\tan\theta - \frac{v_0^2}{L\cos\theta\sin^2\theta} \rightarrow \frac{\sin\theta}{\cos\theta}\left(\ddot{y}\frac{\sin\theta}{\cos\theta} + \frac{v_0^2}{L\cos\theta\sin^2\theta}\right) = \frac{F - m_1\ddot{y}}{m_2} \rightarrow$$

$$\ddot{y}\tan^2\theta + \frac{v_0^2}{L\sin\theta\cos^2\theta} = \frac{F}{m_2} - \frac{m_1}{m_2}\ddot{y} \rightarrow \ddot{y}\left(\frac{m_2\tan^2\theta + m_1}{m_2}\right) = \frac{F L\cos^2\theta\sin\theta - m_2 v_0^2}{m_2 L\cos^2\theta\sin\theta} \rightarrow \boxed{\ddot{y} = \frac{F L\cos^2\theta\sin\theta - m_2 v_0^2}{\sin\theta(m_1 + m_2\tan^2\theta)L\cos^2\theta}}$$

$$\ddot{x} = \frac{-F\sin\theta}{(m_1 + m_2\tan^2\theta)\cos\theta} + \frac{m_2 v_0^2}{(m_1 + m_2\tan^2\theta)L\cos^3\theta} - \frac{v_0^2}{L\cos\theta\sin^2\theta} \rightarrow$$

$$\ddot{x} = \frac{m_2 v_0^2\sin^2\theta - F L\sin^3\theta\cos^2\theta - (m_1 + m_2\tan^2\theta)\cos^2\theta\, v_0^2}{(m_1 + m_2\tan^2\theta)L\sin^2\theta\cos^3\theta} \rightarrow \boxed{\ddot{x} = -\frac{F L\sin^3\theta + m_1 v_0^2}{(m_1 + m_2\tan^2\theta)L\sin^2\theta\cos\theta}} \rightarrow$$

$$\boxed{T = \frac{m_2\left(F L\sin^3\theta + m_1 v_0^2\right)}{(m_1 + m_2\tan^2\theta)L\sin^2\theta\cos^2\theta}} \xrightarrow{\theta = \frac{\pi}{6}} \boxed{T = \frac{2m_2\left(F L + 8 m_1 v_0^2\right)}{(3m_1 + m_2)L}}$$

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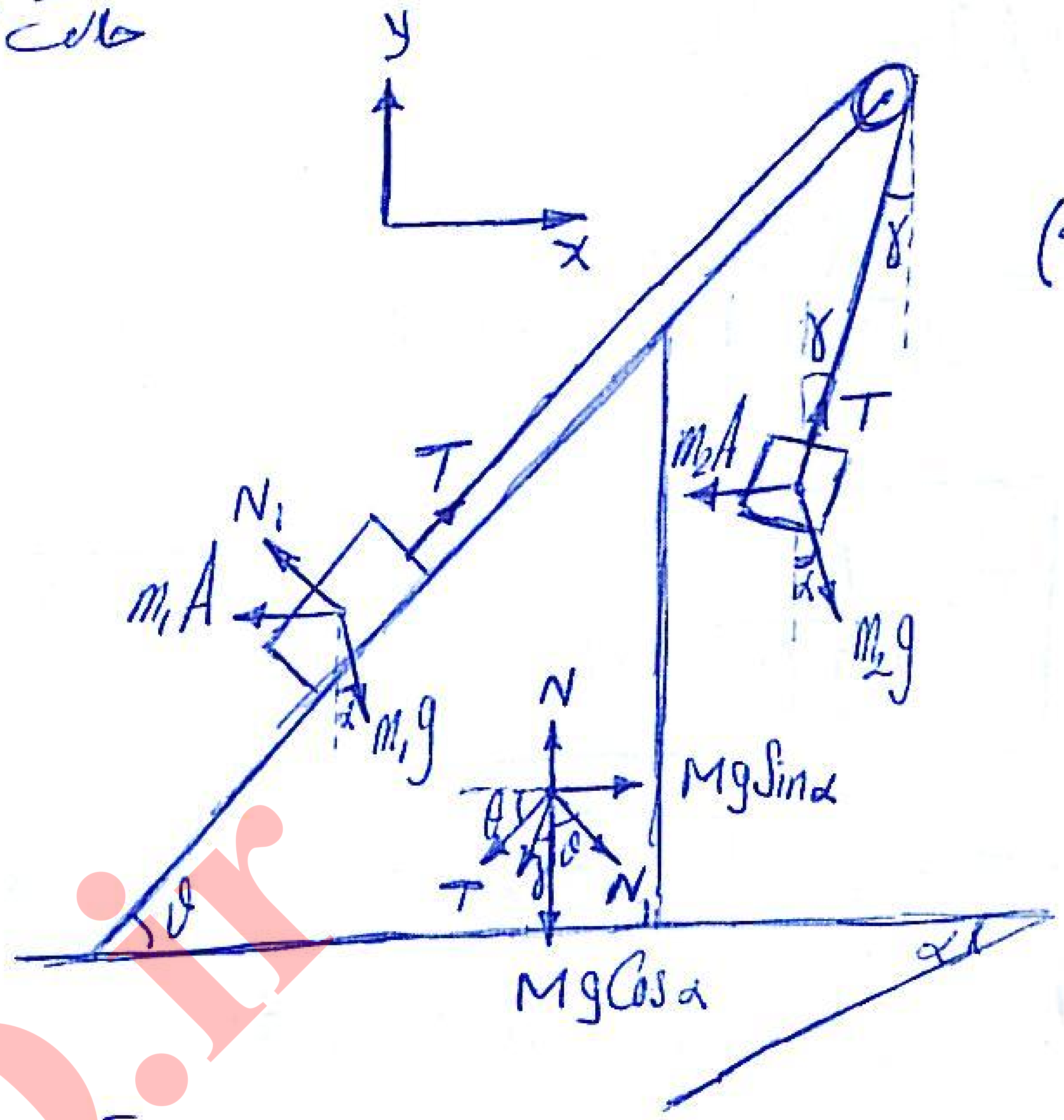
$$\boxed{\ddot{x} = -\frac{\sqrt{3}\,(FL+8m_1V_0)}{(3m_1+m_2)L}}$$

$$\boxed{\ddot{y} = \frac{3FL-8m_2V_0^2}{(3m_1+m_2)L}}$$

این سوال بدون داشتن یا هم باید حل شود.

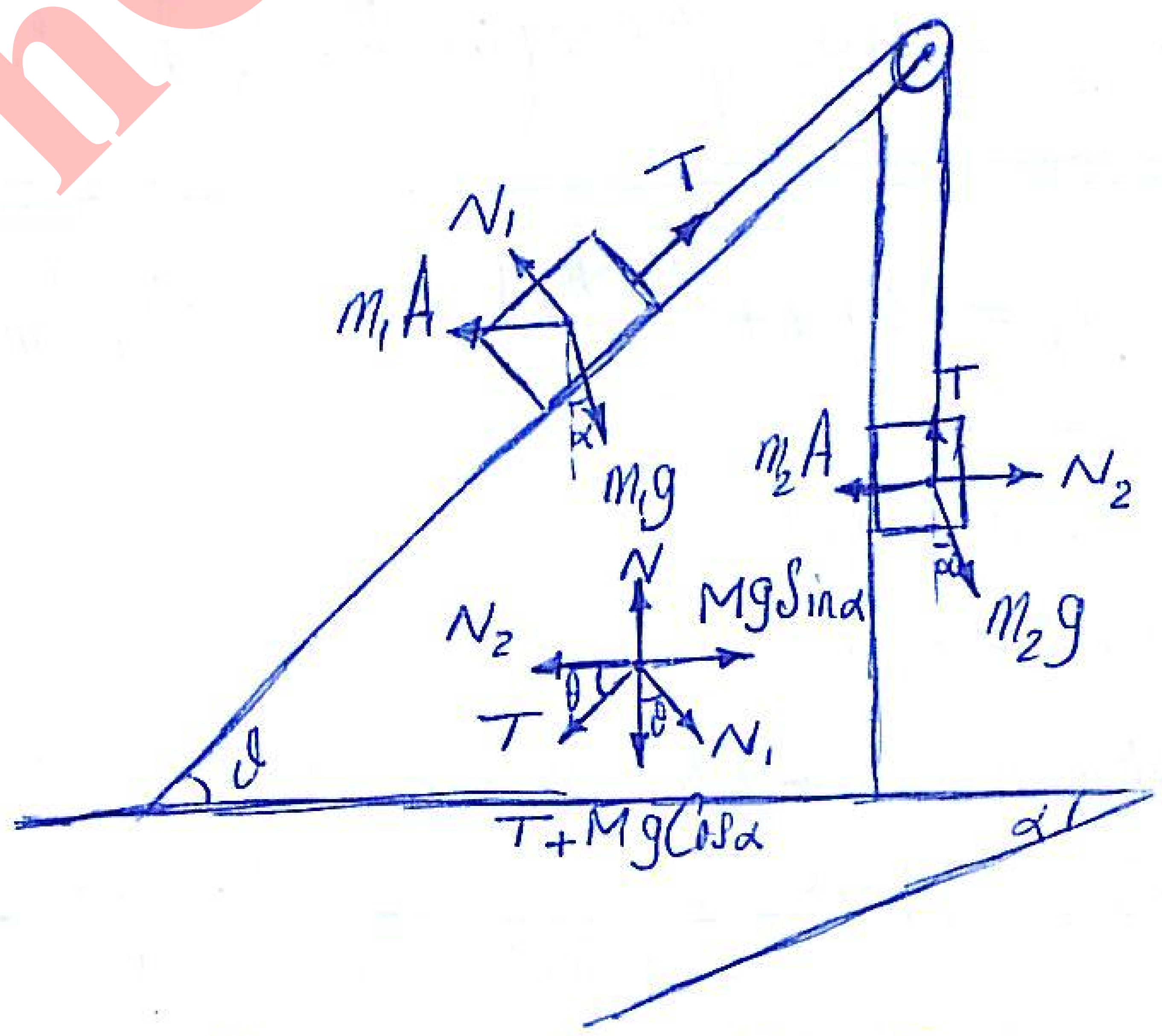
4)

حالت اول:




$$\begin{cases} Mg\sin\alpha + N_1\sin\theta - T(\cos\theta+\sin\gamma) = MA \\ T\cos\gamma - m_2 g\cos\alpha = m_2\ddot{y}_2 \\ T\sin\gamma + m_2 g\sin\alpha - m_2A = m_2\ddot{x}_2 \\ T\sin\theta + N_1\cos\theta - m_1 g\cos\alpha = m_1\ddot{y}_1 \\ T\cos\theta - N_1\sin\theta - m_1A - m_1 g\sin\alpha = m_1\ddot{x}_1 \\ \ddot{y}_1 = \ddot{x}_1\tan\theta \\ \ddot{y}_1^2+\ddot{x}_1^2 = \ddot{y}_2^2+\ddot{x}_2^2 \\ \ddot{x}_2 = \ddot{y}_2\tan\gamma \end{cases}$$

حالت دوم:




$$\begin{cases} Mg\sin\alpha - N_2 - T\cos\theta + N_1\sin\theta = MA \\ T - m_2 g\cos\alpha = m_2\ddot{y}_2 \\ N_2 + m_2 g\sin\alpha = m_2A \\ T\sin\theta + N_1\cos\theta - m_1 g\cos\alpha = m_1\ddot{y}_1 \\ T\cos\theta - N_1\sin\theta - m_1A + m_1 g\sin\alpha = m_1\ddot{x}_1 \\ \ddot{y}_1 = \ddot{x}_1\tan\theta \\ \ddot{y}_2^2 = \ddot{x}_1^2+\ddot{y}_1^2 \end{cases}$$

امتحان دوم - تابستان 87 - دوره ی 21

1) الف)

$$Q_{in} = 4\pi\rho_0\int_0^a\left(1-\frac{r^2}{a^2}\right)r^2dr = 4\pi\rho_0\left(\frac{a^3}{3}-\frac{a^3}{5}\right) \longrightarrow \boxed{Q_{in} = \frac{8\pi\rho_0 a^3}{15}}$$

ب)

ج)

$$E_{out} = \frac{Q_{in}}{\varepsilon_0\times 4\pi r^2} \longrightarrow \boxed{E_{out} = \left(\frac{2\rho_0 a^3}{15\varepsilon_0}\right)\left(\frac{1}{r^2}\right)}$$

د)

$$dV_0 = \rho_0(1-r^2)\times 4\pi r^2 dr\times\frac{1}{\ldots} \longrightarrow V_0 = \frac{\rho_0 a^2}{\ldots}\left(\frac{1}{2}-\frac{1}{4}\right) \longrightarrow \left|V_0 = \frac{\rho_0 a^2}{\ldots}\right|$$

باسمه تعالی

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ادامه ه)

$$x=(C+1)\sqrt{\frac{l^2(1+C^2+2C)}{4(1-C)^2}+\frac{Z_0^2(1+C^2-2C)}{(1+C)^2}-Z_0 l}\pm\left[\frac{l(C+1)}{2}+Z_0(1-C)\right]\longrightarrow$$

$$x=(C+1)\sqrt{\left(\frac{l(1+C)}{2(1-C)}-\frac{Z_0(1-C)}{(1+C)}\right)^2}\pm\left[\frac{l}{2}(C+1)+Z_0(1-C)\right]=\frac{l(C+1)}{2}\left(1+\frac{C+1}{1-C}\right)=\frac{l(1+C)}{-C+1}\longrightarrow\boxed{x=\frac{l(e^{\frac{lV_0}{kq}}+1)}{-e^{-\frac{lV_0}{kq}}+1}}$$

$$\longrightarrow x=\frac{l(1+e^{\frac{lV_0}{kq}})}{(e^{\frac{lV_0}{kq}}-1)}=l\frac{\frac{e^{\frac{lV_0}{2kq}}+e^{-\frac{lV_0}{2kq}}}{2}}{\frac{e^{\frac{lV_0}{2kq}}-e^{-\frac{lV_0}{2kq}}}{2}}\longrightarrow\boxed{x=l\coth\left(\frac{lV_0}{2kq}\right)}$$

و)

$$dF=\frac{kq_1q_2}{l_1l_2d}\left(\frac{\frac{l_1}{2}-x_2}{\sqrt{d^2+(\frac{l_1}{2}-x_2)^2}}dx_2+\frac{\frac{l_1}{2}+x_2}{\sqrt{d^2+(\frac{l_1}{2}+x_2)^2}}dx_2\right)\longrightarrow F=\frac{kq_1q_2}{2l_1l_2d}\left(-\int\frac{d\left(d^2+(\frac{l_1}{2}-x_2)^2\right)}{\sqrt{d^2+(\frac{l_1}{2}-x_2)^2}}+\int\frac{d\left((\frac{l_1}{2}+x_2)^2+d^2\right)}{\sqrt{d^2+(\frac{l_1}{2}+x_2)^2}}\right)$$

$$\longrightarrow F=\frac{kq_1q_2}{l_1l_2d}\left(\left.\sqrt{d^2+(\tfrac{l_1}{2}-x_2)^2}\right|_{l_2/2}^{-l_2/2}+\left.\sqrt{d^2+(\tfrac{l_1}{2}+x_2)^2}\right|_{-l_2/2}^{l_2/2}\right)\longrightarrow\boxed{F=\frac{2kq_1q_2}{l_1l_2d}\left(\sqrt{d^2+\left(\frac{l_1+l_2}{2}\right)^2}-\sqrt{d^2+\left(\frac{l_1-l_2}{2}\right)^2}\right)}$$

ز)

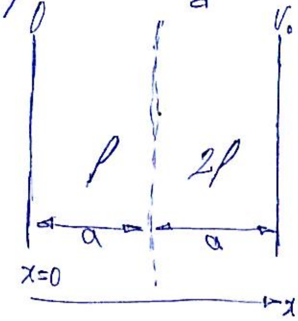
$$F=\frac{2kq_1q_2}{l_1l_2d}\left(\sqrt{l_2^2+2l_1l_2}-\sqrt{l_2^2-2l_1l_2}\right)=\frac{kq_1q_2}{l_1d}\left(1+\frac{l_1}{l_2}-1+\frac{l_1}{l_2}\right)\longrightarrow\boxed{F=\frac{2kq_1q_2}{l_2d}}$$

ح)

$$F=\frac{2kq_1q_2}{l_1l_2}\left(\sqrt{1+\left(\frac{l_1+l_2}{2d}\right)^2}-\sqrt{1+\left(\frac{l_1-l_2}{2d}\right)^2}\right)\longrightarrow F=\frac{2kq_1q_2}{l_1l_2}\left(\frac{(2l_1l_2+2l_1l_2)}{2\times4d^2}\right)\longrightarrow F=\frac{kq_1q_2}{d^2}$$

3)

$$\nabla\cdot E=\begin{cases}\frac{\rho}{\varepsilon_0} & 0<x<a\\ \frac{2\rho}{\varepsilon_0} & a<x<2a\\ 0 & x<0 \text{ یا } x>2a\end{cases}\longrightarrow E=\begin{cases}\frac{\rho}{\varepsilon_0}x+A & 0<x<a\\ \frac{2\rho}{\varepsilon_0}x+B & a<x<2a\\ C & x<0\\ D & x>2a\end{cases}\longrightarrow$$

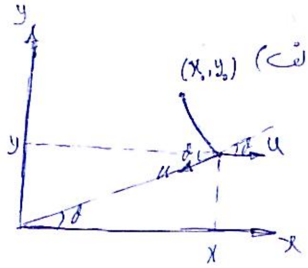



$$V=\begin{cases}\frac{\rho x^2}{2\varepsilon_0}+Ax+A' & 0\le x\le a\\ \frac{\rho x^2}{\varepsilon_0}+Bx+B' & a\le x\le 2a\\ Cx+C' & x\le 0\\ Bx+B' & x\ge 2a\end{cases}$$

شرایط مرزی: 1) $V_{(0)}=0$ ، 2) $V_{(2a)}=V_0$ 3) $V_{(2a)}=V_{(2a)}$ 4) $V_{(0)}=V_{(0)}$ 5) $V_{(a)}=V_{(a)}$

بسمه تعالی

4) الف) $v_{(x)} = -u\cos\theta + u$ , $v_y = -u\sin\theta \to \boxed{v_y = -u\sin\theta}$ , $\boxed{v_x = u(1-\cos\theta)}$




ب) $\boxed{v_{(r)} = -u(1-\cos\theta)}$ , $\boxed{v_\theta = -u\sin\theta}$

ج) $\frac{dr}{r\,d\theta} = \frac{1-\cos\theta}{\sin\theta} \to \int\frac{dr}{r} = -\int\frac{d\cos\theta}{1-\cos^2\theta} - \int\frac{d\sin\theta}{\sin\theta} \to \ln\left(\frac{r}{r_0}\right) = -\left.\tanh^{-1}(\cos\theta)\right|_{\theta_0}^{\theta} - \left.\ln(\sin\theta)\right|_{\theta_0}^{\theta}$

$\to \ln\left(\frac{r}{r_0}\right) = -\frac{1}{2}\ln\left(\frac{(1-\cos\theta)(1+\cos\theta_0)}{(1+\cos\theta)(1-\cos\theta_0)}\right) - \ln\left(\frac{\sin\theta}{\sin\theta_0}\right) \to \boxed{r = r_0\frac{\sin\theta_0}{\sin\theta}\sqrt{\frac{(1-\cos\theta)(1+\cos\theta_0)}{(1-\cos\theta_0)(1+\cos\theta)}}} \to$

$r = r_0\frac{\sin\theta_0}{\sin\theta}\times\frac{(1+\cos\theta_0)}{\sin\theta_0}\times\frac{\sin\theta}{1+\cos\theta} \to \boxed{r_{(\theta)} = r_0\frac{1+\cos\theta_0}{1+\cos\theta}}$

د) $\dot r_{(\theta)} = r_0(1+\cos\theta_0)\times\frac{\sin\theta}{(1+\cos\theta)^2}\times\frac{d\theta}{dt} = -u(1-\cos\theta) \to dt = -\frac{r_0(1+\cos\theta_0)}{u}\cdot\frac{\sin\theta}{(1+\cos\theta)^2(1-\cos\theta)}d\theta = -\frac{r_0(1+\cos\theta_0)}{u}\cdot\frac{d\theta}{(1+\cos\theta)\sin\theta}$

$\simeq -\frac{r_0(1+\cos\theta_0)}{u}\cdot\frac{1+\frac{\theta^2}{4}}{2\theta} \to t \simeq -\frac{r_0(1+\cos\theta_0)}{2u}\times\left(\ln\left(\frac{\theta}{\alpha}\right) + \left.\frac{\theta^2}{8}\right|_{\alpha}^{\theta}\right)$ , $\theta\to0$ , $\alpha\ll1 \to \boxed{t=\infty}$

5) الف) ب) ج)

①: $F_{(x)} = N\sin\alpha = mg\sin\alpha\cos\alpha = ma_{(x)} \to \boxed{v_{(x)} = (g\sin\alpha\cos\alpha)t}$ , $\boxed{t_1 = \sqrt{\frac{2b}{g\sin\alpha\cos\alpha}}}$ , $v_{(x)max} = \sqrt{2ga}\cos\alpha$

②: $0 \le t \le \sqrt{\frac{2a}{g}} : \boxed{v_{(x)} = 0}$ , $\sqrt{\frac{2a}{g}} \le t \le \frac{b+2a}{\sqrt{2ag}} : \boxed{v_{(x)} = \sqrt{2ag}}$

③: $0 \le t \le 2\sqrt{\frac{a}{g}} : \boxed{v_{(x)} = 0}$ , $2\sqrt{\frac{a}{g}} \le t \le \sqrt{\frac{2a}{g}}\left(\sqrt2 + \frac{1-\sqrt2}{\sin\alpha}\right) : \boxed{v_{(x)} = (2\sqrt{ag} - g\sin\alpha\, t)\cos\alpha}$ ,

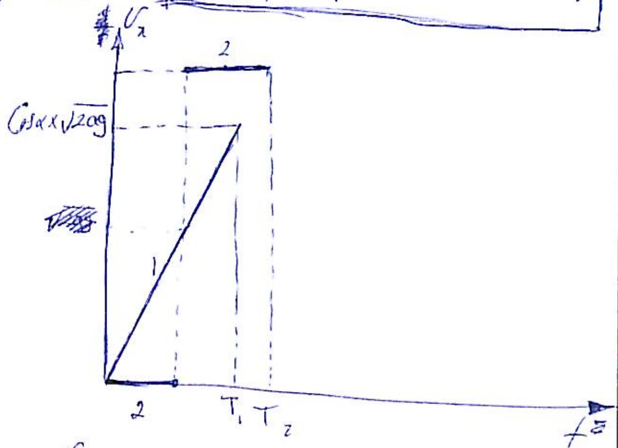
$-g\sin\alpha\cos\alpha\, t' = \sqrt{2ag}\cos\alpha - 2\sqrt{ag}\cos\alpha \to t' = \frac{\sqrt{2ag}(1-\sqrt2)}{g\sin\alpha} = \sqrt{\frac{2a}{g}}\frac{1-\sqrt2}{\sin\alpha} \to \boxed{T_3 = \sqrt{\frac{2a}{g}}\left(\sqrt2 + \frac{1-\sqrt2}{\sin\alpha}\right)}$

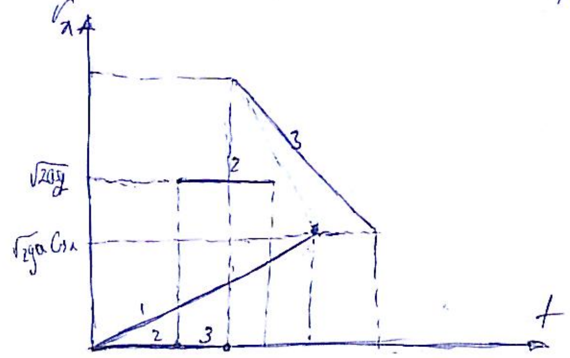
$\boxed{T_2 = \frac{b+2a}{\sqrt{2ag}}}$ ③: $\boxed{v_{(x)} = \left(2\sqrt{ag}(1+\sin\alpha) - g\sin\alpha\, t\right)\cos\alpha}$ , $\sin\alpha = \frac{a}{\sqrt{a^2+b^2}}$ , $\cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$

①: $\boxed{v_{(x)} = \frac{gab}{a^2+b^2}t}$ $v_{(x)} = \left(2\sqrt{ag}\left(1+\frac{a}{\sqrt{a^2+b^2}}\right) - \frac{ag}{\sqrt{a^2+b^2}}t\right)\times\frac{b}{\sqrt{a^2+b^2}} \to \boxed{v_{(x)} = \frac{\sqrt{ag}\,b}{a^2+b^2}\left(2(\sqrt{a^2+b^2}+a) - \sqrt{ag}\,t\right)}$

$\boxed{T_3 = \sqrt{\frac{2a}{g}}\left(\sqrt2 + \frac{(1-\sqrt2)\sqrt{a^2+b^2}}{a}\right)}$ , $\boxed{T_1 = \sqrt{\frac{2(a^2+b^2)}{ag}}}$ ,

$\cancel{T_2 \lessgtr T_1 \to \frac{b+2a}{\sqrt{2ag}} \lessgtr \frac{\sqrt{a^2+b^2}}{\sqrt{2ag}} \to b^2+4a^2+4ab \lessgtr a^2+b^2 \to T_2 > T_1}$



باسمه تعالی

تابستان 87 - آزمون دوم - دوره ی اول

۱) ب) $\frac{1}{r^2}\frac{\partial(r^2E_{in})}{\partial r}=\frac{\rho_0}{\varepsilon_0}\left(1-\frac{r^2}{a^2}\right)\longrightarrow r^2E_{in}=\frac{\rho_0 r^3}{\varepsilon_0}\left(\frac{1}{3}-\frac{r^2}{5a^2}\right)\longrightarrow \boxed{E_{in}=\frac{\rho_0 r}{\varepsilon_0}\left(\frac{1}{3}-\frac{r^2}{5a^2}\right)}$

ج) $V_{(r)}-V_0=-\int_0^r\frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{r^3}{5a^2}\right)dr\longrightarrow V_{(r)}=\frac{\rho_0a^2}{4\varepsilon_0}-\frac{\rho_0r^2}{\varepsilon_0}\left(\frac{1}{6}-\frac{r^2}{20a^2}\right)\longrightarrow\boxed{V_{(r)\,in}=\frac{\rho_0}{\varepsilon_0}\left(\frac{a^2}{4}-\frac{r^2}{6}+\frac{r^4}{20a^2}\right)}$

هـ) $V_{out}-V_{(a)}=-\int_a^r E_{out}dr\longrightarrow V_{out}=\frac{\rho_0a^2}{\varepsilon_0}\left(\frac{1}{4}-\frac{1}{6}+\frac{1}{20}\right)+\frac{2\rho_0a^3}{15\varepsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right)\longrightarrow\boxed{V_{out}=\left(\frac{2\rho_0a^3}{15\varepsilon_0}\right)\frac{1}{r}}$

و) درخارج کره میدان برحسب r نزولی است. پس $E_{max}$ درداخل کره رخ می‌دهد.

$\frac{\partial E_{in}}{\partial r}=0\longrightarrow \cancel{\quad}\ \frac{1}{3}=\frac{3r^2}{5a^2}\longrightarrow\boxed{r=\frac{a\sqrt5}{3}}$

ز) $E_{(a)}=\frac{2\rho_0a}{15\varepsilon_0}\longrightarrow E_{(a)}=\frac{2\times5\times10^{23}\times3.4\times10^{-5}}{15\times8.85\times10^{-12}}\longrightarrow E_{(a)}=\frac{2\times34\times10^{31}}{3\times885}\longrightarrow E_{(a)}=\frac{2\times884\times10^{31}}{3\times26\times885}\longrightarrow E_{(a)}\simeq\frac{10^{31}}{39}\longrightarrow$

$E_{(a)}\simeq0.25\times10^{30}\longrightarrow\boxed{E_{(a)}\simeq2.5\times10^{29}}$

$V_{(a)}=\frac{2\rho_0a^2}{15\varepsilon_0}\longrightarrow V_{(a)}\simeq\frac{34\times10^{-6}\times10^{31}}{39}\longrightarrow V_{(a)}=\frac{272\times7\times10^{25}}{8\times273}\longrightarrow V_{(a)}\simeq\frac{7}{8}\times10^{25}\longrightarrow V_{(a)}\simeq\frac{7\times5^3}{10^3}\times10^{25}\longrightarrow\boxed{V_{(a)}\simeq8.7\times10^{24}}$

$Q_{in}=\frac{8\pi\rho_0a^3}{15}\longrightarrow\cancel{Q_{in}=8\times3.1\times5\times10^{23}\times}\ Q_{in}=\left(\frac{2\rho_0a^2}{15\varepsilon_0}\right)4\pi a\varepsilon_0\longrightarrow Q_{in}=8.7\times10^{24}\times4\times3.1\times3.4\times10^{-5}\times8.85\times10^{-12}\longrightarrow$

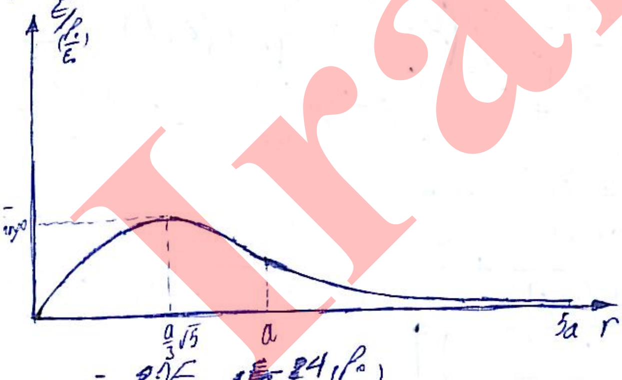
$Q_{in}=87\times4\times31\times34\times885\times10^{2}\longrightarrow Q_{in}\simeq90\times2\times30\times30\times180\times10^{3}\longrightarrow\boxed{Q_{in}\simeq2.9\times10^{10}}$

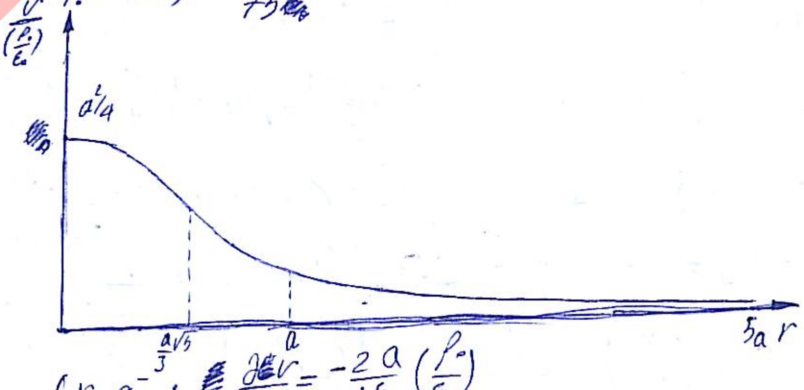
$V_0=\frac{\rho_0a^2}{4\varepsilon_0}\longrightarrow V_0=\left(\frac{2\rho_0a^2}{15\varepsilon_0}\right)\times\frac{15}{8}\longrightarrow V_0=\frac{15}{8}V_{(a)}\longrightarrow V_0\simeq1.9\,V_{(a)}\longrightarrow\boxed{V_0\simeq1.6\times10^{25}}$

ح) $\frac{E}{\left(\frac{\rho_0}{\varepsilon_0}\right)}=\begin{cases}\frac{r}{3}-\frac{r^3}{5a^2}\\ \frac{2a^3}{15r^2}\end{cases}$ $\quad r<a,\ \left(\frac{\varepsilon_0}{\rho_0}\right)\frac{\partial E}{\partial r}=\frac{1}{3}-\frac{3r^2}{5a^2}\longrightarrow$ $\quad r>a,\ \left(\frac{\varepsilon_0}{\rho_0}\right)\frac{\partial^2E}{\partial r^2}=-\frac{3r}{5a^2}$

$\frac{V}{\left(\frac{\rho_0}{\varepsilon_0}\right)}=\begin{cases}\frac{a^2}{4}-\frac{r^2}{6}+\frac{r^4}{20a^2}\\ \left(\frac{2a^3}{15}\right)\frac{1}{r}\end{cases}$ $\quad r<a,\ \left(\frac{\varepsilon_0}{\rho_0}\right)\frac{\partial V}{\partial r}=\frac{r^3}{5a^2}-\frac{r}{3}\longrightarrow$ $\quad r>a\ \left(\frac{\varepsilon_0}{\rho_0}\right)\frac{\partial^2V}{\partial r^2}=-\frac{1}{3}+\frac{3r^2}{5a^2}$

$\left(\frac{\varepsilon_0}{\rho_0}\right)E_{(5a)}=\frac{2a}{375}$ $\qquad\left(\frac{\varepsilon_0}{\rho_0}\right)V_{(5a)}=\frac{2a^2}{75}$




$r=a^-:\ \frac{\partial E}{\partial r}=-\frac{4}{15}\left(\frac{\rho_0}{\varepsilon_0}\right)$ $\qquad r=a^-:\ \frac{\partial V}{\partial r}=-\frac{2a}{15}\left(\frac{\rho_0}{\varepsilon_0}\right)$

$r=a^+:\ \frac{\partial E}{\partial r}=-\frac{4}{15}\left(\frac{\rho_0}{\varepsilon_0}\right)$ $\qquad r=a^+:\ \frac{\partial V}{\partial r}=-\frac{2a}{15}\left(\frac{\rho_0}{\varepsilon_0}\right)$

$r=0:\ \frac{\partial E}{\partial r}=\frac{1}{3}\ ,\ E=0$ $\qquad r=0:\ \frac{\partial V}{\partial r}=0\ ,\ V=\frac{a^2}{4}\left(\frac{\rho_0}{\varepsilon_0}\right)$

2) الف) $dE_z=\frac{-k\lambda dx\ (x-z)}{(\rho^2+(x-z)^2)^{3/2}}\ ,\ \rho^2+(x-z)^2=a$

$dE_\rho=\frac{k\lambda dx\cdot\rho}{\ldots}\ ,\ x-z=u$

پاسخ سوال

الف)

$$dV=\frac{k\lambda dx}{\sqrt{(x-z)^2+\rho^2}}=k\lambda\frac{d(x-z)}{\sqrt{(x-z)^2+\rho^2}}\longrightarrow \boxed{V=-\frac{kq}{l}\ln\left(\frac{\left(-\frac{l}{2}-z\right)+\sqrt{\rho^2+\left(z+\frac{l}{2}\right)^2}}{\left(\frac{l}{2}-z\right)+\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}}\right)}$$

ب)

$$\int dE_z=\frac{-k\lambda\, da}{2a^{3/2}}\longrightarrow E_z=\frac{kq}{l}\left(\frac{1}{\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}}-\frac{1}{\sqrt{\rho^2+\left(\frac{l}{2}+z\right)^2}}\right)\longrightarrow$$

$$dE_\rho=k\lambda\rho\frac{du}{\left[\rho^2+u^2\right]^{3/2}}\longrightarrow E_\rho=\frac{kq}{l\rho}\left.\frac{u}{\sqrt{\rho^2+u^2}}\right|_{-\frac{l}{2}-z}^{\frac{l}{2}-z}\longrightarrow E_\rho=\frac{kq}{l\rho}\left(\frac{\frac{l}{2}-z}{\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}}+\frac{z+\frac{l}{2}}{\sqrt{\rho^2+\left(z+\frac{l}{2}\right)^2}}\right)$$

$$\boxed{\vec{E}=\frac{kq}{l}\left[\frac{1}{\rho}\left(\frac{\frac{l}{2}-z}{\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}}+\frac{\frac{l}{2}+z}{\sqrt{\rho^2+\left(z+\frac{l}{2}\right)^2}}\right)\hat{\rho}+\left(\frac{1}{\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}}-\frac{1}{\sqrt{\rho^2+\left(\frac{l}{2}+z\right)^2}}\right)\hat{z}\right]}$$

ج)

$$E_\rho=0\longrightarrow\vec{E}=\frac{kq}{l}\left(\frac{1}{\sqrt{\left(\frac{l}{2}-z\right)^2}}-\frac{1}{\sqrt{\left(z+\frac{l}{2}\right)^2}}\right)\hat{z}\longrightarrow\vec{E}=\begin{cases}\dfrac{kq}{l}\cdot\dfrac{2z}{\frac{l^2}{4}-z^2} & |z|<\frac{l}{2}\\[2ex] \dfrac{kq}{l}\cdot\dfrac{l}{z^2-\frac{l^2}{4}} & |z|>\frac{l}{2}\end{cases}\longrightarrow$$

$$\boxed{\vec{E}=\begin{cases}\dfrac{2kqz}{l\left(\frac{l^2}{4}-z^2\right)} & |z|<\frac{l}{2}\\[2ex] \dfrac{kq}{z^2-\frac{l^2}{4}} & |z|>\frac{l}{2}\end{cases}}$$

د)

$$e^{-\frac{lV_0}{kq}}=\frac{\pm\left(z+\frac{l}{2}\right)+\sqrt{\rho^2+\left(z+\frac{l}{2}\right)^2}}{\left(\frac{l}{2}-z\right)+\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}}=c\longrightarrow\pm\left(z+\frac{l}{2}\right)\pm c\left(\frac{l}{2}-z\right)=c\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2}-\sqrt{\rho^2+\left(z+\frac{l}{2}\right)^2}\longrightarrow$$

$$\left(z+\frac{l}{2}\right)^2+c^2\left(\frac{l}{2}-z\right)^2+2c\left(\frac{l^2}{4}-z^2\right)=c^2\rho^2+c^2\left(\frac{l}{2}-z\right)^2+\rho^2+\left(z+\frac{l}{2}\right)^2-2c\sqrt{\left(\rho^2+\left(\frac{l}{2}-z\right)^2\right)\left(\rho^2+\left(z+\frac{l}{2}\right)^2\right)}\longrightarrow$$

$$4c^2\rho^2+4c^2\left(\frac{l^2}{4}-z^2\right)^2+4c^2\rho^2\left(2z^2+\frac{l^2}{2}\right)=\rho^4+c^4\rho^4+4c^2\left(\frac{l^2}{4}-z^2\right)^2+2c^2\rho^4-4c\rho^2\left(\frac{l^2}{4}-z^2\right)-4c^3\rho^2\left(\frac{l^2}{4}-z^2\right)$$

$$\longrightarrow 2c^2\rho^2+8c^2z^2+2c^2l^2=\rho^2+c^4\rho^2-cl^2+4cz^2-c^3l^2+4c^3z^2\longrightarrow$$

$$\rho^2\left(1-c^2\right)^2+4cz^2(1-c)^2=cl^2(1+c)^2\longrightarrow\frac{\rho^2(1-c)^2(1+c)^2}{4c(1-c)^2(1+c)^2}+\frac{4cz^2(1-c)^2}{4c(1-c)^2(1+c)^2}=\frac{cl^2(1+c)^2}{4c(1-c)^2(1+c)^2}$$

$$\longrightarrow\left(\frac{\rho}{2\sqrt{c}}\right)^2+\left(\frac{z}{1+c}\right)^2=\left(\frac{l}{2(1-c)}\right)^2\longrightarrow\boxed{\left(\frac{\rho}{2e^{-\frac{lV_0}{2kq}}}\right)^2+\left(\frac{z}{1+e^{-\frac{lV_0}{kq}}}\right)^2=\left(\frac{l}{2\left(1-e^{-\frac{lV_0}{kq}}\right)}\right)^2}$$

بیضی گون

هـ)

$$\cdot=\sqrt{\rho_0^2+\left(z_0+\frac{l}{2}\right)^2}+\sqrt{\rho_0^2+\left(\frac{l}{2}-z_0\right)^2}=(c+1)\sqrt{\rho_0^2+\left(\frac{l}{2}-z_0\right)^2}\pm\left[c\left(\frac{l}{2}-z_0\right)+\left(z_0+\frac{l}{2}\right)\right]$$

$$\rho_0^2=\frac{cl^2(1+c)^2-4cz_0^2(1-c)^2}{(1+c)^2(1-c)^2}\longrightarrow\rho_0^2=\frac{cl^2}{(1-c)^2}-\frac{4cz_0^2}{(1+c)^2}$$

$$=(c+1)\sqrt{\frac{cl^2}{(1-c)^2}-\frac{4cz_0^2}{(1+c)^2}+\frac{l^2}{4}+z_0^2-z_0l}\pm\left[c\left(\frac{l}{2}-z_0\right)+\left(z_0+\frac{l}{2}\right)\right]\longrightarrow$$

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۱) الف)

$$mv_0 = (m_1+m)v_{10} \to \boxed{v_{10} = \frac{m}{m_1+m}v_0} \quad , \quad \boxed{v_{20}=0}$$

ب) حالت حدی:

$$\begin{cases} mv_0 = (m_1+m+m_2)v \\ mv_0^2 = (m_1+m+m_2)v^2+kd^2\end{cases} \to \left(m-\frac{m^2}{m_1+m+m_2}\right)v_0^2 = kd^2 \to \boxed{v_{0max} = d\sqrt{\frac{k(m_1+m_2+m)}{m_1(m_1+m_2)}}}$$

ج)

$$\begin{cases}-k\Delta x = m_2\ddot x_2\\ k\Delta x = m_1'\ddot x_1\end{cases} \to m_1'\ddot x_1+m_2\ddot x_2=0 \to m_1'\dot x_1+m_2\dot x_2 = mv_0 \to m_1'x_1+m_2x_2 = mv_0 t \to \boxed{m_1'=m_1+m}$$

د)

$$x_2 = \frac{-(m_1+m)x_1+mv_0t}{m_2} \quad , \quad \Delta x = ((x_2+l)-x_1)-l \to \boxed{\Delta x = x_2-x_1}$$

$$\to kx_2-kx_1=(m_1+m)\ddot x_1 \to \frac{kmv_0}{m_2}t = kx_1\left(1+\frac{m_1+m}{m_2}\right)+(m_1+m)\ddot x_1 \to$$

$$\ddot x_1+\frac{k(m_2+m_1+m)}{m_2(m_1+m)}x_1 = \frac{kmv_0}{m_2(m_1+m)}t \to x_1 = A\sin(\omega t+\varphi)+\gamma t \quad , \quad t=0: x_1=0 \to \varphi=0\to$$

$$x_1 = A\sin(\omega t)+\gamma t \quad , \quad t=0:\dot x_1 = \frac{mv_0}{m_1+m}\to \dot x_1 = A\omega\cos(\omega t)+\gamma \to \frac{mv_0}{m_1+m}=A\omega+\gamma\to$$

$$A\sqrt{\frac{k(m_2+m_1+m)}{m_2(m_1+m)}}+\frac{mv_0}{m_2+m_1+m}=\frac{mv_0}{m_1+m}\to \boxed{A=\sqrt{\frac{m_2}{k(m_1+m)(m_1+m_2+m)}}\,\frac{mm_2v_0}{m_1+m_2+m}}\to$$

$$\boxed{x_1 = \frac{mv_0}{m_1+m_2+m}\left(m_2\sqrt{\frac{m_2}{k(m_1+m)(m_1+m_2+m)}}\sin\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,t\right)+t\right)}\to$$

$$x_2 = \frac{-m(m_1+m)v_0}{m_1+m_2+m}\sqrt{\frac{m_2}{k(m_1+m)(m_1+m_2+m)}}\sin\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,t\right) - \frac{m(m_1+m)v_0}{m_2(m_1+m_2+m)}t+\frac{mv_0}{m_2}t\to$$

$$\boxed{x_2=\frac{mv_0}{m_1+m_2+m}\left(t-\sqrt{\frac{m_2(m_1+m)}{k(m_1+m_2+m)}}\sin\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,t\right)\right)}$$

$$\tau: \dot x_1=\dot x_2 \to \dot x_1 = \frac{mv_0}{m_1+m_2+m} \quad , \quad \dot x_1 = \frac{mv_0}{m_1+m_2+m}\left(\frac{m_2}{m_1+m}\cos\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,t\right)+1\right)\to$$

$$\cos\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,\tau\right)=0\to\boxed{\tau=\frac{\pi}{2}\sqrt{\frac{m_2(m_1+m)}{k(m_1+m_2+m)}}}$$

ه) $v_{2(t_1)}$

$$\boxed{m_2>m_1+m} \quad t_1: \dot x_1=0\to\boxed{t_1=\sqrt{\frac{m_2(m_1+m)}{k(m_1+m_2+m)}}\cos^{-1}\left(\frac{m_1+m}{m_2}\right)} \quad , \quad \dot x_1=0\to\boxed{\dot x_2=\frac{m}{m_2}v_0}$$

و)

$$x_2-x_1=-d\to \frac{mv_0}{m_1+m_2+m}\sqrt{\frac{m_2}{k(m_1+m)(m_1+m_2+m)}}(m_2+m_1+m)\sin\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,t_2\right) = d \xrightarrow{t=t_2}$$

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$$\sin\left(\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}\,t_2\right)=\frac{d}{mv_0}\sqrt{\frac{k(m_1+m)(m_1+m+m_2)}{m_2}} \longrightarrow \boxed{t_2=\sqrt{\frac{m_2(m_1+m)}{k(m_1+m_2+m)}}\sin^{-1}\left(\frac{d}{mv_0}\sqrt{\frac{k(m_1+m)(m_1+m_2+m)}{m_2}}\right)}$$

$$mv_0=(m_1+m+m_2)v_0' \longrightarrow \boxed{v_0'=\frac{m}{m_1+m_2+m}v_0}$$

ز)

$$x_1=a\sin\left((t-t_2)\omega+\varphi'\right)+\gamma(t-t_2)+b \quad , \quad t=t_2: x_1=b \longrightarrow \boxed{\varphi'=0} \,,$$

$$t=t_2: \dot{x}_1=\frac{m}{m_1+m_2+m}v_0 \longrightarrow a\omega+\gamma=\frac{m}{m_1+m_2+m}v_0 \longrightarrow$$

$$a\sqrt{\frac{k(m_1+m_2+m)}{m_2(m_1+m)}}+$$

غلط است

$$\begin{cases} -k\Delta x=m_2\ddot{x}_2' \\ k\Delta x=m_1'\ddot{x}_1' \end{cases} \longrightarrow m_1'\ddot{x}_1'+m_2\ddot{x}_2'=0 \longrightarrow m_1'\dot{x}_1'+m_2\dot{x}_2'=(m_1'+m_2)v_0'=mv_0$$

$$\longrightarrow (m_1+m)x_1'+m_2x_2'=mv_0T \longrightarrow$$

$$\begin{cases} x_1'=x_1-x_{1(t_2)} \\ x_2'=x_2-x_{2(t_2)} \\ T=t-t_2 \end{cases}$$ (ز

$$x_2'=\frac{mv_0T-(m_1+m)x_1'}{m_2} \quad , \quad \Delta x=(x_2'+l-d)-x_1'-l \longrightarrow \Delta x=x_2'-x_1'-d$$

$$kx_1'-kd+\frac{kmv_0T}{m_2}-\frac{k(m_1+m)x_1'}{m_2}=(m_1+m)\ddot{x}_1' \longrightarrow \ddot{x}_1'+\frac{k(m_1+m+m_2)}{m_2(m_1+m)}x_1'=\frac{kmv_0T}{m_2(m_1+m)}-\frac{kd}{m_1+m}$$

$$\longrightarrow x_1'=a\sin(\omega T+\varphi')+\gamma' T-\frac{kd}{(m_1+m)\omega^2}$$

$$T=0: x_1'=0 \longrightarrow a\sin\varphi'=\frac{kd}{(m_1+m)\omega^2} \longrightarrow a=\frac{kd}{(m_1+m)\omega^2} \longrightarrow \boxed{a=\frac{m_2}{m_1+m+m_2}d}$$

$$T=0: \dot{x}_1'=v_0' \longrightarrow a\cos\varphi'+\gamma'=v_0' \longrightarrow \cos\varphi'=0 \longrightarrow \varphi'=\frac{\pi}{2} \quad , \gamma'=v_0'=\frac{m}{m_1+m_2+m}v_0$$

$$\longrightarrow x_1'=\frac{m_2}{m_1+m+m_2}d\cos(\omega T)+\frac{m}{m_1+m_2+m}v_0T-\frac{m_2}{m_1+m+m_2}d \longrightarrow$$

$$\boxed{x_1=\frac{1}{m_1+m+m_2}\Big(m_2d\left(\cos(\omega(t-t_2))-1\right)+mv_0(t-t_2)\Big)+x_{1(t_2)}} \longrightarrow$$

$$x_2'=\frac{mv_0T}{m_2}+\frac{(m_1+m)d}{m_1+m_2+m}\left(1-\cos(\omega T)\right)-\frac{(m_1+m)mv_0}{m_2(m_1+m_2+m)}T \longrightarrow$$

$$\boxed{x_2=\frac{1}{m_1+m+m_2}\Big((m_1+m)d\left(1-\cos(\omega(t-t_2))\right)+mv_0(t-t_2)\Big)+x_{2(t_2)}}$$

$$\boxed{x_{1(t_2)}=\frac{mv_0}{m_1+m_2+m}\left(\frac{m_2d}{mv_0}+\sqrt{\frac{m_2(m_1+m)}{k(m_1+m_2+m)}}\sin^{-1}\left(\frac{d}{mv_0}\sqrt{\frac{k(m_1+m)(m_1+m_2+m)}{m_2}}\right)\right)}$$

$$\boxed{x_{2(t_2)}=\frac{mv_0}{m_1+m_2+m}\left(\sqrt{\frac{m_2(m_1+m)}{k(m_1+m_2+m)}}\sin^{-1}\left(\frac{d}{mv_0}\sqrt{\frac{k(m_1+m)(m_1+m_2+m)}{m_2}}\right)-\frac{(m_1+m)d}{mv_0}\right)}$$

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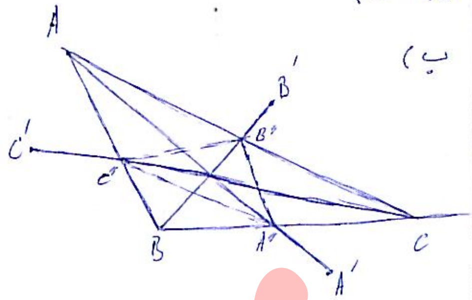
3) الف)

ب)

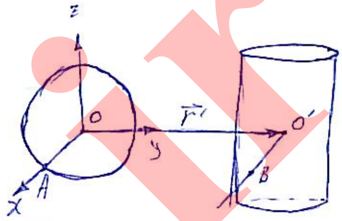



$$\frac{AC''}{BC''}=\frac{AB''}{B''C}=1 \to C''B'' \parallel BC \xrightarrow{\text{وزز}} \overset{\triangle}{AB''C''} \sim \overset{\triangle}{ABC} \xrightarrow[\text{ترتیب}]{\text{به همین}}$$

$$\overset{\triangle}{AB''C''} \cong \overset{\triangle}{A''C''B} \cong \overset{\triangle}{CA''B''}$$

$$\phi_0=\frac{\phi_0}{2}+\frac{\phi_{B'}}{2}+\frac{\phi_{A'}}{2}+\frac{\phi_{C'}}{2} \to \boxed{\phi_{A'}+\phi_{B'}+\phi_{C'}=\phi_0}$$

4)




$$\vec{E}_{os}\times 2\pi r\times h=\frac{\rho\times\pi R^2 h}{\varepsilon_0} \to \vec{E}_{os}=\left(\frac{\rho R^2}{2\varepsilon_0}\right)\frac{\vec{r}}{r^2}\ ,\ E_{ic}=\frac{4}{3}\pi r^3\times\frac{\rho}{\varepsilon_0}\times\frac{1}{4\pi r^2} \to \vec{E}_{ic}=\frac{\rho\vec{r}}{3\varepsilon_0}$$

$$E_{is}\times 2\pi r\times h=\frac{\rho\pi r^2 h}{\varepsilon_0} \to \vec{E}_{is}=\left(\frac{\rho}{2\varepsilon_0}\right)\vec{r}\ ,\ E_{oc}\times 4\pi r^2=\frac{4\pi\rho R^3}{3\varepsilon_0} \to \vec{E}_{oc}=\left(\frac{\rho R^3}{3\varepsilon_0}\right)\frac{\vec{r}}{r^3}$$

$$\vec{E}_{ics}=\vec{E}_{os}+\vec{E}_{ic}=\left(\frac{\rho}{3\varepsilon_0}\right)(\vec{x}+\vec{y}+\vec{z})+\left(\frac{\rho R^2}{2\varepsilon_0}\right)\frac{\vec{x}+\vec{y}-\vec{r}'}{(y-r')^2+x^2} \to \boxed{\vec{E}_{ics}=\frac{\rho}{\varepsilon_0}\left(\frac{\vec{r}}{3}+\frac{R^2\left(\vec{x}+(y-4R)\hat{j}\right)}{2\left(x^2+(y-4R)^2\right)}\right)}$$

الف) برهم نهی می‌کنیم.

$$\vec{E}_{ocs}=\vec{E}_{oc}+\vec{E}_{os}=\left(\frac{\rho R^3}{3\varepsilon_0}\right)\frac{\vec{r}}{r^3}+\frac{\rho R^2}{2\varepsilon_0}\cdot\frac{\vec{x}+\vec{y}-\vec{r}'}{(y-r')^2+x^2} \to \boxed{\vec{E}_{ocs}=\frac{\rho R^2}{\varepsilon_0}\left(\frac{R\vec{r}}{3r^3}+\frac{\vec{x}+(y-4R)\hat{j}}{2\left(x^2+(y-4R)^2\right)}\right)}$$

ب)

$$\vec{E}_{isc}=\vec{E}_{oc}+\vec{E}_{is}=\left(\frac{\rho R^3}{3\varepsilon_0}\right)\frac{\vec{r}}{r^3}+\left(\frac{\rho}{2\varepsilon_0}\right)\cancel{\vec{r}}\cdot\frac{\vec{x}+\vec{y}-\vec{r}'}{\cancel{(y-r')^2+x^2}} \to \boxed{\vec{E}_{isc}=\frac{\rho}{\varepsilon_0}\left(\frac{R^3\vec{r}}{3r^3}+\frac{\vec{x}+(y-4R)\hat{j}}{2\cancel{\left(x^2+(y-4R)^2\right)}}\right)}$$

ج)

$$V_A-V_B=-\int_0^{O'}\vec{E}_{ocs}\cdot\vec{dy}=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{R}{3}\int_0^{4R}\frac{y\,dy}{(x^2+y^2)^{3/2}}+\frac{1}{2}\int_0^{4R}\frac{(y-4R)dy}{\left(x^2+(y-4R)^2\right)}\right)=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{R}{3}\times-\Big|_0^{4R}\frac{1}{\sqrt{x^2+y^2}}+\frac{1}{4}\Big|_0^{4R}\ln\left(x^2+(y-4R)^2\right)\right) \to$$

هـ)

$$V_A-V_B=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{R}{3}\left(\frac{1}{R}-\frac{1}{R\sqrt{17}}\right)+\frac{1}{4}\ln\left(\frac{R^2}{17R^2}\right)\right) \to \boxed{V_A-V_B=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{\sqrt{17}-1}{3\sqrt{17}}-\frac{\ln(17)}{4}\right)}$$

$$V_0-V_{0'}=-\left(\int\vec{E}_{ics}\cdot\vec{dy}+\int\vec{E}_{ocs}\cdot\vec{dy}+\int\vec{E}_{isc}\cdot\vec{dy}\right)=-\frac{\rho}{\varepsilon_0}\left(\frac{1}{3}\int_0^R y\,dy+\frac{R^2}{2}\int_0^R\frac{dy}{y-4R}+\frac{R^3}{3}\int_R^{3R}\frac{dy}{y^2}+\frac{R^2}{2}\int_R^{3R}\frac{dy}{y-4R}+\frac{R^3}{3}\int_{3R}^{4R}\frac{dy}{y^2}+\frac{1}{2}\int_{3R}^{4R}(y-4R)\,dy\right)$$

د)

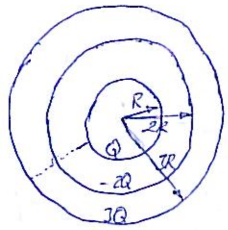
$$=-\frac{\rho}{\varepsilon_0}\left(\frac{R^2}{6}+\frac{R^2}{2}\ln\left(\frac{3}{4}\right)+\frac{R^3}{3}\left(\frac{1}{R}-\frac{1}{3R}\right)+\frac{R^2}{2}\ln\left(\frac{1}{3}\right)+\frac{R^3}{3}\left(\frac{1}{3R}-\frac{1}{4R}\right)+\frac{1}{4}\left(-R^2\right)\right)=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{1}{6}+\frac{2}{9}+\frac{1}{36}-\frac{1}{4}+\frac{\ln\left(\frac{1}{4}\right)}{2}\right) \to$$

$$V_0-V_{0'}=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{6+8+1-9}{36}-\ln(2)\right) \to \boxed{V_0-V_{0'}=-\frac{\rho R^2}{\varepsilon_0}\left(\frac{1}{6}-\ln(2)\right)}$$

5) الف)

$$\vec{E}=\begin{cases}0 & r<R\\ \frac{kQ}{r^2} & R<r<2R\\ -\frac{kQ}{r^2} & 2R<r<3R\\ \frac{2kQ}{r^2} & 3R<r\end{cases}$$

ب)




$$V_{(R)}=\frac{kQ}{R}-\frac{2kQ}{2R}+\frac{3kQ}{3R} \to \boxed{V_{(R)}=\frac{kQ}{R}}$$

$$V_{(2R)}=\frac{kQ}{2R}-\frac{2kQ}{2R}+\frac{3kQ}{3R} \to \boxed{V_{(2R)}=\frac{kQ}{2R}}$$

$$V_{(3R)}=-\frac{kQ}{3R}+\frac{3kQ}{3R} \to \boxed{V_{(3R)}=\frac{2kQ}{3R}}$$

ج)

$$U=\frac{1}{2}\cdot\frac{kQ^2}{R}\left((-1+1)-2\times\left(\frac{1}{2}+1\right)+3\left(-\frac{1}{3}\right)\right)=\frac{kQ^2}{2R}\times(-3-1) \to \boxed{U=-\frac{2kQ^2}{R}}$$

$$q_1+q_2=4Q \to \boxed{q_2=\frac{7}{2}Q} \xrightarrow[\text{صفر است}]{E\text{ داخل رسانا}} -\frac{Q}{18}=-\frac{q_{i2}}{9} \to \boxed{q_{i2}=\frac{Q}{2}} \to \boxed{q_{o2}=3Q}$$

د)، هـ)

$$\frac{q_2}{3R}-\frac{2Q}{2R}+\frac{q_1}{R}=\frac{q_1}{3R}-\frac{2Q}{3R}+\frac{q_2}{3R} \to \frac{Q}{3}=\frac{2q_1}{3} \to \boxed{q_1=\frac{Q}{2}} \to \boxed{q_{i1}=0}\ ,\ \boxed{q_{o1}=\frac{Q}{2}}$$

و)

$$U=\frac{kQ^2}{2R}\left(\left(\frac{7}{6}-1\right)+\left(\frac{1}{4}+1\right)\times-2+3\left(\frac{-\frac{3}{2}}{3}\right)\right) \to U=\frac{kQ^2}{2R}\cdot\frac{1-15-9}{6} \to \boxed{U=-\frac{23kQ^2}{12R}}$$

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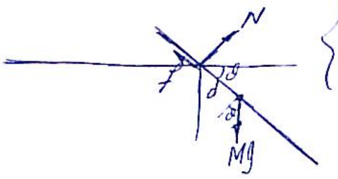



1) الف)

$$\begin{cases} Mgd\sin\theta = \frac{1}{2} I\dot\theta^2 \longrightarrow \omega^2 = \frac{2Mgd\sin\theta}{I} \longrightarrow \\ I = \int r^2 dm = \frac{M}{3L} r^3\Big|_{\frac{L}{2}-d}^{\frac{L}{2}+d} = \frac{M}{3L}\left[2d\left(\frac{L^2}{2}+2d^2+\frac{L^2}{4}-d^2\right)\right] \longrightarrow I = \frac{Md}{3L}\left(d^2+\frac{3L^2}{2}\right) \end{cases}$$

$$\omega^2 = \frac{2Mgd\sin\theta}{\frac{Md}{3L}\left(d^2+\frac{3L^2}{2}\right)} \longrightarrow \omega^2 = \frac{12gL\sin\theta}{2d^2+3L^2} \longrightarrow \boxed{\omega = \sqrt{\frac{12gL\sin\theta}{2d^2+3L^2}}}$$

ب)

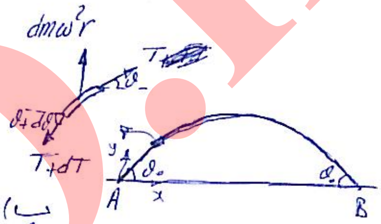
$$Mg\cos\theta - N = Md\ddot\theta \longrightarrow N = M\left(g\cos\theta - \frac{6gLd\cos\theta}{2d^2+3L^2}\right) \longrightarrow \boxed{N = \frac{Mg\cos\theta(2d^2+3L^2-6Ld)}{2d^2+3L^2}}$$

$$Mgd\cos\theta\,\dot\theta = I\dot\theta\ddot\theta \longrightarrow \ddot\theta = \frac{gMd\cos\theta}{\frac{Md}{6L}(2d^2+3L^2)} \longrightarrow \ddot\theta = \frac{6gL\cos\theta}{2d^2+3L^2}$$

ج)

$$f = Mg\sin\theta + M\omega^2 d \longrightarrow f = Mg\sin\theta\left(1+\frac{12Ld}{2d^2+3L^2}\right) \longrightarrow Mg\sin\theta_0(2d^2+3L^2+12Ld) = \mu Mg\cos\theta_0(2d^2+3L^2-6Ld)$$

$$\longrightarrow \boxed{\tan\theta_0 = \mu\frac{3L^2+2d^2-6Ld}{2d^2+3L^2+12Ld}}$$




2) الف)

$$2T_0\sin\theta_0 = M\omega^2 R_{cm} \longrightarrow \boxed{T_0 = \frac{M\omega^2 R_{cm}}{2\sin\theta_0}}$$

ب)

$$\begin{cases} -dT_{(y)} = dm\,\omega^2 r \\ dT_{(x)} = 0 \longrightarrow T\cos\theta = T_0\cos\theta_0 \longrightarrow \boxed{T = \frac{T_0\cos\theta_0}{\cos\theta}} \end{cases}$$

ج)

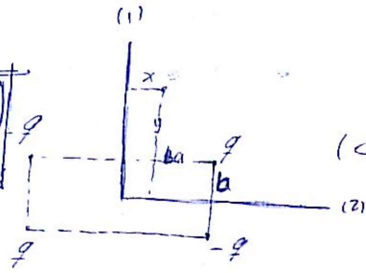
$$-T_0\cos\theta_0\, d(\tan\theta) = \frac{M\omega^2}{L}\times ds\times r \longrightarrow -T_0\cos\theta_0\frac{\sin\theta\,d\theta}{\cos^2\theta} = \frac{M\omega^2}{L} r\,dr \longrightarrow \boxed{r^2 = \frac{2LT_0\cos\theta_0}{M\omega^2}\left(\frac{1}{\cos\theta_0}-\frac{1}{\cos\theta}\right)}$$

د)

$$\frac{M\omega^2 r^2}{2LT_0} = 1 - \frac{\cos\theta_0}{\cos\theta} \longrightarrow \frac{\cos\theta_0}{\cos\theta} = 1 - \frac{M\omega^2}{2LT_0}r^2 \longrightarrow \boxed{T = T_0\left(1-\frac{M\omega^2}{2LT_0}r^2\right)}$$

ه)

$$\boxed{r^2 = \frac{2LT_0}{M\omega^2}(1-\cos\theta_0)}$$




3) الف)

$$\boxed{V = kq\left(\frac{1}{\sqrt{(y-b)^2+(a-x)^2+z^2}} + \frac{1}{\sqrt{(y+b)^2+(x+a)^2+z^2}}\ \frac{1}{\sqrt{(x+a)^2+(y-b)^2+z^2}} - \frac{1}{\sqrt{(y+b)^2+(a-x)^2+z^2}}\right)}$$

ب)

$$\vec E_{(0,y,z)} = kq\left[\left(\frac{x-a}{\left((y-b)^2+(x-a)^2+z^2\right)^{3/2}} + \frac{x+a}{\left((y+b)^2+(x+a)^2+z^2\right)^{3/2}} - \frac{x+a}{\left((x+a)^2+(y-b)^2+z^2\right)^{3/2}} - \frac{x-a}{\left((y+b)^2+(x-a)^2+z^2\right)^{3/2}}\right)\hat x\right] \longrightarrow \boxed{\vec E_{(0,y,z)} = 2kaq\left(\frac{1}{\left((y+b)^2+a^2+z^2\right)^{3/2}} - \frac{1}{\left((y-b)^2+a^2+z^2\right)^{3/2}}\right)\hat x}$$

اصل تقارن

$$\boxed{\sigma_{(1)} = \frac{aq}{2\pi}\left(\frac{1}{\left((y+b)^2+a^2+z^2\right)^{3/2}} - \frac{1}{\left((y-b)^2+a^2+z^2\right)^{3/2}}\right)} \qquad \boxed{\sigma_{(2)} = \frac{bq}{2\pi}\left(\frac{1}{\left((x+a)^2+b^2+z^2\right)^{3/2}} - \frac{1}{\left((x-a)^2+b^2+z^2\right)^{3/2}}\right)}$$

ج)، د)، ه)، و)

$$\sigma_{(2)} = \frac{dQ_2}{dx\,dz} \longrightarrow q_2 = \frac{bq\,dx}{2\pi}\left[\int_{-z}^{z}\frac{z}{\left((x+a)^2+b^2\right)\sqrt{(x+a)^2+b^2+z^2}} - \int_{-z}^{z}\frac{z}{\left((x-a)^2+b^2\right)\sqrt{(x-a)^2+b^2+z^2}}\right] \longrightarrow$$

$$q_2 = \frac{2bq\,dx}{2\pi}\left(\frac{1}{b^2+(x+a)^2} - \frac{1}{b^2+(x-a)^2}\right) \longrightarrow Q_2 = \frac{2bq}{\pi}\left(\int_0^\infty\frac{d(x+a)}{b^2+(x+a)^2} - \int_0^\infty\frac{d(x-a)}{b^2+(x-a)^2}\right) \longrightarrow Q_2 = \frac{2q}{\pi}\left(\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}\left(\frac{-a}{b}\right)\right) \longrightarrow$$

$$\boxed{Q_2 = -\frac{2q}{\pi}\tan^{-1}\left(\frac{a}{b}\right)} \longrightarrow \boxed{Q_1 = -\frac{2q}{\pi}\tan^{-1}\left(\frac{b}{a}\right)} \longrightarrow Q_1+Q_2 = -\frac{2q}{\pi}\left(\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}\left(\frac{b}{a}\right)\right) \longrightarrow \boxed{Q_1+Q_2 = -q}$$

4) (1)

$$-\mu(mg+N) = m(\ddot r - \omega^2 r) \longrightarrow -\ddot r + \omega^2 r = \mu g + 2\mu\omega\dot r \longrightarrow \omega^2 e^{at} - a^2 e^{at} = 2\mu\omega a e^{at} \longrightarrow a^2 + 2\mu\omega a - \omega^2 = 0 \longrightarrow$$

$$N = m\dot r\omega\times 2 \longrightarrow \boxed{a = \left(-\mu\pm\sqrt{\mu^2+1}\right)\omega} \longrightarrow r_{(t)} = Ae^{a_1 t} + Be^{a_2 t}$$ (2)

$$\boxed{N_{(t)} = 2m\omega\dot r_{(t)}}$$ (3) $$\vec L = mr^2\dot\theta\,\hat z \longrightarrow \boxed{\vec L = m\omega r^2\hat z}$$ (4) $$\longrightarrow \vec\tau = \frac{d\vec L}{dt} = 2m\omega r\dot r\,\hat z \longrightarrow \boxed{\vec\tau = 2m\omega r\dot r\,\hat z}$$ (5)