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$$G(u, u_x, u_{xx}, \dots, u_x^{(n)}, u_y, u_{yy}, \dots, u_y^{(m)}, u_{xy}^{(i+j)}, \dots) = 0 \quad ()$$

$$u_x = \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} = u_y \quad u_{xy} = \frac{\partial^2 u}{\partial x \partial y}$$

$$\begin{cases} u_{xxx}(x, y) + u_{yyy}(x, y) = 0 \\ \frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3} = 0 \end{cases} :$$

PDE

(())

PDE

PDE $A, B, \dots, G$

$$Au_{xx}(x, y) + Bu_{xy} + Cu_{yy}(x, y) + Du_x(x, y) + Eu_y(x, y) + Fu(x, y) + G = 0 \quad (1)$$

x, y $A, B, \dots, G$

U $A, B, \dots, G$

$$u_{xx} + (\sin u)_{yy} = 0 \quad :$$

$$zu_{xx}(x, y) + xy^2u_{yy}(x, y) + e^xu(x, y) = 10 \quad :$$

$$u(x, y, z)$$

U

Boundary value

U

Initial Value Problems

() Problems

Homogenous PDE

PDE

Fix

L

$$v(x + \Delta x, t)$$

y

x

$$H(v, t) = H(x + \Delta x, t) \rightarrow H(x, t) = cte \quad \forall t \quad 0 < x < l$$

$$\tan \alpha = \frac{v(x + \Delta x, t)}{H} \quad \tan \beta = \frac{-v(x, t)}{-H}$$

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$$f = -by_t(x, t) \quad : b \quad : ()$$

$$y_{tt}(x, t) = H/\delta y_{xx}(x, t) - by_t(x, t) \quad :$$

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$$y(0, t) = 0$$

$$y(l, t) = 0$$

$$y(x, 0) = f(x) \quad 0 < x < l$$

:

f(x)

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:

$$y(0, t) = 0$$

$$y(l, t) = 0$$

$$y(x, 0) = 0$$

$$y_t(x, 0) = g(x)$$

x

g(x)

.

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y(x, t).

t

X

X + ΔX

X + ΔX

F(X, t)

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$$F = y(x + \Delta x) - y(x, t)$$

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:A

$$F(x, t) = \frac{E}{\Delta x} (y(x + \Delta x, t) - y(x, t))$$

(v) ()

:B

$$F(x, t) = AE y_x(x, t)$$

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$$F(x, t) = -E y_x(x, t)$$

x

$$F(x + \Delta x, t) = AE y_x(x + \Delta x, t)$$

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$$F(x + \Delta x, t) - F(x, t) = \delta A \Delta x y_{tt}(x, t)$$

()

:A

$$\Rightarrow y_{tt}(x, t) = \frac{AE}{\delta A} \left(\frac{y(x + \Delta x, t) - y(x, t)}{\Delta x} \right)$$

:\Delta

\delta

$$y_{tt}(x, t) = \frac{E}{\delta} y_{xx}(x, t)$$

:()

$$Z_{tt}(x, y, t) = H / \delta (Z_{xx}(x, y, t) + Z_{yy}(x, y, t))$$

:

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t

y x

 Z_{tt}

:

.

t

x, y

 $u(x, y, z, t) \in \mathbb{R}$

.

$$\phi = -K \frac{du}{dn}$$

n .

:n

:k

:\phi

:

()

z y x

$$\phi_1 = -K \frac{du}{dx}$$

() :y

$$\phi_2 = -K \frac{du}{dy}$$

$$\phi_3 = -K \frac{du}{dz}$$

.

y

$$J = \phi_1 \hat{I} + \phi_2 \hat{J} + \phi_3 \hat{K}$$

$$J = -k \left(\frac{\partial u}{\partial x} \hat{I} + \frac{\partial u}{\partial y} \hat{J} + \frac{\partial u}{\partial z} \hat{K} \right)$$

$$J = -k(\Delta u)$$

$$\phi = J.n$$

$$\phi$$

. : yz

:

$$\phi = j \quad j = -k \frac{du}{dx}$$

:

 I_1 I_1

$$I_1 = \iiint_{R_0} \delta u dv \quad (\quad)$$

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$$I_1 = \iint_R J.dA$$

$$I_2 = \iiint_R (\text{div} j) dv = \iiint_R \nabla(J) dV$$

:(—):

$$I_2 = \iiint_R -k \nabla.J dv = \iiint_R -k \nabla^2 u dV$$

$$I_1 = I_2 \quad : (\quad)$$

$$I_1 - I_2 = 0 \Rightarrow \iiint_R (\delta(u_t) - k u(vu)) dv = 0$$

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U

R

$$\delta u_t = k u(v_u) = 0 \Rightarrow \Delta U_t = k \nabla^2 u$$

$$\nabla u(x, y, t, z) \cdot u = \frac{\partial u}{\partial x} \hat{i} + \frac{\partial u}{\partial y} \hat{j} + \frac{\partial u}{\partial z} \hat{k}$$

$$u_1 = M_1 \hat{i} + M_2 \hat{j} + M_3 \hat{k}$$

$$U_M = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot (M_1 \hat{i} + M_2 \hat{j} + M_3 \hat{k}) = \frac{\partial M_1}{\partial x} + \frac{\partial M_2}{\partial y} + \frac{\partial M_3}{\partial z}$$

$$\therefore \bar{\mu} = Uu$$

$$K \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = \delta u_t(x, y, z, t) \quad :$$

$$\frac{\partial^2 u(x, y, z, t)}{\partial x^2} + \frac{\partial^2 (x, y, z, t)}{\partial y^2} + \frac{\partial^2 u(x, t, z, t)}{\partial z^2} = \frac{\partial}{k} \frac{\partial u}{\partial t}(x, y, z, t)$$

$$u_{xx} + u_{yy} + u_{zz} = \frac{\delta}{k} u_t$$

PDE

PDE

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$$? \nabla^2 u = \frac{\partial}{k} u_t \quad \nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial x^2} \quad .$$

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$$k \frac{du}{dn} \Big| = \phi_0$$

$$\frac{du}{dx} = 0 \quad :$$

:

$$k \frac{du}{dn} = h(T_0 - u)$$

$$\nabla^2 u = \frac{\delta}{k} \frac{\partial u}{\partial t} \Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\delta}{k} \frac{\partial u}{\partial t}$$

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$$) \quad k \frac{du}{dx} \Big|_0 = 0 \quad u_x(x, 0, t) = 0 \quad 0 < x < l$$

$$0 < x < l, \quad y = 0$$

$$) \quad k \frac{du}{dx} \Big|_0 = \phi_0 \quad Ku_x(0, y, t) = \phi_0 \quad y > 0$$

$$x = 0, \quad y > 0$$

$$) \quad k \frac{du}{dx} \Big|_e = h(0, u) \quad KU_x(0, y, t) = -h(u) \quad y > 0$$

$$x = e, \quad y > 0$$

$$u(x, y, 0) = f(x)$$

$$\nabla^2 u = \partial^2 u / \partial t^2$$

u

$$\nabla^2 u = \delta(u, x, y, z, t)$$

$$c_1 u_1 - c_2 u_2$$

R

$$c_2, c_1$$

$$u_2, u_1$$

$$u_2, u_1$$

$$e\{c_1 u_1 + c_2 u_2\} = c_1 e\{u_1\} + c_2 e\{u_2\}$$

e-

:

$$e\{u\} = \int u dt \rightarrow e\{c_1 u_1 + c_2 u_2\} = \int (c_1 u_1 + c_2 u_2) du =$$

$$\int c_1 u_1 du + \int c_2 u_2 du = c_1 \int u_1 du + c_2 \int u_2 du$$

:

$$e\{0\} = 0$$

$$e\left\{\sum_{k=1}^n c_k u_k\right\} = \sum_{k=1}^n c_k e\{u_k\}$$

:

M

$$e\{M\{u\}\} = M\{e\{u\}\} = eM\{u\}$$

$$(e + M)\{u\} = e\{u\} + M\{u\}$$

:

$$A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial xy} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu(x, y) = G(x, y)$$

.

e

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$$G=0$$

:

$$e = A \frac{\partial^2}{\partial x^2} + B \frac{\partial^2}{\partial xy} + C \frac{\partial^2}{\partial y^2} + D \frac{\partial}{\partial x} + E \frac{\partial}{\partial y} + F$$

e

$$e\{u\} = 0 \quad (\quad u).$$

$$e\{u\} = G$$

$$e\{u_1\} = 0 \quad e\{u_2\} = 0$$

:

 u_1, u_2

$$c_1 e\{u_1\} + c_2 e\{u_2\} = e\{c_1 u_1 + c_2 u_2\} = 0$$

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 u_1, u_2

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 u_2 u_1

$$e\{u_1\} = 0 \quad e\{u_2\} = G$$

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$$u = cu_1 + u_2$$

u

$$e\{cu_1 + u_2\} = ce\{u_1\} + e\{u_2\} = G$$

$$u_t(x, t) = Ku_{xx}(x, t)$$

:

$$\begin{aligned} u_x(o, t) &= 0 \\ u_x(c, t) &= 0 \end{aligned} \quad :$$

$$u_0(x, t) = \frac{1}{2} \quad U_n(x, t) = \exp\left(-\frac{n^2 \pi^2 k}{c^2} t\right) \cos \frac{n \pi x}{c} \quad n = 1, 2, \dots$$

$$u_t(x, t) = \left(-\frac{n^2 \pi^2 k}{c^2}\right) \exp\left(-\frac{n^2 \pi^2 k}{c^2} t\right) \cos \frac{n \pi x}{c} \quad t$$

$$u_{xx} = \left(-\frac{n^2 \pi^2}{c^2}\right) \exp\left(-\frac{n^2 \pi^2}{c^2} k t\right) \cos \frac{n \pi x}{c} \quad k$$

$$k u_{xx} = u(x, t)$$

$$u = c_o u_o + c_1 u_1 + \dots + c_n u_n$$

$$A u_{xx} + B u_{xy} + C u_{yy} + D u_x + E u_y + F u = G$$

$$x, y, t \quad F \quad A$$

$$F-A$$

$$B^2 - 4Ac = 0$$

$$\text{Hyperbolic}$$

$$B^2 - 4Ac > 0$$

$$\text{(Parabolic)}.$$

$$B^2 - 4Ac < 0$$

$$\text{Elliptic}$$

$$:$$

$$u$$

$$u$$

$$u_{tt}, u_t \quad Ku + \frac{du}{dn}$$

$$(\quad)$$

$$u_t \quad u$$

$$:$$

$$\begin{cases} x = p \cos \phi \\ y = p \sin \phi \\ z = z \end{cases}$$

$$\begin{cases} p = \sqrt{x^2 + y^2} \\ \phi = \text{Arc tan} \left(\frac{y}{x} \right) \\ z = z \end{cases}$$



$$r = \sqrt{p^2 + z^2} \quad \theta = \text{Arc tan} \left(\frac{p}{z} \right)$$

$$\phi = \phi$$

$$\begin{cases} x = r \sin \theta \cos \phi \\ y = r \sin \theta \sin \phi \\ z = r \cos \theta \end{cases}$$



$$\begin{cases} p = r \sin \theta \\ \phi = \phi \\ z = r \cos \theta \end{cases}$$



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\phi = \text{ArcTan} \frac{y}{x}$$



$$z = \text{ArcTan} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\nabla^2 u = 0 \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

:

$$u(p, \phi, z)$$

u

:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial \phi} \cdot \frac{\partial \phi}{\partial x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial x}$$

$$= \frac{\partial u}{\partial p} \frac{x}{\sqrt{x^2 + y^2}} + \frac{\partial u}{\partial \phi} \left(\frac{-y}{x^2 + y^2} \right) = \frac{\partial u}{\partial p} \cdot \frac{x}{p} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} = \cos \phi \frac{\partial u}{\partial p} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} *$$

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u

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial u}{\partial x} \right)$$

$$\frac{\partial^2 u}{\partial x^2} = \cos \phi \left(\frac{\partial}{\partial p} \left(\frac{\partial u}{\partial x} \right) \right) - \frac{\sin \phi}{p} \left(\frac{\partial}{\partial \phi} \left(\frac{\partial u}{\partial x} \right) \right)$$

$$\frac{\partial^2 u}{\partial x^2} = \cos \left(\frac{\partial}{\partial p} \left(\cos \phi \frac{\partial u}{\partial p} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} \right) \right) - \frac{\sin \phi}{p} \left(\frac{\partial}{\partial \phi} \cos \phi \frac{\partial u}{\partial \phi} - \frac{\sin \phi}{p} \frac{\partial u}{\partial \phi} \right)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \cos \phi \left(\cos \phi \frac{\partial^2 u}{\partial p^2} + \frac{\sin \phi}{p^2} \frac{\partial u}{\partial \phi} - \frac{\sin \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} \right) -$$

$$\frac{\sin \phi}{p} \left(-\sin \phi \frac{\partial u}{\partial p} + \cos \phi \frac{\partial^2 u}{\partial u \partial \phi} - \frac{\cos \phi}{p} \frac{\partial u}{\partial \phi} - \frac{\sin \phi}{p} \frac{\partial^2 u}{\partial \phi^2} \right)$$

$$= \cos^2 \phi \frac{\partial^2 u}{\partial p^2} + \frac{\cos \phi \sin \phi}{p^2} \frac{\partial u}{\partial \phi} - \frac{\cos \phi \sin \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} + \frac{\sin^2 \phi}{p} \frac{\partial u}{\partial p} -$$

$$\frac{\sin \phi \cos \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} + \frac{\sin \phi \cos \phi}{p^2} \frac{\partial u}{\partial \phi} + \frac{\sin^2 \phi}{p} \frac{\partial^2 u}{\partial \phi^2} \Rightarrow$$

$$\cos^2 \phi \frac{\partial^2 u}{\partial p^2} + \frac{2 \sin \phi \cos \phi}{p^2} \frac{\partial u}{\partial \phi} - \frac{2 \cos \phi \sin \phi}{p} \frac{\partial^2 u}{\partial \phi \partial p} + \frac{\sin^2 \phi}{p} \frac{\partial u}{\partial p} + \frac{\sin^2 \phi}{p^2} \frac{\partial^2 u}{\partial \phi^2}$$

$$\frac{\partial^2 u}{\partial y^2} :$$

$$\frac{\partial u}{\partial y} = \sin \phi \frac{\partial u}{\partial p} + \frac{\cos \phi}{p} \frac{\partial u}{\partial \phi} \Rightarrow \frac{\partial^2 u}{\partial y^2} = \sin \phi \frac{\partial}{\partial p} \left(\frac{\partial u}{\partial y} \right) + \frac{\cos \phi}{p} \frac{\partial}{\partial \phi} \left(\frac{\partial u}{\partial y} \right)$$

$$\frac{\partial^2 u}{\partial y^2} = \sin^2 \phi \frac{\partial^2 u}{\partial p^2} + \frac{2 \sin \phi \cos \phi}{p} \frac{\partial^2 u}{\partial p \partial \phi} + \frac{\cos^2 \phi}{p^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\cos^2 \phi}{p} \frac{\partial u}{\partial p} - \frac{2 \sin \phi \cos \phi}{p^2} \frac{\partial u}{\partial \phi}$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 (x, y, z)}{\partial z^2} \quad \nabla^2 u = 0$$

$$\nabla^2 u = 0 \rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \Rightarrow$$

$$\frac{\partial^2 u}{\partial p^2} + \frac{1}{p} \frac{\partial u}{\partial p} + \frac{1}{p^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

:

$$P^2 \frac{\partial^2 u}{\partial p^2} + p \frac{\partial u}{\partial p} + \frac{\partial^2 p}{\partial \phi^2} p^2 \frac{\partial^2 u}{\partial z^2} = 0$$

: Z

$$P^2 \frac{\partial^2 u}{\partial p^2} + p \frac{\partial u}{\partial p} + \frac{\partial^2 p}{\partial \phi^2} p^2 \frac{\partial^2 u}{\partial z^2} = 0 \quad p \frac{\partial^2 u}{\partial z^2} = 0$$

:

$$x^2 \frac{\partial u}{\partial x^2} + x \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \begin{array}{l} 0 < x < \infty \\ 0 < y < 2\pi \end{array}$$

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:

$$\nabla^2 u = 0 \Rightarrow \frac{\partial^2 u}{\partial r^2} + \frac{2}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \phi^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\cos \theta}{r^2} \frac{\partial u}{\partial \theta} = 0 \quad :$$

:

$$\nabla^2 u = \frac{1}{r} (ru)_{rr} + \frac{1}{r^2 \sin^2 \theta} u_{\phi\phi} + \frac{1}{r^2 \sin \theta} (u \theta \sin \theta)_{\theta}$$

$$(u + ru_r) \rightarrow (u_r + u_r + ru_{rr})$$

:

$$\nabla^2 u = \frac{1}{r^2} (r^2 u_r)_r + \frac{1}{r^2 \sin^2 \theta} u_{\phi\phi} + \frac{1}{r^2 \sin \theta} (u_{\theta} \sin \theta)_{\theta} \quad : \quad \phi$$

$$r \frac{1}{r^2} (r^2 u_r)_r + \frac{1}{r^2 \sin \theta} (u_{\theta} \sin \theta)_{\theta} = \nabla^2 u$$

$$\nabla^2 u = G \quad :$$

(Separation .

of Variable)

$$\vdots$$

$$Ku_{xx}(x,t) = u_t(x,t) \quad 0 < x < c \quad t > 0$$

$$u(x,0) = f(x)$$

$$\begin{cases} Ku_{xx}(x,t) = u_t(x,t) \\ u_x(0,t) = 0 \\ u_x(c,t) = 0 \\ u(x,0) = f(x) \end{cases}$$

 $f(x)$

$$u = \sum_{k=1}^{\infty} c_k u_k$$

$$u_n \dots u_1 : ($$

$$n = \infty$$

$$u : ($$

$$: ($$

$$u(x,t)$$

$$u(x,t) = X(x)T(t)$$

$$KX''(x)T(t) = X(x)T'(t) \rightarrow \frac{KX''(x)}{X(x)} = \frac{T'(t)}{T(t)} = -\lambda$$

$$\frac{X''(x)}{X(x)} = -\frac{1}{K} \frac{T'(t)}{T(t)} = -\lambda \rightarrow$$

$$\frac{X'(x)}{X(x)} = -\lambda \quad \frac{T'(t)}{KT(t)} = -\lambda$$

$$X''(x) - \lambda X(x) = 0$$

$$\leftarrow 1$$

$$T'(t) - \lambda KT(t) = 0$$

$$\leftarrow$$

$$\vdots$$

$$1 \quad X'(o)T(t) = X'(c)T(t) = 0$$

$$\Rightarrow T(t) \neq 0, X'(o) = o_1 X'(c) = 0$$

$$\begin{cases} X'(x) - \lambda x(x) = 0 \\ X(o) = 0 \\ X'(c) = 0 \end{cases} \quad T'(t) - k\lambda t(t) = 0 \Rightarrow \frac{T'(t)}{T(t)} = k\lambda \Rightarrow \cos T(t) = k\lambda t + \cos A$$

$$\Rightarrow \cos \frac{T(t)}{A} = k\lambda t \Rightarrow \frac{T(t)}{A} = e^{k\lambda t} \Rightarrow T(t) = Ae^{k\lambda t}$$

: : —

$$1) \lambda > 0 \quad X''(x) - \lambda X(x) = 0 \Rightarrow X(x) = a_1 e^{\sqrt{\lambda}x} + a_2 e^{-\sqrt{\lambda}x} \quad \lambda > 0$$

فرض $\lambda = a^2 \Rightarrow X''(x) = a^2 \times (x) \Rightarrow X(x) = Ae^{ax} + Be^{-ax}$

$$X(x) = Aae^{ax} - Bae^{-ax}$$

$$\begin{cases} X'(o) = 0 \Rightarrow A - B = 0 \\ X'(c) = 0 \Rightarrow Ae^{ac} - Be^{-ac} = 0 \end{cases} \Rightarrow A = 0, B = 0$$

. $\lambda > 0$ $u(x, t)$ $X(x)$

$$2) \lambda = 0 \quad X''(x) = 0 \Rightarrow X(x) = Ax + B$$

$$X'(o) = 0 \Rightarrow A = 0$$

$$X'(c) = 0 \Rightarrow A = 0$$

$$X(x) = B \quad T(t) = A \quad u(x, t) = X(x)T(t) = A.B$$

$$u(x, o) = f(x) \Rightarrow AB = f(x)$$

. $u(x, t) = f$ $f(x)$

$$3) \lambda < 0 \quad X''(x) = -a^2 X(x) \Rightarrow X(x) = A \cos ax + B \sin ax$$

$$\lambda = -a^2$$

$$X'(o) = 0 \Rightarrow -Aa \sin ac + aB \cos ax = 0 \Big|_{x=0} \quad B = 0$$

$$X'(0) = 0 \Rightarrow -Aa \sin ac = 0 \Rightarrow \sin ac = 0 \rightarrow ac = n\pi \Rightarrow a = \frac{n\pi}{c}$$

$$X(x) = \cos\left(\frac{n\pi}{c}x\right) \quad n = 0, 1, \dots$$

$$T(t) = e^{k\lambda t} = e^{-ka^2 t} = e^{-k\left(\frac{n^2\pi^2}{c^2}\right)t} \quad n = 0, 1, \dots$$

$$ku_{xx} = u_t \quad u_n$$

$$u = \sum_{n=1}^{\infty} a_n u_n(x, t)$$

$$u(x, t) = X(x)T(t) = e^{-k\frac{n^2\pi^2}{c^2}t} + \cos\frac{n\pi x}{c}$$

$$u_n(x, t) = e^{-\frac{kn^2\pi^2}{c^2}t} \cos\left(\frac{n\pi x}{c}\right) \quad n = 0, 1, \dots$$

$$u(x, t) = \sum_{n=1}^{\infty} a_n e^{-\frac{n^2\pi^2}{c^2}kt} \cos\frac{n\pi x}{c}$$

$$u(x, 0) = f(x)$$

$$u(x, 0) = \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{c} = f(x)$$

$$f(x) = \sum a_n \cos\frac{n\pi x}{c}$$

$$\int_0^c \cos\frac{n\pi x}{c} dx = \frac{c}{n\pi} \sin\frac{n\pi x}{c} = 0 \quad \begin{cases} 0 & n = 1, 2, \dots \\ c & n = 0 \end{cases}$$

$$\int_0^c \cos\frac{n\pi x}{c} \cos\frac{m\pi x}{c} dx = \frac{1}{2} \int_0^c \left(\cos(n+m)\frac{\pi x}{c} + \cos(n-m)\frac{\pi x}{c} \right) dx$$

$$\Rightarrow \quad n, m, m \quad \begin{cases} 0 & \forall m, n | m \neq n \\ c/2 & n = 0 \end{cases}$$

$$\sum_{n=0}^{\infty} a_n \cos\frac{n\pi x}{c} = f(x) = a_0 + a_1 \cos\frac{n\pi x}{c} + a_2 \cos\frac{2\pi x}{c} + \dots + a_n \cos\frac{n\pi x}{c} = f(x)$$

$$\vdots \quad \cdot \quad c \quad 0$$

$$a_0 \int_0^c dx + a_1 \int_0^c \cos \frac{n\pi x}{c} dx + \dots + \dots = \int_0^c f(x) dx$$

$$a_0 c = \int_0^c f(x) dx \Rightarrow a_0 = \frac{1}{c} \int_0^c f(x) dx$$

$$\cdot \quad \left(\quad \right) \cos \frac{n\pi x}{c} \quad a_1$$

$$a_0 \int_0^c \cos \frac{n\pi x}{c} dx + a_1 \int_0^c \cos \frac{n\pi x}{c} \cos \frac{n\pi x}{c} dx + \dots$$

$$= \int \cos \frac{n\pi x}{c} f(x) dx = a_1 \frac{c}{2} = \int f(x) c \cos \frac{n\pi x}{c} dx$$

$$\vdots$$

$$a_1 = \frac{2}{c} \int_0^c f(x) \cos \frac{2\pi x}{c} dx$$

$$a_n = \frac{2}{c} \int_0^c f(x) \cos \frac{n\pi x}{c} dx \quad \forall n = 1, 2, \dots$$

$$Ku_{xx} = u_t$$

$$u_x(0, t) = 0 \quad u_x(c, t) = 0$$

$$u(x, 0) = ? \quad \vdots$$

$$f(x) = 10x \quad c = 10x \quad \cdot \quad f(x) \quad \vdots$$

$$\cdot$$

$$u(x, t) = \sum_{n=0}^{\infty} a_n e^{-\frac{n^2 \pi^2}{c^2} kt} \cos \frac{n\pi x}{c}$$

$$k = \frac{\text{ظرفیت} \quad \text{حرارتی} \quad \text{واحد}}{\text{حجم} \quad \text{واحد} \quad \text{جرم}}$$

$$a_0 = \frac{1}{10} \int_0^c 10x dx \quad a_n = \frac{2}{10} \int_0^c f(x) \cos \frac{n\pi x}{c} dx$$

$$a_0 = 50 \text{ متوسط} \quad a_n = \frac{2}{10} \int_0^{10} 10x \cos \frac{n\pi x}{c} dx$$

$$a_n = \frac{2}{10} \int_0^c x \cos \frac{n\pi}{10} x dx = \frac{2}{10} \left(\frac{u10}{n\pi} \sin \frac{n\pi x}{c} \Big|_0^{10} - \frac{10}{n\pi} \int_0^{10} \sin \frac{n\pi x}{c} dx \right)$$

$$\begin{cases} \text{زوج} & a_n = 0 \\ \text{فرد} & a_n = -\frac{400}{n^2 \pi^2} \end{cases} \quad \text{یا} \quad a_n = \frac{200}{n^2 \pi^2} ((-1)^n - 1)$$

$$a_n = \frac{2}{10} \left(\left(\frac{10}{n\pi} \right)^2 \cos \frac{n\pi x}{10} \Big|_0^{10} \right) \Rightarrow a_n = \frac{200}{n^2 \pi^2} (\cos n\pi - 1)$$

$$u(x, t) = 50 + \sum_{n=1}^{\infty} \frac{200}{n^2 \pi^2} ((-1)^n - 1) e^{-\frac{n^2 \pi^2 k}{10} t} \cos \frac{n\pi x}{10}$$

$$f(x)$$

Separation of Variables

PDE

PDE

$$\begin{aligned} u_{xx}(x, y) &= 0 & u(0, y) &= y^2 & u(1, y) &= 1 \\ 0 < x < 1 & & -\infty < y < \infty \end{aligned}$$

$$u_{xx}(x, y) = 0 \Rightarrow u_x(x, y) = \phi(y) \Rightarrow u(x, y) = \phi(y)x + h_y$$

$$u(0, y) = y^2 \Rightarrow h(y) = y^2 \quad (\phi(y) \times 0)$$

$$u(1, y) = \phi(y) + h(y) = 1 \Rightarrow \phi(y) = 1 - y^2$$

$$u(x, y) = (1 - y^2)x + y^2$$

$$a^2 y_{xx}(x, t) = y_{tt}(x, t)$$

$$\begin{aligned} y(x, 0) &= h(x) & -\infty < x < \infty \\ y_t(x, 0) &= 0 & t > 0 \end{aligned}$$

$$\begin{cases} u = x + at \\ v = x - at \end{cases}$$

← :

:

$$(x, t) \rightarrow (v, u)$$

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial y}{\partial v} \cdot \frac{\partial v}{\partial t} \quad y(x, t) \rightarrow y(u, v)$$

$$\frac{\partial y}{\partial u} \cdot a + \frac{\partial y}{\partial v} \cdot (-a) = a \frac{\partial y}{\partial u} - a \frac{\partial y}{\partial v}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t} \left(\frac{\partial y}{\partial t} \right) \Rightarrow \frac{\partial^2 y}{\partial t^2} = a \frac{\partial}{\partial u} \left(a \frac{\partial y}{\partial u} - a \frac{\partial y}{\partial v} \right) - a \frac{\partial}{\partial v} \left(a \frac{\partial y}{\partial u} - a \frac{\partial y}{\partial v} \right)$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial u^2} - a^2 \frac{\partial^2 y}{\partial u \partial v} - a^2 \frac{\partial^2 y}{\partial u \partial v} + a^2 \frac{\partial^2 y}{\partial v^2}$$

$$\Rightarrow a^2 \frac{\partial^2 y}{\partial u^2} - 2a^2 \frac{\partial^2 y}{\partial u \partial v} + a^2 \frac{\partial^2 y}{\partial v^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} \right)$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial y}{\partial v} \cdot \frac{\partial v}{\partial x} = \frac{\partial y}{\partial u} + \frac{\partial y}{\partial v}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} + \frac{\partial y}{\partial v} \right) + \frac{\partial}{\partial u} \left(\frac{\partial y}{\partial u} + \frac{\partial y}{\partial v} \right) = \frac{\partial^2 y}{\partial u^2} + 2 \frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 y}{\partial v^2}$$

PDE

$$a^2 \left(\frac{\partial^2 y}{\partial u^2} + \frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 y}{\partial v^2} \right) = a^2 \left(\frac{\partial^2 y}{\partial u^2} - 2 \frac{\partial^2 y}{\partial u \partial v} + \frac{\partial^2 y}{\partial v^2} \right)$$

$$4 \frac{\partial^2 y}{\partial u \partial v} = 0 \Rightarrow \frac{\partial^2 y}{\partial u \partial v} = 0 \Rightarrow \begin{cases} y_u(u, v) = h(u) \\ y_v(u, v) = q(v) \end{cases}$$

:

$$g(u, v) = f(u) + g(v) \quad f'(u) \rightarrow 0$$

$$y(u, v) = f(u) + g(v)$$

$$x + at = u$$

$$x - at = v$$

:

$$y(x, t) = f(x, at) + g(x - at)$$

$$af'(x) - ag(x) = 0 \Rightarrow f'(x) = g'(x)$$

$$f(x) = g(x) + k \quad c$$

$$2f(x) = h(x) + c \Rightarrow f(x) = \frac{1}{2}(h(x) + c)$$

$$2g(x) = h(x) - c \Rightarrow g(x) = \frac{1}{2}(h(x) - c)$$

:

$$\text{پس } y(x, t) = f(x, at) + g(x - at) = \frac{1}{2}(h(a + at) + h(x - at)) \quad \left(\begin{array}{l} \\ \end{array} \right)$$

:

$$y_{tt}(x, t) = a^2 y_{xx}(x, t)$$

$$y(0, t) = y(c, t) = 0 \quad 1$$

$$y(x, 0) = f(x) \quad y_t(x, 0) = 0 \quad 3$$

$$a, c \quad .$$

.

:

$$y(x, t) = X(x)T(t)$$

$$X(x)T''(t) = a^2 X''(x)T(t) \Rightarrow \frac{X''(x)}{X(x)} = \frac{1}{a^2} \frac{T''(t)}{T(t)} = \begin{cases} \lambda^2 \\ 0 \\ -\lambda^2 \end{cases}$$

$$\text{الف } \frac{X''(x)}{X(x)} = \lambda^2 \Rightarrow x''(X) = \lambda^2 X(X)$$

$$\frac{T''(t)}{T(t)} = \lambda^2 \rightarrow T''(t) = a^2 \lambda^2 T(t)$$

$$\begin{cases} X(t) = Ae^{\lambda x} + Be^{-\lambda x} \\ T(t) = Ce^{a\lambda t} + De^{-a\lambda t} \end{cases}$$

$$1 \Rightarrow X(0) = 0 \Rightarrow A + B = 0 \quad \Rightarrow A = 0, B = 0$$

$$\begin{aligned} X(c) = 0 &\Rightarrow Ae^{\lambda c} + Be^{-\lambda c} = 0 \\ \Rightarrow \lambda = 0 &\Rightarrow \begin{cases} X''(x) = 0 \\ T''(t) = 0 \end{cases} \Rightarrow \begin{cases} X(x) = Ax + B \\ T(t) = Ct + D \end{cases} \end{aligned}$$

غلق

مرزى شرط

$$\begin{cases} X(0) = 0 \Rightarrow B = 0 \\ X(c) = 0 \Rightarrow A = 0 \end{cases}$$

$$\text{ج) } \lambda < 0 \quad \begin{cases} X''(x) = -\lambda^2 X(x) \Rightarrow X(x) = B \sin \lambda x + A \cos \lambda x \\ T''(t) = -a^2 \lambda^2 T(t) \Rightarrow T(t) = C \sin a\lambda t + D \cos a\lambda t \end{cases}$$

$$1 \rightarrow X(0) = 0 \Rightarrow A = 0 \quad X_n(x) = \sin \frac{n\lambda}{c} x$$

$$X(c) = 0 \Rightarrow B \sin \lambda c = 0 \Rightarrow \lambda c = n\pi \Rightarrow \lambda = \frac{n\pi}{c} \quad \forall n = 0, 1, \dots$$

:

$$T_n(t) = C \sin \frac{an\pi}{c} t + D \cos \frac{an\pi}{c} t + *$$

$$3: \quad X(x) T'(0) = 0$$

$$* = 0 \Rightarrow T'_n(0) = 0 \Rightarrow C$$

$$T_n(t) = \cos \frac{an\pi C}{T}$$

بنابر این :

$$X_n(x) = \sin \frac{n\pi x}{c}, \quad T_n(t) = \cos \frac{a\pi x}{c} t$$

پس

$$Y_n(x, t) = \sin \frac{n\pi x}{c} \cos \frac{a\pi x}{c} t$$

$$\sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{c} \quad \text{پس} \quad y(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \cos \frac{a\pi x}{c} t$$

 b_n $y(x, t)$

$$\left(\quad t \quad \right).$$

 $f(x)$

$$g(x, 0) = f(x)$$

$$y(x, 0) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} = f(x)$$

$$\int_0^c \sin \frac{n\pi x}{c} dx = -\frac{c}{n\pi} \cos \frac{n\pi x}{c} \Big|_0^c = -\frac{c}{n\pi} (\cos n\pi - 1) = \frac{c}{n\pi} ((-1)^{n+1} + 1) = \frac{c}{n\pi} (1 - (-1)^n)$$

$$\int_0^c \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx = \begin{cases} 0 & n \neq m \\ c/2 & n = m \end{cases}$$

$$\therefore \sin \frac{n\pi x}{c} \sum b_n \sin \frac{n\pi x}{c} = f(x)$$

$$\sum_{n=1}^{\infty} \int_0^c b_n \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx = \int_0^c f(x) \sin \frac{m\pi x}{c} dx$$

$$c/2 b_m = \int_0^c f(x) \sin \frac{m\pi x}{c} dx \Rightarrow b_m = 2/c \int_0^c f(x) \sin \frac{m\pi x}{c} dx$$

$$\begin{cases} f(x) = 0.2x & 0 < x < 10 \\ f(x) = -0.2x + 4 & 10 < x < 20 \end{cases} \quad : \quad f(x)$$

$$= 2/20 \left(2/10 \left(-x \frac{20}{n\pi} \cos \frac{n\pi x}{20} \right) \Big|_0^{10} + \frac{20}{n\pi} \int_0^{10} \cos \frac{n\pi x}{20} dx \right)$$

$$b_n = 2/20 \int_0^{20} f(x) \sin \frac{n\pi x}{20} dx = 2/20 \left[\int_0^{10} 2/10 x \sin \frac{n\pi x}{20} dx + \int_{10}^{20} (-2/20 + 4) \sin \frac{n\pi x}{20} dx \right]$$

$$+ \left(\left((-2/20 x + 4) \left(\frac{-20}{n\pi} \cos \frac{n\pi x}{20} \right) \Big|_{10}^{20} + \frac{20}{n\pi} \int_{10}^{20} \left(\frac{-2}{10} \cos \frac{n\pi x}{c} \right) dx \right) \right)$$

$$\dots + \int_{10}^{20} \frac{-2}{20} x \sin \frac{n\pi x}{20} dx + \int_{10}^{20} 4 \sin \frac{n\pi x}{20} dx$$

$$= 2/20 \int_{10}^{20} x \sin \frac{n\pi x}{20} dx + \frac{4 \times 20}{n\pi} \left(-\cos \frac{n\pi x}{20} \right) \Big|_{10}^{20} = \dots$$

:

:

[a,b]

f(x)

$$x_1, x_2, \dots, x_n$$

$$a < x_1 < x_2 < \dots < x_n < b$$

$$f(x_1^-)f(x_1^+), f(x_2^-), f(x_2^+), \dots, f(x_n^-), f(x_n^+)$$

:

$$f(x) = \begin{cases} -1 & -1Mx < -0.5 \\ a & -0.5 < x < 0.5 \\ 1 & 0.5 \leq x < 1 \end{cases}$$

$$(c, d) \subset (a, b) \quad (c, d)$$

:

$$\int_c^d f(x) dx = \int_c^{x_M} f(x) dx + \int_{x_M}^{x_{M+1}} f(x) dx + \dots + \int_{x_{r-1}}^d f(x) dx$$

.

$$f(x)$$

$$[a, b] \quad)$$

.

:

$$f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c}$$

.

$$C \quad 0$$

$$f(x)$$

.

$$0 < x < \eta \quad f(x) \quad :$$

$$f(x) \approx a_0/2 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{\pi}$$

$$: \quad a_n$$

$$a_0/2 + \sum_{n=1}^{\infty} a_n \cos nx = f(x) \Rightarrow a_0/2 \int_0^{\eta} dx + \sum_{n=1}^{\infty} a_n \int_0^{\eta} \cos nx dx = \int_0^{\eta} f(x) dx$$

$$= a_0 / 2 \times \pi + 0 = \int_0^{\eta} f(x) dx \Rightarrow a_0 = 2 / \pi \int_0^{\eta} f(x) dx$$

$$a_0 / 2 \int_0^{\eta} \cos mx dx + \sum_{n=1}^{\infty} a_n \int_0^{\eta} \cos nx \cos mx dx = \int_0^{\eta} f(x) \cos mx dx$$

$$\sum_{n=1}^{\infty} a_n \frac{1}{2} \left[\int_0^{\eta} (\cos(n-m)x - \cos(nn+m)x) dx \right] = \int_0^{\eta} f(x) \cos mx dx$$

$$\sum_{n=1}^{\infty} a_n \frac{1}{2} \left[\int_0^{\eta} (\cos(n-m)x - \cos(nn+m)x) dx \right] = \int_0^{\eta} f(x) \cos mx dx$$

$$a_n = 2 / \pi \int_0^{\eta} f(x) \cos nx dx$$

:

$$f(x) \cong \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \quad [0, c] \quad f(x)$$

$$f(x) \cong \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c}$$

n

$$\int_0^c \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx = \int_0^c f(x) \sin \frac{m\pi x}{c} dx$$

:

f(x)

$$b_m \sum_{n=1}^{\infty} \int_0^c \sin \frac{n\pi x}{c} \sin \frac{m\pi x}{c} dx \Rightarrow b_m = \frac{2}{c} \int_0^c f(x) \sin \frac{m\pi x}{c} dx$$

 $\frac{m\pi x}{c}$

:

(0, \pi)

f(x)=x :

$$f(x) \cong \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \Rightarrow f(x) = \sum_{n=1}^{\infty} b_n \sin \pi x$$

$$b_n = 2 / \pi \int_0^{\pi} x \sin nx dx = \frac{-x}{n} \cos nx \Big|_0^{\pi} + \frac{1}{n} \int_0^{\pi} \cos nx dx$$

$$= 2 / \pi \left(\frac{\pi}{n} (-1)^{n+1} - 0 + \frac{1}{n^2} \sin x \Big|_0^{\pi} \right) = \left(\frac{(-1)^{n+1} \times \pi}{n} \right) \times 2 / \pi = \frac{2(-1)^{n+1}}{n}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \times 2}{n} \sin nx = 2 \sin x - \sin 2x + \frac{2}{3} \sin 3x \dots \dots \dots$$

$$f(x) = x \quad [0, \pi] \quad \sum_{n=1}^{\infty} b_n \sin nx \quad \sin nx \quad f(x)$$

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx \quad (\quad) \quad \pi \quad - \quad :$$

$$(-c, c) \quad f(x)$$

$$f(x) = h(x) + g(x) \quad \Leftarrow \quad g(-x) = -g(x) \quad , \quad h(-x) = h(x)$$

$$h(x) = \frac{f(x) + f(-x)}{2} \quad g(x) = \frac{f(x) - f(-x)}{2} \quad h(x)$$

$$(-c, c) \quad h(x)$$

$$h(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} \quad a_n = \frac{2}{c} \int_0^c h(x) \cos \frac{n\pi x}{c} dx$$

$$f(x) = h(x) + g(x) \quad (-c, c)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \quad \Leftarrow \quad x \in (-c, c)$$

$$(-c, c) \quad f(x)$$

$$a_n = \frac{2}{c} \int_0^c \frac{f(x) + f(-x)}{2} \cos \frac{n\pi x}{c} dx = \frac{1}{c} \left[\int_0^c f(x) \cos \frac{n\pi x}{c} dx + \int_0^c f(-x) \cos \frac{n\pi x}{c} dx \right]$$

$$= \frac{1}{c} \left[\int_0^c f(x) \cos \frac{n\pi x}{c} dx + \int_{-c}^0 f(x) \cos \frac{n\pi x}{c} dx \right] = \frac{1}{c} \int_{-c}^c f(x) \cos \frac{n\pi x}{c} dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{c} + b_n \sin \frac{n\pi x}{c} \right)$$

$$a_n = \frac{1}{c} \int_{-c}^c f(x) \cos \frac{n\pi x}{c} dx \quad b_n = \frac{1}{c} \int_{-c}^c f(x) \sin \frac{n\pi x}{c} dx$$

$$f(x) = \begin{cases} 0 & -\pi < x < -\frac{\pi}{2} \\ 1 & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < x < \pi \end{cases} \quad x \in (-\pi, \pi) :$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^N (a_n \cos nx + b_n \sin nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin \frac{n\pi x}{\pi} dx$$

$$\frac{1}{\pi} \int_{-\pi/2}^{\pi} 1 \times \cos nx dx = \frac{1}{n\pi} \sin nx \Big|_{-\pi/2}^{\pi/2} = \frac{1}{n\pi} \left(\sin \frac{n\pi}{2} + \sin \frac{n\pi}{2} \right) = \frac{1}{n\pi} \times 2 \sin \frac{n\pi}{2}$$

$$\text{ح} \quad b_n : \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \sin nx dx = 0$$

$$f(x) = 1 + \sum_{n=1}^{\infty} \left(\frac{2}{n\pi} \sin \frac{n\pi}{2} \right) \cos nx \quad \int_{-\pi/2}^{\pi/2} 1 dx = 1 \quad a_0 =$$

$$f(x) \quad (-\pi, \pi) \quad f(x) :$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

$$f(x) = x \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin nx \, dx = \frac{2}{\pi} \int_0^{\pi} \sin nx \, dx = \frac{2}{\pi} \left(-\frac{x}{n} \cos nx \Big|_0^{\pi} + \frac{1}{n^2} \sin nx \Big|_0^{\pi} \right)$$

$$f(x) \cong \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx = 2 \left(\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x + \dots \right)$$

$$f(-\pi) = 0 \quad f(\pi) = 0 \quad f(x) = f(x)$$

$$\begin{aligned} & \vdots \\ & : \quad S_N(x) = \frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx \\ & \frac{2}{\pi} \int_0^{\pi} S_N(x) \cos nx = a_n \end{aligned}$$

$$S_N(x) = \frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx$$

$$\frac{2}{\pi} \int_0^{\pi} (S_N(x))^2 dx$$

$$\frac{2}{\pi} \int_0^{\pi} \left(\frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx \right) S_N(x) dx = \frac{2}{\pi} \int_0^{\pi} \frac{a_0}{2} S_N(x) dx + \frac{2}{\pi} \sum_{n=1}^N a_n \int_0^{\pi} S_N(x) \cos nx dx$$

$$= \frac{2}{\pi} \frac{a_0^2}{4} \int_0^{\pi} dx + \frac{2}{\pi} \sum_{n=1}^N a_n^2 \times \frac{\pi}{2} = \left(\frac{a_0^2}{2} + \sum_{n=1}^N a_n^2 \right)$$

$$S_N(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$\frac{2}{\pi} \int_0^{\pi} S_N(x) \sin mx dx = b_m$$

$$\begin{aligned} \frac{2}{\pi} \int_0^{\pi} [S_N(x)]^2 dx &= \sum_{n=1}^N \frac{2}{\pi} \int_0^{\pi} S_N(x) b_n \sin nx dx \\ &= \sum_{n=1}^N b_n \times \frac{2}{\pi} \int_0^{\pi} S_N(x) \sin nx dx = \sum_{n=1}^N (b_n)^2 \end{aligned}$$

$$E = \frac{2}{\pi} \int_0^{\pi} (f(x) - S_N(x))^2 dx \geq 0 \quad \vdots$$

$$\frac{2}{\pi} \int_0^{\pi} f^2(x) dx - \frac{4}{\pi} \int_0^{\pi} f(x) S_N(x) + \frac{2}{\pi} \int_0^{\pi} (S_N(x))^2 dx$$

f

$$(0, \pi) \quad f(x)$$

$$\vdots \quad \eta \leq 0$$

$$\int_0^{\pi} f^2(x) dx$$

:

$$\frac{2}{\pi} \int_0^{\pi} f(x) S_N(x) dx$$

$$\left(\quad \right)$$

:

$$\begin{aligned} \frac{2}{\pi} \int_0^{\pi} f(x) \left(\frac{a_0}{2} + \sum_{n=1}^N a_n \cos nx \right) dx &= \frac{2}{\pi} \times \frac{a_0}{2} \int_0^{\pi} f(x) dx + \sum_{n=1}^N a_n \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx \\ &= \frac{a_0^2}{2} + \sum_{n=1}^N a_n^2 \end{aligned}$$

:

$$\frac{2}{\pi} \int_0^{\pi} f(x) \left(\sum_{n=1}^N b_n \sin nx \right) dx = \sum_{n=1}^N b_n \times \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx = \sum_{n=1}^N b_n^2$$

$$\frac{2}{\pi} \int_0^{\pi} f^2(x) dx = \frac{a_0^2}{2} + \sum_{n=1}^{\pi} (a_n^2 + b_n^2)$$

$$\frac{2}{\pi} \int_0^{\pi} f^2(x) dx \geq \left(\frac{a_0^2}{2} + \sum_{n=1}^N a_n^2 \right)$$

$$\sum_{n=1}^N b_n^2 \leq \frac{2}{\pi} \int_0^{\pi} f^2(x) dx$$

$$\left(\sum_{n=1}^N a_n^2 \right) \leq \frac{2}{\pi} \int_0^{\pi} f^2(x) dx$$

:

$$\frac{a_0^2}{2} + \sum_{n=1}^N a_n^2 \geq \frac{2}{\pi} \int_0^{\pi} f^2(x) dx$$

:

$$\frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

$$\lim_{n \rightarrow \infty} a_n = 0$$

:

$$\frac{2}{\pi} \int_0^{\pi} f^2(x) dx \geq 2 \times \left(\sum_{n=1}^N b_n^2 + \sum_{n=1}^N a_n^2 \right) \geq 0$$

$$\sum_{n=1}^N b_n^2 \leq \frac{2}{\pi} \int_0^{\pi} f^2(x) dx$$

$$\lim_{n \rightarrow \infty} b_n = 0$$

.

$$a_n, b_n$$

:

$$(-n, n)$$

$$f(x)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

:

$$a_n = \frac{1}{n} \int_{-n}^n f(x) \cos x dx$$

$$b_n = \frac{1}{n} \int_{-n}^n f(x) \sin x dx$$

$$f(x)$$

$$f(x)$$

$$f(x)$$

$$f(x)$$

$$f(x)$$

$$\lim_{n \rightarrow \infty} a_n = 0, \quad \lim_{n \rightarrow \infty} b_n = 0$$

$$a_0^2/2 + \sum_{n=1}^N a_n^2 \leq \frac{1}{R} \int_{-R}^R f(x)^2 \cos^2 x dx \geq 1$$

$$\sum_{n=1}^N b_n^2 \leq \frac{1}{R} \int_{-R}^R f(x)^2 \sin^2 x dx$$

$$\sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2}$$

$$f(x)$$

$$b_n, a_n$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \beta(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} \beta(x) \sin nx dx$$

$$\alpha_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f'(x) \cos nx dx$$

$$\beta_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f'(x) \sin nx dx$$

$$\alpha_n = a_n \times n, \quad \beta_n = n b_n$$

$$S_N = \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2}$$

$$S_N = \left(\sum_{n=1}^N \sqrt{a_n^2 + b_n^2} \right)^2$$

$$S_N = \left(\sum_{n=1}^N (a_n^2 + b_n^2)^{\frac{1}{2}} \right)^2 = \left(\sum_{n=1}^N \left(\frac{\alpha_n^2}{n^2} + \frac{\beta_n^2}{n^2} \right)^{\frac{1}{2}} \right)^2 =$$

$$S_N = \left(\sum_{n=1}^N \left(\frac{1}{n^2} (a_n^2 + b_n^2)^{\frac{1}{2}} \right) \right)^2 < \left(\sum_{n=1}^N \frac{1}{n^2} \right) \left(\sum_{n=1}^N (a_n^2 + b_n^2) \right)$$

$$\sum_{n=1}^N (\alpha_n^2 + \beta_n^2)$$

$$\sum_{n=1}^N \frac{1}{n^2}$$

$$\begin{aligned}
& \sum_{n=1}^N (\alpha_n^2 + \beta_n^2)^{1/2} \\
& (-\pi, \pi) \quad f(x) \quad : \\
& \frac{1}{\pi} \int_{-\pi}^{\pi} f^2(x) dx = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \left(\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx \right) dx \\
& = \frac{a_0}{2} \times \frac{1}{\eta} \int_{-\eta}^{\eta} f(x) dx + \sum_{n=1}^{\infty} a_n \left(\frac{1}{\eta} \int_{-\eta}^{\eta} f(x) \cos nx dx \right) + \sum_{n=1}^{\infty} b_n \left(\frac{1}{\eta} \int_{-\eta}^{\eta} f(x) \sin nx dx \right) \\
& = \frac{1}{\eta} \int_{-\eta}^{\eta} \frac{a_0}{2} f(x) dx + \sum_{n=1}^{\infty} a_n \frac{1}{\eta} \int_{-\eta}^{\eta} f(x) \cos nx dx + \sum_{n=1}^{\infty} b_n \frac{1}{\eta} \int_{-\eta}^{\eta} f(x) \sin nx dx \\
& = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + \sum_{n=1}^{\infty} b_n^2 \\
& \frac{1}{\eta} \int_{-\eta}^{\eta} f^2(x) dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \quad (\quad) \\
& : \\
& : \quad (-\eta, \eta) \quad f(x) \\
& \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \\
& a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \\
& : \quad f(x) \\
& \int_{-\pi}^{\pi} f(x) dx = \frac{a_0}{2} (x + \pi) + \sum_{n=1}^{\infty} \frac{1}{\pi} (a_n \sin nx - b_n (\cos nx (-1)^{n+1})) \\
& : \quad [-\pi, \pi] \quad f(x) \\
& f'(x) = f(x) \\
& : \quad f''(x) \quad f'(x) \\
& f'(x) = \sum (-n a_n \sin nx + n b_n \cos nx)
\end{aligned}$$

$$y_t = a^2 y_{xx}(x, t)$$

$$y(a, t) = 0 \rightarrow y(c, t) = 0 \quad y(x, 0) = f(x)$$

$$\vdots$$

$$y_t(x, t) = a^2 u_{xx}(x, t)$$

$$\vdots$$

$$f(x)$$

$$u_t(x, t) = a^2 u_{xx}(x, t)$$

$$U(x, t) = u(z, t) + \phi(x)$$

$$U_t(x, t) = a^2 u_{xx}(x, t) + h(x)$$

$$u_t(x, t) a^2 (u_{xx}(x, t) + \phi''(x)) + h(x)$$

$$a^2 \phi''(x) + h(x) = 0 \rightarrow \phi'' = -1/a^2 h(x)$$

$$\phi(x) = \frac{-1}{a^2} \iint h(t) dt dv$$

$$u_t(x, t) = a^2 u_{xx}(x, t)$$

$$(\quad)$$

$$U_{tt}(xyt) = a^2 [(Z_{xx})(xy+) + Z_{yy}(xy_+)]$$

$$0 < x < a \quad 0 < y < b \quad + > 0$$

$$\vdots$$

$$Z(0, y, t) = 0$$

$$Z(a, y, t) = 0$$

$$Z(x, 0, t) = 0$$

$$Z(x, b, t) = 0$$

$$Z(x, y, 0) = f(x, y)$$

$$\vdots$$

$$\vdots$$

$$Z(x, y, t) = X(x).Y(y).T(t)$$

$$XY T'' = a^2 (x'' y t + X Y'' T) \Rightarrow \frac{T''}{a^2 T} = \frac{x''}{x} + \frac{y''}{y}$$

$$\vdots$$

$$\frac{T''}{a^2 T} = -\lambda^2$$

$$\vdots$$

$$\lambda < 0$$

$$T'' = -a^2 \lambda^2 T$$

$$\frac{X''(x)}{X(x)} + \frac{y''}{y} = -\lambda^2 \rightarrow \frac{X''}{X} = \frac{-y''}{y} - \lambda^2 = \begin{cases} +\eta^2 \\ 0 \\ -\eta^2 \end{cases}$$

$$\begin{cases} x(0) = 0 & x(a) = 0 \\ y(0) = 0 & y(b) = 0 \end{cases} :$$

$$\frac{X''}{X} = -\eta^2 \Rightarrow X''(x) = -\eta^2 X \Rightarrow X = B \sin \eta x + A \cos \eta x$$

$$X(a) = 0 \rightarrow A = 0$$

$$X(a) = 0 \rightarrow 0 = B \sin \eta a \Rightarrow \eta a = x \eta \Rightarrow \eta = \frac{n \eta}{a}$$

$$\Rightarrow X_n(x) = \sin \left(\frac{n \eta}{a} x \right)$$

$$\frac{y''}{y} - \lambda^2 = -\eta^2 \Rightarrow \frac{y''}{y} = \lambda^2 - \eta^2 \Rightarrow y'' = (\lambda^2 - \eta^2)y$$

$$\lambda^2 - \eta^2$$

$$y(0) = 0$$

$$y(b) = 0$$

$$y'' = -(\lambda^2 - \eta^2)y$$

$$y = C \sqrt{\eta^2 - \lambda^2} y + D \sin \sqrt{\eta^2 - \lambda^2} y$$

$$\sqrt{\eta^2 - \lambda^2} = \frac{m \eta}{b} \Rightarrow \eta^2 - \lambda^2 = \left(\frac{m \eta}{b} \right)^2 \Rightarrow \lambda^2 = \left(\frac{n \eta}{a} \right)^2 - \left(\frac{m \eta}{b} \right)^2$$

$$\Rightarrow \lambda = \eta \sqrt{\frac{n^2}{a^2} - \frac{m^2}{b^2}}$$

$$y_m = \sin \left(\frac{m \eta}{b} y \right) \quad \eta, \lambda$$

$$\begin{matrix} Z_3 = 3 - 2\lambda & Z_2 = 1 - \lambda & Z_1 = 2\lambda \\ 3i & (1 + \sqrt{3}i) & (-1 + i) \end{matrix}$$

$$3i \quad \left(\quad 1 + \sqrt{3}i \quad \left(\quad -1 + i \quad \left(\right.$$

$$\therefore \quad Z_3 = se^{\frac{i}{u}} \quad , \quad Z_2 = \sqrt{2}e^{\frac{\pi}{3}} \quad , \quad Z_1 = 2e^{i\pi} \quad -$$

$$Z^4 - 2Z^2 + 4 = 0 \quad \left(\quad Z_1 \overline{Z_2} \quad \left(\quad \frac{Z_1}{Z_3} \quad \left(\right. \right. \\ \cos 3\theta \quad , \quad \sin 3\theta \quad -$$

$$\therefore \quad -$$

$$\operatorname{Im}(iz) = \operatorname{Re} z \quad \left(\quad \operatorname{Re}(iz) = -\operatorname{Im} z \quad \left(\right.$$

$$\overline{Z} + 3i = z - 3i \quad \left(\quad \overline{iz} = -i\overline{z} \quad \left(\right.$$

$$\therefore \quad -$$

$$\left. \right) \operatorname{Re}(\overline{z} - i) = z$$

$$\left. \right) |z - i| = |z + i|$$

$$\cdot \quad Z_1 Z_2 = 0 \quad -$$

$$\therefore \quad -$$

$$\left(\quad Z^{\frac{2}{3}} + 3i = 0 \quad \left(\quad Z^2 + 4 = 0 \quad \left(\right.$$

$$\cdot \quad z^2 + z^{-2} = 2 \quad x^2 - y^2 = 1 \quad -$$

$$-$$

$$(1+z)^n = 1 + \frac{n}{i}z + \frac{n(n-1)}{2i}z^2 + \dots + \frac{n(n-1)(n-2) + \dots (n-k+1)}{ki}z^k + \dots + z^n$$

$$\cdot \quad n$$

$$z_1 - z_2 \quad , \quad z_1 + z_2$$

$$\cdot$$

$$\left. \right) z_{1=} = (-3, 1) \quad z_2 = (1, 4)$$

$$\left. \right) z_1 = x_1 + iy_1 \quad z_{2=} = x_1 - iy_1$$

$$\therefore \quad -$$

$$\left| (2\overline{z} + 5)(\sqrt{2} - i) \right| = \sqrt{3} |2z + 5|$$

$$|\operatorname{Re} z| + |\operatorname{Im} z| < \sqrt{2} |z| \quad \therefore \quad -$$

$$\therefore \quad \arg z \quad -$$

$$Z = \frac{-2}{1 + \sqrt{3}i}$$

$$Z = \frac{i}{-2 - 2i}$$

$$\left(\begin{array}{c} \vdots \\ \vdots \end{array} \right)$$

$g, f:$

$$\vec{f} = (a_1, a_2, a_3) \quad \vec{g} = (b_1, b_2, b_3)$$

:

$$\vec{f} \cdot \vec{g} = a_1 b_1 + a_2 b_2 + a_3 b_3 \quad |f|^2 = f \cdot f = a_1^2 + a_2^2 + a_3^2$$

$$\vec{f} \cdot \vec{g} = |\vec{f}| |\vec{g}| \cos \theta \quad \left| \vec{f} - \vec{g} \right| = \left((a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 \right)^{1/2}$$

$$f + g \Rightarrow \vec{f} \cdot \vec{g} = 0 \quad |f| = 1 \Rightarrow f = \text{بردار نرمال}$$

:

$$\phi_n, \dots, \phi_2, \phi_1$$

$$\langle \phi_i, \phi_j \rangle = 0 \quad , \quad \langle \phi_i, \phi_i \rangle = |\phi_i|^2$$

$$\frac{\phi_n}{|\phi_n|}, \dots, \frac{\phi_2}{|\phi_2|}, \frac{\phi_1}{|\phi_1|}$$

$$\phi_n, \dots, \phi_1$$

$$f = c_1 \phi_1 + c_2 \phi_2, \dots, c_n \phi_n$$

:

$$C_K \langle \phi_k \phi_k \rangle = \langle f \cdot \phi_k \rangle \Rightarrow C_k = \frac{1}{\|\phi_k\|} \langle f, \phi_k \rangle$$

:

$$\Delta x \quad (\text{pwc}) \quad - \quad (a, b) \quad f, g$$

:

$$f = (f_1, f_2, \dots, f_n)$$

$$g = (g_1, g_2, \dots, g_n)$$

:f, g

$$(f, g) = \sum f_K g_K \Delta x_K$$

$$\lim_{\Delta x \rightarrow 0} (f, g) = \int_a^b f(x)g(x)dx$$

Fundamental interval

a, b

:

$$(f, g) = (g, f)$$

$$(f, g + h) = (f, g) + (f, h)$$

$$(cf, g) = c(f, g)$$

$$(f, f) = \|f\|^2 \geq 0 \rightarrow \|f\| = \left(\int_a^b f^2(x)dx \right)^{1/2}$$

$$(f, g) = 0 \Rightarrow f \perp g$$

$$\|f - g\|^2 = \int_a^b (f - g)^2 dx$$

$$(f, g) = 1 \Rightarrow f \text{ and } g \text{ are orthonormal}$$

:

$$\langle \Psi_i(x), \Psi_j(x) \rangle = \frac{2}{c} \langle \Psi_i \Psi_j \rangle = \delta(i, j) \quad 0 < x < c \quad \Psi_n(x) = \sin \frac{n\pi x}{c}$$

:

$$\phi_i = \sqrt{\frac{2}{c}} \sin \frac{n\pi x}{c}$$

$$\phi_0(x) = \frac{1}{\sqrt{c}} \quad , \quad \phi_n(x) = \sqrt{\frac{2}{c}} \cos \frac{n\pi x}{c} \quad :$$

$$0 < x < c$$

$$\phi_0(x) = \frac{1}{\sqrt{c}} \sin \frac{n\pi x}{c} \quad , \quad \phi_{2n-1} = \frac{1}{\sqrt{c}} \cos \frac{n\pi x}{c} \quad , \quad \phi_0 = \frac{1}{\sqrt{2c}} \quad :$$

$$(-C, C)$$

:

$$f = c_0 \phi_0 + c_1 \phi_1 + \dots + c_k \phi_k + \dots + c_n \phi_n$$

$$C_k = \langle f, \phi_k \rangle \quad k = 0, 1, \dots, \quad C_k = \int_a^b f(x) \phi_k(x) dx$$

:

$$f(x) = C_0 \frac{1}{\sqrt{2c}} + C_1 \frac{1}{\sqrt{2}} \cos \frac{n\pi x}{c} + C_2 \frac{1}{\sqrt{2}} \sin \frac{n\pi x}{c} + \dots$$

$$C_k = \langle f(x), \frac{1}{\sqrt{c}} \cos \frac{n\pi x}{c} \rangle = \frac{1}{\sqrt{c}} \int_{-c}^c f(x) \cos \frac{n\pi x}{c} dx$$

$$a_k = \frac{ck}{\sqrt{c}} = \frac{1}{c} \int_{-c}^c f(x) \cos \frac{n\pi x}{c} dx.$$

:

$$\phi_n(x), \dots, \phi_2(x), \phi_1(x)$$

$$\langle \phi_m(x), \phi_k(x) \rangle = \begin{cases} 0 & k \neq m \\ c_k^2 = 1 & k = m \end{cases} \quad k, m = 1, 2, \dots$$

$$\phi_n(x), \dots, \phi_1(x)$$

$$\phi(x)$$

$$\phi(x) = \delta_1 \phi_1(x) + \dots + \delta_n \phi_n(x)$$

$$f(x)$$

$$\phi(x)$$

$$k=1, 2, \dots, n$$

$$\phi_k$$

$$f(x)$$

.

$$f(x)$$

$$\phi(x)$$

$$\delta_n, \dots, \delta_2, \delta_1$$

:

$$|f(x) - \phi(x)|$$

$$|f(x) - \phi(x)|$$

$$E = |f(x) - \phi(x)| = \left(\int_a^b f(x) - \phi(x)^2 dx \right)^{1/2}$$

$$E^2 = |f(x) - \phi(x)|^2 = \int_a^b (f(x) - \phi(x))^2 dx$$

$$\int_a^b f^2(x) dx - 2 \int_a^b f(x) dx + \int_a^b \phi^2(x) dx$$

$$= |f(x)|^2 - 2 \int_a^b f(x) \sum_{k=1}^n \delta_k \phi_k(x) dx + \int_a^b \left(\sum_{k=1}^n \delta_k \phi_k(x) \right)^2 dx$$

$$= |f(x)|^2 - 2 \sum_{k=1}^n \delta_k \int_a^b f(x) \phi_k(x) dx + \int_a^b \sum_{k=1}^n \sum_{\ell=1}^n \delta_k \phi_k(x) \delta_\ell \phi_\ell(x) dx$$

.

$$\delta_k$$

.

$$\text{Min}$$

$$E^2$$

$$\text{Min}$$

$$\delta_k$$

$$\delta_x$$

$$\phi(x)$$

$$E^2 = \int_a^b f^2(x) dx - 2 \sum_{k=1}^n \delta_k \int_a^b f(x) \phi_k(x) dx + \sum_{k=1}^n \sum_{\ell=1}^n \delta_k \delta_\ell \int_a^b \phi_k(x) \phi_\ell(x) dx$$

$$\delta_k = ck$$

$$= |f(x)|^2 - 2 \sum_{k=1}^n \delta_k c_k + \sum_{k=1}^n \delta_k^2 (1)$$

$$c_k = \langle f(x) \phi_k(x) \rangle$$

$$\delta_k = c_k \Rightarrow E^2 = \|f\|^2 - \sum_{k=1}^n C_k^2$$

$$: \quad (\quad) \quad E^2 > 0$$

$$|f(x)|^2 \geq \sum_{k=1}^n C_k^2 \quad \forall_n \in N \quad (n = \infty)$$

$$n \rightarrow \infty \quad \sum_{k=1}^n ck^2 \quad \lim C_n = 0 \quad n \rightarrow \infty \quad |f(x)|^2$$

$$: \quad (a, b) \quad f \quad (a, b) \quad \{\phi_k(x)\}$$

$$\sum_{k=1}^n C_k^2 = |f|^2$$

$$0 =$$

$$\phi_0 = \frac{1}{\sqrt{2\pi}}, \quad \phi_{2n-1}(x) = \frac{1}{\sqrt{\pi}} \cos nx, \quad \phi_{2n}(x) = \frac{1}{\sqrt{\pi}} \sin nx$$

$$(-\pi, \pi)$$

$$b_n : C$$

$$a_n : C$$

$$:$$

$$p(x) \quad (a, b) \quad \phi_k(x)$$

$$\int_a^b P(x) \phi_k(x) \phi_m(x) dx = \begin{cases} 0 & m \neq k \\ cte & m = k \end{cases}$$

$$\sqrt{p(x)}\phi_k(x)$$

$$\phi_{mn}(x, y)$$

$$\iint_R \phi_{mn}(x, y) \phi_{ek}(x, y) dx dy = \begin{cases} 0 & m \neq e, n \neq k \\ c_{mn}^2 & m = e, n = k \end{cases}$$

$$\phi_{mn}(x, y)$$

$$\omega = u + iv$$

$$\int_a^b \omega(t) dt = \int_a^b u(t) dt + i \int_a^b v(t) dx$$

$$\overline{\omega}(t) = u(t) - iv(t)$$

$$\langle \omega_m(t), \omega_k(t) \rangle = \int_a^b \omega_m(t) \overline{\omega_k(t)} dt = \begin{cases} 0 & m \neq k \\ \int_a^b (u_m^2(t) + v_m^2(t)) dt & m = k \end{cases}$$

$$(-2, 2)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\eta x}{c} + b_n \sin \frac{n\eta x}{c}$$

$$a_0 = \frac{1}{c} \int_{-c}^c f(x) dx \quad a_n = \frac{1}{c} \int_{-c}^c f(x) \cos \frac{n\eta x}{c} dx$$

$$b_n = \frac{1}{c} \int_{-c}^c f(x) \sin \frac{n\eta x}{c} dx$$

$$\frac{1}{2c} \int_{-c}^c f(s) ds + \sum_{n=1}^{\infty} \frac{1}{c} \left[\int_{-c}^c f(s) \cos \frac{n\pi s}{c} \cos \frac{n\pi x}{c} ds + \int_{-c}^c f(s) \sin \frac{n\pi s}{c} \sin \frac{n\pi x}{c} ds \right]$$

$$= \frac{1}{2c} \int_{-c}^c f(s) ds + \frac{1}{c} \sum_{n=1}^{\infty} \int_{-c}^c f(s) \cos \frac{n\eta}{c} (s-x) dx$$

$$\int_{-\infty}^{\infty} |f(s)| ds \quad \vdots \quad (-\infty, \infty) \quad f(x)$$

$$\Delta\alpha = \frac{\eta}{c}$$

$$\frac{1}{c} \sum_{n=1}^n \int_{-\infty}^{+\infty} f(s) \cos(n\Delta\alpha(x-s)) ds = \frac{1}{\eta} \sum_{n=1}^{\infty} \Delta\alpha \int_{-\infty}^{+\infty} f(s) \cos(n\Delta\alpha(x-s)) ds$$

$$\frac{1}{\eta} \sum_{n=1}^{\infty} f(n\Delta\alpha) \Delta\alpha$$

$$1 = \frac{1}{\eta} \int_0^{\infty} F(\alpha, x) d\alpha = \frac{1}{\eta} \int_0^{\infty} \int_{-\infty}^{\infty} f(s) \cos(\alpha(x-s)) ds d\alpha$$

$$\sum_{n=1}^{\infty} f(n\Delta\alpha) \Delta\alpha = \int_0^x f(x) dx = f(x) = \int_0^{\infty} (A(\alpha) \cos \alpha x + B(\alpha) \sin \alpha x)$$

$$c \rightarrow \infty$$

$$f(x) = \frac{1}{\eta} \int_0^{\infty} \int_{-\infty}^{\infty} f(s) \cos \alpha(s-x) ds d\alpha \quad -\infty < x < \infty$$

$$A(\alpha) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(u) \cos \alpha u du \quad B(\alpha) = \frac{1}{\pi} \int_{-\infty}^{+\infty} f(u) \sin \alpha u du$$

$$B(\alpha), A(\alpha)$$

$$f(x) = e^{-2|x|} \quad \vdots$$

$$A(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(s) \cos \alpha s ds \quad B(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(s) \sin \alpha s ds$$

$$|bn|n^4 < M \rightarrow \left| 4bn \sin \frac{n\eta x}{c} \right| \rightarrow \left| b_n \sin \frac{n\eta x}{c} \right| \leq |b_n|$$

$$\leq \sum |b_n| \leq \frac{M}{n^4} = M \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$f(x) = \int_{-\infty}^{\infty} (A(\alpha) \cos \alpha x + B(\alpha) \sin \alpha x) d\alpha$$

$$A(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(x) \cos \alpha x dx, \quad B(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} f(x) \sin \alpha x dx$$

$$f(x) = e^{-|x|} \quad -\infty < x < \infty \quad B(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} e^{-|x|} \sin \alpha x dx$$

$$f(x) = \int_0^{\infty} (A(\alpha) \cos \alpha x + B(\alpha) \sin \alpha x) d\alpha \quad \text{Re} \int_0^{\infty} e^{-x} e^{i\alpha x} \quad B(\alpha) = 0, \quad A(\alpha) = \frac{2}{\eta} \int_0^{\infty} e^{-x} \cos \alpha x dx$$

$$A(\alpha) = \frac{1}{\eta} \int_{-\infty}^{\infty} e^{-|x|} \cos \alpha x dx$$

$$= \frac{2}{\eta} \times \frac{1}{\eta} \left[\frac{1}{\alpha} - \frac{1}{\alpha} A(\alpha) \times \frac{\eta}{2} \right]$$

$$\Rightarrow A(\alpha) = \frac{2}{\eta} \left(\frac{1}{\alpha^2} (1 - A(\alpha)) \right) \Rightarrow \frac{\eta \alpha^2 (A(\alpha))}{2} = 1 - A(\alpha) \frac{\eta}{2}$$

$$\Rightarrow A(\alpha) = \frac{2}{\eta(\alpha^2 + 1)}$$

$$e^{-|x|} = \int_0^{\infty} \frac{2}{\eta(\alpha^2 + 1)} \cos \alpha x dx \Rightarrow$$

$$\frac{2}{\eta} \int_0^{\infty} \frac{\cos \alpha x}{1 + \alpha^2} d\alpha = \frac{R}{2} e^{-x} \quad x > 0$$

$$\int_0^{\infty} \frac{\cos \alpha x}{1 + \alpha^2} d\alpha = \frac{\pi}{2} e^{-x} \rightarrow \alpha = x$$

$$\int_0^{\infty} \frac{\sin 2\pi x}{1 + x^2} dx$$

$$f(x) = \begin{cases} -e^{-x} & x > 0 \\ -e^{-x} & x < 0 \end{cases} \quad \text{جواب} \quad = \frac{\pi}{2} e^{-2\pi}$$

$$\frac{2}{\eta} \int_0^{\infty} \frac{\cos \alpha x}{1 + \alpha^2} d\alpha = \frac{R}{2} e^{-x} \rightarrow \alpha = x$$

$$f(x_0) = \int_0^{\infty} A(\alpha) \cos \alpha x_0 + B(\alpha) \sin \alpha x_0 d\alpha$$

$$u_t(x, t) = k u_{xx}(x, t)$$

$$u(0, t) = 0 \quad u(x, 0) = F(x) \quad x > 0 \quad t > 0$$

$$\cdot \quad x$$

$$\cdot$$

$$u(x, t) X(x) T(t) \rightarrow \frac{T'(t)}{kt(t)} = \frac{X''(x)}{X(x)} = \begin{cases} x^2 \\ 0 \\ -\lambda^2 \end{cases} \Rightarrow \text{عق عغ}$$

$$U(x_1) = 0 \rightarrow X(0) = 0$$

$$X(x) = A \cos(\lambda x) + B \sin(\lambda x)$$

$$X(x) = \sin \lambda x$$

$$T(t) = e - \lambda zkt$$

$$U(x, t) = \int_0^\infty (B \lambda e^{-k\lambda t} \sin \lambda x) dx \Rightarrow \int_{-\infty}^\infty B(\lambda) \sin \lambda x d\lambda = f(x)$$

$$B(\lambda) = \frac{2}{R} \int_{-\infty}^\infty f(x) \sin \lambda x dx$$

$$\vdots$$

$$\vdots$$

$$\Delta^2 u = p^2 \frac{du^2}{dp^2} + p \frac{du}{dp} + (\lambda p^2 - v^2) y = 0$$

$$\vdots$$

$$x = \sqrt{\lambda} p$$

$$x/\lambda \times \frac{\partial^2 u}{\partial p^2} \cdot \frac{\partial x^2}{\partial p^2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial p^2} = \frac{\partial}{\partial p} \left(\frac{\partial u}{\partial p} \right) = \frac{\partial}{\partial p} \left(\frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial p} \right) \cdot \frac{ex}{ep}$$

$$\vdots$$

$$= \frac{\partial}{\partial x} \left(\sqrt{\lambda} \frac{\partial u}{\partial x} \right) \sqrt{\lambda} = \lambda \frac{\partial^2 u}{\partial x^2}$$

$$\Rightarrow \frac{x^2}{\lambda} \cdot \lambda \frac{d^2 y}{dx^2} + \frac{x}{\lambda} \cdot \sqrt{\lambda} \frac{dy}{dx} + (x^2 - v^2) y = 0$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} \sum_{k=0}^{\infty} a_k x^k (x^2 - v^2) y = 0$$

$$x = 0$$

$$x^2 y'' + xy' + (x^2 - v^2)y = 0 \quad y'' + \frac{1}{x}y' + \frac{x^2 - v^2}{x^2}y = 0$$

$$x=0$$

$$x^2 \sum_{k=0}^{\infty} (r+k)(r+k-1)a_k x^{r+k-2} + x \sum_{k=0}^{\infty} (r+k)a_k x^{r+k-1}$$

$$t(x^2 + v^2) \sum_{k=0}^{\infty} (r+k)(r+k-1)a_k x^{r+k} = 0$$

$$n=1,2,\dots, \quad v=n$$

$$\sum ((r+k)(r+k-1)av x^{r+k} + (r+k)akx^{r+k} - n^2 akx^{r+k}) + \sum akx^{r+k+2} = 0$$

$$\sum_{k=0}^{\infty} x^{r+k} ((r+k)(r+k-1) + (r+k) - n^2 ak + akx^2) = 0$$

$$: (r-n)(r+n)a_0 + (r-n+1)(r+n+1)a_1 x + \sum ((r-n+k)(r+n+k)a_k + a_k - 2)x^k = 0$$

$$\Rightarrow a_0 \quad \text{و} \quad (r+n)(r+n)=0$$

$$(r-n+1)(r-n+1)=0 \quad \text{و} \quad a_1 = 0$$

$$* \Rightarrow a_k = \frac{-1}{(r-n+k)(r+n+k)} a_k - 2$$

$$* \Rightarrow a_k = \frac{-1}{(r-n+k)(r+n+k)} a_k - 2$$

$$r=n \Rightarrow a_k = \frac{-1}{(r-n+k)(r+n+k)} a_k - 2$$

$$a_1, a_3, a_{2m} + 1 = 0$$

$$k$$

$$a_0 \neq 0$$

$$a_2 = \frac{-1}{2(2n+2)} a_0, a_4 = \frac{1}{4(2n+2)(2)(2n+2)}$$

$$a_{2k} = \frac{(-1)^k}{k!(n+1)(n+2)(n+k)2^{2k}}$$

$$a_0$$

$$a_0 = \frac{1}{n!2^n}$$

$$a_{2k} = \frac{(-1)^k}{k!(n+k)2^{n+2k}}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)} \left(\frac{x}{2}\right)^{n+2k} :$$

$$y_n(x)$$

$$y_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)} \left(\frac{x}{2}\right)^{n+2k}$$

$$n \quad y_n(x)$$

$$x^2 y'' + xy' + (x^2 - n^2)y = 0 \rightarrow y = y_n(x)$$

$$x^2 y'' + xy' + x^2 y = 0 \rightarrow y = y_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} \left(\frac{x}{2}\right)^{2k}$$

$$y_0(x)$$

$$y_0(x) = 1 - \left(\frac{x}{2}\right)^2 + \frac{1}{(2!)^2} \left(\frac{x}{2}\right)^4 - \frac{1}{(3!)^2} \left(\frac{x}{2}\right)^6 + \dots y_0$$

:

$$1) \cos(x \sin \theta) = y_0(x) + \sum_{n=1}^{\infty} (1 + (-1)^n) y_n(x) \cos \phi$$

$$2) \sin(x \sin \theta) = \sum_{n=1}^{\infty} (1 - (-1)^n) y_n(x) \sin \phi$$

$$y_{2n}(x) = \frac{1}{R} \int_0^R \sin(x \sin \phi) \sin(2x - 1) \phi d\phi$$

$$) \quad y_n(x) = \frac{1}{R} \int_0^R \cos(n\phi - x \sin \phi) d\phi$$

$$y_n(x) = \sum_{k=0}^{\infty} \frac{-1^k}{K!(x+k)!} \left(\frac{x}{2}\right)^{n+2k}$$

$$x^{-n} y_n(x) = \frac{1}{2^n} \sum_{k=0}^{\infty} \frac{-1^k}{k!(n+k)!} \left(\frac{x}{2}\right)^{2k}$$

$$4) \frac{d}{dx} x^{-n} y_n(x) = -n x^{-n-1} y_n(x) + x^{-n} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \frac{(n+2x)}{2} \left(\frac{x}{2}\right)^{n-2k-1}$$

$$-x^{-n} y_{n+1}(x) \quad \text{بسیار} \quad y'(0) = -y_1(x)$$

y_0(x)

$$Y_0(x) = \frac{2}{\pi} \left[\left(\log \frac{x}{3} - \nu \right) J_0(x) \right] + \frac{x^4}{x^2} - \frac{x^4}{x^2 4^2} \left(1 + \frac{1}{2} \right) + \frac{x^6}{x^2 4^2 6^2} \left(1 + \frac{1}{2} + \frac{1}{3} \right) + \dots$$

$$R(V) = \frac{R(v+1)}{v}$$

$$\frac{V}{V \left(\frac{3}{2} \right)} = \frac{1}{\sqrt{n}} \frac{V}{2} = \min$$

:

$$1) \cos(x \sin \theta) = y_0(x) + \sum_{n=1}^{\infty} (1 + (-1)^n) y_n(x) \cos \phi$$

$$2) \sin(x \sin \theta) = \sum_{n=1}^{\infty} (1 - (-1)^n) y_n(x) \sin \phi$$

$$y_{2n}(x) = \frac{1}{R} \int_0^R \sin(x \sin \phi) \sin(2x - 1) \phi d\phi$$

$$y_n(x) = \frac{1}{R} \int_0^R \cos(n\phi - x \sin \phi) d\phi$$

$$y_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{K!(x+k)!} \left(\frac{x}{2}\right)^{n+2k}$$

$$x^{-n} y_n(x) = \frac{1}{2^n} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(n+k)!} \left(\frac{x}{2}\right)^{2k}$$

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$$-x^{-n} y_{n+1}(x) \quad \text{بجای} \quad y'(0) = -y_1(x)$$

:

$$1) xy_n(x) = ny_n(x) - xy_{n+1}(x)$$

$$2) xy_n(x) = -ny_n(x) + xy_{n-1}(x)$$

$$xy_{n+1}(x) = 2ny_n(x) - xy_{n+1}$$

:

$$\cdot \quad \left(\quad \right) \quad y_n(x)$$

$$y_n(x) = 0 \rightarrow \quad = \{\alpha_{n0}, \alpha_{n1}, \dots, \alpha_{Ai}\}$$

$$\phi a \rightarrow y_n(\alpha ni) = 0$$

.

 y_o

$$b_n = \frac{2}{c} \int_0^c \frac{f(x) - f(-x)}{2} \sin \frac{N\eta x}{c} dx = \frac{1}{c} \left[\int_0^c f(x) \sin \frac{N\eta x}{c} dx - \int_0^c f(x) \sin \frac{N\eta x}{c} dx \right]$$

$$x = -x \quad dx = -dx$$

$$\int_0^c f(x) \sin \frac{N\eta x}{c} (-dx)$$

$$\sum_{n=1}^{\infty} b_n \sin nx \quad f(x)$$

$$f(x) = h(x) + g(x) \quad \Leftrightarrow \quad g(-x) = -g(x) \quad , \quad h(-x) = h(x)$$

$$h(x) = \frac{f(x) + f(-x)}{2} \quad g(x) = \frac{f(x) - f(-x)}{2}$$

$$h(x)$$

$$h(x)$$

$$h(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} \quad a_n = \frac{2}{c} \int_0^c h(x) \cos \frac{n\pi x}{c} dx$$

$$f(x) = h(x) + g(x) \quad (-c, c)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c} \quad \Leftrightarrow \quad x \in (-c, c)$$

$$a_n = \frac{2}{c} \int_0^c \frac{f(x) + f(-x)}{2} \cos \frac{n\pi x}{c} dx = \frac{1}{c} \left[\int_0^c f(x) \cos \frac{n\pi x}{c} dx + \int_0^c f(-x) \cos \frac{n\pi x}{c} dx \right]$$

$$\beta(z)$$

$$\int_B \beta(z) dz = 0 \quad \beta(z)$$

$$\int_C \frac{dz}{z^2(z^2 + q)} = \phi \quad D$$

$$\int \beta(z) dz$$

$$\int_C \beta(z) dz = \int_a^b \beta(z) dz = F(z(b)) - F(z(a))$$

$$\beta(z)$$

⋮

$$\beta(z)z^2 \rightarrow F(z)z^3$$

$$\int_0^{1+i} z^2 z = \frac{1}{3} (1+i)^3 +$$

$$\int_{-2i}^{2i} \frac{dz}{z} = \log z \Big|_{-2i}^{2i} = \log 2i - \log(-2i) = tui$$

C

C

$$\beta(z)$$

$$\int_c \frac{\beta(\bar{z}) dz}{z - z_0} \quad \beta(z_0)$$

⋮

$$\int_c \frac{\beta(\bar{z}) dz}{z - z_0} = 2\pi i \quad \beta(z_0)$$

$$\int_c \frac{z dz}{(q - z)(z - i)} = 2\pi i \frac{i}{q + 1} = \frac{-2\pi}{10}$$

$$\int_c z_0 \beta(z) dz = z_0 \int_c \beta(z) dz$$

$$\int_c (\beta(z + g|t|)) dz = \int_c \beta(z) dz + \int_c g(z) dz$$

$$\left| \int_c \beta(z) dx \right| \leq \int_a^b |\beta(z(t)z')| dt$$

$$I_c \int_{C1} z^2 dz$$

⋮

$$I_1 = \frac{2}{3} + \frac{11}{3}i$$

$$I_2 = \frac{2}{3} + \frac{11}{3}i$$



$$Ic \int_C Z^2 dZ$$

$$\int_C Pdx + Qdy = \int_R \int \left(\frac{\partial Q}{\partial X} - \frac{\partial R}{\partial y} \right) dx dy$$

$$\int_C \beta(Z) d\delta = \int_C Ndx - Vdy + i \int Xdx + udy$$

$$= \int_R \int \left(\frac{-\partial u}{\partial y} - \frac{\partial n}{\partial x} \right) dx dy + i \int_R \int \left(\frac{\partial u}{\partial y} - \frac{\partial x}{\partial y} \right) dx dy$$

$$u_x = v_y \qquad \qquad \qquad \beta(Z) \qquad \qquad \qquad C$$

$$Xy = -V_x$$

$$\beta(Z) \qquad \qquad \qquad C \qquad \qquad \qquad \beta(Z)$$

C

$$\omega(t) = X(t) + iy(t) \qquad \qquad \qquad :$$

$$\int_a^b \omega(t) d + \int_a^b X(t) dt + o \int_a^b y(t) dt$$

$$\int \theta^{i2t} dt = \sqrt{3}/2 + i/4 \qquad \qquad \qquad :$$

:

$$\operatorname{Re} \int_a^b \omega(t) dt = \int_a^b \operatorname{Re} |Z(t)| dt$$

$$\int Z\omega(t)dt = Z\int_a^b \omega(t)$$

$$r_0e^{\theta_0} = \int \omega(t)dt \Rightarrow \int \left(e^{-u\theta_0} \omega(t) \right) dt \nless r_0c \int \operatorname{Re} \left(e^{-1\theta_0} \omega(t) \right) dt !$$

$$\left| \int_a^b X(t) dt \right| \leq \int_a^b |\omega(t)| dt$$

Line Integral

(

$$\int_C \beta(t) dt \quad \text{و} \quad \int_{z_1}^{z_2} \beta(t) dt$$

Z

$$Z = Z(t)$$

$$\int_a^b \beta(Z) dt = \int_{z_1}^{z_2} (u/x(t) + iv(x(t), y(t)))(dX' + 1dy') dt$$

$$= \int_{Z/t_0}^{Z/t_1} (UX' - Vy') + i(uy' + Vx') dt$$

$$= \int_{t_0}^{t_1} (UX' + Vy') dt + \int_{t_0}^{t_1} (Uy' + Vx') dt$$

$$\int_{-C} \beta(Z) dZ - \int_C \beta(Z) dZ$$

:

$$(1 - x^2)y'' - 2xy' + y = 0$$

$$y_1 = a_0 + \sum_{h=1}^{\infty} a_{2h} x^{2h}$$

$$y_2 = a_1 x + \sum_{h=1}^{\infty} a_{2h+1} x^{2h+1}$$

$$\lambda = n(n+1) \quad P_n(x) \quad n$$

$$P_n(x) - \frac{1}{2^n} \sum \frac{(-1)^x (2n-2h)!}{h!(n-2h)! (n+x)} X^{n-2h} \quad n = 0, 1, \dots$$

$$P_0(x) = 1$$

$$P_1(x) = X$$

$$P_2(x) = \frac{1}{2} (3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(3S - 3)$$

$$P_4(x) = \frac{1}{2}(3s \times 4 - 30x^2 + 3)$$

$$P_n(-x) = (-1)^n P_n(x)$$

$$(i - x^g)y^n - 3 \times y' + n(n-1)y = 0 \quad !$$

$$\int_{-1}^1 P_m(x) + P_n(x) dx = \int mn \quad m \neq n$$

$$\phi_n(x) = \frac{P_n(x)}{|P_n(x)|} \rightarrow \{\phi_n(x)\}$$

$$P_n(x) = \frac{1}{2^n} \frac{1}{n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

$$P_{n+1}(x) - xp_n = \frac{n}{2^n n!} D^{n-1} u^n, \quad u = (x^2 - 1)$$

$$P'_{n+1}(x) - P'_{n-1}(x) = (2n+1)P_n(x)$$

$$|P_n(x)| = \sqrt{\frac{2}{2n+1}} \rightarrow \phi_n(x) = \sqrt{\frac{2n+1}{2}} P_n(x)$$

$$\beta(x) = \int_{n=0}^{\infty} A_n P_n(x) \Rightarrow A_n = \frac{2n+1}{2} \int_{-1}^1 \beta(x) P_n(x) dx$$

$$r \frac{\partial^2}{\partial r^2} (ru) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial v}{\partial \theta} \right) = 0 \quad 0 < r < c, \quad 0 < \theta < \pi$$

$$V(c, \partial) = F(\theta)$$

$$V(r, \theta) = \sum_{n=0}^{\infty} A_n \left(\frac{r}{c} \right)^n P_n(\cos \theta)$$

$$= \frac{1}{c} = \left[\int_0^c f(x) \cos \frac{n\eta x}{c} dx + \int_{-c}^0 f(x) \cos \frac{n\eta x}{c} dx \right] = \frac{1}{c} \int_{-c}^c f(x) \cos \frac{n\eta x}{c} dx$$