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Probabilistic load flow methodology for distribution networks including loads uncertainty

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ABSTRACT

Distribution grids probabilistic analysis is an essential step in order to assess the daily network operability under uncertain and stress conditions. It is also functional to the development of new services that require load growth capacity or to the exploitation of new energy resources affected by uncertainty. Efficient numerical tools able to forecast the possible scenarios while accounting for loads and sources uncertainty are thus of paramount importance. The majority of available uncertainty-aware predictive tools are based on Monte Carlo analysis which allows probabilistic evaluations of the network state at the price of time consuming simulations. In this paper, a much more efficient simulation framework is presented. The proposed approach relies on the generalized Polynomial Chaos algorithm and deterministic Power Flow analysis and allows achieving an at least 100× acceleration compared to standard Monte Carlo analysis for the same accuracy.

1. Introduction

Distribution networks probabilistic analysis is key to the development of new services and ways to exploit the electrical infrastructures. Most of such new services are associated to the low-voltage (LV) distribution network. Promising services are those related to the charging of vehicles, the decentralization of resources and the diffusion of new consumption patterns [1]. The widespread diffusion of such facilities is expected to introduce a significant variability/uncertainty of power load profiles compared to those of conventional users. For instance, the increase of electric vehicles, especially considering their usage as storage systems, will presumably stress the physical limits (e.g., the maximum current capability) of the lines. Due to the limited number of monitoring devices that are commonly deployed along the LV feeders, a comprehensive view of the overall network state requires the support of effective computational tools able to predict bus voltages and line currents under variable loading conditions [2–4]. Computational tools should be able to deal with the uncertainty of power loads and the trends of variation [5–7]. These techniques, commonly referred to as Probabilistic Load Flow (PLF) analysis, consist in using appropriate probabilistic models for the power load profiles as well as in replacing deterministic load flow simulation with statistical Monte Carlo (MC) analysis.

In order to account for the interplay of many independent uncertain loads (i.e. variations in the active power demands at the different phases of the network) a great number of MC runs is needed to achieve a satisfactory statistical description. In fact, even though loads uncertainty can commonly be modeled as Gaussian distributed parameters [8] the nonlinear nature of the load flow problem leads to state variable variations, e.g. maximum voltage at nodes or lines current, that are nonGaussian-distributed. In this case, the statistical information about mean value and variance of an electric variable is not enough to describe it properly and the detailed Probability Density Function (PDF) shape is required for further meaningful inferences. The accurate determination of PDF with MC method can require tens of thousands load flow analyses thus becoming very time consuming. Other approximate techniques for PLF analysis exhist, such as the Point Estimate method and the Cumulant Tensor (CT) [9–15]. However, commonly such techniques do not provide the detailed PDF shape which is instead required in our analysis.

In order to address the above issues, in this paper we describe an innovative approach to PLF analysis which is based on generalized Polynomial Chaos (gPC) algorithm and Stochastic Testing (ST) method [16–19]. The relevant features of the proposed approach are: (a) the implementation of the gPC + ST method does not need to modify the code of the deterministic load flow solver employed; (b) gPC + ST method allows handling strongly nonlinear problems, as it is the case for PLF formulated in terms of node voltages and powers, and with

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many independent statistical parameters.

In this paper, we provide the following original contributions:

1. we present in an intuitive way the application of the gPC + ST method to the PLF problem by considering data-based uncertainty in the load profiles;
2. we describe a simulation framework where the gPC + ST algorithm, implemented in Matlab, is interfaced with the deterministic load flow solver OpenDSS [20,21], to prove how the metodology is not-invasive and doesn't require a direct access to the simulation kernel;
3. we show how the proposed method can be exploited to predict the detailed probability distribution of monitor variables (e.g. Voltages, Currents, Voltage Unbalance or whatever else is needed to be observed) in the IEEE European low voltage test feeder while accounting for the interplay of variability in the three phase loads.

The above contributions are organized as follows. In Section 2, we review some background material about deterministic load flow analysis and its probabilistic extension with Monte Carlo method. In Section 3, we describe load uncertainty modeling while in Sections 4 and 5, we illustrate the gPC method and its computation details. The implementation and simulation frame are discussed in Section 6. Finally, Sections 7 and 8 report details about the considered test network and the related numerical results.

2. Background material: Probabilistic load flow with Monte Carlo method

2.1. Deterministic load flow

We consider the model of a distribution network made of N buses, represented by nodes, and connected by N_l lines described by their impedances. At each candidate node (a candidate is a node of the network in wich we assign the power load profile), equipment are connected that may supply or remove power from the electric network. This is described by power profiles assigned to the candidate nodes (e.g it is possible to choose the number of the nodes subjected to the variation). Deterministic load flow analysis consists in calculating Voltages and Currents by solving a set of nonlinear equations of the type:

$$\mathbf{F}_n(\vec{\mathbf{V}}) = \mathbf{S}_n - \mathbf{V}_n \sum_{i=1}^{N} \mathbf{Y}_{ni}\mathbf{V}_i^* = \mathbf{0} \tag{1}$$

for $n = 1, ...,N$. In (1), $\mathbf{S}_n = P_n + jQ_n$ denotes the complex power at node n where P_n and Q_n are the active and reactive powers respectively, $\mathbf{V}_n$ is the node voltage phasor, while $\mathbf{Y}_{ni}$ are the entries of the bus admittance matrix. Node voltage phasors are collected into vector $\vec{\mathbf{V}}$.

Network terminations are specified at the buses by imposing the known active and reactive powers P_n, Q_n absorved or delivered by loads. Load conditions vary in time and thus the associated powers become function of time, $P_n(t)$, $Q_n(t)$. Let us consider a given time window (e.g. a day or a week), that is discretized into a sequence of N_{times} equally-spaced time instants $t_m = m{\cdot}\Delta t$, over which the load profiles are given. Node voltage waveforms $\mathbf{V}_n(t)$ are calculated by repeatedly solving the nonlinear problem (1) over the sequence of time instants t_m. In doing that, the network state computed at time t_m is used as the solver initialization at next time t_{m+1}.

2.2. Monte Carlo analysis

Statistical fluctuations and uncertainty of loads can be accounted for by including into the load flow analysis a set of l stochastic parameters ξ_r that can be collected in the vector $\vec{\xi} = \left[\xi_1, \xi_2, ..., \xi_l\right]$. More details about probabilistic load modeling will be provided in the next section. Mathematically, each ξ_r is a zero-mean stochastic variable described by a given Probability Density Function (PDF) $\rho_r(\xi_r)$ [22]. Due to the uncertainty of the power loads applied to the candidate nodes, each observable variable describing the state of the network at time t, e.g. the magnitude of the nth node voltage $V_n(t) = |\mathbf{V}_n(t)|$, becomes a stochastic variable that depends on the uncertainties vector, i.e. $V_n\left(t, \vec{\xi}\right) = \left|\mathbf{V}_n\left(t, \vec{\xi}\right)\right|$. In conventional MC implementations, the statistical description of $V_n\left(t, \vec{\xi}\right)$ is achieved by generating a very large number N_{mc} of uncertainty vectors $\vec{\xi}^1, \vec{\xi}^2, ..., \vec{\xi}^{N_{mc}}$ according to the joint probability distribution of variables in $\vec{\xi}$. At time instant t_m, For each vector $\vec{\xi}^k$, the physical quantity $V_n\left(t, \vec{\xi}\right)$, sampled in time over the t_m, can be evaluated by running one deterministic LF analysis. As the number N_{mc} of evaluations grows, at limit tending to infinity, the distribution of values provided by LF analyses tends to the statistical distribution of $V_n(t)$. However, due to the slow $1/\sqrt{N_{mc}}$ convergence rate of MC method, the number of repeated LF simulations actually needed to obtain a satisfactory statistical description of $V_n(t)$ (i.e., the detailed shape of its PDF) can be very large. The PLF problem is made particularly critical by the nonlinear nature of Eqs. (1). In this case in fact, Gaussian-distributed parameters can result in network state variables $V_n\left(\vec{\xi}\right)$ being not Gaussian distributed. A qualitative example of non Gaussian-distributed variable is shown in Fig. 1. The statistical inference about the variability interval (with a certain confidence level) of the network observable $V_n\left(\vec{\xi}\right)$ requires the complete information about PDF distribution and it cannot only rely on mean value and variance. As a result, even in the case of a few uncertainty parameters (e.g. 2 or 3), several tens of thousands deterministic LF analyses are required, making MC approach time consuming.

3. Modeling load uncertainty

Current practice in probabilistic load modeling relies on an observational approach where power profile datasets for different type of utilities, area, and time periods are collected and analyzed in order to extract the relevant information that should be reproduced in simulations. The theme of loads or energy resources forecasting is a great issue that goes beyond the scope of this article. There are several approaches to forecast load variation with most of them being based on users behavior analysis [23,24] or on historical data [25], that could be used to improve the probabilistic analysis. Here we will suppose to start from the loads presented in the *IEEE European low voltage test feeder* [26] introducing for each of them, or for groups of loads, a variation described by a statistical parameter. In order to account for the potential growth or reduction in the power demand, we adopt the following expression for the active power at nth node in the network:

$$P_n(t) = p_n^0(t)[1 + \sigma_n^p \xi^p] \tag{2}$$

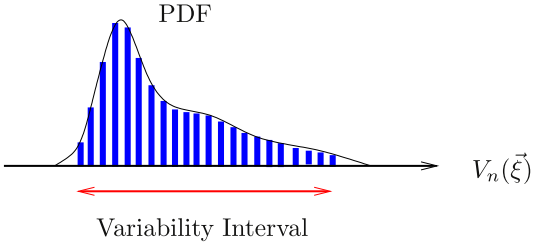

Fig. 1. Qualitative example of a nonGaussian-distributed state variable in the network: the area beneath the curve, over a given interval, provides the probability for that variable of falling within the interval.

where $p_n^0(t)$ is the nominal power profile. In (2), ξ^p denotes a zero-mean Gaussian-distributed statistical parameter having unitary variance, i.e. $\langle \xi^p \rangle = 0$ and $\langle (\xi^p)^2 \rangle = 1$ where $\langle \cdot \rangle$ is the expectation operator. The parameter σ_n^p is a scaling constant that determines the *coefficient of variation*.

As a consequence, the active power is a stochastic process whose mean value and standard deviation are given by [22]:

$$\begin{aligned} \langle P_n(t) \rangle &= p_n^0(t) \\ \sqrt{\langle (P_n(t) - p_n^0(t))^2 \rangle} &= \sigma_n^p p_n^0(t). \end{aligned} \tag{3}$$

In the analysis that follows, we also consider uncertainty in the power factor $\cos(\phi_n)$ by adopting the following statistical model:

$$\cos(\phi_n) = \cos(\phi_n^0)[1 + \sigma_n^\phi \xi^\phi], \tag{4}$$

where $\cos(\phi_n^0)$ is the nominal power factor at nth node while ξ^ϕ is a zero-mean unitary-variance Gaussian-distributed statistical parameter and σ_n^ϕ is the scaling constant that determines the power factor degree of variability.

4. Uncertainty quantification with generalized Polynomial Chaos

We consider a probabilistic problem where the uncertainty in the load power profiles is described by means of l stochastic parameters ξ_r modeling active power and power factor variability as in (2) and (4), respectively. The gPG method consists in adopting generalized polynomial chaos expansions for the node voltages. Depending on the numerical technique used to solve the gPC problem, the variables that have to be expanded can be all of the node voltages (i.e. the complex phasors including magnitude and phase information) in the network or a subset of them. In some cases, the variables that we need to expand are limited to the quantities that we want to monitor: they may be the magnitude of some node voltages or line currents at a given time or the peak or minimum value assumed over the time window. In what follows, we will generically denote as $V\left(t, \vec{\xi}\right)$ one of such variable. Under the mild hypothesis that $V\left(t, \vec{\xi}\right)$ has finite variance (i.e. it is a second-order stochastic process), it can be approximated by an order-β truncated series [16]

$$V\left(t, \vec{\xi}\right) \approx \sum_{i=1}^{N_b} c_i(t) H_i\left(\vec{\xi}\right), \tag{5}$$

formed by N_b multi-variate basis functions $H_i\left(\vec{\xi}\right)$ weighted by unknown polynomial chaos coefficients $c_i(t)$. The main feature in expression (5) is that the dependence of $V(\cdot)$ on the deterministic variable time, which is incorporated into coefficients $c_i(t)$, is separated by its dependence on statistical parameters $\vec{\xi}$ represented by basis functions $H_i\left(\vec{\xi}\right)$.

Each multi-variate basis function is given by the product

$$H_i\left(\vec{\xi}\right) = \prod_{r=1}^{l} \phi_{i_r}(\xi_r) \tag{6}$$

where $\phi_{i_r}(\xi_r)$ is a univariate orthogonal polynomial of degree i_r whose form depends on the density function of the rth parameter ξ_r. For instance, $\phi_{i_r}(\xi_r)$ are Hermite polynomials if ξ_r is a Gaussian-distributed variable, while $\phi_{i_r}(\xi_r)$ are Legendre polynomials if ξ_r is a uniformly distributed variable. A complete list of correspondence between several typical stochastic distributions and associated orthogonal polynomials can be found in [16].

For a given number of parameters l and series expansion truncation order β, the degrees i_r of univariate polynomials in (6) forming $H_i\left(\vec{\xi}\right)$, for $r = 1, \ldots, l$, satisfy the following relation

$$\sum_{r=1}^{l} i_r \leqslant \beta. \tag{7}$$

Univariate Hermite polynomials

$$\phi_0(\xi) = 1, \quad \phi_1(\xi) = \xi, \quad \phi_2(\xi) = \xi^2 - 1, \quad \phi_3(\xi) = \xi^3 - 3\xi$$

Multi-variate gPC basis functions

$$\begin{aligned} H_j(\xi_1, \xi_2) \in \{ & \\ & \phi_0(\xi_1)\,\phi_0(\xi_2), \quad \phi_0(\xi_1)\,\phi_1(\xi_2), \quad \phi_0(\xi_1)\,\phi_2(\xi_2), \quad \phi_0(\xi_1)\,\phi_3(\xi_2), \\ & \phi_1(\xi_1)\,\phi_0(\xi_2), \quad \phi_1(\xi_1)\,\phi_1(\xi_2), \quad \phi_1(\xi_1)\,\phi_2(\xi_2), \\ & \phi_2(\xi_1)\,\phi_0(\xi_2), \quad \phi_2(\xi_1)\,\phi_1(\xi_2), \\ & \phi_3(\xi_1)\,\phi_0(\xi_2) \} \end{aligned}$$

Fig. 2. Univariate Hermite polynomials and the set of multivariate gPC basis functions for the case of two Gaussian-distributed parameters $\vec{\xi} = \left(\xi_1, \xi_2\right)$ and truncation order $\beta = 3$.

As an example, in Fig. 2, we report the case of two independent Gaussian-distributed parameters ξ_1 and ξ_2 (i.e. $l = 2$) and for expansion order $\beta = 3$. In this simple case, the basis functions are the product of couples of Hermite polynomials

$$H_i\left(\vec{\xi}\right) = \phi_{i_1}(\xi_1)\phi_{i_2}(\xi_2) \tag{8}$$

whose degrees are such that $i_1 + i_2 \leqslant 3$. In this example, the number N_b of basis functions $H_i\left(\vec{\xi}\right)$ is ten. For generic truncation order β and number of parameters l, the number of gPC basis functions is given by [17]

$$N_b = \frac{(\beta + l)!}{\beta!\, l!}. \tag{9}$$

Once the coefficients $c_j(t)$ are computed, (using one of the methods described in the next section) the mean value and standard deviation of $V\left(t, \vec{\xi}\right)$ can easily be determined [17]. Furthermore, and even more importantly, the gPC expansion (5) provides a compact model for the $V\left(t, \vec{\xi}\right)$ multi-dimensional dependence. For each realization of the uncertainty vector $\vec{\xi} = \left[\xi_1, \xi_2, \ldots, \xi_l\right]$, generated accordingly to the joint probability distribution of variables in $\vec{\xi}$, the evaluation of polynomials in Fig. 2 and gPC expansion (5) provides a realization of $V\left(t, \vec{\xi}\right)$. This enables repeated evaluations of $V\left(t, \vec{\xi}\right)$ for large numbers of uncertainty vector realizations $\vec{\xi}^k$ in very short times (one million of evaluations take a few seconds on a quad-core computer) and the determination of the detailed PDF.

5. Computing the gPC coefficients

There are two different mainstream approaches for computing the

gPC expansion coefficients in (5): Galerkin projection and collocation method [27].

5.1. Galerkin Projection (GP)

Galerkin projection is an *intrusive* numerical technique that requires modifying the LF code (1). According to this method, a gPC expansion of the type (5) is adopted for each unknown nodal voltage $\mathbf{V}_n(t)$, i.e.

$$\mathbf{V}_n\left(t, \vec{\xi}\right) \approx \sum_{i=1}^{N_b} \mathbf{c}_i^n(t) H_i\left(\vec{\xi}\right), \tag{10}$$

leading to $N_b \times N$ unknown $\mathbf{c}_i^n$ coefficients that are complex variables. Such coefficients are determined by plugging the expansions (10) into (1) and then projecting the resulting nodal equations along the N_b basis functions. This results in a very large nonlinear system of size $N_b \times N$, i.e.

$$\left\langle \mathbf{F}_n\left(\vec{\mathbf{V}}\left(\vec{\xi}\right)\right), H_i\left(\vec{\xi}\right)\right\rangle_{\Omega} = \mathbf{0}, \tag{11}$$

for $n = 1, ..., N$ and $i = 1, ..., N_b$, where $\langle\cdot\rangle_{\Omega}$ denotes the inner product in the stochastic space [17]. The solution of (11) requires a significant computational effort both in terms of time and allocated memory. As an example, consider the case of a distribution network with $N = 100$ nodes, and suppose to perform stochastic Galerkin with $l = 3$ statistical parameters and expansion order $\beta = 3$. In this case, $N_b = 20$, so that the nonlinear system to be solved has size $N_b \times N = 2000$. Due to the problem nonlinearity, equations (11) require a computational time for solving the system that tends to grow as a power of two of the system size, i.e. about $(N_b \times N)^2$.

In our example, the computational time for solving Galerkin problem is about 400× greater than that needed for a single LF analysis. The GP computational time grows very rapidly with the number of statistical parameters thus limiting the applicability of the method to networks of small size and with a few statistical parameters (i.e. 2 − 3).

5.2. Stochastic Collocation (SC)

SC is a *nonintrusive* method that can be combined with any LF formulation (1) without modifying the implementation codes. A second advantage of SC method is that the gPC expansion (5) is adopted limitedly to the set of network variables that we want to evaluate, i.e. the peak value of some monitoring node voltages. In what follows, we will focus on a recently proposed efficient implementation of SC method referred to as Stochastic Testing (ST) method. According to collocation-based Stochastic Testing (ST) [17], the N_b unknown coefficients $c_j(t)$ in the series (5) are calculated by properly selecting $N_s = N_b$ testing points $\vec{\xi}^k$, for $k = 1, ..., N_s$ in the stochastic space where $V_k(t) = V\left(t, \vec{\xi}^k\right)$ is calculated with a deterministic LF analysis.

At each testing point, the series expansion (5) is enforced to fit *exactly* (i.e., the polynomials interpolate the samples) the values $V_k(t)$.

Mathematically, this results in the following linear system

$$\mathbf{M}\vec{c}(t) = \vec{V}(t), \tag{12}$$

where $\vec{c}(t) = [c_1(t), ..., c_{N_b}(t)]^T$ and $\vec{V}(t) = [V_1(t), ..., V_{N_s}(t)]^T$ are the column vectors collecting the unknown coefficients and node voltage values respectively.

The $N_b \times N_b$ square matrix $\mathbf{M} = \{a_{k,i}\} = \left\{H_i\left(\vec{\xi}^k\right)\right\}$ collects the gPC basis functions evaluated at the testing points, i.e.

$$\mathbf{M} = \begin{bmatrix} H_1\left(\vec{\xi}^1\right) & \dots & H_{N_b}\left(\vec{\xi}^1\right) \\ \vdots & \ddots & \vdots \\ H_1\left(\vec{\xi}^{N_s}\right) & \dots & H_{N_b}\left(\vec{\xi}^{N_s}\right) \end{bmatrix}. \tag{13}$$

It is worth noting that the $\mathbf{M}$ only depends on the selected basis functions and testing points, so that it can be precalculated, inverted and used for any $t = t_m$ as follows:

$$\vec{c}(t_m) = \mathbf{M}^{-1}\vec{V}(t_m). \tag{14}$$

The ST method enables handling PLF problems with larger size and larger number of parameters. As an example, for expansion order $\beta = 3$ and number of stochastic parameters $l = 6$, the ST method needs only repeating 84 deterministic LF analysis.

5.3. Testing points selection

The selection of the testing points $\vec{\xi}^k$ in the stochastic space is done so as to ensure the highest numerical accuracy of the gPC-based interpolation scheme and of the associated statistical description. This is achieved by considering the highest order β univariate polynomial $\phi(\xi_r)$ describing the rth parameter ξ_r with PDF $\rho_r(\xi_r)$. For the univariate case, the testing points are selected in correspondence of the $(P + 1)$ Gauss quadrature nodes used in numerical integration ξ_r^k [17]. When the multivariate case with l parameters is considered, the testing points vectors $\vec{\xi}^k = \left[\xi_1^k, \xi_2^k, ..., \xi_l^k\right]$ are determined by considering the multi-dimensional grid of all of the possible combinations (i.e. the tensor product) of the univariate quadrature nodes.

The number $(\beta + 1)^l$ of nodes in the multi-dimensional grid is greater than the number N_b of basis functions defined in (9). For the considered example with $l = 2$ and $\beta = 3$, the number of Gauss nodes is $4 \times 4 = 16$ while the number of basis is $N_b = 10$. To make problem (13) well posed, a subset formed by $N_s = N_b$ quadrature nodes has to be selected as testing points. A possible method for selecting the subset of testing points among the quadrature nodes is presented in [17]. It relies on the criteria of preferring those quadrature nodes with largest associated Gauss weights and that lead to the best (smallest) condition number for the matrix $\mathbf{M}$. Fig. 3 shows the subset of testing points selected for the example with 2 stochastic variables.

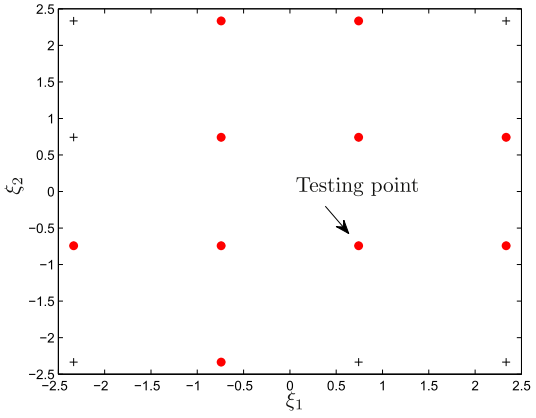

Fig. 3. (Black Cross Marker) The 2-dimensional grid of quadrature nodes. (Red Circle Marker) Subset of nodes used as testing points in ST method. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

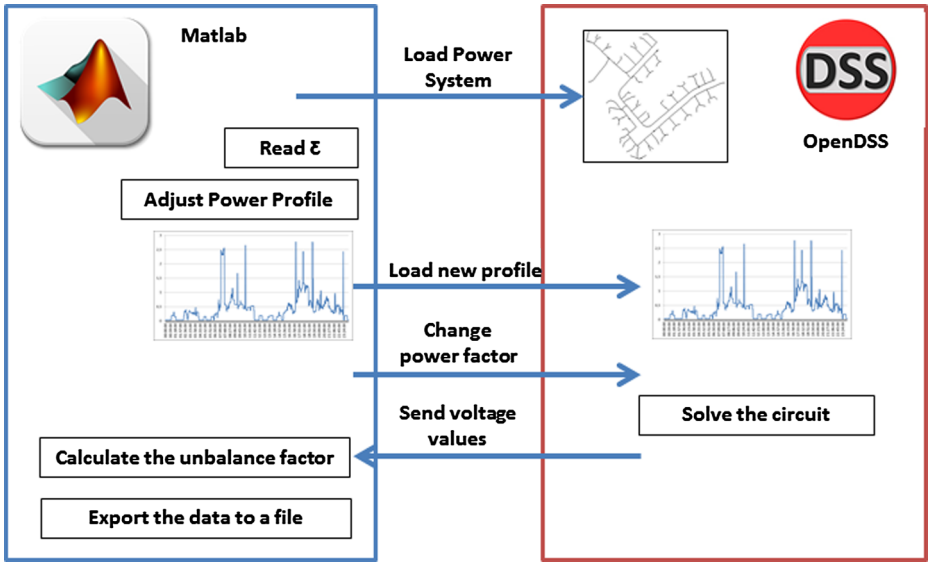

Fig. 4. Flowchart of the simulation framework.

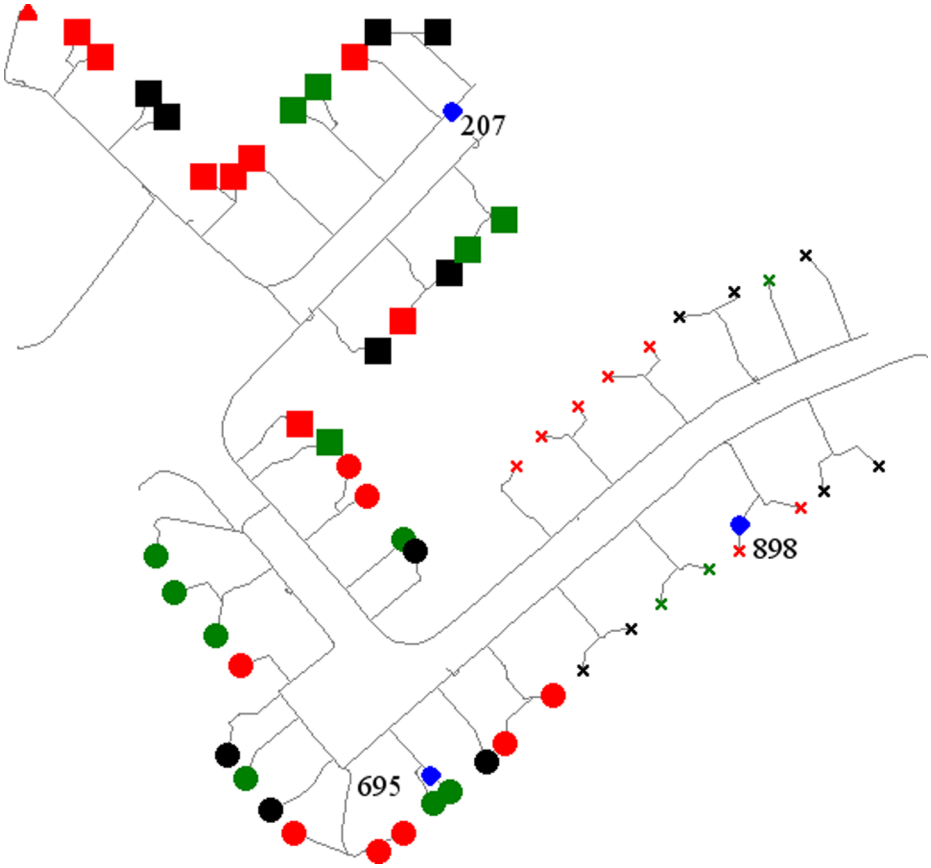

Fig. 5. Topology of the IEEE LV European test feeder. The numbered nodes are some of the nodes under observation in the implemented analysis.

Table 1
Number of loads per phase.

Phase A	21
Phase B	19
Phase C	15

6. Implementation and simulation framework

In our implementation, the uncertainty about active power profiles and power factor are modeled as described in Section 3 by means of l independent Gaussian-distributed random variables ξ_r. Hence, variability analysis is performed for a set of node voltages and line current

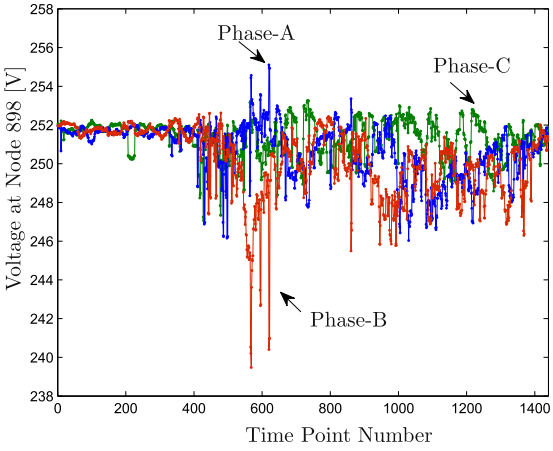

Fig. 6. Daily time evolution of the three phases at node 898 for one of the testing points.

magnitudes, considered here as the *output variables*, by using the non-intrusive ST method described in Section 4 and subsection V-C. This is achieved by interfacing the gPC + ST code developed at *Massachusetts Institute of Technology* [17] and written in Matlab with the Load Flow deterministic solver OpenDSS. Fig. 4 shows the qualitative flowchart of the implemented simulation framework. To this aim, we exploit the OpenDSS internal interface DCOM which allows bidirectional information exchange between OpenDSS and Matlab. Such an information flow is first used to import in OpenDSS the relevant information about the load profiles, i.e. the active power time profiles and $\cos(\phi)$, at all network nodes, generated with gPC + ST in the Matlab workspace. Second, the information about the time waveforms of simulated network voltages and currents are exported to Matlab for further processing. This includes extracting the c_i expansion coefficients and using the compact gPC model (5) to efficiently compute the detailed PDFs of the network variables of interest.

7. Test network

The distribution network adopted in our simulations is the IEEE European low voltage test feeder[26]. Such a test network, published by the Test Feeders Working Group of the Distribution System Analysis Subcommittee of the *Power Systems Analysis, Computing, and Economics Committee* (PSACE), provides a valid benchmark for researchers willing to study low voltage feeders which are common in Europe. The previously published test feeders were mainly focused on North American style systems, which consists in radial, very wide medium voltage networks with a large number of secondary service transformers, each serving a couple of houses. In Europe instead the most common distribution system design is with medium voltage networks still radial, or weekly meshed, but smaller in amplitude and serving few secondary substations, which are larger with respect to the North American style. Each substation supply a radial low voltage network. Residential and small commercial loads are supplied by these networks.

The topology of the proposed network is reported in Fig. 5. The test feeder is a radial distribution feeder with a base frequency of 50 Hz, at 240 V (phase voltage)/416 V (line to line voltage), which is typical of the Italian low voltage distribution systems.

The medium voltage system supplying the substation is modeled as a voltage source with an impedance (Thevenin equivalent). The impedance is specified by short circuit current. The LV test feeder model is composed of 906 low voltage nodes, connected by 905 branches. The network is radial, with 55 load buses. The distribution lines are defined by their codes and lengths. To each code specific line impedance and shunt admittance values are assigned. Due to the short length of lines (all branches are shorter than a hundred meters) the shunt admittance is neglected and just the series impedance is considered.

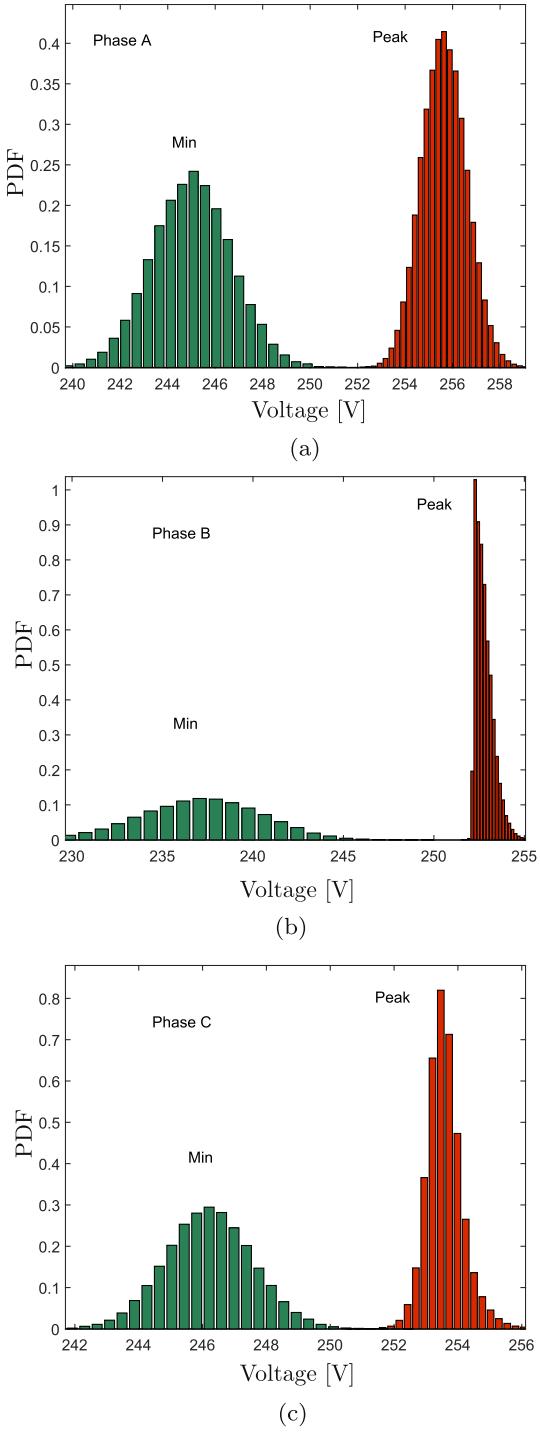

Fig. 7. PDFs for the peak and minimum values of the voltage phases at node 898: subfigures (a), (b) and (c) report phases A, B, C, respectively.

7.1. Load data analysis

The test distribution network is a 3-phase/4 wire network, with the possibility of assigning the terminal powers either as 3-phase or 1-phase loads. In this work, we assume that all of the powers are given as 1-phase loads which are distributed among the three-phase lines, i.e. Phase A, Phase B and Phase C, in the numbers reported in Table 1.

Such loads are assigned to 55 nodes and their shapes are provided by the benchmark, as 1 min time series, for a day [26]. The aim of the

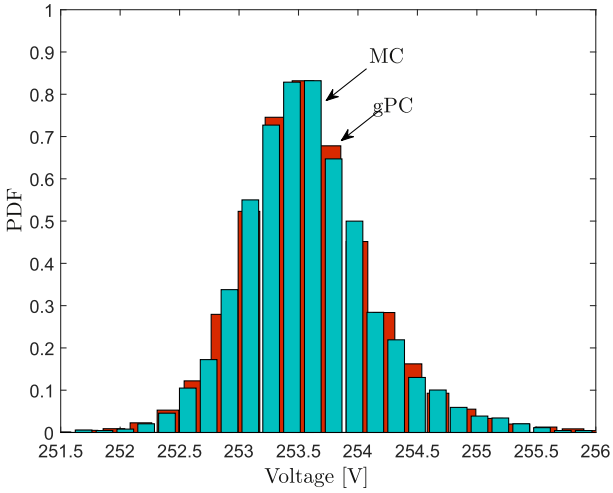

Fig. 8. Detail of the distributions of the Phase-C peak value supplied by gPC and MC (5000 samples) methods.

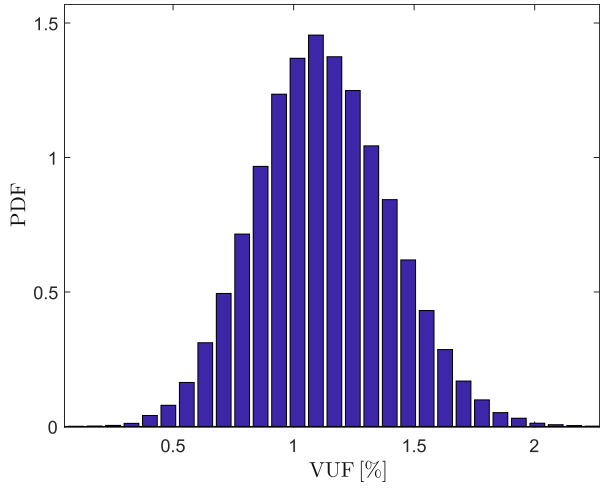

Fig. 9. Statistical distribution of the average value of the voltage unbalance factor.

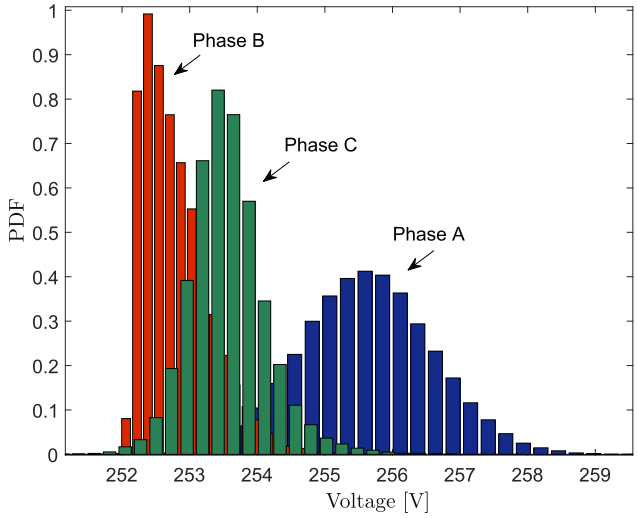

Fig. 10. Case with 6 Gaussian-distributed statistical parameters. PDFs for the peak values of voltage at node 898 for the three phases A, B, C.

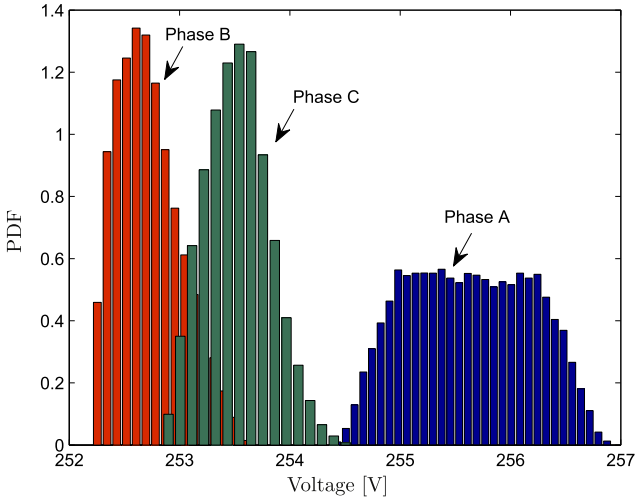

Fig. 11. Case with 6 Uniformly-distributed statistical parameters. PDFs for the peak values of voltage at node 898 for the three phases A, B, C.

Table 2
Simulation times.

	2 variables	3 variables	6 variables
Number of simulations	10	20	84
Total time elapsed [s]	40	80	336
Time per simulation [s]	4	4	4

analysis is to study, in a probabilistic sense, the fluctuations of the node voltages which are induced by power loads variation in order to assess the quality of the network, therefore we focus on the Voltage Unbalance Factor (VUF). The percentage VUF is defined as the ratio of the negative voltage sequence component V_n to the positive voltage sequence component V_p [28], i.e.

$$\text{VFU} = \frac{|V_n|}{|V_p|} \cdot 100, \tag{15}$$

with

$$V_n = \frac{V_{AB} + a^2 \cdot V_{BC} + a \cdot V_{CA}}{3} \tag{16}$$

and

$$V_p = \frac{V_{AB} + a \cdot V_{BC} + a^2 \cdot V_{CA}}{3}, \tag{17}$$

where V_{AB}, V_{BC}, V_{CA} are the phasors of the unbalanced line voltages while $a = \exp(j\,120^{\circ})$ and $a^2 = \exp(j\,240^{\circ})$.

8. Numerical results

In this section, we present fresh results provided by the application of the proposed variability analysis to the IEEE European low voltage test feeder.

Our goal is that of investigating the effects that the variability of the *total power* assigned to a given phase induces on the node voltages. To this aim, and accordingly to load model (2), we assume that the active powers $P_n(t)$ of all of the nodes assigned to a given phase line, e.g. Phase A, are scaled by the same ξ^p Gaussian statistical parameter, e.g. ξ_A^p. As a result, three statistically independent parameters ξ_A^p, ξ_B^p and ξ_C^p are simulated. In simulations the variability degrees $\sigma_A^p = \sigma_B^p = \sigma_C^p = 0.2$ are adopted.This is a simplification that we adopt in order to show the main features of the method. More complex scenarios could be implemented where an independent statistical parameter is introduced for each load.

Assuming gPC expansion order $\beta = 3$, twenty testing points are

generated in the statistical space and for each one of them a deterministic load flow analysis is performed. Fig. 6 reports the waveforms of the three phase voltages at node 898, used here as the monitoring point, simulated with OpenDSS in one of the testing points (i.e. for a given set of parameters $\vec{\xi}$). Such waveforms exhibit sharp fluctuations over the considered time window with peaks (maxima) and valleys (minima) that are significantly affected by the potential growth or reduction of the loads. The probabilistic evaluation of the achievable daily peak and minimum is thus crucial in order to assess the quality of the service provided by the distribution infrastructure. To this aim, with the proposed gPC + ST method we calculate the detailed statistical distribution of the peak and minimum values in each node. As an example, Fig. 7 shows the statistical distribution of the peak value of voltage at node 898 for the three phases. For the assumed loads uncertainty, the peak value of Phase A exhibits wide variability with an almost Gaussian distribution. In fact, the peak of Phase A ranges within the interval (254, 257) V with 90% probability. The peak values of Phase B and Phase C, fluctuate within narrower intervals, i.e. about (252.5, 254) V, however their distributions are nonGaussian. This is due to the nonlinear nature of the LF problem.

Similarly the statistical distributions of minimum voltage at the same node shows as the greatest variability is seen for the minimum value of Phase B that fluctuates into the interval (230, 243) V with 90% probability.

In order to check the accuracy of the gPC + ST method we compare it with a reference MC method that uses 5000 runs (i.e. deterministic load flow analyses) selected with a latin-hypercube sampling. Fig. 8 shows the PDF for the peak value of the Phase C voltage provided by proposed gPC and MC (5000 samples) methods. The two distributions are almost superimposed, e.g. the Kullback Leibler [29] divergence between them is ≈ 0.0161, and the associated standard deviations, i.e. $\sigma_{gPC} = 0.572$ V and $\sigma_{MC} = 0.565$ V respectively, match within a relative accuracy of 2%. As a further check, we also use the point estimate scheme described in [14,30], which adopts a numerically efficient samples selection method, to estimate the raw moments of the output variables of interest. The standard deviation predicted by the numerically-efficient point estimate method for the peak value of Phase-C is $\sigma_{PEM} = 0.410$ V so that the relative error compared to reference MC analysis is about 27%.

Finally, Fig. 9 shows the distribution of the average value (over the day window) of the Voltage Unbalance Factor (VUF) figure of merit: the average VUF is always smaller than 2%.

As a second example, we repeat variability analysis also considering the uncertainty of the power factors for the three phases. We thus add three extra statistically-independent Gaussian distributed parameters ξ_A^ϕ, ξ_B^ϕ and ξ_C^ϕ scaling power factor accordingly to (4). We assume nominal power factor values $\cos(\phi_A^0) = \cos(\phi_B^0) = \cos(\phi_C^0) = 0.925$ and associated scaling constants $\sigma_A^\phi = \sigma_B^\phi = \sigma_C^\phi = 0.01$. Such scaling constants meet the constraint $0.9 \leqslant \cos(\phi) \leqslant 1$. We perform variability analysis considering the mutual effect of the six statistical parameters. The simulated distributions of the peak values at monitoring node are collected in Fig. 10. The distributions for the case with six parameters are quite similar to those for three parameters shown in (7) meaning that, for the considered scenario, the power factor uncertainty is less relevant than active power variability. Even though in this paper we have focused on the case of Gaussian-distributed parameters, since this is the most frequent case in applications, more complex scenarios may arise where statistical parameters are not Gaussian distributed [31,32].

The generalized Polynomial Chaos method allows handling statistical parameters with several nonGaussian statistical distributions as listed in [16], and complex combinations of them [33]. As an example, Fig. 11 shows the distributions of the peak values at monitoring node in the case where the six parameters ξ_A^p, ξ_B^p, ξ_C^p and ξ_A^ϕ, ξ_B^ϕ, ξ_C^ϕ scaling active powers and power factors, respectively, are all uniformly distributed into the interval $[-1, 1]$. In this case, the expansion (5) is made of Legendre-chaos polynomials [16].

Finally, in Table 2 we report the simulation times of the proposed variability analysis for the cases $l = 2$, $l = 3$ and $l = 6$ statistical parameters. The simulation times are dominated by the deterministic load flow simulation with OpenDSS. For the case with $l = 3$ parameters, one deterministic load flow analysis takes about 4 s and the whole variability analysis is completed in about 5 min. By contrast, the same analysis performed with the reference Monte Carlo method require about 5000 load flow analyses and takes more than 5 h.

9. Conclusion

In this paper, we have described an innovative simulation framework for the probabilistic analysis of power distribution networks subject to load uncertainty. Our approach employes generalized Polynomial Chaos (gPC) algorithm and Stochastic Testing (ST) method combined with the deterministic load flow solver OpenDSS. We have shown how the proposed method enables deriving in a very efficient way the detailed information about the variability of a subset of electric variables and figure of merits that are relevant for the quality of service. The speed up factor in computation is about 100× compared to standard Monte Carlo simulations.

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