Nonlinear Assignment 1

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1- The Lorentz equations

$$\frac{d}{dt}x_1 = \sigma(x_2 - x_1)$$
$$\frac{d}{dt}x_2 = rx_1 - x_2 - x_1x_3$$
$$\frac{d}{dt}x_3 = x_1x_2 - bx_3 \qquad \sigma, r, b > 0$$

Where σ, r, b are constants, are often used as example of chaotic motion.

(a) Determine all equilibrium points.

(b) Linearize the equation around $x = 0$ and determine for what σ, r, b this equilibrium is locally asymptotically stable.

2- For each of the following systems, find all the equilibrium points and determine the type of the isolated equilibrium points. Then sketch the phase portrait and discuss the properties of each system.

$$a)\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = -x_1 - x_2 + \frac{x_1^3}{6}\end{cases} \qquad b)\begin{cases}\dot{x}_1 = -x_1 + (1 + x_1)x_2\\ \dot{x}_2 = -(1 + x_1)x_1\end{cases}$$

3- For each of the following systems, find and classify all equilibrium points.

(a)
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + x_1^3/6 - x_2$$

(b)
$$\dot{x}_1 = -x_1 + x_2$$
$$\dot{x}_2 = 0.1x_1 - 2x_2 - x_1^2 - 0.1x_1^3$$

(c)
$$\dot{x}_1 = (1 - x_1)x_1 - 2x_1x_2/(1 + x_1)$$
$$\dot{x}_2 = (1 - x_2/(1 + x_1))x_2$$

(d)
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + x_2(1 - 3x_1^2 - 2x_2^2)$$

(e)
$$\dot{x}_1 = -x_1 + x_2(1 + x_1)$$
$$\dot{x}_2 = -x_1(1 + x_1)$$

(f)
$$\dot{x}_1 = (x_1 - x_2)(x_1^2 + x_2^2 - 1)$$
$$\dot{x}_2 = (x_1 + x_2)(x_1^2 + x_2^2 - 1)$$

4- The following system

$$\dot{x}_1 = (u - x_1)(1 + x_2^2)$$
$$\dot{x}_2 = (x_1 - 2x_2)(1 + x_1^2)$$
$$y = x_2$$

is controlled by the output feedback

$$u = -Ky$$

(a) For all values of the gain K, determine the equilibrium points of the closed loop system.

(b) Determine the equilibrium character of the origin for all values of the parameter K. Determine in particular for what values the closed loop system is (locally) asymptotically stable.

5- Consider the nonlinear system

$$\dot{x} = y + x\,(x^2 + y^2 - 1)\sin\frac{1}{x^2 + y^2 - 1}$$
$$\dot{y} = -x + y(x^2 + y^2 - 1)\sin\frac{1}{x^2 + y^2 - 1}$$

Without solving the above equations explicitly, show that the system has infinite number of limit cycles. Determine the stability of these limit cycles.

6- Consider following nonlinear system:

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} -x + y \\ -xy + y \end{pmatrix}$$

First, plot phase portrait of this system then, determine correctness of the following statement (give overall intuition for correct statements and counterexample for incorrect ones):

a) The unique equilibrium point is (0,0)
b) Any trajectory with initial point $\{(x_0, y_0) |\, y_0 > 0\}$ approach (1; 1) as $t \to \infty$.
c) We can enumerate exactly two trajectories that approach the equilibrium 0 as $t \to \infty$.

Good Luck