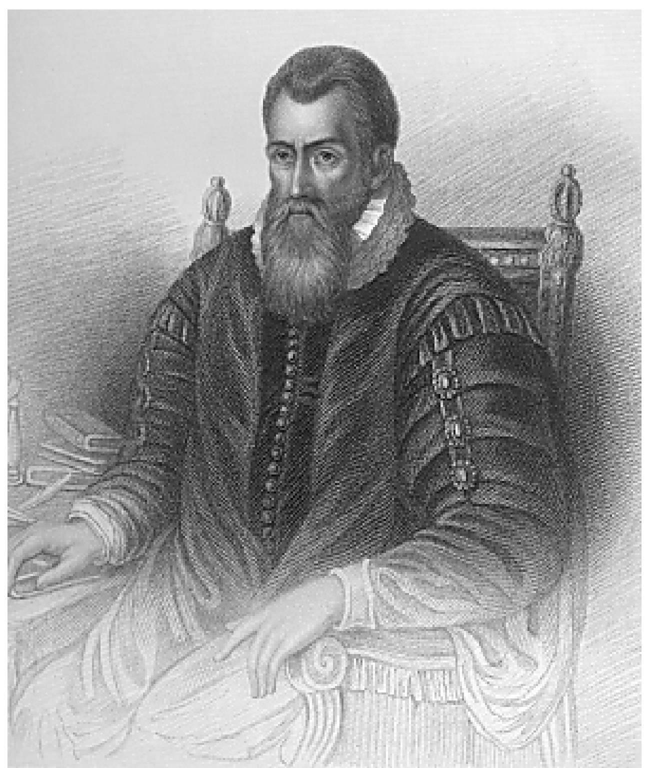
Chapter3

Logarithms

Genesis of the logarithm:

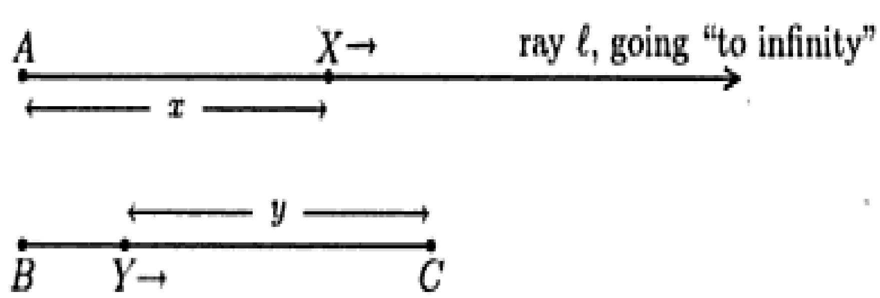
John Napier



Napier: A Description of the Admirable Table of Logarithms

Making Napier logarithm tables

Excerpt: Calculation Techniques



How do Napier logarithms compare with modern logarithms?

$$\text{Nap}\log y = -\ln y = -\log_e y = \log_e 1/y,$$

where $e \approx 2.71828$ is the number known as Euler's number.

* * *

For those familiar with calculus, we shall provide justification for the above claim as follows. Observe that

$$\frac{dx}{dt} = 1, \quad \therefore \ x = t \ \ (\text{since } x = 0 \text{ when } t = 0)\text{; and}$$
$$\frac{dy}{dt} = -y, \quad \therefore \ t = -\ln y \ \ (\text{since } y = 1 \text{ when } t = 0).$$

It follows that $x = -\ln y$.

* * *

$$N = 10^7(1 - 10^{-7})^L.$$

$$L = \log_{(1-10^{-7})}\left(\frac{N}{10^7}\right) \approx 10^7 \log_{\frac{1}{e}}\left(\frac{N}{10^7}\right) = -10^7 \log_e\left(\frac{N}{10^7}\right),$$

$$(1 - 10^{-7})^{10^7} \approx \frac{1}{e}.$$

Brief historical details:

- NAPIER JOHN of Merchiston near Edinburgh the celebrated inven tor of the logarithms was born in the year 1550

- In his head Napier might be wrestling with theological or mathematical complexities but to onlookers his behaviour seemed sinister Rumours spread that he was a warlock after he enlisted the help of the cockeral to discover which if his servants had been stealing from him

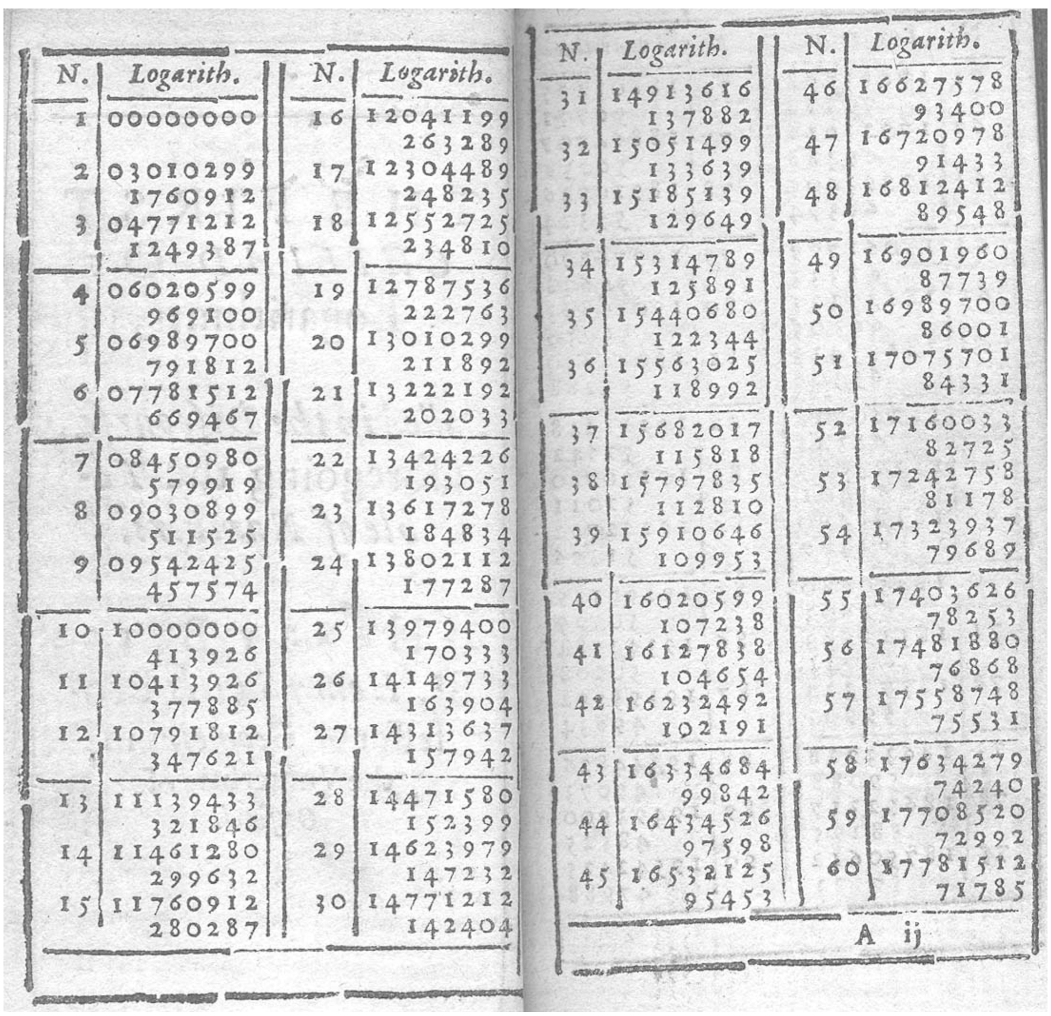
- In his book published in 1614 *Mirifici Logarithmorum Canonis Des criptio* Description of the wonderful canon of logarithms

- It took Napier about 20 years to actually assemble his table of logarit hms but shortly after publishing his book Napier was visited by the English mathematician Henry Briggs

- They both discussed the convenience of setting the logarithm of 1 equ al to 0 rather than the

original 10 000 000 and setting the logarithm of 10 at 1 In this way the more familiar form of the logarithm was born and a common property like log xy log x log y could be used to make a new table

- Finally in 1624 Briggs published his tables in his *Arithmetica Logarithmica* The logarithms of the numbers between 20 000 and 90 000 were calculated by the Dutchman Adrian Vlacq



N.	Logarith.
1	00000000
2	03010299
	1760912
3	04771212
	1249387
4	06020599
	969100
5	06989700
	791812
6	07781512
	669467
7	08450980
	579919
8	09030899
	511525
9	09542425
	457574
10	10000000
	413926
11	10413926
	377885
12	10791812
	347621
13	11139433
	321846
14	11461280
	299632
15	11760912
	280287
16	12041199
	263289
17	12304489
	248235
18	12552725
	234810
19	12787536
	222763
20	13010299
	211892
21	13222192
	202033
22	13424226
	193051
23	13617278
	184834
24	13802112
	177287
25	13979400
	170333
26	14149733
	163904
27	14313637
	157942
28	14471580
	152399
29	14623979
	147232
30	14771212
	142404
31	14913616
	137882
32	15051499
	133639
33	15185139
	129649
34	15314789
	125891
35	15440680
	122344
36	15563025
	118992
37	15682017
	115818
38	15797835
	112810
39	15910646
	109953
40	16020599
	107238
41	16127838
	104654
42	16232492
	102191
43	16334684
	99842
44	16434526
	97598
45	16532125
	95453
46	16627578
	93400
47	16720978
	91433
48	16812412
	89548
49	16901960
	87739
50	16989700
	86001
51	17075701
	84331
52	17160033
	82725
53	17242758
	81178
54	17323937
	79689
55	17403626
	78253
56	17481880
	76868
57	17558748
	75531
58	17634279
	74240
59	17708520
	72992
60	17781512
	71785

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Why do we use logarithms anyway?

- To find the number of payments on a loan or the time to reach an invest ment goal

- To model many natural processes particularly in living systems We per ceive loudness of sound as the logarithm of the actual sound intensity and dB decibels are a logarithmic scale

- To measure the pH or acidity of a chemical solution The pH is the negat ive logarithm of the concentration of free hydrogen ions

- To measure earthquake intensity on the Richter scale

- To analyze exponential processes

Definition:

Consider the table below which gives the power of 2 :

$2^0 = 1$	$2^4 = 16$	$2^8 = 256$	$2^{12} = 4096$
$2^1 = 2$	$2^5 = 32$	$2^9 = 512$	$2^{13} = 8192$
$2^2 = 4$	$2^6 = 64$	$2^{10} = 1024$	$2^{14} = 16384$
$2^3 = 8$	$2^7 = 128$	$2^{11} = 2048$	$2^{15} = 32768$

For example

$$64 \times 256 = 2^6 \times 2^8 = 2^{14} = 16384$$

$$32768 \div 256 = 2^{15} \div 2^8 = 2^7 = 128$$

Let "a" be a fixed positive number $a \neq 1$

and let b, c be numbers such that :

$$a^c = b$$

Then c is called the logarithm of b to base a and we writh :

$$log_a^b = c$$

Consider the following sequence of numbers :

$$\ldots, 2^{-4}, 2^{-3}, 2^{-2}, 2^{-1}, 2^0, 2^1, 2^2, 2^3, 2^4, \ldots$$

Or

$$\ldots, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16, \ldots$$

- $log_a^{b^3} = 3c$, $because\ a^{3c} = (a^c)^3 = b^3$

- $log_a^{b^n} = nc$, $because\ a^{nc} = (a^c)^n = b^n$; $\forall n \in N$

- $log_a^{\frac{1}{b}} = -c$, $because\ a^{-c} = \frac{1}{a^c} = \frac{1}{b}$

$$\downarrow$$

$$log_a^{b^k} = klog_a^b$$

We have :

$$log_a^b + log_a^c = log_a^{bc} \quad , log_a^b - log_a^c = log_a^{\frac{b}{c}}$$

The following statements carry the same meaning:

$$log_a^b = c \quad , \quad a^c = b \quad , \quad antilog_a^c = b$$

The End

Refrences: Mathematical Marvels a primer on logarithms By Shailesh Shirali

The Early History of a Familiar Function

by Kathleen M. Clark (Florida State University) and Clemency Montelle (University of Canterbury)

http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=3495&bodyId=3845