CHAPTER TWO SOLUTIONS

1.	(a) 12 μs	(d) 3.5 Gbits	(g) 39 pA
	(b) 750 mJ	(e) 6.5 nm	(h) 49 kΩ
	(c) 1.13 kΩ	(f) 13.56 MHz	(i) 11.73 pA

CHAPTER TWO SOLUTIONS

2.

(a) 1 MW	(e) 33 μJ	(i) 32 mm
(b) 12.35 mm	(f) 5.33 nW	
(c) 47. kW	(g) 1 ns	
(d) 5.46 mA	(h) 5.555 MW	

3. Motor power = 175 Hp

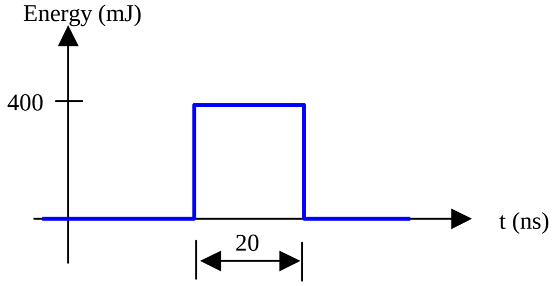
(a) With 100% efficient mechanical to electrical power conversion,
(175 Hp)[1 W/ (1/745.7 Hp)] = $\boxed{130.5 \text{ kW}}$

(b) Running for 3 hours,
Energy = $(130.5\times10^3$ W)(3 hr)(60 min/hr)(60 s/min) = $\boxed{1.409 \text{ GJ}}$

(c) A single battery has 430 kW-hr capacity. We require
(130.5 kW)(3 hr) = 391.5 kW-hr therefore $\boxed{\text{one battery is sufficient.}}$

4. The 400-mJ pulse lasts 20 ns.

(a) To compute the peak power, we assume the pulse shape is square:



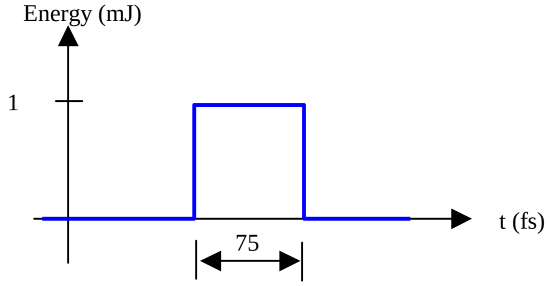
Then $P = 400\times10^{-3}/20\times10^{-9} = \boxed{20 \text{ MW.}}$

(b) At 20 pulses per second, the average power is

P_{avg} = (20 pulses)(400 mJ/pulse)/(1 s) = $\boxed{8 \text{ W.}}$

5. The 1-mJ pulse lasts 75 fs.

(c) To compute the peak power, we assume the pulse shape is square:

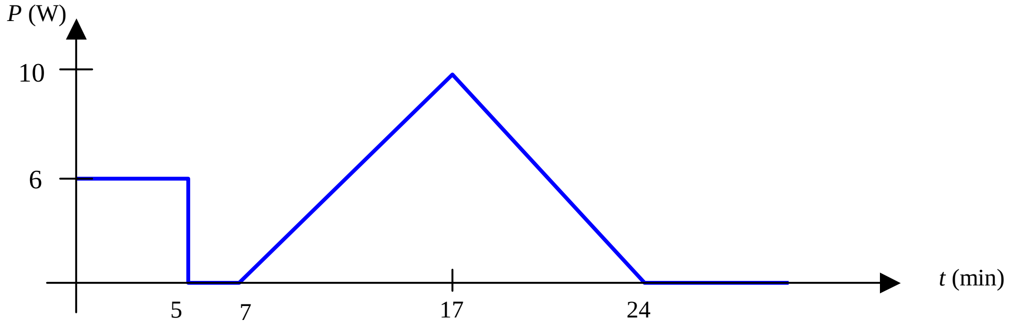


Then $P = 1\times10^{-3}/75\times10^{-15} = \boxed{13.33 \text{ GW.}}$

(d) At 100 pulses per second, the average power is

P_{avg} = (100 pulses)(1 mJ/pulse)/(1 s) = $\boxed{100 \text{ mW.}}$

6. The power drawn from the battery is (not quite drawn to scale):



(a) Total energy (in J) expended is
[6(5) + 0(2) + 0.5(10)(10) + 0.5(10)(7)]60 = $\boxed{\text{6.9 kJ.}}$

(b) The average power in Btu/hr is
(6900 J/24 min)(60 min/1 hr)(1 Btu/1055 J) = $\boxed{\text{16.35 Btu/hr.}}$

7. Total charge $q = 18t^2 - 2t^4$ C.

(a) $q(2\text{ s}) = \boxed{40\text{ C.}}$

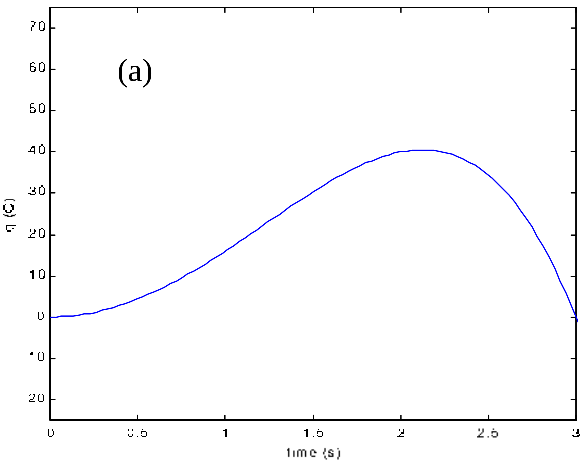
(b) To find the maximum charge within $0 \le t \le 3$ s, we need to take the first and second derivitives:

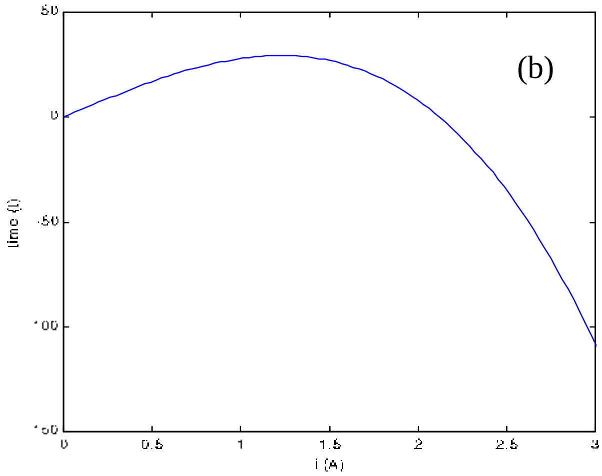
$dq/dt = 36t - 8t^3 = 0$, leading to roots at 0, ± 2.121 s
$d^2q/dt^2 = 36 - 24t^2$

substituting $t = 2.121$ s into the expression for d^2q/dt^2, we obtain a value of –14.9, so that this root represents a maximum.
Thus, we find a maximum charge $q = 40.5$ C at $t = 2.121$ s.

(c) The rate of charge accumulation at $t = 8$ s is
$dq/dt|_{t\,=\,0.8} = 36(0.8) - 8(0.8)^3 = 24.7$ C/s.

(d) See Fig. (a) and (b).





8. Referring to Fig. 2.6*c*,

$$i_1(t) = \begin{cases} -2+3e^{-5t} \text{ A}, & t<0 \\ -2+3e^{3t} \text{ A}, & t>0 \end{cases}$$

Thus,

(a) $i_1(-0.2)$ = $\boxed{6.155 \text{ A}}$

(b) i_1 (0.2) = $\boxed{3.466 \text{ A}}$

(c) To determine the instants at which i_1 = 0, we must consider $t < 0$ and $t > 0$ separately:

for $t < 0$, $- 2 + 3e^{-5t} = 0$ leads to t = -0.2 ln (2/3) = +2.027 s (impossible)

for $t > 0$, $-2 + 3e^{3t} = 0$ leads to t = (1/3) ln (2/3) = -0.135 s (impossible)

Therefore, the current is *never* negative.

(d) The total charge passed left to right in the interval $0.08 < t < 0.1$ s is

$$\begin{aligned} q(t) &= \int_{-0.08}^{0.1} i_1(t)\,dt \\ &= \int_{-0.08}^{0} \left[-2+3e^{-5t}\right]dt + \int_{0}^{0.1} \left[-2+3e^{3t}\right]dt \\ &= -2+3e^{-5t}\Big|_{-0.08}^{0} + -2+3e^{3t}\Big|_{0}^{0.1} \\ &= 0.1351 + 0.1499 \\ &= \boxed{285 \text{ mC}} \end{aligned}$$

9. Referring to Fig. 2.28,

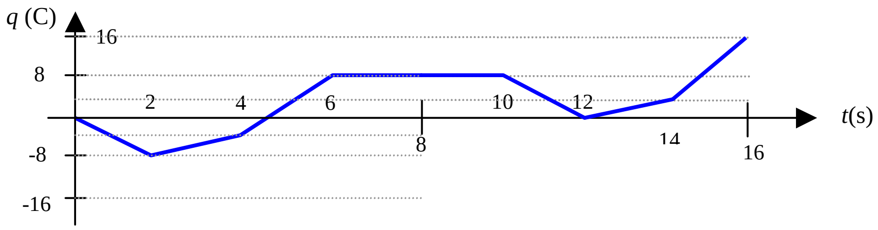
(a) The average current over one period (10 s) is

i_{avg} = [-4(2) + 2(2) + 6(2) + 0(4)]/10 = $\boxed{\text{800 mA}}$

(b) The total charge transferred over the interval 1 < *t* < 12 s is

$$q_{total} = \int_1^{12} i(t)dt = -4(2) + 2(2) + 6(2) + 0(4) - 4(2) = \boxed{0\text{ C}}$$

(d) See Fig. below



CHAPTER TWO SOLUTIONS

10. (a) $P_{abs} = (+3.2\text{ V})(-2\text{ mA}) = \boxed{-6.4\text{ mW}}$ (or +6.4 mW *supplied*)

(b) $P_{abs} = (+6\text{ V})(-20\text{ A}) = \boxed{-120\text{ W}}$ (or +120 W *supplied*)

(d) $P_{abs} = (+6\text{ V})(2\,i_x) = (+6\text{ V})[(2)(5\text{ A})] = \boxed{+60\text{ W}}$

(e) $P_{abs} = (4 \sin 1000t\text{ V})(-8 \cos 1000t\text{ mA})\Big|_{t\,=\,2\text{ ms}} = \boxed{+12.11\text{ W}}$

11. $i = 3te^{-100t}$ mA and $v = [6 - 600t]\, e^{-100t}$ mV

(a) The power absorbed at t = 5 ms is

$$P_{\text{abs}} = \left[(6-600t)e^{-100t} \cdot 3te^{-100t}\right]_{t=5ms} \quad \mu\text{W}$$

$$= 0.01655\ \mu\text{W} \quad = \quad \boxed{16.55\ \text{nW}}$$

(b) The energy delivered over the interval $0 < t < \infty$ is

$$\int_0^\infty P_{abs}\, dt \quad = \quad \int_0^\infty 3t(6-600t)e^{-200t}\, dt \qquad \mu\text{J}$$

Making use of the relationship

$$\int_0^\infty x^n\, e^{-ax}\, dx \quad = \quad \frac{n!}{a^{n+1}}$$ where n is a positive integer and a > 0,

we find the energy delivered to be

$$= 18/(200)^2 \quad - \quad 1800/(200)^3$$

$$\boxed{= 0}$$

12. (a) $P_{\text{abs}} = (40i)(3e^{-100t})\Big|_{t\,=\,8\text{ ms}} = 360\left[e^{-100t}\right]^2_{t=8ms} = \boxed{72.68\text{ W}}$

(b) $P_{\text{abs}} = \left(0.2\dfrac{di}{dt}\right)i = -180\left[e^{-100t}\right]^2_{t=8ms} = \boxed{-36.34\text{ W}}$

(c) $P_{\text{abs}} = \left(30\int_0^t i\,dt + 20\right)\left(3e^{-100t}\right)\Big|_{t=8ms}$

$$= \left(90e^{-100t}\int_0^t 3e^{-100t'}dt' + 60e^{-100t}\right)\Big|_{t=8ms} = \boxed{27.63\text{ W}}$$

13. (a) The short-circuit current is the value of the current at V = 0.

Reading from the graph, this corresponds to approximately 3.0 A.

(b) The open-circuit voltage is the value of the voltage at I = 0.

Reading from the graph, this corresponds to roughly 0.4875 V, estimating the curve as hitting the x-axis 1 mm behind the 0.5 V mark.

(c) We see that the maximum current corresponds to zero voltage, and likewise, the maximum voltage occurs at zero current. The maximum power point, therefore, occurs somewhere between these two points. By trial and error,

Pmax is roughly (375 mV)(2.5 A) = 938 mW, or just under 1 W.

14. Note that in the table below, only the –4-A source and the –3-A source are actually "absorbing" power; the remaining sources are supplying power to the circuit.

Source	Absorbed Power	Absorbed Power
2-V source	(2 V)(-2 A)	- 4 W
8-V source	(8 V)(-2 A)	- 16 W
-4-A source	(10 V)[-(-4 A)]	40 W
10-V source	(10 V)(-5 A)	- 50 W
-3-A source	(10 V)[-(-3 A)]	30 W

$\boxed{\text{The 5 power quantities sum to } -4 - 16 + 40 - 50 + 30 = 0,}$ as demanded from conservation of energy.

15. We are told that $V_x = 1$ V, and from Fig. 2.33 we see that the current flowing through the dependent source (and hence through each element of the circuit) is $5V_x = 5$ A. We will compute *absorbed* power by using the current flowing *into* the positive reference terminal of the appropriate voltage (passive sign convention), and we will compute *supplied* power by using the current flowing *out of* the positive reference terminal of the appropriate voltage.

(a) The power absorbed by element "A" = (9 V)(5 A) = $\boxed{45 \text{ W}}$

(b) The power supplied by the 1-V source = (1 V)(5 A) = $\boxed{5 \text{ W}}$, and
the power supplied by the dependent source = (8 V)(5 A) = $\boxed{40 \text{ W}}$

(c) The sum of the supplied power = 5 + 40 = 45 W
The sum of the absorbed power is 45 W, so

yes, the sum of the power supplied = the sum of the power absorbed, as we expect from the principle of conservation of energy.

16. We are asked to determine the voltage v_s, which is identical to the voltage labeled v_1. The only remaining reference to v_1 is in the expression for the current flowing through the dependent source, $5v_1$.

This current is equal to $-i_2$.
Thus,

$$5\ v_1 = -i_2 = -\ 5\ \text{mA}$$

Therefore $v_1 = -1$ mV

and so $\boxed{v_s = v_1 = -1\ \text{mV}}$

17. The battery delivers an energy of 460.8 W-hr over a period of 8 hrs.

(a) The power delivered to the headlight is therefore (460.8 W-hr)/(8 hr) = $\boxed{57.6\text{ W}}$

(b) The current through the headlight is equal to the power it absorbs from the battery divided by the voltage at which the power is supplied, or

I = (57.6 W)/(12 V) = $\boxed{4.8\text{ A}}$

18. The supply voltage is 110 V, and the maximum dissipated power is 500 W. The fuses are specified in terms of current, so we need to determine the maximum current that can flow through the fuse.

$$P = V\,I \quad \text{therefore } I_{max} = P_{max}/V = (500\text{ W})/(110\text{ V}) = 4.545\text{ A}$$

If we choose the 5-A fuse, it will allow up to (110 V)(5 A) = 550 W of power to be delivered to the application (we must assume here that the fuse absorbs zero power, a reasonable assumption in practice). This exceeds the specified maximum power.

If we choose the 4.5-A fuse instead, we will have a maximum current of 4.5 A. This leads to a maximum power of (110)(4.5) = 495 W delivered to the application.

Although 495 W is less than the maximum power allowed, this fuse will provide adequate protection for the application circuitry. If a fault occurs and the application circuitry attempts to draw too much power, 1000 W for example, the fuse will blow, no current will flow, and the application circuitry will be protected. However, if the application circuitry tries to draw its maximum rated power (500 W), the fuse will also blow. In practice, most equipment will not draw its maximum rated power continuously- although to be safe, we typically assume that it will.

19. (a) $P_{abs} = i^2R \quad = [20e^{-12t}]^2 \ (1200)\ \mu\text{W}$

$= [20e^{-1.2}]^2 \ (1200)\ \mu\text{W}$

$\boxed{= 43.54 \text{ mW}}$

(b) $P_{abs} = v^2/R = [40 \cos 20t]^2 / 1200 \quad \text{W}$

$= [40 \cos 2]^2 / 1200 \quad \text{W}$

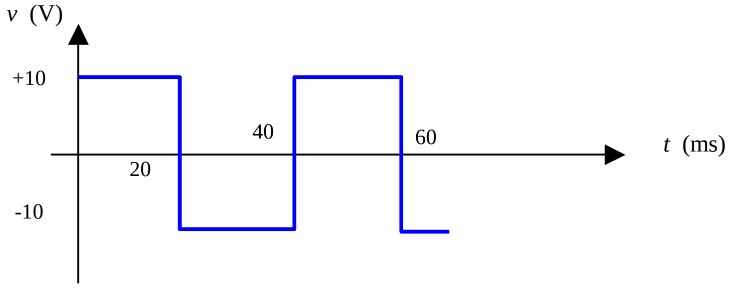
$\boxed{= 230.9 \text{ mW}}$

keep in mind we are using radians

(c) $P_{abs} = v\, i \quad = 8t^{1.5} \quad \text{W}$

$\boxed{= 253.0 \text{ mW}}$

20. It's probably best to begin this problem by sketching the voltage waveform:



(a) $v_{\max} = \boxed{+10\text{ V}}$

(b) $v_{\text{avg}} = [(+10)(20\times10^{-3}) + (-10)(20\times10^{-3})]/(40\times10^{-3}) = \boxed{0}$

(c) $i_{\text{avg}} = v_{\text{avg}}/R = \boxed{0}$

(d) $$p_{abs}\big|_{\max} = \frac{v_{\max}^2}{R} = (10)^2 / 50 = \boxed{2\text{ W}}$$

(e) $$p_{abs}\big|_{avg} = \frac{1}{40}\left[\frac{(+10)^2}{R}\cdot 20 + \frac{(-10)^2}{R}\cdot 20\right] = \boxed{2\text{ W}}$$

21. We are given that the conductivity σ of copper is 5.8×10^{7} S/m.

(a) 50 ft of #18 (18 AWG) copper wire, which has a diameter of 1.024 mm, will have a resistance of $l/(\sigma A)$ ohms, where A = the cross-sectional area and l = 50 ft.

Converting the dimensional quantities to meters,

l = (50 ft)(12 in/ft)(2.54 cm/in)(1 m/100 cm) = 15.24 m

and

$r = 0.5(1.024\text{ mm})(1\text{ m}/1000\text{ mm}) = 5.12\times10^{-4}$ m

so that

$A = \pi r^2 = \pi(5.12\times10^{-4}\text{ m})^2 = 8.236\times10^{-7}\text{ m}^2$

Thus, $R = (15.24\text{ m})/[(5.8\times10^{7})(8.236\times10^{-7})] = \boxed{319.0\text{ m}\Omega}$

(b) We assume that the conductivity value specified also holds true at 50°C.

The cross-sectional area of the foil is

$A = (33\ \mu\text{m})(500\ \mu\text{m})(1\text{ m}/10^{6}\ \mu\text{m})(1\text{ m}/10^{6}\ \mu\text{m}) = 1.65\times10^{-8}\text{ m}^2$

So that

$R = (15\text{ cm})(1\text{ m}/100\text{ cm})/[(5.8\times10^{7})(1.65\times10^{-8})] = \boxed{156.7\text{ m}\Omega}$

A 3-A current flowing through this copper in the direction specified would lead to the dissipation of

$$I^2R = (3)^2\,(156.7)\text{ mW} = \boxed{1.410\text{ W}}$$

22. Since we are informed that the same current must flow through each component, we begin by defining a current *I* flowing out of the positive reference terminal of the voltage source.

The power supplied by the voltage source is $V_s I$.
The power absorbed by resistor R_1 is I^2R_1.
The power absorbed by resistor R_2 is I^2R_2.

Since we know that the total power supplied is equal to the total power absorbed, we may write:

$$V_s I = I^2R_1 + I^2R_2$$

or

$$V_s = I R_1 + I R_2$$
$$V_s = I (R_1 + R_2)$$

By Ohm's law,

$$I = V_{R_2} / R_2$$

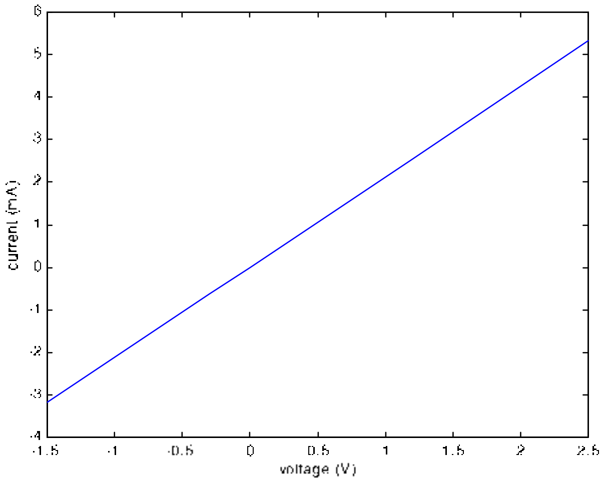
so that

$$V_s = \frac{V_{R_2}}{R_2}(R_1 + R_2)$$

Solving for V_{R_2} we find

$$V_{R_2} = V_s \frac{R_2}{(R_1 + R_2)} \qquad \textit{Q.E.D.}$$

23. (a)

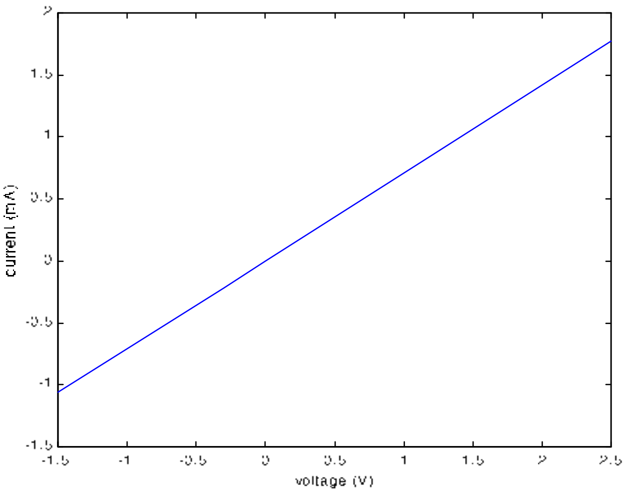


(b) We see from our answer to part (a) that this device has a reasonably linear characteristic (a not unreasonable degree of experimental error is evident in the data). Thus, we choose to estimate the resistance using the two extreme points:

$$R_{\text{eff}} = [(2.5 - (-1.5)]/[5.23 - (-3.19)] \text{ k}\Omega = 475\ \Omega$$

Using the last two points instead, we find $R_{\text{eff}} = 469\ \Omega$, so that we can state with some certainty at least that a reasonable estimate of the resistance is $\boxed{\text{approximately } 470\ \Omega.}$

(c)



24. Top Left Circuit: $I = (5/10)$ mA = 0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

Top Right Circuit: $I = -(5/10)$ mA = -0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

Bottom Left Circuit: $I = (-5/10)$ mA = -0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

Bottom Right Circuit: $I = -(-5/10)$ mA = 0.5 mA, and $P_{10k} = V^2/10$ mW = 2.5 mW

25. The voltage v_{out} is given by

$$v_{out} = -10^{-3}\, v_\pi \ (1000)$$

$$= -\, v_\pi$$

Since $v_\pi = v_s = 0.01 \cos 1000t$ V, we find that

$$v_{out} = -\, v_\pi = \boxed{-0.001 \cos 1000t \quad \text{V}}$$

26. 18 AWG wire has a resistance of 6.39 Ω / 1000 ft.

Thus, we require 1000 (53) / 6.39 = $\boxed{\text{8294 ft of wire.}}$
(Or 1.57 miles. Or, 2.53 km).

27. We need to create a 470-Ω resistor from 28 AWG wire, knowing that the ambient temperature is 108^{o}F, or 42.22^{o}C.

Referring to Table 2.3, 28 AWG wire is 65.3 mΩ/ft at 20^{o}C, and using the equation provided we compute

$$R_2/R_1 = (234.5 + T_2)/(234.5 + T_1) = (234.5 + 42.22)/(234.5 + 20) = 1.087$$

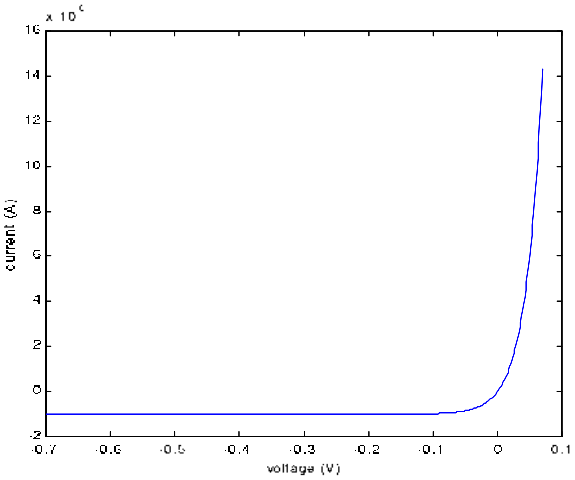
We thus find that 28 AWG wire is (1.087)(65.3) = 71.0 mΩ/ft.

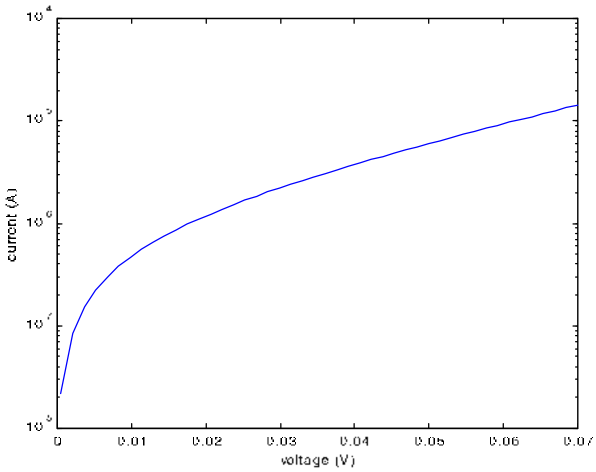
Thus, to repair the transmitter we will need

$(470\ \Omega)/(71.0 \times 10\text{-}3\ \Omega/\text{ft}) = \boxed{6620\text{ ft}}$ (1.25 miles, or 2.02 km).

Note: This seems like a lot of wire to be washing up on shore. We may find we don't have enough. In that case, perhaps we should take our cue from Eq. [6], and try to squash a piece of the wire flat so that it has a very small cross-sectional area…..

28. (a) We need to plot the negative and positive voltage ranges separately, as the positive voltage range is, after all, exponential!





(b) To determine the resistance of the device at V = 550 mV, we compute the corresponding current:

$$I = 10^{-6}\,[e^{39(0.55)} - 1] \quad = \quad 2.068 \text{ A}$$

Thus, R(0.55 V) = 0.55/2.068 = $\boxed{266 \text{ m}\Omega}$

(c) $R = 1\ \Omega$ corresponds to $V = I$. Thus, we need to solve the transcendental equation

$$I = 10^{-6}\,[e^{39I} - 1]$$

Using a scientific calculator or the tried-and-true trial and error approach, we find that

$$\boxed{I = 325.5 \text{ mA}}$$

29. We require a 10-Ω resistor, and are told it is for a portable application, implying that size, weight or both would be important to consider when selecting a wire gauge. We have 10,000 ft of each of the gauges listed in Table 2.3 with which to work. Quick inspection of the values listed eliminates 2, 4 and 6 AWG wire as their respective resistances are too low for only 10,000 ft of wire.

Using 12-AWG wire would require (10 Ω) / (1.59 mΩ/ft) = 6290 ft.
Using 28-AWG wire, the narrowest available, would require

$$(10\ \Omega) / (65.3\ \text{m}\Omega/\text{ft}) = \boxed{153\ \text{ft.}}$$

Would the 28-AWG wire weight less? Again referring to Table 2.3, we see that the cross-sectional area of 28-AWG wire is 0.0804 mm^2, and that of 12-AWG wire is 3.31 mm^2. The volume of 12-AWG wire required is therefore 6345900 mm^3, and that of 28-AWG wire required is only 3750 mm^3.

The best (but not the only) choice for a portable application is clear: 28-AWG wire!

30. Our target is a 100-Ω resistor. We see from the plot that at $N_D = 10^{15}$ cm^{-3}, $\mu_n \sim 2\text{x}10^3$ cm^2/V-s, yielding a resistivity of 3.121 Ω-cm.

At $N_D = 10^{18}$ cm^{-3}, $\mu_n \sim 230$ cm^2/ V-s, yielding a resistivity of 0.02714 Ω-cm.

Thus, we see that the lower doping level clearly provides material with higher resistivity, requiring less of the available area on the silicon wafer.

Since $R = \rho L/A$, where we know $R = 10$ Ω and ρ = 3.121 Ω-cm, we need only define the resistor geometry to complete the design.

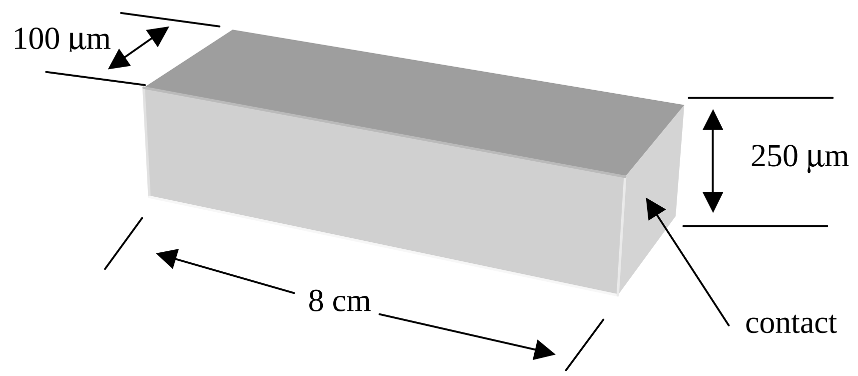
We typically form contacts primarily on the surface of a silicon wafer, so that the wafer thickness would be part of the factor *A*; *L* represents the distance between the contacts. Thus, we may write

$$R = 3.121\, L/(250\text{x}10^{-4}\, Y)$$

where *L* and *Y* are dimensions on the surface of the wafer.

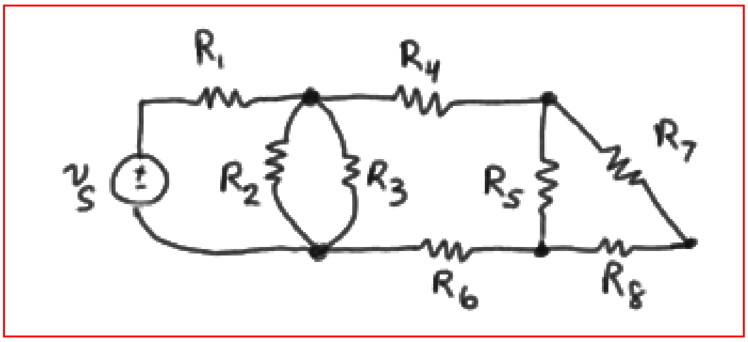
If we make *Y* small (*i.e.* a narrow width as viewed from the top of the wafer), then *L* can also be small. Seeking a value of 0.080103 then for *L*/Y, and choosing *Y* = 100 μm (a large dimension for silicon devices), we find a contact-to-contact length of *L* = 8 cm! While this easily fits onto a 6” diameter wafer, we could probably do a little better. We are also assuming that the resistor is to be cut from the wafer, and the ends made the contacts, as shown below in the figure.

Design summary (one possibility): $N_D = 10^{15}$ cm^{-3}
L = 8 cm
Y = 100 μm



CHAPTER THREE SOLUTIONS

1.



CHAPTER THREE SOLUTIONS

2. (*a*) six nodes; (*b*) nine branches.

CHAPTER THREE SOLUTIONS

3. (*a*) Four nodes; (*b*) five branches; (*c*) path, yes – loop, no.

CHAPTER THREE SOLUTIONS

4. (*a*) Five nodes; (*b*) seven branches; (*c*) path, yes – loop, no.

5. (*a*) 3 A; (*b*) –3 A; (*c*) 0.

6. By KCL, we may write:

$$5 + i_y + i_z = 3 + i_x$$

(*a*) $i_x = 2 + i_y + i_z = 2 + 2 + 0 = \boxed{4\text{ A}}$

(*b*) $i_y = 3 + i_x - 5 - i_z$
$i_y = -2 + 2 - 2\, i_y$

Thus, we find that $\boxed{i_y = 0.}$

(*c*) $\boxed{\textit{This situation is impossible,}}$ since i_x and i_z are in opposite directions. The only possible value (zero), is also disallowed, as KCL will not be satisfied ($5 \neq 3$).

7. Focusing our attention on the bottom left node, we see that $\boxed{i_x = 1\text{ A.}}$

Focusing our attention next on the top right node, we see that $\boxed{i_y = 5\text{A.}}$

8. (*a*) $v_y = 1(3v_x + i_z)$

v_x = 5 V and given that i_z = –3 A, we find that

$$v_y = 3(5) - 3 = \boxed{12 \text{ V}}$$

(*b*) $v_y = 1(3v_x + i_z) = -6 = 3v_x + 0.5$

Solving, we find that $v_x = (-6 - 0.5)/3 = \boxed{-2.167 \text{ V.}}$

9. (*a*) $i_x = v_1/10 + v_1/10 = 5$

$2v_1 = 50$

so $\boxed{v_1 = 25 \text{ V.}}$

By Ohm's law, we see that $i_y = v_2/10$

also, using Ohm's law in combination with KCL, we may write

$$i_x = v_2/10 + v_2/10 = i_y + i_y = 5 \text{ A}$$

Thus, $\boxed{i_y = 2.5 \text{ A.}}$

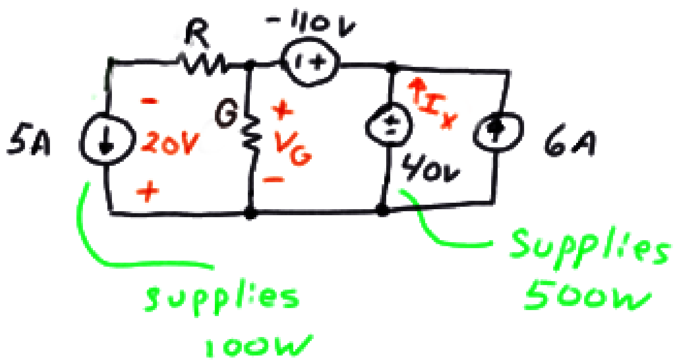
(*b*) From part (*a*), $i_x = 2\, v_1/ 10$. Substituting the new value for v_1, we find that

$\boxed{i_x = 6/10 = 600 \text{ mA.}}$

Since we have found that $i_y = 0.5\, i_x$, $\boxed{i_y = 300 \text{ mA.}}$

(*c*) $\boxed{\text{no value – this is impossible.}}$

10. We begin by making use of the information given regarding the power generated by the 5-A and the 40-V sources. The 5-A source supplies 100 W, so it must therefore have a terminal voltage of 20 V. The 40-V source supplies 500 W, so it must therefore provide a current of 12.5 A. These quantities are marked on our schematic below:



(1) By KVL, $-40 - 110 + R(5) = 0$

Thus, $\boxed{R = 30\ \Omega.}$

(2) By KVL, $-V_G - (-110) + 40 = 0$

So $V_G = 150$ V

Now that we know the voltage across the unknown conductance G, we need only to find the current flowing through it to find its value by making use of Ohm's law.

KCL provides us with the means to find this current: The current flowing into the "+" terminal of the –110-V source is 12.5 + 6 = 18.5 A.

Then, $I_x = 18.5 - 5 = 13.5$ A

By Ohm's law, $I_x = G \cdot V_G$

So $G = 13.5/150$ or $\boxed{G = 90 \text{ mS}}$

11. (*a*) $-1 + 2 + 10i - 3.5 + 10i = 0$

Solving, $\boxed{i = 125 \text{ mA}}$

(*b*) $+10 + 1i - 2 + 2i + 2 - 6 + i = 0$

Solving, we find that $4i = -4$ or $\boxed{i = -1 \text{ A.}}$

12. (a) By KVL, $-2 + v_x + 8 = 0$

so that $\boxed{v_x = -6 \text{ V.}}$

(b) By KCL at the top right node,

$I_S + 4\, v_x = 4 - v_x/4$

So $\boxed{I_S = 29.5 \text{ A.}}$

(c) By KCL at the top left node,

$i_{in} = 1 + I_S + v_x/4 - 6$

or $\boxed{i_{in} = 23 \text{ A}}$

(d) The power provided by the dependent source is $8(4v_x) = \boxed{-192 \text{ W.}}$

13. (a) Working from left to right,

$v_1 = 60$ V
$v_2 = 60$ V
$i_2 = 60/20 = 3$ A
$i_4 = v_1/4 = 60/4 = 15$ A
$v_3 = 5i_2 = 15$ V

By KVL, $-60 + v_3 + v_5 = 0$
$v_5 = 60 – 15 = 45$ V
$v_4 = v_5 = 45$

$i_5 = v_5/5 = 45/5 = 9$ A
$i_3 = i_4 + i_5 = 15 + 9 = 24$ A
$i_1 = i_2 + i_3 = 3 + 24 = 27$

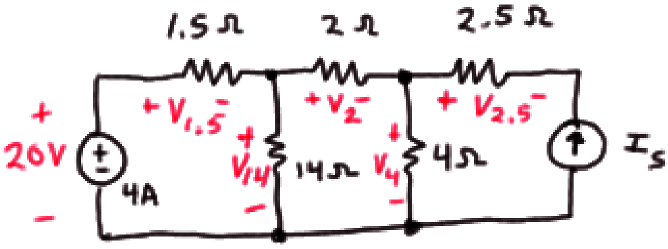
$v_1 = 60$ V	$i_1 = 27$ A
$v_2 = 60$ V	$i_2 = 3$ A
$v_3 = 15$ V	$i_3 = 24$ A
$v_4 = 45$ V	$i_4 = 15$ A
$v_5 = 45$ V	$i_5 = 9$ A

(b) It is now a simple matter to compute the power absorbed by each element:

p_1	$= -v_1 i_1$	= -(60)(27)	= -1.62 kW
p_2	$= v_2 i_2$	= (60)(3)	= 180 W
p_3	$= v_3 i_3$	= (15)(24)	= 360 W
p_4	$= v_4 i_4$	= (45)(15)	= 675 W
p_5	$= v_5 i_5$	= (45)(9)	= 405 W

and it is a simple matter to check that these values indeed sum to zero as they should.

14. Refer to the labeled diagram below.



Beginning from the left, we find

$\boxed{p_{20V} = -(20)(4) = -80 \text{ W}}$

$v_{1.5} = 4(1.5) = 6$ V therefore $\boxed{p_{1.5} = (v_{1.5})^2/ 1.5 = 24 \text{ W.}}$

$v_{14} = 20 - v_{1.5} = 20 - 6 = 14$ V $\boxed{\text{therefore } p_{14} = 14^2/ 14 = 14 \text{ W.}}$

$i_2 = v_2/2 = v_{1.5}/1.5 - v_{14}/14 = 6/1.5 - 14/14 = 3$ A

Therefore $v_2 = 2(3) = 6$ V and $\boxed{p_2 = 6^2/2 = 18 \text{ W.}}$

$v_4 = v_{14} - v_2 = 14 - 6 = 8$ V therefore $\boxed{p_4 = 8^2/4 = 16 \text{ W}}$

$i_{2.5} = v_{2.5}/ 2.5 = v_2/2 - v_4/4 = 3 - 2 = 1$ A

Therefore $v_{2.5} = (2.5)(1) = 2.5$ V and so $\boxed{p_{2.5} = (2.5)^2/2.5 = 2.5 \text{ W.}}$

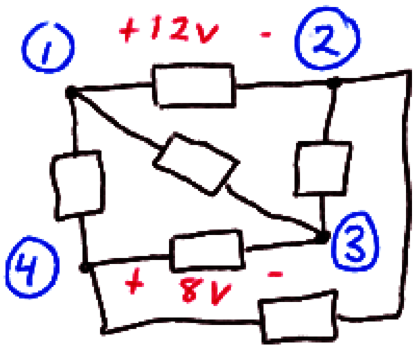
$I_{2.5} = -I_S$, thefore $I_S = -1$ A.

KVL allows us to write $-v_4 + v_{2.5} + v_{IS} = 0$

so $V_{I_S} = v_4 - v_{2.5} = 8 - 2.5 = 5.5$ V and $\boxed{p_{I_S} = -V_{I_S} I_S = 5.5 \text{ W.}}$

A quick check assures us that these power quantities sum to zero.

15. Sketching the circuit as described,



(*a*) $v_{14} = 0$.

$v_{13} = v_{43} = 8$ V
$v_{23} = -v_{12} - v_{34} = -12 + 8 = -4$ V
$v_{24} = v_{23} + v_{34} = -4 - 8 = -12$ V

(*b*) $v_{14} = 6$ V.

$v_{13} = v_{14} + v_{43} = 6 + 8 = 14$ V
$v_{23} = v_{13} - v_{12} = 14 - 12 = 2$ V
$v_{24} = v_{23} + v_{34} = 2 - 8 = -6$ V

(*c*) $v_{14} = -6$ V.

$v_{13} = v_{14} + v_{43} = -6 + 8 = 2$ V
$v_{23} = v_{13} - v_{12} = 2 - 12 = -10$ V
$v_{24} = v_{23} + v_{34} = -10 - 8 = -18$ V

16. (*a*) By KVL, $-12 + 5000I_D + V_{DS} + 2000I_D = 0$

Therefore, $\boxed{V_{DS} = 12 - 7(1.5) = 1.5 \text{ V.}}$

(*b*) By KVL, $- V_G + V_{GS} + 2000I_D = 0$

Therefore, $\boxed{V_{GS} = V_G - 2(2) = -1 \text{ V.}}$

17. Applying KVL around this series circuit,

$$-120 + 30i_x + 40i_x + 20i_x + v_x + 20 + 10i_x = 0$$

where v_x is defined across the unknown element X, with the "+" reference on top. Simplifying, we find that $100i_x + v_x = 100$

To solve further we require specific information about the element X and its properties.

(*a*) if X is a 100-Ω resistor,

$$v_x = 100i_x \text{ so we find that } 100\, i_x + 100\, i_x = 100.$$

Thus

$$i_x = 500 \text{ mA and } \boxed{p_x = v_x\, i_x = 25 \text{ W.}}$$

(*b*) If X is a 40-V independent voltage source such that $v_x = 40$ V, we find that

$$i_x = (100 – 40) / 100 = 600 \text{ mA and } \boxed{p_x = v_x\, i_x = 24 \text{ W}}$$

(*c*) If X is a dependent voltage source such that v_x = 25ix,

$$i_x = 100/125 = 800 \text{ mA and } \boxed{p_x = v_x\, i_x = 16 \text{ W.}}$$

(*d*) If X is a dependent voltage source so that $v_x = 0.8v_1$,
where $v_1 = 40i_x$, we have

$$100\, i_x + 0.8(40i_x) = 100$$

or i_x = 100/132 = 757.6 mA and $\boxed{p_x = v_x\, i_x = 0.8(40)(0.7576)^2 = 18.37 \text{ W.}}$

(*e*) If X is a 2-A independent current source, arrow up,

$$100(-2) + v_x = 100$$

so that v_x = 100 + 200 = 300 V and $\boxed{p_x = v_x\, i_x = -600 \text{ W}}$

18. (*a*) We first apply KVL:

$$-20 + 10i_1 + 90 + 40i_1 + 2v_2 = 0$$

where $v_2 = 10i_1$. Substituting,

$$70 + 70\, i_1 = 0$$

or $\boxed{i_1 = -1\text{ A.}}$

(*b*) Applying KVL,

$$-20 + 10i_1 + 90 + 40i_1 + 1.5v_3 = 0 \qquad [1]$$

where

$$v_3 = -90 - 10i_1 + 20 = -70 - 10\, i_1$$

alternatively, we could write

$$v_3 = 40i_1 + 1.5v_3 = -80i_1$$

Using either expression in Eq. [1], we find $\boxed{i_1 = 1\text{ A.}}$

(*c*) Applying KVL,

$$-20 + 10i_1 + 90 + 40i_1 - 15\, i_1 = 0$$

Solving, $\boxed{i_1 = -2\text{A.}}$

19. Applying KVL, we find that

$$-20 + 10i_1 + 90 + 40i_1 + 1.8v_3 = 0 \qquad [1]$$

Also, KVL allows us to write

$$v_3 = 40i_1 + 1.8v_3$$

$$v_3 = -50i_1$$

So that we may write Eq. [1] as

$$50i_1 - 1.8(50)i_1 = -70$$

or $i_1 = -70/-40 = 1.75$ A.

Since $v_3 = -50i_1 = -87.5$ V, no further information is required to determine its value.

The 90-V source is absorbing $(90)(i_1) = 157.5$ W of power and the dependent source is absorbing $(1.8v_3)(i_1) = -275.6$ W of power.

Therefore, *none of the conditions specified in (a) to (d) can be met by this circuit*.

20. (*a*) Define the charging current i as flowing clockwise in the circuit provided. By application of KVL,

$$-13 + 0.02i + Ri + 0.035i + 10.5 = 0$$

We know that we need a current i = 4 A, so we may calculate the necessary resistance

$$\boxed{R = [13 - 10.5 - 0.055(4)]/ 4 = 570\ \text{m}\Omega}$$

(*b*) The total power delivered to the battery consists of the power absorbed by the 0.035-Ω resistance ($0.035i^2$), and the power absorbed by the 10.5-V ideal battery ($10.5i$). Thus, we need to solve the quadratic equation

$$0.035i^2 + 10.5i = 25$$

which has the solutions i = -302.4 A and i = 2.362 A.

In order to determine which of these two values should be used, we must recall that the idea is to charge the battery, implying that it is absorbing power, or that i as defined is positive. Thus, we choose i = 2.362 A, and, making use of the expression developed in part (*a*), we find that

$$\boxed{R = [13 - 10.5 - 0.055(2.362)]/ 2.362 = 1.003\ \Omega}$$

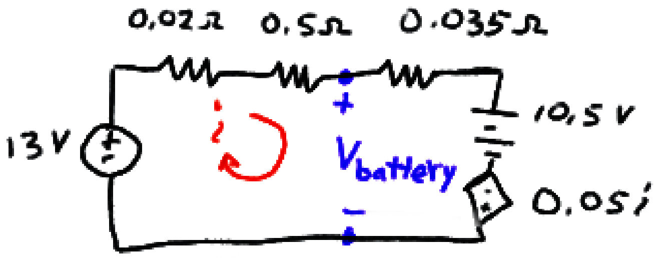
(*c*) To obtain a voltage of 11 V across the battery, we apply KVL:

$$0.035i + 10.5 = 11 \quad \text{so that } i = 14.29 \text{ A}$$

From part (*a*), this means we need

$$\boxed{R = [13 - 10.5 - 0.055(14.29)]/ 14.29 = 119.9\ \text{mW}}$$

21. Drawing the circuit described, we also define a clockwise current *i*.



By KVL, we find that

$$-13 + (0.02 + 0.5 + 0.035)i + 10.5 - 0.05i = 0$$

or that $i = (13 - 10.5)/0.505 = \boxed{4.950\text{ A}}$

and $V_{battery} = 13 - (0.02 + 0.5)i = \boxed{10.43\text{ V.}}$

22. Applying KVL about this simple loop circuit (the dependent sources are still linear elements, by the way, as they depend only upon a sum of voltages)

$$-40 + (5 + 25 + 20)i - (2v_3 + v_2) + (4v_1 - v_2) = 0 \qquad [1]$$

where we have defined i to be flowing in the clockwise direction, and $v_1 = 5i$, $v_2 = 25i$, and $v_3 = 20i$.

Performing the necessary substition, Eq. [1] becomes

$$50i - (40i + 25i) + (20i - 25i) = 40$$

so that $i = 40/-20 = -2$ A

Computing the absorbed power is now a straightforward matter:

p_{40V}	$= (40)(-i)$	$= 80$ W
p_{5W}	$= 5i^2$	$= 20$ W
p_{25W}	$= 25i^2$	$= 100$ W
p_{20W}	$= 20i^2$	$= 80$ W
$p_{depsrc1}$	$= (2v_3 + v_2)(-i) = (40i + 25i)$	$= -260$ W
$p_{depsrc2}$	$= (4v_1 - v_2)(-i) = (20i - 25i)$	$= -20$ W

and we can easily verify that these quantities indeed sum to zero as expected.

23. We begin by defining a clockwise current i.

(a) $i = 12/(40 + R)$ mA, with R expressed in kΩ.

We want $i^2 \cdot 25 = 2$

or
$$\left(\frac{12}{40+R}\right)^2 \cdot 25 = 2$$

Rearranging, we find a quadratic expression involving R:

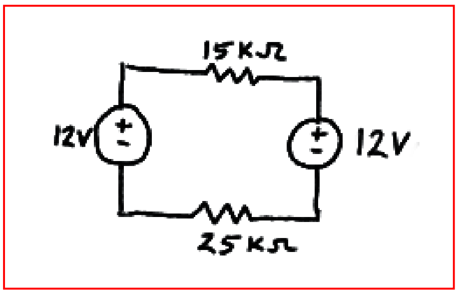
$$R^2 + 80R - 200 = 0$$

which has the solutions R = -82.43 kΩ and R = 2.426 kΩ. Only the latter is a physical solution, so

$$\boxed{R = 2.426 \text{ k}\Omega.}$$

(b) We require $i \cdot 12 = 3.6$ or $i = 0.3$ mA
From the circuit, we also see that $i = 12/(15 + R + 25)$ mA.
Substituting the desired value for i, we find that the required value of R is $\boxed{R = 0.}$

(c)



24. By KVL,

$$-12 + (1 + 2.3 + R_{\text{wire segment}})\, i = 0$$

The wire segment is a 3000–ft section of 28–AWG solid copper wire. Using Table 2.3, we compute its resistance as

$$(16.2\ \text{m}\Omega/\text{ft})(3000\ \text{ft}) = 48.6\ \Omega$$

which is certainly *not* negligible compared to the other resistances in the circuit!

Thus,

$$i = 12/(1 + 2.3 + 48.6) = \boxed{231.2\ \text{mA}}$$

25. We can apply Ohm's law to find an expression for v_o:

$$v_o = 1000(-g_m\, v_\pi)$$

We do not have a value for v_π, but KVL will allow us to express that in terms of v_o, which we *do* know:

$$-10\times10^{-3} \cos 5t + (300 + 50\times10^{3})\, i = 0$$

where i is defined as flowing clockwise.

Thus, $v_\pi = 50\times10^{3}\, i = 50\times10^{3}\,(10\times10^{-3} \cos 5t) / (300 + 50\times10^{3})$

$= 9.940\times10^{-3} \cos 5t$ V

and we by substitution we find that

$$v_o = 1000(-25\times10^{-3})(9.940\times10^{-3} \cos 5t)$$

$$= \boxed{-248.5 \cos 5t \text{ mV}}$$

26. By KVL, we find that

$$-3 + 100\, I_D + V_D = 0$$

Substituting $I_D = 3\times10^{-6}(e^{V_D / 27\times10^{-3}} - 1)$, we find that

$$-3 + 300\times10^{-6}(e^{V_D / 27\times10^{-3}} - 1) + V_D = 0$$

This is a transcendental equation. Using a scientific calculator or a numerical software package such as MATLAB®, we find

$$V_D = \boxed{246.4 \text{ mV}}$$

Let's assume digital assistance is unavailable. In that case, we need to "guess" a value for V_D, substitute it into the right hand side of our equation, and see how close the result is to the left hand side (in this case, zero).

GUESS	RESULT
0	–3
1	3.648×10^{12}
0.5	3.308×10^{4}
0.25	0.4001
0.245	–0.1375
0.248	0.1732
0.246	–0.0377

oops

better

At this point, the error is getting much smaller, and our confidence is increasing as to the value of V_D.

27. Define a voltage v_x, "+" reference on the right, across the dependent current source. Note that in fact v_x appears across each of the four elements. We first convert the 10 mS conductance into a 100–Ω resistor, and the 40–mS conductance into a 25–Ω resistor.

(*a*) Applying KCL, we sum the currents flowing into the right–hand node:

$$5 - v_x / 100 - v_x / 25 + 0.8\, i_x = 0 \quad [1]$$

This represents one equation in two unknowns. A second equation to introduce at this point is

$i_x = v_x /25$ so that Eq. [1] becomes

$$5 - v_x / 100 - v_x / 25 + 0.8\,(v_x / 25) = 0$$

Solving for v_x, we find v_x = 277.8 V. It is a simple matter now to compute the power absorbed by each element:

P_{5A}	$= -5\, v_x$	= –1.389 kW
$P_{100\Omega}$	$= (v_x)^2 / 100$	= 771.7 W
$P_{25\Omega}$	$= (v_x)^2 / 25$	= 3.087 kW
P_{dep}	$= -v_x(0.8\, i_x) = -0.8\,(v_x)^2 / 25$	= –2.470 kW

A quick check assures us that the calculated values sum to zero, as they should.

(*b*) Again summing the currents into the right–hand node,

$$5 - v_x / 100 - v_x / 25 + 0.8\, i_y = 0 \quad [2]$$

where $i_y = 5 - v_x/100$

Thus, Eq. [2] becomes

$$5 - v_x / 100 - v_x / 25 + 0.8(5) - 0.8\,(i_y) / 100 = 0$$

Solving, we find that v_x x = 155.2 V and i_y = 3.448 A

So that

P_{5A}	$= -5\, v_x$	= –776.0 W
$P_{100\Omega}$	$= (v_x)^2 / 100$	= 240.9 W
$P_{25\Omega}$	$= (v_x)^2 / 25$	= 963.5 W
P_{dep}	$= -v_x(0.8\, i_y)$	= –428.1 W

A quick check assures us that the calculated values sum to 0.3, which is reasonably close to zero (small roundoff errors accumulate here).

28. Define a voltage v with the "+" reference at the top node. Applying KCL and summing the currents flowing out of the top node,

$$v/5{,}000 \; + \; 4\times10^{-3} \; + \; 3i_1 \; + \; v/20{,}000 \; = \; 0 \quad [1]$$

This, unfortunately, is one equation in two unknowns, necessitating the search for a second suitable equation. Returning to the circuit diagram, we observe that

$$i_1 \quad = \quad 3\,i_1 \; + \; v/2{,}000$$

or

$$i_1 \quad = \quad -v/40{,}000 \qquad [2]$$

Upon substituting Eq. [2] into Eq. [1], Eq. [1] becomes,

$$v/5{,}000 \; + \; 4\times10^{-3} \; - \; 3v/40{,}000 \; + \; v/20{,}000 \; = \; 0$$

Solving, we find that

$$v = -22.86 \text{ V}$$

and

$$i_1 \; = \; 571.4 \; \mu\text{A}$$

Since $i_x = i_1$, we find that $i_x = \boxed{571.4 \; \mu\text{A}}$.

29. Define a voltage v_x with its "+" reference at the center node. Applying KCL and summing the currents into the center node,

$$8 \; - v_x /6 \; + \; 7 \; - v_x /12 \; - v_x /4 \; = \; 0$$

Solving, $v_x = 30$ V.

It is now a straightforward matter to compute the power absorbed by each element:

P_{8A}	$= -8\, v_x$	$= -240$ W
$P_{6\Omega}$	$= (v_x)^2 / 6$	$= 150$ W
P_{8A}	$= -7\, v_x$	$= -210$ W
$P_{12\Omega}$	$= (v_x)^2 / 12$	$= 75$ W
$P_{4\Omega}$	$= (v_x)^2 / 4$	$= 225$ W

and a quick check verifies that the computed quantities sum to zero, as expected.

30. (*a*) Define a voltage v across the 1–kΩ resistor with the "+" reference at the top node. Applying KCL at this top node, we find that

$$80\times10^{-3} - 30\times10^{-3} = v/1000 + v/4000$$

Solving,

$$v = (50\times10^{-3})(4\times10^{6} / 5\times10^{3}) = 40 \text{ V}$$

and $$P_{4k\Omega} = v^2/4000 = \boxed{400 \text{ mW}}$$

(*b*) Once again, we first define a voltage v across the 1–kΩ resistor with the "+" reference at the top node. Applying KCL at this top node, we find that

$$80\times10^{-3} - 30\times10^{-3} - 20\times10^{-3} = v/1000$$

Solving,

$$v = 30 \text{ V}$$

and $$P_{20mA} = v \cdot 20\times10^{-3} = \boxed{600 \text{ mW}}$$

(*c*) Once again, we first define a voltage v across the 1–kΩ resistor with the "+" reference at the top node. Applying KCL at this top node, we find that

$$80\times10^{-3} - 30\times10^{-3} - 2i_x = v/1000$$

where

$$i_x = v/1000$$

so that

$$80\times10^{-3} - 30\times10^{-3} = 2v/1000 + v/1000$$

and

$$v = 50\times10^{-3}(1000)/3 = 16.67 \text{ V}$$

Thus, $$P_{dep} = v \cdot 2i_x = \boxed{555.8 \text{ mW}}$$

(*d*) We note that i_x = 60/1000 = 60 mA. KCL stipulates that (viewing currents into and out of the top node)

$$80 - 30 + i_s = i_x = 60$$

Thus, i_s = 10 mA

and P_{60V} = 60(–10) mW = $\boxed{-600 \text{ mW}}$

31. (*a*) To cancel out the effects of both the 80-mA and 30-mA sources, i_S must be set to

$\boxed{i_S = -50 \text{ mA.}}$

(*b*) Define a current is flowing out of the "+" reference terminal of the independent voltage source. Interpret "no power" to mean "zero power."

Summing the currents flowing into the top node and invoking KCL, we find that

$$80\times10^{-3} - 30\times10^{-3} - v_S/1\times10^{3} + i_S = 0$$

Simplifying slightly, this becomes

$$50 - v_S + 10^3 i_S = 0 \qquad [1]$$

We are seeking a value for v_S such that $v_S \cdot i_S = 0$. Clearly, setting $v_S = 0$ will achieve this. From Eq. [1], we also see that setting $v_S = 50$ V will work as well.

32. Define a voltage v_9 across the 9-Ω resistor, with the "+" reference at the top node.

(*a*) Summing the currents into the right-hand node and applying KCL,

$$5 + 7 = v_9 / 3 + v_9 / 9$$

Solving, we find that $v_9 = 27$ V. Since $i_x = v_9 / 9$, $\boxed{i_x = 3 \text{ A.}}$

(*b*) Again, we apply KCL, this time to the top left node:

$$2 - v_8 / 8 + 2i_x - 5 = 0$$

Since we know from part (*a*) that $i_x = 3$ A, we may calculate $v_8 = 24$ V.

(*c*) $p_{5A} = (v_9 - v_8) \cdot 5 \boxed{= 15 \text{ W}}$.

33. Define a voltage v_x across the 5-A source, with the "+" reference on top.

Applying KCL at the top node then yields

$$5 + 5v_1 - v_x/(1 + 2) - v_x/5 = 0 \qquad [1]$$

where $v_1 = 2[v_x/(1 + 2)] = 2\,v_x/3$.

Thus, Eq. [1] becomes

$$5 + 5(2\,v_x/3) - v_x/3 - v_x/5 = 0$$

or $75 + 50\,v_x - 5\,v_x - 3\,v_x = 0$, which, upon solving, yields $v_x = -1.786$ V.

The power absorbed by the 5-Ω resistor is then simply $(v_x)^2/5 = \boxed{638.0 \text{ mW.}}$

34. Despite the way it may appear at first glance, this is actually a simple node-pair circuit. Define a voltage v across the elements, with the "+" reference at the top node.

Summing the currents leaving the top node and applying KCL, we find that

$$2 + 6 + 3 + v/5 + v/5 + v/5 = 0$$

or $v = -55/3 = -18.33$ V. The power supplied by each source is then computed as:

$$\boxed{\begin{aligned} p_{2A} &= -v(2) = 36.67 \text{ W} \\ p_{6A} &= -v(6) = 110 \text{ W} \\ p_{3A} &= -v(3) = 55 \text{ W} \end{aligned}}$$

We can check our results by first determining the power absorbed by each resistor, which is simply $v^2/5 = 67.22$ W for a total of 201.67 W, which is the total power supplied by all sources.

35. Defining a voltage V_x across the 10-A source with the "+" reference at the top node, KCL tells us that $10 = 5 + I_{1\Omega}$, where $I_{1\Omega}$ is defined flowing downward through the 1-Ω resistor.

Solving, we find that $I_{1\Omega} = 5$ A, so that $V_x = (1)(5) = 5$ V.

So, we need to solve

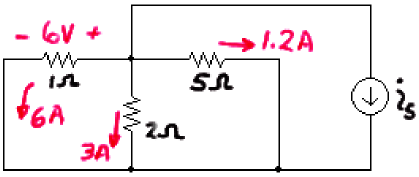
$$V_x = 5 = 5(0.5 + R_{segment})$$

with $R_{segment} = 500$ mΩ.

From Table 2.3, we see that 28-AWG solid copper wire has a resistance of 65.3 mΩ/ft. Thus, the total number of miles needed of the wire is

$$\frac{500 \text{ m}\Omega}{(65.3 \text{ m}\Omega/\text{ft})(5280 \text{ ft/mi})} = \boxed{1.450\times 10^{-3} \text{ miles}}$$

36. Since v = 6 V, we know the current through the 1-Ω resistor is 6 A, the current through the 2-Ω resistor is 3 A, and the current through the 5-Ω resistor is 6/5 = 1.2 A, as shown below:



By KCL, $6 + 3 + 1.2 + i_S = 0$ or $\boxed{i_S = -10.2 \text{ A.}}$

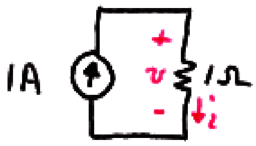
37. (*a*) Applying KCL, $1 - i - 3 + 3 = 0$ so $\boxed{i = 1 \text{ A.}}$

(*b*) The rightmost source should be labeled 3.5 A to satisfy KCL.

Then, looking at the left part of the circuit, we see 1 + 3 = 4 A flowing into the unknown current source, which, by virtue of KCL, must therefore be a 4-A current source. Thus, KCL at the node labeled with the "+" reference of the voltage *v* gives

$$4 - 2 + 7 - i = 0 \qquad \text{or} \qquad \boxed{i = 9 \text{ A}}$$

38. (*a*) We may redraw the circuit as

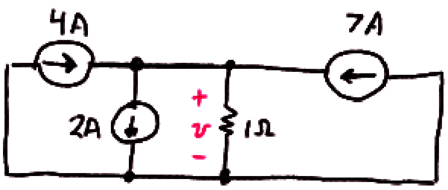


Then, we see that v = (1)(1) = $\boxed{1\text{ V.}}$

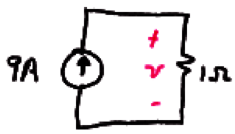
(*b*) The current source at the far right should be labeled 3.5 A, or KCL is violated. In that case, we may combine all sources to the right of the 1-Ω resistor into a single 7-A current source. On the left, the two 1-A sources in sereies reduce to a single 1-A source.

The new 1-A source and the 3-A source combine to yield a 4-A source in series with the unknown current source which, by KCL, must be a 4-A current source.

At this point we have reduced the circuit to



Further simplification is possible, resulting in

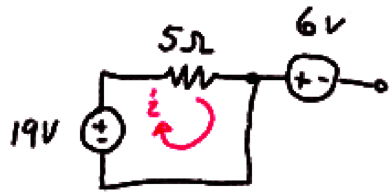


From which we see clearly that v = (9)(1) = $\boxed{9\text{ V.}}$

39. (*a*) Combine the 12-V and 2-V series connected sources to obtain a new 12 – 2 = 10 V source, with the "+" reference terminal at the top. The result is two 10-V sources in parallel, which is permitted by KVL. Therefore,

$$\boxed{i = 10/1000 = 10 \text{ mA.}}$$

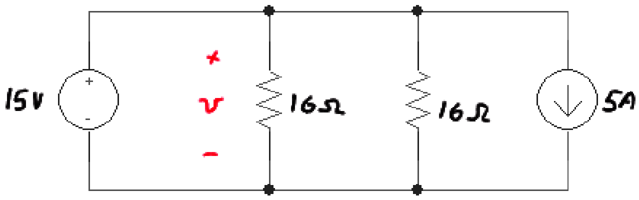
(*b*) No current flows through the 6-V source, so we may neglect it for this calculation. The 12-V, 10-V and 3-V sources are connected in series as a result, so we replace them with a 12 + 10 –3 = 19 V source as shown



Thus, $\boxed{i = 19/5 = 3.8 \text{ A.}}$

40. We first combine the 10-V and 5-V sources into a single 15-V source, with the "+" reference on top. The 2-A and 7-A current sources combine into a 7 – 2 = 5 A current source (arrow pointing down); although these two current sources may not appear to be in parallel at first glance, they actually are.

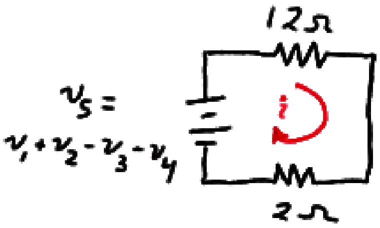
Redrawing our circuit,



we see that v = 15 V (note that we can completely the ignore the 5-A source here, since we have a voltage source directly across the resistor). Thus,

$$p_{16\Omega} = v^2/16 = \boxed{14.06 \text{ W.}}$$

41. We can combine the voltage sources such that



$i = v_S / 14$

(*a*) $v_S = 10 + 10 - 6 - 6 = 20 - 12 = 8$
Therefore

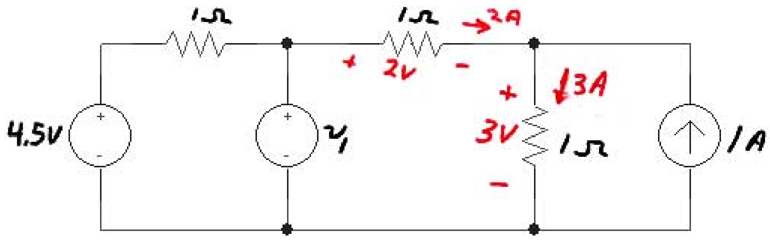
$$i = 8/14 = \boxed{571.4 \text{ mA.}}$$

(*b*) $v_S = 3 + 2.5 - 3 - 2.5 = 0$ Therefore $\boxed{i = 0.}$

(*c*) $v_S = -3 + 1.5 - (-0.5) - 0 = -1$ V
Therefore

$$i = -1/14 = \boxed{-71.43 \text{ mA.}}$$

42. We first simplify as shown, making use of the fact that we are told $i_x = 2$ A to find the voltage across the middle and right-most 1-Ω resistors as labeled.



By KVL, then, we find that $\boxed{v_1 = 2 + 3 = 5 \text{ V.}}$

43. We see that to determine the voltage v we will need v_x due to the presence of the dependent current soruce. So, let's begin with the right-hand side, where we find that

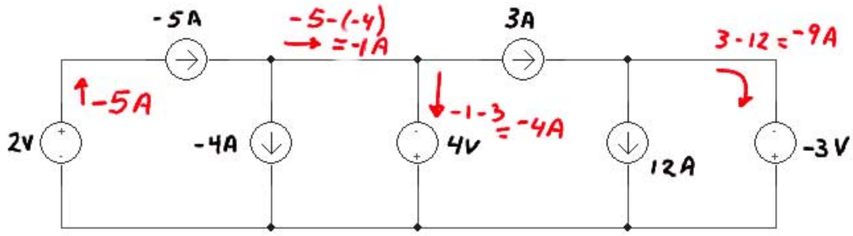
$$v_x = 1000(1 - 3) \times 10^{-3} = -2 \text{ V}.$$

Returning to the left-hand side of the circuit, and summing currents into the top node, we find that

$$(12 - 3.5) \times 10^{-3} + 0.03\, v_x = v/10\times10^3$$

or $\boxed{v = -515 \text{ V}.}$

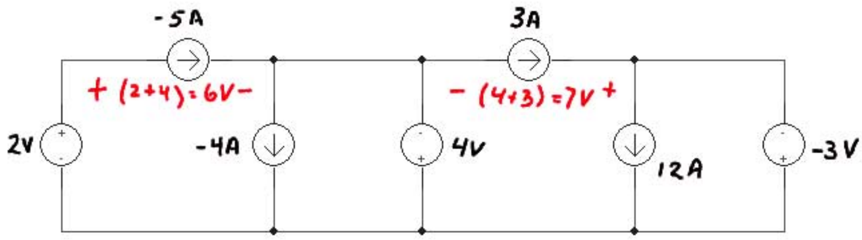
44. (*a*) We first label the circuit with a focus on determining the current flowing through each voltage source:



Then the power absorbed by each voltage source is

P_{2V}	$= -2(-5)$	= 10 W
P_{4V}	$= -(-4)(4)$	= 16 W
P_{-3V}	$= -(-9)(-3)$	= 27 W

For the current sources,



So that the absorbed power is

P_{-5A}	$= +(-5)(6)$	= -30 W
P_{-4A}	$= -(-4)(4)$	= 16 W
P_{3A}	$= -(3)(7)$	= -21 W
P_{12A}	$= -(12)(-3)$	= 36 W

A quick check assures us that these absorbed powers sum to zero as they should.

(*b*) We need to change the 4-V source such that the voltage across the –5-A source drops to zero. Define V_x across the –5-A source such that the "+" reference terminal is on the left. Then,

$$-2 + V_x - V_{needed} = 0$$

or $\boxed{V_{needed} = -2 \text{ V.}}$

45. We begin by noting several things:
(1) The bottom resistor has been shorted out;
(2) the far-right resistor is only connected by one terminal and therefore does not affect the equivalent resistance *as seen from the indicated terminals*;
(3) All resistors to the right of the top left resistor have been shorted.

Thus, from the indicated terminals, we only see the single 1-kΩ resistor, so that $\boxed{R_{eq} = 1\ \text{k}\Omega.}$

46. (*a*) We see $1\Omega \parallel (1\ \Omega + 1\ \Omega) \parallel (1\ \Omega + 1\ \Omega + 1\ \Omega)$

$= 1\Omega \parallel 2\ \Omega \parallel 3\ \Omega$

$\boxed{= 545.5\ \text{m}\Omega}$

(*b*) $1/R_{eq} = 1 + 1/2 + 1/3 + \ldots 1/N$

Thus, $\boxed{R_{eq} = [1 + 1/2 + 1/3 + \ldots 1/N]^{-1}}$

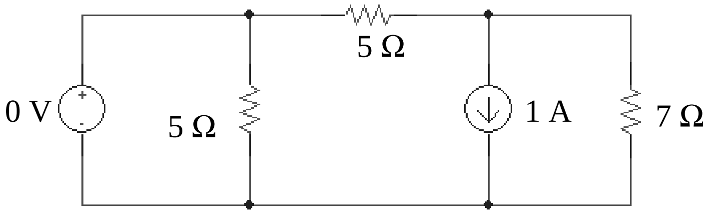
47. (*a*) 5 kΩ = 10 kΩ ‖ 10 kΩ

(*b*) 57 333 Ω = 47 kΩ + 10 kΩ + 1 kΩ ‖ 1kΩ ‖ 1kΩ

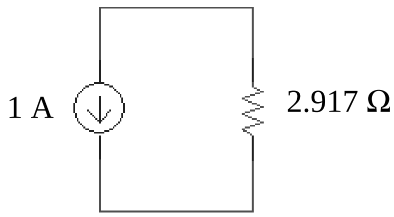
(*c*) 29.5 kΩ = 47 kΩ ‖ 47 kΩ + 10 kΩ ‖ 10 kΩ + 1 kΩ

48. (*a*) no simplification is possible using only source and/or resistor combination techniques.

(*b*) We first simplify the circuit to



and then notice that the 0-V source is shorting out one of the 5-Ω resistors, so a further simplification is possible, noting that 5 Ω || 7 Ω = 2.917 Ω:

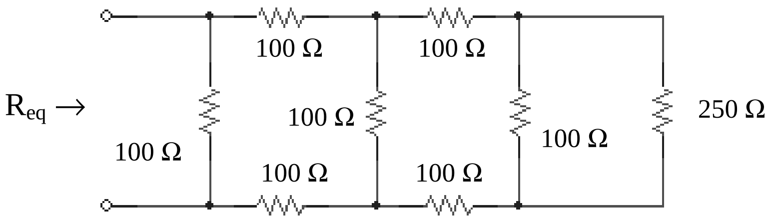


49. R_{eq} $= 1\ \text{k}\Omega + 2\ \text{k}\Omega \parallel 2\ \text{k}\Omega + 3\ \text{k}\Omega \parallel 3\ \text{k}\Omega + 4\ \text{k}\Omega \parallel 4\ \text{k}\Omega$

$= 1\ \text{k}\Omega + 1\ \text{k}\ \Omega + 1.5\ \text{k}\Omega + 2\ \text{k}\Omega$

$\boxed{= 5.5\ \text{k}\Omega.}$

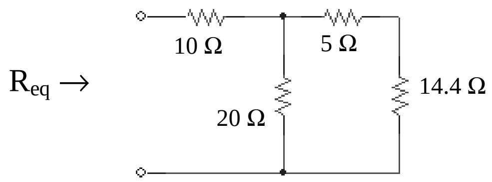
50. (a) Working from right to left, we first see that we may combine several resistors as 100 Ω + 100 Ω || 100 Ω + 100 Ω = 250 Ω, yielding the following circuit:



Next, we see 100 Ω + 100 Ω || 250 Ω + 100 Ω = 271.4 Ω,
and subsequently 100 Ω + 100 Ω || 271.4 Ω + 100 Ω = 273.1 Ω,
and, finally,

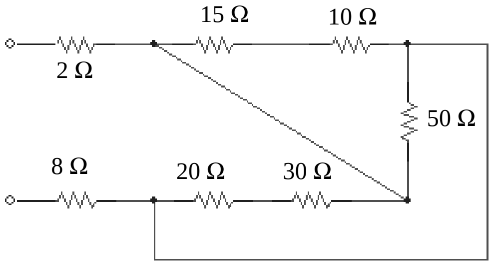
$$R_{eq} = 100\ \Omega \parallel 273.1\ \Omega = \boxed{73.20\ \Omega.}$$

(b) First, we combine 24 Ω || (50 Ω + 40 Ω) || 60 Ω = 14.4 Ω, which leaves us with

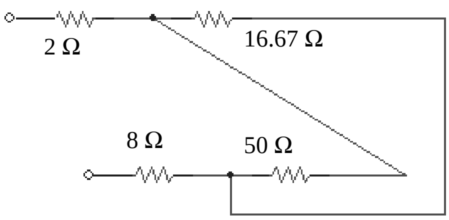


Thus, R_{eq} = 10 Ω + 20 Ω || (5 + 14.4 Ω) = $\boxed{19.85\ \Omega.}$

(c) First combine the 10-Ω and 40-Ω resistors and redraw the circuit:



We now see we have (10 Ω + 15 Ω) || 50 Ω = 16.67 Ω. Redrawing once again,



where the equivalent resistance is seen to be 2 Ω + 50 Ω || 16.67 Ω + 8 Ω = $\boxed{22.5\ \Omega.}$

51. (*a*) $R_{eq} = [(40\ \Omega + 20\ \Omega) \parallel 30\ \Omega + 80\ \Omega] \parallel 100\ \Omega + 10\ \Omega = \boxed{60\ \Omega.}$

(*b*) $R_{eq} = 80\ \Omega = [(40\ \Omega + 20\ \Omega) \parallel 30\ \Omega + R] \parallel 100\ \Omega + 10\ \Omega$

$$70\ \Omega = [(60\ \Omega \parallel 30\ \Omega) + R] \parallel 100\ \Omega$$

$$1/70 = 1/(20 + R) + 0.01$$

$$20 + R = 233.3\ \Omega \qquad \text{therefore} \qquad \boxed{R = 213.3\ \Omega.}$$

(*c*) $R = [(40\ \Omega + 20\ \Omega) \parallel 30\ \Omega + R] \parallel 100\ \Omega + 10\ \Omega$

$$R - 10\ \Omega = [20 + R] \parallel 100$$

$$1/(R - 10) = 1/(R + 20) + 1/100$$

$$3000 = R^2 + 10R - 200$$

Solving, we find $R = -61.79\ \Omega$ or $R = 51.79\ \Omega$.
Clearly, the first is not a physical solution, so $\boxed{R = 51.79\ \Omega.}$

52. (*a*) $25\ \Omega = \boxed{100\ \Omega \parallel 100\ \Omega \parallel 100\ \Omega}$

(*b*) $60\ \Omega = \boxed{[(100\ \Omega \parallel 100\ \Omega) + 100\ \Omega] \parallel 100\ \Omega}$

(*c*) $40\ \Omega = \boxed{(100\ \Omega + 100\ \Omega) \parallel 100\ \Omega \parallel 100\ \Omega}$

53. $R_{eq} = [(5\ \Omega \parallel 20\ \Omega) + 6\ \Omega] \parallel 30\ \Omega + 2.5\ \Omega = 10\ \Omega$
The source therefore provides a total of 1000 W and a current of 100/10 = 10 A.

$P_{2.5\Omega} = (10)^2 \cdot 2.5 = \boxed{250\text{ W}}$

$V_{30\Omega} = 100 - 2.5(10) = 75\text{ V}$
$P_{30\Omega} = 75^2/30 = \boxed{187.5\text{ W}}$

$I_{6\Omega} = 10 - V_{30\Omega}/30 = 10 - 75/30 = 7.5\text{ A}$
$P_{6\Omega} = (7.5)^2 \cdot 6 = \boxed{337.5\text{ W}}$

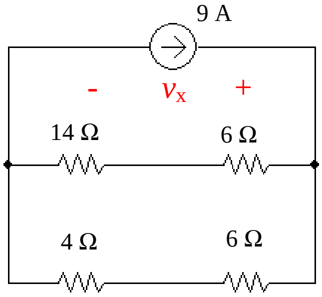
$V_{5\Omega} = 75 - 6(7.5) = 30\text{ V}$
$P_{5\Omega} = 30^2/5 = \boxed{180\text{ W}}$

$V_{20\Omega} = V_{5\Omega} = 30\text{ V}$
$P_{20\Omega} = 30^2/20 = \boxed{45\text{ W}}$

We check our results by verifying that the absorbed powers in fact add to 1000 W.

54. To begin with, the 10-Ω and 15-Ω resistors are in parallel (= 6 Ω), and so are the 20-Ω and 5-Ω resistors (= 4 Ω).

Also, the 4-A, 1-A and 6-A current sources are in parallel, so they can be combined into a single 4 + 6 – 1 = 9 A current source as shown:



Next, we note that (14 Ω + 6 Ω) || (4 Ω + 6 Ω) = 6.667 Ω

so that

$$v_x = 9(6.667) = 60 \text{ V}$$

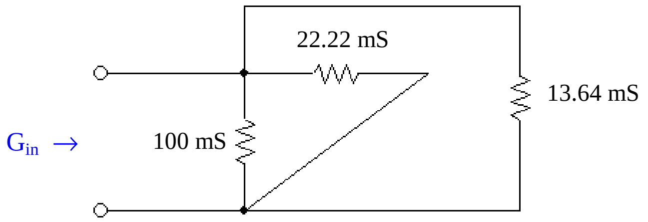
and

$$i_x = -60/10 = -6 \text{ A.}$$

55. (*a*) Working from right to left, and borrowing $x \parallel y$ notation from resistance calculations to indicate the operation $xy/(x + y)$,

$$G_{in} = \{[(6 \parallel 2 \parallel 3) + 0.5] \parallel 1.5 \parallel 2.5 + 0.8\} \parallel 4 \parallel 5 \text{ mS}$$
$$= \{[(1) + 0.5] \parallel 1.5 \parallel 2.5 + 0.8\} \parallel 4 \parallel 5 \text{ mS}$$
$$= \{1.377\} \parallel 4 \parallel 5$$
$$= 0.8502 \text{ mS} \quad = \boxed{850.2 \text{ mS}}$$

(*b*) The 50-mS and 40-mS conductances are in series, equivalent to (50(40)/90 = 22.22 mS. The 30-mS and 25-mS conductances are also in series, equivalent to 13.64 mS. Redrawing for clarity,



we see that $G_{in} = 10 + 22.22 + 13.64 = \boxed{135.9 \text{ mS.}}$

56. The bottom four resistors between the 2-Ω resistor and the 30-V source are shorted out. The 10-Ω and 40-Ω resistors are in parallel (= 8 Ω), as are the 15-Ω and 60-Ω (=12 Ω) resistors. These combinations are in series.

Define a clockwise current I through the 1-Ω resistor:

$$I = (150 - 30)/(2 + 8 + 12 + 3 + 1 + 2) = 4.286 \text{ A}$$

$$P_{1\Omega} = I^2 \cdot 1 = \boxed{18.37 \text{ W}}$$

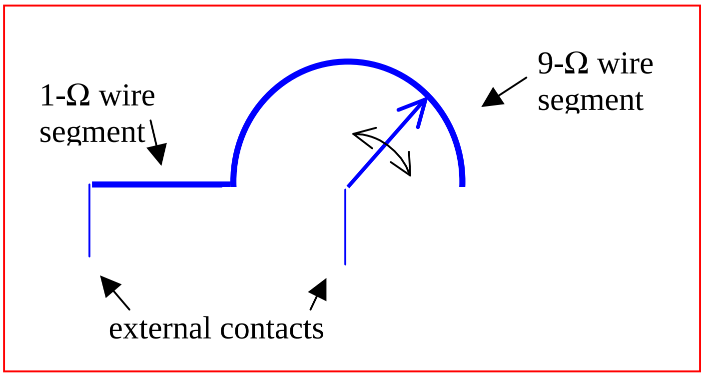
To compute $P_{10\Omega}$, consider that since the 10-Ω and 40-Ω resistors are in parallel, the same voltage V_x ("+" reference on the left) appears across both resistors. The current I = 4.286 A flows into this combination. Thus, $V_x = (8)(4.286) = 34.29$ V and

$$P_{10\Omega} = (V_x)^2 / 10 = \boxed{117.6 \text{ W.}}$$

$\boxed{P_{13\Omega} = 0}$ since no current flows through that resistor.

57. One possible solution of many:

The basic concept is as shown



If we use 28-AWG soft copper wire, we see from Table 2.3 that 9-Ω would require 138 feet, which is somewhat impractical. Referring to p. 4-48 of the *Standard Handbook for Electrical Engineers* (this should be available in most engineering/science libraries), we see that 44-AWG soft copper wire has a resistance of 2590 Ω per 1000 ft, or 0.08497 Ω/cm.

Thus, 1-Ω requires 11.8 cm of 44-AWG wire, and 9-Ω requires 105.9 cm. We decide to make the wiper arm and leads out of 28-AWG wire, which will add a slight resistance to the total value, but a negligible amount.

The radius of the wiper arm should be (105.9 cm)/π = 33.7 cm.

58. One possible solution of many:

$v_S = 2(5.5) = 11$ V
$R_1 = R_2 = 1$ kΩ.

59. One possible solution of many:

$i_S = 11$ mA
$R_1 = R_2 = 1$ kΩ.

60. $p_{15\Omega} = (v_{15})^2 / 15\times10^3$ A

$v_{15} = 15\times10^3 (-0.3\ v_1)$

where $v_1 = [4 (5)/ (5 + 2)] \cdot\ 2 = 5.714$ V

Therefore $v_{15} = -25714$ V and $\boxed{p_{15} = 44.08 \text{ kW.}}$

61. Replace the top 10 kΩ, 4 kΩ and 47 kΩ resistors with 10 kΩ + 4 kΩ || 47 kΩ = 13.69 kΩ.

Define v_x across the 10 kΩ resistor with its "+" reference at the top node: then

v_x = 5 · (10 kΩ || 13.69 kΩ) / (15 kΩ + 10 || 13.69 kΩ) = 1.391 V

$i_x = v_x / 10$ mA = $\boxed{139.1\ \mu\text{A}}$

v_{15} = 5 – 1.391 = 3.609 V and $p_{15} = (v_{15})^2 / 15\times10^3$ = $\boxed{868.3\ \mu\text{W}.}$

62. We may combine the 12-A and 5-A current sources into a single 7-A current source with its arrow oriented upwards. The left three resistors may be replaced by a 3 + 6 || 13 = 7.105 Ω resistor, and the right three resistors may be replaced by a 7 + 20 || 4 = 10.33 Ω resistor.

By current division, $i_y = 7\ (7.105)/(7.105 + 10.33) = \boxed{2.853\text{ A}}$

We must now return to the original circuit. The current into the 6 Ω, 13 Ω parallel combination is $7 - i_y = 4.147$ A. By current division,

$$i_x = 4.147\ .\ 13/\ (13 + 6) = \boxed{2.837\text{ A}}$$

$$\text{and } p_x = (4.147)^2 \cdot 3 = \boxed{51.59\text{ W}}$$

63. The controlling voltage v_1, needed to obtain the power into the 47-kΩ resistor, can be found separately as that network does not depend on the left-hand network.
The right-most 2 kΩ resistor can be neglected.

By current division, then, in combination with Ohm's law,

$$v_1 = 3000[5\times10^{-3}\,(2000)/\,(2000 + 3000 + 7000)] = 2.5\text{ V}$$

Voltage division gives the voltage across the 47-kΩ resistor:

$$0.5v_1 \frac{47}{47 + 100 \parallel 20} = \frac{0.5(2.5)(47)}{47 + 16.67} = 0.9228\text{ V}$$

So that $p_{47\text{k}\Omega} = (0.9928)^2 / 47\times10^3 = \boxed{18.12\ \mu\text{W}}$

64. The temptation to write an equation such as

$$v_1 = 10\frac{20}{20+20}$$

must be fought!

Voltage division only applies to resistors connected in series, meaning that the *same* current must flow through *each* resistor. In this circuit, $i_1 \neq 0$, so we do not have the same current flowing through both 20 kΩ resistors.

65. (*a*) $$v_2 = V_S \frac{R_2 \parallel (R_3 + R_4)}{R_1 + [R_2 \parallel (R_3 + R_4)]}$$

$$= V_S \frac{R_2 (R_3 + R_4)/(R_2 + R_3 + R_4)}{R_1 + R_2 (R_3 + R_4)/(R_2 + R_3 + R_4)}$$

$$\boxed{= V_S \frac{R_2 (R_3 + R_4)}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}}$$

(*b*) $$v_1 = V_S \frac{R_1}{R_1 + [R_2 \parallel (R_3 + R_4)]}$$

$$= V_S \frac{R_1}{R_1 + R_2 (R_3 + R_4)/(R_2 + R_3 + R_4)}$$

$$\boxed{= V_S \frac{R_1 (R_2 + R_3 + R_4)}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}}$$

(*c*) $$i_4 = \left(\frac{v_1}{R_1}\right)\left(\frac{R_2}{R_2 + R_3 + R_4}\right)$$

$$= V_S \frac{R_1 (R_2 + R_3 + R_4) R_2}{R_1[R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)(R_2 + R_3 + R_4)]}$$

$$\boxed{= V_S \frac{R_2}{R_1(R_2 + R_3 + R_4) + R_2(R_3 + R_4)}}$$

66. (*a*) With the current source open-circuited, we find that

$$v_1 = -40\frac{500}{500 + 3000 \| 6000} = \boxed{-8\text{ V}}$$

(*b*) With the voltage source short-circuited, we find that

$$i_2 = \left(3\times10^{-3}\right)\frac{1/3000}{1/500+1/3000+1/6000} = \boxed{400\text{ mA}}$$

$$i_3 = \left(3\times10^{-3}\right)\frac{500}{500+3000 \| 6000} = \boxed{600\text{ mA}}$$

67. (*a*) The current through the 5-Ω resistor is 10/5 = 2 A. Define R as 3 || (4 + 5) = 2.25 Ω. The current through the 2-Ω resistor then is given by

$$I_S \frac{1}{1+(2+R)} = \frac{I_S}{5.25}$$

The current through the 5-Ω resistor is

$$\frac{I_S}{5.25}\left(\frac{3}{3+9}\right) = 2\text{ A}$$

so that $\boxed{I_S = 42\text{ A.}}$

(*b*) Given that I_S is now 50 A, the current through the 5-Ω resistor becomes

$$\frac{I_S}{5.25}\left(\frac{3}{3+9}\right) = 2.381\text{ A}$$

Thus, $v_x = 5(2.381) = \boxed{11.90\text{ V}}$

(*c*) $$\frac{v_x}{I_S} = \frac{\left[\frac{5I_S}{5.25}\left(\frac{3}{3+9}\right)\right]}{I_S} = \boxed{0.2381}$$

68. First combine the 1 kΩ and 3 kΩ resistors to obtain 750 Ω.
By current division, the current through resistor R_x is

$$I_{R_x} = 10\times10^{-3}\frac{2000}{2000+R_x+750}$$

and we know that $R_x \cdot I_{R_x} = 9$

so $$9 = \frac{20\,R_x}{2750+R_x}$$

$9\,R_x + 24750 = 20\,R_x$ or $R_x = 2250$ W. Thus,

$$\boxed{P_{R_x} = 9^2/\,R_x = 36 \text{ mW.}}$$

69. Define $R = R_3 \parallel (R_4 + R_5)$

Then $v_R = V_S\left(\frac{R}{R+R_2}\right)$

$$= V_S\left(\frac{R_3\,(R_4+R_5)/(R_3+R_4+R_5)}{R_3\,(R_4+R_5)/(R_3+R_4+R_5)+R_2}\right)$$

$$= V_S\left(\frac{R_3\,(R_4+R_5)}{R_2\,(R_3+R_4+R_5)+R_3(R_4+R_5)}\right)$$

Thus,

$$v_5 = v_R\left(\frac{R_5}{R_4+R_5}\right)$$

$$\boxed{= V_S\left(\frac{R_3\,R_5}{R_2\,(R_3+R_4+R_5)+R_3(R_4+R_5)}\right)}$$

70. Define $R_1 = 10 + 15 \| 30 = 20\ \Omega$ and $R_2 = 5 + 25 = 30\ \Omega$.

(*a*) $I_x = I_1 \,.\, 15 / (15 + 30) = \boxed{4 \text{ mA}}$

(*b*) $I_1 = I_x \,.\, 45/15 = \boxed{36 \text{ mA}}$

(*c*) $I_2 = I_S R_1 / (R_1 + R_2)$ and $I_1 = I_S R_2 / (R_1 + R_2)$
So $I_1/I_2 = R_2/R_1$
Therefore

$$I_1 = R_2 I_2/R_1 = 30(15)/20 = 22.5 \text{ mA}$$

Thus, $I_x = I_1 \,.\, 15/45 = \boxed{7.5 \text{ mA}}$

(*d*) $I_1 = I_S R_2/(R_1 + R_2) = 60(30)/50 = 36 \text{ A}$

Thus, $I_x = I_1\, 15/45 = \boxed{12 \text{ A.}}$

71. $v_{out} = -g_m v_\pi (100 \text{ k}\Omega \parallel 100 \text{ k}\Omega) = -4.762\times10^3 g_m v_\pi$

where $v_\pi = (3 \sin 10t) \cdot \; 15/(15 + 0.3) = 2.941 \sin 10t$

Thus, $\boxed{v_{out} = -56.02 \sin 10t \text{ V}}$

72. $v_{out} = -1000 g_m v_\pi$

where $v_\pi = 3\sin 10t \dfrac{15\,\|\,3}{(15\,\|\,3) + 0.3} = 2.679\sin 10t$ V

therefore

$$v_{out} = -(2.679)(1000)(38\times10^{-3})\sin 10t = \boxed{-101.8\sin 10t \text{ V.}}$$

CHAPTER FOUR SOLUTIONS

1. (*a*)

$$\begin{bmatrix} 0.1 & -0.3 & -0.4 \\ -0.5 & 0.1 & 0 \\ -0.2 & -0.3 & 0.4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator, $\boxed{v_2 = -8.387 \text{ V}}$

(*b*) Using a scientific calculator, the determinant is equal to $\boxed{32.}$

2. (*a*) $$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 27 \\ -16 \\ -6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator,

$$\boxed{\begin{aligned} v_A &= 19.57 \\ v_B &= 18.71 \\ v_C &= -11.29 \end{aligned}}$$

(*b*) Using a scientific calculator,

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 0 & 4 \end{vmatrix} = \boxed{16}$$

3. The bottom node has the largest number of branch connections, so we choose that as our reference node. This also makes v_P easier to find, as it will be a nodal voltage. Working from left to right, we name our nodes 1, P, 2, and 3.

NODE 1: $10 = v_1/20 + (v_1 - v_P)/40$ [1]

NODE P: $0 = (v_P - v_1)/40 + v_P/100 + (v_P - v_2)/50$ [2]

NODE 2: $-2.5 + 2 = (v_2 - v_P)/50 + (v_2 - v_3)/10$ [3]

NODE 3: $5 - 2 = v_3/200 + (v_3 - v_2)/10$ [4]

Simplifying,

$$\begin{aligned} 60v_1 - 20v_P &= 8000 && [1] \\ -50v_1 + 110v_P - 40v_2 &= 0 && [2] \\ -v_P + 6v_2 - 5v_3 &= -25 && [3] \\ -200v_2 + 210v_3 &= 6000 && [4] \end{aligned}$$

Solving,

$$\boxed{v_P = 171.6 \text{ V}}$$

4. The logical choice for a reference node is the bottom node, as then v_x will automatically become a nodal voltage.

NODE 1: $4 = v_1/100 + (v_1 - v_2)/20 + (v_1 - v_x)/50$ [1]

NODE x: $10 - 4 - (-2) = (v_x - v_1)/50 + (v_x - v_2)/40$ [2]

NODE 2: $-2 = v_2/25 + (v_2 - v_x)/40 + (v_2 - v_1)/20$ [3]

Simplifying,

$$4 = 0.0800v_1 - 0.0500v_2 - 0.0200v_x \quad [1]$$
$$8 = -0.0200v_1 - 0.02500v_2 + 0.04500v_x \quad [2]$$
$$-2 = -0.0500v_1 + 0.1150v_2 - 0.02500v_x \quad [3]$$

Solving,

$$\boxed{v_x = 397.4 \text{ V.}}$$

5. Designate the node between the 3-Ω and 6-Ω resistors as node X, and the right-hand node of the 6-Ω resistor as node Y. The bottom node is chosen as the reference node.

(*a*) Writing the two nodal equations, then

NODE X: $-10 = (v_X - 240)/3 + (v_X - v_Y)/6$ [1]

NODE Y: $0 = (v_Y - v_X)/6 + v_Y/30 + (v_Y - 60)/12$ [2]

Simplifying, $-180 + 1440 = 9\,v_X - 3\,v_Y$ [1]

$10800 = -360\,v_X + 612\,v_Y$ [2]

Solving, $v_X = 181.5$ V and $v_Y = 124.4$ V

Thus, $v_1 = 240 - v_X = \boxed{58.50 \text{ V}}$ and $v_2 = v_Y - 60 = \boxed{64.40 \text{ V}}$

(*b*) The power absorbed by the 6-Ω resistor is

$$(v_X - v_Y)^2 / 6 = \boxed{543.4 \text{ W}}$$

6. Only one nodal equation is required: At the node where three resistors join,

$$0.02v_1 = (v_x - 5\,i_2) / 45 + (v_x - 100) / 30 + (v_x - 0.2\,v_3) / 50 \qquad [1]$$

This, however, is one equation in four unknowns, the other three resulting from the presence of the dependent sources. Thus, we require three additional equations:

$$i_2 = (0.2\,v_3 - v_x) / 50 \qquad [2]$$

$$v_1 = 0.2\,v_3 - 100 \qquad [3]$$

$$v_3 = 50i_2 \qquad [4]$$

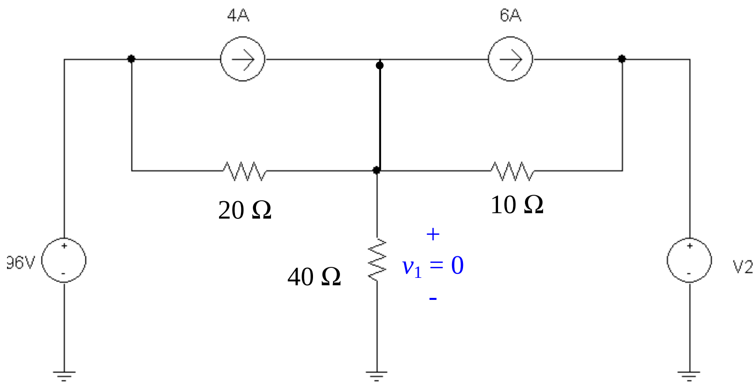
Simplifying,

$$\begin{array}{llllll} & v_1 - 0.2v_3 & & & = -100 & [3] \\ & & -v_3 & +50\,i_2 & = 0 & [4] \\ -v_x & & +0.2v_3 & -50\,i_2 & = 0 & [2] \end{array}$$

$$0.07556v_x - 0.02v_1 - 0.004v_3 - 0.111i_2 = 33.33 \qquad [1]$$

Solving, we find that $\boxed{v_1 = -103..8 \text{ V}}$ and $\boxed{i_2 = -377.4 \text{ mA.}}$

7. If $v_1 = 0$, the dependent source is a short circuit and we may redraw the circuit as:



At NODE 1: $4 - 6 = v_1/40 + (v_1 - 96)/20 + (v_1 - V_2)/10$

Since $v_1 = 0$, this simplifies to

$$-2 = -96/20 - V_2/10$$

so that $\boxed{V_2 = -28 \text{ V.}}$

8. We choose the bottom node as ground to make calculation of i_5 easier. The left-most node is named "1", the top node is named "2", the central node is named "3" and the node between the 4-Ω and 6-Ω resistors is named "4."

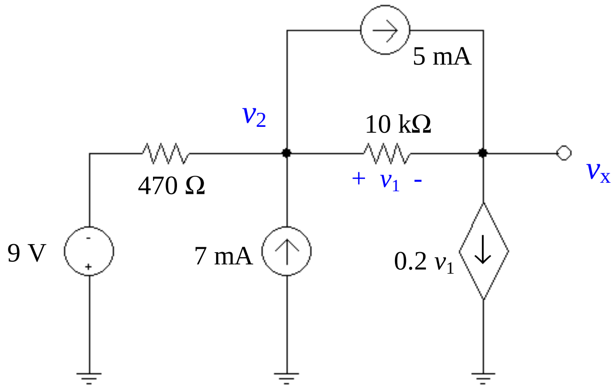
NODE 1:	$-3 = v_1/2 + (v_1 - v_2)/1$	[1]
NODE 2:	$2 = (v_2 - v_1)/1 + (v_2 - v_3)/3 + (v_2 - v_4)/4$	[2]
NODE 3:	$3 = v_3/5 + (v_3 - v_4)/7 + (v_3 - v_2)/3$	[3]
NODE 4:	$0 = v_4/6 + (v_4 - v_3)/7 + (v_4 - v_2)/4$	[4]

Rearranging and grouping terms,

$$
\begin{aligned}
3v_1 - 2v_2 &= -6 && [1]\\
-12v_1 + 19v_2 - 4v_3 - 3v_4 &= 24 && [2]\\
-35v_2 + 71v_3 - 15v_4 &= 315 && [3]\\
-42v_2 - 24v_3 + 94v_4 &= 0 && [4]
\end{aligned}
$$

Solving, we find that $v_3 = 6.760$ V and so $\boxed{i_5 = v_3/5 = 1.352 \text{ A.}}$

9. We can redraw this circuit and eliminate the 2.2-kΩ resistor as no current flows through it:



At NODE 2: $7\times10^{-3} - 5\times10^{-3} = (v_2 + 9)/ 470 + (v_2 - v_x)/ 10\times10^{-3}$ [1]

At NODE x: $5\times10^{-3} - 0.2v_1 = (v_x - v_2)/ 10\times10^{3}$ [2]

The additional equation required by the presence of the dependent source and the fact that its controlling variable is not one of the nodal voltages:

$$v_1 = v_2 - v_x \qquad [3]$$

Eliminating the variable v1 and grouping terms, we obtain:

$$10{,}470\ v_2 - 470\ v_x = -89{,}518$$

and

$$1999\ v_2 - 1999\ v_x = 50$$

Solving, we find $\boxed{v_x = -8.086 \text{ V.}}$

10. We need concern ourselves with the bottom part of this circuit only. Writing a single nodal equation,

$$-4 + 2 = v/50$$

We find that $\boxed{v = -100 \text{ V.}}$

11. We choose the center node for our common terminal, since it connects to the largest number of branches. We name the left node "A", the top node "B", the right node "C", and the bottom node "D". We next form a supernode between nodes A and B.

At the supernode: $5 = (V_A - V_B)/ 10 + V_A/ 20 + (V_B - V_C)/ 12.5$ [1]

At node C: $V_C = 150$ [2]

At node D: $-10 = V_D/ 25 + (V_D - V_A)/ 10$ [3]

Our supernode-related equation is $V_B - V_A = 100$ [4]

Simplifiying and grouping terms,

$$
\begin{array}{lr}
0.15\, V_A + 0.08\, V_B - 0.08\, V_C - 0.1\, V_D = 5 & [1] \\
V_C = 150 & [2] \\
-25\, V_A + 35\, V_D = -2500 & [3] \\
- V_A + V_B = 100 & [4]
\end{array}
$$

Solving, we find that $V_D = -63.06$ V. Since $v_4 = - V_D$,

$$\boxed{v_4 = 63.06 \text{ V.}}$$

12. Choosing the bottom node as the reference terminal and naming the left node "1", the center node "2" and the right node "3", we next form a supernode about nodes 1 and 2, encompassing the dependent voltage source.

At the supernode, $5 - 8 = (v_1 - v_2)/2 + v_3/2.5$ [1]
At node 2, $8 = v_2/5 + (v_2 - v_1)/2$ [2]

Our supernode equation is $v_1 - v_3 = 0.8\, v_A$ [3]
Since $v_A = v_2$, we can rewrite [3] as $v_1 - v_3 = 0.8 v_2$

Simplifying and collecting terms,

$$0.5\, v_1 - 0.5\, v_2 + 0.4\, v_3 = -3 \qquad [1]$$
$$-0.5\, v_1 + 0.7\, v_2 = 8 \qquad [2]$$
$$v_1 - 0.8\, v_2 - v_3 = 0 \qquad [3]$$

(*a*) Solving for $v_2 = v_A$, we find that $\boxed{v_A = 25.91 \text{ V}}$

(*b*) The power absorbed by the 2.5-Ω resistor is
$(v_3)^2/2.5 = (-0.4546)^2/2.5$ $\boxed{= 82.66 \text{ mW.}}$

13. Selecting the bottom node as the reference terminal, we name the left node "1", the middle node "2" and the right node "3."

NODE 1: $5 = (v_1 - v_2)/ 20 + (v_1 - v_3)/ 50$ [1]

NODE 2: $v_2 = 0.4\, v_1$ [2]

NODE 3: $0.01\, v_1 = (v_3 - v_2)/ 30 + (v_3 - v_1)/ 50$ [3]

Simplifying and collecting terms, we obtain

$$
\begin{aligned}
0.07\, v_1 - 0.05\, v_2 - 0.02\, v_3 &= 5 \quad [1]\\
0.4\, v_1 - v_2 &= 0 \quad [2]\\
-0.03\, v_1 - 0.03333\, v_2 + 0.05333\, v_3 &= 0 \quad [3]
\end{aligned}
$$

Since our choice of reference terminal makes the controlling variable of both dependent sources a nodal voltage, we have no need for an additional equation as we might have expected.

Solving, we find that $\boxed{v_1 = 148.2\text{ V},}$ $v_2 = 59.26$ V, and $v_3 = 120.4$ V.

The power supplied by the dependent current source is therefore

$$(0.01\, v_1) \bullet v_3 = \boxed{177.4\text{ W.}}$$

14. At node x: $v_x/ 4 + (v_x - v_y)/ 2 + (v_x - 6)/ 1 = 0$ [1]
At node y: $(v_y - kv_x)/ 3 + (v_y - v_x)/ 2 = 2$ [2]

Our additional constraint is that $v_y = 0$, so we may simplify Eqs. [1] and [2]:

$14\ v_x = 48$ [1]
$-2k\ v_x - 3\ v_x = 12$ [2]

Since Eq. [1] yields $v_x = 48/14 = 3.429$ V, we find that

$$k = (12 + 3\ v_x)/ (-2\ v_x) = \boxed{-3.250}$$

15. Choosing the bottom node joining the 4-Ω resistor, the 2-A current sourcee and the 4-V voltage source as our reference node, we next name the other node of the 4-Ω resistor node "1", and the node joining the 2-Ω resistor and the 2-A current source node "2." Finally, we create a supernode with nodes "1" and "2."

At the supernode:	$-2 = v_1/4 + (v_2 - 4)/2$	[1]
Our remaining equations:	$v_1 - v_2 = -3 - 0.5i_1$	[2]
and	$i_1 = (v_2 - 4)/2$	[3]

Equation [1] simplifies to	$v_1 + 2v_2 = 0$	[1]
Combining Eqs. [2] and [3,	$4v_1 - 3v_2 = -8$	[4]

Solving these last two equations, we find that $v_2 = 727.3$ mV. Making use of Eq. [3], we therefore find that

$$\boxed{i_1 = -1.636 \text{ A.}}$$

16. We first number the nodes as 1, 2, 3, 4, and 5 moving left to right. We next select node 5 as the reference terminal. To simplify the analysis, we form a supernode from nodes 1, 2, and 3.

At the supernode,

$$-4 - 8 + 6 = v_1/\,40 + (v_1 - v_3)/\,10 + (v_3 - v_1)/\,10 + v_2/\,50 + (v_3 - v_4)/\,20 \qquad [1]$$

Note that since both ends of the 10-Ω resistor are connected to the supernode, the related terms cancel each other out, and so could have been ignored.

At node 4: $v_4 = 200$ [2]

Supernode KVL equation: $v_1 - v_3 = 400 + 4v_{20}$ [3]

Where the controlling voltage $v_{20} = v_3 - v_4 = v_3 - 200$ [4]

Thus, Eq. [1] becomes $-6 = v_1/\,40 + v_2/\,50 + (v_3 - 200)/\,20$ or, more simply,

$4 = v_1/\,40 + v_2/\,50 + v_3/\,20$ [1']

and Eq. [3] becomes $v_1 - 5\,v_3 = -400$ [3']

Eqs. [1'], [3'], and [5] are not sufficient, however, as we have four unknowns. At this point we need to seek an additional equation, possibly in terms of v_2. Referring to the circuit,

$v_1 - v_2 = 400$ [5]

Rewriting as a matrix equation,

$$\begin{bmatrix} 1/40 & 1/50 & 1/20 \\ 1 & 0 & -5 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -400 \\ 400 \end{bmatrix}$$

Solving, we find that
$v_1 = 145.5$ V, $v_2 = -254.5$ V, and $v_3 = 109.1$ V. Since $v_{20} = v_3 - 200$, we find that

$$\boxed{v_{20} = -90.9 \text{ V.}}$$

17. We begin by naming the top left node "1", the top right node "2", the bottom node of the 6-V source "3" and the top node of the 2-Ω resistor "4." The reference node has already been selected, and designated using a ground symbol.

By inspection, $\boxed{v_2 = 5 \text{ V.}}$

Forming a supernode with nodes 1 & 3, we find

At the supernode: $-2 = v_3/1 + (v_1 - 5)/10$ [1]

At node 4: $2 = v_4/2 + (v_4 - 5)/4$ [2]

Our supernode KVL equation: $v_1 - v_3 = 6$ [3]

Rearranging, simplifying and collecting terms,

$$v_1 + 10\,v_3 = -20 + 5 = -15 \qquad [1]$$

and

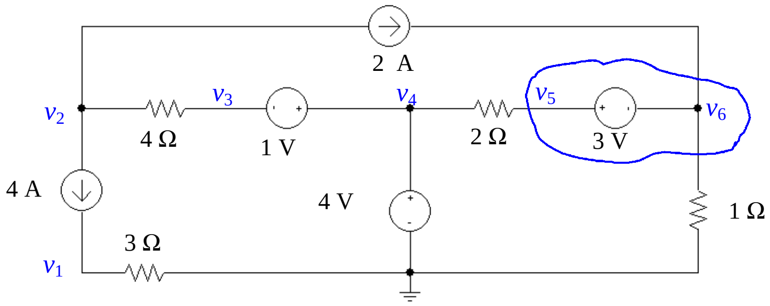
$$v_1 - v_3 = 6 \qquad [2]$$

Eq. [3] may be directly solved to obtain $\boxed{v_4 = 4.333 \text{ V.}}$

Solving Eqs. [1] and [2], we find that

$$\boxed{v_1 = 4.091 \text{ V}} \quad \text{and} \quad \boxed{v_3 = -1.909 \text{ V.}}$$

18. We begin by selecting the bottom node as the reference, naming the nodes as shown below, and forming a supernode with nodes 5 & 6.



By inspection, $v_4 = 4$ V.

By KVL, $v_3 - v_4 = 1$ so $v_3 = -1 + v_4 = -1 + 4$ or $v_3 = 3$ V.

At the supernode, $2 = v_6/1 + (v_5 - 4)/2$ [1]

At node 1, $4 = v_1/3$ therefore, $v_1 = 12$ V.

At node 2, $-4 - 2 = (v_2 - 3)/4$

Solving, we find that $v_2 = -21$ V

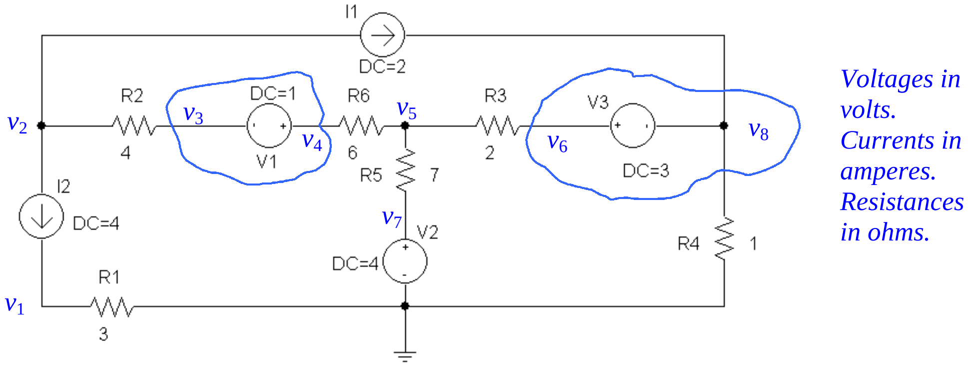
Our supernode KVL equation is $v_5 - v_6 = 3$ [2]

Solving Eqs. [1] and [2], we find that

$$v_5 = 4.667 \text{ V} \quad \text{and} \quad v_6 = 1.667 \text{ V}.$$

The power supplied by the 2-A source therefore is $(v_6 - v_2)(2) = \boxed{45.33 \text{ W.}}$

19. We begin by selecting the bottom node as the reference, naming each node as shown below, and forming two different supernodes as indicated.



By inspection, $v_7 = 4$ V and $v_1 = (3)(4) = 12$ V.

At node 2: $-4 - 2 = (v_2 - v_3)/4$ or $v_2 - v_3 = -24$ [1]

At the 3-4 supernode:
$0 = (v_3 - v_2)/4 + (v_4 - v_5)/6$ or $-6v_2 + 6v_3 + 4v_4 - 4v_5 = 0$ [2]

At node 5:
$0 = (v_5 - v_4)/6 + (v_5 - 4)/7 + (v_5 - v_6)/2$ or $-14v_4 + 68v_5 - 42v_6 = 48$ [3]

At the 6-8 supernode: $2 = (v_6 - v_5)/2 + v_8/1$ or $-v_5 + v_6 + 2v_8 = 4$ [4]

3-4 supernode KVL equation: $v_3 - v_4 = -1$ [5]
6-8 supernode KVL equation: $v_6 - v_8 = 3$ [6]

Rewriting Eqs. [1] to [6] in matrix form,

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -6 & 6 & 4 & -4 & 0 & 0 \\ 0 & 0 & -14 & 68 & -42 & 0 \\ 0 & 0 & 0 & -1 & 1 & 2 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_8 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 48 \\ 4 \\ -1 \\ 3 \end{bmatrix}$$

Solving, we find that
$v_2 = -68.9$ V, $v_3 = -44.9$ V, $v_4 = -43.9$ V, $v_5 = -7.9$ V, $v_6 = 700$ mV, $v_8 = -2.3$ V.

The power generated by the 2-A source is therefore $(v_8 - v_6)(2) = \boxed{133.2 \text{ W.}}$

20. With the reference terminal already specified, we name the bottom terminal of the 3-mA source node "1," the right terminal of the bottom 2.2-kΩ resistor node "2," the top terminal of the 3-mA source node "3," the "+" reference terminal of the 9-V source node "4," and the "-" terminal of the 9-V source node "5."

Since we know that 1 mA flows through the top 2.2-kΩ resistor, $v_5 = -2.2$ V.
Also, we see that $v_4 - v_5 = 9$, so that $v_4 = 9 - 2.2 = 6.8$ V.
Proceeding with nodal analysis,

At node 1: $-3\times10^{-3} = v_1/ 10\text{x}10^3 + (v_1 - v_2)/ 2.2\times10^3$ [1]

At node 2: $0 = (v_2 - v_1)/ 2.2\times10^3 + (v_2 - v_3)/ 4.7\times10^3$ [2]

At node 3: $1\times10^3 + 3\times10^3 = (v_3 - v_2)/ 4.7\times10^3 + v_3/3.3\times10^3$ [3]

Solving, $\boxed{v_1 = -8.614 \text{ V}, v_2 = -3.909 \text{ V and } v_3 = 6.143 \text{ V.}}$

Note that we could also have made use of the supernode approach here.

21. Moving from left to right, we name the bottom three meshes, mesh "1", mesh "2," and mesh "3." In each of these three meshes we define a clockwise current. The remaining mesh current is clearly 8 A. We may then write:

MESH 1: $12\,i_1 - 4\,i_2 = 100$

MESH 2: $-4\,i_1 + 9\,i_2 - 3\,i_3 = 0$

MESH 3: $-3\,i_2 + 18\,i_3 = -80$

Solving this system of three (independent) equations in three unknowns, we find that

$$i_2 = \boxed{i_x = 2.791 \text{ A.}}$$

22. We define four clockwise mesh currents. The top mesh current is labeled i_4. The bottom left mesh current is labeled i_1, the bottom right mesh current is labeled i_3, and the remaining mesh current is labeled i_2. Define a voltage "v_{4A}" across the 4-A current source with the "+" reference terminal on the left.

By inspection, $i_3 = 5$ A and $i_a = i_4$.

MESH 1: $-60 + 2i_1 - 2i_4 + 6i_4 = 0$ or $2i_1 + 4i_4 = 60$ [1]

MESH 2: $-6i_4 + v_{4A} + 4i_2 - 4(5) = 0$ or $4i_2 - 6i_4 + v_{4A} = 30$ [2]

MESH 4: $2i_4 - 2i_1 + 5i_4 + 3i_4 - 3(5) - v_{4A} = 0$ or $-2i_1 + 10i_4 - v_{4A} = 15$ [3]

At this point, we are short an equation. Returning to the circuit diagram, we note that

$i_2 - i_4 = 4$ [4]

Collecting these equations and writing in matrix form, we have

$$\begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & 0 & 10 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_4 \\ v_{4A} \end{bmatrix} = \begin{bmatrix} 60 \\ 20 \\ 15 \\ 4 \end{bmatrix}$$

Solving, $i_1 = 16.83$ A, $i_2 = 10.58$ A, $i_4 = 6.583$ A and $v_{4A} = 17.17$ V.
Thus, the power dissipated by the 2-Ω resistor is

$$(i_1 - i_4)^2 \bullet (2) = \boxed{210.0 \text{ W}}$$

23. We begin our analysis by defining three clockwise mesh currents. We will call the top mesh current i_3, the bottom left mesh current i_1, and the bottom right mesh current i_2.

By inspection, $i_1 = 5$ A [1] and $i_2 = -0.01\ v_1$ [2]

MESH 3: $50\ i_3 + 30\ i_3 - 30\ i_2 + 20\ i_3 - 20\ i_1 = 0$

or $-20\ i_1 - 30\ i_2 + 100\ i_3 = 0$ [3]

These three equations are insufficient, however, to solve for the unknowns. It would be nice to be able to express the dependent source controlling variable v_1 in terms of the mesh currents. Returning to the diagram, it can be seen that KVL around mesh 1 will yield

$$- v_1 + 20\ i_1 - 20\ i_3 + 0.4\ v_1 = 0$$

or $v_1 = 20\ i_1/ 0.6 - 20\ i_3/ 0.6$ or $v_1 = (20(5)/ 0.6 - 20\ i_3/ 0.6$ [4]

Substituting Eq. [4] into Eq. [2] and then the modified Eq. [2] into Eq. [3], we find

$$-20(5) - 30(-0.01)(20)(5)/0.6 + 30(-0.01)(20)\ i_3/ 0.6 + 100\ i_3 = 0$$

Solving, we find that $i_3 = (100 - 50)/ 90 = 555.6$ mA

Thus, $v_1 = 148.1$ V, $i_2 = -1.481$ A, and the power generated by the dependent voltage source is

$$0.4\ v_1\ (i_2 - i_1) = \boxed{-383.9 \text{ W.}}$$

24. We begin by defining four clockwise mesh currents i_1, i_2, i_3 and i_4, in the meshes of our circuit, starting at the left-most mesh. We also define a voltage v_{dep} across the dependent current source, with the "+" on the top.

By inspection, $i_1 = 2A$ and $i_4 = -5$ A.

At Mesh 2: $10\, i_2 - 10(2) + 20\, i_2 + v_{dep} = 0$ [1]

At Mesh 3: $- v_{dep} + 25\, i_3 + 5\, i_3 - 5(-5) = 0$ [2]

Collecting terms, we rewrite Eqs. [1] and [2] as

$30\, i_2 + v_{dep} = 20$ [1]

$30\, i_3 - v_{dep} = -25$ [2]

This is only two equations but three unknowns, however, so we require an additional equation. Returning to the circuit diagram, we note that it is possible to express the current of the dependent source in terms of mesh currents. (We might also choose to obtain an expression for v_{dep} in terms of mesh currents using KVL around mesh 2 or 3.)

Thus, $1.5 i_x = i_3 - i_2$ where $i_x = i_1 - i_2$ so $-0.5\, i_2 - i_3 = -3$ [3]

In matrix form,

$$\begin{bmatrix} 30 & 0 & 1 \\ 0 & 30 & -1 \\ -0.5 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_{dep} \end{bmatrix} = \begin{bmatrix} 20 \\ -25 \\ -3 \end{bmatrix}$$

Solving, we find that $i_2 = -6.333$ A so that $\boxed{i_x = i_1 - i_2 = 8.333 \text{ A.}}$

25. We define a clockwise mesh current i_1 in the bottom left mesh, a clockwise mesh current i_2 in the top left mesh, a clockwise mesh current i_3 in the top right mesh, and a clockwise mesh current i_4 in the bottom right mesh.

MESH 1: $-0.1\ v_a + 4700\ i_1 - 4700\ i_2 + 4700\ i_1 - 4700\ i_4 = 0$ [1]

MESH 2: $9400\ i_2 - 4700\ i_1 - 9 = 0$ [2]

MESH 3: $9 + 9400\ i_3 - 4700\ i_4 = 0$ [3]

MESH 4: $9400\ i_4 - 4700\ i_1 - 4700\ i_3 + 0.1\ i_x = 0$ [4]

The presence of the two dependent sources has led to the introduction of two additional unknowns (i_x and v_a) besides our four mesh currents. In a perfect world, it would simplify the solution if we could express these two quantities in terms of the mesh currents.

Referring to the circuit diagram, we see that $i_x = i_2$ (easy enough) and that $v_a = 4700\ i_3$ (also straightforward). Thus, substituting these expressions into our four mesh equations and creating a matrix equation, we arrive at:

$$\begin{bmatrix} 9400 & -4700 & -470 & -4700 \\ -4700 & 9400 & 0 & 0 \\ 0 & 0 & 9400 & -4700 \\ -4700 & 0.1 & -4700 & 9400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ -9 \\ 0 \end{bmatrix}$$

Solving,

$$\boxed{i_1 = 239.3\ \mu\text{A},\ i_2 = 1.077\ \text{mA},\ i_3 = -1.197\ \text{mA and } i_4 = -478.8\ \mu\text{A.}}$$

26. We define a clockwise mesh current i_3 in the upper right mesh, a clockwise mesh current i_1 in the lower left mesh, and a clockwise mesh current i_2 in the lower right mesh.

MESH 1: $-6 + 6\,i_1 - 2 = 0$ [1]

MESH 2: $2 + 15\,i_2 - 12\,i_3 - 1.5 = 0$ [2]

MESH 3: $i_3 = 0.1\,v_x$ [3]

Eq. [1] may be solved directly to obtain $\boxed{i_1 = 1.333 \text{ A.}}$

It would help in the solution of Eqs. [2] and [3] if we could express the dependent source controlling variable v_x in terms of mesh currents. Referring to the circuit diagram, we see that $v_x = (1)(i_1) = i_1$, so Eq. [3] reduces to

$$\boxed{i_3 = 0.1\,v_x = 0.1\,i_1 = 133.3 \text{ mA.}}$$

As a result, Eq. [1] reduces to $\boxed{i_2 = [-0.5 + 12(0.1333)]/ 15 = 73.31 \text{ mA.}}$

27. (a) Define a mesh current i_2 in the second mesh. Then KVL allows us to write:

MESH 1: $-9 + R\, i_1 + 47000\, i_1 - 47000\, i_2 = 0$ [1]

MESH 2: $67000\, i_2 - 47000\, i_1 - 5 = 0$ [2]

Given that i_1 = 1.5 mA, we may solve Eq. [2] to find that

$$i_2 = \frac{5 + 47(1.5)}{67} \text{ mA} = 1.127 \text{ mA}$$

and so

$$R = \frac{9 - 47(1.5) + 47(1.127)}{1.5\times10^{-3}} = \boxed{-5687\ \Omega.}$$

(b) This value of *R* is unique; no other value will satisfy **both** Eqs. [1] **and** [2].

28. Define three clockwise mesh currents i_1, i_2 and i_3. The bottom 1-kΩ resistor can be ignored, as no current flows through it.

MESH 1: $-4 + (2700 + 1000 + 5000)\, i_1 - 1000\, i_2 = 0$ [1]

MESH 2: $(1000 + 1000 + 4400 + 3000)\, i_2 - 1000\, i_1 - 4400\, i_3 + 2.2 - 3 = 0$ [2]

MESH 3: $(4400 + 4000 + 3000)\, i_3 - 4400\, i_2 - 1.5 = 0$ [3]

Combining terms,

$$\begin{aligned} 8700\, i_1 - 1000\, i_2 \quad\quad\quad\quad &= 4 \quad [1] \\ -1000\, i_1 + 9400\, i_2 - 4400\, i_3 &= 0.8 \quad [2] \\ -4400\, i_2 + 11400\, i_3 &= 1.5 \quad [3] \end{aligned}$$

Solving,

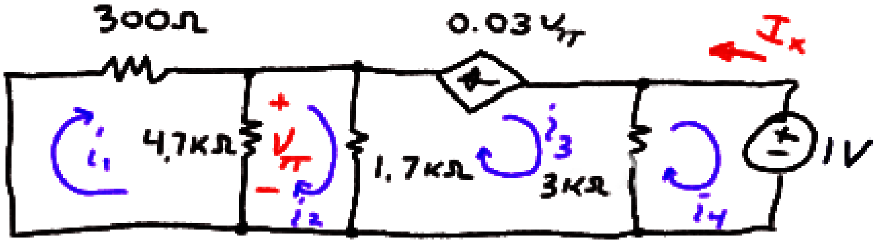
$i_1 = 487.6$ mA, $i_2 = 242.4$ mA and $i_3 = 225.1$ mA.

The power absorbed by each resistor may now be calculated:

P_{5k}	=	$5000\, (i_1)^2$	=	1.189 mW
$P_{2.7k}$	=	$2700\, (i_1)^2$	=	641.9 μW
P_{1ktop}	=	$1000\, (i_1 - i_2)^2$	=	60.12 μW
$P_{1kmiddle}$	=	$1000\, (i_2)^2$	=	58.76 μW
$P_{1kbottom}$	=	0	=	0
$P_{4.4k}$	=	$4400\, (i_2 - i_3)^2$	=	1.317 μW
P_{3ktop}	=	$3000\, (i_3)^2$	=	152.0 μW
P_{4k}	=	$4000\, (i_3)^2$	=	202.7 μW
$P_{3kbottom}$	=	$3000\, (i_2)^2$	=	176.3 μW

Check: The sources supply a total of
4(487.6) + (3 – 2.2)(242.4) + 1.5(225.1) = 2482 μW.
The absorbed powers add to 2482 μW.

29. (a) We begin by naming four mesh currents as depicted below:



Proceeding with mesh analysis, then, keeping in mind that $I_x = -i_4$,

MESH 1: $(4700 + 300)\, i_1 - 4700\, i_2 = 0$ [1]

MESH 2: $(4700 + 1700)\, i_2 - 4700\, i_1 - 1700\, i_3 = 0$ [2]

Since we have a current source on the perimeter of mesh 3, we do not require a KVL equation for that mesh. Instead, we may simply write

$i_3 = -0.03\, v_\pi$ [3*a*] where $v_\pi = 4700(i_1 - i_2)$ [3*b*]

MESH 4: $3000\, i_4 - 3000\, i_3 + 1 = 0$ [4]

Simplifying and combining Eqs. 3*a* and 3*b*,

$$\begin{aligned}
5000\, i_1 - 4700\, i_2 &= 0 \\
-4700\, i_1 + 6400\, i_2 - 1700\, i_3 &= 0 \\
-141\, i_1 + 141\, i_2 - i_3 &= 0 \\
-3000\, i_3 + 3000\, i_4 &= -1
\end{aligned}$$

Solving, we find that i_4 = -333.3 mA, so $\boxed{I_x = 333.3\ \mu A.}$

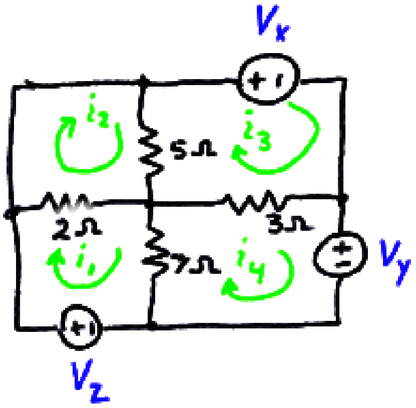
(b) At node "π" : $0.03\, v_\pi = v_\pi / 300 + v_\pi / 4700 + v_\pi / 1700$

Solving, we find that $v_\pi = 0$, therefore no current flows through the dependent source.

Hence, $\boxed{I_x = 333.3\ \mu\text{A as found in part (a).}}$

(c) $\boxed{V_s / I_x \text{ has units of resistance}}$. It can be thought of as the resistance "seen" by the voltage source V_s…. more on this in Chap. 5….

30. We begin by naming each mesh and the three undefined voltage sources as shown below:



MESH 1: $-V_z + 9i_1 - 2i_2 - 7i_4 = 0$

MESH 2: $-2i_1 + 7i_2 - 5i_3 = 0$

MESH 3: $V_x - 5i_2 + 8i_3 - 3i_4 = 0$

MESH 4: $V_y - 7i_1 - 3i_3 + 10i_4 = 0$

Rearranging and setting $i_1 - i_2 = 0$, $i_2 - i_3 = 0$, $i_1 - i_4 = 0$ and $i_4 - i_3 = 0$,

$$\begin{aligned}
9i_1 - 2i_2 \qquad\quad -7i_4 &= V_z \\
-2i_1 + 7i_2 - 5i_3 \qquad\quad &= 0 \\
-5i_2 + 8i_3 - 3i_4 &= -V_x \\
-7i_1 \qquad -3i_3 + 10i_4 &= -V_y
\end{aligned}$$

Since $i_1 = i_2 = i_3 = i_4$, these equations produce:

$$\begin{aligned}
V_z &= 0 \\
0 &= 0 \\
-V_x &= 0 \\
-V_y &= 0
\end{aligned}$$

This is a unique solution. Therefore, the request that nonzero values be found cannot be satisfied.

31. The "supermesh" concept is not required (or helpful) in solving this problem, as there are no current sources shared between meshes. Starting with the left-most mesh and moving right, we define four clockwise mesh currents i_1, i_2, i_3 and i_4. By inspection, we see that $i_1 = 2$ mA.

MESH 2: $-10 + 5000i_2 + 4 + 1000i_3 = 0$ [1]

MESH 3: $-1000i_3 + 6 + 10{,}000 - 10{,}000i_4 = 0$ [2]

MESH 4: $i_4 = -0.5i_2$ [3]

Reorganising, we find

$$\begin{array}{rcll} 5000\, i_2 + 1000\, i_3 & & = 6 & [1] \\ 9000\, i_3 - 10{,}000\, i_4 & & = -6 & [2] \\ 0.5\, i_2 \qquad + \qquad i_4 & & = 0 & [3] \end{array}$$

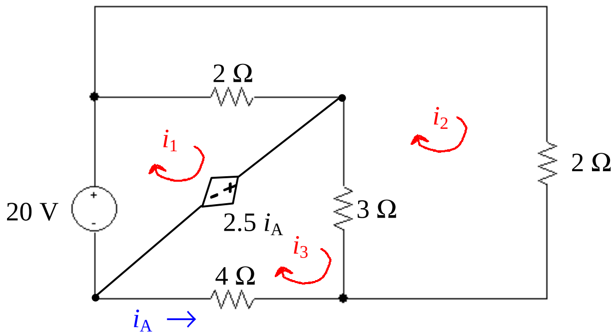
We could either subtitute Eq. [3] into Eq. [2] to reduce the number of equations, or simply go ahead and solve the system of Eqs. [1-3]. Either way, we find that

$$i_1 = 2 \text{ mA},\ i_2 = 1.5 \text{ mA},\ i_3 = -1.5 \text{ mA and } i_4 = -0.75 \text{ mA}.$$

The power generated by each source is:

$$\boxed{\begin{array}{lll} P_{2mA} & = 5000(i_1 - i_2)(i_1) & = 5 \text{ mW} \\ P_{4V} & = 4\,(-i_2) & = -6 \text{ mW} \\ P_{6V} & = 6\,(-i_3) & = 9 \text{ mW} \\ P_{depV} & = 1000\, i_3\,(i_3 - i_2) & = 4.5 \text{ mW} \\ P_{dep\mathit{I}} & = 10{,}000(i_3 - i_4)(0.5\, i_2) & = -5.625 \text{ mW} \end{array}}$$

32. This circuit does not require the supermesh technique, as it does not contain any current sources. Redrawing the circuit so its planar nature and mesh structure are clear,



MESH 1: $-20 + 2\, i_1 - 2\, i_2 + 2.5\, i_A = 0$ [1]

MESH 2: $2\, i_2 + 3\, i_2 - 3\, i_3 + 2\, i_2 - 2\, i_1 = 0$ [2]

MESH 3: $-2.5\, i_A + 7\, i_3 - 3\, i_2 = 0$ [3]

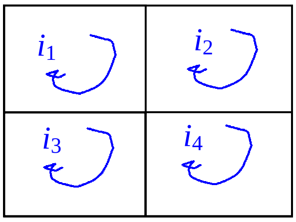
Combining terms and making use of the fact that $i_A = -\, i_3$,

$$2\, i_1 - 2\, i_2 - 2.5\, i_3 = 20 \quad [1]$$
$$-2\, i_1 + 7 i_2 - 3\, i_3 = 0 \quad [2]$$
$$-3\, i_2 + 9.5\, i_3 = 0 \quad [3]$$

Solving, $i_1 = 18.55$ A, $i_2 = 6.129$ A, and $i_3 = 1.935$ A. Since $i_A = -\, i_3$,

$$\boxed{i_A = -1.935 \text{ A.}}$$

33. Define four mesh currents



By inspection, i_1 = -4.5 A.

We form a supermesh with meshes 3 and 4 as defined above.

MESH 2: $2.2 + 3\,i_2 + 4\,i_2 + 5 - 4\,i_3 = 0$ [1]

SUPERMESH: $3\,i_4 + 9\,i_4 - 9\,i_1 + 4\,i_3 - 4\,i_2 + 6\,i_3 + i_3 - 3 = 0$ [2]

Supermesh KCL equation: $i_4 - i_3 = 2$ [3]

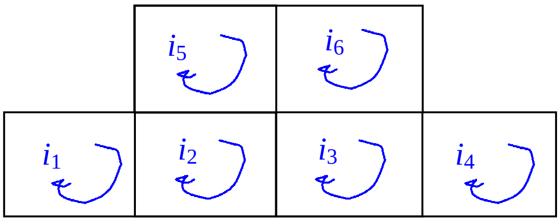
Simplifying and combining terms, we may rewrite these three equations as:

$$\begin{aligned} 7\,i_2 - 4\,i_3 \quad &= -7.2 \qquad [1] \\ -4\,i_2 + 11\,i_3 + 12\,i_4 &= -37.5 \qquad [2] \\ -\,i_3 + i_4 &= 2 \qquad [3] \end{aligned}$$

Solving, we find that i_2 = -2.839 A, i_3 = -3.168 A, and i_4 = -1.168 A.

The power supplied by the 2.2-V source is then $2.2\,(i_1 - i_2) = \boxed{-3.654 \text{ W.}}$

34. We begin by defining six mesh currents as depicted below:



- We form a supermesh with meshes 1 and 2 since they share a current source.
- We form a *second* supermesh with meshes 3 and 4 since they also share a current source.

1, 2 Supermesh:
$(4700 + 1000 + 10{,}000)\ i_1 - 2200\ i_5 + (2200 + 1000 + 4700)\ i_2 - 1000\ i_3 = 0$ [1]

3, 4 Supermesh:
$(4700 + 1000 + 2200)\ i_3 - 1000\ i_2 - 2200\ i_6 + (4700 + 10{,}000 + 1000)\ i_4 = 0$ [2]

MESH 5: $(2200 + 4700)\ i_5 - 2200\ i_2 + 3.2 - 1.5 = 0$ [3]

MESH 6: $1.5 + (4700 + 4700 + 2200)\ c - 2200\ i_3 = 0$ [4]

1, 2 Supermesh KCL equation: $i_1 - i_2 = 3\times10^{-3}$ [5]

3, 4 Supermesh KCL equation: $i_4 - i_3 = 2\times10^{-3}$ [6]

We can simplify these equations prior to solution in several ways. Choosing to retain six equations,

$$
\begin{array}{lllllllr}
15{,}700\ i_1 & +\ 7900\ i_2 & -\ 1000\ i_3 & & -2200\ \mathrm{i5} & & = 0 & [1] \\
 & -\ 1000\ i_2 & +\ 7900\ i_3 & +\ 15{,}700\ i_4 & & -2200\ i_6 & = 0 & [2] \\
 & -\ 2200\ i_2 & & & +\ 6900\ i_5 & & = -1.7 & [3] \\
 & & -\ 2200\ i_3 & & & +\ 11{,}600\ i_6 & = -1.5 & [4] \\
i_1 & -\ i_2 & & & & & = 3\times10^{-3} & [5] \\
 & & -\ i_3 & +\ i_4 & & & = 2\times10^{-3} & [6]
\end{array}
$$

Solving, we find that $i_4 = 540.8$ mA. Thus, the voltage across the 2-mA source is

$$(4700 + 10{,}000 + 1000)\ (540.8\times10^{-6}) = \boxed{8.491\ \text{V}}$$

35. We define a mesh current i_a in the left-hand mesh, a mesh current i_1 in the top right mesh, and a mesh current i_2 in the bottom right mesh.

The left-most mesh can be analysed separately to determine the controlling voltage v_a, as KCL assures us that no current flows through either the 1-Ω or 6-Ω resistor.

Thus, $-1.8 + 3\ i_a - 1.5 + 2\ i_a = 0$, which may be solved to find $i_a = 0.66$ A. Hence, $v_a = 3\ i_a = 1.98$ V.

Forming one supermesh from the remaining two meshes, we may write:

$$-3 + 2.5\ i_1 + 3\ i_2 + 4\ i_2 = 0$$

and the supermesh KCL equation: $i_2 - i_1 = 0.05\ v_a = 0.05(1.98) = 99\times10^{-3}$

Thus, we have two equations to solve:

$$2.5\ i_1 + 7\ i_2 = 3$$
$$-i_1 + i_2 = 99\times10^{-3}$$

Solving, we find that $i_1 = 242.8$ mA and the voltage across the 2.5-Ω resistor (arbitrarily assuming the left terminal is the "+" reference) is $2.5\ i_1 =$ $\boxed{607 \text{ mV.}}$

36. UNDEFINED RESISTOR VALUE IN FIGURE. Set to 10 mΩ.

There are only three meshes in this circuit, as the botton 22-mΩ resistor is not connected connected at its left terminal. Thus, we define three mesh currents, i_1, i_2, and i_3, beginning with the left-most mesh.

We next create a supermesh from meshes 1 and 2 (note that mesh 3 is independent, and can be analysed separately).

Thus, $$-11.8 + 10\times10^{-3}\ i_1 + 22\times10^{-3}\ i_2 + 10\times10^{-3}\ i_2 + 17\times10^{-3}\ i_1 = 0$$

and applying KCL to obtain an equation containing the current source,

$$i_1 - i_2 = 100$$

Combining terms and simplifying, we obtain

$$\begin{aligned} 27\times10\text{-}3\ i_1 + 32\times10^{-3}\ i_2 &= 11.8 \\ i_1 - i_2 &= 100 \end{aligned}$$

Solving, we find that $\boxed{i_1 = 254.2\text{ A}}$ and $\boxed{i_2 = 154.2\text{ A.}}$

The final mesh current is easily found: $\boxed{i_3 = 13\times10^3/\,(14 + 11.6 + 15) = 320.2\text{ A.}}$

37. MESH 1: $-7 + i_1 - i_2 = 0$ [1]
MESH 2: $i_2 - i_1 + 2i_2 + 3i_2 - 3i_3 = 0$ [2]
MESH 3: $3i_3 - 3i_2 + xi_3 + 2i_3 - 7 = 0$ [3]

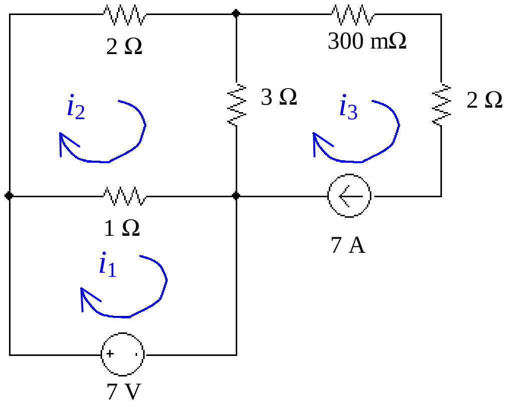
Grouping terms, we find that

$$i_1 - i_2 = 7 \quad [1]$$
$$-i_1 + 6i_2 - 3i_3 = 0 \quad [2]$$
$$-3i_2 + (5 + x)i_3 = 7 \quad [3]$$

This, unfortunately, is four unknowns but only three equations. However, we have not yet made use of the fact that we are trying to obtain $i_2 = 2.273$ A. Solving these "four" equations, we find that

$$\boxed{x = (7 + 3\,i_2 - 5\,i_3)/\,i_3 = 4.498\ \Omega.}$$

38. We begin by redrawing the circuit as instructed, and define three mesh currents:



By inspection, $i_3 = 7$ A.

MESH 1: $-7 + i_1 - i_2 = 0$ or $i_1 - i_2 = 7$ [1]

MESH 2: $(1 + 2 + 3)\, i_2 - i_1 - 3(7) = 0$ or $-i_1 + 6i_2 = 21$ [2]

There is no need for supermesh techniques for this situation, as the only current source lies on the outside perimeter of a mesh- it is not shared between meshes.

Solving, we find that $\boxed{i_1 = 12.6 \text{ A},\ i_2 = 5.6 \text{ A and } i_3 = 7 \text{ A.}}$

39. (a) We are asked for a voltage, and have one current source and one voltage source. Nodal analysis is probably best then- the nodes can be named so that the desired voltage is a nodal voltage, or, at worst, we have one supernode equation to solve.

Name the top left node "1" and the top right node "x"; designate the bottom node as the reference terminal. Next, form a supernode with nodes "1" and "x."

At the supernode: $11 = v_1/2 + v_x/9$ [1]

and the KVL Eqn: $v_1 - v_x = 22$ [2]

Rearranging, $11(18) = 9v_1 + 2v_x$ [1]
$22 = v_1 - v_x$ [2]

Solving, $\boxed{v_x = 0}$

(b) We are asked for a voltage, and so may suspect that nodal analysis is preferrable; with two current sources and only one voltage source (easily dealt with using the supernode technique), nodal analysis does seem to have an edge over mesh analysis here.

Name the top left node "x," the top right node "y" and designate the bottom node as the reference node. Forming a supernode from nodes "x" and "y,"

At the supernode: $6 + 9 = v_x/10 + v_y/20$ [1]
and the KVL Eqn: $v_y - v_x = 12$ [2]

Rearranging, $15(20) = 2v_x + v_y$ [1]
and $12 = -v_x + v_y$ [2]

Solving, we find that $\boxed{v_x = 96\text{ V.}}$

(c) We are asked for a voltage, but would have to subtract two nodal voltages (not much harder than invoking Ohm's law). On the other hand, the dependent current source depends on the desired unknown, which would lead to the need for another equation if invoking mesh analysis. Trying nodal analysis,

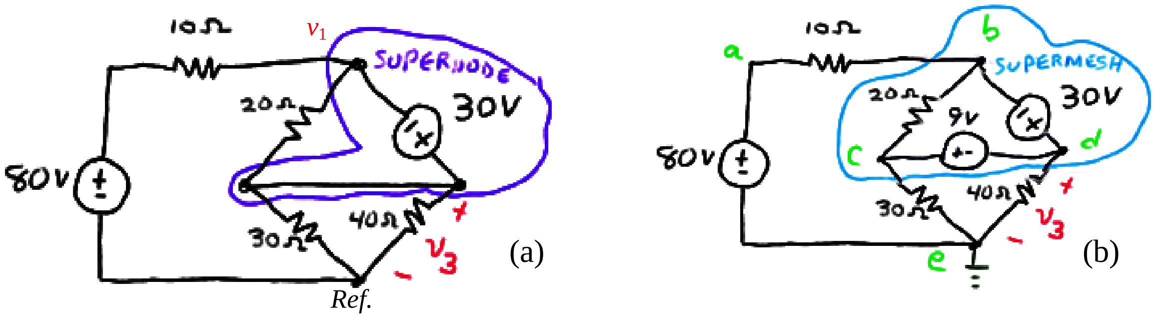
$$0.1\,v_x = (v_1 - 50)/2 + v_x/4 \qquad [1]$$

referring to the circuit we see that $v_x = v_1 - 100$. Rearranging so that we may eliminate v_1 in Eq. [1], we obtain $v_1 = v_x + 100$. Thus, Eq. [1] becomes

$$0.1\,v_x = (v_x + 100 - 50)/2 + v_x/4$$

and a little algebra yields $\boxed{v_x = -38.46\text{ V.}}$

40.



(*a*) We begin by noting that it is a voltage that is required; no current values are requested. This is a three-mesh circuit, or a four-node circuit, depending on your perspective. Either approach requires three equations…. Except that applying the supernode technique reduces the number of needed equations by one.

At the 1, 3 supernode:

$$0 = (v_1 - 80)/10 + \cancel{(v_1 - v_3)/20} + \cancel{(v_3 - v_1)/20} + v_3/40 + v_3/30$$

and $$v_3 - v_1 = 30$$

Both ends of the resistor are connected to the supernode, so we could actually just ignore it…

We simplify these two equations and collect terms, yielding

$$0.1\, v_1 + 0.05833\, v_3 = 8$$
$$-v_1 + v_3 = 30$$

Solving, we find that $\boxed{v_3 = 69.48 \text{ V}}$

(*b*) Mesh analysis would be straightforward, requiring 3 equations and a (trivial) application of Ohm's law to obtain the final answer. Nodal analysis, on the other hand, would require only two equations, and the desired voltage will be a nodal voltage.

At the b, c, d supernode: $\quad 0 = (v_b - 80)/10 + v_d/40 + v_c/30$

and: $\quad v_d - v_b = 30 \qquad v_c - v_d = 9$

Simplify and collect terms:

$$0.1\, v_b + 0.03333\, v_c + 0.025\, v_d = 80$$
$$-v_b + v_d = 30$$
$$v_c - v_d = 9$$

Solving, $\boxed{v_d\ (= v_3) = 67.58 \text{ V}}$

(c) We are now faced with a dependent current source whose value depends on a mesh current. Mesh analysis in this situation requires 1 supermesh, 1 KCL equation and Ohm's law. Nodal analysis requires 1 supernode, 1 KVL equation, 1 other nodal equation, and one equation to express i_1 in terms of nodal voltages. Thus, mesh analysis has an edge here. Define the left mesh as "1," the top mesh as "2", and the bottom mesh as "3."

Mesh 1: $\quad -80 + 10\, i_1 + 20\, i_1 - 20\, i_2 + 30\, i_1 - 30\, i_3 = 0$

2, 3 supermesh: $\quad 20\, i_2 - 20\, i_1 - 30 + 40\, i_3 + 30\, i_3 - 30\, i_1 = 0$

and: $\quad i_2 - i_3 = 5\, i_1$

Rewriting,

$$60\, i_1 - 20\, i_2 - 30\, i_3 = 80$$
$$-50\, i_1 + 20\, i_2 + 70\, i_3 = 30$$
$$5\, i_1 - i_2 + i_3 = 0$$

Solving, $\quad i_3 = 4.727$ A $\quad$ so $\quad \boxed{v_3 = 40\, i_3 = 189 \text{ V.}}$

41. This circuit consists of 3 meshes, and no dependent sources. Therefore 3 simultaneous equations and 1 subtraction operation would be required to solve for the two desired currents. On the other hand, if we use nodal analysis, forming a supernode about the 30-V source would lead to 5 – 1 – 1 = 3 simulataneous equations as well, plus several subtraction and division operations to find the currents. Thus, mesh analysis has a slight edge here.

Define three clockwise mesh currents: i_a in the left-most mesh, i_b in the top right mesh, and i_c in the bottom right mesh. Then our mesh equations will be:

Mesh *a*: $-80 + (10 + 20 + 30)\, i_a - 20\, i_b - 30\, i_c = 0$ [1]
Mesh *b*: $-30 + (12 + 20)\, i_b - 12\, i_c - 20\, i_a = 0$ [2]
Mesh *c*: $(12 + 40 + 30)\, i_c - 12\, i_b - 30\, i_a = 0$ [3]

Simplifying and collecting terms,

$$60\, i_a - 20\, i_b - 30\, i_c = 80 \quad [1]$$
$$-20\, i_a + 32\, i_b - 12\, i_c = 30 \quad [2]$$
$$-30\, i_a - 12\, i_b + 82\, i_c = 0 \quad [3]$$

Solving, we find that i_a = 3.549 A, i_b = 3.854 A, and i_c = 1.863 A. Thus,

$$i_1 = i_a = \boxed{3.549 \text{ A}} \qquad \text{and} \quad i_2 = i_a - i_c = \boxed{1.686 \text{ A.}}$$

42. Approaching this problem using nodal analysis would require 3 separate nodal equations, plus one equation to deal with the dependent source, plus subtraction and division steps to actually find the current i_{10}. Mesh analysis, on the other hand, will require 2 mesh/supermesh equations, 1 KCL equation, and one subtraction step to find i_{10}. Thus, mesh analysis has a clear edge. Define three clockwise mesh currents: i_1 in the bottom left mesh, i_2 in the top mesh, and i_3 in the bottom right mesh.

MESH 1: $i_1 = 5$ mA *by inspection* [1]

SUPERMESH:
$i_1 – i_2 = 0.4\, i_{10}$
$i_1 – i_2 = 0.4(i_3 – i_2)$
$i_1 – 0.6\, i_2 – 0.4\, i_3 = 0$ [2]

MESH 3: $-5000\, i_1 – 10000\, i_2 + 35000\, i_3 = 0$ [3]

Simplify:
$0.6\, i_2 + 0.4\, i_3 = 5\times10^{-3}$ [2]
$-10000\, i_2 + 35000\, i_3 = 25$ [3]

Solving, we find $i_2 = 6.6$ mA and $i_3 = 2.6$ mA. Since $i_{10} = i_3 – i_2$, we find that

$$\boxed{i_{10} = -4 \text{ mA.}}$$

43. For this circuit problem, nodal analysis will require 3 simultaneous nodal equations, then subtraction/ division steps to obtain the desired currents. Mesh analysis requires 1 mesh equation, 1 supermesh equation, 2 simple KCL equations and one subtraction step to determine the currents. If either technique has an edge in this situation, it's probably mesh analysis. Thus, define four clockwise mesh equations: i_a in the bottom left mesh, i_b in the top left mesh, i_c in the top right mesh, and i_d in the bottom right mesh.

At the *a*, *b*, *c* supermesh: $-100 + 6\, i_a + 20\, i_b + 4\, i_c + 10\, i_c - 10\, i_d = 0$ [1]

Mesh d: $100 + 10 \text{ id} - 10\, i_c + 24\, i_d = 0$ [2]

KCL: $-\, i_a + i_b = 2$ [3]

and $-\, i_b + i_c = 3\, i_3 = 3\, i_a$ [4]

Collecting terms & simplifying,

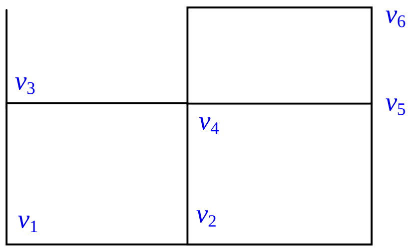
$$6\, i_a + 20\, i_b + 14\, i_c - 10\, i_d = 100 \quad [1]$$
$$-10\, i_c + 34\, i_d = -100 \quad [2]$$
$$-\, i_a + i_b = 2 \quad [3]$$
$$-3\, i_a - i_b + i_c = 0 \quad [4]$$

Solving,

$i_a = 0.1206$ A, $i_b = 2.121$ A, $i_c = 2.482$ A, and $i_d = -2.211$ A. Thus,

$$i_3 = i_a = \boxed{120.6 \text{ mA}} \quad \text{and} \quad i_{10} = i_c - i_d = \boxed{4.693 \text{ A.}}$$

44. With 7 nodes in this circuit, nodal analysis will require the solution of three simultaneous nodal equations (assuming we make use of the supernode technique) and one KVL equation. Mesh analysis will require the solution of three simultaneous mesh equations (one mesh current can be found by inspection), plus several subtraction and multiplication operations to finally determine the voltage at the central node. Either will probably require a comparable amount of algebraic manoeuvres, so we go with nodal analysis, as the desired unknown is a direct result of solving the simultaneous equations. Define the nodes as:



NODE 1: $-2\times10^{-3} = (v_1 - 1.3)/\ 1.8\times10^3 \quad \rightarrow \quad v_1 = -2.84$ V.

2, 4 Supernode:
$2.3\times10^{-3} = (v_2 - v_5)/\ 1\text{x}10^3 + (v_4 - 1.3)/\ 7.3\times10^3 + (v_4 - v_5)/\ 1.3\times10^3 + v_4/\ 1.5\times10^3$

KVL equation: $-v_2 + v_4 = 5.2$

Node 5: $0 = (v_5 - v_2)/\ 1\text{x}10^3 + (v_5 - v_4)/\ 1.3\text{x}10^3 + (v_5 - 2.6)/\ 6.3\text{x}10^3$

Simplifying and collecting terms,

$$\begin{aligned} 14.235\, v_2 + 22.39\, v_4 - 25.185\, v_5 &= 35.275 \quad [1] \\ -v_2 + v_4 &= 5.2 \quad [2] \\ -8.19\, v_2 - 6.3\, v_4 + 15.79\, v_5 &= 3.38 \quad [3] \end{aligned}$$

Solving, we find the voltage at the central node is $\boxed{v_4 = 3.460 \text{ V.}}$

45. Mesh analysis yields current values directly, so use that approach. We therefore define four clockwise mesh currents, starting with i_1 in the left-most mesh, then i_2, i_3 and i_4 moving towards the right.

Mesh 1: $-0.8i_x + (2 + 5)\, i_1 - 5\, i_2 = 0$ [1]

Mesh 2: $i_2 = 1$ A by inspection [2]

Mesh 3: $(3 + 4)\, i_3 - 3(1) - 4(i_4) = 0$ [3]

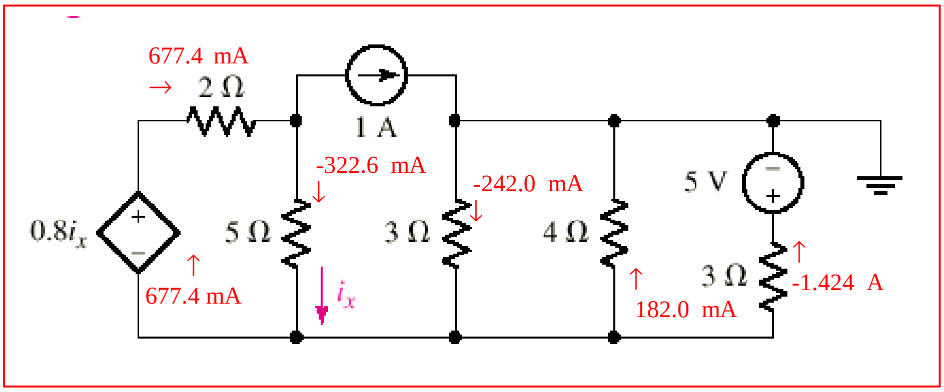
Mesh 4: $(4 + 3)\, i_4 - 4\, i_3 - 5 = 0$ [4]

Simplify and collect terms, noting that $i_x = i_1 - i_2 = i_1 - 1$

$-0.8(i_1 - 1) + 7\, i_1 - 5(1) = 0$ yields $i_1 = 677.4$ mA

Thus, [3] and [4] become: $7\, i_3 - 4\, i_4 = 3$ [3]
$-4\, i_3 + 7\, i_4 = 5$ [4]

Solving, we find that $i_3 = 1.242$ A and $i_4 = 1.424$ A. A map of individual branch currents can now be drawn:



46. If we choose to perform mesh analysis, we require 2 simultaneous equations (there are four meshes, but one mesh current is known, and we can employ the supermesh technique around the left two meshes). In order to find the voltage across the 2-mA source we will need to write a KVL equation, however. Using nodal analysis is less desirable in this case, as there will be a large number of nodal equations needed. Thus, we define four clockwise mesh currents i_1, i_2, i_3 and i_4 starting with the left-most mesh and moving towards the right of the circuit.

At the 1,2 supermesh: $2000\ i_1 + 6000\ i_2 - 3 + 5000\ i_2 = 0$ [1]

and $i_1 - i_2 = 2\times10^{-3}$ [2]

by inspection, $i_4 = -1$ mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and i_1 and i_2 are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain $2000\ i_1 + 11000\ i_2 = 3$ [1]

$1000\ i_1 - 1000\ i_2 = 2$ [2]

Solving, $i_1 = 1.923$ mA and $i_2 = -76.92$ μA.

Thus, the voltage across the 2-mA source ("+" reference at the top of the source) is

$$v = -2000\ i_1 - 6000\ (i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

47. Nodal analysis will require 2 nodal equations (one being a "supernode" equation), 1 KVL equation, and subtraction/division operations to obtain the desired current. Mesh analysis simply requires 2 "supermesh" equations and 2 KCL equations, with the desired current being a mesh current. Thus, we define four clockwise mesh currents i_a, i_b, i_c, i_d starting with the left-most mesh and proceeding to the right of the circuit.

At the *a*, *b* supermesh: $-5 + 2\,i_a + 2\,i_b + 3\,i_b - 3\,i_c = 0$ [1]

At the *c*, *d* supermesh: $3\,i_c - 3\,i_b + 1 + 4\,i_d = 0$ [2]

and $i_a - i_b = 3$ [3]

$i_c - i_d = 2$ [4]

Simplifying and collecting terms, we obtain

$$
\begin{aligned}
2\,i_a + 5\,i_b - 3\,i_c \qquad &= 5 \quad [1]\\
-3\,i_b + 3\,i_c + 4\,i_d &= -1 \quad [2]\\
i_a - i_b \qquad &= 3 \quad [3]\\
i_c - i_d &= 2 \quad [4]
\end{aligned}
$$

Solving, we find i_a = 3.35 A, i_b = 350 mA, i_c = 1.15 A, and i_d = -850 mA. As $i_1 = i_b$,

$$\boxed{i_1 = 350 \text{ mA.}}$$

48. Define a voltage v_x at the top node of the current source I_2, and a clockwise mesh current i_b in the right-most mesh.

We want 6 W dissipated in the 6-Ω resistor, which leads to the requirement $i_b = 1$ A. Applying nodal analysis to the circuit,

$I_1 + I_2 = (v_x - v_1)/ 6 = 1$

so our requirement is $I_1 + I_2 = 1$. There is no constraint on the value of v_1 other than we are told to select a nonzero value.

Thus, $\boxed{\text{we choose } I_1 = I_2 = 500 \text{ mA and } v_1 = 3.1415 \text{ V.}}$

49. Inserting the new 2-V source with "+" reference at the bottom, and the new 7-mA source with the arrow pointing down, we define four clockwise mesh currents i_1, i_2, i_3, i_4 starting with the left-most mesh and proceeding towards the right of the circuit.

Mesh 1: $(2000 + 1000 + 5000)\, i_1 – 6000\, i_2 – 2 = 0$ [1]

2, 3 Supermesh:
$2 + (5000 + 5000 + 1000 + 6000)\, i_2 – 6000\, i_1 + (3000 + 4000 + 5000)\, i_3 – 5000\, i_4 = 0$ [2]

and $i_2 - i_3 = 7\times10^{-3}$ [3]

Mesh 4: $i_4 = -1$ mA by inspection [4]

Simplifying and combining terms,

$$8000\, i_1 – 6000\, i_2 = 2 \quad [1]$$
$$1000\, i_2 – 1000\, i_3 = 7 \quad [4]$$
$$-6000\, i_1 + 17000\, i_2 + 12000\, i_3 = -7 \quad [2]$$

Solving, we find that

$$\boxed{i_1 = 2.653 \text{ A},\ i_2 = 3.204 \text{ A},\ i_3 = -3.796 \text{ A},\ i_4 = -1 \text{ mA}}$$

50. Define node 1 as the top left node, and node 2 as the node joining the three 2-Ω resistors. Place the "+" reference terminal of the 2-V source at the right. The right-most 2-Ω resistor has therefore been shorted out. Applying nodal analysis then,

Node 1: $-5\, i_1 = (v_1 - v_2)/\ 2$ [1]

Node 2: $0 = (v_2 - v_1)/\ 2 + v_2/\ 2 + (v_2 - 2)/\ 2$ [2]

and, $i_1 = (v_2 - 2)/\ 2$ [3]

Simplifying and collecting terms,

$v_1 + v_2 = 10$ [1]
$-v_1 + 3\, v_2 = 2$ [2]

Solving, we find that $\boxed{v_1 = 3.143 \text{ V} \quad \text{and} \quad v_2 = 1.714 \text{ V.}}$

Defining clockwise mesh currents i_a, i_b, i_c, i_d starting with the left-most mesh and proceeding right, we may easily determine that

$$\boxed{\begin{array}{l} i_a = -5\, i_1 = 714.3 \text{ mA} \\ i_b = -142.9 \text{ mA} \\ i_c = i_1 - 2 = -2.143 \text{ A} \\ i_d = 3 + i_c = 857.1 \text{ mA} \end{array}}$$

51. **Hand analysis:**

Define three clockwise mesh currents: i_1 in the bottom left mesh, i_2 in the top mesh, and i_3 in the bottom right mesh.

MESH 1: $i_1 = 5$ mA *by inspection* [1]

SUPERMESH:
$$i_1 - i_2 = 0.4\, i_{10}$$
$$i_1 - i_2 = 0.4(i_3 - i_2)$$
$$i_1 - 0.6\, i_2 - 0.4\, i_3 = 0 \quad [2]$$

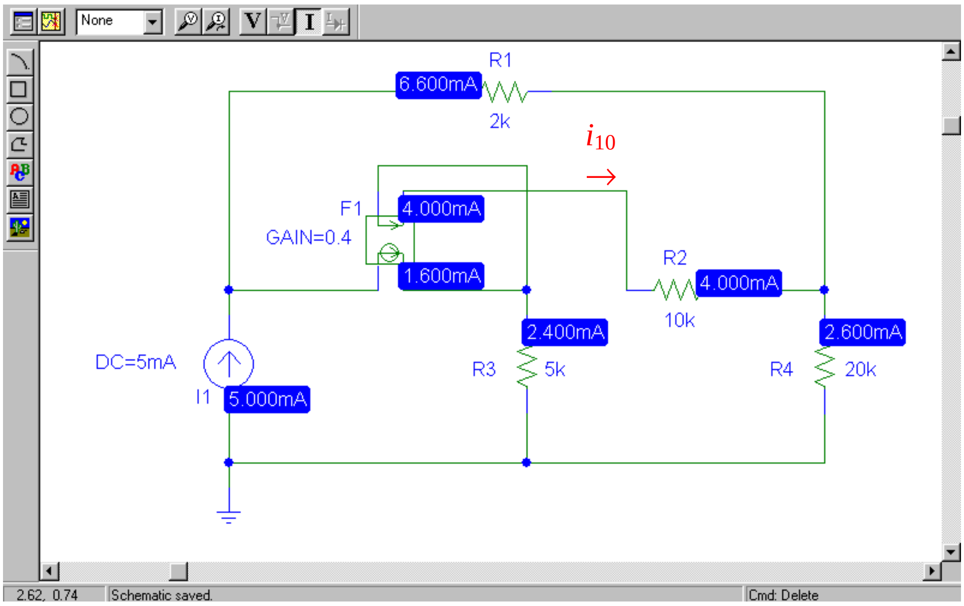
MESH 3: $-5000\, i_1 - 10000\, i_2 + 35000\, i_3 = 0$ [3]

Simplify:
$$0.6\, i_2 + 0.4\, i_3 = 5\times10^{-3} \quad [2]$$
$$-10000\, i_2 + 35000\, i_3 = 25 \quad [3]$$

Solving, we find $i_2 = 6.6$ mA and $i_3 = 2.6$ mA. Since $i_{10} = i_3 - i_2$, we find that

$$\boxed{i_{10} = -4 \text{ mA.}}$$

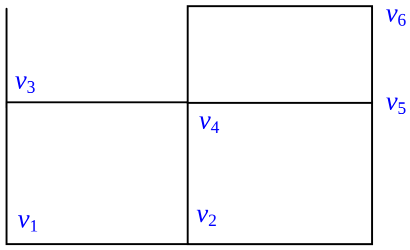
PSpice simulation results:



Summary: The current entering the right-hand node of the 10-kΩ resistor R2 is equal to 4.000 mA. Since this current is $-i_{10}$, $i_{10} = -4.000$ mA as found by hand.

52. **Hand analysis:**

Define the nodes as:



NODE 1: $-2\times10^{-3} = (v_1 - 1.3)/\ 1.8\times10^3 \quad \rightarrow \quad v_1 = -2.84$ V.

2, 4 Supernode:

$2.3\times10^{-3} = (v_2 - v_5)/\ 1\text{x}10^3 + (v_4 - 1.3)/\ 7.3\times10^3 + (v_4 - v_5)/\ 1.3\times10^3 + v_4/\ 1.5\times10^3$

KVL equation: $-v_2 + v_4 = 5.2$

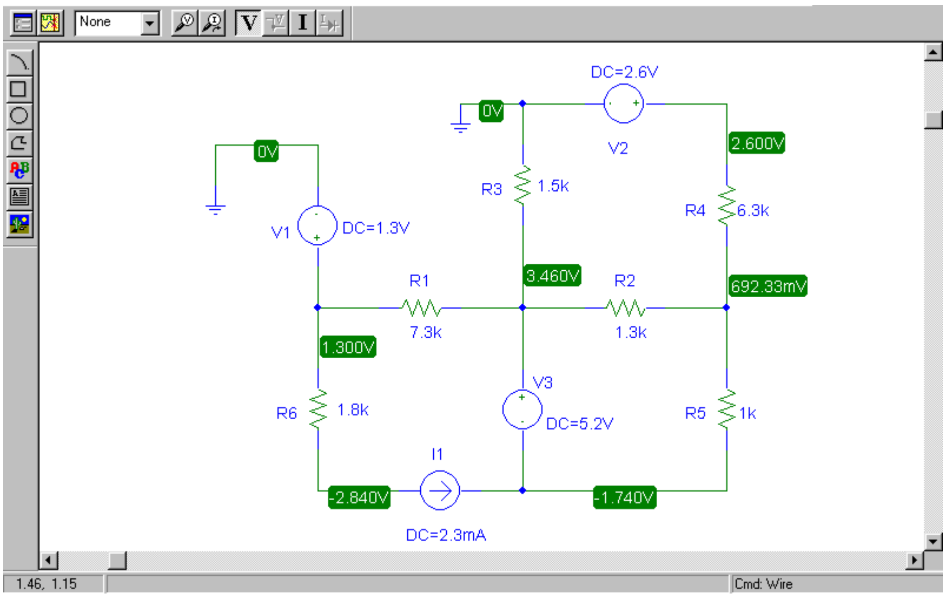
Node 5: $0 = (v_5 - v_2)/\ 1\text{x}10^3 + (v_5 - v_4)/\ 1.3\text{x}10^3 + (v_5 - 2.6)/\ 6.3\text{x}10^3$

Simplifying and collecting terms,

$$
\begin{aligned}
14.235\, v_2 + 22.39\, v_4 - 25.185\, v_5 &= 35.275 \quad [1] \\
-v_2 + \qquad v_4 \qquad\qquad &= 5.2 \quad [2] \\
-8.19\, v_2 \qquad - 6.3\, v_4 + 15.79\, v_5 &= 3.38 \quad [3]
\end{aligned}
$$

Solving, we find the voltage at the central node is $\boxed{v_4 = 3.460 \text{ V.}}$

PSpice simulation results:



Summary: The voltage at the center node is found to be 3.460 V, which is in agreement with our hand calculation.

53. **Hand analysis:**

At the 1,2 supermesh: $2000\, i_1 + 6000\, i_2 - 3 + 5000\, i_2 = 0$ [1]

and $i_1 - i_2 = 2\times10^{-3}$ [2]

by inspection, $i_4 = -1$ mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and i_1 and i_2 are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain

$2000\, i_1 + 11000\, i_2 = 3$ [1]

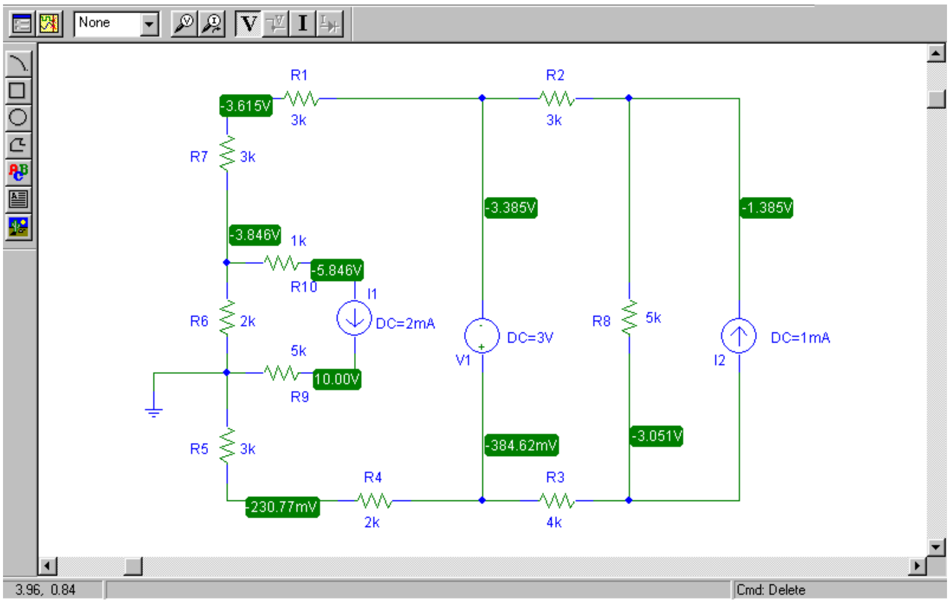
$1000\, i_1 - 1000\, i_2 = 2$ [2]

Solving, $i_1 = 1.923$ mA and $i_2 = -76.92$ μA.

Thus, the voltage across the 2-mA source ("+" reference at the top of the source) is

$$v = -2000\, i_1 - 6000\,(i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

PSpice simulation results:



Summary: Again arbitrarily selecting the "+" reference as the top node of the 2-mA current source, we find the voltage across it is –5.846 – 10 = -15.846 V, in agreement with our hand calculation.

54. **Hand analysis:**
Define a voltage v_x at the top node of the current source I_2, and a clockwise mesh current i_b in the right-most mesh.

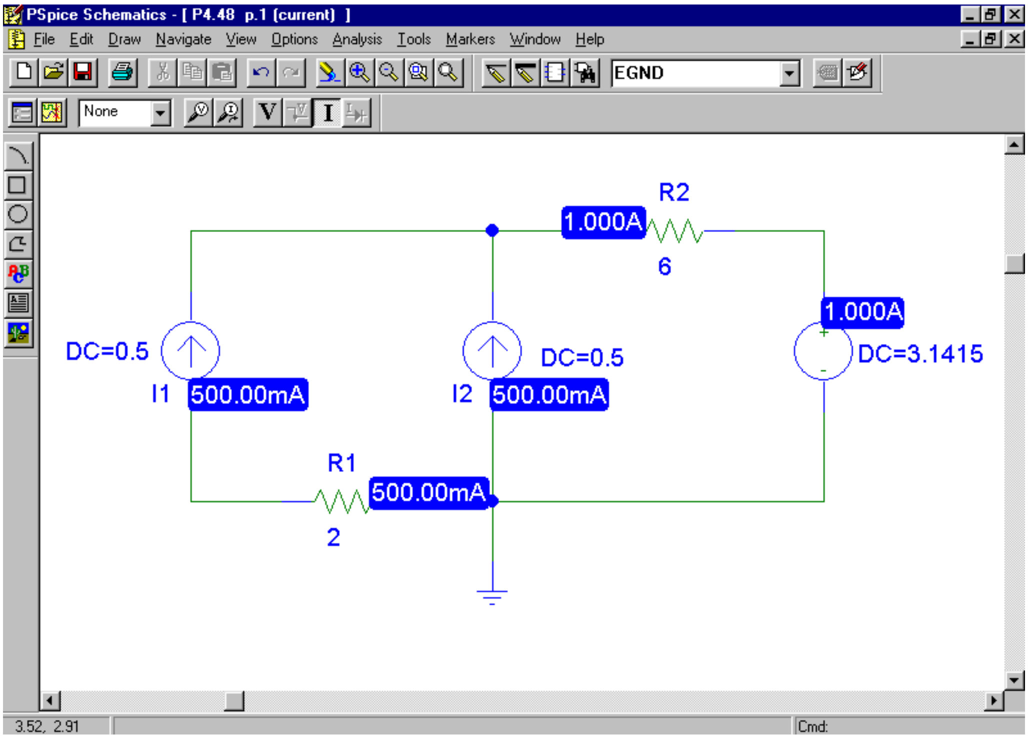
We want 6 W dissipated in the 6-Ω resistor, which leads to the requirement i_b = 1 A. Applying nodal analysis to the circuit,

$$I_1 + I_2 \;=\; (v_x - v_1)/\,6 \;=\; 1$$

so our requirement is $I_1 + I_2 = 1$. There is no constraint on the value of v_1 other than we are told to select a nonzero value.

Thus, $\boxed{\text{we choose } I_1 = I_2 = 500 \text{ mA and } v_1 = 3.1415 \text{ V.}}$

PSpice simulation results:



Summary: We see from the labeled schematic above that our choice for I_1, I_2 and V_1 lead to 1 A through the 6-Ω resistor, or 6 W dissipated in that resistor, as desired.

55. **Hand analysis:**

Define node 1 as the top left node, and node 2 as the node joining the three 2-Ω resistors. Place the "+" reference terminal of the 2-V source at the right. The right-most 2-Ω resistor has therefore been shorted out. Applying nodal analysis then,

Node 1: $-5\ i_1 = (v_1 - v_2)/\ 2$ [1]

Node 2: $0 = (v_2 - v_1)/\ 2 + v_2/\ 2 + (v_2 - 2)/\ 2$ [2]

and, $i_1 = (v_2 - 2)/\ 2$ [3]

Simplifying and collecting terms,

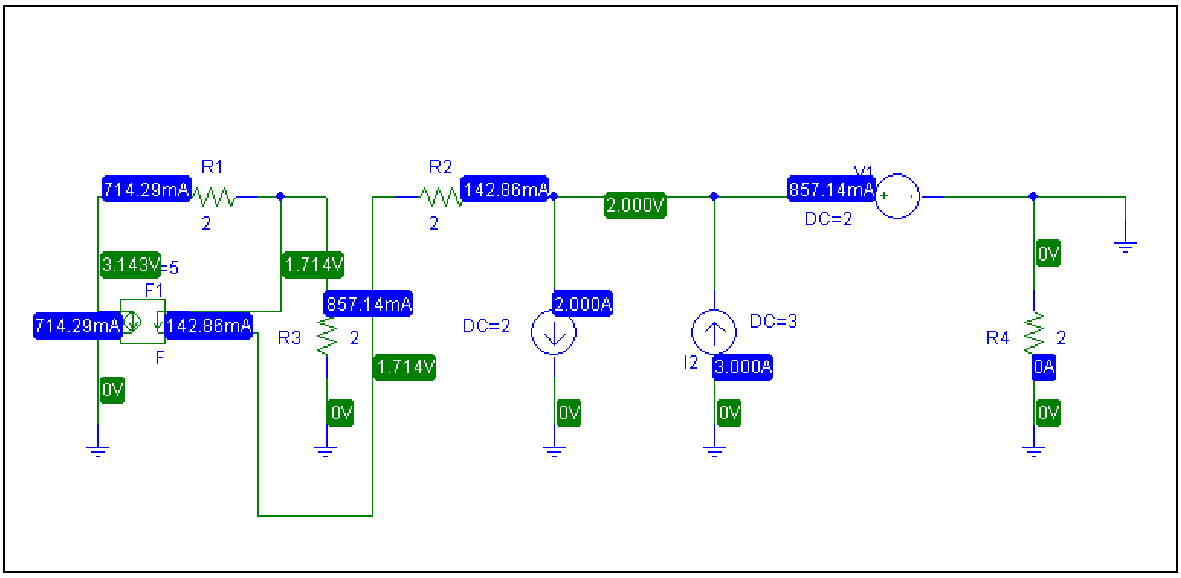
$v_1 + \quad v_2 = 10$ [1]
$-v_1 + 3\ v_2 = 2$ [2]

Solving, we find that $\boxed{v_1 = 3.143 \text{ V} \quad \text{and} \quad v_2 = 1.714 \text{ V}.}$

Defining clockwise mesh currents i_a, i_b, i_c, i_d starting with the left-most mesh and proceeding right, we may easily determine that

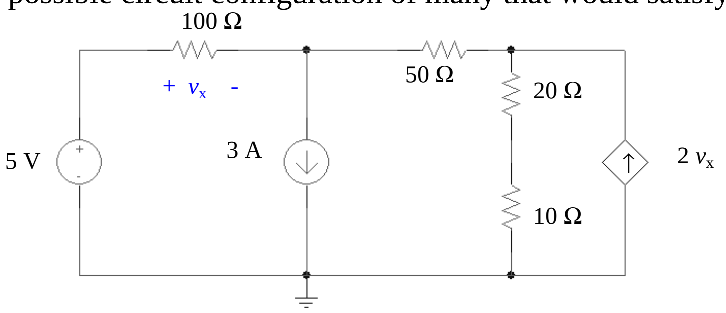
$$\boxed{\begin{aligned} i_a &= -5\ i_1 = 714.3 \text{ mA} \\ i_b &= -142.9 \text{ mA} \\ i_c &= i_1 - 2 = -2.143 \text{ A} \\ i_d &= 3 + i_c = 857.1 \text{ mA} \end{aligned}}$$

PSpice simulation results:



Summary: The simulation results agree with the hand calculations.

56. (a) One possible circuit configuration of many that would satisfy the requirements:



At node 1: $-3 = (v_1 - 5)/ 100 + (v_1 - v2)/ 50$ [1]

At node 2: $2\ v_x = (v2 - v_1)/ 50 + v_2/ 30$ [2]

and, $v_x = 5 - v_1$ [3]

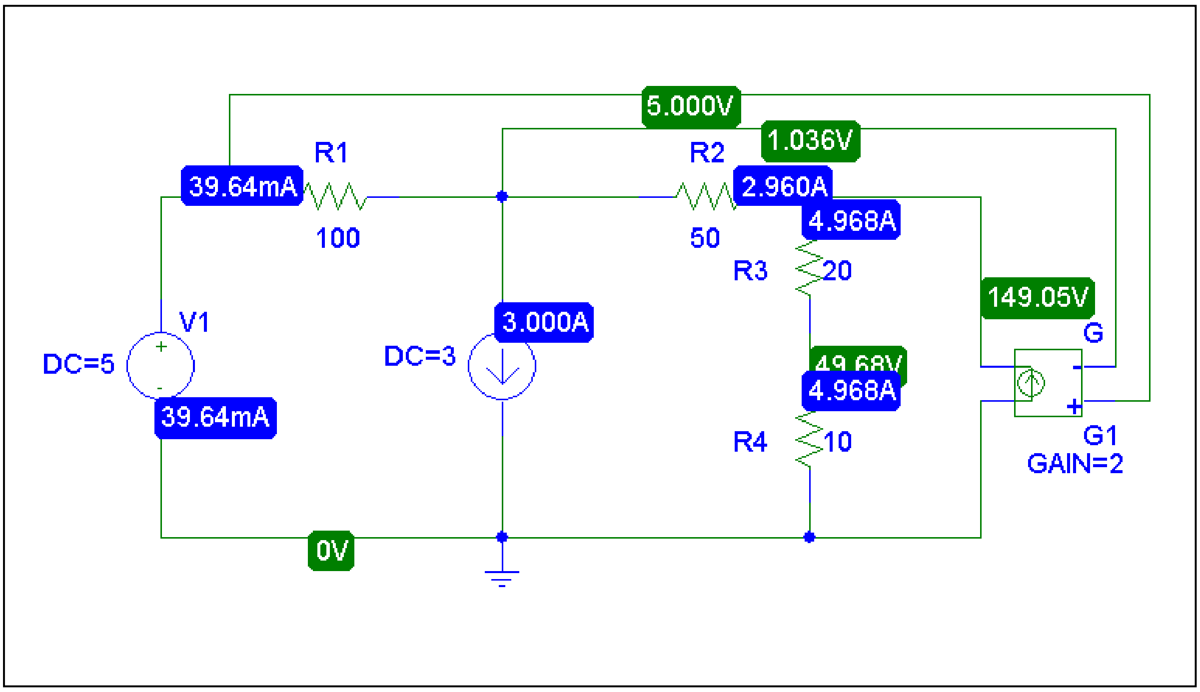
Simplifying and collecting terms,

$150\ v_1 - 100\ v_2 = -14750$ [1]

$2970\ v_1 + 80\ v_2 = 15000$ [2]

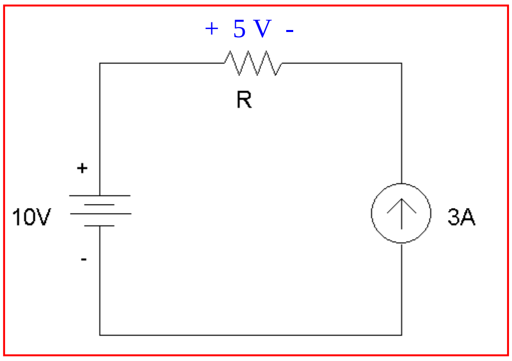
Solving, we find that $\boxed{v_1 = 1.036\text{ V}}$ and $\boxed{v_2 = 149.1\text{ V.}}$
The current through the 100-Ω resistor is simply $(5 - v_1)/100 = \boxed{39.64\text{ mA}}$
The current through the 50-Ω resistor is $(v_1 - v_2)/ 50 = \boxed{-2.961\text{ A,}}$
and the current through the 20-Ω and 10-Ω series combination is $v_2/ 30 = \boxed{4.97\text{ A.}}$
Finally, the dependent source generates a current of $2\ v_x = \boxed{7.928\text{ A.}}$

(b) PSpice simulation results



Summary: The simulated results agree with the hand calculations.

57. One possible solution of many:



Choose R so that 3R = 5; then the voltage across the current source will be 5 V, and so will the voltage across the resistor R.

R = 5/3 Ω. To construct this from 1-Ω resistors, note that

$$\boxed{5/3\ \Omega = 1\ \Omega + 2/3\ \Omega = 1\ \Omega + 1\ \Omega \parallel 1\Omega \parallel 1\Omega + 1\Omega \parallel 1\Omega \parallel 1\Omega}$$

```
* Solution to Problem 4.57

.OP

V1 1 0 DC 10
I1 0 4 DC 3
R1 1 2 1
R2 2 3 1
R3 2 3 1
R4 2 3 1
R5 3 4 1
R6 3 4 1
R7 3 4 1

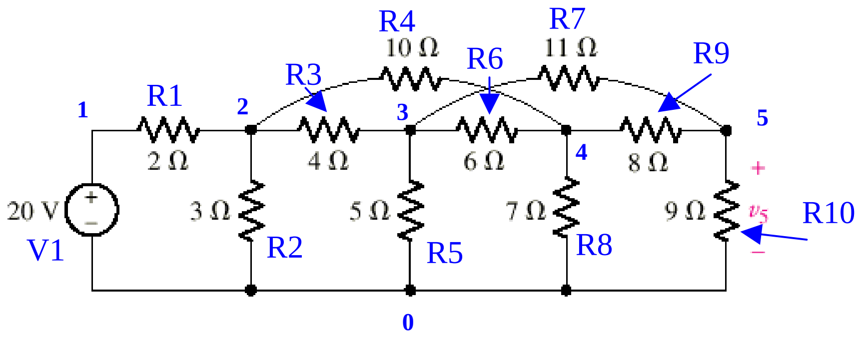
.END
```

```
****    SMALL SIGNAL BIAS SOLUTION      TEMPERATURE =  27.000 DEG C
******************************************************************************
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(   1)  10.0000  (   2)   7.0000  (   3)   6.0000  (   4)   5.0000

   VOLTAGE SOURCE CURRENTS
   NAME         CURRENT

   V1          -3.000E+00
```


58. We first name each node, resistor and voltage source:



We next write an appropriate input deck for SPICE:

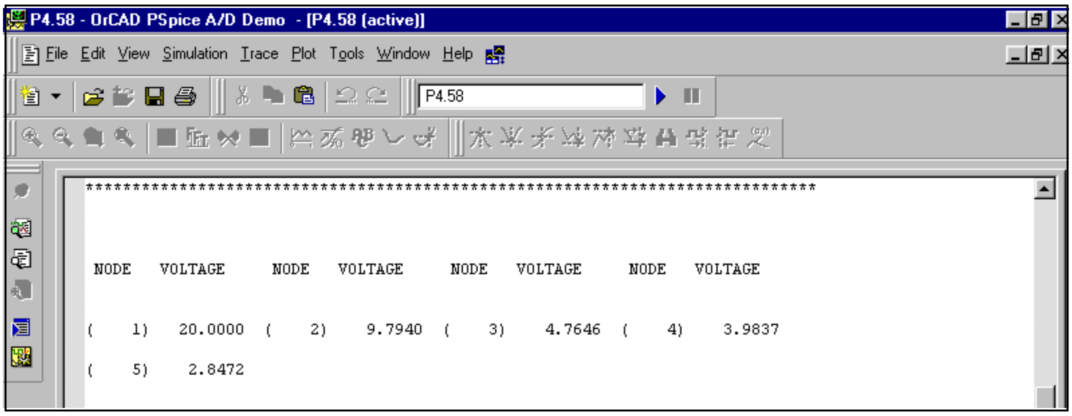
```
* Solution to Problem 4.58

.OP

V1 1 0 DC 20
R1 1 2 2
R2 2 0 3
R3 2 3 4
R4 2 4 10
R5 3 0 5
R6 3 4 6
R7 3 5 11
R8 4 0 7
R9 4 5 8
R10 5 0 9

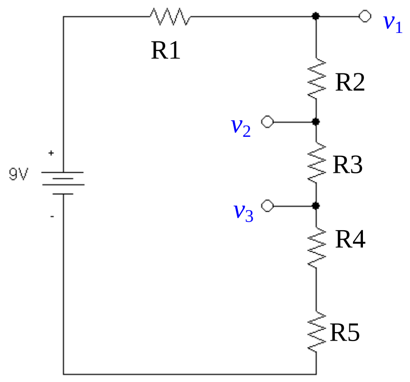
.END
```

And obtain the following output:



We see from this simulation result that the voltage v_5 = 2.847 V.

59. One possible solution of many:



All resistors are 1 Ω, except R1, which represents 5 1-Ω resistors in series.

Verify:

$v_1 = 9(4/9) = 4$ V
$v_2 = 9(3/9) = 3$ V
$v_3 = 9(2/9) = 2$ V

SPICE INPUT DECK:

```
* Solution to Problem 4.59

.OP

V1 1 0 DC 9
R1 1 2 5
R2 2 3 1
R3 3 4 1
R4 4 5 1
R5 5 0 1

.END
```

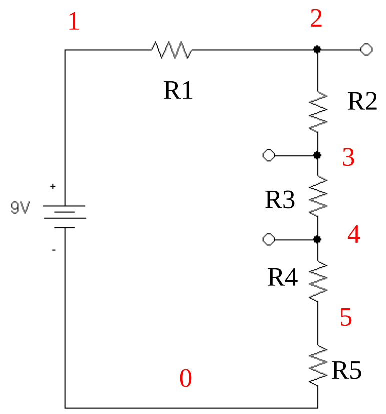
```
**** 07/29/01 21:36:26 *********** Evaluation PSpice (Nov 1999) **************

 * Solution to Problem 4.59
 ****     SMALL SIGNAL BIAS SOLUTION        TEMPERATURE =   27.000 DEG C
*********************************************************************
*******

 NODE   VOLTAGE      NODE    VOLTAGE      NODE    VOLTAGE      NODE
VOLTAGE

(   1)    9.0000  (    2)    4.0000  (    3)    3.0000  (    4)    2.0000

(   5)    1.0000
```



60. (a) If only two bulbs are not lit (and thinking of each bulb as a resistor), the bulbs must be in parallel- otherwise, the burned out bulbs, acting as short circuits, would prevent current from flowing to the "good" bulbs.

(b) In a parallel connected circuit, each bulb "sees" 115 VAC. Therefore, the individual bulb current is 1 W/ 115 V = 8.696 mA. The resistance of each "good" bulb is V/I = 13.22 kΩ. A simplified, electrically-equivalent model for this circuit would be a 115 VAC source connected in parallel to a resistor R_{eq} such that

$1/R_{eq} = 1/13.22\times10^3 + 1/13.22\times10^3 + \ldots + 1/13.22\times10^3$ (400 – 2 = 398 terms)
or $R_{eq} = 33.22\ \Omega$. We expect the source to provide 398 W.

```
* Solution to Problem 4.60

.OP

V1 1 0 AC 115 60
R1 1 0 33.22

.AC LIN 1 60 60
.PRINT AC VM(1)IM(V1)

.END
```

```
**** 07/29/01 21:09:32 *********** Evaluation PSpice (Nov 1999) **************

 * Solution to Problem 4.60

 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
******************************************************************************

 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(    1)    0.0000

   VOLTAGE SOURCE CURRENTS
   NAME         CURRENT

   V1           0.000E+00

   TOTAL POWER DISSIPATION   0.00E+00  WATTS

**** 07/29/01 21:09:32 *********** Evaluation PSpice (Nov 1999)
 * Solution to Problem 4.60


 ****     AC ANALYSIS                  TEMPERATURE =   27.000 DEG C
******************************************************************************

  FREQ        VM(1)       IM(V1)
  6.000E+01   1.150E+02   3.462E+00
```

This calculated power is not the value sought. It is an artifact of the use of an ac source, which requires that we perform an ac analysis. The supplied power is then separately computed as $(1.15\times10^2)(3.462) = 398.1$ W.

(c) The inherent series resistance of the wire connections leads to a voltage drop which increases the further one is from the voltage source. Thus, the furthest bulbs actually have less than 115 VAC across them, so they draw slightly less current and glow more dimly.

CHAPTER FIVE SOLUTIONS

1. Define percent error as 100 $[e^x - (1 + x)]/ e^x$

x	$1 + x$	e^x	% error
0.001	1.001	1.001	5×10^{-5}
0.005	1.005	1.005	1×10^{-3}
0.01	1.01	1.010	5×10^{-3}
0.05	1.05	1.051	0.1
0.10	1.10	1.105	0.5
0.50	1.50	1.649	9
1.00	2.00	2.718	26
5.00	6.00	148.4	96

Of course, "reasonable" is a very subjective term. However, if we choose $x < 0.1$, we ensure that the error is less than 1%.

2. i_A, v_B "on", $v_C = 0$: $i_x = 20$ A
i_A, v_C "on", $v_B = 0$: $i_x = -5$ A
i_A, v_B, v_C "on" : $i_x = 12$ A

so, we can write
$$i_x' + i_x'' + i_x''' = 12$$
$$i_x' + i_x'' = 20$$
$$i_x' + i_x''' = -5$$

In matrix form,
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x' \\ i_x'' \\ i_x''' \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ -5 \end{bmatrix}$$

(a) with i_A on only, the response $i_x = i_x' = \boxed{3\text{ A.}}$
(b) with v_B on only, the response $i_x = i_x'' = \boxed{17\text{ A.}}$
(c) with v_C on only, the response $i_x = i_x''' = \boxed{-8\text{ A.}}$
(d) i_A and v_C doubled, v_B reversed: $2(3) + 2(-8) + (-1)(17) = \boxed{-27\text{ A.}}$

3. One source at a time:
The contribution from the 24-V source may be found by shorting the 45-V source and open-circuiting the 2-A source. Applying voltage division,

$$v_x\text{'} = 24\frac{20}{10+20+45 \parallel 30} = 24\frac{20}{10+20+18} = 10\text{ V}$$

We find the contribution of the 2-A source by shorting both voltage sources and applying current division:

$$v_x\text{''} = 20\left[2\frac{10}{10+20+18}\right] = 8.333\text{ V}$$

Finally, the contribution from the 45-V source is found by open-circuiting the 2-A source and shorting the 24-V source. Defining v_{30} across the 30-Ω resistor with the "+" reference on top:

$$0 = v_{30}/\,20 + v_{30}/\,(10 + 20) + (v_{30} - 45)/\,45$$

solving, v_{30} = 11.25 V and hence v_x''' = -11.25(20)/(10 + 20) = -7.5 V

Adding the individual contributions, we find that $v_x = v_x' + v_x'' + v_x'''$ = $\boxed{10.83\text{ V}}$.

4. The contribution of the 8-A source is found by shorting out the two voltage sources and employing simple current division:

$$i_3' = -8\frac{50}{50+30} = -5\text{ A}$$

The contribution of the voltage sources may be found collectively or individually. The contribution of the 100-V source is found by open-circuiting the 8-A source and shorting the 60-V source. Then,

$$i_3'' = \frac{100}{(50+30)\parallel 60 \parallel 30} = 6.25\text{ A}$$

The contribution of the 60-V source is found in a similar way as $i_3''' = -60/30 = -2$ A.

The total response is $i_3 = i_3' + i_3'' + i_3''' = \boxed{-750\text{ mA.}}$

CHAPTER FIVE SOLUTIONS

5. (a) By current division, the contribution of the 1-A source i_2' is
$i_2' = 1\,(200)/\,250 = 800$ mA.

The contribution of the 100-V source is $i_2'' = 100/\,250 = 400$ mA.

The contribution of the 0.5-A source is found by current division once the 1-A source is open-circuited and the voltage source is shorted. Thus,

$$i_2''' = 0.5\,(50)/\,250 = 100 \text{ mA}$$

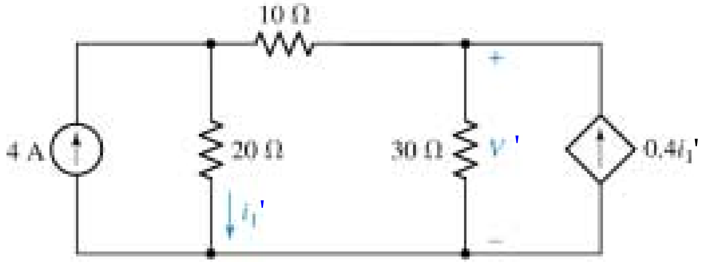
Thus, $i_2 = i_2' + i_2'' + i_2''' = \boxed{1.3 \text{ A}}$

(b)

$$\boxed{\begin{array}{l} P_{1A} = (1)\,[(200)(1 - 1.3)] = 60 \text{ W} \\ P_{200} = (1 - 1.3)^2\,(200) = 18 \text{ W} \\ P_{100V} = -(1.3)(100) = -130 \text{ W} \\ P_{50} = (1.3 - 0.5)^2\,(50) = 32 \text{ W} \\ P_{0.5A} = (0.5)\,[(50)(1.3 - 0.5)] = 20 \text{ W} \end{array}}$$

Check: 60 + 18 + 32 + 20 = +130.

6. We find the contribution of the 4-A source by shorting out the 100-V source and analysing the resulting circuit:



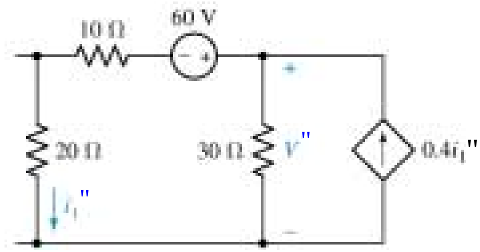
$4 = V_1'/20 + (V_1' - V')/10$ [1]

$0.4\, i_1' = V_1'/30 + (V' - V_1')/10$ [2]

where $i_1' = V_1'/20$

Simplifying & collecting terms, we obtain $30\, V_1' - 20\, V' = 800$ [1]
$-7.2\, V_1' + 8\, V' = 0$ [2]

Solving, we find that V' = 60 V. Proceeding to the contribution of the 60-V source, we analyse the following circuit after defining a clockwise mesh current i_a flowing in the left mesh and a clockwise mesh current i_b flowing in the right mesh.



$30\, i_a - 60 + 30\, i_a - 30\, i_b = 0$ [1]
$i_b = -0.4\, i_1'' = +0.4\, i_a$ [2]

Solving, we find that $i_a = 1.25$ A and so $V'' = 30(i_a - i_b) = 22.5$ V.

Thus, $V = V' + V'' = \boxed{82.5 \text{ V.}}$

7. (a) Linearity allows us to consider this by viewing each source as being scaled by 25/ 10. This means that the response (v_3) will be scaled by the same factor:

$$25\, i_A'/ 10 + 25\, i_B'/ 10 = 25\, v_3'/ 10$$

$$\therefore\ v_3 = 25v_3'/ 10 = 25(80)/ 10 = \boxed{200 \text{ V}}$$

(b) $i_A' = 10$ A, $i_B' = 25$ A $\rightarrow v_4' = 100$ V
$i_A'' = 10$ A, $i_B'' = 25$ A $\rightarrow v_4'' = -50$ V
$i_A = 20$ A, $i_B = -10$ A $\rightarrow v_4 = ?$

We can view this in a somewhat abstract form: the currents i_A and i_B multiply the same circuit parameters regardless of their value; the result is v_4.

Writing in matrix form, $\begin{bmatrix} 10 & 25 \\ 25 & 10 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \end{bmatrix}$, we can solve to find

a = -4.286 and b = 5.714, so that 20a – 10b leads to $\boxed{v_4 = -142.9 \text{ V}}$

8. With the current source open-circuited and the 7-V source shorted, we are left with 100k || (22k + 4.7k) = 21.07 kΩ.

Thus, V_{3V} = 3 (21.07)/ (21.07 + 47) = 0.9286 V.

In a similar fashion, we find that the contribution of the 7-V source is:

V_{7V} = 7 (31.97) / (31.97 + 26.7) = 3.814 V

Finally, the contribution of the current source to the voltage V across it is:

$V_{5mA} = (5\times10^{-3})$ (47k || 100k || 26.7k) = 72.75 V.

Adding, we find that V = 0.9286 + 3.814 + 72.75 = $\boxed{77.49 \text{ V.}}$

9. We must find the current through the 500-kΩ resistor using superposition, and then calculate the dissipated power.

The contribution from the current source may be calculated by first noting that 1M || 2.7M || 5M = 636.8 kΩ. Then,

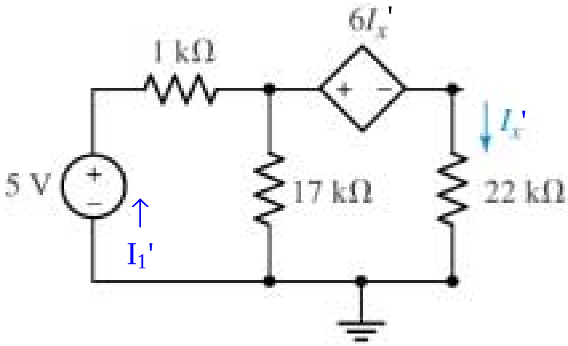
$$i_{60\mu A} = 60\times10^{-6}\left(\frac{3}{0.5+3+0.6368}\right) = 43.51\,\mu A$$

The contribution from the voltage source is found by first noting that 2.7M || 5M = 1.753 MΩ. The total current flowing from the voltage source (with the current source open-circuited) is –1.5/ (3.5 || 1.753 + 1) μA = -0.6919 μA. The current flowing through the 500-kΩ resistor due to the voltage source acting alone is then

$$i_{1.5V} = 0.6919\ (1.753)/\ (1.753 + 3.5)\ \text{mA} = 230.9\ \text{nA}.$$

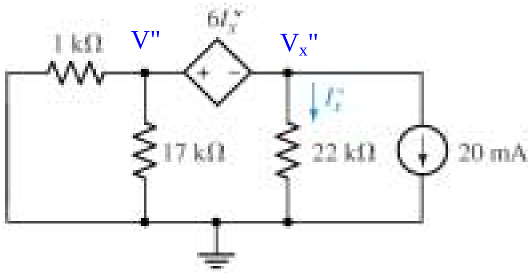
The total current through the 500-kΩ resistor is then $i_{60\mu A} + i_{1.5V}$ = 43.74 μA and the dissipated power is $(43.74\times10^{-9})^2\ (500\times10^3)$ = $\boxed{956.6\ \mu W.}$

10. We first determine the contribution of the voltage source:



Via mesh analysis, we write: $5 = 18000\ I_1' - 17000\ I_x'$
$-6\ I_x' = -17000\ I_x' + 39000\ I_x'$

Solving, we find $I_1' = 472.1$ mA and $I_x' = 205.8$ mA, so $V' = 17\times10^3\ (I_1' - I_x')$ $= 4.527$ V. We proceed to find the contribution of the current source:



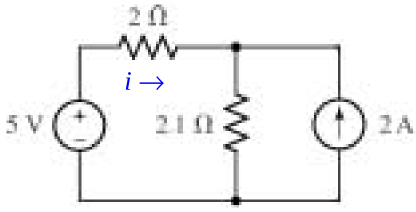
Via supernode: $-20\times10^{-3} = V_x''/\ 22\times10^3 + V''/\ 0.9444\times10^3$ [1]
and $V'' - V_x'' = 6I_x''$ or $V'' - V_x'' = 6\ V_x''/\ 22\times10^3$ [2]

Solving, we find that $V'' = -18.11$ V. Thus, $V = V' + V'' = \boxed{-13.58\text{ V.}}$

The maximum power is $V^2/\ 17\times10^3 = V^2/\ 17$ mW $= 250$ mW, so
$V = \sqrt{(17)(250)} = 65.19 = V' - 13.58$. Solving, we find $V'_{max} = 78.77$ V.
The 5-V source may then be increased by a factor of 78.77/ 4.527, so that its maximum positive value is $\boxed{87\text{ V}}$; past this value, and the resistor will overheat.

11. It is $\boxed{\text{impossible}}$ to identify the individual contribution of each source to the power dissipated in the resistor; superposition cannot be used for such a purpose.

Simplifying the circuit, we may at least determine the total power dissipated in the resistor:



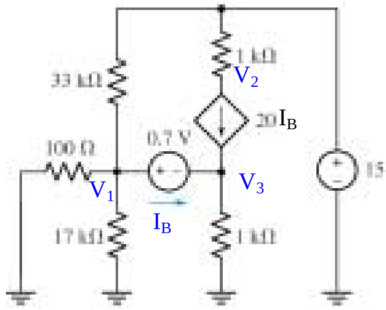
Via superposition in one step, we may write

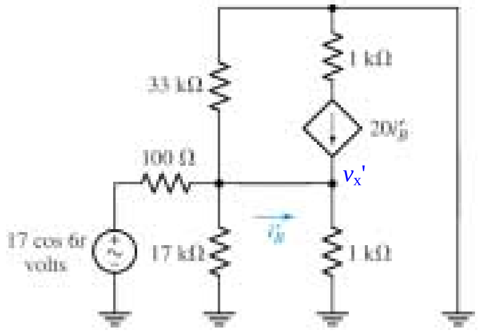
$$i = \frac{5}{2+2.1} - 2\frac{2.1}{2+2.1} = 195.1\,\text{mA}$$

Thus,

$$P_{2\Omega} = i^2 \cdot 2 = \boxed{76.15\text{ mW}}$$

12. We will analyse this circuit by first considering the combined effect of both dc sources (left), and then finding the effect of the single ac source acting alone (right).





1, 3 supernode: $V_1/ 100 + V_1/ 17\times10^3 + (V_1 - 15)/ 33\times10^3 + V_3/ 10^3 = 20\, I_B$ [1]

and: $V_1 - V_3 = 0.7$ [2]

Node 2: $-20\, I_B = (V_2 - 15)/ 1000$ [3]

We require one additional equation if we wish to have I_B as an unknown:

$20\, I_B + I_B = V_3/ 1000$ [4]

Simplifying and collecting terms,

$10.08912\, V_1 + V_3 - 20\times10^3\, I_B = 0.4545$ [1]

$V_1 - V_3 = 0.7$ [2]

$V_2 + 20\times10^3\, I_B = 15$ [3]

$-V_3 + 21\times103\, I_B = 0$ [4]

Solving, we find that $I_B = -31.04\ \mu A$.

To analyse the right-hand circuit, we first find the Thévenin equivalent to the left of the wire marked i_B', noting that the 33-kΩ and 17-kΩ resistors are now in parallel. We find that $V_{TH} = 16.85 \cos 6t$ V by voltage division, and $R_{TH} = 100 \parallel 17k \parallel 33k = 99.12\ \Omega$. We may now proceed:

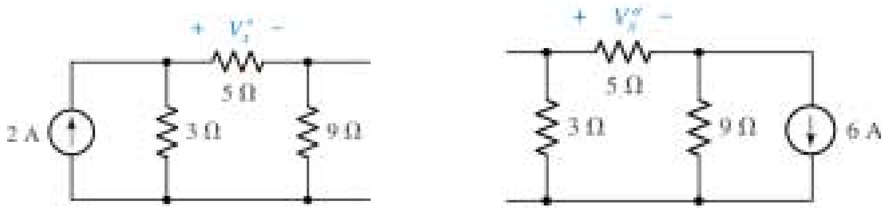
$20\, i_B' = v_x' / 1000 + (v_x' - 16.85 \cos 6t)/ 99.12$ [1]

$20\, i_B' + i_B'' = v_x'/ 1000$ [2]

Solving, we find that $i_B' = 798.6 \cos 6t$ mA. Thus, adding our two results, we find the complete current is

$$\boxed{i_B = i_B' + I_B = -31.04 + 798.6 \cos 6t\ \ \mu A.}$$

13.



We first consider the effect of the 2-A source separately, using the left circuit:

$$V_x' \;=\; 5\left[2\,\frac{3}{3+14}\right] = 1.765\text{ V}$$

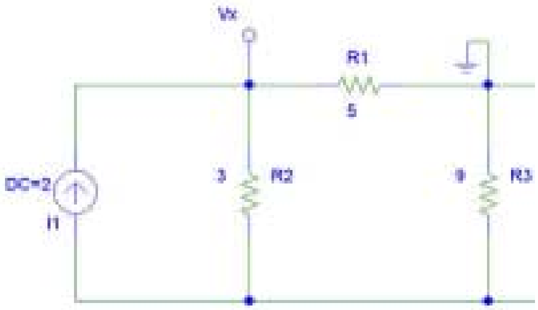
Next we consider the effect of the 6-A source on its own using the right circuit:

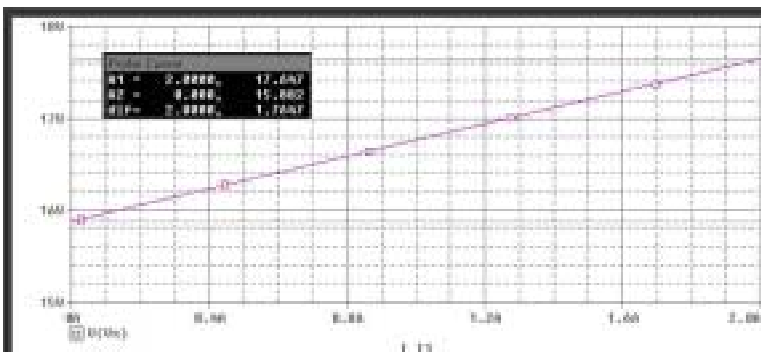
$$V_x'' \;=\; 5\left[6\,\frac{9}{9+8}\right] = 15.88\text{ V}$$

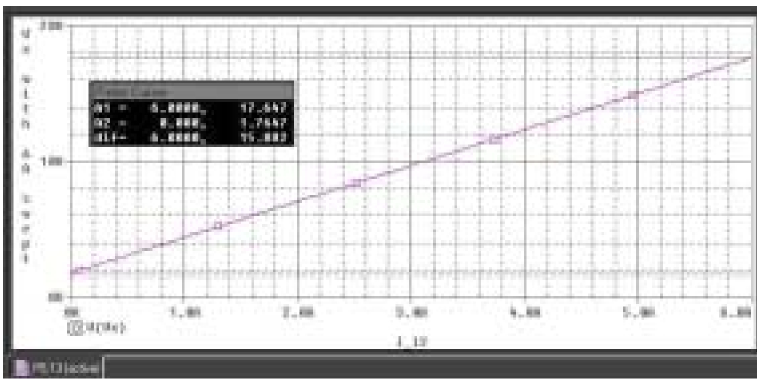
Thus, $V_x = V_x' + V_x''$ = $\boxed{17.65\text{ V}}$.

(b) PSpice verification (DC Sweep)

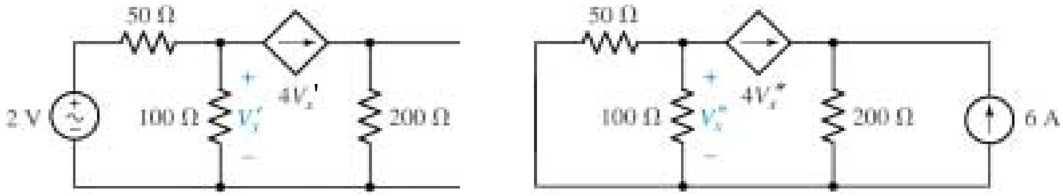
The DC sweep results below confirm that V_x' = 1.765 V







14.



(a) Beginning with the circuit on the left, we find the contribution of the 2-V source to V_x:

$$-4V_x' = \frac{V_x'}{100} + \frac{V_x' - 2}{50}$$

which leads to V_x' = 9.926 mV.

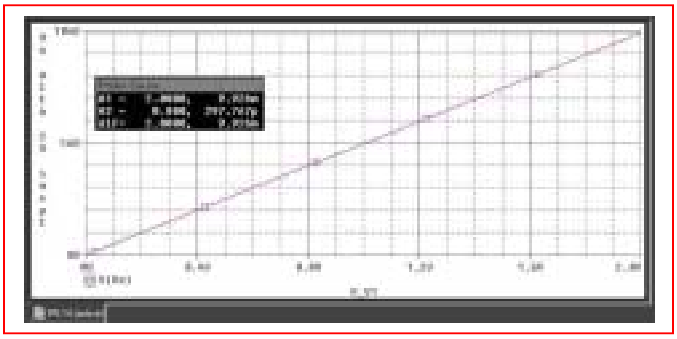
The circuit on the right yields the contribution of the 6-A source to Vx:

$$-4V_x'' = \frac{V_x''}{100} + \frac{V_x''}{50}$$

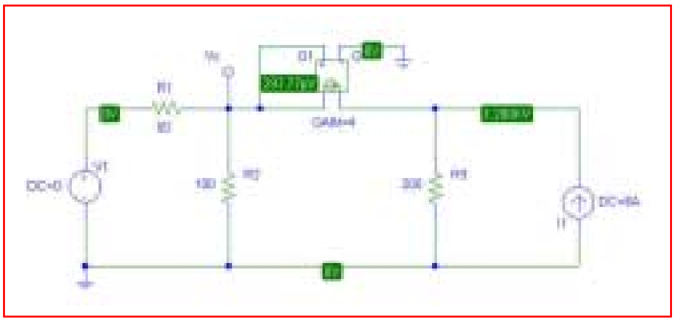
which leads to V_x'' = 0.

Thus, $V_x = V_x' + V_x''$ = $\boxed{9.926 \text{ mV.}}$

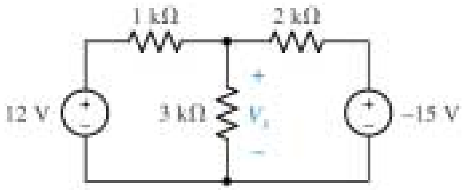
(b) PSpice verification.



As can be seen from the two separate PSpice simulations, our hand calculations are correct; the pV-scale voltage in the second simulation is a result of numerical inaccuracy.

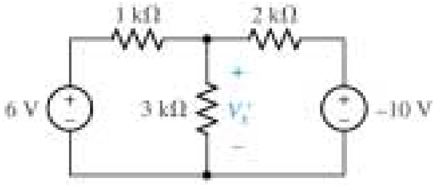


15.



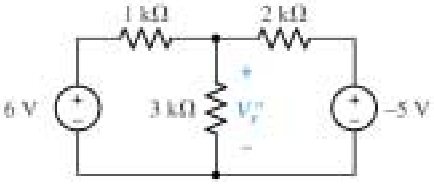
$$\frac{V_x - 12}{1} + \frac{V_x}{3} + \frac{V_x + 15}{2} = 0$$

so $\boxed{V_x = 2.455 \text{ V}}$



$$\frac{V_x' - 6}{1} + \frac{V_x'}{3} + \frac{V_x' + 10}{2} = 0$$

so $\boxed{V_x' = 0.5455 \text{ V}}$



$$\frac{V_x'' - 6}{1} + \frac{V_x''}{3} + \frac{V_x'' + 5}{2} = 0$$

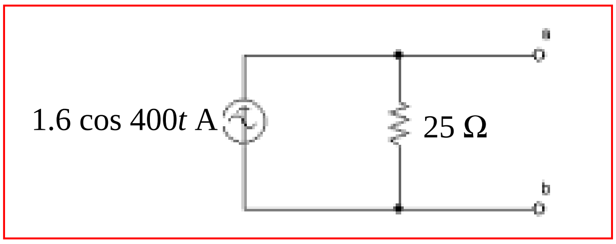
so $\boxed{V_x'' = 1.909 \text{ V}}$

Adding, we find that $V_x' + V_x'' = 2.455$ V $= V_x$ as promised.

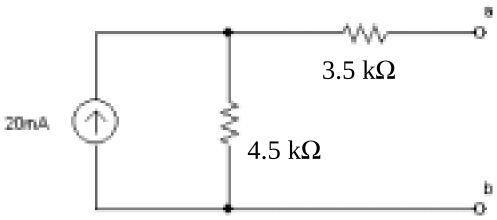
16. (a) $[120 \cos 400t] / 60 = 2 \cos 400t$ A. $60 \parallel 120 = 40\ \Omega$.

$[2 \cos 400t]\ (40) = 80 \cos 400t$ V. $40 + 10 = 50\ \Omega$.

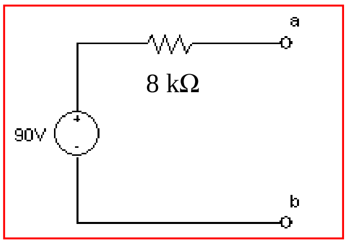
$[80 \cos 400t]/ 50 = 1.6 \cos 400t$ A. $50 \parallel 50 = 25\ \Omega$.



(b) $2k \parallel 3k + 6k = 7.2$ kΩ. $7.2k \parallel 12k = 4.5$ kΩ



$(20)(4.5) = 90$ V.

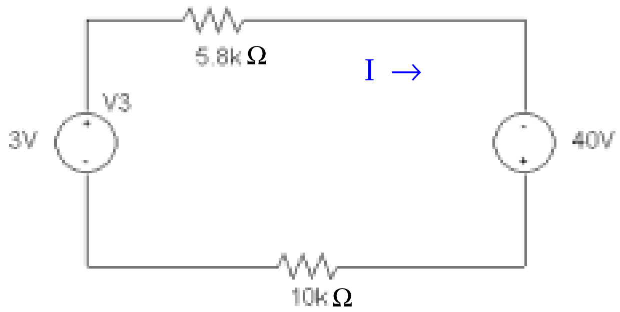


17. We can ignore the 1-kΩ resistor, at least when performing a source transformation on this circuit, as the 1-mA source will pump 1 mA through *whatever* value resistor we place there. So, we need only combine the 1 and 2 mA sources (which are in parallel once we replace the 1-kΩ resistor with a 0-Ω resistor). The current through the 5.8-kΩ resistor is then simply given by voltage division:

$$i \;=\; 3\times10^{-3} \; \frac{4.7}{4.7+5.8} \;=\; 1.343\,\text{mA}$$

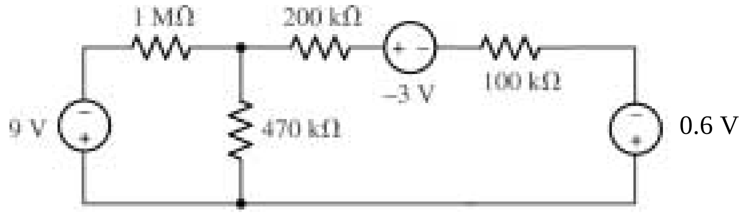
The power dissipated by the 5.8-kΩ resistor is then $i^2 \cdot 5.8\times10^3 = \boxed{10.46 \text{ mW.}}$

18. We may ignore the 10-kΩ and 9.7-kΩ resistors, as 3-V will appear across them regardless of their value. Performing a quick source transformation on the 10-kΩ resistor/ 4-mA current source combination, we replace them with a 40-V source in series with a 10-kΩ resistor:

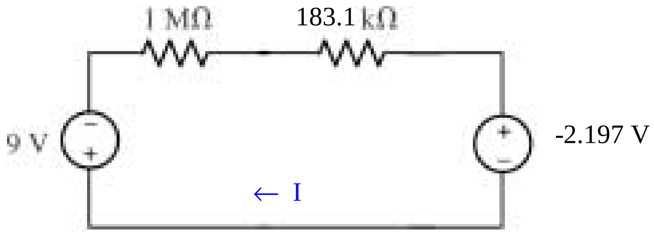


$I = 43/\ 15.8$ mA = 2.722 mA. Therefore, $P_{5.8\Omega} = I^{2} \cdot 5.8\times10^{3} = \boxed{42.97 \text{ mW.}}$

19. (100 kΩ)(6 mA) = 0.6 V



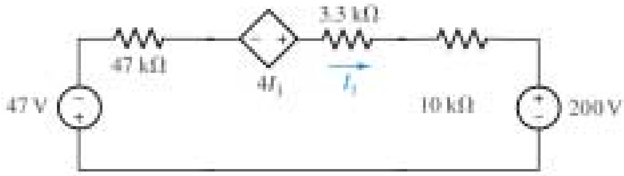
470 k || 300 k = 183.1 kΩ
$(-3 - 0.6)/\, 300\times10^3 = -12\ \mu A$
(183.1 kΩ)(-12 μA) = -2.197 V



Solving, $9 + 1183.1\times10^3\, I - 2.197 = 0$, so I = -5.750 μA. Thus,

$$P_{1M\Omega} = I^2 \cdot 10^6 = \boxed{33.06\ \mu W.}$$

20. (1)(47) = 47 V. (20)(10) = 200 V. Each voltage source "+" corresponds to its corresponding current source's arrow head.

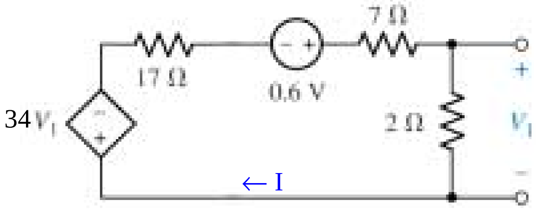


Using KVL on the simplified circuit above,

$$47 + 47\times10^3\ I_1 - 4\ I_1 + 13.3\times10^3\ I_1 + 200 = 0$$

Solving, we find that $I_1 = -247/\left(60.3\times10^3 - 4\right) = \boxed{-4.096\text{ mA.}}$

21. $(2\ V_1)(17) = 34\ V_1$



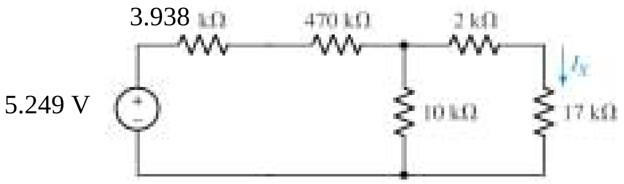
Analysing the simplified circuit above,

$34\ V_1 - 0.6 + 7\ I + 2\ I + 17\ I = 0$ [1] and $V_1 = 2\ I$ [2]

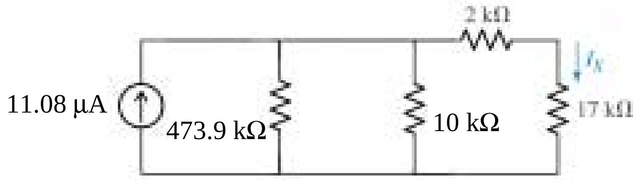
Substituting, we find that $I = 0.6/\ (68 + 7 + 2 + 17) = 6.383$ mA. Thus,

$$V_1 = 2\ I = \boxed{12.77\ \text{mV}}$$

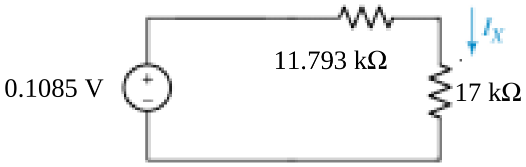
22. 12/ 9000 = 1.333 mA. 9k || 7k = 3.938 kΩ. → (1.333 mA)(3.938 kΩ) = 5.249 V.



5.249/ 473.938×10^3 = 11.08 μA

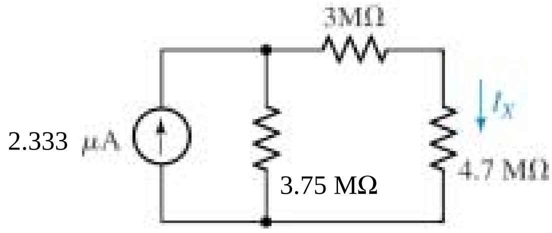


473.9 k || 10 k = 9.793 kΩ. (11.08 mA)(9.793 kΩ) = 0.1085 V



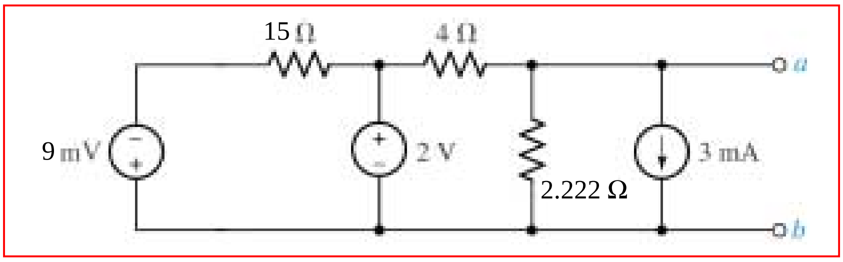
I_x = 0.1085/ 28.793×10^3 = $\boxed{3.768\ \mu A.}$

23. First, (-7 μA)(2 MΩ) = -14 V, "+" reference down. 2 MΩ + 4 MΩ = 6 MΩ.
+14 V/ 6 MΩ = 2.333 μA, arrow pointing up; 6 M || 10 M = 3.75 MΩ.



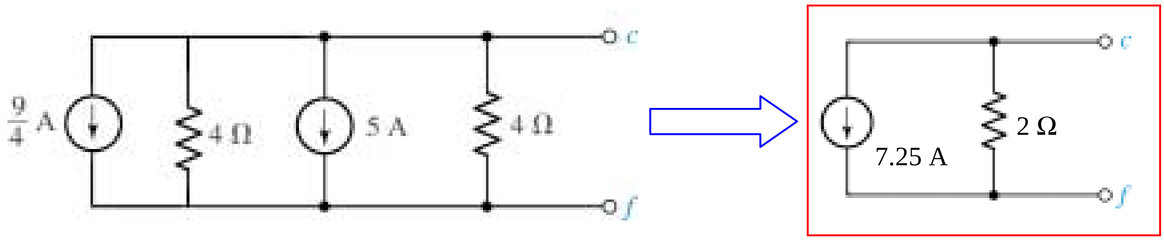
(2.333)(3.75) = 8.749 V. R_{eq} = 6.75 MΩ
∴ I_x = 8.749/ (6.75 + 4.7) μA = $\boxed{\text{764.1 nA.}}$

24. To begin, note that (1 mA)(9 Ω) = 9 mV, and 5 || 4 = 2.222 Ω.



The above circuit may not be further simplified using only source transformation techniques.

25. Label the "-" terminal of the 9-V source node **x** and the other terminal node **x'**. The 9-V source will force the voltage across these two terminals to be –9 V regardless of the value of the current source and resistor to its left. These two components may therefore be neglected from the perspective of terminals **a** & **b**. Thus, we may draw:



26. Beware of the temptation to employ superposition to compute the dissipated power- *it won't work*!

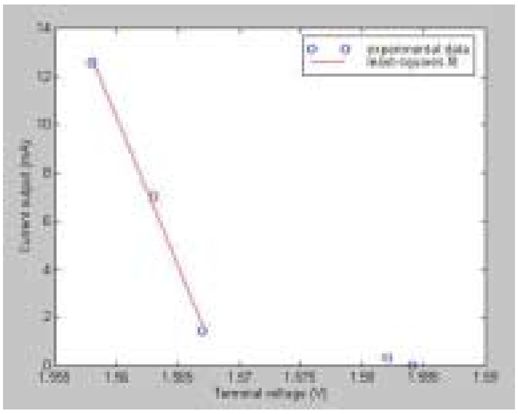
Instead, define a current I flowing into the bottom terminal of the 1-MΩ resistor. Using superposition to compute this current,

$$I = 1.8/\ 1.840 + 0 + 0\ \ \mu A = 978.3 \text{ nA.}$$

Thus,

$$P_{1M\Omega} = (978.3\times10^{-9})^2\,(10^6) = \boxed{957.1 \text{ nW.}}$$

27. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:



Least-squares fit results:

Voltage (V)	**Current (mA)**
1.567	1.6681
1.563	6.599
1.558	12.763

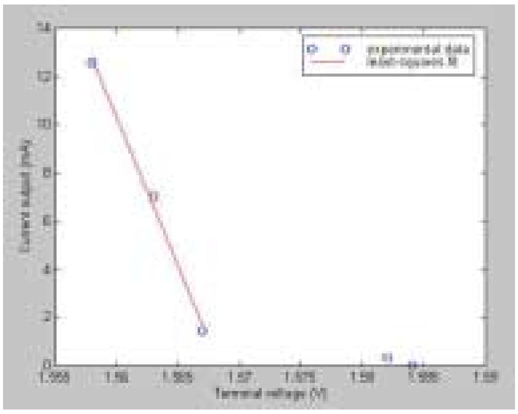
We see from the figure that we cannot draw a very good line through all data points representing currents from 1 mA to 20 mA. We have therefore chosen to perform a linear fit for the three lower voltages only, as shown. Our model will not be as accurate at 1 mA; there is no way to know if our model will be accurate at 20 mA, since that is beyond the range of the experimental data.

Modeling this system as an ideal voltage source in series with a resistance (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$1.567 = V_{src} - R_s\,(1.6681\times10^{-3})$$
$$1.558 = V_{src} - R_s\,(12.763\times10^{-3})$$

Solving, $\boxed{V_{src} = 1.568\text{ V}}$ and $\boxed{R_s = 811.2\text{ m}\Omega}$. It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.

28. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:



Least-squares fit results:

Voltage (V)	**Current (mA)**
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Modeling this system as an ideal current source in parallel with a resistance R_p (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$1.6681\times10^{-3} \;=\; I_{src} - 1.567/\,R_p$$

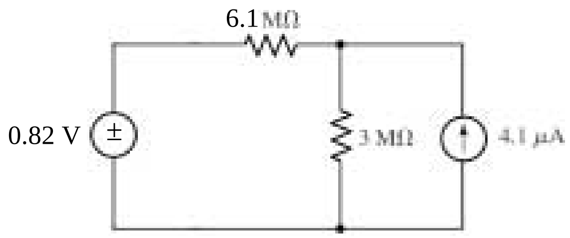
$$12.763\times10^{-3} \;=\; I_{src} - 1.558/\,R_p$$

Solving, $\boxed{I_{src} = 1.933\text{ A}}$ and $\boxed{R_s = 811.2\text{ m}\Omega}$. It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.

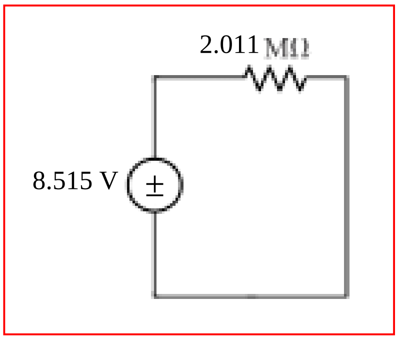
29. Reference terminals are required to avoid ambiguity: *depending on the sources with which we begin the transformation process, we will obtain entirely different answers*. Working from left to right in this case,

2 μA – 1.8 μA = 200 nA, arrow up.
1.4 MΩ + 2.7 MΩ = 4.1 MΩ

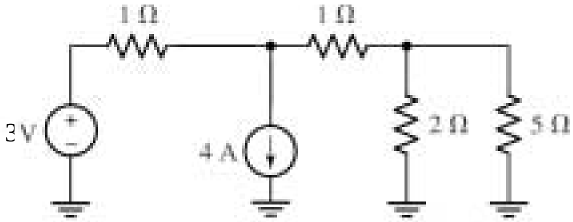
An additional transformation back to a voltage source yields (200 nA)(4.1 MΩ) = 0.82 V in series with 4.1 MΩ + 2 MΩ = 6.1 MΩ, as shown below:



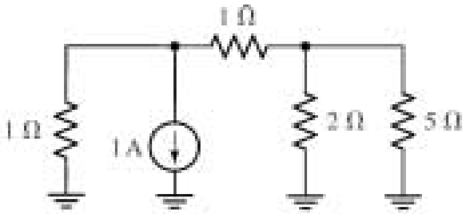
Then, 0.82 V/ 6.1 MΩ = 134.4 nA, arrow up.
6.1 MΩ || 3 MΩ = 2.011 MΩ
4.1 μA + 134.4 nA = 4.234 mA, arrow up.
(4.234 μA) (2.011 MΩ) = 8.515 V.



30. To begin, we note that the 5-V and 2-V sources are in series:



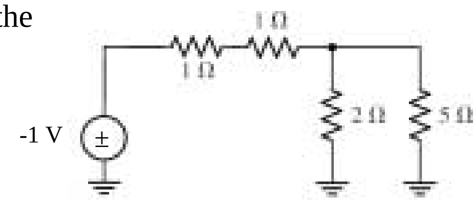
Next, noting that 3 V/ 1 Ω = 3 A, and 4 A – 3 A = +1 A (arrow down), we obtain:



The left-hand resistor and the current source are easily transformed into a 1-V source in series with a 1-Ω resistor:

By voltage division, the voltage across the 5-Ω resistor in the circuit to the right is:

$$(-1)\ \frac{2 \parallel 5}{2 \parallel 5\ +\ 2} = -0.4167 \text{ V.}$$



Thus, the power dissipated by the 5-Ω resistor is $(-0.4167)^2$ / 5 = $\boxed{34.73 \text{ mW.}}$

31. (a) $R_{TH} = 25 \parallel (10 + 15) = 25 \parallel 25 = \boxed{12.5\ \Omega.}$

$$V_{TH} = V_{ab} = 50\left(\frac{25}{10+15+25}\right) + 100\left(\frac{15+10}{15+10+25}\right) = \boxed{75\text{ V.}}$$

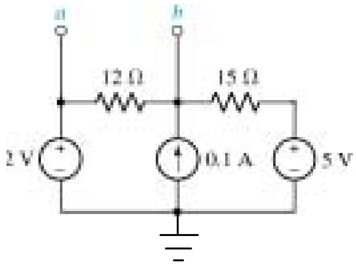
(b) If $R_{ab} = 50\ \Omega$,

$$P_{50\Omega} = \left[75\left(\frac{50}{50+12.5}\right)\right]^2\left(\frac{1}{50}\right) = \boxed{72\text{ W}}$$

(c) If $R_{ab} = 12.5\ \Omega$,

$$P_{12.5\Omega} = \left[75\left(\frac{12.5}{12.5+12.5}\right)\right]^2\left(\frac{1}{12.5}\right) = \boxed{112.5\text{ W}}$$

32. (a) Removing terminal **c**, we need write only one nodal equation:



$0.1 = \frac{V_b - 2}{12} + \frac{V_b - 5}{15}$, which may be solved to yield V_b = 4 V. Therefore, $V_{ab} = V_{TH}$ = 2 – 4 = -2 V.

R_{TH} = 12 || 15 = $\boxed{6.667\ \Omega.}$ We may then calculate I_N as $I_N = V_{TH}/ R_{TH}$

= $\boxed{\text{-300 mA (arrow pointing upwards).}}$

(b) Removing terminal **a**, we again find $\boxed{R_{TH} = 6.667\ \Omega,}$ and only need write a single nodal equation; in fact, it is identical to that written for the circuit above, and we once again find that V_b = 4 V. In this case, $V_{TH} = V_{bc}$ = 4 – 5 = -1 V, so I_N = -1/ 6.667 = $\boxed{\text{-150 mA (arrow pointing upwards).}}$

33. (a) Shorting out the 88-V source and open-circuiting the 1-A source, we see looking into the terminals x and x' a 50-Ω resistor in parallel with 10 Ω in parallel with (20 Ω + 40 Ω), so

$$R_{TH} = 50 \parallel 10 \parallel (20 + 40) = \boxed{7.317\ \Omega}$$

Using superposition to determine the voltage $V_{xx'}$ across the 50-Ω resistor, we find

$$V_{xx'} = V_{TH} = \left[88\frac{50 \parallel (20+40)}{10+[50 \parallel (20+40)]}\right] + (1)(50 \parallel 10)\left[\frac{40}{40+20+(50 \parallel 10)}\right]$$

$$= \left[88\frac{27.27}{37.27}\right] + (1)(8.333)\left[\frac{40}{40+20+8.333}\right] = \boxed{69.27\text{ V}}$$

(b) Shorting out the 88-V source and open-circuiting the 1-A source, we see looking into the terminals y and y' a 40-Ω resistor in parallel with [20 Ω + (10 Ω || 50 Ω)]:

$$R_{TH} = 40 \parallel [20 + (10 \parallel 50)] = \boxed{16.59\ \Omega}$$

Using superposition to determine the voltage $V_{yy'}$ across the 1-A source, we find

$$V_{yy'} = V_{TH} = (1)(R_{TH}) + \left[88\frac{27.27}{10+27.27}\right]\left(\frac{40}{20+40}\right)$$

$$= \boxed{59.52\text{ V}}$$

34. (a) Select terminal **b** as the reference terminal, and define a nodal voltage V_1 at the top of the 200-Ω resistor. Then,

$$0 = \frac{V_1 - 20}{40} + \frac{V_1 - V_{TH}}{100} + \frac{V_1}{200} \qquad [1]$$

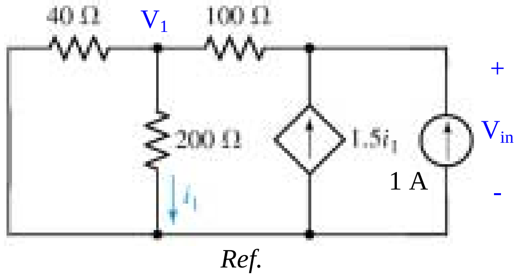
$$1.5\, i_1 = (V_{TH} - V_1)/\,100 \qquad [2]$$

where $i_1 = V_1/\,200$, so Eq. [2] becomes $\quad 150\, V_1/\,200 + V_1 - V_{TH} = 0 \quad$ [2]

Simplifying and collecting terms, these equations may be re-written as:

$$(0.25 + 0.1 + 0.05)\, V_1 - 0.1\, V_{TH} = 5 \qquad [1]$$
$$(1 + 15/\,20)\, V_1 - V_{TH} = 0 \qquad [2]$$

Solving, we find that $\boxed{V_{TH} = 38.89\text{ V}}$. To find R_{TH}, we short the voltage source and inject 1 A into the port:



$$0 = \frac{V_1 - V_{in}}{100} + \frac{V_1}{40} + \frac{V_1}{200} \qquad [1]$$

$$1.5\, i_1 + 1 = \frac{V_{in} - V_1}{100} \qquad [2]$$

$$i_1 = V_1/\,200 \qquad [3]$$

Combining Eqs. [2] and [3] yields $\quad 1.75\, V_1 - V_{in} = -100 \quad$ [4]

Solving Eqs. [1] & [4] then results in V_{in} = 177.8 V, so that $R_{TH} = V_{in}/\,1\text{ A} = \boxed{177.8\ \Omega.}$

(b) Adding a 100-Ω load to the original circuit or our Thévenin equivalent, the voltage across the load is

$$V_{100\Omega} = V_{TH}\left(\frac{100}{100 + 177.8}\right) = 14.00\text{ V}, \text{ and so } P_{100\Omega} = (V_{100\Omega})^2 / 100 = \boxed{1.96\text{ W.}}$$

35. We inject a current of 1 A into the port (arrow pointing up), select the bottom terminal as our reference terminal, and define the nodal voltage V_x across the 200-Ω resistor.

Then,
$$1 = V_1/ 100 + (V_1 - V_x)/ 50 \quad [1]$$
$$-0.1\ V_1 = V_x/ 200 + (V_x - V_1)/ 50 \quad [2]$$

which may be simplified to

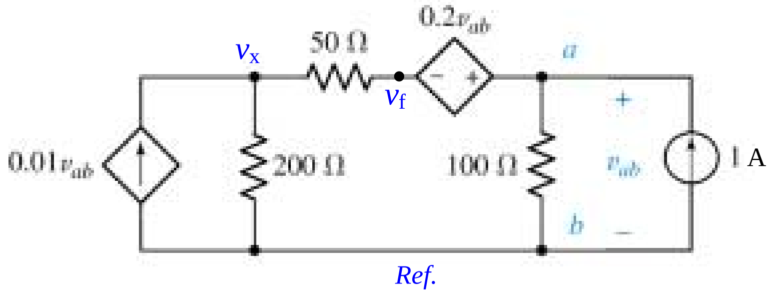
$$3\ V_1 - 2\ V_x = 100 \quad [1]$$
$$16\ V_1 + 5\ V_x = 0 \quad [2]$$

Solving, we find that $V_1 = 10.64$ V, so $R_{TH} = V_1/ (1\text{ A}) = \boxed{10.64\ \Omega.}$

Since there are no independent sources present in the original network, $\boxed{I_N = 0.}$

36. With no independent sources present, $\boxed{V_{TH} = 0.}$

We decide to inject a 1-A current into the port:



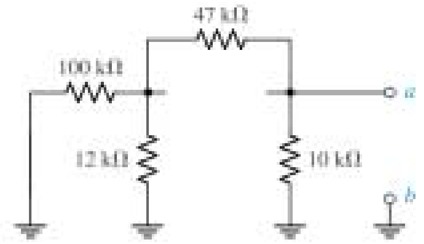
Node '**x**': $0.01\ v_{ab} = v_x/200 + (v_x - v_f)/50$ [1]
Supernode: $1 = v_{ab}/100 + (v_f - v_x)$ [2]
and: $v_{ab} - v_f = 0.2\ v_{ab}$ [3]

Rearranging and collecting terms,

$$-2\ v_{ab} + 5\ v_x - 4\ v_f = 0 \quad [1]$$
$$v_{ab} - 2\ v_x + 2\ v_f = 100 \quad [2]$$
$$0.8\ v_{ab} \qquad - v_f = 0 \quad [3]$$

Solving, we find that v_{ab} = 192.3 V, so $R_{TH} = v_{ab}/(1\text{ A}) = \boxed{192.3\ \Omega.}$

37. We first find R_{TH} by shorting out the voltage source and open-circuiting the current source.



Looking into the terminals **a** & **b**, we see
$R_{TH} = 10 \parallel [47 + (100 \parallel 12)]$
$= \boxed{8.523\ \Omega.}$

Returning to the original circuit, we decide to perform nodal analysis to obtain V_{TH}:

$-12\times10^3 = (V_1 - 12)/ 100\times10^3 + V_1/ 12\times10^3 + (V_1 - V_{TH})/ 47\times10^3$ [1]

$12\times10^3 = V_{TH}/ 10\times10^3 + (V_{TH} - V_1)/ 47\times10^3$ [2]

Rearranging and collecting terms,

$0.1146\ V_1 - 0.02128\ V_{TH} = -11.88$ [1]
$-0.02128\ V_1 + 0.02128\ V_{TH} = 12$ [2]

Solving, we find that $V_{TH} = \boxed{83.48\ \text{V.}}$

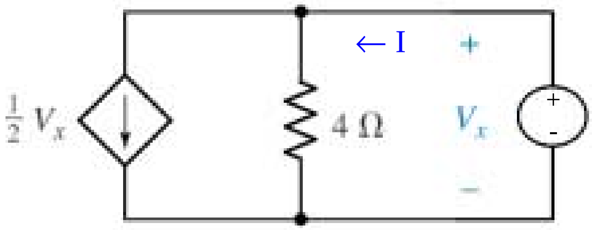
38. (a) $R_{TH} = 4 + 2 \parallel 2 + 10 = \boxed{15\ \Omega.}$
(b) same as above: $\boxed{15\ \Omega.}$

39. For Fig. 5.78a, $I_N = 12/\sim 0 \rightarrow$ $\boxed{\infty \text{ A in parallel with} \sim 0\ \Omega.}$

For Fig. 5.78b, $V_{TH} = (2)(\sim\infty) \rightarrow$ $\boxed{\infty \text{ V in series with} \sim\infty\ \Omega.}$

40. With no independent sources present, $\boxed{V_{TH} = 0.}$

Connecting a 1-V source to the port and measuring the current that flows as a result,



I = 0.5 V_x + 0.25 V_x = 0.5 + 0.25 = 0.75 A.

R_{TH} = 1/ I = 1.333 Ω.

> The Norton equivalent is 0 A in parallel with 1.333 Ω.

41. Performing nodal analysis to determine V_{TH},

$$100\times10^{-3} = V_x/ 250 + V_{oc}/ 7.5\times10^3 \qquad [1]$$

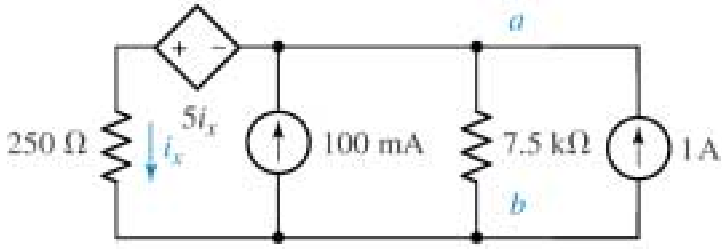
and $V_x - V_{oc} = 5\, i_x$

where $i_x = V_x/ 250$. Thus, we may write the second equation as

$$0.98\, V_x - V_{oc} = 0 \qquad [2]$$

Solving, we find that $V_{oc} = \boxed{V_{TH} = 23.72 \text{ V.}}$

In order to determine R_{TH}, we inject 1 A into the port:



$$V_{ab}/ 7.5\times10^3 + V_x/ 250 = 1 \qquad [1]$$

and $V_x - V_{ab} = 5\, i_x = 5V_x/ 250$ or

$$-V_{ab} + (1 - 5/ 250)\, V_x = 0 \qquad [2]$$

Solving, we find that V_{ab} = 237.2 V. Since $R_{TH} = V_{ab}/ (1 \text{ A})$, $\boxed{R_{TH} = 237.2\ \Omega.}$

Finally, $I_N = V_{TH}/ R_{TH} = \boxed{100 \text{ mA.}}$

42. We first note that $V_{TH} = V_x$, so performing nodal analysis,

$$-5\,V_x = V_x/\,19 \qquad \text{which has the solution } V_x = 0\text{ V.}$$

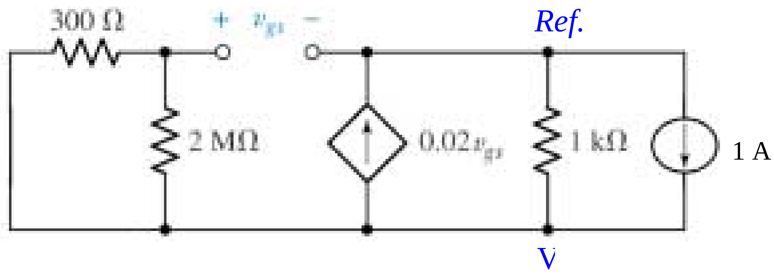
Thus, $\boxed{V_{TH} \text{ (and hence } I_N) = 0.}$ (Assuming $R_{TH} \neq 0$)

To find R_{TH}, we inject 1 A into the port, noting that $R_{TH} = V_x/\,1$ A:

$$-5\,V_x + 1 = V_x/\,19$$

Solving, we find that $V_x = 197.9$ mV, so that $\boxed{R_{TH} = R_N = 197.9\text{ mV.}}$

43. Shorting out the voltage source, we redraw the circuit with a 1-A source in place of the 2-kΩ resistor:



Noting that 300 Ω || 2 MΩ ≈ 300 Ω,

$$0 = (v_{gs} - \text{V})/\ 300 \qquad [1]$$

$$1 - 0.02\ v_{gs} = \text{V}/\ 1000 + (\text{V} - v_{gs})/\ 300 \qquad [2]$$

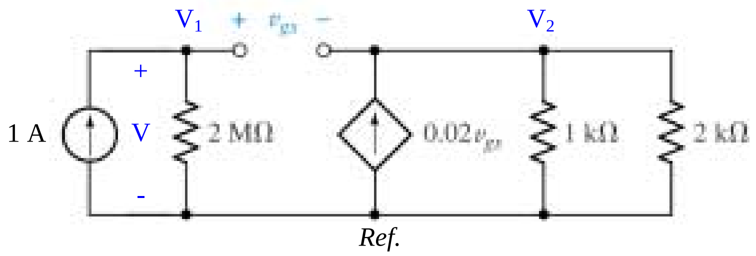
Simplifying & collecting terms,

$$v_{gs} \qquad - \text{V} = 0 \qquad [1]$$

$$0.01667\ v_{gs} + 0.00433\ \text{V} = 1 \qquad [2]$$

Solving, we find that v_{gs} = V = 47.62 V. Hence, R_{TH} = V/ 1 A = $\boxed{47.62\ \Omega.}$

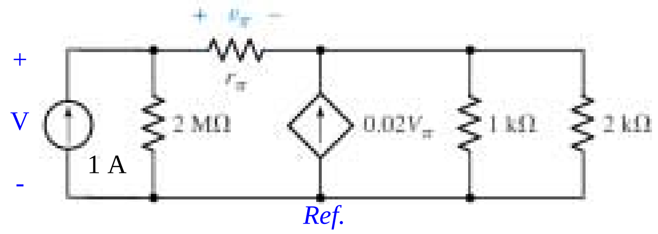
44. We replace the source v_s and the 300-Ω resistor with a 1-A source and seek its voltage:



By nodal analysis, $1 = V_1 / 2\times10^6$ so $V_1 = 2\times10^6$ V.

Since $V = V_1$, we have $R_{in} = V / 1\text{ A} = \boxed{2\text{ M}\Omega.}$

45. Removing the voltage source and the 300-Ω resistor, we replace them with a 1-A source and seek the voltage that develops across its terminals:



We select the bottom node as our reference terminal, and define nodal voltages V_1 and V_2. Then,

$$1 = V_1 / 2\times10^6 + (V_1 - V_2)/ r_\pi \qquad [1]$$

$$0.02\, v_\pi = (V_2 - V_1)/ r_\pi + V_2/ 1000 + V_2/ 2000 \qquad [2]$$

where $v_\pi = V_1 - V_2$

Simplifying & collecting terms,

$$(2\times10^6 + r_\pi)\, V_1 - 2\times10^6\, V_2 = 2\times10^6\, r_\pi \qquad [1]$$

$$-(2000 + 40\, r_\pi)\, V_1 + (2000 + 43\, r_\pi)\, V_2 = 0 \quad [2]$$

Solving, we find that $V_1 = V = 2\times10^6 \left(\dfrac{666.7 + 14.33\, r_\pi}{2\times10^6 + 666.7 + 14.33\, r_\pi} \right)$.

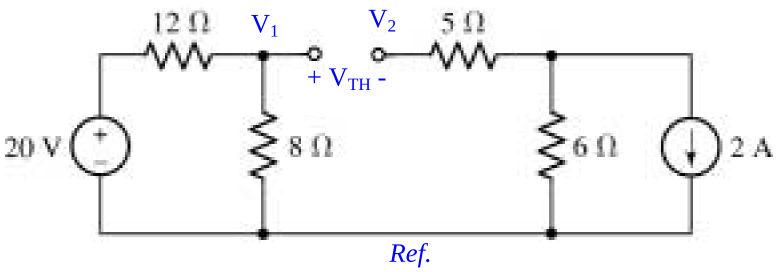
Thus, $\boxed{R_{TH} = 2\times10^6 \,\|\, (666.7 + 14.33\, r_\pi)\ \Omega.}$

46. Such a scheme probably would lead to maximum or at least near-maximum power transfer to our home. Since we pay the utility company based on the power we use, however, this might not be such a hot idea…

47. We need to find the Thévenin equivalent resistance of the circuit connected to R_L, so we short the 20-V source and open-circuit the 2-A source; by inspection, then

$$R_{TH} = 12 \parallel 8 \quad + \quad 5 \quad + 6 = \boxed{15.8\ \Omega}$$

Analyzing the original circuit to obtain V_1 and V_2 with R_L removed:



V_1 = 20 8/ 20 = 8 V; V_2 = -2 (6) = -12 V.

We define $V_{TH} = V_1 - V_2 = 8 + 12 = 20$ V. Then,

$$P_{R_L}\Big|_{max} = \frac{V_{TH}^2}{4\,R_L} = \frac{400}{4(15.8)} = \boxed{6.329\ \text{W}}$$

48. (a) $R_{TH} = 25 \,||\, (10 + 15) = \boxed{12.5\ \Omega}$

Using superposition, $V_{ab} = V_{TH} = 50\frac{25}{15+10+25} + 100\frac{15+10}{50} = \boxed{75\text{ V.}}$

(b) Connecting a 50-Ω resistor,

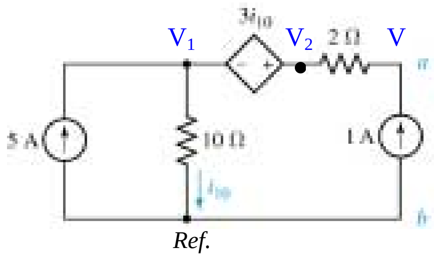
$$P_{load} = \frac{V_{TH}^2}{R_{TH} + R_{load}} = \frac{75^2}{12.5+50} = \boxed{90\text{ W}}$$

(c) Connecting a 12.5-Ω resistor,

$$P_{load} = \frac{V_{TH}^2}{4\,R_{TH}} = \frac{75^2}{4\,(12.5)} = \boxed{112.5\text{ W}}$$

49. (a) By inspection, we see that i_{10} = 5 A, so
$V_{TH} = V_{ab} = 2(0) + 3\,i_{10} + 10\,i_{10} = 13\,i_{10} = 13(5) = \boxed{65\text{ V.}}$

To find R_{TH}, we connect a 1-A source between terminals **a** & **b**:



$5 = V_1/10 + (V_2 - V)/2$ [1] $\rightarrow$ $V_1 + 5\,V_2 - 5\,V = 50$ [1]

$1 = (V - V_2)/2$ [2] $\rightarrow$ $-V_2 + V = 2$ [2]

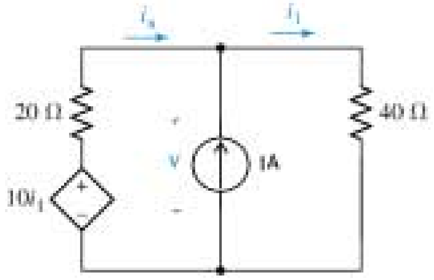
and $V_2 - V_1 = 3\,i_{10}$ [3]
where $i_{10} = V_1/10$ $\rightarrow$ $-13\,V_1 + 10\,V_2 = 0$ [3]

Solving, we find that V = 80 V, so that $R_{TH} = V / 1\text{ A} = \boxed{80\ \Omega.}$

(b) $$P_{max} = \frac{V_{TH}^2}{4\,R_{TH}} = \frac{65^2}{4(80)} = \boxed{13.20\text{ W}}$$

50.

(a) Replacing the resistor R_L with a 1-A source, we seek the voltage that develops across its terminals with the independent voltage source shorted:



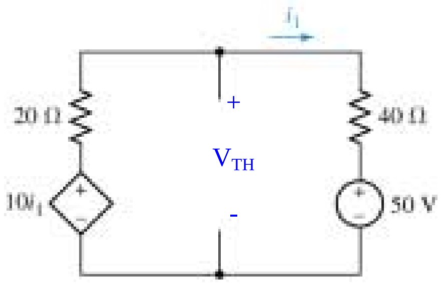
$-10i_1 + 20i_x + 40i_1 = 0 \quad [1] \Rightarrow 30i_1 + 20i_x = 0 \; [1]$

and $i_1 - i_x = 1 \quad [2] \Rightarrow \; i_1 - i_x = 1 \quad [2]$

Solving, $i_1 = 400\,\text{mA}$

So $V = 40i_1 = 16\,\text{V}$ and $R_{TH} = \dfrac{V}{1\text{A}} = \boxed{16\,\Omega}$

(b) Removing the resistor R_L from the original circuit, we seek the resulting open-circuit voltage:



$$0 = \frac{V_{TH} - 10i_1}{20} + \frac{V_{TH} - 50}{40} \qquad [1]$$

where $i_1 = \dfrac{V_{TH} - 50}{40}$

so [1] becomes $0 = \dfrac{V_{TH}}{20} - \dfrac{1}{2}\left(\dfrac{V_{TH} - 50}{40}\right) + \left(\dfrac{V_{TH} - 50}{40}\right)$

$$0 = \frac{V_{TH}}{20} + \frac{V_{TH} - 50}{80}$$

$$0 = 4V_{TH} + V_{TH} - 50$$

$$5V_{TH} = 50$$

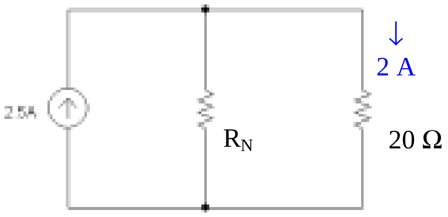
or $V_{TH} = 10\,\text{V}$

Thus, if $R_L = R_{TH} = 16\,\Omega$,

$$V_{R_L} = V_{TH}\frac{R_L}{R_L + R_{TH}} = \frac{V_{TH}}{2} = \boxed{5\,\text{V}}$$

51.

(a) $I_N = 2.5\,\text{A}$



$20i^2 = 80$

$i = 2\,\text{A}$

By current division,

$$2 = 2.5\frac{R_N}{R_N + 20}$$

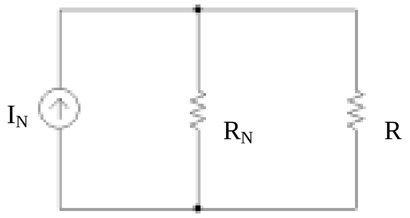
Solving, $R_N = R_{TH} = 80\,\Omega$

Thus, $V_{TH} = V_{OC} = 2.5\times 80 = \boxed{200\,\text{V}}$

(b) $P_{\max} = \dfrac{V_{TH}^{\;2}}{4R_{TH}} = \dfrac{200^2}{4\times 80} = \boxed{125\,\text{W}}$

(c) $R_L = R_{TH} = \boxed{80\,\Omega}$

52.



$10\,\text{W}$ to 250Ω corresp to $200\,\text{mA}$.
$20\,\text{W}$ to 80Ω corresp to $500\,\text{mA}$.

By Voltage ÷, $\quad I_R = I_N \dfrac{R_N}{R + R_N}$

So $\quad 0.2 = I_N \dfrac{R_N}{250 + R_N} \qquad [1]$

$0.5 = I_N \dfrac{R_N}{80 + R_N} \qquad [2]$

Solving, $I_N = 1.7\,\text{A}$ and $R_N = 33.33\Omega$

(a) If $v_L i_L$ is a maximum,

$R_L = R_N = \boxed{33.33\Omega}$

$i_L = 1.7 \times \dfrac{33.33}{33.33 + 33.33} \boxed{= 850\,\text{mA}}$

$v_L = 33.33 i_L = \boxed{28.33\,\text{V}}$

(b) If v_L is a maximum

$V_L = I_N (R_N \| R_L)$

So v_L is a maximum when $R_N \| R_L$ is a maximum, which occurs at $\boxed{R_L = \infty.}$

Then $\boxed{i_L = 0}$ and $v_L = 1.7 \times R_N = \boxed{56.66\,\text{V}}$

(c) If i_L is a maximum

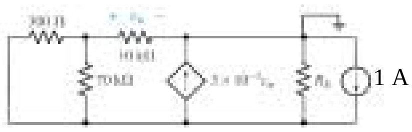
$i_L = i_N \dfrac{R_N}{R_N + R_L}$; max when $\boxed{R_L = 0\,\Omega}$

So $\boxed{i_L = 1.7\text{A}}$

$\boxed{v_L = 0\,\text{V}}$

53. There is no conflict with our derivation concerning maximum power. While a dead short across the battery terminals will indeed result in maximum current draw from the battery, and power is indeed proportional to i^2, the power delivered to the load is $i^2R_{LOAD} = i^2(0) = 0$ watts. This is the *minimum*, not the maximum, power that the battery can deliver to a load.

54. Remove R_E: $R_{TH} = R_E \| R_{in}$



bottom node: $1 - 3\times10^{-3} v_\pi = \dfrac{V - v_\pi}{300} + \dfrac{V - v_\pi}{70\times10^3}$ [1]

at other node: $0 = \dfrac{v_\pi}{10\times10^3} + \dfrac{v_\pi - V}{300} + \dfrac{v_\pi - V}{70\times10^3}$ [2]

Simplifying and collecting terms,

$210\times10^5 = 70\times10^3\, V + 300V + 63000\, v_\pi - 70\text{x}10^3\, v_\pi - 300\, v_\pi$

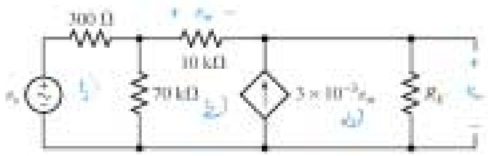
or $70.3\times10^3 V - 7300 v_\pi = 210\times10^5$ [1]

$0 = 2100 v_\pi + 70\times10^3 v_\pi - 70\times10^3 V + 300 v_\pi - 300V$

or $-69.7\times10^3 V + 72.4\times10^3 v_\pi = 0$ [2]

solving, $V = 331.9V$ So $R_{TH} = R_E \| 331.9\Omega$

Next, we determine v_{TH} using mesh analysis:



$-v_s + 70.3\times10^3 i_1 - 70\times10^3 i_2 = 0$ [1]

$80\times10^3 i_2 - 70\times10^3 i_1 + R_E i_3 = 0$ [2]

and: $i_3 - i_2 = 3\times10^{-3} v_\pi$ [3]

or $i_3 - i_2 = 3\times10^{-3}(10\times10^3) i_2$

or $i_3 - i_2 = 30 i_2$

or

$-31 i_2 + i_3 = 0$ [3]

Solving : $\begin{bmatrix} 70.3\times10^3 & -70\times10^3 & 0 \\ -70\times10^3 & 80\times10^3 & R_E \\ 0 & -31 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$

We seek i_3:

$$i_3 = \frac{-21.7\times10^3 v_s}{7.24\times10^6 + 21.79\times10^3 R_E}$$

So $V_{OC} = V_{TH} = R_E\, i_3 = \dfrac{-21.7\times10^3 R_E}{7.24\times10^6 + 21.79\times10^3 R_E} v_s$

$$P_{8\Omega} = 8\left[\frac{V_{TH}}{R_{TH} + 8}\right]^2 = \left[\frac{-21.7\times10^3 R_E}{7.24\times10^6 + 21.79\times10^3 R_E}\right]^2 \frac{8\ vs^2}{\left[\dfrac{331.9\, R_E}{331.9 + R_E}\right]^2}$$

$$= \frac{11.35\times10^6 (331.9 + R_E)^2}{(7.24\times10^6 + 21.79\times10^3 R_E)^2} vs^2$$

This is maximized by setting $R_E = \infty$.

55. Thévenize the left-hand network, assigning the nodal voltage V_x at the free end of right-most 1-kΩ resistor.

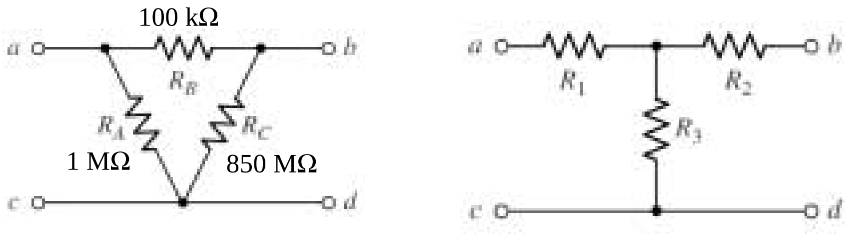
A single nodal equation: $40\times10^{-3} = \dfrac{V_x\big|_{oc}}{7\times10^3}$

So $V_{TH} = V_x\big|_{oc} = 280\,\text{V}$

R_{TH} = 1 k + 7 k = 8 kΩ

$\boxed{\text{Select } R_1 = R_{TH} = 8\text{ k}\Omega.}$

56.



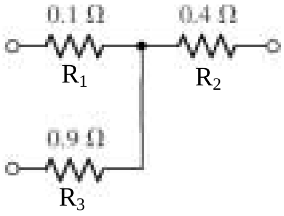
$$D = R_A + R_B + R_C = 1 + 850 + 0.1 = 851.1\times10^6$$

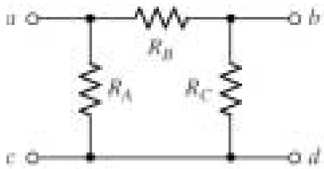
$$R_1 = \frac{R_A R_B}{D} = \frac{10^6\times10^5}{D} = \boxed{117.5\,\Omega}$$

$$R_2 = \frac{R_B R_C}{D} = \frac{10^5\times850\times10^6}{851.1\times10^6} = \boxed{99.87\,k\Omega}$$

$$R_3 = \frac{R_C R_A}{D} = \frac{850\times10^6\times10^6}{851.1\times10^6} = \boxed{998.7\,k\Omega}$$

57.





$$N = R_1R_2 + R_2R_3 + R_3R_1$$
$$= 0.1\times0.4 + 0.4\times0.9 + 0.9\times0.1$$
$$= 0.49\Omega$$

$$R_A = \frac{N}{R_2} = \boxed{1.225\,\Omega}$$

$$R_B = \frac{N}{R_3} = \boxed{544.4\,\text{m}\Omega}$$

$$R_C = \frac{N}{R_1} = \boxed{4.9\,\Omega}$$

58.

$\Delta_1 : 1+6+3 = 10\,\Omega$

$$\frac{6\times 1}{10} = 0.6,\ \frac{6\times 3}{10} = 1.8\ ,\ \frac{3\times 1}{10} = 0.3$$

$\Delta_2 : 5+1+4 = 10\,\Omega$

$$\frac{5\times 1}{10} = 0.5\ ,\ \frac{1\times 4}{10} = 0.4\ ,\ \frac{5\times 4}{10} = 2$$

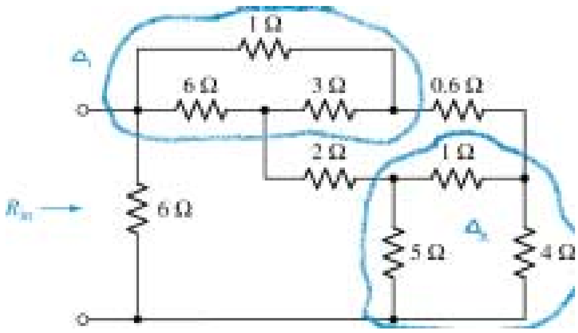
$1.8+2+0.5 = 4.3\Omega$

$0.3+0.6+0.4 = 1.3\Omega$

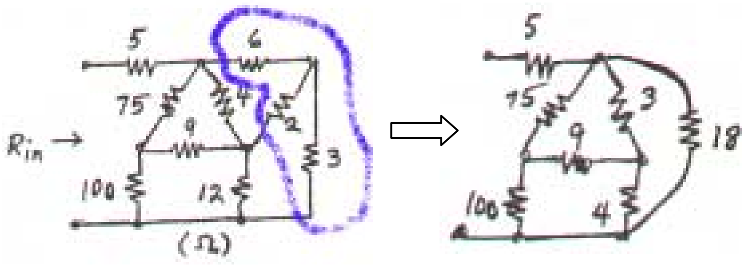
$1.3\|4.3 = 0.9982\,\Omega$

$0.9982+0.6+2 = 3.598\,\Omega$

$3.598\|6 = \boxed{2.249\,\Omega}$



59.

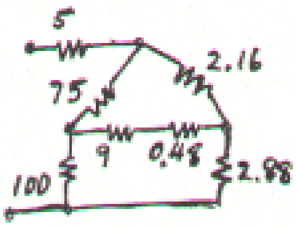


$$6\times2+2\times3+3\times6=36\,\Omega^2$$

$$\frac{36}{6}=6\,\Omega,\ \frac{36}{2}=18\,\Omega,\ \frac{36}{3}=12\,\Omega$$

$$12\|4=3\,\Omega,\ 6\|12\,\Omega=4\,\Omega$$

$$4+3+18=25\,\Omega$$

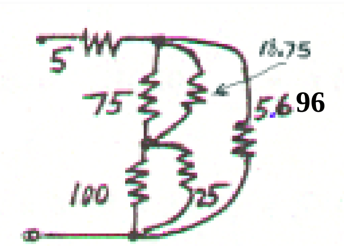


$$3\times\frac{18}{25}=2.16\,\Omega$$

$$4\times\frac{18}{25}=2.88\,\Omega$$

$$4\times\frac{3}{25}=0.48\,\Omega$$

$$9.48\times2.16+9.48\times2.88+2.88\times2.16=54\,\Omega^2$$



$$\frac{54}{2.88}=18.75\,\Omega \qquad \frac{54}{9.48}=5.696\,\Omega$$

$$\frac{54}{2.16}=25\,\Omega$$

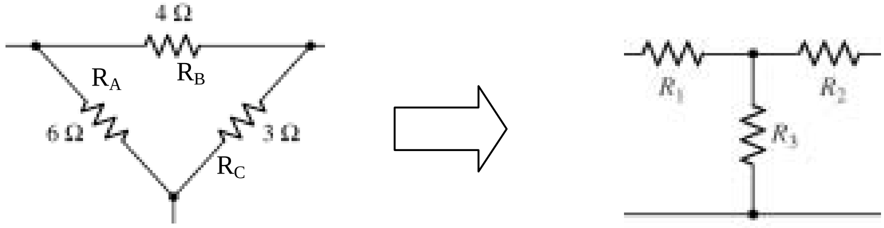
$$75\|18.75=15\,\Omega$$

$$100\|25=20\,\Omega$$

$$(15+20)\|5.696=4.899\,\Omega$$

$$\therefore R_{in}=5+4.899=\boxed{9.899\,\Omega}$$

60. We begin by converting the Δ-connected network consisting of the 4-, 6-, and 3-Ω resistors to an equivalent Y-connected network:



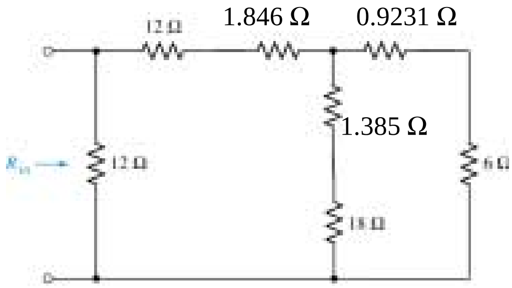
$$D = 6 + 4 + 3 = 13\,\Omega$$

$$R_1 = \frac{R_A R_B}{D} = \frac{6\times 4}{13} = 1.846\,\Omega$$

$$R_2 = \frac{R_B R_C}{D} = \frac{4\times 3}{13} = 923.1\,\text{m}\Omega$$

$$R_3 = \frac{R_C R_A}{D} = \frac{3\times 6}{13} = 1.385\,\Omega$$

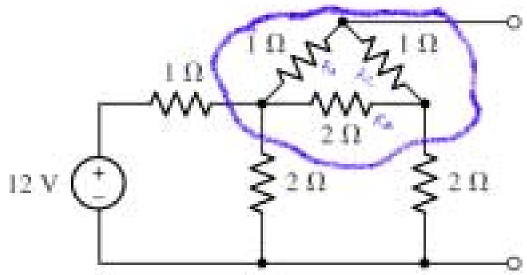
Then network becomes:



Then we may write

$$R_{in} = 12 \,\|\, [13.846 + (19.385 \,\|\, 6.9231)]$$
$$= \boxed{7.347\,\Omega}$$

61.

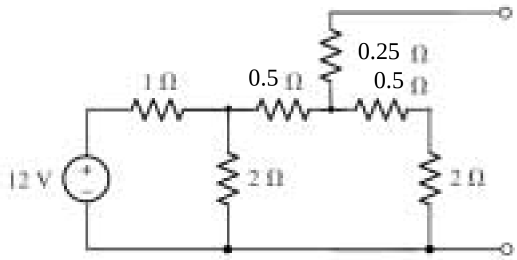


$$1+1+2=4\,\Omega$$

$$R_1=\frac{1\times 2}{4}=\frac{1}{2}\,\Omega$$

$$R_2=\frac{2\times 1}{4}=\frac{1}{2}\,\Omega$$

$$R_3=\frac{1\times 1}{4}=0.25\,\Omega$$



Next, we convert the Y-connected network on the left to a Δ-connected network:

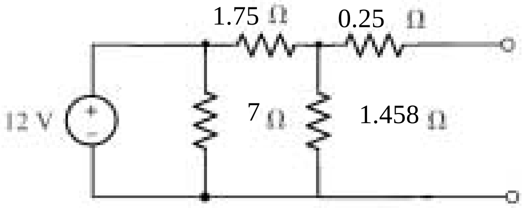
$$1\times 0.5+0.5\times 2+2\times 1=3.5\,\Omega^2$$

$$R_A=\frac{3.5}{0.5}=7\,\Omega$$

$$R_B=\frac{3.5}{2}=1.75\,\Omega$$

$$R_C=\frac{3.5}{1}=3.5\,\Omega$$

After this procedure, we have a 3.5-Ω resistor in parallel with the 2.5-Ω resistor. Replacing them with a 1.458-Ω resistor, we may redraw the circuit:



This circuit may be easily analysed to find:

$$V_{oc}=\frac{12\times 1.458}{1.75+1.458}=\boxed{5.454\,\text{V}}$$

$$R_{TH}=0.25+1.458\,\|\,1.75$$

$$\boxed{=1.045\,\Omega}$$

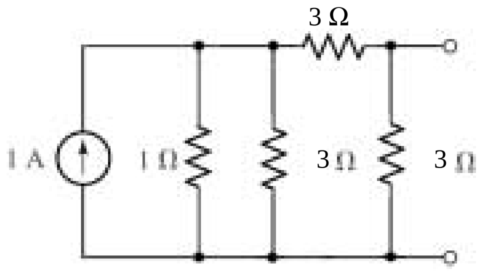
62. We begin by converting the Y-network to a Δ-connected network:

$N = 1.1 + 1.1 + 1.1 = 3\Omega^2$

$R_A = \frac{3}{1} = 3\Omega$

$R_B = \frac{3}{1} = 3\Omega$

$R_C = \frac{3}{1} = 3\Omega$



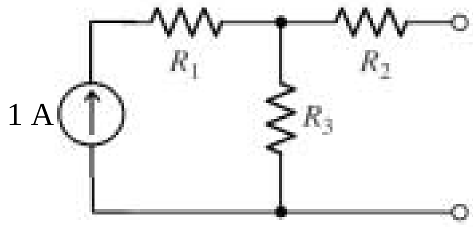
Next, we note that $1\|3 = 0.75\,\Omega$, and hence have a simple Δ-network. This is easily converted to a Y-connected network:

$0.75 + 3 + 3 = 6.75\Omega$

$R_1 = \frac{0.75 \times 3}{6.75} = 0.3333\Omega$

$R_2 = \frac{3 \times 3}{6.75} = 1.333\Omega$

$R_3 = \frac{3 \times 0.75}{6.75} = 0.3333\Omega$

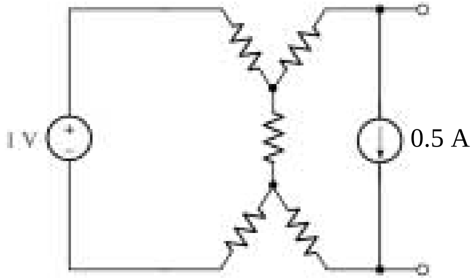


Analysing this final circuit,

$$R_N = 1.333 + 0.3333$$
$$\boxed{= 1.667\Omega}$$

$$I_N = I_{SC} = 1 \times \frac{1/3}{1/3 + 1 + 1/3}$$
$$= \frac{1}{1+3+1} = \frac{1}{5}$$
$$= 0.2\,\text{A}$$
$$\boxed{= 200\,\text{mA}}$$

63. Since 1 V appears across the resistor associated with I_1, we know that I_1 = 1 V/ 10 Ω = 100 mA. From the perspective of the open terminals, the 10-Ω resistor in parallel with the voltage source has no influence if we replace the "dependent" source with a fixed 0.5-A source:



Then, we may write:

$$-1 + (10 + 10 + 10)\, i_a - 10\,(0.5) = 0$$

so that i_a = 200 mA.

We next find that $V_{TH} = V_{ab} = 10(-0.5) + 10(i_a - 0.5) + 10(-0.5) = \boxed{-13 \text{ V.}}$

To determine R_{TH}, we first recognise that with the 1-V source shorted, I_1 = 0 and hence the dependent current source is dead. Thus, we may write R_{TH} from inspection:

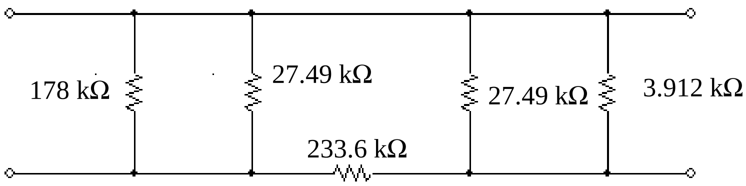
$$R_{TH} = 10 + 10 + 10 \parallel 20 = \boxed{26.67\ \Omega.}$$

64. (a) We begin by splitting the 1-kΩ resistor into two 500-Ω resistors in series. We then have two related Y-connected networks, each with a 500-Ω resistor as a leg. Converting those networks into Δ-connected networks,

$$\Sigma = (17)(10) + (1)(4) + (4)(17) = 89\times106\ \Omega^2$$

$$89/0.5 = 178\text{ k}\Omega; \qquad 89/\,17 = 5.236\text{ k}\Omega; \qquad 89/4 = 22.25\text{ k}\Omega$$

Following this conversion, we find that we have two 5.235 kΩ resistors in parallel, and a 178-kΩ resistor in parallel with the 4-kΩ resistor. Noting that 5.235 k || 5.235 k = 2.618 kΩ and 178 k || 4 k = 3.912 kΩ, we may draw the circuit as:



We next attack the Y-connected network in the center:

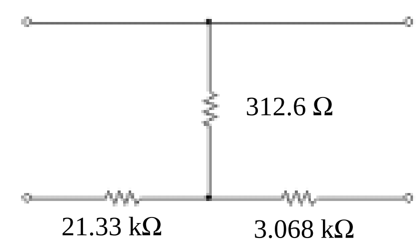
$$\Sigma = (22.25)(22.25) + (22.25)(2.618) + (2.618)(22.25) = 611.6\times106\ \Omega^2$$

$$611.6/\,22.25 = 27.49\text{ k}\Omega;\ 611.6/\,2.618 = 233.6\text{ k}\Omega$$

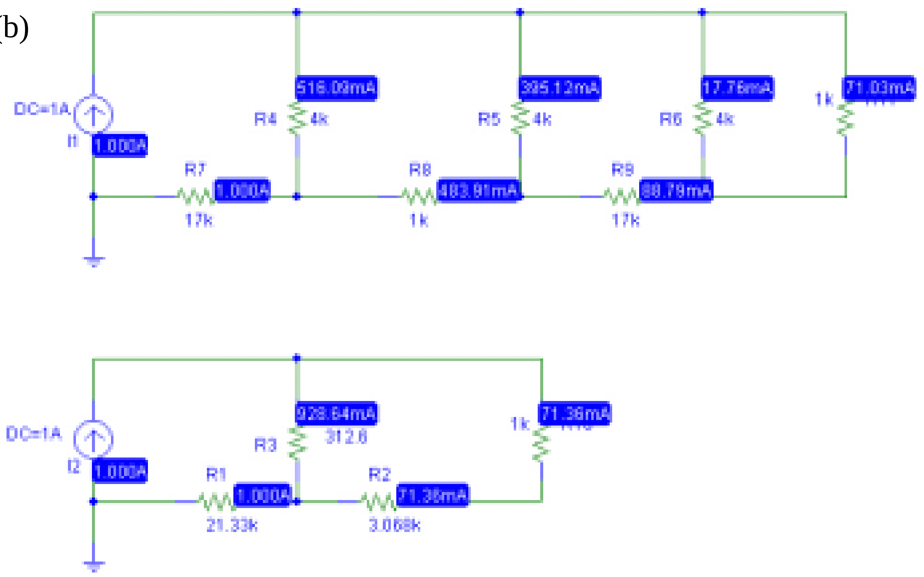
Noting that 178 k || 27.49 k = 23.81 kΩ and 27.49 || 3.912 = 3.425 kΩ, we are left with a simple Δ-connected network. To convert this to the requested Y-network,

$$\Sigma = 23.81 + 233.6 + 3.425 = 260.8\text{ k}\Omega$$

$$(23.81)(233.6)/\,260.8 = 21.33\text{ k}\Omega$$
$$(233.6)(3.425)/\,260.8 = 3.068\text{ k}\Omega$$
$$(3.425)(23.81)/\,260.8 = 312.6\ \Omega$$



(b)



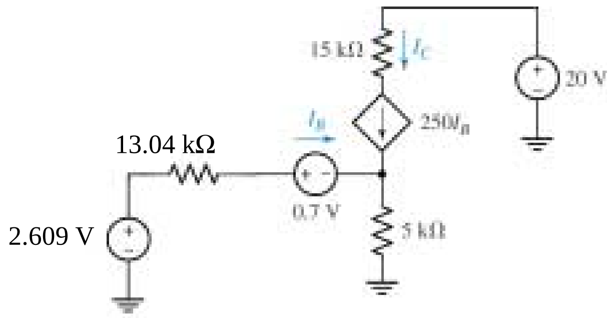
65. (a) Although this network may be simplified, it is not possible to replace it with a three-resistor equivalent.

    (b) See (a).

66. First, replace network to left of the 0.7-V source with its Thévenin equivalent:

$$V_{TH} = 20 \times \frac{15}{100+15} = 2.609\,\text{V}$$
$$R_{TH} = 100k \| 15k = 13.04\,k\Omega$$

Redraw:



Analysing the new circuit to find I_B, we note that $I_C = 250\ I_B$:

$$-2.609 + 13.04 \times 10^3 I_B + 0.7 + 5000(I_B + 250 I_B) = 0$$
$$I_B = \frac{2.609 - 0.7}{13.04 \times 10^3 + 251 \times 5000} = 1.505\,\mu\text{A}$$
$$I_C = 250 I_B = 3.764 \times 10^{-4}\,\text{A}$$
$$= \boxed{376.4\,\mu\text{A}}$$

67. (a) Define a nodal voltage V_1 at the top of the current source I_S, and a nodal voltage V_2 at the top of the load resistor R_L. Since the load resistor can safely dissipate 1 W, and we know that

$$P_{R_L} = \frac{V_2^2}{1000}$$

then $V_2\big|_{max} = 31.62\text{ V}$. This corresponds to a load resistor (and hence lamp) current of 32.62 mA, so we may treat the lamp as a 10.6-Ω resistor.

Proceeding with nodal analysis, we may write:

$$I_S = V_1/200 + (V_1 - 5V_x)/200 \qquad [1]$$

$$0 = V_2/1000 + (V_2 - 5V_x)/10.6 \qquad [2]$$

$$V_x = V_1 - 5V_x \qquad \text{or} \qquad V_x = V_1/6 \qquad [3]$$

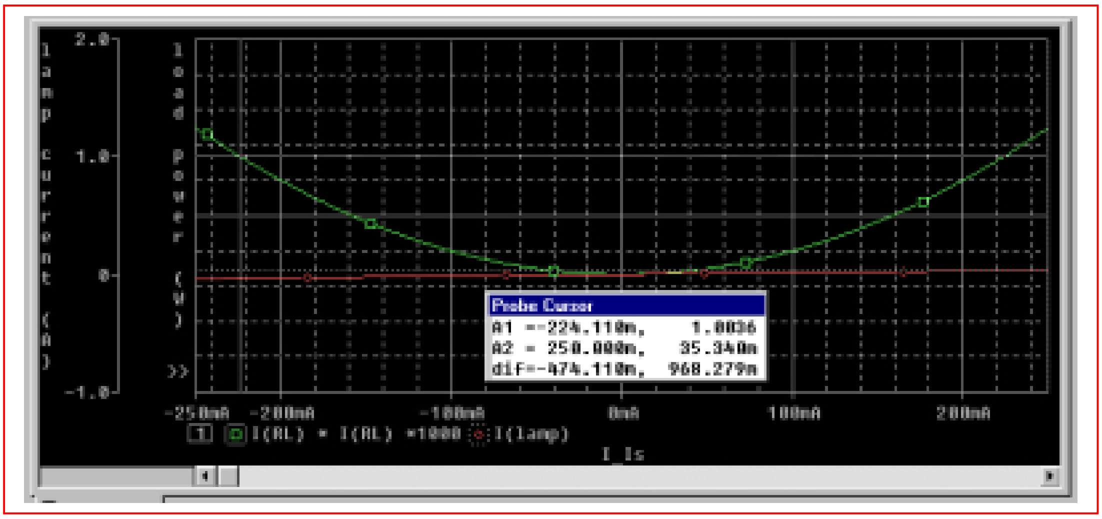
Substituting Eq. [3] into Eqs. [1] and [2], we find that

$$7V_1 = 1200 I_S \qquad [1]$$
$$-5000 V_1 + 6063.6 V_2 = 0 \qquad [2]$$

Substituting $V_2\big|_{max} = 31.62\text{ V}$ into Eq. [2] then yields $V_1 = 38.35$ V, so that

$I_S\big|_{max} = (7)(38.35)/1200 = 223.7$ mA.

(b) PSpice verification.



The lamp current does not exceed 36 mA in the range of operation allowed (*i.e.* a load power of < 1 W.) The simulation result shows that the load will dissipate slightly more than 1 W for a source current magnitude of 224 mA, as predicted by hand analysis.

68. Short out all but the source operating at 10^4 rad/s, and define three clockwise mesh currents i_1, i_2, and i_3 starting with the left-most mesh. Then

$$
\begin{aligned}
608\, i_1 - 300\, i_2 &= 3.5 \cos 10^4\, t & [1]\\
-300\, i_1 + 316\, i_2 - 8\, i_3 &= 0 & [2]\\
-8\, i_2 + 322\, i_3 &= 0 & [3]
\end{aligned}
$$

Solving, we find that
$$
\begin{aligned}
i_1(t) &= 10.84 \cos 10^4\, t \text{ mA}\\
i_2(t) &= 10.29 \cos 10^4\, t \text{ mA}\\
i_3(t) &= 255.7 \cos 10^4\, t\ \mu\text{A}
\end{aligned}
$$

Next, short out all but the 7 sin 200t V source, and and define three clockwise mesh currents i_a, i_b, and i_c starting with the left-most mesh. Then

$$
\begin{aligned}
608\, i_a - 300\, i_b &= -7 \sin 200t & [1]\\
-300\, i_a + 316\, i_b - 8\, i_c &= 7 \sin 200t & [2]\\
-8\, i_b + 322\, i_c &= 0 & [3]
\end{aligned}
$$

Solving, we find that
$$
\begin{aligned}
i_a(t) &= -1.084 \sin 200t \text{ mA}\\
i_b(t) &= 21.14 \sin 200t \text{ mA}\\
i_c(t) &= 525.1 \sin 200t\ \mu\text{A}
\end{aligned}
$$

Next, short out all but the source operating at 10^3 rad/s, and define three clockwise mesh currents i_A, i_B, and i_C starting with the left-most mesh. Then

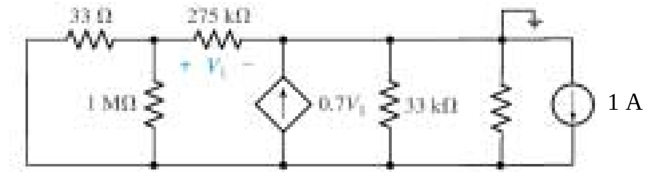
$$
\begin{aligned}
608\, i_A - 300\, i_B &= 0 & [1]\\
-300\, i_A + 316\, i_B - 8\, i_C &= 0 & [2]\\
-8\, i_B + 322\, i_C &= -8 \cos 10^4\, t & [3]
\end{aligned}
$$

Solving, we find that
$$
\begin{aligned}
i_A(t) &= -584.5 \cos 10^3\, t\ \mu\text{A}\\
i_B(t) &= -1.185 \cos 10^3\, t \text{ mA}\\
i_C(t) &= -24.87 \cos 10^3\, t \text{ mA}
\end{aligned}
$$

We may now compute the power delivered to each of the three 8-Ω speakers:

$$
\boxed{
\begin{aligned}
p_1 = 8[i_1 + i_a + i_A]^2 &= 8[10.84\times10^{-3} \cos 10^4\, t - 1.084\times10^{-3} \sin 200t - 584.5\times10^{-6} \cos 10^3\, t]^2\\
p_2 = 8[i_2 + i_b + i_B]^2 &= 8[10.29\times10^{-3} \cos 10^4\, t + 21.14\times10^{-3} \sin 200t - 1.185\times10^{-3} \cos 10^3\, t]^2\\
p_3 = 8[i_3 + i_c + i_C]^2 &= 8[255.7\times10^{-6} \cos 10^4\, t + 525.1\times10^{-6} \sin 200t - 24.87\times10^{-3} \cos 10^3\, t]^2
\end{aligned}
}
$$

69. Replacing the DMM with a possible Norton equivalent (a 1-MΩ resistor in parallel with a 1-A source):



We begin by noting that 33 Ω || 1 MΩ ≈ 33 Ω. Then,

$$0 = (V_1 - V_{in})/\,33 + V_1/\,275\times10^3 \qquad [1]$$

and

$$1 - 0.7\,V_1 = V_{in}/\,10^6 + V_{in}/\,33\times10^3 + (V_{in} - V_1)/\,33 \qquad [2]$$

Simplifying and collecting terms,

$$(275\times10^3 + 33)\,V_1 - 275\times10^3\,V_{in} = 0 \qquad [1]$$

$$22.1\,V_1 + 1.001\,V_{in} = 33 \qquad [2]$$

Solving, we find that V_{in} = 1.429 V; in other words, the DMM sees 1.429 V across its terminals in response to the known current of 1 A it's supplying. It therefore thinks that it is connected to a resistance of $\boxed{1.429\ \Omega.}$

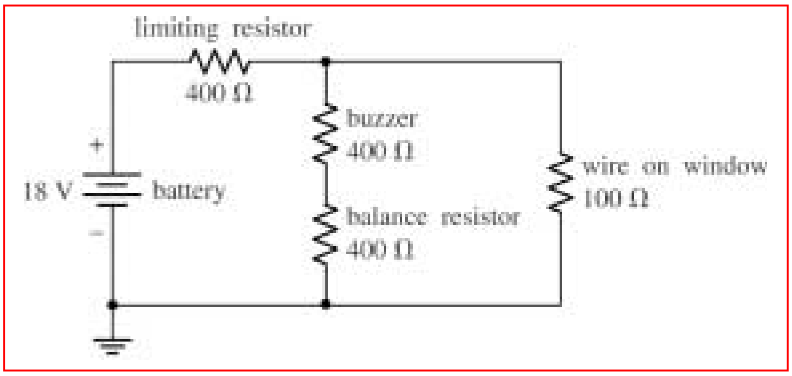
70. We know that the resistor R is absorbing maximum power. We might be tempted to say that the resistance of the cylinder is therefore 10 Ω, but this is wrong: The larger we make the cylinder resistance, the small the power delivery to R:

$$P_R = 10\, i^2 = 10 \left[\frac{120}{R_{cylinder} + 10} \right]^2$$

Thus, if we are in fact delivering the maximum possible power to the resistor from the 120-V source, the resistance of the cylinder must be zero.

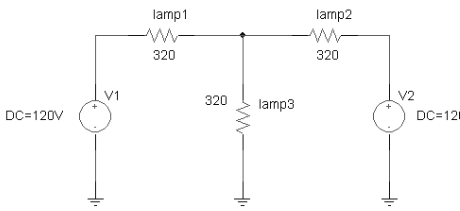
This corresponds to a temperature of absolute zero using the equation given.

71. We note that the buzzer draws 15 mA at 6 V, so that it may be modeled as a 400-Ω resistor. One possible solution of many, then, is:

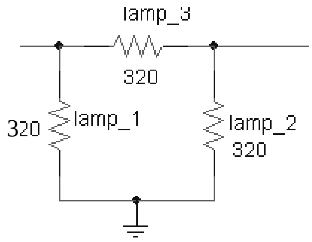


Note: construct the 18-V source from 12 1.5-V batteries in series, and the two 400-Ω resistors can be fabricated by soldering 400 1-Ω resistors in series, although there's probably a much better alternative…

72. To solve this problem, we need to assume that "45 W" is a designation that applies when 120 Vac is applied directly to a particular lamp. This corresponds to a current draw of 375 mA, or a light bulb resistance of 120/ 0.375 = 320 Ω.



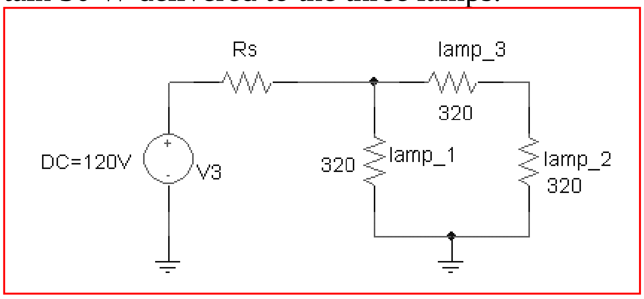
Original wiring scheme



New wiring scheme

In the original wiring scheme, Lamps 1 & 2 draw (40)2 / 320 = 5 W of power each, and Lamp 3 draws (80)2 / 320 = 20 W of power. Therefore, none of the lamps is running at its maximum rating of 45 W. We require a circuit which will deliver the same intensity after the lamps are reconnected in a Δ configuration. Thus, we need a total of 30 W from the new network of lamps.

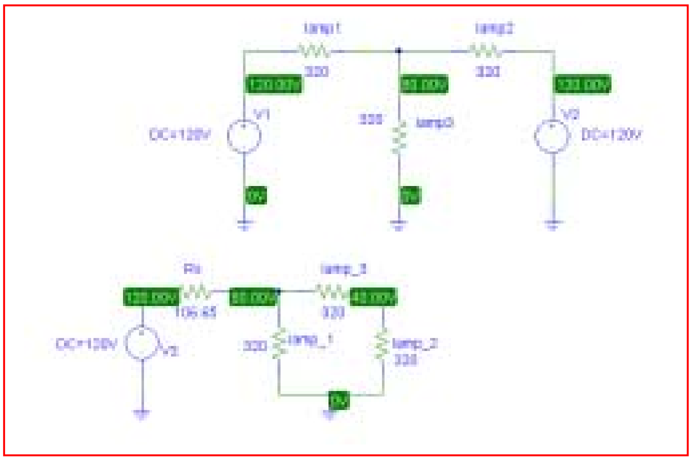
There are several ways to accomplish this, but the simplest may be to just use one 120-Vac source connected to the left port in series with a resistor whose value is chosen to obtain 30 W delivered to the three lamps.



In other words,

$$\frac{\left[120\,\dfrac{213.3}{Rs+213.3}\right]^2}{320}+2\,\frac{\left[60\,\dfrac{213.3}{Rs+213.3}\right]^2}{320} = 30$$

Solving, we find that we require $\boxed{Rs = 106.65\ \Omega}$, as confirmed by the PSpice simulation below, which shows that both wiring configurations lead to one lamp with 80-V across it, and two lamps with 40 V across each.



73.

- Maximum current rating for the LED is 35 mA.
- Its resistance can vary between 47 and 117 Ω.
- A 9-V battery must be used as a power source.
- Only standard resistance values may be used.

One possible current-limiting scheme is to connect a 9-V battery in series with a resistor $R_{limiting}$ and in series with the LED.

From KVL,

$$I_{LED} = \frac{9}{R_{limiting} + R_{LED}}$$

The maximum value of this current will occur at the minimum LED resistance, 47 Ω. Thus, we solve

$$35\times10^{-3} = \frac{9}{R_{limiting} + 47}$$

to obtain $R_{limiting} \geq 210.1\ \Omega$ to ensure an LED current of less than 35 mA. This is not a standard resistor value, however, so we select

$$\boxed{R_{limiting} = 220\ \Omega.}$$

CHAPTER SIX SOLUTIONS

1. The first step is to perform a simple source transformation, so that a 0.15-V source in series with a 150-Ω resistor is connected to the inverting pin of the ideal op amp.

Then, $v_{\text{out}} = -\frac{2200}{150}(0.15) = \boxed{-2.2\text{ V}}$

CHAPTER SIX SOLUTIONS

2. In order to deliver 150 mW to the 10-kΩ resistor, we need v_{out} = $\sqrt{(0.15)(10\times10^3)}\ =\ 38.73\,\text{V}$. Writing a nodal equation at the inverting input, we find

$$0\ =\ \frac{5}{R}+\frac{5-v_{out}}{1000}$$

Using v_{out} = 38.73, we find that $\boxed{R = 148.2\ \Omega.}$

3. Since the 670-Ω switch requires 100 mA to activate, the voltage delivered to it by our op amp circuit must be (670)(0.1) = 67 V. The microphone acts as the input to the circuit, and provides 0.5 V. Thus, an amplifier circuit having a gain = 67/0.5 = 134 is required.

One possible solution of many: a non-inverting op amp circuit with the microphone connected to the non-inverting input terminal, the switch connected between the op amp output pin and ground, a feedback resistor R_f = 133 Ω, and a resistor R_1 = 1 Ω.

4. We begin by labeling the nodal voltages v_- and v_+ at the inverting and non-inverting input terminals, respectively. Since no current can flow into the non-inverting input, no current flows through the 40-kΩ resistor; hence, $v_+ = 0$. Therefore, we know that $v_- = 0$ as well.

Writing a single nodal equation at the non-inverting input then leads to

$$0 = \frac{(v_- - v_S)}{100} + \frac{(v_- - v_{out})}{22000}$$

or

$$0 = \frac{-v_S}{100} + \frac{-v_{out}}{22000}$$

Solving,

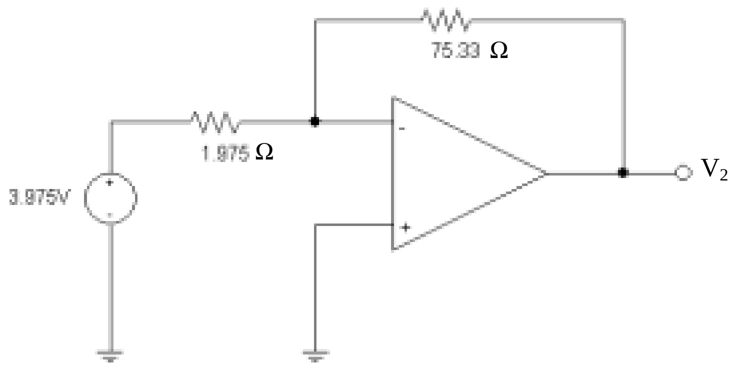
$$\boxed{v_{out} = -220\, v_S}$$

5. We first label the nodal voltage at the output pin V_o. Then, writing a single nodal equation at the inverting input terminal of the op amp,

$$0 = \frac{4 - 3}{1000} + \frac{4 - V_o}{17000}$$

Solving, we find that $V_o = 21$ V. Since no current can flow through the 300-kΩ resistor, $\boxed{V_1 = 21}$ as well.

6. A source transformation and some series combinations are well worthwhile prior to launching into the analysis. With 5 kΩ || 3 kΩ = 1.875 kΩ and (1 mA)(1.875 kΩ) = 1.875 V, we may redraw the circuit as



This is now a simple inverting amplifier with gain $– R_f/ R_1$ = -75.33/ 1.975 = -38.14.

Thus, $V_2 = -38.14(3.975) = \boxed{-151.6 \text{ V}}$.

7. This is a simple inverting amplifier, so we may write

$$v_{\text{out}} = \frac{-2000}{1000}(2 + 2\sin 3t) = \boxed{-4(1 + \sin 3t)\ \text{V}}$$

$\boxed{v_{\text{out}}(t = 3\ \text{s}) = -5.648\ \text{V}.}$

8. We first combine the 2 MΩ and 700 kΩ resistors into a 518.5 kΩ resistor.

We are left with a simple non-inverting amplifier having a gain of
1 + 518.5/ 250 = 3.074. Thus,

v_{out} = (3.074) v_{in} = 18 so $\boxed{v_{in} = 5.856 \text{ V.}}$

9. This is a simple non-inverting amplifier circuit, and so it has a gain of $1 + R_f / R_1$.
We want $v_{out} = 23.7 \cos 500t$ V when the input is $0.1 \cos 500t$ V, so a gain of 23.7/0.1 = 237 is required.
One possible solution of many: $\boxed{R_f = 236\ \text{k}\Omega \text{ and } R_1 = 1\ \text{k}\Omega.}$

10. Define a nodal voltage V_- at the inverting input, and a nodal voltage V_+ at the non-inverting input. Then,

At the non-inverting input: $-3\times10^{-6} = \dfrac{V_+}{1.5\times10^{6}}$ [1]

Thus, $V_+ = -4.5$ V, and we therefore also know that $V_- = -4.5$ V.

At the inverting input: $0 = \dfrac{V_-}{R_6} + \dfrac{V_- - V_{out}}{R_7}$ [2]

Solving and making use of the fact that $V_- = -4.5$ V,

$$v_{out} = -\frac{R_7}{R_6}(4.5) - 4.5 = \boxed{-4.5\left(\frac{R_7}{R_6} + 1\right)\ \text{V}}$$

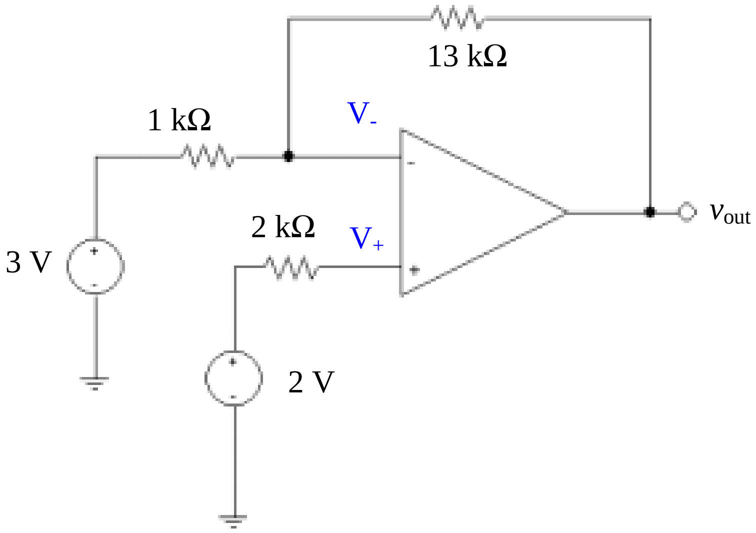
11. (a) **B** must be the non-inverting input: that yields a gain of 1 + 70/10 = 8 and an output of 8 V for a 1-V input.

(b) $R_1 = \infty$, $R_A = 0$. We need a gain of 20/10 = 2, so choose $R_2 = R_B = 1\ \Omega$.

(c) **A** is the inverting input since it has the feedback connection to the output pin.

12. It is probably best to first perform a simple source transformation:

$(1\text{ mA})(2\text{ k}\Omega) = 2\text{ V}.$



Since no current can flow into the non-inverting input pin, we know that $V_+ = 2$ V, and therefore also that $V_- = 2$ V. A single nodal equation at the inverting input yields:

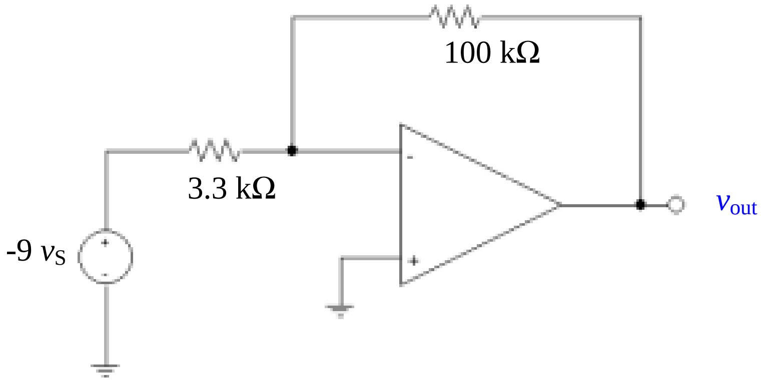
$$0 = \frac{2-3}{1000} + \frac{2-v_{out}}{13000}$$

which yields $\boxed{v_{out} = -11\text{ V.}}$

13. We begin by find the Thévenin equivalent to the left of the op amp:

$$V_{th} = -3.3(3)\ v_\pi \ = \ -9.9\ v_\pi \ = \ -9.9\frac{1000\ v_S}{1100} \ = \ -9\ v_S$$

R_{th} = 3.3 kΩ, so we can redraw the circuit as:

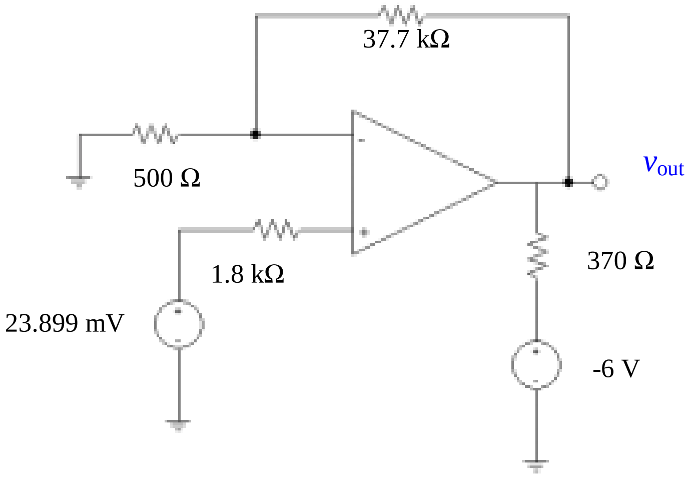


which is simply a classic inverting op amp circuit with gain of -100/3.3 = -30.3.

Thus, v_{out} = (-30.3)(-9 v_S) = 272.7 v_S

For v_S = 5 sin 3t mV, v_{out} = 1.364 sin 3t V, and $\boxed{v_{out}(0.25\ \text{s}) = 0.9298\ \text{V}.}$

14. We first combine the 4.7 MΩ and 1.3 kΩ resistors: 4.7 MΩ || 1.3 kΩ = 1.30 kΩ. Next, a source transformation yields $(3\times10^{-6})(1300)$ = 3.899 mV which appears in series with the 20 mV source and the 500-Ω resistor. Thus, we may redraw the circuit as



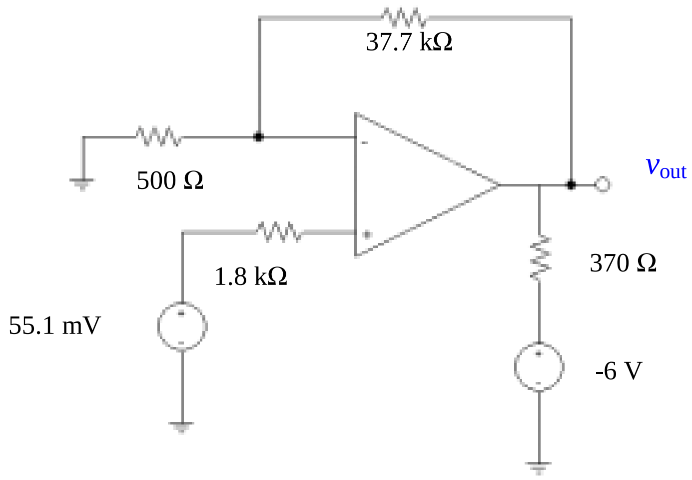
Since no current flows through the 1.8 kΩ resistor, V_+ = 23.899 mV and hence V_- = 23.899 mV as well. A single nodal equation at the inverting input terminal yields

$$0 = \frac{23.899\times10^{-3}}{500} + \frac{23.899\times10^{-3} - v_{out}}{37.7\times10^{3}}$$

Solving,

$$\boxed{v_{out} = 1.826 \text{ V}}$$

15. We first combine the 4.7 MΩ and 1.3 kΩ resistors: 4.7 MΩ || 1.3 kΩ = 1.30 kΩ. Next, a source transformation yields $(27\times10^{-6})(1300)$ = 35.1 mV which appears in series with the 20 mV source and the 500-Ω resistor. Thus, we may redraw the circuit as



Since no current flows through the 1.8 kΩ resistor, V_+ = 55.1 mV and hence V_- = 55.1 mV as well. A single nodal equation at the inverting input terminal yields

$$0 = \frac{55.1\times10^{-3}}{500} + \frac{55.1\times10^{-3} - v_{out}}{37.7\times10^{3}}$$

Solving,

$$\boxed{v_{out} = 4.21 \text{ V}}$$

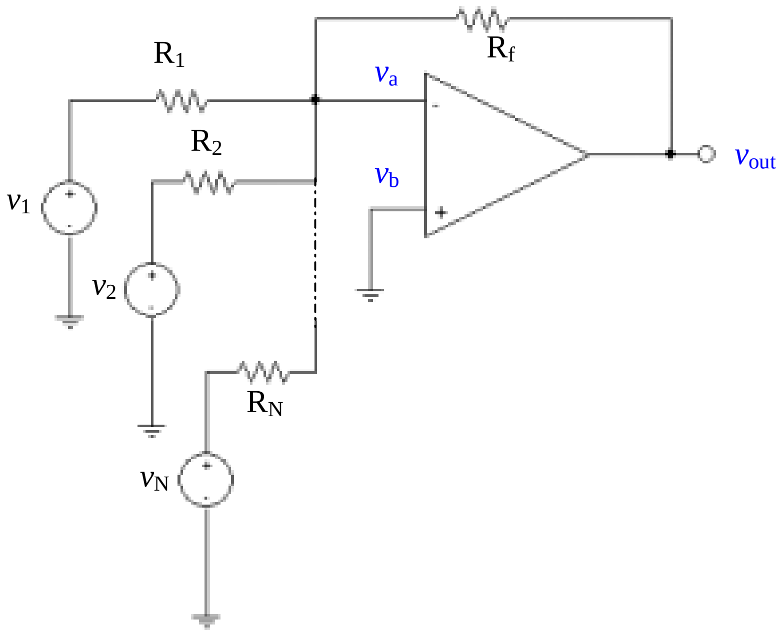
16. The 3 mA source, 1 kΩ resistor and 20 kΩ resistor may be replaced with a –3 V source ("+" reference up) in series with a 21 kΩ resistor. No current flows through either 1 MΩ resistor, so that the voltage at each of the four input terminals is identically zero. Considering each op amp circuit separately,

$$v_{\text{out}}\Big|_{\text{LEFT OPAMP}} = -(-3)\frac{100}{21} = 14.29\text{ V}$$

$$v_{\text{out}}\Big|_{\text{RIGHT OPAMP}} = -(5)\frac{100}{10} = -50\text{ V}$$

$$v_x = v_{\text{out}}\Big|_{\text{LEFT OPAMP}} - v_{\text{out}}\Big|_{\text{RIGHT OPAMP}} = 14.29 + 50 = \boxed{64.29\text{ V.}}$$

17. A general summing amplifier with N input sources:



1. $v_a = v_b = 0$
2. A single nodal equation at the inverting input leads to:

$$0 = \frac{v_a - v_{out}}{R_f} + \frac{v_a - v_1}{R_1} + \frac{v_a - v_2}{R_2} + \ldots + \frac{v_a - v_N}{R_N}$$

Simplifying and making use of the fact that $v_a = 0$, we may write this as

$$\left[-\frac{1}{R_f}\prod_{i=1}^{N} R_i\right] v_{out} = \frac{v_1}{R_1}\prod_{i=1}^{N} R_i + \frac{v_2}{R_2}\prod_{i=1}^{N} R_i + \ldots + \frac{v_N}{R_N}\prod_{i=1}^{N} R_i$$

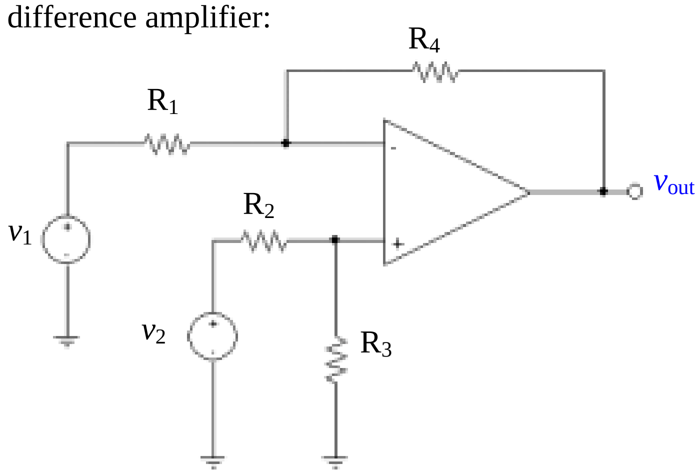
or simply

$$-\frac{v_{out}}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \ldots + \frac{v_N}{R_N}$$

Thus,

$$\boxed{v_{out} = -R_f \sum_{i=1}^{N} \frac{v_i}{R_i}}$$

18. A general difference amplifier:



Writing a nodal equation at the inverting input,

$$0 = \frac{v_a - v_1}{R_1} + \frac{v_a - v_{out}}{R_f}$$

Writing a nodal equation at the non-inverting input,

$$0 = \frac{v_b}{R_3} + \frac{v_b - v_2}{R_2}$$

Simplifying and collecting terms, we may write

$$(R_f + R_1)\, v_a - R_1\, v_{out} = R_f\, v_1 \qquad [1]$$

$$(R_2 + R_3)\, v_b = R_3\, v_2 \qquad [2]$$

From Eqn. [2], we have $v_b = \frac{R_3}{R_2 + R_3} v_2$

Since va = vb, we can now rewrite Eqn. [1] as

$$-R_1\, v_{out} = R_f v_1 - \frac{(R_f + R_1)R_3}{R_2 + R_3} v_2$$

and hence

$$\boxed{v_{out} = -\frac{R_f}{R_1} v_1 + \frac{R_3}{R_1}\left(\frac{R_f + R_1}{R_2 + R_3}\right) v_2}$$

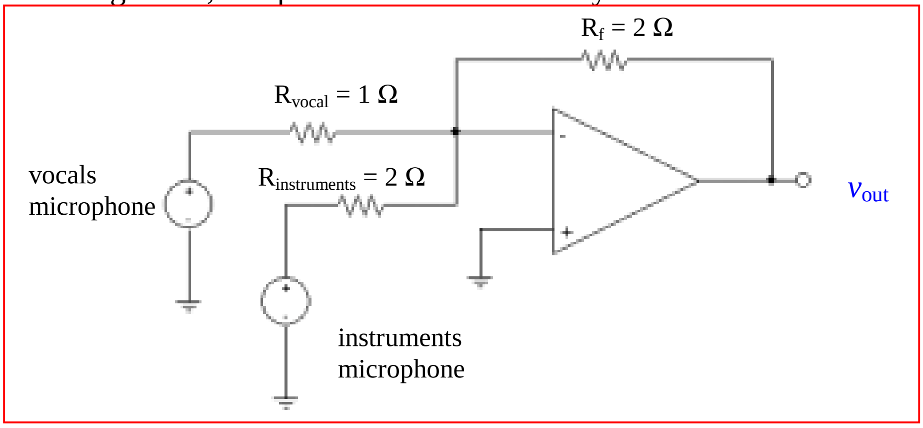
19. In total darkness, the CdS cell has a resistance of 100 kΩ, and at a light intensity L of 6 candela it has a resistance of 6 kΩ. Thus, we may compute the light-dependent resistance (assuming a linear response in the range between 0 and 6 candela) as $R_{CdS} = -15L + 100\ \Omega$.

Our design requirement (using the standard inverting op amp circuit shown) is that the voltage across the load is 1.5 V at 2 candela, and less than 1.5 V for intensities greater than 2 candela.

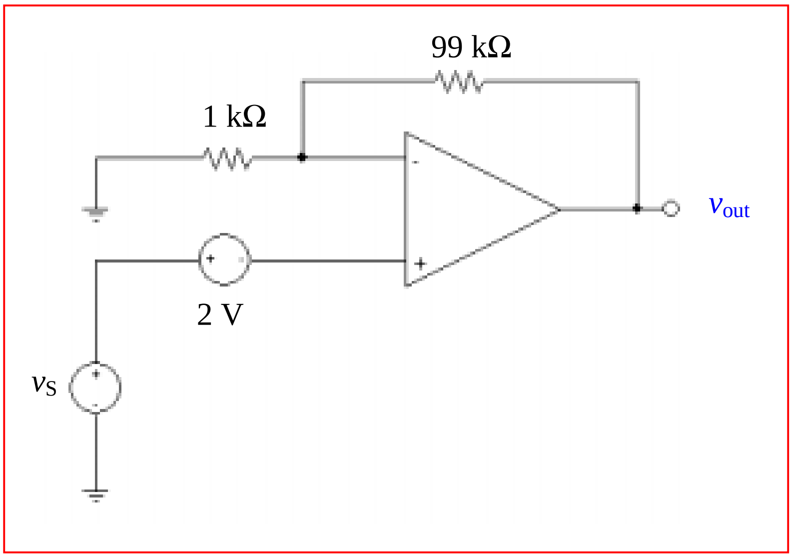
Thus, v_{out}(2 candela) = $-R_{CdS}\, v_S / R_1 = -70\ V_S / R_1 = 1.5$ (R_1 in kΩ).

$\boxed{\text{Pick } R_1 = 10\text{ k}\Omega.\ \text{Then } v_S = -0.2143\text{ V.}}$

20. We want $R_f / R_{instrument} = 2K$, and $R_f / R_{vocal} = 1K$, where K is a constant not specified. Assuming K = 1, one possible solution of many is:



21. One possible solution of many:



22. v_{out} of stage 1 is (1)(-20/ 2) = -10 V.

v_{out} of stage 2 is (-10)(-1000/ 10) = $\boxed{1000 \text{ V}}$

Note: in reality, the output voltage will be limited to a value less than that used to power the op amps.

23. We have a difference amplifier as the first amplifier stage, and a simple voltage follower as the second stage. We therefore need only to find the output voltage of the first stage: v_{out} will track this voltage. Using voltage division, then, we vind that the voltage at the non-inverting input pin of the first op amp is

$$V_2\left(\frac{R_3}{R_2 + R_3}\right)$$

and this is the voltage at the inverting input terminal also. Thus, we may write a single nodal equation at the inverting input of the first op amp:

$$0 \;=\; \frac{1}{R_1}\left[V_2\left(\frac{R_3}{R_2 + R_3}\right) - V_1\right] \;+\; \frac{1}{R_f}\left[V_2\left(\frac{R_3}{R_2 + R_3}\right) - V_{out}\Big|_{Stage\,1}\right]$$

which may be solved to obtain:

$$\boxed{V_{out} \;=\; V_{out}\Big|_{Stage\,1} \;=\; \left(\frac{R_f}{R_1} + 1\right)\frac{R_3}{R_2 + R_3}V_2 \;-\; \frac{R_f}{R_1}V_1}$$

24. The output of the first op amp stage may be found by realising that the voltage at the non-inverting input (and hence the voltage at the *inverting* input) is 0, and writing a ingle nodal equation at the inverting input:

$$0 = \frac{0 - V_{out}\big|_{stage\,1}}{47} + \frac{0 - 2}{1} + \frac{0 - 3}{7} \quad \text{which leads to } V_{out}\big|_{steage\,1} = -114.1 \text{ V}$$

This voltage appears at the input of the second op amp stage, which has a gain of –3/ 0.3 = 10. Thus, the output of the second op amp stage is –10(-114.1) = 1141 V. This voltage appears at the input of the final op amp stage, which has a gain of –47/ 0.3 = -156.7.

Thus, the output of the circuit is –156.7(1141) = $\boxed{-178.8 \text{ kV}}$, which is completely and utterly ridiculous.

25. The output of the top left stage is –1(10/ 2) = -5 V.
The output of the middle left stage is –2(10/ 2) = -10 V.
The output of the bottom right stage is –3(10/ 2) = -15 V.

These three voltages are the input to a summing amplifier such that

$$V_{out} \;=\; -\frac{R}{100}\left(-5-10-15\right) \;=\; 10$$

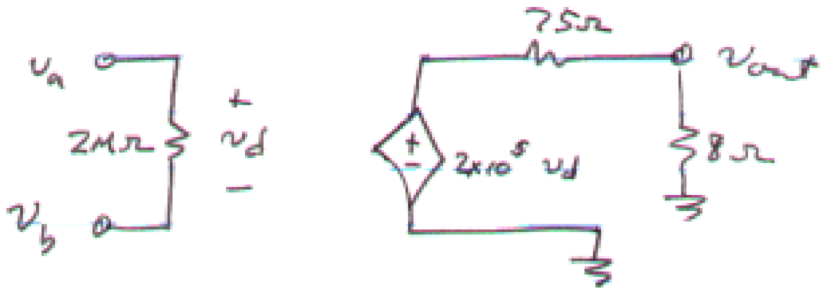
Solving, we find that $\boxed{R = 33.33\ \Omega.}$

26. Stage 1 is configured as a voltage follower: the output voltage will be equal to the input voltage. Using voltage division, the voltage at the non-inverting input (and hence at the inverting input, as well), is

$$5\frac{50}{100+50} = 1.667\text{ V}$$

The second stage is wired as a voltage follower also, so $\boxed{v_{out} = 1.667\text{ V.}}$

27.



(a) $v_a = v_b = 1$ nV $\therefore$ $v_d = 0$ and $v_{out} = 0$. Thus, $\boxed{P_{8\Omega} = 0 \text{ W.}}$

(b) $v_a = 0$, $v_b = 1$ nV $\therefore$ $v_d = -1$ nV

$v_{out} = (2\times10^5)(-1\times10^{-9})\dfrac{8}{75+8} = -19.28$ μV. Thus, $P_{8\Omega} = \dfrac{v_{out}^2}{8} = \boxed{46.46 \text{ pW.}}$

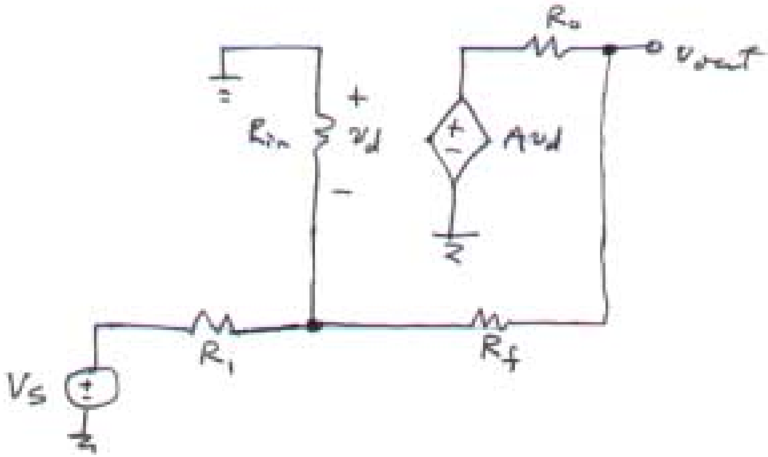
(c) $v_a = 2$ pV, $v_b = 1$ fV $\therefore$ $v_d = 1.999$ pV

$v_{out} = (2\times10^5)(1.999\times10^{-12})\dfrac{8}{75+8} = 38.53$ nV. Thus, $P_{8\Omega} = \dfrac{v_{out}^2}{8} = \boxed{185.6 \text{ aW}}.$

(c) $v_a = 50$ μV, $v_b = -4$ μV $\therefore$ $v_d = 54$ μV

$v_{out} = (2\times10^5)(54\times10^{-6})\dfrac{8}{75+8} = 1.041$ V. Thus, $P_{8\Omega} = \dfrac{v_{out}^2}{8} = \boxed{135.5 \text{ mW}}.$

28.



Writing a nodal equation at the "-v_d" node,

$$0 = \frac{-v_d}{R_{in}} + \frac{-v_d - V_S}{R_1} + \frac{-v_d - v_{out}}{R_f} \qquad [1]$$

or $\quad (R_1R_f + R_{in}R_f + R_{in}R_1)\, v_d + R_{in}R_1 v_{out} = -R_{in}R_fV_S \qquad [1]$

Writing a nodal equation at the "v_{out}" node,

$$0 = \frac{-v_{out} - Av_d}{R_o} + \frac{v_{out} - (-v_d)}{R_f} \qquad [2]$$

Eqn. [2] can be rewritten as:

$$v_d = \frac{-(R_f + R_o)}{R_o - AR_f}\, v_{out} \qquad [2]$$

so that Eqn. [1] becomes:

$$v_{out} = -\frac{R_{in}(AR_f - R_o)V_S}{AR_{in}R_1 + R_fR_1 + R_{in}R_f + R_{in}R_1 + R_oR_1 + R_oR_{in}}$$

where for this circuit, $A = 10^6$, R_{in} = 10 TΩ, R_o = 15 Ω, R_f = 1000 kΩ, R_1 = 270 kΩ.

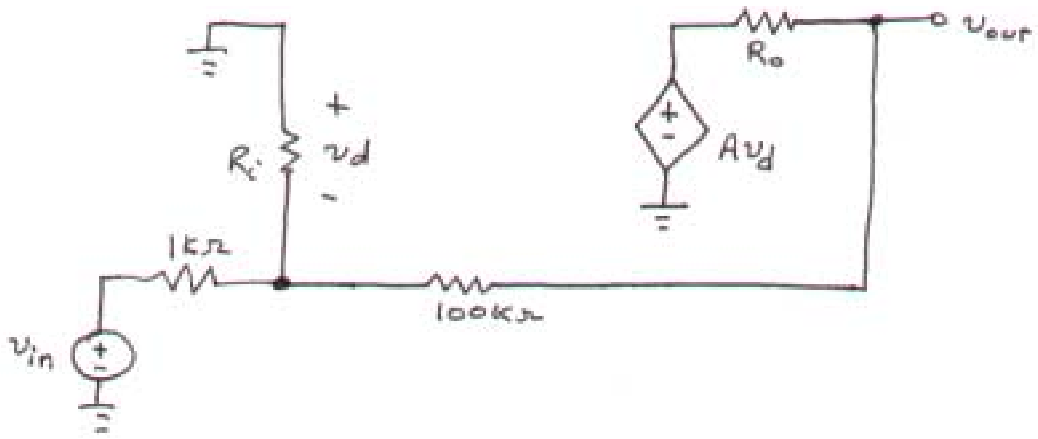
$\boxed{\text{(a) –3.704 mV; \quad (b) 27.78 mV; \quad (c) –3.704 V.}}$

29. $v_{out} = Av_d = A\frac{R_i}{16 + R_i}\left(80 \times 10^{15}\right) \sin 2t$ V

(a) A = 10^5, R_i = 100 MΩ, R_o value irrelevant. $\boxed{v_{out} = 8 \sin 2t \text{ nV}}$

(b) A = 10^6, R_i = 1 TΩ, R_o value irrelevant. $\boxed{v_{out} = 80 \sin 2t \text{ nV}}$

30.



(a) Find v_{out}/ v_{in} if $R_i = \infty$, $R_o = 0$, and A is finite.

The nodal equation at the inverting input is

$$0 = \frac{-v_d - v_{in}}{1} + \frac{-v_d - v_{out}}{100} \qquad [1]$$

At the output, with $R_o = 0$ we may write $v_{out} = Av_d$ so $v_d = v_{out}/ A$. Thus, Eqn. [1] becomes

$$0 = \frac{v_{out}}{A} + v_{in} + \frac{v_{out}}{100A} + \frac{v_{out}}{100}$$

from which we find

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{-100A}{101 + A}} \qquad [2]$$

(b) We want the value of A such that $v_{out}/ v_{in} = -99$ (the "ideal" value would be –100 if A were infinite). Substituting into Eqn. [2], we find

$$\boxed{A = 9999}$$

31. (a) $\delta = 0$ V $\therefore$ $v_d = 0$, and $\boxed{P_{8\Omega} = 0 \text{ W.}}$

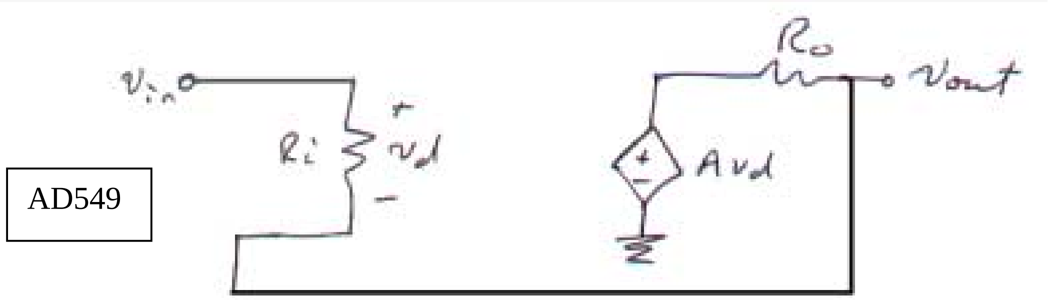
(b) $\delta = 1$ nV, so $v_d = 5 - (5 + 10^{-9}) = -10^{-9}$ V
Thus,

$$v_{out} = (2\times10^5)v_d \frac{8}{8+75} = -19.28\ \mu\text{V and } P_{8\Omega} = (v_{out})^2/\ 8 = \boxed{46.46 \text{ pW.}}$$

(c) $\delta = 2.5\ \mu$V, so $v_d = 5 - (5 + 2.5\times10^{-6}) = -2.5\times10^{-6}$V
Thus,

$$v_{out} = (2\times10^5)v_d \frac{8}{8+75} = -48.19 \text{ mV and } P_{8\Omega} = (v_{out})^2/\ 8 = \boxed{290.3\ \mu\text{W.}}$$

32.



Writing a single nodal equation at the output, we find that

$$0 \;=\; \frac{v_{out} \;-\; v_{in}}{R_i} + \frac{v_{out} \;-\; Av_d}{R_o} \qquad [1]$$

Also, $v_{in} - v_{out} = v_d$, so Eqn. [1] becomes

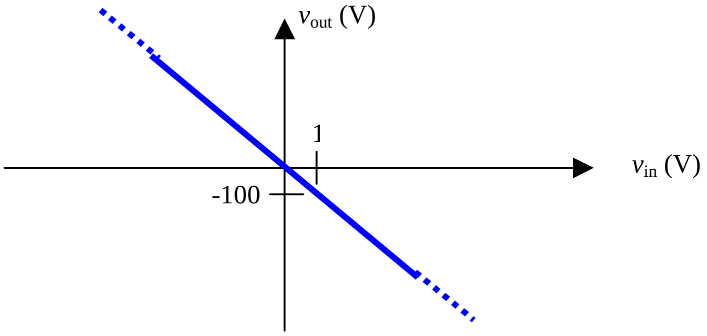
$$0 = (v_{out} - v_{in})\,R_o \;+\; (v_{out} - Av_{in} + Av_{out})\,R_i$$

and

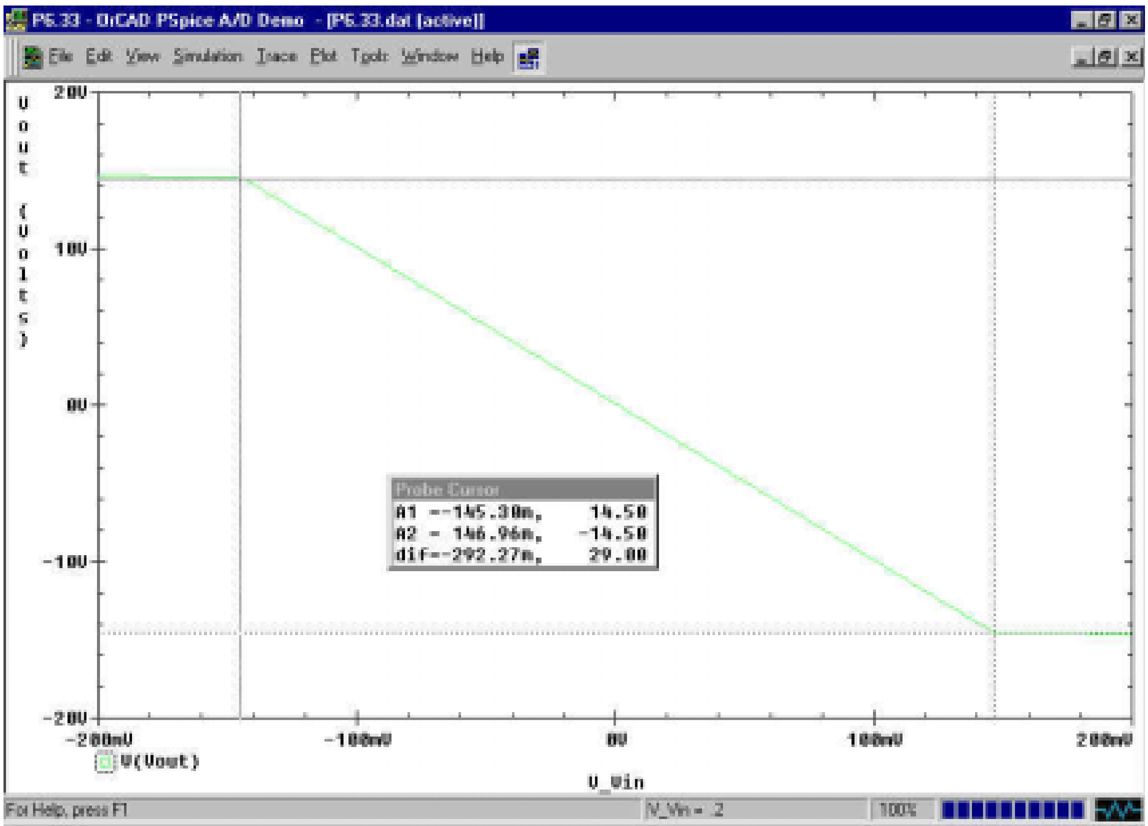
$$v_{out} \;=\; \frac{(R_o + AR_i)}{R_o + (A+1)\,R_i}\,v_{in}$$

To within 4 significant figures (and more, actually), when v_{in} = -16 mV, $\boxed{v_{out} = -16\text{ mV}}$ (this is, after all, a voltage follower circuit).

33. The ideal op amp model predicts a gain $v_{out}/\, v_{in}$ = -1000/ 10 = -100, regardless of the value of v_{in}. In other words, it predicts an input-output characteristic such as:



From the PSpice simulation result shown below, we see that the ideal op amp model is reasonably accurate for $|v_{in}| \times 100 < 15$ V (the supply voltage, assuming both have the same magnitude), but the onset of saturation is at ±14.5 V, or $|v_{in}| \sim 145$ mV. Increasing $|v_{in}|$ past this value does not lead to an increase in $|v_{out}|$.

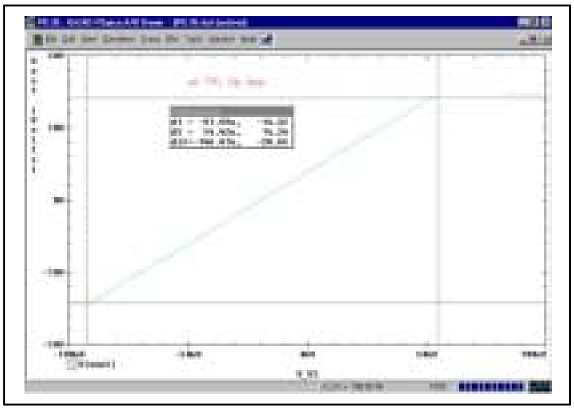


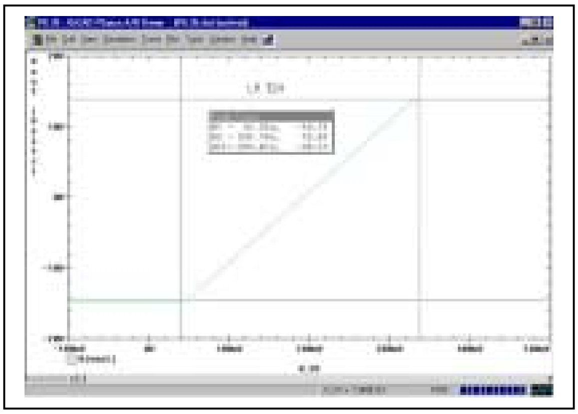
34. $\boxed{\text{Positive voltage supply, negative voltage supply, inverting input, ground, output pin.}}$

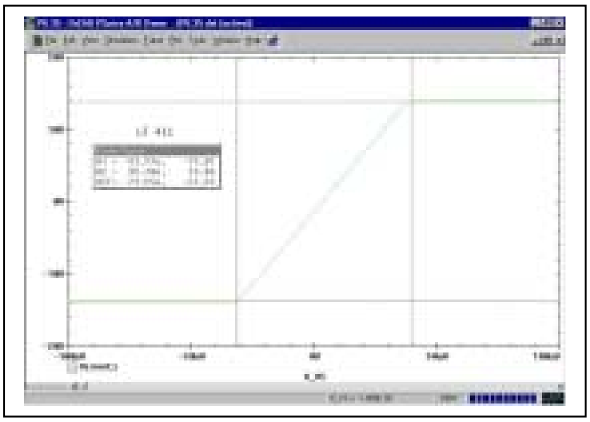
35. This op amp circuit is an open-loop circuit; there is no external feedback path from the output terminal to either input. Thus, the output should be the open-loop gain times the differential input voltage, minus any resistive losses.

From the simulation results below, we see that all three op amps saturate at a voltage magnitude of approximately 14 V, corresponding to a differential input voltage of 50 to 100 μV, except in the interest case of the LM 324, which may be showing some unexpected input offset behavior.

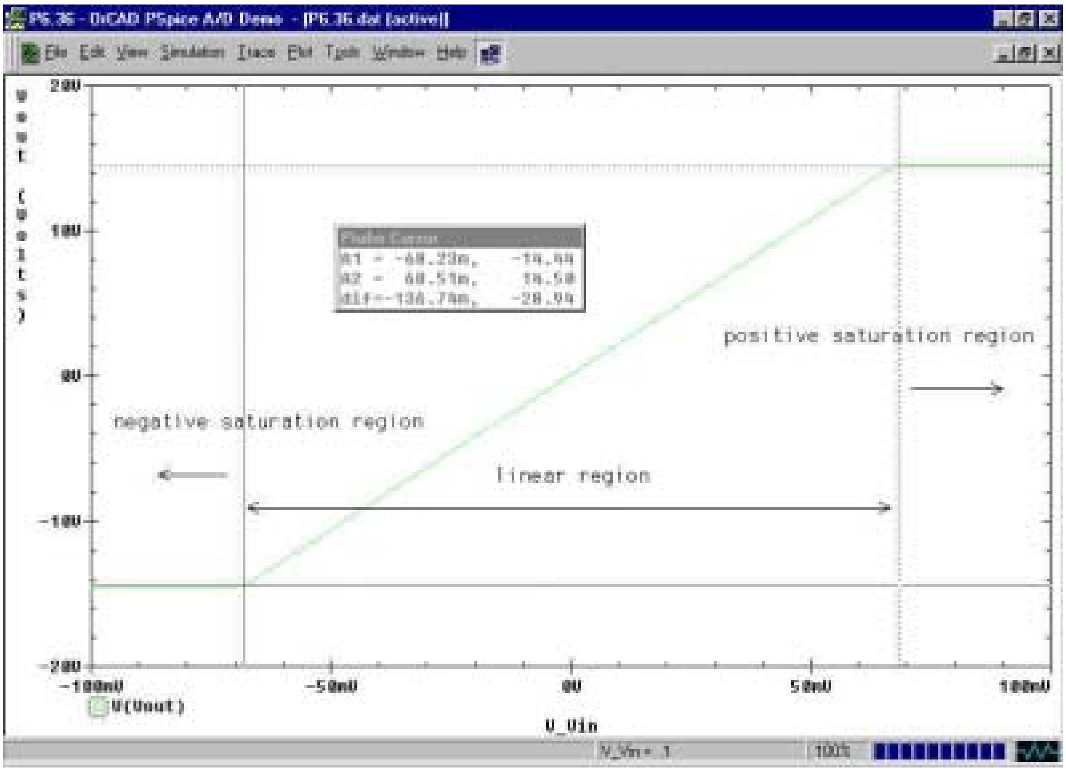
op amp	**onset of negative saturation**	**negative saturation voltage**	**onset of positive saturation**	**positive saturation voltage**
μA 741	-92 μV	-14.32 V	54.4 mV	14.34 V
LM 324	41.3 μV	-14.71 V	337.2 mV	13.87 V
LF 411	-31.77 μV	-13.81 V	39.78 mV	13.86 V







36. This is a non-inverting op amp circuit, so we expect a gain of 1 + 1000/4.7 = 213.8. With ±15 V DC supplies, we need to sweep the input just above and just below this value divided by the gain to ensure that we see the saturation regions. Performing the indicated simulation and a DC sweep from –0.1 V to +0.1 V with 0.001 V steps, we obtain the following input-output characteristic:



Using the cursor tool, we see that the linear region is in the range of
–68.2 mV < V_{in} < 68.5 mV.
The simulation predicts a gain of 7.103 V/ 32.87 mV = 216.1, which is reasonably close to the value predicted using the ideal op amp model.

37. Referring to the detailed model of an op amp, shorting the input terminals together shorts the dependent source to ground. Therefore, any 1-V source connected to the output through a 1-Ω resistor should "see" 1 Ω + R_o. For the µA 741, we expect 1 + 75 = 76 Ω. For the LF 411, we expect ~1 + 1 Ω or ~ 2 Ω.

Simulating the µA741 circuit, we find:

Supply voltages	Current into output	Total resistance	Output resistance
±15 V	-42.5 mA	-23.53 Ω	-22.53 Ω
±5 V	-40.55 mA	-24.66 Ω	-24.66 Ω
±2 V	-40.55 mA	-24.66 Ω	-24.66 Ω
0 V	579.2 mA	1.727 Ω	727 mΩ

Conclusion: as we might expect from previous experience in determining Thévenin equivalent resistances, we must short out the voltage supplies to the op amp when performing such an experiment (hence the negative resistance values obtained above). However, we obtained 0.727 Ω instead of the expected 75 Ω, which leads to two possible conclusions: (1) The PSpice model is not designed to represent the op amp behavior accurately in such a circuit configuration or (2) such an experimental connection is not adequate for measuring the output resistance.

Simulating the LF411 circuit, we find:

Supply voltages	Current into output	Total resistance	Output resistance
±15 V	25.46 mA	39.28 Ω	38.28 Ω
±5 V	25.43 mA	39.32 Ω	38.32 Ω
±2 V	25.48 mA	39.24 Ω	28.24 Ω
0 V	1000 mA	1 Ω	0 Ω

Conclusion: as we might expect from previous experience in determining Thévenin equivalent resistances, we must short out the voltage supplies to the op amp when performing such an experiment. However, we obtained ~0 Ω instead of the expected 1 Ω, which leads to two possible conclusions: (1) The PSpice model is not designed to represent the op amp behavior accurately in such a circuit configuration or (2) such an experimental connection is not adequate for measuring the output resistance. However, it is interesting that PSpice did predict a much lower output resistance for the LF 411 than the µA 741, as we would expect.

38. Based on the detailed model of **the LF 411 op amp**, we can write the following nodal equation at the inverting input:

$$0 = \frac{-v_d}{R_{in}} + \frac{v_x - v_d}{10^4} + \frac{Av_d - v_d}{10^6 + R_o}$$

Substituting values for the LF 411 and simplifying, we make appropriate approximations and then solve for v_d in terms of v_x, finding that

$$v_d = \frac{-10^6}{199.9 \times 10^6} v_x = -\frac{v_x}{199.9}$$

With a gain of –1000/10 = -100 and supply voltage magnitudes of 15 V, we are effectively limited to values of $|v_x| < 150$ mV.

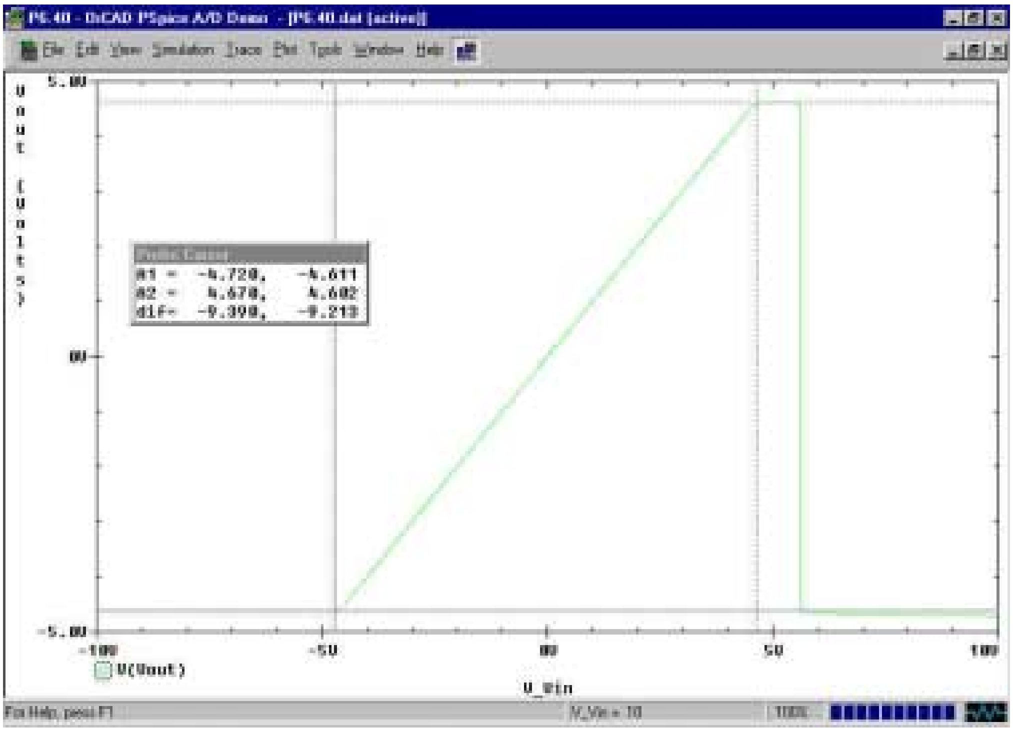
For v_x = -10 mV, PSpice predicts v_d = 6 μV, where the hand calculations based on the detailed model predict 50 μV, which is about one order of magnitude larger. For the same input voltage, PSpice predicts an input current of -1 μA, whereas the hand calculations predict $99.5v_x$ mA = -995 nA (which is reasonably close).

(a) The gain of the inverting amplifier is –1000. At a sensor voltage of –30 mV, the predicted output voltage (assuming an ideal op amp) is +30 V. At a sensor voltage of +75 mV, the predicted output voltage (again assuming an ideal op amp) is –75 V. Since the op amp is being powered by dc sources with voltage magnitude equal to 15 V, the output voltage range will realistically be limited to the range

$\boxed{-15 < V_{out} < 15 \text{ V.}}$

(b) The peak input voltage is 75 mV. Therefore, $15/\,75\times10^{-3} = 200$, and we should set the resistance ratio $\boxed{R_f/\,R_1 < 199}$ to ensure the op amp does not saturate.

39. (a)

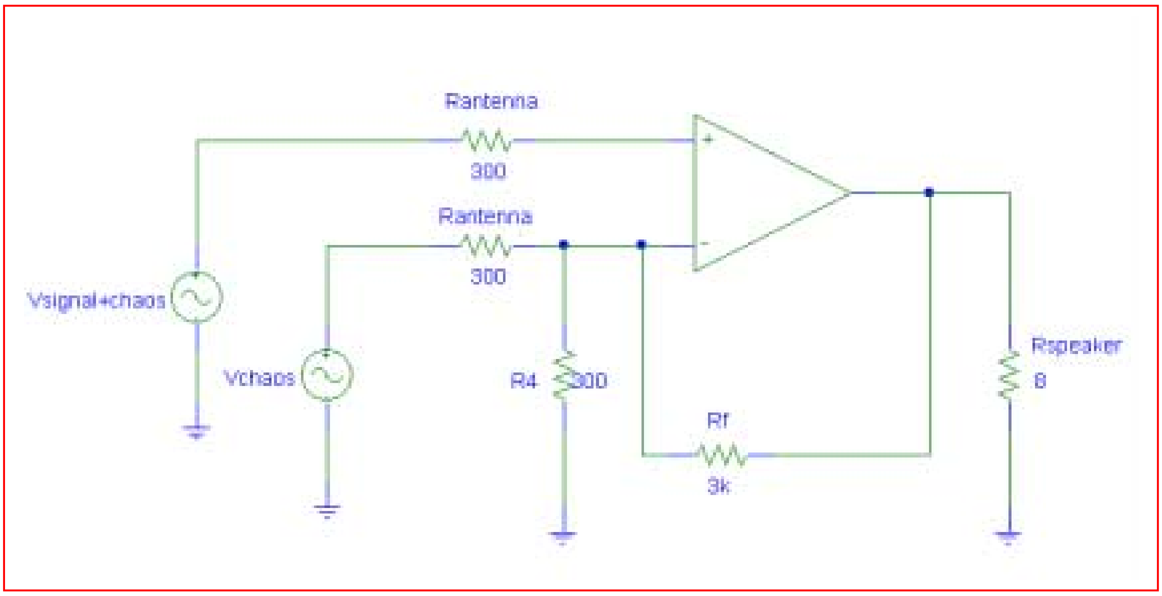


We see from the simulation result that negative saturation begins at V_{in} = –4.72 V, and positive saturation begins at V_{in} = +4.67 V.

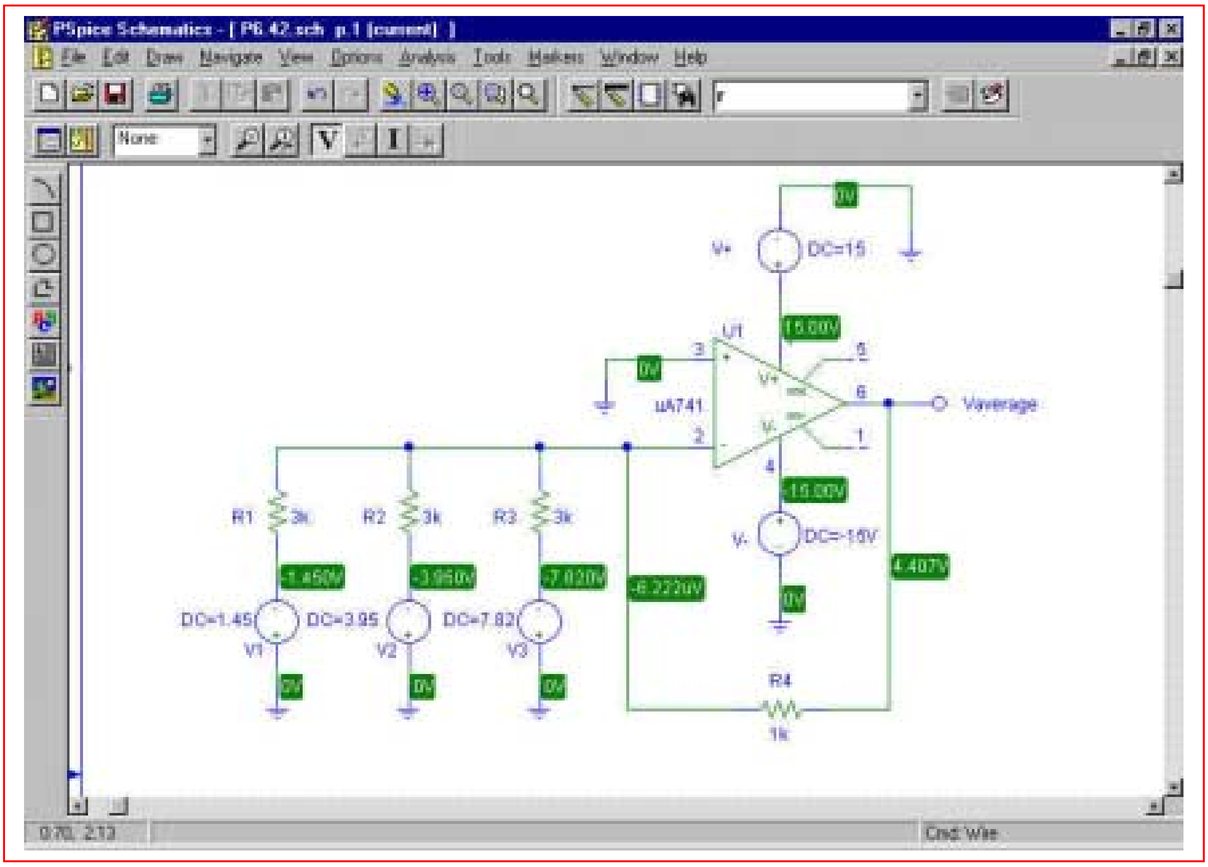
(b) Using a 1 pΩ resistor between the output pin and ground, we obtain an output current of 40.61 mA, slightly larger than the expected 35 mA, but not too far off.

41. We assume that the strength of the separately-broadcast chaotic “noise” signal is received at the appropriate intensity such that it may precisely cancel out the chaotic component of the total received signal; otherwise, a variable-gain stage would need to be added so that this could be adjusted by the user. We also assume that the signal frequency is separate from the “carrier” or broadcast frequency, and has already been separated out by an appropriate circuit (in a similar fashion, a radio station transmitting at 92 MHz is sending an audio signal of between 20 and 20 kHz, which must be separated from the 92 MHz frequency.)

One possible solution of many (all resistances in ohms):



41. One possible solution of many:



This circuit produces an output equal to the average of V_1, V_2, and V_3, as shown in the simulation result: $V_{average} = (1.45 + 3.95 + 7.82)/ 3 = 4.407$ V.

CHAPTER SEVEN SOLUTIONS

1. (a) $C = \frac{\varepsilon A}{d} = \frac{8.854\times10^{-12}\left(78.54\times10^{-6}\right)}{100\times10^{-6}} = \boxed{6.954\, pF}$

(b) $Energy, E = \frac{1}{2}CV^2 \therefore V = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2\left(1\times10^{-3}\right)}{6.954\times10^{-12}}} = \boxed{16.959 kV}$

(c) $E = \frac{1}{2}CV^2 \therefore C = \frac{2E}{V^2} = \frac{2\left(2.5\times10^{-6}\right)}{\left(100^2\right)} = 500\, pF$

$C = \frac{\varepsilon A}{d} \therefore \varepsilon = \frac{Cd}{A} = \frac{\left(500\times10^{-12}\right)\left(100\times10^{-6}\right)}{\left(78.54\times10^{-6}\right)} = 636.62\, pF.m^{-1}$

$\therefore \mathrm{Re}\, lative\_\, permittivity, \frac{\varepsilon}{\varepsilon_0} = \frac{636.62\times10^{-12}}{8.854\times10^{-12}} = \boxed{71.9}$

2. (a) For V_A = -1V, $W = \sqrt{\frac{2K_s\varepsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(1\times10^{24})}(0.57+1)}$

$$= 45.281\times10^{-9}\,m$$

$$C_j = \frac{11.8(8.854\times10^{-12})(1\times10^{-12})}{45.281\times10^{-9}} = \boxed{2.307\,fF}$$

(b) For V_A = -5V, $W = \sqrt{\frac{2K_s\varepsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(1\times10^{24})}(0.57+5)}$

$$= 85.289\times10^{-9}\,m$$

$$C_j = \frac{11.8(8.854\times10^{-12})(1\times10^{-12})}{85.289\times10^{-9}} = \boxed{1.225\,fF}$$

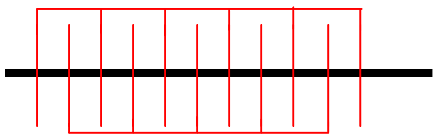
(c) For V_A = -10V,

$$W = \sqrt{\frac{2K_s\varepsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854\times10^{-12})}{(1.6\times10^{-19})(1\times10^{24})}(0.57+10)}$$

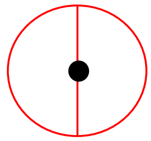
$$= 117.491\times10^{-9}\,m$$

$$C_j = \frac{11.8(8.854\times10^{-12})(1\times10^{-12})}{117.491\times10^{-9}} = \boxed{889.239\,aF}$$

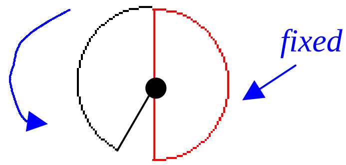
3. We require a capacitor that may be manually varied between 100 and 1000 pF by rotation of a knob. Let's choose an air dielectric for simplicity of construction, and a series of 11 half-plates:



Top view



Side view with no overlap between plates



Side view with a small overlap between plates.

Constructed as shown, the half-plates are in parallel, so that each of the 10 pairs must have a capacitance of 1000/ 10 = 100 pF when rotated such that they overlap completely. If we arbitrarily select an area of 1 cm^2 for each half-plate, then the gap spacing between each plate is $d = \varepsilon A/C = (8.854\times10^{-14}$ F/cm)(1 cm^2)/ $(100\times10^{-12}$ F) = 0.8854 mm. This is tight, but not impossible to achieve. The final step is to determine the amount of overlap which corresponds to 100 pF for the total capacitor structure. A capacitance of 100 pF is equal to 10% of the capacitance when all of the plate areas are aligned, so we need a pie-shaped wedge having an area of 0.1 cm^2. If the middle figure above corresponds to an angle of 0^o and the case of perfect alignment (maximum capacitance) corresponds to an angle of 180^o, we need to set out minimum angle to be 18^o.

4. (a) Energy stored $= \int_{t_0}^{t} v.C\frac{dv}{dt} = C\int_0^{2\times10^{-3}} 3e^{-\frac{t}{5}}.\left(-\frac{3}{5}e^{-\frac{t}{5}}\right)dt = \boxed{-1.080\mu J}$

(b) V_{max} = 3V

Max. energy at t=0, $= \frac{1}{2}CV^2 = 1.35mJ \therefore 37\% E_{max} = 499.5\mu J$

V at 37% E_{max} = 1.825V

$v(t) = 1.825 = 3e^{-\frac{t}{5}} \therefore t = 2.486s \Rightarrow \approx \boxed{2s}$

(c) $i = C\frac{dv}{dt} = 300\times10^{-6}\left(-\frac{3}{5}e^{-\frac{1.2}{5}}\right) = \boxed{-141.593\mu A}$

(d) $P = vi = 2.011(-120.658\times10^{-6}) = \boxed{-242.6\mu W}$

5. (a) $$v = \frac{1}{C}\cdot\frac{\pi}{2}\left(1\times10^{-3}\right)^2 = \frac{1}{47\times10^{-6}}\cdot\frac{(3.14159)}{2}\left(1\times10^{-3}\right)^2 = \boxed{33.421mV}$$

(b) $$v = \frac{1}{C}\cdot\left(\frac{\pi}{2}\left(1\times10^{-3}\right)^2 + 0\right) = \frac{1}{47\times10^{-6}}\cdot\frac{(3.14159)}{2}\left(1\times10^{-3}\right)^2 = \boxed{33.421mV}$$

(c) $$v = \frac{1}{C}\cdot\left(\frac{\pi}{2}\left(1\times10^{-3}\right)^2 + \frac{\pi}{4}\left(1\times10^{-3}\right)^2\right) = \frac{1}{47\times10^{-6}}\cdot\left(\frac{3\pi}{4}\left(1\times10^{-3}\right)^2\right) = \boxed{50.132mV}$$

6. $$V = \frac{1}{C}\int_0^{200ms} idt = \frac{1}{C}\left[\left(-\frac{7\times10^{-3}}{\pi}\cos\pi t\right)\right]_0^{200ms} = \frac{0.426}{C}$$

$$E = \frac{1}{2}CV^2 = 3\times10^{-6} = \frac{181.086\times10^{-9}}{2C} \therefore C = \frac{181.086\times10^{-9}}{2\left(3\times10^{-6}\right)} = \boxed{30181\mu F}$$

7.

(a) $c = 0.2\mu\text{F},\ v_c = 5 + 3\cos^2 200t\text{V};\ \therefore i_c = 0.2\times10^{-6}(3)(-2)\,200\sin 200t\cos 200t$

$\therefore i_c = \boxed{-0.12\sin 400t\text{mA}}$

(b) $w_c = \frac{1}{2}cv_c^2 = \frac{1}{2}\times2\times10^{-7}(5+3\cos^2 200t)^2 \therefore w_{c\max} = 10^{-7}\times64 = \boxed{6.4\mu\text{J}}$

(c) $v_c = \frac{1}{0.2}\times10^6\int_0^t 8e^{-100t}\times10^{-3}dt = 10^3\times40(-0.01)(e^{-100t}-1) = \boxed{400(1-e^{100t})\text{V}}$

(d) $\boxed{v_c = 500 - 400e^{-100t}\text{ V}}$

8. $v_c(0) = 250\text{V},\ c = 2\text{mF}\ (a)\ v_c(0.1) = 250 + 500\int_0^{0.1} 5dt$

$\therefore v_c(0.1) = 500\text{V}; v_c(0.2) = 500\int_{0.1}^{0.2} 10dt = 1000\text{V}$

$\therefore v_c(0.6) = 1750\text{V}, v_c(0.9) = 2000\text{V}$

$\therefore 0.9 < t < 1: v_c = 2000 + 500\int_{0.9}^{t} 10dt = 2000 + 5000(t - 0.9)$

$\therefore v_c = 2100 = 2000 + 5000(t_2 - 0.9) \therefore t_2 = 0.92 \therefore \boxed{0.9 < t < 0.92s}$

9.

(a)

$$w_c = \frac{1}{2}\mathrm{C}v^2 = \frac{1}{2}\times 10^{-6} v^2 = 2\times 10^{-2} e^{-1000t} \;\therefore\; v = \pm 200 e^{-500t}\,\mathrm{V}$$

$$i = \mathrm{C}v' = 10^{-6}(\pm 200)(-500)e^{-500t} = \mp 0.1 e^{-500t}$$

$$\therefore \mathrm{R} = \frac{-v}{i} = \frac{200}{0.1} = \boxed{2k\Omega}$$

(b)

$$\mathrm{P}_R = i^2\mathrm{R} = 0.01\times 2000 e^{-1000t} = 20 e^{-1000t}\,\mathrm{W}$$

$$\therefore \mathrm{W_R} = \int_0^\infty 20 e^{-1000t} dt = -0.02 e^{-1000t}\Big|_0^\infty = \boxed{0.02\mathrm{J}}$$

10. (a) Left circuit:

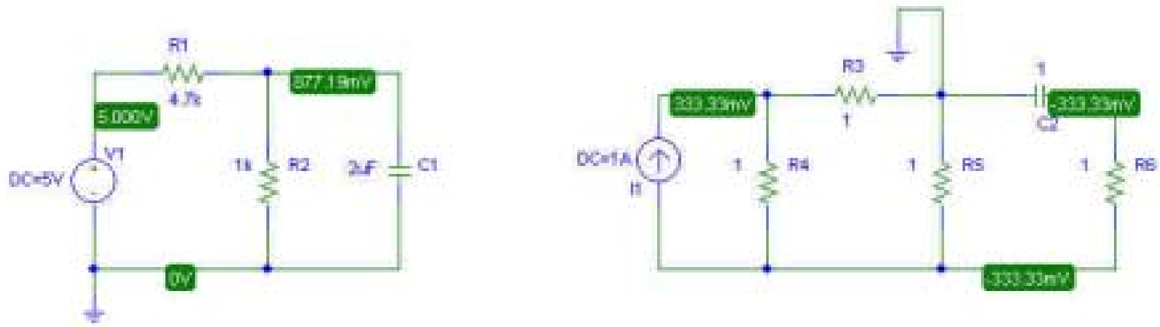
By Voltage division, $V_C = \frac{1k}{4.7k + 1k}(5) = \boxed{0.877V}$

Right circuit:

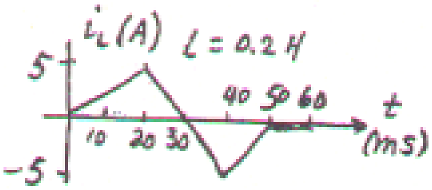
$V_1 = 1(1 // 2) = \frac{2}{3}V$

By Voltage Division, $V_2 = \frac{1}{3}V \therefore \boxed{V_C = -\frac{1}{3}V}$

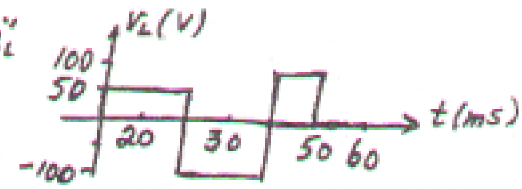
(b)



11.



(a) $v_L = 0.2 i_L'$



(b) $P_L = v_{Li_L} \therefore P_{L\max} = (-100)(-5) = 500\text{W}$ at $t = \boxed{40^- \text{ ms}}$

(c) $P_{L\min} = 100(-5) = -500\text{W}$ at $\boxed{t = 20^+ \text{ and } 40^+ \text{ ms}}$

(d) $W_L = \frac{1}{2}\text{L}i_L^2 \therefore W_L(40\text{ms}) = \frac{1}{2} \times 0.2(-5)^2 = \boxed{2.5\text{J}}$

12.

$\mathrm{L} = 50\times10^{-3}, t < 0: \; i = 0; \; t > 0 \;\; i = 80te^{-100t}\,\mathrm{mA} = 0.08te^{-100t}\,\mathrm{A}$

$\therefore \mathrm{i}' = 0.08\mathrm{e}^{-100\mathrm{t}} - 8te^{-100t} \;\therefore 0.08 = 8t, \boxed{t_m, = 0.01s}, |i|_{\max} = 0.08\times0.01e^{-1}$

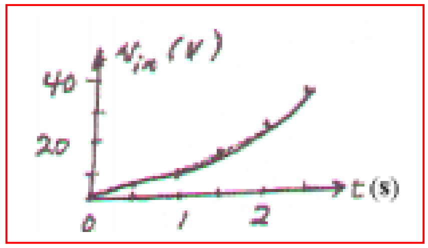
$\therefore \boxed{|i|_{\max} = 0.2943\mathrm{mA}}; \; v = 0.05i' = e^{-100t}(0.004 - 0.4t)$

$\therefore v' = e^{-100t}(-0.4) - 100e^{-100t}(0.004 - 0.4t) \therefore -0.4 = 0.4 - 40t, \; t = \frac{0.8}{40} = 0.02s$

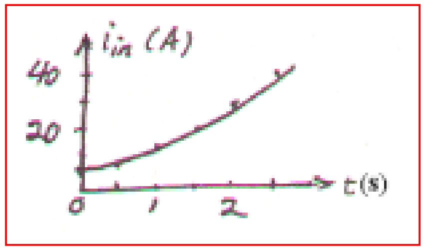
$v = e^{-2}(0.004 - 0.008) = -0.5413\mathrm{mV}$ this is minimum $\therefore |v|_{\max} = \boxed{0.004\mathrm{V}\text{ at } \mathrm{t}{=}0}$

13.

(a) $t > 0:\ i_s = 0.4t^2 \text{A} \therefore v_{in} = 10i_s + 5i_s' = \boxed{4t^2 + 4t\text{V}}$



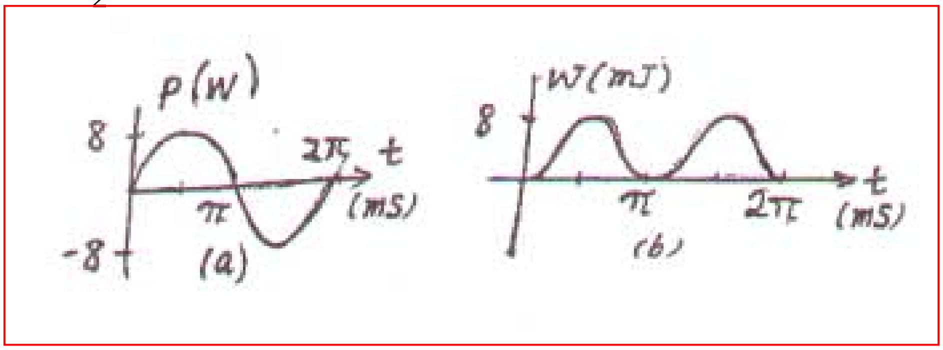
(b) $i_{in'} = 0.1v_s + \frac{1}{5}\int_0^t 40t\,dt + 5 = \boxed{4t + 4t^2 + 5\text{A}}$



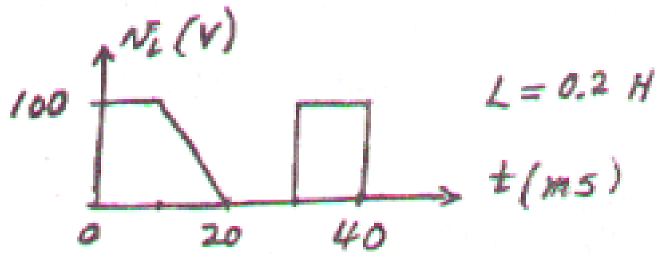
14. $v_L = 20\cos 1000t\text{V},\ \text{L} = 25\text{mH},\ i_L(0) = 0$

(a) $i_L = 40\int_0^t 20\ \cos\ 1000t dt = 0.8\sin\ 1000t\text{A} \therefore \boxed{p = 8\sin 2000t\ \text{W}}$

(b) $w = \frac{1}{2}\times 25\times 10^{-3}\times 0.64\sin^2 1000t = \boxed{8\sin^2 1000t\ \text{mJ}}$



15.



(a) $0 < t < 10$ ms: $i_L = -2 + 5\int_0^t 100dt = -2 + 500t \therefore i_L(10\text{ms}) = 3\text{A}, i_L(8\text{ms}) = \boxed{2\text{A}}$

(b) $i_L(0) = 0 \therefore i_L(10\text{ms}) = 500 \times 0.01 = 5\text{A} \therefore i_L(20\text{ms}) = 5 + 5\int_{0.01}^{0.02} 10^4(0.02 - t)dt$

$\therefore i_L(20\text{ms}) = 5 + 5\times 10^4 (0.02t - 0.5t)_{0.01}^{0.02} = 5 + 5\times 10^4(0.0002 - 0.00015) = 7.5\text{A}$

$\therefore w_L = \frac{1}{2} \times 0.2 \times 7.5^2 = \boxed{5.625\text{J}}$

(c) If the circuit has been connected for a long time, L appears like short circuit.

$$V_{8\Omega} = \frac{8}{2+8}(100V) = 80V$$

$$I_{2\Omega} = \frac{20V}{2\Omega} = 10A$$

$$\therefore \boxed{i_x = \frac{80V}{80\Omega} = 1A}$$

16. $\mathrm{L} = 5\mathrm{H}, \mathrm{V}_L = 10(e^{-t} - e^{-2t})\mathrm{V}, i_L(0) = 0.08\mathrm{A}$

(a) $v_L(1) = 10(e^{-1} - e^{-2}) = \boxed{2.325^{+}\ \mathrm{V}}$

(b) $i_L = 0.08 + 0.2\int_0^t 10(e^{-t} - e^{-2t})dt = 0.08 + 2(-e^{-t} + 0.5e^{-2t})_0^t$

$i_L = 0.08 + 2(-e^{-t} + 0.5e^{-2t} + 1 - 0.5) = 1.08 + e^{-2t} - 2e^{-t} \therefore i_L(1) = \boxed{0.4796\mathrm{A}}$

(c) $i_L(\infty) = \boxed{1.08\mathrm{A}}$

17.

(a)

$$v_x = 120 \times \frac{40}{12+20+40} + 40 \times 5 \times \frac{12}{12+20+40}$$

$$= \frac{200}{3} + \frac{100}{3} = \boxed{100\text{V}}$$

(b)

$$v_x = \frac{120}{12+15\|60} \times \frac{15}{15+60} \times 40 + 40 \times 5 \frac{15\|12}{15\|12+60}$$

$$= \frac{120}{12+12} \times \frac{1}{5} \times 40 + 200 \frac{6.667}{66.667}$$

$$= 40 + 20 = \boxed{60\text{V}}$$

18.

(a) $w_L = \frac{1}{2} \times 5 \times 1.6^2 = \boxed{6.4\text{J}}$

(b) $w_c = \frac{1}{2} \times 20 \times 10^{-6} \times 100^2 = \boxed{0.1\text{J}}$

(c) Left to right (magnitudes): 100, 0, 100, 116, 16, 16, 0 (V)

(d) Left to right (magnitudes): 0, 0, 2, 2, 0.4, 1.6, 0 (A)

19.

(a) $v_s = 400t^2\text{V}, t > 0;\ i_L(0) = 0.5\text{A};\ t = 0.4s$

$$v_c = 400 \times 0.16 = 64\text{V},\ w_c\ \frac{1}{2} \times 10^{-5} \times 64^2 = \boxed{20.48\text{mJ}}$$

(b) $i_L = 0.5 + 0.1\int_0^{0.4} 400t^2 dt = 0.5 + 40 \times \frac{1}{3} \times 0.4^3 = 1.3533\text{A}$

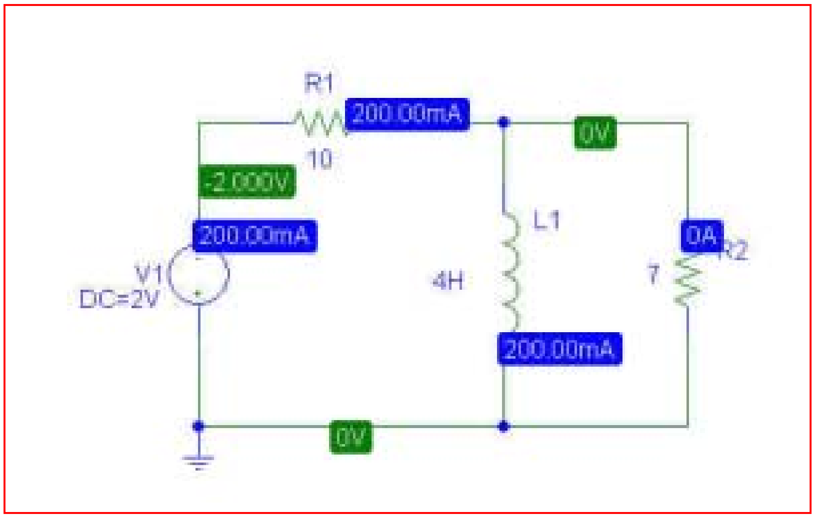
$$\therefore w_L = \frac{1}{2} \times 10 \times 1.3533^2 = \boxed{9.1581\text{J}}$$

(c) $i_R = 4t^2,\ \text{P}_R = 100 \times 16t^4\ \therefore w_R = \int_0^{0.4} 1600t^4 dt = 320 \times 0.4^5 = \boxed{3.277\text{J}}$

20. (a) $\boxed{P_{7\Omega} = 0W}; P_{10\Omega} = \frac{V^2}{R} = \frac{(2)^2}{10} = \boxed{0.4W}$

(b) PSpice verification

We see from the PSpice simulation that the voltage across the 10-Ω resistor is –2 V, so that it is dissipating 4/10 = 400 mW.



The 7-Ω resistor has zero volts across its terminals, and hence dissipates zero power.

Both results agree with the hand calculations.

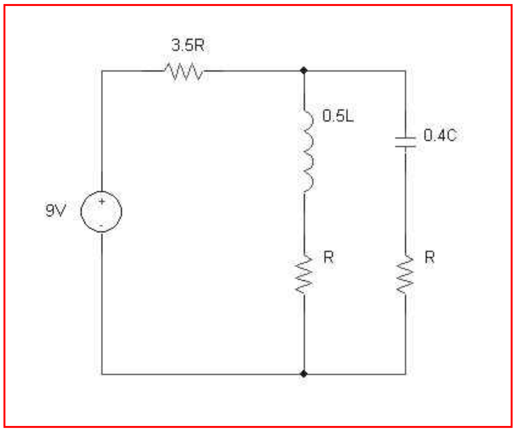
21.

$$C_{equiv} \equiv 10\mu + \left(\frac{1}{\frac{1}{10\mu} + \frac{1}{10\mu}} \right) \text{in series with } 10\mu \text{ in series with } 10\mu + \left(\frac{1}{\frac{1}{10\mu} + \frac{1}{10\mu}} \right)$$

$$\equiv \boxed{4.286\mu F}$$

22. $L_{equiv} \equiv \left(77p \,//\, (77p + 77p)\right) + 77p + \left(77p \,//\, (77p + 77p)\right) = \boxed{179.6\,pH}$

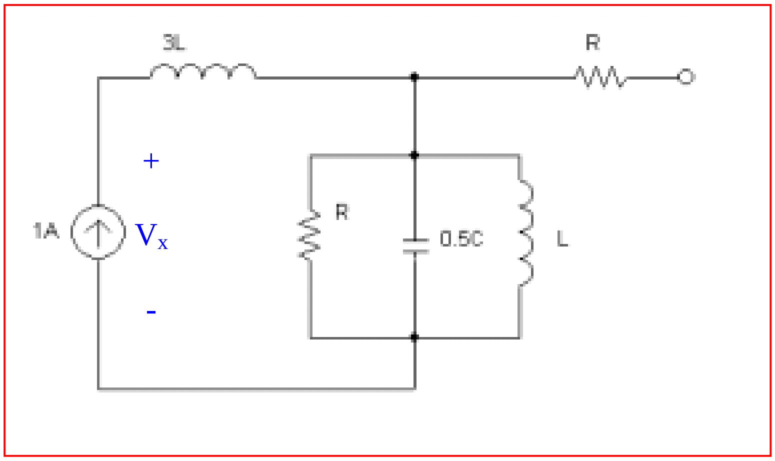
23. (a) Assuming all resistors have value R, all inductors have value L, and all capacitors have value C,



(b) At dc, 20μF is open circuit; 500μH is short circuit.

Using voltage division, $V_x = \dfrac{10k}{10k + 15k}(9) = \boxed{3.6V}$

24. (a) Assuming all resistors have value R, all inductors value L, and all capacitors value C,



(b) $\boxed{V_x = 0V}$ as L is short circuit at dc.

25. C_{equiv} = { [(100 n + 40 n) || 12 n] + 75 n} || {7 μ + (2 μ || 12 μ)}

$$\boxed{C_{equiv} \equiv 85.211 nF}$$

26. $L_{equiv} = \{[(17\text{ p} \parallel 4\text{ n}) + 77\text{ p}] \parallel 12\text{ n}\} + \{1\text{ n} \parallel (72\text{ p} + 14\text{ p})\}$

$$\boxed{L_{equiv} \equiv 172.388\, pH}$$

27. $C_T - C_x = (7 + 47 + 1 + 16 + 100) = 171\mu F$

$$E_{C_T - C_x} = \frac{1}{2}(C_T - C_x)V^2 = \frac{1}{2}(171\mu)(2.5)^2 = 534.375\mu J$$

$$E_{C_x} = E_{C_T} - E_{C_T - C_x} = (534.8 - 534.375)\mu J = 425nJ$$

$$\therefore E_{C_x} = 425n = \frac{1}{2}C_x V^2 \Rightarrow C_x = \frac{425n(2)}{(2.5)^2} = \boxed{136nF}$$

28.

(a) For all L = 1.5H, $L_{equiv} = 1.5 + \left(\dfrac{1}{\dfrac{1}{1.5} + \dfrac{1}{1.5}} \right) + \left(\dfrac{1}{\dfrac{1}{1.5} + \dfrac{1}{1.5} + \dfrac{1}{1.5}} \right) = \boxed{2.75H}$

(b) For a general network of this type, having N stages (and all L values equiv),

$$\boxed{L_{equiv} = \sum_{N=1}^{n} \frac{L^N}{NL^{N-1}}}$$

29.

(a)
$$L_{equiv} = 1 + \left(\frac{1}{\frac{1}{2} + \frac{1}{2}} \right) + \left(\frac{1}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \right) = \boxed{3H}$$

(b) For a network of this type having 3 stages,

$$L_{equiv} = 1 + \frac{1}{\frac{2+2}{(2)^2}} + \frac{1}{\frac{3+3}{(3)^2} + \frac{1}{3}} = 1 + \frac{(2)^2}{2(2)} + \frac{(3)^3}{3(3)^2}$$

Extending for the general case of N stages,

$$\begin{aligned} L_{equiv} &= 1 + \frac{1}{\frac{1}{2} + \frac{1}{2}} + \frac{1}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} + \mathrm{K} + \frac{1}{\frac{1}{\mathrm{N}} + \mathrm{K} \; \frac{1}{\mathrm{N}}} \\ &= 1 + \frac{1}{2(1/2)} + \frac{1}{3(1/3)} + \mathrm{K} + \frac{1}{\mathrm{N}(1/\mathrm{N})} = \boxed{\mathrm{N}} \end{aligned}$$

30. $C_{equiv} = \dfrac{(3p)(0.25p)}{3p + 0.25p} = \boxed{0.231pF}$

31. $L_{equiv} = \frac{(2.3\&n)(0.3\&n)}{2.6\&n} = \boxed{0.2916\&nH}$

32. (a) $\boxed{\text{Use 2 x 1}\mu\text{H in series with 4 x 1}\mu\text{H in parallel.}}$

(b) $\boxed{\text{Use 2 x 1}\mu\text{H in parallel, in series with 4 x 1}\mu\text{H in parallel.}}$

(c) $\boxed{\text{Use 5 x 1}\mu\text{H in parallel, in series with 4 x 1}\mu\text{H in parallel.}}$

33.

(a) $R = 10\Omega : 10 \| 10 \| 10 = \frac{10}{3},\ \frac{10}{3} + 10 + 10 \| 10 = \frac{55}{3}$

$\therefore R_{eq} = \frac{55}{3} \| 30 = \boxed{11.379\Omega}$

(b) $L = 10\text{H} \therefore L_{eq} = \boxed{11.379\text{H}}$

(c) $C = 10\text{F} : \frac{1}{1/30 + 1/10 + 1/20} = 5.4545$

$\therefore C_{eq} = 5.4545 + \frac{10}{3} = \boxed{8.788\text{F}}$

34.

(a) $oc: \mathrm{L}_{eq} = 6\|1 + 3 = \boxed{3.857\mathrm{H}}$

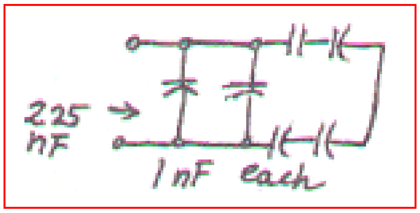
$sc: \mathrm{L}_{eq} = (3\|2+1)\|4 = 2.2\|4 = \boxed{1.4194\mathrm{H}}$

(b) $oc: 1 + \dfrac{1}{1/4 + 1/2} = \dfrac{7}{3}, c_{eq} = \dfrac{1}{3/7 + 1/2} = \boxed{1.3125\mathrm{F}}$

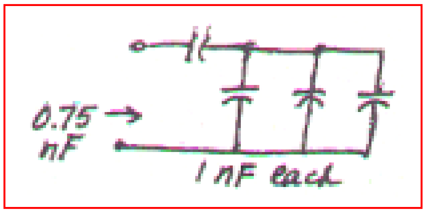
$sc: \dfrac{1}{1/5+1} = \dfrac{5}{6}, \mathrm{C}_{eq} = 4 + \dfrac{5}{6} = \boxed{4.833\mathrm{F}}$

35.

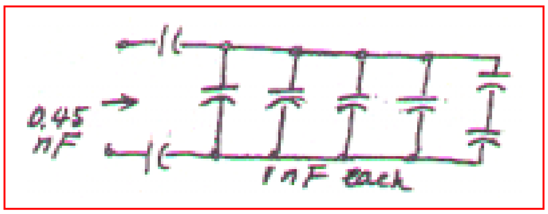
(a)



(b)



(c)



36. $i_s = 60e^{-200t}\text{mA},\ i_1(0) = 20\text{mA}$

(a) $6\|4 = 2.4\text{H} \therefore v = \text{L}_{eq} i_s' = 2.4 \times 0.06(-200)e^{-200t}$

or $\boxed{v = -28.8e^{-200t}\text{V}}$

(b) $i_1 = \frac{1}{6}\int_o^t -28.8e^{-200t}dt + 0.02 = \frac{4.8}{200}(e^{-200t} - 1) + 0.02$

$= \boxed{24e^{-200t} - 4\text{mA}(t > 0)}$

(c) $i_2 = i_s - i_1 = 60e^{-200t} - 24e^{-200t} + 4 = \boxed{36e^{-200t} + 4\text{mA}(t > 0)}$

37. $v_s = 100e^{-80t}V, v_1(0) = 20\text{V}$

(a) $i = C_{eq} v_s' = 0.8\times10^{-6}(-80)100e^{-80t} = \boxed{-6.4\times10^{-3}e^{-80t}\text{A}}$

(b) $v_1 = 10^6(-6.4\times10^{-3})\int_o^t e^{-80t}dt + 20 = \dfrac{6400}{80}(e^{-80t} - 1) + 20$

$\therefore v_1 = \boxed{80e^{-80t} - 60\text{V}}$

(c) $v_2 \dfrac{10^6}{4}(-6.4\times10^{-3})\int_o^t e^{-80t}dt + 80 = \dfrac{1600}{80}(e^{-80t} - 1) + 80$

$= \boxed{20e^{-80t} + 60\text{V}}$

38.

(a)

$$\boxed{\begin{array}{l} \dfrac{v_c - v_s}{20} + 5\times10^{-6} v_c' + \dfrac{v_c - v_L}{10} = 0 \\ \dfrac{v_L - v_c}{10} + \dfrac{1}{8\times10^{-3}} \int_o^t v_L\, dt + 2 = 0 \end{array}}$$

(b)

$$\boxed{\begin{array}{l} 20 i_{20} + \dfrac{1}{5\times10^{-6}} \int_o^t (i_{20} - i_L)\, dt + 12 = v_s \\ \dfrac{1}{5\times10^{-6}} \int_o^t (i_L - i_{20})\, dt - 12 + 10 i_L + 8\times10^{-3} i_L' = 0 \end{array}}$$

39.

$v_c(t)$: 30mA: $0.03\times 20 = 0.6\text{V}, v_c = 0.6\text{V}$

9V: $v_c = 9\text{V}$, 20mA: $v_c = -0.02\times 20 = 0.4\text{V}$

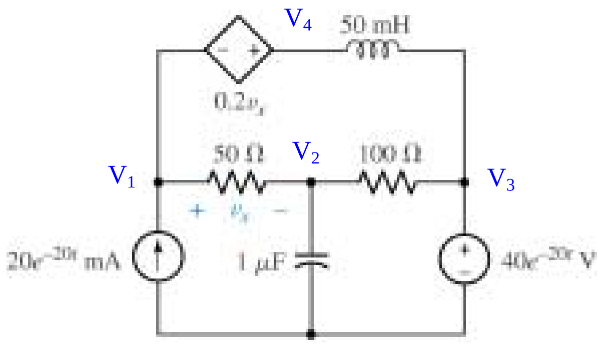
$0.04\cos 10^3 t$: $v_c = 0$

$\therefore \boxed{v_c(t) = 9.2\text{V}}$

$v_L(t)$: 30mA, 20mA,

9V: $v_L = 0$; $0.04\cos 10^3 t$: $v_L = -0.06\times 0.04(-1000)\sin 10^3 t = \boxed{2.4\sin 10^3 t\text{V}}$

40. We begin by selecting the bottom node as the reference and assigning four nodal voltages:



Ref.

1, 4 Supernode: $$\boxed{20\times10^{-3}\,e^{-20t} = \frac{V_1 - V_2}{50} + 0.02\times10^{3}\int_0^t \left(V_4 - 40e^{-20t}\right)dt'} \quad [1]$$

and: $V_1 - V_4 = 0.2\,V_x$ or $\boxed{0.8V_1 + 0.2\,V_2 - V_4 = 0}$ [2]

Node 2: $$\boxed{0 = \frac{V_2 - V_1}{50} + \frac{V_2 - 40e^{-20t}}{100} + 10^{-6}\frac{dV_2}{dt}} \quad [3]$$

41. (a) $R_i = \infty,\ R_o = 0,\ A = \infty \therefore v_i = 0 \therefore i = Cv_s'$

also $0 + Ri + v_o = 0 \therefore v_o = -RCv_s'$

$-v_i + Ri - Av_i = 0,\ \boxed{v_s = \frac{1}{c}\int idt + v_i}$

(b) $v_o = -Av_i \therefore v_i = \frac{-1}{A}v_o \therefore i = \frac{1+A}{R}v_i$

$\therefore v_s = \frac{1}{c}\int idt - \frac{1}{A}v_o = -\frac{1}{A}v_o + \frac{1+A}{RC}\int -\frac{v_o}{A}dt$

$\therefore Av_s' = -v_o' - \frac{1+A}{RC}v_o$ or $\boxed{v_o' + \frac{1+A}{RC}v_o + Av_s' = 0}$

42. Place a current source in parallel with a 1-MΩ resistor on the positive input of a buffer with output voltage, *v*. This feeds into an integrator stage with input resistor, R_2, of 1-MΩ and feedback capacitor, C_f, of 1 μF.

$$i = C_f \frac{dv_{c_f}}{dt} = 1.602 \times 10^{-19} \times \frac{ions}{\text{sec}}$$

$$0 = \frac{V_a - V}{1 \times 10^6} + C_f \frac{dv_{c_f}}{dt} = \frac{V_a - V}{1 \times 10^6} + 1.602 \times 10^{-19} \frac{ions}{\text{sec}}$$

$$0 = \frac{-V}{R_2} + C_f \frac{dv_{c_f}}{dt} = \frac{-V}{1 \times 10^6} + 1.602 \times 10^{-19} \frac{ions}{\text{sec}}$$

Integrating current with respect to t, $\frac{1}{R_2} \int_0^t v dt' = C_f \left(V_{c_f} - V_{c_f}(0) \right)$

$$\frac{1.602 \times 10^{-19} \times ions}{R_2} = C_f V_{c_f}$$

$$V_{c_f} = V_a - V_{out} \Rightarrow V_{out} = \frac{-R_1}{R_2 C_f} \times 1.602 \times 10^{-19} \times ions \Rightarrow V_{out} = \frac{-1}{C_f} \times 1.602 \times 10^{-19} \times ions$$

$R_1 = 1$ MΩ, $C_f = 1$μF

43. $R = 0.5M\Omega,\ C = 2\mu F,\ R_i = \infty,\ R_o = 0,\ v_o = \cos 10t - 1V$

(a) Eq. (16) is: $\left(1+\frac{1}{A}\right)v_o = -\frac{1}{RC}\int_o^t \left(v_s + \frac{v_o}{A}\right)dt - v_c(0)$

$\therefore \left(1+\frac{1}{A}\right)v_o' = -\frac{1}{RC}\left(v_s + \frac{v_o}{A}\right) \therefore \left(1+\frac{1}{A}\right)(-10\sin 10t) = -1\left(v_s + \frac{1}{A}\cos 10t - \frac{1}{A}\right)$

$\therefore v_s = \left(1+\frac{1}{A}\right)10\sin 10t + \frac{1}{A} - \frac{1}{A}\cos 10t$ Let $A = 2000$

$\therefore \boxed{v_s = 10.005\sin 10t + 0.0005 - 0.0005\cos 10t}$

(b) Let $A = \infty \therefore v_s = \boxed{10\sin 10t \text{V}}$

44. Create a op-amp based differentiator using an ideal op amp with input capacitor C_1 and feedback resistor R_f followed by inverter stage with unity gain.

$$V_{out} = +\frac{R}{R} R_f C_1 \frac{dvs}{dt} = 60 \times \frac{1mV}{rpm} / \min$$

$R_f C_1$=60 so choose R_f = 6 MΩ and C_1 = 10 μF.

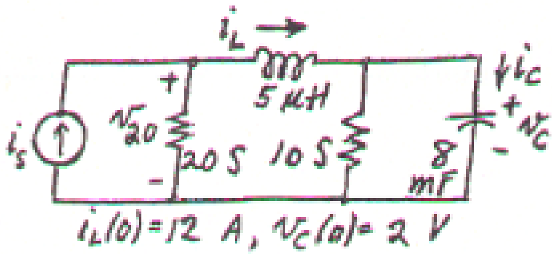
45. (a) $0 = \frac{1}{L}\int v dt + \frac{V_a - V_{out}}{R_f}$

$$V_a = V = 0, \therefore \frac{1}{L}\int v_L dt = \frac{V_{out}}{R_f} \Rightarrow \boxed{V_{out} = \frac{-R_f}{L}\int_0^t v_s dt'}$$

(b) In practice, capacitors are usually used as capacitor values are more readily available than inductor values.

46.

(a)



(b)

$$20v_{20} + \frac{1}{5\times10^{-6}}\int_o^t (v_{20} - v_c)dt + 12 = i_s$$

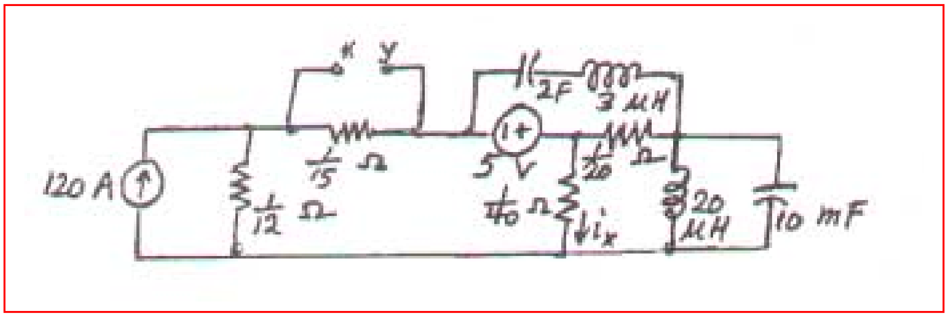
$$\frac{1}{5\times10^{-6}}\int_o^t (v_c - v_{20})dt - 12 + 10v_c + 8\times10^{-3}v_c' = 0$$

(c)

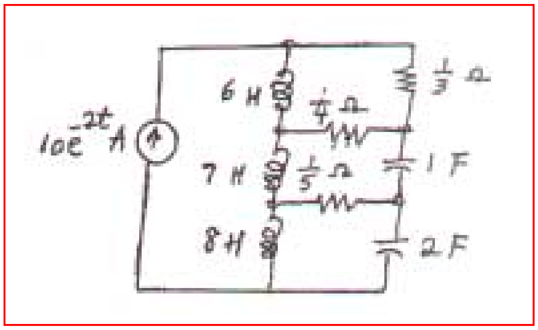
$$\frac{i_L - i_s}{20} + 5\times10^{-6}\, i_L' + \frac{i_L - i_c}{10} = 0$$

$$\frac{i_c - i_L}{10} + \frac{1}{8\times10^{-3}}\int_o^t i_c dt + 2 = 0$$

47.

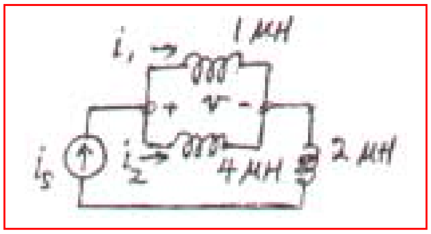


48.



49.

(a)



(b) "Let $i_s = 100e^{-80t}$ A and $i_1(0) = 20$ A in the circuit of (new) Fig. 7.62.

(a) Determine $v(t)$ for all t.
(b) Find $i_1(t)$ for $t \geq 0$.
(c) Find $v_2(t)$ for $t \geq 0$."

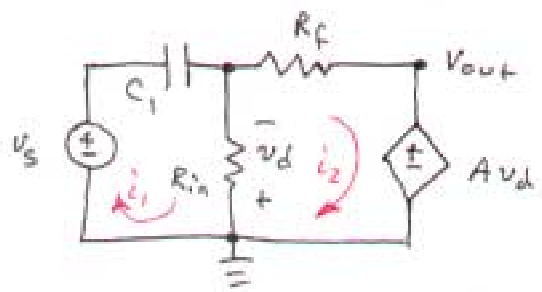
(c) (a) $\mathrm{L}_{eq} = 1 \| 4 = 0.8\mu\mathrm{H} \therefore v(t) = \mathrm{L}_{eq}\, i_s' = 0.8 \times 10^{-6} \times 100(-80)r^{-80t}\mathrm{V}$

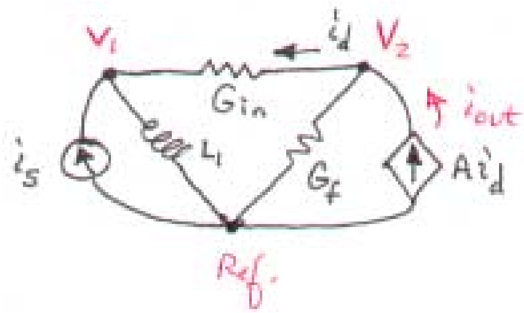
$\therefore \mathrm{v(t)} = \boxed{-6.43^{-80\mathrm{t}}\mathrm{mV}}$

(b) $i_1(t) = 10^6 \int_o^t -6.4 \times 10^{-3} e^{-80t} dt + 20 \therefore i_1(t) = \dfrac{6400}{80}(e^{-80t} - 1) = \boxed{80e^{-80t} - 60\mathrm{A}}$

(c) $i_2(t) = i_s - i_1(t) \therefore i_2(t) = \boxed{20e^{-80t} + 60\mathrm{A}}$

50.





In creating the dual of the original circuit, we have lost both v_s and v_{out}. However, we may write the dual of the original transfer function: $i_{out}/\ i_S$. Performing nodal analysis,

$$i_S = \frac{1}{L_1}\int_0^t V_1 dt' + G_{in}(V_1 - V_2) \qquad [1]$$

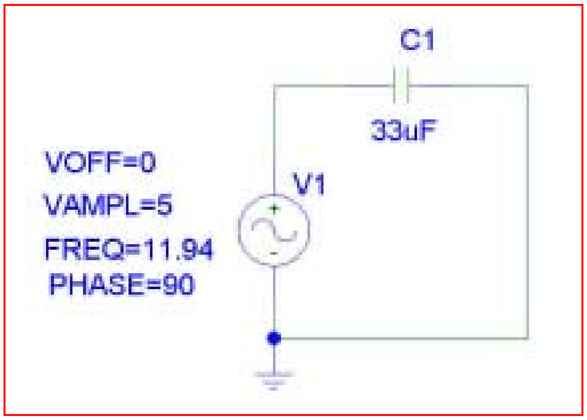
$$i_{out} = Ai_d = G_f V_2 + G_{in}(V_2 - V_1) \qquad [2]$$

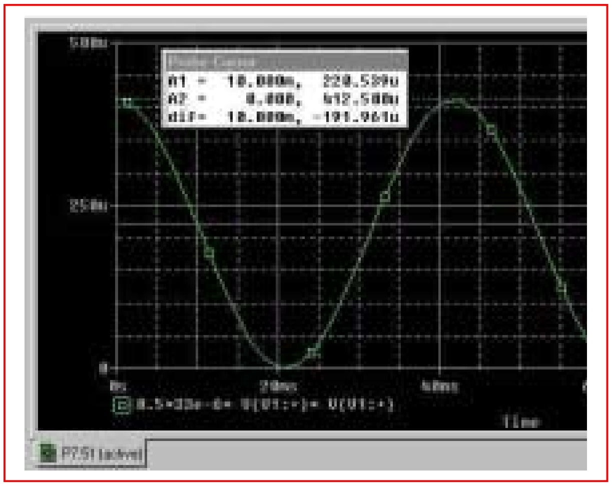
Dividing, we find that

$$\boxed{\frac{i_{out}}{i_S} = \frac{G_{in}(V_2 - V_1) + G_f V_2}{\frac{1}{L_1}\int_0^t V_1 dt' + G_{in}(V_1 - V_2)}}$$

51. PSpice verification

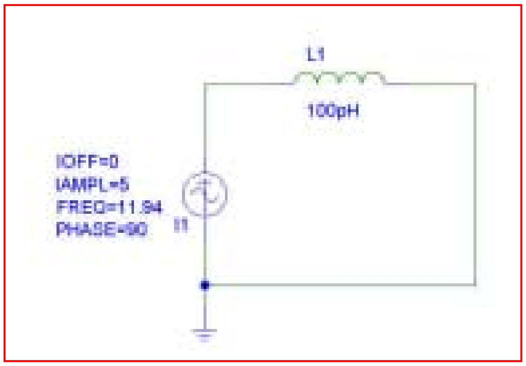
$w = \frac{1}{2} Cv^2 = 0.5\,(33\times10^{-6})[5\cos(75\times10^{-2})]^2 = 220.8\ \mu\text{J}$. This is in agreement with the PSpice simulation results shown below.

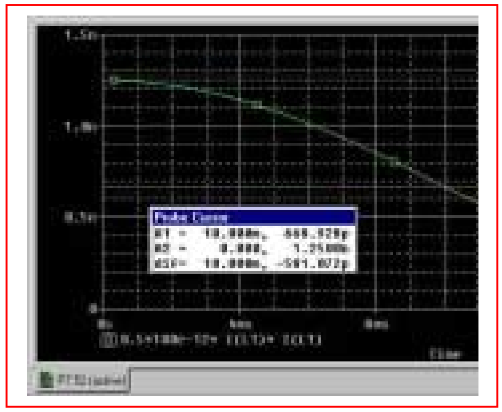




52. PSpice verification

$w = \frac{1}{2} L i^2 = 0.5\,(100\times10^{-12})[5 \cos(75\times10^{-2})]^2 = 669.2$ pJ. This is in agreement with the PSpice simulation results shown below.





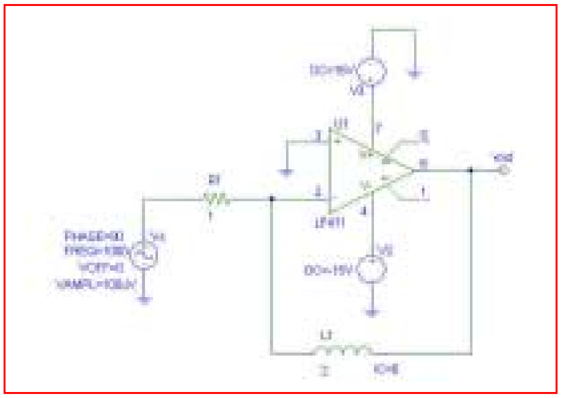
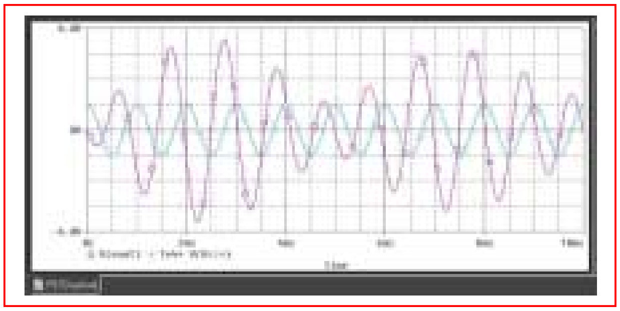
53. $$0 = \frac{V_a - V_s}{R_1} + \frac{1}{L}\int v_{L_f}\,dt$$

$$V_a = V_b = 0, \qquad 0 = \frac{-V_s}{R_1} + \frac{1}{L}\int v_{L_f}\,dt$$

$$V_{L_f} = V_a - V_{out} = 0 - V_{out} = \frac{L}{R_1}\frac{dVs}{dt}$$

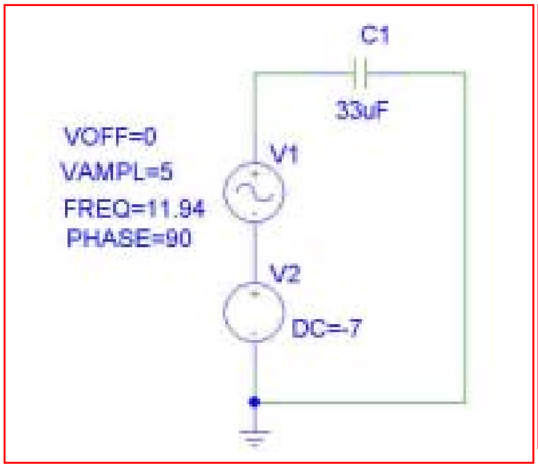
$$V_{out} = -\frac{L_f}{R_1}\frac{dVs}{dt} = -\frac{L_f}{R_1}\frac{d}{dt}\left(A\cos 2\pi 10^3 t\right) \Rightarrow L_f = 2R_1; \boxed{Let\ \text{R} = 1\ \Omega \text{ and L} = 1\text{ H.}}$$

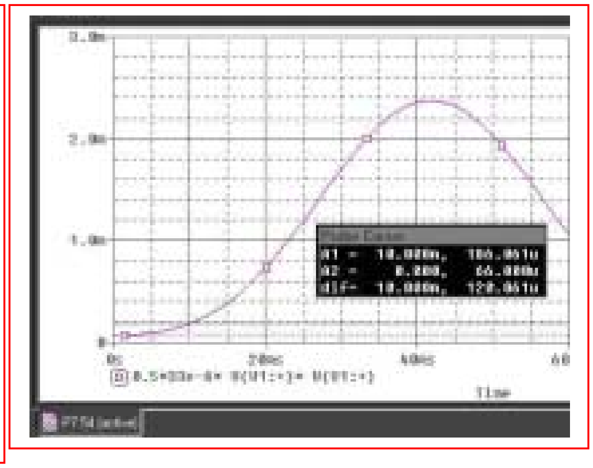
PSpice Verification: clearly, something rather odd is occuring in the simulation of this particular circuit, since the output is not a pure sinusoid, but a combination of several sinusoids.

54. PSpice verification

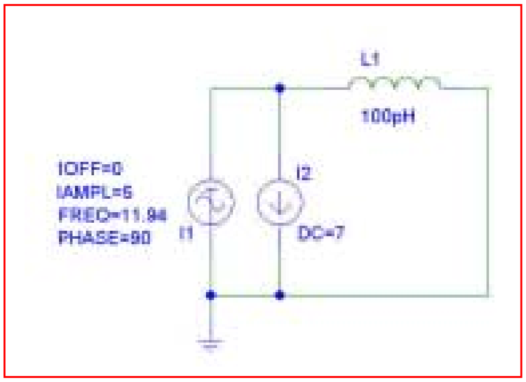
$w = \frac{1}{2} Cv^2 = 0.5\,(33\times10^{-6})[5 \cos (75\times10^{-2}) - 7]^2 = 184.2\ \mu\text{J}$. This is in reasonable agreement with the PSpice simulation results shown below.

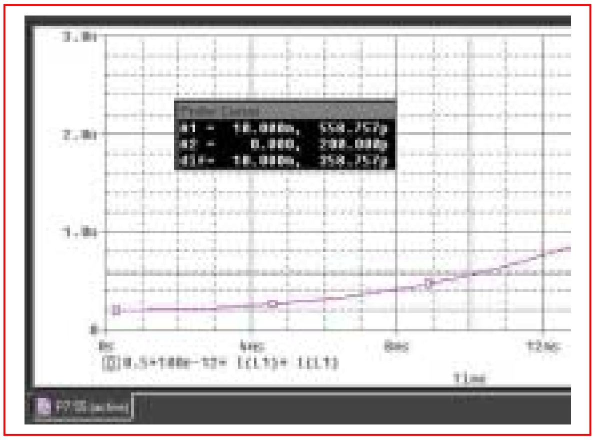




55. PSpice verification

$w = \frac{1}{2} L i^2 = 0.5\,(100\times10^{-12})[5\cos(75\times10^{-2}) - 7]^2 = 558.3$ pJ. This is in agreement with the PSpice simulation results shown below.





CHAPTER EIGHT SOLUTIONS

1.

(a) $i_L(0) = \dfrac{100}{50} = 2\text{A} \therefore i_L(t) = 2e^{-80t/0.2}$

$= \boxed{2e^{-400t}\text{A}, t > 0}$

(b) $i_L(0.01) = 2e^{-4} = \boxed{36.63\text{mA}}$

(c) $2e^{-400t_1} = 1,\ e^{400t_1} = 2,\ t_1 = \boxed{1.7329\text{ms}}$

CHAPTER EIGHT SOLUTIONS

2.

(a) $i_L(0^-) = \frac{1}{2} \times 60 = 30\text{mA},\ i_x(0^-) = \frac{2}{3} \times 30 = \boxed{20\text{mA}}$

(b) $i_L(0^+) = 30\text{mA},\ i_x(0^+) = \boxed{-30\text{mA}}$

(c) $i_L(t) = 30e^{-250t/0.05} = 30e^{-5000t}\text{mA},\ i_L(0.3\text{ms})$
$= 30e^{-1.5} = \boxed{6.694\text{mA}} = -i_x$

3.

(a) $i_L(0) = 4.5\text{mA},\ \text{R/L} = \dfrac{10^3}{4\times10^{-3}} = \dfrac{10^6}{4}$

$\therefore i_L = 4.5e^{-10^6 t/4}\text{mA} \therefore i_L(5\mu\, s) = 4.5e^{-1.25}$

$\boxed{= 1.289 \text{ mA.}}$

(b) $i_{SW}(5\ \mu\text{s}) = 9 - 1.289 = \boxed{7.711 \text{ mA.}}$

4.

(a) Since the inductor current can't change instantaneously, we simply need to find i_L while the switch is closed. The inductor is shorting out both of the resistors, so $\boxed{i_L(0^+) = 2 \text{ A.}}$

(b) The instant after the switch is thrown, we know that 2 A flows through the inductor. By KCL, the simple circuit must have 2 A flowing through the 20-Ω resistor as well. Thus,

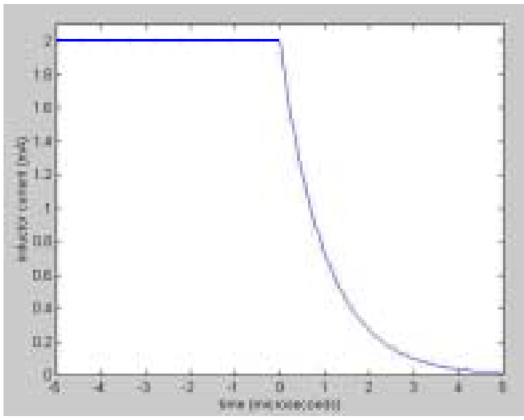
$$\boxed{v = 4(20) = 80 \text{ V.}}$$

5. (a) Prior to the switch being thrown, the 12-Ω resistor is isolated and we have a simple two-resistor current divider (the inductor is acting like a short circuit in the DC circuit, since it has been connected in this fashion long enough for any transients to have decayed). Thus, the current i_L through the inductor is simply 5(8)/ (8 + 2) = $\boxed{4\text{ A.}}$ The voltage v must be $\boxed{0\text{ V.}}$

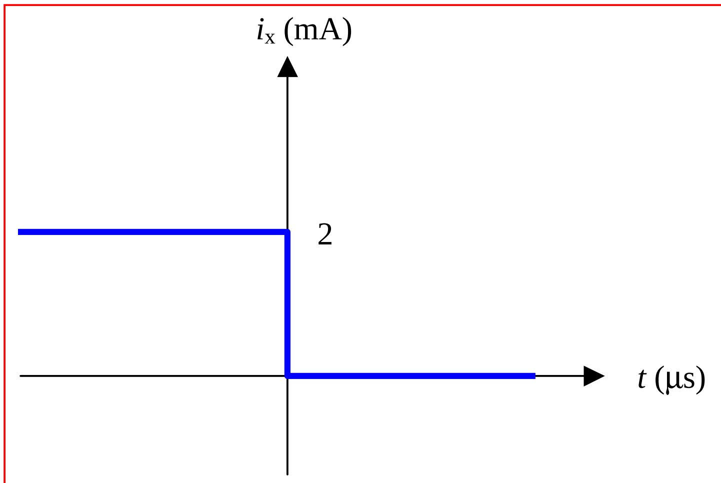
(b) The instant just after the switch is thrown, the inductor current must remain the same, so $\boxed{i_L = 4\text{ A.}}$ KCL requires that the same current must now be flowing through the 12-Ω resistor, so v = 12(-4) = $\boxed{-48\text{ V.}}$

6. For $t < 0$, we have a current divider with $i_L(0^-) = i_x(0^-) = 0.5$ [10 (1/ (1 + 1.5)] mA = 2 mA. For $t > 0$, the resistor through which i_x flows is shorted, so that $i_x(t > 0) = 0$. The remaining 1-kΩ resistor and 1-mH inductor network exhibits a decaying current such that $i_L(t) = 2e^{-t/\tau}$ mA where $\tau = L/R = 1$ μS.

(a)



(b)



7.

(a) $\dfrac{i}{\text{I}_o} = e^{-t/\tau}, \dfrac{t}{\tau} = \ln\dfrac{\text{I}_o}{i}, \dfrac{\text{I}_o}{i} = 10 \therefore \dfrac{t}{\tau} = \ln 10 = 2.303;$

$\dfrac{\text{I}_o}{i} = 100, \dfrac{t}{\tau} = 4.605;\ \dfrac{\text{I}_o}{i} = 1000, \dfrac{t}{\tau} = \boxed{6.908}$

(b) $\dfrac{i}{\text{I}_o} = e^{-t/\tau}, \dfrac{d(i/\text{I}_o)}{d(t/\tau)} = -e^{t/\tau};$ at $t/\tau = 1, \dfrac{d()}{d()} = -e^{-1}$

Now, $y = m(x-1) + b = -e^{-1}(x-1) + e^{-1}\left(\dfrac{t}{\tau} = x, \dfrac{i}{\text{I}_o} = y\right)$

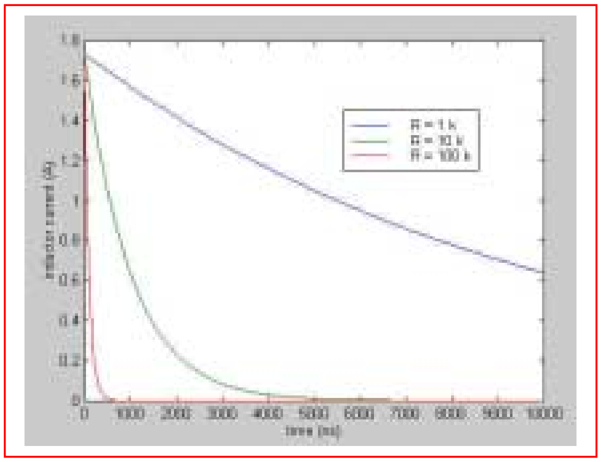
At $y = 0, e^{-1}(x-1) = e^{-1} \therefore x = 2 \therefore t/\tau = \boxed{2}$

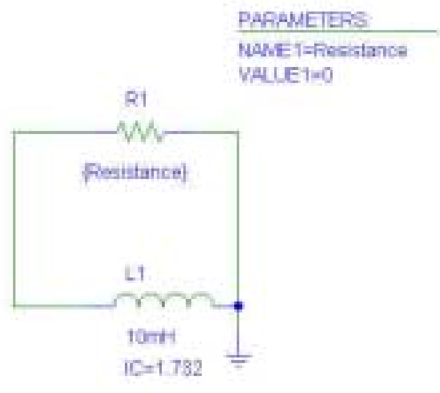
8. Reading from the graph current is at 0.37 at 2 ms

$$\boxed{\begin{aligned} &\therefore\ \tau = 2\text{ ms} \\ &I_0 = 10\text{ A} \end{aligned}}$$

9. $w = \frac{1}{2} Li^2$, so an initial energy of 15 mJ in a 10-mH inductor corresponds to an initial inductor current of 1.732 A. For R = 1 kΩ, τ = L/R = 10 μs, so $i_L(t) = 1.732\, e^{-0.1t}$ A. For R = 10 kΩ, τ = 1 μs so $i_L(t) = 1.732\, e^{-t}$. For R = 100 kΩ, τ = 100 ns or 0.1 μs so $i_L(t) = 1.732\, e^{-10t}$ A. For each current expression above, it is assumed that time is expressed in microseconds.

To create a sketch, we first realise that the maximum current for any of the three cases will be 1.732 A, and after one time constant (10, 1, or 0.1 μs), the current will drop to 36.79% of this value (637.2 mA); after approximately 5 time constants, the current will be close to zero.

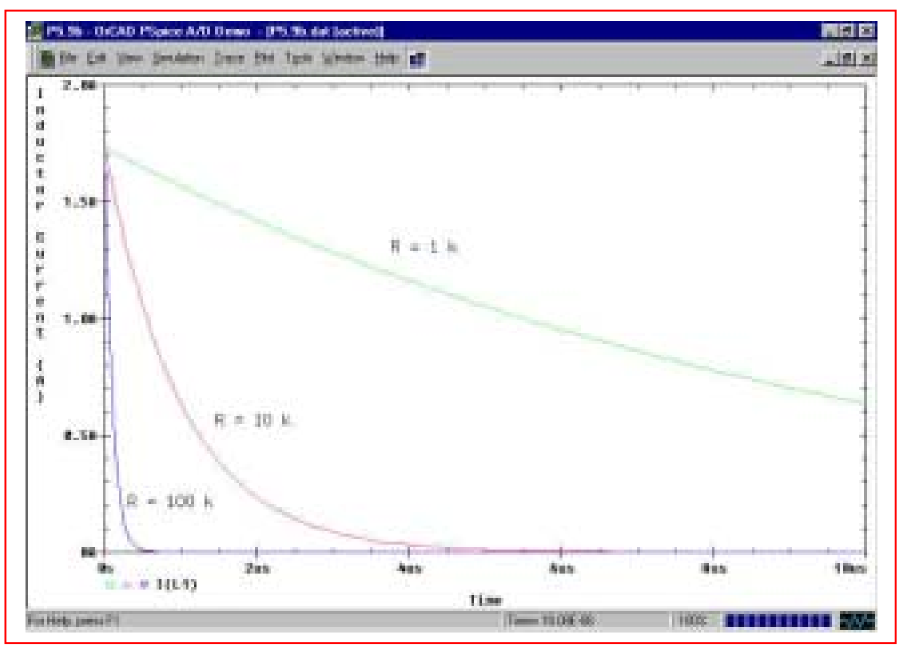




Sketch based on hand analysis

Circuit used for PSpice verification

As can be seen by comparing the two plots, which probably should have the same x-axis scale labels for easier comparison, the PSpice simulation results obtained using a parametric sweep do in fact agree with our hand calculations.



10.

(a) $$\tau = \frac{3.3\times10^{-6}}{1\times10^{6}} = \boxed{3.3\times10^{-12}}$$

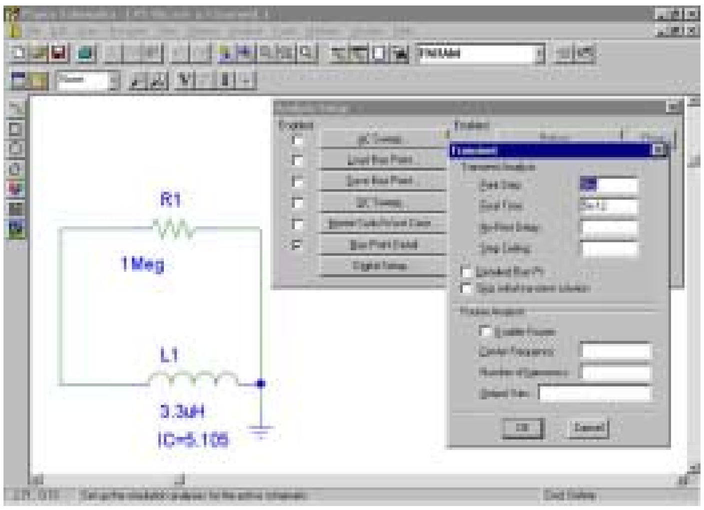
(b)

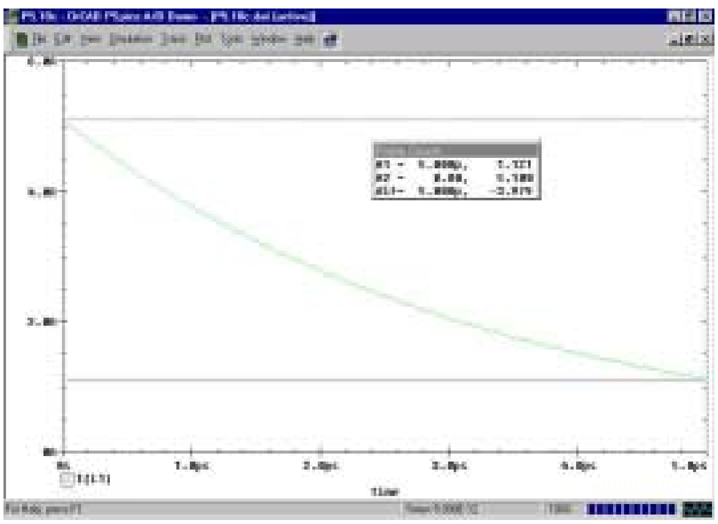
$$\omega = \frac{1}{2}.L.I_0^{\;2}$$

$$I_0 = \sqrt{\frac{2\times 43\times10^{-6}}{3.3\times10^{-6}}} = 5.1\text{ A}$$

$$i(5\,ps) = 5.1e^{-1\times10^{6}\times5\times10^{-12}\;/\;3.3\times10^{-6}} = \boxed{1.12\text{ A}}$$

(c)





From the PSpice simulation, we see that the inductor current is 1.121 A at t = 5 ps, in agreement with the hand calculation.

11. Assume the source Thévenin resistance is zero, and assume the transient is measured to 5τ. Then,

$$\tau = \frac{L}{R} \quad \therefore 5\tau = \frac{5L}{R} = 100\times10^{-9} \text{ secs}$$

$$\therefore R > \frac{(5)(125.7)10^{-6}}{10^{-7}}$$

so R must be greater than $\boxed{6.285 \text{ k}\Omega.}$

(If 1τ assumed then $R > \dfrac{6.285}{5} = 125.7\Omega$)

The film acts as an intensity integrator. Assuming that we may model the intensity as a simple decaying exponential,

$$\phi(t) = \phi_o e^{-t/\tau}$$

where the time constant τ represents the effect of the Thévenin equivalent resistance of the equipment as it drains the energy stored in the two capacitors, then the intensity of the image on the film Φ is actually proportional to the integrated exposure:

$$\Phi = K \int_0^{\text{exposure time}} \phi_o e^{-t/\tau} dt$$

where K is some constant. Solving the integral, we find that

$$\Phi = -K \phi_o \tau \left[e^{-(\text{exposure time})/\tau} - 1 \right]$$

The maximum value of this intensity function is $-K\phi_o\tau$.

With 150 ms yielding an image intensity of approximately 14% of the maximum observed and the knowledge that at 2 s no further increase is seen leads us to estimate that $1 - e^{-150\times10^{-3}/\tau} = 0.14$, assuming that we are observing single-exponential decay behavior and that the response speed of the film is not affecting the measurement. Thus, we may extract an estimate of the circuit time constant as $\tau = 994.5$ ms.

This estimate is consistent with the additional observation that at $t = 2$ s, the image appears to be saturated.

With two 50-mF capacitors connected in parallel for a total capacitance of 100 mF, we may estimate the Thévenin equivalent resistance from $\tau = RC$ as $R_{th} = \tau / C$ $\boxed{= 9.945\ \Omega.}$

13.

(a) $v_c(0) = 8(50 \| 200) \times \frac{30}{50} = 192\text{V}$

$v_c(t) = 192e^{-3000t/24} = \boxed{192e^{-125t}\text{V}}$

(b) $0.1 = e^{-125t} \therefore \boxed{t = 18.421\text{ms}}$

14.

(a) $v_c = 80e^{-10^6 t/100} = 80e^{-10^4 t}\text{V},\ t > 0;\ 0.5 = e^{-10^4 t}\ \therefore \boxed{t = 69.31\mu\, s}$

(b) $w_c = \frac{1}{2} C80^2 e^{-20{,}000t} = \frac{1}{4} C80^2\ \therefore \boxed{t = 34.66\mu\, s}$

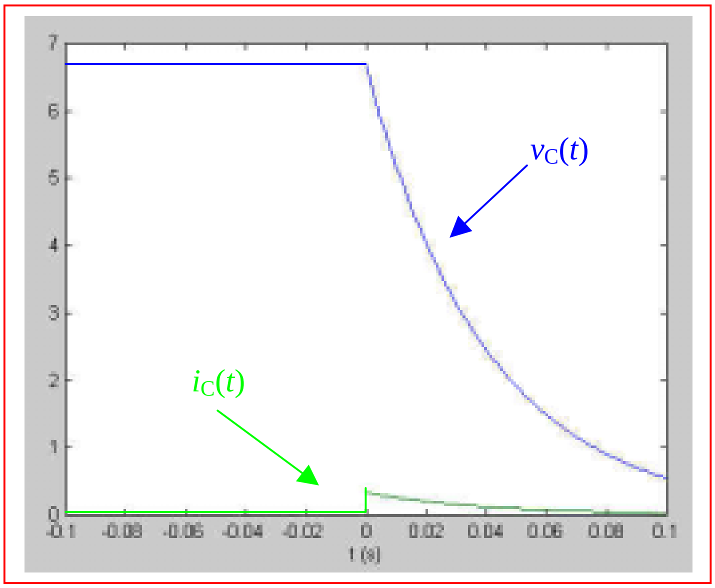
15.

$$t < 0:\ \boxed{i_c(t) = 0,}\ 10 = 5000 i_s + 10^4\ i_s \ \therefore i_s = \frac{2}{3}\text{mA}$$

$$\therefore v_c(t) = \frac{20}{3} = \boxed{6.667\text{V}}$$

$$t > 0:\ i_s = 0 \therefore v_c(t) = 6.667 e^{-t/2\times10^4\times2\times10^{-6}}$$

$$\therefore \boxed{v_c(t) = 6.667 e^{-25t}\text{V}} \therefore i_c(t) = \frac{-6.667}{20\times10^3} e^{-25t} = \boxed{0.3333 e^{-25t}\text{mA}}$$



16.

$$v(0^+) = 20V$$
$$i(0^+) = 0.1A$$

$$v(1.5ms) = 20e^{-1.5\times10^{-3}/50\times20\times10^{-6}} = 4.5V$$
$$i(1.5ms) = 0A$$

$$v(3ms) = 20e^{-3\times10^{-3}/50\times20\times10^{-6}} = 1V$$
$$i(3ms) = 0A$$

17.

(a) $i_L(0) = 4\text{A} \therefore i_L(t) = 4e^{-500t}\text{A} \ (0 \le t \le 1\text{ms})$

$i_L(0.8\text{ms}) = 4e^{-0.4} = \boxed{2.681\text{A}}$

(b) $i_L(1\text{ms}) = 4e^{-0.5} = 2.426\text{A}$

$\therefore i_L(t) = 2.426e^{-250(t-0.001)}$

$\therefore i_L(2\text{ms}) = 2.426e^{-0.25} = \boxed{1.8895^{-}\text{A}}$

18.

(a) $i_L = 40e^{-50,000t}\text{mA} \therefore 10 = 40e^{-50,000t}, \ \therefore t_1 = \boxed{27.73\mu\, s}$

(b) $i_L(10\mu\, s) = 40e^{-0.5} = 24.26\text{mA} \therefore i_L$

$= 24.26e^{-(1000+R)50t}(t > 10\,\mu\, s)$

$\therefore 10 = 24.26e^{-(1000+R)5\times10^{-6}} \ \therefore \ \ln 2.426 = 0.8863$

$= 0.25(1000 + \text{R})10^{-3}, 1000 + \text{R} = 0.8863 \times 4 \times 10^{3} \therefore \text{R} = \boxed{2545^{+}\,\Omega}$

19.

(a) $i_1(0) = 20\text{mA},\ i_2(0) = 15\text{mA}$

$\therefore v(t) = 40e^{-50000t} + 45e^{-100000t}\text{V} \therefore \boxed{v(0) = 85\text{V}}$

(b) $v(15\mu\, s) = 40e^{-0.75} + 45e^{-1.5} = \boxed{28.94\text{V}}$

(c) $\dfrac{85}{10} = 40e^{-50000t} + 45e^{-100000t}.$ Let $e^{-50000t} = x$

$\therefore 45x^2 + 40x - 8.5 = 0$

$\therefore x = \dfrac{-40 \pm \sqrt{1600 + 1530}}{90} = 0.17718, < 0$

$\therefore e^{-50000t} = 0.17718, \boxed{t = 34.61\mu\, s}$

20. $t < 0: v_R = \dfrac{2R_1R_2}{R_1+R_2}, \downarrow i_L(0) = \dfrac{2R_1}{R_1+R_2}$

$t > 0: i_L(t) = \dfrac{2R_1}{R_1+R_2}\, e^{-50R_2t} \;\therefore v_R = \dfrac{2R_1R_2}{R_1+R_2}\, e^{-50R_2t}$

$\therefore v_R(0^+) = 10 = \dfrac{2R_1R_2}{R_1+R_2} \therefore R_1 \| R_2 = 5\Omega.$ Also, $v_R(1\text{ms})$

$= 5 = 10e^{-50R_2/1000} \therefore 0.05R_2 = 0.6931 \therefore \boxed{R_2 = 13.863\Omega}$

$\therefore \dfrac{1}{13.863} + \dfrac{1}{R_1} = \dfrac{1}{5} \therefore \boxed{R_1 = 7.821\Omega}$

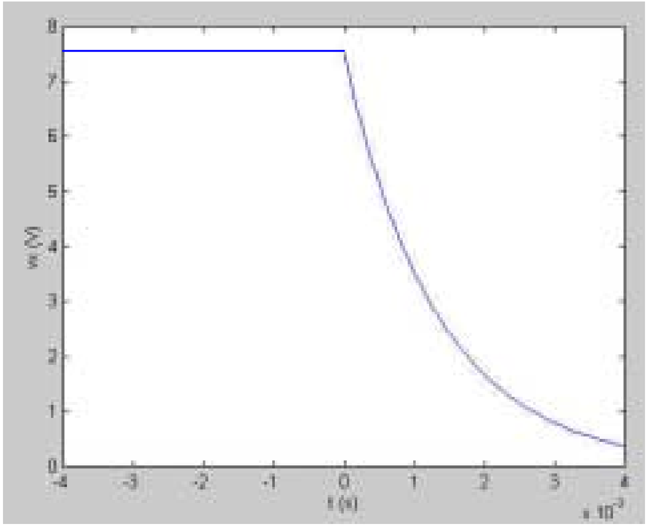
21.

(a) $i_L(0) = \dfrac{24}{60} = 0.4\text{A} \therefore \boxed{i_L(t) = 0.4e^{-750t}\text{A},\ t > 0}$

(b) $v_x = \dfrac{5}{6} \times 24 = 20\text{V},\ t < 0$

$v_x(0^+) = 50 \times 0.4 \times \dfrac{3}{8} = 7.5\text{V}$

$\therefore v_x(t) = 7.5e^{-750t}\text{V},\ t > 0$



22.
$$v_{in} = \frac{3i_L}{4} \times 20 + 10i_L = 25i_L$$

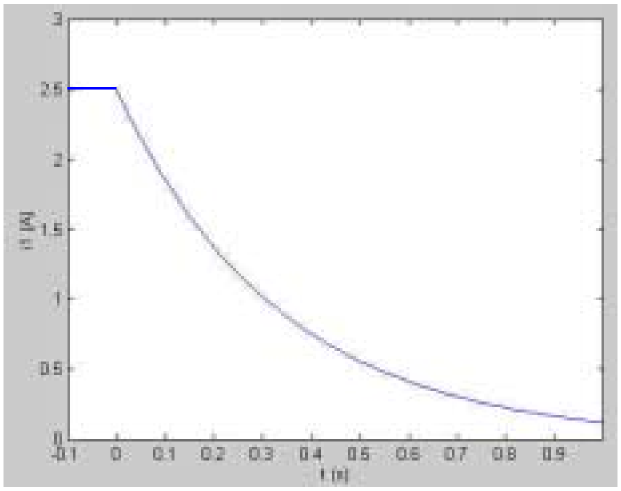
$$v_{in} \therefore \frac{v_{in}}{i_L} = 25\Omega \therefore i_L = 10e^{-25t/0.5} = \boxed{10e^{-50t}\text{A},\ t > 0}$$

23. $$i_L(0) = \frac{64}{4 + 40 \| 8} \times \frac{40}{48} = 5\text{A}$$

$$\therefore i_L = 5e^{-24t/8} = 5e^{-3t}\text{A}$$

$$\therefore i_1(t) = 2.5e^{-3t}\text{A},\ t>0;\ \boxed{i_1(-0.1) = 2.5\text{ A}}$$

$$\boxed{i_1(0.03) = 2.285^{-}\text{A},\ i_1(0.1) = 1.852\text{ A}}$$



24.

(a) $i_L(0) = 4\text{A} \therefore i_L = 4e^{-100t}\text{A},\ 0 < t < 15\,\text{ms}$

$\therefore i_L(15\text{ms}) = 4e^{-1.5} = \boxed{0.8925^{+}\,\text{A}}$

(b) $t > 15\,\text{ms}:\ i_L = 0.8925^{+} e^{-20(t-0.015)}\text{A}$

$\therefore i_L(30\text{ms}) = 0.8925^{+} e^{-0.3} = \boxed{0.6612\text{A}}$

25.

(a) $i_1(0^+) = i_1(0^-) = 10\text{A},\ i_2(0^+) = i_2(0^-) = 20\text{A} \therefore i(0^+) = \boxed{30\text{A}}$

(b) $\tau = \text{L}_{eq} / \text{R}_{eq} = \dfrac{0.08}{48} = \dfrac{5}{3}\text{ms} = \boxed{1.6667\text{ms};}$

(c) $i_1(0^-) = 10\text{A},\ i_2(0^-) = 20\text{A}\ ;\ \boxed{i(t) = 30e^{-600t}\text{A}}$

(d) $v = -48i = \boxed{-1440e^{-600t}\text{V}}$

(e) $i_1 = 10(-440)\int_0^t e^{-600t}dt + 10 = 24e^{-600t}\Big|_0^t + 10 = 24e^{-600t} - 14\text{A}$

$$i_2 = 2.5(-1440)\int_0^t e^{-600t}dt + 20$$

$$= 6e^{-600t}\Big|_0^t + 20 = \boxed{6e^{-600t} + 14\text{A}}$$

(f) $\text{W}_L(0) = \dfrac{1}{2}\times 0.1\times 10^2 + \dfrac{1}{2}\times 0.4\times 20^2 = 5 + 80 = 85\text{J}$

$$\text{W}_\text{L}(\infty) = \frac{1}{2}\times 0.1\times 14^2 + \frac{1}{2}\times 0.4\times 14^2 = 9.8 + 39.2 = 49\text{J}$$

$$\text{W}_\text{R} = \int_0^\infty i^2 48dt = \int_0^\infty 900\times 48e^{-1200t}dt = \frac{900\times 48}{-1200}(-1) = 36\text{J}$$

$\boxed{\therefore 49 + 36 = 85 \text{ checks}}$

26.

(a) $$v_c(0) = 100 \times \frac{2}{2+2} \times \frac{2}{3} = 33.33\text{V};\ i_1(0^-) = \frac{100}{2+2} \times \frac{2}{3} = 16.667\text{mA}$$

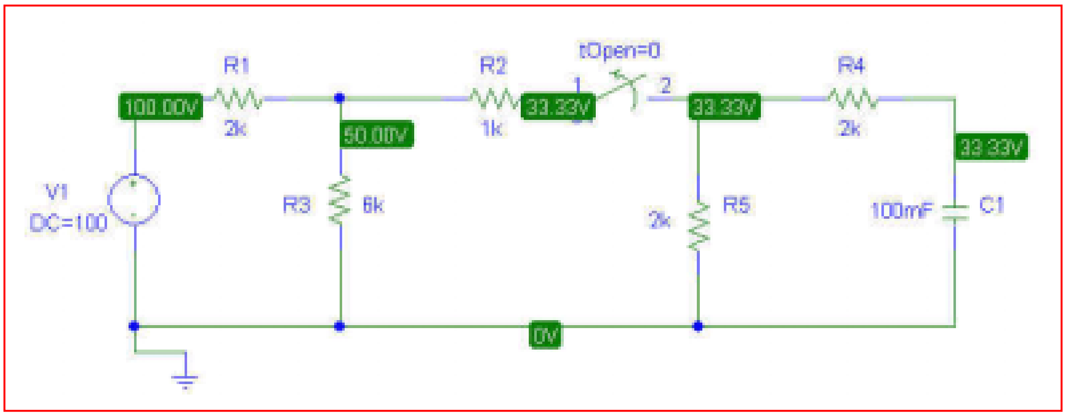
$$\therefore \boxed{\mathrm{v_c}(9:59) = 33.33\text{V},\ i_1(9:59) = 16.667\text{mA}}$$

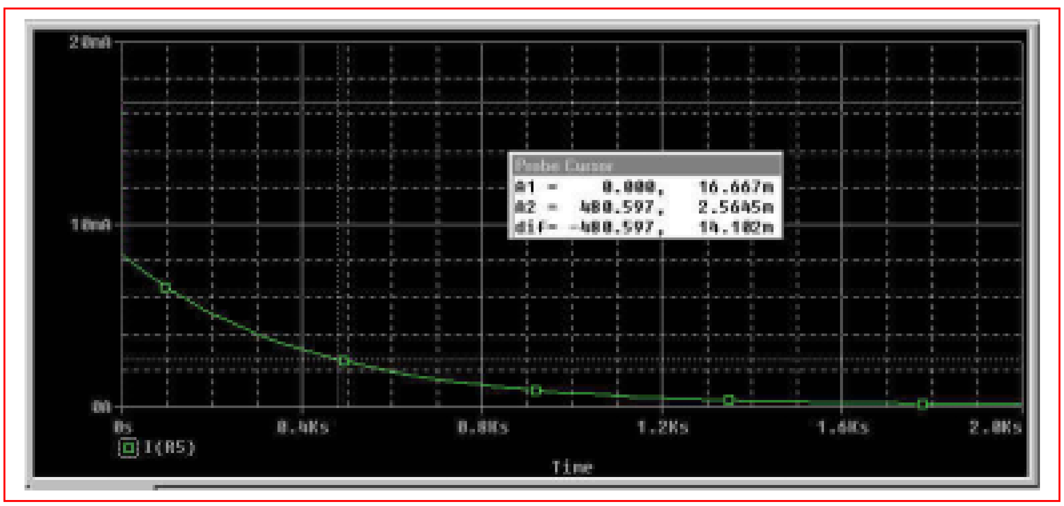
(b) $$v_c(t) = 33.33e^{-t/400},\ t > 10:00 \therefore v_c(10:05) = 33.33e^{-300/400}$$

$$\boxed{= 15.745^+\ \text{V},}\ i_1(10:05) = \frac{15.745}{4000} = \boxed{3.936\text{mA}}$$

(c) $\tau = 400$ s, so $1.2\tau = 480$ s. $v_C(1.2\tau) = 33.33\ e^{-1.2} = 10.04$ V.
Using Ohm's law, we find that $i_1(1.2\tau) = v_C(1.2\tau)/\ 4000 = \boxed{2.51 \text{ mA.}}$

(d) PSpice Verification:





We see from the DC analysis of the circuit that our initial value is correct; the Probe output confirms our hand calculations, especially for part (c).

27.

$$t > 0: \ \frac{25 i_x}{20} = 1.25 i_x \ \therefore 34 = 100(1.25 i_x - 0.8 i_x + i_x) + 25 i_x \ \therefore i_x = 0.2\text{A}$$

(a) $i_s(0^-) = (1.25 - 0.8 + 1)0.2 = \boxed{0.290\,\text{A}}$

(b) $i_x(0^-) = \boxed{0.2\text{A}}$

(c) $v_c(t) = 25 \times 0.2 e^{-t} = 5 e^{-t}\text{V} \ \therefore i_x(0^+) = \dfrac{5}{100} = \boxed{0.05\text{A}}$

(d) $0.8 i_x(0^+) = 0.04\text{A} \ \therefore i_s(0^+) = \dfrac{34}{120} - 0.04 \times \dfrac{20}{120} = \dfrac{33.2}{120} = \boxed{0.2767\text{A}}$

(e) $i_x(0.4) = \dfrac{1}{100} \times 5 e^{-0.4} = \boxed{0.03352\text{A}}$

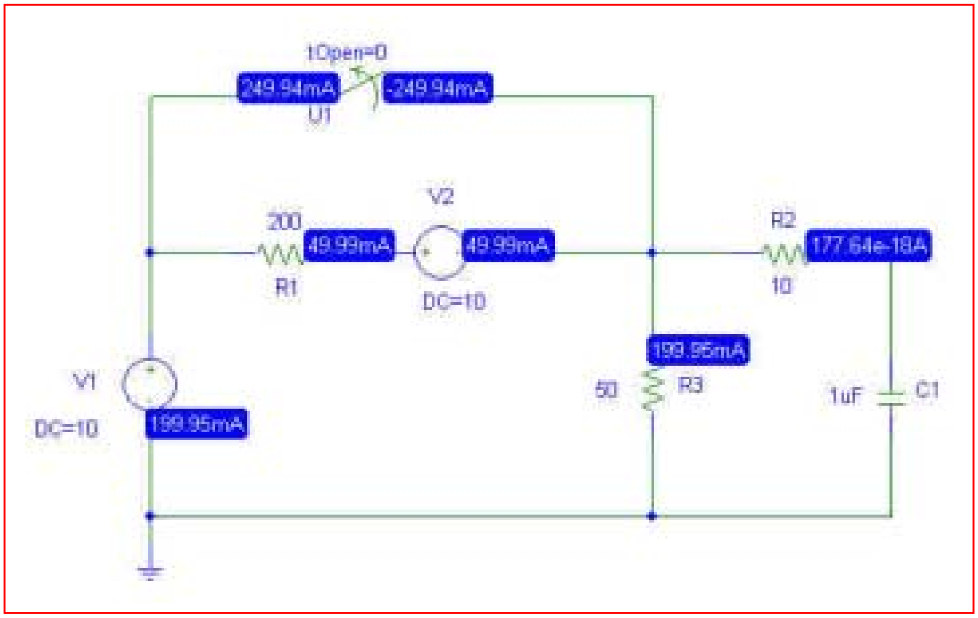
28.

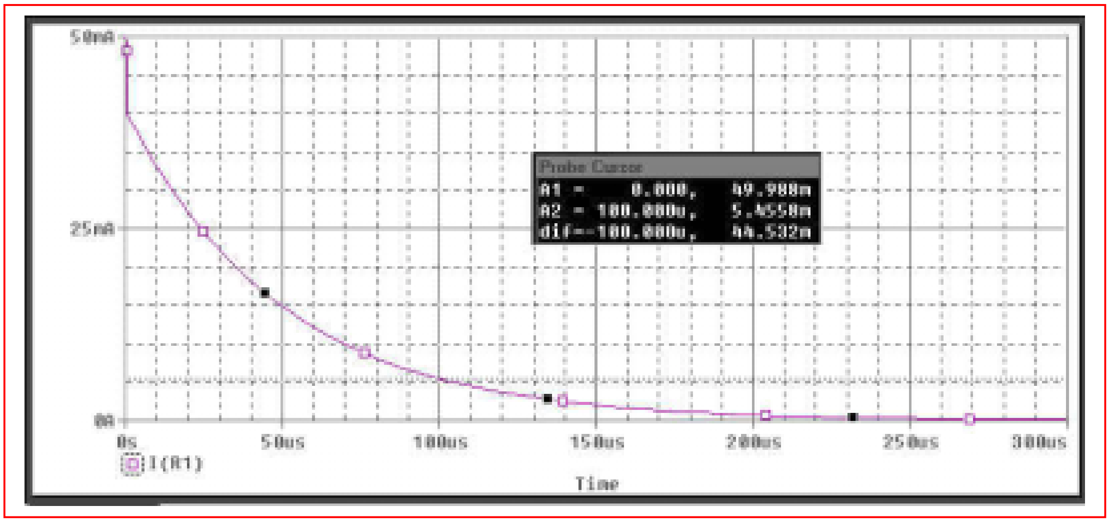
(a) $v_c(0) = 10\text{V} \therefore v_c(t) = 10e^{-10^6 t/(10+50\|200)} = \boxed{10e^{-20000t}\text{V}}$

(b) $i_A(-100\mu s) = i_A(0^-) = \dfrac{10}{200} = \boxed{50\text{mA}}$

$$i_A(100\mu s) = 10e^{-2}\left(\frac{1}{10+40}\right)\frac{50}{250} = \boxed{5.413\text{mA}}$$

(c) PSpice Verification.





From the DC simulation, we see that PSpice verifies our hand calculation of i_A = 50 mA. The transient response plotted using Probe indicates that at 100 μs, the current is approximately 5.46 mA, which is within acceptable round-off error compared to the hand calculated value.

29.

(a) $$i_1(t) = 8(-1)\frac{12}{12+4} = \boxed{-6\text{mA } (t<0)}$$

(b) $$4 \| 12 \| 6 = 2k\Omega,\ v_c(0) = 48\text{V}$$

$$\therefore v_c(t) = 48e^{-10^6 t/5\times 2\times 10^3} = 48e^{-100t}\text{V},\ t>0$$

$$\therefore i(t) = \boxed{12e^{-100t}\text{mA},\ t>0}$$

30.

(a) $v_{CLeft}(0) = 20\text{V},\ v_{CRIGHT}(0) = 80\text{V}$

$\therefore v_{CL} = 20e^{-10^6 t/8}, v_{CR} = 80e^{-10^6 t/0.8}$

$\therefore v_{out} = v_{CR} - v_{CL} = \boxed{80e^{-1,250,000t} - 20e^{-125,000t}\text{V},\ t>0}$

(b) $v_{out}(0^+) = \boxed{60\text{V}};\ v_{out}(1\mu\, s) = 80e^{-1.25} - 20e^{-0.125} = \boxed{5.270\text{V}}$

$v_{out}(5\mu\, s) = 80e^{-6.25} - 20e^{-0.625} = \boxed{-10.551\text{V}}$

31. (a) $t<0: \dfrac{v_c - 0.25v_c}{5} + \dfrac{v_c}{10} + \dfrac{v_c - 40}{4} = 0 \therefore v_c = 20\text{V}\,(t<0)$

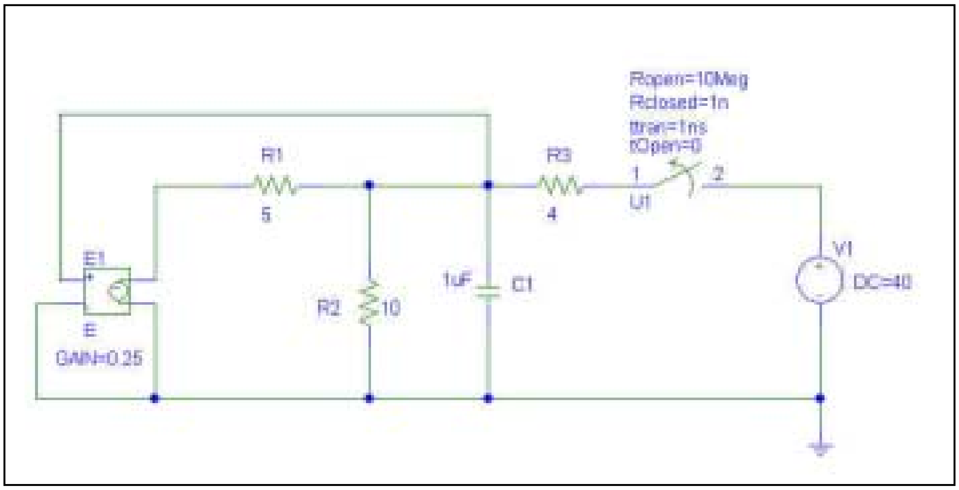
$$t>0: \text{ Apply } v_c = 1\text{V} \therefore \frac{1-0.25}{5} + 0.1 - i_{in} = 0.25\text{A}$$

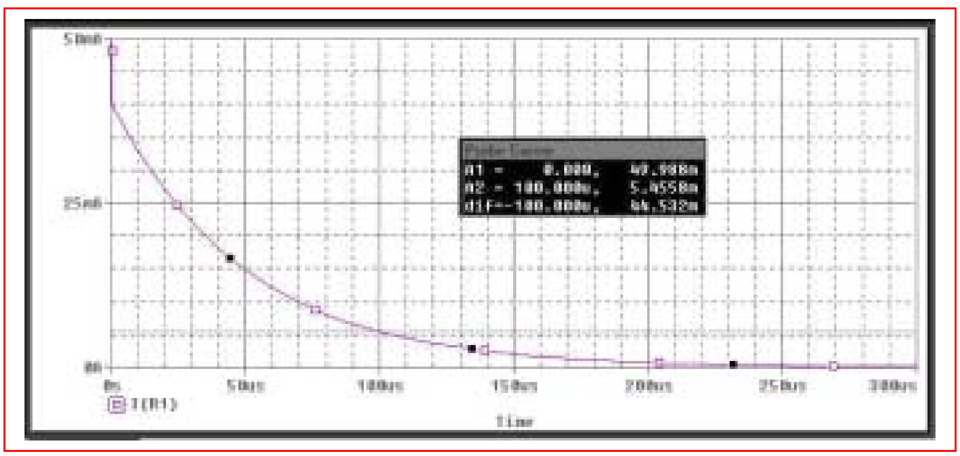
$$\therefore \text{R}_{eq} = \frac{1}{0.25} = 4\Omega$$

$$\therefore v_c(t) = 20e^{-10^6 t/4} = \boxed{20e^{-250{,}000t}\,\text{V}\,(t>0)}$$

(b) v_C(3 μs) = $\boxed{9.447 \text{ V}}$

(c) PSpice verification. Note that the switch parameters had to be changed in order to perform this simulation.





As can be seen from the simulation results, our hand calculations are accurate.

32. $t < 0: \ v_c(0) = 60\text{V}$

$$0 < t < 1\text{ms}: \ v_c = 60e^{-10^6 t/(R_o+1000)} \therefore \frac{50}{60} e^{-500/(R_o+1000)}$$

$$\therefore \frac{500}{\text{R}_o + 1000} = \ln 1.2 = 0.18232 \therefore \frac{\text{R}_o}{500} + 2 = 5.4848, R_o = \boxed{1742.4\Omega}$$

$$\therefore v_c(1\text{ms}) = 60e^{-1000/2742.4} = 41.67\text{V}$$

$$t > 1\text{ms}: v_c = 41.67e^{-10^6(t-10^{-3})} / (1742.4 + \text{R}_1 \| 1000)$$

$$\therefore 25 = 41.67e^{-1000()} \therefore 0.5108 = \frac{.1000}{1742.4 + \text{R}_1 \| 1000}, 1742.4 + \text{R}_1 \| 1000$$

$$= 1957.6, \ \text{R}_1 \| 1000 = 215.2 \frac{1}{\text{R}_1} + 10^{-3} = \frac{1}{215.2} \therefore \text{R}_1 = \boxed{274.2\Omega}$$

33.

(a) With the switch closed, define a nodal voltage V_1 at the top of the 5-kΩ resistor. Then,

$0 = (V_1 - 100)/ 2 + (V_1 - V_C)/ 3 + V_1/ 5$ [1]
$0 = V_C/ 10 + (V_C - V_1)/ 3 + (V_C - 100)$ [2]

Solving, we find that $V_C = v_C(0^-) = \boxed{99.76 \text{ V.}}$

(b) $t > 0:\ R_{eq} = 10 \| 6.5 = 3.939 k\Omega \therefore v_c = 87.59 e^{-10^7 t/3939} = \boxed{87.59 e^{-2539 t} \text{V} \ (t > 0)}$

34. $t < 0$:

$12 = 4i_1 + 20i_1 \therefore i_1 = 0.5\text{mA} \therefore v_c(0) = 6i_1 + 20i_1 = 26i_1$

$v_c(0) = 13\text{V}$

$t > 0$: Apply $\leftarrow 1\text{mA} \therefore 1 + 0.6i_1 = i_1 \therefore i_1 = 2.5\text{mA}; \pm v_{in} = 30i_1 = 75\text{V} \therefore \text{R}_{eq} = 75k\Omega$

$\therefore v_c(t) = 13e^{-t/75\times10^3\times2\times10^{-9}} = 13e^{-10^6 t/150} = 13e^{-6667t}$

$\therefore i_1(t) = \dfrac{v_o}{3\times10^4} = \boxed{0.4333e^{-6667t}\text{mA} \;\; (t > 0)}$

35.

(a) $v_1(0^-) = \boxed{100\text{V.}}\ v_2(0^-) = 0,\ v_R(0^-) = \boxed{0}$

(b) $v_1(0^-) = \boxed{100\text{V.}}\ v_2(0^+) = 0,\ v_R(0^+) = \boxed{100\text{V}}$

(c) $\tau = \dfrac{20\times 5}{20+5}\times 10^{-6}\times 2\ 10^4 = \boxed{8\times 10^{-2}\,s}$

(d) $v_R(t) = \boxed{100e^{-12.5t}\text{V},\ t>0}$

(e) $i(t) = \dfrac{v_R(t)}{2\times 10^4} = \boxed{5e^{-12.5t}\text{mA}}$

(f) $v_1(t) = \dfrac{10^6}{20}\int_o^t -5\times 10^{-3} e^{-12.5t} dt + 100 = \dfrac{10^3}{50} e^{-12.5t}\Big|_o^t + 100 = \boxed{-20e^{-12.5t} + 80\text{V}}$

$v_2(t) = \dfrac{1000}{5}\int_o^t 5e^{-12.5t} dt + 0 = -80e^{-12.5t}\Big|_o^t + 0 = \boxed{-80e^{-12.5t} + 80\text{V}}$

(g) $w_{c1}(\infty) = \dfrac{1}{2}\times 20\times 10^{-6}\times 80^2 = 64\text{mJ},\ w_{c2}(\infty)\dfrac{1}{2}\times 5\times 10^{-6}\times 80^2 = \boxed{16\text{mJ}}$

$w_{c1}(0) = \dfrac{1}{2}\times 20\times 10^{-6}\times 100^2 = \boxed{100\text{mJ}},\ w_{c2}(0) = 0$

$w_R = \int_o^\infty 25\times 10^{-6} e^{-25t}\times 2\times 10^4 dt = \dfrac{25}{-25}\times 2\times 10^4(-1)10^{-6} = \boxed{20\text{mJ}}$

$\boxed{64+16+20 = 100 \text{ checks}}$

36.

(a) $t < 0:\ i_s = 1\text{mA} \therefore \pm\ v_c(0) = 10\text{V},\ \downarrow i_L(0) = -1\text{mA} \therefore v_x(0) = \boxed{10\text{V}, t < 0}$

(b) $t > 0:\ v_c(t) = 10e^{-t/10^4 \times 20 \times 10^{-9}} = 10e^{-5000t}\text{V}$

$\text{i}_\text{L}(t) = -10^{-3}e - 10^{3t/0.1} = -10^{-3}e^{-10000t}\text{A} \therefore \pm v_L(t) = e^{-10000t}\text{V},\ t > 0$

$\therefore v_x = v_c - v_L(t) = \boxed{10e^{-5000t} - e^{-10000t}\text{V},\ t > 0}$

37.

(a) $t<0: \; v_s = 20\text{V} \therefore v_c = 20\text{V}, \; i_L = 20\text{mA} \therefore \boxed{i_x(t) = 20\text{mA}, \; t<0}$

(b) $t>0: \; v_s = 0 \therefore i_L(t) = 0.02e^{-10000t}\text{A}; \; v_c(t) = 20e^{-t/2\times10^{-8}10^4} = 20e^{-5000t}\text{V}$

$\downarrow i_c(t) = 2\times10^{-8}\times20(-5000)e^{-5000t} = -2e^{-5000t}\text{mA}$

$i_x(t) = i_L(t) + i_c(t) = 0.02e^{-10000t} - 0.002e^{-5000t}\text{A} = \boxed{20e^{-10000t} - 2e^{-5000t}\text{mA}}$

38.

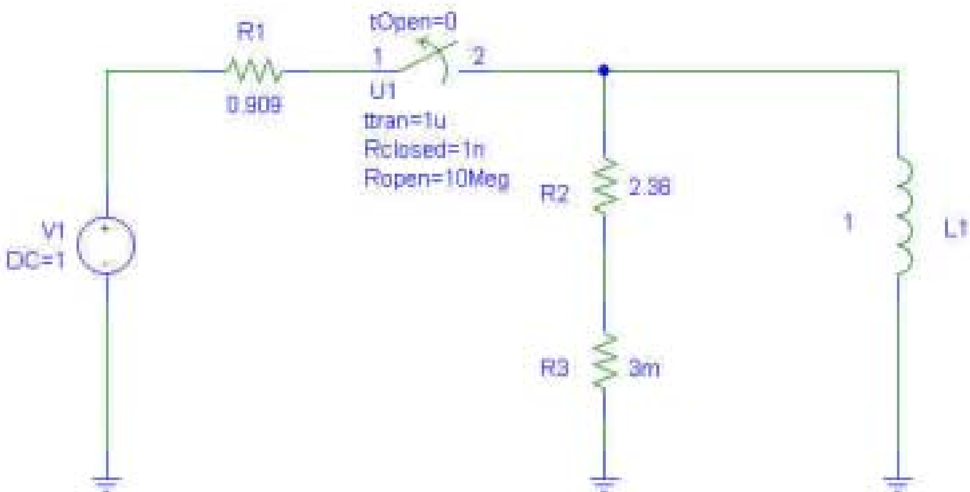
$$i_L(0^-) = \frac{\text{V}}{\text{R}} = \frac{1}{0.909} = 1.1\ \text{A}$$

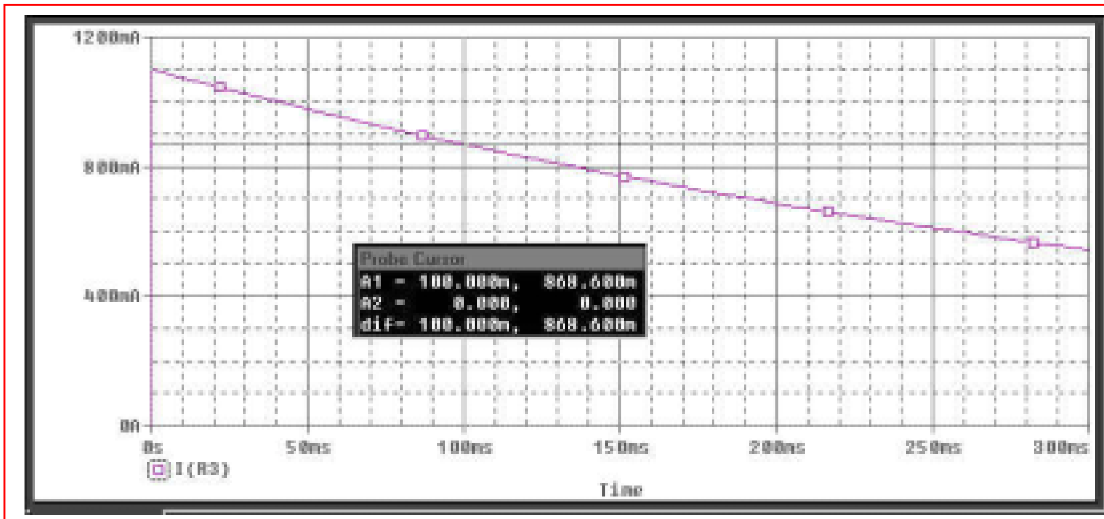
$$t > 0: \ i_L(t) \ = \ e^{-2.363t} \ \ \text{A}$$

$$i_L(0.1s) = 1.1e^{-2.363\times 0.1} = 0.8685\ \text{A}$$

$\therefore$ since the current has dropped to less than 1 A prior to $t = 100$ ms, the fuse does not blow.

PSpice verification: Note that the switch properties were changed.





We see from the simulation result that the current through the fuse (R3) is 869 mA, in agreement with our hand calculation.

39. (a) $v_A = 300u(t-1)\,\text{V},\; v_B = -120u(t+1)\,\text{V};\; i_c = 3u(-t)\,\text{A}$

$$t = -1.5:\; i_1(-1.5) = 3\times\frac{100}{300} = \boxed{1\text{A}}$$

$$t = 0.5:\; i_1(-0.5) = \frac{-120}{300} + 1 = \boxed{0.6\text{A};}$$

$$t = 0.5:\; i_1 = -\frac{120}{300} = -0.4\text{A};\; t = 1.5:\; i_1 = \frac{300}{300} - \frac{120}{300} = \boxed{0.6\text{A}}$$

40.

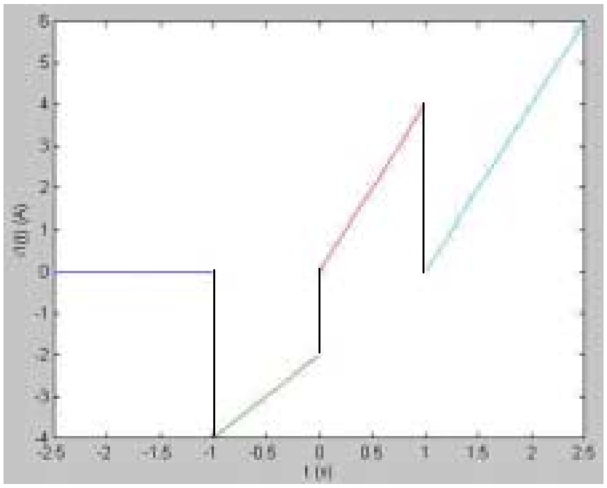
` $v_A = 600tu(t+1)\,v,\ v_B = 600(t+1)u(t)\,\text{V},\ i_c = 6(t-1)u(t-1)\text{A}$

(a) $t = -1.5: i_1 = 0;\ t = -0.5: i_1 = 600(-0.5)/300 = \boxed{-1\text{A}}$

$t = 0.5: i_1 = \dfrac{600(0.5)}{300} + \dfrac{600(1.5)}{300} = \boxed{4\text{A}}$

$t = 1.5: i_1 = \dfrac{600(1.5)}{300} + \dfrac{600(2.5)}{300} + \dfrac{1}{3}\times 6\times 0.5 = 3+5+1 = \boxed{9\text{A}}$

(b)



41.

(a) $2u(-1) - 3u(1) + 4u(3) = -3 + 4 = \boxed{1}$

(b) $[5 - u(2)]\,[2 + u(1)]\,[1 - u(-1)]$
$= 4 \times 3 \times 1 = \boxed{12}$

(c) $4e^{-u(1)}u(1) = 4e^{-1} = \boxed{1.4715^{+}}$

42.

(a) $t < 0$: $i_x = \frac{100}{50} + 0 + 10 \times \frac{20}{50} = \boxed{6\text{A}}$

$t > 0$: $i_x = 0 + \frac{60}{30} + 0 = \boxed{2\text{A}}$

(b) $t < 0$: The voltage source is shorting out the 30-Ω resistor, so $\boxed{i_x = 0.}$

$t > 0$: $i_x = 60/\ 30 = \boxed{2\text{ A}}$.

43. $t = -0.5:$ $50 \| 25 = 16.667,\ i_x = \frac{200}{66.67} - 2\frac{1/50}{1/50 + 1/25 + 1/50} = 3 - \frac{1}{2} = \boxed{2.5\text{A}}$

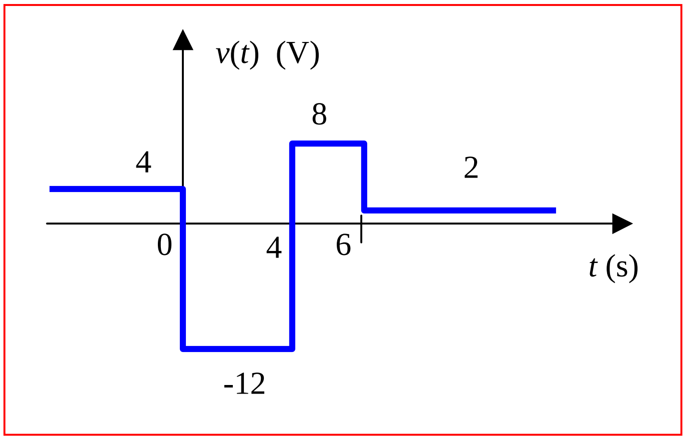
$t = 0.5:$ $i_x = \frac{200}{66.67} = \boxed{3\text{A}}$

$t = 1.5:$ $i_x = 3 - \frac{100}{66.67} \times \frac{1}{3} = \boxed{2.5\text{A}}$

$t = 2.5:$ $i_x = \frac{200 - 100}{50} = \boxed{2\text{A}}$

$t = 3.5:$ $i_x = -\frac{100}{50} = \boxed{-2\text{A}}$

44. $\boxed{v(t) = 4 - 16u(t) + 20u(t-4) - 6u(t-6)\text{V}}$



45. (a) $7\,u(t) - 0.2\,u(t) + 8(t-2) + 3$

$\boxed{v(1) = 9.8 \text{ volts}}$

(b) $\boxed{\text{Resistor of value } 2\Omega}$

46.

(a) $\boxed{i_L(t) = (2 - 2e^{-200000t})\,u(t) \;\; \text{mA}}$

(b) $v_L(t) = \text{L}i_L' = 15\times10^{-3}\times10^{-3}(-2)$

$(-200000e^{-200000t})\; u(t) = \boxed{6e^{-200000t}u(t)\text{V}}$

47.

(a) $i_L(t) = 2 + 2(1 - e^{-2.5t})u(t)\text{ A} \therefore i_1(-0.5) = \boxed{2\text{A}}$

(b) $i_L(0.5) = 2 + 2(1 - e^{-1.25}) = \boxed{3.427\text{A}}$

(c) $i_L(1.5) = 2 + 2(1 - e^{-3.75}) = \boxed{3.953\text{A}}$

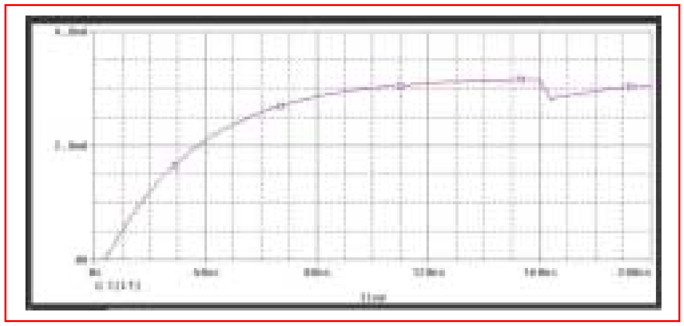
48.

$$R_{th} = \frac{10\times10^3 \times 4.7\times10^3}{10\times10^3 + 4.7\times10^3} = 3.2\times10^3\,\Omega$$

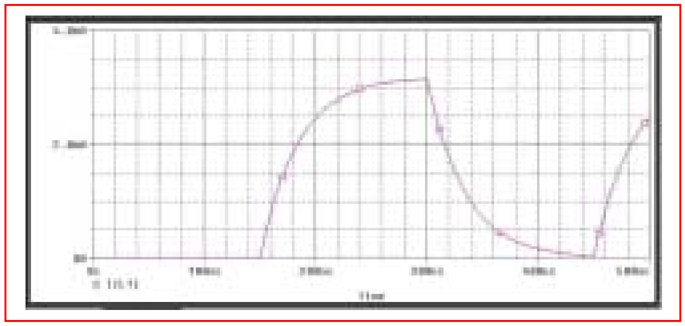
$$\tau = \frac{L}{R} = \frac{500\times10^{-6}}{3.2\times10^3} = 156ns$$

$$i_1.4.7\times10^3 = i_L.10\times10^3$$
$$i_1 + i_L = 10\times10^{-3}\,A$$
$$\therefore i_L = 3.2\times10^{-3}\,A$$

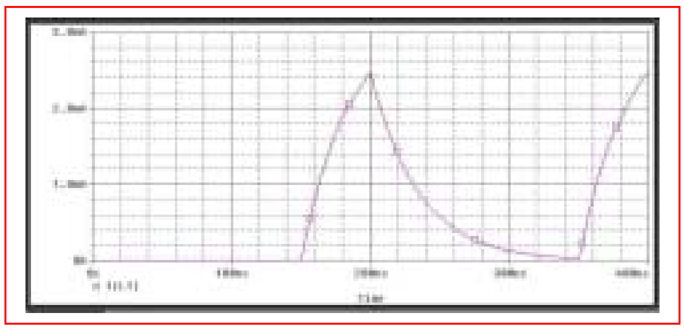
(a)



(b)



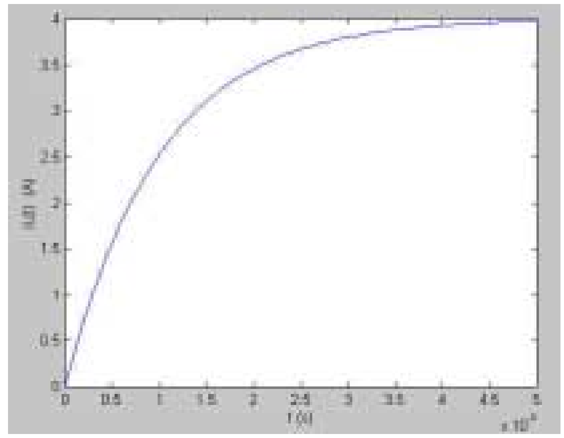
(c)



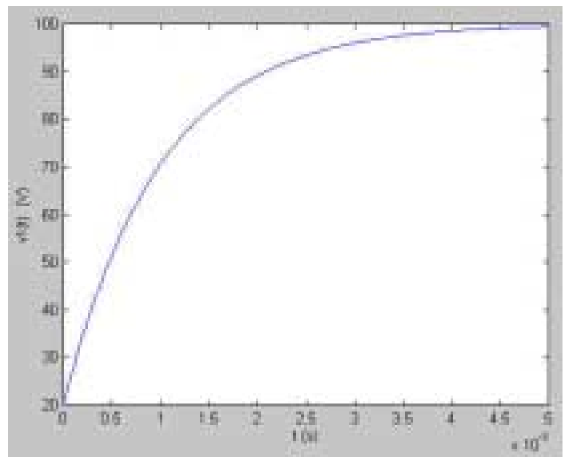
49.

(a) $i_L(t) = (4 - 4e^{-20t/0.02})u(t)$

$\therefore \boxed{i_L(t) = 4(1 - e^{-1000t})u(t)\text{A}}$



(b) $\boxed{v_1(t) = (100 - 80e^{-1000t})u(t)\text{V}}$

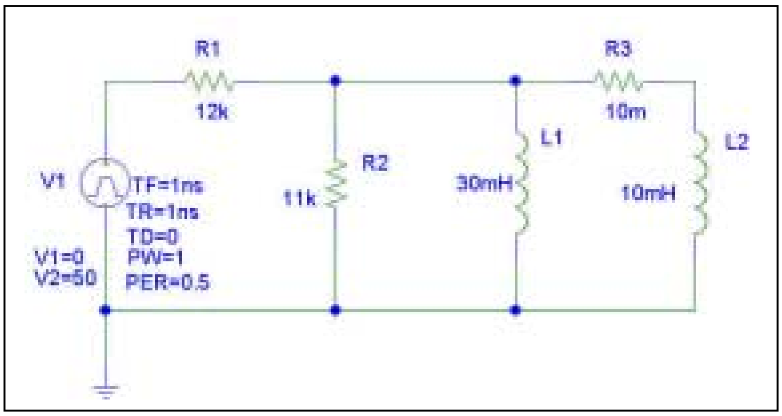


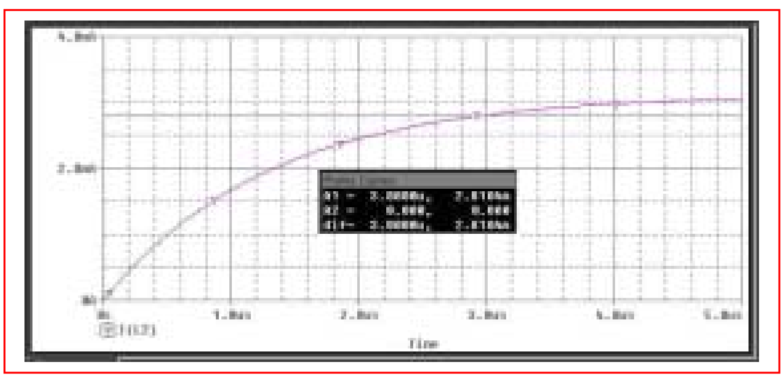
50. (a) $\boxed{0 \text{ W}}$

(b) The total inductance is 30 || 10 = 7.5 mH. The Thévenin equivalent resistance is 12 || 11 = 5.739 kΩ. Thus, the circuit time constant is L/R = 1.307 μs. The final value of the total current flowing into the parallel inductor combination is 50/12 mA = 4.167 mA. This will be divided between the two inductors, so that $i(\infty)$ = (4.167)(30)/ (30 + 10) = 3.125 mA.

We may therefore write $i(t) = 3.125[1 - e^{-10^6 t/1.307}]$ A. Solving at t = 3 μs, we find $\boxed{2.810 \text{ A.}}$

(c) PSpice verification





We see from the Probe output that our hand calculations are correct by verifying using the cursor tool at t = 3 μs.

51.

(a) $\boxed{i_L(t) = 10A,\ t < 0}$

(b) $i_L(t) = 8 + 2e^{-5t/0.5}$

$\therefore \boxed{i_L(t) = 8 + 2e^{-10t}\text{A},\ t > 0}$

52.

(a) $\boxed{i_L(t) = 2\text{A},\ t>0}$

(b) $i_L(t) = 5 - e^{-4t/0.1}$

$\therefore \boxed{i_L(t) = 5 - 3e^{-40t}\ \text{A},\ t>0}$

53. $\dfrac{di}{dt}+\text{P}i=\text{Q},\ i=e^{-Pt}\int \text{Q}e^{Pt}dt+Ae^{-Pt},\ \text{R}=125\Omega,\ \text{L}=5\text{H}$

$\therefore \text{L}\dfrac{di}{dt}\text{LP}i=\text{LQ}\therefore \text{LP}=5\text{P}=\text{R}=125\therefore \text{P}=25$

(a) $\text{Q}(t)=\dfrac{10}{\text{L}}=2\therefore i=e^{-25t}\int_o^t 2e^{25t}dt+\text{A}e^{-25t}=e^{-25t}\times\dfrac{2}{25}e^{25t}\Big|_o^t+\text{A}e^{-25t}$

$\therefore i=\dfrac{2}{25}+\text{A}e^{-25t},\ i(0)=\dfrac{10}{125}=\dfrac{2}{25}\therefore \text{A}=0\therefore i=\dfrac{2}{25}=\boxed{0.08\text{A}}$

(b) $\text{Q}(t)=\dfrac{10u(t)}{5}=2u(t)\therefore i=e^{-25t}\int_o^t 2e^{25t}dt+\text{A}e^{-25t}=\dfrac{2}{25}+\text{A}e^{-25t}$

$i(0)=0\therefore \text{A}=-\dfrac{2}{25}\therefore \boxed{i(t)=0.08(1-e^{-25t})\text{A},\ t>0}$

(c) $\text{Q}(t)=\dfrac{10+10u(t)}{5}=2+2u(t)\therefore \boxed{i=0.16-0.08e^{-25t}\text{A},\ t>0}$

(d) $\text{Q}(t)=\dfrac{10u(t)\cos 50t}{5}=2u(t)\cos 50t\therefore i=e^{-25t}\int_o^t 2\cos 50t\times e^{25t}dt+Ae^{-25t}$

$\therefore i=2e^{-25t}\left[\dfrac{e^{25t}}{50^2+25^2}(25\cos 50t+50\sin 50t)\right]_o^t+\text{A}e^{-25t}$

$=2e^{-25t}\left[\dfrac{e^{25t}}{3125}(25\cos 50t+50\sin 50t)-\dfrac{1}{3125}\times 25\right]+\text{A}e^{-25t}$

$=\dfrac{2}{125}\cos 50t+\dfrac{4}{125}\sin 50t-\dfrac{2}{125}e^{-25t}+\text{A}e^{-25t}$

$i(0)=0\therefore 0=\dfrac{2}{125}-\dfrac{2}{125}+\text{A}\therefore \text{A}=0$

$\therefore \boxed{i(t)=0.016\cos 50t+0.032\sin 50t-0.016e^{-25t}\text{A},\ t>0}$

54.

(a) $\boxed{0,\ 0}$

(b) $\boxed{0,\ 200\text{V}}$

(c) $\boxed{1\text{A},\ 100\text{V}}$

(d) $\tau = \dfrac{50\times10^{-3}}{200} = \dfrac{1}{4}\text{ms} \therefore i_L = 1(1-e^{-4000t})u(t)\text{A},\ i_L(0.2\text{ms}) = \boxed{0.5507\text{A}}$

$v_1(t) = (100+100e^{-4000t})u(t)\text{V},\ v_1(0.2\text{ms}) = \boxed{144.93\text{V}}$

55.

(a) $i_L(t) = \dfrac{100}{20} - \dfrac{100}{5} = \boxed{-15\text{A},\ t<0}$

(b) $i_L(0^+) = i_L(0^-) = \boxed{-15\text{A}}$

(c) $i_L(\infty) = \dfrac{100}{20} = \boxed{5\text{A}}$

(d) $i_L(t) = \boxed{5 - 20e^{-40t}\text{A},\ t>0}$

56. $i_L(0^-) = \frac{18}{60+30} \times \frac{1}{2} = 0.1\text{A} \therefore i_L(0^+) = 0.1\text{A}$

$\mathrm{i_L}(\infty) = 0.1 + 0.1 = 0.2\text{A}$

$\therefore i_L(t) = 0.2 - 0.1e^{-9000t}\text{A},\ t > 0$

$\therefore i_L(t) = 0.1u(-t) + (0.2 - 0.1e^{-9000t})\,u(t)\text{A}$

or, $\boxed{i_L(t) = 0.1 + (0.1 - 0.1e^{-9000t})\,u(t)\text{A}}$

57.

(a) $i_x(0^-) = \frac{30}{7.5} \times \frac{3}{4} = 3\text{A},\ i_L(0^-) = \boxed{4\text{A}}$

(b) $i_x(0^+) = i_L(0^+) = \boxed{4\text{A}}$

(c) $i_x(\infty) = i_L(\infty) = 3\text{A}$

$\therefore i_x(t) = 3 + 1e^{-10t/0.5} = 3 + e^{-20t}\text{A} \therefore i_x(0.04)$

$= 3 + e^{-0.8} = \boxed{3.449\text{A}}$

58.

(a) $i_x(0^-) = i_L(0^-) = \dfrac{30}{10} = \boxed{3\text{A}}$

(b) $i_x(0^+) = \dfrac{30}{30+7.5} \times \dfrac{30}{40} + 3 \times \dfrac{15}{10+15} = \boxed{2.4\text{A}}$

(c) $i_x(\infty) = \dfrac{30}{7.5} \times \dfrac{30}{40} = 3\text{A} \therefore i_x(t) = 3 - 0.6e^{-6t/0.5}$

$= 3 - 0.6e^{-12t} \therefore i_x(0.04) = 3 - 0.6e^{-0.48} = \boxed{2.629\text{A}}$

59. $\text{OC}: v_x = 0,\ v_{oc} = 4u(t)\text{V}$

$$\text{SC}: 0.1u(t) = \frac{v_x - 0.2v_x}{40} + \frac{v_x}{60}, 12u(t) = 0.6v_x + 2v_x$$

$$\therefore v_x = \frac{12u(t)}{2.6} \therefore i_{ab} = \frac{v_x}{60} = \frac{12u(t)}{2.6\times 60} = \frac{u(t)}{13}$$

$$\therefore R_{th} = 4\times 13 = 52\Omega \therefore i_L = \frac{4u(t)}{52}(1 - e^{-52t/0.2})u(t) = \frac{u(t)}{13}(1 - e^{-260t})u(t)$$

$$\therefore v_x = 60i_L = \boxed{4.615^{+}(1 - e^{-260t})u(t)\text{V}}$$

60.

(a) $\text{OC}: -100 + 30i_1 + 20i_1 = 0,\ i_1 = 2\text{A}$

$\therefore v_{oc} = 80u(t)\text{V}$

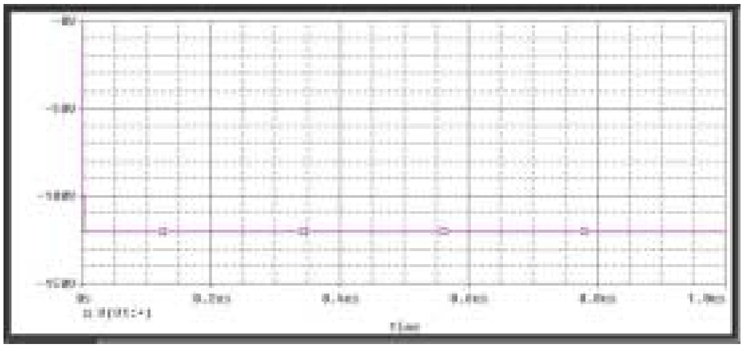
$\text{SC}: i_1 = 10\text{A},\ \downarrow i_{sc} = 10 + \dfrac{20\times 10}{20} = 20\text{A}$

$\therefore \text{R}_{th} = 4\Omega \therefore i_L(t) = \boxed{20(1-e^{-40t})u(t)\text{A}}$

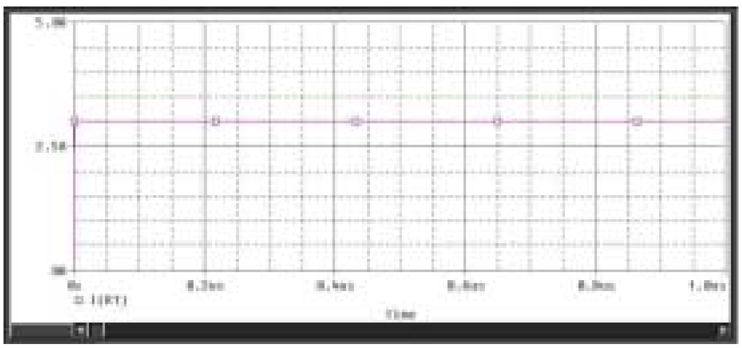
(b) $v_L = 0.1\times 20\times 40e^{-40t}u(t) = 80e^{-40t}u(t)$

$\therefore i_1(t) = \dfrac{100u(t) - 80e^{-40t}u(t)}{10} = \boxed{10 - 8e^{-40t}u(t)\text{A}}$

61. Unfortunately, PSpice will not allow us to use negative time values. Thus, we must perform the simulation starting from $t = 0$, and manually shift the results if needed to account for sources that change value prior to $t = 0$.



Voltage source $v_B(t)$ from Exercise 39.



Current source $i_C(t)$ from Exercise 39.

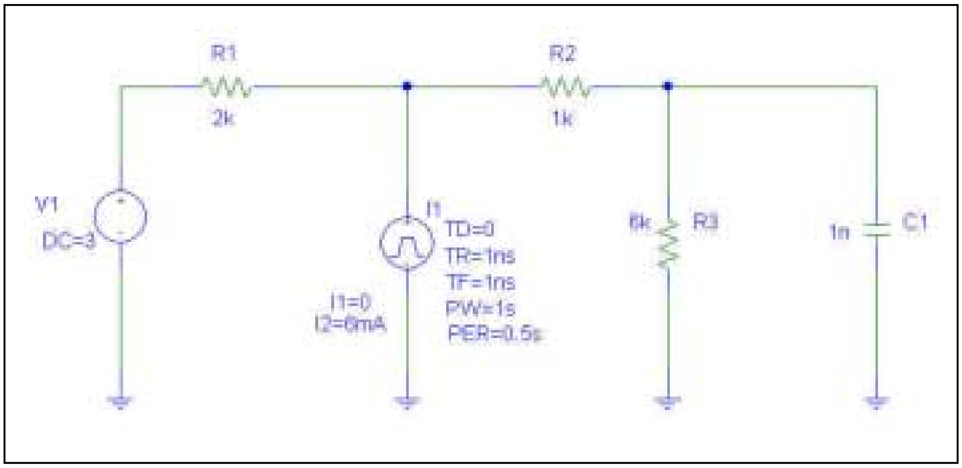
62. (a) $v_c(0^-) = \dfrac{6}{9} \times 3 = 2\text{V} = v_c(0^+)$

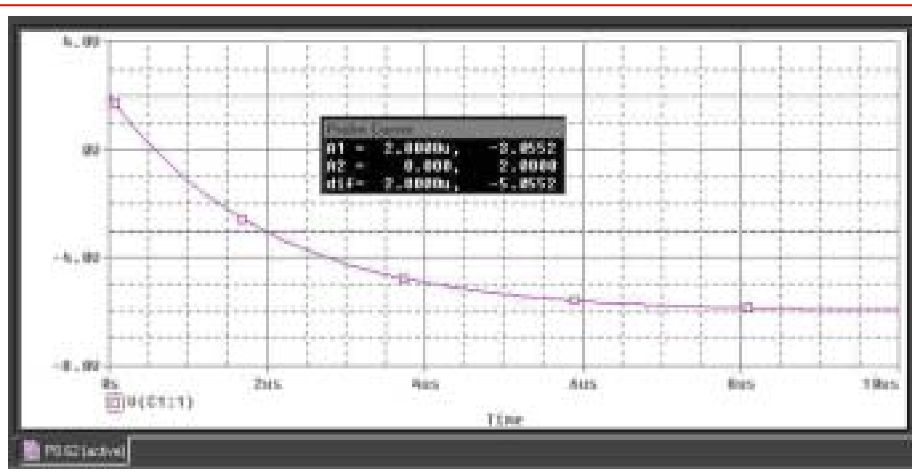
$$v_c(\infty) = 2 - 6(2\|7)\frac{6}{7} = -6\text{V}$$

$$\therefore v_c(t) = -6 + 8e^{-10^9 t/2\times 10^3} = -6 + 8e^{-500000t}\,\text{V},\ t > 0$$

$$v_c(-2\mu s) = v_c(0^-) = 2\text{V},\ v_c(2\mu s) = -6 + 8e^{-1} = \boxed{-3.057\text{V}}$$

(b) PSpice verification.





As can be seen from the plot above, the PSpice simulation results confirm our hand calculations of $v_C(t < 0) = 2$ V and $v_C(t = 2\ \mu s) = -3.06$ V

63. $i_A(0^-) = \frac{10}{4} = 2.5\text{mA},\ i_A(\infty) = 10\text{mA}$

$v_c(0) = 7.5\text{V} \therefore i_A(0^+) = \frac{10}{1} + \frac{7.5}{1} = 17.5\text{mA}$

$\boxed{i_A = 10 + 7.5e^{-10^8 t/10^3} = 10 + 7.5e^{-10^5 t}\text{mA},\ t > 0,\ i_A = 2.5\text{mA}\ \ t < 0}$

64. $i_A(0^-) = \frac{10}{1} = 10\text{mA},\ i_A(\infty) = 2.5\text{mA},\ v_c(0) = 0$

$i_A(0^+) = \frac{10}{1.75} \times \frac{1}{4}\ 1.4286\text{mA} \therefore \boxed{i_A = 10\text{mA},\ t < 0}$

$i_A = 2.5 + (1.4286 - 2.5)\, e^{-10^8 t/1.75\times 10^3} = \boxed{2.5 - 1.0714 e^{-57140t}\text{mA},\ t > 0}$

65.

(a) $v_s = -12u(-t) + 24u(t)\text{V}$

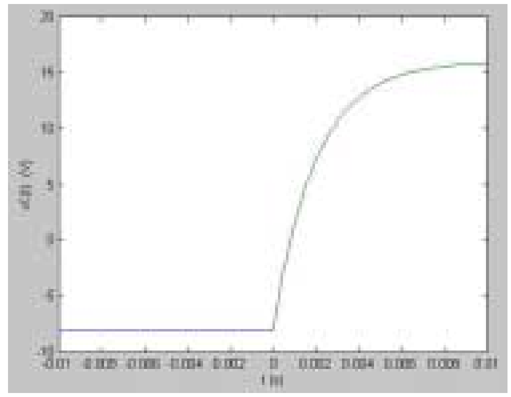
$t < 0:\ v_c(0^-) = -8\text{V} \therefore v_c(0^+) = -8\text{V}$

$t > 0:\ v_c(\infty) = \dfrac{2}{3} \times 24 = 16\text{V}$

$\text{RC} = \dfrac{200}{30} \times 10^3 \times 3 \times 10^{-7} = 2 \times 10^{-3}$

$\therefore v_c(t) = 16 - 24e^{-500t}\text{V},\ t > 0$

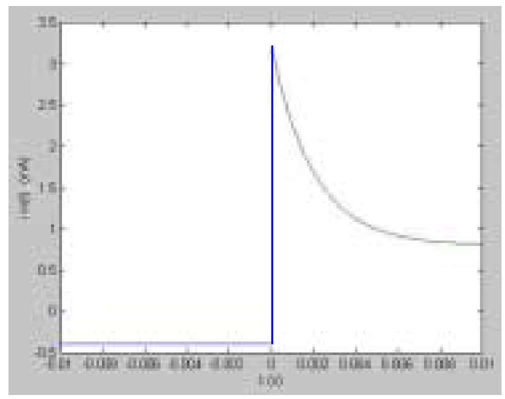
$\therefore \boxed{v_c(t) = -8u(-t) + (16 - 24e^{-500t})u(t)}$



(b) $i_{in}(0^-) = \dfrac{-12}{30} = -0.4\text{mA},\ i_{in}(0^+) = \dfrac{24+8}{10} = 3.2\text{mA}$

$i_{in}(\infty) = \dfrac{24}{30} = 0.8\text{mA}$

$\boxed{i_{in}(t) = -0.4u(t) + (0.8 + 2.4e^{-500t})u(t)\text{mA}}$



66. $v_c(0^-) = 10\text{V} = v_c(0^+),\ i_{in}(0^-) = 0$

$i_{in}(0^+) = 0 \therefore i_{in}(t) = 0$ for all t

(a) $\boxed{i_{in}(-1.5) = 0}$

(b) $\boxed{i_{in}(1.5) = 0}$

67. $0 < t < 0.5s: \; v_c = 10(1 - e^{-2.5t})\text{V}$

$\boxed{v_c(0.4) = 6.321\text{V}}, \; v_c(0.5) = 7.135\text{V}$

$t > 0.5: \; \frac{20-10}{12} = \frac{5}{6}\text{A} \therefore v_c(\infty) = 10 + 8 + \frac{5}{6} = \frac{50}{3}\text{V}, \; 4 \| 8 = \frac{8}{3}\Omega$

$v_c(t) = \frac{50}{3} + \left(7.135 - \frac{50}{3}\right)e^{-0.375\times20(t-0.5)} = 16.667 - 9.532e^{-7.5(t-0.5)}\text{V}$

$\therefore v_c(0.8) = 16.667 - 9.532e^{-7.5(0.3)} = \boxed{15.662\text{V}}$

68. $\text{OC}: \frac{-v_x}{100} - \frac{v_x}{100} + \frac{3-v_x}{100} = 0 \therefore v_x = 1,\ v_{oc} = 3-1 = 2\text{V}$

$\text{SC}: v_x = 3\text{V} \therefore i_{sc} = \frac{v_x}{100} + \frac{v_x}{100} = 0.06\text{A}$

$\therefore \text{R}_{th} = v_{oc} / i_{sc} = 2/0.06 = 33.33\Omega$

$\therefore v_c = v_{oc}(1-e^{-t/R_{th}C}) = 2(1-e^{-10^6 t/33.33})$

$\boxed{= 2(1-e^{-30,000t})\text{ V},\ t>0}$

69.

(a) $t<0:\ 8(10+20)=240\text{V}=\boxed{v_R(t)=80\text{V},\ t<0}$

(b) $t<0:\ v_c(t)=8\times 30=240\text{V}\ \therefore v_c(0^+)=240\text{V}$

$$t=(\infty):\ v_c(\infty)=\frac{1}{2}\times 8(10+10)=80\text{V}$$

$$\therefore v_c(t)=80+160e^{-t/10\times 10^{-6}}=80+160e^{-100000t}\text{V}$$

$$\therefore v_R(t)=\boxed{80+160e^{-100000t}\text{V},\ t>0}$$

(c) $t<0:\ v_R(t)=\boxed{80\text{V}}$

(d)

$$v_c(0^-)=80\text{V},\ v_c(\infty)=240\text{V}\ \therefore v_c(t)=240-160e^{-t/50\times 10^{-6}}=240-160e^{-20000t}\text{V}$$

$$v_R(0^-)=80\text{V},\ v_R(0^+)=8\frac{20}{30+20}\times 10+\frac{80}{50}\times 10=32+16=48\text{V}$$

$$v_R(\infty)=80\text{V}\ \therefore v_R(t)=\boxed{80-32e^{-20000t}\text{V},\ t>0}$$

70.

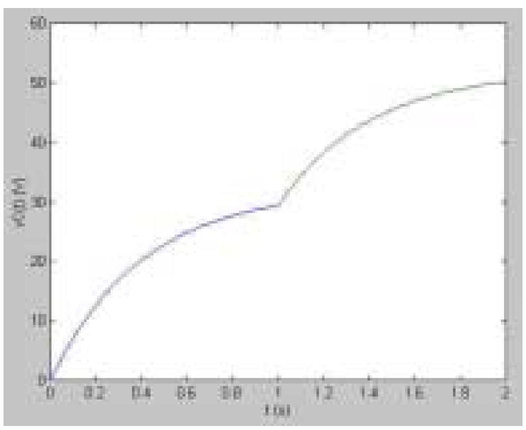
(a) For $t < 0$, there are no active sources, and so $\boxed{v_C = 0.}$

For $0 < t < 1$, only the 40-V source is active. $R_{th} = 5\text{k} \,||\, 20\text{ k} = 4\text{ k}\Omega$ and hence $\tau = R_{th}C = 0.4$ s. The "final" value (assuming no other source is ever added) is found by voltage division to be $v_C(\infty) = 40(20)/(20 + 5) = 32$ V. Thus, we may write $v_C(t) = 32 + [0 - 32]\,e^{-t/0.4}$ V $= \boxed{32(1 - e^{-2.5t})\text{ V.}}$

For $t > 1$, we now have two sources operating, although the circuit time constant remains unchanged. We define a new time axis temporarily: $t' = t - 1$. Then $v_C(t' = 0^+) = v_C(t = 1) = 29.37$ V. This is the voltage across the capacitor when the second source kicks on. The new final voltage is found to be $v_C(\infty) = 40(20)/(20 + 5) + 100(5)/(20 + 5) = 52$ V.

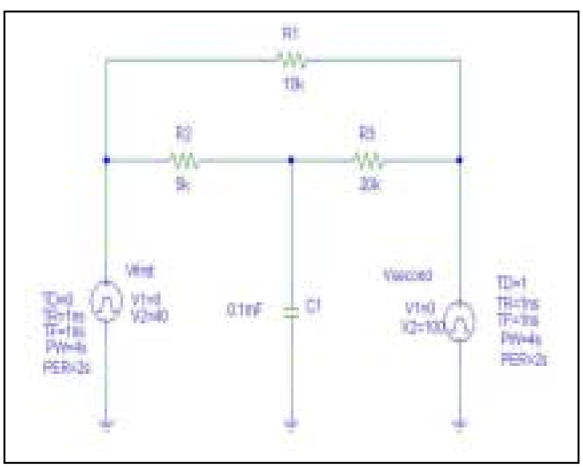
Thus, $v_C(t') = 52 + [29.37 - 52]\,e^{-2.5t'} = \boxed{52 - 22.63\,e^{-2.5(t-1)}\text{ V.}}$

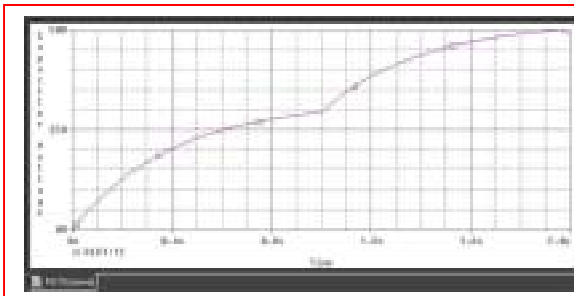
(b)



For $t < 0$, $v_C = 0$.

(c)





We see from the simulation results that our hand calculations and sketch are indeed correct.

71. $v_{x,L} = 200e^{-2000t}\text{V}$

$$v_{x,c} = 100(1 - e^{-1000t})\text{V}$$

$$v_x = v_{x,L} - v_{x,c} = 0$$

$$\therefore 200e^{-2000t} = 100 - 100e^{-1000t}$$

$$\therefore 100e^{-1000t} + 200(e^{-1000t})^2 - 100 = 0,$$

$$e^{-1000t} = \frac{-100 \pm \sqrt{10,000 + 80,000}}{400} = -0.25 \pm 0.75$$

$$\therefore e^{-1000t} = 0.5,\ t = \boxed{0.6931\text{ms}}$$

72. $t < 0:\ v_c = 0$

$0 < t < 1\text{ms}:\ v_c = 9\left(1 - e^{-10^6 t/(R_1+100)}\right)$

$\therefore 8 = 9\left(1 - e^{-1000/(R_1+100)}\right),\ \dfrac{1}{9} = e^{-1000/(R_1+100)}$

$\therefore \dfrac{1000}{R_1 + 100} = 2.197,\ R_1 = \boxed{355.1\Omega}$

$t > 1\text{ms}:\ v_c = 8e^{-10^6 t'/(R_2+100)},\ t' = t - 10^{-3}\ \therefore 1 - 8e^{-1000}(R_2 + 100)$

$\therefore \dfrac{1000}{R_2 + 100} = 2.079,\ R_2 = 480.9 - 100 = \boxed{380.9\Omega}$

73. For $t < 0$, the voltage across all three capacitors is simply 9 (4.7)/ 5.7 = 7.421 V. The circuit time constant is $\tau = RC = 4700\,(0.5455\times10^{-6}) = 2.564$ ms.

When the circuit was first constructed, we assume no energy was stored in any of the capacitors, and hence the voltage across each was zero. When the switch was closed, the capacitors began to charge according to ½ Cv^2. The capacitors charge with the same current flowing through each, so that by KCL we may write

$$C_1\frac{dv_1}{dt} = C_2\frac{dv_2}{dt} = C_3\frac{dv_3}{dt}$$

With no initial energy stored, integration yields the relationship $C_1v_1 = C_2v_2 = C_3v_3$ throughout the charging (*i.e.* until the switch is eventually opened). Thus, just prior to the switch being thrown at what we now call $t = 0$, the total voltage across the capacitor string is 7.421 V, and the individual voltages may be found by solving:

$$\begin{aligned} v_1 + v_2 + v_3 &= 7.421 \\ 10^{-6}\,v_1 - 2\times10^{-6}\,v_2 &= 0 \\ 2\times10^{-6}\,v_2 - 3\times10^{-6}\,v_3 &= 0 \end{aligned}$$

so that $v_2 = 2.024$ V.

With the initial voltage across the 2-uF capacitor now known, we may write

$$v(t) = 2.024\, e^{-t/\,2.564\times10^{-3}}\ \text{V}$$

(a) $v(t = 5.45\text{ ms}) = \boxed{241.6\text{ mV.}}$

(b) The voltage across the entire capacitor string can be written as $7.421\, e^{-t/\,2.564\times10^{-3}}$ V. Thus, the voltage across the 4.7-kΩ resistor at $t = 1.7$ ms = 3.824 V and the dissipated power is therefore $\boxed{3.111\text{ mW.}}$

(c) Energy stored at $t = 0$ is ½ $Cv^2 = 0.5(0.5455\times10^{-6})(7.421)^2 = \boxed{15.02\ \mu\text{J.}}$

74.

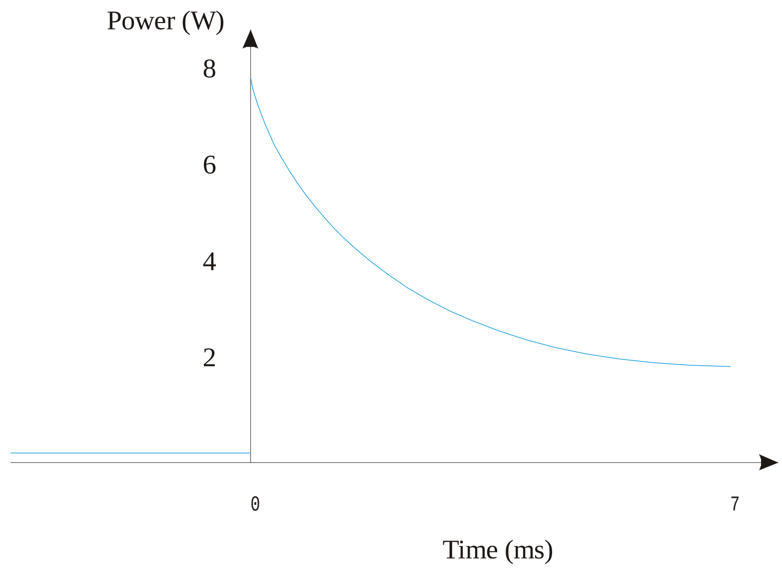
$$P(t<0) = I^2R = 0.001^2 \times 10^3 = 0.001\,\text{W}$$

$$V_{init} = I.R = 7\times10^{-3}\times900 = 6.3\,\text{V}$$

$$P_{init} = \frac{V^2}{R} = 0.08\,\text{W}$$

$$V_{final} = 7\times10^{-3}\times900\Omega//1000\Omega = 3.3\,\text{V}$$

$$P_{final} = \frac{V^2}{R} = 0.02\,\text{W}$$



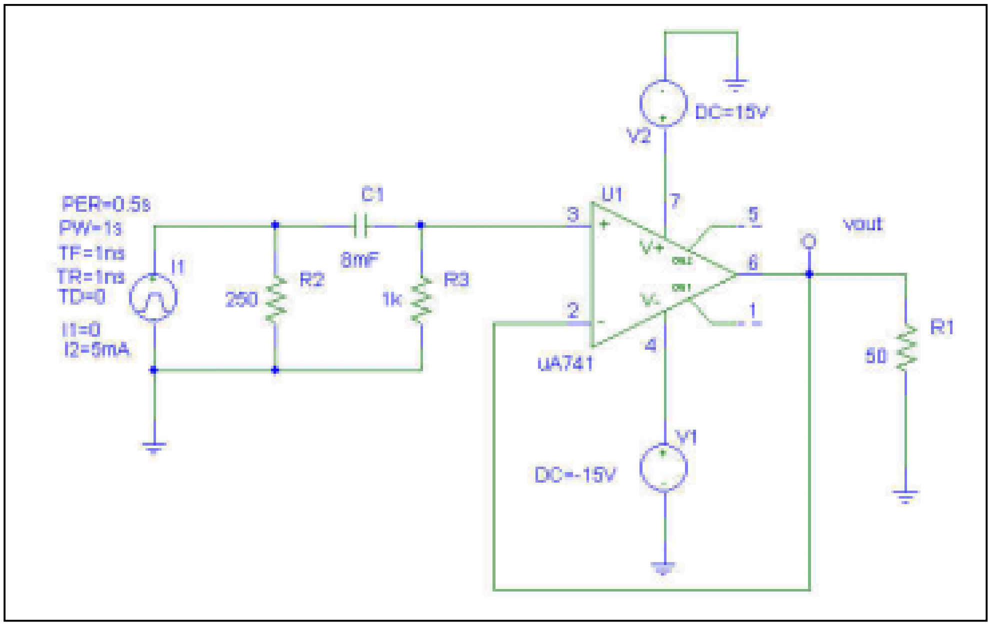
75. voltage follower $\therefore v_o(t) = v_2(t)$

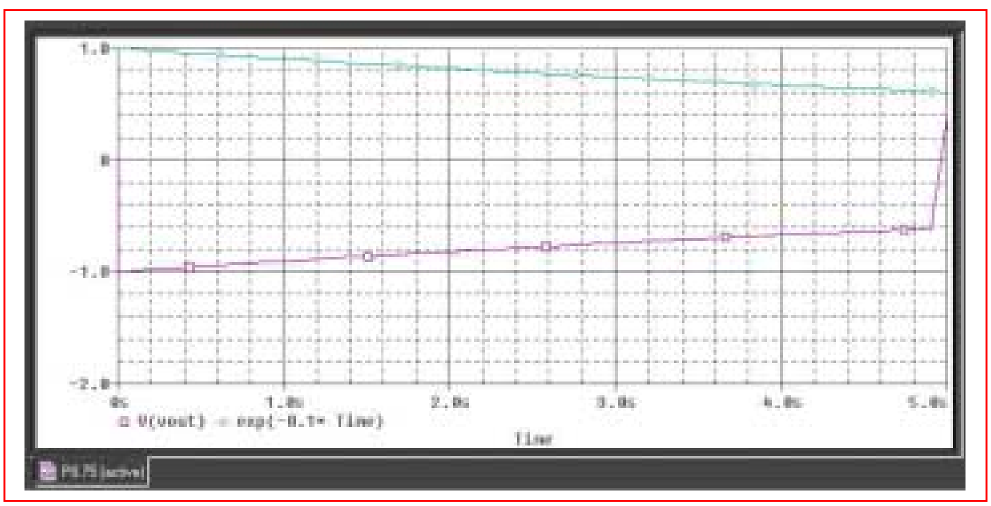
$$v_2(0^+) = 5(0.25\|1) = 1u(t)\text{V}$$

$$\text{v}_2(\infty) = 0,\ \tau = 1.25 \times 8 = 10s$$

$$\therefore v_o(t) = \boxed{e^{-0.1t}u(t)\text{V}}$$

PSpice verification:





From the simulation results, we see that modeling the operation of this circuit using an ideal op amp model does not provide an accurate accounting for the operation of the actual circuit.

76. voltage follower $\therefore v_o(t) = v_2(t)$

$v_2(t) = 1.25u(t)\text{V} = v_o(t)$

$v_x(t) = 1.25e^{-10^6/0.5\times 200}u(t)$

$= \boxed{1.25e^{-10,000t}\,u(t)\text{V}}$

77. (a)

$$v_1 = 0 \text{ (virtual gnd)} \;\; \therefore i = \frac{4}{10^4} e^{-20{,}000t} u(t) \text{A}$$

$$\therefore v_c = 10^7 \int_o^t \frac{4}{10^4} e^{-20{,}000t} dt = -0.2 e^{-20{,}000t} \Big|_o^t$$

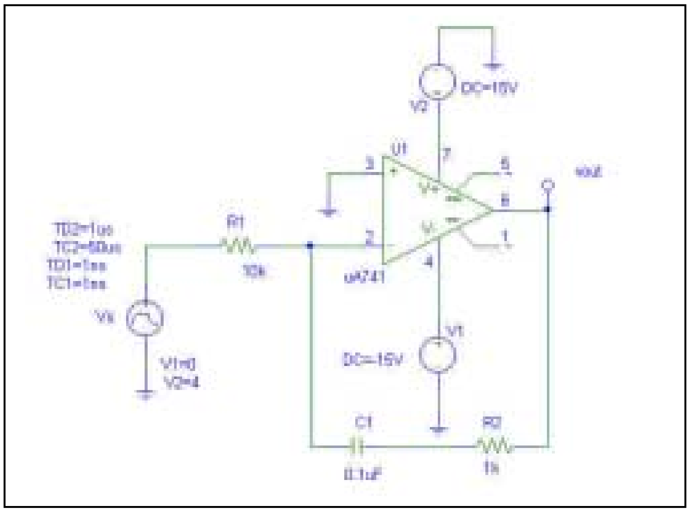
$$\therefore v_c(t) = 0.2(1 - e^{-20{,}000t}) u(t)$$

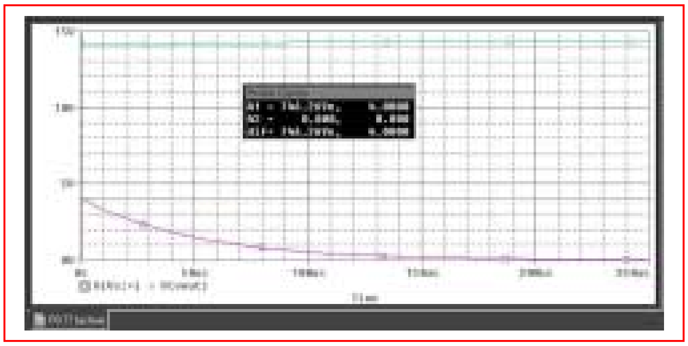
$$\therefore v_R(t) = 10^3 \, i(t) = 0.4 e^{-20{,}000t} u(t) \text{V}$$

$$\therefore v_o(t) = -v_c(t) - v_R(t) = (-0.2 + 0.2e^{-20{,}000t} - 0.4e^{-20{,}000t}) u(t)$$

And we may write $\boxed{v_o(t) \;=\; -0.2[1 + e^{-20\times10^3 t}]u(t) \text{ V.}}$

(b) PSpice verification:

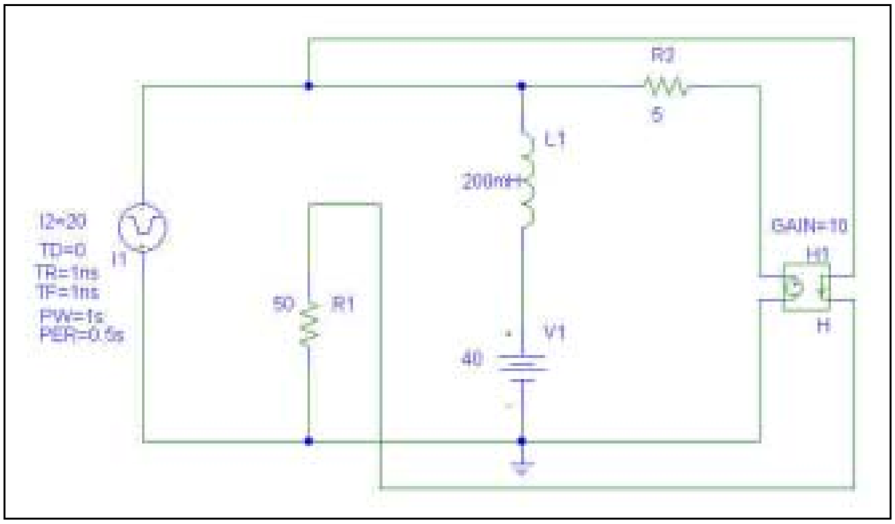


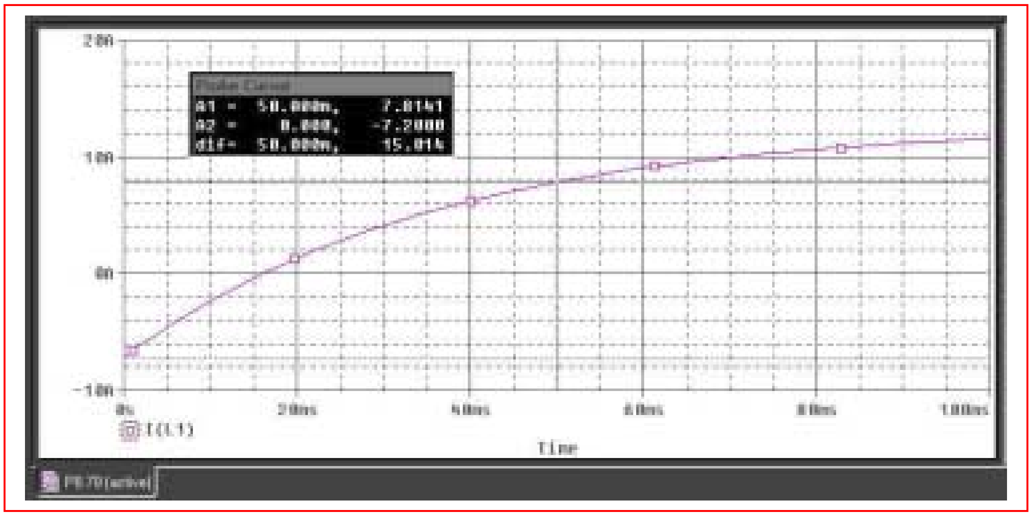


We can see from the simulation result that our ideal op amp approximation is not providing a great deal of accuracy in modeling the transient response of an op amp in this particular circuit; the output was predicted to be negative for $t > 0$.

78. For $t < 0$, the current source is an open circuit and so $i_1 = 40/\ 50 = 0.8$ A.
The current through the 5-Ω resistor is $[40 - 10(0.8)]/\ 5 = 7.2$ A, so the inductor current is equal to $\boxed{-7.2 \text{ A}}$

PSpice Simulation





From the PSpice simulation, we see that our $t < 0$ calculation is indeed correct, and find that the inductor current at $t = 50$ ms is $\boxed{7.82 \text{ A.}}$

79. Assume at least 1 μA required otherwise alarm triggers.

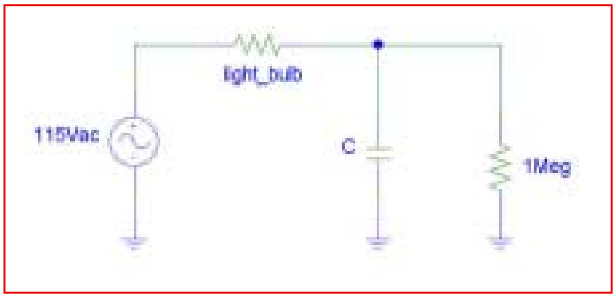
Add capacitor C.

$v_c(1) = 1$ volt

$v_c(0) = \dfrac{1000}{1002.37}.1.5 = 1.496$ volts

$\therefore$ We have $1 = 1.496 e^{-\frac{1}{10^6 C}}$ or $C = \dfrac{1}{10^6 \ln(1.496)} = \boxed{2.48\mu\text{F}}$

80. One possible solution of many: implement a capacitor to retain charge; assuming the light is left on long enough to fully charge the capacitor, the stored charge will run the lightbulb after the wall switch is turned off. Taking a 40-W light bulb connected to 115 V, we estimate the resistance of the light bulb (which changes with its temperature) as 330.6 Ω. We define "on" for the light bulb somewhat arbitrarily as 50% intensity, taking intensity as proportional to the dissipated power. Thus, we need at least 20 W (246 mA or 81.33 V) to the light bulb for 5 seconds after the light switch is turned off.



The circuit above contains a 1-MΩ resistor in parallel with the capacitor to allow current to flow through the light bulb when the light switch is on. In order to determine the required capacitor size, we first recognise that it will see a Thevenin equivalent resistance of 1 MΩ || 330.6 Ω = 330.5 Ω. We want $v_C(t = 5\text{s}) = 81.33 = 115e^{-5/\tau}$, so we need a circuit time constant of t = 14.43 s and a capacitor value of $\tau / R_{th} = \boxed{43.67 \text{ mF.}}$

CHAPTER NINE SOLUTIONS

1. $\omega_o \mathrm{L} = 10\Omega, s_1 = -6s^{-1},\ s_2 = -8s^{-1}$

$$\therefore -6 = \alpha + \sqrt{\alpha^2 - \omega_o^2}, -8 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} \text{ adding,}$$

$-14 = -2\alpha \therefore \alpha = 7s^{-1}$

$$\therefore -6 = -7 + \sqrt{49 - \omega_o^2} \therefore \omega_o^2 = 48\frac{1}{\mathrm{LC}},\ \omega_o = 6.928$$

rad/s $\therefore 6.928\mathrm{L} = 10, \boxed{\mathrm{L} = 1.4434\mathrm{H},}$

$$\mathrm{C} = \frac{1}{48\mathrm{L}} = \boxed{14.434\mathrm{mF}}, \frac{1}{2\mathrm{RC}} = 7 \therefore \mathrm{R} = \boxed{4.949\Omega}$$

2. $i_c = 40e^{-100t} - 30e^{-200t}$ mA, C = 1mF, $v(0) = -0.25$V

(a) $v(t) = \frac{1}{C}\int_o^t i_c dt - 0.25 = \int_o^t (40e^{-100t} - 30e^{-200t})\, dt - 0.25$

$\therefore v(t) = -0.4(e^{-100t} - 1) + 0.15(e^{-200t} - 1) - 0.25$

$\therefore v(t) = \boxed{-0.4e^{-100t} + 0.15e^{-200t}\text{V}}$

(b) $s_1 = -100 = -\alpha + \sqrt{\alpha^2 - \omega_o^2},\ s_2 = -200 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$

$\therefore -300 = -2\alpha,\ \alpha = 150s - 1$

$\therefore 150 + \frac{1}{2\text{R}10^{-3}}, \text{R} = \frac{500}{150} = 3.333\Omega$ Also,

$-200 = -150 - \sqrt{22500 - \omega_o^2}\ \therefore \omega_o^2 = 20000$

$\therefore 20000 = \frac{1}{\text{LC}} = \frac{100}{\text{L}},\ \text{L} = 0.5\text{H}$

$\therefore \text{i}_\text{R}(t) = \frac{v}{\text{R}} = \boxed{0.12e^{-100t} + 0.045e^{-200t}\text{A}}$

(c) $(i)t = -i_R(t) - i_c(t) = (0.12 - 0.04)e^{-100t} + (-0.045 + 0.03)e^{-200t}$

$\therefore i(t) = \boxed{80e^{-100t} - 15e^{-200t}\text{mA},\ t > 0}$

3. Parallel RLC with $\omega_o = 70.71 \times 10^{12}$ rad/s. L = 2 pH.

(a) $$\omega_o^2 = \frac{1}{LC} = (70.71\times10^{12})^2$$

$$\text{So } C = \frac{1}{(70.71\times10^{12})^2(2\times10^{-12})} = \boxed{100.0\,\text{aF}}$$

(b) $$\alpha = \frac{1}{2RC} = 5\times10^9 \;\; s^{-1}$$

$$\text{So } R = \frac{1}{(10^{10})(100\times10^{-18})} = \boxed{1\,\text{M}\Omega}$$

(c) α is the neper frequency: $\boxed{5 \text{ Gs}^{-1}}$

(d) $$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = \boxed{-5\times10^9 + j70.71\times10^{12} \;\; s^{-1}}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = \boxed{-5\times10^9 - j70.71\times10^{12} \;\; s^{-1}}$$

(e) $$\zeta = \frac{\alpha}{\omega_o} = \frac{5\times10^9}{70.71\times10^{12}} = \boxed{7.071\times10^{-5}}$$

4. Given: $L = 4R^2C$, $\alpha = \dfrac{1}{2RC}$

Show that $v(t) = e^{-\alpha t}(A_1 t + A_2)$ is a solution to

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0 \qquad [1]$$

$$\frac{dv}{dt} = e^{-\alpha t}(A_1) - \alpha e^{-\alpha t}(A_1 t + A_2)$$
$$= (A_1 - \alpha A_1 t - \alpha A_2)e^{-\alpha t} \qquad [2]$$
$$\frac{d^2v}{dt^2} = (A_1 - \alpha A_1 t - \alpha A_2)(-\alpha e^{-\alpha t}) - \alpha A_1 e^{-\alpha t}$$
$$= -\alpha(A_1 - \alpha A_2 + A_1 - \alpha A_1 t)e^{-\alpha t}$$
$$= -\alpha(2A_1 - \alpha A_2 - \alpha A_1 t)e^{-\alpha t} \qquad [3]$$

Substituting Eqs. [2] and [3] into Eq. [1], and using the information initially provided,

$$-\frac{1}{2RC}(2A_1)e^{-\alpha t} + \left(\frac{1}{2RC}\right)^2(A_1 t + A_2)e^{-\alpha t} + \frac{1}{RC}(A_1)e^{-\alpha t}$$
$$-\frac{1}{2RC}(A_1 t + A_2)e^{-\alpha t} + \frac{1}{4R^2C^2}(A_1 t + A_2)e^{-\alpha t}$$
$$= 0$$

Thus, $v(t) = e^{-\alpha t}(A_1 t + A_2)$ is in fact a solution to the differential equation.

Next, with $v(0) = \boxed{A_2 = 16}$

and $\left.\dfrac{dv}{dt}\right|_{t=0} = (A_1 - \alpha A_2) = (A_1 - 16\alpha) = 4$

we find that $\boxed{A_1 = 4 + 16\alpha}$

5. Parallel RLC with ω_o = 800 rad/s, and α = 1000 s^{-1} when R = 100 Ω.

$$\alpha = \frac{1}{2RC} \quad \text{so} \quad C = 5\mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = 312.5 \text{ mH}$$

Replace the resistor with 5 meters of 18 AWG copper wire. From Table 2.3, 18 AWG soft solid copper wire has a resistance of 6.39 Ω/1000ft. Thus, the wire has a resistance of

$$(5\,\text{m})\left(\frac{100\,\text{cm}}{1\,\text{m}}\right)\left(\frac{1\,\text{in}}{2.54\,\text{cm}}\right)\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)\left(\frac{6.39\,\Omega}{1000\,\text{ft}}\right)$$
$$= 0.1048\,\Omega \;\text{ or }\; 104.8\,\text{m}\Omega$$

(a) The resonant frequency is unchanged, so $\boxed{\omega_o = 800\,\text{rad/s}}$

(b) $\alpha = \dfrac{1}{2RC} = 954.0\times 10^3\, s^{-1}$

(c) $$\zeta_{old} = \frac{\alpha_{old}}{\omega_o}$$

$$\zeta_{new} = \frac{\alpha_{new}}{\omega_o}$$

Define the percent change as $\dfrac{\zeta_{new} - \zeta_{old}}{\zeta_{old}} \times 100$

$$= \frac{\alpha_{new} - \alpha_{old}}{\alpha_{old}} \times 100$$
$$= \boxed{95300\%}$$

6. $L = 5\text{H},\ \text{R} = 8\Omega,\ \text{C} = 12.5\text{mF},\ v(0^+) = 40\text{V}$

(a) $i(0^+) = 8\text{A}: \alpha = \dfrac{1}{2\text{RC}} = \dfrac{1000}{2\times 8\times 12.5} = 5,\ \omega_o^2 = \dfrac{1}{\text{LC}} = 16,$

$\omega_o = 4\ s_{1,2} = -5 \pm \sqrt{25-16} = -2, -8 \therefore v(t) = \text{A}_1 e^{-2t} + \text{A}_2 e^{-8t}$

$\therefore 40 = \text{A}_1 + \text{A}_2\ v'(0^+) = \dfrac{1000}{12.5}\left(-i_L(0^+) - \dfrac{40}{8}\right) = 80(-8-5) = -1040$

$v/s = -2\text{A}_1 - 8\text{A}_2 \therefore -520 = -\text{A}_1 - 4\text{A}_2 \therefore -3\text{A}_2 = -480,\ \text{A}_2 = 160, \text{A}_1 = -120$

$\therefore v(t) = \boxed{-120e^{-2t} + 160e^{-8t}\text{V},\ t > 0}$

(b) $i_c(0^+) = 8\text{A}$ Let $i(t) = \text{A}_3 e^{-2t} + \text{A}_4 e^{-8t};\ i_R(0^+) = \dfrac{v(0^+)}{\text{R}} = \dfrac{40}{8} = 5\text{A}$

$\therefore i(0^+) = \text{A}_3 + \text{A}_4 = -i_R(0^+) - i_c(0^+) = -8 - 5 = -13\text{A};$

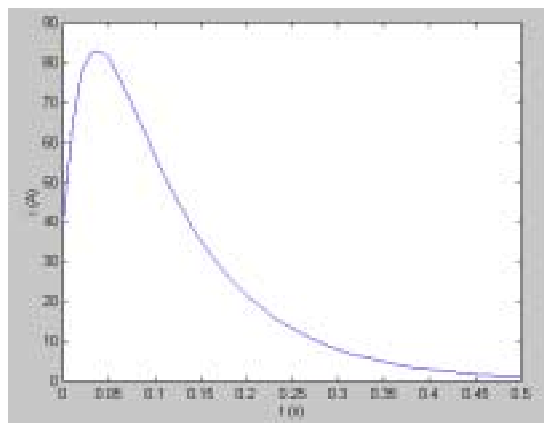
$i(0^+) = -2\text{A}_3 - 8\text{A}_4 = \dfrac{40}{5} = 8\ \text{A}/s \therefore 4 = -\text{A}_3 - 4\text{A}_4$

$\therefore -3\text{A}_4 = -13 + 4,\ \text{A}_4 = 3,\ \text{A}_3 = -16 \therefore i(t) = \boxed{-16e^{-2t} + 3e^{-8t}\text{A},\ t > 0}$

7. $i(0) = 40\text{A},\ v(0) = 40\text{V},\ \text{L} = \frac{1}{80}\text{H},\ \text{R} = 0.1\Omega,\ \text{C} = 0.2\text{F}$

(a)
$$\alpha = \frac{1}{2\times 0.1\times 0.2} = 25,\ \omega_o^2 = \frac{80}{0.2} = 400,$$
$$\omega_o = 20,\ s_{1,2} = -25 \pm \sqrt{625-400} = 10,\ -40$$
$$\therefore v(t) = \text{A}_1 e^{-10t} + \text{A}_2 e^{-40t}\ \therefore 40 = \text{A}_1 + \text{A}_2;$$
$$v'(0^+) = -10\text{A}_1 - 40\text{A}_2\ v'(0^+) = \frac{1}{\text{C}}\left(i(0) - \frac{v(0)}{\text{R}}\right) = -2200$$
$$\therefore -\text{A}_1 - 4\text{A}_2 = -220 \therefore -3\text{A}_2 = -180 \therefore \text{A}_2 = 60,\ \text{A}_1 = -20$$
$$\therefore v(t) = \boxed{-20e^{-10t} + 60e^{-40t}\text{V},\ t > 0}$$

(b)
$$i(t) = -v/\text{R} - \text{C}\frac{dv}{dt} = 200e^{-10t} - 600e^{-40t} - 0.2(-20)(-10)e^{-10t} - (0.2)(60)(-40)e^{-40t}$$
$$= 160e^{-10t} - 120e^{-40t}\ \text{A}$$



8. $i_L(0) = \frac{100}{50} = 2\text{A},\ v_c(0) = 100\text{V}$

$\alpha = \frac{10^6}{2\times 50\times 2.5} = 4000,\ w_o^2 = \frac{3\times 10^{6+3}}{100\times 2.5} = 12\times 10^6$

$\sqrt{16-12}\times 10^3 = 200,\ s_{1,2} = -4000 \pm 2000$

$\therefore i_L(t) = \text{A}_1 e^{-2000t} + \text{A}_2 e^{-6000t},\ t>0\ \therefore \text{A}_1 + \text{A}_s = 2$

$i_L'(0^+) = \frac{-10^3\times 3}{100}\times 100 = -3000 = -2000\text{A}_1 - 6000\text{A}_2\ \therefore -1.5 = -\text{A}_1 - 3\text{A}_2\ \therefore 0.5 = -2\text{A}_2$

$\therefore \text{A}_2 = -0.25,\ \text{A}_1 = 2.25 \therefore i_L(t) = 2.25e^{-2000t} - 0.25e^{-6000t}\text{A},\ t>0$

$t>0:\ i_L(t) = 2\text{A} \therefore i_L(t) = \boxed{2u(-t) + (2.25e^{-2000t} - 0.25e^{-6000t})u(t)\text{A}}$

9.
$$i_L(0) = \frac{12}{5+1} = 2\text{A},\ v_c(0) = 2\text{V}$$

$$\alpha = \frac{1000}{2\times1\times2} = 250,\ \omega_o^2 = \frac{1000\times45}{2} = 22500$$

$$s_{1,2} = -250 \pm \sqrt{250^2 - 22500} = -50, -450\ s^{-1}$$

$$\therefore i_L = \text{A}_1 e^{-50t} + \text{A}_2 e^{-450t}\ \therefore \text{A}_1 + \text{A}_2 = 2;\ i_L'(0^+) = 45(-2) = -50\text{A}_1 - 450\text{A}_2$$

$$\therefore \text{A}_1 + 9\text{A}_2 = 1.8 \therefore -8\text{A}_2 = 0.2 \therefore \text{A}_2 = -0.025,\ \text{A}_1 = 2.025(\text{A})$$

$$\therefore i_L(t) = \boxed{2.025e^{-50t} - 0.025e^{-450t}\text{A},\ t>0}$$

10.

(a) $\alpha = \dfrac{1}{2RC} = \dfrac{1440}{72} = 20,\ \omega_o^2 = \dfrac{1440}{10} = 144$

$s_{1,2} = -20 \pm \sqrt{400 - 144} = -4, -36:\ v = A_1 e^{-4t} + A_2 e^{-36t}$

$v(0) = 18 = A_1 + A_2,\ v'(0) = 1440\left(\dfrac{1}{2} - \dfrac{18}{36}\right) = 0$

$\therefore 0 = -4A_1 - 36A_2 = -A_1 - 9A_2 = \therefore 18 = -8A_2,\ A_2 = -2.25,\ A_1 = 20.25$

$\therefore v(t) = \boxed{20.25e^{-4^{+}} - 2.25e^{-36t}\text{V},\ t > 0}$

(b) $i(t) = \dfrac{v}{36} + \dfrac{1}{1440}v' = 0.5625e^{-4t} - 0.0625e^{-36t} - 0.05625e^{-4t} + 0.05625e^{-36t}$

$\therefore i(t) = \boxed{0.50625e^{-4t} - 0.00625e^{-36t}\text{A},\ t > 0}$

(c) $v_{\max}$ at $t = 0 \therefore v_{\max} = 18\text{V} \therefore 0.18 = 20.25e^{-4t_s} - 2.25e^{-36t_s}$

Solving using a scientific calculator, we find that $\boxed{t_s = 1.181 \text{ s.}}$

11. $L = 1250$ mH

so $\omega_o = {}^{1}\!/\!\sqrt{LC} = 4\,\text{rad/s}$ Since $\alpha > \omega_o$, this circuit is over damped.

$$\alpha = \frac{1}{2RC} = 5\ s^{-1}$$

The capacitor stores 390 J at $t = 0^-$:

$$W_c = \frac{1}{2} C v_c^2$$

$$\text{So } v_c(0^1) = \sqrt{\frac{2W_c}{C}} = 125\ \text{V} = v_c(0^+)$$

The inductor initially stores zero energy,

so $i_L(0^-) = i_L(0^+) = 0$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm 3 = -8,\ -2$$

Thus, $v(t) = Ae^{-8t} + Be^{-2t}$

Using the initial conditions, $v(0) = 125 = A + B$ [1]

$$i_L(0^+) + i_R(0^+) + i_c(0^+) = 0 + \frac{v(0^+)}{2} + i_c(0^+) = 0$$

$$\text{So } i_c(0^+) = -\frac{v(0^+)}{2} = -\frac{125}{2} = -62.5\ \text{V}$$

$$i_c = C\frac{dv}{dt} = 50\times10^{-3}[-8Ae^{-8t} - 2Be^{-2t}]$$

$$i_c(0^+) = -62.5 = -50\times10^{-3}(8A + 2B) \qquad [2]$$

Solving Eqs. [1] and [2], $A = 150$ V
$B = -25$ V

Thus, $\boxed{v(t) = 166.7e^{-8t} - 41.67e^{-2t},\ t > 0}$

12. (a) We want a response $v = Ae^{-4t} + Be^{-6t}$

$$\alpha = \frac{1}{2RC} = 5\,s^{-1}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -4 = -5 + \sqrt{25 - \omega_o^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -6 = -5 - \sqrt{25 - \omega_o^2}$$

Solving either equation, we obtain ω_o = 4.899 rad/s

Since $\omega_o^2 = \frac{1}{LC}$, $L = \frac{1}{\omega_o^2 C} = \boxed{833.3 \text{ mH}}$

(b) If $i_R(0^+) = 10$ A and $i_c(0^+) = 15$ A, find A and B.

with $i_R(0^+) = 10$ A, $v_R(0^+) = v(0^+) = v_c(0^+) = 20$ V

$$v(0) = A + B = 20 \qquad [1]$$

$$i_c = C\frac{dv}{dt} = 50\times10^{-3}(-4Ae^{-4t} - 6Be^{-6t})$$

$$i_c(0^+) = 50\times10^{-3}(-4A - 6B) = 15 \qquad [2]$$

Solving, $\boxed{A = 210 \text{ V}, \ B = -190 \text{ V}}$

Thus, $v = 210e^{-4t} - 190e^{-6t}$, $t > 0$

13. Initial conditions: $i_L(0^-) = i_L(0^+) = 0$ $\qquad i_R(0^+) = \dfrac{50}{25} = 2 \text{ A}$

(a) $v_c(0^+) = v_c(0^-) = 2(25) = \boxed{50 \text{ V}}$

(b) $i_c(0^+) = -i_L(0^+) - i_R(0^+) = 0 - 2 = \boxed{-2 \text{ A}}$

(c) $t > 0$: parallel (source-free) RLC circuit

$$\alpha = \frac{1}{2RC} = 4000\, s^{-1} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 3464 \text{ rad/s} \qquad = -2000,\ -6000$$

Since $\alpha > \omega_0$, this system is overdamped. Thus,

$$v_c(t) = Ae^{-2000t} + Be^{-6000t}$$

$$i_c = C\frac{dv}{dt} = (5\times 10^{-6})(-2000\, Ae^{-2000t} - 6000\, Be^{-6000t})$$

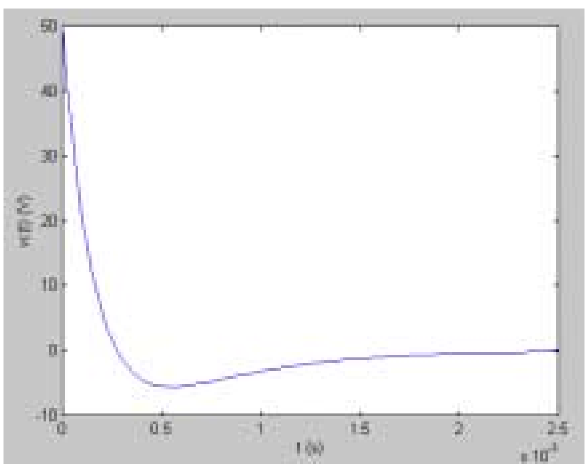
$$i_c(0^+) = -0.01A - 0.03B = -2 \qquad [1]$$

and $v_c(0^+) = A + B = 50$ $\qquad$ [2]

Solving, we find $A = -25$ and $B = 75$

so that $\boxed{v_c(t) = -25e^{-2000t} + 75e^{-6000t},\ t > 0}$

(d)



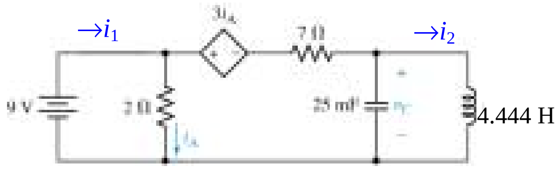
(e) $-25e^{-2000t} + 75e^{-6000t} = 0 \Rightarrow \boxed{t = 274.7\,\mu s}$

using a scientific calculator

(f) $|v_c|_{\max} = -25 + 75 = 50 \text{ V}$

So, solving $|-25e^{-2000t_s} + 75e^{-6000t_s}| = 0.5$ in view of the graph in part (d), we find $\boxed{t_s = 1.955 \text{ ms}}$ using a scientific calculator's equation solver routine.

14. Due to the presence of the inductor, $v_c(0^-) = 0$. Performing mesh analysis,

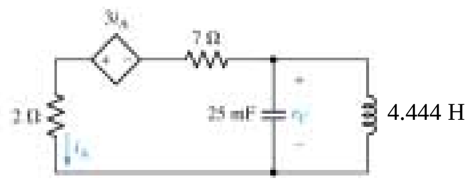


$$-9 + 2i_1 - 2i_2 = 0 \qquad [1]$$
$$2i_2 - 2i_1 + 3i_A + 7i_2 = 0 \qquad [2]$$
and $i_1 - i_2 = i_A$

Rearranging, we obtain $2i_1 - 2i_2 = 0$ and $-4i_1 + 6\,i_2 = 0$. Solving, $i_1 = 13.5$ A and $i_2 = 9$ A.

(a) $i_A(0^-) = i_1 - i_2 = \boxed{4.5 \text{ A}}$ and $i_L(0^-) = i_2 = 9$ A

(b) $t > 0$:



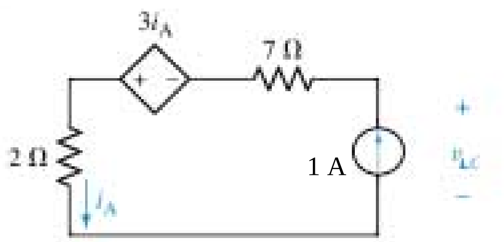
around left mesh:

$$-v_c(0^+) + 7i_A(0^+) - 3i_A(0^+) + 2i_A(0^+) = 0$$

so, $\boxed{i_A(0^+) = 0}$

(c) $\boxed{v_c(0^-) = 0}$ due to the presence of the inductor.

(d)



$$-v_{LC} + 7 - 3(1) + 2 = 0$$

$$v_{LC} = 6 \text{ V} \;\therefore\; \boxed{R_{TH} = \frac{6}{1} = 6\,\Omega}$$

(e)
$$\alpha = \frac{1}{2RC} = 3.333\ s^{-1}$$
$$\omega_o = \frac{1}{\sqrt{LC}} = 3 \text{ rad/s}$$
$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.881,\ -4.785$$

Thus,

$$i_A(t) = Ae^{-1.881t} + Be^{-4.785t}$$
$$i_A(0^+) = 0 = A + B \qquad [1]$$

To find the second equation required to determine the coefficients, we write:

$$i_L = -i_c - i_R$$
$$= -C\frac{dv_c}{dt} - i_A \;=\; -25\times10^{-3}\left[-1.881(6A)e^{-1.881t} - 4.785(6B)e^{-4.785t}\right]$$
$$-\ Ae^{-1.881t} - Be^{-4.785t}$$

$$i_L(0^+) = 9 = -25\times10^{-3}[-1.881(6A) - 4.785(6B)] - A - B$$
or $\quad 9 = -0.7178A - 0.2822B \qquad [2]$

Solving Eqs. [1] and [2], $A = -20.66$ and $B = +20.66$
So that $\boxed{i_A(t) = 20.66[e^{-4.785t} - e^{-1.881t}]}$

15. Diameter of a dime: approximately 8 mm. Area = $\pi r^2 = 0.5027\text{cm}^2$

Capacitance $= \dfrac{\varepsilon_r \varepsilon_o A}{d} = \dfrac{(88)(8.854\times10^{-14}\text{ F/cm})(0.5027\text{cm}^2)}{0.1\text{cm}}$

$= 39.17\text{pF}$

$L = 4\mu\text{H}$

$$\omega_o = \frac{1}{\sqrt{LC}} = 79.89\text{ Mrad/s}$$

For an over damped response, we require $\alpha > \omega_o$.

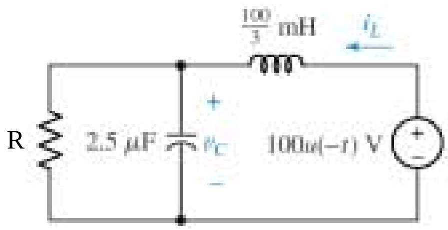
Thus, $\dfrac{1}{2RC} > 79.89\times10^6$

$R < \dfrac{1}{2(39.17\times10^{-12})(79.89\times10^6)}$

or $\boxed{R < 159.8\,\Omega}$

*Note: The final answer depends quite strongly on the choice of ε_r.

16.



crit. damp. (a) $L = 4R^2C = \frac{100}{3}\times 10^{-3} = 4R^2 \times 10^{-6} \therefore \boxed{R = 57.74\Omega}$

(b)

$$\omega_o = \alpha = 10^3 / \sqrt{\frac{1}{30}\times 2.5} = 3464 s^{-1}$$

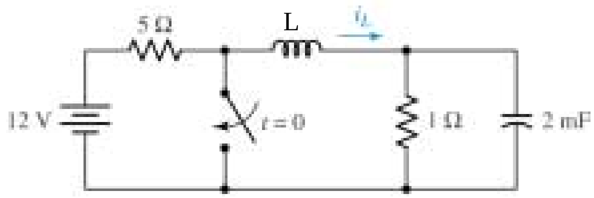
$$\therefore v_c(t) = e^{-3464t}\,(A_1 t + A_2)\;\; v_c(0) = 100\text{V}$$

$$i_L(0) = \frac{100}{57.74} = 1.7321\text{A} \therefore 100 = A_2$$

$$v_c'(0^+) = \frac{10^6}{2.5}\left(1.7321 - \frac{100}{57.74}\right) = 0 = A_1 - 3464A_2 \;\therefore A_1 = 3.464\times 10^5$$

$$\therefore v_c(t) = \boxed{e^{-3464t}(3.464\times 10^5 t + 100)\text{ V},\; t>0}$$

17.



crit. damp. (a) $\mathrm{L} = 4\mathrm{R}^2\mathrm{C} = 4\times1\times2\times10^{-3} = \boxed{8\mathrm{mH}}$

(b)

$$\alpha = \omega_o \frac{1}{2\mathrm{RC}} = \frac{1000}{2\times1\times2} = 250 \therefore i_L = e^{-250t}(\mathrm{A}_1 t + \mathrm{A}_2)$$

$$i_L(0) = 2\mathrm{A},\ v_c(0) = 2\mathrm{V} \therefore i_L = e^{-250t}(\mathrm{A}_1 t + 2)$$

$$\text{Then } 8\times10^{-3}\, i_L'(0^+) = -2 = 8\times10^{-3}(\mathrm{A}_1 - 500),\ = e^{-1.25}\ (1.25 + 2) = \boxed{0.9311\mathrm{A}}$$

(c)

$$i_{L\max}: (250t_m + 2) = 0,\ 1 = 250t_m + 2,\ t_m < 0\ \text{No!}$$

$$\therefore t_m = 0,\ i_{L\max} = 2\mathrm{A} \therefore 0.02 = e^{-250t_s}(250t_s + 2);\ \text{SOLVE: } t_s = \boxed{23.96\mathrm{ms}}$$

18. $\text{L} = 5\text{mH},\ \text{C} = 10^{-8}\ \text{F, crit. damp. } v(0) = -400\text{V},\ i(0) = 0.1\text{A}$

(a) $\text{L} = 4\text{R}^2\text{C} = 5\times10^{-3} = 4\text{R}^2 10^{-8}\ \therefore\ \text{R} = \boxed{353.6\Omega}$

(b) $\alpha = \dfrac{10^8}{2\times353.6} = 141,420\ \therefore\ i = e^{-141,420t}(\text{A}_1 t + \text{A}_2)$

$\therefore\ \text{A}_2 = 0.1 \therefore = e^{-141,421t}(\text{A}_1 t + 0.1),\ 5\times10^{-3}$

$(\text{A}_1 - 141,420\times0.1) = -400\ \therefore\ \text{A}_1 = -65,860$

$\therefore\ i = e^{-141,421t}(-65,860t + 0.1).\ i' = 0$

$\therefore\ e^{-\alpha t}(+65860) + 141,420e^{-\alpha t}(-65,860t_m + 0.1) = 0$

$\therefore\ t_m = 8.590\,\mu s\ \therefore\ i(t_m) = e^{-141,420\times8.590\times10^{-6}}$

$(-65,860\times8.590\times10^{-6} + 0.1) = -0.13821\text{A}$

$\therefore \left|\,\text{i}\,\right|_{\max} = \left|\,i(t_m)\,\right| = \boxed{0.13821\text{A}}$

(c) $\therefore i_{\max} = i(0) = \boxed{0.1\text{A}}$

19. Diameter of a dime is approximately 8 mm. The area, therefore, is πr^2 = 0.5027 cm^2.

The capacitance is $\dfrac{\varepsilon_r \varepsilon_o A}{d} = \dfrac{(88)(8.854\times10^{-14})(0.5027)}{0.1}$

$$= 39.17 \text{ pF}$$

with $L = 4\mu\text{H},\ \omega_o = \dfrac{1}{\sqrt{LC}} = 79.89 \text{ Mrad/s}$

For critical damping, we require $\dfrac{1}{2RC} = \omega_o$

or $R = \dfrac{1}{2\omega_o C} = \boxed{159.8\,\Omega}$

20. Critically damped parallel RLC with $\alpha = 10^{-3}\ s^{-1},\ R = 1\,\text{M}\Omega$.

We know $\dfrac{1}{2RC} = 10^{-3}$, so $C = \dfrac{10^3}{2\times10^6} = 500\,\mu\text{F}$

Since $\alpha = \omega_o$, $\omega_o = \dfrac{1}{\sqrt{LC}} = 10^{-3}$

or $\dfrac{1}{LC} = 10^{-6}$

so $L = 2$ GH (!)

$$L = \frac{\mu N^2 A}{S} = 2\times10^9$$

If So $$\frac{(4\pi\times10^{-7}\,\text{H/m})\left[\left(\dfrac{50\text{ turns}}{\text{cm}}\right).s\right]^2 (0.5\,\text{cm})^2.\pi.\left(\dfrac{1\text{ m}}{100\text{ cm}}\right)}{s}$$

$$= 2\times10^9$$

$$(4\pi^2\times10^{-9})(50)^2(0.5)^2\,s = 2\times10^9$$

So $\boxed{s = 8.106\times10^{13}\,\text{cm}}$

21.
$$\alpha = \frac{1}{2RC} = \frac{10^6}{100 \times 2.5} = 4000,\ \omega_o^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 2 \times 10^7$$

$$\omega_d = \sqrt{20 \times 10^6 - 16 \times 10^6} = 2000$$

$$\therefore i_c = e^{-4000t}(B_1 \cos 2000t + B_2 \sin 2000t)$$

$$i_L(0) = 2A,\ v_c(0) = 0 \therefore i_c(0^+) = -2A;\ i_c'(0^+) = -i_L'(0^+) - i_R'(0^+)$$

$$\therefore i_c'(0^+) = -\frac{1}{L} v_c(0) - \frac{1}{R} v_c'(0^+) = 0 - \frac{1}{RC} i_c(0^+) = \frac{2 \times 10^6}{125}$$

$$\therefore B_1 = -2A,\ \frac{2 \times 10^6}{125} = 16,000 = 2000B_2 + (-2)(-4000) \therefore B_2 = 4$$

$$\therefore \boxed{i_c(t) = e^{-4000t}(-2\cos 2000t + 4\sin 2000t)\text{A},\ t > 0}$$

22.

$$\alpha = \frac{1}{2\text{RC}} = \frac{4}{2\times 2} = 1,\ \omega_o^2 = \frac{1}{\text{LC}} = \frac{4\times 13}{2} = 26,\ \omega_d = \sqrt{26-1} = 5$$

$$\therefore v_c(t) = e^{-t}(B_1 \cos 5t + B_2 \sin 5t)$$

(a) $i_L(0^+) = i_L(0) = \boxed{4\text{A}}$

(b) $v_c(0^+) = v_c(0) = \boxed{0}$

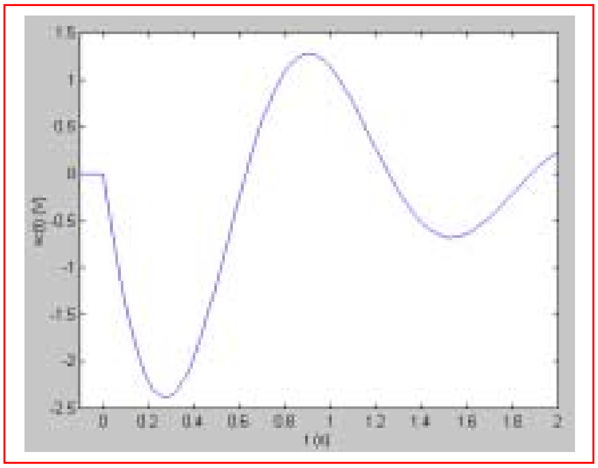
(c) $i_L'(0^+) = \frac{1}{\text{L}} v_c(0^+) = \boxed{0}$

(d) $v_c'(0^+) = \frac{1}{c}[-i_L(0^+) - i_R(0^+)] = 4\left[-4 - \frac{v_c(0^+)}{2}\right] = 4(-4+0) = \boxed{-16\ \text{V/s}}$

(e) $\therefore (e)\, 0 = 1(\text{B}_1) \therefore \text{B}_1 = 0,\ v_c(t) = \text{B}_2 e^{-t}\sin 5t,\ v_c'(0^+) = \text{B}_2(5) = -16$

$\therefore \text{B}_2 = -3.2,\ v_c(t) = \boxed{-3.2e^{-t}\sin 5t\ \text{V},\ t > 0}$

(f)



23.

(a)

$$\alpha\frac{1}{2\text{RC}}=\frac{10^{9-3}}{2\times 20\times 5}=5000,\ \omega_o^2=\frac{1}{\text{LC}}=\frac{10^9}{1.6\times 5}=1.25\times 10^8$$

$$\omega_d=\sqrt{\omega_o^2-\alpha^2}=\sqrt{125\times 10^6-25\times 10^6}=10,000$$

$$\therefore v_c(t)=e^{-5000t}(\text{B}_1\cos 10^4 t+\text{B}_2\sin 10^4 t)$$

$$v_c(0)=200\text{V},\ i_L(0)=10\text{mA}\ \therefore v_c(t)=e^{-5000t}(200\cos 10^4 t+\text{B}_2\sin 10^4 t)$$

$$v_c'(0^+)=\frac{1}{c}i_c(0^+)=\frac{10^9}{5}\left[i_L(0)-\frac{v_c(0)}{20,000}\right]$$

$$=\frac{10^9}{5}\left(10^{-2}-\frac{200}{20,000}\right)=0=10^4\,\text{B}_2-200(5000)$$

$$\therefore \text{B}_2=100\text{V}\ \therefore v_c(t)=\boxed{e^{-5000t}(200\cos 10^4 t+100\sin 10^4 t)\ \text{V},\ t>0}$$

(b)

$$i_{sw}=10^{-2}-i_L,\ i_L=\frac{1}{\text{R}}v_c+\text{C}v_c'$$

$$v_c'=e^{-5000t}[10^4(-200\sin+100\cos]-5000(200\cos+100\sin)]$$

$$=e^{-500t}[10^6(-2\sin-0.5\cos)]=-2.5\times 10^6\,e^{-5000t}\sin 10^4 t\ \ v/s$$

$$\therefore i_L=e^{-5000t}\left[\frac{1}{20,000}(200\cos+100\sin)-5\times 10^{-9}\times 2.5\times 10^6 e^{-5000t}\sin 10^4 t\right]$$

$$=e^{-5000t}(0.01\cos 10^4 t-0.0075\sin 10^4 t)\,\text{A}$$

$$\therefore i_{sw}=\boxed{10-e^{-5000t}(10\cos 10^4 t-7.5\sin 10^4 t)\,\text{mA},\ t>0}$$

24.

(a) $\alpha = \frac{1}{2RC} = \frac{100}{12.5} = 8,\ \omega_o^2 = \frac{1}{LC} = \frac{100}{L},\ \omega_d^2 = 36 = \omega_o^2 - 64$

$\therefore \omega_o^2 = 100 = \frac{100}{L} \therefore \boxed{L = 1H}$

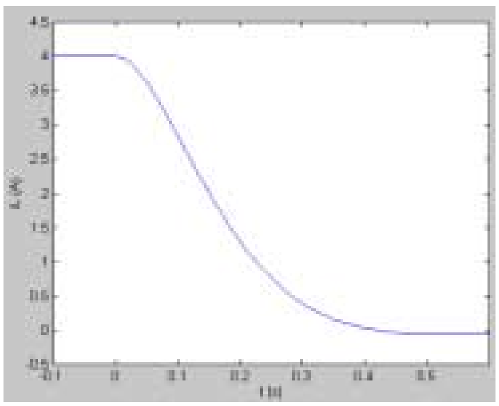
(b) $t<0:\ i_L(t) = 4A;\ t>0:\ i_L(t) = e^{-8t}(B_1 \cos 6t + B_2 \sin 6t)$

$i_L(0) = 4A \therefore B_1 = 4A,\ i_L = e^{-8t}(4\cos 6t + B_2 \sin 6t)\ v_c(0) = 0$

$i_L'(0^+) = t\ v_c(0^+) = 0 \therefore 6B_2 - 8(4) = 0,\ B_2 = 16/3$

$\therefore i_L(t) = \boxed{4u(-t) + e^{-8t}(4\cos 6t + 5.333\sin 6t)\ u(t)\ A}$

(c)



25.

$$\alpha = \frac{1}{2RC} = \frac{10^{6-3}}{2\times 5} = 100s^{-1},\ \omega_o^2 = \frac{1}{LC} = 1.01\times 10^6$$

$$\therefore \omega_d = \sqrt{101\times 10^4 - 10^4} = 100;\ i_L(0) = \frac{60}{10} = 6\text{mA}$$

$$v_c(0) = 0 \therefore v_c(t) = e^{-100t}(A_1 \cos 1000t + A_2 \sin 1000t),\ t > 0$$

$$\therefore A_1 = 0,\ v_c(t) = A_2 e^{-100t} \sin 1000t$$

$$v_c'(0^+) = \frac{1}{C} i_c(0^+) = 10^6[-i_1(0^+) - \frac{1}{5000} v_c(0^+)] = 10^6$$

$$(-6\times 10^{-3}) = -6000 = 1000\,A_2 \therefore A_2 = -6$$

$$\therefore v_c(t) = -6e^{-100t} \sin 1000t\text{V},\ t > 0 \therefore i_1(t) = -\frac{1}{10^4}$$

$$v_c(t) = -10^{-4}(-6)\, e^{-100t} \sin 1000t\text{A}$$

$$\therefore \boxed{i_1(t) = 0.6e^{-100t} \sin 1000t\ \text{mA},\ t > 0}$$

26.

(a)
$$\alpha = \frac{1}{2RC} = \frac{10^6}{2000 \times 25} = 20,\ \omega_o^2 = \frac{1}{LC} = \frac{1.01 \times 10^6}{25} = 40,400$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{40,400 - 400} = 200$$

$$\therefore v = e^{-20t}\,(A_1 \cos 200t + A_2 \sin 200t)$$

$$v(0) = 10\text{V},\ i_L(0) = 9\text{mA} \therefore A_1 = 10\text{V}$$

$$\therefore v = e^{-20t}(10 \cos 200t + A_2 \sin 200t)\ \text{V},\ t > 0$$

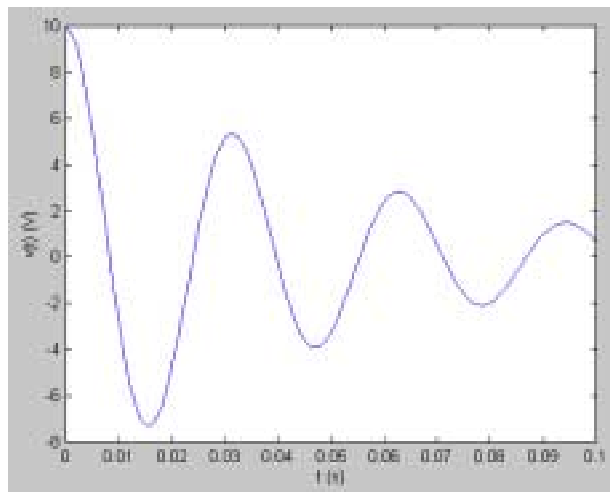
$$v'(0^+) = 200A_2 - 20 \times 10 = 200(A_2 - 1) = \frac{1}{C} i_o(0^+)$$

$$= \frac{10^6}{25}(-10^{-3}) = -40 \therefore A_2 = 1 - 0.2 = 0.8$$

$$\therefore \boxed{v(t) = e^{-20t}(10 \cos 200t + 0.8 \sin 200t)\ \text{V},\ t > 0}$$

(b)
$$v = 10.032e^{-20t} \cos\,(200t - 4.574°)\text{V}$$

$$T = \frac{2\pi}{200} = 3.42\text{ms}$$



27.

$v(0) = 0;\ i(0) = 10\text{A}$

$v = e^{\alpha t}\,(\text{A}\cos\omega_d t + \text{B}\sin\omega_d t) \therefore \text{A} = 0,$

$v = \text{B}e^{-\alpha t}\sin\omega_d t$

$v' = e^{-\alpha t}\ [-\alpha\,\text{B}\sin\omega_d t + \omega_d \text{B}\cos\omega_d t] = 0$

$\therefore \tan\omega_d t = \dfrac{\omega_d}{\alpha},\ t_{m1} = \dfrac{1}{\omega_d}\tan^{-1}\dfrac{\omega_d}{\alpha}$

$t_{m2} = t_{m1} + \dfrac{1}{2}\ \text{T}_d\ = t_{m1} + \dfrac{\pi}{\omega_d};$

$v_{m1} = \text{B}e^{-\alpha t_{m1}}\sin\omega_d t_{m1}\ v_{m2} = -\text{B}e^{-\alpha t_{m1} - \alpha\pi/\omega_d}$

$\sin\omega_d\ \ t_{m1} \therefore \dfrac{v_{m2}}{\text{V}_{m1}} = -e^{-\alpha\pi/\omega_d};\ \text{let}\ \left|\dfrac{v_{m2}}{v_{m1}}\right| = \dfrac{1}{100}$

$\therefore e^{\alpha\pi/\omega_d}\ = 100,\ \alpha = \dfrac{\omega_d}{\pi}\,\ln 100;\ \alpha = \dfrac{1}{2\text{RC}} = \dfrac{21}{\text{R}},$

$\omega_0^2 = \dfrac{1}{\text{LC}} = 6 \therefore \omega_d\ = \sqrt{6 - 441/\text{R}^2}\ \therefore \dfrac{21}{\text{R}}\ \dfrac{\ln 100}{\pi\text{R}}\ \sqrt{6\text{R}^2 - 441}$

$\therefore \text{R} = \sqrt{1/6\left[441 + \left(\dfrac{21\pi}{100}\right)^2\right]} = 10.3781\Omega$ To keep

$\left|\dfrac{v_{m2}}{v_{m1}}\right| < 0.01,$ chose $\boxed{\text{R} = 10.3780\Omega}\ v'(0^+) = \omega_d$

$\text{B} = \text{B}\sqrt{6 - \left(\dfrac{21}{10.378}\right)^2}\ = 4\text{R}\left(10 + \dfrac{0}{10.3780}\right) \therefore \text{B} = 1.380363$

$\alpha = \dfrac{21}{10.378} = 2.02351;\ \omega_d\ = \sqrt{6 - \left(\dfrac{21}{10.378}\right)^2}\ = 1.380363$

$\therefore v = 304.268e^{-2.02351t}\sin 1.380363t\ \ v\ \ t_{m1} = 0.434s,$

$v_{m1} = 71.2926v$ Computed values show

$\boxed{t_s = 2.145\,\text{sec}};\ v_{m2} = 0.7126 < 0.01v_{m1}$

28. We replace the 25-Ω resistor to obtain an underdamped response:

$$\alpha = \frac{1}{2RC} \qquad \text{and} \qquad \omega_0 = \frac{1}{\sqrt{LC}}\text{; we require } \alpha < \omega_0.$$

Thus, $\dfrac{1}{10\times10^{-6}\,R} < 3464$ or $\boxed{R > 34.64\ m\Omega.}$

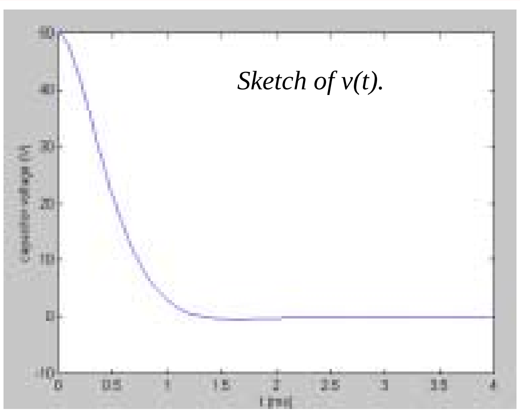
For R = 34.64 Ω (1000× the minimum required value), the response is:

$v(t) = e^{-\alpha t}(A \cos \omega_d t + B \sin \omega_d t)$ where $\alpha = 2887\ s^{-1}$ and $\omega_d = 1914$ rad/s.
$i_L(0^+) = i_L(0^-) = 0$ and $v_C(0^+) = v_C(0^-) = (2)(25) = 50\ V = A$.

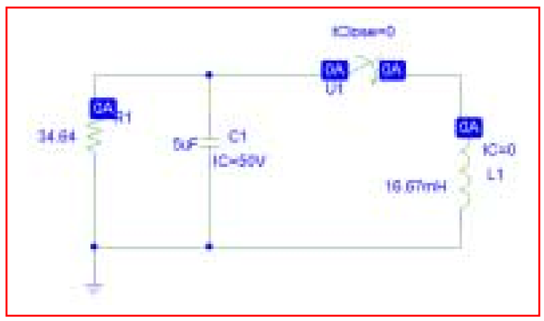
$$i_L(t) = L\frac{dv_L}{dt} = L\frac{dv_C}{dt}$$
$$= L\left[e^{-\alpha t}\left(-A\omega_d t \sin \omega_d t + B\omega_d t \cos \omega_d t\right) - \alpha e^{-\alpha t}\left(A\cos\omega_d t + B\sin\omega_d t\right)\right]$$

$$i_L(0^+) = 0 = \frac{50\times10^{-3}}{3}\left[B\,\omega_d - \alpha A\right]\text{, so that } B = 75.42\ V.$$

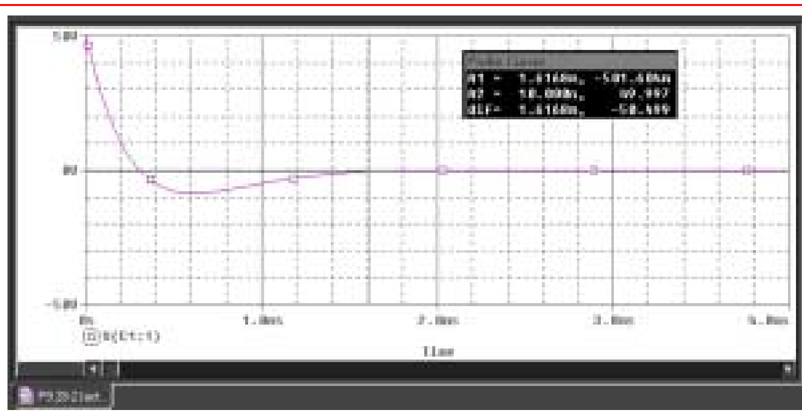
Thus, $v(t) = e^{-2887t}(50 \cos 1914t + 75.42 \sin 1914t)$ V.



PSpice schematic for t > 0 circuit.



From PSpice the settling time using R = 34.64 Ω is approximately 1.6 ms.



29. (a,b) For $t < 0$ s, we see from the circuit below that the capacitor and the resistor are shorted by the presence of the inductor. Hence, $i_L(0^-) = 4$ A and $v_C(0^-) = 0$ V.

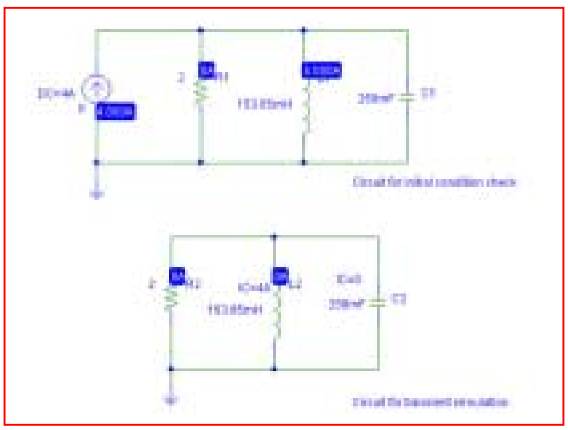
When the 4-A source turns off at $t = 0$ s, we are left with a parallel RLC circuit such that $\alpha = 1/2RC = 1\ \text{s}^{-1}$ and $\omega_0 = 5.099$ rad/s. Since $\alpha < \omega_0$, the response will be underdamped with $\omega_d = 5$ rad/s. Assume the form $i_L(t) = e^{-\alpha t}(C \cos \omega_d t + D \sin \omega_d t)$ for the response.

With $i_L(0^+) = i_L(0^-) = 4$ A, we find C = 4 A. To find D, we first note that

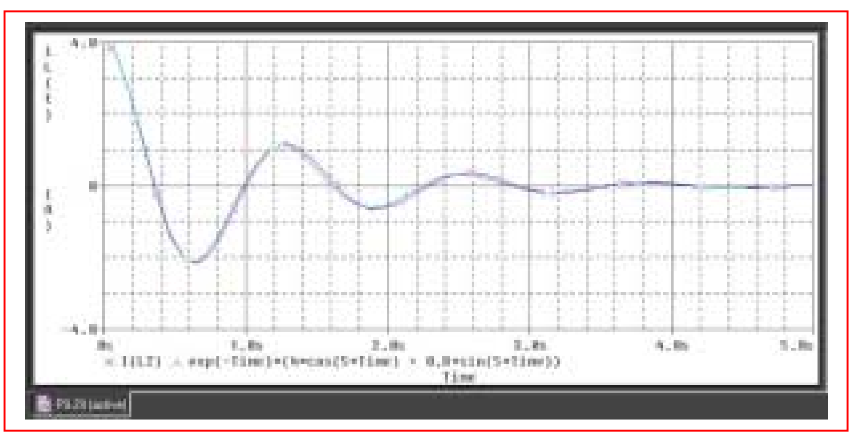
$$v_C(t) = v_L(t) = L\,\frac{di_L}{dt}$$

and so $v_C(t) = (2/13)\,[e^{-\alpha t}(-C\omega_d \sin \omega_d t + D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t}(C \cos \omega_d t + D \sin \omega_d t)]$

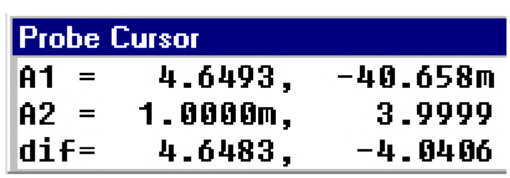
With $v_C(0^+) = 0 = (2/13)(5D - 4)$, we obtain $D = 0.8$ A.
Thus, $i_L(t) = e^{-t}(4 \cos 5t + 0.8 \sin 5t)$ A



We see that the simulation result confirms our hand analysis; there is only a slight difference due to numerical error between the simulation result and our exact expression.



(c) Using the cursor tool, the settling time is approximately 4.65 s.



30. (a) For $t < 0$ s, we see from the circuit that the capacitor and the resistor are shorted by the presence of the inductor. Hence, $i_L(0^-) = 4$ A and $v_C(0^-) = 0$ V.

When the 4-A source turns off at $t = 0$ s, we are left with a parallel RLC circuit such that $\alpha = 1/2RC = 0.4\ s^{-1}$ and $\omega_0 = 5.099$ rad/s. Since $\alpha < \omega_0$, the response will be underdamped with $\omega_d = 5.083$ rad/s. Assume the form $i_L(t) = e^{-\alpha t}$ (C cos $\omega_d t$ + D sin $\omega_d t$) for the response.

With $i_L(0^+) = i_L(0^-) = 4$ A, we find C = 4 A. To find D, we first note that

$$v_C(t) = v_L(t) = L\ \frac{di_L}{dt}$$

and so $v_C(t) = (2/13)\ [e^{-\alpha t}\ (-C\omega_d \sin \omega_d t\ +\ D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t}\ (C \cos \omega_d t\ +\ D \sin \omega_d t)]$

With $v_C(0^+) = 0 = (2/13)\ (5.083D - 0.4C)$, we obtain D = 0.3148 A.

Thus, $i_L(t) = e^{-0.4t}\ (4 \cos 5.083t\ +\ 0.3148 \sin 5.083t)$ A and $\boxed{i_L(2.5) = 1.473\text{ A.}}$

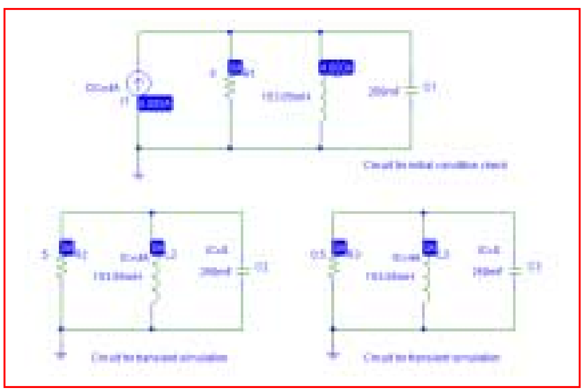
(b) $\alpha = 1/2RC = 4\ s^{-1}$ and $\omega_0 = 5.099$ rad/s. Since $\alpha < \omega_0$, the new response will still be underdamped, but with $\omega_d = 3.162$ rad/s. We still may write

$$v_C(t) = (2/13)\ [e^{-\alpha t}\ (-C\omega_d \sin \omega_d t\ +\ D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t}\ (C \cos \omega_d t\ +\ D \sin \omega_d t)]$$

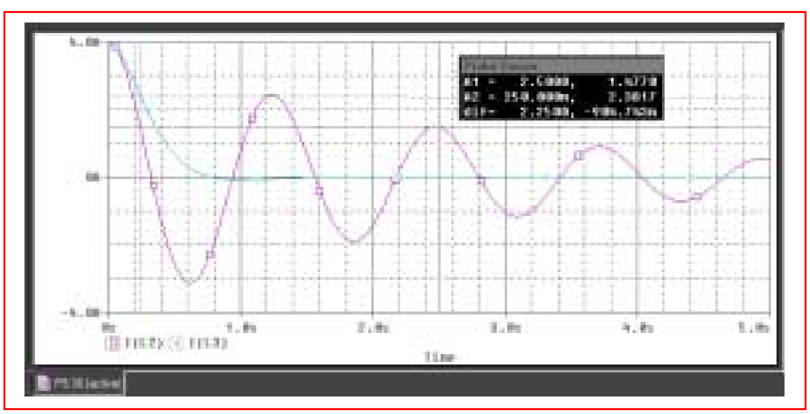
and so with $v_C(0^+) = 0 = (2/13)\ (3.162D - 4C)$, we obtain D = 5.06 A.

Thus, $i_L(t) = e^{-4t}\ (4 \cos 3.162t\ +\ 5.06 \sin 3.162t)$ A and $\boxed{i_L(.25) = 2.358\text{ A.}}$

(c)



We see from the simulation result below that our hand calculations are correct; the slight disagreement is due to numerical inaccuracy. Changing the step ceiling from the 10-ms value employed to a smaller value will improve the accuracy.



31. Series: $\alpha = \frac{R}{2L} = \frac{2}{1/2} = 4,\ \omega_o^2 = \frac{1}{LC} = \frac{4}{0.2} = 20,\ \omega_d = \sqrt{20-16} = 2$

$\therefore i_L = e^{-4t}(A_1 \cos 2t + A_2 \sin 2t);\ i_L(0) = 10A,\ v_c(0) = 20V$

$\therefore A_1 = 10;\ i_L'(0^+) = \frac{1}{L} v_L(0^+) = 4(20-20) = 0$

$\therefore i_L'(0^+) = 2A_2 - 4\times 10 \therefore A_2 = 20$

$\therefore \boxed{i_L(t) = e^{-4t}(10\cos 2t + 20\sin 2t)A,\ t>0}$

32.

$v_c(0) = 50 + 80 \times 2 = 210\,\text{V},\ i_L(0) = 0,\ \alpha = \dfrac{\text{R}}{2\text{L}} = \dfrac{80}{4} = 20$

$\omega_o^2 = \dfrac{100}{2} = 500 : \omega_d = \sqrt{500 - 20^2} = 10$

$\therefore v_c(t) = e^{-20t}\,(\text{A}_1 \cos 10t + \text{A}_2 \sin 10t) \therefore \text{A}_1 = 210\,\text{V}$

$\therefore v_c(t) = e^{-20t}(210 \cos 10t + \text{A}_2 \sin 10t);\ v_c'(0^+) = \dfrac{1}{\text{C}} i_c(0^+) = 0$

$\therefore 0 = 10\text{A}_2 - 20(210),\ \text{A}_2 = 420 \therefore v_c(t) = e^{-20t}(210 \cos 10t + 420 \sin 10t)$

$\therefore v_c(40\text{ms}) = e^{-0.8}(210 \cos 0.4 + 420 \sin 0.4) = \boxed{160.40\,\text{V}}$

Also, $i_L = e^{-20t}(\text{B}_1 \cos 10t + \text{B}_2 \sin 10t)$,

$i_L(0^+) = \dfrac{1}{\text{L}} v_L(0^+) = \dfrac{1}{2}\,[0 - v_c(0^+)] = \dfrac{1}{2} \times 210$

$\therefore i_L'(0^+) = -105 = 10\text{B}_2 \therefore \text{B}_2 = 10.5$

$\therefore i_L(t) = -10.5 e^{-20t} \sin 10t\ \text{A},\ t > 0$

$\therefore v_R(t) = 80 i_L = 840 e^{-20t} \sin 10t\text{V}$

$\therefore v_R(40\text{ms}) = -840 e^{-0.8} \sin 0.4 = \boxed{-146.98\,\text{V}}$

$v_L(t) = -v_c(t) - v_c(t) - v_R(t) \therefore v_L$

$(40\text{ms}) = -160.40 + 146.98 = \boxed{-13.420\,\text{V}}$

[check: $v_L = e^{-20t}(-210 \cos - 420 \sin + 840 \sin)$

$= e^{-20t}(-210 \cos 10t + 420 \sin 10t)\,\text{V},\ t > 0$

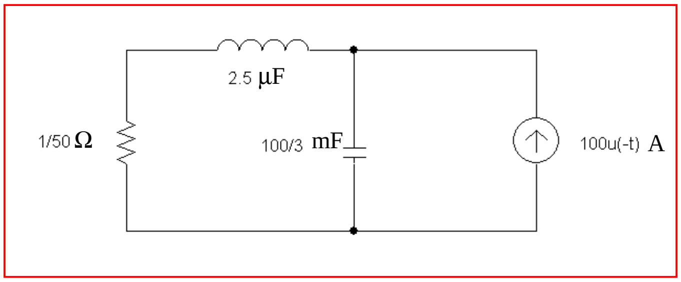
$\therefore v_L(40\text{ms}) = e^{-0.8}(-210 \cos - 420 \sin + 840 \sin) = e^{-20t}$

$(-210 \cos 10t + 420 \sin 10t)\text{V},\ t > 0$

$\therefore \text{V}_L(40\text{ms}) = e^{-0.8}$

$(420 \sin 0.4 - 210 \cos 0.4) = -13.420\,\text{V}$ Checks]

33. "Obtain an expression for $v_c(t)$ in the circuit of Fig. 9.8 (dual) that is valid for all t".



$$\alpha = \frac{R}{2L} = \frac{0.02\times10^6}{2\times2.5} = 4000,\ \omega_o^2 = \frac{10^6\times3}{2.5\times10} = 1.2\times10^7$$

$$\therefore s_{1,2} = -4000 \pm \sqrt{16\times10^6 - 12\times10^6} = -2000, -6000$$

$$\therefore v_c(t) = A_1 e^{-2000t} + A_2 e^{-6000t};\ v_c(0) = \frac{1}{50}\times100 = 2\text{V}$$

$$i_L(0) = 100\text{A} \therefore 2 = A_1 + A_2, v_c'(0^+) = \frac{1}{C}$$

$$(-i_L(0)) = -\frac{3}{100}\times10^3\times100 = -3000v/s$$

$$\therefore -3000 = -200A_1 - 600A_2, -1.5 = -A_1 - 3A_2$$

$$\therefore 0.5 = -2A_2, = -0.25,\ A_1 = 2.25$$

$$\therefore \boxed{v_c(t) = (2.25e^{-200t} - 0.25e^{-6000t})\ u(t) + 2u(-t)\ \text{V}}\ \text{(checks)}$$

34. (a) crit. damp; $\alpha^2 = \dfrac{R^2}{4L^2} = \omega_o^2 = \dfrac{1}{LC} \therefore L = \dfrac{1}{4}R^2C$

$\therefore L = \dfrac{1}{4} \times 4 \times 10^{4-6} = 0.01\text{H},\ \alpha = \dfrac{200}{0.02} = 10^4 = \omega_o$

$\therefore v_c(t) = e^{-10000t}(A_1 t + A_2);\ v_c(0) = -10\text{V},\ i_L(0) = -0.15\text{A}$

$\therefore A_2 = -10,\ v_c(t) = e^{-10000t}(A_1 t - 0);\ v_c'(0^+) = -\dfrac{1}{C}$

$i_L(0) = -10^6(-0.15) = 150{,}000$

Now, $v_c'(0^+) = A_1 + 10^5 = 150{,}000 \therefore A_1 = 50{,}000$

$\therefore \boxed{v_c(t) = e^{-10{,}000t}(50{,}000t - 10)\ \text{V}, t > 0}$

(b) $v_c'(t) = e^{-10{,}000t}[50{,}000 - 10{,}000(50{,}000t - 10)] = \therefore$

$5 = 50{,}000t_m - 10 \therefore t_m = \dfrac{15}{50{,}000} = 0.3\text{ms}$

$\therefore v_c(t_m) = e^{-3}(15 - 10) = 5e^{-3} = 0.2489\text{V}$

$v_c(0) = -10\text{V} \therefore |v_c|_{\max} = \boxed{10\text{V}}$

(c) $\boxed{v_{c,\max} = 0.2489\text{V}}$

35. (a)

$$\alpha = \frac{R}{2L} = \frac{250}{10} = 25,\ \omega_o^2 = \frac{1}{LC} = \frac{10^6}{2500} = 400$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -25 \pm 15 = -10, -40$$

$$\therefore i_L = A_1 e^{-10t} + A_2 e^{-40t},\ i_L(0) = 0.5\text{A},\ v_c(0) = 100\text{V}$$

$$\therefore 0.5 = A_1 + A_2,\ i_L'(0^+) = \frac{1}{5} v_L(0^+) = \frac{1}{5}$$

$$(100 - 25 - 100) = -5\ \text{A}/s = -10A_1 - 40A_2$$

$$\therefore 5 = 10\,A_1 + 40\ (0.5 - A_1) = 10A_1 - 40$$

$$A_1 + 20 \therefore -30A_1 = -15,\ A_1 = 0.5,\ A_2 = 0$$

$$\therefore i_L(t) = \boxed{0.5e^{-10t}\text{A},\ t > 0}$$

(b)

$$v_c = A_3 e^{-10t} + A_4 e^{-40t} \therefore 100 = A_3 + A_4;$$

$$v_c' = \frac{1}{c} i_c'(0^+) \frac{10^6}{500}(-0.5) = -1000$$

$$\therefore -10A_3 - 40A_4 = -1000 \therefore -3A_4 = 0,\ A_4 = 0,\ A_3 = 100$$

$$\therefore v_c(t) = \boxed{100e^{-10t}\text{V}\ t > 0}$$

36. (a) $\alpha = \frac{R}{2L} = \frac{2}{2} = 1,\ \omega_o^2 = \frac{1}{LC} = 5,\ \omega_d = \sqrt{\omega_o^2 - \alpha^2} = 2$

$\therefore i_L = e^{-t}(B_1 \cos 2t + B_2 \sin 2t),\ i_L(0) = 0,\ v_c(0) = 10V$

$\therefore B_1 = 0,\ i_L = B_2\, e^{-t} \sin 2t$

$i_1(0) = \frac{1}{1} v_L(0^+) = v_R(0^+) - V_c(0^+) = 0 - 10 = 2B_2$

$\therefore \boxed{B_2 = 5 \therefore i_L = -5e^{-t} \sin 2t\text{A},\ t > 0}$

(b) $i_L' = -5[e^{-t}(2\cos 2t - \sin 2t)] = 0$

$\therefore 2\cos 2t = \sin 2t, \tan 2t = 2$

$\therefore t_1 = 0.5536s,\ i_L(t_1) = -2.571\text{A}$

$2t_2 = 2 \times 0.5536 + \pi,\ t_2 = 2.124,$

$i_L(t_2) = 0.5345 \therefore |i_L|_{\max} = 2.571\text{A}$

and $i_{L\max} = \boxed{0.5345\text{A}}$

37. We are presented with a series RLC circuit having $\alpha = R/2L = 4700\ s^{-1}$ and $\omega_0 = 1/\sqrt{LC} = 447.2$ rad/s; therefore we expect an overdamped response with $s_1 = -21.32\ s^{-1}$ and $s_2 = -9379\ s^{-1}$.

From the circuit as it exists for $t < 0$, it is evident that $i_L(0^-) = 0$ and $v_C(0^-) = 4.7$ kV

Thus, $v_L(t) = A\,e^{-21.32t} + B\,e^{-9379t}$ [1]

With $i_L(0^+) = i_L(0^-) = 0$ and $i_R(0^+) = 0$ we conclude that $v_R(0^+) = 0$; this leads to $v_L(0^+) = -v_C(0^-) = -4.7$ kV and hence A + B = -4700 [2]

Since $v_L = L\dfrac{di}{dt}$, we may integrate Eq. [1] to find an expression for the inductor current:

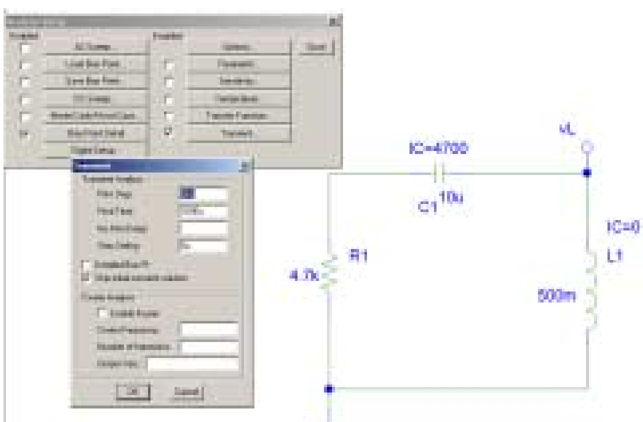
$$i_L(t) = \frac{1}{L}\left[-\frac{A}{21.32}e^{-21.32t} - \frac{B}{9379}e^{-9379t}\right]$$

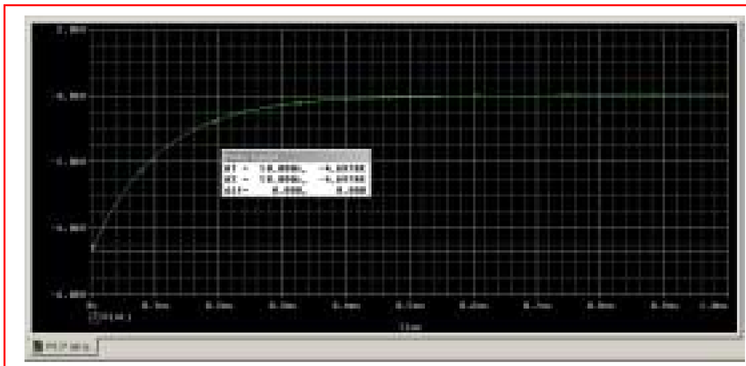
At $t = 0^+$, $i_L = 0$ so we have $\dfrac{1}{500\times10^{-3}}\left[-\dfrac{A}{21.32} - \dfrac{B}{9379}\right] = 0$ [3]

Simultaneous solution of Eqs. [2] and [3] yields A = 10.71 and B = -4711. Thus,

$$v_L(t) = 10.71e^{-21.32t} - 4711\,e^{-9379t}\ \text{V}, \quad t > 0$$

and the peak inductor voltage magnitude is 4700 V.





38. Considering the circuit as it exists for $t < 0$, we conclude that $v_C(0^-) = 0$ and $i_L(0^-) = 9/4 = 2.25$ A. For $t > 0$, we are left with a parallel RLC circuit having $\alpha = 1/2RC = 0.25\ s^{-1}$ and $\omega_o = 1/\sqrt{LC} = 0.3333$ rad/s. Thus, we expect an underdamped response with $\omega_d = 0.2205$ rad/s:

$$i_L(t) = e^{-\alpha t}(A \cos \omega_d t + B \sin \omega_d t)$$

$$i_L(0^+) = i_L(0^-) = 2.25 = A$$

so $i_L(t) = e^{-0.25t}(2.25 \cos 0.2205t + B \sin 0.2205t)$

In order to determine B, we must invoke the remaining boundary condition. Noting that

$$v_C(t) = v_L(t) = L\frac{di_L}{dt}$$

$$= (9)(-0.25)e^{-0.25t}(2.25 \cos 0.2205t + B \sin 0.2205t)$$
$$+ (9)\, e^{-0.25t}[-2.25(0.2205) \sin 0.2205t + 0.2205B \cos 0.2205t]$$

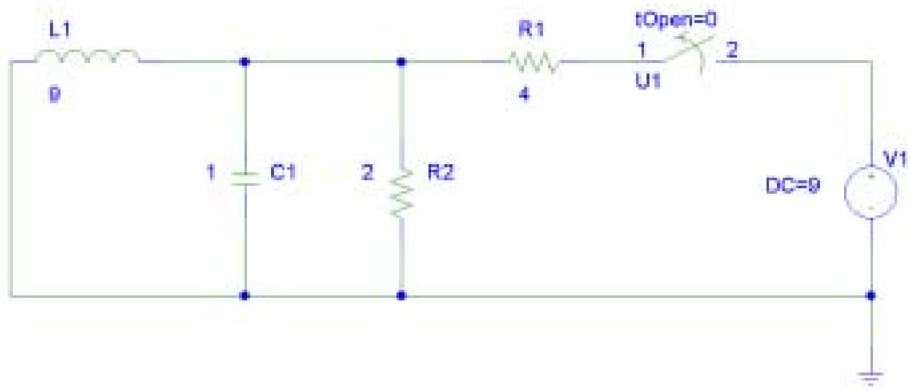
$v_C(0^+) = v_C(0^-) = 0 = (9)(-0.25)(2.25) + (9)(0.2205B)$

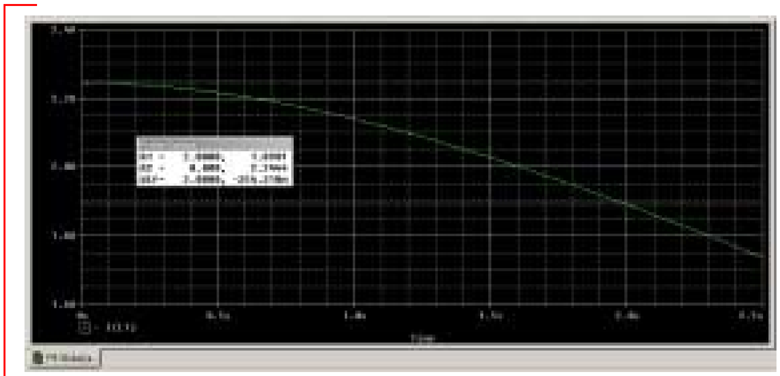
so B = 2.551 and

$$i_L(t) = e^{-0.25t}[2.25 \cos 0.2205t + 2.551 \sin 0.2205t] \text{ A}$$

Thus, $i_L(2) = 1.895$ A

This answer is borne out by PSpice simulation:





39. Considering the circuit at $t < 0$, we note that $i_L(0^-) = 9/4 = 2.25$ A and $v_C(0^-) = 0$.

For a critically damped circuit, we require $\alpha = \omega_o$, or $\dfrac{1}{2RC} = \dfrac{1}{\sqrt{LC}}$, which, with L = 9 H and C = 1 F, leads to the requirement that $\boxed{R = 1.5\ \Omega}$ (so $\alpha = 0.3333\ s^{-1}$).

The inductor energy is given by $w_L = ½\ L\ [i_L(t)]^2$, so we seek an expression for $i_L(t)$:

$$i_L(t) = e^{-\alpha t}\,(At + B)$$

Noting that $i_L(0^+) = i_L(0^-) = 2.25$, we see that B = 2.25 and hence

$$i_L(t) = e^{-0.3333t}\,(At + 2.25)$$

Invoking the remaining initial condition requires consideration of the voltage across the capacitor, which is equal in this case to the inductor voltage, given by:

$$v_C(t) = v_L(t) = L\frac{di_L}{dt} = 9(-0.3333)\,e^{-0.3333t}\,(At + 2.25) + 9A\,e^{-0.3333t}$$

$v_C(0^+) = v_C(0^-) = 0 = 9(-0.333)(2.25) + 9A$ so A = 0.7499 amperes and

$$i_L(t) = e^{-0.3333t}\,(0.7499t + 2.25)\text{ A}$$

Thus, $i_L(100\text{ ms}) = 2.249$ A and so $w_L(100\text{ ms}) = \boxed{22.76\text{ J}}$

40. With the 144 mJ originally stored via a 12-V battery, we know that the capacitor has a value of 2 mF. The initial inductor current is zero, and the initial capacitor voltage is 12 V. We begin by seeking a (painful) current response of the form

$$i_{bear} = \mathrm{A}e^{s_1 t} + \mathrm{B}e^{s_2 t}$$

Using our first initial condition, $i_{bear}(0^+) = i_L(0^+) = i_L(0^-) = 0 = \mathrm{A} + \mathrm{B}$

$$di/dt = \mathrm{A}s_1\, e^{s_1 t} + \mathrm{B}s_2\, e^{s_2 t}$$

$v_L = \mathrm{L}di/dt = \mathrm{AL}s_1\, e^{s_1 t} + \mathrm{BL}s_2\, e^{s_2 t}$

$v_L(0^+) = \mathrm{AL}s_1 + \mathrm{BL}s_2 = v_C(0^+) = v_C(0^-) = 12$

What else is known? We know that the bear stops reacting at t = 18 μs, meaning that the current flowing through its fur coat has dropped just below 100 mA by then (not a long shock).

Thus, $\mathrm{A}\exp[(18\times10^{-6})s_1] + \mathrm{B}\exp[(18\times10^{-6})s_2] = 100\times10^{-3}$

Iterating, we find that R_{bear} = $\boxed{119.9775\ \Omega.}$

This corresponds to A = 100 mA, B = -100 mA, $s_1 = -4.167\ \mathrm{s}^{-1}$ and $s_2 = -24\times10^{6}\ \mathrm{s}^{-1}$

41.

$$v_c(0) = 0,\ i_L(0) = 0,\ \alpha = \frac{\text{R}}{2\text{L}} = \frac{2}{0.5} = 4,\ \omega_o^2 = \frac{1}{\text{LC}} = 4\times 5 = 20$$

$$\therefore \omega_d = \sqrt{20-16} = 2 \therefore i_L(t) = e^{-4t}(\text{A}_1 \cos 2t + \text{A}_2 \sin 2t) + i_{L,f}$$

$$i_{L,f} = 10\text{A} \therefore i_L(t) = 10 + e^{-4t}(\text{A}_1 \cos 2t + \text{A}_2 \sin 2t)$$

$$\therefore 0 = 10 + \text{A}_1,\ \text{A}_1 = -10,\ i_L(t) = 10 + e^{-4t}(\text{A}_2 \sin 2t - 10\cos 2t)$$

$$i_L(0^+) = \frac{1}{\text{L}} v_L(0^+) = 4\times 0 = 0 \therefore i_L(0^+) = 0 = 2\text{A}_2 + 40,\ \text{A}_2 = -20$$

$$\boxed{i_\text{L}(t) = 10 - e^{-4t}(20 \sin 2t + 10 \cos 2t) \text{ A},\ t > 0}$$

42. (a)

Series, driven: $\alpha = \dfrac{R}{2L} = \dfrac{100}{0.2} = 500,$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{10\times10^6}{40} = 250,000$

$\therefore$ Crit. damp $i_L(f) = 3(1-2) = -3,$

$i_L(0) = 3,\ v_c(0) = 300\text{V}$

$\therefore i_L = -3 + e^{-500t}\ (A_1 t + A_2) \therefore 3 = -3 + A_2,\ A_2 = 6\text{A}$

$i_L(0^+) = A_1 - 300 = \dfrac{1}{L}[v_c(0) - v_R(0^+)] = 0$

$\therefore A_1 = 3000\, e^{-5000t}\ \therefore i_L(t) = -3 + e^{-500t}$

$(3000t + 6),\ t > 0$

$\therefore i_L(t) = \boxed{3u(-t) + [-3 + e^{-500t}(3000t + 6)]u(t)\text{A}}$

(b) $e^{-500t_o}(3000t_o + 6) = 3$; by SOLVE, $t_o = \boxed{3.357\text{ms}}$

43.

$$\alpha = \frac{1}{2RC} = \frac{10^6}{100\times 2.5} = 4000,\ \omega_o^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 20\times 10^6$$

$$\therefore \omega_d = \sqrt{\omega_o^2 - \alpha^2} = 2000,\ i_L(0) = 2\text{A},\ v_c(0) = 0$$

$$i_{c,f} = 0,\ (v_{c,f} = 0) \therefore i_c = e^{-400t}\ (\text{A}_1 \cos 2000t + \text{A}_2 \sin 2000t)$$

work with v_c: $v_c(t) = e^{-4000t}(\text{B}_1 \cos 2000t + \text{B}_2 \sin 2000t) \therefore \text{B}_1 = 0$

$$\therefore v_c = \text{B}_2 e^{-4000t} \sin 2000t,\ v_c'(0^+) = \frac{1}{\text{C}} i_c(0^+) = \frac{10^6}{2.5}(2\times 1) = 8\times 10^5$$

$$\therefore 8\times 10^5 = 2000\text{B}_2,\ \text{B}_2 = 400,\ v_c = 400 e^{-4000t} \sin 2000t$$

$$\therefore i_c(t) = \text{C}v_c' = 2.5\times 10^{-6} \times 400 e^{-4000t}\ (-4000 \sin 200^t + 2000 \cos 200t)$$

$$= 10^{-6+3+3} e^{-4000t}\ (-4 \sin 2000t + 2\cos 2000t)$$

$$= \boxed{e^{-4000t}\ (2\cos 2000t - 4\sin 2000t)\ \text{A},\ t > 0}$$

44.

$$\alpha = \frac{R}{2L} = \frac{250}{10} = 25,\ \omega_o^2 = \frac{1}{LC} = \frac{10^6}{2500} = 400$$

$$s_{1,2} = -25 \pm \sqrt{625-400} = -10, -40$$

$$i_L(0) = 0.5\text{A},\ v_c(0) = 100\text{V},\ i_{L,f} = -0.5\text{A}$$

$$\therefore i_L(t) = -0.5 + \text{A}_1 e^{-10t} + \text{A}_2 e^{-40t}\ \text{A}$$

$$t = 0^+:\ v_L(0^+) = 100 - 50\times 1 - 200\times 0.5 = -50\text{V} \therefore -50 = 5 i_L'(0^+)$$

$$\therefore i_L'(0^+) = -10 \therefore -10 = -10\text{A}_1 - 40\text{A}_2,\ 0.5 = -0.5 + \text{A}_1 + \text{A}_2$$

$$\therefore \text{A}_1 + \text{A}_2 = 1 \therefore -10 = -10\text{A}_2 - 40(-1+\text{A}_1) = -50\text{A}_1 + 40,\ \ \text{A}_1 = 1, \text{A}_2 = 0$$

$$\therefore i_L(t) = \boxed{-0.5 + 1e^{-10t}\text{A},}\ t > 0;\ i_L(t) = \boxed{0.5\text{A},\ t > 0}$$

45.

$$\alpha = \frac{R}{2L} = \frac{1}{1} = 1,\ \omega_o^2 = \frac{1}{LC} = 1 \therefore \text{crit. damp}$$

$$v_c(0) = \frac{5}{6} \times 12 = 10\text{V},\ i_L(0) = 2\text{A},\ v_{c,f} = 12\text{V}$$

$$\therefore v_c(t) = 12 + e^{-t}(\text{A}_1 t - 2);\ v_c'(0^+) = \frac{1}{\text{C}}\, i_c(0^+) = \frac{1}{2} \times i_L(0^+) = 1$$

$$\therefore 1 = \text{A}_1 + 2;\ \text{A}_1 = -1 \therefore \boxed{v_c(t) = 12 - e^{-t}(t+2)\,\text{V},\ t > 0}$$

46. (a) $\alpha = \frac{1}{2RC} = \frac{8\times10^6}{2\times4\times10^3} = 1000,\ \omega_o^2 = \frac{8\times10^6\times13}{4} = 26\times10^6$

$\therefore \omega_d = \sqrt{26-1}\times10^3 = 5000,\ v_c(0) = 8\text{V}$

$i_L(0) = 8\text{mA},\ v_{c,f} = 0$

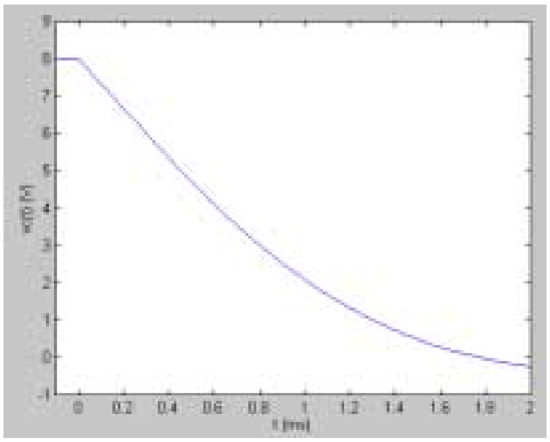
$\therefore v_c = e^{-1000t}(\text{A}_1 \cos 1000t + \text{A}_2 \sin 5000t)$

$\therefore \text{A}_1 = 8;\ v_c'(0^+) = \frac{1}{\text{C}} i_c(0^+) = 8\times10^6(0.01 - \frac{8}{4000} - 0.008) = 0$

$\therefore 5000\text{A}_2 - 1000\times8 = 0,\ \text{A}_2 = 1.6$

So $v_c(t) = e^{-1000t}(8 \cos 1000t + 1.6 \sin 1000t)$ V, $t > 0$

(b)



47. (a)

$$v_s(t) = 10u(-t)\,\text{V}: \alpha = \frac{1}{2\text{RC}} = \frac{10^6}{1000} = 1000$$

$$\omega_o^2 = \frac{1}{\text{LC}} = \frac{10^6 \times 3}{4} \therefore s_{1,2} = -1000 \pm \sqrt{10^6 - \frac{3}{4}\times 10^6} = -500,\ -1500$$

$$v_{c,f} = 0 \therefore v_c = \text{A}_1 e^{-500t} + \text{A}_2 e^{-1500t},\ v_c(0) = 10\text{V},\ i_L(0) = 0$$

$$\therefore 10 = \text{A}_1 + \text{A}_2,\ v_c' = 10^6\ i_c(0^+) = 10^6 \left[0 - \frac{10}{500}\right] = -2\times 10^4$$

$$\therefore -2\times 10^4 = -500\text{A}_1 - 1500\text{A} \therefore 40 = \text{A}_1 + 3\text{A}_2 \therefore 30 = 2\text{A}_2,\ \text{A}_2 = 15,\ \text{A}_1 = -5$$

$$\therefore v_c = -5e^{-500t} + 15e^{-1500t}\text{V},\ t > 0\ \therefore i_s = i_c = \text{C}v_c'$$

$$\therefore i_s = 10^{-6}(2500e^{-500t} - 22,500e^{-1500t})$$

$$= \boxed{2.5e^{-500t} - 22.5e^{-1500t}\text{mA},\ t > 0}$$

(b)

$$v_s(t) = 10u(t)\,\text{V} \therefore v_{c,f} = 10\text{V},\ v_c(0) = 0,\ i_L(0) = 0$$

$$\therefore v_c = 10 + A_3\,e^{-500t} + \text{A}_4 e^{-1500t} \therefore \text{A}_3 + \text{A}_4 = -10$$

$$v_c'(0^+) = 10^6 i_c(0^+) = 10^6\left(0 + \frac{10}{500}\right) = 2\times 10^4 = -500\,\text{A}_3 - 1500\,\text{A}_4$$

$$\therefore -\text{A}_3 - 3\text{A}_4 = 40,\ \text{add:}\ -2\text{A}_4 = 30,\ \text{A}_4 = -15,\ \text{A}_3 = 5,$$

$$v_c = 10 + 5e^{-500t} - 15e^{-1500t}\text{V},\ i_s = i_c =$$

$$10^{-6}(-2500e^{-500t} + 22,500e^{-1500t}) = \boxed{25e^{-500t} + 22.5e^{-1500t}\ \text{mA},\ t > 0}$$

48. (a)

$$v_s = 10u(-t)\ \text{V} : \alpha = \frac{1}{2\text{RC}} = \frac{10^6}{2000\times 0.5} = 1000$$

$$\omega_o^2 = \frac{1}{\text{LC}} = \frac{2\times 10^6 \times 3}{8} = 0.75\times 10^6 \therefore s_{1,2} = -500,\ -1500$$

$$\therefore v_c = \text{A}_1 e^{-500t} + \text{A}_2 e^{-1500t},\ v_o(0) = 10\text{V},\ i_L(0) = 10\text{mA}$$

$$\therefore A_1 + \text{A}_2 = 10,\ v_c'(0^+) = 2\times 10^6 [i_L(0) - i_R(0^+)] = 2\times 10^6$$

$$\left(0.01 - \frac{10}{1000}\right) = 0 \therefore -500\text{A}_1 - 1500\,\text{A}_2 = 0,$$

$$-\text{A}_1 - 3\text{A}_2 = 0;\ \text{add:} -2\,\text{A}_2 = 10,\ \text{A}_2 = -5,\ \text{A}_1 = 15$$

$$\therefore v_c(t) = 15e^{-500t} - 5e^{-1500t}\ \text{V}\ t>0$$

$$\therefore i_R(t) = \boxed{15e^{-500t} - 5e^{-1500t}\text{mA},\ t>0}$$

(b)

$$v_s = 10u(t)\,\text{V},\ v_{c,f} = 10,\ v_c = 10 + \text{A}_3 e^{-500t} + \text{A}_4 e^{-1500t},$$

$$v_c(0) = 0,\ i_L(0) = 0 \therefore \text{A}_3 + \text{A}_4 = -10\text{V},\ v_c'(0^+) = 2\times 10^6$$

$$[i_L(0) - i_R(0^+)] = 2\times 10^6 (0-0) = 0 = -500\text{A}_3 - 1500\text{A}_4$$

$$\therefore -\text{A}_3 - 3\text{A}_4 = 0,\ \text{add:}\ -2\text{A}_4 = -10,\ \text{A}_4 = 5 \therefore \text{A}_3 = -15$$

$$\therefore v_c(t) = 10 - 15e^{-500t} + 5e^{-1500t}\text{V},\ t>0$$

$$\therefore i_R(t) = \boxed{10 - 15e^{-500t} + 5e^{1500t}\text{mA},\ t>0}$$

49. (a) $v_S(0^-) = v_C(0^-) = 2(15) = \boxed{30\text{ V}}$

(b) $i_L(0^+) = i_L(0^-) = 15$ A
Thus, $i_C(0^+) = 22 - 15 = 7$ A and $v_S(0^+) = 3(7) + v_C(0^+) = \boxed{51\text{ V}}$

(c) As $t \to \infty$, the current through the inductor approaches 22 A, so $v_S(t\to \infty,) = \boxed{44\text{ A.}}$

(d) We are presented with a series RLC circuit having $\alpha = 5/2 = 2.5\ \text{s}^{-1}$ and $\omega_o = 3.536$ rad/s. The natural response will therefore be underdamped with $\omega_d = 2.501$ rad/s.

$i_L(t) = 22 + e^{-\alpha t}\,(\text{A} \cos \omega_d t + \text{B} \sin \omega_d t)$
$i_L(0^+) = i_L(0^-) = 15 = 22 + \text{A}$ so A = -7 amperes

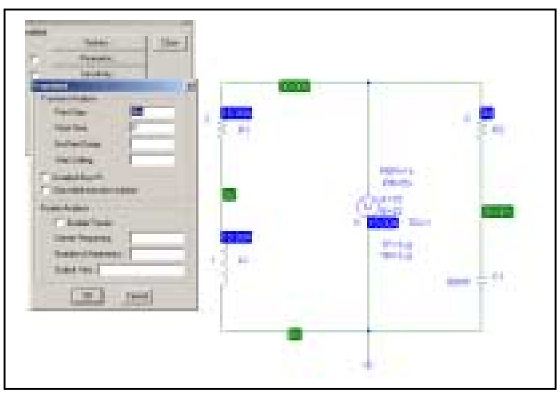
Thus, $i_L(t) = 22 + e^{-2.5t}\,(-7 \cos 2.501\text{t} + \text{B} \sin 2.501\text{t})$

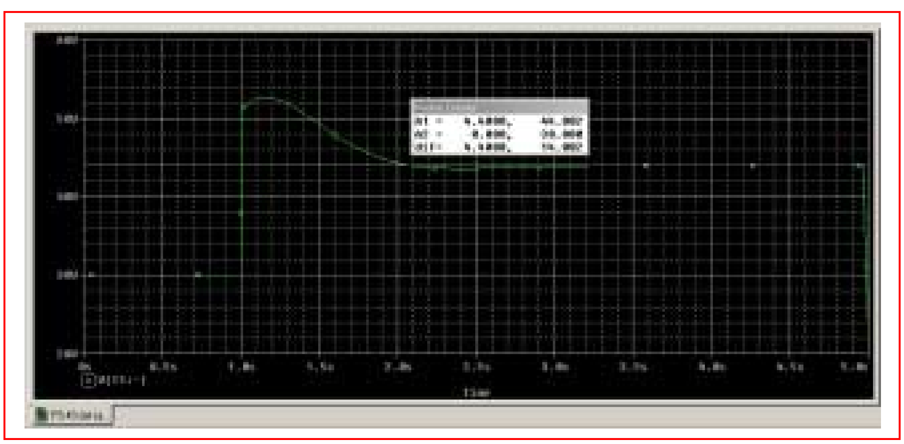
$$v_S(t) = 2\, i_L(t) + L\frac{di_L}{dt} = 2i_L + \frac{di_L}{dt} = 44 + 2e^{-2.5t}\,(-7\cos 2.501t + \text{B}\sin 2.501t)$$
$$- 2.5e^{-2.5t}\,(-7\cos 2.501t + \text{B}\sin 2.501t) + e^{-2.5t}\,[7(2.501) \sin 2.501t + 2.501\text{B} \cos 2.501t)]$$

$v_S(t) = 51 = 44 + 2(-7) - 2.5(-7) + 2.501\text{B}$ so B = 1.399 amperes and hence

$$v_S(t) = 44 + 2e^{-2.5t}\,(-7\cos 2.501t + 1.399\sin 2.501t)$$
$$-2.5e^{-2.5t}\,(-7\cos 2.501t + 1.399\sin 2.501t) + e^{-2.5t}\,[17.51\sin 2.501t + 3.499\cos 2.501t)]$$

and $v_S(t)$ at $t = 3.4$ s = $\boxed{44.002\text{ V.}}$ This is borne out by PSpice simulation:





50. Considering the circuit at $t < 0$, we see that $i_L(0^-) = 15$ A and $v_C(0^-) = 0$.
The circuit is a series RLC with $\alpha = R/2L = 0.375\ s^{-1}$ and $\omega_0 = 1.768$ rad/s. We therefore expect an underdamped response with $\omega_d = 1.728$ rad/s. The general form of the response will be

$$v_C(t) = e^{-\alpha t}(A\cos\omega_d t + B\sin\omega_d t) + 0 \qquad (v_C(\infty) = 0)$$

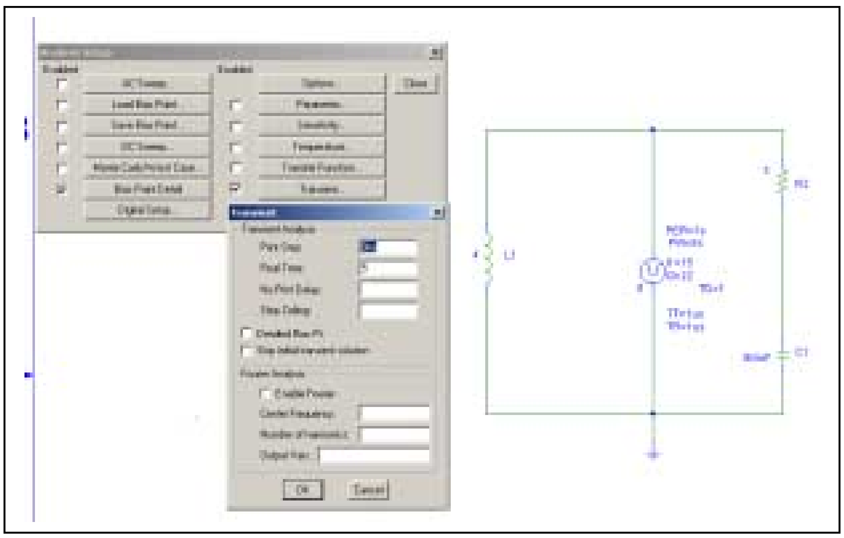
$v_C(0^+) = v_C(0^-) = 0 = A$ and we may therefore write $v_C(t) = Be^{-0.375t}\sin(1.728t)$ V

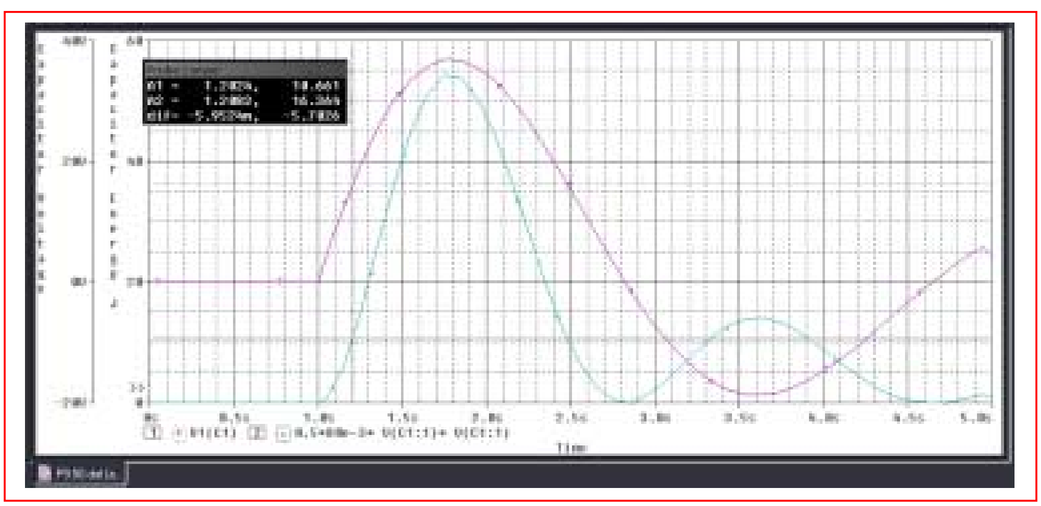
$$i_C(t) = -i_L(t) = C\frac{dv_C}{dt} = (80\times10^{-3})(-0.375B\ e^{-0.375t}\sin 1.728t$$

At $t = 0^+$, $i_C = 15 + 7 - i_L(0^+) = 7 = (80\times10^{-3})(1.728B)$ so that B = 50.64 V.

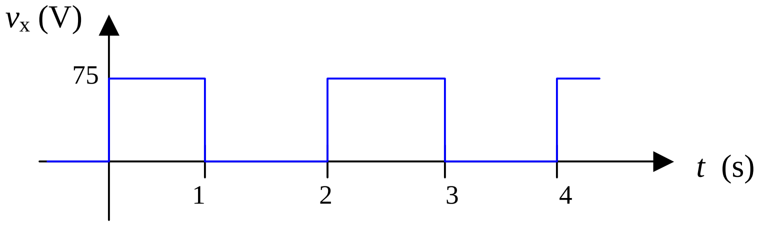
Thus, $v_C(t) = 50.64\ e^{-0.375t}\sin 1.807t$ V and $v_C(t = 200\text{ ms}) = 16.61$ V.

The energy stored in the capacitor at that instant is $\frac{1}{2}Cv_C^2 = \boxed{11.04\text{ J}}$





51. It's probably easiest to begin by sketching the waveform v_x:



(a) The source current (= $i_L(t)$) = $\boxed{0}$ at $t = 0^-$.

(b) $i_L(t)$ = $\boxed{0}$ at $t = 0^+$

(c) We are faced with a series RLC circuit having α = R/2L = 2000 rad/s and ω_0 = 2828 rad/s. Thus, an underdamped response is expected with ω_d = 1999 rad/s.

The general form of the expected response is $i_L(t) = e^{-\alpha t}(\text{A} \cos \omega_d t + \text{B} \sin \omega_d t)$

$i_L(0^+) = i_L(0^-) = 0$ = A so A = 0. This leaves $i_L(t) = \text{B}\, e^{-2000t} \sin 1999t$

$$v_L(t) = \text{L}\frac{di_L}{dt} = \text{B}[(5\times10^{-3})(-2000\, e^{-2000t} \sin 1999t + 1999\, e^{-2000t} \cos 1999t)]$$

$v_L(0^+) = v_x(0^+) - v_C(0^+) - 20\, i_L(0^+) = \text{B}\,(5\times10^{-3})(1999)$ so B = 7.504 A.

Thus, $i_L(t) = 7.504\, e^{-2000t} \sin 1999t$ and i_L(1 ms) = $\boxed{0.9239 \text{ A.}}$

(d) Define $t' = t - 1$ ms for notational convenience. With no source present, we expect a new response but with the same general form:

$$i_L(t') = e^{-2000t'}(\text{A}' \cos 1999t' + \text{B}' \sin 1999t')$$

$v_L(t) = \text{L}\dfrac{di_L}{dt}$, and this enables us to calculate that $v_L(t = 1 \text{ ms}) = -13.54$ V. Prior to the pulse returning to zero volts, $-75 + v_L + v_C + 20\, i_L = 0$ so $v_C(t' = 0) = 69.97$ V.

$i_L(t' = 0) = \text{A}' = 0.9239$ and $-v_x + v_L + v_C + 20\, i_L = 0$ so that B' = -7.925.
Thus, $i_L(t')$ = e-2000 t' (0.9239 cos 1999t' – 7.925 sin 1999t') and
hence $i_L(t = 2 \text{ ms}) = i_L(t' = 1 \text{ ms}) = \boxed{-1.028 \text{ A.}}$

52. For $t < 0$, we have 15 A dc flowing, so that $i_L = 15$ A, $v_C = 30$ V, $v_{3\Omega} = 0$ and $v_S = 30$ V. This is a series RLC circuit with $\alpha = R/2L = 2.5\ s^{-1}$ and $\omega_0 = 3.536$ rad/s. We therefore expect an underdamped response with $\omega_d = 2.501$ rad/s.

<u>$0 < t < 1$</u> $\qquad v_C(t) = e^{-\alpha t}\,(A \cos \omega_d t + B \sin \omega_d t)$

$v_C(0^+) = v_C(0^-) = 30 = A$ so we may write $v_C(t) = e^{-2.5t}\,(30 \cos 2.501t + B \sin 2.501t)$

$$\frac{dv_C}{dt} = -2.5e^{-2.5t}(30 \cos 2.501t + B \sin 2.501t) + e^{-2.5t}\,[-30(2.501)\sin 2.501t + 2.501B \cos 2.501t]$$

$$i_C(0^+) = C\left.\frac{dv_C}{dt}\right|_{t=0^+} = 80\times10^{-3}[-2.5(30) + 2.501B] = -i_L(0^+) = -i_L(0^-) = -15 \text{ so } B = -44.98 \text{ V}$$

Thus, $v_C(t) = e^{-2.5t}\,(30 \cos 2.501t - 44.98 \sin 2.501t)$ and

$i_C(t) = e^{-2.5t}\,(-15 \cos 2.501t + 2.994 \sin 2.501t)$.

Hence, $v_S(t) = 3\,i_C(t) + v_C(t) = e^{-2.5t}\,(-15 \cos 2.501t - 36 \sin 2.501t)$

Prior to switching, $v_C(t = 1) = -4.181$ V and $i_L(t = 1) = -i_C(t = 1) = -1.134$ A.

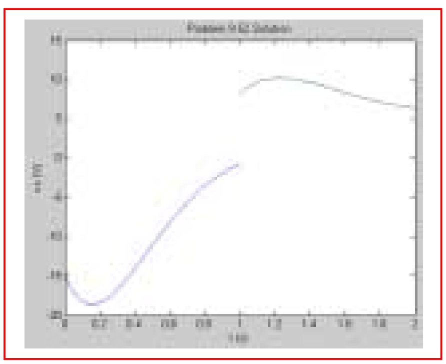
<u>$t > 2$:</u> Define $t' = t - 1$ for notational simplicity. Then, with the fact that $v_C(\infty) = 6$ V, our response will now be $v_C(t') = e^{-\alpha t'}\,(A' \cos \omega_d t' + B' \sin \omega_d t') + 6$.
With $v_C(0^+) = A' + 6 = -4.181$, we find that $A' = -10.18$ V.

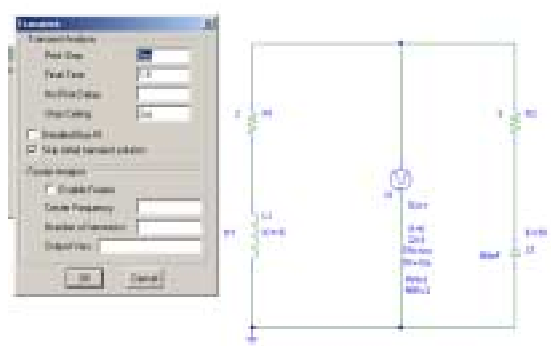
$$i_C(0^+) = C\left.\frac{dv_C}{dt'}\right|_{t'=0^+} = (80\times10^{-3})[(-2.5)(-10.18) + 2.501B')] = 3 - i_L(0^+) \text{ so } B' = 10.48 \text{ V}$$

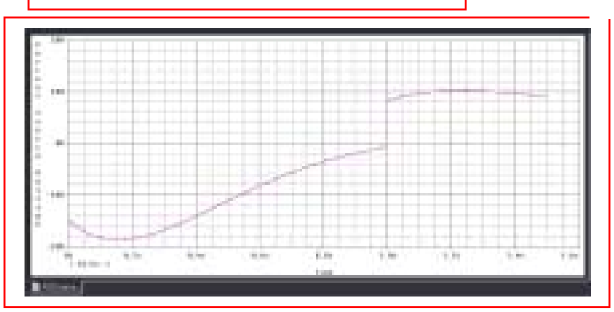
Thus, $v_C(t') = e^{-2.5t}\,(-10.18 \cos 2.501t' + 10.48 \sin 2.501t')$ and

$i_C(t') = e^{-2.5t}\,(4.133 \cos 2.501t' - 0.05919 \sin 2.501t')$.

Hence, $v_S(t') = 3\,i_C(t') + v_C(t') = e^{-2.5t}\,(2.219 \cos 2.501t' + 10.36 \sin 2.501t')$







We see that our hand calculations are supported by the PSpice simulation.

53. The circuit described is a series RLC circuit, and the fact that oscillations are detected tells us that it is an underdamped response that we are modeling. Thus,

$i_L(t) = e^{-\alpha t}(\mathrm{A}\cos\omega_d t + \mathrm{B}\sin\omega_d t)$ where we were given that $\omega_d = 1.825\times10^6$ rad/s.

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 1.914\times10^6$ rad/s, and so $\omega_d^{\,2} = \omega_0^{\,2} - \alpha^2$ leads to $\alpha^2 = 332.8\times10^9$

Thus, $\alpha = R/2L = 576863\ \mathrm{s}^{-1}$, and hence R = $\boxed{1003\ \Omega.}$

Theoretically, this value must include the "radiation resistance" that accounts for the power lost from the circuit and received by the radio; there is no way to separate this effect from the resistance of the rag with the information provided.

54. The key will be to coordinate the decay dictated by α, and the oscillation period determined by ω_d (and hence partially by α). **One possible solution of many:**

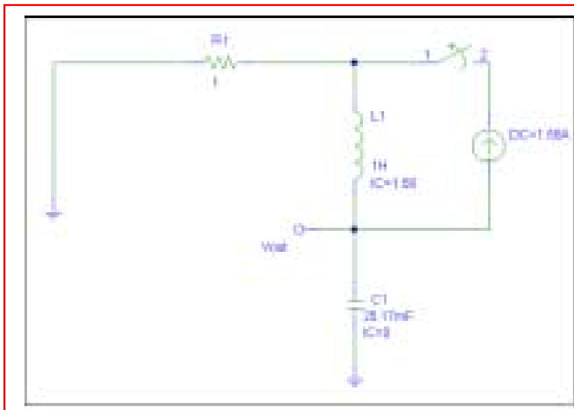
Arbitrarily set $\omega_d = 2\pi$ rad/s.
We want a capacitor voltage $v_C(t) = e^{-\alpha t}$ (A cos $2\pi t$ + B sin $2\pi t$). If we go ahead and decide to set $v_C(0^-) = 0$, then we can force A = 0 and simplify some of our algebra.

Thus, $v_C(t) = \text{B } e^{-\alpha t} \sin 2\pi t$. This function has max/min at t = 0.25 s, 0.75 s, 1.25 s, *etc.* Designing so that there is no strong damping for several seconds, we pick $\alpha = 0.5\ \text{s}^{-1}$. Choosing a series RLC circuit, this now establishes the following:

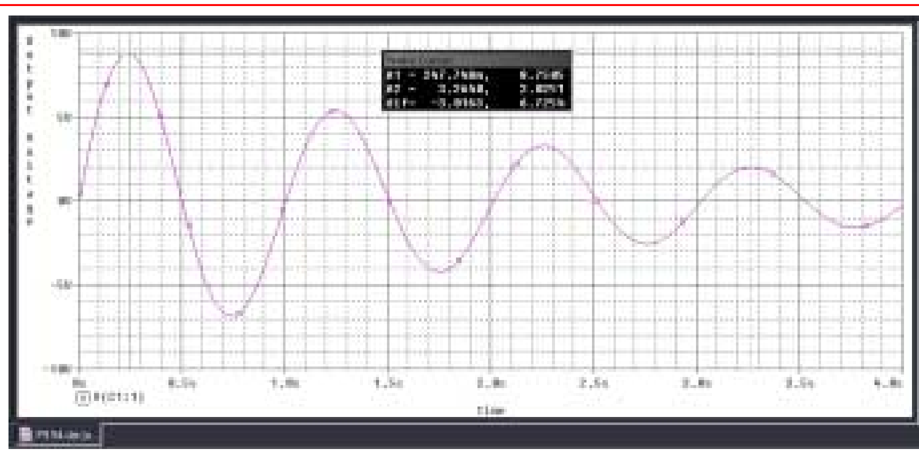
$$R/2L = 0.5 \text{ so } \quad R = L \text{ and}$$

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{1}{2}\right)^2} = 39.73 \text{ rad/s} \quad = \quad \frac{1}{LC}$$

Arbitrarily selecting R = 1 Ω, we find that L = 1 H and C = 25.17 mF. We need the first peak to be at least 5 V. Designing for B = 10 V, we ∴ need $i_L(0^+) = 2\pi(25.17\times10^{-3})(10)$ = 1.58 A. Our final circuit, then is:



And the operation is verified by a simple PSpice simulation:



55. For $t < 0$, $i_L(0^-) = 3$ A and $v_C(0^-) = 25(3) = 75$ V. This is a series RLC circuit with α = R/2L = 5000 s^{-1} and ω_0 = 4000 rad/s. We therefore expect an overdamped response with $s_1 = -2000\ s^{-1}$ and $s_2 = -8000\ s^{-1}$. The final value of v_C = -50 V.

For $t > 0$, $v_C(t) = A\,e^{-2000t} + B\,e^{-8000t} - 50$
$v_C(0^+) = v_C(0^-) = 75 = A + B - 50$
so $\qquad A + B = 125 \qquad$ [1]

$$\frac{dv_C}{dt} = -2000\,Ae^{-2000t} - 8000\,Be^{-8000t}$$

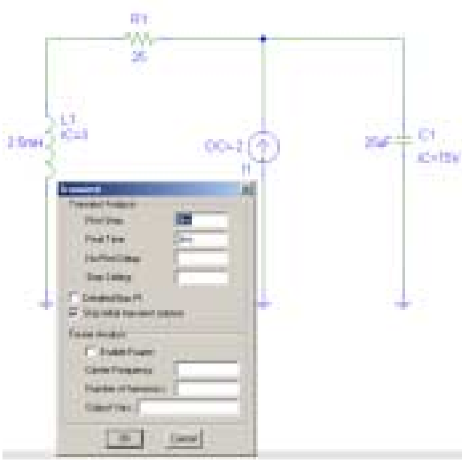
$$i_C(0^+) = C\left.\frac{dv_C}{dt}\right|_{t=0^+} = 3 - 5 - i_L(0^-) = -5 = -25\times10^{-6}\,(2000A + 8000B)$$

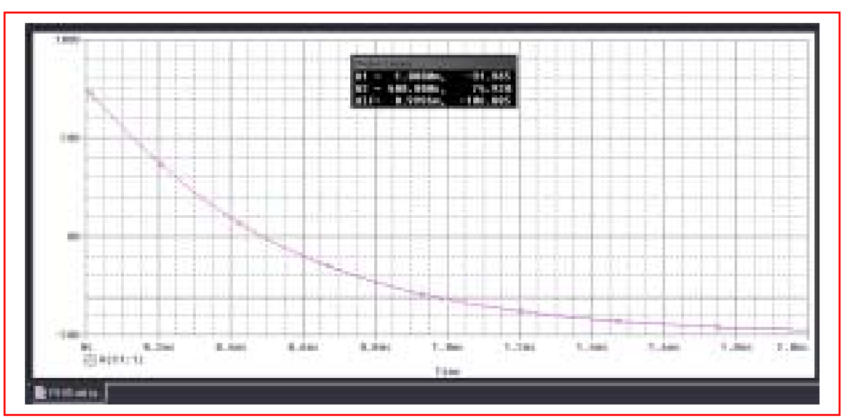
Thus, $2000A + 8000B = 5/25\times10^{-6}$ $\qquad$ [2]

Solving Eqs. [1] and [2], we find that A = 133.3 V and B = -8.333 V. Thus,

$$v_C(t) = 133.3\,e^{-2000t} - 8.333\,e^{-8000t} - 50$$

and $v_C(1\text{ ms})$ = $\boxed{-31.96\text{ V.}}$ This is confirmed by the PSpice simulation shown below.





56. $\alpha = 0$ (this is a series RLC with R = 0, or a parallel RLC with R = ∞)
$\omega_o^2 = 0.05$ therefore ω_d = 0.223 rad/s. We anticipate a response of the form:
$v(t)$ = A cos 0.2236t + B sin 0.2236t

$v(0^+) = v(0^-) = 0 = A$ therefore $v(t)$ = B sin 0.2236t

dv/dt = 0.2236B cos 0.2236t; $i_C(t) = C dv/dt$ = 0.4472B cos 0.2236t

$i_C(0^+) = 0.4472B = -i_L(0^+) = -i_L(0^-) = -1\times10^{-3}$ so B = -2.236×10^{-3} and thus

$v(t)$ = -2.236 sin 0.2236t mV

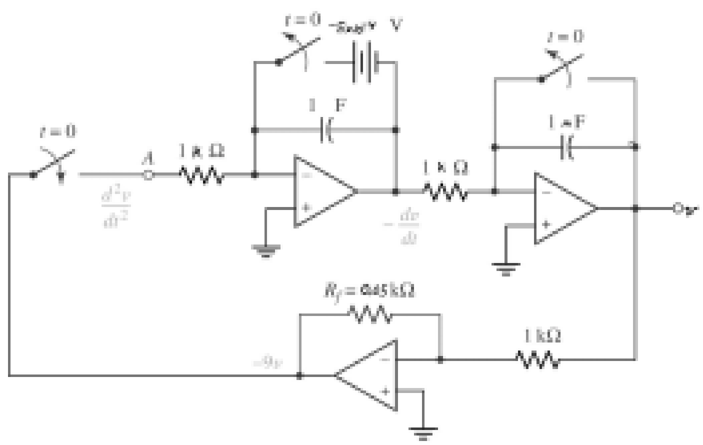
In designing the op amp stage, we first write the differential equation:

$$\frac{1}{10}\int_0^t v\, dt' + 10^{-3} + 2\frac{dv}{dt} = 0 \qquad (i_C + i_L = 0)$$

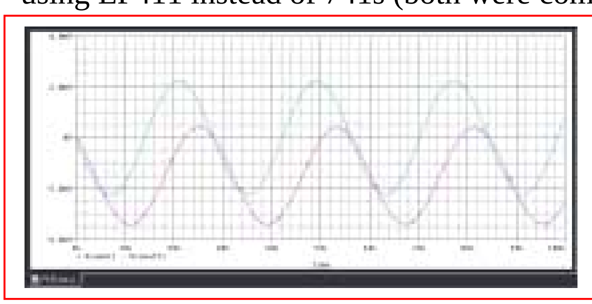
and then take the derivative of both sides:

$$\frac{d^2v}{dt^2} = -\frac{1}{20}v$$

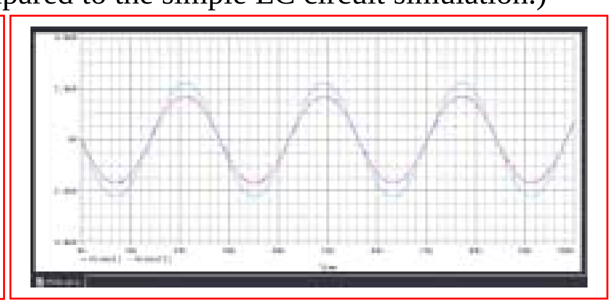
With $\left.\frac{dv}{dt}\right|_{t=0^+} = (0.2236)(-2.236\times10^{-3}) = -5\times10^{-4}$, one possible solution is:



PSpice simulations are very sensitive to parameter values; better results were obtained using LF411 instead of 741s (both were compared to the simple LC circuit simulation.)



Simulation using 741 op amps



Simulation using LF411 op amps

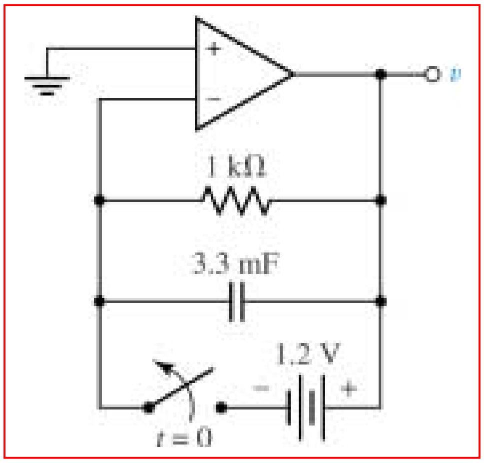
57.

$$\frac{v}{1000} + 3.3\times10^{-3}\frac{dv}{dt} = 0$$

(a) or

$$\boxed{\frac{dv}{dt} = -\frac{1}{3.3}v}$$

(b) One possible solution:



58. $\alpha = 0$ (this is a series RLC with R = 0, or a parallel RLC with R = ∞)
$\omega_o^2 = 50$ therefore $\omega_d = 7.071$ rad/s. We anticipate a response of the form:
$v(t) = A \cos 7.071t + B \sin 7.071t$, knowing that $i_L(0^-) = 2$ A and $v(0^-) = 0$.

$v(0^+) = v(0^-) = 0 = A$ therefore $v(t) = B \sin 7.071t$

$dv/dt = 7.071B \cos 7.071t$; $i_C(t) = C dv/dt = 0.007071B \cos 7.071t$

$i_C(0^+) = 0.007071B = -i_L(0^+) = -i_L(0^-) = -2$ so $B = -282.8$ and thus

$v(t) = -282.8 \sin 7.071t$ V

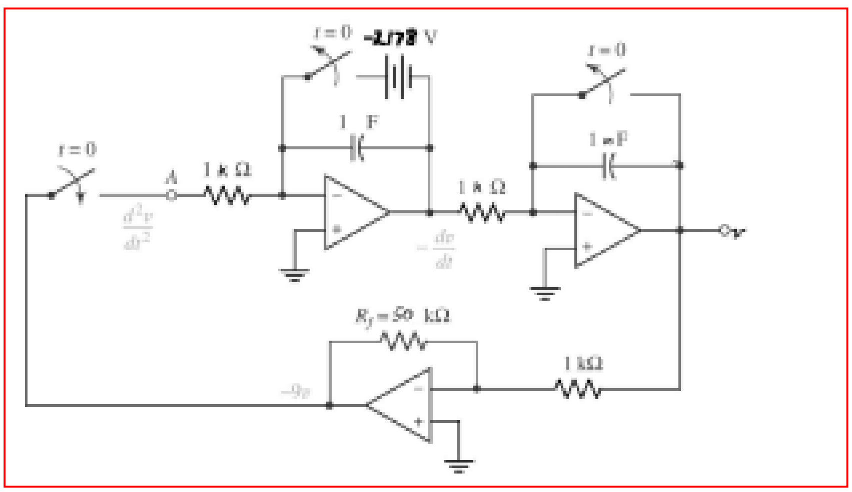
In designing the op amp stage, we first write the differential equation:

$$\frac{1}{20}\int_0^t v\, dt' + 2 + 10^{-3}\frac{dv}{dt} = 0 \qquad (i_C + i_L = 0)$$

and then take the derivative of both sides:

$$\frac{d^2v}{dt^2} = -50v$$

With $\left.\frac{dv}{dt}\right|_{t=0^+} = (7.071)(-282.8) = -2178$, one possible solution is:



59. (a) $v_R = v_L$

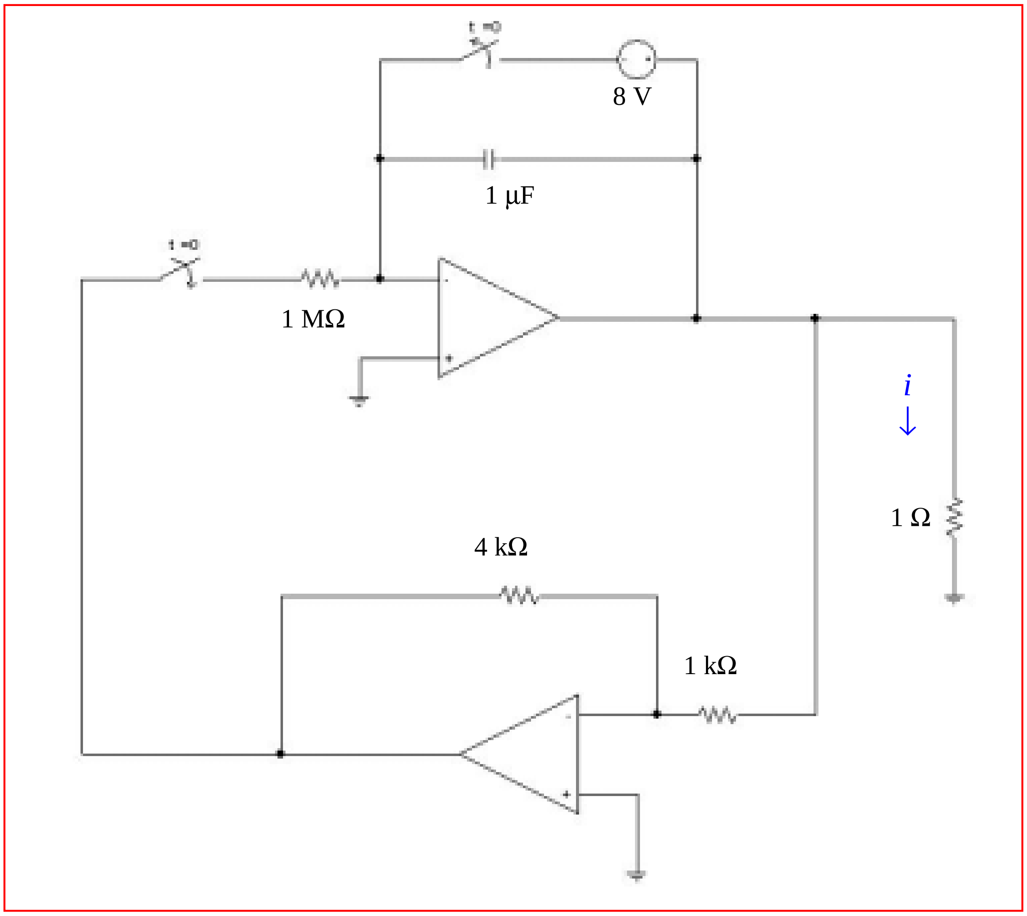
$20(-i_L) = 5\frac{di_L}{dt}$ or $\boxed{\frac{di_L}{dt} = -4i_L}$

(b) We expect a response of the form $i_L(t) = A\, e^{-t/\tau}$ where τ = L/R = 0.25.

We know that $i_L(0^-) = 2$ amperes, so A = 2 and $i_L(t) = 2\, e^{-4t}$

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = -4(2) = -8 \text{ A/s.}$$

One possible solution, then, is



60. We see either a series RLC with R = 0 or a parallel RLC with R = ∞; either way, $\alpha = 0$. $\omega_0^2 = 0.3$ so $\omega_d = 0.5477$ rad/s (combining the two inductors in parallel for the calculation). We expect a response of the form $i(t) = A \cos \omega_d t + B \sin \omega_d t$.

$i(0^+) = i(0^-) = A = 1\times10^{-3}$
$di/dt = -A\omega_d \sin \omega_d t + B\omega_d \cos \omega_d t$
$v_L = 10 di/dt = -10A\omega_d \sin \omega_d t + 10B\omega_d \cos \omega_d t$
$v_L(0^+) = v_C(0^+) = v_C(0^-) = 0 = 10B(0.5477)$ so that B = 0
and hence $i(t) = 10^{-3} \cos 0.5477t$ A

The differential equation for this circuit is

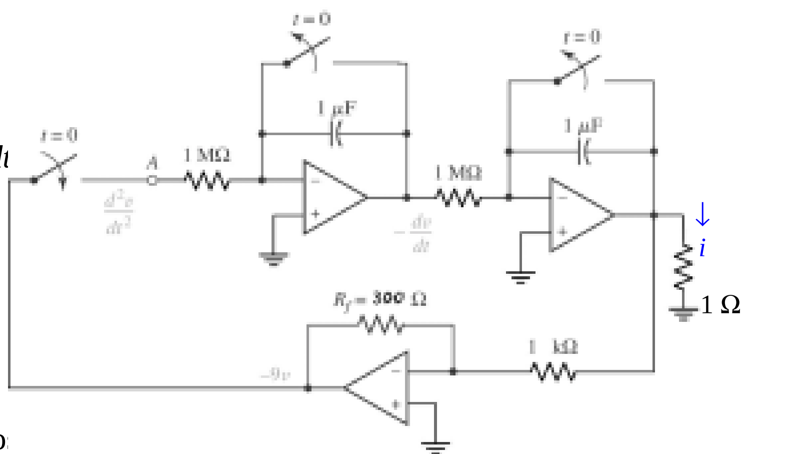
and $\left.\dfrac{di}{dt}\right|_{t=0^+} = 0$

$\dfrac{1}{10}\int_0^t v\,dt$

or

$\dfrac{d^2v}{dt^2} =$

One po



CHAPTER TEN (Phasor Analysis) SOLUTIONS

1.

(a) $T = 4(7.5 - 2.1)10^{-3} = 21.6\times10^{-3},\ \omega = \dfrac{2\pi 10^3}{21.6} = 290.9t \text{ rad/s}$

$\therefore\ f(t) = 8.5\sin(290.9t + \Phi)\ \therefore 0 = 8.5\sin(290.9\times2.1\times10^{-3} + \Phi)$

$\therefore \Phi = -0.6109^{\text{rad}} + 2\pi = 5.672^{\text{rad}}$ or $325.0°$

$\therefore\ f(t) = \boxed{8.5\sin(290.9t + 325.0°)}$

(b) $8.5\sin(290.9t + 325.0°) = 8.5$

$\cos(290.9t + 235°) = \boxed{8.5\cos(290.9t - 125°)}$

(c) $8.5\cos(-125°)\cos\omega t + 8.5\sin 125°$

$\sin\omega t = \boxed{-4.875^{+}\cos 290.9t + 6.963\sin 290.9t}$

2.

(a) $-10\cos\omega t + 4\sin\omega t + \text{ACos}(wt + \Phi),\ \text{A} > 0,\ -180° < \Phi \leq 180°$

$\text{A} = \sqrt{116} = 10.770,\ \text{A}\cos\Phi = -10,\ \text{A}\sin\Phi = -4 \therefore \tan\Phi = 0.4,\ 3^d$ quad

$\therefore \Phi = 21.80° = 201.8°$, too large $\therefore \Phi = 201.8° - 360° = \boxed{-158.20°}$

(b) $200\cos(5t + 130°) = \text{F}\cos 5t + \text{G}\sin 5t \therefore \text{F} = 200\cos 130° = -128.56$

$\text{G} = -200\sin 130° = \boxed{-153.21°}$

(c) $i(t) = 5\cos 10t - 3\sin 10t = 0,\ 0 \leq t \leq 1s \therefore \dfrac{\sin 10t}{\cos 10t} = \dfrac{5}{3},\ 10t = 1.0304,$

$t = 0.10304s;$ also, $10t = 1.0304 + \pi, \boxed{t = 0.4172s;}\ 2\pi : 0.7314s$

(d) $0 < t < 10\text{ms},\ 10\cos 100\pi t \geq 12\sin 100\pi t;$ let $10\cos 100\pi\text{t} = 12\sin 100\pi\text{t}$

$\therefore \tan 100\pi\text{t} = \dfrac{10}{12},\ 100\pi t = 0.6947 \therefore t = 2.211\text{ms} \therefore \boxed{0 < t < 2.211\text{ms}}$

3.

(a) $f(t) = -50\cos\omega t - 30\sin\omega t = 58.31\cos(\omega t + 149.04°)$

$g(t) = 55\cos\omega t - 15\sin\omega t = 57.01\cos(\omega t + 15.255°)$

$\therefore$ ampl. of $f(t) = \boxed{58.31,}$ ampl. of $g(t) = \boxed{57.01}$

(b) $f(t)$ leads $g(t)$ by $149.04° - 15.255° = \boxed{133.78°}$

4.

$$i(t) = \text{A}\cos(\omega t - \theta),\ \text{L}(di/dt) + \text{R}i = \text{V}_m \cos\omega t$$

$$\therefore \text{L}[-\omega\text{A}\sin(\omega t - \theta)] + \text{RA}\cos(\omega t - \theta) = \text{V}_m \cos\omega t$$

$$\therefore -\omega\text{LA}\sin\omega t\cos\theta + \omega\text{LA}\cos\omega t\sin\theta$$

$$+\text{RA}\cos\omega t\cos\theta + \text{RA}\sin\omega t\sin\theta =$$

$$\text{V}_m\cos\omega t \therefore \omega\text{LA}\cos\theta = \text{RA}\sin\theta \text{ and } \omega LA\sin\theta + \text{RA}\cos\theta = \text{V}_m$$

$$\therefore \tan\theta = \frac{\omega\text{L}}{R} \therefore \omega\text{LA}\frac{\omega\text{L}}{\sqrt{\text{R}^2 + \omega^2\text{L}^2}} + \text{RA}\frac{\text{R}}{\sqrt{\text{R}^2 + \omega^2\text{L}^2}} = \text{V}_m$$

$$\therefore \left(\frac{\omega^2\text{L}^2}{\sqrt{\ }} + \frac{\text{R}^2}{\sqrt{\ }}\right)\text{A} = \text{V}_m \therefore \sqrt{\text{R}^2 + \omega^2\text{L}^2}\,\text{A} = \text{V}_m.\ \text{A} = \frac{\text{V}_m}{\sqrt{\text{R}^2 + \omega^2\text{L}^2}}$$

5. f = 13.56 MHz so $\omega = 2\pi f = \boxed{85.20 \text{ Mrad/s.}}$

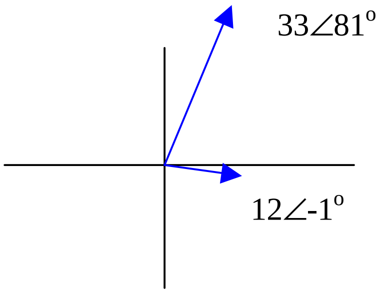
Delivering 300 W (peak) to a 5-Ω load implies that $\frac{V_m^2}{5} = 300$ so $\boxed{V_m = 38.73 \text{ V.}}$

Finally, $(85.2\times10^{6})(21.15\times10^{-3}) + \phi = n\pi$, n = 1, 3, 5, …

Since $(85.2\times10^{6})(21.15\times10^{-3}) = 1801980$, which is $573588^{+}\pi$, we find that

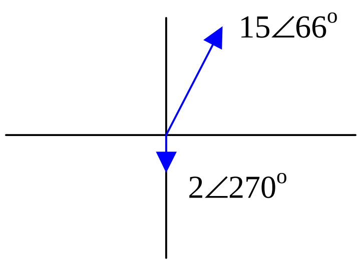
$F = 573587\pi - (85.2\times10^{6})(21.15\times10^{-3}) = -3.295 \text{ rad} = \boxed{-188.8^{o}.}$

6. (a) $-33\sin(8t - 9^o) \rightarrow -33\angle(-9-90)^o = 33\angle 81^o$
$12\cos(8t - 1^o) \rightarrow 12\angle -1^o$



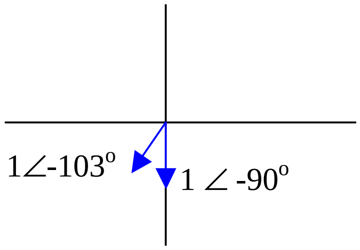
$-33\sin(8t - 9^o)$ leads $12\cos(8t - 1^o)$ by $81 - (-1) = 82^o$.

(b) $15\cos(1000t + 66^o) \rightarrow 15\angle 66^o$
$-2\cos(1000t + 450^o) \rightarrow -2\angle 450^o = -2\angle 90^o = 2\angle 270^o$



$15\cos(1000t + 66^o)$ leads $-2\cos(1000t + 450^o)$ by $66 - -90 = 156^o$.

(c) $\sin(t - 13^o) \rightarrow 1\angle -103^o$
$\cos(t - 90^o) \rightarrow 1\angle -90^o$



$\cos(t - 90^o)$ leads $\sin(t - 13^o)$ by $66 - -90 = 156^o$.

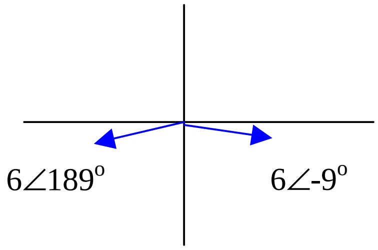
(d) $\sin t \rightarrow 1\angle -90^o$
$\cos(t - 90^o) \rightarrow 1\angle -90^o$

These two waveforms are *in phase*. Neither leads the other.

7. (a) $6\cos(2\pi 60t - 9^\circ) \rightarrow 6\angle{-9^\circ}$

$-6\cos(2\pi 60t + 9^\circ) \rightarrow 6\angle 189^\circ$

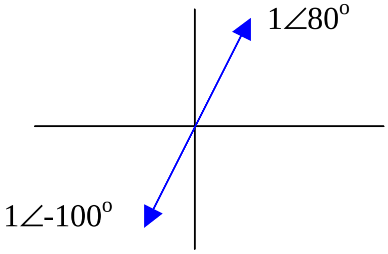
$-6\cos(2\pi 60t + 9^\circ)$ lags $6\cos(2\pi 60t - 9^\circ)$ by $360 - 9 - 189 = 162^\circ$.



(b) $\cos(t - 100^\circ) \rightarrow 1\angle{-100^\circ}$

$-\cos(t - 100^\circ) \rightarrow -1\angle{-100^\circ} = 1\angle 80^\circ$

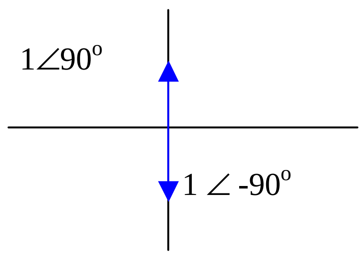
$-\cos(t - 100^\circ)$ lags $\cos(t - 100^\circ)$ by 180°.



(c) $-\sin t \rightarrow -1\angle{-90^\circ} = 1\angle 90^\circ$

$\sin t \rightarrow 1\angle{-90^\circ}$

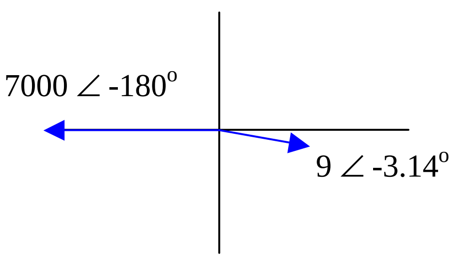
$-\sin t$ lags $\sin t$ by 180°.



(d) $7000\cos(t - \pi) \rightarrow 7000\angle{-\pi} = 7000\angle{-180^\circ}$

$9\cos(t - 3.14^\circ) \rightarrow 9\angle{-3.14^\circ}$

$7000\cos(t - \pi)$ lags $9\cos(t - 3.14^\circ)$ by $180 - 3.14 = 176.9^\circ$.



8. $v(t) = V_1 \cos \omega t \ - \ V_2 \sin \omega t$ [1]

We assume this can be written as a single cosine such that

$$v(t) = V_m \cos(\omega t + \phi) \ = \ V_m \cos \omega t \cos \phi \ - \ V_m \sin \omega t \sin \phi \quad [2]$$

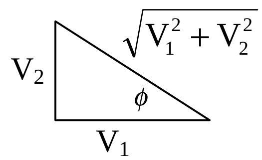
Equating terms on the right hand sides of Eqs. [1] and [2],

$$V_1 \cos \omega t - V_2 \sin \omega t \ = \ (Vm \cos \phi) \cos \omega t - (Vm \sin \phi) \sin \omega t$$

yields

$$V_1 = V_m \cos \phi \text{ and } \quad V_2 = V_m \sin \phi$$

Dividing, we find that $\dfrac{V_2}{V_1} = \dfrac{V_m \sin \phi}{V_m \cos \phi} = \tan \phi$ and $\boxed{\phi = \tan^{-1}(V_2 / V_1)}$



Next, we see from the above sketch that we may write $V_m = V_1 / \cos \phi$ or

$$V_m \ = \ \frac{V_1}{V_1 \Big/ \sqrt{V_1^2 + V_2^2}} \ = \ \boxed{\sqrt{V_1^2 + V_2^2}}$$

Thus, we can write $v(t) = V_m \cos(\omega t + \phi) \ = \ \sqrt{V_1^2 + V_2^2} \ \cos[\ \omega t \ + \tan^{-1}(V_2 / V_1)]$.

9. (a) In the range $0 \leq t \leq 0.5$, $v(t) = t/0.5$ V.
Thus, $v(0.4) = 0.4/0.5 = \boxed{0.8 \text{ V.}}$

(b) Remembering to set the calculator to radians, $\boxed{0.7709 \text{ V.}}$

(c) $\boxed{0.8141 \text{ V.}}$

(d) $\boxed{0.8046 \text{ V.}}$

10. (a) $V_{rms} = \left[\frac{V_m^2}{T}\int_0^T \cos^2 \omega t \, dt\right]^{1/2}$

$$= \left[\frac{V_m^2}{T}\int_0^T \cos^2 \frac{2\pi t}{T} \, dt\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2T}\int_0^T \left(1 + \cos\frac{4\pi t}{T}\right) dt\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2T}\int_0^T dt \; + \; \frac{V_m^2}{2T}\int_0^T \cos\frac{4\pi t}{T} \, dt\right]^{1/2}$$

$$= \left[\frac{V_m^2}{2T} T \; + \; \frac{V_m^2}{8\pi}\cos u \Big|_0^{4\pi}\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}}$$

(b) $\boxed{V_m = 110\sqrt{2} = 155.6 \text{ V}, \; 115\sqrt{2} = 162.6 \text{ V}, \; 120\sqrt{2} = 169.7 \text{ V}}$

11.

At $x - x: v_{oc} = \frac{3}{4} v_s = 15\cos 500t$ V

$\mathrm{R}_{th} = 5 + 20 \| 60 = 20\Omega$

$\therefore \mathrm{V}_m = 15\mathrm{V}, \mathrm{R} = 20\Omega, \ \omega\mathrm{L} = 10\Omega$

$$\therefore i_L = \frac{15}{\sqrt{20^2 + 10^2}} \cos\left(500t - \tan^{-1}\frac{10}{20}\right) = \boxed{0.6708\cos(500t - 26.57)\ \mathrm{A}}$$

12.

At $x - x$: $R_{th} = 80 \| 20 = 16\Omega$

$v_{oc} = -0.4(15\|85)\dfrac{80}{85}\cos 500t$

$\therefore v_{oc} = 4.8\cos 500t$ V

(a) $i_L = \dfrac{4.8}{\sqrt{16^2+10^2}}\cos\left(500t - \tan^{-1}\dfrac{10}{15}\right)$

$= \boxed{0.2544\cos(500t - 32.01°)\text{ A}}$

(b) $v_L = Li_L' = 0.02\times 0.02544(-500)$

$\sin(500t - 32.01°) = -2.544\sin(500t - 32.01°)$ V

$\therefore v_L = 2.544\cos(500t + 57.99°)$ V, i_x

$= \boxed{31.80\cos(500t + 57.99°)\text{ mA}}$

13.

(a) $$i = \frac{100}{\sqrt{500^2 + 800^2}} \cos\left(10^5 t - \frac{800}{500}\right) = 0.10600 \cos(10^5 t - 57.99°) \text{ A}$$

$$p_R = 0 \text{ when } i = 0 \therefore 10^5 t - \frac{57.99°}{180}\pi = \frac{\pi}{2},\ t = \boxed{25.83 \mu s}$$

(b) $$\pm v_L = \text{L}i' = 8\times 10^{-3} \times 0.10600 (-10^5) \sin(10^5 t - 57.99°)$$

$$\therefore v_L = -84.80 \sin(10^5 t - 57.99°)$$

$$\therefore p_L = v_L i = -8.989 \sin(10^5 t - 57.99°)$$

$$\cos(10^5 t - 57.99°) = -4.494 \sin(2\times 16^5 t - 115.989°)$$

$$\therefore p_L = 0 \text{ when } 2\times 10^5 t - 115.989° = 0°, 180°,$$

$$\therefore t = \boxed{10.121 \text{ or } 25.83 \mu s}$$

(c) $$p_s = v_s\, i_L = 10.600 \cos 10^5 t \cos(10^5 t - 57.99°)$$

$$\therefore p_s = 0 \text{ when } 10^5 t = \frac{\pi}{2},\ t = \boxed{15.708 \mu s} \text{ and also } t = \boxed{25.83 \mu s}$$

14. $v_s = 3\cos 10^5 t$ V, $i_s = 0.1\cos 10^5 t$ A

v_s in series with $30\Omega \rightarrow 0.1\cos 10^5 t \text{ A} \| 30\Omega$

Add, getting $0.2\cos 10^5 t \text{ A} \| 30\,\Omega$

change to 6 cos $10^5 t$ V in series with 30Ω; $30\Omega + 20\Omega = 50\Omega$

$$\therefore i_L = \frac{6}{\sqrt{50^2 + 10^2}} \cos\left(10^5 t - \tan^{-1}\frac{10}{50}\right) = 0.11767\cos(10^5 t - 11.310°)\text{ A}$$

At $t = 10\mu s$, $10^5 t = 1 \therefore i_L = 0.1167\cos(1^{\text{rad}} - 11.310°) = \boxed{81.76\text{mA}}$

$\therefore v_L = 0.11767 \times 10\cos(1^{\text{rad}} - 11.30° + 90°) = \boxed{-0.8462\text{V}}$

15.

$$\therefore v_{oc} = \cos 500t \text{ V}$$

$$\therefore \cos 500t = 100\,(0.8 i_{sc})$$

$$\therefore i_{sc} = \frac{1}{80}\cos 500t \text{ A} \therefore \mathrm{R}_{th} = \frac{v_{oc}}{i_{sc}} = 80\Omega$$

$$\therefore i_L = \frac{1}{\sqrt{80^2 + 150^2}}\cos\left(500t - \tan^{-1}\frac{150}{80}\right)$$

$$\boxed{i_L = 5.882\cos(500t - 61.93°)\text{ mA}}$$

16. $v_{s1} = \mathrm{V}_{s2} = 120\cos 120\pi t\ \mathrm{V}$

$$\frac{120}{60} = 2\mathrm{A},\ \frac{120}{12} = 1\mathrm{A},\ 2+1 = 3\mathrm{A},\ 60 \| 120 = 40\Omega$$

$$3\times 40 = 120\,\mathrm{V}, \omega\mathrm{L} = 12\pi = 37.70\Omega$$

$$\therefore i_L = \frac{120}{\sqrt{40^2 + 37.70^2}}\cos\left(120\pi t - \tan^{-1}\frac{37.70}{40}\right)$$

$$= 2.183\cos(120\pi t - 43.30°)\ \mathrm{A}$$

(a) $\therefore \omega_L = \frac{1}{2}\times 0.1\times 2.183^2\cos^2(120\pi t - 43.30°)$

$$= \boxed{0.2383\cos^2(120\pi t - 43.30°)\ \mathrm{J}}$$

(b) $\omega_{L,av} = \frac{1}{2}\times 0.2383 = \boxed{0.11916\ \mathrm{J}}$

17. $\uparrow\ v_{s1} = 120\cos 400t\text{ V},\ v_{s2} = 180\cos 200t\text{ V}$

$$\frac{120}{60} = 2\text{A},\ \frac{180}{120} = 1.5\text{A},\ 60 \| 120 = 40\Omega$$

$$2\times 40 = 80\text{V},\ 1.5\times 40 = 60\text{V}$$

$$i_L = \frac{80}{\sqrt{40^2+40^2}}\cos(400t-45°) + \frac{60}{\sqrt{40^2+20^2}}\cos(200t-26.57°)\text{ A}$$

or $\boxed{i_L = 1.4142\cos(400t-45°) + 1.3416\cos(200t-26.57°)\text{ A}}$

18.
$$R_i = \infty,\ R_o = 0,\ A = \infty,\ \text{ideal},\ R_1 C_1 = \frac{L}{R}$$

$$i_{upper} = -\frac{V_m \cos \omega t}{R},\ i_{lower} = \frac{v_{out}}{R_1}$$

$$\therefore i_{c1} = i_{upper} + i_{lower} = \frac{i}{R_1}(v_{out} - V_m \cos \omega t) = -C_1 v'_{out}$$

$$\therefore V_m \cos \omega t = v_{out} + R_1 C_1 v'_{out} = v_{out} + \frac{L}{R} v'_{out}$$

For RL circuit, $V_m \cos \omega t = v_r + L \dfrac{d}{dt}\left(\dfrac{v_R}{R}\right)$

$$\therefore V_m \cos \omega t = v_R + \frac{L}{R} v'_R$$

By comparison, $v_R = v_{out}$

19.

(a) $V_m \cos \omega t = Ri + \frac{1}{C}\int idt$ (ignore I.C)

$$\therefore \boxed{-\omega V_m \sin \omega t = Ri' + \frac{1}{C} i}$$

(b) Assume $i = A\cos(\omega t + \Phi)$

$$\therefore -\omega V_m \sin \omega t = -R\omega A \sin(\omega t + \Phi) + \frac{A}{C}\cos(\omega t + \Phi)$$

$$\therefore -\omega V_m \sin \omega t = -R\omega A \cos\Phi \sin \omega t \text{-} R\omega A \sin\Phi \cos \omega t + \frac{A}{C}\cos \omega t \cos\Phi - \frac{A}{C}\sin \omega t \sin\Phi$$

Equating terms on the left and right side,

[1] $R\omega A \sin\Phi = \frac{A}{C}\cos\Phi \therefore \tan\Phi = \frac{1}{\omega CR}$ so $\Phi = \tan^{-1}(1/\omega CR)$, and

[2] $-\omega V_m = -R\omega A \frac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} - \frac{A}{C}\frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$

$$\therefore \omega V_m = \frac{A}{C}\left[\frac{R^2\omega^2 C^2 + 1}{\sqrt{1+\omega^2 C^2 R^2}}\right] = \frac{A}{C}\sqrt{1+\omega^2 C^2 R^2} \therefore A = \frac{\omega C V_m}{\sqrt{1+\omega^2 C^2 R^2}}$$

$$\therefore \boxed{i = \frac{\omega C V_m}{\sqrt{1+\omega^2 C^2 R^2}} \cos\left(\omega t + \tan^{-1}\frac{1}{\omega CR}\right)}$$

20. (a) $7 \angle -90^o = \boxed{-j\,7}$

(b) $3 + j + 7 \angle -17^o = 3 + j + 6.694 - j\,2.047 = \boxed{9.694 - j\,1.047}$

(c) $14e^{j15^o} = 14 \angle 15^o = 14 \cos 15^o + j\,14 \sin 15^o = \boxed{13.52 + j\,3.263}$

(d) $1 \angle 0^o = \boxed{1}$

(e) $-2\,(1 + j\,9) = -2 - j\,18 = \boxed{18.11 \angle -96.34^o}$

(f) $3 = \boxed{3 \angle 0^o}$

21. (a) $3 + 15\angle -23^{o} = 3 + 13.81 - j\,5.861 = 16.81 - j\,5.861$

(b) $(j\,12)(17\angle 180^{o}) = (12\angle 90^{o})(17\angle 180^{o}) = 204\angle 270^{o} = -j\,204$

(c) $5 - 16(9 - j\,5)/ (33\angle -9^{o}) = 5 - (164\angle -29.05^{o})/ (33\angle -9^{o})$

$= 5 - 4.992\angle -20.05^{o} = 5 - 4.689 - j\,1.712 = \boxed{0.3109 + j\,1.712}$

22. (a) $5 \angle 9^\circ - 9 \angle -17^\circ = 4.938 + j\,0.7822 - 8.607 + j\,2.631 = -3.669 - j\,1.849$

$= \boxed{4.108 \angle -153.3^\circ}$

(b) $(8 - j\,15)(4 + j\,16) - j = 272 + j\,68 - j = 272 + j\,67 = \boxed{280.1 \angle 13.84^\circ}$

(c) $(14 - j\,9)/\,(2 - j\,8) + 5 \angle -30^\circ = (16.64 \angle -32.74^\circ)/\,(8.246 \angle -75.96^\circ) + 4.330 - j\,2.5$

$= 1.471 + j\,1.382 + 4.330 - j\,2.5 = 5.801 - j\,1.118 = \boxed{5.908 \angle -10.91^\circ}$

(d) $17 \angle -33^\circ + 6 \angle -21^\circ + j\,3 = 14.26 - j\,9.259 + 5.601 - j\,2.150 + j\,3$

$= 19.86 - j\,8.409 = \boxed{21.57 \angle -22.95^\circ}$

23. (a) $e^{j14^o} + 9\angle 3^o - (8 - j\,6)/\,j^2 = 1\angle 14^o + 9\angle 3^o - (8 - j\,6)/\,(-1)$

$= 0.9703 + j\,0.2419 + 8.988 + j\,0.4710 + 8 - j\,6 = 17.96 - j\,5.287 = \boxed{18.72\angle -16.40^o}$

(b) $(5\angle 30^o)/\,(2\angle -15^o) + 2\,e^{j5^o}/\,(2 - j\,2)$

$= 2.5\angle 45^o + (2\angle 5^o)/\,(2.828\angle -45^o) = 1.768 + j\,1.768 + 0.7072\angle 50^o$

$= 1.768 + j\,1.768 + 0.4546 + j\,0.5418$

$= 2.224 + j\,2.310 = \boxed{3.207\angle 46.09^o}$

24.

(a) $5\angle -110° = \boxed{-1.7101 - j4.698}$

(b) $6e^{j160°} = \boxed{-5.638 + j2.052}$

(c) $(3 + j6)(2\angle 50°) = \boxed{-5.336 + j12.310}$

(d) $-100 - j40 = \boxed{107.70\angle -158.20°}$

(e) $2\angle 50° + 3\angle -120° = \boxed{1.0873\angle -101.37°}$

25.

(a) $40\angle-50° - 18\angle 25° = \boxed{39.39\angle -76.20°}$

(b) $3 + \dfrac{2}{j} + \dfrac{2-j5}{1+j2} = \boxed{4.050^{-}\angle -69.78°}$

(c) $(2.1\angle 25°)^3 = 9.261\angle 75° = \boxed{2.397 + j8.945^{+}}$

(d) $0.7e^{j0.3} = 0.7\angle 0.3^{\text{rad}} = \boxed{0.6687 + j0.2069}$

26.

$i_c = 20e^{(40t+30°)}$ A $\therefore v_c = 100\int 20e^{j(40t+30°)}dt$

$v_c = -j50e^{j(40t+30°)}$, $i_R = -j10e^{j(40t+30°)}$ A

$\therefore i_L = (20 - j10)e^{j(40t+30°)}, v_L = j40\times 0.08(20 - j10)e^{j(40t+30°)}$

$\therefore v_L = (32 + j64)e^{j(40t+30°)}$ V $\therefore v_s = (32 + j64 - j50)e^{j(40t+30°)}$

$\therefore v_s = \boxed{34.93e^{j(40t-53.63°)} \text{ V}}$

27.

$$i_L = 20e^{j(10t+25°)} \text{ A}$$

$$v_L = 0.2\frac{d}{dt}[20e^{j(10t+25°)}] = j40e^{(10t=25°)}$$

$$v_R = 80e^{j(10t+25°)}$$

$$v_s = (80+j40)e^{j(10t+25°)}, i_c = 0.08(80+j40)\,j10e^{j(10t+25°)}$$

$$\therefore i_c = (-32+j64)e^{j(10t+25°)} \therefore i_s = (-12+j64)e^{j(10t+25°)}$$

$$\therefore i_s = \boxed{65.12e^{j(10t+125.62°)} \text{ A}}$$

28. $80\cos(500t - 20°)\text{ V} \rightarrow 5\cos(500t + 12°)\text{ A}$

(a) $v_s = 40\cos(500t + 10°) \therefore i_{out} = \boxed{2.5\cos(500t + 42°)\text{ A}}$

(b) $v_s = 40\sin(500t + 10°) = 40\cos(500t - 80°)$

$\therefore i_{out} = \boxed{2.5\cos(500t - 48°)\text{ A}}$

(c) $v_s = 40e^{j(500t+10°)} = 40\cos(500t + 10°)$

$+ j40\sin\ (500t + 10°) \therefore i_{out} = \boxed{2.5e^{j(500t+42°)}\text{ A}}$

(d) $v_s = (50 + j20)e^{j500t} = 53.85^{+}e^{j21.80° + j500t}$

$\therefore i_{out} = \boxed{3.366e^{j(500t+53.80°)}\text{ A}}$

29.

(a) $12\sin(400t + 110°)\text{ A} \rightarrow \boxed{12\angle 20°\text{A}}$

(b) $-7\sin 800t - 3\cos 800t \rightarrow j7 - 3$
$= -3 + j7 = \boxed{7.616\angle 113.20°\text{ A}}$

(c) $4\cos(200t - 30°) - 5\cos(200t + 20°)$
$\rightarrow 4\angle -30° - 5\angle 20° = \boxed{3.910\angle -108.40°\text{ A}}$

(d) $\omega = 600,\ t = 5\text{ms}: 70\angle 30°\text{ V}$
$\rightarrow 70\cos(600 \times 5 \times 10^{-3\text{rad}} + 30°) = \boxed{-64.95^{+}\text{ V}}$

(e) $\omega = 600,\ t = 5\text{ms}: 60 + j40\text{ V} = 72.11\angle 146.3°$
$\rightarrow 72.11\cos(3^{\text{rad}} + 146.31°) = \boxed{53.75^{+}\text{ V}}$

30. $\omega = 4000,\ t = 1\text{ms}$

(a) $\mathbf{I}_x = 5\angle -80°\ \text{A}$

$\therefore i_x = 4\cos(4^{\text{rad}} - 80°) = \boxed{-4.294\ \text{A}}$

(b) $\mathbf{I}_x = -4 + j1.5 = 4.272\angle 159.44°\ \text{A}$

$\therefore i_x = 4.272\cos(4^{\text{rad}} + 159.44°) = \boxed{3.750^{-}\ \text{A}}$

(c) $v_x(t) = 50\sin(250t - 40°)$

$= 50\cos(250t - 130°) \rightarrow \mathbf{V}_x = \boxed{50\angle -103°\ \text{V}}$

(d) $v_x = 20\cos 108t - 30\sin 108t$

$\rightarrow 20 + j30 = \boxed{36.06\angle 56.31°\ \text{V}}$

(e) $v_x = 33\cos(80t - 50°) + 41\cos(80t - 75°)!$

$\rightarrow 33\angle -50° + 41\angle -75° = \boxed{72.27\angle -63.87°\ \text{V}}$

31. $V_1 = 10\angle 90°\,\text{mV},\ \omega = 500;\ V_2 = 8\angle 90°\,\text{mV},$

$\omega = 1200,\ \text{M by } -5,\ t = 0.5\text{ms}$

$$v_{out} = (-5)\,[10\cos(500\times 0.5\times 10^{-3\text{rad}} + 90°)$$

$$+8\cos(1.2\times 0.5 + 90°)]$$

$$= 50\sin 0.25^{\text{rad}} + 40\sin 0.6^{\text{rad}} = \boxed{34.96\text{mV}}$$

32. Begin with the inductor:
$(2.5 \angle 40^o)\ (j500)\ (20\times10^{-3}) = 25\angle 130^o$ V across the inductor and the 25-Ω resistor.
The current through the 25-Ω resistor is then $(25\angle 130^o) / 25 = 1\angle 130^o$ A.

The current through the unknown element is therefore $25\angle 130^o + 1\angle 130^o = 26\angle 130^o$; this is the same current through the 10-Ω resistor as well. Thus, KVL provides that

$$\mathbf{V}_s = 10(26\angle 130^o) + (25 \angle -30^o) + (25\angle 130^o) = 261.6 \angle 128.1^o$$

and so $\boxed{v_s(t) = 261.6 \cos (500t + 128.1^o) \text{ V.}}$

33. $\omega = 5000$ rad/s.

(a) The inductor voltage = $48\angle 30^o = j\omega L\ \mathbf{I}_L = j(5000)(1.2\times 10^{-3})\ \mathbf{I}_L$
So $\mathbf{I}_L = 8\angle -60^o$ and the total current flowing through the capacitor is
$10\angle 0^o - \mathbf{I}_L = 9.165\angle 49.11^o$ A and the voltage $\mathbf{V}_1$ across the capacitor is

$\mathbf{V}_1 = (1/j\omega C)(9.165\angle 49.11^o) = -j2\ (9.165\angle 49.11^o) = 18.33\angle -40.89^o$ V.

Thus, $\boxed{v_1(t) = 18.33 \cos (5000t - 40.89^o) \text{ V.}}$

(b) $\mathbf{V}_2 = \mathbf{V}_1 + 5(9.165\angle 49.11^o) + 60\angle 120^o = 75.88\angle 79.48^o$ V
$\therefore \boxed{v_2(t) = 75.88 \cos (5000t + 79.48°) \text{ V}}$

(c) $\mathbf{V}_3 = \mathbf{V}_2 - 48\angle 30^o = 75.88\angle 79.48^o - 48\angle 30^o = 57.70\angle 118.7^o$ V
$\therefore \boxed{v_3(t) = 57.70 \cos (5000t + 118.70°) \text{ V}}$

34. $\mathbf{V}_R = 1\angle 0^o$ V, $\mathbf{V}_{series} = (1 + j\omega - j/\omega)(1\angle 0^o)$

$V_R = 1$ and $V_{series} = \sqrt{1 + (\omega - 1/\omega)^2}$

We desire the frequency w at which $V_{series} = 2V_R$ or $V_{series} = 2$
Thus, we need to solve the equation $1 + (\omega - 1/\omega)^2 = 4$
or $\omega^2 - \sqrt{3}\,\omega - 1 = 0$

Solving, we find that $\boxed{\omega = 2.189 \text{ rad/s.}}$

35. With an operating frequency of ω = 400 rad/s, the impedance of the 10-mH inductor is $j\omega L = j4\ \Omega$, and the impedance of the 1-mF capacitor is $-j/\omega C = -j2.5\ \Omega$.

$\therefore \mathbf{V}_c = 2\angle 40°(-j2.5) = 5\angle -50°\ \text{A}$

$\therefore \mathbf{I}_L = 3 - 2\angle 40° = 1.9513\angle -41.211°\ \text{A}$

$\therefore \mathbf{V}_L = 4 \times 1.9513\angle 90° - 4.211° = 7.805^{+}\angle 48.79°\ \text{V}$

$\therefore \mathbf{V}_x = \mathbf{V}_L - \mathbf{V}_c = 7.805^{+}\angle 48.79° - 5\angle -50°$

$\therefore \mathbf{V}_x = 9.892\angle 78.76°\ \text{V},\ \boxed{v_x = 9.892\cos(400t + 78.76°)\ \text{V}}$

36.

If $\mathrm{I}_{si} = 2\angle 20°\,\mathrm{A},\ \mathrm{I}_{s2} = 3\angle -30°\ \mathrm{A} \rightarrow \mathrm{V}_{out} = 80\angle 10°\,\mathrm{V}$

$\mathrm{I}_{s1} = \mathrm{I}_{s2} = 4\angle 40°\,\mathrm{A} \rightarrow \mathrm{V}_{out} = 90 - j30\,\mathrm{V}$

Now let $\mathrm{I}_{s1} = 2.5\angle -60°\,\mathrm{A}$ and $\mathrm{I}_{s2} = 2.5\angle 60°\,\mathrm{A}$

Let $\mathrm{V}_{\mathrm{out}} = \mathrm{AI}_{s1} + \mathrm{BI}_{s2}\ \therefore 80\angle 10° = \mathrm{A}(2\angle 20°) + \mathrm{B}(3\angle -30°)$

and $90 - j30 = (\mathrm{A}+\mathrm{B})(4\angle 40°) \therefore \mathrm{A}+\mathrm{B} = \dfrac{90 - j30}{4\angle 40°} = 12.45^{+} - j20.21$

$$\therefore \frac{80\angle 10°}{2\angle 20°} = \mathrm{A} + \mathrm{B}\frac{3\angle -30°}{2\angle 20} \therefore \mathrm{A} = 40\angle -10° - \mathrm{B}(1.5\angle -50°)$$

$$\therefore 12.415^{+} - j20.21 - \mathrm{B} = 40\angle -10° - \mathrm{B}(1.5\angle -50°)$$

$$\therefore 12.415^{+} - j20.21 - 40\angle -10° = \mathrm{B}(1 - 1.5\angle -50°)$$

$$= \mathrm{B}(1.1496\angle -88.21°)$$

$$\therefore \mathrm{B} = \frac{30.06\angle -153.82°}{1.1496\angle -88.21°} = 10.800 - j23.81$$

$$= 26.15\angle -65.61°$$

$$\therefore \mathrm{A} = 12.415^{+} - j20.21 - 10.800 + j23.81$$

$$= 3.952^{-}\angle 65.87°$$

$$\therefore \mathrm{V}_{out} = (3.952\angle 65.87°)(2.5\angle -60°)$$

$$+(26.15^{-}\angle -65.61°)\ (2.5\angle 60°) = \boxed{75.08\angle -4.106°\,\mathrm{V}}$$

37.

(a) $\omega = 800: 2\mu\text{F} \rightarrow -j625,\ 0.6\text{H} \rightarrow j480$

$$\therefore Z_{in} = \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480}$$

$$= \boxed{478.0 + j175.65\Omega}$$

(b) $$\omega = 1600: Z_{in} = \frac{300(-j312.5)}{300 - j312.5}$$

$$+ \frac{600(j960)}{600 + j960} = \boxed{587.6 + j119.79\Omega}$$

38.

(a) $(10+j10)\|(-j5)=\dfrac{50-j50}{10+j5}=\dfrac{10-j10}{2+j1}\;\dfrac{2-j1}{2-j1}$

$=2-j6\,\Omega \therefore \mathrm{Z}_{in}\boxed{=22-j6\,\Omega}$

(b) $\mathrm{SC}a,b:\ 20\|10=6.667,\ (6.667-j5)\|j10$

$=\dfrac{50+j66.67}{6.667+j5}=\dfrac{150+j200}{20+j15}=\dfrac{30+j40}{4+j3}\times\dfrac{4-j3}{4-j3}$

$=Z_{in}\ \therefore Z_{in}(1.2+j1.6)(4-j3)=\boxed{9.6+j2.8\,\Omega}$

39.

$\omega = 800: 2\mu\text{F} \rightarrow -j625,\ 0.6\text{H} \rightarrow j480$

$\therefore \text{Z}_{in} = \dfrac{300(-j625)}{300-j625} + \dfrac{600(j480)}{600+j480}$

$= 478.0 + j175.65\Omega$

$\therefore \text{I} = \dfrac{120}{478.0+j175.65} \times \dfrac{-j625}{300-j625}$

or $\text{I} = 0.2124\angle -45.82°\ \text{A}$

Thus, $\boxed{i(t) = 212.4 \cos(800t - 45.82^{\circ})\ \text{mA.}}$

40.

(a) $3\Omega\;2\text{mH}:\ \mathbf{V} = (3\angle -20°)\ (3 + j4) = 15{,}000\angle 33.13°\ \text{V}$

(b) $3\Omega 125\mu\text{F}:\ \mathbf{V} = (3\angle -20°)\ (3 - j4) = \boxed{15{,}000\angle -73.3°\,\text{V}}$

(c) $3\Omega\;2\text{mH}\ 125\mu\text{F}:\ \mathbf{V} = (3\angle -20°)\ 3 = \boxed{9{,}000\angle -20°\ \text{V}}$

(d) same: $\omega = 4000 \therefore \mathbf{V} = (3\angle -20°)\ (3 + j8 - j2)$

$\therefore \mathbf{V} = (3\angle -20°)\ (3 + j6) = \boxed{20.12\angle 43.43°\,\text{V}}$

41.

(a) $C = 20\mu F,\ \omega = 100$

$$Z_{in} = \frac{1}{\frac{1}{200} + \frac{1}{j1000} + j1000 \times 20 \times 10^{-6}} = \frac{1}{0.005 - j0.01 + j0.002}$$

$$\therefore Z_{in} = \frac{1}{0.005 + j0.001} = \boxed{196.12\angle -11.310°\Omega}$$

(b) $\omega = 100 \therefore Z_{in} = \dfrac{1}{0.005 - j0.001 + j1000C} = 125\angle = \dfrac{1}{0.008\angle}$

$\therefore 0.005^2 + (100C - 0.001)^2 = 0.008^2 \therefore 100C - 0.001 =$

$\pm 6.245 - \times 10^{-3}$, $C = \boxed{72.45^{-}\mu F}$

(c) $C = 20\mu F \therefore Z_{in} = \dfrac{1}{0.0005 - j0.1/\omega + j2\times 10^{-5}\omega} = 100\angle = \dfrac{1}{0.01\angle}$

$$\therefore 0.005^2 + \left(2\times 10^{-5}\omega - \frac{0.1}{\omega}\right)^2 = 0.0001, \left(2\times 10^{-5} - \frac{0.1}{\omega}\right)^2 = 7.5\times 10^{-5}$$

$$\therefore 2\times 10^{-5} - \frac{0.01}{\omega} \mp 866.0\times 10^{-5} = 0 \therefore 2\times 10^{-5}\omega^2 \mp 866.0\times 10^{-5}\omega - 0.1 = 0$$

use $-$ sign: $\omega = \dfrac{866.0\times 10^{-5} \pm \sqrt{7.5\times 10^{-5} + 8\times 10^{-6}}}{4\times 10^{-5}} = 444.3$ and < 0

use + sign: $\omega = \dfrac{-866.0\times 10^{-5} \pm \sqrt{7.5\times 10^{-5} + 8\times 10^{-6}}}{4\times 10^{-5}} = 11.254$ and <0

$\therefore \omega = \boxed{11.254}$ and $\boxed{444.3 \text{ rad/s}}$

42.

(a)

$$\left|\frac{1}{\frac{1}{jx}+\frac{1}{30}}\right| = 25 = \frac{1}{0.04} \therefore \frac{1}{900}+\frac{1}{x^2} = 0.0016$$

$$\therefore \mathrm{X} = 45.23\Omega = 0.002\mathrm{W},\ \omega = \boxed{2261\mathrm{rad/s}}$$

(b)

$$\angle \mathrm{Y}_{in} = -25° = \angle \text{ of } \left(\frac{1}{30} - j\frac{1}{x}\right) = \tan^{-1}\frac{-30}{x}$$

$$\therefore x = 64.34 = 0.02\omega,\ \omega = \boxed{3217\mathrm{rad/s}}$$

(c)

$$\mathrm{Z}_{in} = \frac{30(j0.02\omega)}{30+j0.02\omega}\times\frac{30-j0.092\omega}{30-j0.02\omega} = \frac{0.012\omega^2+j18\omega}{900+0.0004\omega^2}$$

$$\therefore 0.012\omega^2 = 25\ (900+0.0004\omega^2)$$

$$\therefore 0.012\omega^2 = 0.01\omega^2 + 22,500,\ \omega = \boxed{3354\mathrm{rad/s}}$$

(d)

$$18\omega = 10\,(900+0.0004\omega^2),\ 0.004\omega^2 - 18\omega + 9000 = 0,$$

$$\omega^2 - 4500\omega + 2.25\times 10^6 = 0$$

$$\omega = \frac{4500 \pm \sqrt{20.25\times 10^6 - 9\times 10^6}}{2} = \frac{4500 \pm 3354}{2} = \boxed{572.9,\ 3927\mathrm{rad/s}}$$

43. With an operating frequency of ω = 400 rad/s, the impedance of the 10-mH inductor is $j\omega L = j4\ \Omega$, and the impedance of the 1-mF capacitor is $-j/\omega C = -j2.5\ \Omega$.

$$\therefore \mathrm{V}_c = 2\angle 40°(-j2.5) = 5\angle -50°\ \mathrm{A}$$

$$\therefore \mathrm{I}_L = 3 - 2\angle 40° = 1.9513\angle -41.211°\ \mathrm{A}$$

$$\mathrm{I}_L = \frac{2\angle 40°(\mathrm{R}_2 - j2.5)}{\mathrm{R}_1 + j4}$$

$$\therefore \mathrm{R}_1 + j4 = \frac{2\angle 40°(\mathrm{R}_2 - j2.5)}{1.9513\angle -41.21°}$$

$$= 1.0250\angle 81.21°(R_2 - j2.5)$$

$$= \mathrm{R}_2(1.0250\angle 81.21°) + 2.562\angle -8.789°$$

$$= 0.15662\mathrm{R}_2 + j1.0130\,\mathrm{R}_2 + 2.532 - j0.3915$$

$$\therefore \mathrm{R}_1 = 2.532 + 0.15662\mathrm{R}_2,\ 4 = 1.0130\mathrm{R}_2 - 0.395^-$$

$$\therefore \boxed{\mathrm{R}_2 = 4.335^+\,\Omega,\ \mathrm{R}_1 = 3.211\,\Omega}$$

44. $\omega = 1200$ rad/s.

(a) $$Z_{in} = \frac{-j\times(200+j80)}{200+j(80-x)} = \frac{(80x-j200x)[200+j(x-80)]}{40,000+6400-160x+x^2}$$

$$X_{in} = 0 \therefore -40,000x + 80x^2 - 6400x = 0$$

$$\therefore 46,400 = 80x,\ x = 580\,\Omega = \frac{1}{1200c} \therefore \boxed{C = 14.368\mu F}$$

(b) $$Z_{in} = \frac{80X - j200X}{200+j(80-X)} \left|Z_{in}\right| = 100$$

$$\therefore \frac{6400X^2 + 40,000X^2}{40,000+6400-160X+X^2} = 10,000$$

$$\therefore 0.64X^2 + 4X^2 = X^2 - 160X + 46,400$$

$$\therefore 3.64X^2 + 160X - 46,400 = 0,$$

$$X = \frac{-160 \pm \sqrt{25,600+675,600}}{7.28} = \frac{-160 \pm 837.4}{7.28}$$

$$\therefore X = 93.05^{-} (>0) = \frac{1}{1200C} \therefore \boxed{C = 8.956\mu F}$$

45. At ω = 4 rad/s, the 1/8-F capacitor has an impedance of $-j/\omega C$ = $-j2\ \Omega$, and the 4-H inductor has an impedance of $j\omega L = j16\ \Omega$.

(a) $$\text{abOC}: \mathrm{Z}_{in} = \frac{(8+j16)(2-j2)}{10+j14} = \frac{16(3+j1)}{10+j14}$$

$$= \boxed{2.378 - j1.7297\,\Omega}$$

(b) $$\text{abSC}: \mathrm{Z}_{in} = (8 \| -j2) + (2 \| j16) = \frac{-j16}{8-j2} + \frac{j32}{2+j16}$$

$$\therefore Z_{in} = \boxed{2.440 - j1.6362\,\Omega}$$

46. $f = 1$ MHz, $\omega = 2\pi f = 6.283$ Mrad/s

2 μF	$\rightarrow -j0.07958\ \Omega$	$= \mathbf{Z}_1$
3.2 μH	$\rightarrow j20.11\ \Omega$	$= \mathbf{Z}_2$
1 μF	$\rightarrow -j0.1592\ \Omega$	$= \mathbf{Z}_3$
1 μH	$\rightarrow j6.283\ \Omega$	$= \mathbf{Z}_4$
20 μH	$\rightarrow j125.7\ \Omega$	$= \mathbf{Z}_5$
200 pF	$\rightarrow -j795.8\ \Omega$	$= \mathbf{Z}_6$

The three impedances at the upper right, $\mathbf{Z}_3$, 700 kΩ, and $\mathbf{Z}_3$ reduce to $-j0.01592\ \Omega$

Then we form $\mathbf{Z}_2$ in series with $\mathbf{Z}_{eq}$: $\mathbf{Z}_2 + \mathbf{Z}_{eq} = j20.09\ \Omega$.

Next we see $10^6 \parallel (\mathbf{Z}_2 + \mathbf{Z}_{eq}) = j20.09\ \Omega$.

Finally, $\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_4 + j20.09 = \boxed{j26.29\ \Omega.}$

47.

$2\,\text{H} \rightarrow j2$, $1\text{F} \rightarrow -j1$ Let $\text{I}_{\in} = 1\angle 0°\,\text{A}$

$\therefore \text{V}_L = j2\text{V} \therefore \text{I}_c = \text{I}_{in} + 0.5v_L = 1 + j1$

$\therefore \text{V}_{in} = j2 + (1 + j1)(-j1) = 1 + j1$

$$\therefore \text{V}_{in} = \frac{1\angle 0°}{\text{V}_{in}} = \frac{1}{1 + j1}\,\frac{1 - j1}{1 - j1} = 0.5 - j0.55$$

$$\text{Now}\; 0.5\,s \rightarrow \boxed{2\,\Omega,}\; -j0.5\text{S} = \frac{1}{j2} \rightarrow \boxed{2\text{H}}$$

48.

(a) $\omega = 500,\ Z_{inRLC} = 5 + j10 - j1 = 5 + j9$

$$\therefore Y_{inRLC} = \frac{1}{5 + j9} = \frac{5 - j9}{106} \therefore Y_c = \frac{9}{106} = 500C$$

$$\therefore C = \frac{9}{53,000} = \boxed{169.81\mu F}$$

(b) $R_{in,ab} = \frac{106}{5} = \boxed{21.2\,\Omega}$

(c) $\omega = 1000 \therefore Y_{in,ab} = j\frac{9}{53} + \frac{1}{5 + j20 - j0.5}$

$$= \boxed{0.012338 + j0.12169\,S} \text{ or } \boxed{0.12232\angle 84.21°\,S}$$

49.

(a) $R_{in} = 550\,\Omega : Z_{in} = 500 + \dfrac{j0.1\omega}{100 + j0.001\omega}$

$\therefore Z_{in} = \dfrac{50,000 + j0.6\omega}{100 + j0.001\omega} \times \dfrac{100 - j0.001\omega}{100 - j0.001\omega}$

$\therefore Z_{in} = \dfrac{5\times10^6 + 0.0006\omega^2 + j(60\omega - 50\omega)}{10^4 + 10^{-6}\omega^2}$

$\therefore R_{in} = \dfrac{5\times10^6 + 0.006\omega^2}{10^4 + 10^{-6}\omega^2} = 550 \therefore 5.5\times10^6$

$+ 5.5\times10^{-4}\omega^2 = 5\times10^6 \times 10^{-4}\omega^2$

$\therefore 0.5\times10^{-4}\omega^2 = 0.5\times10^6,\ \omega^2 = 10^{10},\ \omega = \boxed{10^5\,\text{rad/s}}$

(b) $X_{in} = 50\,\Omega = \dfrac{10\omega}{10^4 + 10^{-6}\omega^2} = 0.5\times10^6 + 0.5\times10^{-4}\omega^2 - 10\omega$

$= 0,\ \omega^2 - 2\times10^5\omega + 10^{10} = 0$

$\therefore \omega = \dfrac{2\times10^5 \pm \sqrt{4\times10^{10} - 4\times10^{10}}}{2} = 10^5 \therefore \omega = \boxed{10^5\,\text{rad/s}}$

(c) $G_{in} = 1.8\times10^{-3} : Y_{in} = \dfrac{100 + j0.001\omega}{50,000 + j0.6\omega} \times \dfrac{50,000 - j0.6\omega}{50,000 - j0.6\omega}$

$= \dfrac{5\times10^6 + 6\times10^{-4}\omega^2 + j(50\omega - 6\omega)}{25\times10^8 + 0.36\omega^2}$

$\therefore 1.8\times10^3 = \dfrac{5\times10^6 + 6\times10^{-4}\omega^2}{25\times10^8 + 0.36\omega^2}$

$\therefore 5\times10^6 + 6\times10^{-4}\omega^2 = 4.5\times10^6 + 648\times10^{-6}\omega^2$

$\therefore 0.5\times10^6 = 48\times10^{-6}\omega^2 \therefore \omega = \boxed{102.06\,\text{Krad/s}}$

(d) $B_{in} = 1.5\times10^{-4} = \dfrac{-10\omega}{25\times10^8 + 0.36\omega^2}$

$\therefore 10\omega = 37.5\times10^4 + 54\times10^{-6}\omega^2$

$\therefore 54\times10^{-6}\omega^2 - 10\omega + 37.5\times10^4 = 0,$

$\omega = 10 \pm \dfrac{\sqrt{100 - 81}}{108\times10^{-6}} = \boxed{52.23 \text{ and } 133.95\,\text{krad/s}}$

50.

(a) $\mathbf{V}_1 = \dfrac{\mathbf{I}_1}{\mathbf{Y}_1} = \dfrac{0.1\angle 30°}{(3+j4)10^{-3}} = 20\angle -23.13° \therefore |\mathbf{V}_1| = \boxed{20\,\text{V}}$

(b) $\mathbf{V}_2 = \mathbf{V}_1 \therefore |\mathbf{V}_2| = \boxed{20\text{V}}$

(c) $\mathbf{I}_2 = \mathbf{Y}_2\ \mathbf{V}_2 = (5+j2)10^{-3} \times 20\angle -23.13° = 0.10770\angle -1.3286°\,\text{A}$

$\therefore \mathbf{I}_3 = \mathbf{I}_1 + \mathbf{I}_2 = 0.1\angle 30° + 0.10770\angle -1.3286° = 0.2\angle 13.740°\,\text{A}$

$\therefore \mathbf{V}_3 = \dfrac{\mathbf{I}_3}{\mathbf{Y}_3} = \dfrac{0.2\angle 13.740°}{(2-j4)10^{-3}} = 44.72\angle 77.18°\ \text{V} \therefore |\mathbf{V}_3| = \boxed{44.72\text{V}}$

(d) $\mathbf{V}_{in} = \mathbf{V}_1 + \mathbf{V}_3 + 20\angle -23.13° + 44.72\angle 77.18° = 45.60\angle 51.62°$

$\therefore |\mathbf{V}_{in}| = \boxed{45.60\text{V}}$

51.

(a) $50\mu\text{F} \rightarrow -j20\,\Omega \therefore \text{Y}_{in} = 0.1 + j0.05$

$$\text{Y}_{in} = \frac{1}{\text{R}_1 - j\dfrac{1000}{\text{C}}} \therefore \text{R}_1 - j\frac{1000}{\text{C}} = \frac{1}{0.1 + j0.05} = 8 - j4$$

$$\therefore \text{R}_1 = 8\,\Omega \text{ and } \text{C}_1 = \frac{1}{4\omega} = \boxed{250\mu\text{F}}$$

(b) $\omega = 2000 : 50\mu\text{F} \rightarrow -j10\,\Omega \;\therefore \text{Y}_{in} = 0.1 + j0.1 = \dfrac{1}{\text{R}_1 - j\dfrac{500}{\text{C}_1}}$

$$\therefore \text{R}_1 - j\frac{500}{\text{C}_1} = 5 - j5 \therefore \text{R}_1 = 5\,\Omega,\ \text{C}_1 = \boxed{100\mu\text{F}}$$

52.

(a)

$$Z_{in} = 1 + \frac{10}{j\omega} = \frac{10 + j\omega}{j\omega}$$

$$\therefore Y_{in} \frac{j\omega}{10 + j\omega} \times \frac{10 - j\omega}{10 - j\omega}$$

$$\therefore Y_{in} = \frac{\omega^2 + j10\omega}{\omega^2 + 100}$$

$$G_{in} = \frac{\omega^2}{\omega^2 + 100}, \; B_{in} = \frac{10\omega}{\omega^2 + 100}$$

ω	G_{in}	B_{in}
0	0	0
1	0.0099	0.0099
2	0.0385	0.1923
5	0.2	0.4
10	0.5	0.5
20	0.8	0.4
∞	1	0

53.

$$-j5 = \frac{v_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{v_1 - V_2}{j3},\ -j75 = 5V_1 + j3V_1 - j3V_2 - j5V_1 + j5V_2$$

$$\therefore (5 - j2)\,V_1 + j2V_2 = -j75 \qquad (1)$$

$$\frac{v_2 - V_1}{j3} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{6} = 10$$

$$-j10V_2 + j10V_1 + j6V_2 - j6V_1 + 5V_2 = 300 \therefore\ j4V_1 + (5 - j4)\,V_2 = 300 \qquad (2)$$

$$\therefore V_2 = \frac{\begin{vmatrix} 5 - j2 & -j75 \\ j4 & 300 \end{vmatrix}}{\begin{vmatrix} 5 - j2 & j2 \\ j4 & 5 - j4 \end{vmatrix}} = \frac{1500 - j600 - 300}{17 - j30 + 8} = \frac{1200 - j600}{25 - j30} = \boxed{34.36\angle 23.63^\circ\ \text{V}}$$

54.

$$j3\mathrm{I}_B - j5(\mathrm{I}_B - \mathrm{I}_D) = 0 \therefore -2\mathrm{I}_B + j5\mathrm{I}_D = 0$$

$$3(\mathrm{I}_D + j5) - j5(\mathrm{I}_D - \mathrm{I}_B) + 6(\mathrm{I}_D + 10) = 0$$

$$\therefore j5\mathrm{I}_B + (9 - j5)\mathrm{I}_D = -60 - j15$$

$$\mathrm{I}_B = \frac{\begin{vmatrix} 0 & j5 \\ -60 - j15 & 9 - j5 \end{vmatrix}}{\begin{vmatrix} -j2 & j5 \\ j5 & 9 - j5 \end{vmatrix}} = \frac{-75 + j300}{15 - j18}$$

$$= \boxed{13.198\angle 154.23°\ \mathrm{A}}$$

55.

$$v_{s1} = 20\cos 1000t\text{ V}, v_{s2} = 20\sin 1000t\text{ V}$$

$$\therefore \mathrm{V}_{s1} = 20\angle 0°\text{ V},\ \mathrm{V}_{s2} = -j20\text{V}$$

$$0.01\text{H} \rightarrow j10\,\Omega,\ 0.1\text{mF} \rightarrow -j10\,\Omega$$

$$\therefore \frac{v_x - 20}{j10} + \frac{v_x}{25} + \frac{v_x + j20}{-j10} = 0,\ 0.04v_x + j2 - 2 = 0,$$

$$\mathrm{V}_x = 25(2 - j2) = 70.71\angle -45°\text{ V}$$

$$\therefore \boxed{v_x(t) = 70.71\cos(1000t - 45°)\text{ V}}$$

56.

(a) Assume $V_3 = 1V \therefore V_2 = 1 - j0.5V,\ I_2 = 1 - j0.5\,mA$

$\therefore V_1 = 1 - j0.5 + (2 - j0.5)(-j0.5) = 0.75 - j1.5V$

$\therefore I_1 = 0.75 - j1.5\,mA,\ \therefore I_{in} = 0.75 - j1.5 + 2 - j0.5 = 2.75 - j2\,mA$

$\therefore V_{in} = 0.75 - j1.5 - j1.5 + (2.75 - j2)(-j0.5)$

$= -0.25 - j2.875\,V \therefore V_3 = \dfrac{100}{-j0.25 - j2.875} = \boxed{34.65^+ \angle 94.97°V}$

(b) $-j0.5 \to -jx$ Assume $V_3 = 1V \therefore I_3 = 1A,$

$V_2 = 1 - jX,\ I_2 = 1 - jX, \to I_{12} = 2 - jX$

$\therefore V_1 = 1 - jX + (2 - jX)(-jX) = 1 - X^2 - j3X,\ I_1 = 1 - X^2 - j3X,\ I_{in} = 3 - X^2 - j4X$

$\therefore V_{in} = 1 - X^2 - j3X - 4X^2 + jX^3 - j3X = 1 - 5X^2 + j(X^3 - 6X) \therefore X^3 - 6X = 0$

$\therefore X^2 = 6,\ X = \sqrt{6},\ Z_c = \boxed{-j2.449\,K\Omega}$

57. Define three clockwise mesh currents i_1, i_2, i_3 with i_1 in the left mesh, i_2 in the top right mesh, and i_3 in the bottom right mesh.

Mesh 1: $10\angle 0^{\circ} + (1 + 1 - j0.25)\mathbf{I}_1 - (-j0.25)\mathbf{I}_2 = 0$

Mesh 2: $-\mathbf{I}_1 + (1 + 1 + j4)\mathbf{I}_2 - \mathbf{I}_3 = 0$

Mesh 3: $(-j0.25 + 1 + 1)\mathbf{I}_3 - \mathbf{I}_2 - (-j0.25\mathbf{I}_1) = 0$

$$\mathrm{I}_x = \frac{\begin{vmatrix} 2-j0.25 & -1 & 10 \\ -1 & 2+j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2-j0.25 & -1 & j0.25 \\ -1 & 2+j4 & -1 \\ j0.25 & -1 & 2-j0.25 \end{vmatrix}}$$

$$\therefore \mathrm{I}_x = \frac{10(1+1-j0.5)}{j0.25(2-j0.5)+(-2+j0.25+j0.25)+(2-j0.25)(4+1-j0.5+j8-1)}$$

$$= \frac{20-j5}{8+j15} \therefore \mathrm{I}_x = 1.217\angle -75.96^{\circ}\ \mathrm{A},\ \boxed{i_x(t) = 1.2127\cos(100t - 75.96^{\circ})\ \mathrm{A}}$$

58.

$$\mathrm{V}_1 - 10 - j0.25\mathrm{V}_1 + j0.25\mathrm{V}_x + \mathrm{V}_1 - \mathrm{V}_2 = 0$$

$$\therefore (2 - j0.25)\,\mathrm{V}_1 - \mathrm{V}_2 + j0.25\,\mathrm{V}_x = 10$$

$$\mathrm{V}_2 - \mathrm{V}_1 + \mathrm{V}_2 - \mathrm{V}_x + j4\mathrm{V}_2 = 0$$

$$-\mathrm{V}_1 + (2 + j4)\,\mathrm{V}_2 - \mathrm{V}_x = 0$$

$$-j0.25\mathrm{V}_x + j0.25\mathrm{V}_1 + \mathrm{V}_x + \mathrm{V}_x - \mathrm{V}_2$$

$$\therefore\ j0.25\mathrm{V}_1 - \mathrm{V}_2 + (2 - j0.25)\,\mathrm{V}_x = 0$$

$$\mathrm{V}_x = \frac{\begin{vmatrix} 2 - j0.25 & -1 & 10 \\ -1 & 2 + j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -j0.25 & -1 & j0.25 \\ -1 & 2 + j4 & -1 \\ j0.25 & -1 & 2 - j0.25 \end{vmatrix}}$$

$$= \frac{10\,(1 + 1 - j0.5)}{j0.25\,(2 - j0.5) + (-2 + j0.25 + j0.25) + (2 - j0.25)\,(4 + 1 - j0.5 + j8 - 1)}$$

$$= \frac{20 - j5}{8 + j15} = 1.2127\angle -75.96°\ \mathrm{V}$$

$$\therefore \boxed{v_x = 1.2127\cos(100t - 75.96°)\ \mathrm{V}}$$

59.

(a)

$$R_1 = \infty,\ R_o = 0,\ A = -V_o / V_i >> 0$$

$$I = \frac{V_1 + AV_i}{R_f} = j\omega C_1 (V_s - V_i)$$

$$\therefore V_i (1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$V_o = -AV_i \therefore -\frac{V_o}{A} (1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$\therefore \boxed{\frac{V_o}{V_s} = -\frac{j\omega C_1 R_f A}{1 + A + j\omega C_1 R_f}}\ \text{As } A \to \infty, \frac{V_o}{V_s} \to -j\omega C_1 R_f$$

(b)

$$R_f \| C_f = \frac{1}{j\omega C_f + \frac{1}{R_f}} = \frac{R_f}{1 + j\omega C_f R_f}$$

$$I = \frac{(V_1 + AV_i)}{R_f} (1 + j\omega C_f R_f) = (V_s - V_i)\, j\omega C_1,\ V_o = -AV_i$$

$$\therefore V_i (1 + A)(1 + j\omega C_f R_f) = V_s j\omega C_1 R_f - j\omega C_1 R_f V_i,$$

$$V_i [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore -\frac{V_o}{A} [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore \boxed{\frac{V_o}{V_s} = \frac{-j\omega C_1 R_f A}{(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f}}\ \text{As } A \to \infty,\ \frac{V_o}{V_s} \to \frac{-j\omega C_1 R_f}{1 + j\omega C_f R_f}$$

60. Define the nodal voltage $v_1(t)$ at the junction between the two dependent sources. The voltage source may be replaced by a $3\angle{-3^\circ}$ V source, the 600-µF capacitor by a $-j/0.6\ \Omega$ impedance, the 500-µF capacitor by a $-j2\ \Omega$ impedance, and the inductor by a $j2\ \Omega$ impedance.

$$5\mathbf{V}_2 + 3\mathbf{V}_2 = \frac{\mathbf{V}_1 - 3\angle -3^\circ}{100 - j/0.6} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j2} \qquad [1]$$

$$-5\mathbf{V}_2 = \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j2} + \frac{\mathbf{V}_2}{j2} \qquad [2]$$

Simplifying and collecting terms,

$$j2\,\mathbf{V}_1 + (960.1\angle -90.95^\circ)\,\mathbf{V}_2 = 6\angle 87^\circ \qquad [1]$$
$$-j2\,\mathbf{V}_1 + 20\,\mathbf{V}_2 = 0 \qquad [2]$$

Solving, we find that $\mathbf{V}_1 = 62.5\angle 86.76^\circ$ mV and $\mathbf{V}_2 = 6.25\angle 176.8^\circ$ mV. Converting back to the time domain,

$$\boxed{v_2(t) = 6.25\cos(10^3 t + 176.8^\circ)\text{ mV}}$$

61. Define three clockwise mesh currents: $i_1(t)$ in the left-most mesh, $i_2(t)$ in the bottom right mesh, and $i_3(t)$ in the top right mesh. The 15-μF capacitor is replaced with a $-j/\ 0.15\ \Omega$ impedance, the inductor is replaced by a $j20\ \Omega$ impedance, the 74 μF capacitor is replaced by a $-j1.351\ \Omega$ impedance, the current source is replaced by a $2\angle 0^o$ mA source, and the voltage source is replaced with a $5\angle 0^o$ V source.

Around the 1, 2 supermesh: $(100 + j20)\ \mathbf{I}_1 + (13000 - j1.351)\ \mathbf{I}_2 - 5000\ \mathbf{I}_3 = 0$
and

$$-\mathbf{I}_1 + \mathbf{I}_2 = 2\times10^{-3}$$

Mesh 3: $5\angle 0^o + (5000 - j6.667)\ \mathbf{I}_2 - 5000\ \mathbf{I}_3 = 0$

Solving, we find that $\mathbf{I}_1 = 1.22\angle 179.9^o$ mA. Converting to the time domain,

$$i_1(t) = 1.22 \cos (10^4 t + 179.9^o) \text{ mA}$$

Thus, $P_{1000} = [i_1(1 \text{ ms})]2 \cdot 1000$
$= (1.025\times10^{-6})(1000)$ W $=$ $\boxed{1.025 \text{ W.}}$

62. We define an additional clockwise mesh current $i_4(t)$ flowing in the upper right-hand mesh. The inductor is replaced by a $j0.004\ \Omega$ impedance, the 750 μF capacitor is replaced by a $-j/\ 0.0015\ \Omega$ impedance, and the 1000 μF capacitor is replaced by a $-j/\ 2\ \Omega$ impedance. We replace the left voltage source with a a $6 \angle -13^o$ V source, and the right voltage source with a $6 \angle 0^o$ V source.

$$(1 - j/\ 0.0015)\ \mathbf{I}_1 - \mathbf{I}_3 = 6 \angle -13^o \quad [1]$$

$$(0.005 + j/\ 0.0015)\ \mathbf{I}_1 + j0.004\ \mathbf{I}_2 - j0.004\ \mathbf{I}_4 = 0 \quad [2]$$

$$-\mathbf{I}_1 + (1 - j/\ 2)\ \mathbf{I}_3 + j0.5\ \mathbf{I}_4 = -6 \angle 0^o \quad [3]$$

$$-j0.004\ \mathbf{I}_2 + j0.5\ \mathbf{I}_3 + (j0.004 - j0.5)\ \mathbf{I}_4 = 0 \quad [4]$$

Solving, we find that

$\mathbf{I}_1 = 0.00144\angle -51.5^o$ A, $\mathbf{I}_2 = 233.6 \angle 39.65^o$ A, and $\mathbf{I}_3 = 6.64 \angle 173.5^o$ A.

Converting to the time domain,

$$\boxed{\begin{aligned} i_1(t) &= 1.44 \cos(2t - 51.5^o)\ \text{mA} \\ i_2(t) &= 233.6 \cos(2t + 39.65^o)\ \text{A} \\ i_3(t) &= 6.64 \cos(2t + 173.5^o)\ \text{A} \end{aligned}}$$

63. We replace the voltage source with a $115\sqrt{2}\angle 0^{o}$ V source, the capacitor with a $-j/\,2\pi C_1\ \Omega$ impedance, and the inductor with a $j0.03142\ \Omega$ impedance.

Define $\mathbf{Z}$ such that $\mathbf{Z}^{-1} = 2\pi C_1 - j/0.03142 + 1/20$

By voltage division, we can write that $6.014\angle 85.76^{o} = 115\sqrt{2}\dfrac{\mathbf{Z}}{\mathbf{Z}+20}$

Thus, $\mathbf{Z} = 0.7411\angle 87.88^{o}\ \Omega$. This allows us to solve for C_1:

$2\pi C_1 - 1/0.03142 = -1.348$ so that $\boxed{C_1 = 4.85 \text{ F.}}$

64. Defining a clockwise mesh current $i_1(t)$, we replace the voltage source with a $115\sqrt{2}\,\angle 0^\circ$ V source, the inductor with a $j2\pi L$ Ω impedance, and the capacitor with a $-j1.592$ Ω impedance.

Ohm's law then yields $\mathbf{I}_1 = \dfrac{115\sqrt{2}}{20 + j(2\pi L - 1.592)} = 8.132\angle 0^\circ$

Thus, $20 = \sqrt{20^2 + (2\pi L - 1.592)^2}$ and we find that $\boxed{L = 253.4 \text{ mH.}}$

65. (a) By nodal analysis:

$$0 = (\mathbf{V}_\pi - 1)/\, R_s + \mathbf{V}_\pi /\, R_B + \mathbf{V}_\pi /\, r_\pi + j\omega C_\pi \mathbf{V}_\pi + (\mathbf{V}_\pi - \mathbf{V}_{out})\, j\omega C_\mu \quad [1]$$

$$-g_m \mathbf{V}_\pi = (\mathbf{V}_{out} - \mathbf{V}_\pi)\, j\omega C_\mu + \mathbf{V}_{out} /\, R_C + \mathbf{V}_{out} /\, R_L \quad [2]$$

Simplify and collect terms:

$$\left[\left(\frac{1}{R_S} + \frac{1}{R_B} + \frac{1}{r_\pi}\right) + j\omega\left(C_\pi + C_\mu\right)\right]\mathbf{V}_\pi - j\omega C_\mu \mathbf{V}_{out} = \frac{1}{R_S} \quad [1]$$

$$(-g_m + j\omega C_\mu)\, \mathbf{V}_\pi - (j\omega C_\mu + 1/R_C + 1/R_L)\, \mathbf{V}_{out} = 0 \quad [2]$$

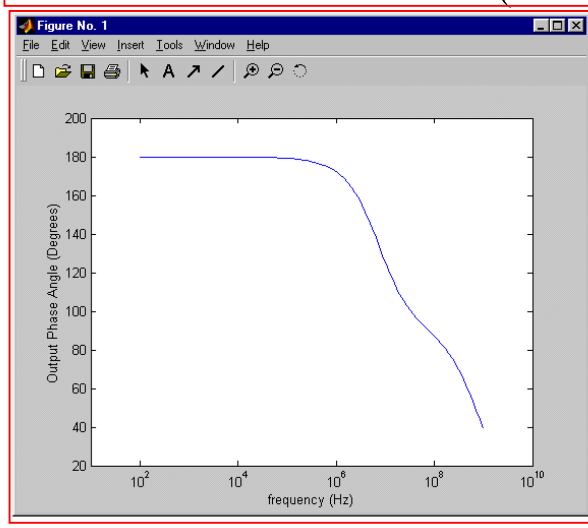
Define $\frac{1}{R_S'} = \frac{1}{R_S} + \frac{1}{R_B} + \frac{1}{r_\pi}$ and $R_L' = R_C \parallel R_L$

Then $\Delta = \frac{-1}{R_S' R_L'} + \omega^2\left(2C_\mu^2 + C_\mu C_\pi\right) - j\omega\left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'}\right)$

And $\mathbf{V}_{out} = \dfrac{g_m R_S - j\omega C_\mu / R_S}{\frac{-1}{R_S' R_L'} + \omega^2\left(2C_\mu^2 + C_\mu C_\pi\right) - j\omega\left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'}\right)}$

Therefore, $\boxed{\text{ang}(\mathbf{V}_{out}) = \tan^{-1}\left(\frac{-j\omega C_\mu}{g_m R_S^2}\right) - \tan^{-1}\left(\frac{-\omega\left(g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_S'}\right)}{\frac{-1}{R_S' R_L'} + \omega^2\left(2C_\mu^2 + C_\mu C_\pi\right)}\right)}$

(b)



(c) The output is ~180^o out of phase with the input for $f < 10^5$ Hz; only for $f = 0$ is it exactly 180^o out of phase with the input.

66.

$$\text{OC}: -\frac{\text{V}_x}{20} + \frac{100 - \text{V}_x}{-j10} - 0.02\text{V}_x = 0$$

$$j10 = (0.05 + j0.1 + 0.02)\ \text{V}_x, \text{V}_x = \frac{j10}{0.07 + j0.1}$$

$$\therefore \text{V}_x = 67.11 + j46.98$$

$$\therefore \text{V}_{ab,oc} = 100 - \text{V}_x = 32.89 - j46.98 = \boxed{57.35\angle -55.01^\circ \text{ V}}$$

$$\text{SC}: \text{V}_x = 100 \therefore \downarrow \text{I}_{SC} = 0.02 \times 100 + \frac{100}{20} = 7\text{A}$$

$$\therefore \text{Z}_{th} = \frac{57.35\angle -55.01^\circ}{7} = \boxed{4.698 - j6.711\Omega}$$

67.

$$V_L = j210 \therefore 0.5V_L = j\omega$$

$$\therefore V_{in} = (1 + j\omega)\frac{1}{j\omega} + j2\omega$$

$$= 1 + \frac{1}{j\omega} + j2\omega$$

$$\therefore Z_{in} = \frac{V_{in}}{1} = 1 + \frac{1}{j\omega} + j2\omega \qquad \text{so } \boxed{\mathbf{Y}_{\text{in}} = \frac{\omega}{\omega + j\left(2\omega^2 - 1\right)}}$$

$$\text{At } \omega = 1,\ Z_{in} = 1 - j1 + j2 = 1 + j$$

$$\therefore Y_{in} = \frac{1}{1 + j1} = 0.5 + j0.5$$

$\boxed{\text{R = 500 m}\Omega\text{, L = 500 mH.}}$

68.

(a) $\mathbf{V}_s: \dfrac{(1-j1)1}{2-j1}\times\dfrac{2+j1}{2+j1}=\dfrac{3-j1}{5}\;\therefore\;\mathbf{V}_1=\dfrac{-15}{j2+0.6-j0.2}\times 0.6-j0.2$

$\therefore\;\mathbf{V}_1=5\angle 90^\circ\;\therefore\;\boxed{v_1(t)=5\cos(1000t+90^\circ)\text{ V}}$

(b) $\mathbf{I}_s$:

$j2\|1=\dfrac{j2}{1+j2}\,\dfrac{1-j2}{1-j2}=0.8+j0.4\;\therefore\;\mathbf{V}_1$

$=j25\dfrac{0.8+j0.4}{1-j1+0.8+j0.4}=\dfrac{-10+j20}{1.8-j0.6}=11.785^{+}\angle 135^\circ\text{ V}$

so $\boxed{v_1(t)\;=\;11.79\cos(1000t+135^\circ)\text{ V.}}$

69.

$\text{OC}: \text{V}_L = 0 \therefore \text{V}_{ab,oc} = 1\angle 0° \text{ V}$

$\text{SC}: \downarrow \text{I}_N \therefore \text{V}_L = j2\text{I}_N \therefore 1\angle 0° = -j1[0.25(j2\text{I}_N) + \text{I}_N] + j2\text{I}_N$

$\therefore 1 = (0.5 - j + j2)\,\text{I}_N = (0.5 + j1)\,\text{I}_N$

$$\therefore \text{I}_N = \frac{1}{0.5 + j1} = 0.4 - j0.8 \therefore Y_N = \frac{\text{I}_N}{1\angle 0°} = 0.4 - j0.8$$

$$\therefore \text{R}_N = \frac{1}{0.4} = \boxed{2.5\Omega,}\ \frac{1}{j\omega\text{L}_N} = \frac{1}{j\text{L}_N} = -j0.8,\ \text{L}_N = \frac{1}{0.8} = \boxed{1.25\text{H}}$$

$$\text{I}_N = 0.4 - j0.8 = \boxed{0.8944\angle -63.43° \text{ A}}$$

70.

$$\mathbf{V}_L = 2(j1) + (-j2)\frac{j1}{1+j1} = j2 + \frac{2}{1+j1} = \frac{1-j1}{1-j1} = 1 + j1$$

$$\therefore \mathbf{V}_{L,200} = 1.4142\cos(200t + 45°)\text{ V}$$

$$\omega = 100 : \mathbf{V}_L = j\frac{1}{2},\ v_{L,100} = 0.5\cos(100t + 90°)\text{ V}$$

so $\boxed{v_L(t) = 1.414\cos(200t + 45^o) + 0.5\cos(100t + 90^o)\text{ V}}$

71.

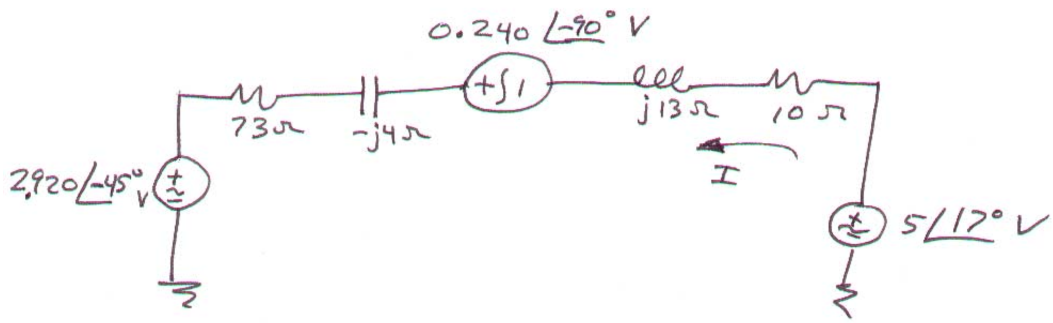
Use superposition. Left: $\mathbf{V}_{ab} = 100\dfrac{j100}{j100 - j300}$

$= -50\angle 0°$ V Right: $\mathbf{V}_{ab} = j100\dfrac{-j300}{-j300 + j100} = j150$ V

$\therefore \boxed{\mathbf{V}_{th} = -50 + j150 = 158.11\angle 108.43°\text{ V}}$

$\mathbf{Z}_{th} = j100 \| -j300 = \dfrac{30,000}{-j200} = \boxed{j150\,\Omega}$

72. This problem is easily solved if we first perform two source transformations to yield a circuit containing only voltage sources and impedances:



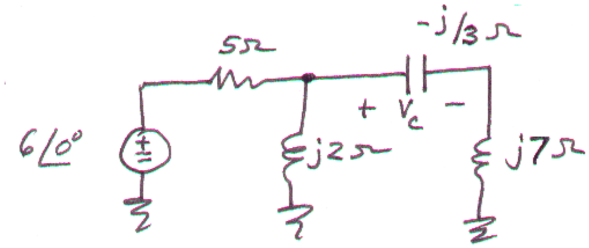
Then $\mathbf{I} = \dfrac{5\angle 17^o + 0.240\angle -90^o + 2.920\angle -45^o}{73 + 10 + j13 - j4}$

$= (6.898\angle -7.019^o) / (83.49 \angle 6.189^o) = 82.62 \angle -13.21$ mA

Converting back to the time domain, we find that

$\boxed{i(t) = 82.62 \cos (10^3 t - 13.21^o) \text{ mA}}$

73.



(a) There are a number of possible approaches: Thévenizing everything to the left of the capacitor is one of them.

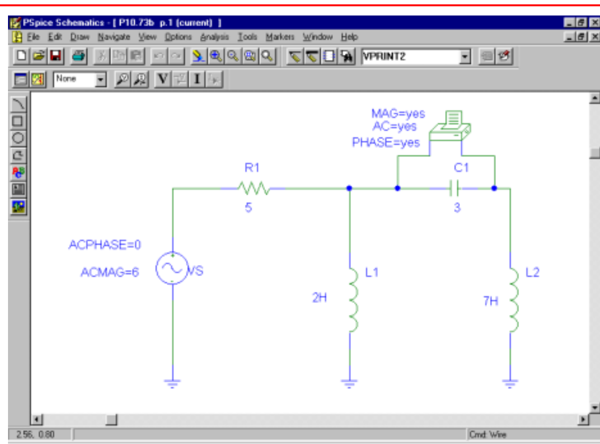
$$\mathbf{V}_{TH} = 6(j2)/(5 + j2) = 2.228 \angle 68.2^{\circ} \text{ V}$$
$$\mathbf{Z}_{TH} = 5 \parallel j2 = j10/(5 + j2) = 1.857 \angle 68.2^{\circ} \ \Omega$$

Then, by simple voltage division, we find that

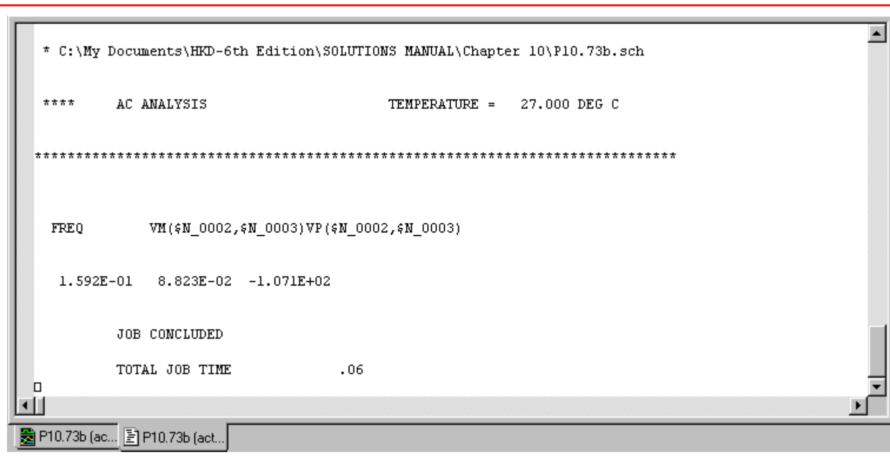
$$\mathbf{V}_C = (2.228 \angle 68.2^{\circ}) \frac{-j/3}{1.857\angle 68.2^{\circ} - j/3 + j7}$$
$$= 88.21 \angle -107.1^{\circ} \text{ mV}$$

Converting back to the time domain, $\boxed{v_C(t) = 88.21 \cos(t - 107.1^{\circ}) \text{ mV}.}$

(b) **PSpice verification**.



Running an ac sweep at the frequency $f = 1/2\pi = 0.1592$ Hz, we obtain a phasor magnitude of 88.23 mV, and a phasor angle of -107.1°, in agreement with our calculated result (the slight disagreement is a combination of round-off error in the hand calculations and the rounding due to expressing 1 rad/s in Hz.



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* C:\My Documents\HKD-6th Edition\SOLUTIONS MANUAL\Chapter 10\P10.73b.sch

****     AC ANALYSIS                    TEMPERATURE =   27.000 DEG C

******************************************************************************

 FREQ        VM($N_0002,$N_0003)VP($N_0002,$N_0003)

  1.592E-01    8.823E-02   -1.071E+02

          JOB CONCLUDED

          TOTAL JOB TIME            .06
```


74. (a) Performing nodal analysis on the circuit,

Node 1: $1 = \mathbf{V}_1/5 + \mathbf{V}_1/(-j10) + (\mathbf{V}_1 - \mathbf{V}_2)/(-j5) + (\mathbf{V}_1 - \mathbf{V}_2)/j10$ [1]

Node 2: $j0.5 = \mathbf{V}_2/10 + (\mathbf{V}_2 - \mathbf{V}_1)/(-j5) + (\mathbf{V}_2 - \mathbf{V}_1)/j10$ [2]

Simplifying and collecting terms,

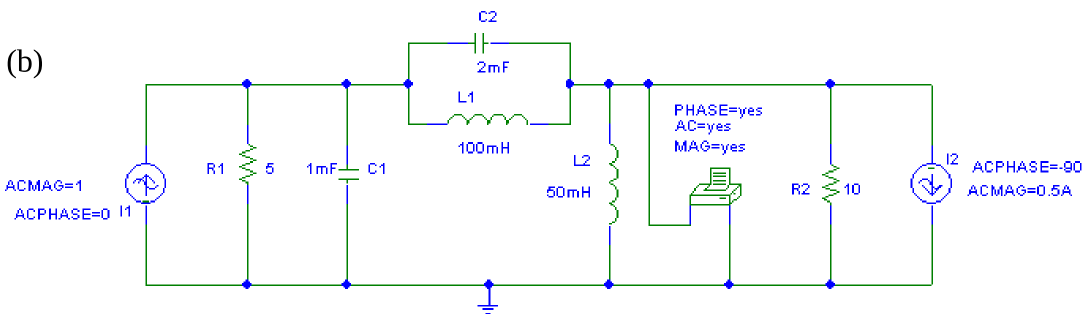
$(0.2 + j0.2)\,\mathbf{V}_1 - j0.1\,\mathbf{V}_2 = 1$ [1]

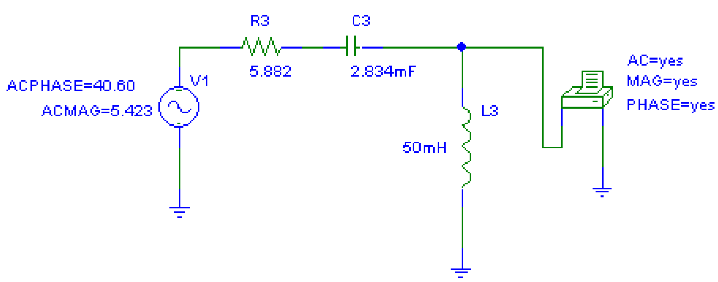
$-j\,\mathbf{V}_1 + (1 + j)\,\mathbf{V}_2 = j5$ [2]

Solving, we find that $\mathbf{V}_2 = \mathbf{V}_{TH} = \boxed{5.423 \angle 40.60^{\circ}\ \text{V}}$

$\mathbf{Z}_{TH} = 10 \parallel [(j10 \parallel -j5) + (5 \parallel -j10)] = 10 \parallel (-j10 + 4 - j2) = \boxed{5.882 - j3.529\ \Omega}.$

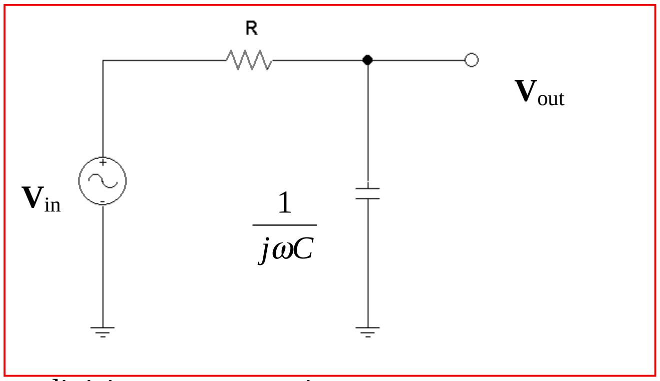
(b)





FREQ	VM($N_0002,0)	VP($N_0002,0)
1.592E+01	4.474E+00	1.165E+02
FREQ	VM($N_0005,0)	VP($N_0005,0)
1.592E+01	4.473E+00	1.165E+02

75. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{out} = \mathbf{V}_{in}\frac{1/j\omega C}{R + 1/j\omega C}, \quad \text{or} \quad \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1}{1 + j\omega RC}$$

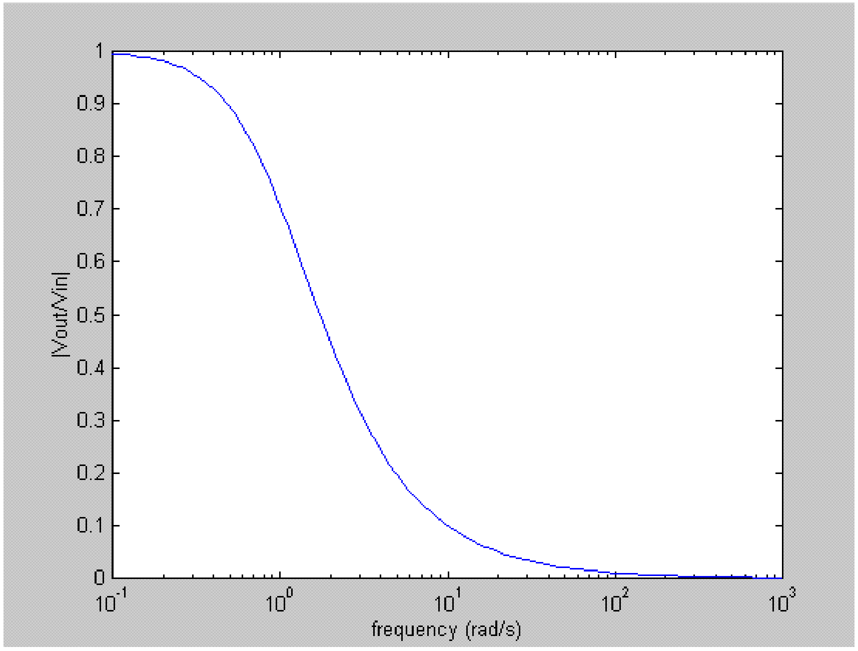
The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

$$\left|\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

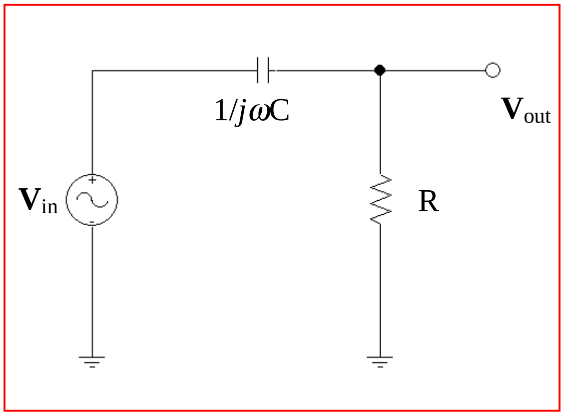
As $\omega \to 0$, this magnitude $\to$ 1, its maximum value.

As $\omega \to \infty$, this magnitude $\to$ 0; the capacitor is acting as a short circuit to the ac signal.

Thus, low frequency signals are transferred from the input to the output relatively unaffected by this circuit, but high frequency signals are attenuated, or "filtered out." This is readily apparent if we plot the magnitude as a function of frequency (assuming R = 1 Ω and C = 1 F for convenience):



76. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \frac{R}{R + 1/j\omega C}, \quad \text{or} \quad \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j\omega RC}{1 + j\omega RC}$$

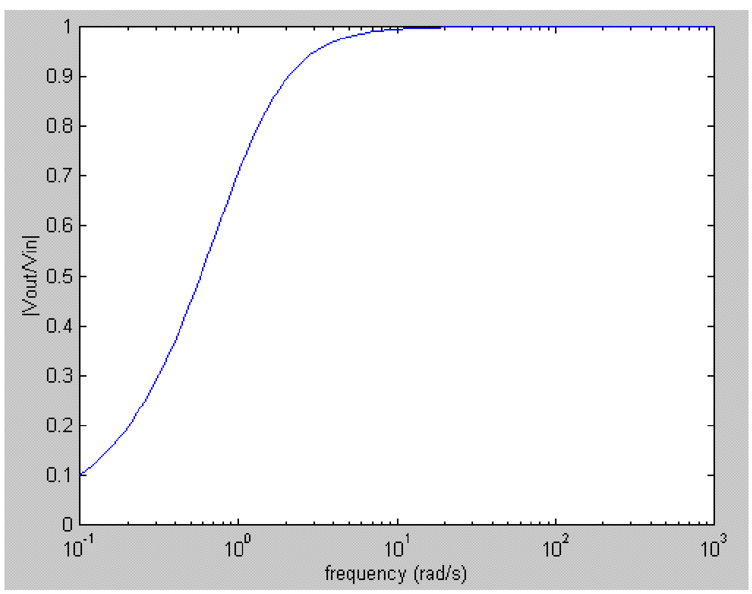
The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

$$\left|\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}\right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

As $\omega \to \infty$, this magnitude $\to 1$, its maximum value.

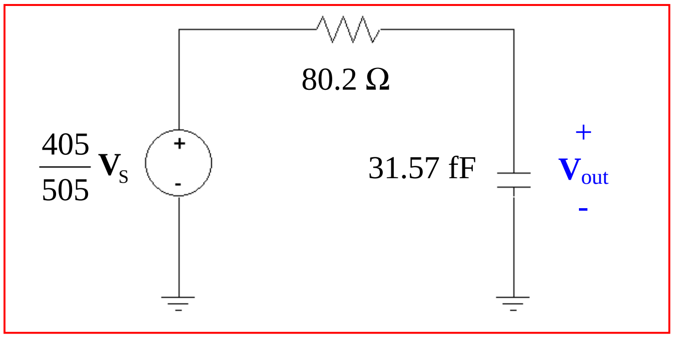
As $\omega \to 0$, this magnitude $\to 0$; the capacitor is acting as an open circuit to the ac signal.

Thus, high frequency signals are transferred from the input to the output relatively unaffected by this circuit, but low frequency signals are attenuated, or "filtered out." This is readily apparent if we plot the magnitude as a function of frequency (assuming R = 1 Ω and C = 1 F for convenience):



77. (a) Removing the capacitor temporarily, we easily find the Thevenin equivalent:

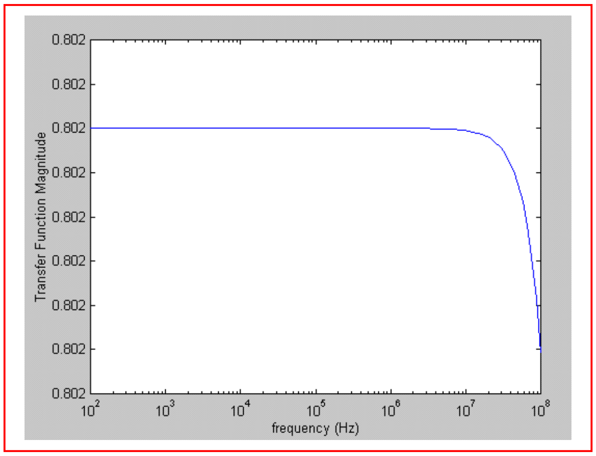
$\mathbf{V}_{th} = (405/505)\,\mathbf{V}_S$ and $R_{th} = 100 \parallel (330 + 75) = 80.2\ \Omega$



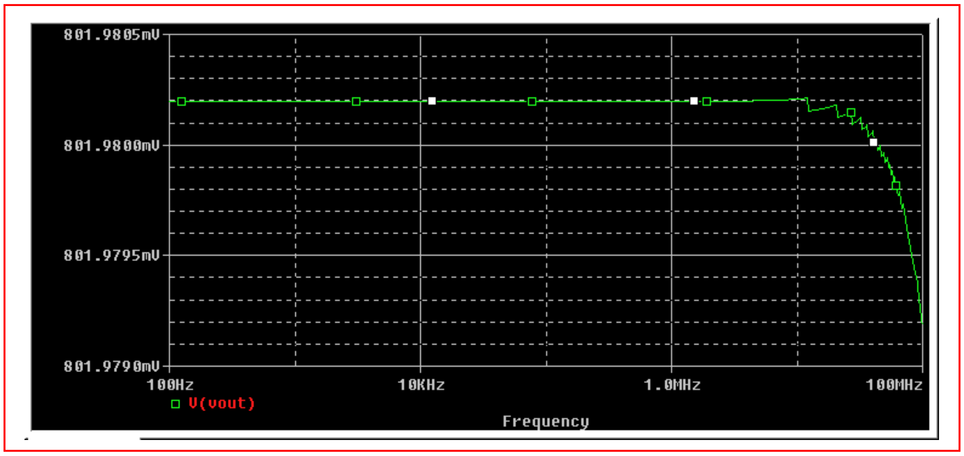
(b) $\mathbf{V}_{out} = \dfrac{405}{505}\mathbf{V}_S \dfrac{1/j\omega C}{80.2 + 1/j\omega C}$ so $\dfrac{\mathbf{V}_{out}}{\mathbf{V}_S} = \left(\dfrac{405}{505}\right)\dfrac{1}{1 + j2.532\times10^{-12}\omega}$

and hence $\boxed{\left|\dfrac{\mathbf{V}_{out}}{\mathbf{V}_S}\right| = \dfrac{0.802}{\sqrt{1 + 6.411\times10^{-24}\omega^2}}}$

(c)



Both the MATLAB plot of the frequency response and the PSpice simulation show essentially the same behavior; at a frequency of approximately 20 MHz, there is a sharp roll-off in the transfer function magnitude.



78. From the derivation, we see that

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{-g_m(R_C \parallel R_L) + j\omega(R_C \parallel R_L)C_\mu}{1 + j\omega(R_C \parallel R_L)C_\mu}$$

so that

$$\left|\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}}\right| = \left[\frac{g_m^{\ 2}\left(\frac{R_C R_L}{R_C + R_L}\right)^2 + \omega^2\left(\frac{R_C R_L}{R_C + R_L}\right)^2 C_\mu^{\ 2}}{1 + \omega^2\left(\frac{R_C R_L}{R_C + R_L}\right)^2 C_\mu^{\ 2}}\right]^{1/2}$$

This function has a maximum value of $g_m (R_C \parallel R_L)$ at $\omega = 0$. Thus, the capacitors reduce the gain at high frequencies; this is the frequency regime at which they begin to act as short circuits. Therefore, the maximum gain is obtained at frequencies at which the capacitors may be treated as open circuits. If we do this, we may analyze the circuit of Fig. 10.25*b* without the capacitors, which leads to

$$\left.\frac{\mathbf{V}_{out}}{\mathbf{V}_S}\right|_{\text{low frequency}} = -g_m\left(\frac{R_C R_L}{R_C + R_L}\right)\frac{(r_\pi \parallel R_B)}{R_S + r_\pi \parallel R_B} = -g_m\left(\frac{R_C R_L}{R_C + R_L}\right)\frac{r_\pi R_B}{R_S(r_\pi + R_B) + r_\pi R_B}$$

The resistor network comprised of r_π, R_S, and R_B acts as a voltage divider, leading to a reduction in the gain of the amplifier. In the situation where $r_\pi \parallel R_B >> R_S$, then it has minimal effect and the gain will equal its "maximum" value of $-g_m (R_C \parallel R_L)$.

(b) If we set $R_S = 100\ \Omega$, $R_L = 8\ \Omega$, $R_C |_{max} = 10\ k\Omega$ and $r_\pi g_m = 300$, then we find that

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_S} = -g_m\,(7.994)\,\frac{r_\pi \parallel R_B}{100 + r_\pi \parallel R_B}$$

We seek to maximize this term within the stated constraints. This requires a large value of g_m, but also a large value of $r_\pi \parallel R_B$. This parallel combination will be less than the smaller of the two terms, so even if we allow $R_B \to \infty$, we are left with

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_S} \approx -(7.994)\,\frac{g_m r_\pi}{100 + r_\pi} = \frac{-2398}{100 + r_\pi}$$

Considering this simpler expression, it is clear that if we select r_π to be small, (*i.e.* $r_\pi << 100$), then g_m will be large and the gain will have a maximum value of approximately –23.98.

(c) Referring to our original expression in which the gain $\mathbf{V}_{out}/\mathbf{V}_{in}$ was computed, we see that the critical frequency $\omega_C = [(R_C \parallel R_L)\, C_\mu]^{-1}$. Our selection of maximum R_C, $R_B \to \infty$, and $r_\pi << 100$ has not affected this frequency.

79. Considering the $\omega = 2\times10^4$ rad/ s source first, we make the following replacements:

$100 \cos (2\times10^4 t + 3^o)$ V $\rightarrow 100 \angle 3^o$ V
33 μF → $-j1.515\ \Omega$ 112 μH → $j2.24\ \Omega$ 92 μF → $-j0.5435\ \Omega$

Then

$(\mathbf{V}_1' - 100 \angle 3^o)/ 47\times10^3 + \mathbf{V}_1'/ (-j1.515) + (\mathbf{V}_1' - \mathbf{V}_2')/ (56\times10^3 + j4.48) = 0$ [1]

$(\mathbf{V}_2' - \mathbf{V}_1')/ (56\times10^3 + j4.48) + \mathbf{V}_2'/ (-j0.5435) = 0$ [2]

Solving, we find that

$$\mathbf{V}_1' = 3.223 \angle -87^o \text{ mV} \quad \text{and } \mathbf{V}_2' = 31.28 \angle -177^o \text{ nV}$$

Thus, $v_1'(t) = 3.223 \cos (2\times10^4 t - 87^o)$ mV and $v_2'(t) = 31.28 \cos(2\times10^4 t - 177^o)$ nV

Considering the effects of the $\omega = 2\times10^5$ rad/ s source next,

$100 \cos (2\times10^5 t - 3^o)$ V $\rightarrow 100 \angle -3^o$ V
33 μF → $-j0.1515\ \Omega$ 112 μH → $j22.4\ \Omega$ 92 μF → $-j0.05435\ \Omega$

Then

$$\mathbf{V}_1''/ -j0.1515 + (\mathbf{V}_1'' - \mathbf{V}_2'')/ (56\times10^3 + j44.8) = 0 \quad [3]$$

$(\mathbf{V}_2'' - \mathbf{V}_1'')/ (56\times10^3 + j44.8) + (\mathbf{V}_2'' - 100 \angle 3^o)/ 47\times10^3 + \mathbf{V}_2''/ (-j0.05435) = 0$ [4]

Solving, we find that

$$\mathbf{V}_1'' = 312.8 \angle 177^o \text{ pV and } \mathbf{V}_2'' = 115.7 \angle -93^o\ \mu\text{V}$$

Thus,
$v_1''(t) = 312.8 \cos (2\times10^5 t + 177^o)$ pV and $v_2''(t) = 115.7 \cos(2\times10^5 t - 93^o)$ μV

Adding, we find

$$\boxed{\begin{aligned} v_1(t) &= 3.223\times10^{-3} \cos (2\times10^4 t - 87^o) + 312.8\times10^{-12} \cos (2\times10^5 t + 177^o) \text{ V and} \\ v_2(t) &= 31.28\times10^{-9} \cos(2\times10^4 t - 177^o) + 115.7\times10^{-12} \cos(2\times10^5 t - 93^o) \text{ V} \end{aligned}}$$

80. For the source operating at $\omega = 4$ rad/s,
$7 \cos 4t \rightarrow 7\angle 0^{\circ}$ V, 1 H $\rightarrow j4\ \Omega$, 500 mF $\rightarrow -j0.5\ \Omega$, 3 H $\rightarrow j12\ \Omega$, and 2 F $\rightarrow -j/8\ \Omega$.

Then by mesh analysis, (define 4 clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, $\mathbf{I}_4$ in the top left, top right, bottom left and bottom right meshes, respectively):

$$(9.5 + j4)\,\mathbf{I}_1 - j4\,\mathbf{I}_2 - 7\,\mathbf{I}_3 - 4\,\mathbf{I}_4 = 0 \quad [1]$$
$$-j4\,\mathbf{I}_1 + (3 + j3.5)\,\mathbf{I}_2 - 3\,\mathbf{I}_4 = -7 \quad [2]$$
$$-7\,\mathbf{I}_1 + (12 - j/8)\,\mathbf{I}_3 + j/8\,\mathbf{I}_4 = 0 \quad [3]$$
$$-3\,\mathbf{I}_2 + j/8\,\mathbf{I}_3 + (4 + j11.875)\,\mathbf{I}_4 = 0 \quad [4]$$

Solving, we find that $\mathbf{I}_3 = 365.3\angle -166.1^{\circ}$ mA and $\mathbf{I}_4 = 330.97\angle 72.66^{\circ}$ mA.

For the source operating at $\omega = 2$ rad/s,
$5.5 \cos 2t \rightarrow 5.5\angle 0^{\circ}$ V, 1 H $\rightarrow j2\ \Omega$, 500 mF $\rightarrow -j\ \Omega$, 3 H $\rightarrow j6\ \Omega$, and 2 F $\rightarrow -j/4\ \Omega$.

Then by mesh analysis, (define 4 clockwise mesh currents $\mathbf{I}_A$, $\mathbf{I}_B$, $\mathbf{I}_C$, $\mathbf{I}_D$ in the top left, top right, bottom left and bottom right meshes, respectively):

$$(9.5 + j2)\,\mathbf{I}_A - j2\,\mathbf{I}_B - 7\,\mathbf{I}_C - 4\,\mathbf{I}_D = 0 \quad [1]$$
$$-j2\,\mathbf{I}_A + (3 + j)\,\mathbf{I}_B - 3\,\mathbf{I}_D = -7 \quad [2]$$
$$-7\,\mathbf{I}_A + (12 - j/4)\,\mathbf{I}_C + j/4\,\mathbf{I}_D = 0 \quad [3]$$
$$-3\,\mathbf{I}_2 + j/4\,\mathbf{I}_C + (4 + j5.75)\,\mathbf{I}_D = 0 \quad [4]$$

Solving, we find that $\mathbf{I}_C = 783.8\angle -4.427^{\circ}$ mA and $\mathbf{I}_D = 134\angle -25.93^{\circ}$ mA.

$\mathbf{V}_1' = -j0.25\,(\mathbf{I}_3 - \mathbf{I}_4) = 0.1517\angle 131.7^{\circ}$ V and $\mathbf{V}_1'' = -j0.25(\mathbf{I}_C - \mathbf{I}_D) = 0.1652\angle -90.17^{\circ}$ V

$\mathbf{V}_2' = (1 + j6)\,\mathbf{I}_4 = 2.013\angle 155.2^{\circ}$ V and $\mathbf{V}_2'' = (1 + j6)\,\mathbf{I}_D = 0.8151\angle 54.61^{\circ}$ V

Converting back to the time domain,

$$\boxed{\begin{aligned} v_1(t) &= 0.1517 \cos(4t + 131.7^{\circ}) + 0.1652 \cos(2t - 90.17^{\circ})\text{ V} \\ v_2(t) &= 2.013 \cos(4t + 155.2^{\circ}) + 0.8151 \cos(2t + 54.61^{\circ})\text{ V} \end{aligned}}$$

81.

(a) $$\mathrm{I}_L = \frac{100}{j2.5 + \frac{-2}{2-j1}} = \frac{100(2-j1)}{2.5+j3} = \boxed{57.26\angle -76.76°\,(2.29in)}$$

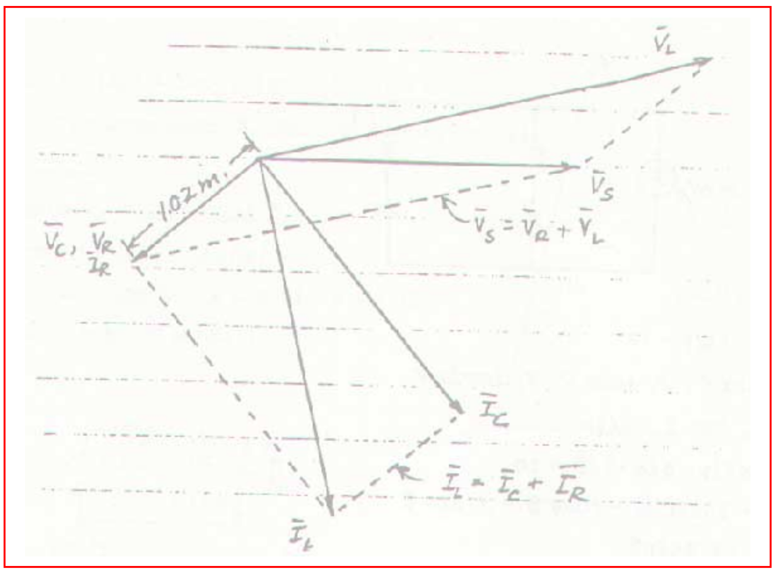
$$\mathrm{I}_R = (57.26\angle -76.76°)\frac{-j1}{2-j1} = \boxed{25.61\angle -140.19°\,(1.02in)}$$

$$\mathrm{I}_c = (57.26\angle -76.76°)\frac{2}{2-j1} = \boxed{51.21\angle -50.19°\,(2.05in)}$$

$$\mathrm{V}_L = 2.5 \times 57.26\angle 90° - 76.76° = \boxed{143.15\angle 13.24°\,(2.86in)}$$

$$\mathrm{V}_R = 2 \times 25.61\angle -140.19° = \boxed{51.22\angle -140.19°\,(1.02in)}$$

$$\mathrm{V}_c = \boxed{51.21\angle -140.19°\,(1.02in)}$$



82.

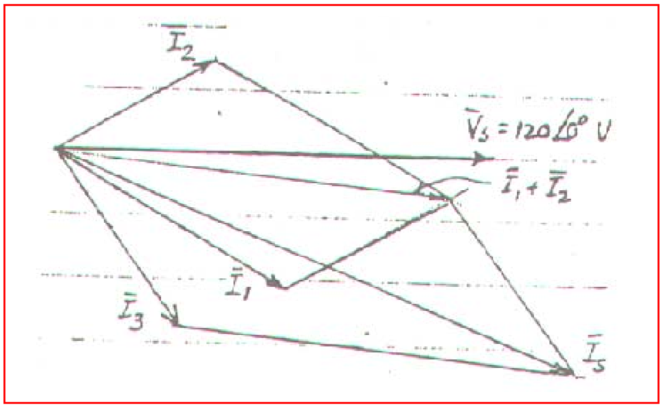
(a)

$$I_1 = \frac{120}{40\angle 30°} = \boxed{3\angle -30°\text{ A}}$$

$$I_2 = \frac{120}{50 - j30} = \boxed{2.058\angle 30.96°\text{ A}}$$

$$I_3 = \frac{120}{30 + j40} = \boxed{2.4\angle -53.13°\text{ A}}$$

(b)



(c)

$$I_s = I_1 + I_2 + I_3$$

$$= \boxed{6.265\angle -22.14°\text{ A}}$$

83.

$|\mathrm{I}_1| = 5\mathrm{A},\ |\mathrm{I}_2| = 7\mathrm{A}$

$\mathrm{I}_1 + \mathrm{I}_2 = 10\angle 0°,\ \mathrm{I}_1$ lags V, I_2 leads V

I_1 lags I_2. Use $2.5\mathrm{A}/in$

[Analytically: $5\angle\alpha + 7\angle\beta = 10$

$= 5\cos\alpha + j5\sin\alpha + 7\cos\beta + j7\sin\beta$

$\therefore \sin\alpha = -1.4\sin\beta$

$\therefore 5\sqrt{1-1.4^2\sin^2\beta} + 7\sqrt{1-1\sin^2\beta} = 10$

By SOLVE, $\boxed{\alpha = -40.54°\ \beta = 27.66°]}$

84. $\mathbf{V}_1 = 100\angle 0^o$ V, $|\mathbf{V}_2| = 140$ V, $|\mathbf{V}_1 + \mathbf{V}_2| = 120$ V.
Let 50 V = 1 inch. From the sketch, for $\angle\mathbf{V}_2$ positive,
$\boxed{\mathbf{V}_2 = 140\angle 122.5^o.}$ We may also have $\boxed{\mathbf{V}_2 = 140\angle -122.5^o \text{ V}}$

[Analytically: $|100 + 140\angle\alpha| = 120$
so $|\,100 + 140 \cos\alpha + j140 \sin\alpha\,| = 120$
Using the "Solve" routine of a scientific calculator,
$\alpha = \pm 122.88^o$.]

CHAPTER 11 SOLUTIONS

1.

$$\mathbf{Z}_c = \frac{10^6}{j500 \times 25} = -j80\,\Omega,\ \frac{50(-j80)}{50-j80} = 42.40\angle -32.01°\,\Omega$$

$$\therefore \mathbf{V} = 84.80\angle -32.01°\,\text{V},\ \mathbf{I}_R = 1.696\angle -32.01°\,\text{A}$$

$$\mathbf{I}_c = 1.0600\angle 57.99°\,\text{A}$$

$$p_s(\pi/2\text{ms}) = 84.80\cos(45° - 32.01°)\,2\cos 45° = \boxed{116.85\,\text{W}}$$

$$p_R = 50 \times 1.696^2 \cos^2(45° - 32.01°) = \boxed{136.55\,\text{W}}$$

$$p_c = 84.80\cos(45° - 32.01°) = 1.060\cos(45° + 57.99°) = \boxed{-19.69\,\text{W}}$$

CHAPTER 11 SOLUTIONS

2.

(a) $4\text{H}: i = 2t^2 - 1 \therefore v = Li' = 4(4t) = 16t, w_L = \frac{1}{2} Li^2 = \frac{1}{2} \times 4(4t^4 - 4t^2 + 1)$

$\therefore w_L = 8t^4 - 8t^2 + 2 \therefore w_L(3) - w_L(1) = 8\times 3^4 - 8\times 3^2 + 2 - 8\times 1 + 8\times 1 - 2 = \boxed{576\text{ J}}$

(b) $0.2\text{F}: v_c = \frac{1}{0.2}\int_1^t (2t^2 - 1)\, dt + 2 = 5\left(\frac{2}{3}t^3 - t\right)\Big|_1^t + 2 = 5\left(\frac{2}{3}t^3 - t\right) - 5\left(\frac{2}{3} - 1\right) + 2$

$\therefore v_c(2) = \frac{10}{3}\times 8 - 10 - \frac{10}{3} + 5 + 2 = \frac{61}{3}\text{V} \therefore \text{P}_c(2) = \frac{61}{3}\times 7 = \boxed{142.33\,\text{W}}$

CHAPTER 11 SOLUTIONS

3. $v_c(0) = -2\text{V},\ i(0) = 4\text{A},\ \alpha = \dfrac{\text{R}}{2\text{L}} = 2,\ \omega_o^2 = \dfrac{1}{\text{LC}} = 3,\ s_{1,2} = -2 \pm 1 = -1, -3$

(a) $i = \text{A}e^{-t} + \text{B}e^{-3t}\ \therefore \text{A} + \text{B} = 4;\ i(0^+) = \dfrac{1}{1} v_L(0^+) = (-4\times 4\times +2) = -14$

$\therefore -\text{A} - 38 = -14 \therefore \text{B} = 5,\ \text{A} = -1,\ i = -e^{-t} + 5e^{-3t}\ \text{A}$

$\therefore +v_c = 3\int_o^t (-e^{-t} + 5e^{-3t})\,dt - 2 = 3(e^{-t} - 5e^{-3t})\,\big|_o^t - 2 = e^{-t} - 3 - 5e^{-3t} + 5 - 2$

$\therefore v_c = 3e^{-t} - 5e^{-3t}\ \therefore \text{P}_c(0^+) = (3-5)(-1+5) = \boxed{-8\,\text{W}}$

(b) $\text{P}_c(0.2) = (3e^{-0.2} - 5e^{-0.6})(-e^{0.2} + 5e^{-0.6}) = \boxed{-0.5542\,\text{W}}$

(c) $\text{P}_c(0.4) = (3e^{-0.4} - 5e^{-1.2})(5e^{-1.2} - e^{-0.4}) = \boxed{0.4220\,\text{W}}$

CHAPTER 11 SOLUTIONS

4. We assume the circuit has already reached sinusoidal steady state by t = 0.
2.5 kΩ → 2.5 kΩ, 1 H → j1000 Ω, 4 μF → -j250 Ω, 10 kΩ → 10 kΩ

$$\mathbf{Z}_{eq} = j1000 \parallel -j250 \parallel 10000 = 11.10 - j333.0\ \Omega$$

$$\mathbf{V}_{eq} = \frac{(20\angle 30)(11.10 - j333.0)}{2500 + 11.10 - j333.0} = 2.631\angle 50.54^{\circ}\ \text{V}$$

$$\mathbf{I}_{10k} = \frac{\mathbf{V}_{eq}}{10000} = 0.2631\angle -50.54^{\circ}\ \text{mA} \qquad \mathbf{I}_{1\,H} = \frac{\mathbf{V}_{eq}}{j1000} = 2.631\angle -140.5^{\circ}\ \text{mA}$$

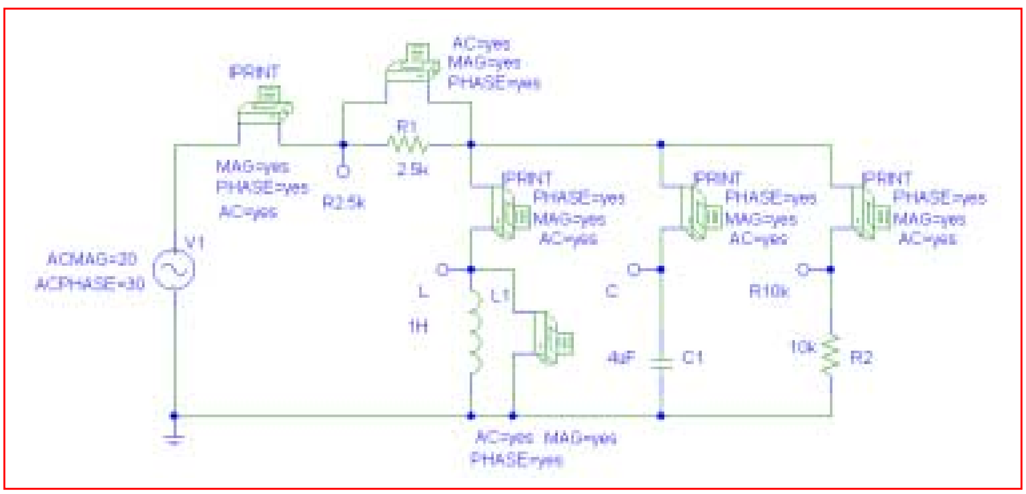
$$\mathbf{I}_{4\,\mu F} = \frac{\mathbf{V}_{eq}}{-j250} = 10.52\angle 39.46^{\circ}\ \text{mA} \quad \mathbf{V}_{2.5k} = \frac{(20\angle 30)(2500)}{2500 + 11.10 - j333.0} = 19.74\angle 37.55^{\circ}\ \text{V}$$

Thus, $$\text{P}_{2.5k} = \frac{[19.74\cos 37.55^{\circ}]^2}{2500} = \boxed{97.97\ \text{mW}}$$

$$\text{P}_{1\,H} = [2.631\cos(-50.54)^{\circ}][2.631\times 10^{-3}\cos(-140.5^{\circ})] = \boxed{-3.395\ \text{mW}}$$

$$\text{P}_{4\,\mu F} = [2.631\cos(-50.54^{\circ})][10.52\times 10^{-3}\cos(39.46^{\circ})] = \boxed{13.58\ \text{mW}}$$

$$\text{P}_{2.5k} = \frac{[2.631\cos(-50.54^{\circ})]^2}{10000} = \boxed{279.6\ \mu\text{W}}$$



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E+02	7.896E-03	3.755E+01

FREQ	VM(R2_5k,$N_0002)	VP(R2_5k,$N_0002)
1.592E+02	1.974E+01	3.755E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E+02	2.628E-03	-1.405E+02

FREQ	VM(L,0)	VP(L,0)
1.592E+02	2.629E+00	-5.054E+01

FREQ	IM(V_PRINT11)	IP(V_PRINT11)
1.592E+02	1.052E-02	3.946E+01

FREQ	IM(V_PRINT12)	IP(V_PRINT12)
1.592E+02	2.629E-04	-5.054E+01

5.

$$i_s \rightarrow 5\angle 0°\text{ A, C} \rightarrow -j4\,\Omega,\ \mathbf{Z}_{in} = 8 \| (3-j4) = \frac{40\angle -53.13°}{11-j4}$$

$$= 3.417\angle -33.15° \therefore \mathbf{V}_s = 17.087\angle -33.15°,$$

$$v_s = 17.087\cos(25t - 33.15°)\text{ V} \therefore$$

$$\text{P}_{s,abs}(0.1) = -17.087\cos(2.5^{\text{rad}} - 33.147°)\times 5\cos 2.5^{\text{rad}} = \boxed{-23.51\text{ W}}$$

$$i_8 = \frac{17.087}{8}\cos(25t - 33.15°) \therefore$$

$$i_8(0.1) = 2.136\cos(2.5^{\text{rad}} - 33.15°) = -0.7338\text{ A}$$

$$\therefore \text{P}_{8,abs} = 0.7338^2 \times 8 = \boxed{4.307\text{ W}};$$

$$\mathbf{I}_3 = \frac{17.087\angle -33.15°}{3-j4} = 3.417\angle 19.98°\text{ A}$$

$$\therefore i_3(0.1) = 3.417\cos(2.5^{\text{rad}} + 19.98°) = -3.272\text{ A} \therefore$$

$$\text{P}_{3,abc} = 3.272^2 \times 3 = \boxed{32.12\text{ W}}$$

$$\mathbf{V}_c = -j4(3.417\angle 19.983°) = 13.67\angle -70.02°,$$

$$v_c(0.1) = 13.670\cos(2.5^{\text{rad}} - 70.02°) = 3.946\text{ V}$$

$$\therefore \text{P}_{c,abc} = 3.946(-3.272) = \boxed{-12.911\text{ W}} \qquad (\Sigma = 0)$$

6.

$$\mathbf{Z}_{in} = 4 + \frac{j5(10 - j5)}{10} = 4 + 2.5 + j5 = 6.5 + j5 \ \ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{6.5 + j5} = 12.194\angle - 37.57° \text{ A}$$

$$\therefore \text{P}_{s,abs} = -\frac{1}{2} \times 100 \times 12.194 \cos 37.57° = -483.3 \text{ W}$$

$$\text{P}_{4,abs} = \frac{1}{2}(12.194)^2\, 4 = \boxed{297.4 \text{ W}},$$

$$\text{P}_{cabs} = \boxed{0}$$

$$\mathbf{I}_{10} = \frac{100}{6.5 + j5}\,\frac{j5}{10} = 6.097\angle 52.43° \ \ \text{so}$$

$$\text{P}_{10,abs} = \frac{1}{2}(6.097)^2 \times 10 = \boxed{185.87 \text{ W}}$$

$$\text{P}_L = \boxed{0} \qquad (\Sigma = 0)$$

7.

$$\mathbf{V} = (10 + j10)\frac{40\angle 30^\circ}{5\angle 50^\circ + 8\angle -20^\circ} = 52.44\angle 69.18^\circ \text{ V}$$

$$P_{10,gen} = \frac{1}{2}\times 10\times 52.44\cos 69.18^\circ = \boxed{93.20 \text{ W}}$$

$$P_{j10,gen} = \frac{1}{2}\times 10\times 52.44\cos(90^\circ - 69.18^\circ) = \boxed{245.08 \text{ W}}$$

$$P_{8\angle -20 abs} = \frac{1}{2}\left(\frac{52.44}{8}\right)^2 8\cos(-20^\circ) = \boxed{161.51 \text{ W}} \qquad (\Sigma_{gen} = \Sigma_{abs})$$

8.

$$\mathbf{Z}_R = 3 + \frac{1}{0.1 - j0.3} = 3 + 1 + j3 = 4 + j3\,\Omega$$

Ignore 30° on $\mathbf{V}_s$, $\mathbf{I}_R = 5\,\dfrac{2 + j5}{6 + j8}$, $|\mathbf{I}_R| = \dfrac{5\sqrt{29}}{10}$

(a) $$\mathrm{P}_{3\Omega} = \frac{1}{2}\left(\frac{5\sqrt{29}}{10}\right)^2 \times 3 = \boxed{10.875\ \mathrm{W}}$$

(b) $$\mathrm{V}_s = 5\angle 0° \frac{(2 + j5)\,(4 + j3)}{6 + j8} = 13.463\angle 51.94°\,\mathrm{V}$$

$$\therefore \mathrm{P}_{s,gen} = \frac{1}{2} \times 13.463 \times 5 \cos 51.94° = \boxed{20.75\ \mathrm{W}}$$

9.

$$\boxed{P_{j10} = P_{-j5} = 0,}$$

$$\frac{\mathbf{V}_{10} - 50}{j10} + \frac{\mathbf{V}_{10}}{10} + \frac{\mathbf{V}_{10} - j50}{-j5} = 0$$

$$\therefore \mathbf{V}_{10}(-j0.1 + 0.1 + j0.2) + j5 + 10 = 0$$

$$\therefore \mathbf{V}_{10} = 79.06\angle 16.57°\text{ V}$$

$$P_{10\Omega} = \frac{1}{2}\frac{79.06^2}{10} = \boxed{312.5\text{ W;}}$$

$$\mathbf{I}_{50} = \frac{79.06\angle 161.57° - 50}{j10} = 12.75\angle 78.69°\text{ A}$$

$$\therefore P_{50V} = \frac{1}{2}\times 50\times 12.748\cos 78.69° = \boxed{62.50\text{ W}}$$

$$\mathbf{I}_{j50} = \frac{79.06\angle 161.57° - j50}{-j5} = 15.811\angle -7.57°:$$

$$P_{j50} = \frac{1}{2}\times 50\times 15.811\cos(90° + 71.57°) = \boxed{-375.0\text{ W}}$$

10.

$$\frac{\mathbf{V}_x - 20}{2} + \frac{\mathbf{V}_x - \mathbf{V}_c}{3} = 2\mathbf{V}_c,$$

$$3\mathbf{V}_x - 60 + 2\mathbf{V}_x - 2\mathbf{V}_c = 12\mathbf{V}_c$$

$$\therefore 5\mathbf{V}_x - 14\mathbf{V}_c = 60, \ \frac{\mathbf{V}_c - \mathbf{V}_x}{3} + \frac{\mathbf{V}_c}{-j2} = 0$$

$$\therefore 2\mathbf{V}_c - 2\mathbf{V}_x + j3\mathbf{V}_c = 0, \ -2\mathbf{V}_x + (2 + j3)\,\mathbf{V}_c = 0$$

$$\mathbf{V}_x = \frac{\begin{vmatrix} 60 & -14 \\ 0 & 2 + j3 \end{vmatrix}}{\begin{vmatrix} 5 & -14 \\ -2 & 2 + j3 \end{vmatrix}} = \frac{120 + j180}{10 + j15 - 28} = 9.233\angle -83.88^\circ\,\text{V}$$

$$\mathbf{V}_c = \frac{\begin{vmatrix} 5 & 60 \\ -2 & 0 \end{vmatrix}}{-18 + j15} = 5.122\angle -140.9^\circ\,\text{V} \therefore$$

$$\text{P}_{gen} = \frac{1}{2} \times 9.233 \times 2 \times 5.122 \cos(-83.88^\circ + 140.19^\circ) = \boxed{26.23\ \text{W}}$$

CHAPTER 11 SOLUTIONS

11.

(a) $X_{in} = 0 \therefore \mathbf{Z}_L = \boxed{R_{th} + j0}$

(b) R_L, X_L independent $\therefore \boxed{\mathbf{Z}_L = \mathbf{Z}^*_{th} = R_{th} - jX_{th}}$

(c) R_L fixed $\therefore P_L = \dfrac{1}{2}\dfrac{|V_{th}|^2}{(R_{th}+R_L)^2 + (X_{th}+X_L)^2} \times R_L \therefore \boxed{\mathbf{Z}_L = R_L - jX_{th}}$

(d) X_L fixed, Let $X_L + X_{th} = a \therefore f = \dfrac{2P_L}{|\mathbf{V}_{th}|^2} = \dfrac{RL}{(R_{th}+R_L)^2 + a^2}$

$$\frac{df}{dR_L} = \frac{R_{th} + R_L^2 + a^2 - 2R_L\,(R_{th}+R_L)}{\left[(R_{th}+R_L)^2 + a^2\right]^2} = 0$$

$$R_{th}^2 + 2R_{th}R_L + R_L^2 + a^2 - 2R_{th}R_L = 2R_L^2 = 0$$

$$\therefore R_L = \sqrt{R_{th}^2 + a^2} = \boxed{\sqrt{R_{th}^2 + (X_{th}+X_L)^2}}$$

(e) $X_L = 0 \therefore R_L = \boxed{\sqrt{R_{th}^2 + X_{th}^2}} = |\mathbf{Z}_{th}|$

12.

$$\mathbf{V}_{th} = 120\frac{-10}{10+j5} = 107.33\angle -116.57^\circ \text{ V}$$

$$\mathbf{Z}_{th} = \frac{-j10\,(10+j15)}{10+j5} = 8-j14\,\Omega$$

(a) $\therefore \mathbf{Z}_L = \boxed{8+j15\,\Omega}$

(b) $\mathbf{I}_L = \dfrac{107.33\angle -116.57^\circ}{16}\ \therefore$

$$\text{P}_{L,\max} = \frac{1}{2}\left(\frac{107.33}{16}\right)^2 \times 8 = \boxed{180\ \text{W}}$$

13.

$$R_L = \left|\mathbf{Z}_{th}\right| \therefore R_L = \sqrt{8^2 + 14^2} = \boxed{16.125\,\Omega}$$

$$P_L = \frac{1}{2}\,\frac{107.33^2}{(8+16.125)^2 + 14^2} \times 16.125 = \boxed{119.38\,\text{W}}$$

14.

$$-j9.6 = -4.8\,\mathrm{I}_x - j1.92\ \mathrm{I}_x - +4.8\mathrm{I}_x$$

$$\therefore\ \mathrm{I}_x = \frac{9.6}{1.92} = 5$$

$$\therefore\ \mathrm{V} = (0.6\times 5)8 = 24\,\mathrm{V}$$

$$\therefore\ \mathrm{P}_o = \frac{1}{2}\times 24\times 1.6\times 5 = \boxed{96\ \mathrm{W}\ (gen)}$$

15.

(a) $\mathbf{Z}_{th} = 80 \| j60 = \dfrac{j480}{80 + j60} \, \dfrac{80 - j60}{80 - j60}$

$= 28.8 + j38.4\,\Omega \therefore \mathbf{Z}_{L\max} = \boxed{28.8 - j38.4\,\Omega}$

(b) $\mathbf{V}_{th} = 5(28.8 + j38.4) = 144 + j192\,\text{V},$

$\therefore \mathbf{I}_L = \dfrac{144 + j192}{2 \times 28.8}$

and $\text{P}_{L,\max} = \dfrac{1}{2} \, \dfrac{144^2 + 192^2}{4 \times 28.8^2} \times 28.8 = \boxed{250\ \text{W}}$

16. $\mathbf{Z}_{eq} = (6 - j8) \parallel (12 + j9) = 8.321 \angle -19.44^o$ W

$\mathbf{V}_{eq} = (5 \angle -30^o)(8.321 \angle -19.44^o) = 41.61 \angle -49.44^o$ V

$P_{total} = ½ (41.61)(5) \cos(-19.44^o) = 98.09$ W

$\mathbf{I}_{6-j8} = \mathbf{V}_{eq} / (6 - j8) = 4.161 \angle 3.69^o$ A
$\mathbf{I}_{4+j2} = \mathbf{I}_{8+j7} = \mathbf{V}_{eq} / 12+j9 = 2.774 \angle -86.31^o$ A

$P_{6-j8} = ½ (41.61)(4.161) \cos(-49.44^o - 3.69^o) = \boxed{51.94 \text{ W}}$

$P_{4+j2} = ½ (2.774)2 (4) = \boxed{15.39 \text{ W}}$
$P_{8+j7} = ½ (2.774)2 (8) = \boxed{30.78 \text{ W}}$

Check: $\Sigma = 98.11$ W (okay)

17.

$$\mathbf{V}_{th} = 100\frac{j10}{20+j10} = 20+j40,\ \mathbf{Z}_{th} = \frac{j10(20)}{20+j10} = 4+j8\,\Omega$$

$$\therefore \mathrm{R}_L = |\mathbf{Z}_{th}|\ \therefore \mathrm{R}_L = \boxed{8.944\,\Omega}$$

$$\therefore \mathrm{P}_{L,\max} = \frac{1}{2}\,\frac{20^2+40^2}{(4+8.944)^2+64}\times 8.944 \boxed{= 38.63\,\mathrm{W}}$$

18. We may write a single mesh equation: $170\angle 0^o = (30 + j10)\,\mathbf{I}_1 - (10 - j50)(-\lambda\mathbf{I}_1)$
Solving,

$$\mathbf{I}_1 = \frac{170\angle 0^o}{30 + j10 + 10\lambda - j50\lambda}$$

(a) $\lambda = 0$, so $\mathbf{I}_1 = \dfrac{170\angle 0^o}{30 + j10} = 5.376\angle\text{-}18.43^o$ A and, with the same current flowing through both resistors in this case,

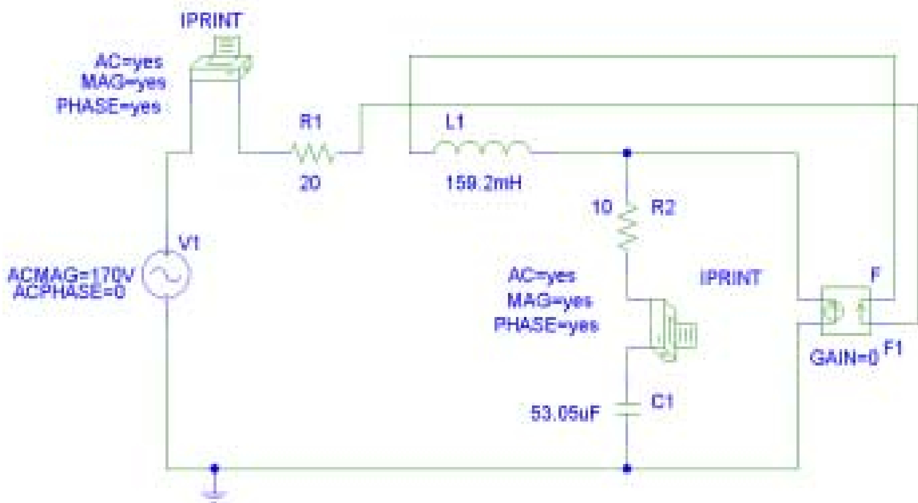
$P_{20} = ½\,(5.376)^2\,(20) = \boxed{289.0\text{ W}}$
$P_{10} = ½\,(5.376)^2\,(10) = \boxed{144.5\text{ W}}$

(b) $\lambda = 1$, so $\mathbf{I}_1 = \dfrac{170\angle 0^o}{40 - j40} = 3.005\angle 45^o$ A

$P_{20} = ½\,(3.005)^2\,(20) = \boxed{90.30\text{ W}}$
The current through the 10-Ω resistor is $\mathbf{I}_1 + \lambda\mathbf{I}_1 = 2\,\mathbf{I}_1 = 6.01 \angle 45^o$ so

$P_{10} = ½\,(6.01)^2\,(10) = \boxed{180.6\text{ W}}$

(c)



(a)

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	5.375E+00	-1.846E+01

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	5.375E+00	-1.846E+01

(b)

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	6.011E+00	4.499E+01

FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	3.006E+00	4.499E+01

19. (a) Waveform (a): $I_{avg} = \dfrac{(10)(1)+(-5)(1)+0(1)}{3} = \boxed{1.667\text{ A}}$

Waveform (b): $I_{avg} = \dfrac{\frac{1}{2}(20)(1)+0(1)}{2} = \boxed{5\text{ A}}$

Waveform (c):

$$I_{avg} = \frac{1}{1\times10^{-3}}\int_0^{10^{-3}} 8\sin\frac{2\pi t}{4\times10^{-3}}\,dt = -\left(8\times10^{3}\right)\left(\frac{4\times10^{-3}}{2\pi}\right)\cos\left(\frac{\pi t}{2\times10^{-3}}\right)\Bigg|_0^{10^{-3}}$$

$$= -\frac{16}{\pi}(0-1) = \boxed{\frac{16}{\pi}\text{ A}}$$

(b) Waveform (a): $I^2_{avg} = \dfrac{(100)(1)+(25)(1)+(0)(1)}{3} = \boxed{41.67\text{ A}^2}$

Waveform (b): $i(t) = -20\times10^{3}\, t + 20$

$i^2(t) = 4\times10^{8}\, t^2 - 8\times10^{5}\, t + 400$

$$I^2_{avg} = \frac{1}{2\times10^{-3}}\int_0^{10^{-3}} \left(4\times10^{8}t^2 - 8\times10^{5}t + 400\right)dt$$

$$= \frac{1}{2\times10^{-3}}\left[\frac{4\times10^{8}}{3}\left(10^{-3}\right)^3 - \frac{8\times10^{5}}{2}\left(10^{-3}\right)^2 + 400\left(10^{-3}\right)\right] = \frac{0.1333}{2\times10^{-3}} = \boxed{66.67\text{ A}^2}$$

Waveform (c):

$$I^2_{avg} = \frac{1}{1\times10^{-3}}\int_0^{10^{-3}} 64\sin^2\frac{2\pi t}{4\times10^{-3}}\,dt = \left(64\times10^{3}\right)\left[\frac{t}{2} - \frac{\sin\pi\times10^{3}t}{2\pi\times10^{3}}\right]\Bigg|_0^{10^{-3}}$$

$$= \left(64\times10^{3}\right)\left[\frac{10^{-3}}{2} - \frac{\sin\pi}{2\pi\times10^{3}}\right] = \boxed{32\text{ A}^2}$$

20. At $\omega = 120\pi$, 1 H $\rightarrow j377\ \Omega$, and 4 μF $\rightarrow -j663.1\ \Omega$

Define $\mathbf{Z}_{\text{eff}} = j377 \parallel -j663.1 \parallel 10\,000 = 870.5\angle 85.01^\circ\ \Omega$

$$\mathbf{V}_{2.5\text{k}} = \frac{\left(400\sqrt{2}\angle -9^\circ\right)2500}{2500 + 870.5\angle 85.01^\circ} = 520.4\angle -27.61^\circ\ \text{V}$$

$$\mathbf{V}_{10\text{k}} = \frac{\left(400\sqrt{2}\angle -9^\circ\right)\left(870.5\angle 85.01^\circ\right)}{2500 + 870.5\angle 85.01^\circ} = 181.2\angle 57.40^\circ\ \text{V}$$

Thus, $P_{2.5\text{k}}$ = ½ (520.4)2 / 2 500 = 54.16 W

$P_{10\text{k}}$ = ½ (181.2)2 / 10 000 = 1.642 W

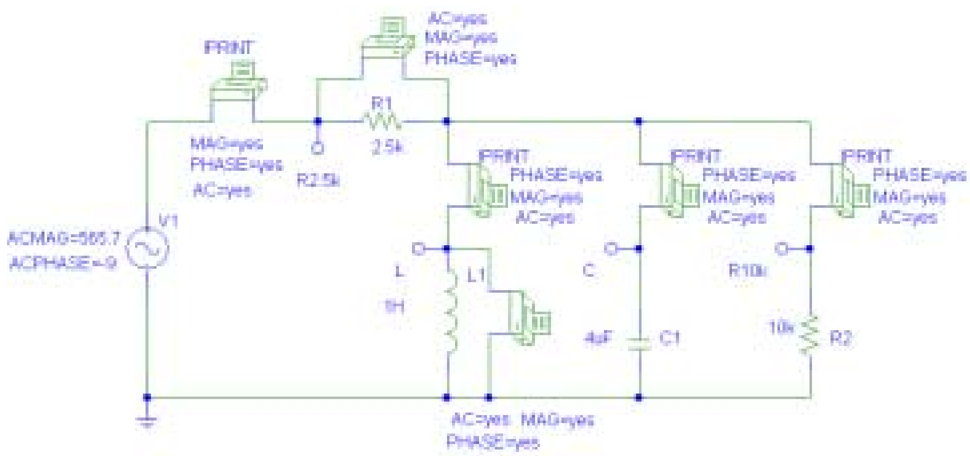
$P_{1\text{H}}$ = 0

$P_{4\mu\text{F}}$ = 0 (A total absorbed power of 55.80 W.)

To check, the average power delivered by the source:

$$\mathbf{I}_{\text{source}} = \frac{400\sqrt{2}\angle -9^\circ}{2500 + 870.5\angle 85.01^\circ} = 0.2081\angle -27.61^\circ\ \text{A}$$

and $P_{\text{source}} = ½\,(400\sqrt{2})(0.2081)\cos(-9^\circ + 27.61^\circ) =$ 55.78 W (checks out).



FREQ	IM(V_PRINT1)	IP(V_PRINT1)	FREQ	VM(L,0)	VP(L,0)
6.000E+01	2.081E-01	-2.760E+01	6.000E+01	1.812E+02	5.740E+01
FREQ	VM(R2_5k,$N_0002)	VP(R2_5k,$N_0002)	FREQ	IM(V_PRINT11)	IP(V_PRINT11)
6.000E+01	5.204E+02	-2.760E+01	6.000E+01	2.732E-01	1.474E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)	FREQ	IM(V_PRINT12)	IP(V_PRINT12)
6.000E+01	4.805E-01	-3.260E+01	6.000E+01	1.812E-02	5.740E+01

21.

(a) $v = 10 + 9\cos 100t + 6\sin 100t$

$$\therefore \mathrm{V}_{eff} = \sqrt{100 + \frac{1}{2}\times 81 + \frac{1}{2}\times 36} = \sqrt{158.5} = \boxed{12.590\ \mathrm{V}}$$

(b) $$\mathrm{F}_{eff} = \sqrt{\frac{1}{4}(10^2 + 20^2 + 10^2)} = \sqrt{150} = \boxed{12.247}$$

(c) $$\mathrm{F}_{\mathrm{avg}} = \frac{(10)(1) + (20)(1) + (10)(1)}{4} = \frac{40}{4} = \boxed{10}$$

22.

(a) $g(t) = 2 + 3\cos 100t + 4\cos(100t - 120^o)$

$3\angle 0 + 4\angle{-120^o} = 3.606\angle{-73.90^o}$ so $G_{eff} = \sqrt{4+\frac{3.606^2}{2}} = \boxed{3.240}$

(b) $h(t) = 2 + 3\cos 100t + 4\cos(101t - 120°)$

$\therefore H_{eff} = \sqrt{2+\frac{1}{2}3^2+\frac{1}{2}4^2} = \sqrt{16.5} = \boxed{4.062}$

(c) $f(t) = 100t,\ 0 < t < 0.1 \therefore F_{eff} = \sqrt{\frac{1}{0.3}\int_0^{0.1} 10^6 t^2 dt}$

$= \sqrt{\frac{10}{3}\times 10^6 \times \frac{1}{3}\times 10^{-3}} = \boxed{33.33}$

23. $f(t) = (2 - 3\cos 100t)^2$

(a) $f(t) = 4 - 12\cos 100t + 9\cos^2 100t$

$\therefore f(t) = 4 - 12\cos 100t + 4.5 + 4.5\cos 200t \therefore F_{av} = 4 + 4.5 = \boxed{8.5}$

(b) $F_{eff} = \sqrt{8.5^2 + \frac{1}{2}\times 12^2 + \frac{1}{2}\times 4.5^2} = \boxed{12.43}$

24. (a) $i_{\text{eff}} = \left[\frac{1}{3}\left(10^2 + (-5)^2\right) + 0\right]^{1/2} = \boxed{6.455 \text{ A}}$

(b) $i_{\text{eff}} = \left[\frac{1}{2}\left(\int_0^1 [-20t + 20]\,dt\right) + 0\right]^{1/2} = \sqrt{5} = \boxed{2.236 \text{ A}}$

(c) $i_{\text{eff}} = \left[\frac{1}{1}\left(\int_0^1 8\sin\left(\frac{2\pi}{4}t\right)dt\right)\right]^{1/2} = \sqrt{\left[-8\left(\frac{2}{\pi}\right)\cos\left(\frac{\pi t}{2}\right)\right]_0^1} = \boxed{2.257 \text{ A}}$

25.

(a) $A = B = 10\text{V},\ C = D = 0 \therefore 10\angle 0° + 10\angle -45° = 18.48\angle -22.50^{o}$

$$\therefore P = \frac{1}{2}\times\frac{1}{4}\times 18.48^2 = \boxed{42.68\text{ W}}$$

(b) $A = C = 10\text{V},\ B = D = 0,\ v_s = 10\cos 10t + 10\cos 40t,$

$$P = \frac{1}{2}\frac{10^2}{4} + \frac{1}{2}\frac{10^2}{4} = \boxed{25\text{ W}}$$

(c) $v_s = 10\cos 10t - 10\sin\ (10t + 45°) \rightarrow 10 - 10\angle -45° = 7.654\angle 67.50^{o}$

$$\therefore P = \frac{1}{2}\frac{7.654^2}{4} = \boxed{7.322\text{ W}}$$

(d) $v = 10\cos 10t + 10\sin(10t + 45°) + 10\cos 40t;$

$10\angle 0° + 10\angle -45° = 18.48\angle -22.50^{o}$

$$\therefore P = \frac{1}{2}\times 18.48^2\times\frac{1}{4} + \frac{1}{2}\times 10^2\times\frac{1}{4} = \boxed{55.18\text{ W}}$$

(e) $// + 10dc \therefore P_{av} = 55.18 + \frac{10^2}{4} = \boxed{80.18\text{ W}}$

26. $\mathbf{Z}_{eq} = \text{R} \parallel j0.3\omega = \dfrac{j0.3R\omega}{R + j0.3R\omega}$. By voltage division, then, we write:

$$\mathbf{V}_{100\text{mH}} = 120\angle 0 \; \frac{j0.1\omega}{j0.1\omega + \dfrac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{-0.03\omega^2 + j0.1\omega R}{-0.03\omega^2 + j0.4R\omega}$$

$$\mathbf{V}_{300\text{mH}} = 120\angle 0 \; \frac{\dfrac{j0.3R\omega}{R + j0.3\omega}}{j0.1\omega + \dfrac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{j36R\omega}{-0.03\omega^2 + j0.4R\omega}$$

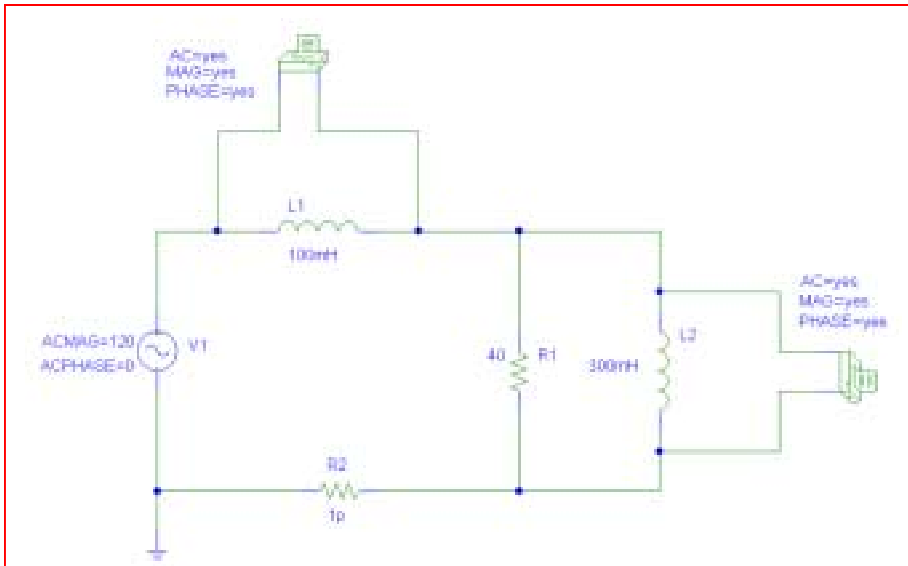
(a) We're interested in the value of R that would lead to equal voltage magnitudes, or

$$\left| \, j36R\omega \, \right| = \left| (120)\left(-0.03\omega^2 + j0.1\omega \text{R}\right) \right|$$

Thus, $36\text{R}\omega = \sqrt{12.96\omega^4 + 144\omega^2 R^2}$ or $\boxed{\text{R} = 0.1061\ \omega}$

(b) Substituting into the expression for $\mathbf{V}_{100\text{mH}}$, we find that $\text{V}_{100\text{mH}} = \boxed{73.47\text{ V},}$ independent of frequency.

To verify with PSpice, simulate the circuit at 60 Hz, or $\omega = 120\pi$ rad/s, so R = 40 Ω. We also include a miniscule (1 pΩ) resistor to avoid inductor loop warnings. We see from the simulation results that the two voltage magnitudes are indeed the same.




```
FREQ        VM($N_0002,$N_0003)VP($N_0002,$N_0003)
6.000E+01   7.349E+01  -3.525E+01

FREQ        VM($N_0001,$N_0002)VP($N_0001,$N_0002)
6.000E+01   7.347E+01   3.527E+01
```


27.

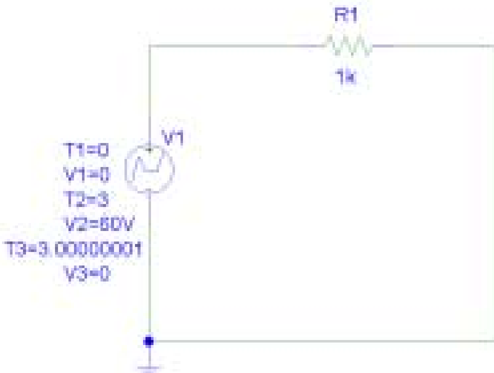
(a) $V_{av,1} = \boxed{30\text{V}}$

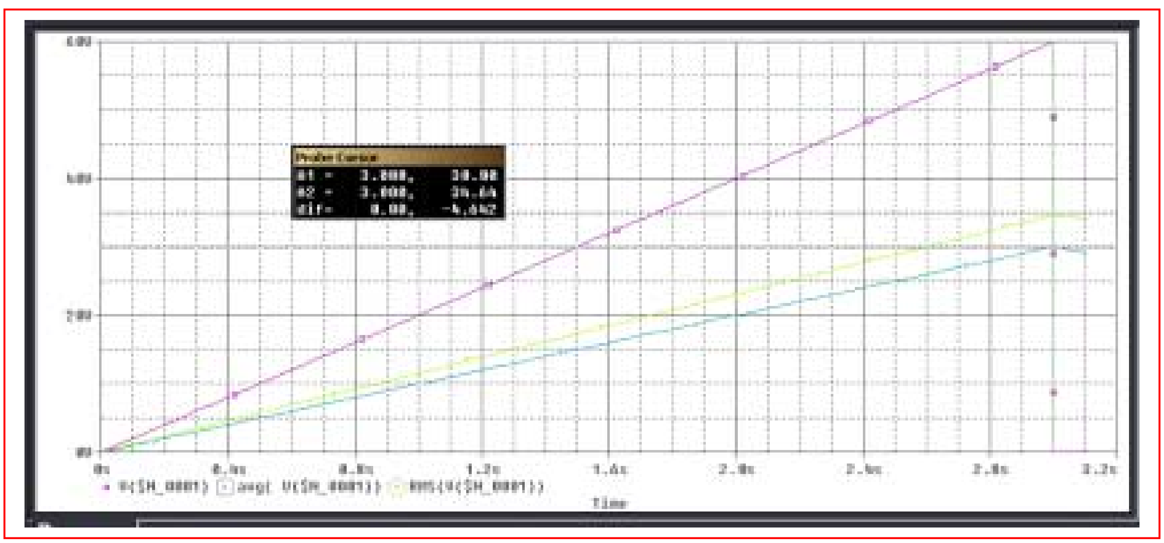
$$V_{av,2} = \frac{1}{3}(10+30+50) = \boxed{30\text{V}}$$

(b) $V_{eff,1} = \sqrt{\frac{1}{3}\int_0^3 (20t)^2\,dt} = \sqrt{\frac{1}{3}\times 400\times\frac{1}{3}\times 27} = \sqrt{1200} = \boxed{34.64\text{V}}$

$$V_{eff,2} = \sqrt{\frac{1}{3}(10^2+30^2+50^2)} = \sqrt{\frac{1}{3}\times 3500} = \boxed{34.16\text{ V}}$$

(c) PSpice verification for Sawtooth waveform of Fig. 11.40*a*:





28. $\mathbf{Z}_{\text{eff}} = \text{R} \,||\, \left(\dfrac{-j10^6}{3\omega}\right) = \dfrac{-jR10^6}{3\omega R - j10^6}$

$$\mathbf{I}_{\text{SRC}} = \frac{120\angle 0}{-j\dfrac{10^6}{\omega} - j\dfrac{R10^6}{3\omega R - j10^6}} = \frac{120\omega\left(3\omega \text{R - } j10^6\right)}{-j10^6\left(3\omega R - j10^6\right) - j\omega R10^6}$$

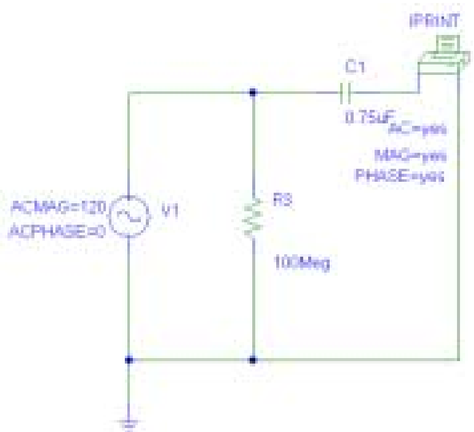
$$\mathbf{I}_{3\mu\text{F}} = \mathbf{I}_{\text{SRC}} \frac{R}{R - j\dfrac{10^6}{3\omega}}$$

(a) For the two current magnitudes to be equal, we must have $\left|\dfrac{R}{R - j\dfrac{10^6}{3\omega}}\right| = 1$. This is only true when $\boxed{\text{R} = \infty;}$ otherwise, current is shunted through the resistor and the two capacitor currents will be unequal.

(b) In this case, the capacitor current is

$$120\angle 0 \frac{1}{-j\dfrac{10^6}{\omega} - j\dfrac{10^6}{3\omega}} = j90\omega\ \mu\text{A, or} \qquad \boxed{90\omega\cos(\omega t + 90^\circ)\ \mu\text{A}}$$

(c) PSpice verification: set f = 60 Hz, simulate a single 0.75-µF capacitor, and include a 100-MΩ resistor in parallel with the capacitor to prevent a floating node. This should resit in a rms current amplitude of 33.93 mA, which it does.



FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	3.393E-02	9.000E+01

29.

$$v(t) = 10t[u(t) - u(t-2)] + 16e^{-0.5(t-3)}\ [u(t-3) - u(t-5)]\ \text{V}$$

Find eff. value separately

$$\text{V}_{1,eff} = \sqrt{\frac{1}{5}\int_0^2 100t^2 dt} = \sqrt{\frac{20}{3}\times 8} = 7.303$$

$$\text{V}_{2,eff} = \sqrt{\frac{1}{5}\int_3^5 256e^{-(t-3)} dt} = \sqrt{\frac{256}{5}e^3(-e^{-t})_3^5} = 6.654$$

$$\therefore \text{V}_{eff} = \sqrt{7.303^2 + 6.654^2} = \boxed{9.879}$$

$$\text{V}_{eff} = \sqrt{\frac{1}{5}\left[\int_0^2 100t^2 dt + \int_3^5 256e^3 e^{-t} dt\right]}$$

$$= \sqrt{\frac{1}{5}\left[\frac{100}{3}\times 8 + 256e^3\,(e^{-3} - e^{-5})\right]}$$

$$= \sqrt{\frac{1}{5}\left[\frac{800}{3} + 256\,(1 - e^{-2})\right]} = 9.879\ \text{V OK}$$

30. The peak instantaneous power is 250 mW. The combination of elements yields

$\mathbf{Z} = 1000 + j1000\ \Omega = 1414\ \angle 45^{o}\ \Omega$.

Arbitrarily designate $\mathbf{V} = V_m \angle 0$, so that $\mathbf{I} = \dfrac{V_m \angle 0}{\mathbf{Z}} = \dfrac{V_m \angle -45^{o}}{1414}$ A.

We may write $p(t) = ½\, V_m\, I_m \cos\phi + ½\, V_m\, I_m \cos(2\omega t + \phi)$ where ϕ = the angle of the current (-45^{o}). This function has a maximum value of $½\, V_m I_m \cos\phi + ½\, V_m I_m$.

Thus, $0.250 = ½\, V_m I_m (1 + \cos\phi) = ½\,(1414)\, I_m^{2}\,(1.707)$
and $I_m = 14.39$ mA.

In terms of rms current, the largest rms current permitted is $14.39 / \sqrt{2} = \boxed{10.18 \text{ mA rms.}}$

31. $\mathbf{I} = 4\angle 35° \text{ A rms}$

(a) $\mathbf{V} = 20\mathbf{I} + 80\angle 35° \text{ Vrms}, \ P_{s,gen} = 80 \times 10 \cos 35° = \boxed{655.3 \text{ W}}$

(b) $P_R = |\mathbf{I}|^2 R = 16 \times 20 = \boxed{320 \text{ W}}$

(c) $P_{Load} = 655.3 - 320 = \boxed{335.3 \text{ W}}$

(d) $AP_{s,gen} = 80 \times 10 = \boxed{800 \text{ VA}}$

(e) $AP_R = P_R = \boxed{320 \text{ VA}}$

(f) $\mathbf{I}_L = 10\angle 0° - 4\angle 35° = 7.104\angle -18.84° \text{ A rms}$

$\therefore AP_L = 80 \times 7.104 = \boxed{568.3 \text{ VA}}$

(g) $PF_L = \cos\theta_L = \dfrac{P_L}{AP_L} = \dfrac{335.3}{568.3} = \boxed{0.599}$

since I_L lags V, PF_L is $\boxed{\text{lagging}}$

32.

(a) $\mathbf{I}_s = \dfrac{120}{4 + \dfrac{j192}{12 + j16}} = 9.214\angle -26.25° \text{ A rms}$

$\therefore \text{PF}_s = \cos 26.25 = \boxed{0.8969 \text{ lag}}$

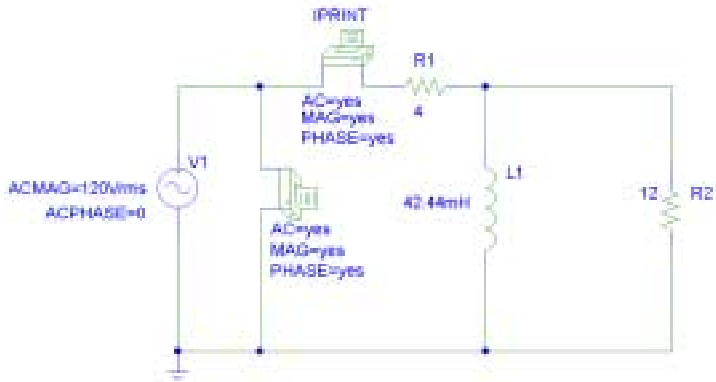
(b) $P_s = 120 \times 9.214 \times 0.8969 = \boxed{991.7\text{W}}$

(c) $\mathbf{Z}_L = 4 + \dfrac{j48}{3 + j4} = 4 + \dfrac{1}{25}(192 + j144)$

$\therefore \mathbf{Z}_L = 11.68 + j5.76\,\Omega,\ \mathbf{Y}_L = \dfrac{11.68 - j5.76}{11.68^2 + 5.76^2}$

$\therefore j120\pi\,C = \dfrac{j5.76}{11.68^2 + 5.76^2},\ C = \boxed{90.09\mu\text{F}}$

(d) PSpice verification



FREQ VM($N_0003,0) VP($N_0003,0)
6.000E+01 1.200E+02 0.000E+00

FREQ IM(V_PRINT1) IP(V_PRINT1)
6.000E+01 9.215E+00 -2.625E+01 ; (a) and (b) are correct

Next, add a 90.09-μF capacitor in parallel with the source:

FREQ IM(V_PRINT1) IP(V_PRINT1)
6.000E+01 8.264E+00 -9.774E-05 ;(c) is correct (-9.8×10^{-5} degrees is essentially zero, for unity PF).

33.

$\mathbf{Z}_A = 5 + j2\,\Omega,\ \mathbf{Z}_B = 20 - j10\,\Omega,\ \mathbf{Z}_c = 10\angle 30°\ \Omega = 8.660 + j5\ \Omega$

$\mathbf{Z}_D = 10\angle -60° = 5 - j8.660\ \ \Omega$

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 200 & -20 + j10 \\ 0 & 33.66 - j13.660 \end{vmatrix}}{\begin{vmatrix} 25 - j8 & -20 + j10 \\ -20 + j10 & 33.66 - j13.660 \end{vmatrix}} = \frac{7265\angle 22.09°}{480.9\angle -26.00°} = 15.11\angle 3.908°\ \text{A rms}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 25 - j8 & 200 \\ -20 + j10 & 0 \end{vmatrix}}{480.9\angle -26.00°} = \frac{200(20 - j10)}{480.9\angle 20.00°} = 9.300\angle -0.5681°\ \text{A rms}$$

$$\text{AP}_A = |\mathbf{I}_1|^2\,|\mathbf{Z}_A| = 15.108^2\sqrt{29} = \boxed{1229\ \ \text{VA}}$$

$$\text{AP}_B = |\mathbf{I}_1 - \mathbf{I}_2|^2\,|\mathbf{Z}_B| = 5.881^2 \times 10\sqrt{5} = \boxed{773.5\,\text{VA}}$$

$$\text{AP}_C = |\mathbf{I}_2|2|\mathbf{Z}_C| = 9.3^2 \times 10 = \boxed{86.49\,\text{VA}}$$

$$\text{AP}_D = |\mathbf{I}_2|^2\,|\mathbf{Z}_1| = 9.3^2 \times 10 = \boxed{864.9\,\text{VA}}$$

$$\text{AP}_S = 200|\mathbf{I}_1| = 200 \times 15.108 = \boxed{3022\,\text{VA}}$$

34. Perhaps the easiest approach is to consider the load and the compensation capacitor separately. The load draws a complex power $\mathbf{S}_{load} = P + jQ$. The capacitor draws a purely reactive complex power $\mathbf{S}_C = -jQ_C$.

$\theta_{load} = \tan^{-1}(Q/P)$, or $Q = P \tan \theta_{load}$

$$Q_C = S_C = V_{rms} \left| \frac{\mathbf{V}_{rms}}{(-j/\omega C)} \right| = \left| \omega C \mathbf{V}_{rms}^2 \right| = \omega C V_{rms}^2$$

$$\mathbf{S}_{total} = \mathbf{S}_{load} + \mathbf{S}_C = P + j(Q - Q_C)$$

$$\theta_{new} = \text{ang}(\mathbf{S}_{total}) = \tan^{-1}\left(\frac{Q\text{-}Q_C}{P}\right), \text{ so that } Q - Q_C = P \tan \theta_{new}$$

Substituting, we find that $Q_C = P \tan \theta_{load} - P \tan \theta_{new}$
or

$$\omega C V_{rms}^2 = P (\tan \theta_{load} - \tan \theta_{new})$$

Thus, noting that $\theta_{old} = \theta_{load}$,

$$\boxed{C = \frac{P\left(\tan \theta_{old} - \tan \theta_{new}\right)}{\omega V_{rms}^2}}$$

35. $\mathbf{Z}_1 = 30\angle 15°\,\Omega,\ \mathbf{Z}_2 = 40\angle 40°\,\Omega$

(a) $\mathbf{Z}_{tot} = 30\angle 15° + 40\angle 40° = 68.37\angle 29.31°\,\Omega$

$\therefore \text{PF} = \cos 29.3° = \boxed{0.8719 \text{ lag}}$

(b) $\mathbf{Z}_{tot} = 68.37\angle 29.31° = 59.62 + j33.48$

$\text{PF}_{new} = 0.9 \text{ lag}$

$\therefore \theta_{new} = \cos^{-1} 0.9 = 25.84°$

$\tan 25.84° = 0.4843 = \dfrac{\text{X}_{new}}{59.62} \therefore \text{X}_{new} = 28.88\,\Omega$

$\therefore 33.48 - \dfrac{1}{100\pi \text{C}} = 28.88,$

$\text{C} = \boxed{691.8\,\mu\text{F}}$

36. $\theta_1 = \cos^{-1}(0.92) = 23.07^\text{o}$, $\theta_2 = \cos^{-1}(0.8) = 36.87^\text{o}$, $\theta_3 = 0$

$$\mathbf{S}_1 = \frac{100\angle 23.07^\text{o}}{0.92} = 100 + j42.59 \text{ VA}$$

$$\mathbf{S}_2 = \frac{250\angle 36.87^\text{o}}{0.8} = 250 + j187.5 \text{ VA}$$

$$\mathbf{S}_3 = \frac{500\angle 0^\text{o}}{1} = 500 \text{ VA}$$

$$\mathbf{S}_{\text{total}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 500 + j230.1 \text{ VA} = 550.4\angle 24.71^\text{o} \text{ VA}$$

(a) $\mathbf{I}_{\text{eff}} = \dfrac{S_{\text{total}}}{V_{\text{eff}}} = \dfrac{550.4}{115} = \boxed{4.786 \text{ A rms}}$

(b) PF of composite load = $\cos(24.71^\text{o}) = \boxed{0.9084 \textit{ lagging}}$

37.

$\text{AP}_L = 10{,}000\ \text{VA},\ \text{PF}_L = 0.8\,\text{lag},\ |\mathbf{I}_L| = 40\,\text{A rms}$

Let $\mathbf{I}_L = 40\angle 0°\,\text{A rms};\ \text{P}_L = 10{,}000 \times 0.8 = 8000\ \text{W}$

Let $\mathbf{Z}_L = \text{R}_L + j\text{X}_L \therefore \text{R}_L = \dfrac{8000}{40^2} = 5\ \Omega$

$\cos\theta_L = 0.8\,\text{lag} \therefore \theta_L = \cos^{-1} 0.8 = 36.87°$

$\therefore \text{X}_L = 5\tan 36.87° = 3.75\,\Omega,\ \mathbf{Z}_L = 5 + j3.75,\ \mathbf{Z}_{tot} = 5.2 + j3.75\,\Omega$

$\therefore \mathbf{V}_s = 40(5.2 + j3.75) = 256.4\angle 35.80°\,\text{V};\ \mathbf{Y}_{tot} = \dfrac{1}{5.2 + j3.75}$

$= 0.12651 - j0.09124\text{S},\ \mathbf{Y}_{new} = 0.12651 + j(120\pi\text{C} - 0.09124),$

$\text{PF}_{new} = 0.9\ \text{lag}, \theta_{new} = 25.84° \therefore \tan 25.84° = 0.4843$

$= \dfrac{0.09124 - 120\pi\text{C}}{0.12651} \therefore$

$\text{C} = \boxed{79.48\mu\text{F}}$

38. $\mathbf{Z}_{eff} = j100 + j300 \parallel 200 = 237 \angle 54.25^o$. PF = cos 54.25° = 0.5843 *lagging*.

(a) Raise PF to 0.92 *lagging* with series capacitance

$$\mathbf{Z}_{new} = j100 + jX_C + j300 \parallel 200 = 138.5 + j(192.3 + X_C)\ \Omega$$

$$\tan^{-1}\left(\frac{192.3 + X_C}{138.5}\right) = \cos^{-1} 0.92 = 23.07^o$$

Solving, we find that $X_C = -133.3\ \Omega = -1/\omega C$, so that C = $\boxed{7.501\ \mu F}$

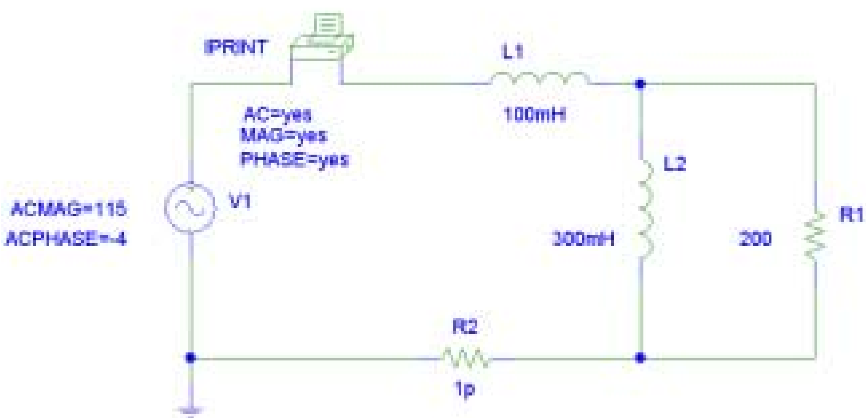
(b) Raise PF to 0.92 *lagging* with parallel capacitance

$$\mathbf{Z}_{new} = j100 \parallel jX_C + j300 \parallel 200 = \frac{-100\,X_C}{j(100 + X_C)} + 138.5 + j92.31\ \Omega$$

$$= 138.5 + j\left(92.31 + \frac{100X_C}{100 + X_C}\right)\ \Omega$$

$$\tan^{-1}\left(\frac{92.31 + \frac{100X_C}{100 + X_C}}{138.5}\right) = \cos^{-1} 0.92 = 23.07^o$$

Solving, we find that $X_C = -25\ \Omega = -1/\omega C$, so that C = $\boxed{40\ \mu F}$



General circuit for simulations. Results agree with hand calculations

	FREQ	IM(V_PRINT1)	IP(V_PRINT1)	θ	PF
With no compensation:	1.592E+02	4.853E-01	-5.825E+01	54.25°	0.5843 lag
With series compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07°	0.9200 lag
With parallel compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07°	0.9200 lag

39.

(a) $P_{s,tot} = 20 + 25\times 0.8 + 30\times 0.75 = \boxed{70\text{ kW}}$

(b)
$$\mathbf{I}_1 = \frac{20,000}{250} = 80\angle 0°\text{ A rms}$$
$$\left|\mathbf{I}_2\right| = 25,000/250 = 100\text{ A rms}$$
$$\angle\mathbf{I}_2 = -\cos^{-1}0.8 = -36.87 \therefore \mathbf{I}_2 = 100\angle -36.87°\text{ A rms}$$
$$AP_3 = \frac{30,000}{0.75} = 40,000\text{ VA},\quad \left|\mathbf{I}_3\right| = \frac{40,000}{250} = 160\text{ A rms}$$
$$\angle\mathbf{I}_3 = -\cos^{-1}0.75 = -41.41° \therefore \mathbf{I}_3 = 160\angle -41.41°\text{ A rms}$$
$$\therefore \mathbf{I}_s = 80\angle 0° + 100\angle -36.87° + 160\angle -41.41° = 325.4\angle -30.64°\text{ A rms}$$
$$\therefore AP_s = 250\times 325.4 = \boxed{81,360\text{ VA}}$$

(c) $PF_3 = \dfrac{70,000}{81,360} = \boxed{0.8604\text{ lag}}$

40. 200 kW average power and 280 kVAR reactive result in a power factor of PF = cos ($\tan^{-1}$ (280/200) = 0.5813 *lagging*, which is pretty low.

(a) 0.65 peak = 0.65(200) = 130 kVAR
Excess = 280 – 130 = 150 kVAR, for a cost of (12)(0.22)(150) = $\boxed{\$396 \text{ / year.}}$

(b) Target = **S** = P + *j*0.65 P
$\theta = \tan^{-1}(0.65P/P) = 33.02^{o}$, so target PF = $\cos\theta = \boxed{0.8385}$

(c) A single 100-kVAR increment costs $200 to install. The excess kVAR would then be 280 – 100 – 130 = 50 kVAR, for an annual penalty of $332. This would result in a first-year savings of $64.

A single 200-kVAR increment costs $395 to install, and would remove the entire excess kVAR. The savings would be $1 (wow) in the first year, but $396 each year thereafter.

The single 200-kVAR increment is the most economical choice.

41.

$$\mathbf{Z}_{in} = -j10 + \frac{20(1+j2)}{3+j2} = 10.769 - j3.846 = 11.435^{+} \angle -19.65° \ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{11.435\angle -19.654°} = 8.745\angle 19.65°$$

$$\therefore \mathbf{S}_s = -\mathbf{V}_s\mathbf{I}_s^* = -100 \times 8.745\angle -19.65° = \boxed{-823.5 + j294.1\text{VA}}$$

$$\mathbf{I}_{20} = 8.745\angle 19.65° \times \frac{10+j20}{30+j20} = 5.423\angle 49.40°$$

$$\therefore \mathbf{S}_{20} = 20 \times 5.432^2 = \boxed{588.2 + j0 \ \text{VA}}$$

$$\mathbf{I}_{10} = \frac{20 \times 5.423\angle 49.40}{10+j20} = 4.851\angle -14.04°$$

$$\mathbf{S}_{10} = 10 \times 4.851^2 = \boxed{235.3 + j0 \ \text{VA}}$$

$$\mathbf{S}_{j20} = j20 \times 4.851^2 = \boxed{j470.6 \ \text{VA}},$$

$$\mathbf{S}_{-j10} = -j10 \times 8.745^2 = \boxed{-j764.7 \ \text{VA}}, \quad \Sigma = 0$$

42.

$$\frac{\mathbf{V}_x - 100}{6 + j4} + \frac{\mathbf{V}_x}{-j10} + \frac{\mathbf{V}_x - j100}{5} = 0$$

$$\therefore \mathbf{V}_x \left(\frac{1}{6 + j4} + j0.1 + 0.2 \right) = \frac{100}{6 + j4} + j20$$

$$\therefore \mathbf{V}_x = 53.35^- \angle 42.66^\circ \text{ V}$$

$$\therefore \mathbf{I}_1 = \frac{100 - 53.35^- \angle 42.66^\circ}{6 + j4} = 9.806 \angle -64.44^\circ \text{ A}$$

$$\therefore \mathbf{S}_{1.gen} = \frac{1}{2} \times 100 \times 9.806 \angle 64.44^\circ = \boxed{211.5 + j4423 \text{ VA}}$$

$$\mathbf{S}_{6,abs} = \frac{1}{2} \times 6 \times 9.806^2 = \boxed{288.5 + j0 \text{ VA}}$$

$$\mathbf{S}_{j4,abs} = \frac{1}{2} (j4) 9.806^2 = \boxed{0 + j192.3 \text{ VA}}$$

$$\mathbf{I}_2 = \frac{j100 - 53.35^- \angle 42.66^\circ}{5} = 14.99 \angle 121.6^\circ,$$

$$\mathbf{S}_{5abs} = \frac{1}{2} \times 5 \times 14.99^2 = \boxed{561.5 + j0 \text{ VA}}$$

$$\mathbf{S}_{2,gen} = \frac{1}{2} (j100) 14.99 \angle -121.57^\circ = \boxed{638.4 - j392.3 \text{ VA}}$$

$$\mathbf{S}_{-j10,abs} = \frac{1}{2} \left(\frac{53.35}{10} \right) (-j10) = 0 - j142.3 \text{ VA} = \boxed{142.3 \angle 0 \ \ \text{VA} \qquad \Sigma = 0}$$

43.

(a) $500\,\text{VA},\ \text{PF} = 0.75\ \text{lead} \therefore$

$\mathbf{S} = 500\angle -\cos^{-1} 0.75 = \boxed{375 - j330.7\ \text{VA}}$

(b) $500\text{W},\ \text{PF} = 0.75\ \text{lead} \therefore$

$\mathbf{S} = 500 - \dfrac{500}{j.075}\sin(\cos^{-1} 0.75) = \boxed{500 - j441.0\ \text{VA}}$

(c) $-500\ \text{VAR},\ \text{PF} = 0.75(\text{lead}) \therefore \theta = -\cos^{-1} 0.75 = -41.41°$

$\therefore \text{P}\,500 / \tan 41.41° = 566.9\text{W},$

$\boxed{\mathbf{S} = 566.9 - j500\ \text{VA}}$

44. $\mathbf{S}_s = 1600 + j500 \text{ VA (gen)}$

(a) $\mathbf{I}_s^* = \dfrac{1600 + j500}{400} = 4 + j1.25 \therefore \mathbf{I}_s = 4 - j1.25$

$\mathbf{I}_c = \dfrac{400}{-j120} = j3.333 \text{ A rms} \therefore \mathbf{I}_L = \mathbf{I}_s - \mathbf{I}_c = 4 - j1.25 - j3.333$

$\therefore \mathbf{I}_L = 4 - j4.583 \text{ A rms} \therefore$

$\mathbf{S}_L = 400(4 + j4.583) = \boxed{1600 + j1833 \text{ VA}}$

(b) $\text{PF}_L = \cos\left(\tan^{-1}\dfrac{1833.3}{1600}\right) = \boxed{0.6575^+ \text{ lag}}$

(c) $\mathbf{S}_s = 1600 + j500 = 1676\angle 17.35° \text{ VA} \therefore \text{PF}_s = \cos 17.35° = \boxed{0.9545 \text{ lag}}$

45. $(\cos^{-1} 0.8 = 36.87°,\ \cos^{-1} 0.9 = 25.84°)$

(a)

$$\begin{aligned}\mathbf{S}_{tot} &= 1200\angle 36.87° + 1600\angle 25.84° + 900 \\ &= 960 + j720 + 1440 + j697.4 + 900 \\ &= 3300 + j1417.4 = 3592\angle 23.25°\ \text{VA}\end{aligned}$$

$$\therefore \mathbf{I}_s = \frac{3591.5}{230} = \boxed{15.62\ \text{A rms}}$$

(b) $\text{PF}_s = \cos 23.245° = \boxed{0.9188}$

(c) $\mathbf{S} = \boxed{3300 + j1417\ \text{VA}}$

46. $\mathbf{V} = 339 \angle{-66^o}$ V, $\omega = 100\pi$ rad/ s, connected to $\mathbf{Z} = 1000\ \Omega$.

(a) $V_{eff} = \dfrac{339}{\sqrt{2}} = \boxed{239.7 \text{ V rms}}$

(b) $p_{max} = 339^2 / 1000 = \boxed{114.9 \text{ W}}$

(c) $p_{min} = \boxed{0 \text{ W}}$

(d) Apparent power $= V_{eff}\, I_{eff} = \left(\dfrac{339}{\sqrt{2}}\right)\left(\dfrac{339/\sqrt{2}}{1000}\right) = \dfrac{V_{eff}^2}{1000} = \boxed{57.46 \text{ VA}}$

(e) Since the load is purely resistive, it draws $\boxed{\text{zero reactive power.}}$

(f) $S = \boxed{57.46 \text{ VA}}$

47. $\mathbf{V} = 339 \angle\text{-}66^\circ$ V, $\omega = 100\pi$ rad/s to a purely inductive load of 150 mH ($j47.12\ \Omega$)

(a) $\mathbf{I} = \dfrac{\mathbf{V}}{\mathbf{Z}} = \dfrac{339\angle -66^\circ}{j47.12} = 7.194\angle -156^\circ$ A

so $\mathbf{I}_{eff} = \dfrac{7.194}{\sqrt{2}} = \boxed{5.087 \text{ A rms}}$

(b) $p(t) = ½\, V_m I_m \cos\phi + ½\, V_m I_m \cos(2\omega t + \phi)$

where ϕ = angle of current – angle of voltage

$p_{max} = ½\, V_m I_m \cos\phi + ½\, V_m I_m = (1 + \cos(-90^\circ))\,(339)(7.194)/\,2 = \boxed{1219 \text{ W}}$

(c) $p_{min} = ½\, V_m I_m \cos\phi - ½\, V_m I_m = \boxed{-1219 \text{ W}}$

(d) apparent power $= V_{eff}\, I_{eff} = \dfrac{339}{\sqrt{2}}(5.087) = \boxed{1219 \text{ VA}}$

(e) reactive power = Q $= V_{eff}\, I_{eff} \sin(\theta - \phi) = \boxed{1219 \text{ VA}}$

(f) complex power $= \boxed{j1219 \text{ VA}}$

48. $1\text{ H} \rightarrow j\ \Omega$, $4\ \mu\text{F} \rightarrow -j250\ \text{k}\Omega$

$$\mathbf{Z}_{eff} = j \parallel -j250\times10^3 \parallel 10^3\ \Omega = 1\angle 89.99^o\ \Omega$$

$$\mathbf{V}_{10k} = \frac{(5\angle 0)(1\angle 89.99^o)}{2500 + (1\angle 89.99^o)} = \boxed{0.002\angle 89.97^o\ \text{V}}$$

(a) $p_{max} = (0.002)^2 / 10\times10^3 = \boxed{400\ \text{pW}}$

(b) $\boxed{0\ \text{W}}$ (purely resistive elements draw no reactive power)

(c) apparent power $= V_{eff}I_{eff} = ½\, V_m I_m = ½\,(0.002)^2 / 10000 = \boxed{200\ \text{pW}}$

(d) $$\mathbf{I}_{source} = \frac{5\angle 0}{2500 + 1\angle 89.99^o} = 0.002\angle -0.02292^o\ \text{A}$$

$$\mathbf{S} = ½\, V_m I_m \angle(89.99^o + 0.02292^o) = \boxed{0.005\angle 90.01^o\ \text{VA}}$$

49. (a) At $\omega = 400$ rad/s, 1 μF → $-j2500\ \Omega$, 100 mH → $j40\ \Omega$
Define $\mathbf{Z}_{eff} = -j2500 \parallel (250 + j40) = 256 \angle 3.287^o\ \Omega$

$$\mathbf{I}_S = \frac{12000\angle 0}{20 + 256\angle 3.287^o} = 43.48 \angle -3.049^o \text{ A rms}$$

$$\mathbf{S}_{source} = (12000)(43.48) \angle 3.049^o = \boxed{521.8 \angle 3.049^o \text{ kVA}}$$

$$\mathbf{S}_{20\Omega} = (43.48)^2 (20) \angle 0 = \boxed{37.81 \angle 0 \quad \text{kVA}}$$

$$\mathbf{V}_{eff} = \frac{(12000\angle 0)(256\angle 3.287^o)}{20 + 256\angle 3.287^o} = 11130 \angle 0.2381^o \text{ V rms}$$

$$\mathbf{I}_{1\mu F} = \frac{\mathbf{V}_{eff}}{-j2500} = 4.452 \angle 90.24^o \text{ A rms}$$

so $\mathbf{S}_{1\mu F} = (11130)(4.452) \angle -90^o = \boxed{49.55 \angle -90^o \text{ kVA}}$

$$\mathbf{V}_{100mH} = \frac{(11130\angle 0.2381^o)(j40)}{250 + j40} = 1758 \angle 81.15^o \text{ V rms}$$

$$\mathbf{I}_{100mH} = \frac{\mathbf{V}_{100mH}}{j40} = 43.96 \angle -8.852^o \text{ A rms}$$

so $\mathbf{S}_{100\mu H} = (1758)(4.43.96) \angle 90^o = \boxed{77.28 \angle 90^o \text{ kVA}}$

$$\mathbf{V}_{250\Omega} = \frac{(11130\angle 0.2381^o)(250)}{250 + j40} = 10990 \angle -8.852^o \text{ V rms}$$

so $\mathbf{S}_{250\Omega} = (10990)^2 / 250 = \boxed{483.1 \angle 0^o \text{ kVA}}$

(b) $37.81 \angle 0 + 49.55 \angle -90^o + 77.28 \angle 90^o + 483.1 \angle 0^o = \boxed{521.6 \angle 3.014^o \text{ kVA},}$ which is within rounding error of the complex power delivered by the source.

(c) The apparent power of the source is 521.8 kVA. The apparent powers of the passive elements sum to 37.81 + 49.55 + 77.28 + 483.1 = 647.7 kVA, so $\boxed{\text{NO!}}$ Phase angle is important!

(d) $P = \mathbf{V}_{eff}\, \mathbf{I}_{eff} \cos(\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \cos(3.049^o) = \boxed{521 \text{ kW}}$

(e) $Q = \mathbf{V}_{eff}\, \mathbf{I}_{eff} \sin(\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \sin(3.049^o) = \boxed{27.75 \text{ kVAR}}$

50. (a) Peak current $= 28\sqrt{2} = \boxed{39.6 \text{ A}}$

(b) $\theta_{load} = \cos^{-1}(0.812) = +35.71^o$ (since lagging PF). Assume ang (**V**) = 0^o.

$$p(t) = \left(2300\sqrt{2}\right)\left(39.60\sqrt{2}\right)\cos(120\pi t)\ \cos\ (120\pi t - 35.71^o)$$

at t = 2.5 ms, then, $p(t) = \boxed{147.9 \text{ kW}}$

(c) $P = V_{eff}\, I_{eff} \cos\theta = (2300)(28) \cos(35.71^o) = \boxed{52.29 \text{ kW}}$

(d) $\mathbf{S} = V_{eff}\, I_{eff} \angle\theta = \boxed{64.4 \angle 35.71^o \text{ kVA}}$

(e) apparent power = $|\mathbf{S}| = \boxed{64.4 \text{ kVA}}$

(f) $|\mathbf{Z}_{load}| = |\mathbf{V}/\mathbf{I}| = 2300/28 = 82.14\ \Omega$. Thus, $\mathbf{Z}_{load} = \boxed{82.14 \angle 35.71^o\ \Omega}$

(g) $Q = V_{eff}\, I_{eff} \sin\theta = \boxed{37.59 \text{ kVAR}}$

CHAPTER TWELVE SOLUTIONS

1. $V_{bc} = V_{be} + V_{ec} = 0.7 – 10 = \boxed{-9.3 \text{ V}}$

   $V_{eb} = - V_{be} = \boxed{-0.7 \text{ V}}$

   $V_{cb} = V_{ce} + V_{eb} = 10 – 0.7 = \boxed{9.3 \text{ V}}$

CHAPTER TWELVE SOLUTIONS

2. (a) $V_{gd} = V_{gs} + V_{sd} = -1 - 5 \quad = \boxed{-6\text{ V}}$

(b) $V_{sg} = V_{sd} + V_{dg} = -4 - 2.5 \quad = \boxed{-6.5\text{ V}}$

CHAPTER TWELVE SOLUTIONS

3. (a) positive phase sequence

$$\boxed{\begin{array}{ll} \mathbf{V}_{an} = |\mathbf{V}_p| \angle 0^o & \mathbf{V}_{dn} = |\mathbf{V}_p| \angle -180^o \\ \mathbf{V}_{bn} = |\mathbf{V}_p| \angle -60^o & \mathbf{V}_{en} = |\mathbf{V}_p| \angle -240^o \\ \mathbf{V}_{cn} = |\mathbf{V}_p| \angle -120^o & \mathbf{V}_{fn} = |\mathbf{V}_p| \angle -300^o \end{array}}$$

(b) negative phase sequence

$$\boxed{\begin{array}{ll} \mathbf{V}_{an} = |\mathbf{V}_p| \angle 0^o & \mathbf{V}_{dn} = |\mathbf{V}_p| \angle 180^o \\ \mathbf{V}_{bn} = |\mathbf{V}_p| \angle 60^o & \mathbf{V}_{en} = |\mathbf{V}_p| \angle 240^o \\ \mathbf{V}_{cn} = |\mathbf{V}_p| \angle 120^o & \mathbf{V}_{fn} = |\mathbf{V}_p| \angle 300^o \end{array}}$$

4. (a) $\mathbf{V}_{yz} = \mathbf{V}_{yx} + \mathbf{V}_{xz}$ $= -110 \angle 20^o + 160 \angle -50^o$
$= -103.4 - j37.62 + 102.8 - j122.6 = -0.6 - j160.2$
$= \boxed{160.2 \angle -90.21^o \text{ V}}$

(b) $\mathbf{V}_{az} = \mathbf{V}_{ay} + \mathbf{V}_{yz}$ $= 80 \angle 130^o + 160.2 \angle -90.21^o$
$= -51.42 + j61.28 - 0.6 - j160.2 = -52.02 - j98.92$
$= \boxed{111.8 \angle -117.7^o \text{ V}}$

(c) $$\frac{\mathbf{V}_{zx}}{\mathbf{V}_{xy}} = \frac{-160\angle -50^o}{110\angle 20^o} = \frac{160\angle 130^o}{110\angle 20^o} = \boxed{1.455\angle 110^o}$$

5. (a) $\mathbf{V}_{25} = \mathbf{V}_{24} + \mathbf{V}_{45}$ $= -80 \angle 120^o + 60 \angle 75^o$

$= 40 - j69.28 + 15.53 + j57.96 = 55.53 - j11.32$

$= \boxed{56.67 \angle -11.52^o \text{ V}}$

(b) $\mathbf{V}_{13} = \mathbf{V}_{12} + \mathbf{V}_{25} + \mathbf{V}_{53}$ $= 100 + 55.53 - j11.32 + j120$

$= 155.53 + j108.7$

$= \boxed{189.8 \angle 34.95^o \text{ V}}$

6.

$230/460\,\text{V rms}\ \bar{Z}_{AN}:\ \bar{S} = 10\angle 40°\,\text{kVA};\ \bar{Z}_{NB}:\ 8\angle 10°\,\text{kVA};$

$\bar{Z}_{AB}:\ 4\angle -80°\,\text{kVA}\quad \text{Let}\ \bar{V}_{AN} = 230\angle 0°\,\text{V}\ \therefore \bar{S}_{AN} = \bar{V}_{AN}\bar{I}^*_{AN}, \bar{I}^*_{AN} = \dfrac{10,000\angle 40°}{230} = 43.48\angle 40°\,\text{A}$

$\therefore \bar{I}_{AN} = 43.48\angle -40°\,\text{A},\ \bar{S}_{AB} = \bar{V}_{AB}\bar{I}^*_{AB}\ \therefore \bar{I}^*_{AB} = \dfrac{4000\angle -80°}{460} = 8.696\angle -80°, \bar{I}_{AB} = 8.696\angle 80° \therefore \bar{I}_{aA} = \bar{I}_{AN} + \bar{I}_{AB}$

$\therefore \bar{I}_{aA} = 43.48\angle 40° + 8.696\angle 80° = 39.85^{-}\angle -29.107°\ \therefore I_{aA} = \boxed{39.85^{-}\ \text{A}}$

$\bar{I}^*_{NB} = \dfrac{8000\angle 10°}{230} = 34.78\angle 10°,\ \bar{I}_{NB} = 34.78\angle -10°\,\text{A}$

$\therefore \bar{I}_{bB} = -34.78\angle -10° - 8.696\angle 80° = 35.85^{+}\angle -175.96°,\ \therefore I_{bB} = \boxed{35.85^{+}\ \text{A}}$

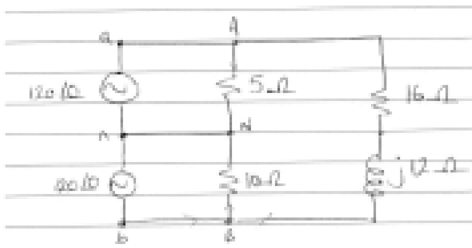
$\bar{I}_{nN} = -43.48\angle -40° + 34.78\angle -10° = 21.93\angle 87.52°,\ I_{nN} = \boxed{21.93\,\text{A}}$

7. (a) $\mathbf{I}_{nN} = 0$ since the circuit is balanced.

$$\mathbf{I}_{AN} = 12\angle 0 \qquad \mathbf{I}_{AB} = \frac{240\angle 0}{16 + j12} = 12\angle -36.9^{o}$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = 12 + 9.596 - j7.205 = \boxed{22.77\angle -18.45^{o}\text{ A}}$$

(b)



$\mathbf{I}_{AN} = 24\angle 0^{o}$ A
$\mathbf{I}_{BN} = -12\angle 0^{o}$ A
$\mathbf{I}_{nN} = -12\angle 0^{o}$ A

The voltage across the 16-Ω resistor and j12-Ω impedance has not changed, so $\mathbf{I}_{AB}$ has not changed from above.

$$\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = 24\angle 0^{o} + 12\angle -36.9^{o} = \boxed{34.36\angle -12.10^{o}\text{ A}}$$
$$\mathbf{I}_{bB} = \mathbf{I}_{BN} - \mathbf{I}_{AB} = -12\angle 0^{o} - 12\angle -36.9^{o} = \boxed{7.595\angle -108.5^{o}\text{ A}}$$
$$\mathbf{I}_{nN} = \mathbf{I}_{BN} - \mathbf{I}_{AN} = -12 - 24 = \boxed{36\angle 180^{o}\text{ A}}$$

8.

(a) $$\Delta = \begin{vmatrix} 21+j3 & -10 & -10-j3 \\ -10 & 19+j2 & -8-j2 \\ -10-j3 & -8-j2 & 36+j5 \end{vmatrix} = (21+j3)(674+j167-60-j32)$$

$$+10(-360-j50-74-j44)-(10+j3)(80+j20+184+j77)$$

$$\therefore \Delta = 5800+j1995 = 6127\angle 18.805^\circ$$

$$\begin{vmatrix} 720 & -10 & -10-j3 \\ 720 & 19+j2 & -8-j2 \\ 0 & -8-j2 & 36+j5 \end{vmatrix} = 720(614+j135+434+j94) = 720\times 1072.7\angle 12.326^\circ$$

$$\therefore \bar{\mathrm{I}}_{aA} = \frac{720\times 1072.7\angle 12.326^\circ}{6127\angle 18.805^\circ} = \boxed{126.06\angle -6.479^\circ \text{ A}}$$

(b) $$\begin{vmatrix} 21+j3 & 720 & -10-j3 \\ -10 & 720 & -8-j2 \\ -10-j3 & 0 & 36+j5 \end{vmatrix} = 720(1084+j247) \quad \therefore \bar{\mathrm{I}}_{Bb} = \frac{720(1084+j247)}{6127\angle 18.805^\circ} = 130.65^{-}\angle -5.968^\circ \text{ A}$$

$$\therefore \mathrm{I}_{nN} = 130.65^{-}\angle -5.968^\circ - 126.06\angle -6.479^\circ = \boxed{4.730\angle 7.760^\circ \text{ A}}$$

(c) $$\mathrm{P}_{\omega,tot} = 126.06^2\times 1 + 130.65^2\times 1 + 4.730^2\times 10 = 15.891+17.069+0.224 = \boxed{33.18\text{ kW}}$$

(d) $$\mathrm{P}_{gen,tot} = 720\times 126.06\cos 6.479^\circ + 720\times 130.65^{-}\cos 5.968^\circ = 90.18+93.56 = \boxed{183.74\text{ kW}}$$

9. $\overline{V}_{AN} = 220\,\text{Vrms},\ 60\,\text{Hz}$

(a) $\text{PF} = 1\ \therefore\ \overline{I}_{AN} = \dfrac{220\angle 0^\circ}{5 + j2} = 40.85^{+}\angle -21.80^\circ\,\text{A};\ \overline{I}_{AB} = j377\text{C}\times 440$

$\therefore\ \overline{I}_{aA} = 40.85\cos 21.80^\circ + j(377\text{C}440 - 40.85\sin 21.80^\circ)$

$\therefore\ \text{C} = \dfrac{40.85\sin 21.80^\circ}{377\times 440} = \boxed{91.47\,\mu\text{F}}$

(b) $\overline{I}_{AB} = 377\times 91.47\times 10^{-6}\times 440 = 15.172\,\text{A}\ \therefore\ \text{VA} = 440\times 15.172 = \boxed{6.676\,\text{kVA}}$

10. (a) $\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = \dfrac{200\angle 0}{12 + j3} + \dfrac{400\angle 0}{R_{AB}} = 15.69 - j3.922 + \dfrac{400}{R_{AB}}$

Since we know that $|\mathbf{I}_{aA}|$ = 30 A rms = 42.43 A,

$$42.43 = \sqrt{\left(15.69 + \frac{400}{R_{AB}}\right)^2 + 3.922^2}$$

or $R_{AB} = \boxed{15.06\ \Omega}$

(b) $\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = \dfrac{200\angle 0}{12 + j3} + \dfrac{400\angle 0}{-jX_{AB}} = 15.69 - j3.922 + \dfrac{j400}{X_{AB}}$

In order for the angle of $\mathbf{I}_{aA}$ to be zero, $\dfrac{400}{X_{AB}}$ = 3.922, so that X_{AB} = $\boxed{102\ \Omega \text{ capacitive.}}$

11. + seq. $\overline{V}_{BC} = 120\angle 60°$ V rms, $R_w = 0.6\,\Omega$ $\quad P_{load} = 5\,$kVA, 0.6 lag

(a) $\overline{V}_{AN} = \dfrac{120}{\sqrt{3}}\angle 150°\text{ V} \therefore \overline{S}_{AN} = \dfrac{5000}{3}\times 0.8 + j\dfrac{5000}{3}\,0.6$

$\therefore \overline{S}_{AN} = \dfrac{120}{\sqrt{3}}\angle 150°\ \overline{I}^*_{aA} \therefore \overline{I}^*_{aA} = 24.06\ \angle -113.13°$ A

$\therefore \overline{I}_{aA} = 24.06\angle 113.13° \therefore P_{wire} = 3\times 24.06^2\times 0.6 = \boxed{1041.7\text{ W}}$

(b) $\overline{V}_{aA} = 0.6\times 24.06\angle 113.13° = 14.434\angle 113.13°\text{ V}$

$\therefore \overline{V}_{an} = \overline{V}_{aA} + \overline{V}_{AN} = 14.434\angle 113.13° + \dfrac{120}{\sqrt{3}}\angle 158° = \boxed{81.29\angle 143.88°\text{ V}}$

12. $\uparrow \bar{V}_{an} = 2300\angle 0°\ V_{\text{rms}},\ R_w = 2\ \Omega, +seq.,\ \bar{S}_{tot} = 100 + j30\,\text{kVA}$

(a) $\frac{1}{3}(100,000 + j30,000) = 2300\,I^*_{aA}\ \therefore\ \bar{I}_{aA} = \boxed{15.131\angle -16.699°\ A}$

(b) $\bar{V}_{AN} = 2300 - 2\times 15.131\angle -16.699° = \boxed{2271\angle 0.2194°\ V}$

(c) $\bar{Z}_p = \bar{V}_{AN} / \bar{I}_{aA} = \frac{2271\angle 0.2194°}{15.131\angle -16.699°} = \boxed{143.60 + j43.67\,\Omega}$

(d) trans. eff. $= \frac{143.60}{145.60} = 0.9863$, or $\boxed{98.63\%}$

13. $\uparrow \overline{Z}_p = 12 + j5\Omega,\ \overline{I}_{bB} = 20\angle 0°$ A rms, +seq., PF $= 0.935$

(a) $\theta = \cos^{-1} 0.935 = 20.77° \therefore \dfrac{5}{12 + R_w} = \tan\ 20.77°,\ \ R_w = \boxed{1.1821\Omega}$

(b) $\overline{V}_{BN} = I_{bB} Z_p = 20(12 + j5) = 240 + j100\text{ V} \ \ \therefore \overline{V}_{bn} = 20(13.1821 + j5) = \boxed{281.97\angle 20.77°\text{ V}}$

(c) $\overline{V}_{AB} = \sqrt{3}\left|\overline{V}_{BN}\right| / \angle V_{BN} + 150° = \boxed{450.3\angle 172.62°\text{ V}}$

(d) $\overline{S}_{source} = 3\ \overline{V}_{Bn} \overline{I}_{bB}^{*} = 3 \times 281.97\angle -20.77°(20)$

$= \boxed{15.819 - j6.000\text{ kVA}}$

14. 125 mH → $j(2\pi)(60)(0.125) = j47.12\ \Omega$ 75 Ω → 75 Ω
55 μF → $-j/(2\pi)(60)(55\times10^{-6}) = -j48.23\ \Omega$

The per-phase current magnitude |**I**| is then $I = \dfrac{125}{\sqrt{75^2 + (47.12 - 48.23)^2}} = \boxed{1.667\text{ A.}}$

The power in each phase = $(1.667)^2$ (75) = 208.4 W, so that the total power taken by the load is 3(208.4) = $\boxed{625.2\text{ W.}}$

The power factor of the load is $\cos\left(\dfrac{47.12 - 48.23}{75}\right) = \boxed{1.000}$

This isn't surprising, as the impedance of the inductor and the impedance of the capacitor essentially cancel each other out as they have approximately the same magnitude but opposite sign and are connected in series.

15.

$\uparrow$ Bal.,+ seq. $Z_{AN} = 8 + j6\,\Omega,\ \bar{Z}_{BN} = 12 - j16\ \Omega,\ \bar{Z}_{CN} = 5 + j0,\ \bar{V}_{AN} = 120\angle 0°\,\text{V rms}$

$$R_w = 0.5\,\Omega\ (a) - \bar{I}_{nN} = \frac{120\angle 0°}{8.5 + j6} + \frac{120\angle -120°}{12.5 - j16} + \frac{120\angle 120°}{5.5} = 6.803\angle 83.86°\,\text{A}$$

$$\therefore\ \bar{I}_{nN} = \boxed{6.803\angle -96.14°\,\text{A rms}}$$

16. Working on a per-phase basis, the line current magnitude is simply

$$|\mathbf{I}| = \frac{40}{\sqrt{(R_w + 5)^2 + 10^2}}$$

(a) $R_W = 0$

Then $|\mathbf{I}| = \dfrac{40}{\sqrt{25 + 10^2}} = \boxed{3.578\text{ A}}$, and the power delivered to each phase of the load is $(3.578)^2(5) = 64.01$ W. The total power of the load is therefore $3(64.01) = \boxed{192.0\text{ W.}}$

(b) $R_W = 3\ \Omega$

Then $|\mathbf{I}| = \dfrac{40}{\sqrt{64 + 10^2}} = \boxed{3.123\text{ A}}$, and the power delivered to each phase of the load is $(3.123)^2(5) = 48.77$ W. The total power of the load is therefore $3(48.77) = \boxed{146.3\text{ W.}}$

17. $\uparrow \bar{Z}_p = 75\angle 25^\circ\ \Omega \| 25\,\mu\text{F},\ \bar{V}_{an} = 240\angle 0^\circ\ \text{V rms},\ 60\,\text{Hz},\ R_w = 2\,\Omega$

(a) $\bar{Z}_{cap} = -j\dfrac{10^6}{377\times 25} = -j\,106.10\,\Omega\ \therefore \bar{Z}_p = \dfrac{75\angle 25^\circ(-j106.10)}{75\angle 25^\circ - j106.10} = 75.34 - j23.63\,\Omega$

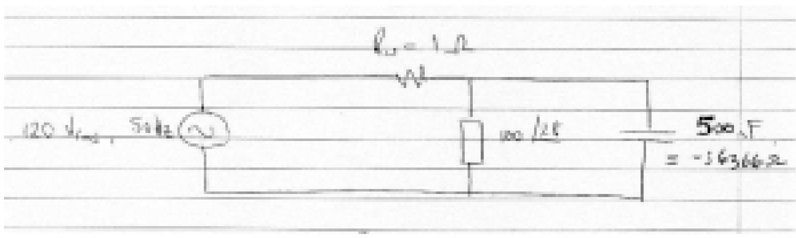
$\therefore \bar{Z}_{p+w} = 77.34 - j23.63\ \therefore \bar{I}_{aA} = \dfrac{240}{77.34 - j23.63} = \boxed{2.968\angle 16.989^\circ\,\text{A}}$

(b) $P_w = 3(2.968)^2 \times 2 = \boxed{52.84\,\text{W}}$

(c) $P_{load} = 3(2.968)^2\, 75.34 = \boxed{1990.6\,\text{W}}$

(d) $PF_{source} = \cos 16.989^\circ = \boxed{0.9564\ \text{lead}}$

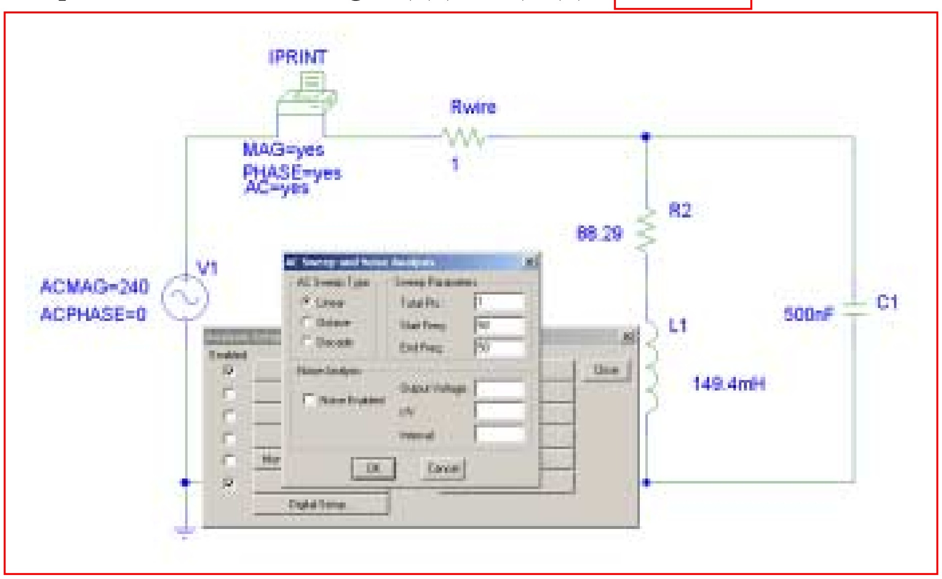
18. Working on a per-phase basis and noting that the capacitor corresponds to a $-j6366$-Ω impedance,



$-j6366 \parallel 100\angle 28^{\circ} = 89.59 + j46.04\ \Omega$ so that the current flowing through the combined load is

$$|\mathbf{I}| = \frac{240}{\sqrt{90.59^2 + 46.04^2}} = \boxed{2.362 \text{ A rms}}$$

The power in each phase is $(2.362)^2(90.59) = 505.4$ W, so that the power deliverd to the total load is $3(505.4) = \boxed{1.516 \text{ kW}}$.
The power lost in the wiring is $(3)(2.362)^2(1) = \boxed{16.74 \text{ W.}}$



Simulation Result:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
5.000E+01	1.181E+00	-2.694E+01

19. $\uparrow$ Bal., $R_w = 0,\ \bar{Z}_p = 10 + j5\,\Omega,\ f = 60$ Hz

(a) $10 + j5 = 11.180\angle 26.57° \ \therefore \mathrm{PF} = \cos 26.57° = \boxed{0.8944}$

(b) $\mathrm{PF} = 0.93 \text{ lag},\ \theta = 21.57°,\ \bar{Y}_p = \dfrac{1}{11.180\angle 26.57°} = 0.08 - j0.04\,\mathrm{S}$

$\bar{Y}p' = 0.08 + j(377\mathrm{C} - 0.04) \ \therefore \dfrac{377\mathrm{C} - 0.04}{0.08} = -\tan 21.57° = -0.3952$

$\therefore 377\mathrm{C} = 0.04 - 0.08 \times 0.3952 = 0.00838 \therefore \mathrm{C} = \boxed{22.23\,\mu\mathrm{F}}$

(c) $V_{L,load} = 440\,\mathrm{V\ rms},\ \bar{Z}_c = \dfrac{-j10^6}{120\pi 22.23} = -j119.30\,\Omega,\ I_c = \dfrac{440/\sqrt{3}}{119.30} = 2.129\,\mathrm{A}$

$\therefore \mathrm{VAR} = 2.129 \times \dfrac{440}{\sqrt{3}} = \boxed{540.9\ \mathrm{VAR}\,(cap.)}$

20. Working from the single-phase equivalent,

$$\mathbf{V}_{an} \text{ rms} = \frac{1}{\sqrt{3}}\left(\frac{115\angle 0^o}{\sqrt{2}}\right) = 46.9 \angle 0^o \text{ V rms}$$

1.5 H $\rightarrow$ j565 Ω, 100 μF $\rightarrow$ -j26.5 Ω and 1 kΩ $\rightarrow$ 1 kΩ.
These three impedances appear in parallel, with a combined value of 27.8 $\angle$ -88.4^o Ω.

Thus, $|\mathbf{I}_{rms}|$ = 46.9/ 27.8 = $\boxed{\text{1.69 A rms}}$

$\mathbf{Z}_{load}$ = 27.8 $\angle$88.4^o = 0.776 – j 27.8 Ω, so P_{load} = (3)(1.69)^2 (0.776) = $\boxed{\text{2.22 W.}}$

21.

$R_w = 0,\ \overline{V}_{an} = 200\angle 60°\ \text{V rms. } S_p = 2 - j1\ \text{kVA} + \text{seq.}$

(a) $\overline{V}_{bc} = 220\sqrt{3}\angle -30° = \boxed{346.4\angle -30°\ \text{V}}$

(b) $\overline{S}_{BC} = 2000 - j1000 = \overline{V}_{BC}\overline{I}^*_{BC} = 346.4\angle -30°\overline{I}^*_{BC}$

$\therefore\ \overline{I}^*_{BC} = 6.455^{-}\angle 3.435°,\ \overline{I}_{BC} = 6.455^{-}\angle -3.435°$

$$\therefore\ \overline{Z}_p = \frac{200\sqrt{3}\angle -30°}{6.455^{-}\angle -3.435°} = 53.67\angle -26.57° = \boxed{48 - j24\ \Omega}$$

(c) $\overline{I}_{aA} = \overline{I}_{AB} - \overline{I}_{CA} = 6.455^{-}\angle 120° - 3.43° - 6.455^{-}\angle -120° - 3.43° = \boxed{11.180\angle 86.57°\ \text{A rms}}$

22. $\uparrow 15\text{kVA},\ 0.8\,\text{lag},\ +\text{seq.},\ \overline{V}_{BC} = 180\angle 30^\circ\ \text{V rms},\ R_w = 0.75\,\Omega$

(a) $\overline{V}_{BC} = 180\angle 30^\circ \;\therefore\; \overline{V}_{AB} = 180\angle 150^\circ\,\text{V},\ \overline{S}_p = 5000\angle\cos^{-1}0.8 = 5000\angle 36.87^\circ = 180\angle 30^\circ\,\overline{I}_{BC}^*$

$\therefore\; \overline{I}_{BC} = 27.78\angle -6.87^\circ$ and $\overline{I}_{AB} = 27.78\angle 113.13^\circ\,\text{A} \;\therefore\; \overline{I}_{bB} = \overline{I}_{BC} - \overline{I}_{AB}$

$\therefore\; \overline{I}_{bB} = 27.78(1\angle -6.87^\circ - 1\angle 113.13^\circ) = 48.11\angle -36.87^\circ\,\text{A} \;\therefore\; \overline{V}_{bC} = 0.75(\overline{I}_{bB} - \overline{I}_{cC})$

$\therefore\; \overline{V}_{bC} = 0.75\times 48.11(1\angle -36.87^\circ - 1\angle -156.87^\circ) + 180\angle 30^\circ = \boxed{233.0\angle 20.74^\circ\,\text{V}}$

(b) $P_{wire} = 3\times 48.11^2\times 0.75 = 5208\,\text{W}$

$\overline{S}_{gen} = 5208 + 15{,}000\angle 36.87^\circ = \boxed{17.208 + j9.000\,\text{kVA}}$

23. $\uparrow$ Bal., $\overline{S}_L = 3 + j1.8$ kVA, $\overline{S}_{gen} = 3.45 + j1.8$ kVA, $R_w = 5\Omega$

(a) $P_w = 450\,\text{W} \therefore \frac{1}{3}\times 450 = I_{aA}^2 \times 5 \;\therefore I_{aA} = \boxed{5.477\text{ A rms}}$

(b) $I_{AB} = \frac{1}{\sqrt{3}}\times 5.477 = \boxed{3.162\text{ A rms}}$

(c) Assume $\overline{I}_{AB} = 3.162\angle 0°$ and +seq. $\therefore \frac{1}{3}(3000 + j1800) = \overline{V}_{AB}\overline{I}_{AB}^* = \overline{V}_{AB}(3.162\angle 0°)$

$\therefore \overline{V}_{AB} = 368.8\angle 30.96°\,\text{V} \;\therefore \overline{V}_{an} = \overline{V}_{aA} + \overline{V}_{AB} - \overline{V}_{bB} + \overline{V}_{bn}$

$\overline{V}_{aA} = 5\overline{I}_{aA} = 5\times 5.477\angle -30° = 27.39\angle -30°, \overline{V}_{bB} = 27.39\angle -150°$

$\therefore \overline{V}_{an} = 27.39\angle -30° - 27.39\angle -150° + 368.8\angle 30.96° + V_{an}(1\angle -120°)$

$\therefore \overline{V}_{an} = \frac{27.39\angle -30° - 27.39\angle -150° + 368.8\angle 30.96°}{1 - 1\angle -120°} = 236.8\angle -2.447° \;\therefore V_{an} = \boxed{236.8\text{ V rms}}$

24. If a total of 240 W is lost in the three wires marked R_w, then 80 W is lost in each 2.3-Ω segment. Thus, the line current is $\sqrt{\frac{80}{2.3}} = 5.898 \text{ A rms}$. Since this is a D-connected load, the phase current is $1/\sqrt{3}$ times the line current, or $\boxed{3.405 \text{ A rms.}}$

In order to determine the phase voltage of the source, we note that

$$P_{total} = \sqrt{3}\left|\mathbf{V}_{line}\right| \cdot \left|\mathbf{I}_{line}\right| \cdot PF \quad = \sqrt{3}\ \left|\mathbf{V}_{line}\right| (5.898)\left(\frac{\sqrt{2}}{2}\right) = 1800$$

where $|\mathbf{V}_{line}| \; = \; \frac{(1800)(2)}{\sqrt{2}\sqrt{3}\,(5.898)} \; = \; 249.2 \text{ V}$

This is the voltage at the load, so we need to add the voltage lost across the wire, which (taking the load voltage as the reference phase) is $\left[5.898\angle - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right](R_W)$

$= 13.57 \angle\text{-}45^o$ V. Thus, the line voltage magnitude of the source is
$|249.2 \angle 0^o + 13.57 \angle \text{-}45^o| \;\; = \;\; \boxed{259.0 \text{ V rms.}}$

25. Bal., +seq.

(a) $\bar{V}_{an} = 120\angle 0° \therefore \bar{V}_{ab} = 120\sqrt{3}\angle 30°, etc., \bar{I}_{AB} = \dfrac{120\sqrt{3}\angle 30°}{10} = 20.78\angle 30°\ \text{A}$

$\bar{I}_{BC} = \dfrac{120\sqrt{3}\angle -90°}{j5} = -41.57\ A;\ \bar{I}_{CA} = \dfrac{120\sqrt{3}\angle 150°}{-j10} = 20.78\angle -120°\ \text{A}$

$\bar{I}_{aA} = \bar{I}_{AB} - \bar{I}_{CA} = 20.78(1\angle 30° - 1\angle -120°) = \boxed{40.15\angle 45°\ \text{A rms}}$

(b) $\bar{I}_{bB} = -41.57 - 20.78\angle 30° = \boxed{60.47\angle -170.10°\ \text{A rms}}$

(c) $I_{cC} = 20.78\angle -120° + 41.57 = \boxed{36.00\angle -30°\ \text{A rms}}$

(d) $\bar{S}_{tot} = \bar{V}_{AB}\bar{I}^*_{AB} + \bar{V}_{BC}\bar{I}^*_{BC} + \bar{V}_{CA}\bar{I}^*_{CA} = 120\sqrt{3}\angle 30° \times 20.78\angle -30° + 120\sqrt{3}\angle -90°(-41.57) +$
$120\sqrt{3}\angle 150° \times 20.78\angle 120° = 4320 + j0 + 0 + j8640 + 0 - j4320 = \boxed{4320 + j4320\ \text{VA}}$

26. $\mathbf{I}_{AB} = \dfrac{200\angle 0}{10 \parallel j30} = \dfrac{200\angle 0}{9.49\angle 18.4^o} = 21.1\angle\text{-}18.4^o$ A

$|\mathbf{I}_A| = \sqrt{3}\, I_{AB} = 36.5$ A

The power supplied by the source = (3) $|I_A|^2$ (0.2) + (3) $(200)^2$ / 10 = $\boxed{12.8 \text{ kW}}$

Define transmission efficiency as $\eta = 100 \times P_{load}/ P_{source}$. Then η = $\boxed{93.8\%.}$

$\mathbf{I}_A$ leads $\mathbf{I}_{AB}$ by 30^o, so that $\mathbf{I}_A$ = 36.5 $\angle$ 11.6^o.

$\mathbf{V}_{R_w} = (0.2)(36.5\angle 11.6^o) = 7.3\angle 11.6^o$ V

With $\mathbf{V}_{AN} = \dfrac{200}{\sqrt{3}}\angle 30^o$, and noting that $\mathbf{V}_{an} = \mathbf{V}_{AN} + \mathbf{V}_{R_w}$ = 122 $\angle$ 28.9^o, we may now compute the power factor of the source as

$$\text{PF} = \cos(\text{ang}(\mathbf{V}_{an}) - \text{ang}(\mathbf{I}_A)) = \cos(28.9^o - 11.6^o) = \boxed{0.955.}$$

27. $\uparrow$ Bal., $\overline{\text{V}}_{an} = 140\angle 0°\ \text{V}_{rms}$, +seq., $\text{R}_w = 0$, $\overline{\text{S}}_L = 15 + j9$ kVA

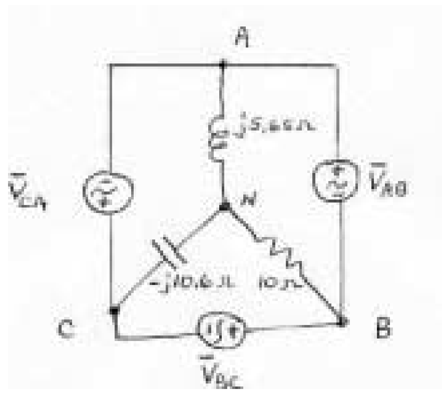
(a) $\overline{\text{V}}_{ab} = \overline{\text{V}}_{AB} = \sqrt{3}\ 140\angle 30° = \boxed{242.5^{-}\angle 30°\ \text{V}}$

(b) $\overline{\text{V}}_{AB}\overline{\text{I}}^{*}_{AB} = 5000 + j3000 = 242.5^{-}\angle 30°\overline{\text{I}}^{*}_{AB}\ \therefore\ \overline{\text{I}}_{AB} = \boxed{24.05^{-}\angle -0.9638°\ \text{A rms}}$

(c) $\overline{\text{I}}_{aA} = \overline{\text{I}}_{AB} - \overline{\text{I}}_{CA} = 24.05^{-}\angle -0.9638° - 24.05^{-}\angle 119.03° = \boxed{41.65^{-}\angle -30.96°\ \text{A rms}}$

28. 15 mH $\rightarrow j5.65\ \Omega$, 0.25 mF $\rightarrow -j10.6\ \Omega$

$$\mathbf{V}_{AB} = 120\sqrt{3}\ \angle 30^\circ \text{ V}$$
$$\mathbf{V}_{BC} = 120\sqrt{3}\ \angle -90^\circ \text{ V}$$
$$\mathbf{V}_{CA} = 120\sqrt{3}\ \angle -210^\circ \text{ V}$$

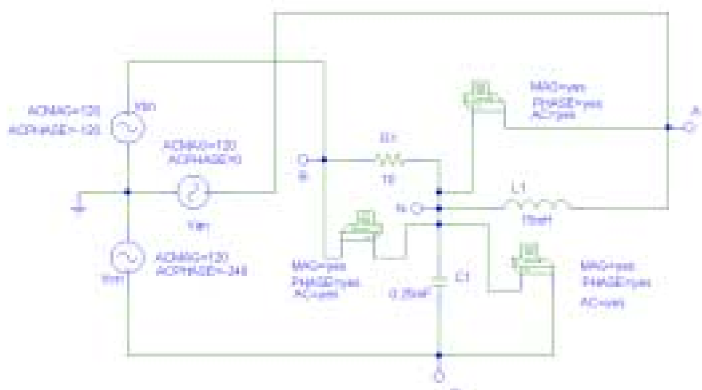


Defining three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ corresponding to sources $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$ and $\mathbf{V}_{CA}$, respectively, we may write:

$\mathbf{V}_{AB} = (10 + j5.65)\ \mathbf{I}_1 - 10\ \mathbf{I}_2 + j5.65\ \mathbf{I}_3$ [1]
$\mathbf{V}_{BC} = -10\ \mathbf{I}_1 + (10 - j10.6)\ \mathbf{I}_2 + j10.6\ \mathbf{I}_3$ [2]
$\mathbf{V}_{CA} = -j5.65\ \mathbf{I}_1 + j10.6\ \mathbf{I}_2 + (j5.65 - j10.6)\ \mathbf{I}_3$ [3]

Solving using MATLAB or a scientific calculator, we find that
$\mathbf{I}_1 = 53.23\ \angle -5.873^\circ$ A, $\mathbf{I}_2 = 40.55\ \angle 20.31^\circ$ A, and $\mathbf{I}_3 = 0$

(a) $\mathbf{V}_{AN} = j5.65(\mathbf{I}_1 - \mathbf{I}_3) = 300.7\ \angle 84.13^\circ$ V, so $V_{AN} = 300.7$ V
(b) $\mathbf{V}_{BN} = 10(\mathbf{I}_2 - \mathbf{I}_1) = 245.7\ \angle 127.4^\circ$ V, so $V_{BN} = 245.7$ V
(c) $\mathbf{V}_{CN} = -j10.6\ (-\mathbf{I}_2) = 429.8\ \angle 110.3^\circ$ V, so $V_{CN} = 429.8$ V



PSpice Simulation Results
(agree with hand calculations)

FREQ	VM(A,N)	VP(A,N)
6.000E+01	3.007E+02	8.410E+01

FREQ	VM(B,N)	VP(B,N)
6.000E+01	2.456E+02	1.274E+02

FREQ	VM(C,N)	VP(C,N)
6.000E+01	4.297E+02	1.103E+02

29. $\uparrow R_{line} = 1\Omega$

(a)

$$120\sqrt{3} = 207.8\ \bar{I}_1 = \frac{\begin{vmatrix} 207.8\angle 30° & -1 & -j10 \\ 207.8\angle -90° & 2+j5 & -j5 \\ 0 & -j5 & 10-j5 \end{vmatrix}}{\begin{vmatrix} 12 & -1 & -j10 \\ -1 & 2+j5 & -j5 \\ -10 & -j5 & 10-j5 \end{vmatrix}} = \frac{207.8\begin{vmatrix} 1\angle 30° & -1 & -10 \\ -j1 & 2+j5 & -j5 \\ 0 & -j5 & 10-j5 \end{vmatrix}}{12(70+j40)+(-10-j45)-10(20+j55)}$$

$$\therefore \bar{I}_1 = \frac{207.8[1\angle 30°(70+j40)+j1(-10-j45)]}{630-j115} = \frac{21.690\angle 34.86°}{630-j115} = \boxed{33.87\angle 45.20° = \bar{I}_{aA}}$$

$$\therefore \bar{I}_2 = \frac{\begin{vmatrix} 12 & 1\angle 30° & -10 \\ -1 & -j1 & -j5 \\ -10 & 0 & 10-j5 \end{vmatrix} 207.8}{630-j115} = \frac{207.8[-1\angle 30°(-10-j45)-j1(20-j60)]}{630-j115}$$

$$= \frac{16{,}136\angle 162.01°}{630-j115} = 25.20\angle 172.36°\text{ A}$$

(b) $\therefore \bar{I}_{cC} = \boxed{25.20\angle -7.641°\text{ A}}$

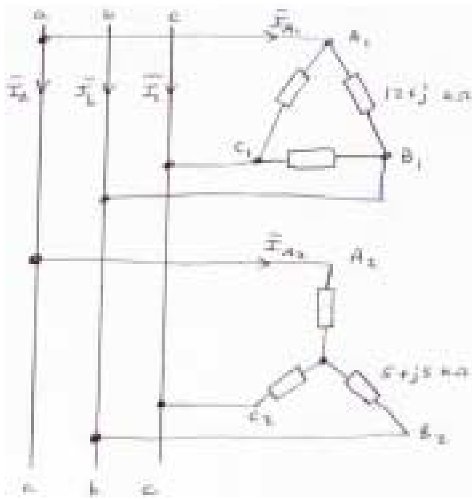
(c) $\therefore \bar{I}_{bB} = -\bar{I}_{aA} - \bar{I}_{CC} = -33.87\angle 45.20° - 25.20\angle -7.641° = \boxed{53.03\angle -157.05°\text{ A rms}}$

(d)
$$\bar{S} = 120\sqrt{3}\angle 30°(33.87\angle -45.20°) + 120\sqrt{3}\angle 90°(25.20\angle 7.641°)$$
$$= 6793 - j1846.1 - 696.3 + j5190.4 = \boxed{6096 + j3344\text{ VA}}$$

30. $|\mathbf{V}_{line}|$ = 240 V. Set $\mathbf{V}_{ab} = 240\angle 0^\circ$ V. Then $\mathbf{V}_{an} = \dfrac{240}{\sqrt{3}}\angle -30^\circ$.

$$\mathbf{I}_{A2} = \frac{\frac{240}{\sqrt{3}}\angle -30^\circ}{5 + j3} = 23.8\angle -61.0^\circ \text{ A}$$

$$\mathbf{I}_{A1B1} = \frac{\frac{240}{\sqrt{3}}\angle 0^\circ}{(12 + j)\times 10^3} = 20.0\angle -4.76^\circ \text{ mA}$$



$\mathbf{I}_{phase}$ leads $\mathbf{I}_{line}$ by 30°, so

$\mathbf{I}_{A1} = 20\sqrt{3}\angle -34.8^\circ$ mA $= 34.6\angle -34.8^\circ$ mA

$\mathbf{I}_a = \mathbf{I}_{A1} + \mathbf{I}_{A2} = 11.5 - j20.8 + 28.4 - j19.7$ mA = $\boxed{56.9\angle -45.4^\circ \text{ mA}}$

The power factor at the source = cos (45.4° – 30°) = $\boxed{\text{0.964 lagging.}}$

The power taken by the load = $(3)(20\times 10^{-3})^2\,(12\times 10^3) + (3)(23.8\times 10^{-3})^2\,(5000)$ = $\boxed{\text{22.9 W.}}$

31. Define **I** flowing from the '+' terminal of the source. Then,

$$\mathbf{I} \;=\; \frac{200\angle 0}{10+(j10\,\|\,20)} = \frac{200\angle 0}{16.12\angle 29.74^{\circ}} = 12.41\angle -29.74^{\circ}$$

(a) $\mathbf{V}_{xy} = 10\,\mathbf{I} = 124.1\angle{-29.74^{\circ}}$ V. Thus, $P_{xy} = (12.41)(124.1) = \boxed{1.54\text{ kW}}$

(b) $P_{xz} = (200)(12.41)\cos(29.74^{\circ}) = \boxed{2.155\text{ kW}}$

(c) $\mathbf{V}_{yz} = 200\angle 0 - 124.1\angle{-29.74^{\circ}} = 110.9\angle 33.72^{\circ}$ V
Thus, $P_{yz} = (110.9)(12.41)\cos(33.72^{\circ} + 29.74^{\circ}) = \boxed{614.9\text{ W}}$

$\boxed{\text{No reversal of meter leads is required for any of the above measurements.}}$

32. 1 H $\rightarrow$ j377 Ω, 25 μF $\rightarrow$ -j106 Ω

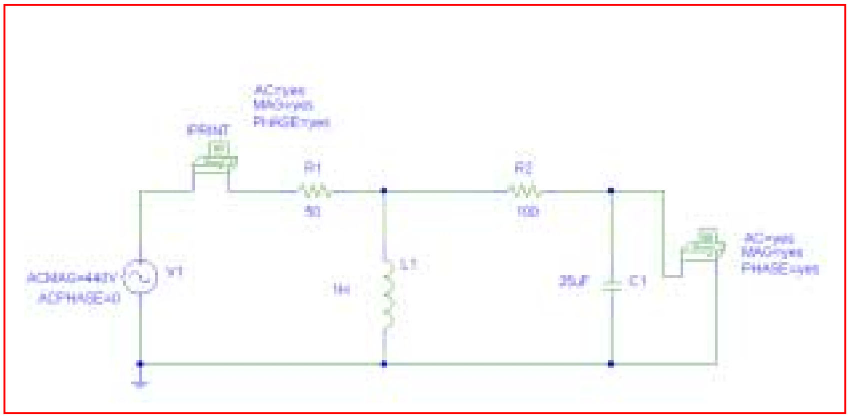
$$\mathbf{I}_1 = \frac{440\angle 0}{50 + [j377 \| (100\text{-}j106)]} = 1.86\angle 21^\circ \text{ A}$$

$$\mathbf{I}_C = \mathbf{I}\ \frac{j377}{j377 + 100 - j106} = 2.43\angle 41.3^\circ \text{ A}$$

$\mathbf{V}_2 = (106\angle\text{-}90^\circ)(2.43\angle\text{-}41.3^\circ) = 257\angle\text{-}48.7^\circ$ V

$P_{measured} = (257)(1.86)\cos(21^\circ + 48.7^\circ) = \boxed{166 \text{ W.}}$

No reversal of meter leads is needed. PSpice verification:



FREQ	VM($N_0002,0)	VP($N_0002,0)
6.000E+01	2.581E+02	-4.871E+01

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000E+01	1.863E+00	2.103E+01

33. 2.5 A peak = 1.77 A rms. 200 V peak = 141 V rms. 100 μF → $-j20\ \Omega$.

Define the clockwise mesh current $\mathbf{I}_1$ in the bottom mesh, and the clockwise mesh current $\mathbf{I}_2$ in the top mesh. $\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2$.

Since $\mathbf{I}_2 = -177\angle{-90^o}$, we need write only one mesh equation:

$$141\angle 0^o = (20 - j40^o)\ \mathbf{I}_1 + (-20 + j20)\ \mathbf{I}_2$$

so that $\mathbf{I}_1 = \dfrac{141\angle 0 + (-20 + j20)(1.77\angle -90^o)}{20 - j40} = 4.023\angle 74.78^o$ A

and $\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2 = 2.361\ \angle\ 63.43^o$ A. $\mathbf{I}_{meter} = -\mathbf{I}_1 = 4.023\angle{-105.2^o}$
$\mathbf{V}_{meter} = 20\ \mathbf{I}_C = 47.23\ \angle 63.43^o$ V

Thus, Pmeter = $(47.23)(4.023)\cos(63.43^o + 105.2^o) = \boxed{-186.3\text{ W.}}$
$\boxed{\text{Since this would result in pegging the meter, we would need to swap the potential leads.}}$

34. (a) Define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ in the top left, bottom left and right-hand meshes, respectively. Then we may write:

$$
\begin{array}{rlll}
100\angle 0 = & (10 - j10)\,\mathbf{I}_1 & & -\,(10 - j10)\,\mathbf{I}_3 \\
50\angle 90^\circ = & & (8 + j6)\,\mathbf{I}_2 & -\,(8 + j6)\,\mathbf{I}_3 \\
0 = & -(10 - j10)\,\mathbf{I}_1 & -\,(8 + j6)\,\mathbf{I}_2 & +\,(48 + j6)\,\mathbf{I}_3
\end{array}
$$

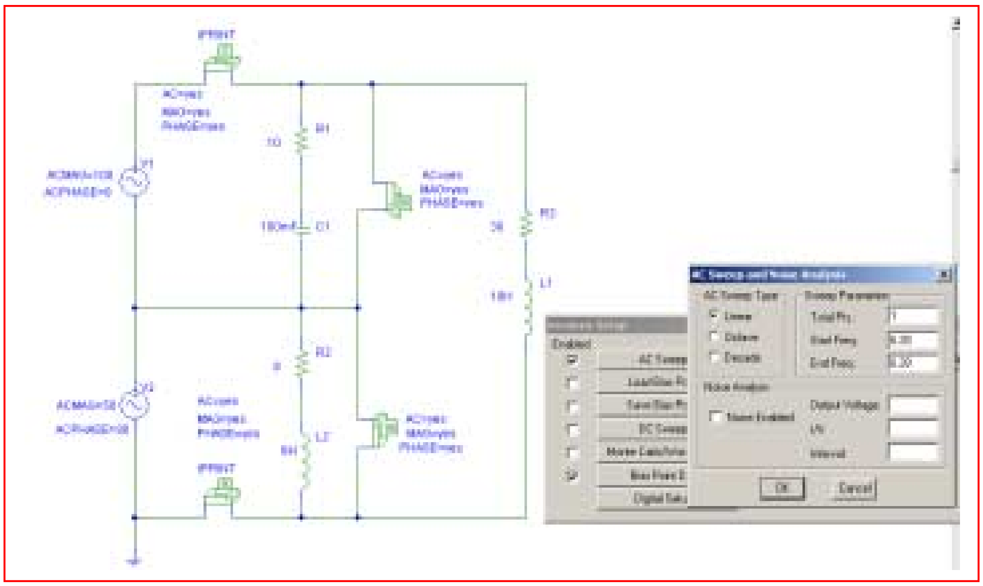
Solving, we find that $\mathbf{I}_1 = 10.12\angle 32.91^\circ$ A, $\mathbf{I}_2 = 7.906\angle 34.7^\circ$ and $\mathbf{I}_3 = 3.536\angle 8.13^\circ$ A.

Thus, $P_A = (100)(10.12)\cos(-32.91^\circ) = \boxed{849.6 \text{ W}}$

and $P_B = (5)(7.906)\cos(90^\circ - 34.7^\circ) = \boxed{225.0 \text{ W}}$

(b) Yes, the total power absorbed by the combined load (1.075 kW) is the sum of the wattmeter readings.

PSpice verification:



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.280E+00	1.014E+01	6.144E-02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.280E+00	4.268E-01	1.465E+02
FREQ	VM($N_0002,$N_0006)	VP($N_0002,$N_0006)
6.280E+00	1.000E+02	0.000E+00
FREQ	VM($N_0004,$N_0006)	VP($N_0004,$N_0006)
6.280E+00	5.000E+01 -	9.000E+01

35. This circuit is equivalent to a Y-connected load in parallel with a Δ-connected load.

For the Y-connected load, $\mathbf{I}_{line} = \dfrac{\frac{200}{\sqrt{3}}\angle -30^o}{25\angle 30^o} = 4.62\angle -60^o$ A

$P_Y = (3)\left(\dfrac{200}{\sqrt{3}}\right)(4.62)\cos 30^o = 1.386$ kW

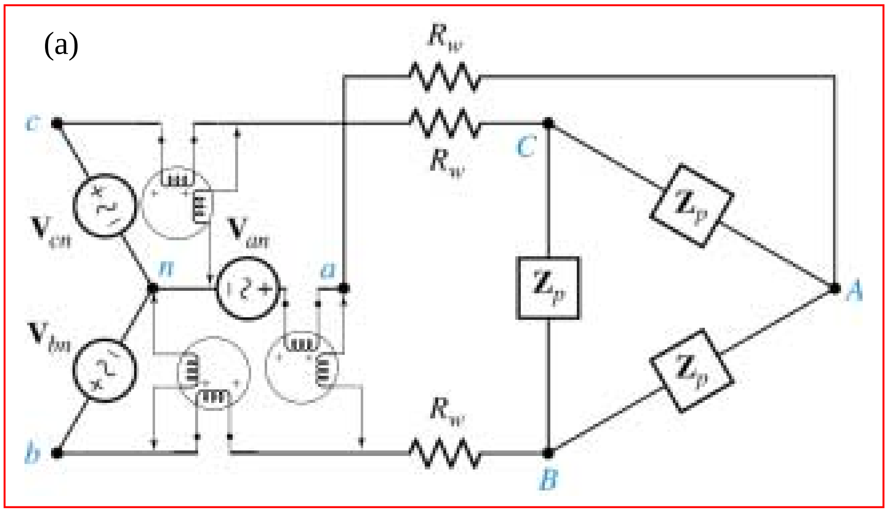
For the Δ-connected load, $\mathbf{I}_{line} = \dfrac{200\angle 0}{50\angle -60^o} = 4\angle 60^o$ A

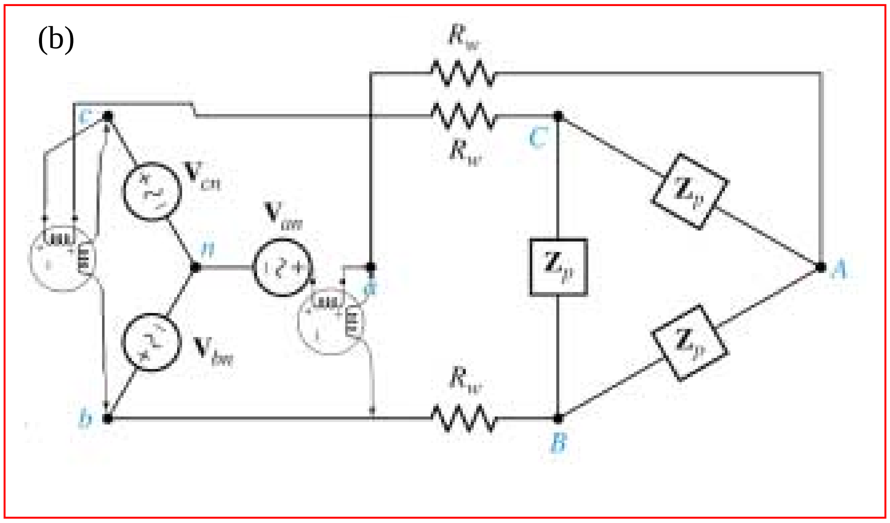
$P_\Delta = (3)(200)(4 \cos 60^o) = 1.2$ kW

$P_{total} = P_Y + P_\Delta = 2.586$ kW

$P_{wattmeter} = P_{total} / 3 = \boxed{862 \text{ W}}$

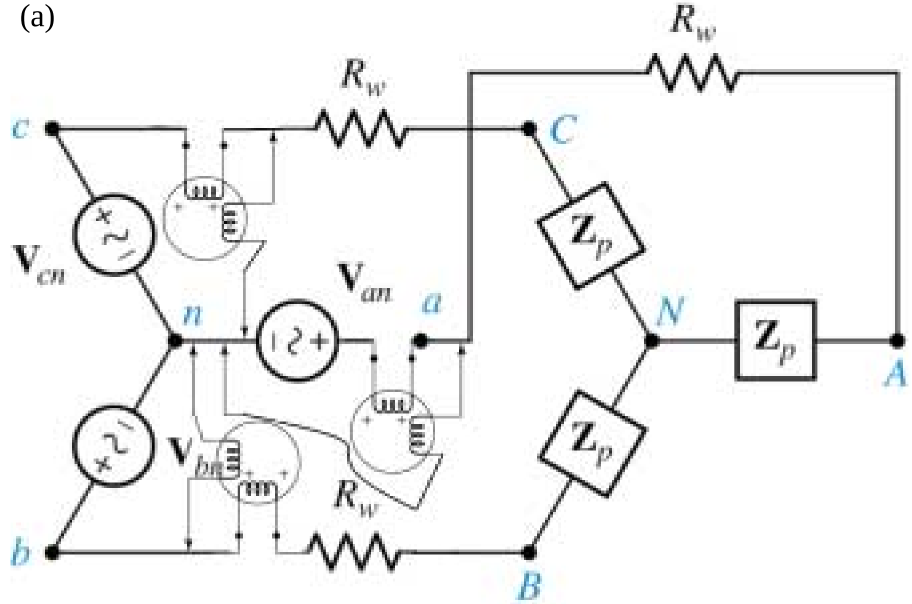
36. We assume that the wire resistance cannot be separated from the load, so we measure from the source connection:



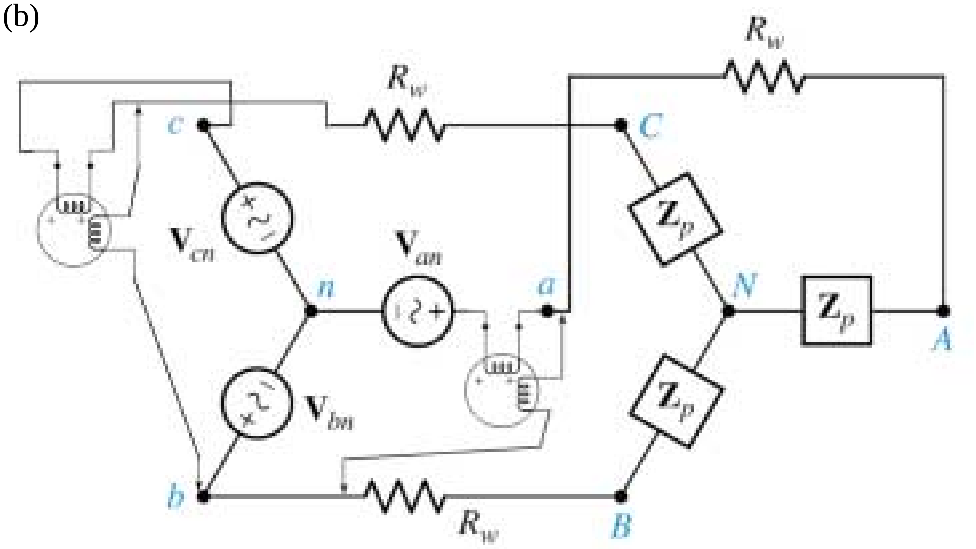


37. We assume that the wire resistance cannot be separated from the load, so we measure from the source connection:

(a)



(b)



CHAPTER THIRTEEN SOLUTIONS

1. $\boxed{\begin{array}{l}\text{1 and 3, 2 and 4}\\ \text{1 and 4, 2 and 3}\\ \text{3 and 1, 2 and 4}\end{array}}$

2. $i_{S1} = 4t\ \text{A},\ i_{S2} = 10t\ \text{A}$

(a) $v_{AG} = 20\times4 + 4\times10 = \boxed{120\ \text{V}}$

(b) $v_{CG} = -4\times6 = \boxed{-24\ \text{V}}$

(c) $v_{BG} = 3\times10 + 4\times4 - 6\times4 = 30 + 16 - 24 = \boxed{22\ \text{V}}$

3.

(a) $100 = (50 + j200)\bar{I}_1 + j300\bar{I}_2, (2000 + j500)\bar{I}_2 + j300\bar{I}_1 = 0$

$\therefore \bar{I}_2 = \dfrac{-j3}{20 + j5}, 100 = \left(50 + j200 + \dfrac{900}{20 + j5}\right)\bar{I}_1$

$\therefore 100 = \dfrac{900 + j4250}{20 + j5}\bar{I}_1 \therefore \bar{I}_1 = 0.4745^1 \angle -64.01° \text{ A}$

$\therefore P_{S,abS} = -\dfrac{1}{2} \times 100 \times 0.4745 \cos 64.01° = \boxed{-10.399 \text{ W}}$

(b) $P_{50} = \dfrac{1}{2} \times 50 \times 0.4745^2 = 5.630 \text{ W}, P_{2000} = \dfrac{1}{2} \times 2000 \times 0.4745^2 \times \left|\dfrac{-j3}{20 + j5}\right|^2 = \boxed{4.769 \text{ W}}$

(c) $\boxed{\text{0 each}}$

(d) $\boxed{0}$

4.

KVL Loop 1 $\quad 100\angle 0 = 2(\mathbf{I}_1 - \mathbf{I}_2) + j\omega 3\,(\mathbf{I}_1 - \mathbf{I}_3) + j\omega 2\,(\mathbf{I}_2 - \mathbf{I}_3)$

KVL Loop 2 $\quad 2(\mathbf{I}_2 - \mathbf{I}_1) + 10\mathbf{I}_2 + j\omega 4\,(\mathbf{I}_2 - \mathbf{I}_3) + j\omega 2\,(\mathbf{I}_1 - \mathbf{I}_3) = 0$

KVL Loop 3 $\quad 5\mathbf{I}_3 + j\omega 3\,(\mathbf{I}_3 - \mathbf{I}_1) + j\omega 2\,(\mathbf{I}_3 - \mathbf{I}_2) + j\omega 4\,(\mathbf{I}_3 - \mathbf{I}_2) + j\omega 2\,(\mathbf{I}_3 - \mathbf{I}_1) = 0$

$\therefore$ LINEAR EQUATIONS

$$\begin{bmatrix} 2+j\omega 3 & -2+j\omega 2 & -j\omega 5 \\ -2+j\omega 2 & 12+j\omega 4 & -j\omega 6 \\ -j\omega 5 & j\omega 2 & 5+j11 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \\ 0 \end{bmatrix}$$

Since $\omega = 2\pi f = 2\pi(50) = 314.2$ rad/s, the matrix becomes

$$\begin{bmatrix} 2+j942.6 & -2+j628.4 & -j1571 \\ -2+j628.4 & 12+j1257 & -j1885 \\ -j1571 & j628.4 & 5+j3456 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving using a scientific calculator or MATLAB, we find that

$\mathbf{I}_1 = 278.5\angle -89.65^{\circ}$ mA, $\mathbf{I}_2 = 39.78\angle -89.43^{\circ}$ mA, $\mathbf{I}_3 = 119.4\angle -89.58^{\circ}$ mA.

Returning to the time domain, we thus find that

$i_1(t) = 278.5\cos(100\pi t - 89.65^{\circ})$ mA, $i_2(t) = 39.78\cos(100\pi t - 89.43^{\circ})$ mA, and $i_3(t) = 119.4\cos(100\pi t - 89.58^{\circ})$ mA.

5.

(a) $\bar{V}_{ab,oc} = \frac{100}{50 + j200}(-j300) = 145.52\angle -165.96°\ \text{V}$

$100 = (50 + j200)\,\bar{I}_1 + j300\,\bar{I}_{2SC},\ j500\,\bar{I}_{2SC} + j300\,\bar{I}_1 = 0$

$\therefore\ \bar{I}_1 = -\frac{5}{3}\bar{I}_{2SC},\ 100 = \left[(50 + j200)\left(-\frac{5}{3}\right) + j300\right]\bar{I}_{2SC}\ \therefore\ \bar{I}_{2SC} = 1.1142\angle 158.199°\ \text{A}$

$\therefore\ \bar{Z}_{th} = \bar{V}_{ab,bc} / \bar{I}_{2SC} = \frac{145.52\angle -165.96°}{1.1142\angle 158.199°} = 130.60\angle 35.84° = \boxed{105.88 + j76.47\,\Omega}$

(b) $\bar{Z}_L = 105.88 - j76.47\,\Omega\ \ \therefore \left|\bar{I}_L\right| = \frac{145.52}{2\times 105.88} = 0.6872\ \text{A}$

$\therefore\ P_{L\max} = \frac{1}{2}\times 0.6872^2 \times 105.88 = \boxed{25.00\ \text{W}}$

6.

(a) $\boxed{v_A(t) = \mathrm{L}_1 i_1' - \mathrm{M} i_2',\ v_B(t) = \mathrm{L}_1 i_1' - \mathrm{M} i_2' + \mathrm{L}_2 i_2' - \mathrm{M} i_1'}$

(b)

$$\boxed{\begin{aligned} \mathbf{V}_1(j\omega) &= j\omega \mathrm{L}_1\, \mathbf{I}_\mathrm{A} + j\omega \mathrm{M}(\mathbf{I}_\mathrm{B} + \mathbf{I}_\mathrm{A}) \\ \mathbf{V}_2(j\omega) &= j\omega \mathrm{L}_2\, (\mathbf{I}_\mathrm{B} + \mathbf{I}_\mathrm{A}) + j\omega \mathrm{M}\mathbf{I}_\mathrm{A} \end{aligned}}$$

7.

$$v_s = \frac{10t^2 u(t)}{t^2+0.01} = 0.01 i'_S \ \therefore i'_S = \frac{1000t^2}{t^2+0.01} u(t)$$

$$v_x = 0.015 i'_S = \frac{15t^2}{t^2+0.01} u(t),\ 100 v_x = \frac{1500t^2}{t^2+0.01} u(t)$$

$$\therefore i_C = 100\times 10^{-6}\, v'_x = 10^{-4} \frac{d}{dt}\left(\frac{15t^2}{t^2+0.01} u(t) \right) = 15\times 10^{-4} \frac{(t^2+0.01)2t - t^2\times 2t}{(t^2+0.01)^2} u(t)$$

$$\therefore i_C = 15\times 10^{-4} \frac{0.02t}{(t^2+0.01)^2} \quad \therefore \boxed{i_C(t) = \frac{30t}{(t^2+0.01)^2}\ \mu\text{A}, \qquad t>0}$$

8.

(a) $-\boldsymbol{V}_2 = j\omega 0.4 \ 1\angle 0$

$$\boldsymbol{V}_2 = -j100\pi\times 0.4\times 1\angle 0 \ = 126 \angle 90^o \text{ V}$$

Thus, $\boxed{v(t) = 126 \cos(100\pi t + 90^o) \text{ V}}$

(b) Define $\mathbf{V}_2$ across the 2-H inductor with + reference at the dot, and a clockwise currents $\mathbf{I}_1$ and $\mathbf{I}_2$, respectively, in each mesh. Then,

$\mathbf{V} = -\mathbf{V}_2$ and we may also write

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + j\omega M\mathbf{I}_1 \quad \text{or} \quad -\mathbf{V} = j\omega L_2 \frac{\mathbf{V}}{10} + j\omega M$$

Solving for $\mathbf{V}$,

$$\mathbf{V} = \frac{-(j100\pi)(0.4)}{1+(j100\pi)(2)} = \frac{125.7\angle -90^o}{1+j62.83} = \frac{125.7\angle -90^o}{62.84\angle 89.09^o} = 2.000\angle -179.1^o$$

Thus,

$$\boxed{v(t) = 2 \cos(100\pi t - 179.1^o) \text{ V.}}$$

(c) Define $\mathbf{V}_1$ across the left inductor, and $\mathbf{V}_2$ across the right inductor, with the "+" reference at the respective dot; also define two clockwise mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Then,

$$\boldsymbol{V}_1 = j\omega L_1 \ \boldsymbol{I}_1 + j\omega M \ \boldsymbol{I}_2$$

$$\boldsymbol{V}_2 = j\omega L_2 \ \boldsymbol{I}_2 + j\omega M \ \boldsymbol{I}_1$$

Now $\boldsymbol{I}_1 = \dfrac{1\angle 0 - \boldsymbol{V}_1}{4}$ and $\boldsymbol{V}_{out} = -\boldsymbol{V}_2$

and $\boldsymbol{I}_2 = \dfrac{\boldsymbol{V}_{out}}{10}$

$$\Rightarrow \boldsymbol{V}_1 = j\omega L_1\left[\frac{1\angle 0 - \boldsymbol{V}_1}{4}\right] + j\omega M\frac{\boldsymbol{V}_{out}}{10} \quad \text{EQN 1}$$

$$-\boldsymbol{V}_{out} = j\omega L_2\frac{\boldsymbol{V}_{out}}{10} + j\omega M\left[\frac{1\angle 0 - \boldsymbol{V}_1}{4}\right] \quad \text{EQN 2}$$

$$\begin{bmatrix} 1-\dfrac{j\omega L_1}{4} & \dfrac{-j\omega M}{10} \\ \dfrac{j\omega M}{4} & -1+\dfrac{j\omega L_2}{10} \end{bmatrix}\begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_{out} \end{bmatrix} = \begin{bmatrix} \dfrac{j\omega L_1 \ 1\angle 0}{4} \\ \dfrac{j\omega M \ 1\angle 0}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1-j39 & -j12.6 \\ j31.4 & -1+j62.8 \end{bmatrix}\begin{bmatrix} \boldsymbol{V}_1 \\ \boldsymbol{V}_{out} \end{bmatrix} = \begin{bmatrix} 39.3j \\ 31.4j \end{bmatrix}$$

Solving, we find that $\mathbf{V}_{out}$ (= $\mathbf{V}$) = 1.20 $\angle$ -2.108^o V and hence

$$\boxed{v(t) = 1.2 \cos(100\pi t - 2.108^o) \text{ V.}}$$

9.

(a) $100 = j5\omega\,(\mathbf{I}_1 - \mathbf{I}_2) + j3\omega\mathbf{I}_2 + 6(\mathbf{I}_1 - \mathbf{I}_3)$ [1]

$(4 + j4\omega)\mathbf{I}_2 + j3\omega\,(\mathbf{I}_1 - \mathbf{I}_2) + j2\omega\,(\mathbf{I}_3 - \mathbf{I}_2) + j6\omega\,(\mathbf{I}_2 - \mathbf{I}_3) - j2\omega\,\mathbf{I}_2 + j5\omega\,(\mathbf{I}_2 - \mathbf{I}_1) - j3\omega\,\mathbf{I}_2 = 0$ [2]

$6\,(\mathbf{I}_3 - \mathbf{I}_1) + j6\omega\,(\mathbf{I}_3 - \mathbf{I}_2) + j2\omega\,\mathbf{I}_2 + 5\,\mathbf{I}_3 = 0$ [3]

Collecting terms,

$$\boxed{\begin{aligned} (6 + j5\omega)\,\mathbf{I}_1 - j2\omega\,\mathbf{I}_2 - 6\,\mathbf{I}_3 &= 100 \\ -j2\omega\,\mathbf{I}_1 + (4 + j5\omega)\,\mathbf{I}_2 - j4\omega\,\mathbf{I}_3 &= 0 \\ -6\,\mathbf{I}_1 - j4\omega\,\mathbf{I}_2 + (11 + j6\omega)\,\mathbf{I}_3 &= 0 \end{aligned}}$$

[1]

[2]

[3]

(b) For $\omega = 2$ rad/s, we find

$(6 + j10)\,\mathbf{I}_1 - j4\,\mathbf{I}_2 - 6\,\mathbf{I}_3 = 100$

$-j4\,\mathbf{I}_1 + (4 + j10)\,\mathbf{I}_2 - j8\,\mathbf{I}_3 = 0$

$-6\,\mathbf{I}_1 - j8\,\mathbf{I}_2 + (11 + j12)\,\mathbf{I}_3 = 0$

Solving, $\boxed{\mathbf{I}_3 = 4.32\angle -54.30^\circ\text{ A}}$

10.

(a)

$$\mathbf{V}_a = j\omega L_1\,\mathbf{I}_a + j\omega M\,\mathbf{I}_b \qquad \mathbf{I}_a = \mathbf{I}_1$$
$$\mathbf{V}_b = j\omega L_2\,\mathbf{I}_b + j\omega M\,\mathbf{I}_a \qquad \mathbf{I}_b = -\mathbf{I}_2$$

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{I}_1\,R_1 + \mathbf{V}_a \\ &= \mathbf{I}_1\,R_1 + j\omega L_1\,\mathbf{I}_a + j\omega M\,\mathbf{I}_b \\ &= \boxed{\mathbf{I}_1\,R_1 + j\omega L_1\,\mathbf{I}_1 - j\omega M\,\mathbf{I}_2}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_2 &= \mathbf{I}_2\,R_2 - \mathbf{V}_b \\ &= \mathbf{I}_2\,R_2 - j\omega L_2\,\mathbf{I}_b - j\omega M\,\mathbf{I}_a \\ &= \boxed{\mathbf{I}_2\,R_2 + j\omega L_2\,\mathbf{I}_2 - j\omega M\,\mathbf{I}_1}\end{aligned}$$

(b) Assuming that the systems connecting the transformer are fully isolated.

$$\mathbf{V}_a = j\omega L_1\,\mathbf{I}_a + j\omega M\mathbf{I}_b \qquad \mathbf{I}_a = -\mathbf{I}_1$$
$$\mathbf{V}_b = j\omega L_2\mathbf{I}_b + j\omega M\mathbf{I}_a \qquad \mathbf{I}_b = -\mathbf{I}_2$$

$$\begin{aligned}\mathbf{V}_1 &= \mathbf{I}_1\,R - \mathbf{V}_a \\ &= \mathbf{I}_1\,R - j\omega L_1\,\mathbf{I}_a - j\omega M\,\mathbf{I}_b \\ &= \boxed{\mathbf{I}_1\,R + j\omega L_1\,\mathbf{I}_1 + j\omega M\,\mathbf{I}_2}\end{aligned}$$

$$\begin{aligned}\mathbf{V}_2 &= \mathbf{V}_b + \mathbf{I}_b\,R_2 \\ &= -\mathbf{I}_2\,R_2 + j\omega L_2\mathbf{I}_b + j\omega M\,\mathbf{I}_a \\ &= \boxed{-\mathbf{I}_2\,R_2 - j\omega L_2\,\mathbf{I}_2 - j\omega M\,\mathbf{I}_1}\end{aligned}$$

11.

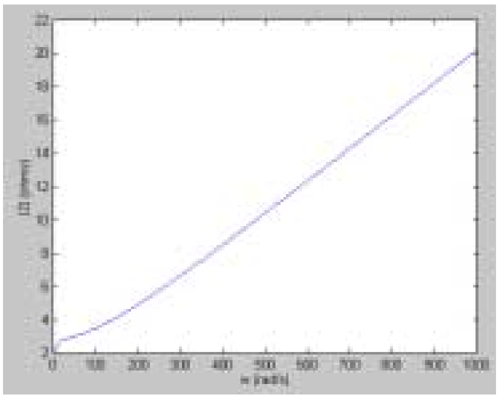
(a)

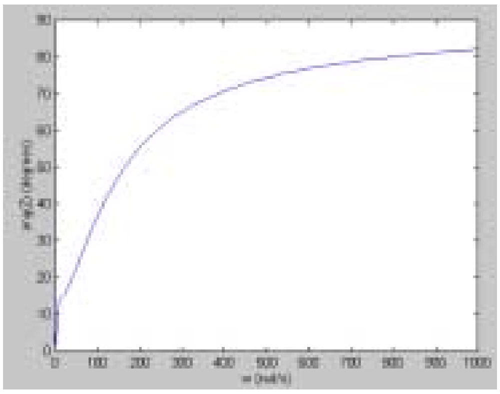
$$\mathbf{Z} = 2 + j\omega 0.1 + \frac{\omega^2 (0.2)^2}{5 + j\omega 0.5}$$

$$= 2 + j\omega 0.1 + \frac{5\omega^2 (0.2)^2}{5^2 + (\omega 0.5)^2} - \frac{j\omega 0.5\ \omega^2 (0.2)^2}{5^2 + (\omega 0.5)^2}$$

$$\boxed{= 2 + \frac{0.2\omega^2}{25 + 0.25\omega^2} + j\omega\left[0.1 - \frac{0.02\omega^2}{25 + 0.25\omega^2}\right]}$$

(b)





(c) $\mathbf{Z}_{in}(j\omega)$ at $\omega = 50$ is equal to $2 + 0.769 + j(50)(0.023) = \boxed{2.77 + j1.15\ \Omega.}$

12.

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2}{\mathbf{Z}_{22}}$$

$$= j\omega 50\times 10^{-3} + \frac{\omega^2 M^2}{8 + j\omega 10\times 10^{-3}}$$

$$\Rightarrow \mathbf{Z}_{in} = j\omega 50\times 10^{-3} + \frac{\omega^2 M^2 8}{8^2 + (\omega 10\times 10^{-3})^2} - \frac{j\omega 10\times 10^{-3}\omega^2 M}{8^2 + (\omega 10\times 10^{-3})^2}$$

$$= \frac{\omega^2 M^2 8}{8^2 + (\omega 10\times 10^{-3})^2} + j\omega\left[50\times 10^{-3} - \frac{10\times 10^{-3}\omega^2 M^2}{8^2 + (\omega 10\times 10^{-3})^2}\right]$$

In this circuit the real power delivered by the source is all consumed at the speaker, so

$$P = \frac{V_{rms}{}^2}{R} \Rightarrow 3.2 = \left(\frac{20}{\sqrt{2}}\right)^2 \times \frac{1}{\frac{\omega^2 M^2 8}{8^2 (\omega 10\times 10^{-3})^2}}$$

$$\Rightarrow \frac{\omega^2 M^2 8}{8^2 + (\omega 10\times 10^{-3})^2} = \frac{20^2}{2\times 3.2} \quad \boxed{= 62.5\text{ W}}$$

13. $i_{S1} = 2\cos 10t \text{ A}, \; i_{S2} = 1.2\cos 10t \text{ A}$

(a) $v_1 = 0.6(-20\sin 10t) - 0.2(-12\sin 10t) + 0.5(-32\sin 10t) + 9.6\cos 10t$

$\therefore v_1 = 9.6\cos 10t - 25.6\sin 10t = \boxed{27.34\cos(10t + 69.44°) \text{ V}}$

(b) $v_2 = 0.8(-12\sin 10t) - 0.2(-20\sin 10t) - 16\sin 10t + 9.6\cos 10t$

$\therefore v_2 = 9.6\cos 10t - 21.6\sin 10t = \boxed{23.64\cos(10t + 66.04°) \text{ V}}$

(c) $\text{P}_{S1} = \frac{1}{2} \times 27.34 \times 2\cos 69.44° = \boxed{9.601 \text{ W}}, \; \text{P}_{S2} = \frac{1}{2} \times 23.64 \times 1.2\cos 66.04° = \boxed{5.760 \text{ W}}$

14.

$$\mathbf{V}_a = j\omega 8\mathbf{I}_a + j\omega 4\mathbf{I}_b$$

$$* \quad \mathbf{V}_b = j\omega 10\mathbf{I}_b + j\omega 4\mathbf{I}_a = j\omega 10\mathbf{I}_b + j\omega 5\mathbf{I}_c$$

$$\mathbf{V}_c = j\omega 6\mathbf{I}_c + j\omega 5\mathbf{I}_b$$

Also $\mathbf{I} = -\mathbf{I}_a = -\mathbf{I}_b = \mathbf{I}_c$

Now examine equation *.

$$-j\omega 10\mathbf{I} - j\omega 4\mathbf{I} = -j\omega 10\mathbf{I} + j\omega 5\mathbf{I}_c$$

$\therefore$ the only solution to this circuit is $\mathbf{I}$ = and hence

$\boxed{v(t) = 120\cos\omega t \text{ V.}}$

15.

$$100 = j10\,\overline{\mathrm{I}}_1 - j15\,\overline{\mathrm{I}}_2$$

$$0 = j200\,\overline{\mathrm{I}}_2 - j15\,\overline{\mathrm{I}}_1 - j15\,\overline{\mathrm{I}}_L$$

$$0 = (5 + j10)\,\overline{\mathrm{I}}_L - j15\,\overline{\mathrm{I}}_2$$

$$\therefore\ \overline{\mathrm{I}}_2 = \frac{5 + j10}{j15}\,\overline{\mathrm{I}}_L = \frac{1 + j2}{j3}\,\overline{\mathrm{I}}_L \quad \therefore 0 = j200\left(\frac{1 + j2}{j3} - j15\right)\overline{\mathrm{I}}_L - j15\,\overline{\mathrm{I}}_1$$

$$\therefore 0 = \left(j\frac{400}{3} - j15 + \frac{200}{3}\right)\overline{\mathrm{I}}_L - j15\,\overline{\mathrm{I}}_1 \quad \therefore\ \overline{\mathrm{I}}_1 = \frac{j118.33 + 66.67}{j15}\,\overline{\mathrm{I}}_L$$

$$\therefore 100 = \left[\frac{2}{3}(66.67 + j118.33) - 5 - j10\right]\overline{\mathrm{I}}_L = (39.44 + j68.89)\,\overline{\mathrm{I}}_L$$

$$\therefore\ \overline{\mathrm{I}}_L = \boxed{1.2597\angle -60.21^\circ\ \mathrm{A}}$$

16. $i_s = 2\cos 10t$ A, $t = 0$

(a) $a-b$ O.C. $\therefore w(0) = \frac{1}{2}\times 5\times 2^2 + \frac{1}{2}\times 4\times 2^2 = 10+8 = \boxed{18 \text{ J}}$

(b) $a-b$ S.C. $\omega = 10$, $\bar{I}_S = 2\angle 0°$ A, $M = \frac{1}{2}\sqrt{12} = \sqrt{3}$ H

$(j30+5)\,\bar{I}_2 - j10\sqrt{3}\times 2$, $\therefore \bar{I}_2 = \frac{j20\sqrt{3}}{5+j30} = 1.1390\angle 9.462°\text{A}$ $\therefore i_2 = 1.1390\cos(10t + 9.462°)$ A

$\therefore i_2(0) = 1.1235^-$ $\therefore w(0) = 10+8-\sqrt{3}\times 2\times 1.1235 + \frac{1}{2}\times 3\times 1.1235^2 = \boxed{16.001 \text{ J}}$

17.

$\bar{V}_s = 12\angle 0°$ V rms, $\omega = 100$ rad/s

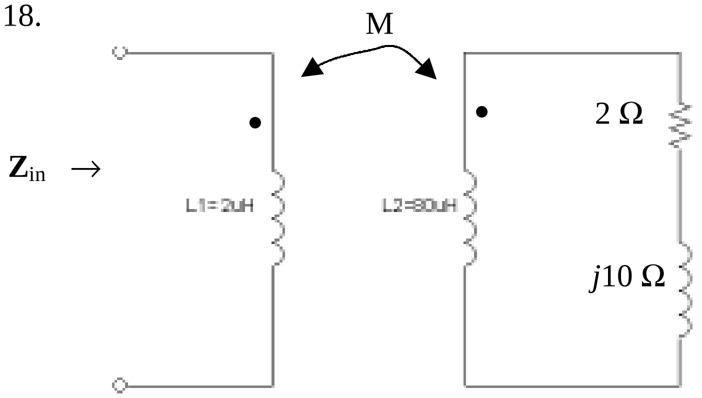
$12 = (6 + j20)\ \bar{I}_1 + j100(0.4K)\ \bar{I}_2,\ (24 + j80)\ \bar{I}_2 + j40K\bar{I}_1 = 0$

$$\therefore \bar{I}_1 = \frac{3 + j10}{-j5K}\ \bar{I}_2 \quad \therefore 12 = \left[(6 + j20)\frac{3 + j10}{-j5K} + j40K\right]\bar{I}_2$$

$$\therefore 12 = \frac{18 - 200 + j60 + j60 + 200K^2}{-j5K}\ \bar{I}_2 \quad \therefore \bar{I}_2 = \frac{-j60K}{-182 + 200K^2 + j120}$$

$$\therefore P_{24} = \frac{60^2 K^2 24}{(200K^2 - 182)^2 + 120^2} = \frac{86,400\ K^2}{40,000K^4 - 72,800K^2 + 47,524} = \boxed{\frac{2.16K^2}{K^4 - 1.82K^2 + 1.1881}\ \text{W}}$$

18.



$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\omega = 250k \ rad/s$$

$$M = \sqrt{L_1 L_2} = \sqrt{2 \times 80} \times 10^{-6}$$
$$= 12.6 \mu\text{H}$$

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} + \frac{-jM^2\omega^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

$$\mathbf{Z}_{11} = j \times 250 \times 10^3 \times 2 \times 10^{-6}$$
$$= j0.5$$

$$R_{22} = 2\Omega$$
$$X_{22} = (250 \times 10^3)(80 \times 10^{-6})$$
$$= 20$$

Thus, $\mathbf{Z}_{in} = j0.5 + 19.8/404 - j198/404$

$$= \boxed{0.049 + j0.010 \ \Omega}.$$

19. $\omega = 100$ rad/s

(a) $K_1 \to j50\Omega,\ K_2 \to j20\Omega,\ 1\text{H} \to j100\,\Omega$

$100 = j200\ \bar{I}_1 - j50\bar{I}_2 - j20\bar{I}_3$

$0 = (10 + j100)\bar{I}_2 - j50\bar{I}_1$

$0 = (20 + j100)\ \bar{I}_3 - j20\bar{I}_1$

$\therefore\ \bar{I}_3 = \dfrac{j2}{2 + j10}\bar{I}_1,\ \bar{I}_2 = \dfrac{j5}{1 + j10}\bar{I}_1\ \ \therefore 10 = \left[j20 - j5\dfrac{j5}{1 + j10} - j2\dfrac{j2}{2 + j10}\right]\bar{I}_1$

$\therefore 10 = \left(j20 + \dfrac{25}{1 + j10} + \dfrac{4}{2 + j10}\right)\bar{I}_1\ \ \therefore\ \bar{I}_1 = 0.5833\angle -88.92°\text{ A},\ \bar{I}_2 = 0.2902\angle -83.20°\text{ A},$

$\bar{I}_3 = 0.11440\angle -77.61°\text{ A}\quad \therefore\ P_{10\Omega} = 0.2902^2 \times 10 = \boxed{0.8422\text{ W}}$

(b) $P_{20} = 0.1144^2 \times 20 = \boxed{0.2617\text{ W}}$

(c) $P_{gen} = 100 \times 0.5833\cos 88.92° = \boxed{1.1039\text{ W}}$

20.

(a)

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\Rightarrow M = 0.4\sqrt{5 \times 1.8}$$

$$= \boxed{1.2\text{H}}$$

(b)

$$\boldsymbol{I}_1 + \boldsymbol{I}_2 = \boldsymbol{I}_3$$

$$\Rightarrow \boldsymbol{I}_2 = \boldsymbol{I}_3 - \boldsymbol{I}_1$$

$$= \boxed{5 \times 10^{\frac{-t}{5}} - 4 \times 10^{\frac{-t}{10}}}$$

(c) The total energy stored at $t = 0$.

$$\boldsymbol{I}_1 = 4A \qquad \boldsymbol{I}_2 = 1A$$

$$W\,total = \frac{1}{2} L_1 \boldsymbol{I}_1^2 + \frac{1}{2} L_2 \boldsymbol{I}_2^2 + M_{12} \boldsymbol{I}_1 \boldsymbol{I}_2$$

$$= \frac{1}{2} \times 5 \times 16 + \frac{1}{2} \times 1.8 \times 1 - 1.2 \times 4 \times 1$$

$$= 40 + 0.9 - 4.8$$

$$= \boxed{36.1\text{J}}$$

21.

$$\mathrm{K} \rightarrow j1000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2},\ \mathrm{L}_1 \rightarrow j1000\mathrm{L}_1,\ \mathrm{L}_2 \rightarrow j1000\mathrm{L}_2$$

$$\therefore \bar{\mathrm{V}}_s = (2 + j1000\mathrm{L}_1)\,\bar{\mathrm{I}}_1 - j1000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2}\ \bar{\mathrm{I}}_2$$

$$0 = -j1000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2}\ \bar{\mathrm{I}}_1 + (40 + j1000\mathrm{L}_2)\,\bar{\mathrm{I}}_2$$

$$\omega = 1000 \text{ rad/s} \quad \therefore \bar{\mathrm{I}}_1 = \frac{40 + j1000\mathrm{L}_2}{j1000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2}}\bar{\mathrm{I}}_2$$

$$\therefore \bar{\mathrm{V}}_s = \frac{(2 + j1000\mathrm{L}_1)(40 + j1000\mathrm{L}_2) + 10^6\mathrm{K}^2\mathrm{L}_1\mathrm{L}_2}{j1000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2}}\bar{\mathrm{I}}_2$$

$$\therefore \bar{\mathrm{I}}_2 = \frac{j1000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2}}{80 + j40{,}000\mathrm{L}_1 + j2000\mathrm{L}_2 - 10^6\mathrm{L}_1\mathrm{L}_2(1-\mathrm{K}^2)}$$

$$\therefore \frac{\bar{\mathrm{V}}_2}{\bar{\mathrm{V}}_s} = \frac{j40{,}000\mathrm{K}\sqrt{\mathrm{L}_1\mathrm{L}_2}}{80 - 10^6\mathrm{L}_1\mathrm{L}_2(1-\mathrm{K}^2) + j(40{,}000\mathrm{L}_1 + 2000\mathrm{L}_2)}$$

(a) $\mathrm{L}_1 = 10^{-3},\ \mathrm{L}_2 = 25\times 10^{-3},\ \mathrm{K} = 1 \ \therefore \dfrac{\bar{\mathrm{V}}_2}{\bar{\mathrm{V}}_s} = \dfrac{j40\times 5}{80 - 0 + j(40+50)} = \dfrac{j200}{80 + j90} = \boxed{1.6609\angle 41.63°}$

(b) $\mathrm{L}_1 = 1,\ \mathrm{L}_2 = 25,\ \mathrm{K} = 0.99 \ \therefore \dfrac{\mathrm{V}_2}{\mathrm{V}_3} = \dfrac{j40{,}000\times 0.99\times 5}{80 - 25\times 10^6(1-0.99^2) + j(40{,}000 + 50{,}000)}$

$$\therefore \frac{\bar{\mathrm{V}}_2}{\bar{\mathrm{V}}_S} = \frac{j198{,}000}{80 - 497{,}500 + j90{,}000} = \boxed{0.3917\angle -79.74°}$$

(c) $\mathrm{L}_1 = 1,\ \mathrm{L}_2 = 25,\ \mathrm{K} = 1 \ \therefore \dfrac{\mathrm{V}_2}{\mathrm{V}_s} = \dfrac{j40{,}000\times 5}{80 - 0 + j90{,}000} = \dfrac{j200{,}000}{80 + j90{,}000} = \boxed{2.222\angle 0.05093°}$

22.

(a) $L_{AB,CDOC} = 10 \text{ mH}, \; L_{CD,ABOC} = 5 \text{ mH}$

$L_{AB,CDSC} = 8 \text{ mH}$

$\therefore L_1 = 10 \text{ mH}, \; L_2 = 5 \text{ mH}, \; 8 = 10 - M + M \| (5-M) \text{ (mH)}$

$\therefore 8 = 10 - M + \dfrac{M(5-M)}{5}, \; \therefore 5M = (10-8)5 + 5M - M^2 \;\; \therefore M = 3.162 \text{ mH } (=\sqrt{10})$

$\therefore K = \dfrac{3.162}{\sqrt{50}} \;\; \therefore \boxed{K = 0.4472}$

(b) Dots at A and D, $i_1 = 5$ A, $w_{tot} = 100$ mJ

$\therefore 100 \times 10^{-3} = \dfrac{1}{2} \times 10 \times 10^{-3} \times 25 + \dfrac{1}{2} \times 5 \times 10^{-3} i_2^2 - \sqrt{10} \times 5 i_2 \times 10^{-3}$

$100 = 125 + 2.5 i_2^2 - 5\sqrt{10}\, i_2 \;\; \therefore i_2^2 - 2\sqrt{10}\, i_2 + 10 = 0, \; i_2 = \dfrac{2\sqrt{10} \pm \sqrt{40-40}}{2} = \sqrt{10}$

$\therefore i_2 = \boxed{3.162 \text{ A}}$

23. Define coil voltages v_1 and v_2 with the "+" reference at the respective dot. Also define two clockwise mesh currents i_1 and i_2. We may then write:

$$v_1 = L_1 \frac{d\boldsymbol{I}_1}{dt} + M\frac{d\boldsymbol{I}_2}{dt} \qquad\qquad M = k\sqrt{L_1 L_2}$$

$$v_2 = L_2 \frac{d\boldsymbol{I}_2}{dt} + M\frac{d\boldsymbol{I}_1}{dt} \qquad \omega = 2\pi 60\, rad/s$$

or, using phasor notation,

$$\boldsymbol{V}_1 = j\omega L_1\,\boldsymbol{I}_1 + j\omega M\,\boldsymbol{I}_2$$
$$\boldsymbol{V}_2 = j\omega L_2\,\boldsymbol{I}_2 + j\omega M\,\boldsymbol{I}_1$$

$$100\angle 0 = 50\boldsymbol{I}_1 + j\omega L_1\boldsymbol{I}_1 + j\omega M\,\boldsymbol{I}_2$$
$$-25\boldsymbol{I}_2 = j\omega L_2\boldsymbol{I}_2 + j\omega M\,\boldsymbol{I}_1$$

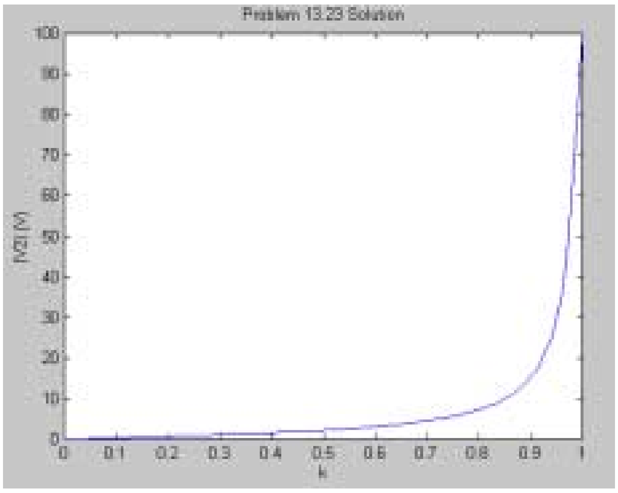
Rearrange: $[50 + j\omega L_1]\,\boldsymbol{I}_1 + j\omega M\boldsymbol{I}_2 = 100\angle 0$

$j\omega M\boldsymbol{I}_1\,[-25 + j\omega L_2]\,\boldsymbol{I}_2 = 0$

or
$$\begin{bmatrix} 50 + j\omega L_1 & j\omega M \\ j\omega M & -25 + j\omega L_2 \end{bmatrix}\begin{bmatrix} \boldsymbol{I}_1 \\ \boldsymbol{I}_2 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \end{bmatrix}$$

We can solve for $\mathbf{I}_2$ and $\mathbf{V}_2 = -25\mathbf{I}_2$:

$$\boxed{\mathbf{V}_2 = -\frac{j1.658}{k\sqrt{\mathrm{L}_1\mathrm{L}_2} + 1}}$$



24.

$$i_1 = 2\cos 500t \text{ A} \quad \text{W}_{\text{max}} \text{ at } t = 0$$

$$\therefore w_{\text{max}} = \frac{1}{2}\times 4\times 2^2 + \frac{1}{2}\times 6\times 2^2 + \frac{1}{2}\times 5\times 2^2 + 3\times 2^2$$

$$= 8 + 12 + 10 + 12 = \boxed{42 \text{ J}}$$

25.

(a) All DC: $L_{1-2} = 2 - 1 = 1 \text{ H}$

(b) AB SC: $L_{1-2} = -1 + 2 \| 8 = \boxed{0.6 \text{ H}}$

(c) BC SC: $L_{1-2} = 2 + (-1) \| 9 = 2 - 9/8 = \boxed{0.875 \text{ H}}$

(d) AC SC: $L_{1-2} = (2-1) \| (1+2) = 1 \| 3 = \boxed{0.750 \text{ H}}$

26.

(a)
$$\frac{\mathbf{I}_L}{\mathbf{V}_S} = \frac{1}{15 + j3\omega + \dfrac{j2\omega(20 + j\omega)}{20 + j3\omega}}\left(\frac{j2\omega}{20 + j3\omega}\right)$$

$$\boxed{= \frac{j2\omega}{300 - 11\omega^2 + j145\omega}}$$

(b)
$$v_s(t) = 100u(t),\ i_s(0) = 0,\ i_L(0) = 0,\ s_{1,2} = \frac{-145 \pm \sqrt{145^2 - 13,200}}{22} = -2.570,\ -10.612$$

$$i_L = i_{Lf} + i_{Ln},\ i_{Lf} = 0,\ \ \therefore i_L = \mathrm{A}e^{-2.57t} + \mathrm{B}e^{-10.61t},\ \ \therefore 0 = \mathrm{A} + \mathrm{B}$$

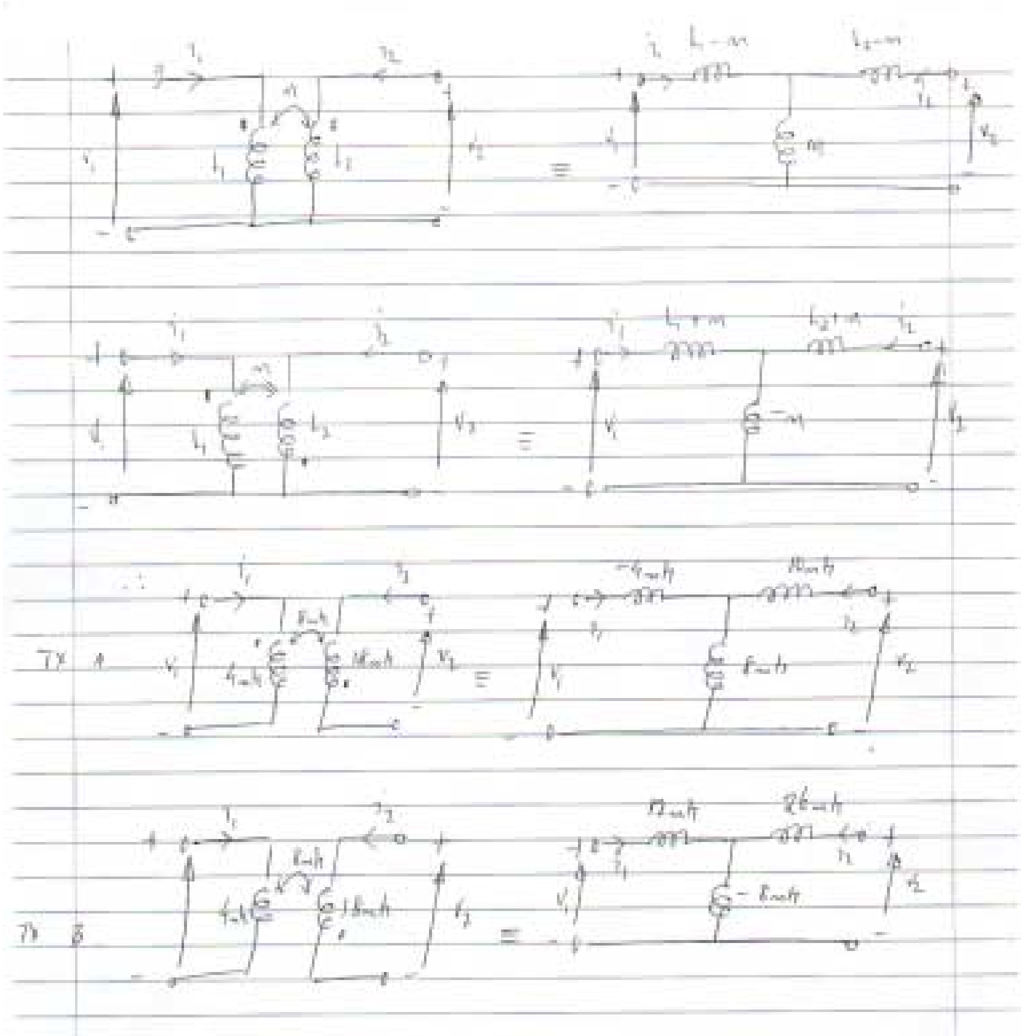
$100 = 15i_s + 5i'_s - 2i'_L,\ 0 = 20i_L + 3i'_L - 2i'_s$ At $t = 0^+$: $100 = 0 + 5i'_s(0^+) - 2i'_L(0^+)$ and

$$0 = 0 + 3i'_L(0^+) - 2i'_s(0^+)\ \ \therefore i'_s(0^+) = 1.5i'_L(0^+)\ \ \therefore 100 = 7.5i'_L(0^+) - 2i'_L(0^+) = 5.5i'_L(0^+)$$

$$\therefore i'_L(0^+) = 18.182\ \text{A/s}\quad \therefore 18.182 = -2.57\mathrm{A} - 10.61\mathrm{B} = -2.57\mathrm{A} + 10.61\mathrm{A} = 8.042\mathrm{A}$$

$$\therefore \mathrm{A} = 2.261,\ \mathrm{B} = -2.261,\ \boxed{i_L(t) = 2.261(e^{-2.57t} - e^{-10.612t})\ \text{A},\ t > 0}$$

27.



(a) Open-Circuit

$$\boxed{\begin{aligned} \mathbf{Z}_{oc}^{T\times A} &= j\omega 4\ M\Omega \\ \mathbf{Z}_{oc}^{T\times B} &= j\omega 4\ M\Omega \end{aligned}}$$

(b) Short-Circuit

$$\boxed{\mathbf{Z}_{SS}^{T\times A} = \mathbf{Z}_{SS}^{T\times B} = -j\omega 4\,M\Omega + j\omega 8 \| j\omega 10\,M\Omega}$$

(c) If the secondary is connected in parallel with the primary

$$\boxed{\begin{aligned} \mathbf{Z}_{in}^{T\times A} &= -j\omega 4 \| -j\omega 10 + j\omega 8\,M\Omega \\ \mathbf{Z}_{in}^{T\times B} &= j\omega 26 \| j\omega 12 - j\omega 8\,M\Omega \end{aligned}}$$

28. Define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ beginning with the left-most mesh.

$\mathbf{V}_s = j8\omega\,\mathbf{I}_1 - j4\omega\,\mathbf{I}_2$
$0 = -4j\omega\,\mathbf{I}_1 + (5 + j6\omega)\,\mathbf{I}_2 - j2\omega\,\mathbf{I}_3$
$0 = -j2\omega\,\mathbf{I}_2 + (3 + j\omega)\,\mathbf{I}_3$

Solving, $\mathbf{I}_3 = j\omega / (15 + j17\omega)$. Since $\mathbf{V}_o = 3\,\mathbf{I}_3$,

$$\boxed{\frac{\mathbf{V}_o}{\mathbf{V}_S} = \frac{j3\omega}{15 + j17\omega}}$$

29. $L_{eq} = 2/3 + 1 + 2 + 6/5 = 4.867$ H

$Z(j\omega) = 10\, j\omega\, (4.867) / (10 + j\omega 4.867)$

$\boxed{= j4.867\omega / (1 + j0.4867\omega)\ \Omega.}$

30.

$\omega = 100$ rad/s

$\bar{V}_s = 100\angle 0°$ V rms

(a) $$Z_{ina-b} = 20 + j600 + \frac{j400(10 - j200)}{10 + j200} = 20 + j600 + \frac{80,000 + j4,000}{10 + j200}$$

$$= \boxed{210.7\angle 73.48^{\circ}\text{ V and } \mathbf{V}_{oc} = 0.}$$

(b) $$\bar{V}_{OC,cd} = \frac{100(j400)}{20 + j1000} = \boxed{39.99\angle 1.146°\text{ V rms}}$$

$$\bar{Z}_{incd},\ \bar{V}_S = 0 = -j200 + \frac{j400(20 + j600)}{20 + j1000} = -j200 + \frac{-240,000 + j8,000}{20 + j1,000} = \boxed{40.19\angle 85.44°\,\Omega}$$

31. $L_1 = 1$ H, $L_2 = 4$ H, $K = 1$, $\omega = 1000$ rad/s

(a) $\overline{Z}_L = 1000\,\Omega \quad \therefore \overline{Z}_{in} = j1000 + \dfrac{10^6 \times 1 \times 4}{j4000 + 100} = \boxed{24.98 + j0.6246\,\Omega}$

(b) $\overline{Z}_L = j1000 \times 0.1 \quad \therefore \overline{Z}_{in} = j1000 + \dfrac{4 \times 10^6}{j4000 + j100} = \boxed{j24.39\,\Omega}$

(c) $\overline{Z}_L = -j100 \quad \therefore \overline{Z}_{in} = j1000 + \dfrac{4 \times 10^6}{j4000 - j100} = \boxed{-j25.46\,\Omega}$

32.

$L_1 = 6 \text{ H}, L_2 = 12 \text{ H}, M = 5 \text{ H}$

$\#1, \ L_{inAB,CDOC} = \boxed{6 \text{ H}}$

$\#2, \ L_{inCD,ABOC} = \boxed{12 \text{ H}}$

$\#3, \ L_{inAB,CDSC} = 1 + 7 \| 5 = \boxed{3.917 \text{ H}}$

$\#4, \ L_{inCD,ABSC} = 7 + 5 \| 1 = \boxed{7.833 \text{ H}}$

$\#5, \ L_{inAC,BDSC} = 7 + 1 = \boxed{8 \text{ H}}$

$\#6, \ L_{inAB,ACSC,BDSC} = 7 \| 1 + 5 = \boxed{5.875 \text{ H}}$

$\#7, \ L_{inAD,BCSC} = 11 + 17 = \boxed{28 \text{ H}}$

$\#8, \ L_{inAB,ADSC} = -5 + 11/17 = \boxed{1.6786 \text{ H}}$

33.

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2}{R_{22} + jX_{22}}$$

$$\frac{1}{\omega C} = 31.83 \Rightarrow \omega = \frac{1}{31.83 \times C} = 314\, rad/s$$

ie. a 50Hz system

$$\mathbf{Z}_{in} = 20 + j\omega 100 \times 10^{-3} + \frac{\omega^2 k^2 L_1 L_2}{2 - j31.83}$$

$$\mathbf{Z}_{in} = 20 + j\omega 100 \times 10^{-3} + \frac{\omega^2 k^2 L_1 L_2 2}{2^2 + 31.83^2} - \frac{j\omega^2 k^2 L_1 L_2 31.83}{2^2 + 31.83^2}$$

$$= 20 + j31.4 + \left[\frac{493}{1020} - j\frac{7840}{1020}\right] k^2$$

$$= 20 + j31.4 + [0.483 - j7.69] k^2$$

(a) $\mathbf{Z}_{in}(k=0) = \boxed{20 + j31.4 \quad \Omega}$

(b) $\mathbf{Z}_{in}(k=0.5) = \boxed{20.2 + j27.6 \;\; \Omega}$

(c) $\mathbf{Z}_{in}(k=0.9) = \boxed{20.4 + j24.5 \;\; \Omega}$

(d) $\mathbf{Z}_{in}(k=1.0) = \boxed{20.5 + j23.7 \;\; \Omega}$

34. $\uparrow \mathrm{L}_1 \rightarrow 125\ \mathrm{H},\ \mathrm{L}_2 \rightarrow 20\ \mathrm{H},\ \mathrm{K}=1,\ \therefore \mathrm{M}=\sqrt{2500}=50\ \mathrm{H},\ j\omega\mathrm{M}=j5000\,\Omega$

(a) $\bar{\mathrm{Z}}_{ina-b} = 20 + j7500 + \dfrac{j5000(10 - j3000)}{10 + j2000}$

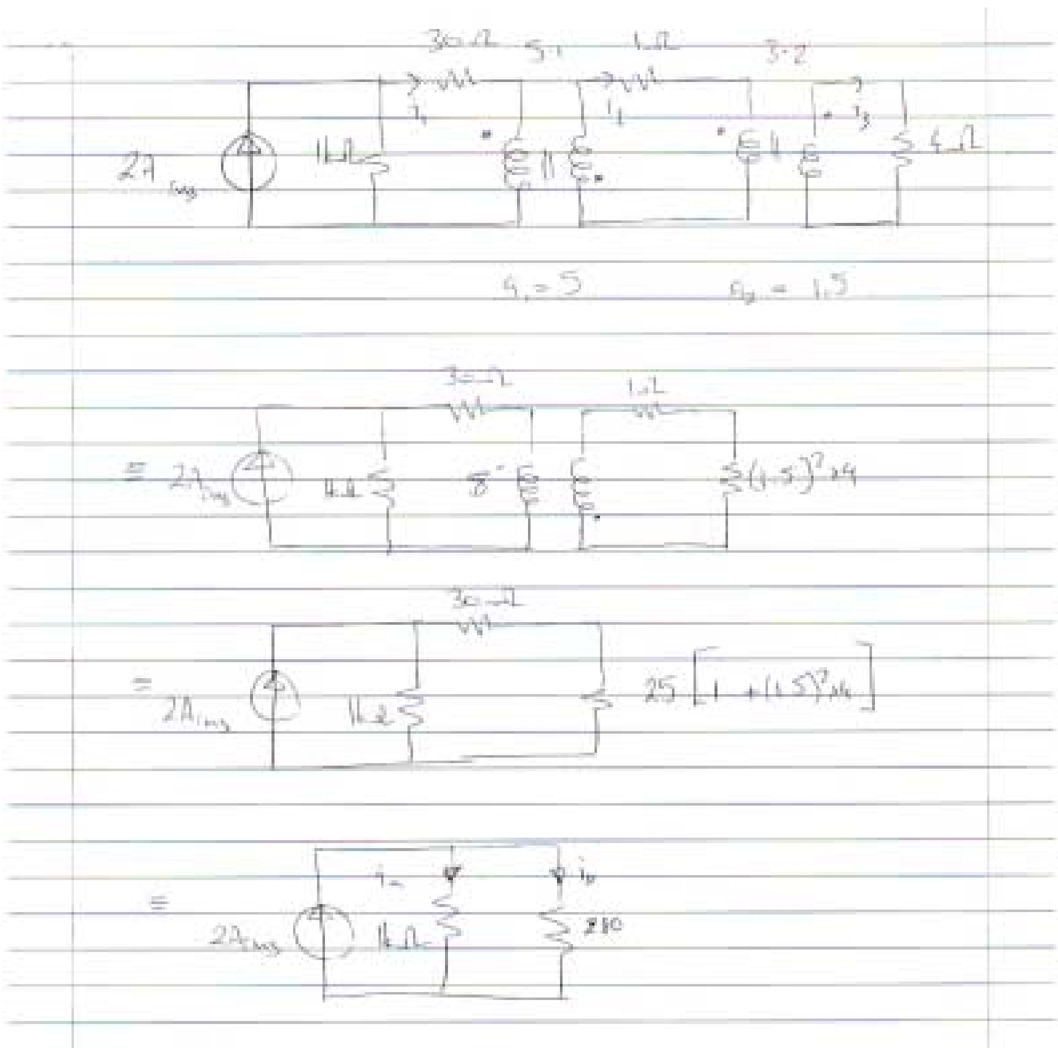
$= 20 + j7500 + \dfrac{15\times 10^6 + j50,000}{10 + j2000} = 82.499\angle 0.2170°\,\Omega$

$= 82.498 + j0.3125^{-}\ \Omega\ \mathrm{V}_{OC} = \boxed{0}$

(b) $\bar{\mathrm{V}}_{OC,cd} = \dfrac{100(j5000)}{20 + j12,500} = \boxed{39.99995\angle 0.09167°\ \mathrm{V\ rms}}$

$\bar{\mathrm{Z}}_{incd},\ \mathrm{V}_S = 0 = -j3000 + \dfrac{j5000(20 + j7500)}{20 + j12,500} = \boxed{3.19999 + j0.00512\,\Omega}$

35.



$\therefore \boldsymbol{I}_1 = 1.56\text{A}$

$\Rightarrow \boldsymbol{I}_2 = 5 \times 1.56 = 7.8\text{A}$

$\Rightarrow \boldsymbol{I}_3 = 1.5 \times 7.8A = 11.7\text{A}$

$$\Rightarrow P(1k) = I_a^2 R$$

$$= 0.438^2 \times 1 \times 10^3$$

$$= \boxed{192\text{W}}$$

$$\Rightarrow P(30\Omega) = I_1^2 R = (1.56)^2 \times 30$$

$$= \boxed{73\text{W}}$$

$$\Rightarrow P(1\Omega) = I_2^2 R \quad = 7.8^2 \times 1$$

$$= \boxed{60.8\text{W}}$$

$$\Rightarrow P(4\Omega) = I_3^2 R \quad = 11.7^2 \times 4$$

$$= \boxed{548\text{W}}$$

36.

(a) R_L sees $10 \times 4^2 = 160\,\Omega \quad \therefore$ use $R_L = 160\,\Omega$

$$P_{L\max} = \left(\frac{100}{20}\right)^2 \times 10 = \boxed{250 \text{ W}}$$

(b) $R_L = 100\,\Omega$

$$I_2 = I_1/4,\ V_2 = 4\ V_1 \ \therefore I_X = \frac{V_2 - V_1}{40} = \frac{3V_1}{40}$$

$$\therefore 100 = 10\left(I_1 \frac{3V_1}{40}\right) + V_1,\ \frac{I_1}{4} = \frac{3V_1}{40} + \frac{4V_1}{100}$$

$$\therefore I_1 = 0.46V_1 \ \therefore 100 = 10(0.46V_1 - 0.075V_1) + V_1 = 4.85\ V_1 \ \therefore V_1 = \frac{100}{4.85}$$

$$\therefore V_2 = 4V_1 = \frac{400}{4.85} = 82.47\ \text{V} \quad \therefore P_L = \frac{82.47^2}{100} = \boxed{68.02\ \text{W}}$$

37.

$$\bar{I}_2 = \frac{\bar{V}_2}{8} \quad \therefore \bar{I}_1 = \frac{\bar{V}_2}{40}, \quad \bar{V}_1 = 5V_2$$

$$\therefore 100 = 300(C + 0.025)\,\bar{V}_2 + 5\bar{V}_2$$

$$\therefore \bar{V}_2 = \frac{100}{12.5 + 300C}$$

(a) $C = 0 \;\therefore \bar{V}_2 = 8\,\text{V} \quad \therefore P_L = \frac{8^2}{8} = \boxed{8\text{ W}}$

(b) $C = 0.04 \;\therefore \bar{V}_2 = \frac{100}{24.5} \quad \therefore P_L = \left(\frac{100}{24.5}\right)^2 \frac{1}{8} = \boxed{2.082\text{ W (neg. } fdbk\text{)}}$

(c) $C = -0.04 \;\therefore \bar{V}_2 = \frac{100}{0.5} = 200\text{ V} \quad \therefore P_L = \frac{200^2}{8} = \boxed{5000\text{ W (pos. } fdbk\text{)}}$

38.

Apply $\overline{\mathrm{V}}_{ab} = 1\ \mathrm{V} \quad \therefore \overline{\mathrm{I}}_x = 0.05\ \mathrm{A},\ \overline{\mathrm{V}}_2 = 4\ \mathrm{V}$

$$\therefore 4 = 60\ \overline{\mathrm{I}}_2 + 20 \times 0.05 \ \therefore \overline{\mathrm{I}}_2 = \frac{4-1}{60} = 0.05\ \mathrm{A}$$

$$\therefore \overline{\mathrm{I}}_1 = 0.2\ \mathrm{A} \quad \therefore \overline{\mathrm{I}}_{in} = 0.25\ \mathrm{A} \quad \therefore \boxed{\mathrm{R}_{th} = 4\ \Omega,\quad \overline{\mathrm{V}}_{th} = 0}$$

39.

$$P_{gen} = 1000 \text{ W}, P_{100} = 500 \text{ W}$$

$$\therefore I_L = \sqrt{\frac{500}{100}} = \sqrt{5} \text{ A}, \ V_L = 100\sqrt{5} \text{ V}$$

$$I_S = \frac{1000}{100} = 10 \text{ A} \quad \therefore V_1 = 100 - 40 = 60 \text{ V}$$

$$\text{Now, } P_{25} = 1000 - 500 - 10^2 \times 4 = 100\text{W} \quad \therefore I_X = \sqrt{\frac{100}{25}} = 2 \text{ A; also}$$

$$Ix = b\sqrt{5} = 2, b = \frac{2}{\sqrt{5}} = \boxed{0.8944}$$

$$\text{Around center mesh: } 60a = 2 \times 25 + 100\sqrt{5}\,\frac{1}{0.8944} \therefore a = \frac{300}{60} = \boxed{5}$$

40.

(a) $3\times\left(\frac{4}{3}\right)^2 = \frac{16}{3}\ \Omega,\ \frac{16}{3}+2=\frac{22}{3}\ \Omega,\ \frac{22}{3}(3)^2 = 66\,\Omega$

$66+25=91\Omega\ \frac{100}{91} = \boxed{1.0989\angle 0°\ \text{A} = \bar{\text{I}}_1}$

(b) $\bar{\text{I}}_2 = 3\bar{\text{I}}_1 = \boxed{3.297\angle 0°\ \text{A}}$

(c) $\bar{\text{I}}_3 = -\frac{4}{3}\times 3.297 = \boxed{4.396\angle 180°\ \text{A}}$

(d) $\text{P}_{25} = 25\times 1.0989^2 = \boxed{30.19\ \text{W}}$

(e) $\text{P}_2 = 3.297^2\times 2 = \boxed{21.74\ \text{W}}$

(f) $\text{P}_3 = 4.396^2\times 3 = \boxed{57.96\ \text{W}}$

41.

$$\bar{V}_1 = 2.5\ \bar{V}_2,\ \bar{I}_1 = 0.4\bar{I}_2,\ \bar{I}_{50} = \bar{I}_2 + 0.1\ \bar{V}_2$$

$$\therefore 60 = 40(0.4\ \bar{I}_2) - 2.5\bar{V}_2\quad \therefore \bar{I}_2 = \frac{60 + 2.5\ \bar{V}_2}{16}$$

$$\text{Also, } 60 = 50(\bar{I}_2 + 0.1\ \bar{V}_2) + \bar{V}_2 = 50\bar{I}_2 + 6\bar{V}_2$$

$$\therefore 60 = 50\left(\frac{60 + 2.5\ V_2}{16}\right) + 6\ \bar{V}_2 = 187.5 + (7.8125 + 6)\ \bar{V}_2$$

$$\therefore \bar{V}_2 = \frac{60 - 187.5}{13.8125} = \boxed{-9.231\text{ V}}$$

42.

$$\frac{400}{5^2} = 16\ \Omega,\ 16 \| 48 = 12\Omega,\ 12 + 4 = 16\,\Omega$$

$$\frac{16}{2^2} = 4\Omega\quad \therefore I_s = \frac{10}{4+1} = 2\text{ A}\quad \therefore P_1 = 4\text{ W}$$

$$\frac{2}{2} = 1\text{ A}\quad \therefore P_4 = 4\text{ W},\ 10 - 2\times 1 = 8\text{ V}$$

$$8\times 2 = 16\text{ V},\ 16 - 4\times 1 = 12\text{ V},\ 12^2/48 = 3\text{ W} = P_{48},\ 12\times 5 = 60\text{ V}$$

$$P_{400} = \frac{60^2}{400} = \boxed{9\text{ W}}$$

43.

$$I_1 = 2I_2,\ 2I_2 = I_s + I_x \quad \therefore I_x + I_s - 2I_2 = 0$$

$$100 = 3I_s + \frac{1}{2}(4I_2 + 20I_2 - 20I_x)$$

$$\therefore 10I_x - 3I_s - 12I_2 = -100$$

$$100 = 3\ I_s - 5I_x + 20I_2 - 20I_x$$

$$\therefore 25I_x - 3I_s - 20I_2 = -100$$

$$\therefore I_X = \frac{\begin{vmatrix} 0 & 1 & -2 \\ -100 & -3 & -12 \\ -100 & -3 & -20 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -2 \\ 10 & -3 & -12 \\ 25 & -3 & -20 \end{vmatrix}} = \frac{0 + 100(-26) - 100(-18)}{1(60-36) - 10(-20-6) + 25(-12-6)} = \frac{-800}{-166} = \boxed{4.819\ \text{A}}$$

44.

(a) $50 \| 10 = \frac{25}{3}\ \Omega \quad \therefore\ V_{AB} = 1\times 4\times \frac{25}{3} = \frac{100}{3}\ \text{V}$

$$\therefore P_{10AB} = \left(\frac{100}{3}\right)^2 \frac{1}{10} = \frac{1000}{9} = \boxed{111.11\ \text{W}}$$

$$V_{CD} = 1\times 3\times \frac{25}{3} = 25\ \text{V},\ P_{10CD} = \frac{25^2}{10} = \boxed{62.5\ \text{W}}$$

(b) Specify 3 A and 4 A in secondaries

$$I_{AB} = I_f + 4$$

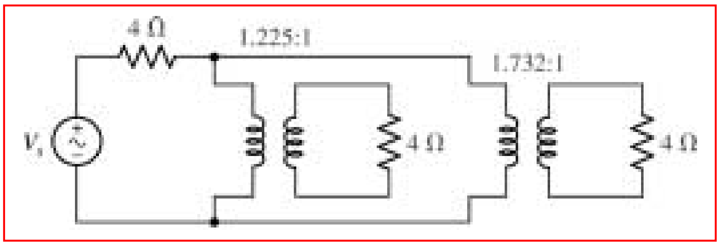
$$I_{CD} = -I_b - 3 \quad \therefore \frac{25}{3}(I_f + 4) = \frac{25}{3}(-I_f - 3)$$

$$\therefore 2I_f = -7,\ I_f = -3.5\ \text{A}$$

$$\therefore V_{AB} = V_{CD} = \frac{25}{3}(-3.5 + 4) = \frac{25}{6}\text{V}$$

$$\therefore P_{10AB} = P_{10CD} = \left(\frac{25}{6}\right)^2 \frac{1}{10} = \boxed{1.7361\ \text{W}}$$

45. **Corrections required to the problem text**: both speakers that comprise the load are 4-Ω devices. We desire a circuit that will connect the signal generator (whose Thévenin resistance is 4 Ω) to the individual speakers such that one speaker receives twice the power delivered to the other. One possible solution of many:



We can see from analysing the above circuit that the voltage across the right-most speaker will be $\frac{1.732}{1.225}$ *or* $\sqrt{2}$ times that across the left speaker. Since power is proportional to voltage *squared*, twice as much power is delivered to the right speaker.

46. (a) We assume $\mathbf{V}_{\text{secondary}} = 230\angle 0^\circ$ V as a phasor reference. Then,

$$\mathbf{I}_{\text{unity PF load}} = \frac{8000}{230}\angle 0^\circ = 34.8\angle 0^\circ \text{ A} \qquad \text{and}$$

$$\mathbf{I}_{0.8\text{ PF load}} = \frac{15000}{230}\angle\left(-\cos^{-1} 0.8\right) = 65.2\angle -36.9^\circ \text{ A}$$

$$\text{Thus, } \mathbf{I}_{\text{primary}} = \frac{230}{2300}\left(34.8\angle 0^\circ + 65.2\angle -36.9^\circ\right)$$

$$= 0.1\,(86.9 - j39.1) = \boxed{9.5\angle -24.3^\circ \text{ A}}$$

(b) The magnitude of the secondary current is limited to $25\times10^3/230 = 109$ A. If we include a new load operating at 0.95 PF lagging, whose current is

$$\mathbf{I}_{0.95\text{ PF load}} = |\,\mathbf{I}_{0.95\text{ PF load}}\,|\angle(-\cos^{-1} 0.95) = |\,\mathbf{I}_{0.95\text{ PF load}}\,|\angle -18.2^\circ \text{ A},$$

then the new total secondary current is

$$86.9 - j39.1 + |\,\mathbf{I}_{0.95\text{ PF load}}\,|\cos 18.2^\circ - j\,|\,\mathbf{I}_{0.95\text{ PF load}}\,|\sin 18.2^\circ \text{ A}.$$

Thus, we may equate this to the maximum rated current of the secondary:

$$109 = \sqrt{\left(86.9 + |\,\mathbf{I}_{0.95\text{ PF load}}\,|\cos 18.2^\circ\right)^2 + \left(39.1 + |\,\mathbf{I}_{0.95\text{ PF load}}\,|\sin 18.2^\circ\right)^2}$$

Solving, we find

$$|\,\mathbf{I}_{0.95\text{ PF load}}\,|^2 = \frac{-189 \pm \sqrt{189^2 + (4)(2800)}}{2}$$

So, $|\mathbf{I}_{0.95\text{ PF load}}\,| =$ 13.8 A (or –203 A, which is nonsense).

This transformer, then, can deliver to the additional load a power of

$$13.8\times0.95\times230 = \boxed{3 \text{ kW.}}$$

47. After careful examination of the circuit diagram, we (fortunately or unfortunately) determine that the meter determines individual IQ based on age alone. A simplified version of the circuit then, is simply a 120 V ac source, a 28.8-kΩ resistor and a $(24^2)R_A$ resistor all connected in series. The IQ result is equal to the power (W) dissipated in resistor R_A divided by 1000.

$$P = \left(\frac{120}{28.8\times 10^3 + 576R_A} \right)^2 \times 576R_A$$

Thus, $$\boxed{IQ = \frac{1}{1000}\left(\frac{120}{28.8\times 10^3 + 576\times \text{Age}} \right)^2 \times 576\times \text{Age}}$$

(a) Implementation of the above equation with a given age will yield the "measured" IQ.

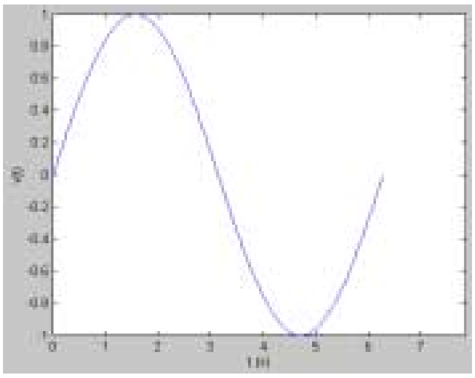
(b) The maximum IQ is achieved when maximum power is delivered to resistor R_A, which will occur when $576R_A = 28.8\times 10^3$, or the person's age is $\boxed{\text{50 years.}}$

(c) Well, now, this arguably depends on your answer to part (a), and your own sense of ethics. Hopefully you'll do the right thing, and simply write to the Better Business Bureau. And watch less television.

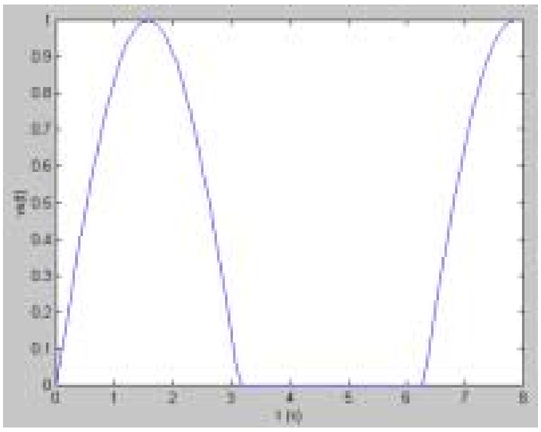
48. We require a transformer that converts 240 V ac to 120 V ac, so that a turns ratio of 2:1 is needed. We attach a male european plug to the primary coil, and a female US plug to the secondary coil. Unfortunately, we are not given the current requirements of the CD writer, so that we will have to over-rate the transformer to ensure that it doesn't overheat. Checking specifications on the web for an example CD writer, we find that the power supply provides a dual DC output: 1.2 A at 5 V, and 0.8 A at 12 V. This corresponds to a total DC power delivery of 15.6 W. Assuming a moderately efficient ac to DC converter is being used (*e.g.* 80% efficient), the unit will draw approximately 15.6/0.8 or 20 W from the wall socket. Thus, the secondary coil should be rated for at least that (let's go for 40 W, corresponding to a peak current draw of about 333 mA). Thus, we include a 300-mA fuse in series with the secondary coil and the US plug for safety.

49. You need to purchase (and wire in) a three-phase transformer rated at $\left(\sqrt{3}\right)(208)(10) = 3.6\,\text{kVA}$. The turns ratio for each phase needs to be 400:208 or 1.923.

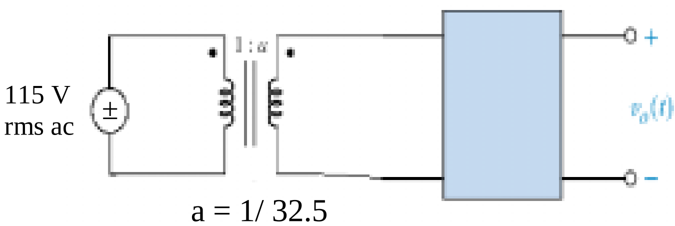
50. (a) The input to the left of the unit will have the shape:



and the output voltage will be:



We need to reduce the magnitude from 115-V (rms) to a peak voltage of 5 V. The corresponding peak voltage at the input will be $115\sqrt{2}$ = 162.6 V, so we require a transformer with a turns ratio of 162.6:5 or about 32.5:1, connected as shown:



(b) If we wish to reduce the “ripple” in the output voltage, we can connect a capacitor in parallel with the output terminals. The necessary size will depend on the maximum allowable ripple voltage and the minimum anticipated load resistance. When the input voltage swings negative and the output voltage tries to reduce to follow, current will flow out of the capacitor to reduce the amount of voltage drop that would otherwise occur.

CHAPTER FOURTEEN SOLUTIONS

1. (a) $\boxed{\mathbf{s} = 0;}$

(b) $\boxed{\mathbf{s} = \pm j9 \text{ s}^{-1};}$

(c) $\boxed{\mathbf{s} = -8 \text{ s}^{-1};}$

(d) $\boxed{\mathbf{s} = -1000 \pm j1000 \text{ s}^{-1};}$

(e) $v(t) = 8 + 2 \cos 2t$ mV cannot be attributed a single complex frequency. In a circuit analysis problem, superposition will need to be invoked, where the original function $v(t)$ is expressed as $v(t) = v_1(t) + v_2(t)$, with $v_1(t) = 8$ mV and $v_2(t) = 2 \cos 2t$ mV. The complex frequency of $v_1(t)$ is $\boxed{\mathbf{s} = 0,}$ and the complex frequency of $v_2(t)$ is $\boxed{\mathbf{s} = \pm j2 \text{ s}^{-1}.}$

2. (a) $(6 - j)^* = \boxed{6 + j}$

(b) $(9)^* = \boxed{9}$

(c) $(-j30)^* = \boxed{+j30}$

(d) $(5\,e^{-j6})^* = \boxed{5\,e^{+j6}}$

(e) $(24 \angle -45^{\circ})^* = \boxed{24 \angle 45^{\circ}}$

(f) $$\left(\frac{4 - j18}{3.33 + j}\right)^* = \boxed{\left(\frac{4 + j18}{3.33 - j}\right)} = \frac{18.44 \angle 77.47^{\circ}}{3.477 \angle -16.72^{\circ}} = 5.303 \angle 94.19^{\circ}$$

(g) $$\left(\frac{5 \angle 0.1^{\circ}}{4 - j7}\right)^* = \left(\frac{5 \angle 0.1^{\circ}}{8.062 \angle -60.26^{\circ}}\right)^* = \left(0.6202 \angle 60.36^{\circ}\right)^* = \boxed{0.6202 \angle -60.36^{\circ}}$$

(h) $(4 - 22 \angle 92.5^{\circ})^* = (4 + 0.9596 - j21.98)^* = (4.9596 - j21.98)^* = \boxed{4.9596 + j21.98}$

3. $\operatorname{Re}\, \bar{i}(t) = i(t)$. No units provided.

(a) $\bar{i}_x(t) = (4 - j7)\,e^{(-3+j15)t} = (8.062\angle -60.26°)\,e^{-3t}e^{j15t} = 8.062e^{-3t}e^{j(15t-60.26°)}$

$\therefore i_x(t) = \operatorname{Re}\bar{i}_x(t) = \boxed{8.062e^{-3t}\cos(15t - 60.26°)}$

(b) $\bar{i}_y(t) = (4 + j7)e^{-3t}(\cos 15t - j\sin 15t) = 8.062e^{-3t}e^{-j15t + j60.26°}$

$\therefore i_y(t) = \boxed{8.062e^{-3t}\cos(15t - 60.26°)}$

(c) $\bar{i}_A(t) = (5 - j8)e^{(-1.5t + j12)t} = 9.434e^{-j57.99°}e^{-1.5t}e^{j12t} = 9.434e^{-1.5t}e^{j(125-57.99°)}$

$\therefore \operatorname{Re}\ \bar{i}_A(0.4) = 9.434e^{-0.6}\cos(4.8^{rad} - 57.99°) = \boxed{-4.134}$

(d) $\bar{i}_B(t) = (5 + j8)e^{(-1.5 + j12)t} = 9.434e^{j57.99°}e^{-1.5t}e^{-j12t} = 9.434e^{-1.5t}e^{-j(12t-57.99°)}$

$\therefore \operatorname{Re}\ \bar{i}_B(0.4) = \boxed{-4.134}$

4. (a) $\omega = 279$ Mrad/s, and $\omega = 2\,\pi f$. Thus, $f = \omega/2\pi = \boxed{44.4 \text{ MHz}}$

(b) If the current $i(t) = 2.33 \cos(279\times10^6\, t)$ fA flows through a precision 1-TΩ resistor, the voltage across the resistor will be $10^{12}\, i(t) = 2.33 \cos(279\times10^6\, t)$ mV. We may write this as $0.5(2.33) \cos(279\times10^6\, t) + j\,(0.5)2.33 \sin(279\times10^6\, t) + 0.5(2.33) \cos(279\times10^6\, t) - j\,(0.5)2.33 \sin(279\times10^6\, t)$ mV

$= \boxed{1.165\, e^{j279\times106\, t} + 1.165\, e^{-j279\times106\, t} \text{ mV}}$

5. (a) $\mathbf{v}_s(0.1) = (20 - j30)\, e^{(-2 + j50)(0.1)} = (36.06 \angle -56.31^o)\, e^{(-0.2 + j5)}$

$= 36.06e^{-0.2} \angle [-56.31^o + j5(180)/\pi] = \boxed{29.52 \angle 230.2^o \text{ V}}$ (or $29.52 \angle -129.8^o$ V).

(b) $\text{Re}\{\, \mathbf{v}_s \,\} = \boxed{36.06\, e^{-2t} \cos(50t - 56.31^o) \text{ V.}}$

(c) $\text{Re}\{\, \mathbf{v}_s(0.1) \,\} = 29.52 \cos(230.2^o) = \boxed{-18.89 \text{ V}}$.

(d) The complex frequency of this waveform is $\boxed{\mathbf{s} = -2 + j50 \text{ s}^{-1}}$

(e) $\mathbf{s}^* = (-2 + j50)^* = \boxed{-2 - j50 \text{ s}^{-1}}$

6. (a) $\mathbf{s} = 0 + j120\pi = \boxed{+\,j120\pi}$

(b) We first construct an s-domain voltage $\mathbf{V}(\mathbf{s}) = 179\ \angle\ 0^{\circ}$ with **s** given above. The equation for the circuit is

$$v(t) \ = \ 100\ i(t) + \mathrm{L}\frac{di}{dt} = \ 100\ i(t) + 500\times10^{-6}\frac{di}{dt}$$

and we assume a response of the form $\mathbf{I}e^{\mathbf{s}t}$.

Substituting, we write $(179\ \angle\ 0^{\circ})\ e^{\mathbf{s}t} = \ 100\ \mathbf{I}e^{\mathbf{s}t}\ + \mathbf{s}\mathrm{L}\ \mathbf{I}e^{\mathbf{s}t}$

Supressing the exponential factor, we may write

$$\mathbf{I} \ = \frac{179\angle0^{\circ}}{100+\mathbf{s}500\times10^{-6}} \ = \ \frac{179\angle0^{\circ}}{100+\ j120\pi\left(500\times10^{-6}\right)} = \frac{179\angle0^{\circ}}{100\angle0.108^{\circ}} = 1.79\ \angle\ \text{-}0.108^{\circ}\ \mathrm{A}$$

Converting back to the time domain, we find that

$$\boxed{i(t) \ = \ 1.79\cos\left(120\pi t - 0.108^{\circ}\right)\ \mathrm{A.}}$$

7.

(a) $v_s = 10e^{-2t}\cos(10t + 30°)\,\text{V} \;\; \therefore s = -2 + j10,\; \bar{\text{V}}_s = 10\angle 30°\ \text{V}$

$$\bar{\text{Z}}_c = \frac{10}{-2+j10} = \frac{5}{-1+j5}\,\frac{-1-j5}{26} = \frac{-5-j25}{26},\; \bar{\text{Z}}_c \parallel 5 = \frac{(-25-j125)/26}{(-5-j25+130)/26}$$

$$\therefore \bar{\text{Z}}_c \parallel 5 = \frac{-25-j125}{125-j25} = \frac{-1-j5}{5-j1} = -j1 \;\; \therefore \bar{\text{Z}}_{in} = 5 + 0.5(-2+j10) - j1 = 4 + j4\,\Omega$$

$$\therefore \bar{\text{I}}_x = \frac{10\angle 30°}{4+j4} \times \frac{(-5-j25)/26}{5+(-5-j25)/26} = \frac{10\angle 30°}{4+j4}\,\frac{-5-j25}{130-5-j25} = \frac{5\angle 30°}{2+j2}\,\frac{-5-j25}{125-j25} = \frac{1\angle 30°}{2+j2}\,\frac{-1-j5}{5-j1}$$

$$\therefore \bar{\text{I}}_x = \frac{1\angle 30°}{2\sqrt{2}\angle 45°}(-j1) = \boxed{0.3536\angle -105°\,\text{A}}$$

(b) $i_x(t) = \boxed{0.3536e^{-2t}\cos(10t - 105°)\ \text{A}}$

8. (a) $\mathbf{s} = 0 + j100\pi = \boxed{+j100\pi}$

(b) We first construct an s-domain voltage $\mathbf{V}(\mathbf{s}) = 339 \angle 0^{o}$ with $\mathbf{s}$ given above. The equation for the circuit is

$$v(t) \;=\; 2000\, i(t) + v_C(t) \;=\; 2000\,\mathrm{C}\frac{dv_C}{dt} + v_C(t) = \; 0.2\frac{dv_C}{dt} + v_C(t)$$

and we assume a response of the form $\mathbf{V}_C e^{st}$.

Substituting, we write $(339 \angle 0^{o})\, e^{st} \;=\; 0.2\mathbf{s}\,\mathbf{V}_C e^{st} + \mathbf{V}_C e^{st}$

Supressing the exponential factor, we may write

$$\mathbf{V}_C \;=\; \frac{339\angle 0^{o}}{1+0.2\mathbf{s}} \;=\; \frac{339\angle 0^{o}}{1+j100\pi(0.2)} \;=\; \frac{339\angle 0^{o}}{62.84\angle 89.09^{o}} = 5.395 \angle -89.09^{o} \text{ A}$$

Converting back to the time domain, we find that

$$v_C(t) \;=\; 5.395 \cos (100\pi t - 89.09^{o}) \text{ V.}$$

and so the current is $i(t) \;=\; \mathrm{C}\frac{dv_C}{dt} = -0.1695 \sin(100\pi t) \text{ A} = \boxed{169.5 \sin(100\pi t) \text{ mA.}}$

9. $i_{S1} = 20e^{-3t}\cos 4t$ A, $i_{S2} = 30e^{-3t}\sin 4t$ A

(a) $\bar{I}_{S1} = 20\angle 0°,\ \bar{I}_{S2} = -j30, \bar{s} = -3 + j4$

$$\therefore \bar{Z}_c = \frac{10}{-3+j4}\,\frac{-3-j4}{-3-j4} = 0.4(-3-j4) = -1.2 - j1.6,\ \bar{Z}_L = -6 + j8$$

$$\therefore \bar{V}_x = 20\frac{5(7.2+j6.4)}{-2.2+j6.4}\times\frac{-6+j8}{-7.2+j6.4} - j30\frac{(-6+j8)(3.8-j1.6)}{-2.2+j6.4}$$

$$= \frac{-600+j800-j30(-22.8+12.8+j30.4+j9.6)}{-2.2+j6.4} = \frac{-600+j800-j30(-10+j40)}{-2.2+j6.4}$$

$$= \frac{-600+1200+j1000}{-2.2+j6.4} = \frac{600+j1000}{-2.2+j6.4} = \boxed{185.15^{-}\angle -47.58°\text{ V}}$$

(b) $v_x(t) = \boxed{185.15^{-}e^{-3t}\cos(4t - 47.58°)\text{ V}}$

CHAPTER FOURTEEN SOLUTIONS

10. (a) If $v(t) = 240\sqrt{2}\, e^{-2t} \cos 120\pi t$ V, then $\mathbf{V} = 240\sqrt{2} \angle 0^{\circ}$ V where $\mathbf{s} = -2 + j120\pi$.

Since R = 3 mΩ, the current is simply $\mathbf{I} = \dfrac{240\sqrt{2}\angle 0^{\circ}}{3\times 10^{-3}} = 113.1 \angle 0^{\circ}$ kA. Thus,

$i(t) = \boxed{113.1 e^{-2t} \cos 120\pi t \text{ kA}}$

(b) Working in the time domain, we may directly compute

$i(t) = v(t) / 3\times 10^{-3} = (240\sqrt{2}\, e^{-2t} \cos 120\pi t) / 3\times 10^{-3} = \boxed{113.1 e^{-2t} \cos 120\pi t \text{ kA}}$

(c) A 1000-mF capacitor added to this circuit corresponds to an impedance

$$\frac{1}{\mathbf{s}C} = \frac{1}{(-2 + j120\pi)(1000\times 10^{-3})} = \frac{1}{-2 + j120\pi}\ \Omega$$ in parallel with the 3-mΩ

resistor. However, since the capacitor has been added in parallel (it would have been more interesting if the connection were in series), the same voltage still appears across its terminals, and so

$\boxed{i(t) = 113.1 e^{-2t} \cos 120\pi t \text{ kA}}$ as before.

11. $$\mathcal{L}\{K\,u(t)\} = \int_{0^-}^{\infty} Ke^{-st}u(t)dt = K\int_{0^-}^{\infty} e^{-st}u(t)dt = K\int_{0}^{\infty} e^{-st}dt = \left.\frac{-K}{\mathbf{s}}e^{-st}\right|_{0}^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-K}{\mathbf{s}}e^{-st}\right) + \lim_{t\to 0}\left(\frac{K}{\mathbf{s}}e^{-st}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{K\,u(t)\} = \boxed{\frac{K}{\mathbf{s}}}$$

12. (a) $\mathcal{L}\{3\,u(t)\} = \int_{0^-}^{\infty} 3e^{-st}u(t)dt = 3\int_{0^-}^{\infty} e^{-st}u(t)dt = 3\int_{0}^{\infty} e^{-st}dt = \left.\frac{-3}{s}e^{-st}\right|_{0}^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-3}{s}e^{-st}\right) + \lim_{t\to 0}\left(\frac{3}{s}e^{-st}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. s must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{3\,u(t)\} = \boxed{\frac{3}{s}}$$

(b) $\mathcal{L}\{3\,u(t-3)\} = \int_{0^-}^{\infty} 3e^{-st}u(t-3)dt = 3\int_{3}^{\infty} e^{-st}dt = \left.\frac{-3}{s}e^{-st}\right|_{3}^{\infty}$

$$= \lim_{t\to\infty}\left(\frac{-3}{s}e^{-st}\right) + \left(\frac{3}{s}e^{-3s}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. s must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{3\,u(t-3)\} = \boxed{\frac{3}{s}e^{-3s}}$$

(c)

$$\mathcal{L}\{3\,u(t-3)-3\} = \int_{0^-}^{\infty}[3u(t-3)-3]e^{-st}dt = 3\int_{3}^{\infty} e^{-st}dt - 3\int_{0^-}^{\infty} e^{-st}dt$$

$$= \left.\frac{-3}{s}e^{-st}\right|_{3}^{\infty} - \left.\frac{-3}{s}e^{-st}\right|_{0^-}^{\infty}$$

Based on our answers to parts (a) and (b), we may write

$$\mathcal{L}\{3\,u(t-3)-3\} = \frac{3}{s}e^{-3s} - \frac{3}{s} = \boxed{\frac{3}{s}\left(e^{-3s} - 1\right)}$$

(d)

$$\mathcal{L}\{3\,u(3-t)\} = 3\int_{0^-}^{\infty} e^{-st}u(3-t)dt = 3\int_{0^-}^{3} e^{-st}dt = \left.\frac{-3}{s}e^{-st}\right|_{0^-}^{3}$$

$$= \frac{-3}{s}\left(e^{-3s} - 1\right) = \boxed{\frac{3}{s}\left(1 - e^{-3s}\right)}$$

13. (a) $$\mathcal{L}\{2+3u(t)\} = \int_{0^-}^{\infty} e^{-st}[2+3u(t)]dt = \int_{0}^{\infty} 5e^{-st}dt = \left.\frac{-5}{\mathbf{s}}e^{-st}\right|_0^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-5}{\mathbf{s}}e^{-st}\right) + \lim_{t\to 0}\left(\frac{5}{\mathbf{s}}e^{-st}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{2+3u(t)\} = \boxed{\frac{5}{\mathbf{s}}}$$

(b) $$\mathcal{L}\{3e^{-8t}\} = \int_{0^-}^{\infty} 3e^{-8t}e^{-st}dt = \int_{0^-}^{\infty} 3e^{-(8+\mathbf{s})t}dt = \left.\frac{-3}{\mathbf{s}+8}e^{-(8+\mathbf{s})t}\right|_{0^-}^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-3}{\mathbf{s}+8}e^{-(\mathbf{s}+8)t}\right) + \lim_{t\to 0}\left(\frac{3}{\mathbf{s}+8}e^{-(\mathbf{s}+8)t}\right) = 0 + \frac{3}{\mathbf{s}+8} = \boxed{\frac{3}{\mathbf{s}+8}}$$

(c) $$\mathcal{L}\{u(-t)\} = \int_{0^-}^{\infty} e^{-st}u(-t)dt = \int_{0^-}^{0} e^{-st}u(-t)dt = \int_{0^-}^{0^-} (0)\,e^{-st}dt = \boxed{0}$$

(d) $$\mathcal{L}\{K\} = \int_{0^-}^{\infty} Ke^{-st}dt = K\int_{0^-}^{\infty} e^{-st}dt = K\int_{0}^{\infty} e^{-st}dt = \left.\frac{-K}{\mathbf{s}}e^{-st}\right|_0^{\infty}$$

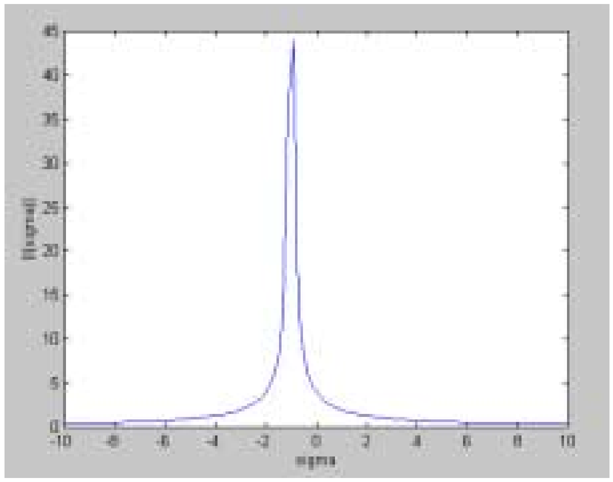
$$= \lim_{t\to\infty}\left(\frac{-K}{\mathbf{s}}e^{-st}\right) + \lim_{t\to 0}\left(\frac{K}{\mathbf{s}}e^{-st}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{K\} = \boxed{\frac{K}{\mathbf{s}}}$$

14. (a) The frequency-domain representation of the voltage across the resistor is (1)**I**(**s**) where **I**(**s**) $= \mathcal{L}\{4e^{-t}\, u(t)\} = \dfrac{4}{\mathbf{s}+1}$ A. Thus, the voltage is $\boxed{\dfrac{4}{\mathbf{s}+1} \text{ V}}$.

(b)



15. (a)

$$\mathcal{L}\{5\,u(t) - 5\,u(t-2)\} = \int_{0^-}^{\infty} [5\,u(t) - 5\,u(t-2)]e^{-st}\,dt$$

$$= 5\int_{0}^{\infty} e^{-st}\,dt - 5\int_{2}^{\infty} e^{-st}\,dt = \left.\frac{-5}{\mathbf{s}}e^{-st}\right|_{0}^{\infty} + \left.\frac{5}{\mathbf{s}}e^{-st}\right|_{2}^{\infty}$$

$$= \lim_{t\to\infty}\left(\frac{-5}{\mathbf{s}}e^{-st}\right) + \lim_{t\to 0}\left(\frac{5}{\mathbf{s}}e^{-st}\right) + \lim_{t\to\infty}\left(\frac{-5}{\mathbf{s}}e^{-st}\right) + \left(\frac{5}{\mathbf{s}}e^{-2\mathbf{s}}\right)$$

If the integral is going to converge, then $\lim_{t\to\infty}\left(e^{-st}\right) = 0$ (i.e. $\mathbf{s}$ must be finite). This leads to the first and third terms dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{5\,u(t) - 5\,u(t-2)\} = \boxed{\frac{5}{\mathbf{s}}\left(1 + e^{-2\mathbf{s}}\right)}$$

(b) The frequency domain current is simply one ohm times the frequency domain voltage, or

$$\boxed{\frac{5}{\mathbf{s}}\left(1 + e^{-2\mathbf{s}}\right)}$$

16.

(a) $f(t) = t + 1 \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} (t+1)\, e^{-(\sigma + j\omega)t} dt \therefore \boxed{\sigma > 0}$

(b) $f(t) = (t+1)\, u(t) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} (t+1)\, e^{-(\sigma + j\omega)t} dt \therefore \boxed{\sigma > 0}$

(c) $f(t) = e^{50t} u(t) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} e^{50t}\, e^{-(\sigma + j\omega)t} dt \therefore \boxed{\sigma > 50}$

(d) $f(t) = e^{50t} u(t-5) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} e^{50t} u(t-5)\, e^{-(\sigma + j\omega)t} dt \therefore \boxed{\sigma > 50}$

(e) $f(t) = e^{-50t} u(t-5) \therefore \mathrm{F}(s) = \int_{0^-}^{\infty} e^{-50t} u(t-5)\, e^{-(\sigma + j\omega)t} dt \therefore \boxed{\sigma > 0}$

17.

(a) $f(t) = 8e^{-2t}[u(t+3) - u(t-3)] \ \therefore \ \text{F}(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{j\omega t}\, dt$

$\therefore \text{F}(j\omega) = \int_{-3}^{3} 8e^{(2+j\omega)t} dt = \dfrac{8}{2+j\omega}[e^{6+j3\omega} - e^{-6-j3\omega}]$

$\text{F}_{(2)}(s) = \int_{\infty}^{\infty} f(t)e^{-st} dt = \int_{-3}^{3} 8e^{-(2+s)t} dt = \dfrac{8}{2+s}[e^{6+3s} - e^{-6-3s}]$

$\text{F}(s) = \int_{0}^{\infty} f(t)e^{-st} dt = \int_{0^-}^{\infty} 8e^{(-2+s)t} dt = \boxed{\dfrac{8}{2+s}[1 - e^{-6-3s}]}$

(b) $f(t) = 8e^{2t}[u(t+3) - u(t-3)] \ \text{F}(j\omega) = \int_{-3}^{3} 8e^{(2-j\omega)t} dt$

$= \dfrac{8}{2-j\omega}[e^{6-j3\omega} - e^{-6+j3\omega}] \ \text{F}_{(2)}(s) = \int_{-3}^{3} 8e^{(2-s)t} dt$

$= \dfrac{8}{s-2}[e^{-6+3s} - e^{6-3s}], \ \text{F}(s) = \int_{0^-}^{\infty} 8e^{(2-s)t} dt$

$= \dfrac{8}{2-s}[e^{6-3s} - 1] = \boxed{\dfrac{8}{s-2}[1 - e^{6-32}]}$

(c) $f(t) = 8e^{-2|t|}[u(t-3) - u(t-3)] \therefore \text{F}(j\omega) = \int_{-3}^{3} 8e^{-2|t|} e^{-j\omega t} dt$

$\therefore \text{F}(j\omega) = \int_{-3}^{0} 8e^{(2-j\omega)t} dt + \int_{0}^{3} 8e^{(-2-j\omega)t} dt = \dfrac{8}{2-j\omega}[1 - e^{6+j3\omega}] + \dfrac{8}{2+j\omega}[1 - e^{-6-j3\omega}]$

$\text{F}_{(2)}(s) = \int_{-3}^{3} 8e^{-2|t|-st} dt = \int_{-3}^{0} 8e^{(2-s)t} dt$

$+ \int 8e^{(-2-s)t} dt \ \therefore \text{F}_{(2)}^{(s)} = \dfrac{8}{2-s}[-e^{-6+3s}] + \dfrac{8}{2+s}[1 - e^{-6-3s}]$

$\text{F}(s) = \int_{0^-}^{3} 8e^{(-2-s)t} dt = \boxed{\dfrac{8}{s+2}[1 - e^{-6-3s}]}$

18. (a) $\mathcal{L}\left\{\mathcal{L}^{-1}\left(\frac{1}{\mathbf{s}}\right)\right\} = \boxed{\frac{1}{\mathbf{s}}}$

(b) $\mathcal{L}\left\{1 + u(t) + [u(t)]^2\right\} = \frac{1}{\mathbf{s}} + \frac{1}{\mathbf{s}} + \frac{1}{\mathbf{s}} = \boxed{\frac{3}{\mathbf{s}}}$

(c) $\mathcal{L}\left\{t\,u(t) - 3\right\} = \boxed{\frac{1}{\mathbf{s}^2} - \frac{3}{\mathbf{s}}}$

(d) $\mathcal{L}\left\{1 - \delta(t) + \delta(t-1) - \delta(t-2)\right\} = \boxed{\frac{1}{\mathbf{s}} - 1 + e^{-\mathbf{s}} - e^{-2\mathbf{s}}}$

19. (a) $f(t) = \boxed{e^{-3t}\, u(t)}$

(b) $f(t) = \boxed{\delta(t)}$

(c) $f(t) = \boxed{t\, u(t)}$

(d) $f(t) = \boxed{275\, u(t)}$

(e) $f(t) = \boxed{u(t)}$

20.

$$\boxed{\begin{aligned} \mathcal{L}\{f_1(t) + f_2(t)\} &= \int_{0^-}^{\infty}[f_1(t) + f_2(t)]e^{-st}dt = \int_{0^-}^{\infty} f_1(t)e^{-st}dt + \int_{0^-}^{\infty} f_2(t)e^{-st}dt \\ &= \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\} \end{aligned}}$$

21.

(a) $f(t) = 2u(t-2) \;\therefore\; \mathrm{F}(s) = 2\int_2^{\infty} e^{-st}\,dt + \frac{-2}{s} e^{st}\Big|_2^{\infty} = \frac{2}{s}e^{-2s};\; s = 1 + j2$

$$\therefore \mathrm{F}(1+j2) = \frac{2}{1+j2} e^{-2}\; e^{-j4} = \boxed{0.04655^{+} + j0.11174}$$

(b) $f(t) = 2\delta(t-2) \;\therefore\; \mathrm{F}(s) = 2e^{-2s},\; \mathrm{F}(1+j2) = 2e^{-2}\,e^{-j4} = \boxed{-0.17692 + j0.2048}$

(c) $f(t) = e^{-t}\,u(t-2) \;\therefore\; \mathrm{F}(s) = \int_2^{-\infty} e^{-(s+1)t}\,dt = \frac{1}{-s+1} e^{-(s+1)t}\Big|_2^{\infty} = \frac{1}{s+1} e^{-2s-2}$

$$\therefore \mathrm{F}(1+j2) = \frac{1}{2+j2} e^{-2}\, e^{-2}\, e^{-j^4} = \boxed{(0.4724 + j6.458)10^{-3}}$$

22. (a) $\int_{-\infty}^{\infty} 8\sin 5t\, \delta(t-1)\, dt \quad = \quad 8\sin 5\times 1 = \boxed{-7.671}$

(b) $\int_{-\infty}^{\infty} (t-5)^2 \delta(t-2)dt \quad = \quad (2-5)^2 = \boxed{9}$

(c) $\int_{-\infty}^{\infty} 5e^{-3000t} \delta(t-3.333\times 10^{-4})dt \quad = \quad 5e^{-3000(3.333\times 10^{-4})} = \boxed{1.840}$

(d) $\int_{-\infty}^{\infty} K\delta(t-2)dt \quad = \quad \boxed{K}$

23.

(a) $f(t) = [u(5-t)]\,[u(t-2)]\,u(t), \;\therefore\; \mathrm{F}(s) \int_{0^-}^{\infty} [u(5-t)]\,[u(t-2)]\,u(t)\,e^{-st}\,dt$

$$\therefore \mathrm{F}(s) = \int_2^5 e^{-st}\,dt = -\frac{1}{s} e^{-st}\Big|_2^5 = \boxed{\frac{1}{s}\left(e^{-2s} - e^{-5s}\right)}$$

(b) $f(t) = 4u(t-2) \;\therefore\; \mathrm{F}(s) = 4\int_2^{\infty} e^{-st}\,dt = \boxed{\frac{4}{s} e^{-2s}}$

(c) $f(t) = 4e^{-3t}u(t-2) \;\therefore\; \mathrm{F}(s) = 4\int_2^{\infty} e^{-(s+3)t}\,dt = \frac{-4}{s+3} e^{-(s+3)t}\Big|_2^{\infty}$

$$\therefore \boxed{\mathrm{F}(s) = \frac{4}{s+3} e^{-2s-6}}$$

(d) $f(t) = 4\delta(t-2) \;\therefore\; \mathrm{F}(s) = 4\int_{0^-}^{\infty} \delta(t-2)\,e^{-st}\,dt = 4\int_2^{2^+} e^{-2s}\,\delta(t-2)\,dt = \boxed{4e^{-2s}}$

(e) $f(t) = 5\delta(t)\sin(10t + 0.2\pi) \;\therefore\; \mathrm{F}(s) = 5\int_{0^-}^{0^+} \delta(t)\;[\sin 0.2\pi]\;\mathrm{X}\;1dt = 5\sin 36°$

$$\therefore \mathrm{F}(s) = \boxed{2.939}$$

24. (a) $\int_{-\infty}^{\infty} \cos 500t\, \delta(t)\, dt \quad = \quad \cos 500\times 0 = \boxed{1}$

(b) $\int_{-\infty}^{\infty} (t)^5 \delta(t-2)dt \quad = \quad (2)^5 = \boxed{32}$

(c) $\int_{-\infty}^{\infty} 2.5e^{-0.001t}\delta(t-1000)dt \quad = \quad 2.5e^{-0.001(1000)} = \boxed{0.9197}$

(d) $\int_{-\infty}^{\infty} -K^2\delta(t-c)dt \quad = \quad \boxed{-K^2}$

25.

(a) $f(t) = 2\,u(t-1)\,u(3-t)\,u(t^3)$

$$\mathbf{F(s)} = \int_1^3 e^{-\mathbf{s}t}\,dt = -\frac{2}{\mathbf{s}}e^{-\mathbf{s}t}\Big|_1^3 = \boxed{\frac{2}{\mathbf{s}}\left(e^{-\mathbf{s}} - e^{-3\mathbf{s}}\right)}$$

(b) $$f(t) = 2u(t-4) \therefore \mathrm{F}(s) = 2\int_4^\infty e^{-st}\,dt = \frac{-2}{s}(0-e^{-4s}) = \boxed{\frac{2}{s}e^{-4s}}$$

(c) $$f(t) = 3e^{-2t}\,u(t-4) \therefore \mathrm{F}(s) = 3\int_4^\infty e^{-(s+2)t}\,dt = \boxed{\frac{3}{s+2}e^{-4s-8}}$$

(d) $$f(t) = 3\delta(t-5) \therefore \mathrm{F}(s) = 3\int_{0^-}^\infty \delta(t-5)\,e^{-st}\,dt = \boxed{3e^{-5s}}$$

(e) $$f(t) = 4\delta(t-1)\,[\cos\pi t - \sin\pi t]$$

$$\therefore \mathrm{F}(s) = 4\int_{0^-}^\infty \delta(t-1)\,[\cos\pi t - \sin\pi t]\,e^{-st}\,dt \therefore \mathrm{F}(s) = \boxed{-4e^{-s}}$$

26. (a) $f(t) = \boxed{5\,u(t) - 16\,\delta(t) + e^{-4.4t}\,u(t)}$

(b) $f(t) = \boxed{\delta(t) + u(t) + t\,u(t)}$

(c) $\mathbf{F}(\mathbf{s}) = \dfrac{5}{\mathbf{s}+7} + \dfrac{88}{\mathbf{s}} + \dfrac{a}{\mathbf{s}+6} + \dfrac{b}{\mathbf{s}+1}$

where $a = \left.\dfrac{17}{\mathbf{s}+1}\right|_{\mathbf{s}=-6} = -3.4$ and $b = \left.\dfrac{17}{\mathbf{s}+6}\right|_{\mathbf{s}=-1} = 3.4$.

Thus,

$$f(t) = \boxed{5\,e^{-7t}\,u(t) + 88\,u(t) - 3.4\,e^{-6t}\,u(t) + 3.4\,e^{-t}\,u(t)}$$

Check with MATLAB:

```
EDU» T1 = '5/(s+7)';
EDU» T2 = '88/s';
EDU» T3 = '17/(s^2 + 7*s + 6)';
EDU» T = symadd(T1,T2);
EDU» P = symadd(T,T3);
EDU» p = ilaplace(P)

p =

5*exp(-7*t)+88-17/5*exp(-6*t)+17/5*exp(-t)

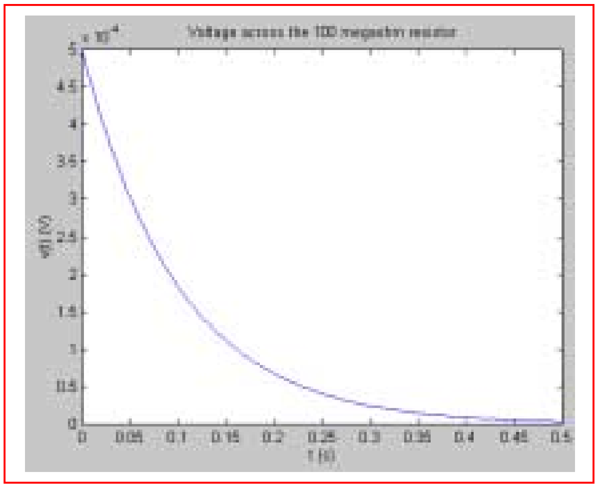
EDU» pretty(p)

          5 exp(-7 t) + 88 - 17/5 exp(-6 t) + 17/5 exp(-t)
```


27. If $\mathbf{V}(\mathbf{s}) = \frac{5}{\mathbf{s}}$, then $v(t) = 5\,u(t)$ V. The voltage at $t = 1$ ms is then simply 5 V, and the current through the 2-kΩ resistor at that instant in time is $\boxed{2.5 \text{ mA.}}$

28. $\mathbf{I(s)} = \dfrac{5}{\mathbf{s}+10}$ pA, so $i(t) = 5\,e^{-10t}\,u(t)$ pA. The voltage across the 100-MΩ resistor is therefore $500\,e^{-10t}\,u(t)$ μV.

(a) The voltage as specified has zero value for $t < 0$, and a peak value of 500 μV.



(b) $i(0.1 \text{ s}) = 1.839$ pA, so the power absorbed by the resistor at that instant = i^2R = $\boxed{338.2 \text{ aW.}}$ (A pretty small number).

(c) $500\,e^{-10t_{1\%}} = 5$
Taking the natural log of both sides, we find $t_{1\%} = \boxed{460.5 \text{ ms}}$

29.

(a) $\mathrm{F}(s) = \dfrac{s+1}{s} + \dfrac{2}{s+1} = 1 + \dfrac{1}{s} + \dfrac{2}{s+1} \leftrightarrow \boxed{\delta(t) + u(t) + 2e^{-t}u(t)}$

(b) $\mathrm{F}(s) = (e^{-s}+1)^2 = e^{-2s} + 2e^{-s} + 1 \leftrightarrow \boxed{\delta(t-2) + 2\delta(t-1) + \delta(t)}$

(c) $\mathrm{F}(s) = 2e^{-(s+1)} = \boxed{2e^{-1}e^{-2s} + 2e^{-1}\delta(t-1)}$

(d) $\mathbf{F}(\mathbf{s}) = 2e^{-3\mathbf{s}} \cosh 2\mathbf{s} = e^{-3\mathbf{s}}(e^{2\mathbf{s}} + e^{-2\mathbf{s}}) = e^{-\mathbf{s}} + e^{-5\mathbf{s}} \leftrightarrow \boxed{\delta(t-1) + \delta(t-5)}$

30. $\mathbf{N(s)} = 5\mathbf{s}.$

(a) $\mathbf{D(s)} = \mathbf{s}^2 - 9$ so $\dfrac{\mathbf{N(s)}}{\mathbf{D(s)}} = \dfrac{5\mathbf{s}}{\mathbf{s}^2 - 9} = \dfrac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}-3)} = \dfrac{a}{(\mathbf{s}+3)} + \dfrac{b}{(\mathbf{s}-3)}$

where $a = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}-3)}\right|_{\mathbf{s}=-3} = \dfrac{-15}{-6} = 2.5$ and $b = \left.\dfrac{5\mathbf{s}}{(\mathbf{s}+3)}\right|_{\mathbf{s}=3} = \dfrac{15}{6} = 2.5$. Thus,

$f(t) = \boxed{[2.5\, e^{-3t} + 2.5\, e^{3t}]\, u(t)}$

(b) $\mathbf{D(s)} = (\mathbf{s}+3)(\mathbf{s}^2 + 19\mathbf{s} + 90) = (\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \frac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)} = \frac{a}{(\mathbf{s}+3)} + \frac{b}{(\mathbf{s}+10)} + \frac{c}{(\mathbf{s}+9)}$$

$$a = \left.\frac{5\mathbf{s}}{(\mathbf{s}+10)(\mathbf{s}+9)}\right|_{\mathbf{s}=-3} = \frac{-15}{(7)(6)} = -0.3571,\ b = \left.\frac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+9)}\right|_{\mathbf{s}=-10} = \frac{-50}{(-7)(-1)} = -7.143$$

$$c = \left.\frac{5\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+10)}\right|_{\mathbf{s}=-9} = \frac{-45}{(-6)(1)} = 7.5. \quad \therefore f(t) = \boxed{[-0.3571\, e^{-3t} - 7.143\, e^{-10t} + 7.5\, e^{-9t}]\, u(t)}$$

(c) $\mathbf{D(s)} = (4\mathbf{s} + 12)(8\mathbf{s}^2 + 6\mathbf{s} + 1) = 32(\mathbf{s}+3)(\mathbf{s}+0.5)(\mathbf{s}+0.25)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \left(\frac{5}{32}\right)\frac{\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+0.5)(\mathbf{s}+0.25)} = \frac{a}{(\mathbf{s}+3)} + \frac{b}{(\mathbf{s}+0.5)} + \frac{c}{(\mathbf{s}+0.25)}$$

$$a = \left(\frac{5}{32}\right)\left.\frac{\mathbf{s}}{(\mathbf{s}+0.5)(\mathbf{s}+0.25)}\right|_{\mathbf{s}=-3} = -0.06818,\ b = \left(\frac{5}{32}\right)\left.\frac{\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+0.25)}\right|_{\mathbf{s}=-0.5} = 0.125$$

$$c = \left(\frac{5}{32}\right)\left.\frac{\mathbf{s}}{(\mathbf{s}+3)(\mathbf{s}+0.5)}\right|_{\mathbf{s}=--0.25} = -0.05682$$

$$\therefore f(t) = \boxed{[-0.06818\, e^{-3t} + 0.125\, e^{-0.5t} - 0.05682 e^{-0.25t}]\, u(t)}$$

(d) Part (a):

```
EDU» N = [5 0];
EDU» D = [1 0 -9];
EDU» [r p y] = residue(N,D)

r =
    2.5000
    2.5000

p =
     3
    -3

y =
     []
```

Part (b):

```
EDU» N = [5 0];
EDU» D = [1 22 147 270];
EDU» [r p y] = residue(N,D)
r =
   -7.1429
    7.5000
   -0.3571
p =
  -10.0000
   -9.0000
   -3.0000
y =
     []
```

Part (c):

```
EDU» N = [5 0];
EDU» D = [32 120 76 12];
EDU» [r p y] = residue(N,D)
r =
   -0.0682
    0.1250
   -0.0568
p =
   -3.0000
   -0.5000
   -0.2500
y =
     []
```

31.

(a) $$\mathrm{F}(s) = \frac{5}{s+1} \leftrightarrow \boxed{5e^{-t}u(t)}$$

(b) $$\mathrm{F}(s) = \frac{5}{s+1} - \frac{2}{s+4} \leftrightarrow \boxed{(5e^{-t} - 2e^{-4t})\,u(t)}$$

(c) $$\mathrm{F}(s) = \frac{18}{(s+1)(s+4)} = \frac{6}{s+1} - \frac{6}{s+4} \leftrightarrow \boxed{6\,(e^{-t} - e^{-4t})\,u(t)}$$

(d) $$\mathrm{F}(s) = \frac{18s}{(s+1)(s+4)} = \frac{-6}{s+1} + \frac{24}{s+4} \leftrightarrow \boxed{6\,(4e^{-4t} - e^{-t})\,u(t)}$$

(e) $$\mathrm{F}(s) = \frac{18s^2}{(s+1)(s+4)} = 18 + \frac{6}{s+1} - \frac{96}{s+4} \leftrightarrow \boxed{18\delta(t) + 6\,(e^{-t} - 16e^{-4t})\,u(t)}$$

32. $\mathbf{N(s)} = 2\mathbf{s}^2$.

(a) $\mathbf{D(s)} = \mathbf{s}^2 - 1$ so $\dfrac{\mathbf{N(s)}}{\mathbf{D(s)}} = \dfrac{2\mathbf{s}^2}{\mathbf{s}^2 - 1} = \dfrac{2\mathbf{s}^2}{(\mathbf{s}+1)(\mathbf{s}-1)} = \dfrac{a}{(\mathbf{s}+1)} + \dfrac{b}{(\mathbf{s}-1)} + 2$

where $a = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}-1)}\right|_{\mathbf{s}\,=\,-1} = \dfrac{2}{-2} = -1$ and $b = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}+1)}\right|_{\mathbf{s}\,=\,1} = \dfrac{2}{2} = 1$. Thus,

$$\boxed{f(t) = [2\delta(t) + e^{-t} + e^{t}]\, u(t)}$$

(b) $\mathbf{D(s)} = (\mathbf{s} + 3)(\mathbf{s}^2 + 19\mathbf{s} + 90) = (\mathbf{s} + 3)(\mathbf{s} + 10)(\mathbf{s} + 9)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \frac{2\mathbf{s}^2}{(\mathbf{s}+3)(\mathbf{s}+10)(\mathbf{s}+9)} = \frac{a}{(\mathbf{s}+3)} + \frac{b}{(\mathbf{s}+10)} + \frac{c}{(\mathbf{s}+9)}$$

$a = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}+10)(\mathbf{s}+9)}\right|_{\mathbf{s}\,=\,-3} = \dfrac{18}{(7)(6)} = 0.4286$, $b = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}+3)(\mathbf{s}+9)}\right|_{\mathbf{s}\,=\,-10} = \dfrac{200}{(-7)(-1)} = 28.57$

$c = \left.\dfrac{2\mathbf{s}^2}{(\mathbf{s}+3)(\mathbf{s}+10)}\right|_{\mathbf{s}\,=\,-9} = \dfrac{162}{(-6)(1)} = -27.$ $\quad\therefore \boxed{f(t) = [0.4286\, e^{-3t} + 28.57\, e^{-10t} - 27\, e^{-9t}]\, u(t)}$

(c) $\mathbf{D(s)} = (8\mathbf{s} + 12)(16\mathbf{s}^2 + 12\mathbf{s} + 2) = 128(\mathbf{s} + 1.5)(\mathbf{s} + 0.5)(\mathbf{s} + 0.25)$ so

$$\frac{\mathbf{N(s)}}{\mathbf{D(s)}} = \left(\frac{2}{128}\right)\frac{\mathbf{s}^2}{(\mathbf{s}+1.5)(\mathbf{s}+0.5)(\mathbf{s}+0.25)} = \frac{a}{(\mathbf{s}+1.5)} + \frac{b}{(\mathbf{s}+0.5)} + \frac{c}{(\mathbf{s}+0.25)}$$

$a = \left(\dfrac{2}{128}\right)\left.\dfrac{\mathbf{s}^2}{(\mathbf{s}+0.5)(\mathbf{s}+0.25)}\right|_{\mathbf{s}\,=\,-1.5} = 0.02813$, $b = \left(\dfrac{2}{128}\right)\left.\dfrac{\mathbf{s}^2}{(\mathbf{s}+1.5)(\mathbf{s}+0.25)}\right|_{\mathbf{s}\,=\,-0.5} = -0.01563$

$c = \left(\dfrac{2}{128}\right)\left.\dfrac{\mathbf{s}^2}{(\mathbf{s}+1.5)(\mathbf{s}+0.5)}\right|_{\mathbf{s}\,=\,-0.25} = 0.003125$

$$\therefore \boxed{f(t) = 0.02813\, e^{-1.5t} - 0.01563\, e^{-0.5t} + 0.003125 e^{-0.25t}]\, u(t)}$$

(d) Part (a):

```
EDU» N = [2 0 0];
EDU» D = [1 0 -1];
EDU» [r p y] = residue(N,D)

r =
  -1.0000
   1.0000

p =
  -1.0000
   1.0000

y =
    2
```

Part (b):

```
EDU» N = [2 0 0];
EDU» D = [1 22 147 270];
EDU» [r p y] = residue(N,D)

r =
   28.5714
  -27.0000
    0.4286

p =
  -10.0000
   -9.0000
   -3.0000

y =
    []
```

Part (c):

```
EDU» N = [2 0 0];
EDU» D = [128 288 160 24];
EDU» [r p y] = residue(N,D)

r =
    0.0281
   -0.0156
    0.0031

p =
   -1.5000
   -0.5000
   -0.2500

y =
    []
```

33.

(a) $\mathrm{F}(s) = \frac{2}{s} - \frac{3}{s+1}$ so $f(t) = \boxed{2\,u(t) - 3\,e^{-t}\,u(t)}$

(b) $\mathrm{F}(s) = \frac{2s+10}{s+3} = 2 + \frac{4}{s+3} \leftrightarrow \boxed{2\delta(t) + 4e^{-3t}u(t)}$

(c) $\mathrm{F}(s) = 3e^{-0.8s} \leftrightarrow \boxed{3\delta(t-0.8)}$

(d) $\mathrm{F}(s) = \frac{12}{(s+2)(s+6)} = \frac{3}{s+2} - \frac{3}{s+6} \leftrightarrow \boxed{3(e^{-2t} - e^{-6t})u(t)}$

(e) $\mathrm{F}(s) = \frac{12}{(s+2)^2(s+6)} = \frac{3}{(s+2)^2} + \frac{\mathrm{A}}{s+2} + \frac{0.75}{s+6}$

Let $s = 0$ $\therefore$ $\frac{12}{4\times 6} = \frac{3}{4} + \frac{\mathrm{A}}{2} + \frac{0.75}{6}$ $\therefore$ $\mathrm{A} = -0.75$

$\therefore \mathrm{F}(s) = \frac{3}{(s+2)^2} - \frac{0.75}{s+2} + \frac{0.75}{s+6} \leftrightarrow \boxed{(3te^{-2t} - 0.75e^{-2t} + 0.75e^{-6t})u(t)}$

34. $$\mathbf{F(s)} = 2 - \frac{1}{\mathbf{s}} + \frac{\pi}{\mathbf{s}^3 + 4\mathbf{s}^2 + 5\mathbf{s} + 2}$$

$$= 2 - \frac{1}{\mathbf{s}} + \frac{\pi}{(\mathbf{s}+2)(\mathbf{s}+1-j7.954)(\mathbf{s}+1+j7.954)}$$

$$= 2 - \frac{1}{\mathbf{s}} + \frac{a}{(\mathbf{s}+2)} + \frac{b}{(\mathbf{s}+1-j7.954)} + \frac{b^*}{(\mathbf{s}+1+j7.954)}$$

where $$a = \left.\frac{\pi}{(\mathbf{s}+1-j7.954)(\mathbf{s}+1+j7.954)}\right|_{\mathbf{s}\,=\,-2} = 0.04888$$

$$b = \left.\frac{\pi}{(\mathbf{s}+2)(\mathbf{s}+1+j7.954)}\right|_{\mathbf{s}\,=\,-1+j7.954} = -0.02444 + j0.003073$$

and hence b* = -0.02444 – j0.003073

Thus, we may write

$$f(t) = 2\,\delta(t) - u(t) + 0.04888\,e^{-2t}\,u(t) + [(-0.02444 + j0.003073)\,e^{(-1+j7.954)t} + (-0.02444 + j0.003073)\,e^{(-1+j7.954)t}\,]\,u(t)$$

This may be further simplified by expressing $(-0.02444 + j0.003073)\,e^{(-1+j7.954)t}$ as $0.02463\,e^{j172.83^{\circ}}\,e^{(-1+j7.954)t}$. This term, plus its complex conjugate above, add to the purely real expression $0.02463\,e^{-t}\cos(7.954t + 172.8^{\circ})$.

Thus, $f(t) = \boxed{2\,\delta(t) - u(t) + 0.04888\,e^{-2t}\,u(t) + 0.02463\,e^{-t}\cos(7.954t + 172.8^{\circ}).}$

35. (a) $\mathbf{F}(\mathbf{s}) = \dfrac{(\mathbf{s}+1)(\mathbf{s}+2)}{\mathbf{s}(\mathbf{s}+3)} = \dfrac{a}{\mathbf{s}} + \dfrac{b}{(\mathbf{s}+3)}$

$a = \left.\dfrac{(\mathbf{s}+1)(\mathbf{s}+2)}{(\mathbf{s}+3)}\right|_{\mathbf{s}=0} = \dfrac{2}{3}$ and $b = \left.\dfrac{(\mathbf{s}+1)(\mathbf{s}+2)}{\mathbf{s}}\right|_{\mathbf{s}=-3} = \dfrac{(-2)(-1)}{-3} = -\dfrac{2}{3}$

so

$$f(t) = \frac{2}{3}u(t) - \frac{2}{3}e^{-3t}\,u(t) = \boxed{\frac{2}{3}\left(1-e^{-3t}\right)u(t)}$$

(b) $\mathbf{F}(\mathbf{s}) = \dfrac{(\mathbf{s}+2)}{\mathbf{s}^2(\mathbf{s}^2+4)} = \dfrac{a}{\mathbf{s}^2} + \dfrac{b}{\mathbf{s}} + \dfrac{c}{(\mathbf{s}+j2)} + \dfrac{c^*}{(\mathbf{s}-j2)}$

$a = \left.\dfrac{(\mathbf{s}+2)}{(\mathbf{s}^2+4)}\right|_{\mathbf{s}=0} = \dfrac{2}{4} = 0.5$

$b = \dfrac{d}{d\mathbf{s}}\left[\dfrac{(\mathbf{s}+2)}{(\mathbf{s}^2+4)}\right]_{\mathbf{s}=0} = \left[\dfrac{(\mathbf{s}^2+4)-2\mathbf{s}(\mathbf{s}+2)}{(\mathbf{s}^2+4)^2}\right]_{\mathbf{s}=0} = \dfrac{4}{4^2} = 0.25$

$c = \left.\dfrac{(\mathbf{s}+2)}{\mathbf{s}^2(\mathbf{s}-j2)}\right|_{\mathbf{s}=-j2} = \dfrac{2-j2}{4(-j4)} = 0.125 + j0.125 = 0.1768\angle 45^{o}$ $(c^* = 0.1768\angle{-45^{o}})$

so

$$f(t) = 0.5\,t\,u(t) + 0.25\,u(t) + 0.1768\,e^{j45^{o}}\,e^{-j2t}\,u(t) + 0.1768\,e^{-j45^{o}}\,e^{j2t}\,u(t)$$

The last two terms may be combined so that

$$\boxed{f(t) = 0.5\,t\,u(t) + 0.25\,u(t) + 0.3536\cos(2t - 45^{o})}$$

36.

(a) $\boxed{5[\mathbf{sI(s)} - i(0^-)] - 7[\mathbf{sI(s)} - \mathbf{s}i(0^-) - i'(0^-)] + 9\mathbf{I(s)} \;=\; \frac{4}{\mathbf{s}}}$

(b) $\boxed{m[\mathbf{sP(s)} - \mathbf{s}p(0^-) - p'(0^-)] + \mu_f\,[\mathbf{sP(s)} - p(0^-)] + k\mathbf{P(s)} \;=\; 0}$

(c) $\boxed{[\mathbf{s}\,\mathbf{\Delta N_p(s)} - \Delta n_p(0^-)] = -\frac{\Delta \mathbf{N}_p(\mathbf{s})}{\tau} + \frac{G_L}{\mathbf{s}}}$

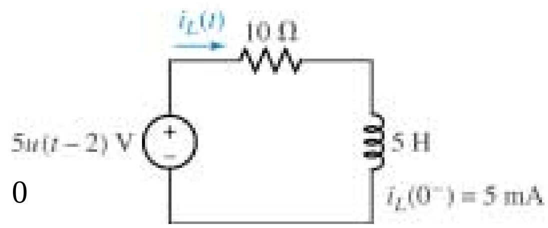
37.

$$15u(t) - 4\delta(t) = 8f(t) + 6f'(t), \quad f(0) = -3$$

$$\therefore \frac{15}{s} - 4 = 8\mathrm{F}(s) + 6s\mathrm{F}(s) + 18 = \frac{15 - 4s}{s} \;\; \therefore \mathrm{F}(s)\;\;(6s + 8) = 18 + \frac{15 - 4s}{s}$$

$$\therefore \mathrm{F}(s) = \frac{-22s + 15}{6s(s + 4/30)} = \frac{15/8}{s + 4/3} \;\; \therefore \boxed{f(t) = (1.875 - 5.542e^{-4t/3})u(t)}$$

38.



(a) $\boxed{-5\,u(t-2) + 10\,i_L(t) + 5\,\frac{di_L}{dt} = 0}$

(b) $\frac{-5}{\mathbf{s}}e^{-2\mathbf{s}} + 10\,\mathbf{I}_L(\mathbf{s}) + 5[\mathbf{s}\mathbf{I}_L(\mathbf{s}) - i_L(0^-)] = 0$

$$\mathbf{I}_L(\mathbf{s}) = \frac{\frac{5}{\mathbf{s}}e^{-2\mathbf{s}} + 5\,i_L(0^-)}{5\mathbf{s}+10} = \boxed{\frac{e^{-2\mathbf{s}} + 5\times10^{-3}\,\mathbf{s}}{\mathbf{s}(\mathbf{s}+2)}}$$

(c) $\mathbf{I}_L(\mathbf{s}) = e^{-2\mathbf{s}}\left[\frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+2}\right] + \frac{5\times10^{-3}}{\mathbf{s}+2}$

where $a = \left.\frac{1}{\mathbf{s}+2}\right|_{\mathbf{s}=0} = \frac{1}{2}$, and $b = \left.\frac{1}{\mathbf{s}}\right|_{\mathbf{s}=-2} = -\frac{1}{2}$, so that we may write

$$\mathbf{I}_L(\mathbf{s}) = \frac{1}{2}e^{-2\mathbf{s}}\left[\frac{1}{\mathbf{s}} - \frac{1}{\mathbf{s}+2}\right] + \frac{5\times10^{-3}}{\mathbf{s}+2}$$

Thus, $i_L(t) = \frac{1}{2}\left[u(t-2) - e^{-2(t-2)}\,u(t-2)\right] + 5\times10^{-3}e^{-2t}\,u(t)$

$$\boxed{= \frac{1}{2}\left[1 - e^{-2(t-2)}\right]u(t-2) + 5\times10^{-3}e^{-2t}\,u(t)}$$

39.

(a) $v_c(0^-) = 50\,\text{V},\; v_c(0^+) = \boxed{50\ \text{V}}$

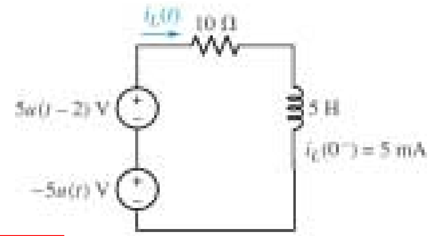
(b) $\boxed{0.1v_c' + 0.2v_c + 0.1(v_c - 20) = 0}$

(c) $\therefore 0.1v_c' + 0.3v_c = 2,\; 0.1s\text{V}_c - 5 + 0.3\text{V}_c = \dfrac{2}{s}$

$$\therefore \text{V}_c(0.1s + 0.3) = 5 + \frac{2}{s} = \frac{5s+2}{s}$$

$$\therefore \text{V}_c(s) = \frac{5s+2}{s(0.1s+0.3)} = \frac{20/3}{s} + \frac{130/3}{s+3} \;\therefore \boxed{v_c(t) = \left(\frac{20}{3} + \frac{130}{3}e^{-3t}\right)u(t)\ \text{V}}$$

40.



(a) $$\boxed{5\,u(t) - 5\,u(t-2) + 10\,i_L(t) + 5\,\frac{di_L}{dt} = 0}$$

(b) $$\frac{5}{\mathbf{s}} - \frac{5}{\mathbf{s}}e^{-2\mathbf{s}} + 10\,\mathbf{I}_L(\mathbf{s}) + 5[\mathbf{s}\mathbf{I}_L(\mathbf{s}) - i_L(0^-)] = 0$$

$$\mathbf{I}_L(\mathbf{s}) = \frac{\frac{5}{\mathbf{s}}e^{-2\mathbf{s}} - \frac{5}{\mathbf{s}} + 5\,i_L(0^-)}{5\mathbf{s}+10} = \boxed{\frac{e^{-2\mathbf{s}} - 5 + 5\times10^{-3}\,\mathbf{s}}{\mathbf{s}(\mathbf{s}+2)}}$$

(c) $$\mathbf{I}_L(\mathbf{s}) = e^{-2\mathbf{s}}\left[\frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+2}\right] + \frac{c}{\mathbf{s}} + \frac{d}{\mathbf{s}+2}$$ where

$$a = \left.\frac{1}{\mathbf{s}+2}\right|_{\mathbf{s}=0} = \frac{1}{2},\ b = \left.\frac{1}{\mathbf{s}}\right|_{\mathbf{s}=-2} = -\frac{1}{2},\ c = \left.\frac{5\times10^{-3}\mathbf{s}-5}{\mathbf{s}+2}\right|_{\mathbf{s}=0} = -\frac{5}{2} = -2.5,$$ and

$$d = \left.\frac{5\times10^{-3}\mathbf{s}-5}{\mathbf{s}}\right|_{\mathbf{s}=-2} = \frac{-10\times10^{-3}-5}{-2} = 2.505,$$

so that we may write

$$\mathbf{I}_L(\mathbf{s}) = \frac{1}{2}e^{-2\mathbf{s}}\left[\frac{1}{\mathbf{s}} - \frac{1}{\mathbf{s}+2}\right] - \frac{2.5}{\mathbf{s}+2} + \frac{2.505}{\mathbf{s}}$$

Thus, $$i_L(t) = \boxed{\frac{1}{2}\left[u(t-2) - e^{-2(t-2)}\,u(t-2)\right] - 2.5e^{-2t}\,u(t) + 2.505\,u(t)}$$

41.

$$12\,u(t) = 20\,f_2'(t) + 3\,f_2(0^-) = 2 \;\therefore\; \frac{12}{s} = 20s\mathrm{F}_2 - 20(2) + 3\mathrm{F}_2$$

$$\therefore \frac{12}{s} + 40 = (20s+3)\ \mathrm{F}_2 = \frac{12+40s}{s} \;\therefore\; \mathrm{F}_2(s) = \frac{2s+0.6}{s(s+0.15)}$$

$$\therefore \mathrm{F}_2(s) = \frac{4}{s} - \frac{2}{s+0.15} \leftrightarrow \boxed{(4-2e^{-0.15t})\,u(t)}$$

42. (a) $f(t) = \boxed{2\,u(t) - 4\delta(t)}$

(b) $f(t) = \boxed{\cos\left(\sqrt{99}\,t\right)}$

(c) $\mathbf{F}(\mathbf{s}) = \dfrac{1}{\mathbf{s}^2 + 5\mathbf{s} + 6} - 5 = \dfrac{a}{\mathbf{s}-3} + \dfrac{b}{\mathbf{s}-2} - 5$

where $a = \left.\dfrac{1}{\mathbf{s}-2}\right|_{\mathbf{s}=3} = 1$ and $b = \left.\dfrac{1}{\mathbf{s}-3}\right|_{\mathbf{s}=2} = -1$

Thus,

$$f(t) = \boxed{e^{-3t}\,u(t) - e^{-2t}\,u(t) - 5\delta(t)}$$

(d) $f(t) = \boxed{\delta'(t)}$ (a "doublet")

(e) $f(t) = \boxed{\delta''(t)}$

43.

$$x' + y = 2u(t),\ \ y' - 2x + 3y = 8u(t),\ \ x(0^-) = 5,\ \ y(0^-) = 8$$

$$s\mathrm{X} - 5 + \mathrm{Y} = \frac{2}{s},\ s\mathrm{Y} - 8 - 2\mathrm{X} + 3\mathrm{Y} = \frac{8}{s}\ \ \therefore\ \ \mathrm{X} = \frac{1}{s}\left(\frac{2}{s} + 5 - \mathrm{Y}\right) = \frac{2}{s^2} + \frac{5}{s} - \frac{\mathrm{Y}}{s}$$

$$\therefore s\mathrm{Y} + 3\mathrm{Y} - \frac{4}{s^2} - \frac{10}{s} + \frac{2\mathrm{Y}}{s} = 8 + \frac{8}{s}\ \ \therefore\ \ \mathrm{Y}\left(s + 3 + \frac{2}{s}\right) = \frac{4}{s^2} + \frac{18}{s} + 8$$

$$\mathrm{Y}\left(\frac{s^2 + 3s + 2}{s}\right) = \frac{4 + 18s + 8s^2}{s^2},\ \ \mathrm{Y}(s) + \frac{8s^2 + 18s + 4}{s(s+1)(s+2)} = \frac{2}{s} + \frac{6}{s+1} + \frac{0}{s+2}$$

$$\therefore \boxed{y(t) = (2 + 6e^{-t})\ u(t);}\ \ x(t) = \frac{1}{2}\ [y' + 3y - 8u(t)] = \frac{1}{2}y' + 1.5y - 4u(t)$$

$$\therefore x(t) = \frac{1}{2}\,[-6e^{-t}u(t)] + 1.5\,[2 + 6e^{-t}]\ u(t) - 4u(t)$$

$$\therefore x(t) = 6e^{-t}u(t) - u(t) = \boxed{(6e^{-t} - 1)\,u(t)}$$

44. (a) $\mathbf{F(s)} = 8\mathbf{s} + 8 + \frac{8}{\mathbf{s}}$, with $f(0^-) = 0$. Thus, we may write:

$$\boxed{f(t) = 8\,\delta(t) + 8\,u(t) + 8\delta'(t)}$$

(b) $\mathbf{F(s)} = \frac{\mathbf{s}^2}{(\mathbf{s}+2)} - \mathbf{s} + 2$.

$$f(t) = \delta'(t) - 2\delta(t) + 4e^{-2t}\,u(t) - \delta'(t) + 2\delta(t) = \boxed{4e^{-2t}\,u(t)}$$

45.

(a) $i_c(0^-) = 0,\ v_c(0) = 100\text{ V},\ \therefore i_c(0^+) = \dfrac{40-100}{100} = \boxed{-0.6\text{ A}}$

(b) $\boxed{40 = 100 i_c + 50\int_{0^-}^{\infty} i_c dt + 100}$

(c) $-\dfrac{60}{s} = 100\ \text{I}_c(s) + \dfrac{50}{s}\ \text{I}_c(s)$

$\therefore \dfrac{6}{s} = \text{I}_c\ \dfrac{10s+5}{s},\ \text{I}_c(s) = \dfrac{-6}{10s+5} = \dfrac{0.6}{s+0.5} \leftrightarrow i_c(t) = \boxed{0.6e^{-0.5t}u(t)}$

46. (a) $4 \cos 100t \leftrightarrow \boxed{\dfrac{4\mathbf{s}}{\mathbf{s}^2 + 100^2}}$

(b) $2 \sin 10^3 t - 3 \cos 100t \leftrightarrow \boxed{\dfrac{2\times10^3}{\mathbf{s}^2 + 10^6} - \dfrac{3}{\mathbf{s}^2 + 100^2}}$

(c) $14 \cos 8t - 2 \sin 8^{o} \leftrightarrow \boxed{\dfrac{14\mathbf{s}}{\mathbf{s}^2 + 64} - \dfrac{2\sin 8^{o}}{\mathbf{s}}}$

(d) $\delta(t) + [\sin 6t\,]u(t) \leftrightarrow \boxed{1 + \dfrac{6}{\mathbf{s}^2 + 36}}$

(e) $\cos 5t \sin 3t = ½ \sin 8t + ½ \sin(-2t) = ½(\sin 8t - \sin 2t) \leftrightarrow \boxed{\dfrac{4}{\mathbf{s}^2 + 64} - \dfrac{1}{\mathbf{s}^2 + 4}}$

47. $i_s = 100e^{-5t}u(t)$ A; $i_s = v' + 4v + 3\int_{0^-}^{t} v\,dt$

(a) $i_s = \dfrac{v}{\text{R}} + \text{C}v' + \dfrac{1}{\text{L}}\int_{0^-}^{t} v\,dt;\ \boxed{\text{R} = \dfrac{1}{4}\,\Omega,\ \ \text{C} = 1\text{F},\ \ \text{L} = \dfrac{1}{3}\ \text{H}}$

(b) $$\frac{100}{s+5} = s\text{V}(s) + 4\text{V}(s) + \frac{3}{s}\ \text{V}(s)$$

$$\text{V}(s)\left(s + 4 + \frac{3}{4}\right) = \text{V}(s)\frac{s^2 + 4s + 3}{s} = \frac{100}{s+5},\ \ \text{V}(s) = \frac{100s}{(s+1)(s+3)(s+5)}$$

$$\therefore \text{V}(s) = \frac{-12.5}{s+1} + \frac{75}{s+3} - \frac{62.5}{s+5},\ \boxed{v(t) = (75e^{-3t} - 12.5e^{-t} - 62.5e^{-5t})\,u(t)\ \text{V}}$$

48.

(a) $\mathbf{V}(\mathbf{s}) = \boxed{\frac{7}{\mathbf{s}} + \frac{e^{-2\mathbf{s}}}{\mathbf{s}}\ \text{V}}$

(b) $\mathbf{V}(\mathbf{s}) = \boxed{\frac{e^{-2\mathbf{s}}}{\mathbf{s}+1}\ \text{V}}$

(c) $\mathbf{V}(\mathbf{s}) = \boxed{48e^{-\mathbf{s}}\ \text{V}}$

49.

$$4u(t) + i_c + 10\int_{0^-}^{\infty} i_c dt + 4\,[i_c - 0.5\delta(t)] = 0$$

$$\therefore \frac{4}{s} + \mathrm{I}_c + \frac{10}{s}\,\mathrm{I}_c + 4\mathrm{I}_c = 2,\ \mathrm{I}_c\left(5 + \frac{10}{s}\right) = 2 - \frac{4}{s} + \frac{2s-4}{s}$$

$$\therefore \mathrm{I}_c = \frac{2s-4}{5s+10} = 0.4 - \frac{1.6}{s+2}$$

$$\therefore \boxed{i_c(t) + 0.4\delta(t) - 1.6e^{-2t}u(t)\ \mathrm{A}}$$

50.

$$v' + 6v + 9\int_{0^-}^{t} v(z)\ dz = 24(t-2)\ u(t-2),\ v'(0) = 0$$

$$\therefore sV(s) - 0 + 6\ V(s) + \frac{9}{s}\ V(s) = 24e^{-2s}\ \frac{1}{s^2} = V(s)\frac{s^2+6s+9}{s} = V(s)\ \frac{(s+3)^2}{s}$$

$$\therefore\ V(s) = 24e^{-2s}\ \frac{1}{s^2}\ \frac{s}{(s+3)^2} = 24e^{-2s}\left[\frac{1/9}{s} - \frac{1/9}{s+3} - \frac{1/3}{(s+3)^2}\right]$$

$$\therefore\ V(s) = e^{-2s}\left[\frac{8/3}{s} - \frac{8}{s+3} - \frac{8}{(s+3)^2}\right] \leftrightarrow \frac{8}{3}\ [u(t-2) - e^{-3(t-2)}\ u(t-2)]$$

$$-8(t-2)\,e^{-3(t-2)}\ u\,(t-2)\ \therefore\ \boxed{v(t) = \left[\frac{8}{3} - \frac{8}{3}e^{-3(t-2)} - 8\,(t-2)\,e^{-3(t-2)}\right] u\,(t-2)}$$

51.

(a) $F(s) = \dfrac{5(s^2+1)}{(s^3+1)}$ $\therefore$ $f(0^+) = \lim\limits_{s\to\infty} \dfrac{5s(s^2+1)}{s^3+1} = \boxed{5}$

$f(\infty) = \lim\limits_{s\to 0} \dfrac{5s(s^2+1)}{s^3+1}$, but 1 pole in RHP $\therefore$ indeterminate

(b) $F(s) = \dfrac{5(s^2+1)}{s^3+16}$ $\therefore$ $f(0^+) = \lim\limits_{s\to\infty} \dfrac{5s(s^2+1)}{s^4+16} = \boxed{0}$

$f(\infty)$ is indeterminate since poles on $j\omega$ axis

(c) $F(s) = \dfrac{(s+1)(1+e^{-4s})}{s^2+2}$ $\therefore$ $f(0^+) = \lim\limits_{s\to\infty} \dfrac{s(s+1)(1+e^{-4s})}{s^2+2} = \boxed{1}$

$f(\infty)$ is indeterminate since poles on $j\omega$ axis

52. (a) $f(0^+) = \lim_{\mathbf{s}\to\infty}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{\mathbf{s}\to\infty}\left(\frac{2\mathbf{s}^2+6}{\mathbf{s}^2+5\mathbf{s}+2}\right) = \boxed{2}$

$f(\infty) = \lim_{\mathbf{s}\to 0}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{\mathbf{s}\to 0}\left(\frac{2\mathbf{s}^2+6}{\mathbf{s}^2+5\mathbf{s}+2}\right) = \frac{6}{2} = \boxed{3}$

(b) $f(0^+) = \lim_{\mathbf{s}\to\infty}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{\mathbf{s}\to\infty}\left(\frac{2\mathbf{s}e^{-\mathbf{s}}}{\mathbf{s}+3}\right) = \boxed{0}$

$f(\infty) = \lim_{\mathbf{s}\to 0}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{\mathbf{s}\to 0}\left(\frac{2\mathbf{s}e^{-\mathbf{s}}}{\mathbf{s}+3}\right) = \boxed{0}$

(c) $f(0^+) = \lim_{\mathbf{s}\to\infty}[\mathbf{s}\,\mathbf{F}(\mathbf{s})] = \lim_{\mathbf{s}\to\infty}\left[\frac{\mathbf{s}(\mathbf{s}^2+1)}{\mathbf{s}^2+5}\right] = \boxed{\infty}$

$f(\infty)$: This function has poles on the $j\omega$ axis, so we may not apply the final value theorem to determine $f(\infty)$.

53.

(a) $\mathrm{F}(s)=\dfrac{5(s^2+1)}{(s+1)^3}$ $\therefore$ $f(0^+)=\lim\limits_{s\to\infty}\dfrac{5s(s^2+1)}{(s+1)^3}=\boxed{5}$

(b) $\mathrm{F}(s)=\dfrac{5(s^2+1)}{s(s+1)^3}$ $\therefore$ $f(0^+)=\lim\limits_{s\to\infty}\dfrac{5(s^2+1)}{(s+1)^3}=\boxed{0}$

$f(\infty)=\lim\limits_{s\to 0}\dfrac{5(s^2+1)}{(s+1)^3}=5$ (pole OK)

(c) $\mathrm{F}(s)=\dfrac{(1-e^{-3s})}{s^2}$ $\therefore$ $f(0^+)=\lim\limits_{s\to\infty}\dfrac{1-e^{-3s}}{s}=\boxed{0}$

$f(\infty)=\lim\limits_{s\to 0}\dfrac{1-e^{-3s}}{s}=\lim\limits_{s\to 0}\dfrac{1}{2}\left(1-1+3s-\dfrac{1}{2}\times 9s^2+\ldots\right)=\boxed{3}$ (no poles)

54.

$$f(t)=\frac{1}{t}(e^{at}-e^{-bt})u(t)$$

(a) Now, $\frac{1}{t}f(t)\leftrightarrow\int_s^\infty \mathrm{F}(s)\,ds \therefore e^{-at}u(t)\leftrightarrow\frac{1}{s+a},\ -e^{-bt}u(t)\leftrightarrow-\frac{1}{s+b}$

$$\therefore\frac{1}{t}(e^{-at}-e^{-bt}u(t)\leftrightarrow\int_s^\infty\left(\frac{1}{s+a}-\frac{1}{s+b}\right)ds=\ln\left.\frac{s+a}{s+b}\right|_s^\infty=\ln\left.\frac{s+a}{s+b}\right|_s^\infty=\boxed{\ln\frac{s+b}{s+a}}$$

(b)

$$\lim_{t\to0^+}\frac{1}{t}(e^{-at}-e^{-bt})\ u(t)=\lim_{t\to0^+}\frac{1-at+\ldots-1+bt}{t}=\boxed{b-a}$$

$$\lim_{s\to\infty}s\ln\frac{s+b}{s+a}=\lim_{s\to\infty}\frac{\ln(s+b)-\ln(s+a)}{1/s}$$

Use l' Hopital. $\therefore\ \lim_{s\to\infty}s\mathrm{F}(s)=\frac{1/(s+b)-1/(s+a)}{-1/s^2}=\lim_{s\to\infty}\left[-s^2\frac{(a-b)}{(s+b)(s+a)}\right]=\boxed{b-a}$

55.

(a) $\mathrm{F}(s) = \dfrac{8s-2}{s^2+6s+10} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \dfrac{s(8s-2)}{s^2+6s+10} = \boxed{8}$

$$f(\infty) = \lim_{s\to 0} \frac{s(8s-2)}{s^2+6s+10} = 0 \left(\text{poles: } s = \frac{-6 \pm \sqrt{36-40}}{2}, \text{ LHP}, \therefore \text{ OK} \right)$$

(b) $\mathrm{F}(s) = \dfrac{2s^3 - s^2 - 3s - 5}{s^3 + 6s^2 + 10s} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \dfrac{2s^3 - s^2 - 3s - 5}{s^2+6s+10} = \boxed{\infty}$

$$f(\infty) = \lim_{s\to 0} \frac{2s^3 - s^2 - 3s - 5}{s^2+6s+10} = -0.5 \text{ (poles OK)}$$

(c) $\mathrm{F}(s) = \dfrac{8s-2}{s^2-6s+10} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \dfrac{s(8s-2)}{s^2-6s+10} = \boxed{8}$

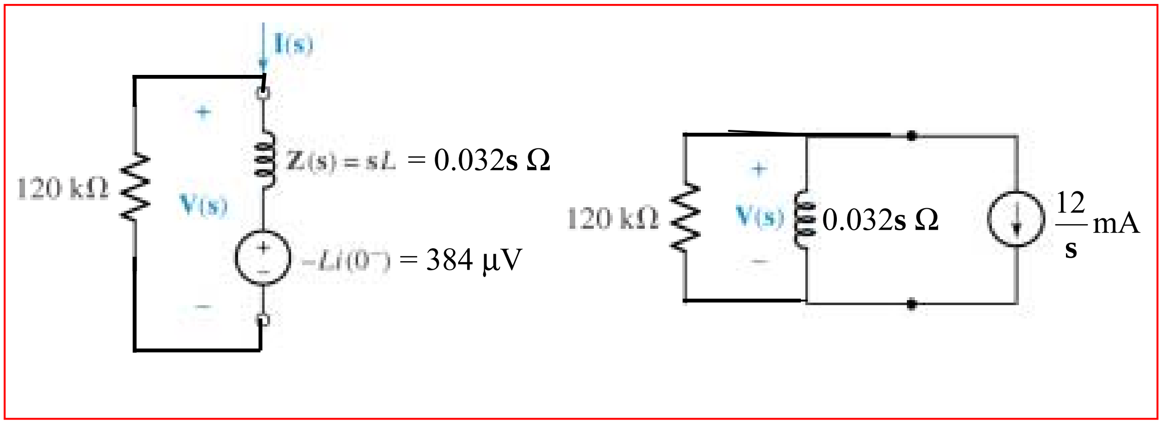
$$f(\infty) = \lim_{s\to 0} \frac{s(8s-2)}{s^2-6s+10}, \; s = \frac{6 \pm \sqrt{36-40}}{2} \text{ RHP } \therefore \text{ indeterminate}$$

(d) $\mathrm{F}(s) = \dfrac{8s^2-2}{(s+2)^2(s+1)(s^2+6s+10)} \quad \therefore \quad f(0^+) = \lim_{s\to\infty} \mathrm{F}(s) = \boxed{0}$

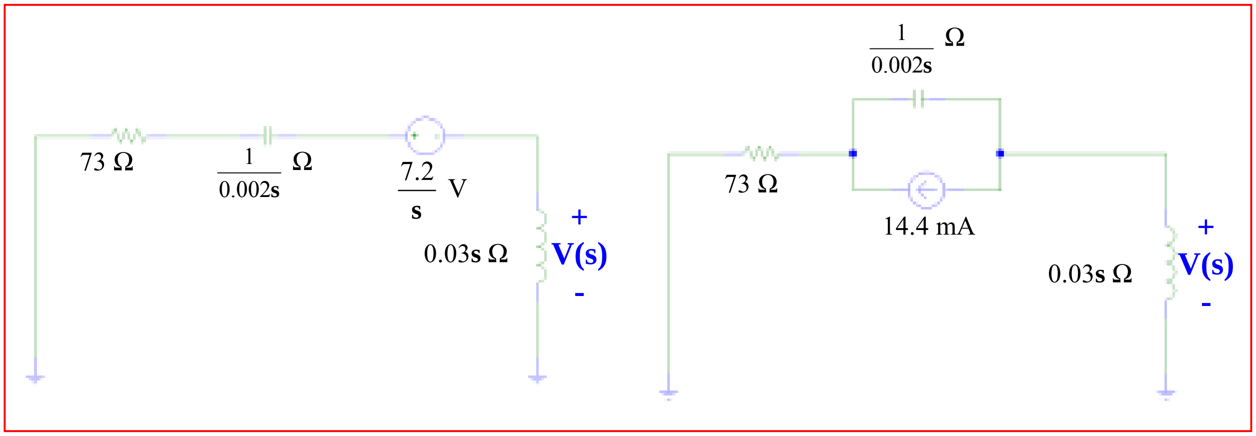
$$f\infty = \lim_{s\to 0} \frac{s(8s^2-2)}{(s+2)^2(s+1)(s^2+6s+10)} = \boxed{10} \text{ (pole OK)}$$

CHAPTER FIFTEEN SOLUTIONS

1. Note that $i_L(0^+) = 12$ mA. We have two choices for inductor model:



2. $i_L(0^-) = 0$, $v_C(0^+) = 7.2$ V ('+' reference on left). There are two possible circuits, since the inductor is modeled simply as an impedance:



3.

(a) $$\mathbf{Z}_m(\mathbf{s}) = \frac{2\mathbf{s}}{20+0.1\mathbf{s}} + \frac{2000/\mathbf{s}}{2+1000/\mathbf{s}} = \frac{20\mathbf{s}}{\mathbf{s}+200} + \frac{1000}{\mathbf{s}+500}$$
$$= \frac{20\mathbf{s}^2 + 10{,}000\mathbf{s} + 1000\mathbf{s} + 200{,}000}{\mathbf{s}^2 + 700\mathbf{s} + 100{,}000} = \boxed{\frac{20\mathbf{s}^2 + 11{,}000\mathbf{s} + 200{,}000}{\mathbf{s}^2 + 700\mathbf{s} + 100{,}000}}$$

(b) $\mathbf{Z}_{\text{in}}(-80) = \boxed{-10.95\ \Omega}$

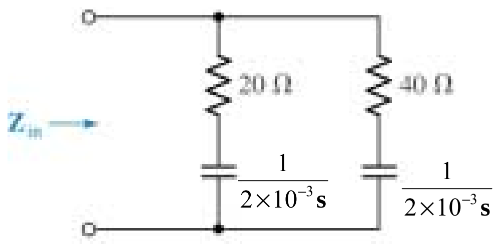
(c) $$\mathbf{Z}_{in}(j80) = \frac{-128{,}000 + j880{,}000 + 200{,}000}{-6400 + j56{,}000 + 100{,}000} = \boxed{8.095\angle 54.43^\circ\ \Omega}$$

(d) $$\mathbf{Y}_{RL} = \frac{1}{20} + \frac{10}{\mathbf{s}} = \boxed{\frac{\mathbf{s}+200}{20\mathbf{s}}}$$

(e) $$\mathbf{Y}_{RC} = \frac{1}{2} + 0.001\mathbf{s} = \boxed{\frac{\mathbf{s}+500}{1000}}$$

(f) $$\boxed{\begin{aligned}\frac{\mathbf{Y}_{RL}+\mathbf{Y}_{RC}}{\mathbf{Y}_{RL}\mathbf{Y}_{RC}} &= \frac{\frac{\mathbf{s}+200}{20s} + 0.5 + 0.001\mathbf{s}}{\frac{(\mathbf{s}+200)}{20\mathbf{s}}(0.001\mathbf{s}+0.5)} = \frac{\mathbf{s}+200+10\mathbf{s}+0.02\mathbf{s}^2}{0.001\mathbf{s}^2+0.7\mathbf{s}+100} \\ &= \frac{20\mathbf{s}^2 + 11{,}000\mathbf{s} + 200{,}000}{\mathbf{s}^2 + 700\mathbf{s} + 100{,}000} = \mathbf{Z}(\mathbf{s})\end{aligned}}$$

4.



$$\mathbf{Z}_{\text{in}} = \left(20 + \frac{1}{2\times10^{-3}\mathbf{s}}\right) \| \left(40 + \frac{1}{2\times10^{-3}\mathbf{s}}\right) = (20 + 500\mathbf{s}^{-1}) \ \| \ (40 + 500\mathbf{s}^{-1})$$

$$\boxed{= \frac{80\mathbf{s}^2 + 3000\mathbf{s} + 25000}{6\mathbf{s}^2 + 100\mathbf{s}}}$$

5. (a) $$\mathbf{Z}_{in} = \frac{50}{\mathbf{s}} + \frac{16(0.2\mathbf{s})}{16+0.2\mathbf{s}} = \frac{50}{\mathbf{s}} + \frac{16\mathbf{s}}{\mathbf{s}+80} = \boxed{\frac{16\mathbf{s}^2 + 50\mathbf{s} + 4000}{\mathbf{s}^2 + 80\mathbf{s}}}$$

(b) $$\mathbf{Z}_{in}(j8) = \frac{-1024 + 4000 + j400}{-64 + j640} = \boxed{0.15842 - j4.666\ \Omega}$$

(c) $$\mathbf{Z}_{in}(-2+j6) = \frac{16(4-36-j24) - 100 + j300 + 4000}{-32 - j24 - 160 + j480} = \boxed{6.850\angle -114.3°\,\Omega}$$

(d) $$\mathbf{Z}_{in} = \frac{50}{\mathbf{s}} + \frac{0.2\,\mathbf{s}\mathrm{R}}{\mathrm{R}+0.2\mathbf{s}} = \frac{0.2\mathrm{R}\mathbf{s}^2 + 10\mathbf{s} + 50\mathrm{R}}{0.2\mathbf{s}^2 + \mathrm{R}\mathbf{s}},$$

$$\mathbf{Z}_{in}(-5) = \frac{5\mathrm{R} - 50 + 50R}{5 - 5\mathrm{R}} \quad \therefore 55\mathrm{R} = 50, \qquad \mathrm{R} = \boxed{0.9091\ \Omega}$$

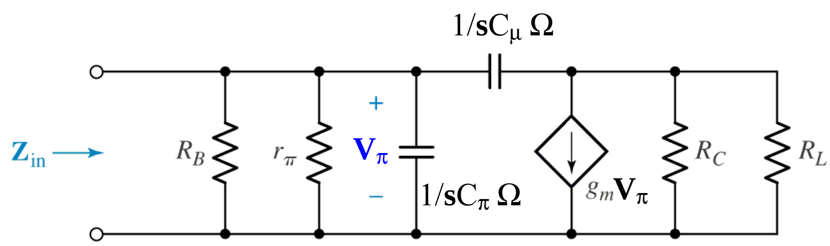
(e) $\mathrm{R} = \boxed{1\Omega}$

6. $2 \text{ mF} \rightarrow \dfrac{1}{2\times10^{-3}\mathbf{s}}\,\Omega,\;\; 1 \text{ mH} \rightarrow 0.001\mathbf{s}\;\Omega,$

$\mathbf{Z}_{\text{in}} = (55 + 500/\,\mathbf{s}) \parallel (100 + \mathbf{s}/\,1000) =$

$$\frac{\left(55+\dfrac{500}{\mathbf{s}}\right)\left(100+\dfrac{\mathbf{s}}{1000}\right)}{155+\dfrac{500}{\mathbf{s}}+\dfrac{\mathbf{s}}{1000}} = \boxed{\frac{55\mathbf{s}^2 + 5.5005\times10^6\mathbf{s} + 5\times10^7}{\mathbf{s}^2 + 5\times10^5\mathbf{s} + 1.55\times10^5}}$$

7. We convert the circuit to the s-domain:



Defining $\mathbf{Z}_\pi = R_B \,||\, r_\pi \,||\, (1/\mathbf{s}C_\pi) = \dfrac{r_\pi R_B}{r_\pi + R_B + r_\pi R_B C_\pi \mathbf{s}}$ and

$\mathbf{Z}_L = R_C \,||\, R_L = R_C R_L / (R_C + R_L)$, we next connect a 1-A source to the input and write two nodal equations:

$$1 = \mathbf{V}_\pi / \mathbf{Z}_\pi + (\mathbf{V}_\pi - \mathbf{V}_L) C_\mu \mathbf{s} \qquad [1]$$

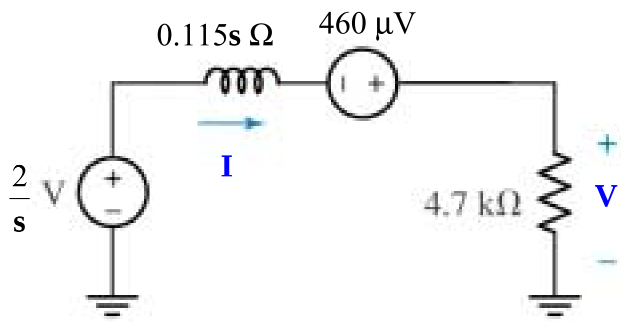
$$-g_m \mathbf{V}_\pi = \mathbf{V}_L / \mathbf{Z}_L + (\mathbf{V}_L - \mathbf{V}_\pi) C_\mu \mathbf{s} \qquad [2]$$

Solving,

$$\mathbf{V}_\pi = \boxed{\frac{r_\pi R_B \left(1 + \mathbf{Z}_L C_\mu \mathbf{s}\right)}{\mathbf{Z}_L r_\pi R_B C_\pi C_\mu \mathbf{s}^2 + (g_m \mathbf{Z}_L r_\pi R_B C_\mu + r_\pi R_B C_\pi + r_\pi R_B C_\mu + \mathbf{Z}_L r_\pi C_\mu + \mathbf{Z}_L R_B C_\mu)\mathbf{s} + r_\pi + R_B}}$$

Since we used a 1-A 'test' source, this is the input impedance. Setting both capacitors to zero results in $r_\pi \,||\, R_B$ as expected.

8.



$$\mathbf{V}(\mathbf{s}) \;=\; 4700\,\frac{\frac{2}{\mathbf{s}}+460\times10^{-6}}{4700+0.115\mathbf{s}} \;=\; \frac{2.162}{0.115\mathbf{s}+4700} \;+\; \frac{9400}{\mathbf{s}\,(0.115\mathbf{s}+4700)}$$

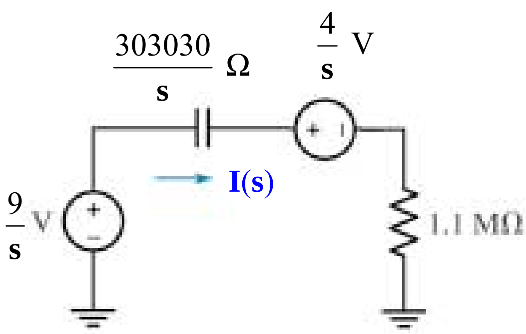
$$=\; \frac{18.8}{\mathbf{s}+40870} \;+\; \frac{81740}{\mathbf{s}\,(\mathbf{s}+40870)} \;=\; \frac{18.8}{\mathbf{s}+40870} \;+\; \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+40870}$$

where $a = \left.\dfrac{81740}{\mathbf{s}+40870}\right|_{\mathbf{s}=0} = 2$ and $b = \left.\dfrac{81740}{\mathbf{s}}\right|_{\mathbf{s}=-40870} = -2$

Thus, $\mathbf{V}(\mathbf{s}) \;=\; \dfrac{18.8}{\mathbf{s}+40870} \;+\; \dfrac{2}{\mathbf{s}} - \dfrac{2}{\mathbf{s}+40870}$. Taking the inverse transform of each term,

$$\boxed{v(t) \;=\; [16.8\, e^{-40870t} + 2]\, u(t)\ \text{V}}$$

9. $v(0^-) = 4$ V

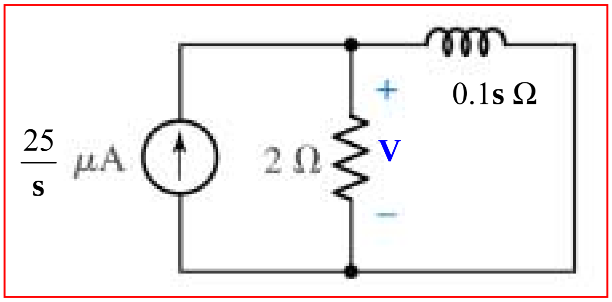


$$\mathbf{I}(\mathbf{s}) = \frac{\frac{9}{\mathbf{s}} - \frac{4}{\mathbf{s}}}{\frac{303030}{\mathbf{s}} + 1.1\times 10^6} = \frac{5}{1.1\times 10^6 + 303030} = \frac{4.545\times 10^{-6}}{\mathbf{s}+0.2755}$$

Taking the inverse transform, we find that $i(t) = \boxed{4.545\ e^{-0.2755t}\ u(t)\ \mu\text{A}}$

10. From the information provided, we assume no initial energy stored in the inductor.

(a) Replace the 100 mH inductor with a 0.1**s**-Ω impedance, and the current source with a $\frac{25\times10^{-6}}{\mathbf{s}}$ A source.



(b) $\mathbf{V}(\mathbf{s}) = \frac{25\times10^{-6}}{\mathbf{s}}\left[\frac{2\,(0.1\mathbf{s})}{2+0.1\mathbf{s}}\right] = \frac{5\times10^{-6}}{0.1\mathbf{s}+2} = \frac{5\times10^{-5}}{\mathbf{s}+20}$ V

Taking the inverse transform,

$v(t) = 50\ e^{-20t}$ mV

The power absorbed in the resistor R is then $p(t) = 0.5\ v^2(t) = \boxed{1.25\ e^{-40t}\ \text{nW}}$

11. $v(t) = 10e^{-2t}\cos(10t + 30^\circ)$ V

$$\cos(10t + 30^\circ) \Leftrightarrow \frac{\mathbf{s}\cos 30^\circ - 10\sin 30^\circ}{\mathbf{s}^2 + 100} = \frac{0.866\mathbf{s} - 5}{\mathbf{s}^2 + 100}$$

$\Lambda\ \{f(t)e^{-at}\} \Leftrightarrow \mathrm{F}(\mathbf{s} + a)$, so

$$\mathbf{V}(\mathbf{s}) = 10\,\frac{0.866(\mathbf{s}+2) - 5}{(\mathbf{s}+2)^2 + 100} = \frac{8.66\mathbf{s} - 16.34}{\mathbf{s} + 100}$$

The voltage across the 5-Ω resistor may be found by simple voltage division. We first note that $\mathbf{Z}_{\text{eff}} = (10/\mathbf{s}) \parallel 5 = \dfrac{50}{5\mathbf{s}+10}\ \Omega$. Thus,

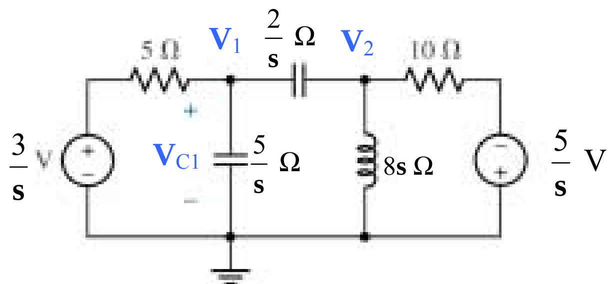
$$\mathbf{V}_{5\Omega} = \frac{\left(\dfrac{50}{5\mathbf{s}+10}\right)\mathbf{V}_s}{0.5\mathbf{s} + 5 + \left(\dfrac{50}{5\mathbf{s}+10}\right)} = \frac{50\,\mathbf{V}_s}{(0.5\mathbf{s}+5)\,(5\mathbf{s}+10) + 50} = \frac{50\,\mathbf{V}_s}{2.5\mathbf{s}^2 + 30\mathbf{s} + 100}$$

(a) $$\mathbf{I}_x = \frac{\mathbf{V}_{\text{eff}}}{5} = 40\,\frac{0.866\mathbf{s} - 3.268}{\left[(\mathbf{s}+2)^2 + 100\right]\left[\mathbf{s}^2 + 12\mathbf{s} + 40\right]} = \boxed{\frac{34.64\mathbf{s} - 130.7}{\left[(\mathbf{s}+2)^2 + 100\right]\left[(\mathbf{s}+6)^2 + 100\right]}}$$

(b) Taking the inverse transform using MATLAB, we find that

$$\boxed{i_x(t) = \mathrm{e}^{-6t}\,[0.0915\cos 2t - 1.5245\sin 2t] - \mathrm{e}^{-2t}\,[0.0915\cos 10t - 0.3415\sin 10t]\ \text{A}}$$

12.



Node 1: $0 = 0.2\,(\mathbf{V}_1 - 3/\,\mathbf{s}) + 0.2\,\mathbf{V}_1\,\mathbf{s} + 0.5\,(\mathbf{V}_1 - \mathbf{V}_2)\,\mathbf{s}$

Node 2: $0 = 0.5\,(\mathbf{V}_2 - \mathbf{V}_1)\,\mathbf{s} + 0.125\,\mathbf{V}_2\,\mathbf{s} + 0.1\,(\mathbf{V}_2 + 5/\,\mathbf{s})$

Rewriting, $(3.5\,\mathbf{s}^2 + \mathbf{s})\,\mathbf{V}_1 + 2.5\,\mathbf{s}^2\,\mathbf{V}_2 = 3$ [1]
$-4\,\mathbf{s}^2\,\mathbf{V}_1 + (4\,\mathbf{s}^2 + 0.8\,\mathbf{s} + 1)\,\mathbf{V}_2 = -4$ [2]

Solving using MATLAB or substitution, we find that

$$\mathbf{V}_1(\mathbf{s}) = \frac{-20\mathbf{s}^2 + 16\mathbf{s} + 20}{40\mathbf{s}^4 + 68\mathbf{s}^3 + 43\mathbf{s}^2 + 10\mathbf{s}}$$

$$= \left(\frac{1}{40}\right)\frac{-20\mathbf{s}^2 + 16\mathbf{s} + 20}{\mathbf{s}(\mathbf{s} + 0.5457 - j0.3361)(\mathbf{s} + 0.5457 + j0.3361)(\mathbf{s} + 0.6086)}$$

which can be expanded:

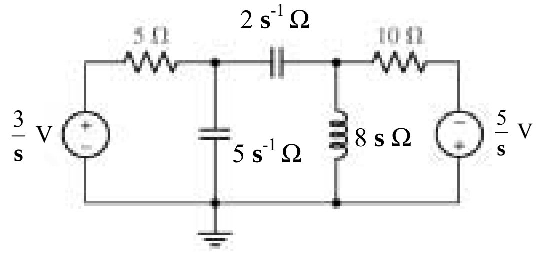
$$\mathbf{V}_1(\mathbf{s}) = \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s} + 0.5457 - j0.3361} + \frac{b^*}{\mathbf{s} + 0.5457 + j0.3361} + \frac{c}{\mathbf{s} + 0.6086}$$

Using the method of residues, we find that
a = 2, b = 2.511 $\angle 101.5^\circ$, b* = 2.511$\angle -101.5^\circ$ and c = -1.003.

Thus,taking the inverse transform,

$$\boxed{v_1(t) = [2 - 1.003\,e^{-0.6086t} + 5.022\,e^{-0.5457t}\cos(0.3361t - 101.5^\circ)]\,u(t)\ \mathrm{V}}$$

13. With zero initial energy, we may draw the following circuit:



Define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ in the left, centre and right meshes, respectively.

Mesh 1: $-3/\mathbf{s} + 5\mathbf{I}_1 + (5/\mathbf{s})\mathbf{I}_1 - (5/\mathbf{s})\mathbf{I}_2 = 0$
Mesh 2: $-(5/\mathbf{s})\mathbf{I}_1 + (8\mathbf{s} + 7/\mathbf{s})\mathbf{I}_2 - 8\mathbf{s}\,\mathbf{I}_3 = 0$
Mesh 3: $-8\mathbf{s}\mathbf{I}_2 + (8\mathbf{s} + 10)\mathbf{I}_3 - 5/\mathbf{s} = 0$

Rewriting,

$$\begin{array}{llll} (5\mathbf{s} + 5)\,\mathbf{I}_1 - 5\,\mathbf{I}_2 & & = 3 & [1] \\ -5\,\mathbf{I}_1 & + (8\mathbf{s}^2 + 7)\,\mathbf{I}_2 - 8\mathbf{s}^2\,\mathbf{I}_3 & = 0 & [2] \\ & -\,8\mathbf{s}^2\,\mathbf{I}_2 + (8\mathbf{s}^2 + 10\mathbf{s})\,\mathbf{I}_3 & = 5 & [3] \end{array}$$

Solving, we find that

$$\mathbf{I}_2(\mathbf{s}) = \frac{20\mathbf{s}^2 + 32\mathbf{s} + 15}{40\mathbf{s}^3 + 68\mathbf{s}^2 + 43\mathbf{s} + 10} = \left(\frac{1}{40}\right)\frac{20\mathbf{s}^2 + 32\mathbf{s} + 15}{(\mathbf{s} + 0.6086)(\mathbf{s} + 0.5457 - j0.3361)(\mathbf{s} + 0.5457 + j0.3361)}$$

$$= \frac{a}{(\mathbf{s} + 0.6086)} + \frac{b}{(\mathbf{s} + 0.5457 - j0.3361)} + \frac{b^*}{(\mathbf{s} + 0.5457 + j0.3361)}$$

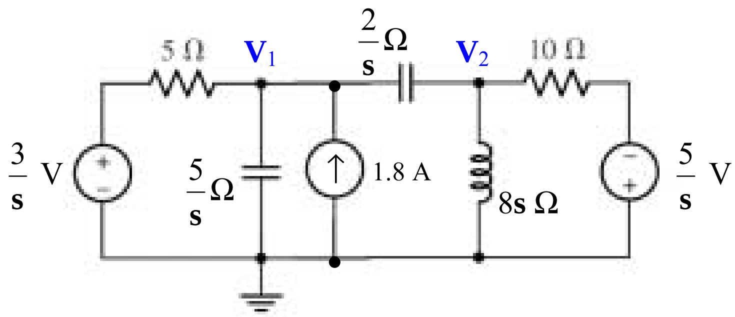
where $a = 0.6269$, $b = 0.3953\angle{-99.25^\circ}$, and $b^* = 0.3955\angle{+99.25^\circ}$

Taking the inverse tranform, we find that

$$i_2(t) = [0.6271e^{-0.6086t} + 0.3953e^{-j99.25^\circ}\,e^{(-0.5457 + j0.3361)t} + 0.3953e^{j99.25^\circ}\,e^{(-0.5457 - j0.3361)t}\,]u(t)$$

$$\boxed{= [0.6271e^{-0.6086t} + 0.7906\,e^{-0.5457t}\cos(0.3361t + 99.25^\circ)]\,u(t)}$$

14. We choose to represent the initial energy stored in the capacitor with a current source:



Node 1: $$1.8 = \frac{\mathbf{V}_1 - \frac{3}{\mathbf{s}}}{5} + \frac{\mathbf{s}}{5}\mathbf{V}_1 + \frac{\mathbf{s}}{2}(\mathbf{V}_1 - \mathbf{V}_2)$$

Node 2: $$0 = \frac{\mathbf{s}}{2}(\mathbf{V}_2 - \mathbf{V}_1) + \frac{1}{8\mathbf{s}}\mathbf{V}_2 + \frac{\mathbf{V}_2 + \frac{5}{\mathbf{s}}}{10}$$

Rewriting,
$$(5\mathbf{s}^2 + 4\mathbf{s})\,\mathbf{V}_1 - 5\mathbf{s}^2\,\mathbf{V}_2 = 18\mathbf{s} + 6 \qquad [1]$$
$$-4\mathbf{s}^2\,\mathbf{V}_1 + (4\mathbf{s}^2 + 0.8\mathbf{s} + 1)\mathbf{V}_2 = -4 \qquad [2]$$

Solving, we find that $$\mathbf{V}_1(\mathbf{s}) = \frac{360\mathbf{s}^3 + 92\mathbf{s}^2 + 114\mathbf{s} + 30}{\mathbf{s}(40\mathbf{s}^3 + 68\mathbf{s}^2 + 43\mathbf{s} + 10)}$$

$$= \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s} + 0.6086} + \frac{c}{\mathbf{s} + 0.5457 - j0.3361} + \frac{c^*}{\mathbf{s} + 0.5457 + j0.3361}$$

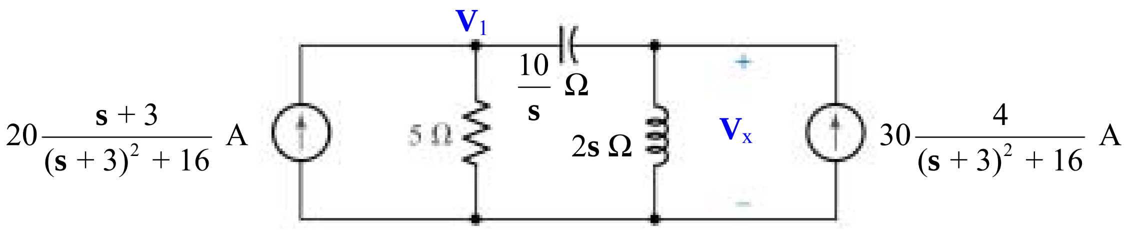
where $a = 3$, $b = 30.37$, $c = 16.84\ \angle 136.3^\circ$ and $c^* = 16.84\ \angle{-136.3^\circ}$

Taking the inverse transform, we find that

$$v_1(t) = [3 + 30.37e^{-0.6086t} + 16.84\,e^{j136.3^\circ}\,e^{-0.5457t}\,e^{j0.3361t} + 16.84\,e^{-j136.3^\circ}\,e^{-0.5457t}\,e^{-j0.3361t}]u(t)\text{ V}$$

$$= \boxed{[3 + 30.37e^{-0.6086t} + 33.68e^{-0.5457t}\cos(0.3361t + 136.3^\circ]u(t)\text{ V}}$$

15. We begin by assuming no initial energy in the circuit and transforming to the s-domain:



(a) via nodal analysis, we write:

$$\frac{20\mathbf{s}+60}{(\mathbf{s}+3)^2+16} = \frac{\mathbf{s}}{10}(\mathbf{V}_1 - \mathbf{V}_x) + \frac{\mathbf{V}_1}{5} \qquad [1] \quad \text{and}$$

$$\frac{120}{(\mathbf{s}+3)^2+16} = \frac{\mathbf{V}_x}{2\mathbf{s}} + \frac{\mathbf{s}}{10}(\mathbf{V}_x - \mathbf{V}_1) \qquad [2]$$

Collecting terms and solving for $\mathbf{V}_x(\mathbf{s})$, we find that

$$\mathbf{V}_x(\mathbf{s}) = \frac{200\mathbf{s}(\mathbf{s}^2+9\mathbf{s}+12)}{2\mathbf{s}^4+17\mathbf{s}^3+90\mathbf{s}^2+185\mathbf{s}+250}$$

$$= \boxed{\frac{200\mathbf{s}(\mathbf{s}^2+9\mathbf{s}+12)}{(\mathbf{s}+3-j4)(\mathbf{s}+3+j4)(\mathbf{s}+1.25-j1.854)(\mathbf{s}+1.25+j1.854)}}$$

(b) Using the method of residues, this function may be rewritten as

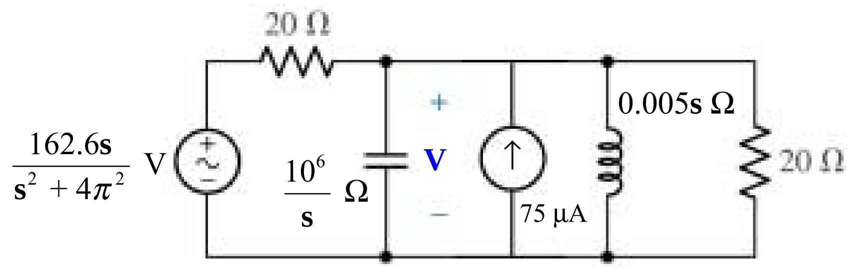
$$\frac{a}{(\mathbf{s}+3-j4)} + \frac{a^*}{(\mathbf{s}+3+j4)} + \frac{b}{(\mathbf{s}+1.25-j1.854)} + \frac{b^*}{(\mathbf{s}+1.25+j1.854)}$$

with a = 92.57 ∠ -47.58°, a^* = 92.57 ∠ 47.58°, b = 43.14 ∠106.8°, b^* = 43.14 ∠-106.8°
Taking the inverse transform, then, yields

$$v_x(t) = [92.57\, e^{-j47.58^\circ}\, e^{-3t}\, e^{j4t} + 92.57\, e^{j47.58^\circ}\, e^{-3t}\, e^{-j4t} + 43.14 e^{j106.8^\circ}\, e^{-1.25t}\, e^{j1.854t} + 43.14 e^{-j106.8^\circ}\, e^{-1.25t}\, e^{-j1.854t}]\, u(t)$$

$$= \boxed{[185.1\, e^{-3t} \cos(4t - 47.58^\circ) + 86.28\, e^{-1.25t} \cos(1.854t + 106.8^\circ)]\, u(t)}$$

16. We model the initial energy in the capacitor as a 75-μA independent current source:



First, define $\mathbf{Z}_{\text{eff}} = 10^6/\mathbf{s} \parallel 0.005\mathbf{s} \parallel 20 = \dfrac{\mathbf{s}}{10^{-6}\mathbf{s}^2 + 0.005\mathbf{s} + 200}\ \Omega$

Then, writing a single KCL equation, $75\times10^{-6} = \dfrac{\mathbf{V}(\mathrm{s})}{\mathbf{Z}_{\text{eff}}} + \dfrac{1}{20}\left(\mathbf{V}(\mathbf{s}) - \dfrac{162.6\mathbf{s}}{\mathbf{s}^2 + 4\pi^2}\right)$

which may be solved for $\mathbf{V}(\mathbf{s})$:

$$\mathbf{V}(\mathbf{s}) = \frac{75\mathbf{s}\left(\mathbf{s}^2 + 1.084\times10^5\mathbf{s} + 39.48\right)}{\mathbf{s}^4 + 5.5\times10^4\mathbf{s}^3 + 2\times10^8\mathbf{s}^2 + 2.171\times10^6\mathbf{s} + 7.896\times10^9}$$

$$= \frac{75\mathbf{s}\left(\mathbf{s}^2 + 1.084\times10^5\mathbf{s} + 12.57\right)}{(\mathbf{s}+51085)(\mathbf{s}+3915)(\mathbf{s}-j6.283)(\mathbf{s}+j6.283)}$$

(NOTE: factored with higher-precision denominator coefficients using MATLAB to obtain accurate complex poles: otherwise, numerical error led to an exponentially growing pole i.e. real part of the pole was positive)

$$= \frac{a}{(\mathbf{s}+51085)} + \frac{b}{(\mathbf{s}+3915)} + \frac{c}{(\mathbf{s}-j2\pi)} + \frac{c^*}{(\mathbf{s}+j2\pi)}$$

where a = -91.13, b = 166.1, c = 0.1277∠89.91° and c* = 0.1277∠-89.91°.
Thus, consolidating the complex exponential terms (the imaginary components cancel),

$$\boxed{v(t) = [-91.13e^{-51085t} + 166.1e^{-3915t} + 0.2554\cos(2\pi t + 89.91^\circ)]\,u(t)\ \text{V}}$$

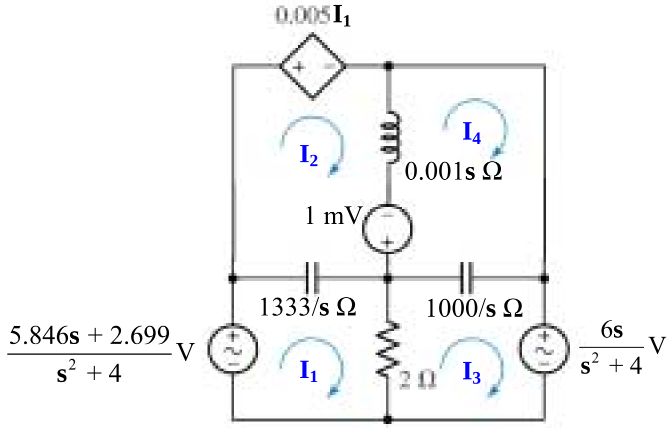
(b) The steady-state voltage across the capacitor is $\mathbf{V} = [255.4\cos(2\pi t + 89.91^\circ)]$ mV This can be written in phasor notation as 0.2554 ∠89.91° V. The impedance across which this appears is $\mathbf{Z}_{\text{eff}} = [j\omega C + 1/j\omega L + 1/20]^{-1} = 0.03142\ \angle 89.91^\circ\ \Omega$, so $\mathbf{I}_{\text{source}} = \mathbf{V}/\mathbf{Z}_{\text{eff}} = 8.129\angle{-89.91^\circ}$ A.

Thus, $i_{\text{source}} = \boxed{8.129\cos 2\pi t\ \text{A.}}$

(c) By phasor analysis, we can use simple voltage division to find the voltage division to find the capacitor voltage:

$$\mathbf{V}_C(j\omega) = \frac{(162.6\angle 0)(0.03142\angle 89.91^\circ)}{20 + 0.03142\angle 89.91^\circ} = 0.2554\angle 89.92^\circ\ \text{V}$$ which agrees with our answer to (a), assuming steady state. Dividing by 0.03142 ∠89.91° Ω, we find $i_{\text{source}} = 8.129\cos 2\pi t$ A.

17. Only the inductor appears to have initial energy, so we model that with a voltage source:



Mesh 1: $\frac{5.846\mathbf{s} + 2.699}{\mathbf{s}^2 + 4} = \left(2 + \frac{1333}{\mathbf{s}}\right)\mathbf{I}_1 - \frac{1333}{\mathbf{s}}\mathbf{I}_2 - 2\mathbf{I}_3$

Mesh 2: $0 = 0.005\mathbf{I}_1 - 0.001 + (0.001\mathbf{s} + 1333/\mathbf{s})\,\mathbf{I}_2 - (1333/\mathbf{s})\mathbf{I}_1 - 0.001\mathbf{s}\mathbf{I}_4$

Mesh 3: $0 = (2 + 1000/\mathbf{s})\mathbf{I}_3 - 2\mathbf{I}_1 - (1000/\mathbf{s})\mathbf{I}_4 + \frac{6\mathbf{s}}{\mathbf{s}^2 + 4}$

Mesh 4: $0 = (0.001\mathbf{s} + 1000/\mathbf{s})\,\mathbf{I}_4 - 0.001\mathbf{s}\mathbf{I}_2 - (1000/\mathbf{s})\mathbf{I}_3 + 0.001$

Solving, we find that $\mathbf{I}_1 = -0.2\frac{154\mathbf{s} - 2699}{\mathbf{s}^2 + 4}$ and

$$\mathbf{I}_2 = 0.001\frac{154\mathbf{s}^4 - 7.378\times10^7\mathbf{s}^3 - 1.912\times10^{10}\mathbf{s}^2 - 4.07\times10^{13}\mathbf{s} + 7.196\times10^{14}}{2333\mathbf{s}^4 + 6.665\times10^5\mathbf{s}^3 + 1.333\times10^9\mathbf{s}^2 + 5.332\times10^9}$$

$$= \frac{0.4328\angle-166.6^\circ}{\mathbf{s} + 142.8 + j742} + \frac{0.4328\angle+166.6^\circ}{\mathbf{s} + 142.8 - j742}$$

$$+ \frac{135.9\angle-96.51^\circ}{\mathbf{s} - j2} + \frac{135.9\angle+96.51^\circ}{\mathbf{s} + j2} + 6.6\times10^{-5}$$

Taking the inverse transform of each,

$i_1(t) = 271.7 \cos(2t - 96.51^\circ)$ A and
$i_2(t) = 0.8656\, e^{-142.8t} \cos(742.3t + 166.6^\circ) + 271.8 \cos(2t - 96.51^\circ) + 6.6\times10^{-5}\,\delta(t)$ A

Verifying via phasor analysis, we again write four mesh equations:

$6\angle-13^\circ = (2 - j666.7)\mathbf{I}_1 + j667\mathbf{I}_2 - 2\mathbf{I}_3$

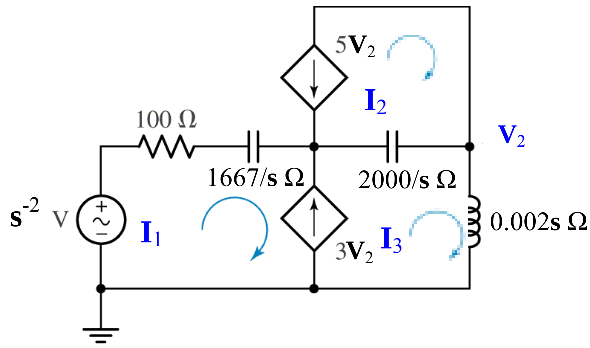
$0 = (0.005 + j666.7)\mathbf{I}_1 + (j2\times10^{-3} - j666.7)\mathbf{I}_2 - j2\times10^{-3}\mathbf{I}_4$

$-6\angle0 = -2\mathbf{I}_1 + (2 - j500)\mathbf{I}_3 + j500\mathbf{I}_4$

$0 = -j2\times10^{-3}\mathbf{I}_2 + j500\mathbf{I}_3 + (j2\times10^{-3} - j500)\mathbf{I}_4$

Solving, we find $\mathbf{I}_1 = 271.7\angle-96.5^\circ$ A and $\mathbf{I}_2 = 272\angle-96.5^\circ$ A. From the Laplace analysis, we see that this agrees with our expression for $i_1(t)$, and as $t \to \infty$, our expression for $i_2(t)$ $\to 272 \cos(2t - 96.5^\circ)$ in agreement with the phasor analysis.

18. With no initial energy storage, we simply convert the circuit to the s-domain:



Writing a supermesh equation,

$$\frac{1}{\mathbf{s}^2} = 100\mathbf{I}_1 + \frac{1}{6\times10^{-4}\mathbf{s}}\mathbf{I}_1 + \frac{2000}{\mathbf{s}}\mathbf{I}_3 + 0.002\mathbf{s}\mathbf{I}_3 - \frac{2000}{\mathbf{s}}\mathbf{I}_2$$

we next note that $\mathbf{I}_2 = -5\mathbf{V}_2 = -5(0.002\mathbf{s})\mathbf{I}_3 = -0.01\mathbf{s}\mathbf{I}_3$
and $\mathbf{I}_3 - \mathbf{I}_1 = 3\mathbf{V}_2 = 0.006\mathbf{s}\mathbf{I}_3$, or $\mathbf{I}_1 = (1 - 0.006\mathbf{s})\mathbf{I}_3$, we may write

$$\mathbf{I}_3 = \frac{1}{-0.598\mathbf{s}^3 + 110\mathbf{s}^2 + 3666\mathbf{s}}$$

$$\mathbf{V}_2(\mathbf{s}) = \mathbf{I}_3/\,0.002\mathbf{s} = \frac{1}{-0.0012\mathbf{s}^4 + 0.22\mathbf{s}^3 + 7.332\mathbf{s}^2}$$

$$= -\frac{7.645\times10^{-5}}{\mathbf{s} - 212.8} + \frac{4.167\times10^{-3}}{\mathbf{s} + 28.82} - \frac{4.091\times10^{-3}}{\mathbf{s}} + \frac{0.1364}{\mathbf{s}^2}$$

Taking the inverse transform,

$$v_2(t) = -7.645\times10^{-5}\, e^{212.8t} + 4.167\times10^{-3}\, e^{-28.82t} - 4.091\times10^{-3} + 0.1364\, t]\, u(t) \text{ V}$$

(a) $v_2(1 \text{ ms}) =$ -5.58×10^{-7} V
(b) $v_2(100 \text{ ms}) =$ -1.334×10^{5} V
(c) $v_2(10 \text{ s}) =$ -1.154×10^{920} V. This is pretty big- best to start running.

19. We need to write three mesh equations:

Mesh 1: $$\frac{5.846\mathbf{s} + 2.699}{\mathbf{s}^2 + 4} = \left(2 + \frac{1333}{\mathbf{s}}\right)\mathbf{I}_1 - 2\mathbf{I}_3$$

Mesh 3: $$0 = (2 + 1000/\mathbf{s})\mathbf{I}_3 - 2\mathbf{I}_1 - (1000/\mathbf{s})\mathbf{I}_4 + \frac{6\mathbf{s}}{\mathbf{s}^2 + 4}$$

Mesh 4: $$0 = (0.001\mathbf{s} + 1000/\mathbf{s})\,\mathbf{I}_4 - (1000/\mathbf{s})\mathbf{I}_3 + 10^{-6}$$

Solving,

$$\mathbf{I}_1 = -0.001\mathbf{s}\frac{\left(154\mathbf{s}^3 - 2.925\times10^6\mathbf{s}^2 + 1.527\times10^8\mathbf{s} - 2.699\times10^9\right)}{2333\mathbf{s}^4 + 6.665\times10^5\mathbf{s}^3 + 1.333\times109\mathbf{s}^2 + 2.666\times10^6\mathbf{s} + 5.332\times10^9}$$

$$= \frac{0.6507\angle 12.54^\circ}{\mathbf{s} + 142.8 - j742.3} + \frac{0.6507\angle -12.54^\circ}{\mathbf{s} + 142.8 + j742.3} + \frac{0.00101\angle -6.538^\circ}{\mathbf{s} - j2} + \frac{0.00101\angle 6.538^\circ}{\mathbf{s} + j2} - 6.601\times10^{-5}$$

which corresponds to
$i_1(t) = 1.301\ e^{-142.8t}\cos(742.3t + 12.54^\circ) + 0.00202\cos(2t - 6.538^\circ) - 6.601\times10^{-5}\,\delta(t)$ A

and

$$\mathbf{I}_3 = -0.001\frac{\left(154\mathbf{s}^4 + 3.997\times10^6\mathbf{s}^3 + 1.547\times10^8\mathbf{s}^2 + 3.996\times10^{12}\mathbf{s} - 2.667\times10^6\right)}{\left(\mathbf{s}^2 + 4\right)\left(2333\mathbf{s}^2 + 6.665\times10^5\mathbf{s} + 1.333\times10^9\right)}$$

$$= \frac{0.7821\angle -33.56^\circ}{\mathbf{s} + 142.8 - j742.3} + \frac{0.7821\angle 33.56^\circ}{\mathbf{s} + 142.8 + j742.3} + \frac{1.499\angle 179.9^\circ}{\mathbf{s} - j2} + \frac{1.499\angle -179.9^\circ}{\mathbf{s} + j2}$$

which corresponds to
$i_3(t) = 1.564\ e^{-142.8t}\cos(742.3t - 33.56^\circ) + 2.998\cos(2t + 179.9^\circ)$ A

The power absorbed by the 2-Ω resistor, then, is $2\left[i_1(t) - i_3(t)\right]^2$ or

$$p(t) = \boxed{\begin{array}{c}2[1.301\ e^{-142.8t}\cos(742.3t + 12.54^\circ) + 0.00202\cos(2t - 6.538^\circ) - 6.601\times10^{-5}\,\delta(t) \\ - 1.564\ e^{-142.8t}\cos(742.3t - 33.56^\circ) - 2.998\cos(2t + 179.9^\circ)]^2\ \text{W}\end{array}}$$

20. (a) We first define $\mathbf{Z}_{eff} = R_B \parallel r_\pi \parallel (1/\mathbf{s}C_\pi) = \dfrac{r_\pi R_B}{r_\pi + R_B + r_\pi R_B C_\pi \mathbf{s}}$. Writing two nodal equations, then, we obtain:

$$0 = (\mathbf{V}_\pi - \mathbf{V}_S)/R_S + \mathbf{V}_\pi (r_\pi + R_B + r_\pi R_B C_\pi \mathbf{s})/r_\pi R_B + (\mathbf{V}_\pi - \mathbf{V}_o)C_\mu \mathbf{s}$$

and

$$-g_m \mathbf{V}_\pi = \mathbf{V}_o(R_C + R_L)/R_C R_L + (\mathbf{V}_o - \mathbf{V}_p) C_\mu \mathbf{s}$$

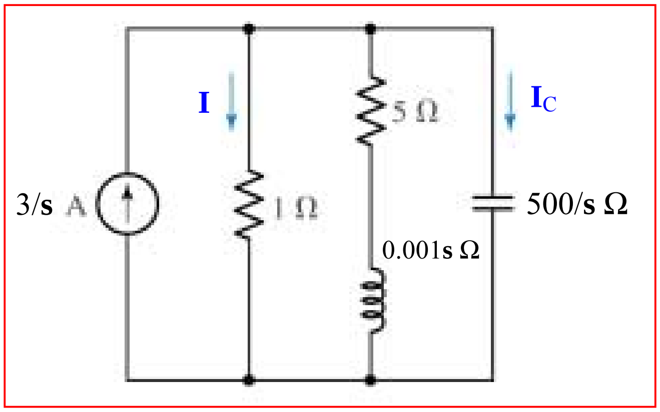
Solving using MATLAB, we find that

$$\boxed{\begin{aligned}\frac{\mathbf{V}_o}{\mathbf{V}_s} = r_\pi R_B R_C R_L(-g_m + C_\mu \mathbf{s})\,[&R_s r_\pi R_B R_C R_L C_\pi C_\mu \mathbf{s}^2 + (R_s r_\pi R_B R_C C_\pi + R_s r_\pi R_B R_C C_\mu \\ &+ R_s r_\pi R_B R_L C_\pi + R_s r_\pi R_B R_L C_\mu + r_\pi R_B R_C R_L C_\mu + R_s r_\pi R_C R_L C_\mu \\ &+ R_s R_B R_C R_L C_\mu + g_m R_s r_\pi R_B R_C R_L C_\mu)\mathbf{s} \\ &+ r_\pi R_B R_C + R_s r_\pi R_C + R_s R_B R_C + r_\pi R_B R_L + R_s r_\pi R_L + R_s R_B R_L]^{-1}\end{aligned}}$$

(b) Since we have only two energy storage elements in the circuit, the maximum number of poles would be two. The capacitors cannot be combined (either series or in parallel), so we expect a second-order denominator polynomial, which is what we found in part (a).

21.

(a)



(b) $\mathbf{Z}_{TH} = (5 + 0.001\mathbf{s}) \,||\, (500/\,\mathbf{s}) = \boxed{\dfrac{2500\mathbf{s} + 0.5}{0.001\mathbf{s}^2 + 5\mathbf{s} + 500}\ \Omega}$

$$\mathbf{V}_{TH} = (3/\,\mathbf{s})\mathbf{Z}_{TH} = \boxed{\frac{7.5\times10^6\mathbf{s} + 1500}{\mathbf{s}\left(\mathbf{s}^2 + 5000\mathbf{s} + 5\times10^5\right)}\ \text{V}}$$

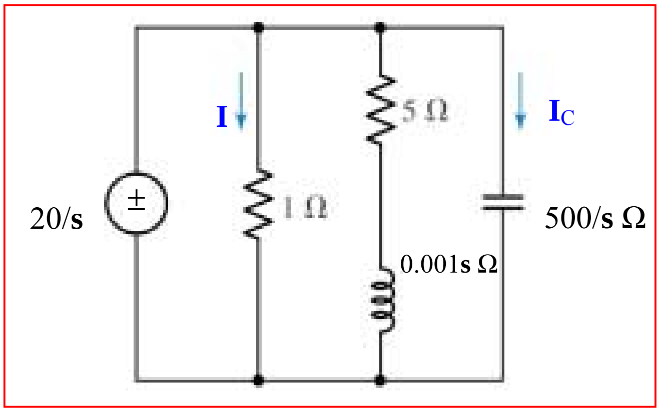
(c) $$\mathbf{V}_{1\Omega} = \mathbf{V}_{TH}\frac{1}{1 + \mathbf{Z}_{TH}} = \frac{7.5\times10^6\mathbf{s} + 1500}{\mathbf{s}\left(\mathbf{s}^2 + 505000\right)\left(1 + \dfrac{2500\mathbf{s} + 0.5}{0.001\mathbf{s}^2 + 5\mathbf{s} + 500}\right)}$$

$$= -\frac{2.988}{\mathbf{s} + 2.505\times10^6} + \frac{10.53\angle{-89.92^\circ}}{\mathbf{s} + j710.6} + \frac{10.53\angle{+89.92^\circ}}{\mathbf{s} - j710.6} + \frac{2.956}{\mathbf{s} + 0.1998} + \frac{2.967\times10^{-3}}{\mathbf{s}}$$

Thus, $i_{1\Omega} = v_{1\Omega}(t) = \boxed{[-2.988\, e^{-2.505\times10^6 t} + 2.956\, e^{-0.1998t} + 2.967\times10^{-3} + 21.06\cos(710.6t + 89.92^\circ)]\, u(t)}$

22.

(a)



(b) $\boxed{\mathbf{Z}_{TH} = 0}$, $\mathbf{V}_{TH} = 20/\mathbf{s}$ V so $\boxed{\mathbf{I}_N = \infty}$

(c) $\mathbf{I}_C = \dfrac{\left(\dfrac{20}{\mathbf{s}}\right)}{\left(\dfrac{500}{\mathbf{s}}\right)} = 0.04$ A. Taking the inverse transform, we obtain a delta function:

$$\boxed{i_C(t) = 40\delta(t) \text{ mA.}}$$

This "unphysical" solution arises from the circuit above attempting to force the voltage across the capacitor to change in zero time.

23. Beginning with the source on the left (10/**s** V) we write two nodal equations:

$$\left(\mathbf{V}_1' - \frac{10}{\mathbf{s}}\right)\frac{1}{47000} + \frac{\mathbf{s}}{30303}\mathbf{V}_1' + \frac{\mathbf{V}_1' - \mathbf{V}_2'}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

$$\frac{\mathbf{V}_2'}{47000} + \frac{\mathbf{s}}{10870}\mathbf{V}_2' + \frac{\mathbf{V}_2' - \mathbf{V}_1'}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

Solving,

$$\mathbf{V}_1' = \frac{303030(0.3197\times10^{13} + 0.1645\times10^{11}\mathbf{s} + 98700\mathbf{s}^2)}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

$$\mathbf{V}_2' = \frac{0.9676\times10^{18}}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

Shorting out the left source and activating the right-hand source (5 – 3/**s**) V:

$$\frac{1}{47000}\mathbf{V}_1'' + \frac{\mathbf{s}}{30303}\mathbf{V}_1'' + \frac{\mathbf{V}_1'' - \mathbf{V}_2''}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

$$\frac{\mathbf{V}_2'' - 5 + \frac{3}{\mathbf{s}}}{47000} + \frac{\mathbf{s}}{10870}\mathbf{V}_2'' + \frac{\mathbf{V}_2'' - \mathbf{V}_1''}{56 + 336\times10^{-6}\mathbf{s}} = 0$$

Solving,

$$\mathbf{V}_1'' = \frac{0.9676\times10^{17}(5\mathbf{s} - 3)}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

$$\mathbf{V}_2'' = \frac{7609(705000\mathbf{s}^3 + 0.1175\times10^{12}\mathbf{s}^2 + 0.6359\times10^{14}\mathbf{s} - 0.3819\times10^{14})}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}$$

Adding, we find that

$$\boxed{\begin{aligned}\mathbf{V}_1 &= \frac{30303(0.2239\times10^{13} + 0.1613\times10^{13}\mathbf{s} + 98700\mathbf{s}^2)}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}\\ \mathbf{V}_2 &= \frac{7609(705000\mathbf{s}^3 + 0.1175\times10^{12}\mathbf{s}^2 + 0.6359\times10^{14}\mathbf{s} + 0.8897\times10^{14})}{\mathbf{s}(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s} + 0.1936\times10^{18})}\end{aligned}}$$

(b) Using the *ilaplace*() routine in MATLAB, we take the inverse transform of each:

$$\boxed{\begin{aligned}v_1(t) &= [3.504 + 0.3805\times10^{-2}\,e^{-165928t} - 0.8618\,e^{-739t} - 2.646\,e^{-0.3404t}]\,u(t)\ \text{V}\\ v_2(t) &= [3.496 - 0.1365\times10^{-2}\,e^{-165928t} + 0.309\,e^{-739t} - 2.647\,e^{-0.3404t}]\,u(t)\ \text{V}\end{aligned}}$$

24. $(10/\mathbf{s})(1/47000) = 2.128\times10^{-4}/\,\mathbf{s}$ A
$(5 - 3/\mathbf{s})/\,47000 = (1.064 - 0.6383/\,\mathbf{s})\times10^{-4}$ A

$$\mathbf{Z}_L = 47000 \,\|\, (30303/\,\mathbf{s}) = \frac{1.424\times10^9}{47000\mathbf{s} + 30303}\,\Omega$$

$$\mathbf{Z}_R = 47000 \,\|\, (10870/\,\mathbf{s}) = \frac{5.109\times10^8}{47000\mathbf{s} + 10870}\,\Omega$$

Convert these back to voltage sources, one on the left ($\mathbf{V}_L$) and one on the right ($\mathbf{V}_R$):

$$\mathbf{V}_L = (2.128\times10^{-4}/\,\mathbf{s}\,)\left(\frac{1.424\times10^9}{47000\mathbf{s} + 30303}\right) = \frac{3.0303\times10^5}{\mathbf{s}(47000\mathbf{s} + 30303)}\ \text{V}$$

$$\mathbf{V}_R = (1.064 - 0.6383/\,\mathbf{s})\times10^{-4}\left(\frac{5.109\times10^8}{47000\mathbf{s} + 10870}\right)$$

$$= \frac{54360}{47000\mathbf{s} + 10870} - \frac{32611}{\mathbf{s}(47000\mathbf{s} + 10870)}$$

Then, $$\mathbf{I}_{56\Omega} = \frac{\mathbf{V}_L - \mathbf{V}_R}{\mathbf{Z}_L + \mathbf{Z}_R + 336\times10^{-6}\mathbf{s} + 56}$$

$$= -6250\frac{2.555\times10^9\mathbf{s}^2 - 1.413\times10^{10}\mathbf{s} - 4.282\times10^9}{\mathbf{s}\left(4.639\times10^9\mathbf{s}^3 + 7.732\times10^{14}\mathbf{s}^2 + 5.691\times10^{17}\mathbf{s} + 1.936\times10^{17}\right)}$$

$$= \frac{0.208}{\mathbf{s} + 1.659\times10^5} - \frac{0.0210}{\mathbf{s} + 739} - \frac{1.533\times10^{-18}}{\mathbf{s} + 0.6447} + \frac{2.658\times10^{-5}}{\mathbf{s} + 0.3404} + \frac{2.755\times10^{-18}}{\mathbf{s} + 0.2313} + \frac{1.382\times10^{-4}}{\mathbf{s}}$$

Thus,

$i_{56\Omega}(t) = [0.208 \exp(-1.659\times10^5 t) - 0.0210 \exp(-739t) - 1.533\times10^{-18} \exp(-.06447t)$
$+ 2.658\times10^{-5} \exp(-0.3404t) + 2.755\times10^{-18} \exp(-0.2313t) + 1.382\times10^{-4}]\, u(t)$ A.

The power absorbed in the 56-Ω resistor is simply $56\,[i_{56\Omega}(t)]^2$ or

$$\boxed{\begin{aligned}&56\,[0.208 \exp(-1.659\times10^5 t) - 0.0210 \exp(-739t) - 1.533\times10^{-18} \exp(-.06447t)\\ &+ 2.658\times10^{-5} \exp(-0.3404t) + 2.755\times10^{-18} \exp(-0.2313t) + 1.382\times10^{-4}]^2 \text{ W}\end{aligned}}$$

25. (a) Begin by finding $\mathbf{Z}_{TH} = \mathbf{Z}_N$:

$$\mathbf{Z}_{TH} = 47000 + (30303/\mathbf{s}) \parallel [336\times10^{-6}\,\mathbf{s} + 56 + (10870/\mathbf{s}) \parallel 47000]$$

$$= \boxed{\frac{4.639\times109\mathbf{s}^3 + 7.732\times10^{14}\mathbf{s}^2 + 5.691\times10^{17}\mathbf{s} + 1.936\times10^{17}}{98700\mathbf{s}^3 + 1.645\times10^{10}\mathbf{s}^2 + 1.21\times10^{13}\mathbf{s} + 2.059\times10^{12}}\ \Omega}$$

To find the Norton source value, define three clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ in the left, centre and right hand meshes, such that $\mathbf{I}_N(\mathbf{s}) = -\mathbf{I}_1(\mathbf{s})$ and the 10/**s** source is replaced by a short circuit.

$$(47000 + 30303/\mathbf{s})\,\mathbf{I}_1 - (30303/\mathbf{s})\,\mathbf{I}_2 = 0$$
$$(10870/\mathbf{s} + 56 + 336\times10^{-6}\,\mathbf{s} + 30303/\mathbf{s})\,\mathbf{I}_2 - (30303/\mathbf{s})\,\mathbf{I}_1 - (10870/\mathbf{s})\mathbf{I}_3 = 0$$
$$(47000 + 10870/\mathbf{s})\,\mathbf{I}_3 - (10870/\mathbf{s})\mathbf{I}_2 = -5 + 3/\mathbf{s}$$

Solving,

$$\mathbf{I}_N = -\mathbf{I}_1 = \boxed{\frac{2.059\times10^{12}(5\mathbf{s} - 3)}{\mathbf{s}(4.639\times10^{9}\mathbf{s}^3 + 7.732\times10^{14}\mathbf{s}^2 + 5.691\times10^{17}\mathbf{s} + 1.936\times10^{17})}}$$

(b) $\mathbf{I}_{source} = (10/\mathbf{s})\,(1/\mathbf{Z}_{TH}) - \mathbf{I}_N(\mathbf{s})$

$$= 0.001(0.4579\times10^{13}\mathbf{s}^6 + 0.1526\times10^{19}\mathbf{s}^5 + 0.1283\times10^{24}\mathbf{s}^4 + 0.1792\times10^{27}\mathbf{s}^3$$
$$+ 0.6306\times10^{29}\mathbf{s}^2 + 0.3667\times10^{29}\mathbf{s} + 0.5183\times10^{28})[\mathbf{s}(4639\mathbf{s}^3 + 0.7732\times10^{9}\mathbf{s}^2$$
$$+ 0.5691\times1012\mathbf{s} + 0.1936\times10^{12})(0.4639\times10^{10}\mathbf{s}^3 + 0.7732\times10^{15}\mathbf{s}^2 + 0.5691\times10^{18}\mathbf{s}$$
$$+ 0.1936\times10^{18})]^{-1}$$

Taking the inverse transform using the MATLAB *ilaplace*() routine, we find that

$$i_{source}(t) = 0.1382\times10^{-3} + 0.8607\times10^{-8}\exp(-165930t) + 0.8723\times10^{-7}\exp(-739t)$$
$$+ 0.1063\times10^{-3}\exp(-0.3403t) - 0.8096\times10^{-7}\exp(-165930t)$$
$$+ 0.1820\times10^{-4}\exp(-739t) - 0.5\times10^{-4}\exp(-0.3404t)$$

$$i_{source}(1.5\text{ ms}) = 2.0055\times10^{-4}\text{ A} = \boxed{200.6\ \mu\text{A}}$$

26. We begin by shorting the 7 cos 4t source, and replacing the 5 cos 2t source with $\dfrac{5\mathbf{s}}{\mathbf{s}^2+4}$.

(a) Define four clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_x$ in the top left, top right, bottom left and bottom right meshes, respectively. Then,

$$\frac{5\mathbf{s}}{\mathbf{s}^2+4} = (12 + 1/2\mathbf{s})\,\mathbf{I}_3 - 7\,\mathbf{I}_1 - (1/2\mathbf{s})\,\mathbf{I}_x \qquad [1]$$

$$0 = -4\,\mathbf{I}_x + (9.5 + \mathbf{s})\,\mathbf{I}_1 - \mathbf{s}\,\mathbf{I}_2 - 7\,\mathbf{I}_3 \qquad [2]$$

$$0 = (3 + \mathbf{s} + 2/\mathbf{s})\,\mathbf{I}_2 - \mathbf{s}\,\mathbf{I}_1 - 3\,\mathbf{I}_x \qquad [3]$$

$$0 = (4 + 3\mathbf{s} + 1/2\mathbf{s})\,\mathbf{I}_x - 3\,\mathbf{I}_2 - (1/2\mathbf{s})\,\mathbf{I}_3 \qquad [4]$$

$$\mathbf{V}_1' = (\mathbf{I}_3 - \mathbf{I}_x)\,(2\mathbf{s}) \qquad [5]$$

Solving all five equations simultaneously using MATLAB, we find that

$$\mathbf{V}_1' = \frac{20\mathbf{s}^3(75\mathbf{s}^3 + 199\mathbf{s}^2 + 187\mathbf{s} + 152)}{1212\mathbf{s}^6 + 3311\mathbf{s}^5 + 7875\mathbf{s}^4 + 15780\mathbf{s}^3 + 12408\mathbf{s}^2 + 10148\mathbf{s} + 1200}$$

Next we short the 5 cos 2t source, and replace the 7 cos 4t source with $\dfrac{7\mathbf{s}}{\mathbf{s}^2+16}$.

Define four clockwise mesh currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$ and $\mathbf{I}_x$ in the bottom left, top left, top right and bottom right meshes, respectively (*note order changed from above*). Then,

$$0 = (12 + 1/2\mathbf{s})\,\mathbf{I}_1 - 7\,\mathbf{I}_2 - (1/2\mathbf{s})\,\mathbf{I}_x \qquad [1]$$

$$0 = -4\,\mathbf{I}_x + (9.5 + \mathbf{s})\,\mathbf{I}_2 - \mathbf{s}\,\mathbf{I}_3 - 7\,\mathbf{I}_1 \qquad [2]$$

$$-\frac{7\mathbf{s}}{\mathbf{s}^2+16} = (3 + \mathbf{s} + 2/\mathbf{s})\,\mathbf{I}_3 - \mathbf{s}\,\mathbf{I}_2 - 3\,\mathbf{I}_x \qquad [3]$$

$$0 = (4 + 3\mathbf{s} + 1/2\mathbf{s})\,\mathbf{I}_x - 3\,\mathbf{I}_3 - (1/2\mathbf{s})\,\mathbf{I}_1 \qquad [4]$$

$$\mathbf{V}_1'' = (\mathbf{I}_1 - \mathbf{I}_x)\,(2\mathbf{s}) \qquad [5]$$

Solving all five equations simultaneously using MATLAB, we find that

$$\mathbf{V}_1'' = \frac{-56\mathbf{s}^4(21\mathbf{s}^2 - 8\mathbf{s} - 111)}{\left(1212\mathbf{s}^6 + 3311\mathbf{s}^5 + 22420\mathbf{s}^4 + 55513\mathbf{s}^3 + 48730\mathbf{s}^2 + 40590\mathbf{s} + 4800\right)}$$

The next step is to form the sum $\mathbf{V}_1(\mathbf{s}) = \mathbf{V}_1' + \mathbf{V}_1''$, which is accomplished in MATLAB using the function *symadd*(): V1 = symadd(V1prime, V1doubleprime);

$$\mathbf{V}_1(\mathbf{s}) = \frac{4\mathbf{s}^3(81\mathbf{s}^5 + 1107\mathbf{s}^4 + 7313\mathbf{s}^3 + 17130\mathbf{s}^2 + 21180\mathbf{s} + 12160)}{\left(\mathbf{s}^2+4\right)\left(1212\mathbf{s}^6 + 3311\mathbf{s}^5 + 22420\mathbf{s}^4 + 55513\mathbf{s}^3 + 48730\mathbf{s}^2 + 40590\mathbf{s} + 4800\right)}$$

(b) Using the *ilaplace*() routine from MATLAB, we find that

$$\boxed{\begin{aligned} v_1(t) = [&0.2673\,\delta(t) + 6.903\times10^{-3}\cos 2t - 2.403\sin 2t - 0.1167\,e^{-1.971t} \\ &- 0.1948\,e^{-0.3315t}\cos 0.903t + 0.1611\,e^{-0.3115t}\sin 0.903t - 0.823\times10^{-3}\,e^{-0.1376t} \\ &+ 3.229\cos 4t + 3.626\sin 4t]\,u(t)\text{ V}\end{aligned}}$$

27. (a) We can combine the two sinusoidal sources in the time domain as they have the same frequency. Thus, there is really no need to invoke source transformation as such to find the current.

$$65 \cos 10^3 t \;\Leftrightarrow\; \frac{65\mathbf{s}}{\mathbf{s}^2 + 10^6}, \text{ and } 13 \text{ mH} \rightarrow 0.013\mathbf{s}\ \Omega$$

We may therefore write

$$\mathbf{I}(\mathbf{s}) = \left(\frac{65\mathbf{s}}{\mathbf{s}^2 + 10^6}\right)\left(\frac{1}{83 + 0.013\mathbf{s}}\right) = \boxed{\frac{5000\mathbf{s}}{\left(\mathbf{s}^2 + 10^6\right)\left(\mathbf{s} + 6385\right)}}$$

$$= -\frac{0.7643}{\left(\mathbf{s} + 6385\right)} + \frac{0.3869\angle -8.907^\circ}{\left(\mathbf{s} - j10^3\right)} + \frac{0.3869\angle 8.907^\circ}{\left(\mathbf{s} + j10^3\right)}$$

(b) Taking the inverse transform,

$$i(t) = \boxed{[-0.7643\, e^{-6385t} + 0.7738 \cos (10^3 t - 8.907^\circ)]\, u(t) \text{ A}}$$

(c) The steady-state value of $i(t)$ is simply $\boxed{0.7738 \cos (10^3 t - 8.907^\circ) \text{ A.}}$

28.

(a) $$\mathbf{Z}_{in} = \frac{\left(5+\frac{5}{\mathbf{s}}\right)(2+5\mathbf{s})}{5\mathbf{s}+7+5/\mathbf{s}} = \frac{(5\mathbf{s}+5)(2+5\mathbf{s})}{5\mathbf{s}^2+7\mathbf{s}+5} = \frac{25\mathbf{s}^2+35\mathbf{s}+10}{5\mathbf{s}^2+7\mathbf{s}+5}$$

$$\therefore \boxed{\mathbf{Y}_{in}(\mathbf{s}) = \frac{5\mathbf{s}^2+7\mathbf{s}+5}{25\mathbf{s}^2+35\mathbf{s}+10}}$$

(b) Poles: $\mathbf{s}^2+1.4\mathbf{s}+0.2=0,\ \mathbf{s} = \dfrac{-1.4 \pm \sqrt{1.96-0.8}}{2} = \boxed{-0.1615, -1.239\,\mathrm{s}^{-1}}$

Zeros: $\mathbf{s}^2+1.4\mathbf{s}+1=0,\ \mathbf{s} = \dfrac{-1.4 \pm \sqrt{1.96-4}}{2} = \boxed{-0.7 \pm j0.7141\mathrm{s}^{-1}}$

(c) Poles: same; $\mathbf{s} = \boxed{-0.1615,\ -1.239\ \mathrm{s}^{-1}}$

(d) Zeros: same; $\mathbf{s} = \boxed{-0.7 \pm j0.7141\ \mathrm{s}^{-1}}$

29. (a) Regarding the circuit of Fig. 15.45, we replace each 2-mF capacitor with a 500/ **s** Ω impedance. Then,

$$\mathbf{Z}_{in}(\mathbf{s}) = \frac{\left(20 + \frac{500}{\mathbf{s}}\right)\left(40 + \frac{500}{\mathbf{s}}\right)}{60 + \frac{100}{\mathbf{s}}} = 13.33\frac{(\mathbf{s}+25)(\mathbf{s}+12.5)}{\mathbf{s}(\mathbf{s}+1.667)}$$

Reading from the transfer function, we have

$$\boxed{\begin{array}{l}\text{zeros at } \mathbf{s} = -25 \text{ and } -12.5\ \text{s}^{-1}, \text{ and} \\ \text{poles at } \mathbf{s} = 0 \text{ and } \mathbf{s} = -1.667\ \text{s}^{-1}.\end{array}}$$

(b) Regarding the circuit of Fig. 15.47, we replace the 2-mF capacitor with a 500/ **s** Ω impedance and the 1-mH inductor with a 0.001**s**-Ω impedance. Then,

$$\mathbf{Z}_{in}(\mathbf{s}) = \frac{\left(55 + \frac{500}{\mathbf{s}}\right)(100 + 0.001\mathbf{s})}{155 + \frac{500}{\mathbf{s}} + 0.001\mathbf{s}} = 55\frac{(\mathbf{s} + \frac{500}{55})(\mathbf{s} + 10^5)}{\left(\mathbf{s} + 1.55\times10^5\right)(\mathbf{s} + 3.226)}$$

Reading from the transfer function, we have

$$\boxed{\begin{array}{l}\text{zeros at } \mathbf{s} = -9.091 \text{ and } -10^5\ \text{s}^{-1}, \quad \text{and} \\ \text{poles at } \mathbf{s} = -1.55\times10^5 \text{ and } \mathbf{s} = -3.226\ \text{s}^{-1}.\end{array}}$$

30. $\mathbf{Y}(\mathbf{s})$: zeros at $\mathbf{s} = 0; -10;$ *poles* at $\mathbf{s} = -5, -20\ \text{s}^{-1}$; $\mathbf{Y}(\mathbf{s}) \rightarrow 12$ S as $\mathbf{s} \rightarrow \infty$

(a)
$$\mathbf{Y}(\mathbf{s}) = \frac{K\mathbf{s}(\mathbf{s}+10)}{(\mathbf{s}+5)(\mathbf{s}+20)},\ K = 12 \therefore$$

$$\mathbf{Y}(\mathbf{s}) = \frac{12\mathbf{s}(\mathbf{s}+10)}{(\mathbf{s}+5)(\mathbf{s}+20)} = \frac{12\mathbf{s}^2 + 120\mathbf{s}}{\mathbf{s}^2 + 25\mathbf{s} + 100}$$

$$\therefore \mathbf{Y}(j10) = \frac{-1200 + j1200}{-100 + j250 + 100} = 4.800 + j4.800 = \boxed{6.788\angle 45°\ \text{S}}$$

(b) $\mathbf{Y}(-j10) = \boxed{6.788\angle -45°\ \text{S}}$

(c)
$$\mathbf{Y}(-15) = \frac{12(-15)(-5)}{(-10)5} = \boxed{-18\ \text{S}}$$

(d)
$$5 + \mathbf{Y}(\mathbf{s}) = 5 + \frac{12\mathbf{s}^2 + 120\mathbf{s}}{\mathbf{s}^2 + 25\mathbf{s} + 100} = \frac{17\mathbf{s}^2 + 245\mathbf{s} + 500}{(\mathbf{s}+5)(\mathbf{s}+20)},\ \mathbf{s} = \frac{-245 \pm \sqrt{245^2 - 68(500)}}{34}$$

Zeros: $\mathbf{s} = \boxed{-2.461 \text{ and } -11.951\ \text{s}^{-1}}$; Poles: $\mathbf{s} = \boxed{-5, -20\ \text{s}^{-1}}$

31.

(a) $\mathbf{Y}_{in} = \frac{1}{4+\mathbf{s}} + \frac{1}{5+5\mathbf{s}} = \frac{0.2(6\mathbf{s}+9)}{(4+\mathbf{s})(1+\mathbf{s})} \;\therefore\; \boxed{\mathbf{Z}_{in} = \frac{5(\mathbf{s}+1)(\mathbf{s}+4)}{6(\mathbf{s}+1.5)}}$

(b) Poles: $\mathbf{s} = \boxed{-1.5, \infty}$; Zeros: $\mathbf{s} = \boxed{-1,\ -4\ \text{s}^{-1}}$

32. $\mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{s}+2}{(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}$

(a) $d(t) \Leftrightarrow 1$, so the output is $\boxed{\dfrac{\mathbf{s}+2}{(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(b) $e^{-4t}\,u(t) \Leftrightarrow 1\,/\,(\mathbf{s}+4)$, so the output is $\boxed{\dfrac{\mathbf{s}+2}{(\mathbf{s}+4)(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(c) $2\cos 15t\,u(t) \Leftrightarrow \dfrac{2\mathbf{s}}{\mathbf{s}^2+225}$, so the output is $\boxed{\dfrac{2\mathbf{s}(\mathbf{s}+2)}{(\mathbf{s}^2+225)(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(d) $t\,e^{-t}\,u(t) \Leftrightarrow 1/\,(\mathbf{s}+1)$, so the output is $\boxed{\dfrac{\mathbf{s}+2}{(\mathbf{s}+1)(\mathbf{s}+5)(\mathbf{s}^2+6\mathbf{s}+25)}}$

(e) poles and zeros of each:

(a): zero at $\mathbf{s} = -2$, poles at $\mathbf{s} = -5, -3 \pm j4$
(b): zero at $\mathbf{s} = -2$, poles at $\mathbf{s} = -4, -5, -3 \pm j4$
(c): zeros at $\mathbf{s} = 0, -2$, poles at $\mathbf{s} = \pm j15, -5, -3 \pm j4$
(d): zero at $\mathbf{s} = -2$, poles at $\mathbf{s} = -1, -5, -3 \pm j4$

33. $h(t) = 5\,[u(t) - u(t-1)]\sin \pi t \qquad x(t) = 2[u(t) - u(t-2)]$

$$y(t) = \int_{0^-}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda$$

$t < 0$: $y(t) = \boxed{0}$

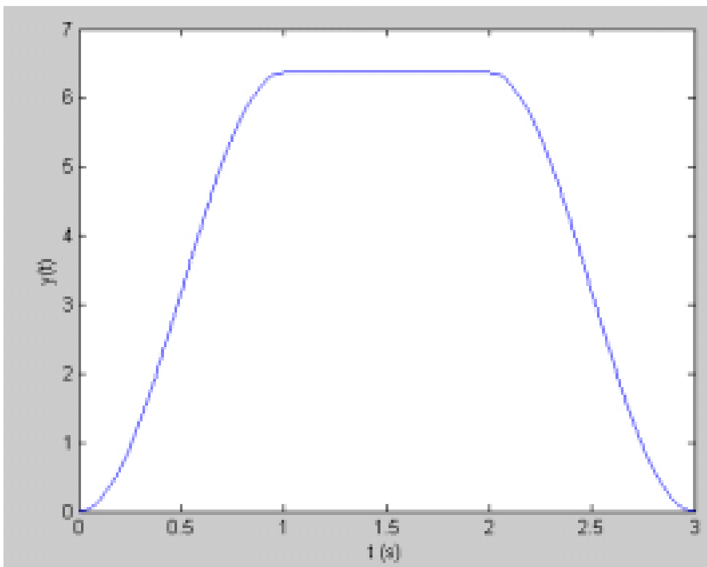
$$0 < t < 1:\ y(t) = \int_0^t 10\sin\pi\lambda\,d\lambda = -\frac{10}{\lambda}\cos\pi\lambda\Big|_0^t = \boxed{\frac{10}{\pi}(1-\cos\pi t)}$$

$$1 < t < 2:\ y(t) = \int_0^1 10\sin\pi\lambda\,d\lambda = \boxed{\frac{20}{\pi}}$$

$$2 < t < 3:\ y(t) = \int_{t-2}^1 10\sin\pi\lambda\,d\lambda = -\frac{10}{\pi}\cos\pi\lambda\Big|_{t-2}^1 = -\frac{10}{\pi}[-1-\cos(\pi t - 2\pi)]$$

$$= \boxed{(10/\pi)\,(1 + \cos \pi t)}$$

$t > 3$: $y(t) = \boxed{0}$



34. $f_1(t) = e^{-5t}\, u(t)$, $f_2(t) = (1 - e^{-2t})\, u(t)$

(a) $f_1 * f_2 = \int_{0^-}^{\infty} f_1(\lambda)\, f_2(t-\lambda)\, d\lambda$

$t < 0$: $f_1 * f_2 = 0$

$$t > 0:\; f_1 * f_2 = \int_{0^-}^{t} e^{-5\lambda}\left(1 - e^{2\lambda-2t}\right) d\lambda = \int_{0^-}^{t}\left(e^{-5\lambda} - e^{-2t}e^{-3\lambda}\right) d\lambda$$

$$= -\frac{1}{5}e^{-5\lambda}\Big|_0^t + \frac{1}{3}e^{-2t}e^{-3\lambda}\Big|_0^t = \boxed{\left(\frac{1}{5}+\frac{2}{15}e^{-5t}-\frac{1}{3}e^{-2t}\right)u(t)}$$

(b) $\mathbf{F}_1(\mathbf{s}) = 1/(\mathbf{s}+5)$, $\mathbf{F}_2(\mathbf{s}) = 1/\mathbf{s} - 1/(\mathbf{s}+2)$

$$\mathbf{F}_1(\mathbf{s})\, F_2(\mathbf{s}) = \frac{1}{\mathbf{s}(\mathbf{s}+5)} - \frac{1}{(\mathbf{s}+5)(\mathbf{s}+2)} = \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+2} + \frac{c}{\mathbf{s}+5}$$

Where a = 0.2, b = -1/3, and c = -1/5 + 1/3 = 2/15.

Taking the inverse transform, we find that $f_1 * f_2 = \boxed{\left(\frac{1}{5}+\frac{2}{15}e^{-5t}-\frac{1}{3}e^{-2t}\right)u(t)}$

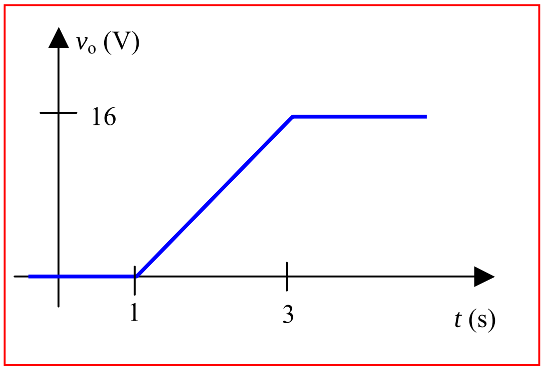
35. The impulse response is $v_o(t) = 4u(t) - 4u(t-2)$ V,
so we know that $h(t) = 4u(t) - 4u(t-2)$. $v_i(t) = 2u(t - 1)$, and $v_o(t) = h(t) * v_i(t)$.

Thus, $v_o(t) = \int_{0^-}^{\infty} h(\lambda) v_i(t-\lambda)\, d\lambda$

$t < 0$: $v_o = \boxed{0}$

$1 < t < 3$: $v_o = \int_{0^-}^{t-1} 8\, d\lambda = \boxed{8t - 8}$

$t > 3$: $v_o = \int_{0}^{2} 8\, d\lambda = \boxed{16}$



36. $h(t) = 2e^{-3t}\,u(t)$, $x(t) = u(t) - \delta(t)$

(a) $y(t) = \int_{0^-}^{\infty} h(\lambda)x(t-\lambda)d\lambda$

$$t < 0: y(t) = 0$$

$$t > 0: y(t) = 2\int_{0^-}^{t} e^{-3\lambda}\left[1-\delta(t-\lambda)\right]d\lambda \;=\; 2\left[-\frac{1}{3}e^{-3\lambda}u(t)\Big|_0^t \;-\; e^{-3t}u(t)\right]$$

$$= \frac{2}{3}(1-e^{-3t})u(t) \;-\; 2e^{-3t}u(t) \;=\; \boxed{\left(\frac{2}{3}-\frac{8}{3}e^{-3t}\right)u(t)}$$

(b) $\mathbf{H}(\mathbf{s}) = \dfrac{2}{\mathbf{s}+3}$ $\qquad \mathbf{X}(\mathbf{s}) = \dfrac{1}{\mathbf{s}} - 1$

thus, $\mathbf{Y}(\mathbf{s}) = \dfrac{2(1-\mathbf{s})}{\mathbf{s}(\mathbf{s}+3)} = \dfrac{2}{3}\left(\dfrac{1}{\mathbf{s}}\right) - \dfrac{8}{3}\left(\dfrac{1}{\mathbf{s}+3}\right)$

Taking the inverse transform, we find that $y(t) = \boxed{\frac{2}{3}u(t) \;-\; \frac{8}{3}e^{-3t}u(t)}$

37. $h(t) = 5\,u(t) - 5\,u(t-2)$, so $\mathbf{H(s)} = \frac{5}{\mathbf{s}} - 5e^{-2s}$

(a) $v_{in}(t) = 3\delta(t)$, so $\mathbf{V}_{in}(\mathbf{s}) = 3$

$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \boxed{\frac{15}{\mathbf{s}} - 15e^{-2s}}$. $v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = \boxed{15\,u(t) - 15\,u(t-2)}$

(b) $v_{in}(t) = 3u(t)$, so $\mathbf{V}_{in}(\mathbf{s}) = \frac{3}{\mathbf{s}}$

$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \left(\frac{3}{\mathbf{s}}\right)\left(\frac{5}{\mathbf{s}} - 5e^{-2s}\right) = \boxed{\frac{15}{\mathbf{s}^2} - \frac{15}{\mathbf{s}}e^{-2s}}$.

$v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = 15\,t\,u(t) - 15\,u^2(t-2) = \boxed{15\,t\,u(t) - 15\,u\,(t-2)}$

(c) $v_{in}(t) = 3u(t) - 3u(t-2)$, so $\mathbf{V}_{in}(\mathbf{s}) = \frac{3}{\mathbf{s}} - 3e^{-2s}$

$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \left(\frac{3}{\mathbf{s}} - 3e^{-2s}\right)\left(\frac{5}{\mathbf{s}} - 5e^{-2s}\right) = \boxed{\frac{15}{\mathbf{s}^2} - \frac{30}{\mathbf{s}}e^{-2s} + 15e^{-4s}}$.

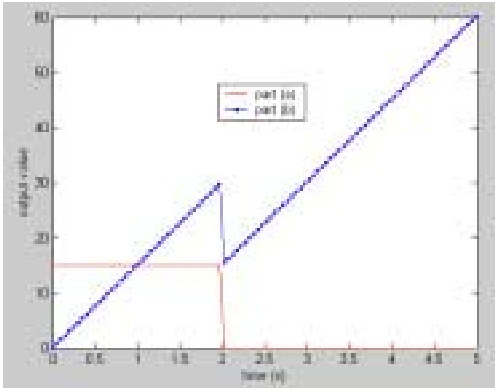
$v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = 15\,t\,u(t) - 30\,u^2(t-2) + 15\,u^2(t-4)$

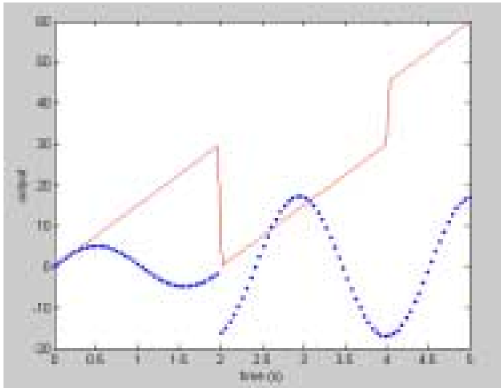
$= \boxed{15\,t\,u(t) - 30\,u\,(t-2) + 15\,u(t)}$

(d) $v_{in}(t) = 3\cos 3t$, so $\mathbf{V}_{in}(\mathbf{s}) = \frac{3\mathbf{s}}{\mathbf{s}^2 + 9}$

$\mathbf{V}_{out}(\mathbf{s}) = \mathbf{V}_{in}(\mathbf{s})\,\mathbf{H}(\mathbf{s}) = \boxed{\frac{15}{\mathbf{s}^2 + 9} - \frac{15\mathbf{s}}{\mathbf{s}^2 + 9}e^{-2s}}$.

$v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(\mathbf{s})\} = \boxed{5\sin 3t\,u(t) - 15\cos[3(t-2)]\,u(t-2)}$





38.

$$\mathbf{I}_{in} = \frac{\mathbf{V}_{in}}{\frac{10}{\mathbf{s}} + 20 \| 20\left(20 + \frac{10}{\mathbf{s}}\right)} = \frac{\mathbf{V}_{in}}{\frac{10}{\mathbf{s}} + \frac{20(20 + 10/\mathbf{s})}{40 + 10/\mathbf{s}}}$$

$$= \frac{\mathbf{V}_{in}}{\frac{10}{\mathbf{s}} + \frac{40\mathbf{s} + 20}{4\mathbf{s} + 1}} = \frac{\mathbf{V}_{in}}{\frac{40\mathbf{s}^2 + 60\mathbf{s} + 10}{4\mathbf{s}^2 + \mathbf{s}}} = \mathbf{V}_{in} \frac{40\mathbf{s}^2 + \mathbf{s}}{40\mathbf{s}^2 + 60\mathbf{s} + 10}$$

$$\therefore \mathbf{I}_{top} = \mathbf{I}_{in} \frac{20}{40 + \frac{10}{s}} = \mathbf{I}_{in} \frac{2\mathbf{s}}{4\mathbf{s} + 1} = \mathbf{V}_{in} \frac{2\mathbf{s}^2}{40\mathbf{s}^2 + 60\mathbf{s} + 10};$$

$$\mathbf{V}_{out} = \frac{10}{s}\mathbf{I}_{in} + 20\mathbf{I}_{top} = \mathbf{V}_{in}\left[\frac{4\mathbf{s} + 1}{4\mathbf{s}^2 + 6\mathbf{s} + 1} + \frac{4\mathbf{s}^2}{4\mathbf{s}^2 + 6\mathbf{s} + 1}\right] \therefore$$

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{4\mathbf{s}^2 + 4\mathbf{s} + 1}{4\mathbf{s}^2 + 6\mathbf{s} + 1} = \frac{\mathbf{s}^2 + \mathbf{s} + 0.25}{\mathbf{s}^2 + 1.5\mathbf{s} + 0.25} = \frac{(\mathbf{s} + 0.5)^2}{(\mathbf{s} + 0.19098)(\mathbf{s} + 1.3090)} \ \therefore$$

$$\boxed{\text{zeros: } \mathbf{s} = -0.5, s = -0.5; \text{ poles: } \mathbf{s} = -1.3090, -0.19098}$$

39.

(a)

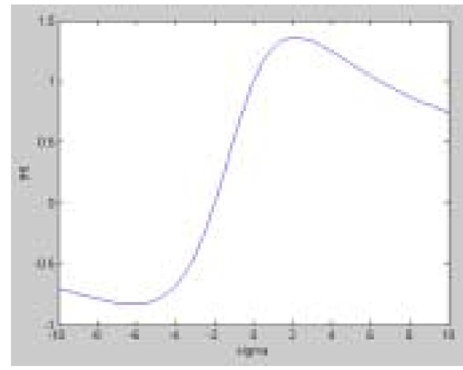
$$\mathbf{H}(\mathbf{s}) = \mathbf{V}_2(\mathbf{s})/\mathbf{V}_1(\mathbf{s}),\ \mathbf{H}(0) = 1$$

$$\therefore \mathbf{H}(\mathbf{s}) = \frac{\mathrm{K}(\mathbf{s}+2)}{(\mathbf{s}+1+j4)(\mathbf{s}+1-j4)} = \frac{\mathrm{K}(\mathbf{s}+2)}{\mathbf{s}^2+2\mathbf{s}+17}$$

$$1 = 2\frac{\mathrm{K}}{17},\ \text{so}\ \ \mathrm{K}{=}8.5$$

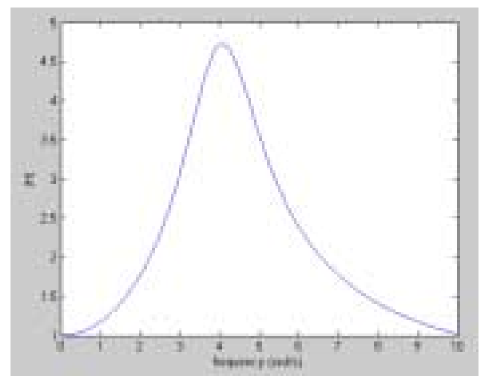
$$Thus,\ \ \mathbf{H}(\mathbf{s}) = \frac{8.5(\mathbf{s}+2)}{\mathbf{s}^2+2\mathbf{s}+17}$$

$$\text{Let}\ \omega = 0\ \ \therefore \mathbf{H}(\sigma) = \frac{8.5(\sigma+2)}{\sigma^2+2\sigma+17}$$



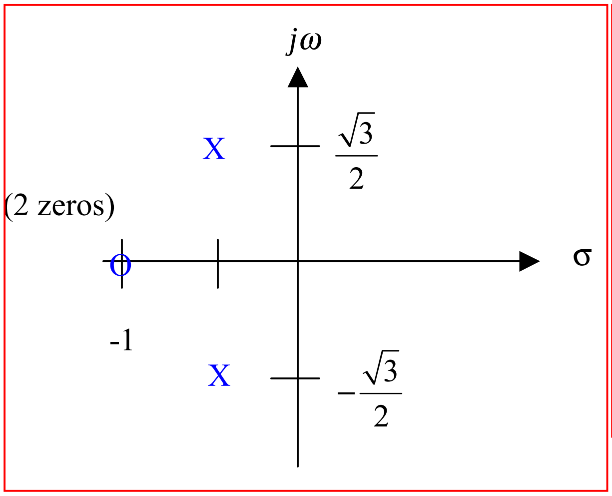
(b)

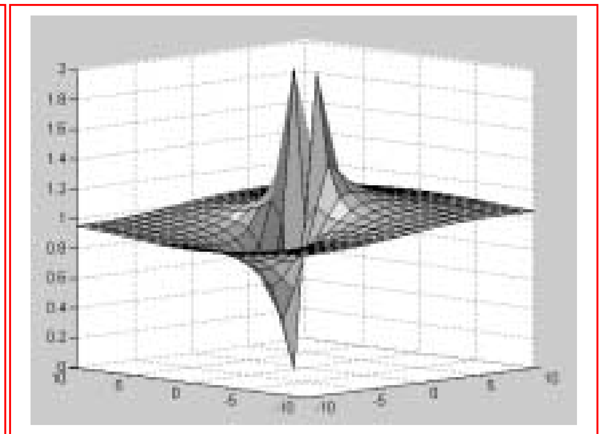
$$|H(j\omega)| = 8.5\sqrt{\frac{\omega^2+4}{(17-\omega^2)^2+4\omega^2}}$$



(c) By trial & error: $|\mathbf{H}(j\omega)|_{\max} = \boxed{4.729 \text{ at } \omega = 4.07 \text{ rad/s}}$

40. (a) pole-zero constellation (b) elastic-sheet model



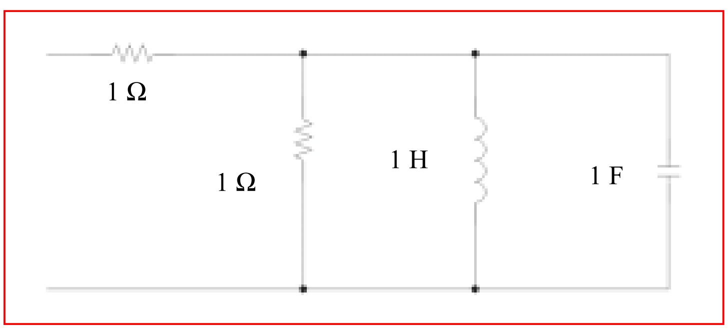


(c) $$\mathbf{H}(\mathbf{s}) = \frac{(\mathbf{s}+1)^2}{\left(\mathbf{s}+0.5+j\frac{\sqrt{3}}{2}\right)\left(\mathbf{s}+0.5-j\frac{\sqrt{3}}{2}\right)} = \frac{(\mathbf{s}+1)^2}{\mathbf{s}^2+\mathbf{s}+1}$$

$$= \frac{\mathbf{s}^2+2\mathbf{s}+1}{\mathbf{s}^2+\mathbf{s}+1} = 1 + \frac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+1}$$

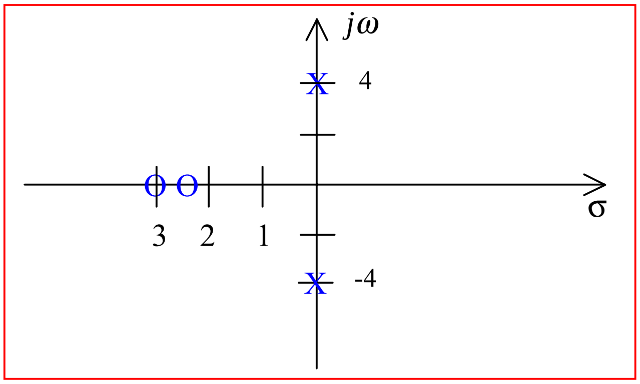
We can implement this with a 1-Ω resistor in series with a network having the impedance given by the second term. There are two energy storage elements in that network (the denominator is order 2). That network impedance can be rewritten as

$\frac{\mathbf{s}}{\mathbf{s}^2+\mathbf{s}+1} = \frac{1}{\mathbf{s}+1+\frac{1}{\mathbf{s}}}$, which can be seen to be equal to the parallel combination of a 1-Ω resistor, a 1-H inductor, and a 1-F capacitor.



41. $\mathbf{H}(\mathbf{s}) = (10\mathbf{s}^2 + 55\mathbf{s} + 75)/(\mathbf{s}^2 + 16)$

(a) $\mathbf{H}(\mathbf{s}) = 10\dfrac{(\mathbf{s}+3)(\mathbf{s}+2.5)}{(\mathbf{s}+j4)(\mathbf{s}-j4)}$. Critical frequencies: zeros at –3, -2.5; poles at $\pm j4$.

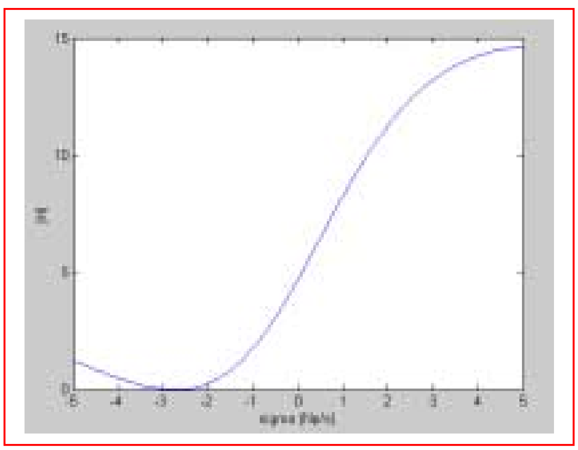


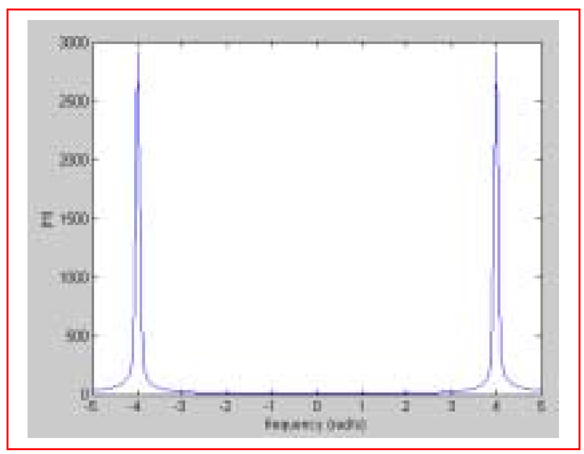
(b) $\mathbf{H}(0) = \dfrac{75}{16} = \boxed{4.688,}\ \mathbf{H}(\infty) = \boxed{10}$

(c) $\mathbf{H}(0) = 4.679\ \text{K} = 3, \text{ so }\ \text{K} = 0.64$

$$\therefore \mathbf{H}(j3) = 0.64\left|\frac{-90+75+j165}{7}\right| = \frac{0.64}{7}\left|-15+j165\right| = \boxed{15.15\ \ cm}$$

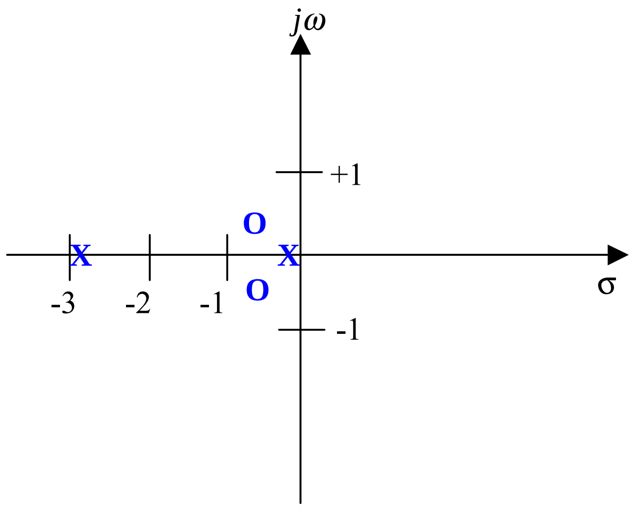
(d)



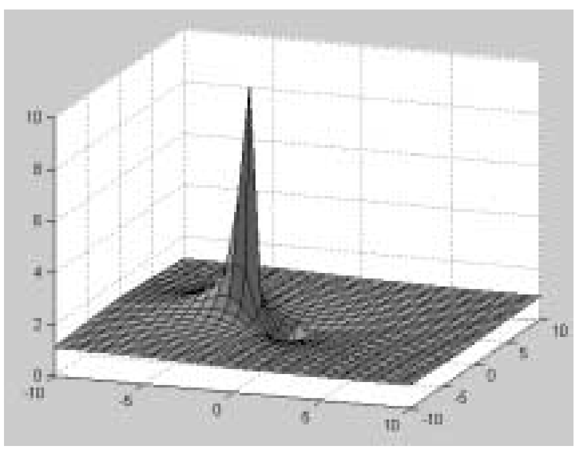


42. (a) $\mathbf{Y(s)} = \dfrac{5\mathbf{s}^2 + 5\mathbf{s} + 2}{5\mathbf{s}^2 + 15\mathbf{s} + 2} = \dfrac{(\mathbf{s} + 0.5 + j0.3873)(\mathbf{s} + 0.5 - j0.3873)}{(\mathbf{s} + 2.86)(\mathbf{s} + 0.1399)}$

Zeros: $\mathbf{s} = -0.5 \pm j0.3873$
Poles: $\mathbf{s} = -2.86$, $\mathbf{s} = -0.1399$



(b) elastic sheet model



(c) lattitude 5°5’2”, longitude 5°15’2” puts it a little off the coast of Timbuktu.

43. $\mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{I}_0}{\mathbf{I}_M}$; $\mathbf{H}(-2) = 6$

(a) $$\mathbf{H}(\mathbf{s}) = \text{K}\frac{(\mathbf{s}-1)(\mathbf{s}+1)(\mathbf{s}+3)}{(\mathbf{s}+3+j2)(\mathbf{s}+3-j2)}$$

$$\mathbf{H}(-2) = 6 = \frac{(-3)(-1)\text{K}}{(1+j2)(1-j2)} = \frac{3\text{K}}{5} \quad \therefore \text{K} = 10,$$

$$\textit{Thus,}\quad \mathbf{H}(\mathbf{s}) = 10\frac{(\mathbf{s}^2-1)(\mathbf{s}+3)}{\mathbf{s}^2+6\mathbf{s}+13} = \boxed{\frac{10\mathbf{s}^3+30\mathbf{s}^2-10\mathbf{s}-30}{\mathbf{s}^2+6\mathbf{s}+13}}$$

(b) $$\mathbf{H}(0) = -\frac{30}{13} = \boxed{-2.308,}\ \mathbf{H}(\infty) = \boxed{\infty}$$

(c) $$1:(s-1) = (j2-1) = \boxed{2.236\angle 116.57^\circ}$$

$$-1:(s+1) = (j2+1) = \boxed{2.236\angle 63.43^\circ}$$

$$-3:(s+3) = j2+3 = \boxed{3.606\angle 33.69^\circ}$$

$$-3-j2:\ j2+3+j2 = \boxed{5.000\angle 53.13^\circ}$$

$$-3+j2:\ j2+3-j2 = \boxed{3\angle 0^\circ}$$

44.

$\mathbf{Z}_A$: zero at $\mathbf{s} = -10 + j0$; $\mathbf{Z}_A + 20$: zero at $\mathbf{s} = -3.6 + j0$

$$\therefore \mathbf{Z}_A = 5 + \frac{\text{R}/\mathbf{s}\text{C}}{\text{R} + 1/\text{SC}} = 5 + \frac{\text{R}}{\mathbf{s}\text{CR} + 1} = 5 + \frac{1/\text{C}}{\mathbf{s} + 1/\text{RC}} = \frac{5\mathbf{s} + 5/\text{RC} + 1/\text{C}}{\mathbf{s} + 1/\text{RC}}$$

$$\therefore \mathbf{Z}_A = \frac{5(\mathbf{s} + 1/\text{RC} + 1/5\text{C})}{\mathbf{s} + 1/\text{RC}}$$

Thus, using the fact that $\mathbf{Z}_A = 0$ at $\mathbf{s}$ = -10, we may write $\frac{1}{\text{RC}} + \frac{1}{5\text{C}} = 10$

$$\text{Also, } \mathbf{Z}_B = 25 + \frac{1/\text{C}}{\mathbf{s} + 1/\text{RC}} = \frac{25\mathbf{s} + \frac{25}{\text{RC}} + \frac{1}{\text{C}}}{\mathbf{s} + 1/\text{RC}} = \frac{25\left(\mathbf{s} + \frac{1}{\text{RC}} + \frac{1}{25\text{C}}\right)}{\mathbf{s} + \frac{1}{\text{RC}}}$$

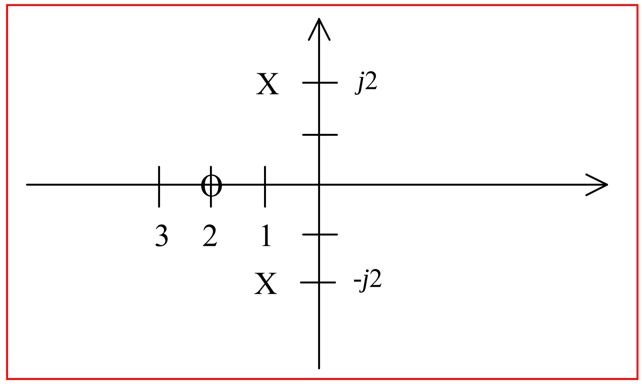
$$\therefore \frac{1}{\text{RC}} + \frac{1}{25\text{C}} = 3.6 \quad \text{or} \quad \frac{4}{25\text{C}} = 6.4,$$

$$\text{C} = \frac{1}{40} = \boxed{25 \text{ mF},}$$

$$\frac{40}{\text{R}} + \frac{40}{5} = 10, \; \frac{40}{\text{R}} = 2, \quad \text{so} \quad \text{R} = \boxed{20 \;\Omega}$$

45. $\mathbf{H}(\mathbf{s}) = 100(\mathbf{s}+2)/(\mathbf{s}^2+2\mathbf{s}+5)$

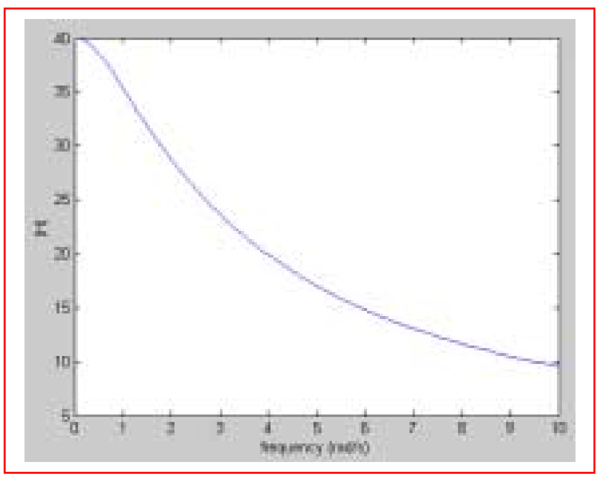
(a) zero at $\mathbf{s}$ = -2, poles at $\mathbf{s} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2$



(b) $\mathbf{H}(j\omega) = \boxed{\dfrac{100(2+j\omega)}{(5-\omega^2)+j2\omega}}$

(c) $|\mathbf{H}(j\omega)| = \boxed{100\sqrt{\dfrac{\omega^2+4}{\omega^4-6\omega^2+25}}}$

(d)



(e) $\dfrac{|\mathbf{H}(j\omega)|^2}{10\ 000} = \dfrac{\omega^2+4}{\omega^4-6\omega^2+25}, \quad \dfrac{d\,|\mathbf{H}(j\omega)|^2}{d\omega} = \dfrac{(\omega^4-6\omega^2+25)2\omega-(\omega^2+4)(4\omega^3-12\omega)}{etc}$

$\therefore \omega^4-6\omega^2+25 = (\omega^2+4)(2\omega^2-6), \ \omega^4-6\omega^2+25 = 2\omega^4+2\omega^2-24, \ \omega^4+8\omega^2-49=0$

$\therefore \omega^2 = \dfrac{-8 \pm \sqrt{64+196}}{2} = 4.062 \ \therefore \omega_{mar} = \boxed{2.016\text{ rad/s}}, \ |\mathrm{H}(j2.016)| = \boxed{68.61}$

46. $\mathbf{Z}_{in}(\mathbf{s}) = \dfrac{5\mathbf{s}+20}{\mathbf{s}+2}\ \ \Omega$

(a) $v_{ab}(0) = 25\,\text{V};\ \mathbf{Z}_{in}(\mathbf{s}) = \dfrac{5(\mathbf{s}+4)}{\mathbf{s}+2},\ \ \mathbf{V}_{ab} = \mathbf{Z}_{in}\mathbf{I}_{in}$

$\therefore \mathbf{H}(\mathbf{s}) = \dfrac{5(\mathbf{s}+4)}{\mathbf{s}+2},\quad \text{single pole at } \mathbf{s} = -2\ \therefore v_{ab}(t) = \boxed{25e^{-2t}\,\text{V},\ t > 0}$

(b) $i_{ab}(0) = 3\,\text{A}\ \ \therefore \mathbf{I}_{ab} = \dfrac{\mathbf{V}_s}{\mathbf{Z}_{in}}\ \ \therefore \mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{I}_{ab}}{\mathbf{V}_{in}} = \dfrac{1}{\mathbf{Z}_{in}} = \dfrac{\mathbf{s}+2}{5(\mathbf{s}+4)}$ single pole at $\mathbf{s} = -4$

$\therefore i_{ab}(t) = \boxed{3e^{-4t}\,\text{A},\ t > 0}$

47. $\mathbf{Z}_{in}(\mathbf{s}) = 5(\mathbf{s}^2 + 4\mathbf{s} + 20)/(\mathbf{s}+1)$

(a) $v_{ab} = 160e^{-6t}\text{V} \;\therefore \mathbf{V}_{ab} = 160\,\text{V},\ \mathbf{s} = -6$

$$\mathbf{I}_a = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{in}} = \frac{160(\mathbf{s}+1)}{5(\mathbf{s}^2 + 4\mathbf{s} + 20)} = \frac{32(-5)}{3b - 24 + 20} = -5\,\text{A} \;\therefore i_a(t) = \boxed{-5e^{-6t}\text{A (all}\,t)}$$

(b) $v_{ab} = 160e^{-6t}u(t),\ i_a(0) = 0, i_a'(0) = 32\,\text{A/s} \;\therefore \mathbf{H}(\mathbf{s}) = \dfrac{\mathbf{I}_a}{\mathbf{V}_s} = \dfrac{1}{\mathbf{Z}_{in}} = \dfrac{\mathbf{s}+1}{5(\mathbf{s}^2 + 4\mathbf{s} + 20)}$

$$\mathbf{s} = \frac{-4 \pm \sqrt{16-80}}{2} = -2 \pm j4 \;\therefore i_a(t) = -5e^{-6t} + e^{-2t}(\text{A}\cos 4t + \text{B}\sin 4t) \;\therefore 0 = -5 + \text{A},\ \text{A} = 5$$

$$i_a'(0) = 32 = 30 - 10 + 4\text{B} \;\therefore \text{B} = 3 \;\therefore i_a(t) = \boxed{[-5e^{-6t} + e^{-2t}(5\cos 4t + 3\sin 4t)]u(t)\,\text{A}}$$

48.

(a) $\mathbf{H}(\mathbf{s}) = \mathbf{I}_c / \mathbf{I}_s = \dfrac{0.5}{0.5 + 0.002\mathbf{s} + 500/\mathbf{s}} = \boxed{\dfrac{250\mathbf{s}}{\mathbf{s}^2 + 250\mathbf{s} + 25\ 000}}$

(b) $\mathbf{s} = \dfrac{1}{2}(-250 \pm \sqrt{62\ 500 - 10^6}) = \boxed{-125 \pm j484.1\text{s}^{-1}}$

(c) $\alpha = \dfrac{\text{R}}{2\text{L}} = \dfrac{0.5}{0.004} = \boxed{125\ \text{s}^{-1}}$, $\omega_o = \sqrt{10^6/4} = \boxed{500\ \text{s}^{-1}}$, $\omega_d = \sqrt{25\times10^4 - 15{,}625} = \boxed{484.1\ \text{s}^{-1}}$

(d) $\mathbf{I}_s = 1, \mathbf{s} = 0 \ \therefore \text{I}_c = 0 \ \therefore i_{cf} = \boxed{0}$

(e) $i_{c,n} = \boxed{e^{-125t}(A\cos 484t + \text{B}\sin 484t)}$

(f) $i_L(0) = 0 \ \therefore i_c(0^+) = 0, \ v_c(0) = 0 \ \therefore 1\times\dfrac{1}{2} = 2\times10^{-3} i(0^+) + 0 \ \therefore i(0^+) = \boxed{250\text{ A/s}}$

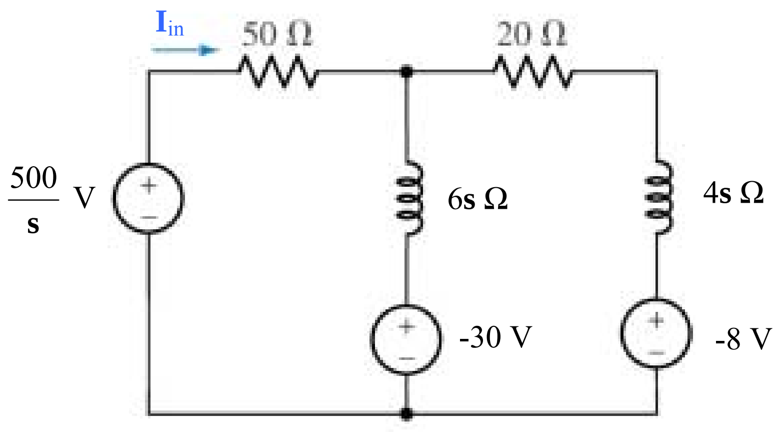
(g) $\therefore \text{A} = 0, \ 484\text{B} = 250, \ \text{B} = 0.5164 \ \therefore i_c(t) = \boxed{(0.5164 e^{-125t} \sin 484.1t)\ u(t)\text{ A}}$

49.

(a) $$\mathbf{H}(\mathbf{s}) = \mathbf{I}_{in} / \mathbf{V}_{in} = \frac{1}{\mathbf{Z}_{in}} = \frac{1}{50 + \dfrac{6\mathbf{s}(4\mathbf{s}+20)}{10\mathbf{s}+20}} = \frac{10\mathbf{s}+20}{24\mathbf{s}^2 + 620\mathbf{s} + 1000}$$

$$\therefore \mathbf{s} = \frac{1}{48}(-620 \pm \sqrt{620^2 - 96,000}) = \boxed{-1.729 \text{ and } -24.10 \text{ s}^{-1}}$$

(b) Note that the element labeled 6 H should be an inductor, as is suggested by the context of the text (i.e. initial condition provided). Convert to s-domain and define a clockwise mesh current $\mathbf{I}_2$ in the right-hand mesh.



Mesh 1: $0 = -500/\mathbf{s} + (50 + 6\mathbf{s})\,\mathbf{I}_{in} - 30 - 6\mathbf{s}\,\mathbf{I}_2$ [1]
Mesh 2: $0 = 30 + (20 + 10\mathbf{s})\,\mathbf{I}_2 - 6\mathbf{s}\,\mathbf{I}_{in} - 8$ [2]

Solving, we find that $$\mathbf{I}_{in} = \frac{42\mathbf{s}^2 + 1400\mathbf{s} + 2500}{\mathbf{s}(6\mathbf{s}^2 + 155\mathbf{s} + 250)} = \frac{7\mathbf{s}^2 + 233.3\mathbf{s} + 416.7}{\mathbf{s}(\mathbf{s} + 24.10)(\mathbf{s} + 1.729)}$$

$$= \frac{a}{\mathbf{s}} + \frac{b}{(\mathbf{s} + 24.10)} + \frac{c}{(\mathbf{s} + 1.729)}$$

where a = 10, b = -2.115 and c = -0.8855. Thus, we may write

$$\boxed{i_{in}(t) = [10 - 2.115\, e^{-24.10t} - 0.885\, e^{-1.729t}]\, u(t) \text{ A}}$$

50.

(a) $\mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}}{\mathbf{I}_s} = \frac{50(1000/\mathbf{s})}{50 + (1000/\mathbf{s})} = \boxed{\frac{1000}{\mathbf{s}+20}}$

(b)

$$\mathbf{I}_s = \frac{2}{\mathbf{s}} \quad \text{so} \quad \mathbf{V}(\mathbf{s}) = \left(\frac{2}{\mathbf{s}}\right)\left(\frac{1000}{(\mathbf{s}+20)}\right) = \frac{2000}{\mathbf{s}(\mathbf{s}+20)} = \frac{a}{s} + \frac{\text{b}}{\mathbf{s}+20}$$

$$a = \left.\frac{2000}{(\mathbf{s}+20)}\right|_{\mathbf{s}=0} = 100; \quad \text{b} = a = \left.\frac{2000}{(\mathbf{s})}\right|_{\mathbf{s}=-20} = -100$$

$$\textit{Thus,} \quad \mathbf{V}(\mathbf{s}) = \frac{100}{s} - \frac{100}{\mathbf{s}+20} \quad \text{and} \quad \boxed{v(t) = 100\left[1 - \text{e}^{-20t}\right]u(t)\ \text{V}}$$

(c) This function as written is technically valid for all time (although that can't be possible physically). Therefore, we can't use the one-sided Laplace technique we've been studying. We can, however, use simple s-domain/ complex frequency analysis:

$$i_s = 4e^{-10t}\ \text{A} \ \therefore \mathbf{I}_s = 4\,\text{A},\ \mathbf{s} = 10 \ \therefore \mathbf{V} = 4\mathbf{H}(-10) = 4 \times \frac{1000}{10} = 400\,\text{V} \ \therefore$$

$$\boxed{v(t) = 400e^{-10t}\ \text{V (all } t)}$$

(d) $$4\text{e}^{-10t}\,u(t) \Leftrightarrow \frac{4}{\mathbf{s}+10}, \text{ so } \mathbf{V}(\mathbf{s}) = \left(\frac{4}{\mathbf{s}+10}\right)\left(\frac{1000}{\mathbf{s}+20}\right) = \frac{\text{a}}{\mathbf{s}+10} + \frac{\text{b}}{\mathbf{s}+20}$$

a = 400 and b = -400, so $v(t) = \boxed{400\,[\text{e}^{-10t} - \text{e}^{-20t}]\,u(t)\ \text{V}}$

51.

(a)

$$\mathbf{H(s)} = \frac{\mathbf{V}_{c2}}{\mathbf{V}_s} = \frac{\dfrac{100}{\mathbf{s}}}{20+\dfrac{100}{\mathbf{s}}} \times \frac{\left(20+\dfrac{100}{\mathbf{s}}\right)\dfrac{25}{\mathbf{s}}}{20+\dfrac{125}{\mathbf{s}}} \Big/ \left[50+\frac{(20\mathbf{s}+100)25}{\mathbf{s}(20\mathbf{s}+125)}\right]$$

$$\therefore \mathbf{H(s)} = \frac{2500}{\mathbf{s}(20\mathbf{s}+125)}\ \frac{\mathbf{s}(20\mathbf{s}+125)}{1000\mathbf{s}^2+6250\mathbf{s}+500\mathbf{s}+2500}$$

$$\therefore \mathbf{H(s)} = \boxed{\frac{2.5}{\mathbf{s}^2+6.75\mathbf{s}+2.5}}$$

(b) No initial energy stored in either capacitor. With $v_s = u(t)$, $\mathbf{V}_s(\mathbf{s}) = \dfrac{1}{\mathbf{s}}$, so

$$\mathbf{V}_{C2} = \frac{2.5}{\mathbf{s}(\mathbf{s}+6.357)(\mathbf{s}+0.3933)} = \frac{a}{\mathbf{s}} + \frac{b}{\mathbf{s}+6.357} + \frac{c}{\mathbf{s}+0.3933}$$

Where a = 1, b = 0.06594 and c = -1.066. Thus,

$$v_{C2}(t) = \boxed{[1 + 0.06594\ e^{-6.357t} - 1.066\ e^{-0.3933t}]\ u(t)\ \text{V}}$$

52.

$$\mathbf{Z}_{in}(\mathbf{s}) = \frac{1}{0.1 + 0.025\mathbf{s} + \dfrac{1}{20 + (80/\mathbf{s})}} = \frac{1}{0.1 + 0.025\mathbf{s} + \dfrac{0.05\mathbf{s}}{\mathbf{s}+4}}$$

$$= \frac{\mathbf{s}+4}{0.025\mathbf{s}^2 + 0.25\mathbf{s} + 0.4} = \frac{40(\mathbf{s}+4)}{\mathbf{s}^2 + 10\mathbf{s} + 16} = \frac{40(\mathbf{s}+4)}{(\mathbf{s}+2)(\mathbf{s}+8)}\,\Omega$$

$20u(t) \Leftrightarrow \dfrac{20}{\mathbf{s}}$, so $\mathbf{V}_{\text{in}}(\mathbf{s}) = \left(\dfrac{20}{\mathbf{s}}\right)\left[\dfrac{40(\mathbf{s}+4)}{(\mathbf{s}+2)(\mathbf{s}+8)}\right] = \dfrac{\text{a}}{\mathbf{s}} + \dfrac{b}{\mathbf{s}+2} + \dfrac{c}{\mathbf{s}+8}$

a = 200, b = -133.3 and c = -66.67, so $\boxed{v_{\text{in}}(t) = [200 - 133.3\ e^{-2t} - 66.67\ e^{-8t}]\ u(t)\ \text{V}}$

53.

$$\mathbf{H}(\mathbf{s}) = -\frac{\mathbf{Z}_f}{\mathbf{Z}_1}$$

(a) $\mathbf{Z}_1 = 10^3 + \dfrac{10^8}{\mathbf{s}}$, $\mathbf{Z}_f = 5000 \;\therefore \mathbf{H}(\mathbf{s}) = -\dfrac{5000}{1000 + (10^8/\mathbf{s})} = -\dfrac{5000\mathbf{s}}{1000\mathbf{s} + 10^8}$

$$\therefore \boxed{\mathbf{H}(s) = \frac{-5\mathbf{s}}{\mathbf{s} + 10^5}}$$

(b) $\mathbf{Z}_1 = 5000$, $\mathbf{Z}_f = 10^3 + 10^8/\mathbf{s} \;\therefore \mathbf{H}(\mathbf{s}) = -\dfrac{10^3 + 10^8/\mathbf{s}}{5000} = -\dfrac{1000\mathbf{s} + 10^8}{5000\mathbf{s}} = \boxed{-\dfrac{\mathrm{R} + 10^5}{5\mathbf{s}}}$

(c) $\mathbf{Z}_1 = 10^3 + 10^8/\mathbf{s}$, $\mathbf{Z}_f = 10^4 + 10^8/\mathbf{s} \;\therefore \mathbf{H}(\mathbf{s}) = -\dfrac{10^4 + 10^8/\mathbf{s}}{1000 + 10^8/\mathbf{s}} = -\dfrac{10^4\mathbf{s} + 10^8}{1000\mathbf{s} + 10^8} = \boxed{-\dfrac{10\mathbf{s} + 10^5}{\mathbf{s} + 10^5}}$

54.

$$\mathrm{R}_f = 20\ \mathrm{k}\Omega,\ \mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = -\mathrm{R}_f \mathrm{C}_1\left(\mathbf{s} + \frac{1}{\mathrm{R}_1\mathrm{C}_1}\right)$$

$$\therefore \mathbf{H}(\mathbf{s}) = -2\times10^4\mathrm{C}_1\left(\mathbf{s} + \frac{1}{\mathrm{R}_1\mathrm{C}_1}\right)$$

(a) $\mathbf{H}(\mathbf{s}) = -50 \therefore \boxed{\mathrm{C}_1 = 0,}\ \dfrac{2\times10^4}{\mathrm{R}_1} = 50,\quad \mathrm{R}_1 = \boxed{400\,\Omega}$

(b) $\mathbf{H}(\mathbf{s}) = -10^{-3}(\mathbf{s}+10^4) = -2\times10^4\,\mathrm{C}_1\left(\mathbf{s} + \dfrac{1}{\mathrm{R}_1\mathrm{C}_1}\right) \therefore 2\times10^4\mathrm{C}_1 = 10^{-3}$

$\therefore \boxed{\mathrm{C}_1 = 50\,\mathrm{nF};}\quad \dfrac{1}{50\times10^{-9}\mathrm{R}_1} = 10^4$, so $\boxed{\mathrm{R}_1 = 2\ \mathrm{k}\Omega}$

(c) $\mathbf{H}(\mathbf{s}) = -10^{-4}(\mathbf{s}+1000) = -2\times10^4\mathrm{C}_1\left(\mathbf{s} + \dfrac{1}{\mathrm{R}_1\mathrm{C}_1}\right) \therefore 2\times10^4\mathrm{C}_1 = 10^{-4},\ \mathrm{C}_1 = 5\,\mathrm{nF}$

$\therefore v_{in}(t) = 200 + \mathrm{A}e^{-2t} + \mathrm{B}e^{-8t}, v_{in}(0) = 0\ \therefore 0 = 200 + \mathrm{A} + \mathrm{B}$

$i_{25}(0^+) = 20 = 0.025v'_{in}(0^+) = 0.025(-2\mathrm{A} - 8\mathrm{B}) \therefore -\mathrm{A} - 4\mathrm{B} = 400 \therefore -3\mathrm{B} = 200$

$\mathrm{B} = -\dfrac{200}{3},\ \therefore \mathrm{A} = -\dfrac{400}{3}\ \therefore \boxed{v_{in}(t) = \left(200 - \dfrac{400}{3}e^{-2t} - \dfrac{200}{3}e^{-8t}\right)u(t)\ \mathrm{V}}$

55.

(a)

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = -50,$$

$$\mathbf{H}(\mathbf{s}) = \frac{-1/R_1C_f}{\mathbf{s}+1/R_fC_f},\ R_f = 20\ \text{k}\Omega$$

set $\boxed{C_f = 0}$ $\therefore -50 = -\frac{R_f}{R_1}\ \therefore R_1 = \frac{20\times10^3}{50} = \boxed{400\ \Omega}$

(b)

$$\mathbf{H}(\mathbf{s}) = -\frac{1000}{\mathbf{s}+10\ 000} = \frac{1/R_1C_f}{\mathbf{s}+1/20\ 000C_f}\ \therefore 10\ 000 = \frac{1}{20\ 000C_f}$$

$C_f = \frac{1}{2\times10^8} = \boxed{5\ \text{nF}}$ We may then find R_1: $1000 = \frac{1}{5\times10^{-9}R_1}$ $\therefore \boxed{R_1 = 200\ \text{k}\Omega}$

(c)

$$\mathbf{H}(\mathbf{s}) = -\frac{10\ 000}{\mathbf{s}+1000} = \frac{1/R_1C_f}{\mathbf{s}+1/20\ 000C_f}\ \therefore 1000 = \frac{1}{20\ 000C_f}\ C_f = \boxed{50\ \text{nF}}$$

$$\frac{1}{5\times10^{-9}R_1} = 1000,\ R_1 = \boxed{200\ \text{k}\Omega}$$

(d)

$$\mathbf{H}(\mathbf{s}) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{100}{\mathbf{s}+10^5}$$

$$= \left[-\frac{1/R_{1A}C_{fA}}{s+1/R_{fA}C_{fA}}\right]\left[-\frac{1/R_{1B}C_{fB}}{s+1/R_{fB}C_{fB}}\right] = \left[-\frac{1/R_{1A}C_{fA}}{s+1/R_{fA}C_{fA}}\right]\left[-\frac{R_{fB}}{R_{1B}}\right]$$

We may therefore set $\frac{R_{fB}}{R_{1B}R_{1A}C_{fA}} = 100$

and $1/R_{fA}C_{fA} = 10^5$. Arbitrarily choosing $\boxed{R_{fA} = 1\ \text{k}\Omega}$, we find that $\boxed{C_{fA} = 10\ \text{nF}}$.

Arbitrarily selecting $\boxed{R_{fB} = 100\ \Omega}$, we may complete the design by choosing $\boxed{R_{1B} = R_{1A} = 10\ \text{k}\Omega}$

56.

$$\mathbf{H}(\mathbf{s}) = \frac{-10^{-4}\mathbf{s}(\mathbf{s}+100)}{\mathbf{s}+1000} = \frac{[-K_A\mathbf{s}][-K_B(\mathbf{s}+100)]}{\left(-\dfrac{K_C}{\mathbf{s}+1000}\right)}$$

Let $\mathbf{H}_A(\mathbf{s}) = -K_A\mathbf{s}$. Choose inverting op amp with parallel RC network at inverting input.

$$0 = \frac{-V_i}{R_{1A}}(1+\mathbf{s}C_{1A}) - \frac{V_o}{R_{fa}}$$

$$\therefore \mathbf{H}_A(\mathbf{s}) = -\frac{R_{fA}}{R_{1A}}(1+\mathbf{s}R_{1A}C_{1A}) = -\frac{R_{fA}}{R_{1A}} - \mathbf{s}R_{fA}C_{1A} = -K_A\mathbf{s}. \text{ Set } \boxed{R_{1A} = \infty.} \text{ Then}$$

$$-R_{fA}C_{1A}\mathbf{s} = -10^4 C_{1A}\mathbf{s}$$

Same configuration for $\mathbf{H}_B(\mathbf{s})$ $\therefore \mathbf{H}_B(\mathbf{s}) = -K_B(\mathbf{s}+100) = -\dfrac{R_{fB}}{R_{1B}}(1+\mathbf{s}R_{1B}C_{1B})$

For the last stage, choose an inverting op amp circuit with a parallel RC circuit in the feedback loop.

Let $\mathbf{H}_C(\mathbf{s}) = -K_C\dfrac{1}{\mathbf{s}+1000} = -\dfrac{R_{fC}}{R_{1C}}\dfrac{1}{(1+\mathbf{s}R_{FC}C_{FC})}$

Cascading these three tranfer functions, we find that

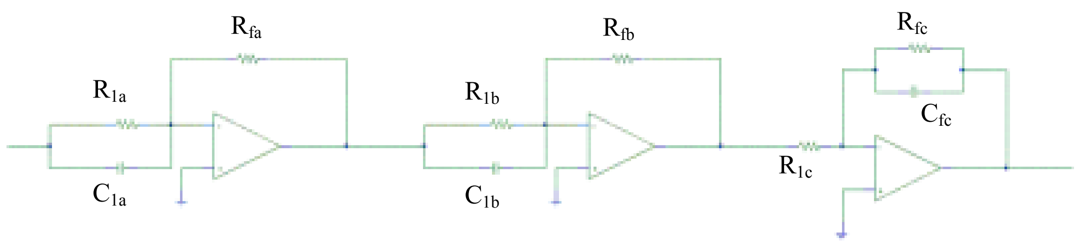
$$\mathbf{H}_A\,\mathbf{H}_B\,\mathbf{H}_C = \left[-\mathbf{s}R_{fA}C_{1A}\right]\left[-\left(R_{fB}C_{1B}\mathbf{s} + \frac{R_{fB}}{R_{1B}}\right)\right]\left[-\left(\frac{R_{fC}}{R_{1C}}\right)\frac{1}{R_{fc}C_{fc}\mathbf{s}+1}\right]$$

Choosing $\boxed{\text{all remaining resistors to be 10 k}\Omega\text{,}}$ we compare this to our desired transfer function.

$(R_{fc}\,C_{fc})^{-1} = 1000$ so $\boxed{C_{fc} = 100 \text{ nF}}$

Next, $\dfrac{R_{fB}}{R_{1B}\;R_{fB}\;C_{1B}} = 100$ so $\boxed{C_{1B} = 1\ \mu\text{F.}}$

Finally, $R_{fA}C_{1A}R_{fB}C_{1B}R_{fC}\,(R_{1C}R_{fC}C_{fC}) = 10^{-4}$, so $\boxed{C_{1A} = 1 \text{ nF}}$



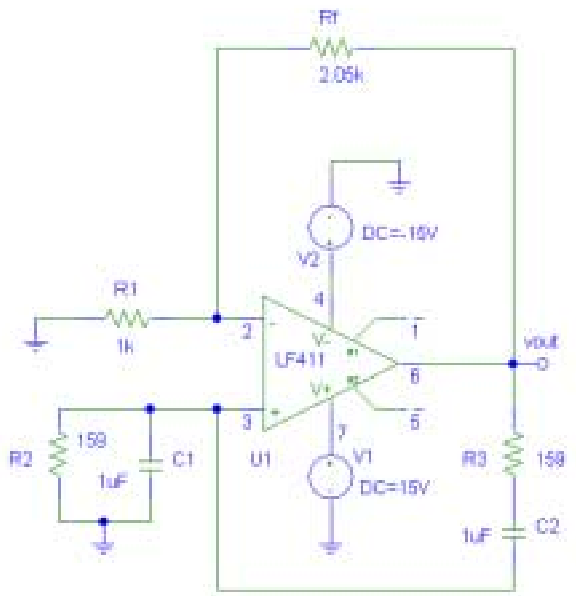
57. Design a Wien-bridge oscillator for operation at 1 kHz, using only standard resistor values. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 1000$$

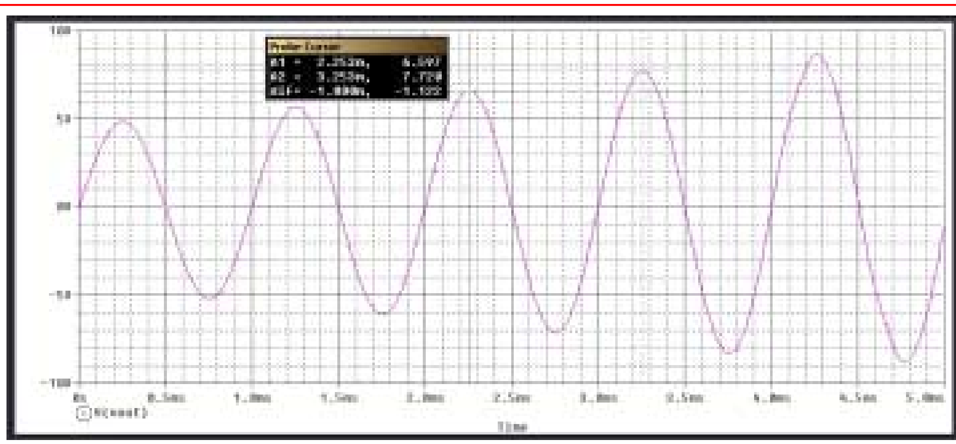
If we use a 1-μF capacitor, then R = 159 Ω. To construct this using standard resistor values, connect a 100-Ω, 56-Ω and 3-Ω in series.

To complete the design, select R_f = 2 kΩ and R_1 = 1 kΩ.

PSpice verification:



The feedback resistor was set to 2.05 kΩ to initiate oscillations in the simulation. The output waveform shown below exhibits a frequency of 1 kHz as desired.



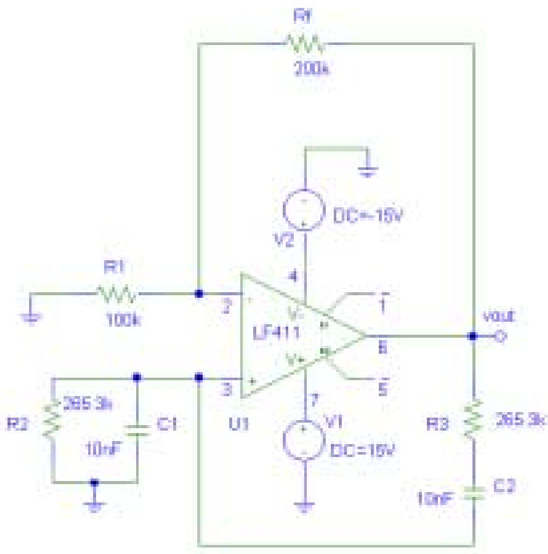
58. Design a Wien-bridge oscillator for operation at 60 Hz. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 60$$

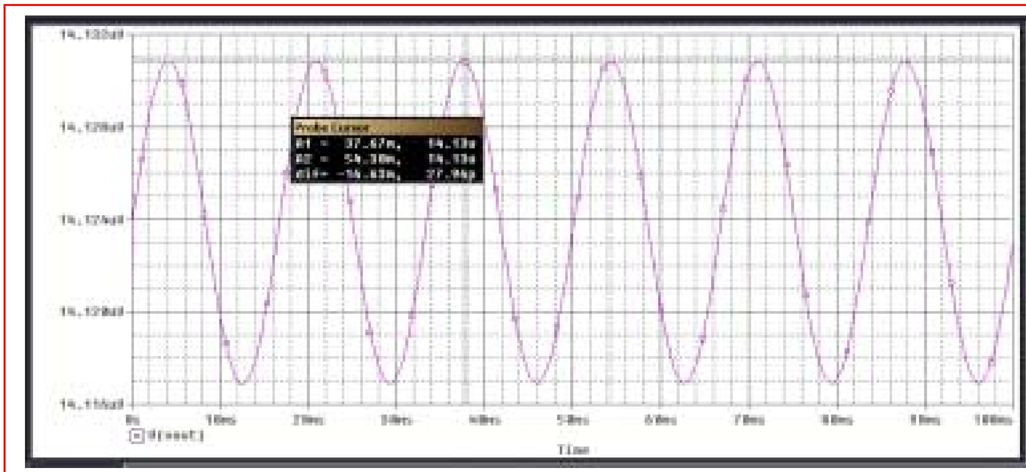
If we use 10-nF capacitors, then R = 265.3 kΩ.

To complete the design, select R_f = 200 kΩ and R_1 = 100 kΩ.

PSpice verification:



The simulated output of the circuit shows a sinusoidal waveform having period 54.3 ms – 37.67 ms = 0.01663 ms, which corresponds to a frequency of 60.13 Hz, as desired.



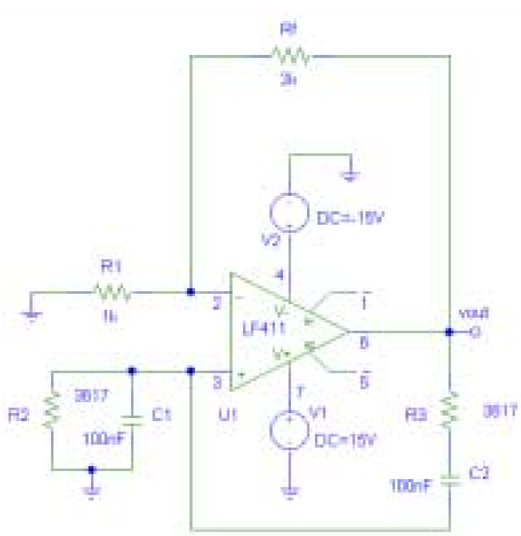
59. Design a Wien-bridge oscillator for operation at 440 Hz, using only standard resistor values. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 440$$

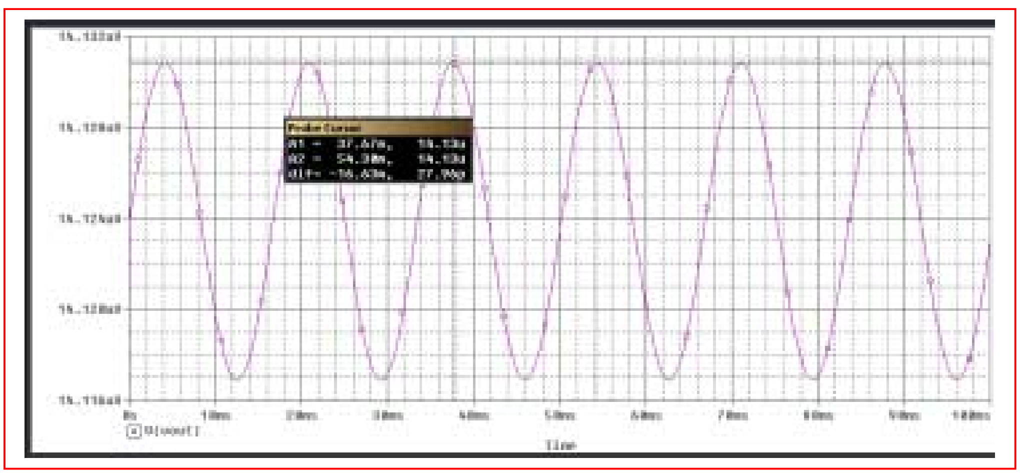
If we use 100-nF capacitors, then R = 3.167 kΩ. To construct this using standard resistor values, connect a 3.6-kΩ, 16-Ω and 1-Ω in series. (May not need the 1-Ω, as we're using 5% tolerance resistors!). This circuit will produce the musical note, 'A.'

To complete the design, select R_f = 2 kΩ and R_1 = 1 kΩ.

PSpice verification:



Simulation results show a sinusoidal output having a period of approximately 5.128 – 2.864 = 2.264 ms, or a frequency of approximately 442 Hz. The error is likely to uncertainty in cursor placement; a higher-resolution time simulation would enable greater precision.



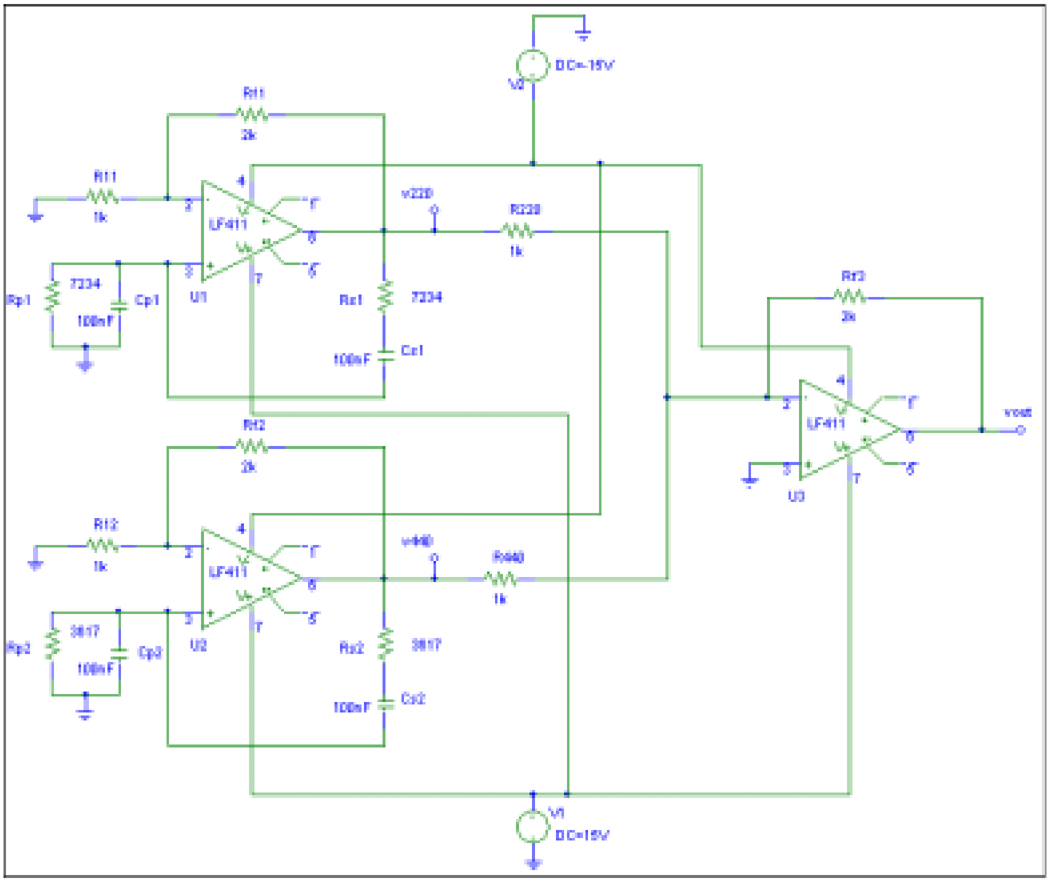
60. Design a Wien-bridge oscillator for 440 Hz: $\omega = 2\pi f = 1/RC$, so set $(2\pi RC)^{-1} = 440$

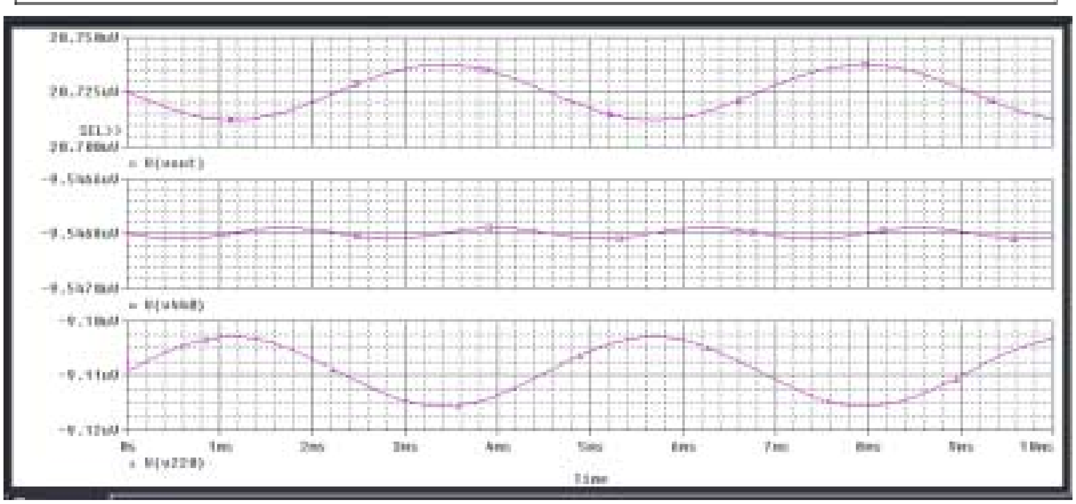
$\boxed{\text{If we use 100-nF capacitors, then R = 3.167 k}\Omega\text{.}}$

Design a Wien-bridge oscillator for 220 Hz: $\omega = 2\pi f = 1/RC$, so set $(2\pi RC)^{-1} = 220$

$\boxed{\text{If we use 100-nF capacitors, then R = 7.234 k}\Omega\text{.}}$

Using a summing stage to add the two waveforms together:



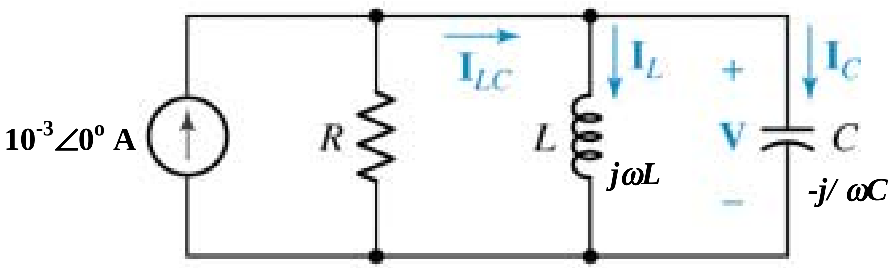


1. We have a parallel RLC with R = 1 kΩ, C = 47 μF and L = 11 mH.

(a) $Q_o = R(C/L)^{½} = \boxed{65.37}$

(b) $f_o = \omega_o / 2\pi = (LC)^{-½} / 2\pi = \boxed{221.3 \text{ Hz}}$

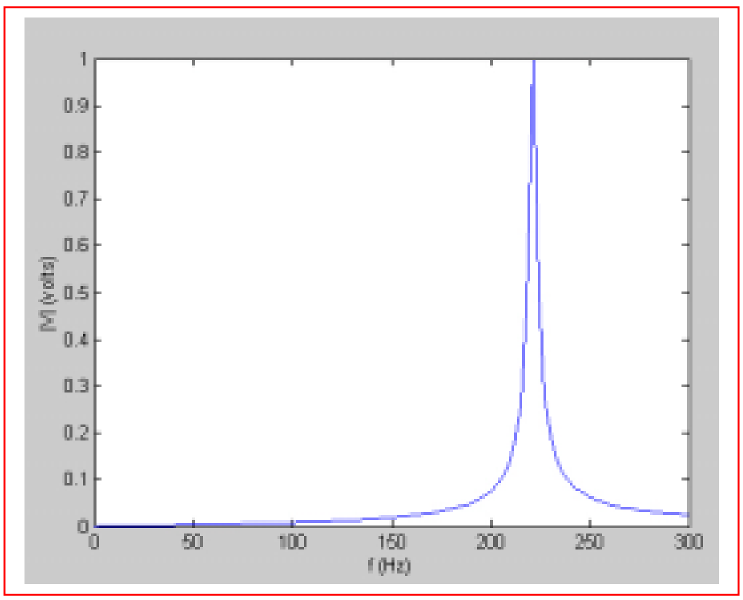
(c) The circuit is excited by a steady-state 1-mA sinusoidal source:



The admittance **Y**(**s**) facing the source is **Y**(**s**) = 1/R + 1/**s**L + **s**C

= C($\mathbf{s}^2$ + **s**/RC + 1/LC)/ **s** so **Z**(**s**) = (**s**/C) / ($\mathbf{s}^2$ + **s**/RC + 1/LC) and
Z($j\omega$) = (1/C) ($j\omega$) / (1/LC – ω^2 + $j\omega$/RC).

Since **V** = 10^{-3} **Z**, we note that |**V**| > 0 as $\omega \to 0$ and also as $\omega \to \infty$.



CHAPTER SIXTEEN SOLUTIONS

2. (a) $R = 1000\ \Omega$ and $C = 1\ \mu F$.

$$Q_o = R(C/L)^{1/2} = 200 \quad \text{so} \quad L = C(R/Q_o)^2 = \boxed{25\ \mu H}$$

(b) $L = 12$ fH and $C = 2.4$ nF

$$R = Q_o\,(L/C)^{1/2} = \boxed{447.2\ m\Omega}$$

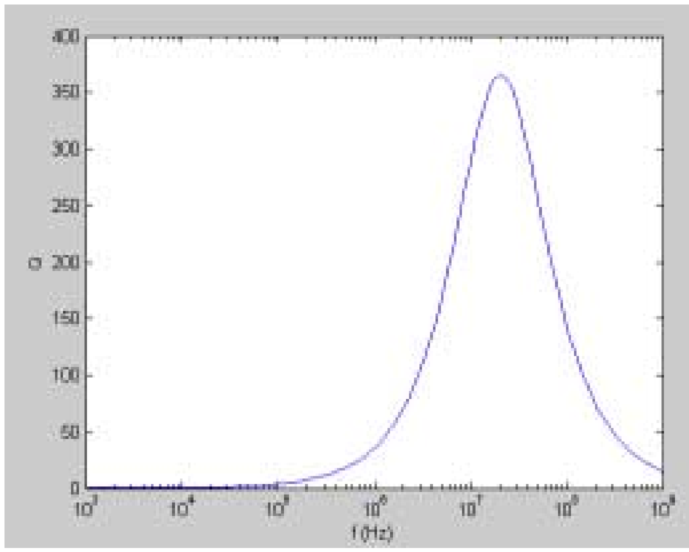
(c) $R = 121.7\ k\Omega$ and $L = 100$ pH

$$C = (Q_o / R)^2\,L = \boxed{270\ aF}$$

3. We take the approximate expression for Q of a varactor to be

$$Q \approx \omega C_j R_p / (1 + \omega^2 C_j^2 R_p R_s)$$

(a) $C_j = 3.77$ pF, $R_p = 1.5$ MΩ, $R_s = 2.8$ Ω



(b) $dQ/d\omega = [(1 + \omega^2 C_j^2 R_p R_s)(C_j R_p) - \omega C_j R_p (2\omega C_j^2 R_p R_s)] / (1 + \omega^2 C_j^2 R_p R_s)$

Setting this equal to zero, we may subsequently write

$$C_j R_p (1 + \omega^2 C_j^2 R_p R_s) - \omega C_j R_p (2\omega C_j^2 R_p R_s) = 0$$

Or $1 - \omega^2 C_j^2 R_p R_s = 0$. Thus, $\omega_o = (C_j^2 R_p R_s)^{-½}$ = 129.4 Mrad/s = $\boxed{21.00 \text{ MHz}}$

$Q_o = Q(\omega = \omega_o) = \boxed{366.0}$

4. Determine Q for (*dropping onto a smooth concrete floor*):

(a) A ping pong ball: Dropped twice from 121.1 cm (arbitrarily chosen). Both times, it bounced to a height of 61.65 cm.

$$Q = 2\pi h_1 / (h_1 - h_2) = \boxed{12.82}$$

(b) A quarter (25 ¢). Dropped three times from 121.1 cm.
Trial 1: bounced to 13.18 cm
Trial 2: bounced to 32.70 cm
Trial 3: bounced to 16.03 cm. *Quite a bit of variation, depending on how it struck.*

Average bounce height = 20.64 cm, so

$$Q_{avg} = 2\pi h_1 / (h_1 - h_2) = \boxed{7.574}$$

(c) Textbook. Dropped once from 121.1 cm. Didn't bounce much at all- only 2.223 cm. Since the book bounced differently depending on angle of incidence, only one trial was performed.

$$Q = 2\pi h_1 / (h_1 - h_2) = \boxed{6.4}$$

All three items were dropped from the same height for comparison purposes. An interesting experiment would be to repeat the above, but from several different heights, preferrably ranging several orders of magnitude (*e.g.* 1 cm, 10 cm, 100 cm, 1000 cm).

5.

$\alpha = 80\,\text{Np/s},\ \omega_d = 1200\ \text{rad/s},\ \left|\bar{Z}(-2\alpha + j\omega_d)\right| = 400\,\Omega$

$\omega_o = \sqrt{1200^2 + 80^2} = 1202.66\ \text{rad/s} \quad \therefore Q_o = \dfrac{\omega_o}{2\alpha} = 7.517$

$\text{Now, } \bar{Y}(s) = C\dfrac{(s+\alpha - j\omega_d)(s+\alpha + j\omega_d)}{s} \ \therefore \bar{Y}(-2\alpha + j\omega_d) = C\dfrac{(-\alpha)(-\alpha + j2\omega_d)}{-2\alpha + j\omega_d}$

$\therefore \bar{Y}(-160 + j1200) = C\dfrac{-80(-80 + j2400)}{-160 + j1200} \ \therefore \left|\bar{Y}(-160 + j1200)\right| = \dfrac{1}{400} = 80C\left|\dfrac{-1 + j30}{-2 + j15}\right|$

$\therefore C = \dfrac{1}{32,000}\sqrt{\dfrac{229}{901}} = \boxed{15.775^{-}\ \mu\text{F};}\ L = \dfrac{1}{\omega_o^2 C} = \boxed{43.88\,\text{mH};}\ R = \dfrac{1}{2\alpha C} = \boxed{396.7\,\Omega}$

6.

$$\bar{Y}_{in} = \frac{1}{2+j0.1\omega} + 0.2 + \frac{1}{1+1000/j\omega} = \frac{2-j0.1\omega}{4+0.01\omega^2} + 0.2 + \frac{j\omega}{1000+j10}$$

$$= \frac{2-j0.1\omega}{4+0.1\omega^2} + 0.2 + \frac{\omega^2 + j1000\omega}{10^6+\omega^2} \therefore \frac{-0.1\omega}{4+0.01\omega^2} + \frac{1000\omega}{\omega^2+10^6} = 0$$

$$\therefore 0.1\omega^3 + 10^5\omega = 4000\omega + 10\omega^3 \ \therefore 9.9\omega^2 = 96,000 \ \therefore \omega = \boxed{98.47 \text{ rad/s}}$$

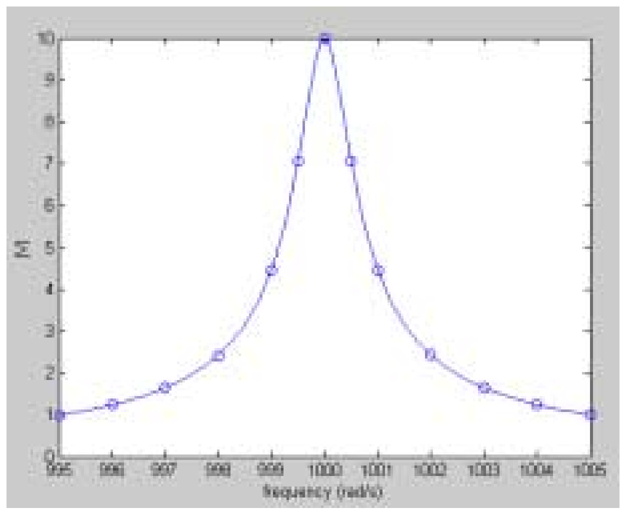
7. Parallel: $R = 10^6,\ L = 1,\ C = 10^{-6},\ \bar{I}_s = 10\angle 0°\ \mu A$

(a) $\omega_o = \dfrac{1}{\sqrt{LC}} = \boxed{1000 \text{ rad/s};}\ Q_o = \omega_o RC = 10^{3+6-6} = \boxed{1000}$

(b) $\bar{Y} = 10^{-6} + j\left(10^{-6} - \dfrac{1}{\omega}\right),\ \bar{V} = \dfrac{\bar{I}}{\bar{Y}} = 10^{-5} / 10^{-3}\left[10^{-3} + j\left(\dfrac{\omega}{1000} - \dfrac{1000}{\omega}\right)\right]$

$$\therefore \bar{V} = \frac{10^{-2}}{0.001 + j\left(\dfrac{\omega}{1000} - \dfrac{1000}{\omega}\right)},\ \left|\bar{V}\right| = \frac{10^{-2}}{\sqrt{10^{-6} + \left(\dfrac{\omega}{1000} - \dfrac{1000}{\omega}\right)^2}}$$

ω	$\lvert\bar{V}\rvert$
995	0.993
996	1.238
997	1.642
998	2.423
999	4.47
1000	10.0
1001	4.47
1002	2.428
1003	1.646
1004	1.243
1005	0.997
999.5	7.070
1000.5	7.072



8.

(a) $$\bar{Z}_{in} = \frac{5(100/j\omega)}{5+(100/j\omega)} + 2 + \frac{j0.1\omega}{10+j0.01\omega}$$

$$= \frac{500}{100+j5\omega} + 2 + \frac{j10\omega}{1000+j\omega} = \frac{100}{20+j\omega} + 2 + \frac{j10\omega}{1000+j\omega} = \frac{100(20-j\omega)}{\omega^2+400} + 2 + \frac{j10\omega(1000-j)}{\omega^2+10^6}$$

$$\therefore \frac{-100\omega}{\omega^2+400} + \frac{10^4\omega}{\omega^2+10^6} = 0 \ \therefore \omega^2+10^6 = 100\omega^2+40,000, \ 99\omega^2 = 960,000$$

$$\therefore \omega_o = \sqrt{960,000/99} = \boxed{98.47 \text{ rad/s}}$$

(b) $$\bar{Z}_{in}(\omega_o) = \frac{2000}{\omega_o^2+400} + 2 + \frac{10\omega_o^2}{\omega_o^2+10^6} = \boxed{2.294\,\Omega}$$

9.

(a) $\alpha = 50\,s^{-1}, \omega_d = 1000s^{-1}\ \therefore \omega_o^2 = \alpha^2 + \omega_d^2 = 1,002,500\ \therefore \omega_o = 1001.249$

$$\text{L} = \frac{1}{\omega_o^2 \text{C}} = \frac{10^6}{1,002,500} = \boxed{0.9975^+\,\text{H};}\ \ \text{R} = \frac{1}{2\alpha\text{C}} = \frac{10^6}{100} = \boxed{10\,k\Omega}$$

(b) $$\bar{\text{Y}} = 10^{-4} + j\left(10^{-6}\omega - \frac{1}{0.9975\omega}\right),\ \omega = 1000\ \therefore \bar{\text{Z}} = \frac{1}{\text{Y}} = \boxed{9997\angle 1.4321^\circ\,\Omega}$$

10.

$f_{\min} = 535\,\text{kHz},\ f_{\max} = 1605\,\text{kHz},\ \text{Q}_o = 45$ at one end and

$\text{Q}_o \leq 45$ for $535 \leq f \leq 1605$ kHz

$$f_o = 1/2\pi\sqrt{\text{LC}}\ \ \therefore 535\times10^3 = \frac{1}{2\pi\sqrt{\text{L}_{\max}\text{C}}}, 1605\times10^3 = \frac{1}{2\pi\sqrt{\text{L}_{\min}\text{C}}}$$

$$\therefore \sqrt{\text{L}_{\max}/\text{L}_{\min}} = 3;\ \text{L}_{\max}\text{C} = \left(\frac{1}{2\pi\times535\times10^3}\right)^2 = 8.8498\times10^{-14}$$

$$\omega_o\text{RC} \leq 45, 535\times10^3 \leq \frac{\omega_o}{2\pi} \leq 1605\times10^3.\ \text{Use } \omega_{o\max}$$

$$\therefore 2\pi\times1605\times10^3\times20\times10^3\text{C} = 45\ \therefore \boxed{\text{C} = 223.1\text{pF}}$$

$$\therefore \text{L}_{\max} = \frac{8.8498\times10^{-14}}{223.1\times10^{-12}} = \boxed{397.6\ \mu\text{H}},\ \text{L}_{\min} = \frac{\text{L}_{\max}}{9} = \boxed{44.08\,\mu\text{H}}$$

11.

(a) Apply ± 1V. $\therefore \bar{I}_R = -10^{-4}$ A

$$\therefore \bar{Y}_{in} = \bar{I}_{in} = \frac{1}{4.4\times10^{-3}s} + 10^{-4} + (1-[10^5(-10^{-4})])10^{-8}s$$

$$\therefore \bar{Y}_{in} = \frac{1000}{4.4s} + 10^{-4} + 11\times10^{-8}s = \frac{48.4\times10^{-8}s^2 + 4.4\times10^{-4}s + 1000}{4.4s}$$

$$\therefore \boxed{\bar{Y}_{in}(j\omega) = \frac{1000 - 48.4\times10^{-8}\omega^2 + j4.4\times10^{-4}\omega}{j4.4\omega}}$$

(b) At $\omega = \omega_o$, $1000 = 48.4\times10^{-8}\omega_o^2$, $\omega_o = \boxed{45.45^{-}\ \text{krad/s}}$

$$\bar{Z}_{in}(j\omega_o) = \left(\frac{j4.4\times10^{-4}\omega_o}{j4.4\omega_o}\right)^{-1} = \boxed{10\,k\Omega}$$

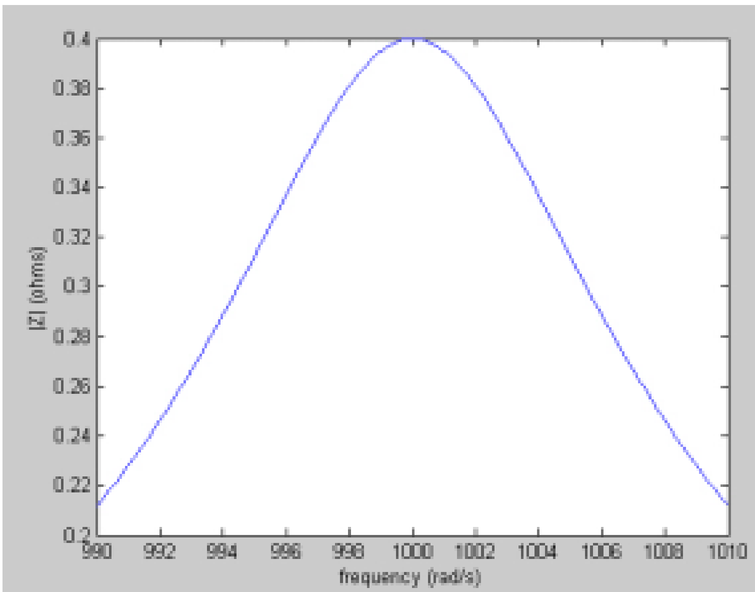
12. $\omega_o = 1000$ rad/s, $Q_o = 80$, $C = 0.2\,\mu$F

(a) $L = \dfrac{1}{\omega_o^2 C} = \dfrac{10^6}{0.2\times 10^6} = \boxed{5\,\text{H}}$, $Q_o = \omega_o RC \;\therefore R = \dfrac{80}{10^3 \times 0.2\times 10^{-6}} = \boxed{400\,k\Omega}$

(b) $B = \omega_o / Q_o = 1000/80 = 12.5$

$\therefore \dfrac{1}{2}B = 6.25$ rad/s

$\therefore \left|\bar{Z}\right| = R \Big/ \left|1 + j\dfrac{\omega - \omega_o}{B/2}\right| = 400\times 10^3 \Big/ \sqrt{1 + \left(\dfrac{\omega - \omega_o}{6.25}\right)^2}$



13.

$$\omega_1 = 103\,\text{rad/s},\ \omega_2 = 118,$$
$$\left|\overline{\text{Z}}(j105)\right| = 10\,\Omega$$
$$\omega_o^2 = \omega_1\omega_2 = 103 \times 118$$
$$\therefore \omega_o = 110.245^+,\ \text{B} = 118 - 103 = 15\ \text{rad/s},\ \text{Q}_o = \frac{\omega_o}{\text{B}} = \frac{110.245^+}{15} = 7.350$$
$$\therefore 7.350 = \omega_o \text{RC} \quad \therefore \text{RC} = \frac{7.350}{110.245_1^+} = 66.67 \times 10^{-3}, \text{LC} = \frac{1}{\omega_o^2} = \frac{1}{12{,}154}$$
$$\left|\overline{\text{Y}}(j105)\right| = 0.1 = \left|\frac{1}{\text{R}} + j\left(105\text{C} - \frac{1}{105\text{L}}\right)\right| = \left|15\text{C} + j\left(105\text{C} - \frac{12{,}154}{105}\text{C}\right)\right| = 18.456\,\text{C}$$
$$\therefore \text{C} = \frac{0.1}{18.456} = \boxed{5.418\,\text{mF},}\ \text{R} = \frac{1}{15}\text{C} = \boxed{12.304\,\Omega,}\ \text{L} = \frac{1}{12{,}154\text{C}} = \boxed{15.185^-\,\text{mH}}$$

14. $\omega_o = 30$ krad/s, $Q_o = 10$, $R = 600\,\Omega$,

(a) $B = \dfrac{\omega_o}{Q_o} = \boxed{3\text{ krad/s}}$

(b) $N = \dfrac{\omega - \omega_o}{B/2} = \dfrac{28-30}{1.5} = \boxed{-1.3333}$

(c) $\mathbf{Z}_{\text{in}}(j28\,000) = 600 / (1 - j1.333) = \boxed{360 \angle 53.13^\circ\ \Omega}$

(d) $\bar{Z}_{in}(j28{,}000) = \left[\dfrac{1}{600} + j28{,}000C - j\dfrac{1}{28{,}000L}\right]^{-1}, C = \dfrac{Q_o}{\omega_o R} = \dfrac{10}{30{,}000 \times 600}$

$L = \dfrac{R}{\omega_o Q_o} = \dfrac{600}{30{,}000 \times 10}, \dfrac{1}{L} = \dfrac{30{,}000 \times 10}{600}\ \therefore \bar{Z}_{in} = \left[\dfrac{1}{600} + j\left(\dfrac{28}{30} \times \dfrac{10}{600} - \dfrac{30}{28}\,\dfrac{10}{600}\right)\right]^{-1}$

$\bar{Z}_{in} = \dfrac{600}{1 + j10\left(\dfrac{28}{30} - \dfrac{30}{28}\right)} = \boxed{351.906\angle 54.0903^\circ\,\Omega}$

(e) magnitude: $100\% \dfrac{\text{approx-true}}{\text{true}} = 100\% \dfrac{360 - 351.906}{351.906} = \boxed{2.300\%}$

angle: $100\% \dfrac{53.1301^\circ - 54.0903^\circ}{54.0903^\circ} = \boxed{-1.7752\%}$

15. $f_o = 400\,\text{Hz},\ Q_o = 8,\ R = 500\,\Omega,\ \bar{I}_S = 2\times10^{-3}\,\text{A}\ \therefore B = 50\,\text{Hz}$

(a) $\left|\bar{V}\right| = 2\times10^{-3}\times500/\sqrt{1+N^2} = 0.5\quad \therefore 1+N^2 = 4,\ N = \pm\sqrt{3} = \dfrac{f-400}{50/2}$

$\therefore f = 400 \pm 25\sqrt{3} = \boxed{443.3 \text{ and } 356.7\,\text{Hz}}$

(b) $\left|\bar{I}_R\right| = \dfrac{|v|}{R} = \dfrac{1}{\sqrt{1+N^2}}\times\dfrac{1}{500} = 0.5\times10^{-3}\quad \therefore \sqrt{1+N^2} = 4,\ N^2 = 15,\ N = \pm\sqrt{15}$

$\therefore f = 400 \pm 25\sqrt{15} = \boxed{496.8 \text{ and } 303.2\text{ Hz}}$

16. $\omega_o = 10^6,\ Q_o = 10,\ R = 5\times10^3,\ p.r.$

(a) $Q_o = \dfrac{R}{\omega_o L}\quad \therefore L = \dfrac{5\times10^3}{10\times10^6} = \boxed{0.5\,\text{mH}}$

(b) Approx: $2 = 5/\sqrt{1+N^2}\quad \therefore N = 2.291 = \dfrac{\omega - 10^6}{10^6/20}\quad \therefore \omega = \boxed{1.1146\ \text{Mrad/S}}$

Exact: $\bar{Y} = \dfrac{1}{R}\left[1 + jQ_o\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)\right]\ \therefore 0.5 = 0.2\sqrt{1 + 100\left(\omega - \dfrac{1}{\omega}\right)^2}\quad (\omega \text{ in Mrad/S})$

$\therefore 6.25 = 1 + 100(\omega^2 - 2 + 1/\omega^2),\ \omega^2 - 2 + \dfrac{1}{\omega^2} = 0.0525,\ \omega^2 + \dfrac{1}{\omega^2} = 2.0525$

$\omega^4 - 2.0525\omega^2 + 1 = 0,\ \omega^2 = \dfrac{1}{2}\left(2.0525 + \sqrt{2.0525^2 - 4}\right) = 1.2569, \omega = \boxed{1.1211\ \text{Mrad/s}}$

(c) Approx: $\angle Y = 30°\quad \therefore \tan^{-1} N = 30°,\ N = 0.5774 = \dfrac{\omega - 1}{1/20},\ \omega = \boxed{1.0289\ \text{Mrad/s}}$

Exact: $\bar{Y} = \dfrac{1}{5000}\left[1 + j10\left(\omega - \dfrac{1}{\omega}\right)\right]$ (in Mrad/s) $\ \therefore \tan 30° = 0.5774 = 10\left(\omega - \dfrac{1}{\omega}\right)$

$\therefore \omega - \dfrac{1}{\omega} = 0.05774,\ \omega^2 - 0.05774\omega - 1 = 0, \omega = \dfrac{0.05774 + \sqrt{0.05774^2 + 4}}{2} = \boxed{1.0293\ \text{Mrad/s}}$

17.

(a) $C = 3 + 7 = 10\,\text{nF} \;\; \therefore \omega_o = \dfrac{1}{\sqrt{10^{-4}10^{-8}}} = \boxed{10^6 \text{ rad/s}}$

(b) $Q_o = \omega_o CR = 10^6 10^{-8} 5\ 5\times 10^3 = 50$

$B = \omega_o / Q_o = 20 \text{ krad/s}$

Parallel current source is $\dfrac{1\angle 0°}{\bar{Z}_3} = j\omega 3\times 10^{-9}$ At ω_o, $I_s = j10^{6-9}\times 3$

$\therefore V_{1,0} = j3\times 10^{-3}\times 5\times 10^3 = \boxed{15\angle 90° \text{ V}}$

(c) $\omega - \omega_o = 15\times 10^3 \;\; \therefore N = \dfrac{15\times 10^3}{10\times 10^3} = 1.5 \;\; \therefore \bar{V}_1 = \dfrac{15\angle 90°}{1 + j1.5} = \boxed{8.321\angle 33.69° \text{ V}}$

18.

(a) $$\bar{Z}_{in}(s) = \frac{(5+0.01s)(5+10^6/s)}{10+0.01s+10^6/s} = \frac{(5+0.01s)(5s+10^6)}{0.01s^2+10s+10^6}$$

$$\bar{Z}_{in}(s) = \frac{0.05s^2+25s+10^4 s+5\times10^6}{0.01s^2+10s+10^6}$$

$$\therefore \bar{Z}_{in}(j\omega) = \frac{5\times10^6-0.05\omega^2+j10,025\omega}{10^6-0.01\omega^2+j10\omega}$$

At $\omega=\omega_o$, $\dfrac{10,025\omega_o}{5\times10^6-0.05\omega_o^2} = \dfrac{10\omega_o}{10^6-0.01\omega_o^2}$, $10.025\times10^9-100.25\omega_o^2 = 5\times10^7-0.5\omega_o^2$

$$\therefore 99.75\omega_o^2 = 9.975\times10^9, \omega_o = \boxed{10,000 \text{ rad/s}}$$

(b) $$\bar{Z}_{in}(j\omega_o) = (5+j100)\|(5-j100) = \frac{25+10,000}{10} = \boxed{1002.5\,\Omega}$$

19. $f_o = 1000\,\text{Hz},\ Q_o = 40, \left|\overline{Z}_{in}(j\omega_o)\right| = 2k\Omega \quad \therefore B = 25\,\text{Hz}$

(a) $\mathbf{Z}_{in}(j\omega) = \dfrac{2000}{1 + j\text{N}},\ \text{N} = \dfrac{f - 1000}{12.5},\ f = 1010,\ \therefore \text{N} = 0.8$

$\mathbf{Z}_{in} = 2000 / (1 + j0.8) = \boxed{1562 \angle -38.66^{o}\ \Omega}$

(b) $0.9 f_o < f < 1.1 f_o \quad \boxed{\therefore 900 < f < 1100\ \text{Hz}}$

20. Taking $2^{-½}$ = 0.7, we read from

Fig. 16.48*a*: 1.7 kHz – 0.6 kHz = $\boxed{1.1 \text{ kHz}}$

Fig. 16.48*b*: 2×10^7 Hz – 900 Hz = $\boxed{20 \text{ MHz}}$

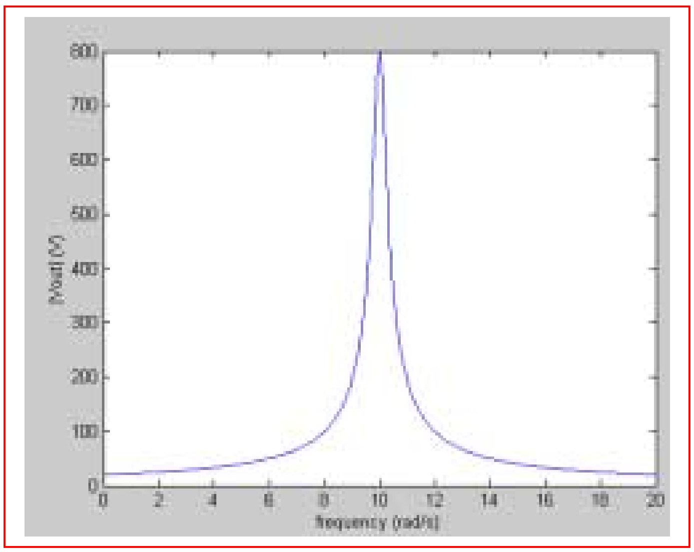
21.

(a) $20\text{A} \| 6\Omega,\ 3\|6 = 2,\ 40\,\text{V}$ in series with $2+1=3\,\Omega$

$$\omega_o = \frac{1}{\sqrt{\text{LC}}} = 10 \text{ rad/s},\ \text{Q}_o = \frac{\omega_o \text{L}}{\text{R}} = \frac{60}{3} = 20\,\Omega$$

$$\text{B} = \frac{10}{20} = 0.5, \frac{1}{2}\text{B} = 0.25, \left|\bar{\text{V}}_{out}(j\omega_o)\right| = 40\text{Q}_o = 800\,\text{V}$$

$$\therefore \left|\bar{\text{V}}_{out}(j\omega)\right| = 800 / \sqrt{1+\left(\frac{\omega-10}{0.25}\right)^2}$$



(b) $\omega = 9$ rad/s

$$\text{(Approx: } \left|\bar{\text{V}}_{out}(j9)\right| = \frac{800}{\sqrt{17}} = 194.03\,\text{V}$$

$$\text{Exact: } \bar{\text{V}}_{out} = \frac{40}{3 + j(6\omega - 600/\omega)} \times \frac{600}{j\omega}$$

$$\therefore \bar{\text{V}}_{out}(j9) = \frac{24{,}000}{9[3+j(54-66.67)]} = \boxed{204.86\angle -13.325^\circ\ \text{V}}$$

22. Series: $\text{R} = 50\,\Omega,\ \text{L} = 4\,\text{mH},\ \text{C} = 10^{-7}$

(a) $\omega_o = 1/\sqrt{4\times10^{-3-7}} = \boxed{50\text{ krad/s}}$

(b) $f_o = 50\times10^3/2\pi = \boxed{7.958\,\text{kHz}}$

(c) $\text{Q}_o = \dfrac{\omega_o\text{L}}{\text{R}} = \dfrac{50\times10^3\times4\times10^{-3}}{50} = \boxed{4}$

(d) $\text{B} = \omega_o/\text{Q}_o = 50\times10^3/4 = \boxed{12.5\text{ krad/s}}$

(e) $\omega_1 = \omega_o\left[\sqrt{1+(1/2Q_o)^2} - 1/2Q_o\right] = 50\left[\sqrt{1+1/64} - 1/8\right] = \boxed{44.14\text{ krad/s}}$

(f) $\omega_2 = 50\left[\sqrt{65/64} + 1/8\right] = \boxed{56.64\text{ krad/s}}$

(g) $\overline{\text{Z}}_{in}(j45,000) = 50 + j(180 - 10^{7-3}/45) = 50 - j42.22 = \boxed{65.44\angle -40.18°\,\Omega}$

(h) $\left|\overline{\text{Z}}_c/\overline{\text{Z}}_R\right|_{45,000} = \left|10^7/j45,000\times50\right| = \boxed{4.444}$

23. Apply 1 A, in at top. $\therefore \overline{\mathrm{V}}_R = 10\,\mathrm{V}$

(a) $\overline{\mathrm{V}}_{in} = \overline{\mathrm{Z}}_{in} = 10^{-3}s + 10 + \dfrac{10^8}{5s}(0.5\times 10 + 1) = 10^{-3}s + 10 + \dfrac{1.2\times 10^8}{s}$

$\overline{\mathrm{Z}}_{in}(j\omega) = 10 + j(10^{-3}\omega - 1.2\times 10^8/\omega)\ \therefore 10^{-3}\omega_o = 1.2\times 10^8/\omega_o$

$\therefore \omega_o^2 = 1.2\times 10^{11},\ \omega_o = \boxed{346.4\ \text{krad/s}}$

(b) $\mathrm{Q}_o = \dfrac{\omega_o \mathrm{L}}{\mathrm{R}} = \dfrac{346.4\times 10^{3-3}}{10} = \boxed{34.64}$

24. Find the Thévenin equivalent seen by the inductor-capacitor combination:

$$\text{SC}: 1.5 = \bar{V}_1 + 10\left(\frac{V_1}{125} - 0.105\,V_1\right) \therefore \bar{V}_1 = 50\,\text{V}$$

$$\therefore \downarrow \bar{I}_{SC} = \frac{50}{125} = 0.4\,\text{A}$$

$$\text{OC}: \bar{V}_1 = 0 \ \therefore \bar{V}_{OC} = 1.5\,\text{V} \quad \therefore R_{th} = \frac{1.5}{0.4} = 3.75\,\Omega$$

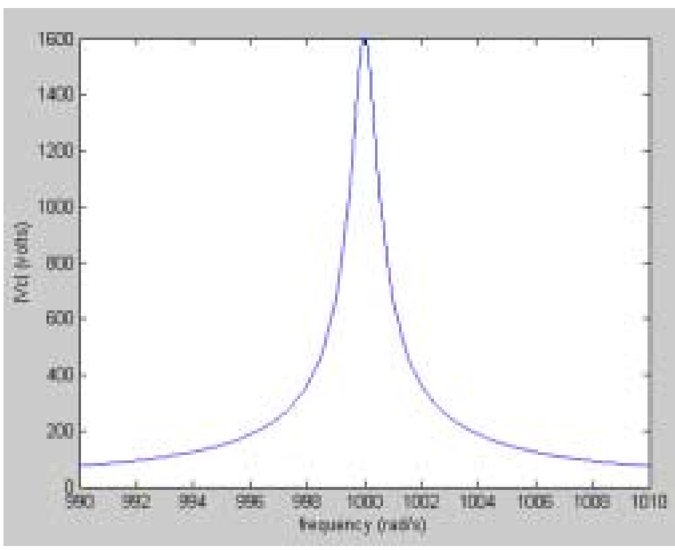
$$\therefore \omega_o = 1/\sqrt{4\times 0.25\times 10^{-6}} = 1000,\ Q_o = \frac{1000\times 4}{3.75} = 1066.7$$

$$B = \omega_o / Q_o = \frac{1000}{1066.7} = 0.9375, \frac{1}{2}B = 0.4688\ \text{rad/s}$$

$$\left|\bar{V}_C\right|_{\max} = Q_o V_{th} = 1066.7 \times 1.5 = 1600\,\text{V}$$

Therefore, keep your hands off!

To generate a plot of $|\mathbf{V}_C|$ vs. frequency, note that $\mathbf{V}_C(j\omega) = 1.5\dfrac{-\dfrac{j}{\omega C}}{3.75 + j\omega L - \dfrac{j}{\omega C}}$



25. Series, $f_o = 500\,\text{Hz},\ Q_o = 10,\ X_{L,0} = 500\,\Omega$

(a) $500 = \omega_o L = 2\pi(500)L \ \therefore \boxed{L = 0.15915^{+}\ \text{H}},\ C = \dfrac{1}{\omega_o^2 L} = \dfrac{2\pi}{(2\pi \times 500)^2} = \boxed{0.6366\,\mu\text{F}}$

$Q_o = 10 = \dfrac{X_{L,0}}{R} = \dfrac{500}{R} \ \therefore \boxed{R = 50\,\Omega}$

(b) $1 = \bar{I}\left(50 + j2\pi f \times \dfrac{1}{2\pi} - j\dfrac{10^6 \times 0.5\pi}{2\pi f}\right) = \bar{I}\left(50 + j f - j\dfrac{250{,}000}{f}\right)$

$\therefore \bar{I} = 1/50 + j(f - 250{,}000/f),\ \bar{V}_c = \dfrac{10^6 \times 0.5\pi}{j2\pi f}\bar{I}$

$\bar{V}_C = \dfrac{-j250{,}000/f}{50 + j(f - 250{,}000/f)} \quad \therefore \left|\bar{V}_c(2\pi \times 450)\right| = \boxed{4.757\,\text{V}}$

$\left|\bar{V}_c(2\pi \times 500)\right| = \boxed{10{,}000\,\text{V}}\ \left|\bar{V}_c(2\pi \times 550)\right| = \boxed{4.218\,\text{V}}$

26.

$\mathrm{X}: s = 0, \infty, 0 : s = -20,000 \pm j80,000\, s^{-1},\ \overline{\mathrm{Z}}_{in}(-10^4) = -20 + j0\,\Omega \quad \therefore \mathrm{SERIES}$

$\alpha = 20,000,\ \omega_d = 80,000 \therefore \omega_o = \sqrt{(64+4)10^8} = 82,462 \text{ rad/s}, \dfrac{1}{\mathrm{LC}} = \omega_o^2 = 68 \times 10^8$

$\dfrac{\mathrm{R}}{2\mathrm{L}} = \alpha = 20,000 \therefore \dfrac{\mathrm{R}}{\mathrm{L}_1} = 40,000, \dfrac{1}{\mathrm{LC}} \times \dfrac{\mathrm{L}}{\mathrm{R}} = \dfrac{68 \times 10^8}{40,000} = 170,000;\ \mathrm{Z}(\sigma) = \mathrm{R} + \sigma \mathrm{L} + \dfrac{1}{\sigma \mathrm{C}}$

$\therefore -20 = \mathrm{R} - 10,000\mathrm{L} - \dfrac{1}{10,000\mathrm{C}} = \mathrm{R} - \dfrac{1}{4}\mathrm{R} - \dfrac{170,000}{10,000}\mathrm{R} \therefore \boxed{\mathrm{R} = 1.2308\,\Omega}$

$\therefore \mathrm{L} = \dfrac{1.2308}{40,000} = \boxed{30.77\,\mu\mathrm{H},}\ \mathrm{C} = \dfrac{1}{170,000 \times 1.2308} = \boxed{4.779\,\mu\mathrm{F}}$

27.

ω_o B $1/\sqrt{10^{-3-7}} = \boxed{10^5 \text{ rad/s,}}$ $Q_L = \dfrac{10^{5-3}}{1} = 100,\ R_{PL} = 10,000\,\Omega$

$Q_c = \dfrac{1}{10^{5-7} \times 0.2} = 500,\ R_{PC} = 500^2 \times 0.2 = 50,000\,\Omega$

$50 \| 10 = 8.333\,k\Omega \quad \therefore Q_o = \omega_o CR = 10^{5-7} \times 8333 = \boxed{83.33}$

$B = \dfrac{100,000}{83.33} = \boxed{1200 \text{ rad/s,}}$ $\overline{Z}_{in}(j\omega_o) = \boxed{8333\,\Omega}$

$\omega = 99,000 \ \therefore N = \dfrac{(99-100)10^3}{600} = -1.6667, \overline{Z}_{in}(j99,000) = \dfrac{8.333}{1 - j1.667}$

$= \boxed{4.287 \angle 59.04^o \text{ k}\Omega}$

28. $R_{eq} = Q_o / \omega_o C = 50 / 10^{5-7} = 5000\ \Omega$.
Thus, we may write $1/5000 = 1/8333 + 1/R_x$ so that

$R_x = \boxed{12.5\ \text{k}\Omega}$.

29.

$$3\,\text{mH} \| 1.5\,\text{mH} = 1\text{mH},\ 2\,\mu\text{F} + 8\,\mu\text{F} = 10\,\mu\text{F},\ \therefore \omega_o = \frac{1}{\sqrt{10^{-3-5}}} = 10 \text{ krad/s}$$

$$Q = \frac{3\times10^{-3}\times10^{4}}{0.3} = 100,\ R_p = 100^2 \times 0.3 = 3\,k\Omega$$

$$Q = \frac{1.5\times10^{-3}\times10^{4}}{0.25} = 60,\ R_p = 60\times0.25 = 900\,\Omega$$

$$900 \| 3000 = 692.3\,\Omega\ \ \therefore Q_L = \frac{692.3}{10^{4-3}} = 69.23$$

$$\therefore R_{LS} = \frac{692.3}{69.23^2} = 0.14444\,\Omega$$

$$Q = \frac{10^6}{10^4\times0.1\times8} = 125,\ R_{pc} = 125^2\times0.1 = 1562.5\ \Omega \| 10\,\mu\text{F}$$

$$\therefore Q_c = 10^4\times10^{-5}\times15625 = 156.25\ \ \therefore R_{SC} = \frac{1562.5}{(156.25)^2} = 0.064\,\Omega$$

$$\therefore R_{S,tot} = 0.14444 + 0.064 = \boxed{0.2084\,\Omega} = \left|\bar{Z}_{in}\right|_{\min},\ \omega_o = \boxed{10 \text{ krad/s}}$$

30.

(a) $\omega_o \, \text{B} \, 1/\sqrt{2\times 0.2\times 10^{-3}} = 50 \text{ rad/s}$

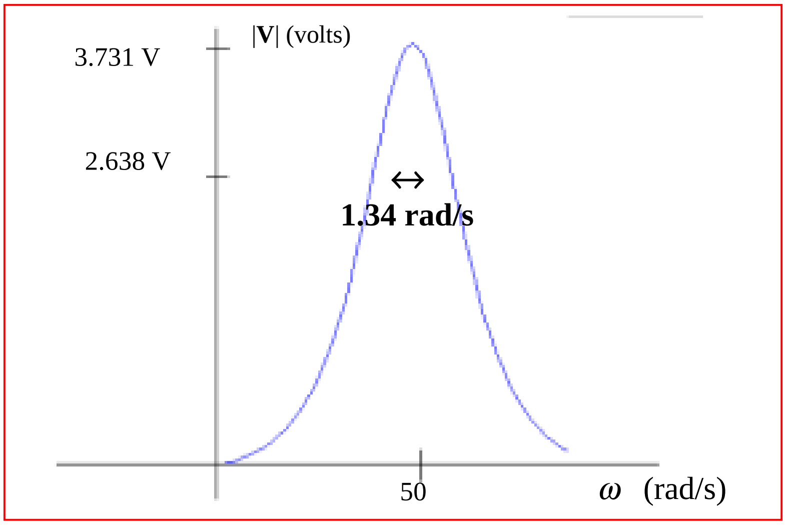
$Q_{leftL} = 50\times 2.5/2 = 62.5,\ 2\times 62.5^2 = 7812.5\,\Omega$

$Q_{rightL} = \dfrac{50\times 10}{10} = 50,\ 10\times 50^2 = 25\,k\Omega$

$Q_c = \dfrac{1000}{50\times 0.2\times 1} = 100,\ 100^2\times 1 = 10\,k\Omega,\ R_p = 7.8125\|25\|10 = 3731\,\Omega$

$Q_o = 50\times 3731\times 0.2\times 10^{-3} = 37.31;\ B = \dfrac{50}{37.31} = 1.3400,\ \dfrac{1}{2}B = 0.6700$

$\therefore |V|_o = 10^{-3}\times 3731 = \boxed{3.731\text{V}}$



(b) $V = 10^{-3}[(2+j125)\|(10+j500)\|(1-j100)]$

$$= \frac{10^{-3}}{\dfrac{1}{2+j125}+\dfrac{1}{10+j500}+\dfrac{1}{1-j100}} = \boxed{3.7321\angle -0.3950^{+\circ}\text{ V}}$$

31.

(a)

$$\omega_o \;\text{B}\; \frac{1000}{\sqrt{0.25}} = 2000 \text{ rad/s, } Q_c = 2000 \times 2 \times 10^{-6} \times 25 \times 10^3 = 100$$

$$\therefore R_{C,S} = 25,000/100^2 = 2.5\,\Omega;\; Q_L = \frac{R}{\omega_o L} = \frac{20 \times 10^4}{2000 \times 0.25} = 40$$

$$\therefore R_{L,S} = \frac{20,000}{1600} = 12.5\,\Omega \;\; \therefore R_{tot} = 12.5 + 2.5 = 15\,\Omega$$

$$\therefore Q_o = \frac{2000 \times 0.25}{15} = 33.33 \;\; \therefore \left|\bar{V}_x\right| = 1 \times 33.33 \times \frac{1}{2} = \boxed{16.667\,\text{V}}$$

(b)

$$20,000 \| j500 = \frac{20,000 \times j500}{20,000 + j500} = 12,4922 + j499.688\,\Omega$$

$$25,000 \| -j250 = \frac{25,000(-j250)}{25,000 - j250} = 2.4998 - j249.975$$

$$\therefore \bar{Z}_{in} = 12.4922 + 2.4998 + j499.688 - j250 - j249.975 = 14.9920 - j0.2870\,\Omega$$

$$\therefore \left|\bar{I}\right| = 1/\left|14.9920 - j0.2870\right| = 66.6902\,\text{mA} \;\; \therefore \left|\bar{V}_x\right| = 250 \times 66.6902 \times 10^{-3} = \boxed{16.6726\,\text{V}}$$

32.

(a) $$\text{K}_m = \frac{50}{100} = 0.5 \ \ \text{K}_f = \frac{20\times10^3}{10^6} = 0.02$$

$$\therefore 9.82\,\mu\text{H} \rightarrow 0.5\times9.82\times\frac{1}{0.02} = 24.55\,\mu\text{H},\ 31.8\,\mu\text{H} \rightarrow \frac{0.5}{0.02}\times31.8 = 795\,\mu\text{H}$$

$$2.57\,\text{nF} \rightarrow \frac{2.57}{0.5\times0.02} = \boxed{257\,\text{nF}}$$

(b) same ordinate; divide numbers on abscissa by $\boxed{50}$

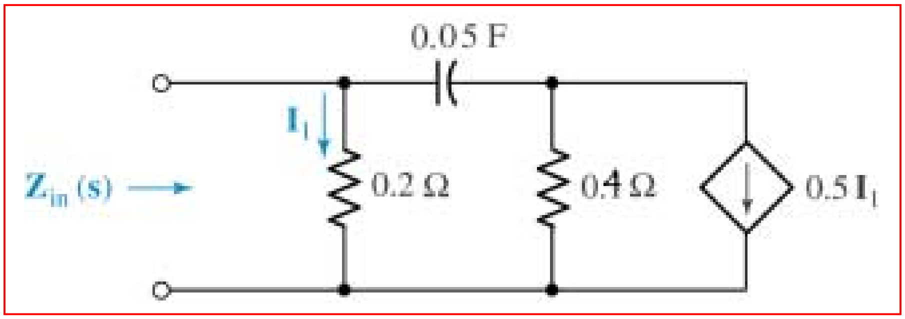
33.

(a) Apply 1 V $\therefore \overline{\mathrm{I}}_1 = 10\text{A} \quad \therefore 0.5\,\overline{\mathrm{I}}_1 = 5\text{A}\downarrow$; $5\text{A}\,\|\,0.2\ \Omega$ can be replaced by 1 V in series with 0.2 Ω

$$\therefore \overline{\mathrm{I}}_{in} \rightarrow = 10 + \frac{1-(-1)}{0.2+2/s} = 10 + \frac{2s}{0.2s+2} = \frac{4s+20}{0.2s+2} = \frac{20(s+5)}{s+10} \quad \therefore \boxed{\overline{\mathrm{Z}}_{in}(s) = \frac{s+10}{20(s+5)}}$$

(b) $$\mathrm{K}_m = 2,\ \mathrm{K}_f = 5 \ \therefore \overline{\mathrm{Z}}_{in}(s) \rightarrow \frac{2(s/5+10)}{20(s/5+5)} = \boxed{\frac{0.1(s+50)}{s+25}}$$

(c) $\boxed{0.1\Omega \rightarrow 0.2\Omega,\ 0.2\Omega \rightarrow 0.4\Omega,\ 0.5\text{F} \rightarrow 0.05\,\text{F},\ 0.5\overline{\mathrm{I}}_1 \rightarrow 0.5\overline{\mathrm{I}}_1}$



34.

(a) ω_o B $1/\sqrt{(2+8)10^{-3}10^{-6}} = \boxed{10^4 \text{ rad/s}}$

$Q_{L,8} = 10^4/8\times10^{-3}10^4 = 125 \;\therefore R_{L,S} = \dfrac{10^4}{125^2} = 0.64\,\Omega$

$2+8 = 10\,\text{mH} \;\;\therefore Q_L = \dfrac{10^4\times10\times10^{-3}}{0.64} = 156.25$

$\therefore R_{L,P} = 0.64\times156.25^2 = 15.625\,k\Omega;\; Q_C = \dfrac{1}{10^4\times10^{-6}} = 100,\; R_{C,P} = 100^2\times1 = 10\,k\Omega$

$\therefore R_P = 20\|15.625\|10 = 4.673\,k\Omega \;\;\therefore Q_o = 10^4\times10^{-6}\times4.673\times10^3 = \boxed{46.73}$

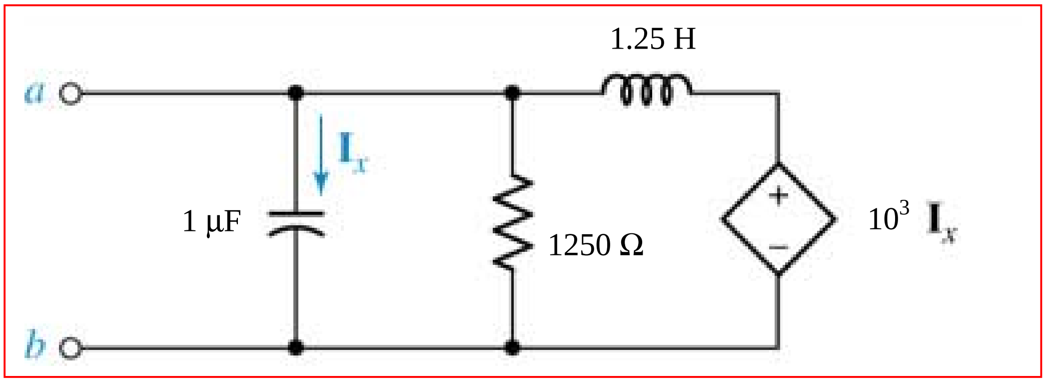
(b) $K_f = 10^6/10^4 = 100,\; K_m = 1 \;\therefore \boxed{\text{R's stay the same; 2 mH} \rightarrow 20\,\mu\text{H, 8 mH} \rightarrow 80\,\mu\text{H, 1}\,\mu\text{F} \rightarrow 10\,\text{nF}}$

(c) $\omega_o = \boxed{10^6 \text{ rad/s,}}\, Q_o$ stays the same, $\therefore B = \dfrac{10^6}{46.73} = \boxed{21.40 \text{ krad/s}}$

35.

(a) $K_m = 250,\ K_f = 400\ \therefore 0.1\text{F} \rightarrow \dfrac{0.1}{250 \times 400} = 1\mu\text{F}$

$5\Omega \rightarrow 1250\,\Omega,\ 2\text{H} \rightarrow \dfrac{2 \times 250}{400} = 1.25\,\text{H},\ 4\bar{\text{I}}_x \rightarrow 10^3\ \bar{\text{I}}_x$



(b) $\omega = 10^3$. Apply 1 V $\therefore \text{I}_x = 10^{-6}s,\ \downarrow \text{I}_{1250} = \dfrac{1}{1250}$

$\therefore 1000\ \text{I}_x = 10^{-3}s\ \ \therefore \rightarrow \text{I}_L = \dfrac{1 - 10^{-3}s}{1.25s}$

$\therefore \text{I}_{in} = 10^{-6}s + \dfrac{1}{1250} + \dfrac{0.8}{s}(1 - 10^{-3}s) = 10^{-6}s + \dfrac{0.8}{s};\ s = j10^3$

$\therefore \text{I}_{in} = j10^{-3} + \dfrac{0.8 \times 10^{-3}}{j} = j0.2 \times 10^{-3}\ \ \therefore \text{Z}_{th} = \dfrac{1}{\text{I}_{in}} = \dfrac{1000}{j0.2} = \boxed{-j5\ k\Omega\ \ \bar{\text{V}}_{oc} = 0}$

36.

(a) $\bar{I}_s = 2\angle 0°\text{ A},\ \omega = 50\ \therefore \bar{V}_{out} = \boxed{60\angle 25°\text{ V}}$

(b) $\bar{I}_s = 2\angle 40°\text{ A},\ \omega = 50\ \therefore \bar{V}_{out} = \boxed{60\angle 65°\text{ V}}$

(c) $\bar{I}_s = 2\angle 40°\text{ A},\ \omega = 200,\ \therefore \boxed{\text{OTSK}}$

(d) $K_m = 30,\ \bar{I}_S = 2\angle 40°\text{ A},\ \omega = 50\ \therefore \bar{V}_{out} = \boxed{1800\angle 65°\text{ V}}$

(e) $K_m = 30,\ K_f = 4,\ \bar{I}_s = 2\angle 40°\text{ A},\ \omega = 200\ \therefore \bar{V}_{out} = \boxed{1800\angle 65°\text{ V}}$

37.

(a) $\bar{\mathrm{H}}/(s) = 0.2 \quad \therefore \mathrm{H}_{dB} = 20 \log 0.2 = \boxed{-13.979\,\mathrm{dB}}$

(b) $\bar{\mathrm{H}}(s) = 50 \quad \therefore \mathrm{H}_{dB} = 20 \log 50 = \boxed{33.98\,\mathrm{dB}}$

(c) $\bar{\mathrm{H}}(j10) = \dfrac{12}{2 + j10} + \dfrac{26}{20 + j10} \quad \therefore \mathrm{H}_{dB} = 20 \log \left| \dfrac{6}{1 + j5} + \dfrac{13}{10 + j5} \right| = 20 \log \left| \dfrac{292 + j380}{-60 + j220} \right| = \boxed{6.451\,\mathrm{dB}}$

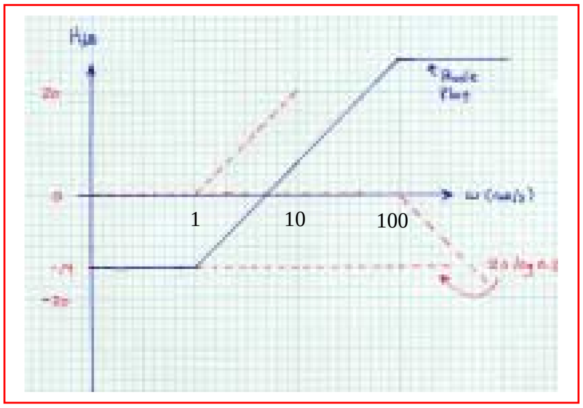
(d) $\mathrm{H}_{dB} = 37.6\,\mathrm{dB} \quad \therefore \left|\bar{\mathrm{H}}(s)\right| = 10^{37.6/20} = \boxed{75.86}$

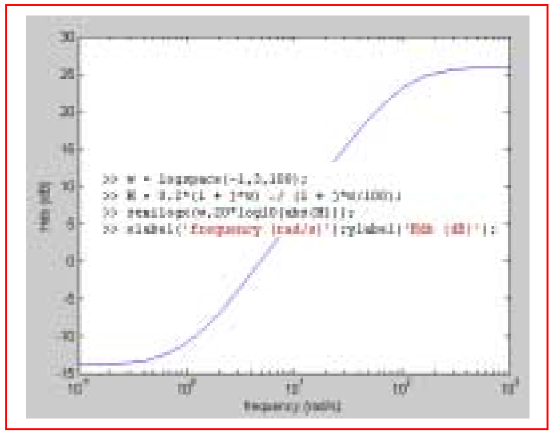
(e) $\mathrm{H}_{dB} = -8\,\mathrm{dB} \quad \therefore \left|\bar{\mathrm{H}}(s)\right| = 10^{-8/20} = \boxed{0.3981}$

(f) $\mathrm{H}_{dB} = 0.01\,\mathrm{dB} \quad \therefore \left|\bar{\mathrm{H}}(s)\right| = 10^{0.01/20} = \boxed{1.0012}$

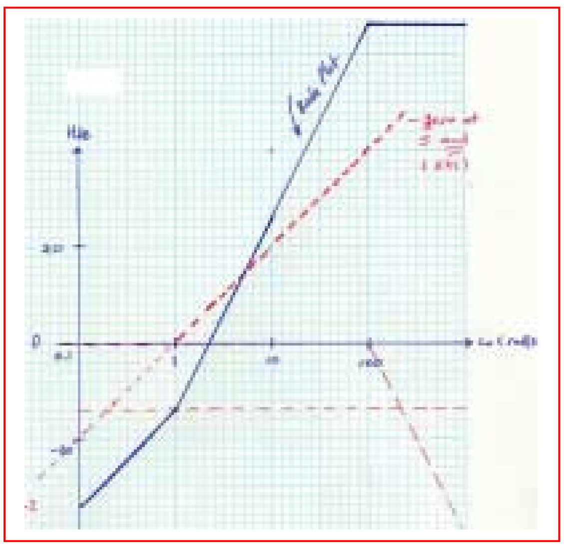
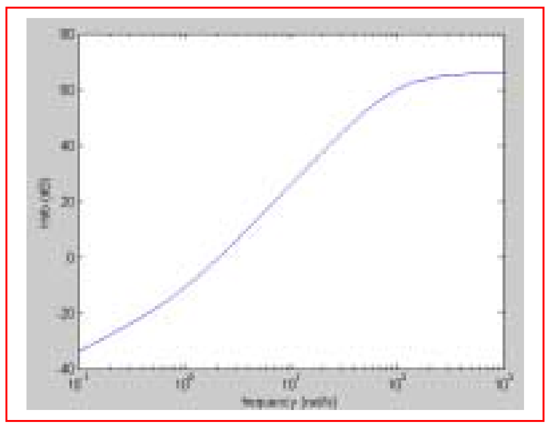
38. (d) MATLAB verification- shown adjacent to Bode plots below.

(a) $\bar{\mathrm{H}}(s) = \dfrac{20(s+1)}{s+100} = \dfrac{0.2(1+s)}{1+s/100}$, $0.2 \to -14\,\mathrm{dB}$

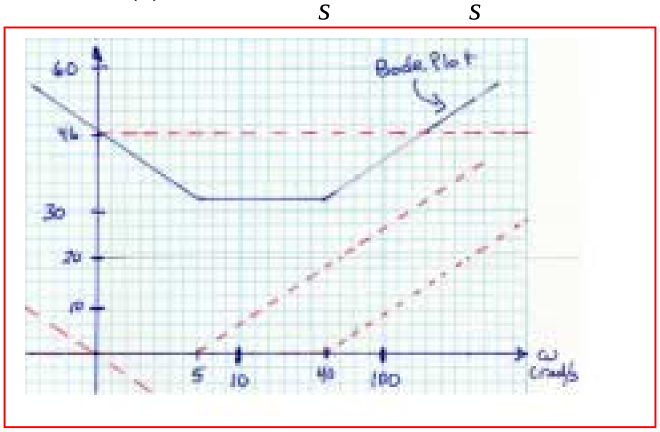


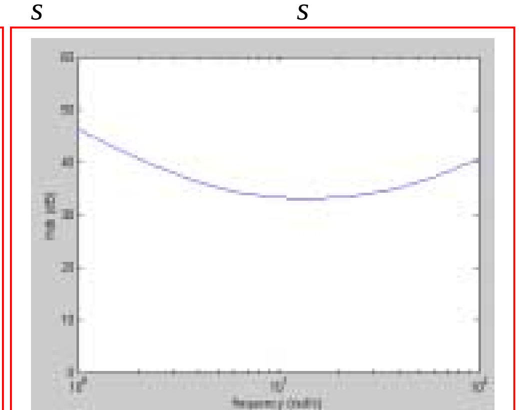


(b) $\bar{\mathrm{H}}(s) = \dfrac{2000(s+1)s}{(s+100)^2} = \dfrac{0.2s(1+s)}{(1+s/100)^2}$, $0.2 \to -14\,\mathrm{dB}$

(c) $\bar{\mathrm{H}}(s) = s + 45 + \dfrac{200}{s} = \dfrac{s^2+45s+200}{s} = \dfrac{(s+5)(s+40)}{s} = \dfrac{200(1+s/5)(1+s/40)}{s}$, $200 \to 46\,\mathrm{dB}$



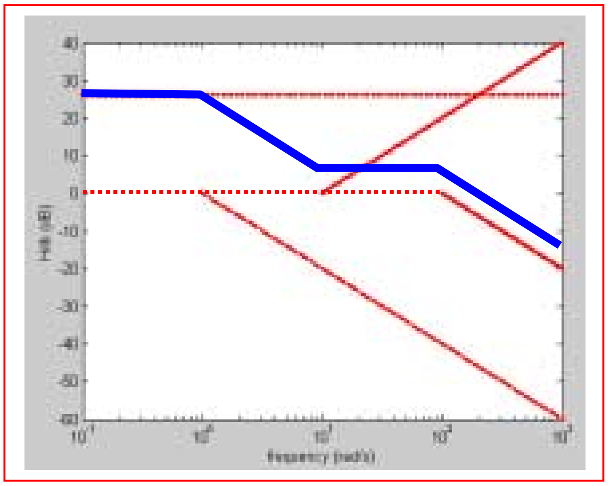


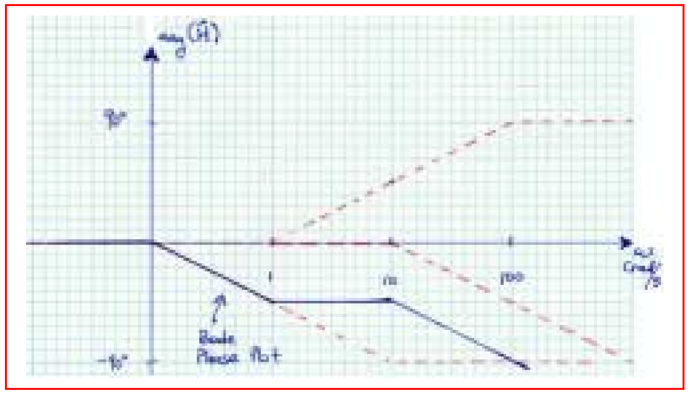
39.

$$\bar{\mathrm{H}}(s) = \frac{\mathrm{V}_C}{I_R} = \frac{(20+2s)(182+200/s)}{202+2s+200/s} \times \frac{200/s}{182+200/s}$$

$$= \frac{400(s+10)}{2(s^2+101s+100)} = \frac{200(10+s)}{(1+s)(100+s)}$$

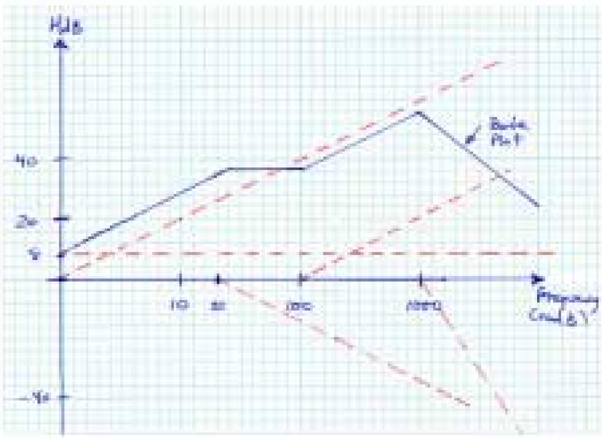
$$\bar{\mathrm{H}}(s) = \frac{20(1+s/10)}{(1+s)(1+s/100)},\ 20 \rightarrow 26\,\mathrm{dB}$$





40.

(a) $\bar{H}(s) = \dfrac{5\times 10^8 s(s+100)}{(s+20)(s+1000)^3} = \dfrac{2.5s(1+s/100)}{(1+s/20)(1+s/1000)^3}$, $2.5 \to 8\,\text{dB}$



(b) Corners: $\omega = \boxed{20,\ 34\,\text{dB};}$

$\omega = \boxed{100,\ 34\,\text{dB};}$

$\omega = \boxed{1000,\ 54\,\text{dB}}$

Intercepts: $0\,\text{dB},\ 2.5\omega = 1,\ \omega = \boxed{0.4}$

$\omega = 1,\ 8\,\text{dB};\ 0\,\text{dB}, \dfrac{2.5\omega(\omega/100)}{(\omega/20)(\omega/1000)^3} = \dfrac{2.5\omega^2(20)10^9}{100\omega\omega^3} = 1 \quad \therefore \omega = \boxed{22,360\ \text{rad/s}}$

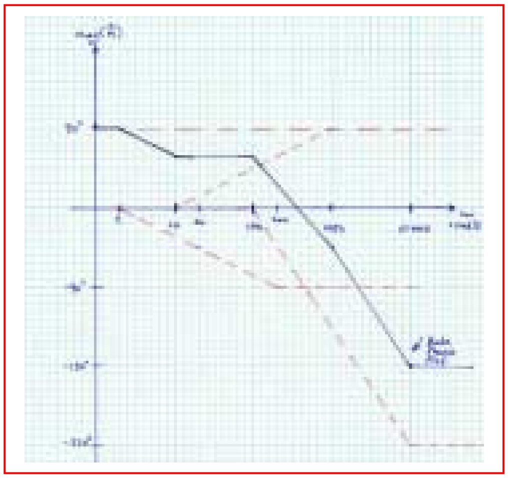
(c) Corners: $\omega = 20,\ \boxed{31.13\,\text{dB}}$

$\omega = 100,\ \boxed{36.69\,\text{dB}} \quad H_{dB} = 20\log 2.5\omega \dfrac{\sqrt{1+(\omega/100)^2}}{\sqrt{[1+(\omega/20)^2][1+(\omega/1000)^2]^3}}$

$\omega = 1000,\ \boxed{44.99\,\text{dB}}$

41.

(a) $$\bar{\mathrm{H}}(s) = \frac{5\times10^{8}\, s(s+100)}{(s+20)(s+1000)^{3}} = \frac{2.5s(1+s/100)}{(1+s/20)(1+s/1000)^{3}},$$



(b) $$\omega = 2 : \angle = \boxed{90°}$$

$$\omega = 10 : \angle = 90° - \left(45° + 45° \log \frac{10}{20}\right) = \boxed{58.5°}$$

$$\omega = 100 : \angle = 90° - \left(45° + 45° \log \frac{100}{20}\right) + \left(45° + 45° \log \frac{100}{100}\right) = \boxed{58.5°}$$

$$\omega = 200 : \angle = 90° - 90° + \left(45° + 45° \log \frac{200}{100}\right) - 3\left(45° + 45° \log \frac{200}{100}\right) = \boxed{17.9°}$$

$$\omega = 1000 : \angle = 90° - 90° + 90° - 3\left(45° + 45° \log \frac{1000}{1000}\right) = \boxed{-45°}$$

$$\omega = 10,000 : \angle = 90° - 90° + 90° - 3\times 90° = \boxed{-180°}$$

(c) $$\omega = 2 : \angle = 90° + \tan^{-1} 0.02 - \tan^{-1} 0.1 - 3\tan^{-1} 0.002 = \boxed{85.09°}$$

$$\omega = 10 : \angle = 90° + \tan^{-1} 0.1 - \tan^{-1} 0.5 - 3\tan^{-1} 0.01 = \boxed{67.43°}$$

$$\omega = 100 : \angle = 90° + \tan^{-1} 1 - \tan^{-1} 5 - 3\tan^{-1} 0.1 = \boxed{39.18°}$$

$$\omega = 200 : \angle = 90° + \tan^{-1} 2 - \tan^{-1} 10 - 3\tan^{-1} 0.2 = \boxed{35.22°}$$

$$\omega = 1000 : \angle = 90° + \tan^{-1} 10 - \tan^{-1} 50 - 3\tan^{-1} 1 = \boxed{-49.56°}$$

$$\omega = 10,000 : \angle = 90° + \tan^{-1} 100 - \tan^{-1} 500 - 3\tan^{-1} 10 = \boxed{-163.33°}$$

42.

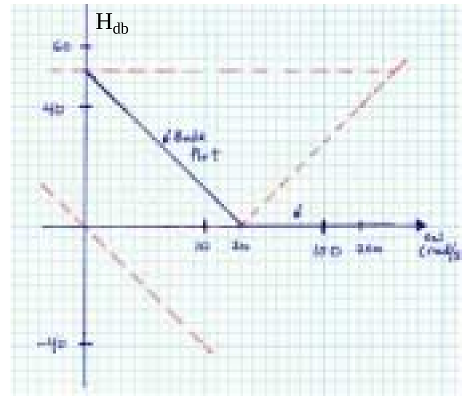
(a) $$\mathrm{H}(s) = 1 + \frac{20}{s} + \frac{400}{s^2} = \frac{s^2 + 20s + 400}{s^2}$$

$$= 400\frac{1 + 2\times 0.5(s/20) + (s/20)^2}{s^2}$$

$$\therefore \omega_o = 20,\ \zeta = 0.5$$

$$20 \log 400 = 52\text{dB}$$

Correction at ω_o is $20 \log 2\zeta = 0$ dB



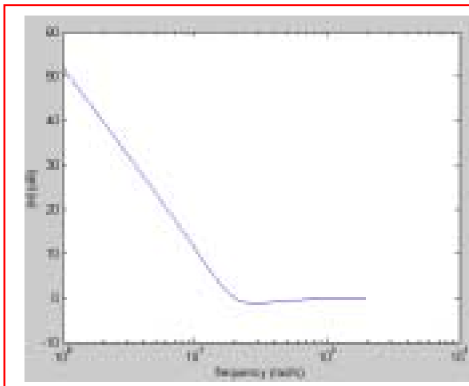
(b) $\omega = 5: \mathrm{H}_{dB} = 52 - 2\times 20 \log 5 = \boxed{24.0\,\text{dB}}$ (plot)

$\mathrm{H}_{dB} = 20 \log |1 - 16 + j4| = \boxed{23.8\,\text{dB}}$ (exact)

$\omega = 100: \mathrm{H}_{dB} = \boxed{0\,\text{dB}}$ (plot)

$\mathrm{H}_{dB} = 20 \log |1 - 0.04 + j0.2| = \boxed{-0.170\text{ dB}}$ (exact)

(c)

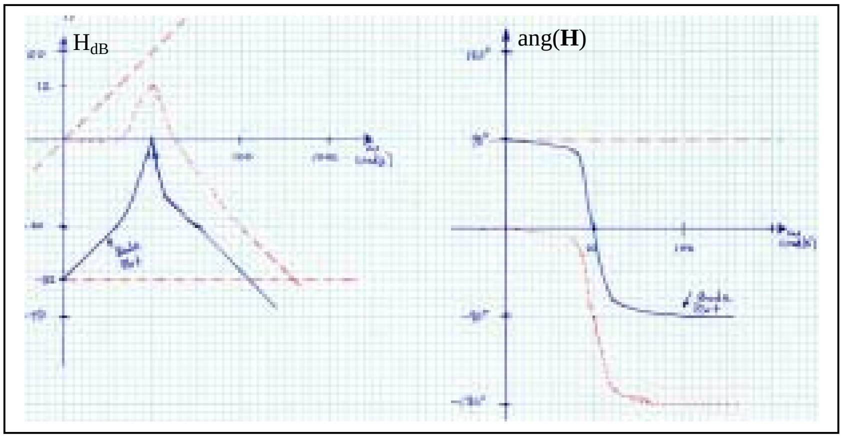


43.

(a) $$\bar{H}(s) = \frac{V_R}{V_5} = \frac{25}{10s + 25 + 1000/s} = \frac{25s}{10s^2 + 25s + 1000} = \boxed{\frac{0.025s}{1 + 2\left(\frac{1}{8}\right)\left(\frac{s}{10}\right) + \left(\frac{s}{10}\right)^2}}$$

(b) $$\therefore \omega_o = 10,\ \zeta = 1/8 \quad \therefore \text{correction} = -20 \log \left(2 \times \frac{1}{8}\right) = 12 \text{ dB}$$

$$0.025 \rightarrow -32 \text{ dB}$$



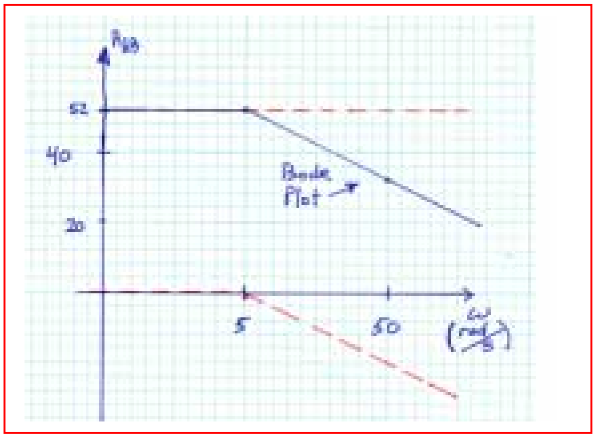
(c) $$\omega = 20,\ \bar{H}(j20) = \frac{j0.5}{1 - 4 + j0.5} \quad \therefore H_{dB} = \boxed{-15.68 \text{ dB}} \quad \angle H(j20) = \boxed{-80.54°}$$

44.

1st two stages, $\bar{H}_1(s) = \bar{H}_2(s) = -10;\ \bar{H}_3(s) = \dfrac{-1/(50\times10^3\times10^{-6})}{s+1/(200\times10^3\times10^{-6})} = \dfrac{-20}{s+5}$

$$\therefore \bar{H}(s) = (-10)(-10)\left(\frac{-20}{s+5}\right) = \boxed{\frac{-400}{1+s/5}}$$

$-400 \rightarrow 52\text{ dB}$



45.

(a) 1st stage: $C_{1A} = 1\ \mu\text{F},\ R_{1A} = \infty,\ R_{fA} = 10^5 \quad \therefore \bar{H}_A(S) = -R_{fA}C_{1A}s = -0.1s$

2nd stage: $R_{1B} = 10^5,\ R_{fB} = 10^5,\ C_{fB} = 1\ \mu\text{F} \quad \therefore \bar{H}_B(s) = \dfrac{-1/R_{1B}C_{fB}}{s + 1/R_{fB}C_{fB}}$

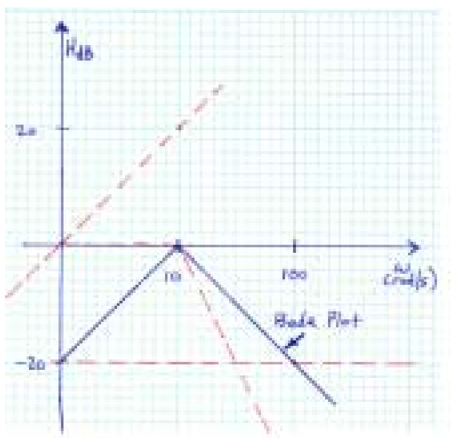
$$\therefore \bar{H}_B(s) = \frac{1/(10^5 \times 10^{-6})}{s + 1/(10^5 \times 10^{-6})} = -\frac{10}{s+10}$$

3rd stage: same as 2nd

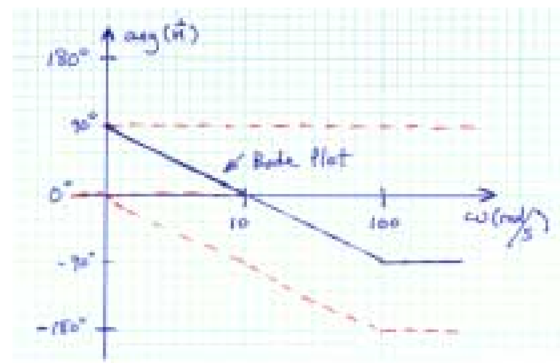
$$\therefore \bar{H}(s) = (-0.1s)\left(\frac{-10}{s+10}\right)\left(\frac{-10}{s+10}\right) = \boxed{-\frac{0.1s}{(1+s/10)^2}}$$

$20\log_{10}(0.1)$ = -20 dB

(b)



(c)



46. An amplifier that rejects high-frequency signals is required. There is some ambiguity in the requirements, as social conversations may include frequencies up to 50 kHz, and echolocation sounds, which we are asked to filter out, may begin below this value. Without further information, we decide to set the filter cutoff frequency at 50 kHz to ensure we do not lose information. However, we note that *this decision is not necessarily the only correct one*.

Our input source is a microphone modeled as a sinusoidal voltage source having a peak amplitude of 15 mV in series with a 1-Ω resistor. Our output device is an earphone modeled as a 1-kΩ resistor. A voltage of 15 mV from the microphone should correspond to about 1 V at the earphone according to the specifications, requiring a gain of 1000/15 = 66.7.

If we select a non-inverting op amp topology, we then need $\frac{R_f}{R_1} = 66.7 - 1 = 65.7$

Arbitrarily choosing $R_1 = 1$ kΩ, we then need $R_f = 65.7$ kΩ. This completes the amplification part. Next, we need to filter out frequencies greater than 50 kHz.

Placing a capacitor across the microphone terminals will "short out" high frequencies. We design for $\omega_c = 2\pi f_c = 2\pi(50\times10^3) = \frac{1}{R_{mic}C_{filter}}$. Since $R_{mic} = 1$ Ω, we require

$C_{filter} = 3.183$ μF.

47. We choose a simple series RLC circuit. It was shown in the text that the "gain" of the circuit with the output taken across the resistor is $\left|A_V\right| = \dfrac{\omega RC}{\left[\left(1-\omega^2 LC\right)^2+\omega^2R^2C^2\right]^{1/2}}$.

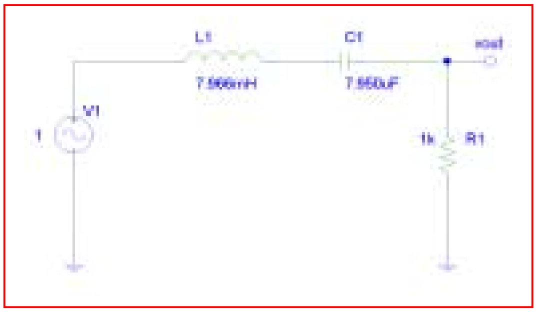
This results in a bandpass filter with corner frequencies at

$$\omega_{c_L} = \frac{-RC+\sqrt{R^2C^2+4LC}}{2LC} \quad \text{and} \quad \omega_{c_H} = \frac{RC+\sqrt{R^2C^2+4LC}}{2LC}$$

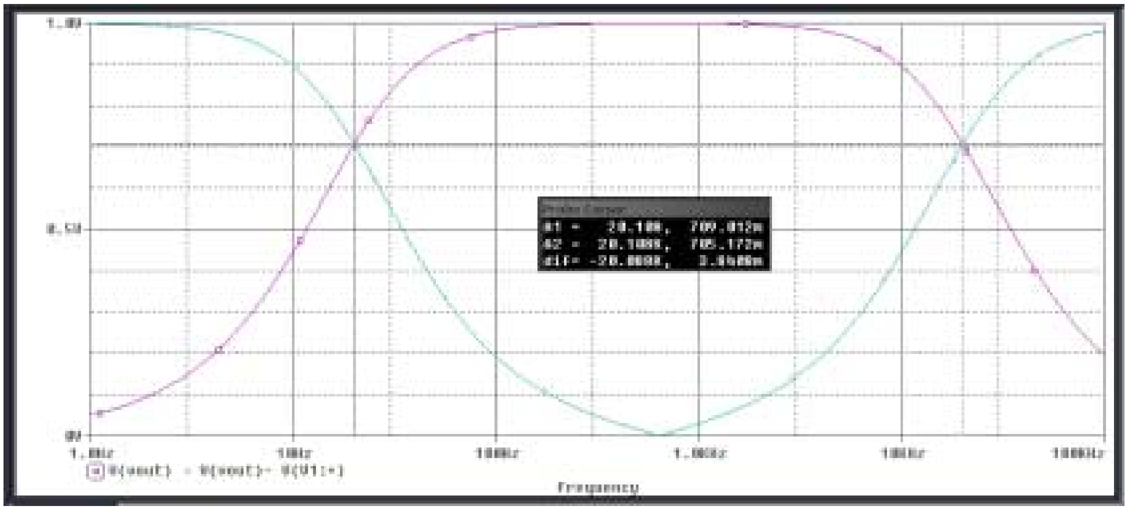
If we take our output across the inductor-capacitor combination instead, we obtain the opposite curve- *i.e.* a bandstop filter with the same cutoff frequencies. Thus, we want

$$2\pi(20) = \frac{-RC+\sqrt{R^2C^2+4LC}}{2LC} \text{ and } 2\pi(20\times10^3) = \frac{RC+\sqrt{R^2C^2+4LC}}{2LC}$$

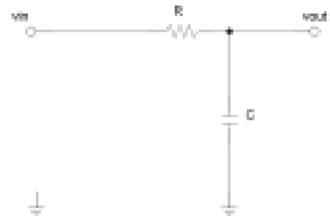
Noting that $\omega_{c_H} - \omega_{c_L}$ = R/L = 125.5 krad/s, we arbitrarily select R = 1 kΩ, so that L = 7.966 mH. Returning to either cutoff frequency expression, we then find C = 7.950 μF



PSpice verification. The circuit performs as required, with a lower corner frequency of about 20 Hz and an upper corner frequency of about 20 kHz.



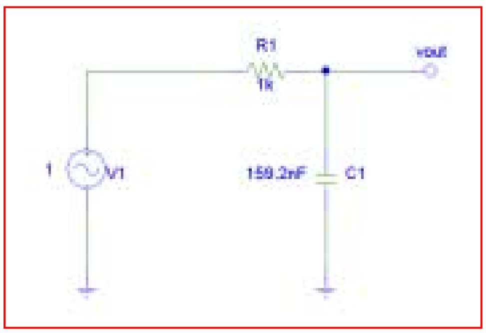
48. We choose a simple RC filter topology:

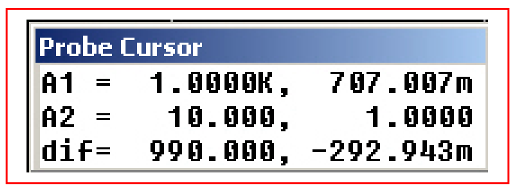


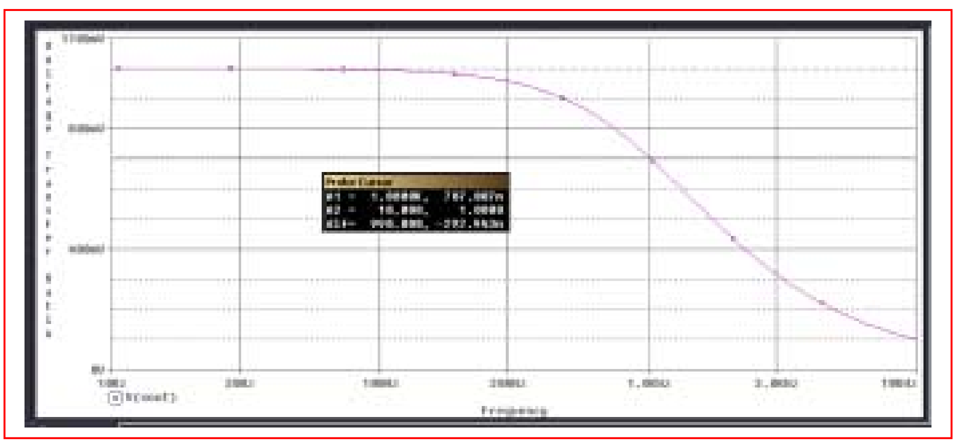
Where $\dfrac{V_{out}}{V_{in}} = \dfrac{1}{1 + j\omega RC}$ and hence $\left|\dfrac{V_{out}}{V_{in}}\right| = \dfrac{1}{\sqrt{1+(\omega RC)^2}}$. We desire a cutoff frequency of 1 kHz, and note that this circuit does indeed act as a low-pass filter (higher frequency signals lead to the capacitor appearing more and more as a short circuit). Thus,

$$= \frac{1}{\sqrt{1+(\omega_c RC)^2}} \quad = \quad \frac{1}{\sqrt{2}} \qquad\qquad \text{where } \omega_c = 2\pi f_c = 2000\pi \text{ rad/s.}$$

A small amount of algebra yields $1 + [2\pi(1000)RC]^2 = 2$ or $2000\pi RC = 1$. Arbitrarily setting R = 1 kΩ, we then find that C = 159.2 nF. The operation of the filter is verified in the PSpice simulation below:

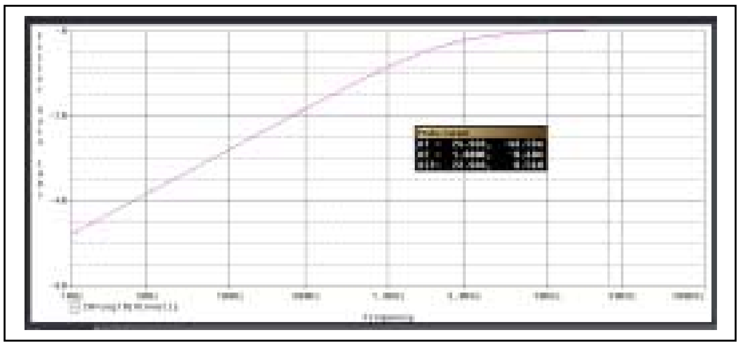


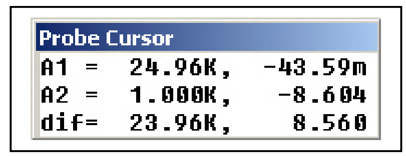




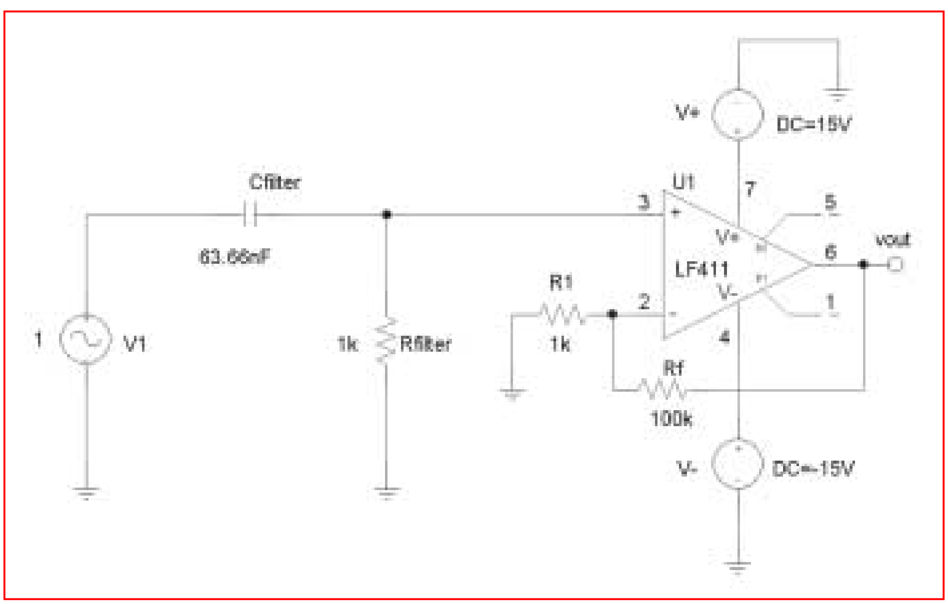
49. We are not provided with the actual spectral shape of the noise signal, although the reduction to 1% of its peak value (a drop of 40 dB) by 1 kHz is useful to know. If we place a simple high-pass RC filter at the input of an op amp stage, designing for a pole at 2.5 kHz should ensure an essentially flat response above 25 kHz, and a 3 dB reduction at 2.5 kHz. If greater tolerance is required, the 40 dB reduction at 1 kHz allows the pole to be moved to a frequency even closer to 1 kHz. The PSpice simulation below shows a filter with R = 1 kΩ (arbitrarily chosen) and C = $\dfrac{1}{2\pi(2.5\times10^3)(1000)} = 63.66\text{ nF}$.

At a frequency of 25 kHz, the filter shows minimal gain reduction, but at 1 kHz any signal is reduced by more than 8 dB.





We therefore design a simple non-inverting op amp circuit such as the one below, which with R_f = 100 kΩ and R_1 = 1 kΩ, has a gain of 100 V/V. In simulating the circuit, a gain of approximately 40 dB at 25 kHz was noted, although the gain dropped at higher frequencies, reaching 37 dB around 80 kHz. Thus, to completely assess the suitability of design, more information regarding the frequency spectrum of the "failure" signals would be required.



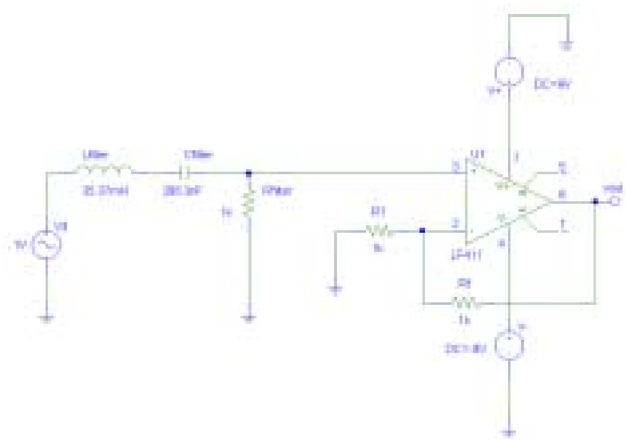
50. We select a simple series RLC circuit with the output taken across the resistor to serve as a bandpass filter with 500 Hz and 5000 Hz cutoff frequencies. From Example 16.12, we know that

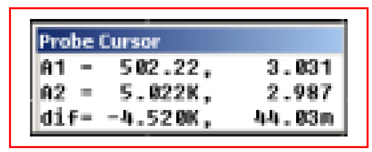
$$\omega_{c_L} = -\frac{R}{2L} + \frac{1}{2LC}\sqrt{R^2C^2 + 4LC} = 2\pi(500)$$

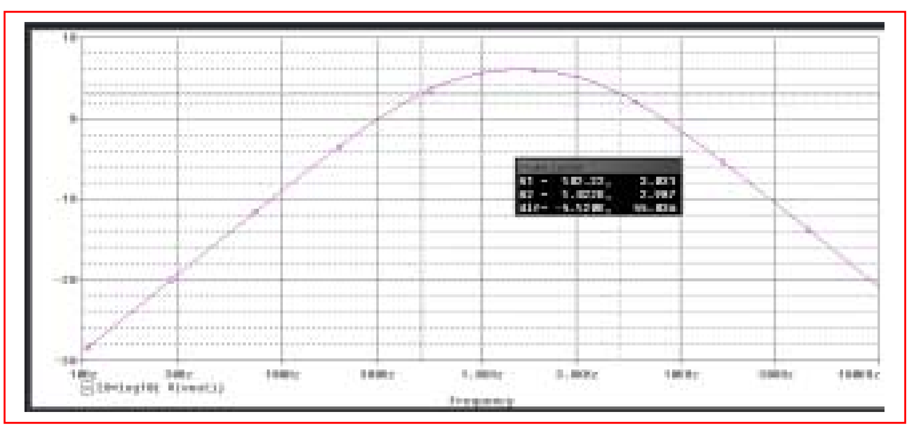
and

$$\omega_{c_H} = \frac{R}{2L} + \frac{1}{2LC}\sqrt{R^2C^2 + 4LC} = 2\pi(5000)$$

With $\omega_{c_H} - \omega_{c_L} = 2p(5000 - 500) = R/L$, we (arbitrarily) select R = 1 kΩ, so that L = 35.37 mH. Substituting these two values into the equation for the high-frequency cutoff, we find that C = 286.3 nF. We complete the design by selecting R_1 = 1 kΩ and R_f = 1 kΩ for a gain of 2 (no value of gain was specified). As seen in the PSpice simulation results shown below, the circuit performs as specified at maximum gain (6 dB or 2 V/V), with cutoff frequencies of approximately 500 and 5000 KHz and a peak gain of 6 dB.



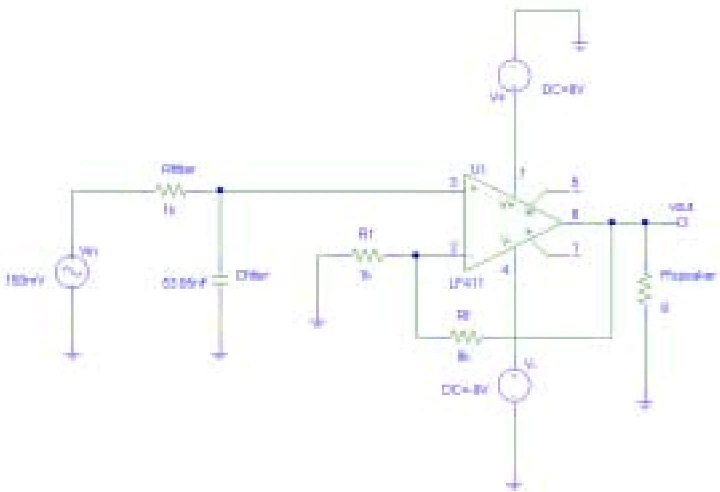


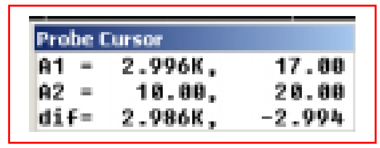


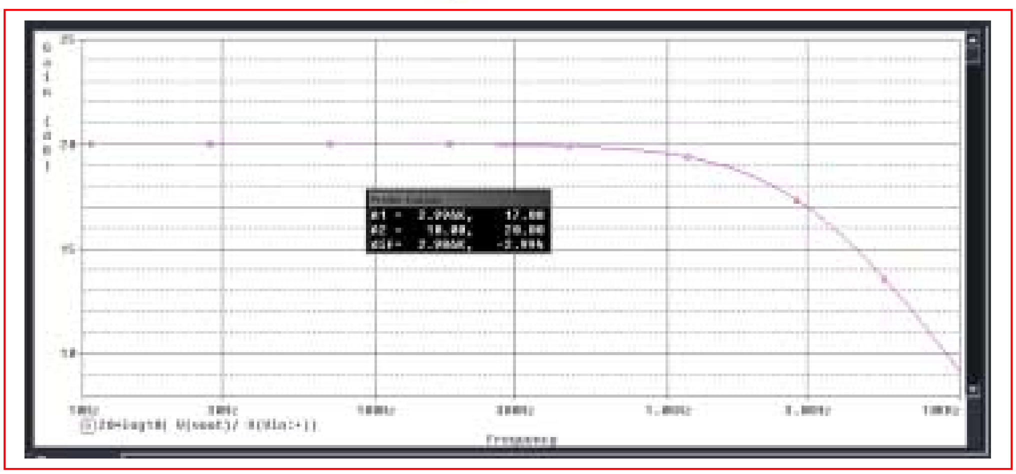
51. For this circuit, we simply need to connect a low-pass filter to the input of a non-inverting op amp having $R_f/R_1 = 9$ (for a gain of 10). If we use a simple RC filter, the cutoff frequency is

$$\omega_c = \frac{1}{\mathrm{RC}} = 2\pi(3000)$$

Selecting (arbitrarily) R = 1 kΩ, we find C = 53.05 nF. The PSpice simulation below shows that our design does indeed have a bandwidth of 3 kHz and a peak gain of 10 V/V (20 dB).

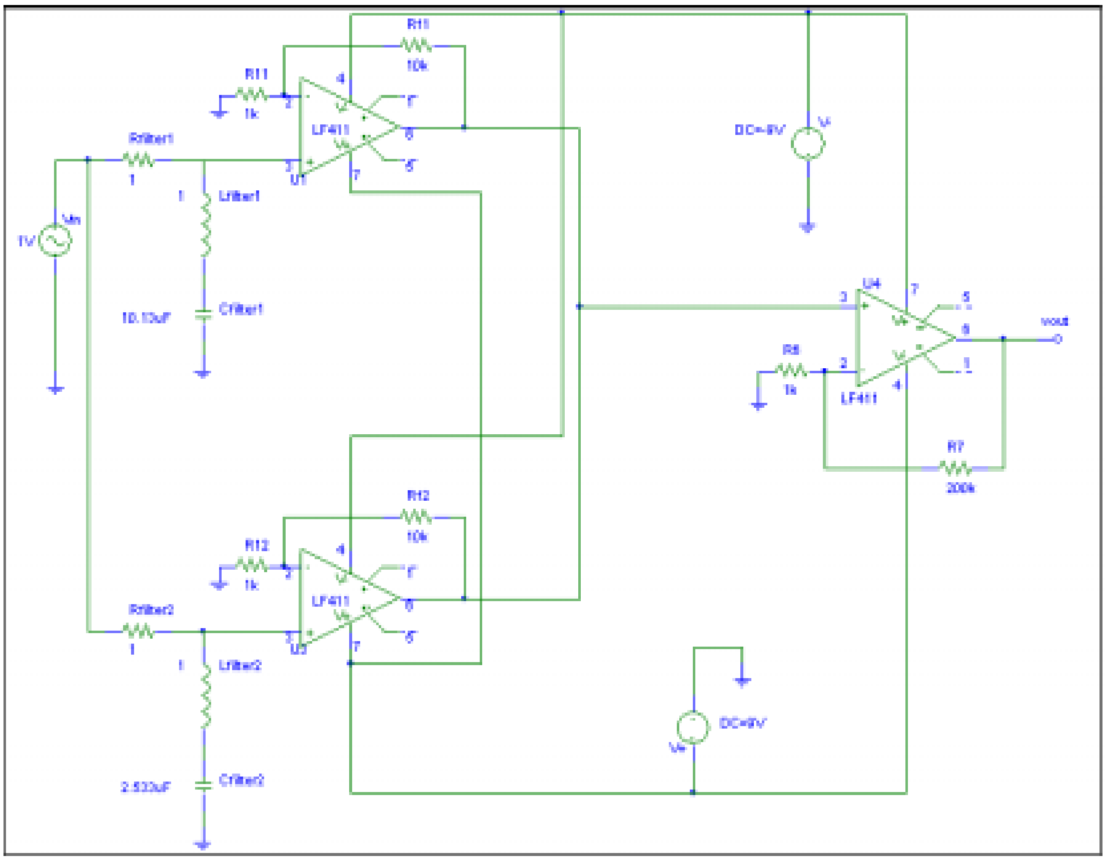


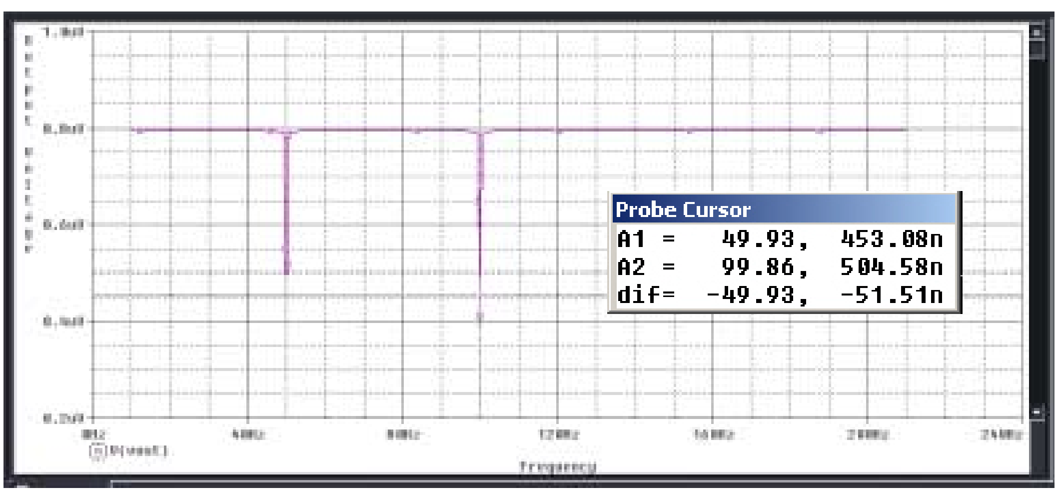




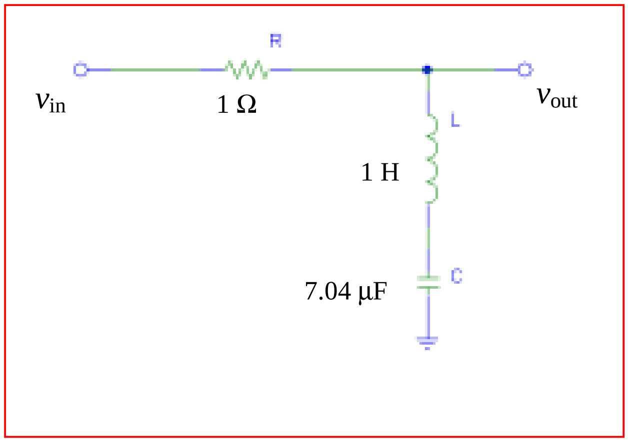
52. We require four filter stages, and choose to implement the circuit using op amps to isolate each filter sub-circuit. Selecting a bandwidth of 1 rad/s (no specification was given) and a simple RLC filter as suggested in the problem statement, a resistance value of 1 Ω leads to an inductor value of 1 H (bandwidth for this type of filter = $\omega_H - \omega_L$ = R/L). The capacitance is found by designing each filter's respective resonant frequency ($1/\sqrt{LC}$) at the desired "notch" frequency. Thus, we require C_{F1} = 10.13 µF, C_{F2} = 2.533 µF, C_{F3} = 1.126 µF and C_{F4} = 633.3 nF.

The Student Version of PSpice® will not permit more than 64 nodes, so that the total solution must be simulated in two parts. The half with the filters for notching out 50 and 100 Hz components is shown below; an additional two op amp stages are required to complete the design.





53. Using the series RLC circuit suggested, we decide to design for a bandwidth of 1 rad/ s (as no specification was provided). With $\omega_H - \omega_L = R/L$, we arbitrarily select R = 1 Ω so that L = 1 H. The capacitance required is obtained by setting the resonant frequency of the circuit ($1/\sqrt{LC}$) equal to 60 Hz (120π rad/s). This yields C = 7.04 μF.



CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

1.

$$\Delta_Z = \begin{vmatrix} 17 & -8 & -3 \\ -8 & 17 & -4 \\ -3 & -4 & 17 \end{vmatrix} = 17(273) + 8(-148) - 3(83) = 3208\,\Omega^3$$

(a) $$Z_{in1} = \frac{\Delta_Z}{\Delta_{11}} = \frac{3208}{273} = 11.751\,\Omega \quad \therefore P_1 = \frac{100^2}{11.751} = \boxed{851.0 \text{ W}}$$

(b) $$Z_{in2} = \frac{\Delta_Z}{\Delta_{22}} = \frac{3208}{280} = 11.457\,\Omega \quad \therefore P_2 = \frac{100^2}{11.457} = \boxed{872.8 \text{ W}}$$

(c) $$Z_{in3} = \frac{\Delta_Z}{\Delta_{33}} = \frac{3208}{225} = 14.258\,\Omega \quad \therefore P_3 = \frac{100^2}{14.258} = \boxed{701.4\,\Omega}$$

2.

$$\Delta_Y = \begin{vmatrix} 0.35 & -0.1 & -0.2 \\ -0.1 & 0.5 & -0.15 \\ -0.2 & -0.15 & 0.75 \end{vmatrix} = 0.35(0.3525) + 0.1(-0.105) - 0.2(0.115) = 0.089875 \text{ S}^3$$

(a) $$\text{Y}_{in1} = \frac{\Delta_Y}{\Delta_{11}} = \frac{0.089875}{0.3525} = 0.254965 \quad \therefore \text{P}_1 = \frac{10^2}{0.254965} = \boxed{392.2 \text{ W}}$$

(b) $$\text{Y}_{in2} = \frac{\Delta_Y}{\Delta_{22}} = \frac{0.089875}{0.2225} = 0.403933 \quad \therefore \text{P}_2 = \frac{10^2}{0.403933} = \boxed{247.6 \text{ W}}$$

(c) $$\text{Y}_{in3} = \frac{0.089875}{0.165} = 0.544697 \text{ S} \quad \therefore \text{P}_3 = \frac{100}{0.544697} = \boxed{183.59 \text{ W}}$$

3.

$$[\mathrm{R}] = \begin{bmatrix} 3 & -1 & -2 & 0 \\ -1 & 4 & 1 & 3 \\ -2 & 2 & 5 & 2 \\ 0 & -3 & -2 & 6 \end{bmatrix} (\Omega) = 3\begin{vmatrix} 4 & 1 & 3 \\ 2 & 5 & 2 \\ -3 & -2 & 6 \end{vmatrix} + 1\begin{vmatrix} -1 & -2 & 0 \\ 2 & 5 & 2 \\ -3 & -2 & 6 \end{vmatrix} - 2\begin{vmatrix} -1 & -2 & 0 \\ 4 & 1 & 3 \\ -3 & -2 & 6 \end{vmatrix}$$

$$= 3[4(34) - 2(12) - 3(-13)] + [-1(34) - 2(-12) - 3(-4)] = 2[-1(12) - 4(-12) - 3(-6)]$$

$$= 3(73) + (-22) - 2(18) = 161\ \Omega^4 \quad \therefore \mathrm{R}_{in} = \frac{\Delta_R}{\Delta_{11}} = \frac{161}{73} = \boxed{2.205^+\ \Omega}$$

4. Define a counter-clockwise current $\mathbf{I}_2$ in the left-most mesh, and a counter-clockwise current $\mathbf{I}_1$ flowing in the right-most mesh. Then,

$$\overline{V}_1 = 4\overline{I}_2 \quad \therefore 0.2\overline{V}_1 = 0.8\overline{I}_2$$

$$\overline{V}_{in} = \overline{I}_1 s + 5(\overline{I}_1 + 0.8\overline{I}_2 - \overline{I}_2) = (s+5)\overline{I}_1 - \overline{I}_2$$

Also, $\overline{I}_2(2s+4) - 5(\overline{I}_1 + 0.8\overline{I}_2 - \overline{I}_2) = 0$

or $0 = -5\overline{I}_1 + (5+2s)\overline{I}_2$

$$\therefore \Delta_Z = (s+5)(5+2s) - 5 = 2s^2 + 15s + 20,\ \Delta_{11} = 5 + 2s$$

$$\therefore \boxed{Z_{th} = \frac{2s^2 + 15s + 20}{2s+5}}$$

5. Define a clockwise mesh current $\mathbf{I}_1$ flowing in the bottom left mesh, a clockwise mesh current $\mathbf{I}_2$ flowing in the top mesh, and a clockwise mesh current $\mathbf{I}_3$ flowing in the bottom right mesh. Then,

(a)

$$\bar{V}_{in} = 10(\bar{I}_1 - \bar{I}_2) - 0.6\times 8\bar{I}_2 = 10\bar{I}_1 - 14.8\bar{I}_2$$

$$0 = 50\bar{I}_2 - 10\bar{I}_1 - 12\bar{I}_3 = -10\bar{I}_1 + 50\bar{I}_2 - 12\bar{I}_3$$

$$0 = 4.8\bar{I}_2 + 17\bar{I}_3 - 12\bar{I}_2 = -7.2\bar{I}_2 + 17\bar{I}_3$$

$$\therefore \Delta_Z = \begin{vmatrix} 10 & -14.8 & 0 \\ -10 & 50 & -12 \\ 0 & -7.2 & 17 \end{vmatrix} = 10(763.6) + 10(-251.6) = 5120 \;\; \therefore Z_{in} = \frac{5120}{763.6} = \boxed{6.705^+\ \Omega}$$

(b)

$$\bar{I}_{in} = \frac{\bar{V}_1 - \bar{V}_2}{28} + \frac{\bar{V}_1 - 0.6\bar{V}_x}{10} = 0.13571\ \bar{V}_1 - 0.03571\bar{V}_2 - 0.06\bar{V}_x$$

$$0 = \frac{\bar{V}_2 - \bar{V}_1}{28} + \frac{\bar{V}_2 - 0.6\bar{V}_x}{12} + \frac{\bar{V}_2}{5} = -0.03571\bar{V}_1 + 0.31905\bar{V}_2 - 0.05\bar{V}_x$$

$$0 = -\frac{\bar{V}_x}{8} + \frac{\bar{V}_2 - \bar{V}_x - \bar{V}_1}{20} = -0.05\bar{V}_1 + 0.05\bar{V}_2 - 0.175\bar{V}_x$$

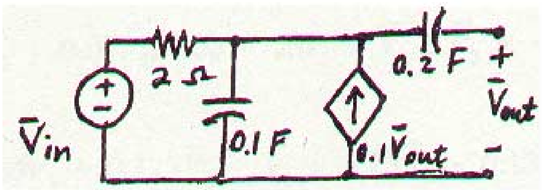
$$\therefore \Delta_y = \begin{vmatrix} 0.13571 & -0.03571 & -0.06 \\ -0.03571 & 0.31905^- & -0.05 \\ -0.05 & 0.05 & -0.175 \end{vmatrix} = 0.13571(-0.05583 + 0.0025) + 0.03571(0.00625 + 0.003)$$

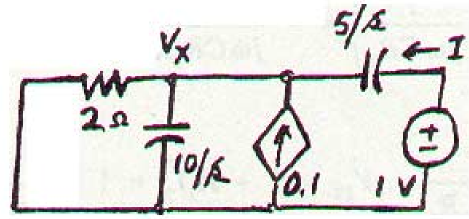
$$-0.05(0.00179 + 0.01914) = -0.00724$$

$$\therefore \Delta_y = 0.007954,\ \Delta_{11} = -0.05333 \;\; \therefore \bar{Y}_m = \frac{\Delta_y}{\Delta_{11}} = \frac{-0.007954}{-0.05333} = 0.14926\ \text{S}$$

$$\therefore \bar{Z}_{in} = \frac{1}{0.14926} = \boxed{6.705^+\ \Omega}$$

6.





$$\frac{\bar{V}_x}{2}+\frac{s\bar{V}_x}{10}-0.1+\frac{s}{5}(\bar{V}_x-1)=0$$

$$\therefore \bar{V}_x(0.5+0.3s)=0.1+0.2s$$

$$\therefore \bar{V}_x=\frac{0.2s+0.1}{0.3s+0.5}$$

$$\therefore \bar{I}=(1-\bar{V}_x)\frac{s}{5}=\left(1-\frac{0.2s+0.1}{0.3s+0.5}\right)0.2s=0.2s\frac{0.1s+0.4}{0.3s+0.5}$$

$$\therefore \bar{Y}_{out}=\bar{I}=\frac{s(0.1s+0.4)}{1.5s+2.5},\ \bar{Z}_{out}=\frac{1.5s+2.5}{s(0.1s+0.4)}=\boxed{\frac{15s+25}{s(s+4)}}$$

7.

$$\overline{V}_{in} = 1\text{ V},\ \overline{V}_i = 0 \quad \therefore \overline{V}_x + \overline{V}_{in} = 0,\ \overline{V}_x = -1\text{ V}$$

$$\overline{I}_x = \frac{\overline{V}_x}{R_x} = -\frac{1}{R_x};\ 2\times10^4 I_{in} + 2\times10^4 I_x = 0$$

$$\therefore \overline{I}_{in} = -I_x = \frac{1}{R_x} \quad \therefore R_{in} = -V_{in} / I_{in} = \boxed{-R_x}$$

8.

(a) Assume 1 V at input. Since V_i = 0 at each op-amp input, 1 V is present between R_2 and R_3, and also C and R_4.

$$\therefore \bar{V}_4 = \frac{1}{R_4}\left(R_4 + \frac{1}{j\omega C}\right) = 1 + \frac{1}{j\omega CR_4}$$

$$\therefore \bar{I}_3 = \frac{1}{R_3}\left(1 - 1 - \frac{1}{j\omega CR_4}\right) = -\frac{1}{j\omega CR_3R_4}$$

$$\therefore \bar{I}_2 = \bar{I}_3 = -\frac{1}{j\omega CR_3R_4} \quad \therefore \bar{V}_{12} = 1 + R_2\bar{I}_2 = 1 - \frac{R_2}{j\omega CR_3R_4}$$

$$\bar{I}_1 = \frac{1-\bar{V}_{12}}{R_1} = \frac{R_2}{j\omega CR_1R_3R_4} = \bar{I}_{in} \quad \therefore \bar{Z}_{in} = \frac{1}{I_{in}} = \boxed{j\omega C\frac{R_1R_3R_4}{R_2}}$$

(b) $$R_1 = 4\times10^3,\ R_2 = 10\times10^3,\ R_3 = 10\times10^3,\ R_4 = 10^3,\ C = 2\times10^{-10}$$

$$\therefore \bar{Z}_{in} = j\omega 2\times10^{-10}\frac{4\times10\times1}{10}\times10^6 = \boxed{j\omega 0.8\times10^3\ \Omega\ (L_{in} = 0.8\text{ mH})}$$

9. Define a clockwise mesh current $\mathbf{I}_1$ in the left-most mesh, a clockwise mesh current $\mathbf{I}_x$ in the center mesh, and a counter-clockwise mesh current $\mathbf{I}_2$ in the right-most mesh. Then,

$$\bar{V}_1 = 13\bar{I}_1 - 10\bar{I}_2$$

$$0 = -10\bar{I}_1 + 35\bar{I}_x + 20\bar{I}_2 \quad \therefore\ \bar{I}_1 = \frac{\begin{vmatrix} \bar{V}_1 & -10 & 0 \\ 0 & 35 & 20 \\ \bar{V}_2 & 20 & 22 \end{vmatrix}}{\begin{vmatrix} 13 & -10 & 0 \\ -10 & 35 & 20 \\ 0 & 20 & 22 \end{vmatrix}}$$

$$\bar{V}_2 = 20\bar{I}_x + 22\bar{I}_2$$

$$\therefore\ \bar{I}_1 = \frac{\bar{V}_1(370) + \bar{V}_2(-200)}{13(370) + 10(-220)} = \frac{37}{261}\ \bar{V}_1 - \frac{20}{261}\ \bar{V}_2$$

$$\therefore\ \bar{y}_{11} = \frac{37}{261} = \boxed{141.76 \text{ mS},}\ \ \bar{y}_{12} = \frac{-20}{261} = \boxed{-76.63 \text{ mS}}$$

10.

$$[y] = \begin{bmatrix} 10 & -5 \\ 50 & 20 \end{bmatrix} \text{ (mS)} \quad \therefore \bar{I}_1 = 0.01\bar{V}_1 - 0.005\bar{V}_2,$$

$$\bar{I}_2 = 0.05\bar{V}_1 + 0.02\bar{V}_2,\ 100 = 25\bar{I}_1 + \bar{V}_1,\ \bar{V}_2 = -100\bar{I}_2$$

$$\therefore 100 = 0.25\bar{V}_1 - 0.125\bar{V}_2 + \bar{V}_1 = 1.25\bar{V}_1 - 0.125\bar{V}_2$$

$$\bar{I}_2 = -0.01\bar{V}_2 = 0.05\bar{V}_1 + 0.02\bar{V}_2 \quad \therefore -0.03\bar{V}_2 = 0.05\bar{V}_1 \quad \therefore \bar{V}_2 = -\frac{5}{3}\bar{V}_1$$

$$\therefore 100 = 1.25\bar{V}_1 + \frac{0.625}{3}\bar{V}_1 = \frac{4.375}{2}\bar{V}_1 \quad \therefore \bar{V}_1 = \frac{300}{4.375} = \boxed{68.57\text{ V},}\ \bar{V}_2 = -\frac{5}{3}\bar{V}_1 = \boxed{-114.29\text{ V}}$$

11.

$$\bar{I}_1 = \frac{\bar{V}_1 - \bar{V}_2}{25} = 0.04\bar{V}_1 - 0.04\bar{V}_2$$

$$\bar{I}_2 = 2I_1 + \frac{\bar{V}_2}{100} - \bar{I}_1 = \bar{I}_1 + 0.01\bar{V}_2 = 0.04\bar{V}_1 - 0.03\bar{V}_2$$

$$\therefore \bar{y}_{11} = \boxed{0.04\,\text{S},}\ \bar{y}_{12} = \boxed{-0.04\,\text{S}},\ \bar{y}_{21} = \boxed{0.04\,\text{S},}\ \bar{y}_{22} = \boxed{-0.03\ \text{S}}$$

12.

$$\therefore \overline{V}_1 = 100(\overline{I}_1 - 0.5\overline{I}_1) = 50\overline{I} \quad \therefore \overline{I}_1 = 0.02\ \overline{V}_1$$

$$\overline{V}_2 = 300\overline{I}_2 + 200(\overline{I}_2 + 0.5\overline{I}_1) = 100\overline{I}_1 + 500\overline{I}_2$$

$$\therefore \overline{V}_2 = 2\overline{V}_1 + 500\overline{I}_2, \ \overline{I}_2 = -0.004\overline{V}_1 + 0.002\overline{V}_2$$

$$\therefore \boxed{[\overline{y}] = \begin{bmatrix} 0.02 & 0 \\ -0.004 & 0.002 \end{bmatrix} \text{(S)}}$$

13.

$$[y] = \begin{bmatrix} 0.1 & -0.0025 \\ -8 & 0.05 \end{bmatrix} \text{ (S)}$$

(a)

$$\bar{I}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2,\ \ \bar{I}_2 = -8\bar{V}_1 + 0.05\bar{V}_2$$

$$1 = 2\bar{I}_1 + \bar{V},\ \bar{V}_2 = -5\bar{I}_2$$

$$\therefore \bar{I}_2 = -0.2\bar{V}_2 = -8\bar{V}_1 + 0.05\bar{V}_2 \quad \therefore 0.25\bar{V}_2 = 8\bar{V}_1,\ \ \bar{V}_2 / \bar{V}_1 = 32$$

$$\bar{I}_2 = -8\bar{V}_1 + 0.05\times 32\bar{V}_1,\ \ \bar{I}_1 = 0.1\bar{V}_1 - 0.0025\times 32\bar{V}_1 \ \ \therefore \bar{I}_2 = -6.4\bar{V}_1,\ \ \bar{I}_1 = 0.02\bar{V}_1$$

$$\therefore \bar{I}_2 / \bar{I}_1 = \frac{-6.4}{0.02} = -320,\ \ \bar{V}_1 / \bar{I}_1 = \boxed{50\Omega}$$

(b)

$$\bar{V}_1 = -2\bar{I}_1,\ \ \bar{I}_1 = 0.1\bar{V}_1 - 0.0025V_2,\ \ \bar{I}_2 = -8\bar{V}_1 + 0.05\bar{V}_2$$

$$\therefore \bar{I}_1 = -0.5\bar{V}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2 \therefore 0.6\bar{V}_1 = 0.0025\bar{V}_2$$

$$\therefore \bar{V}_1 = \bar{V}_2 / 240,\ \ \bar{I}_2 = -8\times\bar{V}_2 / 240 + \frac{1}{20}\ \bar{V}_2 = \frac{1}{60}\bar{V}_2$$

$$\therefore \frac{\bar{V}_2}{\bar{I}_2} = \boxed{60\,\Omega}$$

14. $$[\bar{y}] = \begin{bmatrix} 10 & -5 \\ -20 & 2 \end{bmatrix} \text{ (mS)}$$

(a)

$$\overline{I}_1 = 0.01\overline{V}_1 - 0.005\overline{V}_2,\ \ \overline{I}_2 = -0.02\overline{V}_1 + 0.002\overline{V}_2$$

$$\overline{V}_1' = 100\,\overline{I}_1 + \overline{V}_1$$

$$\therefore\ \overline{V}_1 = \overline{V}1 - 100\,\overline{I}_1\ \therefore\ \overline{I}_1 = 0.01\overline{V}1 - \overline{I}_1 - 0.005\overline{V}_2\quad \therefore\ \overline{I}_1 = 0.005\overline{V}1 - 0.0025\overline{V}_2$$

$$\overline{I}_2 = -0.02\overline{V}1 + 2\,\overline{I}_1 + 0.002\overline{V}_2 = -0.02\overline{V}1 + 0.01\overline{V}1 - 0.005\overline{V}_2 + 0.002\overline{V}_2 = -0.01\overline{V}1 - 0.003\overline{V}_2$$

$$\therefore \boxed{[y]_{new} = \begin{bmatrix} 0.005 & -0.0025 \\ -0.01 & -0.003 \end{bmatrix} \text{ (S)}}$$

(b)

$$\overline{V}2 = 100\,\overline{I}_2 + \overline{V}_2,\ \therefore\ \overline{V}_2 = \overline{V}2 - 100\,\overline{I}_2$$

$$\therefore\ \overline{I}_2 = -0.02\overline{V}_1 + 0.002\overline{V}2 - 0.2\,\overline{I}_2$$

$$\therefore 1.2\,\overline{I}_2 = -0.02\overline{V}_1 + 0.002\overline{V}2\ \ \therefore\ \overline{I}_2 = -\frac{1}{60}\ \overline{V}_1 + \frac{1}{600}\ \overline{V}2$$

$$\overline{I}_1 = 0.01\overline{V}_1 - 0.005(\overline{V}2 - 100\,\overline{I}_2) = 0.01\overline{V}_1 - 0.005\overline{V}2 + 0.5\left(-\frac{1}{60}\ \overline{V}_1 + \frac{1}{600}\ \overline{V}2\right)$$

$$\therefore\ \overline{I}_1 = \left(\frac{1}{100} - \frac{1}{120}\right)\overline{V}_1 - \left(\frac{1}{200} - \frac{1}{1200}\right)\overline{V}_2' = \frac{1}{600}\ \overline{V}_1 - \frac{1}{240}\ \overline{V}_2'$$

$$\therefore \boxed{[\bar{y}]_{new} = \begin{bmatrix} 1/600 & -1/240 \\ -1/60 & 1/600 \end{bmatrix} \text{ (S)}}$$

15.

	$\overline{V}_{S1}$	$\overline{V}_{S2}$	$\overline{I}_1$	$\overline{I}_2$
Exp #1	100 V	50 V	5 A	-32.5 A
Exp #2	50	110	-20	-5
Exp #3	20	0	4	-8
Exp #4	-8.333	-22.22	5	0
Exp #5	-58.33	-55.56	5	15

$$\overline{I}_1 = \overline{y}_{11}\overline{V}_1 + \overline{y}_{12}\overline{V}_2$$
$$\overline{I}_2 = \overline{y}_{21}V_1 + y_{22}\overline{V}_2$$

Use 1st 2 rows to find y's

$\therefore 5 = 100\overline{y}_{11} + 50\overline{y}_{12}, -32.5 = 100\overline{y}_{21} + 50\overline{y}_{22}$

$-20 = 50\overline{y}_{11} + 100\overline{y}_{12,}\ -5 = 50\overline{y}_{21} + 100\overline{y}_{22} \rightarrow \therefore -10 = 100\overline{y}_{21} + 200\overline{y}_{22}$

$\therefore -40 = 100\overline{y}_{11} + 200\overline{y}_{12}$ Substracting, $150\overline{y}_{12} = -45\ \therefore \boxed{\overline{y}_{12} = -0.3\text{ S}}$

$\therefore 5 = 100\overline{y}_{11} - 15\ \therefore \boxed{\overline{y}_{11} = 0.2\text{ S}}$ Subtracting $22.5 = 150\overline{y}_{22}$

$\therefore \boxed{\overline{y}_{22} = 0.15\text{ S}}\ \therefore -32.5 = 100\overline{y}_{21} + 7.5\ \therefore \boxed{\overline{y}_{21} = -0.4\text{ S}}\ \therefore [\overline{y}] = \begin{bmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{bmatrix}$ (S)

Completing row 3: $\overline{I}_1 = 0.2 \times 20 = 4$ A, $\overline{I}_2 = -0.4 \times 20 = -8$ A

Completing row 4: $5 = 0.2\overline{V}_{S1} - 0.3\overline{V}_{S2},\ 0 = -0.4\overline{V}_{S1} + 0.15\overline{V}_{S2}\ \therefore \overline{V}_{S2} = \frac{8}{3}\overline{V}_{S1}$

$\therefore 5 = 0.2\overline{V}_{S1} - 0.8\overline{V}_{S1} = -0.6\overline{V}_{S1}\ \therefore \overline{V}_{S1} = -\frac{50}{6} = -8.333\text{ V},\ \overline{V}_{S2} = -22.22\text{ V}$

Completing row 5: $5 = 0.2\overline{V}_{S1} - 0.3\overline{V}_{S2},\ 15 = -0.4\overline{V}_{S1} + 0.15\overline{V}_{S2}$

$$\therefore \overline{V}_{S1} = \frac{\begin{vmatrix} 5 & -0.3 \\ 15 & 0.15 \end{vmatrix}}{\begin{vmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{vmatrix}} = \frac{0.75 + 4.5}{0.03 - 0.12} = \frac{5.25}{-0.09} = -58.33\text{ V},\ \overline{V}_{S2} = \frac{\begin{vmatrix} 0.2 & 5 \\ -0.4 & 15 \end{vmatrix}}{-0.09} = -55.56\text{ V}$$

16.

$\Delta_1: 1+6+3=10\,\Omega \rightarrow \dfrac{6\times 1}{10}=0.6,\ \dfrac{6\times 3}{10}=1.8,\ \dfrac{3\times 1}{10}=0.3$

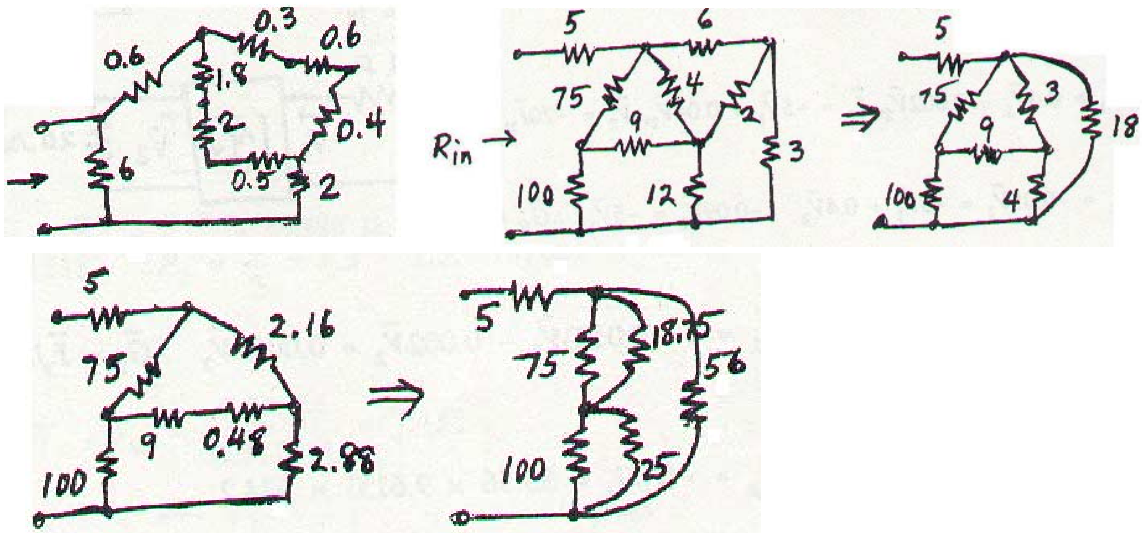
$\Delta_2:\ 5+1+4=10\,\Omega \rightarrow \dfrac{5\times 1}{10}=0.5,\ \dfrac{1\times 4}{10}=0.4,\ \dfrac{5\times 4}{10}=2$

$1.8+2+0.5=4.3\,\Omega,\ 0.3+0.6+0.4=1.3\,\Omega$

$1.3\|4.3=0.99821\,\Omega,\ 0.9982+0.6+2=3.598\,\Omega$

$3.598\|6=\boxed{2.249\,\Omega}$

17.



$$6\times2+2\times3+3\times6=36\,\Omega^2$$

$$36/6=6,\ 36/2=18,\ 36/3=12$$

$$12\|4=3,\ 6\|12=4$$

$$4+3+18=25\,\Omega$$

$$3\times18/25=2.16,\ 4\times18/25=2.88,\ \frac{4\times3}{25}=0.48$$

$$9.48\times2.16+9.48\times2.88+2.88\times2.16=54\,\Omega^2$$

$$\frac{54}{2.88}=18.75,\ \frac{54}{2.16}=25,\ \frac{54}{9.48}=5.6962,\ 75\|18.75=15,\ 100\|25=20$$

$$(15+20)\|5.696=4.899\ \ \therefore\ \mathrm{R}_{in}=5+4.899=\boxed{9.899\,\Omega}$$

18.

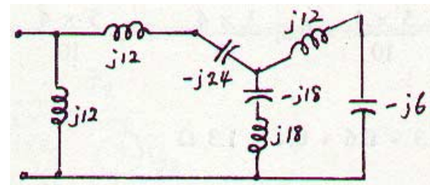
$\Delta: -j6 + j4 + j3 = j1$

$\frac{24}{j1} = -j24, \frac{-12}{j1} = j12, \frac{18}{j1} = -j18, j18 - j18 = 0\,(\text{S.C})$

$\therefore$ ignore $j12, -j6$

$-j24 + j12 = -j12$

$-j12 \| j12 = \infty \therefore \boxed{\bar{Z}_{in} = \infty}$



19. $[\bar{y}] = \begin{bmatrix} 0.4 & -0.002 \\ -5 & 0.04 \end{bmatrix}$ (S)

(a) $\bar{I}_1 = 0.4\bar{V}_1 - 0.002\bar{V}_2,\ \bar{I}_2 = -5\bar{V}_1 + 0.04\bar{V}_2, \bar{V}_2 = -20\bar{I}_2,\ \bar{V}_S = \bar{V}_1 + 2\bar{I}_1$

$\bar{I}_2 = -0.05\bar{V}_2 = -5V_1 + 0.4\bar{V}_2\ \therefore -0.09\bar{V}_2 = -5\bar{V}_1\ \therefore \bar{G}_V = \bar{V}_2/\bar{V}_1 = \frac{500}{9} = \boxed{55.56}$

(b) $\bar{I}_1 = 0.4(0.018)\bar{V}_2 - 0.002\bar{V}_2 = 0.0052\bar{V}_2\ \therefore \bar{G}_I = \bar{I}_2/\bar{I}_1 = \frac{-0.05V_2}{0.0052V_2} = \boxed{-9.615^+}$

(c) $G_p = -G_V G_I = 55.56 \times 9.615^+ = \boxed{534.2}$

(d) $\bar{I}_1 = 0.0052\bar{V}_2 = 0.0052 \times 55.56\bar{V}_1\ \therefore \bar{Z}_{in} = \bar{V}_1/\bar{I}_1 = \frac{1}{0.0052 \times 55.56} = \boxed{3.462\,\Omega}$

(e) $\bar{V}_1 = -2\bar{I}_1,\ \bar{V}_S = 0\ \therefore \bar{I}_1 = -0.5\bar{V}_1 = 0.4\bar{V}_1 - 0.002\bar{V}_2\ \therefore \bar{V}_1 = \frac{0.002}{0.9}\bar{V}_2$

$\bar{I}_2 = -5\left(\frac{0.002}{0.9}\right)\bar{V}_2 + 0.04\bar{V}_2 = 0.02889\bar{V}_2\ \therefore \bar{Z}_{out} = \bar{V}_2/\bar{I}_2 = \boxed{34.62\,\Omega}$

20. $[\bar{y}] = \begin{bmatrix} 0.1 & -0.05 \\ -0.5 & 0.2 \end{bmatrix}$ (S)

(a) $\bar{I}_1 = 0.1\bar{V}_1 - 0.05\bar{V}_2$

$\bar{I}_2 = -0.5\bar{V}_1 + 0.2\bar{V}_2,\ 1 = 10\bar{I}_1 + \bar{V}_1,\ \bar{I}_2 = -0.2\bar{V}_2$

$\therefore -0.2\bar{V}_2 = -0.5\bar{V}_1 + 0.2\bar{V}_2 \quad \therefore \bar{G}_V = \bar{V}_2 / \bar{V}_1 = \boxed{1.25}$

(b) $\bar{G}_I = \bar{I}_2 / \bar{I}_1 = \dfrac{(-0.5 + 0.2 \times 1.25)V_1}{(0.1 - 0.005 \times 1.25)V_1} = \boxed{-6.667}$

(c) $G_P = 1.25 \times 6.667 = \boxed{8.333}$

(d) $\bar{I}_1 = (0.1 - 0.05 \times 1.25)\bar{V}_1 \quad \therefore \bar{Z}_{in} = \bar{V}_1 / \bar{I}_1 = \boxed{26.67\,\Omega}$

(e) $\bar{V}_S = 0,\ \bar{V}_1 = -10\bar{I}_1 \ \therefore \bar{I}_1 = -0.1\bar{V}_1 = 0.1\bar{V}_1 - 0.05\bar{V}_2$

$\therefore \bar{V}_1 = 0.25\bar{V}_2,\ \therefore \bar{I}_2 = -0.05(0.25\bar{V}_2) + 0.2\bar{V}_2 = 0.075\bar{V}_2 \quad \therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 + 13.333\Omega$

(f) $\bar{G}_{V,rev} = \bar{V}_1 / \bar{V}_2 = \boxed{0.25}$

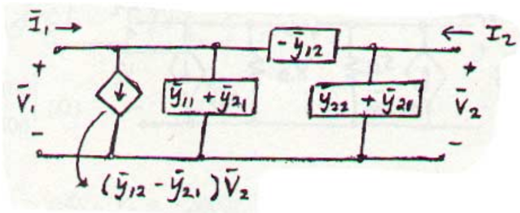
(g) With 2 port: $1 = 10\bar{I}_1 + 26.67I_1$

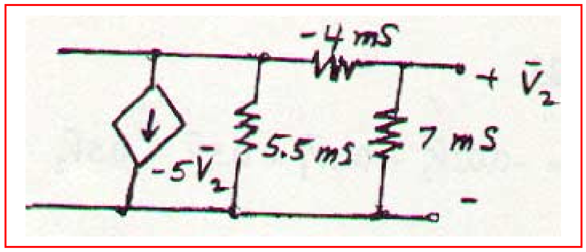
$\therefore 1 = 36.67\bar{I}_1,\ \bar{I}_1 = 1/36.67 \quad \therefore \bar{I}_2 = \dfrac{-6.667}{36.67} = -0.15182 \quad \therefore P_L = \dfrac{1}{2} \times I_2^2 5 = 2.5(0.15182)^2 = 0.08264 \text{ W}$

Without 2 port: $P_L = \dfrac{1}{2}\left(\dfrac{1}{15}\right)^2 \times 5 = 0.011111 \text{ W} \quad \therefore G_{ins} = \dfrac{0.08264}{0.011111} = \boxed{7.438}$

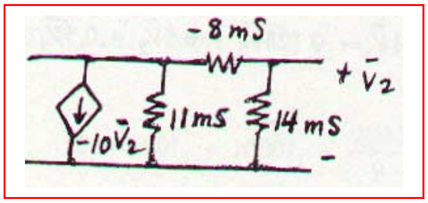
21.

(a)

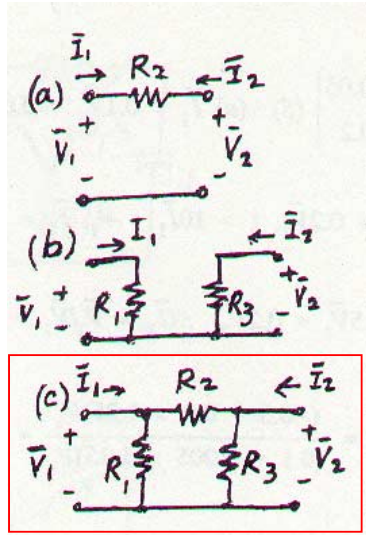

$\bar{I}_1 \rightarrow$
$\leftarrow I_2$
$-\bar{y}_{12}$
$\bar{y}_{11}+\bar{y}_{21}$
$\bar{y}_{22}+\bar{y}_{21}$
+
$\bar{V}_1$
-
+
$\bar{V}_2$
-
$(\bar{y}_{12}-\bar{y}_{21})\bar{V}_2$


-4 mS
$+\ \bar{V}_2$
$-5\bar{V}_2$
5.5 mS
7 mS
-

(b) 2 in ‖:


-8 mS
$+\ \bar{V}_2$
$-10\bar{V}_2$
11mS
14 mS
-

22.



(a) $\bar{I}_1 = \dfrac{\bar{V}_1 - \bar{V}_2}{R_2}$, $\bar{I}_2 = \dfrac{\bar{V}_2 - V_1}{R_2}$ $\boxed{[\bar{y}]_a = \begin{bmatrix} 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_2 \end{bmatrix}}$

(b) $\bar{I}_1 = \bar{V}_1 / R_1$, $\bar{I}_2 = \bar{V}_2 / R_3$ $\therefore \boxed{[\bar{y}]_B = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_3 \end{bmatrix}}$

(c) $\bar{I}_1 = \dfrac{\bar{V}_1}{R_1} + \dfrac{\bar{V}_1 - \bar{V}_2}{R_2}$ $\therefore [\bar{y}] = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_3 + 1/R_2 \end{bmatrix}$

$\bar{I}_2 = \dfrac{\bar{V}_2}{R_3} + \dfrac{\bar{V}_2 - V_1}{R_2}$, $[\bar{y}]_a + [\bar{y}]_b = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_3 + 1/R_2 \end{bmatrix}$

23.

$$\overline{V}_1 = 8\overline{I}_1 + 0.1\overline{V}_2 \quad \therefore \overline{V}_2 = 10\overline{V}_1 - 80\overline{I}_1$$

$$\overline{I}_2 = \overline{V}_2 / 12 + 0.05\overline{V}_1 \quad \therefore \overline{I}_2 = \frac{1}{12}(10\overline{V}_1 - 80\overline{I}_1) + 0.05\overline{V}_1$$

$$\therefore \overline{I}_2 = \left(\frac{5}{6} + \frac{1}{20}\right)\overline{V}_1 - \frac{20}{3}\overline{I}_1 = \frac{53}{60}\overline{V}_1 - \frac{20}{3}\overline{I}_1$$

$$\therefore \overline{V}_1 = \frac{60}{53}\left(\frac{20}{3}\overline{I}_1 + \overline{I}_2\right) = \frac{400}{53}\overline{I}_1 + \frac{60}{53}\overline{I}_2 \quad \therefore \overline{V}_2 = \frac{4000}{53}\overline{I}_1 + \frac{600}{53}\overline{I}_2 - 80\overline{I}_1$$

$$\therefore \overline{V}_2 = -\frac{240}{53}\overline{I}_1 + \frac{600}{53}\overline{I}_2 \quad \boxed{\therefore [\overline{z}] = \begin{bmatrix} 7.547 & 1.1321 \\ -4.528 & 11.321 \end{bmatrix} (\Omega)}$$

24.

(a) $\bar{I}_1 = -0.02\bar{V}_2 + 0.2\bar{V}_1 + 0.5\bar{V}_1 - 0.5\bar{V}_2$

$\therefore \bar{I}_1 = 0.7\bar{V}_1 - 0.52\bar{V}_2 \quad \bar{I}_2 = 0.1\bar{V}_1 + 0.125\bar{V}_2 + 0.5\bar{V}_2 - 0.5\bar{V}_1$

$\therefore \bar{I}_2 = -0.4\bar{V}_1 + 0.625\bar{V}_2$

$$\therefore \bar{V}_1 = \frac{\begin{vmatrix} \bar{I}_1 & -0.52 \\ \bar{I}_2 & 0.625 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.52 \\ -0.4 & 0.625 \end{vmatrix}} = \frac{0.625\bar{I}_1 + 0.52\bar{I}_2}{0.2295} = 2.723\bar{I}_1 + 2.266\bar{I}_2, \ \bar{V}_2 = \frac{\begin{vmatrix} 0.7 & \bar{I}_1 \\ -0.4 & \bar{I}_2 \end{vmatrix}}{0.2295}$$

$$\therefore \bar{V}_2 = \frac{0.4\bar{I}_1 + 0.7\bar{I}_2}{0.2295} = 1.7429\bar{I}_1 + 3.050\bar{I}_2 \quad \therefore \boxed{[\bar{z}] = \begin{bmatrix} 2.723 & 2.266 \\ 1.7429 & 3.050 \end{bmatrix} (\Omega)}$$

(b) $\bar{I}_1 = \bar{I}_2 = 1 \text{ A} \quad \therefore \dfrac{\bar{V}_2}{\bar{V}_1} = \dfrac{1.7429 + 3.050}{2.723 + 2.266} = \boxed{0.9607}$

25. $[\bar{z}] = \begin{bmatrix} 4 & 1.5 \\ 10 & 3 \end{bmatrix} (\Omega),\ \mathrm{R}_S = 5\Omega,\ \mathrm{R}_L = 2\Omega$

(a) $\bar{\mathrm{V}}_1 = 4\bar{\mathrm{I}}_1 + 1.5\bar{\mathrm{I}}_2,\ \bar{\mathrm{V}}_2 = 10\bar{\mathrm{I}}_1 + 3\bar{\mathrm{I}}_2,\ \bar{\mathrm{V}}_2 = -2\bar{\mathrm{I}}_2 = 10\bar{\mathrm{I}} + 3\bar{\mathrm{I}}_2\ \therefore \bar{\mathrm{G}}_1 = \bar{\mathrm{I}}_2 / \bar{\mathrm{I}}_1 = \boxed{-2}$

(b) $\bar{\mathrm{G}}_v = \bar{\mathrm{V}}_2 / \bar{\mathrm{V}}_1 = \dfrac{10\mathrm{I}_1 - 6\bar{\mathrm{I}}_1}{4\bar{\mathrm{I}} - 3\bar{\mathrm{I}}_1} = \boxed{4}$

(c) $\mathrm{G}_p = -\bar{\mathrm{G}}_V \bar{\mathrm{G}}_I = \boxed{8}$

(d) $\bar{\mathrm{V}}_1 = 4\bar{\mathrm{I}}_1 - 3\bar{\mathrm{I}}_1 = \bar{\mathrm{I}}_1\ \therefore \bar{\mathrm{Z}}_{in} = \dfrac{\bar{\mathrm{V}}_1}{\bar{\mathrm{I}}_1} = \boxed{1\ \Omega}$

(e) $\bar{\mathrm{V}}_1 = -5\bar{\mathrm{I}}_1 = 4\bar{\mathrm{I}}_1 + 1.5\bar{\mathrm{I}}_2\ \therefore \bar{\mathrm{I}}_1 = -\dfrac{1}{6}\bar{\mathrm{I}}_2\ \therefore \bar{\mathrm{V}}_2 = -\dfrac{10}{6}\bar{\mathrm{I}}_2 + 3\bar{\mathrm{I}}_2 = \dfrac{8}{6}\bar{\mathrm{I}}_2\ \therefore \bar{\mathrm{Z}}_{out} = \boxed{1.3333\ \Omega}$

26. $[\bar{z}] = \begin{bmatrix} 1000 & 100 \\ -2000 & 400 \end{bmatrix} (\Omega)$

(a) $\bar{V}_1 = 1000\bar{I}_1 + 100\bar{I}_2,\ \bar{V}_2 = -2000\bar{I}_1 + 400\bar{I}_2,\ 10 = 200\bar{I}_1 + \bar{V}_1, \bar{V}_2 = -500\bar{I}_2$

$\therefore -500\bar{I}_2 = -2000\bar{I}_1 + 400\bar{I}_2,\ \bar{I}_2 = \frac{20}{9}\ \bar{I}_1;\ \therefore 10 = 200\bar{I}_1 + 1000\bar{I}_1 + \frac{2000}{9}\ \bar{I}_1$

$\therefore\ \bar{I}_1 = 7.031 \text{ mA},\quad \therefore\ \bar{I}_2 = \frac{20}{9}\ \bar{I}_1 = 15.625 \text{ mA} \quad \therefore P_{200} = 7.031^2 \times 200 \times 10^{-6} = \boxed{9.888 \text{ mW}}$

(b) $P_{500} = 15.625^2 \times 500 \times 10^{-6} = \boxed{122.07 \text{ mW}}$

(c) $P_S = 10I_1 = 70.31 \text{ mW(gen)} \quad \therefore P_{2\,port} = P_S - P_{200} - P_{500} = 70.31 - 9.89 - 122.07\ \therefore$

$\therefore \boxed{P_{2\,port} = -61.65^{-}\ \text{mW}}$

27.

$\omega = 10^8, \ \bar{I}_1 = 10^{-5}\bar{V}_1 + j5\times10^{-4}\bar{V}_1 + j10^{-4}(\bar{V}_1 - \bar{V}_2)$

$\therefore \bar{I}_1 = (10^{-5} + j6\times10^{-4})\ \bar{V}_1 - j10^{-4}\bar{V}_2$

$\bar{I}_2 = 10^{-4}\bar{V}_2 + 0.01\bar{V}_1 + j10^{-4}\ (\bar{V}_2 - \bar{V}_1)$

$\therefore \bar{I}_2 = (0.01 - j10^{-4})\ \bar{V}_1 + (10^{-4} + j10^{-4})\bar{V}_2$

$$\therefore \bar{V}_1 = \frac{\begin{vmatrix} \bar{I}_1 & -j10^{-4} \\ \bar{I}_2 & 10^{-4} + j10^{-4} \end{vmatrix}}{\begin{vmatrix} 10^{-5} + j6\times10^{-4} & -j10^{-4} \\ 10^{-2} - j10^{-4} & 10^{-4} + 10^{-4} \end{vmatrix}} = \frac{(10^{-4} + j10^{-4})\bar{I}_1 + j10^{-4}\bar{I}_2}{1.0621\times10^{-6}\angle 92.640} \ \therefore \boxed{\begin{array}{l} \bar{z}_{11} = 133.15^{-}\angle -47.64^\circ\,\Omega \\ \bar{z}_{12} = 94.15^{+}\angle -2.642^\circ\,\Omega \end{array}}$$

$$\bar{V}_2 = \frac{\begin{vmatrix} 10^{-5} + j6\times10^{-4} & \bar{I}_1 \\ 10^{-2} - j10^{-4} & \bar{I}_2 \end{vmatrix}}{1.0621\times10^{-6}\angle 92.64^\circ} \ \boxed{\begin{array}{l} \therefore \bar{z}_{21} = 9416\angle 86.78^\circ\,\Omega \\ \bar{z}_{22} = 565.0\angle -3.60^\circ\,\Omega \end{array}}$$

28.

$$[\bar{z}] = \begin{bmatrix} 20 & 2 \\ 40 & 10 \end{bmatrix} (\Omega),\ \bar{V}_S = 100\angle 0°\ \text{V},\ R_S = 5\ \Omega,\ R_L = 25\ \Omega$$

$$100 = 5\bar{I}_1 + \bar{V}_1,\ \bar{V}_1 = 20\bar{I}_1 + 2\bar{I}_2\ \ \therefore 100 = 25\bar{I}_1 + 2\bar{I}_2$$

$$\bar{V}_2 = 40\bar{I}_1 + 10\bar{I}_2\ \ \therefore \bar{I}_1 = \frac{1}{40}\bar{V}_2 - \frac{1}{4}\bar{I}_2\ \ \therefore 100 = \frac{25}{40}\bar{V}_2 - \frac{25}{4}\bar{I}_2 + 2\bar{I}_2$$

$$\therefore 100 = \frac{5}{8}\bar{V}_2 - \frac{17}{4}\bar{I}_2\ \ \therefore \bar{V}_2 = 160 + \frac{8}{5} \times \frac{17}{4}\bar{I}_2 = 160 + 6.8\bar{I}_2$$

$$\boxed{\therefore \bar{V}_{th} = 160\ \text{V},\ R_{th} = 6.8\,\Omega}$$

29. $[\bar{h}] = \begin{bmatrix} 9\Omega & -2 \\ 20 & 0.2\ \text{S} \end{bmatrix}$

(a) $\bar{V}_1 = 9I_1 - 2\bar{V}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2,\ \bar{V}_1' = 1\bar{I}_1 + \bar{V}_1$ Eliminate $\bar{V}_1$

$\therefore \bar{V}_1 = \bar{V}_1' - \bar{I}_1\ \ \therefore \bar{V}_1' - \bar{I}_1 = 9\bar{I}_1 - 2\bar{V}_2,\ \bar{V}_1' = 10\bar{I} - 2\bar{V}_2\ \ \therefore \boxed{[\bar{h}]_{new} = \begin{bmatrix} 10\Omega & -2 \\ 20 & 0.2\ \text{S} \end{bmatrix}}$

(b) $\bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2,\ \bar{V}_2' = 1\bar{I}_2 + \bar{V}_2$

Eliminate $\bar{V}_2\ \ \therefore \bar{V}_2 = \bar{V}_2' - \bar{I}_2$

$\bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2 + 2\bar{I}_2,\ \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2' - 0.2\bar{I}_2\ \ \therefore 1.2\bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2'$

$\therefore \bar{I}_2 = 16.667\bar{I}_1 + 0.16667\bar{V}_2'\ \ \bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2' + 2(16.667\bar{I}_1 + 0.1667\ \bar{V}_2')$

$\therefore \bar{V}_1 = 42.38\ \bar{I}_1 - 1.6667\bar{V}_2'\ \ \therefore \boxed{[h]_{new} = \begin{bmatrix} 42.33\Omega & -1.6667 \\ 16.667 & 0.16667\ \text{S} \end{bmatrix}}$

30.

$$R_S = 100\,\Omega,\ R_L = 500\,\Omega\ \ [\bar{h}] = \begin{bmatrix} 100\Omega & 0.01 \\ 20 & 1\text{ mS} \end{bmatrix}$$

$$\bar{Z}_{in}:\ \ \bar{V}_1 = 100\bar{I}_1 + 0.01\bar{V}_2,\ \ \bar{I}_2 = 20\bar{I}_1 + 0.001\bar{V}_2 = 20\bar{I}_1 - 0.5\bar{I}_2\ \ \ \therefore 1.5\bar{I}_2 = 20\bar{I}_1$$

$$\therefore \bar{V}_1 = 100\bar{I}_1 + 0.01(-500)\frac{20}{1.5}\ \bar{I}_1 = 33.33\bar{I}_1\ \ \ \therefore \boxed{\bar{Z}_{in} = 33.33\Omega}$$

$$\bar{Z}_{out}:\ \ \bar{V}_1 = -100\bar{I}_1 = 100\bar{I}_1 + 0.01\bar{V}_2\ \ \ \therefore \bar{I}_1 = \frac{0.01}{-200}\ \bar{V}_2$$

$$\bar{I}_2 = 20\left(\frac{0.01}{-200}\ \bar{V}_2\right) + 0.001\ \bar{I}_2 = 0\ \ \ \ \therefore \boxed{\bar{Z}_{out} = \infty}$$

31.

(a) $\bar{h}_{12} = \bar{V}_1 / \bar{V}_2 |_{I_1=0}$ Let $\bar{V}_2 = 1$ V

$\therefore \bar{I}_{10} \downarrow = 0.1$ A, $\bar{I}_1 = 0$ $\therefore \bar{I}_{4\Omega} \leftarrow = 0.2\bar{I}_2$

$\therefore 0.1 = \bar{I}_2 - 0.2\bar{I}_2 = 0.8\bar{I}_2$, $\bar{I}_2 = 0.125$ A

$\therefore \bar{V}_1 = 0.3 - 4(0.2)(0.125) + 1 = 1.2$ V $\therefore \boxed{\bar{h}_{12} = 1.2}$

(b) $\bar{z}_{12} = \left.\frac{\bar{V}_1}{\bar{I}_2}\right|_{I_1=0}$ From above, $\bar{z}_{12} = \frac{1.2}{0.125} = \boxed{9.6\,\Omega}$

(c) $\bar{y}_{12} = \bar{I}_1 / \bar{V}_2 \big|_{V_1=0}$ SC input Let $\bar{V}_2 = 1$ V

$\bar{I}_2 = 0.1 + \frac{1.3}{4} = 0.425$ A, $\bar{I}_1 = 0.2(0.425) - \frac{1.3}{4}$

$\therefore \bar{I}_1 = -0.24$ A $\therefore \boxed{\bar{y}_{12} = 0.24 \text{ S}}$

32. $[\bar{h}] = \begin{bmatrix} 1000\Omega & -1 \\ 4 & 500\mu\text{S} \end{bmatrix}$

(a) $100 = 200\ \bar{I}_1 + 1000\bar{I}_1 - \bar{V}_2 = 1200\bar{I}_1 - \bar{V}_2$

$\bar{I}_2 = 4\bar{I}_1 + 5\times10^{-4}\ \bar{V}_2 = -10^{-3}\bar{V}_2 \quad \therefore 4\bar{I}_1 = -1.5\times10^{-3}\ \bar{V}_2$

$\therefore \bar{V}_2 = -\dfrac{4000}{1.5}\ \bar{I}_1 \quad \therefore 100 = 1200\bar{I}_1 + \dfrac{4000}{1.5}\ \bar{I}_1 \quad \therefore \bar{I}_1 = 25.86\text{ mA}$

$\therefore \text{P}_{200} = 25.86^2\times10^{-6}\times200 = \boxed{133.77\text{ mW}}$

(b) $\bar{V}_2 = \dfrac{4000}{1.5}\times25.86\times10^{-3} = 68.97\text{ V} \quad \therefore \text{P}_{1K} = \dfrac{68.97^2}{1000} = \boxed{4.756\text{ W}}$

(c) $\text{P}_S = 100\times25.86\times10^{-3} = 2.586\text{ W (gen)}$

$\therefore \text{P}_{2\,port} = 2.586 - 0.1338 - 4.756 = \boxed{-2.304\text{W}}$

33.

(a)

$$\overline{V}_1 = 1000\left(\overline{I}_1 + 10^{-5}\overline{V}_2\right) = 1000\overline{I}_1 + 0.01\overline{V}_2$$

$$\overline{V}_2 = 10^4\overline{I}_2 - 100\overline{V}_1 \quad \therefore \overline{I}_2 = 10^{-4}(100\overline{V}_1 + \overline{V}_2)$$

$$\therefore \overline{I}_2 = 10^{-2}(1000\overline{I}_1 + 0.01\overline{V}_2) + 10^{-4}\overline{V}_2$$

$$\therefore \overline{I}_2 = 10\overline{I}_1 + 2\times10^{-4}\overline{V}_2 \quad \therefore \boxed{[\overline{h}] = \begin{bmatrix} 1000\Omega & 0.01 \\ 10 & 2\times10^{-4}\text{S} \end{bmatrix}}$$

(b)

$$\overline{V}_1 = -200\overline{I}_1 = 1000\overline{I}_1 + 0.01\overline{V}_2$$

$$\therefore \overline{I}_1 = \frac{-1}{12,000}\overline{V}_2 \quad \therefore \overline{I}_2 = 10\overline{I}_1 + 2\times10^{-4}\overline{V}_2 = \frac{-1}{12,000}\overline{V}_2 + \frac{1}{5000}\overline{V}_2 + 116.67\times10^{-6}\overline{V}_2$$

$$\therefore \overline{Z}_{out} = \overline{V}_2 / \overline{I}_2 = 10^6 / 116.67 = \boxed{8.571k\Omega}$$

34.

(a)

$$\bar{V}_1 = \bar{I}_1 R + \bar{V}_2 \qquad \therefore \bar{I}_1 = \frac{\bar{V}_1}{R} - \frac{\bar{V}_2}{R} \qquad [\bar{y}] = \begin{bmatrix} 1/R & -1/R \\ -1/R & 1/R \end{bmatrix}$$

$$\bar{I}_1 = -\bar{I}_2 \qquad \bar{I}_2 = -\frac{\bar{V}_1}{R} + \frac{\bar{V}_2}{R}$$

$[z]$ parameters are all ∞

$$\bar{V}_1 = \bar{I}_1 R + \bar{V}_2 \qquad \therefore \boxed{[\bar{h}] = \begin{bmatrix} R & 1 \\ -1 & 0 \end{bmatrix}}$$

$$\bar{I}_2 = -\bar{I}_1$$

(b) $[\bar{y}]$ parameters are ∞

$$\bar{V}_1 = \bar{V}_2 \qquad \bar{V}_1 = R\bar{I}_1 + R\bar{I}_2 \qquad \therefore [\bar{z}] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

$$\bar{I}_1 = \frac{\bar{V}_1}{R} - \bar{I}_2 \qquad \bar{V}_2 = R\bar{I}_1 + R\bar{I}_2$$

$$\bar{V}_1 = \bar{V}_2$$

$$\bar{I}_2 = -\bar{I}_1 + \frac{\bar{V}_2}{R} \qquad \therefore \boxed{[\bar{h}] = \begin{bmatrix} 0 & 1 \\ -1 & 1/R \end{bmatrix}}$$

35. $[\bar{y}] = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, $[\bar{b}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}$, $[\bar{c}] = \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix}$, $[\bar{d}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix}$

(a) $$[\bar{y}][\bar{b}] = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix} = \boxed{\begin{bmatrix} 6 & -4 \\ 8 & 38 \end{bmatrix}}$$

(b) $$[\bar{b}][\bar{y}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = \boxed{\begin{bmatrix} 22 & 16 \\ 14 & 22 \end{bmatrix}}$$

(c) $$[\bar{b}][\bar{c}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 26 & 46 & -4 \\ -13 & 13 & 21 & 1 \end{bmatrix}}$$

(d) $$[\bar{c}][\bar{d}] = \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} -3 & -2 & 9 \\ -3 & -19 & 22 \end{bmatrix}}$$

(e) $$[\bar{y}][\bar{b}][\bar{c}][\bar{d}] = \begin{bmatrix} 6 & -4 \\ 8 & 38 \end{bmatrix}\begin{bmatrix} -3 & -2 & 9 \\ -3 & -19 & 22 \end{bmatrix} = \boxed{\begin{bmatrix} -6 & 64 & -34 \\ -138 & -738 & -908 \end{bmatrix}}$$

36.

(a)

$$\overline{V}_1 = \overline{t}_{11}\overline{V}_2 - \overline{t}_{12}\overline{I}_2,\ \overline{I}_1 = \overline{t}_{21}\overline{V}_2 - \overline{t}_{22}\overline{I}_2$$

$$\overline{V}_1 = 10\overline{I}_1 + \overline{V}_2 - 1.5\overline{V}_1,\ \overline{I}_2 = \frac{\overline{V}_2}{20} + \frac{\overline{V}_2 - 1.5\overline{V}_1}{25} + \frac{\overline{V}_2 - 1.5\overline{V}_1 - \overline{V}_1}{10}$$

$$\therefore\ \overline{I}_2 = 0.19\overline{V}_2 - 0.31\overline{V}_1,\ \overline{V}_1 = \frac{0.19}{0.31}\ \overline{V}_2 - \frac{1}{0.31}\ \overline{I}_2$$

$$\therefore\ \overline{V}_1 = 0.6129\overline{V}_2 - 3.226\ \overline{I}_2$$

Then, $10\overline{I}_1 = \overline{V}_1 - (\overline{V}_2 - 1.5\overline{V}_1) = 2.5(0.6129\overline{V}_2 - 3.226\overline{I}_2) - \overline{V}_2$

$$\therefore\ \overline{I}_1 = 0.05323\overline{V}_2 - 0.8065^{-}\ \overline{I}_2 \qquad \therefore\ \boxed{[\overline{t}\,] = \begin{bmatrix} 0.6129 & 3.226\Omega \\ 0.05323\text{S} & 0.8065^{-} \end{bmatrix}}$$

(b)

Let $R_S = 15\ \Omega$

$$\therefore\ \overline{V}_1 = 0.06129\overline{V}_2 - 3.226\overline{I}_2,\ \overline{I}_1 = 0.05323\overline{V}_2 - 0.8065\overline{I}_2,\ \overline{V}_1 = -15\overline{I}_1$$

$$\therefore -15\overline{I}_1 = -15(0.05323\overline{V}_2 - 0.8065^{-}\ \overline{I}_2) = 0.6129\overline{V}_2 - 3.226\overline{I}_2$$

$$\therefore 1.4114\overline{V}_2 = 15.324\overline{I}_2 \quad \therefore\ \overline{Z}_{out} = \overline{V}_2 / \overline{I}_2 = \boxed{10.857\,\Omega}$$

37.

$\bar{V}_1 = 5\bar{I}_1 - 0.3\bar{V}_1 + \bar{V}_2 \quad \therefore 1.3\bar{V}_1 = 5\bar{I}_1 + \bar{V}_2$

$\bar{I}_1 = 0.1\bar{V}_2 + \bar{V}_2 / 4 - \bar{I}_2 \quad \therefore \bar{I}_1 = 0.35\bar{V}_2 - \bar{I}_2$

$\therefore 1.3\bar{V}_1 = 5(0.35\bar{V}_2 - \bar{I}_2) + \bar{V}_2 = 2.75\,\bar{V}_2 - 5\bar{I}_2$

$$\therefore \bar{V}_1 = 2.115^{+}\bar{V}_2 - 3.846\bar{I}_2 \quad \therefore \boxed{[\bar{t}\,] = \begin{bmatrix} 2.115^{+} & 3.846\Omega \\ 0.35\,\text{S} & 1 \end{bmatrix}}$$

38.

(a)

$$\bar{V}_1 = 2\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = 0.2\bar{V}_2 - \bar{I}_2$$

$$\bar{I}_2 = 0.2\bar{V}_2 - \bar{I}_1 \quad \therefore \bar{V}_1 = 1.4\bar{V}_2 - 2\bar{I}_2$$

$$\bar{V}_1 = 3\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{1}{6}\bar{V}_2 - \bar{I}_2 \quad \therefore \boxed{[\bar{t}\,]_A = \begin{bmatrix} 1.4 & 2\Omega \\ 0.2\text{ S} & 1 \end{bmatrix}}$$

$$\bar{I}_2 = \frac{1}{6}\bar{V}_2 - \bar{I}_1 \quad \therefore \bar{V}_1 = 1.5\bar{V}_2 - 3\bar{I}_2 \quad \therefore \boxed{[\bar{t}\,]_B = \begin{bmatrix} 1.5 & 3\Omega \\ \frac{1}{6}\text{ S} & 1 \end{bmatrix}}$$

$$\bar{V}_1 = 4\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{1}{7}\bar{V}_2 - \bar{I}_2 \quad \boxed{[\bar{t}\,]_C = \begin{bmatrix} 11/7 & 4\Omega \\ 1/7\text{ S} & 1 \end{bmatrix}}$$

$$\bar{I}_R = \frac{1}{7}\bar{V}_2 - \bar{I}_1 \quad \bar{V}_1 = \frac{11}{7}\bar{V}_2 - 4\bar{I}_2$$

(b)

$$[\bar{t}\,] = [\bar{t}\,]_A[\bar{t}\,]_B[\bar{t}\,]_C = \begin{bmatrix} 1.4 & 2 \\ 0.2 & 1 \end{bmatrix}\begin{bmatrix} 1.5 & 3 \\ 1/6 & 1 \end{bmatrix}\begin{bmatrix} 11/7 & 4 \\ 1/7 & 1 \end{bmatrix} = \begin{bmatrix} 2.433 & 6.2 \\ 0.4667 & 1.6 \end{bmatrix}\begin{bmatrix} 11/7 & 4 \\ 1/7 & 1 \end{bmatrix}$$

$$\therefore \boxed{[\bar{t}\,] = \begin{bmatrix} 4.710 & 15.933\Omega \\ 0.9619\text{ S} & 3.467 \end{bmatrix}}$$

39.

(a) $\bar{V}_1 = 2\bar{I}_1 + \bar{V}_2 = -2\bar{I}_2 + \bar{V}_2 = \bar{V}_2 - 2\bar{I}_2 \quad \therefore \boxed{[\bar{t}\,]_A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}}$

$\bar{I}_1 = -\bar{I}_2$

(b) $$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10\Omega \\ 0 & 1 \end{bmatrix} = \left(\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}\right)^5 \quad \text{Also, } 10 \rightarrow \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

40.

(a) $\bar{V}_1 = \bar{V}_2 \quad \therefore \boxed{[\bar{t}]_a = \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}}$

$\bar{I}_1 = \bar{V}_2 / R - \bar{I}_2$

$\bar{V}_1 = \bar{V}_2 - R\bar{I}_2 \quad \therefore \boxed{[\bar{t}]_b = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}}$

$\bar{I}_1 = -\bar{I}_2$

$\bar{V}_1 = \bar{V}_2 / a \quad \therefore \boxed{[\bar{t}]_c = \begin{bmatrix} 1/a & 0 \\ 0 & a \end{bmatrix}}$

$\bar{I}_1 = -a\bar{I}_2$

(b) $[\bar{t}] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix}$

$\therefore [\bar{t}] = \begin{bmatrix} 1.2 & 2 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 5 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} = \begin{bmatrix} 0.3 & 14 \\ 0.025 & 4.5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0.58 & 14\,\Omega \\ 0.115\text{ S} & 4.5 \end{bmatrix}}$

41.

(a)

$$\bar{I}_1 = 0.1\bar{V}_x,\ -0.1\bar{V}_x + 0.02(\bar{V}_1 - \bar{V}_x) + 0.2(\bar{V}_1 - \bar{V}_x - \bar{V}_2) = 0$$

$$\bar{I}_2 = 0.08\bar{V}_x + 0.2(\bar{V}_2 - \bar{V}_1 + \bar{V}_x)$$

$$\therefore 0.32\bar{V}_x = 0.22\bar{V}_1 - 0.2\ \bar{V}_2 \quad \therefore \bar{V}_x = \frac{11}{16}\ \bar{V}_1 - \frac{5}{8}\ \bar{V}_2$$

$$\therefore \bar{I}_1 = \frac{11}{160}\ \bar{V}_1 - \frac{1}{16}\ \bar{V}_2 \quad \text{Also, } \bar{I}_2 = 0.28\left(\frac{11}{16}\ \bar{V}_1 - \frac{5}{8}\ \bar{V}_2\right) + 0.2\bar{V}_2 - 0.2\ \bar{V}_1$$

$$\therefore \bar{I}_2 = -\frac{3}{400}\ \bar{V}_1 + \frac{1}{40}\ \bar{V}_2 \quad \therefore \bar{V}_1 = \frac{10}{3}\ \bar{V}_2 - \frac{400}{3}\ \bar{I}_2 \quad [\bar{t}\,] = \begin{bmatrix} 3.333 & 133.33\Omega \\ 0.16667\text{S} & 9.17 \end{bmatrix}$$

$$\therefore \bar{I}_1 = \frac{11}{160}\left(\frac{10}{3}\ \bar{V}_2 - \frac{400}{3}\ \bar{I}_2\right) - \frac{1}{16}\ \bar{V}_2 = \frac{1}{6}\ \bar{V}_2 - \frac{55}{6}\bar{I}_2 \quad \boxed{\therefore [\bar{t}\,] = \begin{bmatrix} 3.333 & 133.33\Omega \\ 0.16667\text{S} & 9.167 \end{bmatrix}}$$

(b)

$$\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix} \quad \therefore [\bar{t}\,]_{new} = \begin{bmatrix} 10/3 & 400/3 \\ 1/6 & 55/6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 10 & 133.33\Omega \\ 0.625\ \text{S} & 9.167 \end{bmatrix}}$$

CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

1.

$$v(t) = 3 - 3\cos(100\pi t - 40°) + 4\sin(200\pi t - 10°) + 2.5\cos 300\pi t \text{ V}$$

(a) $\quad V_{av} = 3 - 0 + 0 + 0 = \boxed{3.000 \text{ V}}$

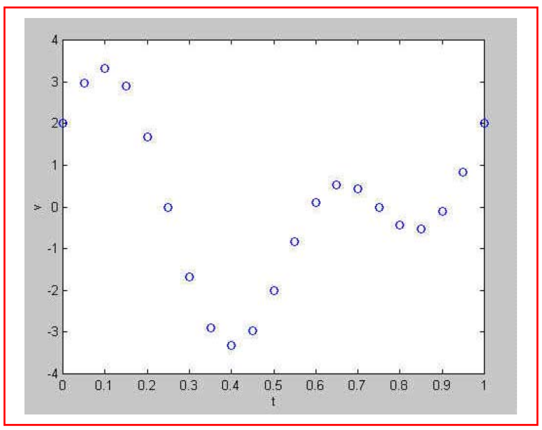
(b) $\quad V_{eff} = \sqrt{3^2 + \frac{1}{2}(3^2 + 4^2 + 2.5^2)} = \boxed{4.962 \text{ V}}$

(c) $\quad T = \frac{2\pi}{\omega_o} = \frac{2\pi}{100\pi} = \boxed{0.02\, s}$

(d) $\quad v(18ms) = 3 - 3\cos(-33.52°) + 4\sin(2.960°) + 2.5\cos(19.440°) = \boxed{-2.459 \text{ V}}$

2. (a)

t	v	t	v
0	2	0.55	-0.844
0.05	2.96	0.6	0.094
0.1	3.33	0.65	0.536
0.15	2.89	0.7	0.440
0.2	1.676	0.75	0
0.25	0	0.8	-0.440
0.3	-1.676	0.85	-0.536
0.35	-2.89	0.9	-0.094
0.4	-3.33	0.95	0.844
0.45	-2.96	1	2
0.5	-2		



(b) $v' = -4\pi \sin 2\pi t + 7.2\pi \cos 4\pi t = 0$

$\therefore 4\sin 2\pi t = 7.2(\cos^2 2\pi t - \sin^2 2\pi t)$

$\therefore 4\sin 2\pi t = 7.2(1 - 2\sin^2 2\pi t) \;\; \therefore x = \dfrac{-4 \pm \sqrt{16 + 414.72}}{28.8} = 0.5817, -0.8595 = \sin 2\pi t$

$\therefore t = 0.09881, 0.83539 \;\; \therefore v_{max} = \boxed{3.330}$ (0.5593 for smaller max)

(c) $|v_{min}| = \boxed{3.330}$

3.

(a) $T = 10\ s,\ F_{av} = a_o = 0.1(2\times 4 + 2\times 2) = \boxed{1.200}$

(b) $$F_{eff} = \sqrt{\frac{1}{5}\int_0^2 (4-t)^2\, dt} = \sqrt{0.2\int_0^2 (16 - 8t + t^2)\, dt}$$

$$= \sqrt{0.2\left[16t\Big|_0^2 - 4t^2\Big|_0^2 + \frac{1}{3}t^3\Big|_0^2\right]} = \sqrt{0.2\left(32 - 16 + \frac{8}{3}\right)} = \boxed{1.9322}$$

(c) $$a_3 = \frac{2}{10}\times 2\int_0^2 (4-t)\cos 3\times\frac{2\pi t}{10}\, dt = 0.4\int_0^2 4\cos 0.6\pi t\ dt - 0.4\int_0^2 t\cos 0.6\pi t\ dt$$

$$= 1.6\frac{1}{0.6\pi}\sin 0.6\pi t\Big|_0^2 - 0.4\left(\frac{1}{0.36\pi^2}\cos 0.6\pi t + \frac{t}{0.6\pi}\sin 0.6\pi t\right)_0^2$$

$$= \frac{8}{3\pi}\sin 1.2\pi - \frac{10}{9\pi^2}(\cos 1.2\pi - 1) - \frac{4}{3\pi}\sin 1.2\pi = \boxed{-0.04581}$$

4.

(a) $T = 8 - 2 = \boxed{6 \text{ s}}$

(b) $f_o = \boxed{\frac{1}{6} \text{ Hz}}$

(c) $\omega_o = 2\pi f_o = \boxed{\frac{\pi}{3} \text{ rad/s}}$

(d) $a_o = \frac{1}{6}(10\times1 + 5\times1) = \boxed{2.5}$

(e)
$$b_2 = \frac{2}{6}\left[\int_2^3 10\sin\frac{2\pi t}{3}dt + \int_3^4 5\sin\frac{2\pi t}{3}dt\right]$$

$$= \frac{1}{3}\left[-\frac{30}{2\pi}\cos\frac{2\pi t}{3}\Bigg|_2^3 - \frac{15}{2\pi}\cos\frac{2\pi t}{3}\Bigg|_3^4\right]$$

$$\therefore b_2 = \frac{1}{3}\left[-\frac{15}{\pi}\left(\cos 2\pi - \cos\frac{4\pi}{3}\right) - \frac{7.5}{\pi}\left(\cos\frac{8\pi}{3} - \cos 2\pi\right)\right] = \frac{1}{3}\left[-\frac{15}{\pi}(1.5) - \frac{7.5}{\pi}(-1.5)\right] = \boxed{-1.1937}$$

5.

$$a_3 = \frac{2}{6}\left[\int_2^3 10\cos\frac{6\pi t}{6}\,dt + \int_3^4 5\cos\frac{6\pi t}{6}\,dt\right] = \frac{1}{3}\left[\frac{10}{\pi}\sin\pi t\Big|_2^3 - \frac{5}{\pi}\sin\pi t\Big|_3^4\right]$$

$$= \frac{10}{3\pi}\left(\sin 3\pi - \sin 2\pi + \frac{1}{2}\sin 4\pi - \frac{1}{2}\sin 3\pi\right)\boxed{= 0}$$

$$b_3 = \frac{1}{3}\left[\int_2^3 10\sin\pi t dt + \int_3^4 5\sin\pi t\,dt\right] = \frac{1}{3}\left[-\frac{10}{\pi}\cos\pi t\Big|_2^3 - \frac{5}{\pi}\cos\pi t\Big|_3^4\right]$$

$$= -\frac{10}{3\pi}\left(\cos 3\pi - \cos 2\pi + \frac{1}{2}\cos 4\pi - \frac{1}{2}\cos 3\pi\right) = -\frac{10}{3\pi}(-1) = \boxed{1.0610}$$

$$\sqrt{a_3^2 + b_3^2} = \boxed{1.0610}$$

6.

(a) $3.8\cos^2 80\pi t = 1.9 + 1.9\cos 160\pi t,\ \mathrm{T} = \dfrac{2\pi}{160\pi} = 12.5 \text{ ms, ave value} = \boxed{1.9}$

(b) $3.8\cos^3 80\pi t = (3.8\cos 80\pi t)(0.5 + 0.5\cos 160\pi t)$

$= 1.9\cos 80\pi t + 0.95\cos 240\pi t + 0.95\cos 80\pi t = 2.85\cos 80\pi t + 0.95\cos 240\pi t$

$\mathrm{T} = \dfrac{2\pi}{80\pi} = 25 \text{ ms, ave value } = \boxed{0}$

(c) $3.8\cos 70\pi t - 3.8\sin 80\pi t;\ \omega_o t = \pi t,\ \omega_o = \pi,\ \mathrm{T} = \dfrac{2\pi}{\pi} = 2s;\ \text{ave value } = \boxed{0}$

7. T = 2 s

(a) $$b_4 = \frac{2}{2}\int_0^{t_1} \sin\frac{4\times 2\pi t}{2}\,dt = -\frac{1}{4\pi}\cos 4\pi t\Big|_0^{t_1}$$

$$\therefore b_4 = \frac{1}{4\pi}\left(1-\cos 4\pi t_1\right)$$

$$\text{max when } 4\pi t_1 = \frac{\pi}{2},\ t_1 = \boxed{0.125\,s}$$

(b) $$\boxed{b_4 = \frac{1}{4\pi}}$$

8.

$$g(t) = 5 + 8\cos 10t - 5\cos 15t + 3\cos 20t - 8\sin 10t - 4\sin 15t + 2\sin 20t$$

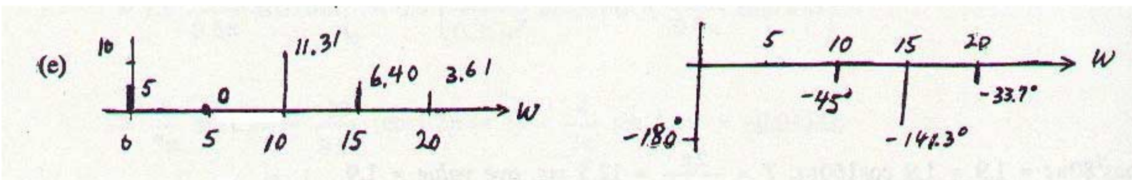
(a) $\omega_o = 5 \quad \therefore \mathrm{T} = \dfrac{2\pi}{5} = \boxed{1.2566\,s}$

(b) $f_o = \dfrac{5}{2\pi} \beta = 4 f_o = \dfrac{10}{\pi} = \boxed{3.183\ \text{Hz}}$

(c) $\mathrm{G}_{av} = -5$

(d) $\mathrm{G}_{eff} = \sqrt{(-5)^2 + \dfrac{1}{2}(8^2 + 5^2 + 3^2 + 8^2 + 4^2 + 2^2)} = \sqrt{116} = \boxed{10.770}$

(e)



9.

$\mathrm{T} = 0.2,\ f(t) = \mathrm{V}_m \cos 5\pi t, -0.1 < t < 0.1$

$$a_n = \frac{2}{0.2}\int_{-0.1}^{0.1} \mathrm{V}_m \cos 5\pi t \cos 10\, n\pi t\, dt = 5\mathrm{V}_m \int_{-0.1}^{0.1} \left[\cos(5\pi + 10n\pi)t + \cos(10n\pi - 5\pi)t\right] dt$$

$$= 5\mathrm{V}_m \left[\frac{1}{10n\pi + 5\pi}\sin(10n\pi + 5\pi)\, t + \frac{1}{10n\pi - 5\pi}\ \sin(10n\pi - 5\pi)t\right]_{-0.1}^{0.1}$$

$$= \frac{\mathrm{V}_m}{\pi}\left[\frac{2}{2n+1}\sin(10n\pi + 5\pi)\, 0.1 + \frac{2}{2n-1}\sin(10n\pi - 5\pi)\, 0.1\right]$$

$$= \frac{\mathrm{V}_m}{\pi}\left[\frac{2}{2n+1}\sin(n\pi + 0.5\pi) + \frac{2}{2n-1}\sin(n\pi - 0.5\pi)\right]$$

$$= \frac{\mathrm{V}_m}{\pi}\left[\frac{2}{2n+1}\cos n\pi + \frac{2}{2n-1}(-\cos n\pi)\right] = \frac{2\mathrm{V}_m}{\pi}\cos n\pi\left(\frac{1}{2n+1} - \frac{1}{2n-1}\right)$$

$$= \frac{2\mathrm{V}_m}{\pi}\cos n\pi \frac{2n - 1 - 2n - 1}{4n^2 - 1} = -\frac{4\mathrm{V}_m}{\pi}\frac{\cos n\pi}{4n^2 - 1}$$

$$a_o = \frac{1}{0.2}\int_{-0.1}^{0.1} \mathrm{V}_m \cos 5\pi t\, dt = 5\mathrm{V}_m \frac{1}{5\pi}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{2\mathrm{V}_m}{\pi}$$

$$\therefore \boxed{v(t) = \frac{2\mathrm{V}_m}{\pi} + \frac{4\mathrm{V}_m}{3\pi}\cos 10\pi t - \frac{4\mathrm{V}_m}{15\pi}\cos 20\pi t + \frac{4\mathrm{V}_m}{35\pi}\cos 30\pi t - \frac{4\mathrm{V}_m}{63\pi}\cos 40\pi t + \ldots}$$

10.

(a) $\boxed{\text{even, } \frac{1}{2} - \text{wave}}$

(b) $\boxed{b_n = 0 \text{ for all } n;\ a_{even} = 0;\ a_o = 0}$

(c) $\boxed{b_1 = b_2 = b_3 = 0,\ a_2 = 0}$

$$a_n = \frac{8}{12}\int_1^2 5\cos\frac{n\pi t}{6}\,dt = \frac{10}{3}\,\frac{6}{n\pi}\sin\frac{n\pi t}{6}\bigg|_1^2 = \frac{20}{n\pi}\left(\sin\frac{n\pi}{3} - \sin\frac{n\pi}{6}\right)$$

$$\therefore a_1 = \frac{20}{\pi}\left(\sin\frac{\pi}{3} - \sin\frac{\pi}{6}\right) = \boxed{2.330},\ a_3 = \frac{20}{3\pi}\left(\sin\pi - \sin\frac{\pi}{2}\right) = -\frac{20}{3\pi} = \boxed{-2.122}$$

11.

(a) $a_o = a_n = 0$

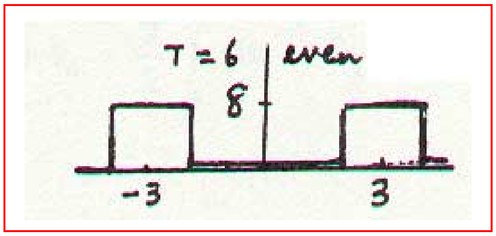
$\therefore \boxed{y(t) = 0.2\sin 1000\pi t + 0.6\sin 2000\pi t + 0.4\sin 3000\pi t}$

(b) $Y_{eff} = \sqrt{0.5(0.2^2 + 0.6^2 + 0.4^2)} = \sqrt{0.5(0.56)} = \boxed{0.5292}$

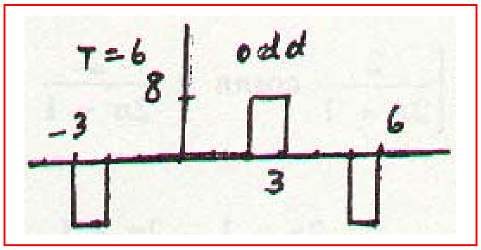
(c) $y(2\text{ms}) = 0.2\sin\ 0.2\pi + 0.6\sin 0.4\pi + 0.4\sin 0.6\pi = \boxed{1.0686}$

12.

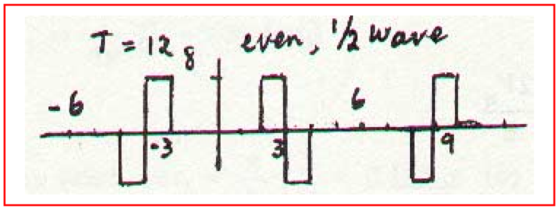
(a)



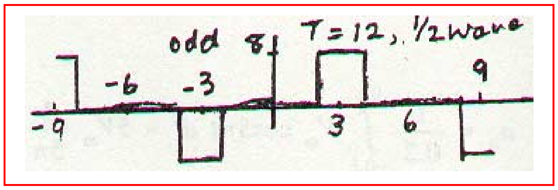
(b)



(c)



(d)



(e)

$$[a] b_5 = 0,\ a_5 = \frac{4}{6}\int_2^3 8\cos\frac{2\pi 5t}{6}\,dt = \frac{32}{6}\,\frac{6}{10\pi}\sin\frac{5\pi t}{3}\bigg|_2^3 = \frac{3.2}{\pi}\left(\sin\frac{15\pi}{3} - \sin\frac{10\pi}{3}\right) = \boxed{0.8821}$$

$$[b] a_5 = 0,\ b_5 = \frac{4}{6}\int_2^3 8\sin\frac{2\pi 5t}{6}\,dt = \frac{32}{6}\left(\frac{-6}{10\pi}\right)\left(\cos\frac{15\pi}{3} - \cos\frac{10\pi}{3}\right) = -\frac{3.2}{\pi}(-0.5) = \boxed{0.5093}$$

$$[c] b_5 = 0,\ a_5 = \frac{8}{12}\int_2^3 8\cos\frac{2\pi 5t}{12}\,dt = \frac{64}{12}\,\frac{12}{10\pi}\left(\sin\frac{15\pi}{6} - \sin\frac{10\pi}{6}\right) = \boxed{3.801}$$

$$[d] a_5 = 0,\ b_5 = \frac{8}{12}\int_2^3 8\sin\frac{10\pi t}{12}\,dt = \frac{64}{12}\left(-\frac{12}{10\pi}\right)\left(\cos\frac{15\pi}{6} - \cos\frac{10\pi}{6}\right) = \boxed{1.0186}$$

13.
T = 4 ms

(a) $$a_o = \frac{1000}{4}\int_0^{0.004} 8\sin 125\pi t \; dt = \frac{250\times 8}{-125\pi}\cos 125\pi t\Big|_0^{0.004}$$

$$= -\frac{16}{\pi}\left(\cos\frac{\pi}{2}-1\right) = \frac{16}{\pi} = \boxed{5.093}$$

(b) $$a_1 = 4000\int_0^{0.004} \sin 125\pi t \cos\frac{2\pi t}{0.004}\,dt$$

$$\therefore a_1 = 4000\int_0^{0.004} \sin 125\pi t\cos 500\pi t \; dt = 2000\int_0^{0.004} (\sin 625\pi t - \sin 375\pi t)\,dt$$

$$= 2000\left(-\frac{\cos 625\pi t}{625\pi}+\frac{\cos 375\pi t}{375\pi}\right)\Bigg|_0^{0.004} = \frac{3.2}{\pi}(1-\cos 2.5\pi) - \frac{5.333}{\pi}(1-\cos 1.5\pi) = \boxed{-0.6791}$$

$$b_1 = 4000\int_0^{0.004} \sin 125\pi t\sin 500\pi t \; dt = 2000\int_0^{0.004} (\cos 375\pi t - \cos 625\pi t)\; dt$$

$$= 2000\left[\frac{1}{375\pi}(\sin 1.5\pi) - \frac{1}{625\pi}(\sin 2.5\pi)\right] = 2000\left(\frac{-1}{375\pi}-\frac{1}{625\pi}\right) = \boxed{-2.716}$$

(c) $-4 < t < 0: \boxed{8\sin 125\pi t}$

(d) $$b_1 = 0,\; a_1 = \frac{4000}{8}\int_0^{0.004} 8\sin 125\pi t\;\cos 250\pi t\; dt$$

$$\therefore a_1 = 2000\int_0^{0.004} [\sin 375\pi t - \sin 125\pi t]\,dt = 2000\left[-\frac{\cos 375\pi t}{375\pi}+\frac{\cos 125\pi t}{125\pi}\right]_0^{0.004}$$

$$= \frac{5.333}{\pi}(1-\cos 1.5\pi) + \frac{16}{\pi}\left(\cos\frac{\pi}{2}-1\right) = \boxed{-3.395^{+}}$$

14.

odd and $\frac{1}{2}-$wave $\therefore a_o = 0,\ a_n = 0,\ b_{even} = 0$

$\mathrm{T} = 10ms = 0.01s$

$$b_{odd} = \frac{8}{0.01}\left[\int_0^{0.001} 10\sin 200n\pi t\ dt\right] = 8000\left(\frac{-1}{200n\pi}\right)\cos 200n\pi t\Big|_0^{0.001}$$

$$\therefore b_{odd} = -\frac{40}{n\pi}(\cos 0.2n\pi - 1) = \frac{40}{n\pi}(1-\cos 0.2n\pi)$$

$$\therefore \boxed{b_1 = 2.432,\ b_3 = 5.556,\ b_5 = 5.093,\ b_7 = 2.381,\ b_9 = 0.2702}$$

15.

odd and $\frac{1}{2}-$wave, $\mathrm{T}=8\,ms \;\; \therefore b_n=\frac{8}{\mathrm{T}}\int_0^{T/4} f(t)\sin n\omega_o t\,dt$

$\omega_o=\frac{2\pi}{\mathrm{T}}=250\pi \;\; \therefore b_n=1000\int_0^{0.001} 1000\,t\sin 250\pi nt\;dt$

Now, $\int x\sin ax\;dx=\frac{1}{a^2}\left(\sin a_x-ax\cos ax\right),\; a=250\,n\pi$

$f(t)=10^3 t \;\; \therefore b_n=\frac{10^6}{250^2 n^2\pi^2}\left(\sin 250n\pi t-250n\pi t\cos 250n\pi t\right)_0^{0.001}$

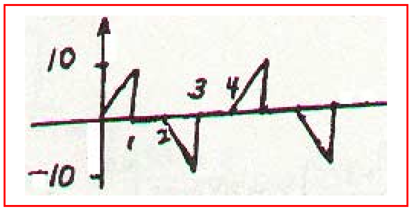
$\therefore b_n=\frac{16}{n^2\pi^2}\left(\sin\frac{n\pi}{4}-0-\frac{n\pi}{4}\cos\frac{n\pi}{4}+0\right) \;\; \therefore b_1=\frac{16}{\pi^2}\left(\sin\frac{\pi}{4}-\frac{\pi}{4}\cos\frac{\pi}{4}\right)=\boxed{0.2460}$

$b_3=\frac{16}{9\pi^2}\left(\sin\frac{3\pi}{4}-\frac{3\pi}{4}\cos\frac{3\pi}{4}\right)=0.4275^-;\; b_5=\frac{16}{25\pi^2}\left(\sin\frac{5\pi}{4}-\frac{5\pi}{4}\cos\frac{5\pi}{4}\right)=\boxed{0.13421}$

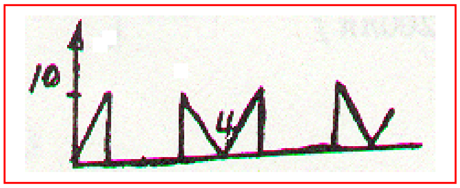
$b_{even}=\boxed{0}$

16.

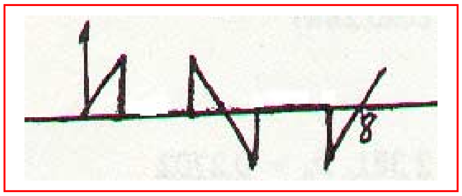
(a) odd, T = 4



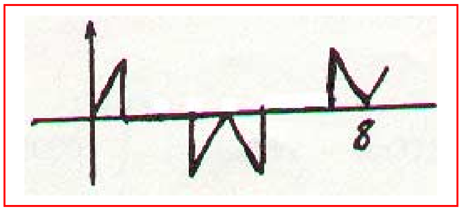
(b) even, T = 4:



(c) odd, $\frac{1}{2}-$wave: $T=8$



(d) even, $\frac{1}{2}-$wave, $T=8$:



17.

(a) $v_s = 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n}\sin\frac{2\pi nt}{0.4\pi} \quad \therefore v_{sn} = \frac{20}{n\pi}\sin 5nt, \ \bar{V}_{sn} = \frac{20}{n\pi}(-j1)$

$$Z_n = 4 + j5n2 = 4 + j10n, \ \bar{I}_{fn} = \frac{\bar{V}_{sn}}{Z_n} = \frac{-j20}{n\pi(4+j10n)} = -\frac{j5}{1+j2.5n}$$

$$\therefore \bar{I}_{fn} = -\frac{j5}{n\pi}\frac{1-j2.5n}{1+6.25n^2} = -\frac{12.5+j5}{n\pi(1+6.25n^2)}$$

$$\therefore i_{fn} = -\frac{12.5}{\pi}\frac{1}{1+6.25n^2}\cos 5nt + \frac{5}{n\pi}\frac{1}{1+6.25n^2}\sin 5nt$$

$$\therefore \boxed{i_f = 1.25 + \sum_{1,odd}^{\infty}\frac{1}{1+6.25n^2}\left[-\frac{12.5}{\pi}\cos 5nt + \frac{5}{n\pi}\sin 5nt\right]}$$

(b) $i_n = \text{A}e^{-2t}, \ i = i_f + i_n, \ i(0) = 0, \ i_f(0) = 1.25 + \sum_{1,odd}^{\infty}\frac{1}{1+6.25n^2}\left(-\frac{12.5}{\pi}\right)$

$$\therefore i_f(0) = 1.25 - \frac{2}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n^2+0.16} = 1.25 - \frac{2}{\pi}\frac{\pi}{4\times 0.4}\tanh 0.2\pi = 0.55388$$

$$\therefore \text{A} = -0.55388, \ \boxed{i = -0.55388e^{-2t} + 1.25 + \sum_{1,odd}^{\infty}\frac{1}{1+6.25n^2}\left[-\frac{12.5}{\pi}\cos 5nt + \frac{5}{n\pi}\sin 5nt\right]}$$

18.

(a) $0 < t < 0.2\pi: i = 2.5(1 - e^{-2t}) \;\; \therefore i(0.2\pi) = 2.5(1 - e^{-0.4\pi}) = \boxed{1.78848 \text{ A}}$

(b) $0.2\pi < t < 0.4\pi: \; i = 1.78848 e^{-2(t-0.2\pi)} \quad \therefore i(0.4\pi) = \boxed{0.50902 \text{ A}}$

(c) $0.4\pi < t < 0.6\pi: i = 2.5 - (2.5 - 0.50902)e^{-2(t-0.4\pi)}, \; i(0.6\pi) = \boxed{1.9335^{-}}$

19.

(a)

$$v_s = 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n}\sin 5nt$$

$$v_{sn} = \frac{20}{n\pi}\sin 5nt$$

$$\bar{V}_{sn} = -j\frac{20}{n\pi}$$

$$\bar{Z}_n = 2 + \frac{1}{j5n2} = 2 + \frac{1}{j10n} \quad \therefore \bar{V}_{cn} = \frac{-j20/n\pi}{2+1/j10n}\times\frac{1}{j10n} = \frac{-j20/n\pi}{1+j20n}\times\frac{1-j20n}{1-j20n}$$

$$\therefore \bar{V}_{cn} = \frac{-20n-j1}{1+400n^2}\times\frac{20}{n\pi},\ v_{cn} = \frac{20}{n\pi}\frac{1}{1+400n^2}\left(-20n\cos 5nt + \sin 5nt\right)$$

$$\therefore \boxed{v_{cf} = 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{1+400n^2}\left(\frac{1}{n}\sin 5nt - 20\cos 5nt\right)}$$

(b)

$$\boxed{v_n = \mathrm{A}e^{-t/4}}$$

(c)

$$v_c(0) = \mathrm{A} + 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{-20}{1+400n^2} = \mathrm{A} + 5 - \frac{1}{\pi}\sum_{1,odd}^{\infty}\frac{1}{n^2+(1/20)^2}$$

$$\sum_{1,odd}^{\infty}\frac{1}{n^2+(1/20)^2} = \frac{\pi}{4(1/20)}\tanh\frac{\pi}{20\times 2} = 5\pi\tanh\frac{\pi}{40} = 1.23117$$

$$\therefore \mathrm{A} = 0 - 5 + \frac{1}{\pi}\times 1.23117 = -4.60811$$

$$\therefore \boxed{v_c(t) = -4.60811e^{-t/4} + 5 + \frac{20}{\pi}\sum_{1,odd}^{\infty}\frac{1}{1+400n^2}\left(\frac{1}{n}\sin 5nt - 20\cos 5nt\right)}$$

20.

$$c_3 = \frac{10^3}{6}\left[\int_0^{0.001} 100e^{-j3\times2\pi t/6\times10^{-3}} - \int_{0.003}^{0.005} 100e^{-j100\pi t}\right]$$

$$= \frac{10^5}{6}\left[\frac{-1}{j1000\pi}e^{-j1000\pi t}\Big|_0^{0.001} + \frac{1}{j1000\pi}e^{-j1000\pi t}\Big|_{0.003}^{0.005}\right]$$

$$= \frac{100}{j6\pi}\left(e^{-j\pi}+1+e^{-j5\pi}-e^{-j3\pi}\right) = \frac{100}{j6\pi}(1+1-1+1) = \boxed{-j10.610}$$

$$\therefore \boxed{c_{-3} = j10.610;\ |c_3| = 10.610}$$

$$a_3 = \frac{2\times10^3}{6}\left[\int_0^{0.001} 100\cos 100\pi t\ dt - \int_{0.003}^{0.005} 100\cos 1000\pi t\ dt\right]$$

$$= \frac{2\times10^5}{6}\,\frac{1}{1000\pi}\left(\sin\pi - 0 - \sin 5\pi + \sin 3\pi\right) = \boxed{0}$$

$$c_3 = \frac{1}{2}(a_3 - jb_3) = -j\frac{1}{2}b_3 \quad \therefore \boxed{b_3 = 21.22} \text{ and } \sqrt{a_3^2+b_3^2} = \boxed{21.22}$$

21.

(a) $$\mathrm{T} = 5\,ms \quad c_m = \frac{1}{0.005}\left[\int_0^{0.001} 10^5 t e^{-j400\pi nt}\,dt + \int_{0.001}^{0.002} 100 e^{-j400\pi nt} dt\right]$$

$$\therefore c_n = 20,000\left[\int_0^{0.001} 1000t\, e^{-j400\pi nt} dt + \int_{0.001}^{0.002} e^{-j400\pi nt} dt\right]$$

$$\therefore c_n = 20,000\left[\frac{e^{-j400\pi nt}}{160n^2\pi^2}(j400\pi nt+1)\Big|_0^{0.001} + \frac{1}{-j400\pi n} e^{-j400\pi nt}\Bigg|_{0.001}^{0.002}\right]$$

(b) $$\therefore c_o = a_o = (50\times10^{-3} + 100\times10^{-3})\frac{1}{0.005} = 0.15\times200 = \boxed{30}$$

$$c_1 = 20,000\left[\frac{1}{160\pi^2}e^{-j0.4\pi}(1+j0.4\pi) - \frac{1}{160\pi^2} - \frac{1}{j400\pi}\left(e^{-j0.8\pi} - e^{-j0.4\pi}\right)\right]$$

$$= \frac{125}{\pi^2}(1\angle-72°)(1.60597\angle 51.488°) - 12.66515 + 15.91548\angle 90°(1\angle-144° - 1\angle-72°)$$

$$= 12.665(1\angle-72°)(1+j1.2566) - 12.665 + j15.915(1\angle-144° - 1\angle-72°)$$

$$= 20.339\angle-20.513° - 12.665 + 18.709\angle-108° = \boxed{24.93\angle-88.61°}$$

$$c_2 = 3.16625\angle-144°(1+j2.5133) - 3.16625 + j7.9575(1\angle-288° - 1\angle-144°)$$

$$= 8.5645\angle-75.697° - 3.16625 + 15.1361\angle 144° = \boxed{13.309\angle 177.43°}$$

22.

Fig. 17-8a: $V_o = 8$ V, $\tau = 0.2\mu s$, $f_o = 6000\,pps$

(a) $T = \frac{1}{6000}$, $f_o = 6000, \tau = 0.2\,\mu s \;\; \therefore\, f = \frac{1}{\tau} = \boxed{5\text{ MHz}}$

(b) $\boxed{f_o = 6000\text{ Hz}}$

(c) $6000\times 3 = 18,000$ (closest) $\;\;\therefore |c_3| = \frac{8\times 0.2\times 10^{-6}}{1/6000}\left|\frac{\sin(1/2\times 3\times 12,000\pi\times 0.2\times 10^{-6}}{0.0036\pi}\right|$

$\therefore |c_3| = \boxed{9.5998\text{ mV}}$

(d) $\frac{2\times 10^6}{6\times 10^3} = 333.3 \;\; \therefore |c_{333}| = \frac{8\times 0.2\times 10^{-6}}{1/6000}\left|\frac{\sin(1/2\times 333\times 12,000\pi\times 0.2\times 10^{-6}}{1/2\times 333\times 12,000\pi\times 0.2\times 10^{-6}}\right| = \boxed{7.270\text{ mV}}$

(e) $\beta = 1/\tau = \boxed{5\text{ MHz}}$

(f) $2 < \omega < 2.2$ Mrad/s $\;\;\therefore \frac{2000}{2\pi} < f < \frac{2200}{2\pi}$ kHz or $318.3 < f < 350.1$ kHz

$f_o = 6$ kHz $\;\;\therefore f = 6\times 53 = 318;\; 324, 330, 336, 342, 348$ kHz $\;\;\therefore \boxed{n = 5}$

(g) $|c_{227}| = \frac{8\times 0.2\times 10^{-6}}{1/6000}\left|\frac{\sin(1/2\times 227\times 12,000\pi\times 0.2\times 10^{-6}}{('')}\right| = \boxed{8.470\text{ mV}}$

$f = 227\times 6 = \boxed{1362\text{ kHz}}$

23.

$\mathrm{T} = 5\,ms;\ \bar{c}_o = 1,\ \bar{c}_1 = 0.2 - j0.2,\ \bar{c}_2 = 0.5 + j0.25,\ \bar{c}_3 = -1 - j2,\ \bar{c}_n = 0,\ |n| \geq 4$

(a) $a_n = -jb_n = 2\bar{c}_n \ \therefore a_o = \bar{c}_o = 1,\ a_1 - jb_1 = 0.4 - jb_1 = 0.4 - j0.4,\ a_2 - jb_2 = 1 + j0.5,\ a_3 - jb_3 = -2$

$\therefore v(t) = \boxed{1 + 0.4\cos 400\pi t + \cos 800\pi t - 2\cos 1200\pi t + 0.4\sin 400\pi t - 0.5\sin 800\pi t + 4\sin 1200\pi t}$

(b) $v(1\,ms) = 1 + 0.4\cos 72° + \cos 144° - 2\cos 216° + 0.4\sin 72° - 0.5\sin 144° + 4\sin 216°$

$\therefore \boxed{v(1\,ms) = -0.332\mathrm{V}}$

24.

(a) $\mathrm{T} = 5\ \mu s \ \therefore \bar{c}_n = \dfrac{10^6}{5} \times 2 \displaystyle\int_{0.4\times10^{-6}}^{0.6\times10^{-6}} 1 \cos 2\pi n \frac{t}{5\times10^{-6}} dt$

$$\therefore \bar{c}_n = 4\times10^5 \frac{5\times10^{-6}}{2\pi n}(\sin 43.2°n - \sin 28.8°n)$$

$$\therefore \boxed{\bar{c}_n = \frac{1}{n\pi}(\sin 43.2°n - \sin 28.8°n)}$$

(b) $\bar{c}_4 = \dfrac{1}{4\pi}(\sin 172.8° - \sin 115.2°) = \boxed{-0.06203}$

(c) $\bar{c}_o = a_o = \dfrac{0.2\times10^{-6} + 0.2\times10^{-6}}{5\times10^{-6}} = \boxed{0.08}$

(d) a little testing shows $|c_o|$ is max $\therefore |\bar{c}_{\max}| = \boxed{0.08}$

(e) $0.01\times0.08 = 0.8\times10^{-3} \ \therefore \left|\dfrac{1}{n\pi}(\sin 43.2°n - \sin 28.8°n)\right| \le 0.8\times10^{-3}$

$$\therefore \left|\frac{125}{n\pi}(\sin 43.2°n - \sin 28.8°n)\right| \le 1$$

$\boxed{\text{ok for } n > 740}$

(f) $\beta = 740 f_o = \dfrac{740\times10^6}{5} = \boxed{148 \text{ MHz}}$

25.

$\text{T} = 1/16,\ \omega_o = 32\pi$

(a) $$\overline{c}_3 = 16\int_0^{1/96} 40e^{-j96\pi t}\, dt - \frac{16\times 40}{-j96\pi}\, e^{-j96\pi}\Big|_0^{1/96}$$

$$\therefore \overline{c}_3 = j\frac{20}{3\pi}(e^{-j\pi} - 1) = -j\frac{40}{3\pi} = \boxed{-j4.244\text{ V}}$$

(b) Near harmonics are $2f_o = 32$ Hz, $3f_o = 48$ Hz

Only 32 and 48 Hz pass filter $a_n - jb_n = 2\overline{c}_n$

$a_3 - jb_3 = 2\overline{c}_3 = -j8.488 \quad \therefore a_3 = 0,\ b_3 = 8.488\text{ V}$

$$\overline{\text{I}}_3 = \frac{8.488}{5 + j0.01\times 96\pi} = 1.4536\ \angle -31.10^\circ\text{ A};\ \text{P}_3 = \frac{1}{2}\times 1.4536^2 \times 5 = 5.283\text{ W}$$

$$\overline{c}_2 = \frac{1}{1/16}\int_0^{1/96} 40e^{-j64\pi t}\, dt = \frac{640}{-j64\pi}(e^{-j64\pi/96} - 1) = 2.7566 - j4.7746\text{ V}$$

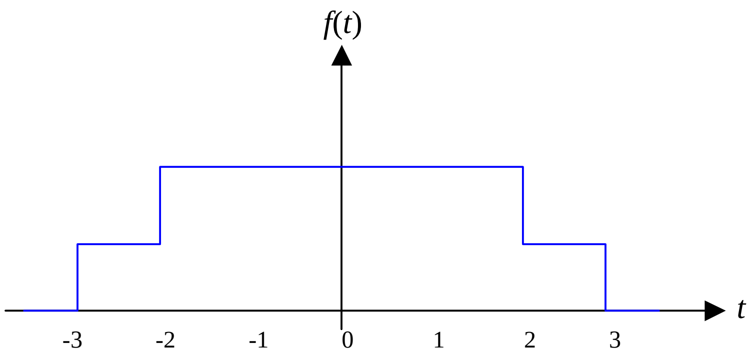
$a_2 - b_2 = 2\overline{c}_2 = 5.5132 - j9.5492 = 11.026\ \angle -60^\circ$

$$\therefore \overline{\text{I}}_2 = \frac{11.026\angle -60^\circ}{5 + j0.01\times 64\pi} = 2.046\angle -65.39^\circ\text{ A}$$

$$\therefore \text{P}_2 = \frac{1}{2}\times 2.046^2 \times 5 = 10.465\text{ W} \quad \therefore \text{P}_{tot} = \boxed{15.748\text{ W}}$$

26. $f(t) = 5[u(t+3) + u(t+2) - u(t-2) - u(t-3)]$

(a)



(b)

$$\mathrm{F}(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = \int_{-3}^{-2} 5e^{-j\omega t}\, dt + \int_{-2}^{2} 10e^{-j\omega t}\, dt + \int_{2}^{3} 5e^{-j\omega t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{5}{-j\omega}(e^{j2\omega} - e^{j3\omega}) + \frac{10}{-j\omega}(e^{-j2\omega} - e^{j2\omega}) + \frac{5}{-j\omega}(e^{-j3\omega} - e^{-j2\omega})$$

$$= \frac{5}{-j\omega}(-e^{j3\omega} + e^{-j3\omega}) + \frac{5}{-j\omega}(e^{j2\omega} - e^{-j2\omega}) + \frac{10}{-j\omega}(-e^{j2\omega} + e^{-j2\omega})$$

$$= \frac{5}{-j\omega}(-j2)\sin 3\omega + \frac{5}{-j\omega}(j2)\sin 2\omega + \frac{10}{-j\omega}(-j2)\sin 2\omega$$

$$\therefore \boxed{\mathrm{F}(j\omega) = \frac{10}{\omega}\sin 3\omega - \frac{10}{\omega}\sin 2\omega + \frac{20}{\omega}\sin 2\omega = \frac{10}{\omega}\,(\sin 3\omega + \sin 2\omega)}$$

27.

(a) $$f(t) = e^{-at}\, u(t),\ a > 0 \quad \therefore \mathrm{F}(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{-1}{a + j\omega} e^{-(a+j\omega)t} \Big|_{0}^{\infty} = \boxed{\frac{1}{a + j\omega}}$$

(b) $$f(t) = e^{at_6} e^{-at}\, u(t - t_o),\ a > 0 \quad \therefore \mathrm{F}(j\omega) = e^{at_o} \int_{t_o}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = e^{at_o} \frac{-1}{a + j\omega} e^{-(a+j\omega)\,t} \Big|_{t_o}^{\infty} = e^{at_o} \frac{-1}{a + j\omega} \left[-e^{-(a+j\omega)t_o} \right] = \boxed{\frac{1}{a + j\omega} e^{-j\omega t_o}}$$

(c) $$f(t) = te^{-at} u(t),\ a > 0 \quad \therefore \mathrm{F}(j\omega) = \int_{0}^{\infty} te^{-(a+j\omega)t} dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{e^{-(a+j\omega)t}}{(a + j\omega)^2} \left[-(a + j\omega)t - 1 \right]_{0}^{\infty} = 0 - \frac{1}{(a + j\omega)^2} [-1] = \boxed{\frac{1}{(a + j\omega)^2}}$$

28.

$$-4 < t < 0:\ f(t) = 2.5(t+4);\ 0 < t < 4:\ f(t) = 2.5(4-t)$$

$$\therefore \mathrm{F}(j\omega) = \int_{-4}^{0} 2.5(t+4)\,e^{-j\omega t}\,dt + \int_{0}^{4} 2.5(4-5)e^{-j\omega t}\,dt$$

$$\text{ln } 1^{st},\ \text{let } t = \tau \quad \therefore \mathrm{I}_1 = \int_{4}^{0} 2.5(4-\tau)e^{j\omega\tau}(-d\tau)$$

$$\therefore \mathrm{I}_1 = \int_{0}^{4} 2.5(4-\tau)e^{j\omega\tau}d\tau \quad \therefore \mathrm{F}(j\omega) = 2.5\int_{0}^{4}(4-t)(e^{j\omega t} + e^{-j\omega t})\,dt$$

$$\therefore \mathrm{F}(j\omega) = 5\int_{0}^{4}(4-t)\cos\,\omega t\ dt = 20 \times \frac{1}{\omega}\sin\omega t\Big|_{0}^{4} - 5\int_{0}^{4}\cos\,\omega t\ dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{20}{\omega}\sin 4\omega - \frac{5}{\omega^2}(\cos\omega t + \omega t\ \sin\ \omega t)_0^4$$

$$= \frac{20}{\omega}\sin 4\omega - \frac{5}{\omega^2}(\cos 4\omega - 1) - \frac{5}{\omega^2}\ 4\omega\sin 4\omega = \frac{5}{\omega^2}(1-\cos 4\omega)$$

$$\text{or, } \mathrm{F}(j\omega) = \frac{2\times 5}{\omega^2}\sin^2 2\omega = \boxed{10\left(\frac{\sin 2\omega}{\omega}\right)^2}$$

29.

$$f(t) = 5\sin t, -\pi < t < \pi \quad \therefore \mathrm{F}(j\omega) = \int_{-\pi}^{\pi} 5\sin t \; e^{-j\omega t}\, dt$$

$$\therefore \mathrm{F}(j\omega) = \frac{5}{j2}\int_{-\pi}^{\pi} (e^{jt} - e^{-jt})\; e^{-j\omega t}\, dt$$

$$= \frac{5}{j2}\int_{-\pi}^{\pi} [e^{jt(1-\omega)} - e^{-jt(1+\omega)}]\, dt$$

$$\mathrm{F}(j\omega) = \frac{5}{j2}\left[\frac{1}{j(1-\omega)}(e^{j\pi(1-\omega)} - e^{-j\pi(1-\omega)}) - \frac{1}{-j(1+\omega)}(e^{-j\pi(1+\omega)} - e^{j\pi(1+\omega)})\right]$$

$$= \frac{-2.5}{1-\omega}(-e^{-j\pi\omega} + e^{j\pi\omega}) - \frac{2.5}{1+\omega}(-e^{-j\pi\omega} + e^{j\pi\omega})$$

$$= \frac{-2.5}{1-\omega}(j2\sin\pi\omega) - \frac{2.5}{1+\omega}(j2\sin\pi\omega) = j5\sin\pi\omega\left(-\frac{1}{1-\omega} - \frac{1}{1+\omega}\right)$$

$$= j5\sin\pi\omega(-1)\left(\frac{1+\omega+1-\omega}{1-\omega^2}\right) = -\frac{j10\sin\pi\omega}{1-\omega^2} = \boxed{\frac{j10\sin\pi\omega}{\omega^2-1}}$$

30.

$$f(t) = 8\cos t[u(t+0.5\pi) - u(t-0.5\pi)]$$

$$\therefore \mathrm{F}(j\omega) = \int_{-\pi/2}^{\pi/2} 8\cos t e^{-j\omega t}\,dt = 4\int_{-\pi/2}^{\pi/2} (e^{jt} + e^{-jt})e^{-j\omega t}\,dt$$

$$= 4\int_{-\pi/2}^{\pi/2} \left[e^{jt(1-\omega)} + e^{-jt(1+\omega)}\right]dt$$

$$= 4\left\{\frac{1}{j(1-\omega)}e^{jt}\ e^{-j\omega t}\Big|_{-\pi/2}^{\pi/2} - \frac{1}{j(1+\omega)}e^{-jt}e^{-j\omega t}\Bigg|_{-\pi/2}^{\pi/2}\right\}$$

$$= 4\left\{\frac{1}{j(1-\omega)}\left[je^{-j\pi\omega/2} - (-j)e^{j\pi\omega/2}\right] - \frac{1}{j(1+\omega)}\left[-je^{-j\pi\omega/2} - je^{j\pi\omega/2}\right]\right\}$$

$$= 4\left\{\frac{1}{1-\omega}\times 2\cos\frac{\pi\omega}{2} + \frac{1}{1+\omega}\times 2\cos\frac{\pi\omega}{2}\right\} = 8\cos\frac{\pi\omega}{2}\left(\frac{1}{1-\omega} + \frac{1}{1+\omega}\right)$$

$$= 8\cos\frac{\pi\omega}{2}\ \frac{2}{1-\omega^2} = 16\frac{\cos\pi\omega/2}{1-\omega^2}$$

(a) $\omega = 0 \quad \therefore \mathrm{F}(j0) = \boxed{16}$

(b) $\omega = 0.8,\ \mathrm{F}(j0.8) = \dfrac{16\cos 72°}{0.36} = \boxed{13.734}$

(c) $\omega = 3.1,\ \mathrm{F}(j3.1) = \dfrac{16\cos(3.1\times 90°)}{1-3.12} = \boxed{-0.2907}$

31.

(a) $\mathrm{F}(j\omega) = 4\left[u(\omega+2) - \omega(\omega-2)\right] \;\therefore\; f(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} e^{j\omega t}\;\mathrm{F}(j\omega)d\omega$

$$\therefore\; f(t) = \frac{4}{2\pi}\int_{-2}^{2} e^{j\omega t}\,d\omega = \frac{2}{\pi}\,\frac{1}{jt}\,e^{j\omega t}\Big|_{-2}^{2} = \frac{2}{j\pi t}\left(e^{j2t} - e^{-j2t}\right)$$

$$\therefore\; f(t) = \frac{2}{2\pi t}\,j2\sin 2t = \frac{4}{\pi t}\sin 2t \;\therefore\; f(0.8) = \frac{5}{\pi}\sin 1.6^{rad} = \boxed{1.5909}$$

(b) $\mathrm{F}(j\omega) = 4e^{-2|\omega|} \;\therefore\; f(t) = \dfrac{4}{2\pi}\displaystyle\int_{-\infty}^{\infty} e^{-2|\omega| + j\omega t}\,d\omega$

$$\therefore\; f(t) = \frac{2}{\pi}\int_{-\infty}^{0} e^{(2+jt)\omega}\,d\omega + \frac{2}{\pi}\int_{0}^{\infty} e^{(-2+j\omega)t}\,d\omega$$

$$= \frac{2}{\pi}\left[\frac{1}{2+jt}(1-0) + \frac{1}{-2+jt}(0-1)\right] = \frac{2}{\pi}\left(\frac{1}{2+jt} + \frac{1}{2-jt}\right) = \frac{2}{\pi}\,\frac{4}{4+t^2}$$

$$\therefore\; f(t) = \frac{8}{\pi(4+t^2)} \;\therefore\; f(0.8) = \frac{8}{\pi\times 4.64} = \boxed{0.5488}$$

(c) $\mathrm{F}(j\omega) = 4\cos\pi\omega\,\left[u(\omega+0.5) - u(\omega-0.5)\right]$

$$\therefore\; f(t) = \frac{4}{2\pi}\int_{-0.5}^{0.5} \cos\pi\omega \times e^{j\omega t}\,d\omega = \frac{2}{\pi}\int_{-0.5}^{0.5} \frac{1}{2}\left(e^{j\pi\omega} + e^{-j\pi\omega}\right) e^{j\omega t}\,d\omega$$

$$= \frac{1}{\pi}\int_{-0.5}^{0.5}\left[e^{(j\pi + jt)\omega} + e^{(-j0.5\pi - j0.5t)\omega}\right]d\omega$$

$$= \frac{1}{\pi}\left[\frac{1}{j(\pi+t)}\left(e^{j0.5\pi + j0.5t} - e^{-j0.5\pi - j0.5t}\right) + \frac{1}{j(-\pi+t)}\left(e^{-j0.5\pi + j0.5t} - e^{j0.5\pi - j0.5t}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{j(\pi+t)}\left(je^{j0.5t} + je^{-j0.5t}\right) + \frac{1}{j(-\pi+t)}\left(-je^{j0.5t} - je^{-j0.5t}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{\pi+t}2\cos 0.5t - \frac{1}{-\pi+t}2\cos 0.5t\right] = \frac{2\cos 0.5t}{\pi}\left(\frac{1}{\pi+t} - \frac{1}{-\pi+t}\right)$$

$$= 2\cos 0.5t\left(\frac{-2}{t^2-\pi^2}\right) = \frac{4}{\pi^2 - t^2}\cos 0.5t \;\therefore\; f(0.8) = \boxed{0.3992}$$

32. $v(t) = 20e^{1.5t}\ u(-t-2)$ V

(a) $\mathrm{F}_v(j\omega) = \int_{-\infty}^{\infty} 20e^{1.5t}\, u(-t-2)e^{-j\omega t}\, dt = \int_{-\infty}^{-2} 20e^{1.5t-j\omega t}\, dt$

$= \frac{20}{1.5-j\omega} e^{(1.5-j\omega)t}\Big|_{-\infty}^{-2} = \frac{20}{1.5-j\omega} e^{-3+j2\omega}\ \therefore \mathrm{F}_v(j0) = \frac{20}{1.5}\, e^{-3} = \boxed{0.6638}$

(b) $\mathrm{F}_v(j\omega) = \mathrm{A}_v(\omega) + \mathrm{B}_v(\omega) = \frac{20}{1.5-j\omega}\, e^{-3}e^{j2\omega}$

$\therefore \mathrm{F}_v(j2) = \frac{20}{1.5-j2} e^{-3}\ e^{j4} = 0.39830\ \angle 282.31° = 0.08494 - j0.38913$

$\therefore \mathrm{A}_v(2) = \boxed{0.08494}$

(c) $\mathrm{B}_v(2) = \boxed{-0.3891}$

(d) $|\mathrm{F}_v(j2)| = \boxed{0.3983}$

(e) $\boxed{\phi_V(j2)\ =\ 282.3^o \text{ or } -77.69^o}$

33. $\left|\mathrm{I}(j\omega)\right| = 3\cos 10\omega\,[u(\omega+0.05\pi) - u(\omega-0.05\pi)]$

(a) $$\mathrm{W} = 4\times\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\mathrm{I}(j\omega)\right|^2 d\omega = \frac{2}{\pi}\int_{-0.05\pi}^{0.05\pi} 9\cos^2 10\omega\, d\omega$$
$$= \frac{18}{\pi}\int_{-\pi/20}^{\pi/20}\left(\frac{1}{2}+\frac{1}{2}\cos 20\omega\right)d\omega = \frac{9}{\pi}\times 0.1\pi + \frac{9}{\pi}\,\frac{1}{20}\sin 20\omega\Big|_{-\pi/20}^{\pi/20} \boxed{= 0.9\text{ J}}$$

(b) $$\frac{9}{\pi}\int_{-\omega_x}^{\omega_x}(1+\cos 20\omega)\, d\omega = 0.45 = \frac{9}{\pi}\left[2\omega_x + \frac{1}{20}\times 2\sin 20\,\omega_x\right]$$
$$\therefore 0.05\pi = 2\omega_x + 0.1\sin 20\omega_x,\ \boxed{\omega_x = 0.04159\text{ rad/s}}$$

34. $f(t) = 10te^{-4t}\, u(t)$

(a) $$W_{1\Omega} = \int_0^{\infty} f^2(t)\,dt = \int_0^{\infty} 100t^2 e^{-8t}\,dt = 100 \times \frac{e^{-8t}}{(-512)}(64t^2 + 16t + 2)\Bigg|_0^{\infty}$$

$$= \frac{100}{512} \times 2 = \boxed{0.3906\text{ J}}$$

(b) $$\mathrm{F}(j\omega) = \mathcal{F}\{10te^{-4t}u(t)\} = 10\int_0^{\infty} t\ e^{-(4+j\omega)t}dt = \frac{10e^{-(4+j\omega)t}}{(4+j\omega)^2}[-(4+j\omega)t - 1\Bigg|_0^{\infty}$$

$$= \frac{10}{(4+j\omega)^2} \quad \therefore \boxed{|\mathrm{F}(j\omega)| = \frac{10}{\omega^2 + 16}}$$

(c) $$|F(j\omega)|^2 = \frac{100}{\left(\omega^2 + 16\right)^2}$$

$$\boxed{|F(j\omega)|^2_{\omega=0} = 390.6\text{ mJ/Hz},\ |F(j\omega)|^2_{\omega=4} = 97.66\text{ mJ/Hz}}$$

35. $v(t) = 8e^{-2|t|}$ V

(a) $W_{1\Omega} = \int_{-\infty}^{\infty} v^2(t)\ dt = 2\times 64 \int_{0}^{\infty} e^{-4t}\ dt = \boxed{32\text{ J}}$

(b) $F_v(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t}\, v(t)\, dt = 8 \int_{-\infty}^{\infty} e^{-2|t|}\, e^{-j\omega t}\ dt$

$\therefore F_v(j\omega) = 8 \int_{-\infty}^{0} e^{(2-j\omega)t}\ dt + 8\int_{0}^{\infty} e^{-(2+j\omega)t}\ dt$

$= \frac{8}{2-j\omega} e^{(2-j\omega)t}\Big|_{-\infty}^{0} - \frac{8}{2+j\omega} e^{-(2+j\omega)t}\Big|_{0}^{\infty} = \frac{8}{2-j\omega} + \frac{8}{2+j\omega} = \boxed{\frac{32}{4+\omega^2} = |F_v(j\omega)|}$

(c) $0.9\times 32 = \frac{1}{2\pi}\int_{-\omega_1}^{\omega_1} \frac{32^2}{(\omega^2+4)^2} d\omega = \frac{32^2}{2\pi}\left[\frac{\omega}{8(\omega_1^2+4)} + \frac{1}{16}\tan^{-1}\frac{\omega_1}{2}\right]$

$\therefore 0.9 = \frac{16}{\pi}\times 2\left[\frac{\omega_1}{8(\omega_1^2+4)} + \frac{1}{16}\,\frac{\omega_1}{2}\right] = \frac{2}{\pi}\left[\frac{2\omega_1}{\omega_1^2+4} + \tan^{-1}\frac{\omega_1}{2}\right]$

$\therefore 0.45\pi = \frac{2\omega_1}{\omega_1^2+4} + \tan^{-1}\frac{\omega_1}{2}$ $\therefore \boxed{\omega_1 = 2.7174\text{ rad/s}}$ (by SOLVE)

CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

36.

(a) Prove: $\mathcal{F}\{f(t-t_o)\} = e^{-j\omega t_o}\,\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t}\,dt$ Let $t - t_o = \tau$

$\therefore \mathcal{F}\{f(t-t_o)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau}e^{-j\omega t_o}\,dt = e^{-j\omega t_o}\mathcal{F}\{f(t)\}$

(b) Prove: $\mathcal{F}\{f(t)\} = j\omega\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t}\frac{df}{dt}\,dt$ Let $u = e^{-j\omega t}$, $du = -j\omega e^{-j\omega t}$,

$dv = df$, $v = f$ $\therefore \mathcal{F}\{f(t)\} = f(t)e^{-j\omega t}\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} j\omega f(t)e^{-j\omega t}\,dt$

We assume $f(\pm\infty) = 0$ $\therefore \mathcal{F}\{f(t)\} = j\omega\mathcal{F}\{f(t)\}$

(c) Prove: $\mathcal{F}\{f(kt)\} = \frac{1}{|k|}\,\mathrm{F}\left(\frac{j\omega}{k}\right) = \int_{-\infty}^{\infty} f(kt)e^{-j\omega t}\,dt$ Let $\tau = kt$, $k > 0$

$\therefore \mathcal{F}\{f(kt)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau/k}\frac{1}{k}\,d\tau = \frac{1}{k}\,\mathrm{F}\left(\frac{j\omega}{k}\right)$

If $k < 0$, limits are interchanged and we get: $-\frac{1}{k}\mathrm{F}\left(\frac{j\omega}{k}\right)$

$\therefore \mathcal{F}\{f(kt)\} = \frac{1}{|k|}\mathrm{F}\left(\frac{j\omega}{k}\right)$

(d) Prove: $\mathcal{F}\{f(-t)\} = \mathrm{F}(-j\omega)$ Let $k = 1$ in (c) above

(e) Prove: $\mathcal{F}\{tf(t)\} = j\,\frac{d}{d\omega}\,\mathrm{F}(j\omega)$ Now, $\mathrm{F}(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$

$\therefore \frac{d\mathrm{F}(j\omega)}{d\omega} = \int_{-\infty}^{\infty} f(t)(-jt)e^{-j\omega t}\,dt = -j\,\mathcal{F}\{tf(t)\}$ $\therefore \mathcal{F}\{tf(f)\} = j\omega\mathcal{F}f(t)\}$

37.

(a) $f(t) = 4[\text{sgn}(t)\delta(t-1)] \quad \therefore \mathcal{F}\{4[\text{sgn}(t)\delta(t-1)] = \mathcal{F}\{4\,\text{sgn}(1)\ \delta(t-1)\} = \mathcal{F}\{4\delta(t-1)\} = \boxed{4e^{-j\omega}}$

(b) $f(t) = 4[\text{sgn}(t-1)\ \ \delta(t)] \quad \therefore \mathcal{F}\{4\,\text{sgn}(-1)\delta(t)\} = \mathcal{F}\{-4\delta(t)\} = \boxed{-4}$

(c) $f(t) = 4\sin(10t - 30°) \quad \therefore \mathcal{F}\{4\sin(10t-30°) = \mathcal{F}\left\{\frac{4}{j2}\left[e^{j(10t-30°)} - e^{-j(10t-30°)}\right]\right\} =$

$$\mathcal{F}\{-j2e^{-j30°}e^{j10t} + j2e^{j30°}e^{-j10t}\} = -j2e^{-j\pi/6}\,2\pi\delta(\omega-10) + j2e^{j\pi/6}2\pi\delta(\omega+10)$$

$$\boxed{= -j4\pi\,[e^{-j\pi/6}\delta(\omega-10) - e^{j\pi/6}\delta(\omega+10)]}$$

38.

(a) $f(t) = \text{A}\cos(\omega_o t + \phi) \quad \therefore \text{F}(j\omega) = \mathcal{F}\{\text{A}\cos\phi\cos\omega_o t - \text{A}\sin\phi\sin\omega_o t\} =$

$$\text{A}\cos\phi\{\pi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)]\} - \text{A}\sin\phi\left\{\frac{\pi}{j}[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]\right\} =$$

$$\pi\text{A}\{\cos\phi[\delta(\omega+\omega_o)+\delta(\omega-\omega_o)] + j\sin\phi[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)]\}$$

$$\therefore \boxed{\text{F}(j\omega) = \pi\text{A}[e^{j\phi}\,\delta(\omega-\omega_o) + e^{-j\phi}\,\delta(\omega+\omega_o)]}$$

(b) $f(t) = 3\,\text{sgn}(t-2) - 2\delta(t) - u(t-1) \quad \therefore \text{F}(j\omega) = e^{-j2\omega}\times 3\times\frac{2}{j\omega} - 2 - e^{-j\omega}\left[\pi\delta(\omega) + \frac{1}{j\omega}\right]$

$$\therefore \boxed{\text{F}(j\omega) = -j\frac{6}{\omega}e^{-j2\omega} - 2 - e^{-j\omega}\left[\pi\delta(\omega) - j\frac{1}{\omega}\right]}$$

(c) $f(t) = \sinh kt\; u(t) \quad \therefore \text{F}(j\omega) = \mathcal{F}\left\{\frac{1}{2}[e^{kt} - e^{-kt}]u(t)\right\}$

$$\therefore \boxed{\text{F}(j\omega) = \frac{1}{2}\,\frac{1}{-k+j\omega} - \frac{1}{2}\,\frac{1}{k+j\omega} = \frac{k+j\omega+k-j\omega}{2(-k^2-\omega^2)} = \frac{-k}{\omega^2+k^2}}$$

39.

(a) $\mathrm{F}(j\omega) = 3u(\omega+3) - 3u(\omega-1) \;\therefore\; f(t) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} [3u(\omega+3) - 3u(\omega-1)]\, e^{j\omega t}\, d\omega$

$$\therefore\; f(t) = \frac{3}{2\pi}\int_{-3}^{1} e^{j\omega t}\, dt = \frac{3}{2\pi}\,\frac{1}{jt}\, e^{j\omega t}\Big|_{-3}^{1} = \frac{3}{j2\pi t}(e^{+jt} - e^{-j3t})$$

$$\therefore\; f(5) = -j\frac{3}{10\pi}\,(1\angle 5^{rad} - 1\angle -15^{rad})\boxed{= 0.10390\ \angle -106.48^\circ}$$

(b) $\mathrm{F}(j\omega) = 3u(-3-\omega) + 3u(\omega-1) \rightarrow$

$\therefore \mathrm{F}(j\omega) = 3 - \mathrm{F}_a(j\omega)$

$$f(t) = 3\delta(t) - \frac{3}{j2\pi t}(e^{jt} - e^{-j3t}) \;\;\therefore\; f(5) = 0 - 0.10390\ \angle -106.48^\circ$$

so $\boxed{f(5) = 0.1039\angle 73.52^{\circ}}$

(c) $\mathrm{F}(j\omega) = 2\delta(\omega) + 3u(-3-\omega) + 3u(\omega-1)$ Now, $\mathcal{F}\{2\delta(\omega)\} = \dfrac{2}{2\pi} = \dfrac{1}{\pi}$

$$\therefore\; f(t) = \frac{1}{\pi} + \left[-\frac{3}{j2\pi t}(e^{jt} - e^{-j3t})\right] \;\therefore\; f(5) = \frac{1}{\pi} - 0.10390\ \angle -106.48^\circ = \boxed{0.3618\ \angle 15.985^{+\circ}}$$

40.

(a) $$\mathrm{F}(j\omega) = \frac{3}{1+j\omega} + \frac{3}{j\omega} + 3 + 3\delta(\omega - 1)$$

$$\therefore \boxed{f(t) = 3e^{-t}u(t) + 1.5\,\mathrm{sgn}(t) + 3\delta(t) + \frac{1.5}{\pi}e^{jt}}$$

(b) $$\mathrm{F}(j\omega) = \frac{1}{\omega}5\sin 4\omega = 8\frac{\sin \omega 8/2}{\omega 8/2}\times 2.5$$

$$\therefore \boxed{f(t) = 2.5[u(t+4) - u(t-4)]}$$

(c) $$\mathrm{F}(j\omega) = \frac{6(3+j\omega)}{(3+j\omega)^2 + 4} = \frac{6(3+j\omega)}{(3+j\omega)^2 + 2^2} \quad \therefore \boxed{f(t) = 3^{-3t}\cos 2t\,u(t)}$$

41.

T = 4, periodic; find exp′l form

$$\therefore c_n = \frac{1}{4}\int_{-1}^{1} 10te^{-jn\pi t/2}\,dt$$

$$\therefore c_n = 2.5\left[e^{-jn\pi t/2}\left(\frac{t}{-jn\pi/2} - \frac{1}{-n^2\pi^2/4}\right)\right]_{-1}^{1}$$

$$\therefore c_n = 2.5\left[e^{-jn\pi/2}\left(\frac{1}{-jn\pi 2} + \frac{1}{n^2\pi^2/4}\right) - e^{jn\pi/2}\left(\frac{1}{jn\pi/2} + \frac{1}{n^2\pi^2/4}\right)\right]$$

$$= 2.5\left[\frac{1}{jn\pi/2}(-e^{-jn\pi/2} - e^{jn\pi/2}) + \frac{4}{n^2\pi^2}(e^{-jn\pi/2} - e^{jn\pi/2})\right]$$

$$= \frac{j5}{n\pi} \times 2\cos\frac{n\pi}{2} + \frac{10}{n^2\pi^2}\left(-j2\sin\frac{n\pi}{2}\right)$$

$$\therefore f(t) = \sum_{-\infty}^{\infty}\left[\frac{j10}{n\pi}\cos\frac{n\pi}{2} - j\frac{20}{n^2\pi^2}\sin\frac{n\pi}{2}\right]e^{jn\pi t/2}$$

$$\boxed{\therefore \mathrm{F}(j\omega) = \sum_{-\infty}^{\infty}\left[\frac{j10}{n\pi}\cos\frac{n\pi}{2} - j\frac{20}{n^2\pi^2}\sin\frac{n\pi}{2}\right]2\pi\delta\left(\omega - \frac{n\pi}{2}\right)}$$

42.

$$\mathrm{T} = 4\ ms,\ f_1(t) = 10u(t) - 6u(t-0.001) - 4u(t-0.003)$$

$$c_n = \frac{1}{\mathrm{T}}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt,\ \omega_o = \frac{2\pi 10^3}{4} = 500\pi$$

$$= 250\int_{-0.002}^{0.002} f(t)e^{-j500n\pi t}\,dt$$

$$= 250\left[\int_{-0.002}^{-0.001} 4e^{-j500n\pi t}\,dt + \int_{0}^{0.001} 10e^{-j500n\pi t}\,dt + \int_{0.001}^{0.002} 4e^{-j500n\pi t}\,dt\right]$$

$$= \frac{j250}{500n\pi}\left[4e^{-j500n\pi t}\Big|_{-0.002}^{0.001} + 10e^{-j500n\pi t}\Big|_{0}^{0.001} + 4e^{-j500n\pi t}\Big|_{0.001}^{0.002}\right]$$

$$= \frac{j}{2n\pi}\left[4(e^{jn\pi/2} - e^{-jn\pi}) + 10(e^{-jn\pi/2} - 1) + 4(e^{-jn\pi} - e^{-jn\pi/2})\right]$$

$$= \frac{j}{2n\pi}\left[4(e^{jn\pi/2} - e^{-jn\pi/2}) - 4(e^{jn\pi} - e^{-jn\pi}) + 10(e^{-jn\pi/2} - 1)\right]$$

$$= \frac{j}{2n\pi}\left[4j2\sin\frac{n\pi}{2} - 4j2\ \sin\ n\pi + 10(e^{-jn\pi/2} - 1)\right]$$

$$= \frac{1}{n\pi}\left[-4\sin\frac{n\pi}{2} + j5(e^{-jn\pi/2} - 1)\right] = \frac{1}{n\pi}\left[j5e^{-jn\pi/4}(e^{-jn\pi/4} - e^{jn\pi/4}) - 4\sin\frac{n\pi}{2}\right]$$

$$= \frac{1}{n\pi}\left[j5e^{-jn\pi/4}\left(-j2\sin\frac{n\pi}{4}\right) - 4\sin\frac{n\pi}{2}\right] = \frac{1}{n\pi}\left[10e^{-jn\pi/4}\sin\frac{n\pi}{4} - 4\sin\frac{n\pi}{2}\right]$$

$$\boxed{\therefore \mathrm{F}(j\omega) = 2\pi\sum_{-\infty}^{\infty}\frac{1}{n\pi}\left(10e^{-jn\pi/4}\sin\frac{n\pi}{4} - 4\sin\frac{n\pi}{2}\right)\delta(\omega - 500n\pi)}$$

43.

$$\mathrm{F}(j\omega) = 20\sum_{-\infty}^{\infty} \frac{1}{|n|!+1}\delta(\omega-20n)$$

$$= 20\left[\frac{1}{1+1}\delta(\omega) + \frac{1}{1+1}\delta(\omega+20) + \frac{1}{1+1}\delta(\omega-20) + \frac{1}{2+1}\delta(\omega+40) + \frac{1}{3}\delta(\omega-40)\right.$$

$$\left. + \frac{1}{7}\delta(\omega+60) + \frac{1}{7}\delta(\omega-60) + \ldots\right]$$

$$= 10\delta(\omega) + \frac{20}{2\pi}[\pi\delta(\omega+20) + \pi\delta(\omega-20)] + \frac{20}{3\pi}[\pi\delta(\omega+40) + \pi\delta(\omega-40)] +$$

$$\frac{20}{7\pi}[\pi\delta(\omega+60) + \pi\delta(\omega-60) + \frac{20}{25\pi}[\pi\delta(\omega+80) + \pi\delta(\omega-80)] + \ldots$$

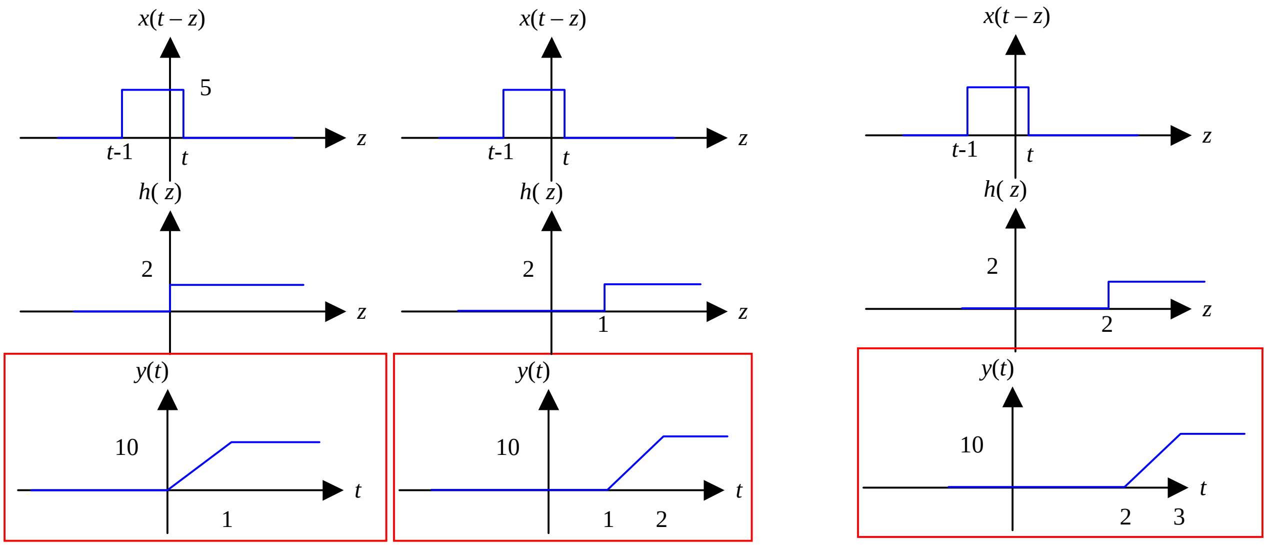
$$\therefore\ f(t) = \frac{10}{2\pi} + \frac{20}{2\pi}\cos 20t + \frac{20}{3\pi}\cos 40t + \frac{20}{7\pi}\cos 60t + \frac{20}{25\pi}\cos 80t + \ldots$$

$$= \frac{20}{\pi}\left[0.25 + \frac{1}{2}\cos 20t + \frac{1}{3}\cos 40t + \frac{1}{7}\cos 60t + \frac{1}{25}\cos 80t + \ldots\right.$$

$$\therefore\ f(0.05) = \frac{20}{\pi}\left[0.25 + \frac{1}{2}\cos 1^{rad} + \frac{1}{3}\cos 2 + \frac{1}{7}\cos 3 + \frac{1}{25}\cos 4 + \ldots\right] = \boxed{1.3858}$$

44. Input $= x(t) = 5[u(t) - u(t-1)]$ $\quad y(t) = \int_{-\infty}^{t} x(z)\, h(t-z)\, dz$

(a) $h(t) = 2u(t)$ (b) $h(t) = 2u(t-1)$ (c) $h(t) = 2u(t-2)$



(a)

$t < 0$:
$y(t) = 0$

$0 < t < 1$:

$$y(t) = \int_0^t 10dz = 10t$$

$t > 1$:

$$y(t) = \int_{t-1}^{t} 10dz = 10$$

(b)

$t < 1$: $y(t) = 0$

$1 < t < 2$:

$$y(t) = \int_1^t 10dz = 10(t-1)$$

$t > 2$:

$$y(t) = \int_{t-1}^{t} 10dz = 10$$

(c)

$t < 2$: $y(t) = 0$

$2 < t < 3$:

$$y(t) = \int_2^t 10dz = 10(t-2)$$

$t > 3$:

$$y(t) = \int_{t-1}^{t} 10dz = 10$$

45. $x(t) = 5[u(t) - u(t-2)];\ h(t) = 2[u(t-1) - u(t-2)]$

$$y(t) = \int_{-\infty}^{t} x(z)\ h(t-z)\, dz$$

$t < 1:\ y(t) = 0$

$$1 < t < 2:\ \ y(t) = \int_{0}^{t-1} 10\, dz = 10(t-1)$$

$2 < t < 3:\ \ y(t) = 10$

$$3 < t < 4:\ \ y(t) = \int_{t-2}^{2} 10\, dz = 10(2 - t + 2) = 10(4 - t)$$

$t > 4:\ \ y(t) = 0$

$$\boxed{\begin{array}{l} \therefore\ y(-0.4) = 0;\quad y(0.4) = 0;\quad y(1.4) = 4 \\ y(2.4) = 10;\ \ y(3.4) = 6;\ \ y(4.4) = 0 \end{array}}$$

or.... $y(t) = \int_{0}^{\infty} x(t-z)\ h(z)\ dz$

$t < 1:\ \ y(t) = 0$

$$1 < t < 2:\ \ y(t) = \int_{1}^{t} 10\, dz = 10(t-1)$$

$2 < t < 3:\ \ y(t) = 10$

$$3 < t < 4:\ \ y(t) = \int_{t-2}^{2} 10\, dz = 10(2 - t + 2) = 10(4 - t)$$

$t > 4:\ \ y(t) = 0$

$\boxed{\text{same answers as above}}$

46.

$$h(t) = 3[e^{-t} - e^{-2t}],\ x(t) = u(t)$$

$$y(t) = \int_{-\infty}^{t} x(z)h(t-z)\,dz$$

$$= \int_{0}^{t} 3[e^{-(t-z)} - e^{-2(t-z)}]\,dz$$

$$= 3e^{-t}[e^{z}]_0^t - 3e^{-2t}\left[\frac{1}{2}e^{2Z}\right]_0^t$$

$$= 3e^{-t}(e^{t} - 1) - 1.5e^{-2t}(e^{2t} - 1)$$

$$\therefore\ y(t) = 3(1 - e^{-t}) - 1.5(1 - e^{-2t}) = \boxed{1.5 - 3e^{-t} + 1.5e^{-2t},\ t > 0}$$

47.

$$y(t) = \int_0^{\infty} x(t-2)h(z)dz$$

$$h(t) = \frac{2}{3}(5-t),\ 2 < t < 5$$

(a) $$y(t) = \int_2^5 10 \times \frac{2}{3}(5-z)\ dz = \boxed{\frac{20}{3}\int_2^5 (5-z)\,dz}$$

Note: $h(z)$ is in window for $4 < t < 6$

(b) $$y(t) = \frac{20}{3}\left(-\frac{1}{2}\right)(5-z)^2\Big|_2^5$$

$$= -\frac{10}{3}(0-9) = \boxed{30 \text{ at } t = 5}$$

48. $x(t) = 5e^{-(t-2)}\,u(t-2),\ h(t) = (4t-16)\,[u(t-4) - u(t-7)],\ y(t) = \int_0^{\infty} x(t-z)\,h(z)\,dz$

(a) $t < 6:\ \ y(t) = 0 \quad \therefore \boxed{y(5) = 0}$

(b) $t = 8:\ \ y(8) = \int_4^6 5e^{-(8-z-2)}\,(4z-16)\,dz$

$$\therefore y(8) = 20e^{-6}\int_4^6 z\,e^z\,dz - 80e^{-6}\int_4^6 e^z\,dz$$

$$= 20e^{-6}\left[\frac{e^z}{1}(z-1)\right]_4^6 - 80e^{-6}(e^6 - e^4)$$

$$= 20e^{-6}(5e^6 - 3e^4) - 80 + 80e^{-2} = 20 + 80e^{-2} - 60e^{-2}$$

$$= 20\,(1 + e^{-2}) = \boxed{22.71}$$

(c) $t = 10:\ \ y(10) = \int_4^7 5e^{-(10-z-2)}\,(4z-16)\,dz$

$$\therefore y(10) = \int_4^7 20e^{-8}e^z\,(z-4)dz$$

$$\therefore y(10) = 20e^{-8}\int_4^7 ze^z\,dz - 80e^{-8}\int_4^7 e^z\,dz = 20e^{-8}[e^z\,(z-1)]_4^7 - 80e^{-8}(e^7 - e^4)$$

$$= 20e^{-8}(6e^7 - 3e^4) - 80(e^{-1} - e^{-4}) = 40e^{-1} + 20e^{-4} = \boxed{15.081}$$

49.

$h(t) = \sin t,\ 0 < t < \pi;\ $ 0 elsewhere, Let $x(t) = e^{-t}u(t)$

$$y(t) = \int_0^\infty x(t-z)\ h(z)\,dz$$

$t < 0:\ y(t) = 0$

$$0 < t < \pi:\quad y(t) = \int_0^t \sin\ z \times e^{-t+z}\ dz = e^{-t}\int_0^t e^z \sin z\ dz$$

$$\therefore y(t) = e^{-t}\left[\frac{1}{2}e^z(\sin z - \cos z)\right]_0^t$$

$$= \frac{1}{2}e^{-t}[e^t(\sin\ t - \cos\ t) + 1]$$

$$= \frac{1}{2}(\sin\ t - \cos\ t + e^{-t})$$

(a) $y(1) = \boxed{0.3345^+}$

(b) $y(2.5) = \boxed{0.7409}$

(c) $y > \pi:\ y(t) = e^{-t}\int_0^\pi e^z \sin z\,dz$

$$y > \pi:\quad y(t) = e^{-t}\left[\frac{1}{2}e^z\ (\sin z - \cos z)\right]_0^\pi = \frac{1}{2}e^{-t}(e^\pi + 1) = 12.070e^{-t}$$

$\therefore y(4) = \boxed{0.2211}$

50.

$$x(t) = 0.8(t-1)[u(t-1) - u(t-3)],$$
$$h(t) = 0.2\ (t-2)[u(t-2) - u(t-3)]$$
$$y(t) = \int_0^\infty x(t-z)\ h(z)\,dz,$$
$$t < 3:\ y(t) = 0$$

(a) $$3 < t < 4:\ \ y(t) = \int_2^{t-1} 0.8(t-z-1)\,0.2(z-2)\,dz$$

$$\therefore y(t) = 0.16\int_2^{t-1} (tz - 2t - z^2 + 2z - z + 2)\,dz$$

$$= 0.16\int_2^{t-1} [-z^2 + (t+1)z + 2 - 2t]\,dz = 0.16\left[-\frac{1}{3}z^3 + \frac{1}{2}(t+1)z^2 + (2-2t)z\right]_2^{t-1}$$

$$= 0.16\left[-\frac{1}{3}(t-1)^3 + \frac{8}{3} + \frac{1}{2}(t+1)(t-1)^2 - \frac{1}{2}(t+1)\,4 + (2-2t)(t-1-2)\right]$$

$$\therefore y(t) = 0.16\left[-\frac{1}{3}t^3 + t^2 - t + \frac{1}{3} + \frac{8}{3} + \frac{1}{2}(t^2-1)(t-1) - 2t - 2 + 2t - 6 - 2t^2 + 6t\right]$$

$$= 0.16\left[\frac{1}{6}t^3 + t^2\left(1 - \frac{1}{2} - 2\right) + t\left(-1 - \frac{1}{2} + 6\right) + 3 + \frac{1}{2} - 8\right] = 0.16\left(\frac{1}{6}t^3 - \frac{3}{2}t^2 + \frac{9}{2}t - \frac{9}{2}\right)$$

$$\therefore \boxed{y(3.8) = 13.653\times10^{-3}}$$

(b) $$4 < t < 5:\ \ y(t) = \int_2^3 0.16(t-z-1)(z-2)\,dz = 0.16\left[-\frac{1}{3}z^3 + \frac{1}{2}(t+1)z^2 + (2-2t)z\right]_2^3$$

$$\therefore y(t) = 0.16\left[-\frac{1}{3}(27-8) + \frac{1}{2}(t+1)5 + (2-2t)1\right]$$

$$= 0.16\left[-\frac{19}{3} + 2.5t + 2.5 + 2 - 2t\right] = 0.16\left(0.5t - \frac{11}{6}\right)$$

$$\therefore \boxed{y(4.8) = 90.67\times10^{-3}}$$

51.

$$x(t) = 10e^{-2t}u(t),\ h(t) = 10e^{-2t}u(t)$$

$$y(t) = \int_0^{\infty} x(t-z)\ h(z)\ dz$$

$$\therefore\ y(t) = \int_0^{t} 10e^{-2(t-z)}\,10e^{-2z}\ dz$$

$$= 100e^{-2t}\int_0^{t} dz = 100\,e^{-2t} \times t$$

$$\therefore \boxed{y(t) = 100t\,e^{-2t}u(t)}$$

52. $h(t) = 5e^{-4t}\,u(t)$

(a) $W_{1\Omega} = 25\int_{0.1}^{0.8} e^{-8t}\,dt = \frac{25}{8}(e^{-0.8} - e^{-6.4}) = 1.3990\ \text{J}$

$\therefore \% = 1.3990 / \left(\frac{25}{8}\right) \times 100\% = \boxed{44.77\%}$

(b) $H(j\omega) = \frac{5}{j\omega + 4}\quad \therefore W_{1\Omega} = \frac{1}{\pi}\int_0^2 \frac{25}{\omega^2 + 16}\,d\omega = \frac{25}{\pi}\,\frac{1}{4}\tan^{-1}\frac{\omega}{4}\Big|_0^2$

$\therefore W_{1\Omega} = \frac{25}{4\pi}\tan^{-1}\frac{1}{2} = 0.9224\ \text{J}\quad \therefore \% = \frac{0.9224}{25/8} \times 100\% = \boxed{29.52\%}$

53.

$$\mathrm{F}(j\omega) = \frac{2}{(1+j\omega)(2+j\omega)} = \frac{2}{1+j\omega} - \frac{2}{2+j\omega} \ \therefore\ f(t) = (2e^{-t} - 2e^{-2t})u(t)$$

(a) $$\mathrm{W}_{1\Omega} = \int_0^\infty (4e^{-2t} - 8e^{-3t} + 4e^{-4t})\ dt = \frac{4}{2} - \frac{8}{3} + \frac{4}{4} = \boxed{\frac{1}{3}\ \mathrm{J}}$$

(b) $$f(t) = -2e^{-t} + 4e^{-2t} = 0,\ -2 + 4e^{-t} = 0,\ e^{t} = 2,\ t = 0.69315$$

$$\therefore\ f_{\max} = 2(e^{-0.69315} - e^{-2\times 0.69315}) = \boxed{0.5}$$

54.

(a)
$$\mathrm{F}(j\omega) = \frac{1}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} - \frac{1/2}{2+j\omega} + \frac{1/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \frac{1}{12}\mathrm{sgn}(t) - \frac{1}{2}e^{-2t}u(t) + \frac{1}{3}e^{-3t}u(t)}$$

(b)
$$\mathrm{F}(j\omega) = \frac{1+j\omega}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} + \frac{1/2}{2+j\omega} - \frac{2/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \frac{1}{12}\mathrm{sgn}(t) + \frac{1}{2}e^{-2t}u(t) - \frac{2}{3}e^{-3t}u(t)}$$

(c)
$$\mathrm{F}(j\omega) = \frac{(1+j\omega)^2}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} - \frac{1/2}{2+j\omega} + \frac{4/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \frac{1}{12}\mathrm{sgn}(t) - \frac{1}{2}e^{-2t}u(t) + \frac{4}{3}e^{-3t}u(t)}$$

(d)
$$\mathrm{F}(j\omega) = \frac{(1+j\omega)^3}{j\omega(2+j\omega)(3+j\omega)} = 1 + \frac{1/6}{j\omega} + \frac{1/2}{2+j\omega} - \frac{8/3}{3+j\omega}$$

$$\therefore \boxed{f(t) = \delta(t) + \frac{1}{12}\mathrm{sgn}(t) + \frac{1}{2}e^{-2t}u(t) - \frac{8}{3}e^{-3t}u(t)}$$

55. $h(t) = 2e^{-t}u(t)$

(a) $\mathrm{H}(j\omega) = 2 \times \dfrac{1}{1+j\omega} = \boxed{\dfrac{2}{1+j\omega}}$

(b) $\dfrac{1}{2}\,\mathrm{H}(j\omega) = \dfrac{1}{1+j\omega} = \dfrac{1}{2}\,\dfrac{\mathrm{V}_o}{\mathrm{V}_i} = \boxed{\dfrac{1/j\omega}{1+1/j\omega}}$

(c) $\boxed{\text{Gain} = 2}$

56.

$$V_o(j\omega) = \frac{\frac{1}{2}j\omega + \frac{1}{j\omega}}{1+\frac{1}{2}j\omega + \frac{1}{j\omega}} = \frac{(j\omega)^2+2}{(j\omega)^2+2(j\omega)+2}$$

$$\therefore V_o(j\omega) = \frac{(j\omega)^2+2(j\omega)+2-2(j\omega)}{(j\omega)^2+2(j\omega)+2} = 1+\frac{-2(j\omega)}{(j\omega)^2+2(j\omega)+2}$$

$$\text{Let } j\omega = x \quad \therefore V_o(x) = 1-\frac{2x}{x+2x+2}; \quad x = \frac{-2\pm\sqrt{4-8}}{2} = -1\pm j1$$

$$\therefore V_o(x) = 1+\frac{A}{x+1+j1}+\frac{B}{x+1-j1} = \quad \text{Let } x=0 \;\; \therefore \frac{A}{1+j1}+\frac{B}{1-j1} = 0$$

$$\text{Let } x=-1 \;\; \therefore \frac{A}{j1}+\frac{B}{-j1} = 2 \;\; \therefore A-B = j2, \;\; A = B+j2 \;\; \therefore \frac{B+j2}{1+j1}+\frac{B}{1-j1} = 0$$

$$\therefore B - jB + j2 + 2 + B + jB = 0 \quad \therefore B = -1-j1 \quad \therefore A = -1+j1$$

$$\therefore V_o(x) = 1+\frac{-1+j1}{x+1+j1}+\frac{-1-j1}{x+1-j1}, \;\; V_o(j\omega) = 1-\frac{1-j1}{(j\omega)+1+j1}-\frac{1+j1}{(j\omega)+1-j1}$$

$$\therefore v_o(t) = \delta(t)-(1-j1)e^{(-1-j1)t}u(t)-(1+j1)e^{(-1+j1)t}u(t)$$

$$= \delta(t)-\sqrt{2}\,e^{-j45^\circ-jt-t}u(t)-\sqrt{2}\,e^{j45^\circ+jt-t}u(t)$$

$$= \boxed{\delta(t)-2\sqrt{2}\,e^{-t}\cos(t+45^\circ)u(t)}$$

57.

$$V_c(j\omega) = 10\frac{5/j\omega}{5/j\omega + 35 + 30(j\omega)} = \frac{10/j\omega}{1/j\omega + 7 + 6(j\omega)}$$

$$\therefore V_c(j\omega) = \frac{10}{6(j\omega)^2 + 7(j\omega) + 1} = \frac{10/6}{(j\omega)^2 + \frac{7}{6}(j\omega) + \frac{1}{6}}$$

$$\therefore\ j\omega = \left(-7/6 \pm \sqrt{\frac{49}{36} - \frac{24}{36}}\right)/2 = -\frac{1}{6}, -1\ \therefore V_c(j\omega) = \frac{10/6}{(j\omega + 1/6)(j\omega + 1)} = \frac{2}{j\omega + 1/6} - \frac{2}{j\omega + 1}$$

$$\therefore \boxed{v_c(t) = 2(e^{-t/6} - e^{-t})u(t)}$$

58. $f(t) = 5e^{-2t}u(t),\ g(t) = 4e^{-3t}u(t)$

(a) $f * g = \int_0^{\infty} f(t-z)\, g(z)\, dz$

$$= \int_0^t 5e^{-2t}\, e^{2z}\, 4e^{-3z}\, dz = 20e^{-2t}\int_0^t e^{-z}\, dz$$

$$= -20\, e^{-2t}(e^t - 1)\ \text{V}$$

$$\therefore \boxed{f * g = (e^{-2t} - e^{-3t})\ u(t)}$$

(b) $\mathrm{F}(j\omega) = \dfrac{5}{j\omega + 2},\ \mathrm{G}(j\omega) = \dfrac{4}{j\omega + 3} \quad \therefore \mathrm{F}(j\omega)\mathrm{G}(j\omega) = \dfrac{20}{(j\omega + 2)(j\omega + 3)}$

$$\therefore \mathrm{F}(j\omega)\mathrm{G}(j\omega) = \frac{20}{j\omega + 2} - \frac{20}{j\omega + 3} \quad \therefore \boxed{f * g = 20(e^{-2t} - 2^{-3t})\ u(t)}$$

1.

Order i_1, i_2, i_3 : $-2i_1' - 6i_3' = 5 + 2\cos 10t - 3i_1 + 2i_2$ (1)

$$4i_2 = 0.05i_1 - 0.15i_2' + 0.25i_3' \quad (2)$$

$$i_2 = -2i_1 - 5i_3 + 0.4\int_0^t (i_1 - i_3)dt + 8 \quad (3)$$

$$(1) \rightarrow -2i_1' - 6i_3' = -3i_1 + 2i_2 + 5 + 2\cos 10t = \text{A}$$

$$(2) \rightarrow 0.05i_1' - 0.15i_2' + 0.25i_3' = 4i_2 = \text{B}$$

$$(3) \rightarrow 2i_1' + i_2' + 5i_3' = 0.4i_1 - 0.4i_3 = \text{C}$$

$$\therefore \boxed{i_1' = 30i_1 + 20i_2 + 50 + 20\cos 10t + 240i_2 + 3.6i_1 - 3.6i_3}$$

$$\therefore i_1' = \frac{\begin{vmatrix} \text{A} & 0 & -6 \\ \text{B} & -0.15 & 0.25 \\ \text{C} & 1 & 5 \end{vmatrix}}{\begin{vmatrix} -2 & 0 & -6 \\ 0.05 & -0.15 & 0.25 \\ 2 & 1 & 5 \end{vmatrix}} = \frac{\text{A}(-1) - \text{B}(6) + \text{C}(-0.9)}{-2(-1) - 6(0.35)} = \frac{-\text{A} - 6\text{B} - 0.9\text{C}}{-0.1} = 10\text{A} + 60\text{B} + 9\text{C}$$

$$i_2' = \frac{\begin{vmatrix} -2 & \text{A} & -6 \\ 0.05 & \text{B} & 0.25 \\ 2 & \text{C} & 5 \end{vmatrix}}{-0.1} = -10[-\text{A}(-0.25) + \text{B}(2) - \text{C}(-0.2)] = -2.5\text{A} - 20\text{B} - 2\text{C}$$

$$i_2' = 7.5i_1 - 5i_2 - 12.5 - 5\cos 10t - 80i_2 - 0.8i_1 + 0.8i_3$$

$$\boxed{\therefore i_2' = 6.7i_1 - 85i_2 + 0.8i_3 - 12.5 - 5\cos 10t}$$

$$i_3' = \frac{\begin{vmatrix} -2 & 0 & \text{A} \\ 0.05 & -0.15 & \text{B} \\ 2 & 1 & \text{C} \end{vmatrix}}{-0.1} = -10[\text{A}(0.35) - \text{B}(-2) + \text{C}(0.3)] = -3.5\text{A} - 20\text{B} - 3\text{C}$$

$$i_3' = 10.5i_1 - 7i_2 - 17.5 - 7\cos 10t - 80i_2 - 1.2i_1 + 1.2i_3$$

$$\boxed{\therefore i_3' = 9.3i_1 - 87i_2 + 1.2i_3 - 17.5 - 7\cos 10t} \leftarrow$$

CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

2. $x' + y' = x + y + 1, \; x' - 2y' = 2x - y - 1$

(a) Order x, y M by 2: $2x' + 2y' = 2x + 2y + 2$; add: $3x' = 4x + y + 1$

$\boxed{\therefore x' = \frac{4}{3}x + \frac{1}{3}y + \frac{1}{3}}$ and $\boxed{y' = -\frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}}$

(b) $x'' = \frac{4}{3}x' + \frac{1}{3}\left(-\frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}\right) = \frac{4}{3}x' - \frac{1}{9}x + \frac{2}{9}y + \frac{2}{9} = \frac{4}{3}x'$

$-\frac{1}{9}x + \frac{2}{9}(3x' - 4x - 1) + \frac{2}{9}$

$\therefore x'' = 2x' - x,$ or $\boxed{x'' - 2x' + x = 0}$

(c) Let $x(0) = 2$ and $y(0) = -5 \;\; \therefore x'(0) = \frac{4}{3}(2) + \frac{1}{3}(-5) + \frac{1}{3} = \frac{8}{3} - \frac{5}{3} + \frac{1}{3} = \frac{4}{3}$

Also, $y'(0) = -\frac{1}{3}(2) + \frac{2}{3}(-5) + \frac{2}{3} = -\frac{2}{3} - \frac{10}{3} + \frac{2}{3} = -\frac{10}{3}$

$\therefore x''(0) = 2x'(0) - x(0) = \frac{8}{3} - 2 = \frac{2}{3}$

$x'''(0) = 2x''(0) - x'(0) = 2\left(\frac{2}{3}\right) - \frac{4}{3} = \boxed{0}$

CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

3.

$x' - 2y - 3z' = f_1(t),\ 2x' + 5z = 3,\ z' - 2y' - x = 0$ Order x, y, z

$$\boxed{\therefore x' = -2.5z + 1.5 \qquad (1)}$$

$$-2.5z + 1.5 - 2y - 3z' = f_1 \ \therefore \boxed{z' = -\frac{2}{3}y - \frac{5}{6}z + 0.5 - \frac{1}{3}f_1 \qquad (3)}$$

$$\therefore 2y' = z' - x = -\frac{2}{3}y - \frac{5}{6}z + \frac{1}{2} - \frac{1}{3}f_1 - x \ \therefore \boxed{y' = -\frac{1}{2}x - \frac{1}{3}y - \frac{5}{12}z + \frac{1}{4} - \frac{1}{6}f_1 \qquad (2)}$$

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4. $x' = -2x - 3y + 4,\ y' = 5x - 6y + 7,\ x(0) = 2,\ y(0) = \frac{1}{3}$

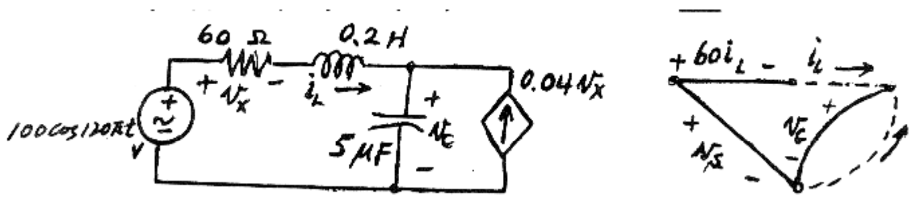
(a) $x'' = -2x' - 3y',\ x'(0) = -2(2) - 3\left(\frac{1}{3}\right) + 4 = -1,\ y'(0) = 5(2) - 6\left(\frac{1}{3}\right) + 7 = 15$

$\therefore x''(0) = -2(-1) - 3(15) = 2 - 45 = \boxed{-43}$

(b) $y'' = 5x' - 6y' \quad \therefore y''(0) = 5(-1) - 6(15) = \boxed{-95}$

(c) $y''' = 5x'' - 6y'' \quad \therefore y'''(0) = 5(-43) - 6(-95) = -215 + 570 = \boxed{355}$

5.



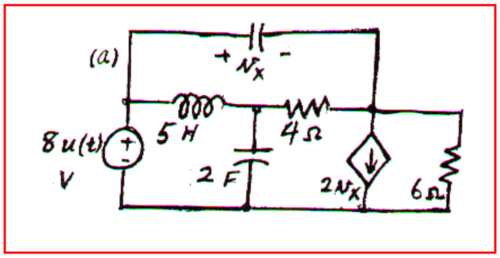
$\therefore 0.2 i'_L = v_s - 60 i_L - v_C$

$\boxed{\therefore i_L = -300 i_L - 5 v_C + 500 \cos 120\pi t \quad (1)}$

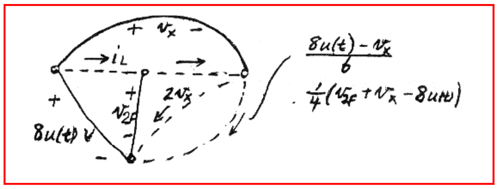
$5\times 10^{-6}\, v'_C = i_L + 2.4 i_L = 3.4 i_L \quad \boxed{\therefore v'_C = 6.8\times 10^{5}\, i_L \qquad (2)}$

6.

(a)



(b)



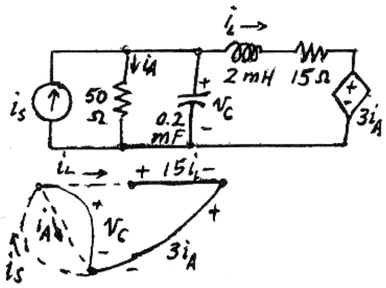
(c) Order: $i_L,\ v_x,\ v_{2F}$ $\quad \therefore 5i'_L = -v_{2F} + 8u(t)$ $\quad \boxed{\therefore i'_L = -0.2v_{2F} + 1.6u(t) \qquad (1)}$

$$3v'_x = -\frac{1}{4}\left(v_{2F} + v_x - 8u(t)\right) + 2v_x + \frac{1}{6}\left(8u(t) - v_x\right) = \frac{19}{12}v_x - \frac{1}{4}v_{2F} + \frac{10}{3}u(t)$$

$$\boxed{\therefore v'_x = \frac{19}{36}v_x - \frac{1}{12}v_{2F} + \frac{10}{9}u(t) \qquad (2)}$$

$$2v'_{2F} = i_L - \frac{1}{4}\left(v_{2F} + v_x - 8u(t)\right) \quad \boxed{\therefore v'_{2F} = \frac{1}{2}i_L - \frac{1}{8}v_x - \frac{1}{8}v_{2F} + u(t) \qquad (3)}$$

7.



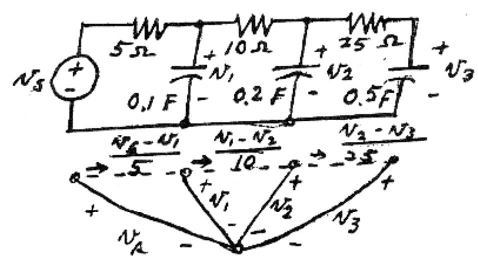
(a) Order: i_L, v_C

$0.002 i'_L = v_C - 3i_A - 15 i_L,\ i_A = 0.02\, v_C$

$\therefore 0.002 i'_L = -15 i_L + 0.94 v_C,\ \boxed{i'_L = -7500 i_L + 470 v_C \qquad (1)}$

$2\times10^{-4}\, v'_C = -i_L - i_A + i_s = -i_L - 0.02 v_C + i_s$

$\boxed{\therefore v_C = -5000 i_L - 100 v_C + 5000 i_s \qquad (2)}$



(b) Order: v_1, v_2, v_3

$0.1 v'_1 = -0.2 v_1 + 0.2 v_s - 0.1 v_1 + 0.1 v_2$

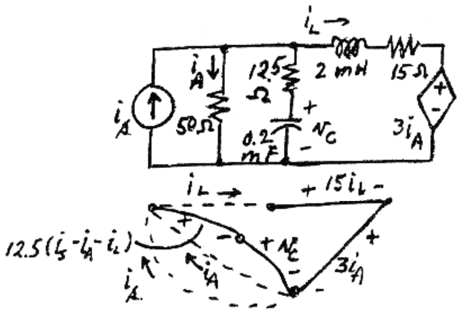
$\boxed{\therefore v'_1 = -3 v_1 + v_2 + 2 v_s \qquad (1)}$

$0.2 v'_2 = 0.1 v_1 - 0.1 v_2 - 0.04 v_2 + 0.04 v_s$

$\boxed{\therefore v'_2 = 0.5 v_1 - 0.7 v_2 + 0.2 v_s \qquad (2)}$

$\boxed{0.5 v'_3 = 0.04 v_2 - 0.04 v_3,\ v'_3 = 0.08 v_2 - 0.08 v_3 \qquad (3)}$

8.



$i_A = 0.02(12.5i_s - 12.5i_A - 12.5i_2 + v_C)$

$\therefore 1.25i_A = -0.25i_L + 0.02v_C + 0.25i_s$

$\therefore i_A = -0.2i_L + 0.016v_C + 0.2v_s$

$0.002i'_L = -15i_L - 3i_A + v_C + 12.5i_s - 12.5i_A - 12.5i_L$

$\therefore 0.002i'_L = -27.5i_L + v_C - 15.5(-0.2i_L + 0.016v_C + 0.2i_s) + 12.5i_s$

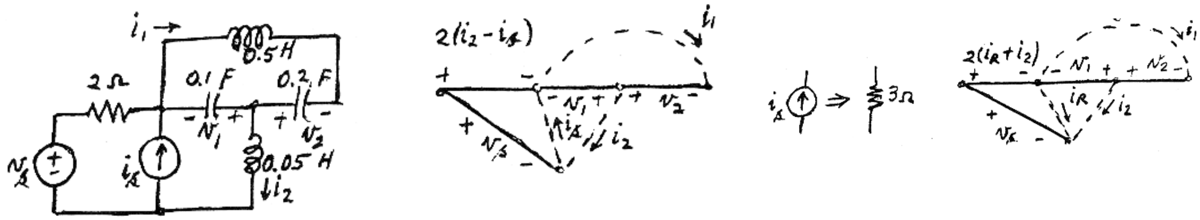
$\therefore 0.002i'_L = -24.4i_L + 0.752v_C + 9.4i_s$

$\boxed{\therefore i'_L = -12,200i_L + 376v_C + 4700i_s \qquad (1)}$

Order: i_L, v_C Also, $0.0002v'_C = i_s - i_A - i_L = i_s + 0.2i_L - 0.016v_C - 0.2i_s - i_L$

$\therefore 0.0002v'_C = -0.8i_L - 0.016v_C + 0.8i_s$ $\boxed{\therefore v'_C = -4000i_L - 80v_C + 4000i_s \qquad (2)}$

9.



$0.1v_1' = i_1 - i_2 \quad \therefore \boxed{v_1' = 10i_1 - 10i_2 \qquad (1)}$

$0.2v_2' = -i_1 \quad \therefore \boxed{v_2' = -5i_1 \qquad (2)}$

$0.5i_1' = -v_1 + v_2, \quad \boxed{\therefore i_1' = -2v_1 + 2v_2 \qquad (3)}$

$0.05i_2' = v_1 + 2i_s - 2i_2 + v_s \quad \therefore \boxed{i_2' = 20v_1 - 40i_2 + 40i_s + 20v_s \qquad (4)}$

Order: v_1, v_2, i_1, i_2

$0.1v_1' = i_1 - i_2 \quad \therefore \boxed{v_1' = 10i_1 - 10i_2 \qquad (1)}$

$0.2v_2' = -i_1 \quad \therefore \boxed{v_2' = -5i_1 \qquad (2)}$

$0.5i_1' = -v_1 + v_2 \quad \therefore \boxed{i_1' = -2v_1 + 2v_2 \qquad (3)}$

$0.05i_2' = v_1 + v_s - 2i_R - 2i_2,\ 3i_R = -2i_R - 2i_2 + v_s \quad \therefore i_R = -0.4i_2 + 0.2v_s$

$\therefore 0.05i_2' = v_1 + v_s + 0.8i_2 - 0.4v_s - 2i_2 \quad \therefore \boxed{i_2' = 20v_1 - 24i_2 + 12v_s \qquad (4)}$

10.

(a) Order: $i_L, v_C \quad \therefore 0.2i'_L = -v_C - 20i_L + 20i_s$

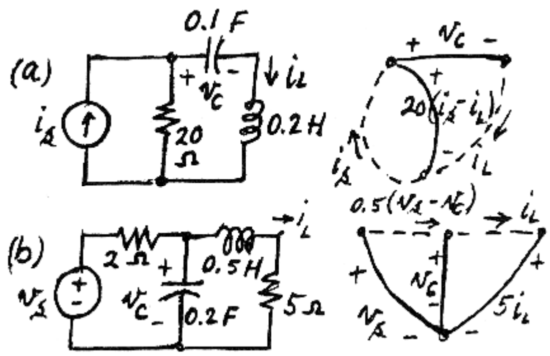
$\therefore \boxed{i'_L = -100i_L - 5v_C + 100i_s \qquad (1)}$

$0.1v'_C = i_L \quad \therefore \boxed{v'_C = 10i_L \qquad (2)}$

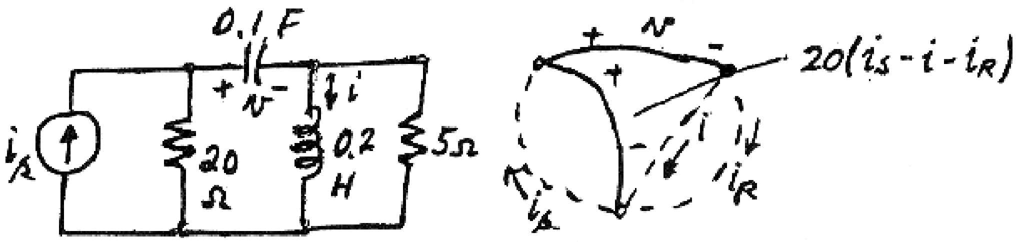
(b) $0.5i'_L = v_C - 5i_L \quad \therefore \boxed{i'_L = -10i_L + 2v_C \qquad (1)}$

$0.2v'_C = 0.5v_s - 0.5v_C - i_L$

$\therefore \boxed{v'_C = -5i_L - 2.5v_C + 2.5v_s \qquad (2)}$



11.



$\therefore 5i_R = -v + 20i_s - 20i - 20i_R$

$\therefore i_R = -0.04v - 0.8i + 0.8i_s$

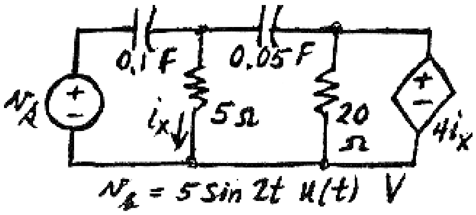
$\therefore 0.1v' = i + i_R = -0.04v + 0.2i + 0.8i_s$

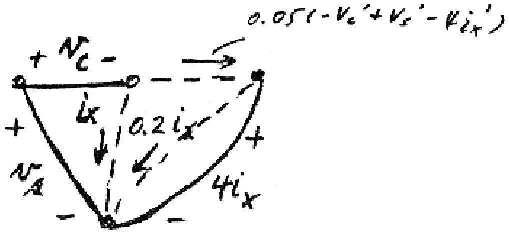
Order: v, i $\quad \therefore \boxed{v' = -0.4v + 2i + 8i_s \qquad (1)}$

$0.2i' = -v + 20i_s + 20i - 20(-0.04v + 0.2i + 0.8i_s)$

$\therefore 0.2i' = -0.2v - 4i + 4i_s \quad \therefore \boxed{i' = -v - 20i + 20i_s \qquad (2)}$

12.





$$\frac{4i_x}{20} = 0.2i_x$$

$$0.1v'_C = i_x - 0.05v'_C + 0.05v'_s - 0.2i'_x$$

$$\therefore 0.15v'_C = i_x - 0.2i'_x + 0.05v'_s$$

$$5i_x = -v_c + v_s,\ i_x = -0.2v_c + 0.2v_s,\ i'_x = -0.2v'_C + 0.2v'_s$$

$$\therefore 0.15v'_C = -0.2v_C + 0.2v_s + 0.04v'_C - 0.04v'_s + 0.05v'_s$$

$$\therefore 0.11v'_C = -0.2v_C + 0.2v_s + 0.01v'_s \quad \therefore 0.11v'_C = -0.2v_C + \sin 2tu(t) + 0.1\cos 2tu(t)$$

$$\boxed{\therefore v'_C = -\frac{20}{11}v_C + \frac{100}{11}\sin 2tu(t) + \frac{10}{11}\cos 2tu(t) \qquad (1)}$$

13.

(a) $\bar{q} = \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_C \end{bmatrix}, \ \bar{a} = \begin{bmatrix} -1 & -2 & -3 \\ 4 & -5 & 6 \\ 7 & -8 & -9 \end{bmatrix}, \ \bar{f} = \begin{bmatrix} 2t \\ 3t^2 \\ 1+t \end{bmatrix} \ \therefore \boxed{i'_{L1} = -i_{L1} - 2i_{L2} - 3v_C + 2t \qquad (1)}$

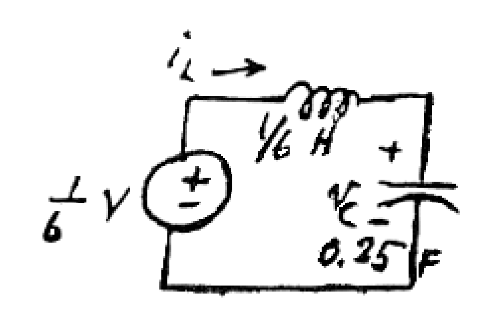
$$\boxed{\begin{aligned} i'_{L2} &= 4i_{L1} - 5i_{L2} + 6v_C + 3t^2 \qquad (2) \\ v'_C &= 7i_{L1} - 8i_{L2} - 9v_C + 1 + t \qquad (3) \end{aligned}}$$

(b) $\bar{q} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}, \ \bar{a} = \begin{bmatrix} 0 & -6 \\ 4 & 0 \end{bmatrix}, \ \bar{f} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

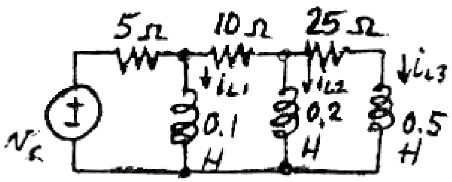
$\therefore i'_L = -6v_C + 1, \ v'_C = 4i_L$

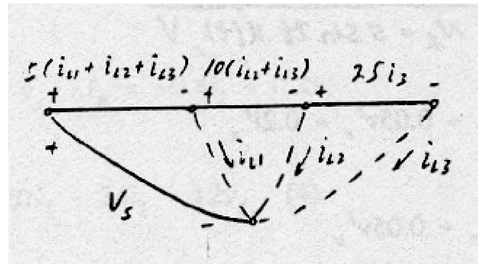
$\therefore 0.25v'_C = i_L \quad \therefore \text{C} = 0.25 \text{ F}$

$\frac{1}{6} i'_L = -v_C + \frac{1}{6} \quad \therefore \text{L} = \frac{1}{6} \text{ H}$



14.





$$0.1i'_L = -5i_{L1} - 5i_{L2} - 5i_{L3} + v_s$$

$$\therefore i'_{L1} = -50i_{L1} - 50i_{L2} - 50i_{L3} + 10v_s \qquad (1)$$

$$0.2i'_{L2} = -5i_{L1} - 15i_{L2} - 15i_{L3} + v_s$$

$$\therefore i'_{L2} = -25i_{L1} - 75i_{L2} - 75i_{L3} + 5v_s \qquad (2) \quad 0.5i'_{L3} = -5i_{L1} - 15i_{L2} - 40i_{L3} + v_s$$

$$\therefore i'_{L3} = -10i_{L1} - 30i_{L2} - 80i_{L3} + 2v_s \qquad (3)$$

$$\boxed{\bar{a} = \begin{bmatrix} -50 & -50 & -50 \\ -25 & -75 & -75 \\ -10 & -30 & -80 \end{bmatrix} \qquad \bar{f} = \begin{bmatrix} 10v_s \\ 5v_s \\ 2v_s \end{bmatrix}}$$

15.

$$\bar{q} = \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix}, \ \bar{a} = \begin{bmatrix} -3 & 3 & 0 \\ 2 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix}, \ \bar{f} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}; \ \bar{w} = \begin{bmatrix} v_{01} \\ v_{02} \\ i_{R1} \\ i_{R2} \end{bmatrix}, \ \bar{b} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix}, \ \bar{d} = \begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \end{bmatrix}$$

$$\bar{w} = \bar{b}\bar{q} + \bar{d} \quad \therefore \bar{q}' = \bar{a}\bar{q} + \bar{f}, \ \bar{w} = \bar{b}\bar{q} + \bar{d}, \ \bar{w}' = \bar{b}\bar{q}' = \bar{b}\bar{a}\bar{q} + \bar{b}\bar{f}$$

$$\bar{b}\bar{a} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} -3 & 3 & 0 \\ 2 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -4 & 2 \\ 0 & 9 & -4 \\ 3 & 0 & 0 \end{bmatrix}, \ \bar{b}\,\bar{f} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\therefore \bar{w}' = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -4 & 2 \\ 0 & 9 & -4 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} v_{C1} + v_{C2} + 10 \\ -v_{C1} - 4v_{C2} + 2i_L \\ 9v_{C2} - 4i_L \\ 3v_{C1} + 10 \end{bmatrix}$$

$$\boxed{\therefore v'_{01} = v_{C1} + v_{C2} + 10, \ v'_{02} = -v_{C1} - 4v_{C2} + 2i_L, \ i'_{R1} = 9v_{02} - 4i_L, \ i'_{R2} = 3v_{C1} + 10}$$

16.

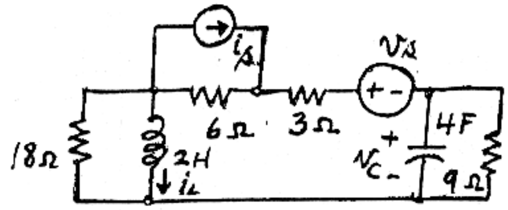
$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}, \ \bar{a} = \begin{bmatrix} -3 & 1 & 2 \\ -2 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}, \ \bar{f} = \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix}, \ \bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \ \bar{b} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 2 & -1 & -1 & 3 \end{bmatrix}$$

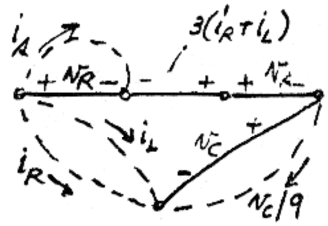
$$\bar{d} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \ \bar{q}/ = \bar{a}\,\bar{q} + \bar{f}, \bar{q} = \bar{b}\,\bar{y} + \bar{d}, \ \bar{y}(0) = \begin{bmatrix} 10 \\ -10 \\ -5 \\ 5 \end{bmatrix}$$

$$\therefore \bar{q} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 2 & -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} y_1 + 2y_2 + 3y_4 + 2 \\ -y_2 + y_3 + y_4 + 1 \\ 2y_1 - y_2 - y_3 + 3y_4 + 3 \end{bmatrix} \ \boxed{\therefore \bar{q}(0) = \begin{bmatrix} 7 \\ 11 \\ 53 \end{bmatrix}}$$

$$\bar{q}'(0) = \begin{bmatrix} -3 & 1 & 2 \\ -2 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 53 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -21 + 11 + 106 + 1 \\ -14 - 22 + 53 \\ -7 + 33 \end{bmatrix} = \boxed{\begin{bmatrix} 97 \\ 17 \\ 26 \end{bmatrix}}$$

17.





$\bar{q} = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$

$4v'_C = -\frac{v_C}{9} - i_L - i_R$, $2i'_L = v_R - 3i_R - 3i_L + v_s + v_C$, $18 i_R = 2i'_L$ $\therefore i_R = \frac{1}{9} i'_L$

$\frac{v_R}{6} = -i_R - i_L - i_s$ $\therefore v_R = -6i_R - 6i_L - 6i_s = -\frac{2}{3} i'_L - 6i_L - 6i_s$, $\therefore 4v'_C = -\frac{v_C}{9} - i_L - \frac{1}{9} i'_L$

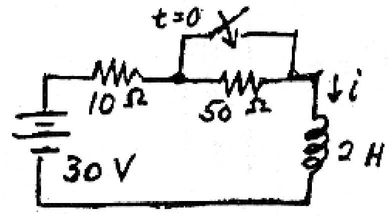
Also, $2i'_L = -\frac{2}{3} i'_L - 6i_L - 6i_s - \frac{1}{3} i'_L - 3i_L + v_C + v_s$ $\therefore 3i'_L = v_C - 9i_L + v_s - 6i_s$

$\therefore i'_L = \frac{1}{3} v_C - 3i_L + \frac{1}{3} v_s - 2i_s$ (2) $4v'_C = -\frac{1}{9} v_C - i_L - \frac{1}{27} v_C + \frac{1}{3} i_L - \frac{1}{27} v_s + \frac{2}{9} i_s$

$\therefore 4v'_C = -\frac{4}{27} v_C - \frac{2}{3} i_L - \frac{1}{27} v_s + \frac{2}{9} i_s$ $\therefore v'_C = -\frac{1}{27} v_C - \frac{1}{6} i_L - \frac{1}{108} v_s + \frac{1}{18} i_s$ (1)

$$\boxed{\therefore [\bar{a}] = \begin{bmatrix} -1/27 & -1/6 \\ 1/3 & -3 \end{bmatrix} \quad [\bar{f}] = \begin{bmatrix} -\frac{1}{108} v_s + \frac{1}{18} i_s \\ \frac{1}{3} v_s - 2i_s \end{bmatrix}}$$

18.



(a) $30 = 10i + 2i' \quad \therefore \boxed{i' = -5i + 15}$

(b) $i(0) = 0.5 \text{ A},\ a = -5 \quad \therefore i = e^{-5t}\,0.5 + e^{-5t}\int_0^t 15e^{5Z}\,dz$

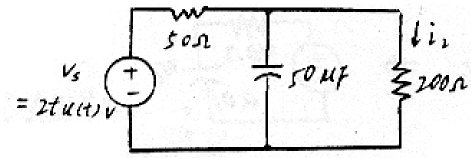
$\therefore i = 0.5e^{-5t} + e^{-5t}3(e^{5t} - 1) = \boxed{3 - 2.5e^{-5t} \text{ A},}\ t > 0$

(c) $i_{zero\ state} = 3(1 - e^{-5t}) \text{ A},\ i_{zero\ input} = \boxed{0.5e^{-5t} \text{ A}}$

(d) $i_f = 3,\ i_n = \text{A}e^{-5t} \quad \therefore i = 3 + \text{A}e^{-5t},\ i(0) = 0.5 \ \therefore i = 3 - 2.5e^{-5t} \text{ A}$

$\therefore i_n = \boxed{-2.5e^{-5t} \text{ A},\ i_f = 3 \text{ A}}$

19.

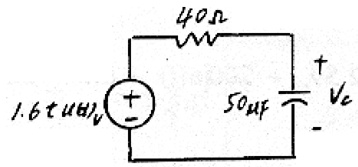


$$\therefore 5\times10^{-5} v'_C = 0.04t\ u(t) - 0.025 v_C$$

$$\therefore v'_C = -500 v_C + 800tu(t) \quad \therefore a = -500$$

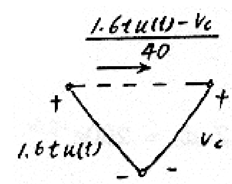
$$\therefore v_C = e^{-500t}\int_0^t e^{500Z}\, 800\text{Z}\, dz$$

$$= 800e^{-500t}\int_0^t \text{Z}e^{500Z}\, dz$$



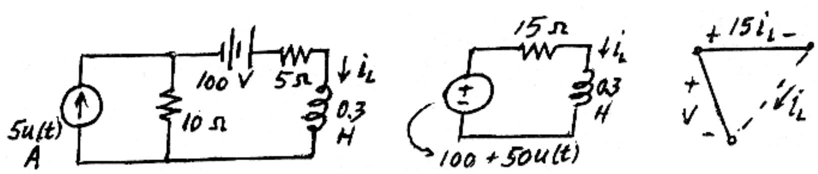
$$\therefore v_C = 800e^{-500t}\left[e^{500Z}\left(\frac{\text{Z}}{500} - \frac{1}{500^2}\right)\right]_0^t$$

$$\therefore v_C = 800e^{-500t}\left[e^{500t}\left(\frac{t}{500} - \frac{1}{500^2}\right) + \frac{1}{500^2}\right]$$



$$\therefore v_C = 1.6t + \frac{1.6}{500}(-1 + e^{-500t}) \quad \therefore i_2 = \frac{1}{200} v_C = \boxed{8\times10^{-3} t - 16\times10^{-6}(1 - e^{-500t})\ \text{A},\ t > 0}$$

20.



(a) $0.3i'_L = -15i_L + 100 + 50u(t),\ i'_L = -50i_L + \frac{1000}{3} + \frac{500}{3}\ u(t),\ i_L(0) = \frac{100}{15} = \frac{20}{3}$ A

$$\therefore i_L = e^{-50t} \times \frac{20}{3} + e^{-50t}\int_0^t e^{50Z}\ 500\,dZ = \frac{20}{3}e^{-50t} + 500e^{-50t}\frac{1}{50}(e^{50t} - 1)$$

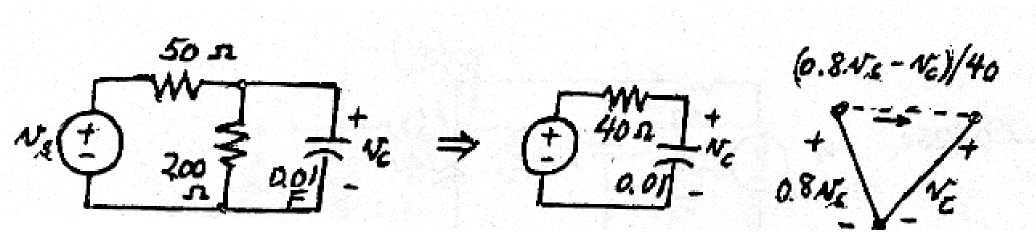
$$\therefore i_L = \frac{20}{3}e^{-50t} + 10(1 - e^{-50t})\ t > 0$$

$$\therefore \boxed{i_L = \frac{20}{3}\ u(-t) + \left[10 - \frac{10}{3}e^{-50t}\right] u(t)\ \text{A}}$$

(b)

$$\boxed{\begin{array}{l} i_{L,zero\ input} = \frac{20}{3}e^{-50t}\ \text{A},\ i_{L,zero\ state} = 10(1 - e^{-50t})\ \text{A} \\ i_{Lf} = 10\ \text{A},\quad i_{Ln} = -\frac{10}{3}e^{-50t}\ \text{A} \end{array}}$$

21.



$$v_s = 100[u(t) - u(t-0.5)]\cos \pi t \text{ V}$$

$$0.01v'_C = \frac{1}{40}(0.8v_s - v_C) \quad \therefore v'_C = -2.5v_C + 200[u(t) - u(t-0.5)]\cos \pi t$$

$$t < 0 \quad v_C = 0$$

$$0 \le t \le 0.5 \quad v_C = e^{-2.5t} \times 200\int_0^t e^{2.5Z} \cos \pi \mathrm{Z}\; dz = 200e^{-2.5t}\left[\frac{e^{2.5Z}}{6.25 + \pi^2}(2.5\cos \pi \mathrm{Z} + \pi \sin \pi \mathrm{Z})\right]_0^t$$

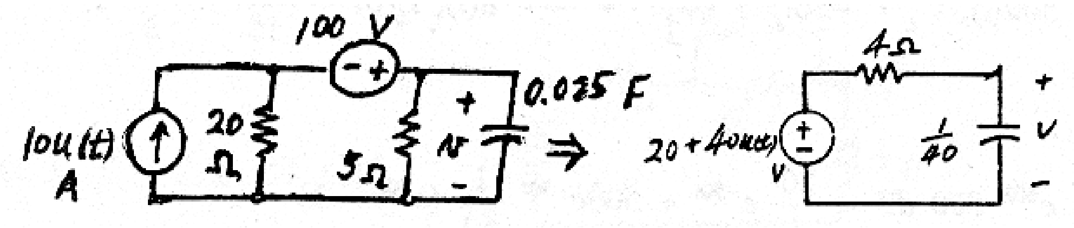
$$\therefore v_C = \frac{200}{6.25+\pi^2}(2.5\cos \pi t + \pi \sin \pi t) - \frac{200}{6.25+\pi^2}2.5e^{-2.5t}$$

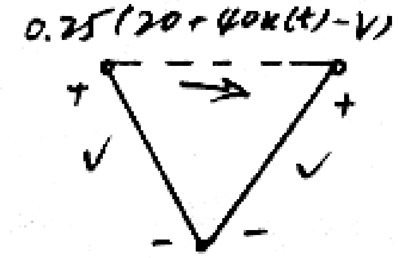
$$\therefore \boxed{v_C = 31.02\cos \pi t + 38.98\sin \pi t - 31.02e^{-2.5t}}$$

$$t > 0.5 \quad v_C = 200e^{-2.5t}\left[\frac{e^{2.5Z}}{6.25+\pi^2}(2.5\cos \pi \mathrm{Z} + \pi \sin \pi \mathrm{Z})\right]_0^{0.5}$$

$$\therefore v_C = 200e^{-2.5t}\left[\frac{e^{1.25}}{6.25+\pi^2}\pi - \frac{1}{6.25+\pi^2}\times 2.5\right] = \boxed{105.03e^{-2.5t} \text{ V}}$$

22.





$$\therefore \frac{1}{40}v' = -\frac{1}{4}v + 5 + 10u(t)$$

(a) $\therefore v' = \boxed{-10v + 200 + 400u(t)}$

(b) $t < 0 \ \therefore v = 20\text{ V}$

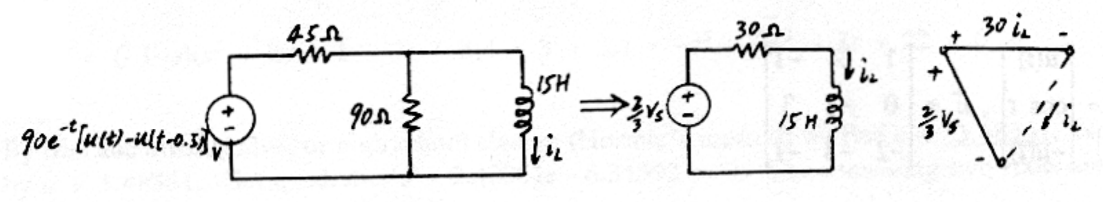
$$t > 0 \ \ v = 20e^{-10t} + 600e^{-10t}\int_0^Z e^{10Z}dz = 20e^{-10t} + 60e^{-10t}(e^{10t} - 1)$$

$$\therefore v = 20e^{-10t} + 60 - 60e^{-10t} \ \ \therefore v = \boxed{60 - 40e^{-10t}\text{ V}}$$

(c) $v_{forced} = 60\text{V} \quad v_{nat} = \boxed{-40e^{-10t}\text{ V}}$

$v_{zero\ state} = 60(1 - e^{-10t})\text{ V} \quad v_{zero\ input} = \boxed{20e^{-10t}\text{ V}}$

23.



$$15i'_L = -30i_L + \frac{2}{3}v_s$$

$$\therefore i'_L = -2i_L + 4e^{-t}[u(t) - u(t-0.5)]$$

$$\therefore i_L = 4e^{-2t}\int_0^t e^{2Z}e^{-Z}[u(\mathrm{Z}) - u(\mathrm{Z}-0.5)]\,dz$$

$$0 \le t \le 0.5 \; i_L = 4e^{-2t}\int_0^t e^Z \mathrm{dZ} = 4e^{-2t}(e^t - 1) = \boxed{4e^{-t} - 4e^{-2t} \text{ A}}$$

$$t \ge 0.5 \;\; i_L = 4e^{-2t}\int_0^{0.5} e^Z \,\mathrm{dZ} + 0 = 4e^{-2t}(e^{0.5} - 1) = \boxed{2.595^{-}e^{-2t} \text{ A}}$$

24.

$$[a] = \begin{bmatrix} -8 & 5 \\ 10 & -10 \end{bmatrix}, \ t = 0.01$$

(a)
$$e^{-t\bar{a}} = \bar{I} - \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix}\begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} - \ldots$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0.0114 & -0.0090 \\ -0.0180 & 0.0150 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} 0.0114 & -0.009 \\ -0.018 & 0.015 \end{bmatrix}\begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \ldots$$
$$= \begin{bmatrix} 1.08 & -0.05 \\ -0.1 & 1.1 \end{bmatrix} + \begin{bmatrix} 0.0057 & -0.0045 \\ -0.009 & 0.0075 \end{bmatrix} - \frac{1}{6}\begin{bmatrix} -0.00181 & 0.00147 \\ 0.00294 & -0.00240 \end{bmatrix} = \boxed{\begin{bmatrix} 1.0860 & -0.0547 \\ -0.1095 & 1.1079 \end{bmatrix}}$$

(b)
$$e^{t\bar{a}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \begin{bmatrix} 0.0057 & -0.0045 \\ -0.009 & 0.0075 \end{bmatrix} + \begin{bmatrix} -0.0003 & 0.0002 \\ 0.0005 & -0.0004 \end{bmatrix} = \boxed{\begin{bmatrix} 0.9254 & 0.0457 \\ 0.0915 & 0.9071 \end{bmatrix}}$$

(c)
$$e^{-t\bar{a}}e^{t\bar{a}} = \begin{bmatrix} 1.0860 & -0.0547 \\ -0.1095 & 1.1079 \end{bmatrix}\begin{bmatrix} 0.9254 & 0.0457 \\ 0.0915 & 0.9071 \end{bmatrix} = \boxed{\begin{bmatrix} 1.0000 & 0 \\ 0 & 1.0000 \end{bmatrix}}$$

25.

(a)

$$\bar{q} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \ \bar{f} = \begin{bmatrix} u(t) \\ \cos t \\ -u(t) \end{bmatrix}, \ \bar{a} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ -2 & -3 & -1 \end{bmatrix}$$

$$\therefore \boxed{x' = x + 2y - z + u(t), \ \ y' = -y + 3z + \cos t, \ z' = -2x - 3y - z - u(t)}$$

(b)

$$\bar{q}(0) = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \ \ \therefore \bar{q}\,/(0) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ -2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ 3 \end{bmatrix} \quad \Delta t = 0.1$$

$$\bar{q}(0.1) = \bar{q}(0) + 0.1\bar{q}'(0) = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.4 \\ 0.7 \\ 0.3 \end{bmatrix} = \boxed{\begin{bmatrix} 1.6 \\ -2.3 \\ 1.3 \end{bmatrix}}$$

(c)

$$\Delta t = 0.05 \ \ \therefore \bar{q}(0.5) = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + 0.05 \begin{bmatrix} -4 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.8 \\ -2.65 \\ 1.15 \end{bmatrix}$$

$$\bar{q}'(0.05) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ -2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1.8 \\ -2.65 \\ 1.15 \end{bmatrix} + \begin{bmatrix} 1 \\ \cos 0.05 \\ -1 \end{bmatrix} = \begin{bmatrix} -3.65 \\ 7.0988 \\ 2.2 \end{bmatrix}$$

$$\bar{q}(0.1) = \bar{q}(0.05) + 0.05\bar{q}'(0.05) = \begin{bmatrix} 1.8 \\ -2.65 \\ 1.15 \end{bmatrix} + 0.05 \begin{bmatrix} -3.65 \\ 7.0988 \\ 2.2 \end{bmatrix} = \boxed{\begin{bmatrix} 1.6175 \\ -2.2951 \\ 1.26 \end{bmatrix}}$$

26.

$$\bar{a} = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix} \text{ Now, } \bar{a} - s\,\bar{\mathrm{I}} = \begin{bmatrix} -1-s & 2 & 3 \\ 0 & -1-s & 2 \\ 3 & 1 & -1-s \end{bmatrix}$$

$$\det[\ ] = (-1-s)(s^2+2s+1-2) + 3(4+3+3s) = -s^3 - 3s^2 + 8s + 22 = 0$$

By trial and error, Solve, or high school algebra (Hormer's method) we find $\boxed{s = -3.48361}$. Now divide polynomial by s + 3.48361. Get quadratic, $s^2 - 0.48361s - 6.31592 = 0$.

$\boxed{\text{The remaining two roots are: } s = -2.28282 \text{ and } s = +2.76641.}$

27.

$$\bar{a} = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix} \ \det(\bar{a} - s\bar{\mathrm{I}}) = \det\begin{bmatrix} -3-s & 2 \\ 1 & -4-s \end{bmatrix} = s^2 + 7s + 12 - 2 = 0$$

Roots are $s_1 = -2,\ s_2 = -5$ Now, $e^{s_1 t} = u_o + u_1 s_1,\ e^{s_2 t} = u_o + u_1 s_2$

$$\therefore e^{-2t} = u_o - 2u_1,\ e^{-5t} = u_o - 5u_1 \ \therefore e^{-2t} - e^{-5t} = 3u_1,\ u_1 = \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t}$$

$$\therefore u_o = e^{-2t} + \frac{2}{3}\, e^{-2t} - \frac{2}{3}\, e^{-5t},\ \text{or } u_o = \frac{5}{3}\, e^{-2t} - \frac{2}{3}\, e^{-5t}$$

$$e^{t\bar{a}} = u_o\bar{\mathrm{I}} + u_1\bar{a} = \begin{bmatrix} \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} & 0 \\ 0 & \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} \end{bmatrix} + \left(\frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t}\right)\begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$$

$$e^{t\bar{a}} = \begin{bmatrix} \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} - e^{-2t} + e^{-5t} & \frac{2}{3}e^{-2t} - \frac{2}{3}e^{-5t} \\ \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t} & \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} - \frac{4}{3}e^{-2t} + \frac{4}{3}e^{-5t} \end{bmatrix}$$

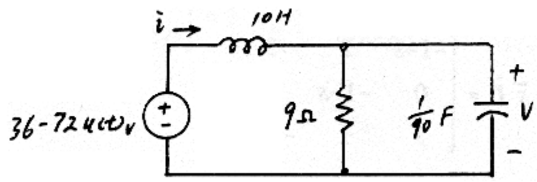
$$\therefore \boxed{e^{t\bar{a}} = \begin{bmatrix} \frac{2}{3}e^{-2t} + \frac{1}{3}e^{-5t} & \frac{2}{3}e^{-2t} - \frac{2}{3}e^{-5t} \\ \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t} & \frac{1}{3}e^{-2t} + \frac{2}{3}e^{-5t} \end{bmatrix}}$$

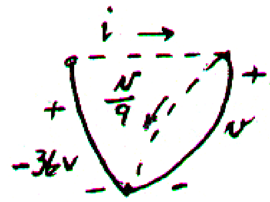
28.

(a) $\bar{q} = \begin{bmatrix} i \\ v \end{bmatrix} \begin{matrix} 10i' = -36 - v \\ \dfrac{1}{90}v' = i - \dfrac{1}{9}v \end{matrix}$

$\therefore \boxed{i' = -0.1v - 3.6,\ v' = 90i - 10v}$

$\therefore \bar{a} = \begin{bmatrix} 0 & -0.1 \\ 90 & -10 \end{bmatrix},\ \bar{f} = \begin{bmatrix} -3.6 \\ 0 \end{bmatrix}$





(b) $\bar{a} - s\bar{\mathrm{I}} = \begin{bmatrix} 0-s & -0.1 \\ 90 & -10-s \end{bmatrix},\ \det(\bar{a} - s\bar{\mathrm{I}}) = s^2 + 10s + 9 \quad \therefore \boxed{s_1 = -1, s_2 = -9}$

(c) $e^{-t} = u_o - u_1,\ e^{-9t} = u_o - 9u_1 \quad \therefore e^{-t} - e^{-9t} = 8u_1$

$\therefore \boxed{u_1 = \frac{1}{8}e^{-t} - \frac{1}{8}e^{-9t}},\ u_o = \frac{1}{8}e^{-t} - \frac{1}{8}e^{-9t} + e^{-t} = \boxed{\frac{9}{8}e^{-t} - \frac{1}{8}e^{-9t}}$

(d) $e^{t\bar{a}} = u_o\bar{\mathrm{I}} + u_1\bar{a} = \frac{1}{8}(9e^{-t} - e^{-9t})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{8}(e^{-t} - e^{-9t})\begin{bmatrix} 0 & -0.1 \\ 90 & -10 \end{bmatrix}$

$$\therefore 8e^{t\bar{a}} = \begin{bmatrix} 9e^{-t} - e^{-9t} & 0 \\ 0 & 9e^{-t} - e^{-9t} \end{bmatrix} + \begin{bmatrix} 0 & -0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -10e^{-t} + 10e^{-9t} \end{bmatrix}$$

$$8e^{t\bar{a}} = \begin{bmatrix} 9e^{-t} - e^{-9t} & -0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -e^{-t} + 9e^{-9t} \end{bmatrix}$$

$$\therefore \boxed{e^{t\bar{a}} = \begin{bmatrix} \frac{9}{8}e^{-t} - \frac{1}{8}e^{-9t} & -\frac{1}{80}e^{-t} + \frac{1}{80}e^{-9t} \\ \frac{90}{8}e^{-t} - \frac{90}{8}e^{-9t} & -\frac{1}{8}e^{-t} + \frac{9}{8}e^{-9t} \end{bmatrix}}$$

(e) $\bar{q} = e^{t\bar{a}}\bar{q}(0) + e^{t\bar{a}}\int_0^t e^{-\mathrm{Z}\bar{a}}\ f(\mathrm{Z})dz \quad \bar{q}(0) = \begin{bmatrix} i(0) \\ v(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 36 \end{bmatrix}$

$$\bar{q} = \frac{1}{2}\begin{bmatrix} 9e^{-t} - e^{-9t} & -0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -e^{-t} + 9e^{-9t} \end{bmatrix}\begin{bmatrix} 1 \\ 9 \end{bmatrix}$$

$$+\frac{1}{8}e^{t\bar{a}}\int_0^t \begin{bmatrix} 9e^{Z} - e^{9Z} & -0.1e^{Z} + 0.1e^{9Z} \\ 90e^{Z} - 90e^{9Z} & -e^{Z} + 9e^{9Z} \end{bmatrix}\begin{bmatrix} -3.6 \\ 0 \end{bmatrix}dz$$

$$\therefore \bar{q} = \frac{1}{2}\begin{bmatrix} 9e^{-t} - e^{-9t} - 0.9e^{-t} + 0.9e^{-9t} \\ 90e^{-t} - 90e^{-9t} - 9e^{-t} + 81e^{-9t} \end{bmatrix} - 0.45e^{t\bar{a}}\int_0^t \begin{bmatrix} 9e^{Z} - e^{9Z} \\ 90e^{Z} - 90e^{9Z} \end{bmatrix}\mathrm{dZ}$$

$$= \begin{bmatrix} 4.05e^{-t} - 0.05e^{9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - 0.45e^{t\bar{a}} \begin{bmatrix} 9e^{t} - \frac{1}{9}e^{9t} - 9 + \frac{1}{9} \\ 90e^{t} - 10e^{9t} - 90 + 10 \end{bmatrix}$$

$$\bar{q} = \begin{bmatrix} 4.05e^{-t} - 0.05e^{-9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - \frac{0.45}{8} \begin{bmatrix} 9e^{-t} - e^{-9t} & 0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -e^{-t} + 9e^{-9t} \end{bmatrix} \begin{bmatrix} 9e^{t} - \frac{1}{9}e^{9t} - \frac{80}{9} \\ 90e^{t} - 10e^{9t} - 80 \end{bmatrix}$$

$$= \begin{bmatrix} 4.05e^{-t} & -0.05e^{-9t} \\ 4.05e^{-t} & -4.5e^{-9t} \end{bmatrix}$$

$$-\frac{0.45}{8} \begin{bmatrix} 81 - e^{8t} - 80e^{-t} - 9e^{-8t} + \frac{1}{9} + \frac{80}{9}e^{-9t} - 9 + e^{8t} + 8e^{-t} + 9e^{-8t} - 1 - 8e^{-9t} \\ 810 - 10e^{8t} - 800e^{-t} - 810e^{-8t} + 10 + 800e^{-9t} - 90 + 10e^{8t} + 80e^{-t} + 810e^{-8t} - 90 - 720e^{-9t} \end{bmatrix}$$

$$= \begin{bmatrix} 4.05e^{-t} - 0.05e^{-9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - \frac{0.45}{7} \begin{bmatrix} 71 + \frac{1}{9} - 72e^{-t} + \frac{8}{9}e^{-9t} \\ 640 - 720e^{-t} + 80e^{-9t} \end{bmatrix}$$

$$= \begin{bmatrix} 4.05e^{-t} - 0.05e^{-9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - \begin{bmatrix} 4 - 4.05e^{-t} + 0.05e^{-9t} \\ 36 - 40.5e^{-t} + 4.5e^{-9t} \end{bmatrix}$$

$$\therefore \boxed{\bar{q} = \begin{bmatrix} -4 + 8.1e^{-t} - 0.1e^{-9t} \\ -36 + 81e^{-t} - 9e^{-9t} \end{bmatrix} (t > 0)}$$

