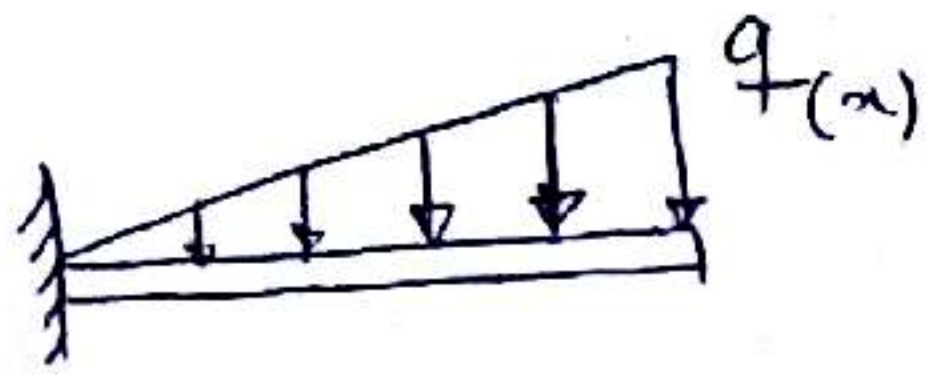




$$N_1(x) = 1 - \frac{rx^r}{L^r} + \frac{rx^r}{L^r}$$

$$F_1 = \int_0^L \frac{q_0 x}{L} N_1(x) dx = \int_0^L \frac{q_0 x}{L} \left(1 - \frac{rx^r}{L^r} + \frac{rx^r}{L^r} \right) dx = \frac{q_0}{L} \left(\frac{x^r}{r} - \frac{r}{L^r} \left(\frac{x^2}{2} \right) + \frac{r}{L^r} \left(\frac{x^2}{2} \right) \right) \Big|_0^L$$

$$= \frac{q_0}{L} \left(\frac{L^r}{r} - \frac{rL^r}{2} + \frac{rL^r}{2} \right) = \frac{q_0}{L} \left(L \left(\frac{1}{r} - \frac{r}{2} + \frac{r}{2} \right) \right) = \frac{r}{r_0} q_0 L$$

$$M_1 = \int_0^L \frac{q_0 x}{L} N_2(x) dx, N_2(x) = x - \frac{rx^r}{L} + \frac{x^r}{L^r}$$

$$M_1 = \int_0^L \frac{q_0 x}{L} \left(x - \frac{rx^r}{L} + \frac{x^r}{L^r} \right) dx = \frac{q_0}{L} \int_0^L \left(x^2 - \frac{rx^r}{L} + \frac{x^r}{L^r} \right) dx = \frac{q_0}{L} \left(\frac{L^r}{r} - \frac{L^r}{r} + \frac{L^r}{\Delta} \right) = \frac{q_0 L^r}{r_0}$$

$$F_r = \int_0^L \frac{q_0 x}{L} N_r(x) dx, N_r(x) = r \frac{x^r}{L^r} - r \frac{x^r}{L^r}$$

$$F_r = \int_0^L \frac{q_0 x}{L} \left(r \frac{x^r}{L^r} - r \frac{x^r}{L^r} \right) dx = \frac{q_0}{L} \int_0^L \left(\frac{rx^r}{L^r} - r \frac{x^r}{L^r} \right) dx = \frac{q_0}{L} \left(\frac{rx^2}{2L^r} - \frac{rx^2}{2L^r} \right) \Big|_0^L$$

$$= \frac{q_0}{L} \left(\frac{rL^r}{2} - \frac{rL^r}{2} \right) = \frac{q_0}{L} \left(\frac{rL}{2} - \frac{rL}{2} \right) = \frac{rL}{r_0} q$$

$$M_r = \int_0^L \frac{q_0 x}{L} N_\varepsilon(x) dx, N_\varepsilon(x) = \frac{x^r}{L^r} - \frac{x^r}{L}$$

$$M_r = \int_0^L \frac{q_0 x}{L} \left(\frac{x^r}{L^r} - \frac{x^r}{L} \right) dx = \frac{q_0}{L} \int_0^L \left(\frac{x^r}{L^r} - \frac{x^r}{L} \right) dx = \frac{q_0}{L} \left(\frac{x^{\Delta}}{\Delta L^r} - \frac{x^{\varepsilon}}{2L} \right) \Big|_0^L$$

$$= \frac{q_0}{L} \left(\frac{L^r}{\Delta} - \frac{L^r}{2} \right) = \frac{q_0}{L} \left(-\frac{L^r}{r_0} \right) = -\frac{qL^r}{r_0}$$

$$\frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} v_1=0 \\ \theta_1=0 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_1 = \frac{3}{80} q_0 L \\ M_1 = \frac{1}{40} q_0 L^2 \\ F_2 = \frac{V}{80} q_0 L \\ M_2 = -\frac{1}{40} q_0 L^2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}^{-1} \begin{bmatrix} F_2 \\ M_2 \end{bmatrix}$$

برای وقتی که اعداد معلوم باشند بهتر است
با سنج حساب ماتریسی حساب کنیم.

اما متوجه این که در اینجا داریم باید معادلات را بنویسیم:

$$\frac{EI}{L^3} (12v_2 - 6L\theta_2) = \frac{V}{80} q_0 L \quad \textcircled{1} \Rightarrow 12v_2 - 6L\theta_2 = \frac{V q_0 L^2}{80EI} \Rightarrow v_2 = \frac{1}{12} \left(\frac{V q_0 L^2}{80EI} + 6L\theta_2 \right) \quad \textcircled{1}$$

$$\frac{EI}{L^3} (-6Lv_2 + 4L^2\theta_2) = -\frac{1}{40} q_0 L^2 \quad \textcircled{2}$$

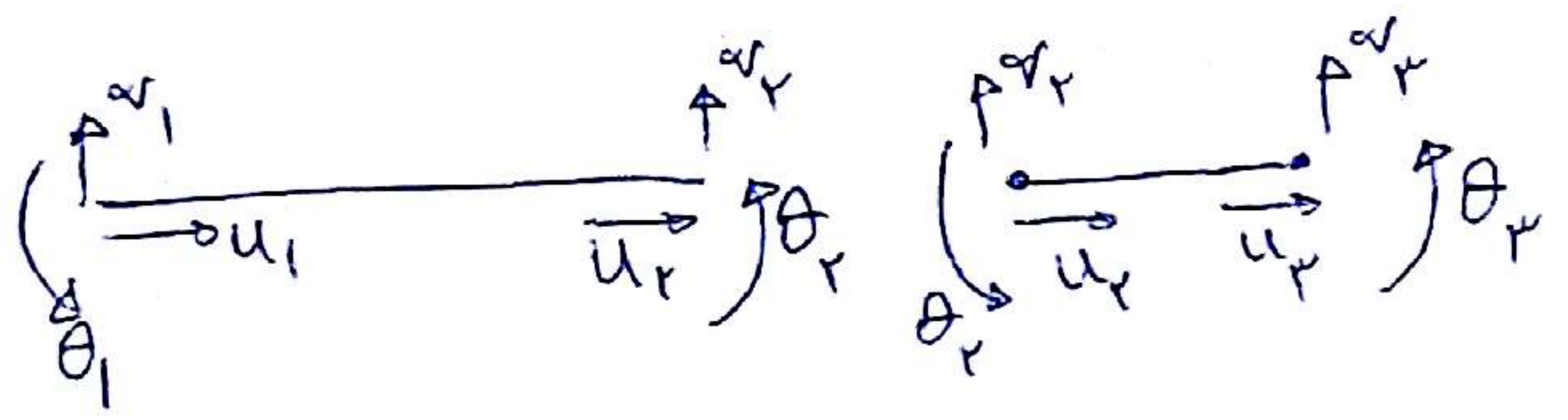
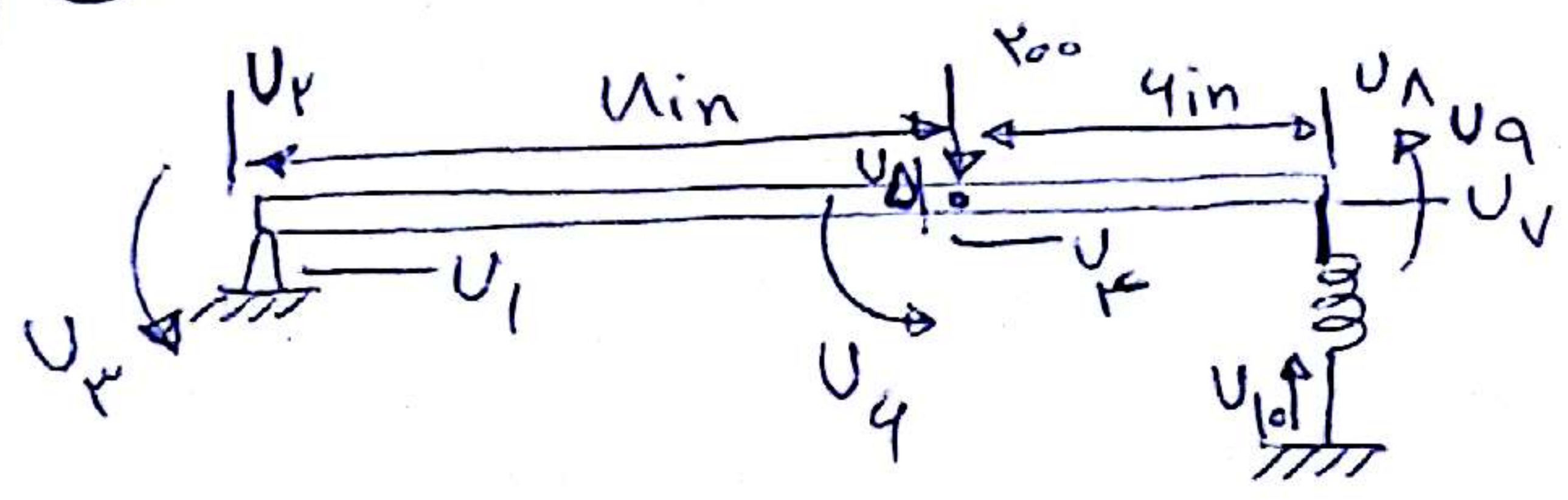
$$\textcircled{1}, \textcircled{2} \Rightarrow \frac{EI}{L^3} \left(-6L \left(\frac{1}{12} \left(\frac{V q_0 L^2}{80EI} + 6L\theta_2 \right) \right) + 4L^2\theta_2 \right) = -\frac{1}{40} q_0 L^2$$

$$\Rightarrow \frac{EI}{L^3} \left(\left(-\frac{V q_0 L^2}{80EI} - 3L^2\theta_2 \right) + 4L^2\theta_2 \right) = -\frac{1}{40} q_0 L^2 \Rightarrow -\frac{V q_0 L^2}{80} - \frac{3EI\theta_2}{L} + \frac{4EI\theta_2}{L} = -\frac{1}{40} q_0 L^2$$

$$\Rightarrow \theta_2 \left(\frac{EI}{L} \right) = -\frac{1}{40} q_0 L^2 + \frac{V}{80} q_0 L^2 = \frac{1}{40} q_0 L^2 \Rightarrow \theta_2 = \frac{q_0 L^2}{40EI}$$

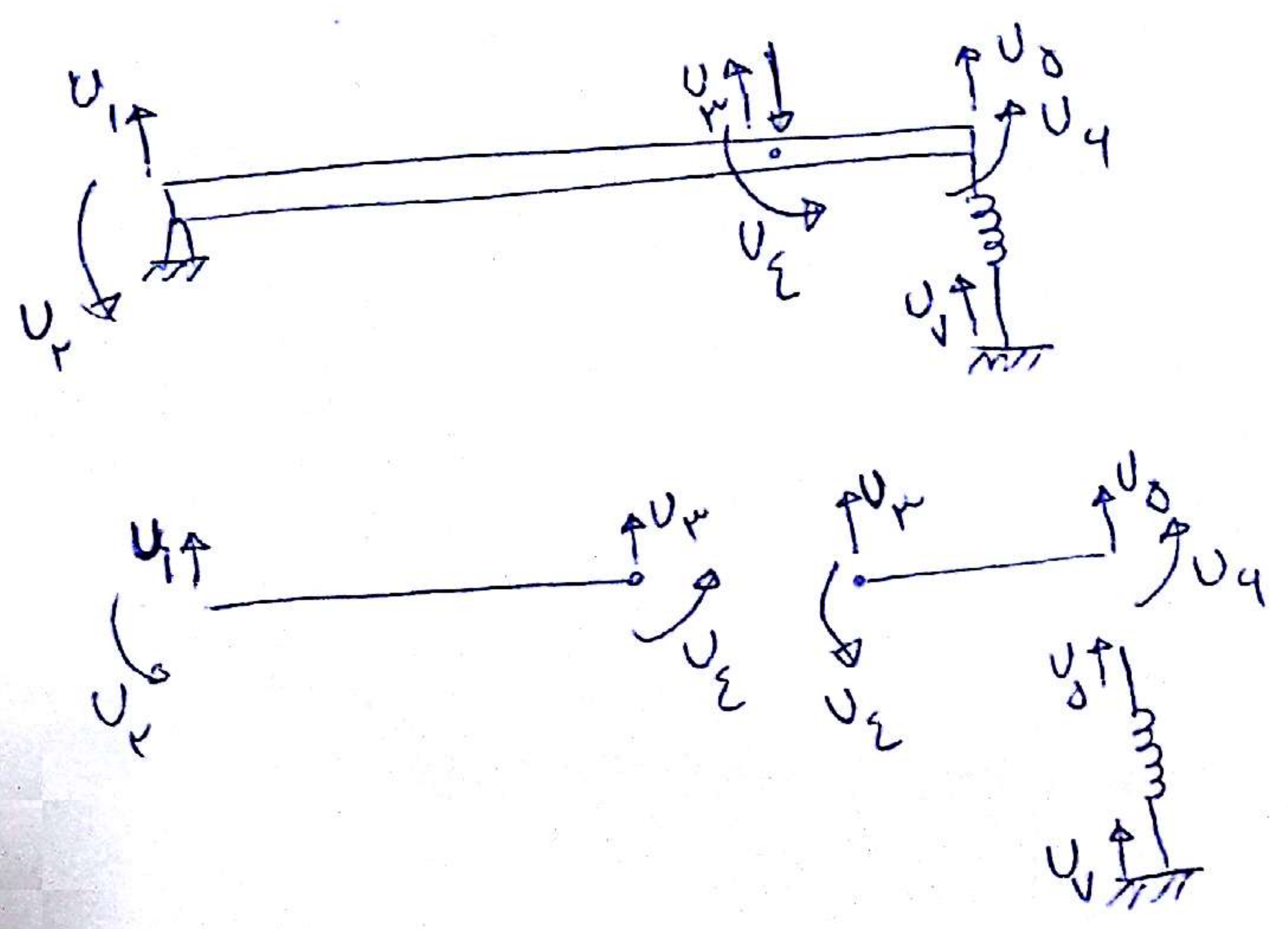
$$\textcircled{1} \Rightarrow v_2 = \frac{1}{12} \left(\frac{V q_0 L^2}{80EI} + 6L \left(\frac{q_0 L^2}{40EI} \right) \right) = \frac{1}{12} \left(\left(\frac{V}{80} + \frac{3}{20} \right) \frac{q_0 L^2}{EI} \right) = \frac{11}{120} \left(\frac{q_0 L^2}{EI} \right)$$

(۲)



شماره	1	2	3	4	5	6	7	8	9	10
1	1	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	0	1

توجه کنید که چون سرعت، انحراف و تغییر شکل برای این دو عضو از ایستگاه به صورت راست در نظر گرفته می شود.



شماره	1	2	3	4	5	6	7
1	1	0	0	0	0	0	0
2	0	1	0	0	0	0	0
3	0	0	1	0	0	0	0
4	0	0	0	1	0	0	0
5	0	0	0	0	1	0	0
6	0	0	0	0	0	1	0
7	0	0	0	0	0	0	1

(12)

$$K_1 = EI \begin{bmatrix} \frac{12}{\delta 12} & \frac{\Sigma \Delta}{\delta 12} & \frac{-12}{\delta 12} & \frac{\Sigma \Delta}{\delta 12} \\ \frac{\Sigma \Delta}{\delta 12} & \frac{284}{\delta 12} & \frac{-\Sigma \Delta}{\delta 12} & \frac{121}{\delta 12} \\ \frac{-12}{\delta 12} & \frac{-\Sigma \Delta}{\delta 12} & \frac{12}{\delta 12} & \frac{-\Sigma \Delta}{\delta 12} \\ \frac{\Sigma \Delta}{\delta 12} & \frac{121}{\delta 12} & \frac{-\Sigma \Delta}{\delta 12} & \frac{284}{\delta 12} \end{bmatrix}$$

$$K_2 = EI \begin{bmatrix} \frac{12}{214} & \frac{44}{214} & \frac{-12}{214} & \frac{44}{214} \\ \frac{44}{214} & \frac{122}{214} & \frac{-44}{214} & \frac{44}{214} \\ \frac{-12}{214} & \frac{-44}{214} & \frac{12}{214} & \frac{-44}{214} \\ \frac{44}{214} & \frac{44}{214} & \frac{-44}{214} & \frac{122}{214} \end{bmatrix}$$

$$K_3 = \begin{bmatrix} \frac{K}{EA} & \frac{-K}{EA} \\ \frac{-K}{EA} & \frac{K}{EA} \end{bmatrix}$$

$$K_{11} = K_{11}^{(1)}$$

$$K_{12} = K_{12}^{(1)}$$

$$K_{13} = K_{13}^{(1)}$$

$$K_{14} = K_{14}^{(1)}$$

$$K_{15} = 0$$

$$K_{16} = 0$$

$$K_{17} = 0$$

$$K_{22} = K_{22}^{(1)}$$

$$K_{23} = K_{23}^{(1)}$$

$$K_{24} = K_{24}^{(1)}$$

$$K_{25} = 0$$

$$K_{26} = 0$$

$$K_{27} = 0$$

$$K_{33} = K_{33}^{(1)} + K_{33}^{(2)}$$

$$K_{34} = K_{34}^{(1)} + K_{34}^{(2)}$$

$$K_{35} = K_{35}^{(2)}$$

$$K_{36} = K_{36}^{(2)}$$

$$K_{37} = 0$$

$$K_{44} = K_{44}^{(1)} + K_{44}^{(2)}$$

$$K_{45} = K_{45}^{(2)}$$

$$K_{46} = K_{46}^{(2)}$$

$$K_{47} = 0$$

$$K_{55} = K_{55}^{(2)} + K_{55}^{(3)}$$

$$K_{56} = K_{56}^{(2)}$$

$$K_{57} = K_{57}^{(2)}$$

$$K_{66} = K_{66}^{(2)}$$

$$K_{67} = 0$$

$$K_{77} = K_{77}^{(3)}$$

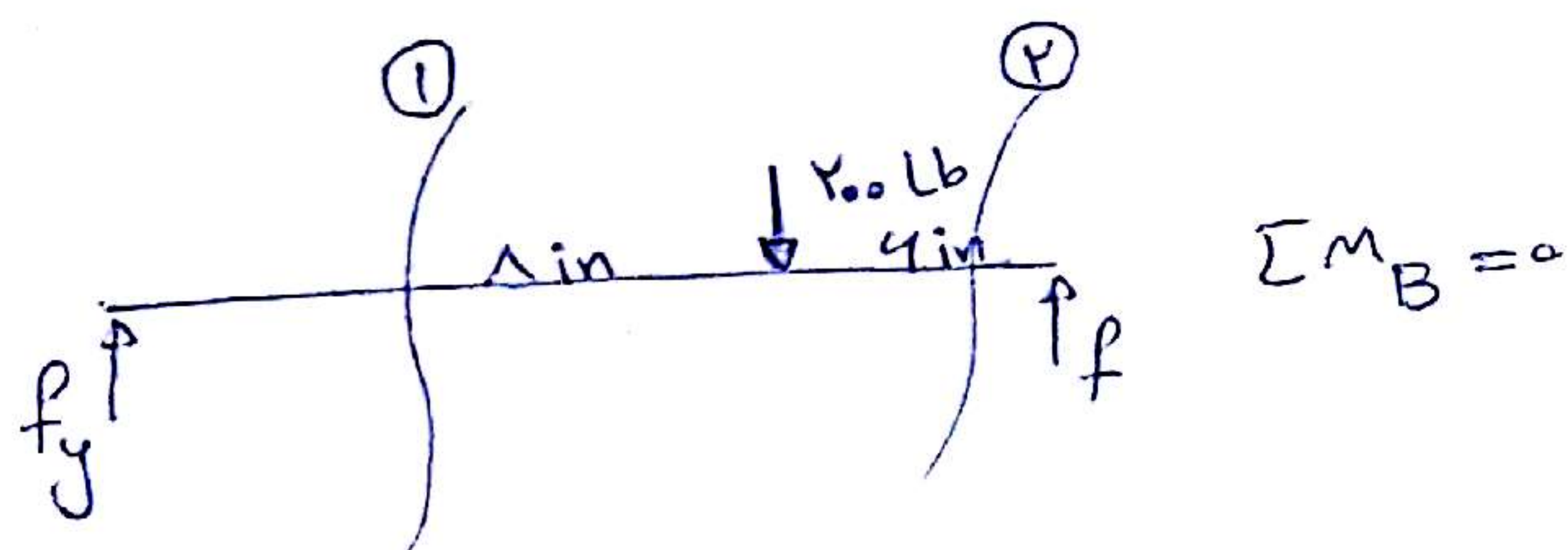
②

EI

$\frac{12}{\delta 12}$	$\frac{24}{\delta 12}$	$\frac{-12}{\delta 12}$	$\frac{24}{\delta 12}$	0	0	0
$\frac{24}{\delta 12}$	$\frac{48}{\delta 12}$	$\frac{-24}{\delta 12}$	$\frac{48}{\delta 12}$	0	0	0
$\frac{-12}{\delta 12}$	$\frac{-24}{\delta 12}$	$\frac{12}{\delta 12} + \frac{12}{214}$	$\frac{-24}{\delta 12} + \frac{48}{214}$	$\frac{-12}{214}$	$\frac{48}{214}$	0
$\frac{24}{\delta 12}$	$\frac{48}{\delta 12}$	$\frac{-24}{\delta 12} + \frac{48}{214}$	$\frac{48}{\delta 12} + \frac{96}{214}$	$\frac{-48}{214}$	$\frac{96}{214}$	0
0	0	$\frac{-12}{214}$	$\frac{-48}{214}$	$\frac{12}{214} + \frac{K}{EA}$	$\frac{-48}{214}$	$-\frac{K}{EA}$
0	0	$\frac{48}{214}$	$\frac{96}{214}$	$\frac{-48}{214}$	$\frac{96}{214}$	0
0	0	0	0	$-\frac{K}{EA}$	0	$\frac{K}{EA}$

$\begin{bmatrix} \Delta_1 = 0 \\ \theta_1 \\ \Delta_2 \\ \theta_2 \\ \Delta_3 \\ \theta_3 \\ U = 0 \end{bmatrix} = \begin{bmatrix} F_1 = 180 \\ M_1 = 0 \\ F_2 = -200 \\ M_2 = 1200 \\ F_3 = 0 \\ M_3 = 900 \\ F \end{bmatrix}$

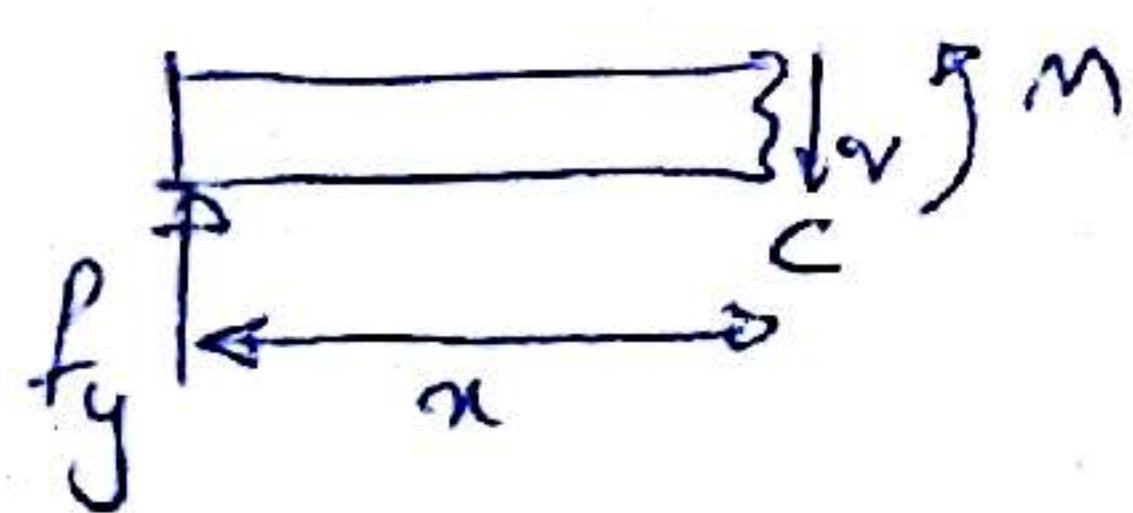
$\left(\begin{matrix} \text{مكتوبه من} \\ \text{مكتوبه من} \end{matrix} \right)$



$$\sum M_B = 0 \Rightarrow 200 \times 7 = f_y \times 1 \Rightarrow f_y = 140 \text{ lb}$$

①

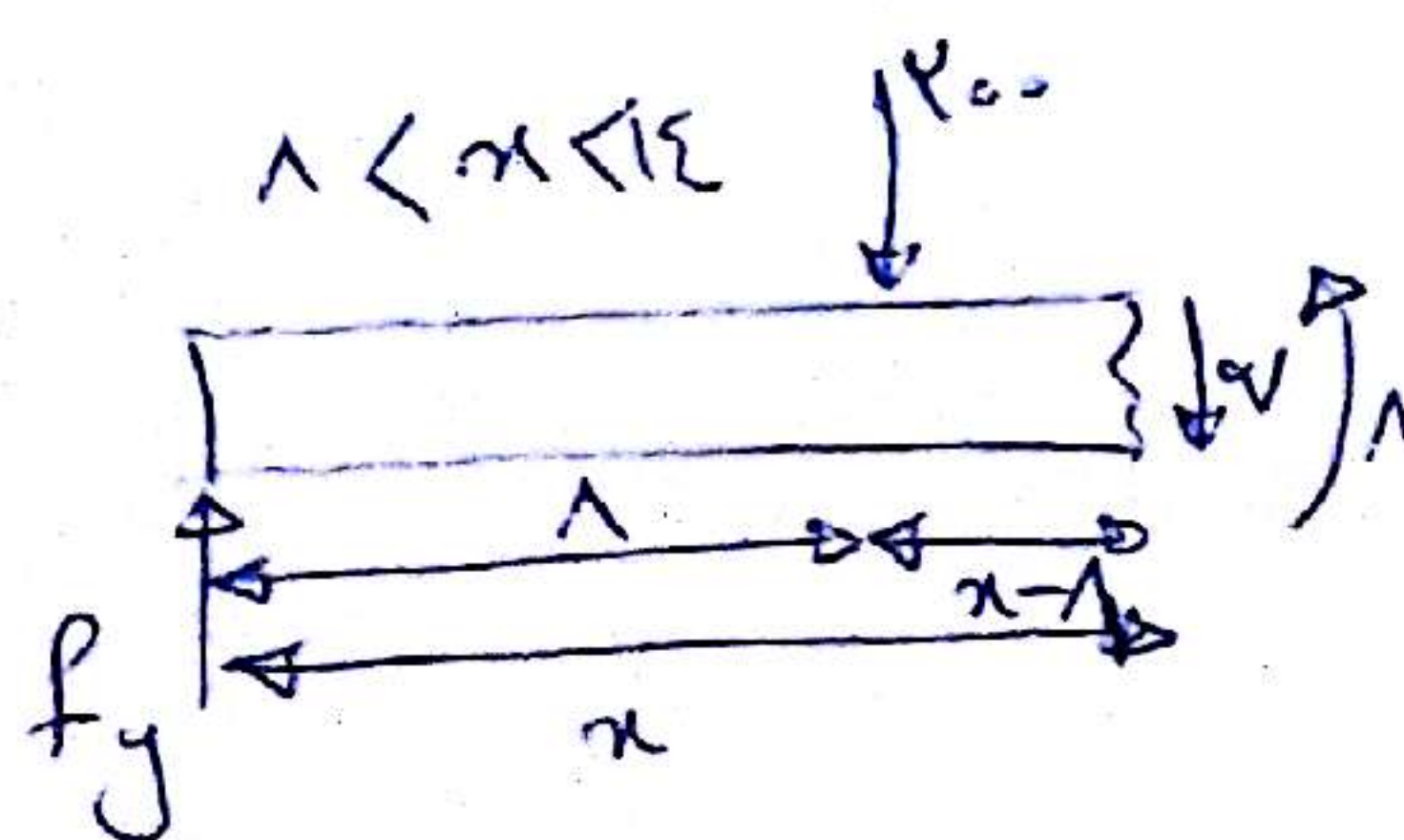
$$0 < x < 1$$



$$\sum M = 0 \Rightarrow M = f_y x = 140x$$

$$M @ x=1 = 140 \times 1 = 140 \text{ lb.in}$$

②



$$\sum M = 0 \Rightarrow M = 140x - 200(x-1) = -60x + 140$$

$$M @ x=1 = 140 \text{ lb.in} \checkmark$$

$$M @ x=12 = -60(12) + 140 = 900 \text{ lb.in}$$

② $[EI[K]]^{-1} [F, M] = [\psi, \theta]$

$$\begin{bmatrix} 0.00012 & 0.00092 & 0.00012 & 0.00142 & 0.00011 \\ 0.00092 & 0.00728 & 0.00092 & 0.01299 & 0.00092 \\ 0.00012 & 0.00092 & 0.00012 & 0.00142 & 0.00012 \\ 0.00142 & 0.01299 & 0.00142 & 0.02264 & 0.00142 \\ 0.00011 & 0.00092 & 0.00012 & 0.00142 & 0.00012 \end{bmatrix} \begin{bmatrix} 0 \\ -100 \\ 100 \\ 0 \\ 900 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \psi_1 \\ \theta_2 \\ \psi_2 \\ \theta_3 \end{bmatrix}$$

$$\theta_1 = 0.000209$$

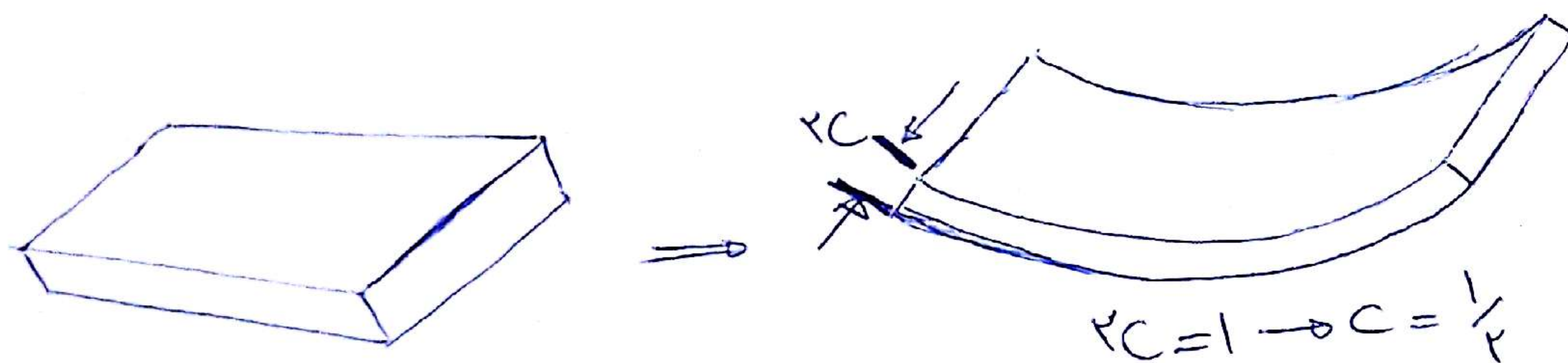
$$\psi_1 = 0.00111$$

$$\theta_2 = 0.000912$$

$$\psi_2 = 0.001149$$

$$\theta_3 = 0.000419$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{M_{\max} = 100}{1} \frac{100 \times \frac{1}{4}}{\frac{1}{4}} = 4 \times 100 = 400 \text{ lb/in}$$



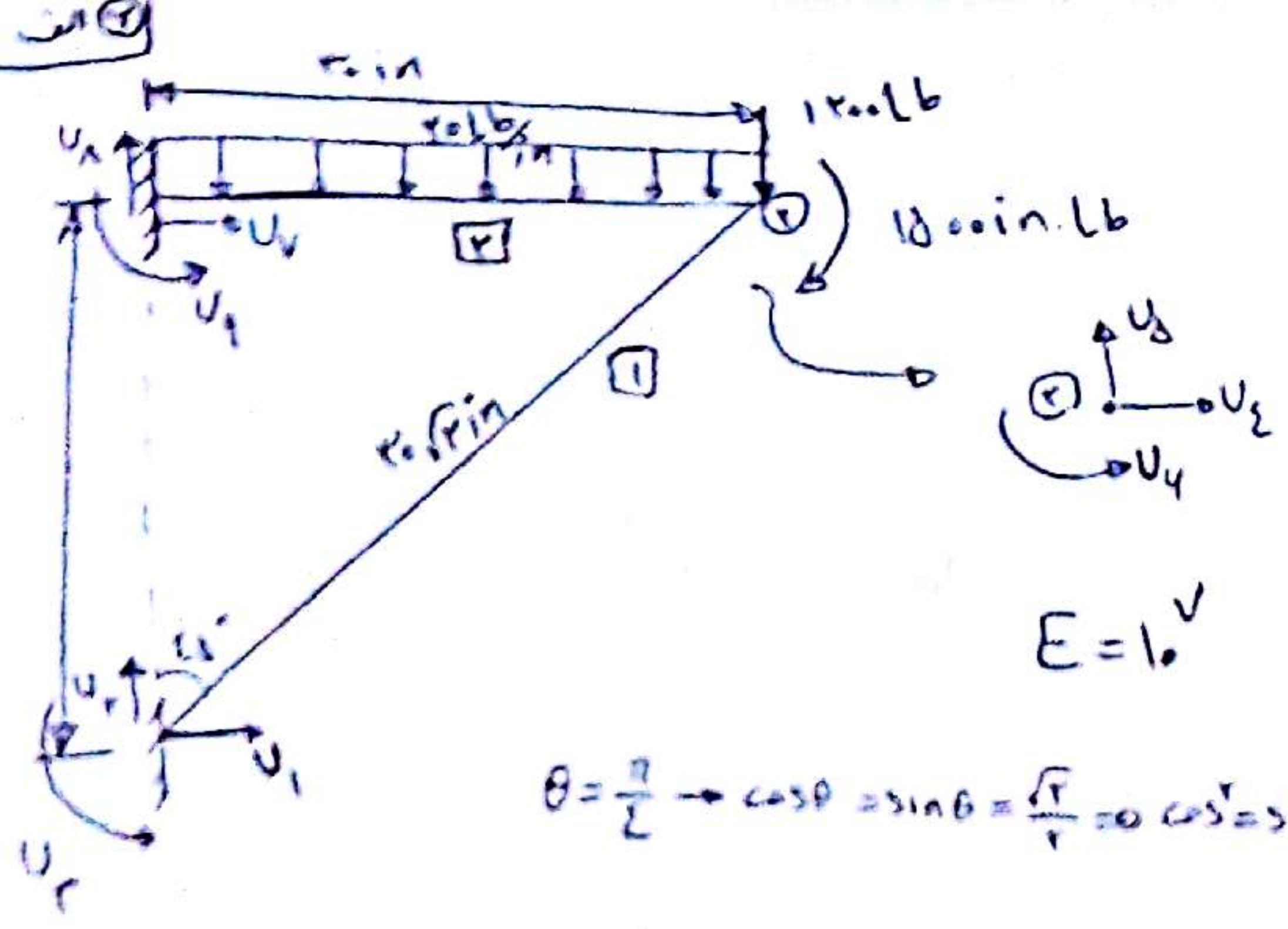
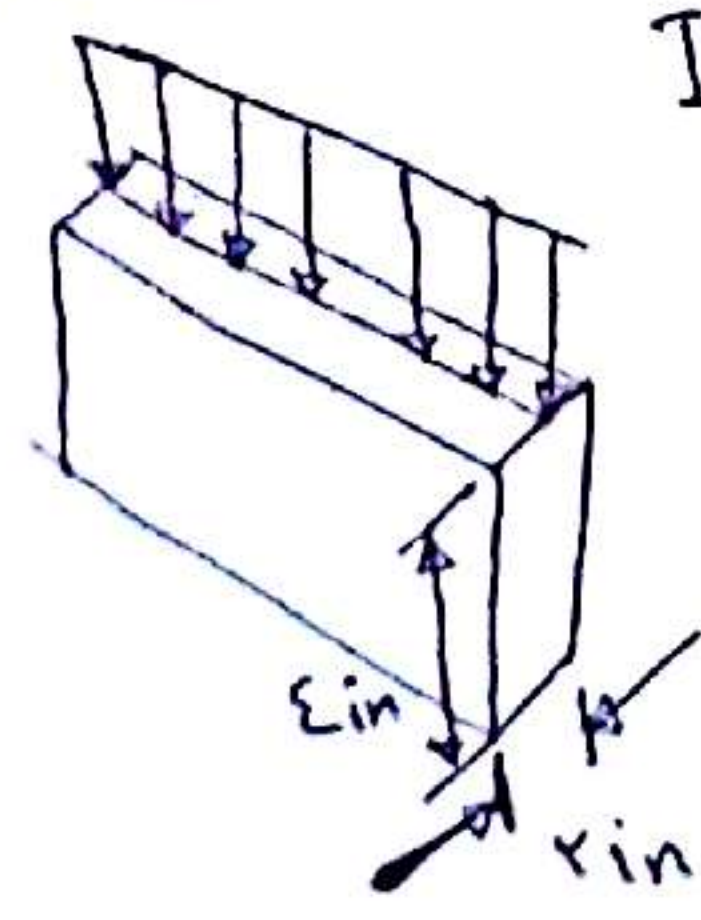
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (r) (1)^3 = \frac{1}{12}$$

$$b = r, h = 1$$

علی رقم فرضیات سؤال چون لان II یک عضو نیروی است
 بنابراین می توان این عضو را به صورت یک خرابه مدل نمود.
 (یکبار در عضو را به صورت خرابه و تیر و یکبار به صورت خرابه و تیر حل کردیم)

$$I = \frac{1}{12} (2)(4)^3 = 10.67 \text{ in}^4$$

$$A = 8 \text{ in}^2$$



$$\theta = \frac{\pi}{2} \rightarrow \cos \theta = \sin \theta = \frac{\sqrt{2}}{2} \Rightarrow \cos^2 = \sin^2 = \sin \cos = \frac{1}{2}$$

چون در لان تیر نیروی عمودی نه ابع می توان برای حرکت از تیر از دو درجه آزادی استفاده کرد:

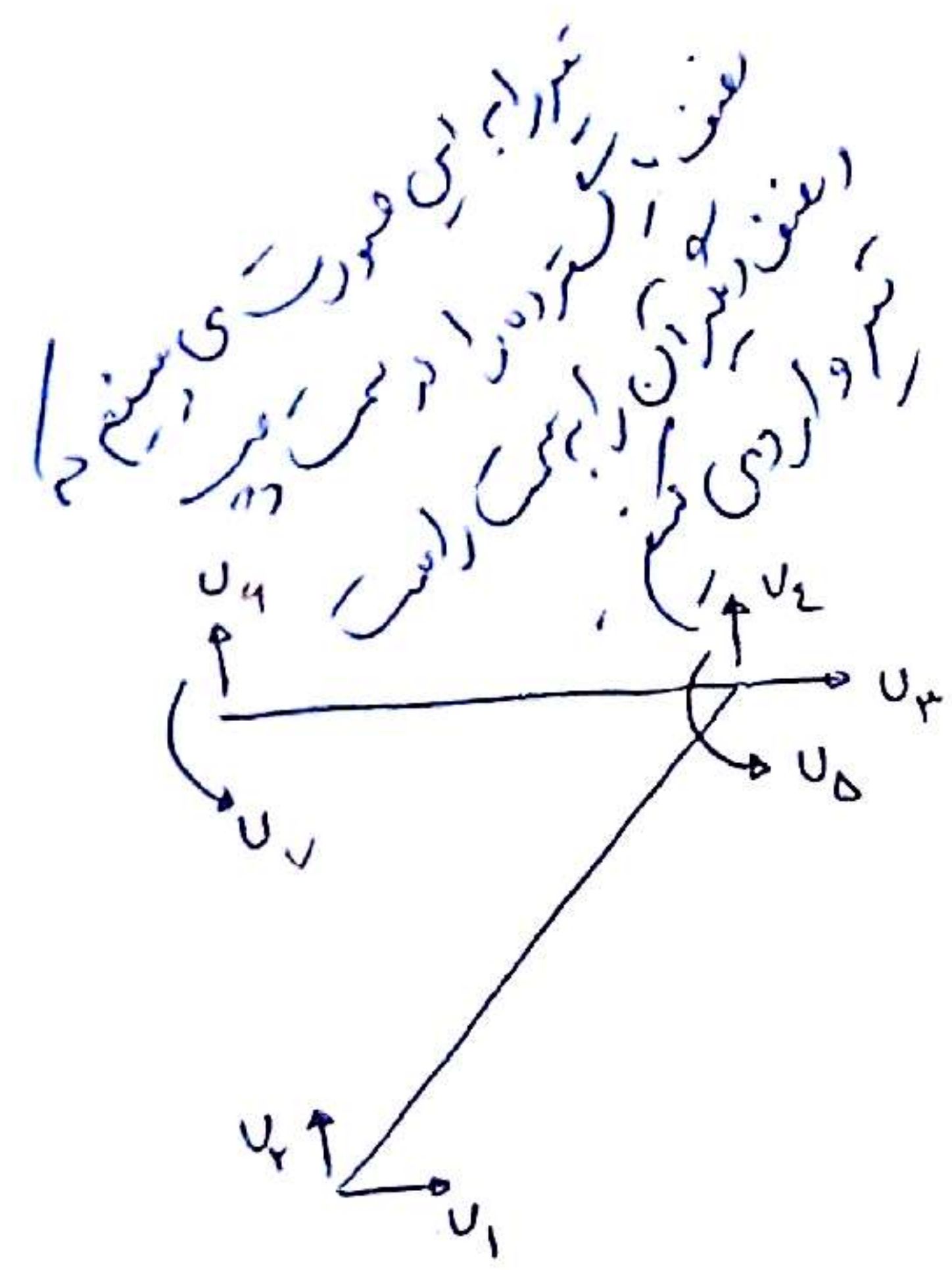
$$K_{II} = \frac{EI}{L^3} \begin{bmatrix} 12 & 4L & -12 & 4L \\ 4L & 4L^2 & -4L & 2L^2 \\ -12 & -4L & 12 & -4L \\ 4L & 2L^2 & -4L & 4L^2 \end{bmatrix}$$

$$K_{II} = \frac{AE}{L} \begin{bmatrix} \cos^2 & \sin \cos & -\cos^2 & -\sin \cos \\ \sin \cos & \sin^2 & -\sin \cos & -\sin^2 \\ -\cos^2 & -\sin \cos & \cos^2 & \sin \cos \\ -\sin \cos & -\sin^2 & \sin \cos & \sin^2 \end{bmatrix}$$

$$[K_{II}][u_{II}] = [F_{II}]$$

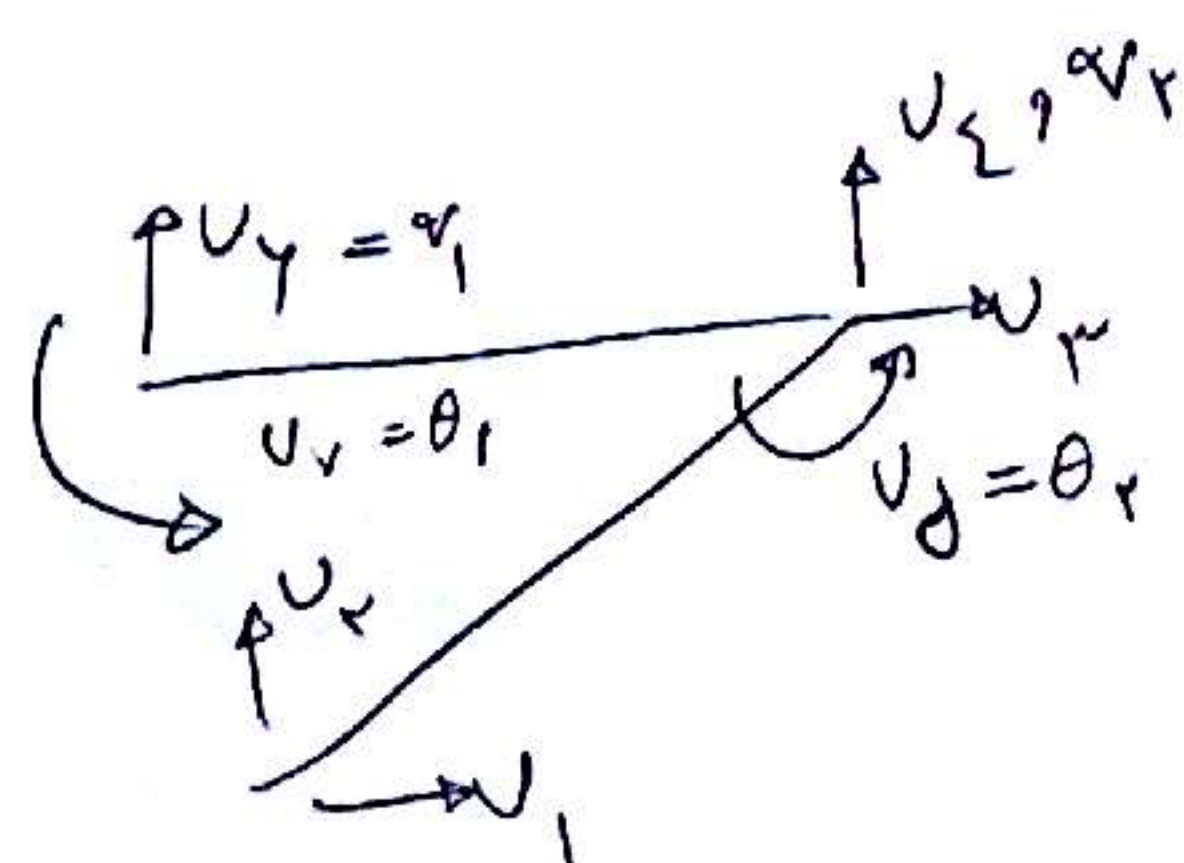
$$K_{II} = E \begin{bmatrix} -0.12 & 2.13 & -0.12 & 2.13 \\ 2.13 & 22.42 & -2.13 & 2.13 \\ -0.12 & -2.13 & 0.12 & -2.13 \\ 2.13 & 2.13 & -2.13 & 22.42 \end{bmatrix}$$

$$K_{II} = E \begin{bmatrix} +0.09 & +0.09 & -0.09 & -0.09 \\ +0.09 & +0.09 & -0.09 & -0.09 \\ -0.09 & -0.09 & +0.09 & +0.09 \\ -0.09 & -0.09 & +0.09 & +0.09 \end{bmatrix}$$



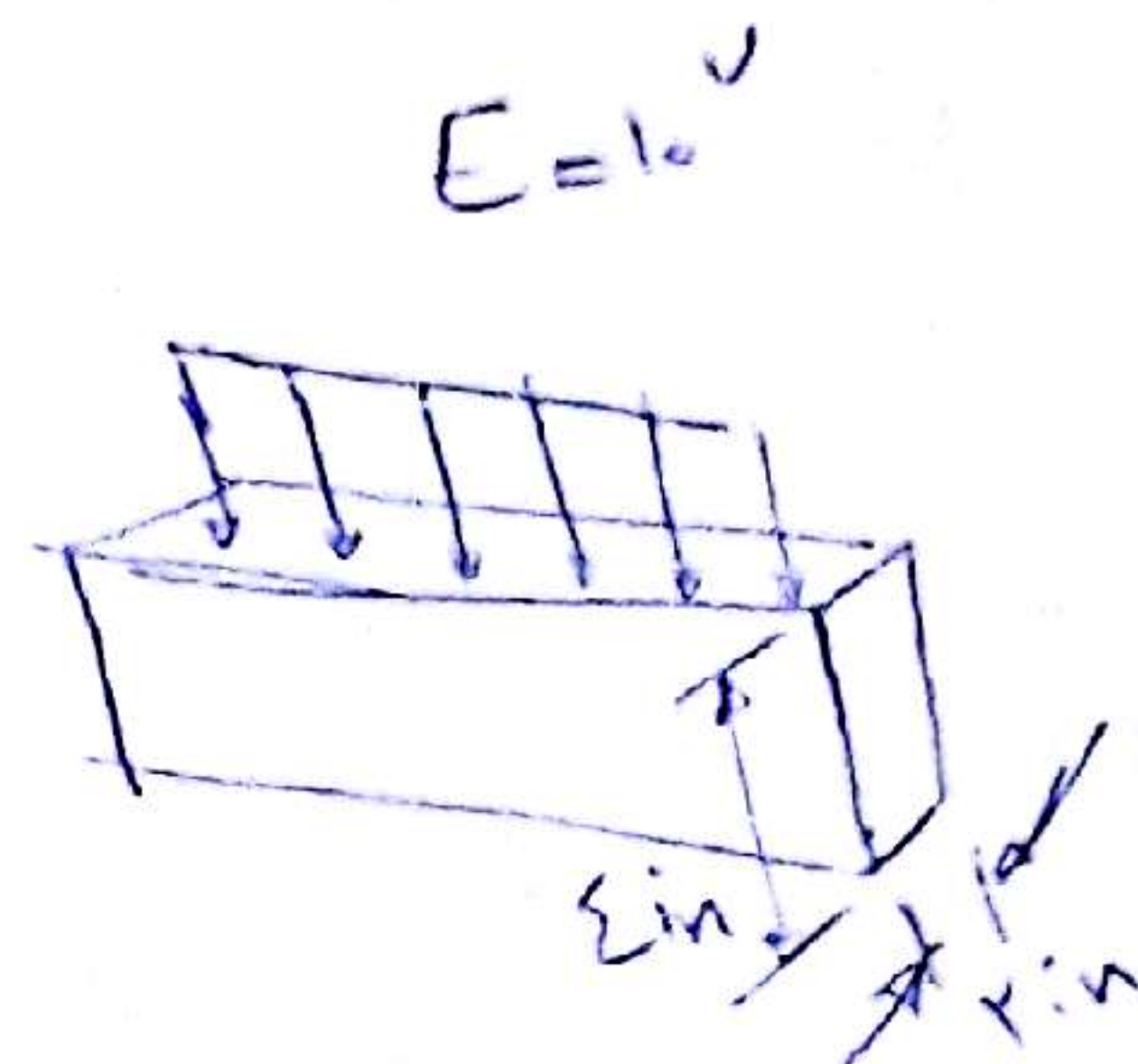
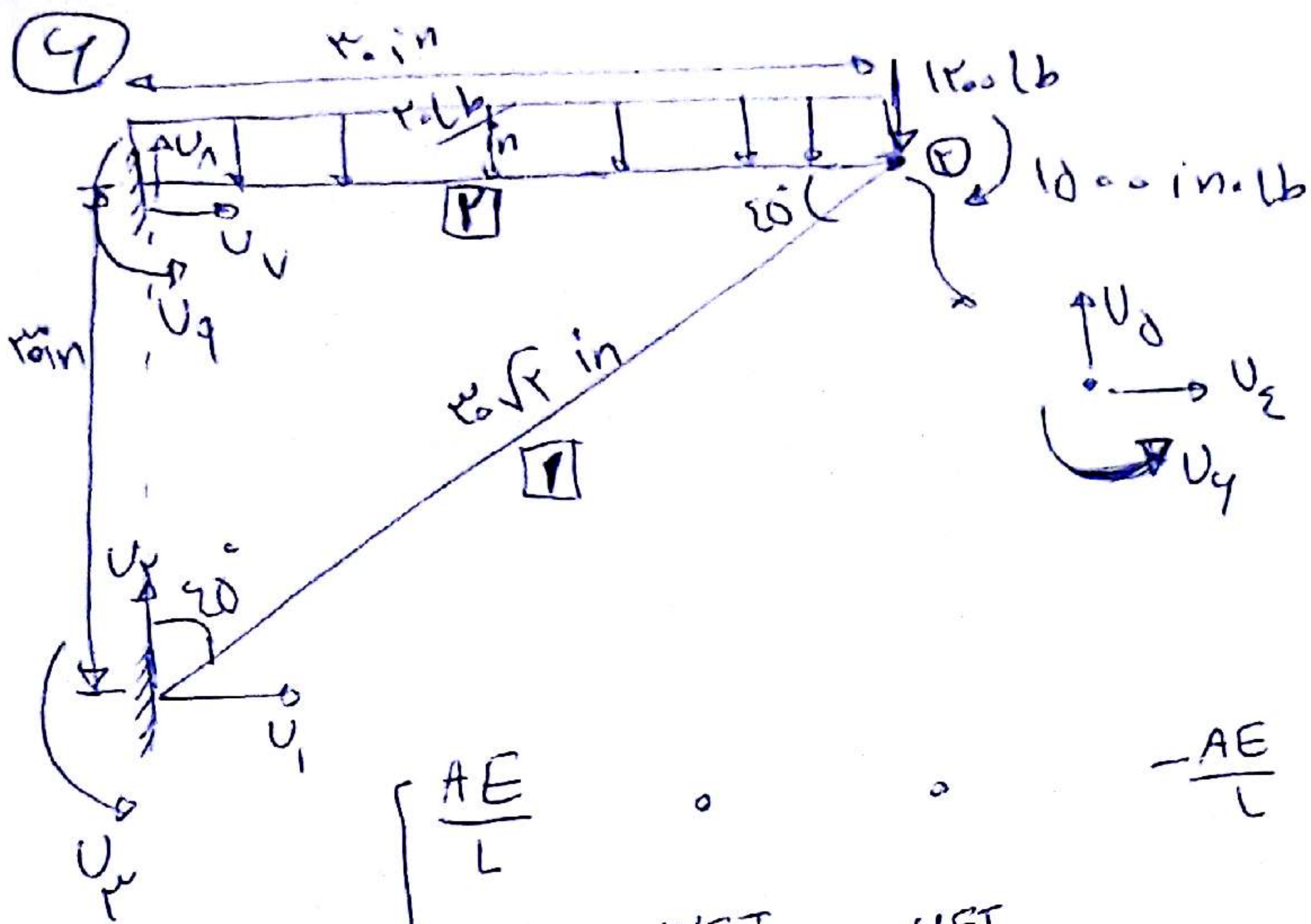
u2 برای تیر می شود u2
 u4 برای تیر می شود u4
 u1 ~ ~ ~ u1
 u3 ~ ~ ~ u3
 u5 ~ ~ ~ u5
 u6 ~ ~ ~ u6

$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$
0	1	1
0	2	2
0	3	3
1	2	2
2	0	0
3	0	0
2	0	0



Σ	Σ	Σ
1	1	1
2	2	2
3	3	3
4	4	4
5	5	5
6	6	6
7	7	7
8	8	8
9	9	9
10	10	10

$$\left[\begin{array}{l} U_Y = 1.4 \times 10^{-4} \\ v_Y = v_Z = -1.4 \times 10^{-4} \\ \theta_Y = 1.01 \times 10^{-2} \end{array} \right]$$



$E = 10^6$
 $I = \frac{1}{12} (b) (h)^3$
 $I = \frac{1}{12} (2) (1)^3 = 1/6 \text{ in}^4$
 $A = 2 \text{ in}^2$

$$K_e = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{AE}{L} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & -\frac{2EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \end{bmatrix}$$

Global stiffness matrix $[K] = [R]^T [K_e] [R]$

$\alpha = 20^\circ$

$$R = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

⑤

$$R^T = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 & 0 & 0 & 0 \\ \sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$K = [R]^T [K_e] [R]$$

ماتریس سفتی الاستیک

$$K_e = \begin{bmatrix} 9118 & 0 & 0 & -9118 & 0 & 0 \\ 0 & 1.47 \times 10^{-3} & 0.103 & 0 & -1.47 \times 10^{-3} & 0.103 \\ 0 & 0.103 & 1 & 0 & -0.103 & 0.18 \\ -9118 & 0 & 0 & 9118 & 0 & 0 \\ 0 & -1.47 \times 10^{-3} & -0.103 & 0 & 1.47 \times 10^{-3} & 0.103 \\ 0 & 0.103 & 0.18 & 0 & 0.103 & 1 \end{bmatrix}$$

$$\Rightarrow K = \begin{bmatrix} 901380 & 191980 & -212132.03 & -901380 & -191980 & -212132.03 \\ 191980 & 901380 & 212132.03 & -901380 & -191980 & 212132.03 \\ -212132.03 & 212132.03 & 100000000 & 212132.03 & -212132.03 & 100000000 \\ -901380 & -191980 & -212132.03 & 901380 & 191980 & 212132.03 \\ -191980 & -901380 & -212132.03 & 191980 & 901380 & 212132.03 \\ -212132.03 & 212132.03 & 100000000 & 212132.03 & -212132.03 & 100000000 \end{bmatrix}$$

(1)

$K_{(r)} = E$

$\frac{A}{L}$	0	0	$-\frac{A}{L}$	0	0
0	$\frac{12I}{L^3}$	$\frac{6I}{L^2}$	0	$-\frac{12I}{L^3}$	$\frac{6I}{L^2}$
0	$\frac{6I}{L^2}$	$\frac{4I}{L}$	0	$-\frac{6I}{L^2}$	$\frac{2I}{L}$
$-\frac{A}{L}$	0	0	$\frac{A}{L}$	0	0
0	$-\frac{12I}{L^3}$	$-\frac{6I}{L^2}$	0	$\frac{12I}{L^3}$	$-\frac{6I}{L^2}$
0	$\frac{6I}{L^2}$	$\frac{2I}{L}$	0	$-\frac{6I}{L^2}$	$\frac{4I}{L}$

$$I = 10,74$$

$$A = 1$$

$$L = 2$$

نوعی الی

$K_{(r)} = E$

0,124	0	0	-0,124	0	0
0	$\sum 1,73 \times 10^{-3}$	0,10V	0	$-\sum 1,73 \times 10^{-3}$	0,10V
0	0,10V	1,22	0	-0,10V	0,10V
-0,124	0	0	0,124	0	0
0	$-\sum 1,73 \times 10^{-3}$	-0,10V	0	$\sum 1,73 \times 10^{-3}$	0,10V
0	0,10V	0,10V	0	0,10V	1,22

$$\begin{aligned} K_{11} &= K_{11}^{(1)} \\ K_{12} &= K_{12}^{(1)} \\ K_{13} &= K_{13}^{(1)} \\ K_{14} &= K_{14}^{(1)} \\ K_{15} &= K_{15}^{(1)} \\ K_{16} &= K_{16}^{(1)} \\ K_{17} &= K_{17}^{(1)} \\ K_{18} &= K_{18}^{(1)} \\ K_{19} &= K_{19}^{(1)} \\ K_{10} &= 0 \end{aligned}$$

$$\begin{aligned} K_{22} &= K_{22}^{(1)} \\ K_{23} &= K_{23}^{(1)} \\ K_{24} &= K_{24}^{(1)} \\ K_{25} &= K_{25}^{(1)} \\ K_{26} &= K_{26}^{(1)} \\ K_{27} &= K_{27}^{(1)} \\ K_{28} &= K_{28}^{(1)} \\ K_{29} &= K_{29}^{(1)} \\ K_{20} &= 0 \end{aligned}$$

$$\begin{aligned} K_{33} &= K_{33}^{(1)} \\ K_{34} &= K_{34}^{(1)} \\ K_{35} &= K_{35}^{(1)} \\ K_{36} &= K_{36}^{(1)} \\ K_{37} &= K_{37}^{(1)} \\ K_{38} &= K_{38}^{(1)} \\ K_{39} &= K_{39}^{(1)} \\ K_{30} &= 0 \end{aligned}$$

$$\begin{aligned} K_{44} &= K_{44}^{(1)} + K_{44}^{(r)} \\ K_{45} &= K_{45}^{(1)} + K_{45}^{(r)} \\ K_{46} &= K_{46}^{(r)} \\ K_{47} &= K_{47}^{(r)} \\ K_{48} &= K_{48}^{(r)} \\ K_{49} &= K_{49}^{(r)} \\ K_{40} &= 0 \end{aligned}$$

$$\begin{aligned} K_{55} &= K_{55}^{(1)} + K_{55}^{(r)} \\ K_{56} &= K_{56}^{(r)} \\ K_{57} &= K_{57}^{(r)} \\ K_{58} &= K_{58}^{(r)} \\ K_{59} &= K_{59}^{(r)} \\ K_{50} &= 0 \end{aligned}$$

$$\begin{aligned} K_{66} &= K_{66}^{(1)} + K_{66}^{(r)} \\ K_{67} &= K_{67}^{(r)} \\ K_{68} &= K_{68}^{(r)} \\ K_{69} &= K_{69}^{(r)} \\ K_{60} &= 0 \end{aligned}$$

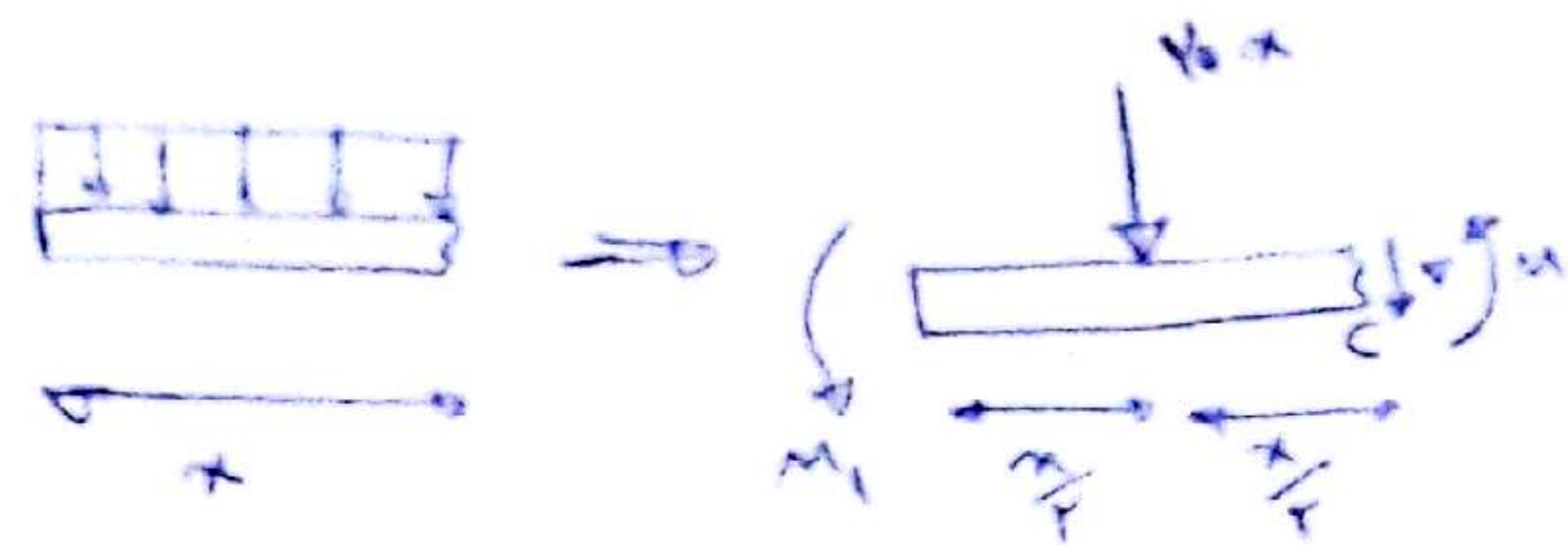
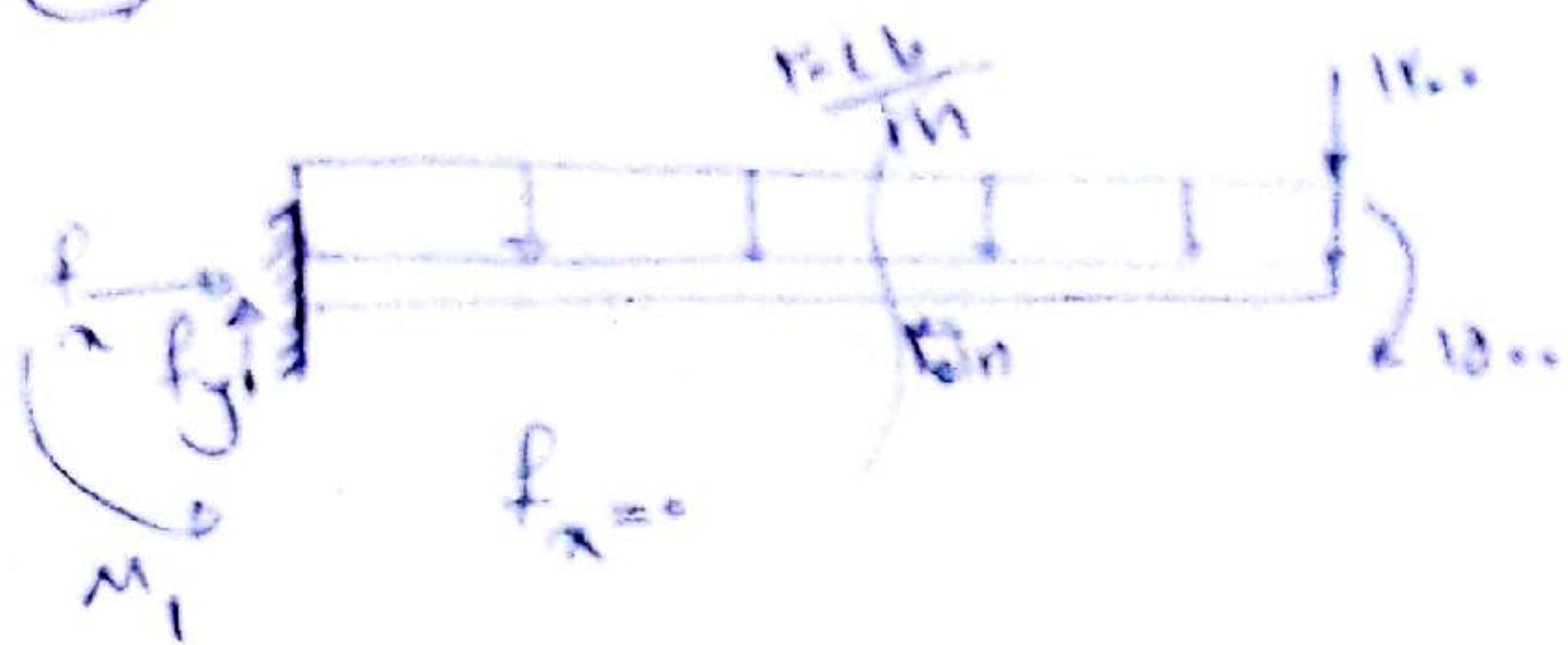
$$\begin{aligned} K_{77} &= K_{77}^{(1)} + K_{77}^{(r)} \\ K_{78} &= K_{78}^{(r)} \\ K_{79} &= K_{79}^{(r)} \\ K_{70} &= 0 \end{aligned}$$

$$\begin{aligned} K_{88} &= K_{88}^{(1)} + K_{88}^{(r)} \\ K_{89} &= K_{89}^{(r)} \\ K_{80} &= 0 \end{aligned}$$

10002	10002	10002
0	1	1
0	2	2
0	3	3
1	2	2
2	0	0
3	0	0
2	0	0
0	0	0
0	0	0

$$\begin{aligned} K_{99} &= K_{99}^{(1)} + K_{99}^{(r)} \\ K_{90} &= K_{90}^{(r)} \\ K_{91} &= K_{91}^{(r)} \\ K_{92} &= K_{92}^{(r)} \\ K_{93} &= K_{93}^{(r)} \\ K_{94} &= K_{94}^{(r)} \\ K_{95} &= K_{95}^{(r)} \\ K_{96} &= K_{96}^{(r)} \\ K_{97} &= K_{97}^{(r)} \\ K_{98} &= K_{98}^{(r)} \\ K_{99} &= K_{99}^{(r)} \end{aligned}$$

9

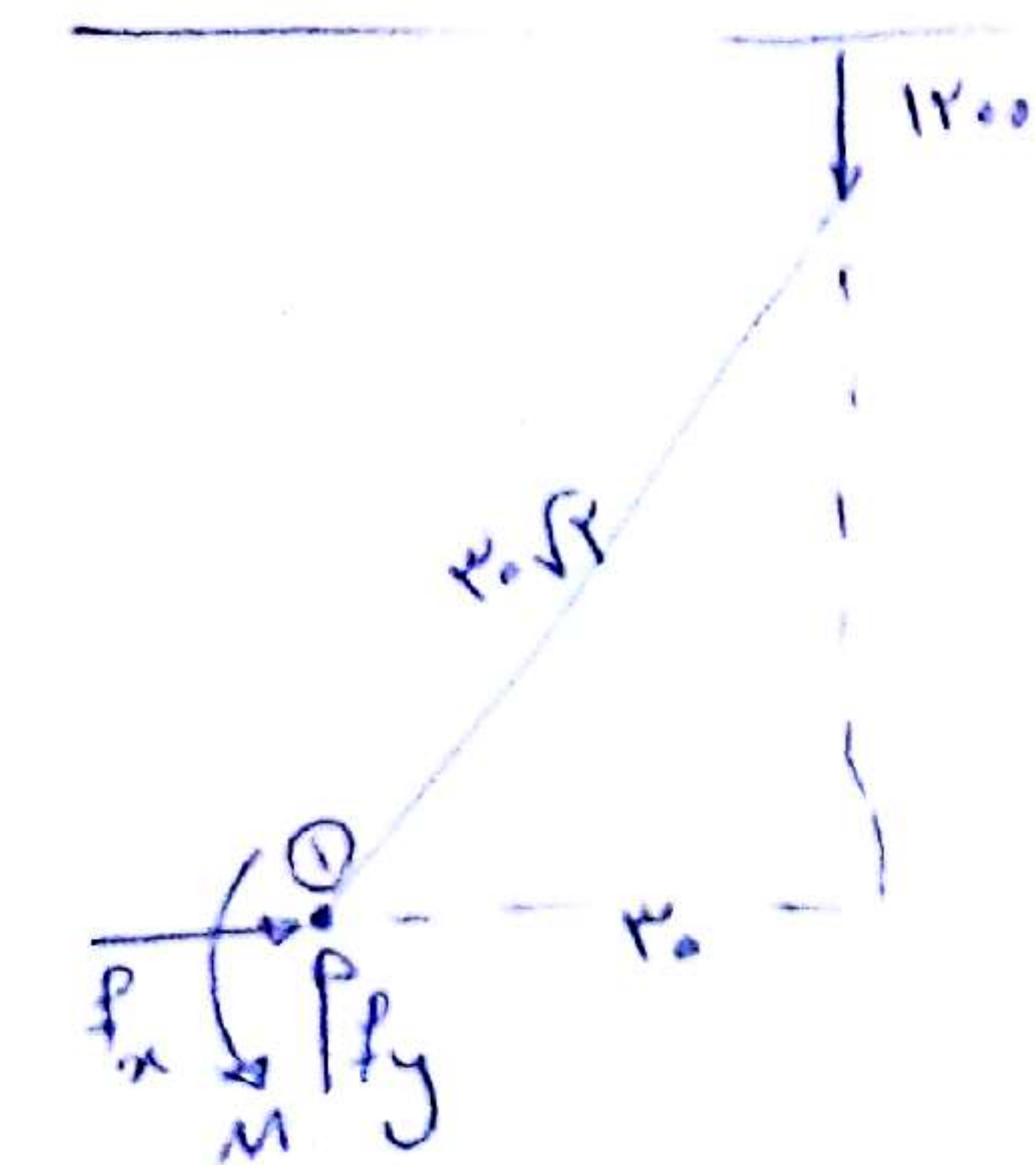


$$0 < x < 4 \quad \sum M_C = 0 \Rightarrow M = -10x\left(\frac{x}{2}\right) = -5x^2$$

$$\sum M_A = 0 \Rightarrow M_P = (10 \times 4)(4) + 12(10) + 12(4) = 248$$

$$\sum F_y = 0 \Rightarrow f_y - (10 \times 4) - 12 = 0 \Rightarrow f_y = 12 + 40 = 52$$

$$\sum F_x = 0 \Rightarrow f_x = 0$$



$$\sum F_x = 0 \Rightarrow f_x = 0$$

$$\sum F_y = 0 \Rightarrow f_y = 12$$

$$\sum M_D = 0 \Rightarrow M_1 = 12(4) = 48$$

↓ [K]

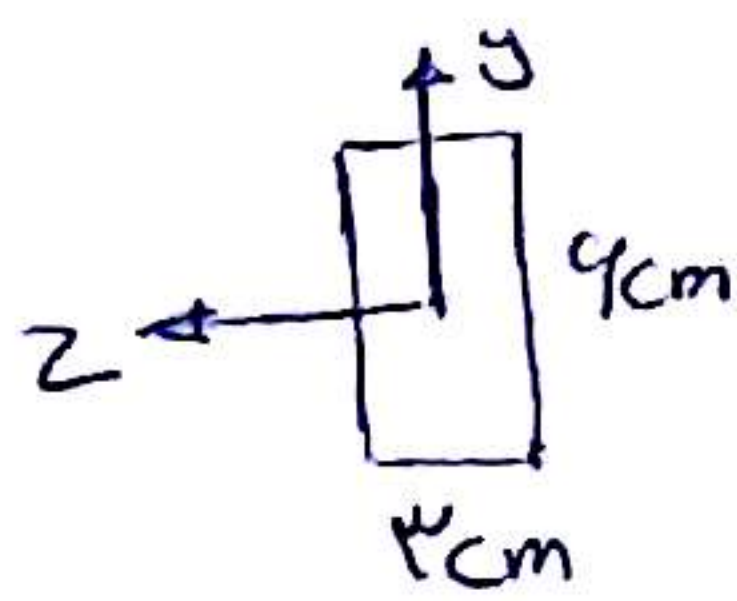
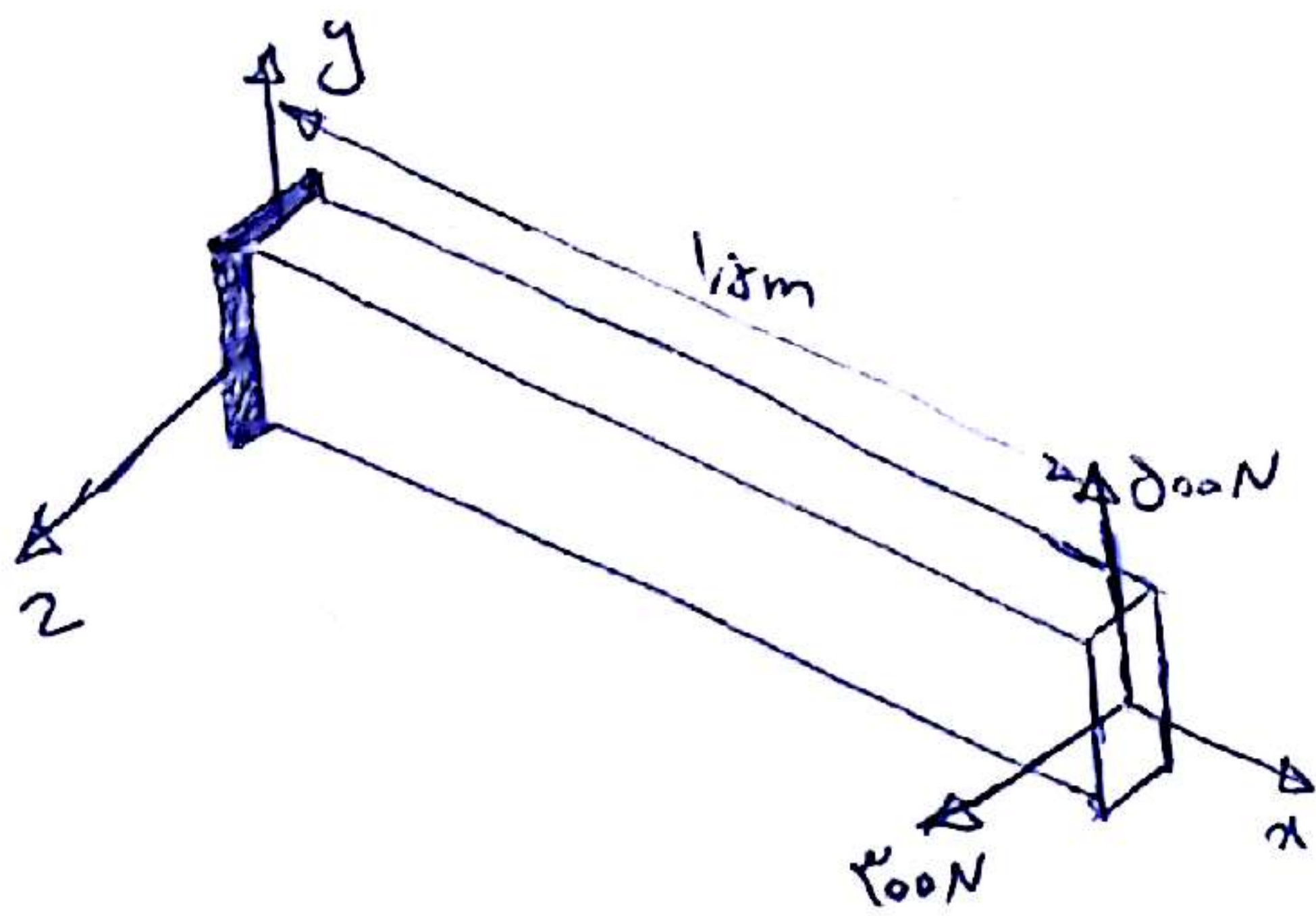
E *

[illegible]

$$E[k] \begin{bmatrix} u_1 = 0 \\ v_1 = 0 \\ \theta_1 \\ u_r \\ v_r \\ \theta_r \\ u_\mu = 0 \\ v_\mu = 0 \\ \theta_\mu = 0 \end{bmatrix} = \begin{bmatrix} F_{x_1} = 0 \\ F_{y_1} = 1200 \\ M_1 = 3900 \\ F_{x_r} = 0 \\ F_{y_r} = -1200 \\ M_r = -1000 \\ F_{x_\mu} = 0 \\ F_{y_\mu} = 1200 \\ M_\mu = 29800 \end{bmatrix}$$

$$E_X \begin{bmatrix} 1 & 0 & -0.104 & 0 \\ 0 & 0.122 & 0 & 0 \\ 0 & 0 & 0.122 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ u_r \\ v_r \\ \theta_r \end{bmatrix} = \begin{bmatrix} 1200 \\ 2400 \\ 0 \\ -1200 \end{bmatrix} \Rightarrow \begin{bmatrix} \theta_1 = -0.100109 \\ u_r = 0.100012 \\ v_r = -0.10221 \\ \theta_r = 0.100109 \end{bmatrix}$$

(11)



$$I_y = \frac{1}{12} (40)(30)^3 = 135000$$

$$I_z = \frac{1}{12} (30)(40)^3 = 266666.67$$

$$A = 30 \times 40 = 1200 \text{ mm}^2$$

$$L = 1.8 \text{ m} = 1800 \text{ mm}$$

$$E = 200 \times 10^9 \text{ MPa}$$

$$\sigma_{\max} = \frac{MC}{I} = ?$$

$$K_{xz} = \frac{EI_y}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

$$9K_{xy} = \frac{EI_z}{L^3} \begin{bmatrix} 12 & 6L & 12 & 6L \\ 6L & 4L^2 & -6L & -2L^2 \\ 12 & -6L & 12 & -6L \\ 6L & -2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$[K] = E \begin{bmatrix} \frac{A}{L} & -\frac{A}{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{A}{L} & \frac{A}{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12I_z}{L^3} & \frac{6I_z}{L^2} & \frac{12I_z}{L^3} & \frac{6I_z}{L^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{6I_z}{L^2} & \frac{4I_z}{L} & -\frac{6I_z}{L^2} & -\frac{2I_z}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{12I_z}{L^3} & -\frac{6I_z}{L^2} & \frac{12I_z}{L^3} & -\frac{6I_z}{L^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{6I_z}{L^2} & -\frac{2I_z}{L} & -\frac{6I_z}{L^2} & \frac{4I_z}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{12I_y}{L^3} & -\frac{6I_y}{L^2} & -\frac{12I_y}{L^3} & -\frac{6I_y}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{6I_y}{L^2} & \frac{4I_y}{L} & \frac{6I_y}{L^2} & \frac{2I_y}{L} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{12I_y}{L^3} & \frac{6I_y}{L^2} & \frac{12I_y}{L^3} & \frac{6I_y}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{6I_y}{L^2} & \frac{2I_y}{L} & \frac{6I_y}{L^2} & \frac{4I_y}{L} \end{bmatrix}$$

$$Y_{\alpha} X_{\beta}$$

[illegible]

$$\begin{bmatrix} u_1 = 0 \\ u_r \\ a_1 = 0 \\ \theta_{z_1} = 0 \\ a_r \\ \theta_{z_r} \\ w_1 = 0 \\ \theta_{y_1} = 0 \\ w_r \\ \theta_{y_r} \end{bmatrix}$$

$$\left[\begin{array}{l} f_{x_1} = 0 \\ f_{x_2} = 0 \\ f_{y_1} = -2000 \\ M_{z_1} = 18000 \\ f_{y_2} = 800 \\ M_{z_2} = 0 \\ f_{z_1} = -100 \\ M_{y_1} = 100000 \\ f_{z_2} = 100 \\ M_{y_2} = 0 \end{array} \right]$$

(13)

$$M_z = 800 \times 10 \times 10^{-3} = 80000 \text{ Nmm}, \quad M_y = 300 \times 10 \times 10^{-3} = 30000 \text{ Nmm}$$

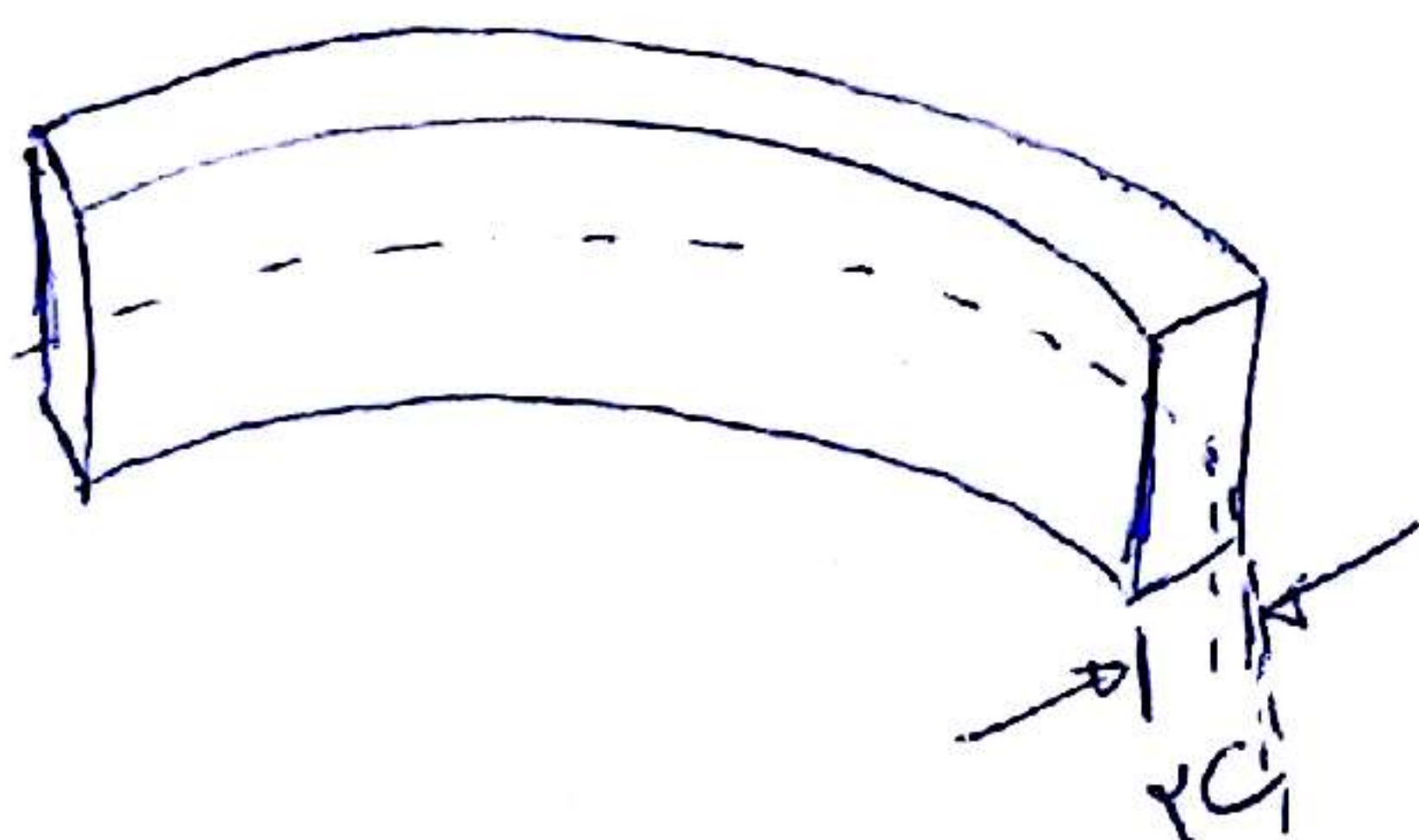
$$f_y = -800$$

$$f_z = -300$$

$$100 \times 10^{-3} \begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1.92 \times 10^{-3} & -1.22 & 0 & 0 \\ 0 & -1.22 & 1220 & 0 & 0 \\ 0 & 0 & 0 & 2.18 \times 10^{-2} & 0.34 \\ 0 & 0 & 0 & 0.34 & 340 \end{bmatrix} \times \begin{bmatrix} u_r \\ v_r \\ \theta_{z_r} \\ w_r \\ \theta_{y_r} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 300 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} u_r = 0 \\ v_r = 0.10 \text{ m} \\ \theta_{z_r} = 0.100 \text{ rad} \\ w_r = 12.10 \text{ mm} \\ \theta_{y_r} = -0.012 \end{bmatrix}$$

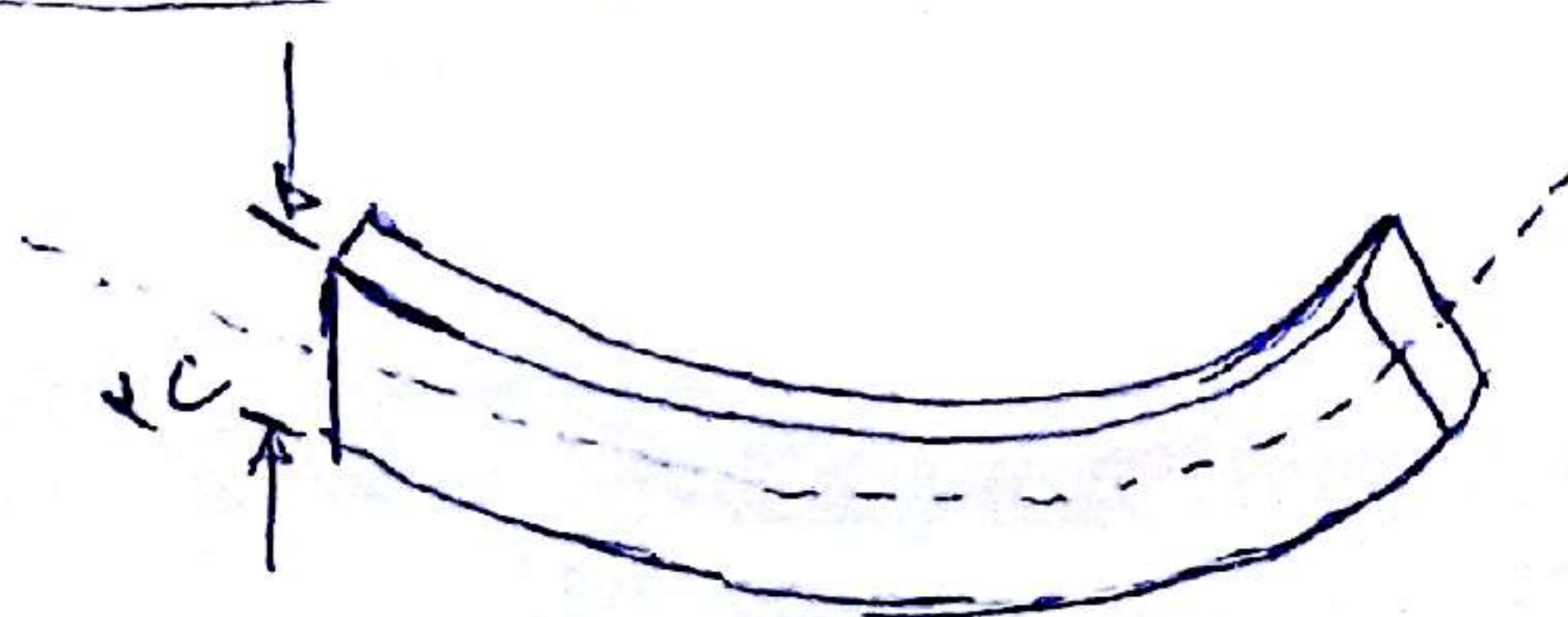
عش ناسی از 300 N, $r_c = 30 \Rightarrow C = 18 \text{ mm}$



$$\sigma_{\max} = \frac{MC}{I} = \frac{80000 \times 10}{138000} = 50 \text{ MPa}$$

$$b = 90 \text{ mm}, h = 30 \text{ mm} \Rightarrow I = \frac{1}{12} (90)(30)^3 = 138000$$

عش ناسی از 800 N



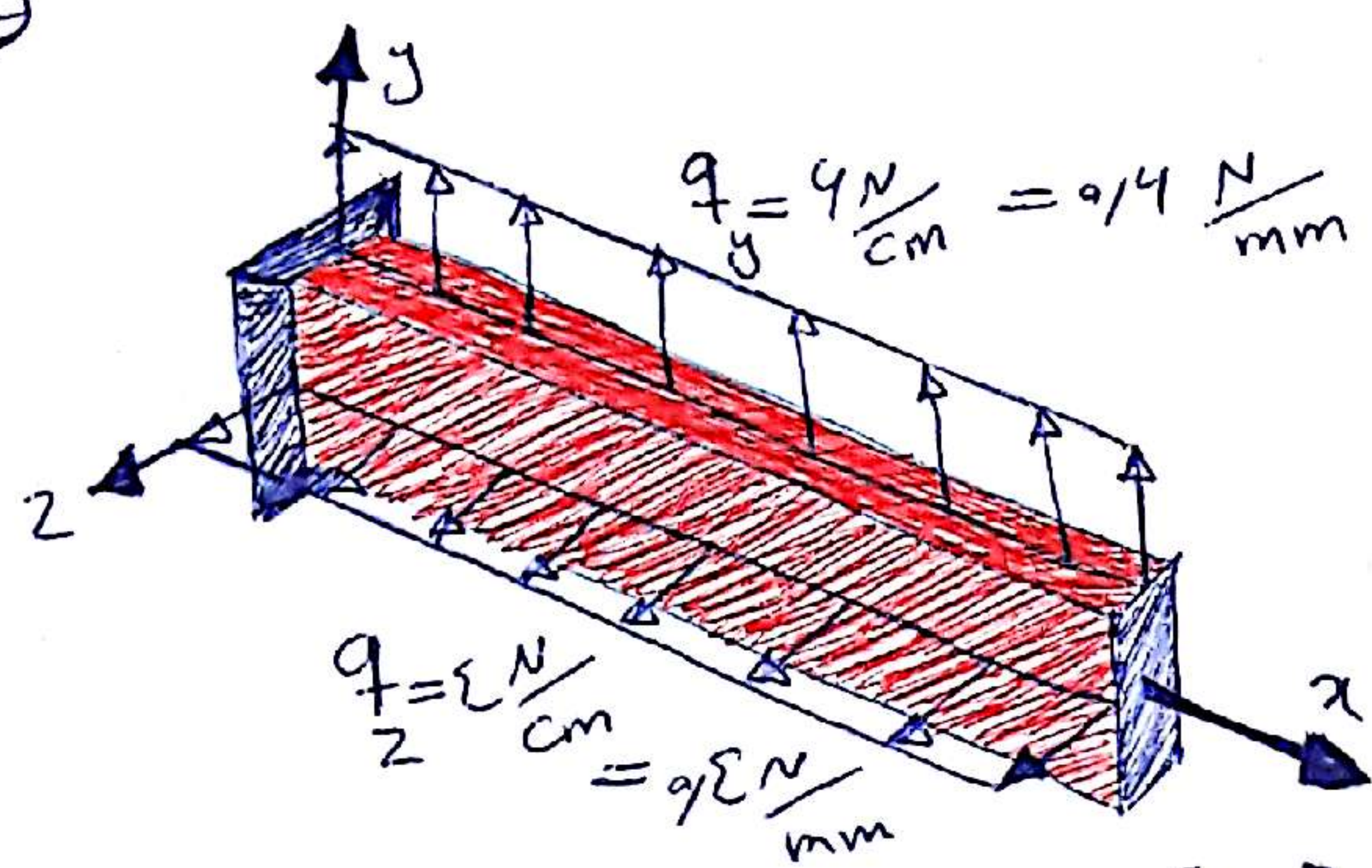
$$r_c = 90 \text{ mm} \Rightarrow C = 30 \text{ mm}$$

$$b = 30 \text{ mm}, h = 90 \text{ mm} \Rightarrow I_z = \frac{1}{12} (30)(90)^3 = 820000$$

$$M_z = 80000$$

$$\sigma = \frac{80000 \times 30}{820000} = 2.94 \text{ MPa}$$

(12)



$$\begin{bmatrix} f_{z1} \\ m_{z1} \\ f_{zr} \\ m_{zr} \end{bmatrix} = \begin{bmatrix} \frac{qL}{r} \\ -\frac{qL^2}{1r} \\ \frac{qL}{r} \\ \frac{qL^2}{1r} \end{bmatrix}, \quad \begin{bmatrix} f_{y1} \\ m_{y1} \\ f_{yr} \\ m_{yr} \end{bmatrix} = \begin{bmatrix} \frac{qL}{r} \\ -\frac{qL^2}{1r} \\ \frac{qL}{r} \\ \frac{qL^2}{1r} \end{bmatrix} \quad (2V \text{ ges})$$

$$[K_e] = \begin{bmatrix} [K_{axial}] & [0] & [0] \\ [0] & [K_{bending}]_{xy} & [0] \\ [0] & [0] & [K_{bending}]_{xz} \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_r \\ v_1 \\ \theta_{z1} \\ v_r \\ \theta_{zr} \\ w_1 \\ \theta_{y1} \\ w_r \\ \theta_{yr} \end{bmatrix} =$$

$$\begin{bmatrix} f_{x1} \\ f_{xr} \\ f_{y1} \\ m_{z1} \\ f_{yr} \\ m_{zr} \\ f_{z1} \\ m_{y1} \\ f_{zr} \\ m_{yr} \end{bmatrix} =$$

$$\begin{bmatrix} f_{x1} \\ f_{xr} \\ \left(\frac{qL}{r}\right)_y \\ \left(-\frac{qL^2}{1r}\right)_z \\ \left(\frac{qL}{r}\right)_y \\ \left(\frac{qL^2}{1r}\right)_z \\ \left(\frac{qL}{r}\right)_z \\ \left(-\frac{qL^2}{1r}\right)_y \\ \left(\frac{qL}{r}\right)_z \\ \left(\frac{qL^2}{1r}\right)_y \end{bmatrix}$$

$$E = 206 \times 10^3 \text{ MPa}$$

$$\sigma_{max} = ?$$

$$I_y = 148000$$

$$I_z = 820000$$

$$A = 1100 \text{ mm}^2$$

$$L = 1800 \text{ mm}$$

$$[K_e]_{xz} = \frac{EI_y}{L^3} \begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

$$9[K_e]_{xy} = \frac{EI_z}{L^3} \begin{bmatrix} 12 & 6L & 12 & 6L \\ 6L & 4L^2 & -6L & -2L^2 \\ 12 & -6L & 12 & -6L \\ 6L & -2L^2 & -6L & 4L^2 \end{bmatrix}$$

۲۰۷ × ۱۰۰

ماتریس معنی دقیقاً سمت چپ تساوی مثل قبل است :

$u_1 = 0$	$f_{x_1} = 0$	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
$u_2 = 0$	$f_{x_2} = 0$	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
$\theta_{z1} = 0$	$\theta_{z2} = 0$	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
v_1	v_2	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
θ_{z1}	θ_{z2}	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
$w_1 = 0$	$w_2 = 0$	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
$\theta_{y1} = 0$	$\theta_{y2} = 0$	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
w_1	w_2	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$
θ_{y1}	θ_{y2}	$\frac{q_{y1}}{2} = \frac{0.14 \times 1000}{2} = 70$	$-\frac{q_{y2}}{2} = -\frac{0.14 \times 1000}{2} = -70$

1.2
 -1.2
 1.2

1.92×10^{-3}
 1.92
 1.92×10^{-3}

1.92×10^{-3}
 1.92
 1.92×10^{-3}

1.92×10^{-3}
 1.92
 1.92×10^{-3}

1.92×10^{-3}
 1.92
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1.92×10^{-3}
 1.92
 1.92×10^{-3}

1.92×10^{-3}
 1.92
 1.92×10^{-3}

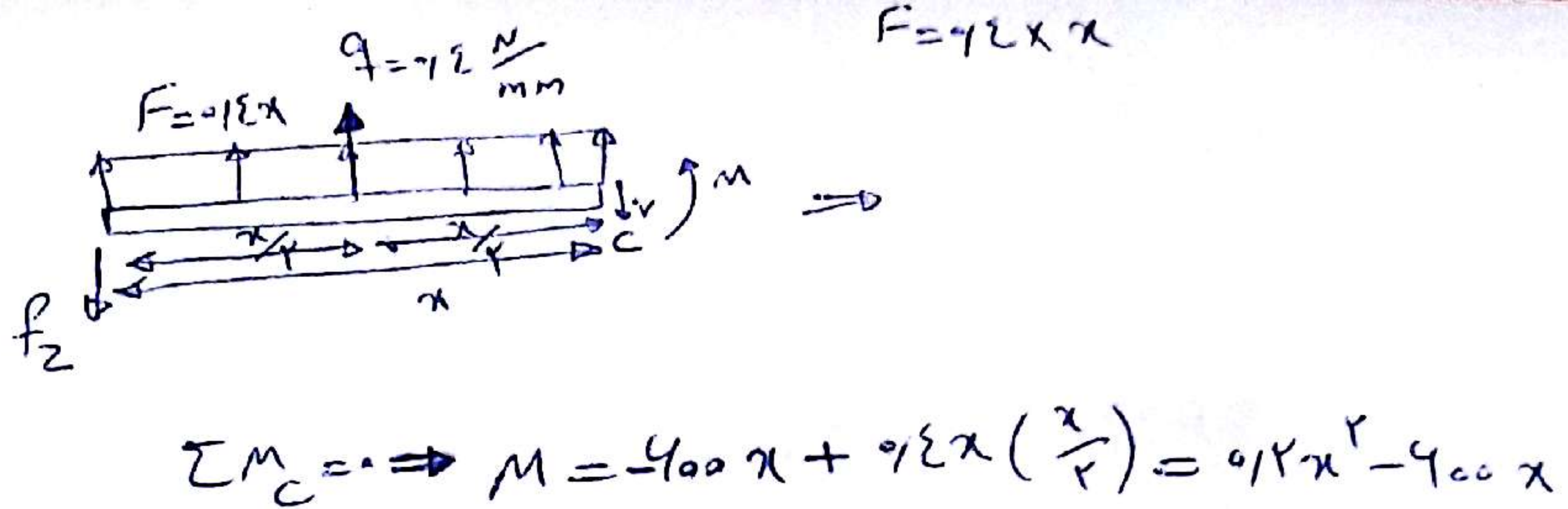
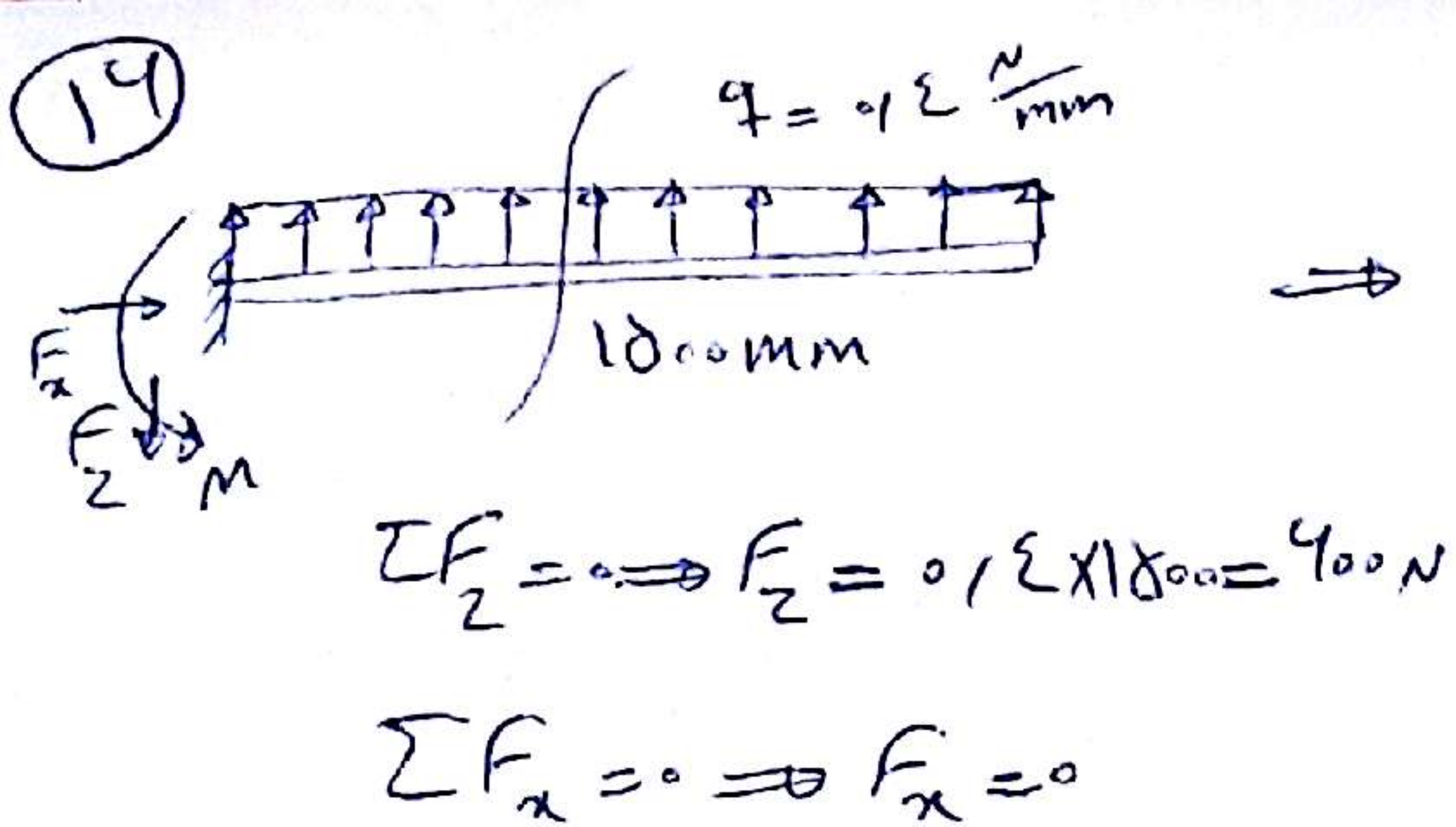
1.92×10^{-3}
 1.92
 1.92×10^{-3}

1.92×10^{-3}
 1.92
 1.92×10^{-3}

1.92×10^{-3}
 1.92
 1.92×10^{-3}

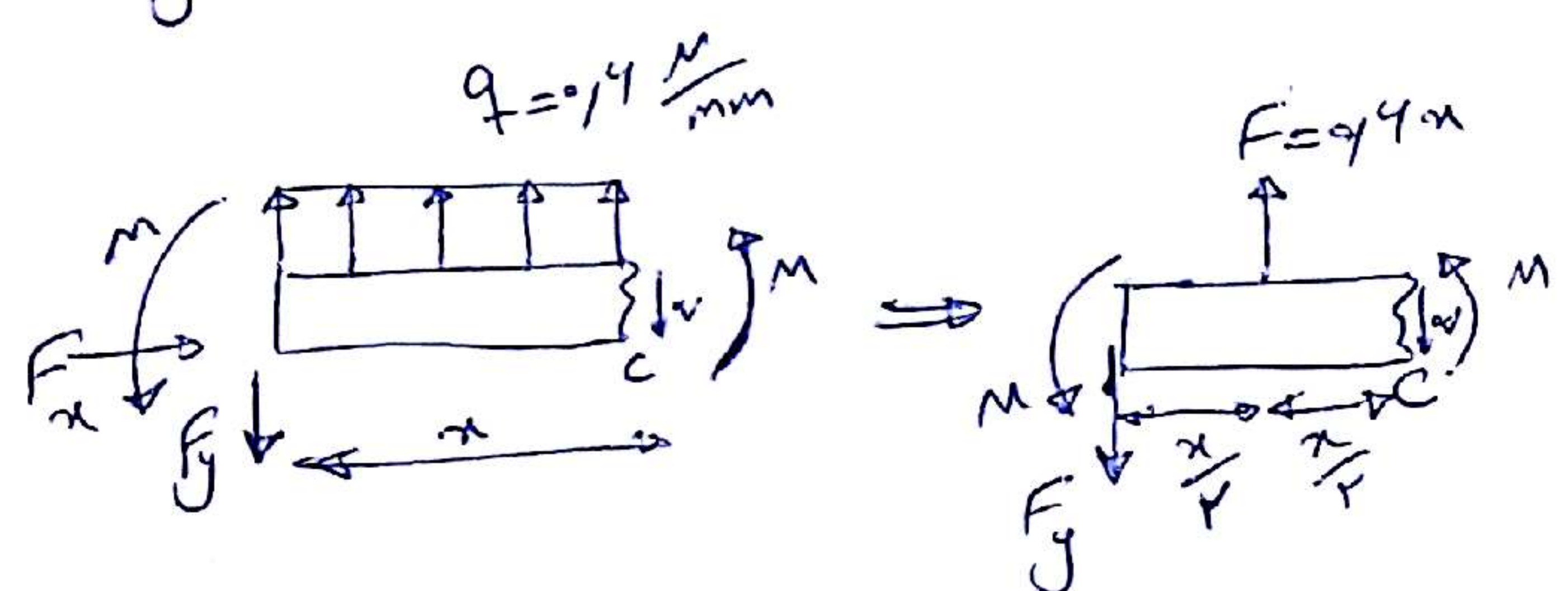
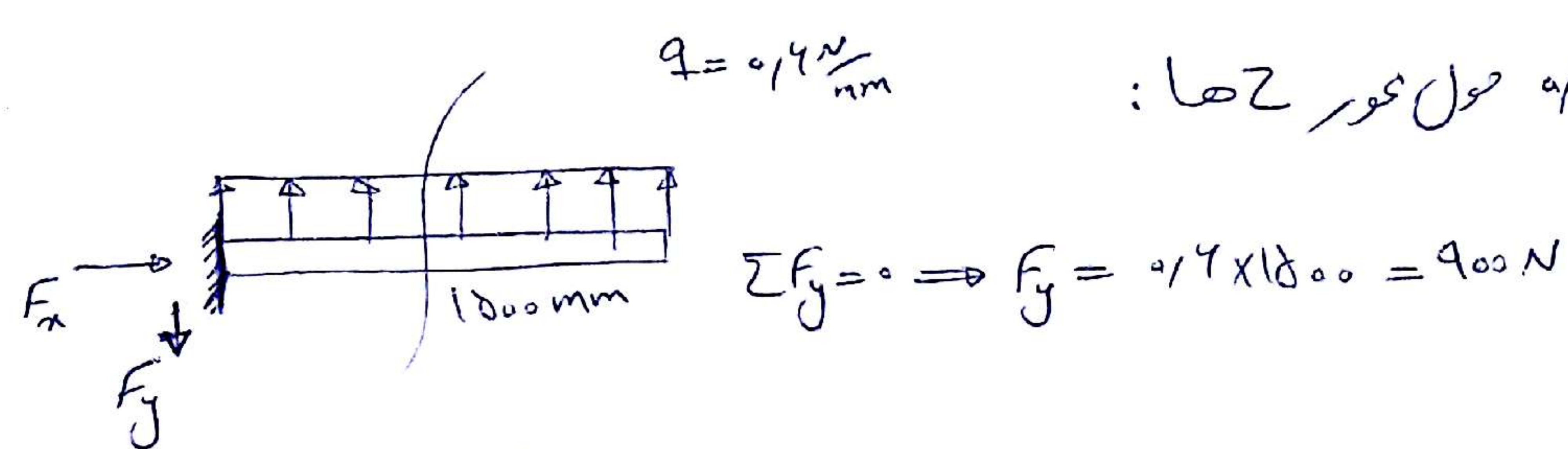
1.92×10^{-3}
 1.92
 1.92×10^{-3}

14



$\textcircled{a} x = 1800 \Rightarrow M = 0.12(1800)^2 - 400(1800) = -280000$
 ← ساعتش برده

حساب بخش ناستی از بار کسره ده $0.12 \frac{N}{mm}$ حول محور Z ها.



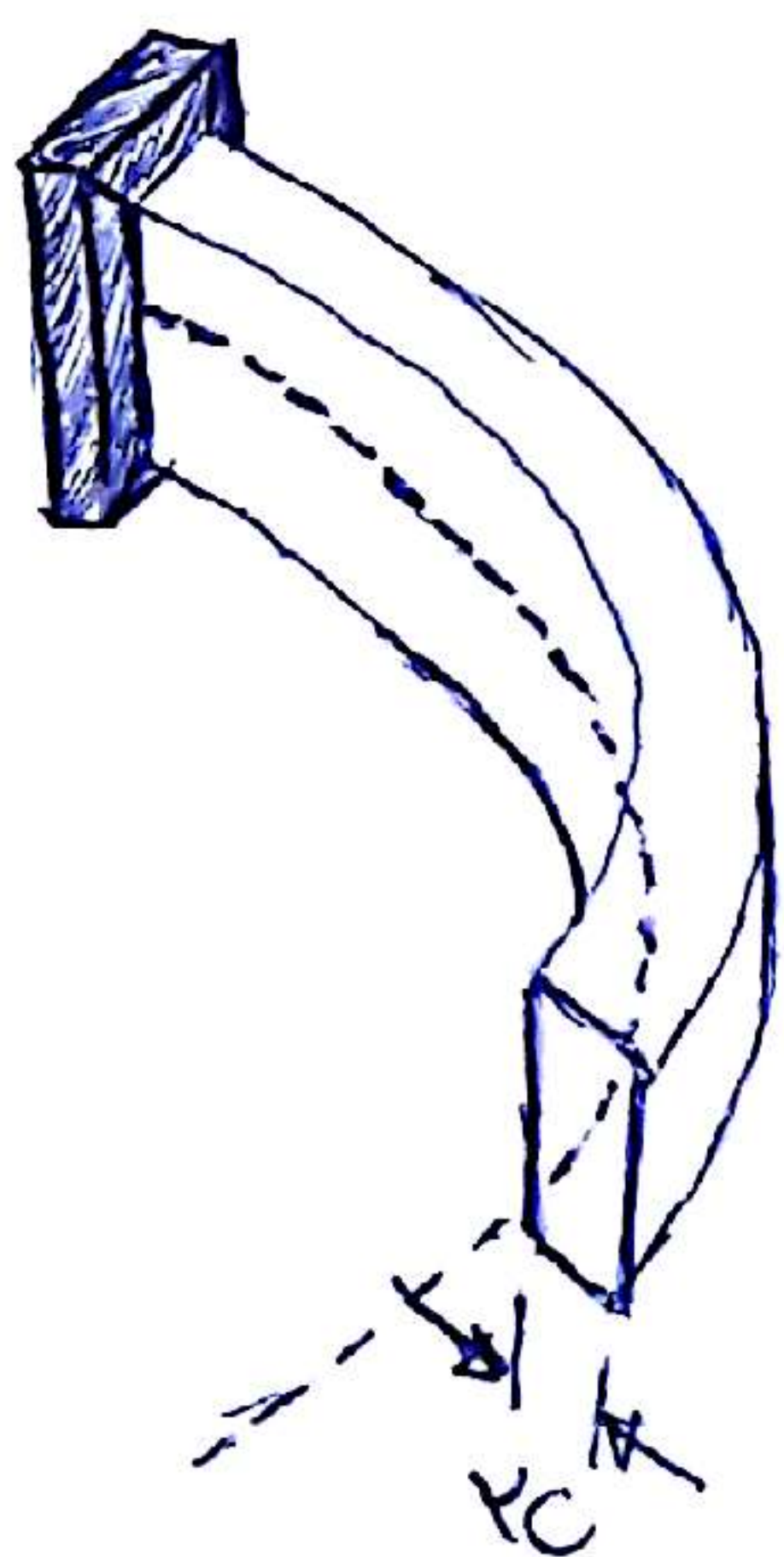
$\textcircled{a} x = 1800 \Rightarrow M = 0.14(1800)^2 - 900(1800) = -270000$
 ← ساعتش برده

(IV)

$$P_0 \times 10^3 \begin{bmatrix} 1.94 \times 10^{-3} & -1.22 & 0 \\ -1.22 & 122 & 0 \\ 0 & 0 & 2.18 \times 10^{-2} \\ 0 & 0 & 0.134 \end{bmatrix} \begin{bmatrix} v_r \\ \theta_{rz} \\ w_r \\ \theta_{yr} \end{bmatrix} = \begin{bmatrix} 200 \\ 7000 \\ 300 \\ 11200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_r = 0.128 \text{ mm} \\ \theta_{rz} = 0.0000882 \\ w_r = 7.89 \text{ mm} \\ \theta_{yr} = -0.00403 \end{bmatrix}$$

تغییراتی از بار گسترده $\frac{\Delta N}{cm}$



$$r_c = 30 \rightarrow C = 8 \text{ mm}$$

$$\sigma_{\max} = \frac{MC}{I} = \frac{280000 \times 8}{138000} = 20 \text{ MPa}$$

$$b = 40, h = 30 \Rightarrow I_y = 138000$$

$$r_c = 90 \Rightarrow C = 30$$

$$I = 30, h = 40 \Rightarrow I_z = 820000$$

$$\sigma = \frac{MC}{I} = \frac{400000 \times 30}{820000} = 14.76 \text{ MPa}$$

