بسم الله الرحمن الرحیم

دانشگاه صنعتی خواجه نصیرالدین طوسی

محاسبات عددی

ابراهیم شاه ابراهیمی

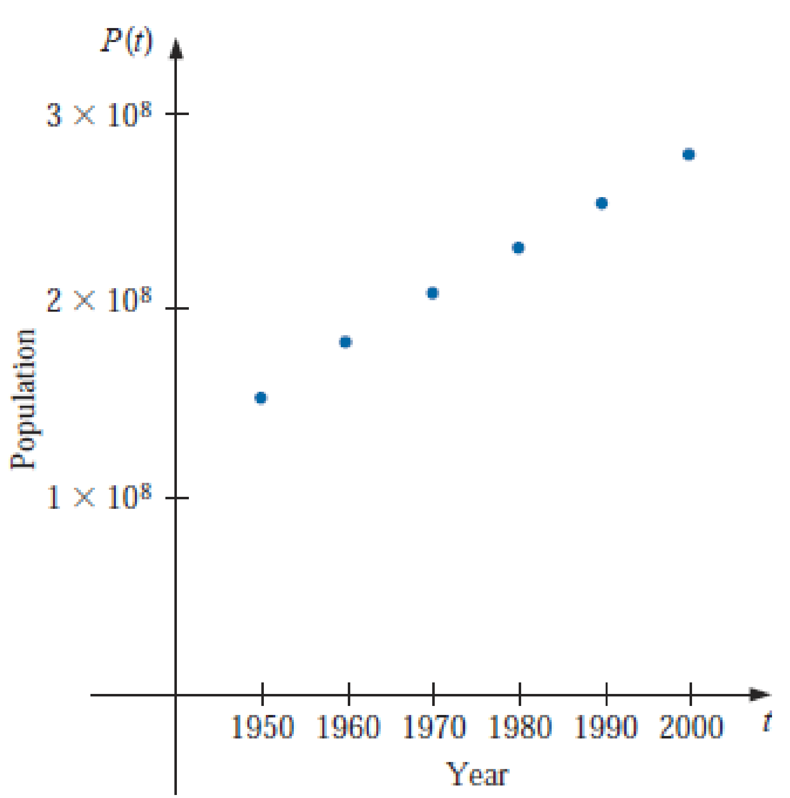
نوروز ۹۹

2 Interpolation and Polynomial Approximation

Introduction

A census of the population of the United States is taken every 10 years.

Year	1950	1960	1970	1980	1990	2000
Population (in thousands)	151,326	179,323	203,302	226,542	249,633	281,422



In reviewing these data,
we might ask whether they could be used
to provide a reasonable estimate of the population
say, in 1975 or even in the year 2020.
This process is called
interpolation
and is the subject of this chapter.

Lagrange Interpolating Polynomials

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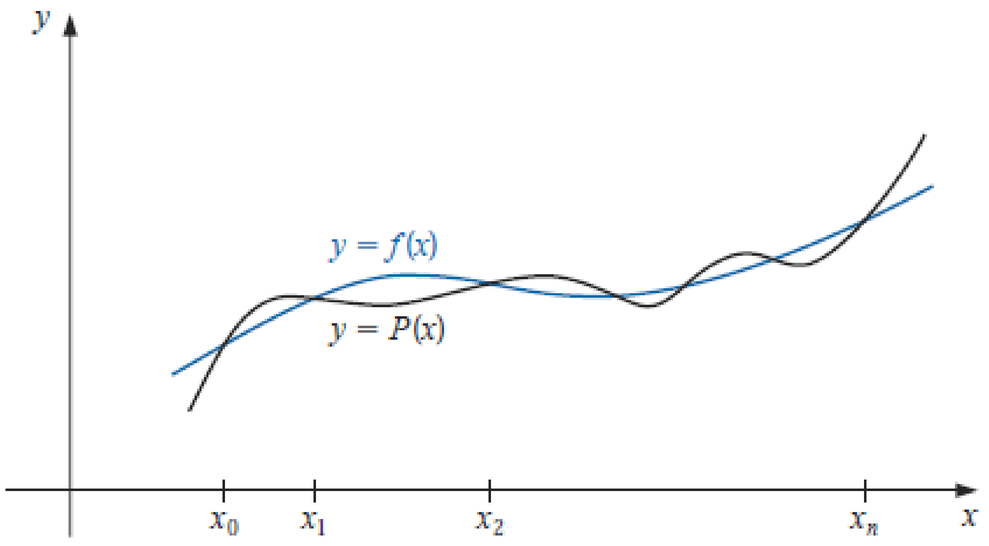


$$(x_0, f(x_0)),\ (x_1, f(x_1)), \ldots, (x_n, f(x_n))$$

where, for each $k = 0, 1, \ldots, n$,

$$L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)} = \prod_{\substack{i=0\\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}$$

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n)L_{n,n}(x) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x)$$

Example 1

(a) Use the numbers (called *nodes*) $x_0 = 2$, $x_1 = 2.75$, and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = 1/x$.

(b) Use this polynomial to approximate $f(3) = 1/3$.

Solution (a) We first determine the coefficient polynomials $L_0(x)$, $L_1(x)$, and $L_2(x)$

$$L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

$$L_0(x) = \frac{(x-2.75)(x-4)}{(2-2.75)(2-4)} = \frac{2}{3}(x-2.75)(x-4)$$

$$L_1(x) = \frac{(x-2)(x-4)}{(2.75-2)(2.75-4)} = -\frac{16}{15}(x-2)(x-4)$$

$$L_2(x) = \frac{(x-2)(x-2.75)}{(4-2)(4-2.75)} = \frac{2}{5}(x-2)(x-2.75)$$

Example 1

(a) Use the numbers (called *nodes*) $x_0 = 2$, $x_1 = 2.75$, and $x_2 = 4$ to find the second Lagrange interpolating polynomial for $f(x) = 1/x$.

(b) Use this polynomial to approximate $f(3) = 1/3$.

Solution Also, $f(x_0) = f(2) = 1/2$ $\quad f(x_1) = f(2.75) = 4/11$ $\quad f(x_2) = f(4) = 1/4$

$$P(x) = f(x_0)L_{n,0}(x) + \cdots + f(x_n) = \sum_{k=0}^{n} f(x_k)L_{n,k}(x)$$

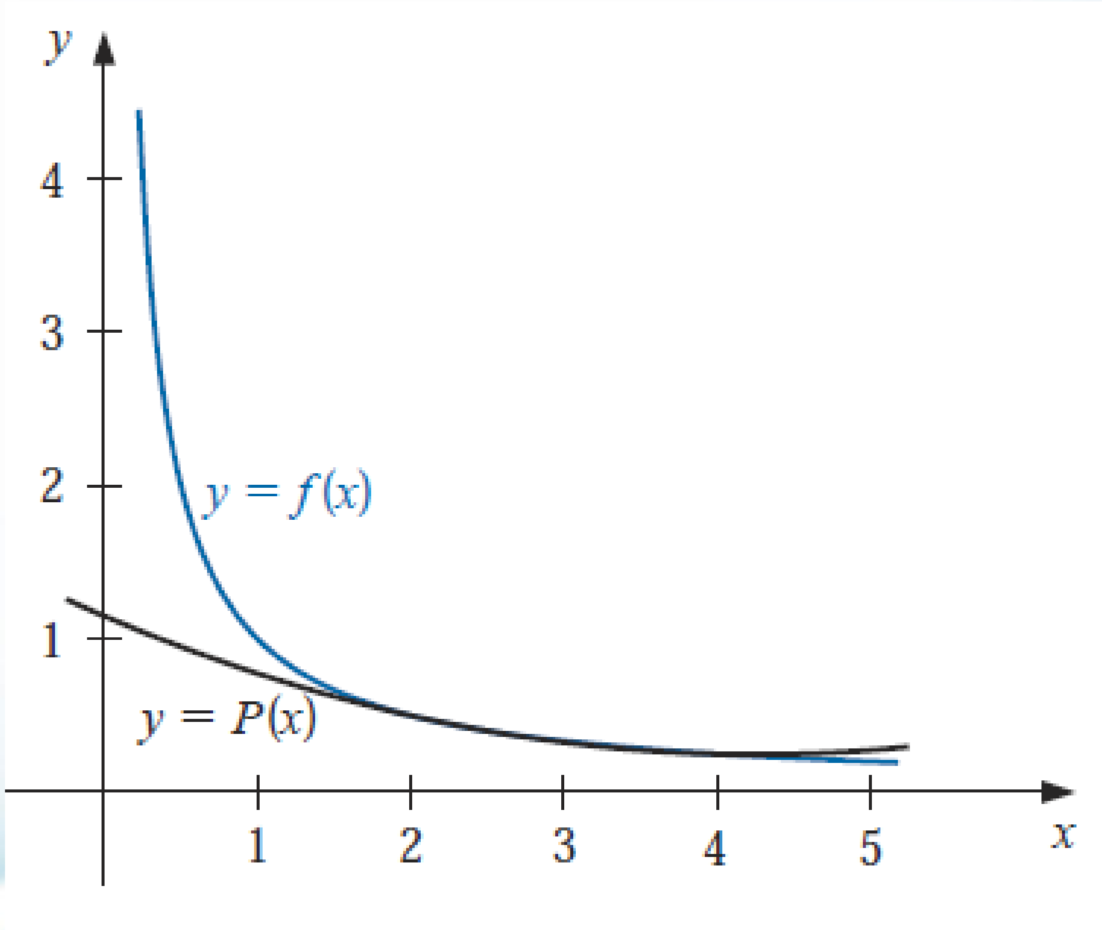
$$P(x) = \sum_{k=0}^{2} f(x_k)L_k(x) = \frac{1}{3}(x-2.75)(x-4) - \frac{64}{165}(x-2)(x-4) + \frac{1}{10}(x-2)(x-2.75)$$

$$= \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44}$$

(b) An approximation to $f(3) = 1/3$ $\quad f(3) \approx P(3) = \frac{9}{22} - \frac{105}{88} + \frac{49}{44} = \frac{29}{88} \approx 0.32955$

$$f(x) = 1/x$$

$$P(x) = \frac{1}{22}x^2 - \frac{35}{88}x + \frac{49}{44}$$



سوال :
چرا از بسط تیلور استفاده نکردیم ؟

EXERCISE SET 2.1

1. Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate each of the following:
   a. $f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$
   b. $f(0)$ if $f(-0.5) = 1.93750$, $f(-0.25) = 1.33203$, $f(0.25) = 0.800781$, $f(0.5) = 0.687500$

2. It is suspected that the high amounts of tannin in mature oak leaves inhibit the growth of the winter moth (*Operophtera bromata L., Geometridae*) larvae that extensively damage these trees in certain years. The following table lists the average weight of two samples of larvae at times in the first 28 days after birth. The first sample was reared on young oak leaves, whereas the second sample was reared on mature leaves from the same tree.

   Use Lagrange interpolation to approximate the average weight curve for each sample.

Day	0	6	10	13	17	20	28
Sample 1 average weight (mg)	6.67	17.33	42.67	37.33	30.10	29.31	28.74
Sample 2 average weight (mg)	6.67	16.11	18.89	15.00	10.56	9.44	8.89

پایان جلسه چهارم
(پایان فصل۲-قسمت۱)

۲۱ فروردین ۹۹

با تشکر از توجه شما