Taiwan International Mathematics Competition 2012 (TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

Individual Contest

1. In how many ways can 20 identical pencils be distributed among three girls so that each gets at least 1 pencil?

【**Solution**】

The first girl can take from 1 to 18 pencils. If she takes 1, the second girl can take from 1 to 18. If she takes 2, the second girl can take from 1 to 17, and so on. The third girl simply takes whatever is left. Hence the total number of ways is $18+17+\ldots+1=\frac{19\times 18}{2}=171$.

ANS: 171

2. On a circular highway, one has to pay toll charges at three places. In clockwise order, they are a bridge which costs \$1 to cross, a tunnel which costs \$3 to pass through, and the dam of a reservoir which costs \$5 to go on top. Starting on the highway between the dam and the bridge, a car goes clockwise and pays toll-charges until the total bill amounts to \$130. How much does it have to pay at the next place?

【**Solution**】

Since 1+3+5=9, it takes \$9 to go once around the highway. When 130 is divided by 9, the quotient is 14, but more importantly the remainder is 4. This means that after completing a number of round which happens to be 14, the car has pay an extra \$4, which means it has crossed the bridge and passed through the tunnel. At the next place, it will have to pay \$5 to go on top of the dam of the reservoir.

ANS: \$5

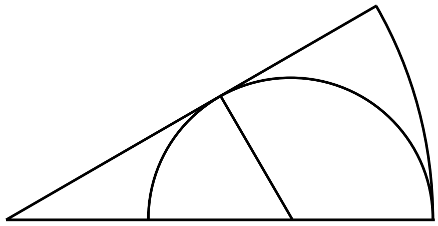
3. When a two-digit number is increased by 4, the sum of its digits is equal to half of the sum of the digits of the original number. How many possible values are there for such a two-digit number?

【**Solution**】

Clearly, a carrying occurs when the number is increased by 4. Hence its units digit is 6, 7, 8 or 9. Suppose it is 6. With this 6 turning into a 1 in the tens dight, there is a net loss of 5. Hence the sum of the digits of the original number must be $2\times 5=10$, so that it is 46. Indeed, the sum of the digits of 46 is 10 and the sum of the digits of 46+4=50 is 5, which is half of 10. Using the same reasoning, we see that if the units digit is 7, 8 or 9, the original number must be 37, 28 and 19. Hence there are 4 possible values.

ANS: 4

4. In the diagram below, OAB is a circular sector with $OA = OB$ and $\angle AOB = 30°$. A semicircle passing through A is drawn with centre C on OA, touching OB at some point T. What is the ratio of the area of the semicircle to the area of the circular sector?

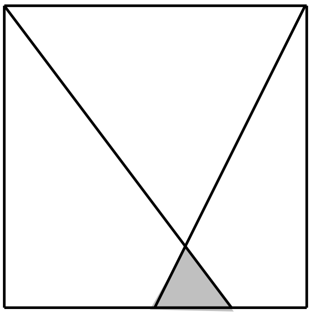


【Solution】

We may take the length of $OA = OB$ to be 6 cm. The area of a circle with radius 6 cm is $\pi \times 6^2 = 36\pi\ \text{cm}^2$, so that the area of a $30°$ sector is $36\pi \times \frac{30°}{360°} = 3\pi\ \text{cm}^2$. Note that COT is half an equilalteral triangle. Hence OC=2CT=2CA so that OA=3CT. Since OA=6, CT=2 and the area of the semicircle is $\frac{1}{2}\pi \times 2^2 = 2\pi\ \text{cm}^2$. The desired ratio is therefore 2:3.

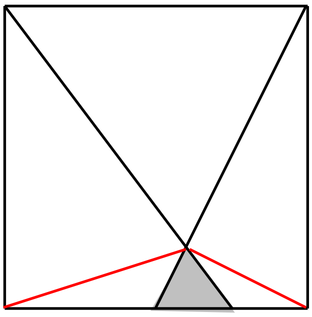
ANS: 2:3

5. *ABCD* is a square with total area 36 cm^2. *F* is the midpoint of *AD* and *E* is the midpoint of *FD*. *BE* and *CF* intersect at *G*. What is the area, in cm^2, of triangle *EFG*?



【Solution】

The area of triangle *GED* is equal to that of triangle *EFG* because *ED*=*EF*. The area of triangle *GAF* is twice that of triangle *EFG* because *AF*=2*FE*. The area of triangle *GBC* is sixteen times that of triangle *EFG* because the two triangles are similar and *BC*=4*EF*. Hence the area of triangle *EFG* is $\frac{1}{1+1+2+16} = \frac{1}{20}$ that of the total area of triangles *ADG* and *BCG*, which is half that of the square *ABCD*. It follows that the area of triangle *EFG* is $\frac{1}{40}$ that of the square *ABCD*, so the area of triangle *EFG* is 0.9.



ANS: 0.9

6. In a village, friendship among girls is mutual. Each girl has either exactly one friend or exactly two friends among themselves. One morning, all girls with two friends wear red hats and the other girls all wear blue hats. It turns out that any two friends wear hats of different colours. In the afternoon, 10 girls change their

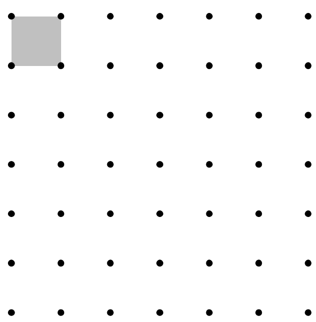
red hats into blue hats and 12 girls change their blue hats into red hats. Now it turns out that any two friends wear hats of the same colour. How many girls are there in the village?

【Solution】

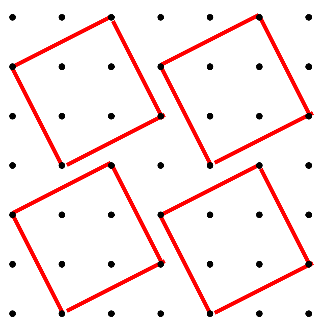
Consider a girl with only one friend. She will be wearing a blue hat in the morning, and her only friend will be wearing a red hat. This friend has a second friend who must be wearing a blue hat, and the friendship network stops there. It follows that the girls in the village may be divided into groups of three, with one being friends with the other two, but the other two are not friends of each other. In the afternoon, in order for any two friends to be wearing hats of the same colour, all three girls in each group must wear hats of the same colour. This can be accomplished by either the girl in red changing her hat, or the two girls both changing their hats. It follows that in 10 groups, the girl in red hat changes, and in $12 \div 2 = 6$ groups, the girls in blue hats change. The total number of groups is 10+6=16 and the total number of girls is $16 \times 3 = 48$.

ANS: 48

7. The diagram below shows a 7×7 grid in which the area of each unit cell (one of which is shaded) is 1 cm^2. Four congruent squares are drawn on this grid. The vertices of each square are chosen among the 49 dots, and two squares may not have any point in common. What is the maximum area, in cm^2, of each of these four squares?



【Solution】



Because the vertices of the squares are chosen among the dots, the side-length of each square, in cm, is either a positive integer, or by Pythagoras' Theorem, the square root of the sum of two positive integers. In either case, its area in cm^2 is a positive integer. Since the area of the whole grid is 36 cm^2, this integer is at most 9 cm^2, and cannot be 6 or 7 cm^2. It cannot be 9 cm^2, as otherwise the squares will have boundary points in common. It cannot be 8 cm^2 as we cannot fit even two copies of such a square without them having common points. Hence the maximum area of each square is 5 cm^2, and the diagram below shows that this can be attained.

ANS: 5

8. The sum of 1006 different positive integers is 1019057. If none of them is greater than 2012, what is the minimum number of these integers which must be odd?

【Solution】

Suppose we take the first 1006 even numbers. Then their sum is $\dfrac{1006\times(2+2012)}{2}=$ 1013042. Now $1019057 - 1013042 = 6015$. So we have to trade some even numbers for an equal amount of odd numbers. Clearly, trading only one number can raise the total by at most 2011−2=2009. Hence we have to trade more than one number. However, if we trade exactly two numbers, the total will increase by an even amount, which is not what we want. Hence we must trade at least three numbers. If we trade 2, 4 and 6 for 2007, 2009 and 2011, the total will increase by 2007+2009+2011−2−4−6 =3×(2009−4)=6015 exactly. Hence the minimum number of the 1006 integers which must be odd is 3.

ANS: 3

9. The desks in the TAIMC contest room are arranged in a 6 × 6 configuration. Two contestants are neighbours if they occupy adjacent seats along a row, a column or a diagonal. Thus a contestant in a seat at a corner of the room has 3 neighbours, a contestant in a seat on an edge of the room has 5 neighbours, and a contestant in a seat in the interior of the room has 8 neighbours. After the contest, a contestant gets a prize if at most one neighbour has a score greater than or equal to the score of the contestant. What is maximum number of prize-winners?

【Solution】

Divide the contest room into 9 sections each of which is a 2 × 2 configuration. In each section, arrange the four contestants in order of their scores starting with the highest. Then the contestant third or fourth in the line cannot be prize-winners because each would have at least two neighbours whose scores are not lower. Hence the maximum number of prize-winners is 2 × 9 = 18. The diagram below shows the marks of the contestants in the respective seats, and each contestant in the first, third or fifth row get a prize.

40	50	60	70	80	90
10	10	10	10	10	10
40	50	60	70	80	90
10	10	10	10	10	10
40	50	60	70	80	90
10	10	10	10	10	10

ANS: 18

10. The sum of two positive integers is 7 times their difference. The product of the same two numbers is 36 times their difference. What is the larger one of these two numbers?

【Solution】

Suppose the difference is 1. Then the sum is 7. If we add twice the smaller number to the difference, we will get the sum. Hence the smaller number is 3, and the larger number is 4. Now the product is 12, which is 12 times the difference. However, it is given that the product is 36 times the difference. Since 36 ÷ 12 = 3, the smaller number is 3 × 3 = 9 and the larger number is 3 × 4 = 12.

ANS: 12

11. In a competition, every student from school A and from school B is a gold medalist, a silver medalist or a bronze medalist. The number of gold medalist from each school is the same. The ratio of the percentage of students who are

gold medalist from school A to that from school B is 5:6. The ratio of the number of silver medalists from school A to that from school B is 9:2. The percentage of students who are silver medalists from both school is 20%. If 50% of the students from school A are bronze medalists, what percentage of the students from school B are gold medalists?

【Solution】

Suppose school A has 9 silver medalists. Then school B has 2, for a total of 9+2=11. Hence the total population of the two schools is $11 \div 20\% = 55$. For gold medalists, the numbers are the same but the percentages are in the ratio 5:6. This means that the ratio of the populations are in the ratio 6:5. It follows that the population of school A is $55 \times \frac{6}{6+5} = 30$ and that of school B is $55-30 = 25$. Now the number of bronze medalists from school A is $30 \times 50\% = 15$, so that the number of gold medalists from each school is 30−15−9=6. It follows that the percentage of students from school B who are gold medalists is $6 \div 25 = 24\%$.

ANS: 24%

12. We start with the fraction $\frac{5}{6}$. In each move, we can either increase the numerator by 6 or increases the denominator by 5, but not both. What is the minimum number of moves to make the value of the fraction equal to $\frac{5}{6}$ again?

【Solution】

Since the value of the fraction is unchanged, the ratio of the amount added to the numerator and the amount added to the denominator is also 5:6. Since we are adding 6s to the numerator and 5s to the denominator, the number of 6s added must be a multiple of 25 and the number of 5s added must be the same multiple of 36. Hence the minimum number of moves is 25+36= 61.

ANS: 61

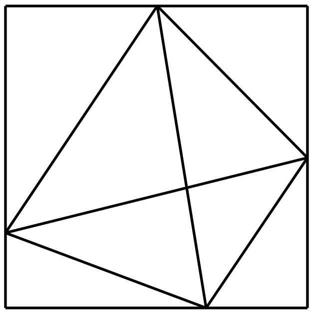
13. Five consecutive two-digit numbers are such that 37 is a divisor of the sum of three of them, and 71 is also a divisor of the sum of three of them. What is the largest of these five numbers?

【Solution】

Among five consecutive numbers, the sum of the largest three is only 6 more than the sum of the smallest three. Thus we are looking for a multiple of 37 and a multiple of 71 which differ by at most 6. Now $37 \times 2 - 71 = 3$ and $37 \times 4 - 71 \times 2 = 6$. In the latter case, 148 must be the sum of the largest three numbers, which are consecutive. Hence the sum is divisible by 3, but 148 is not a multiple of 3. In the former case, note that $71 < 23 + 24 + 25 = 72 < 74$. Hence the smallest of the five numbers cannot be 23 or more, and the largest cannot be 25 or less. This means that the five numbers must be 22, 23, 24, 25 and 26, the largest of which is 26. There is no need to consider higher multiples of 37 and 71 as the sums will be too large to allow the five numbers to have only two digits.

ANS: 26

14. *ABCD* is a square. *M* is the midpoint of *AB* and *N* is the midpoint of *BC*. *P* is a point on *CD* such that *CP* = 4 cm and *PD* = 8 cm, *Q* is a point on *DA* such that *DQ* = 3 cm. *O* is the point of intersection of *MP* and *NQ*. Compare the areas of the two triangles in each of the pairs (*QOM*, *QAM*), (*MON*, *MBN*), (*NOP*, *NCP*) and (*POQ*, *PDQ*). In cm^2, what is the maximum value of these four differences?



【Solution】

We have *AM* =*MB* =*BN* =*NC* =6 cm and *AQ* = 9 cm. It follows that the area of *ABNQ* is $\frac{1}{2}\times 12\times(9+6)$ = 90 cm^2. Hence the area of *QMN* is $90-\frac{1}{2}\times 9\times 6-\frac{1}{2}\times 6\times 6=$ 45 cm^2. The area of *QNCD* is 12×12−90 = 54 cm^2. Hence the area of *NPQ* is $54-\frac{1}{2}\times 3\times 8-\frac{1}{2}\times 6\times 4=$ 30 cm^2. Similarly, the area of *PQM* is 45 cm^2 and the area of *MNP* is 30 cm^2. It follows that *QO* : *ON* = 45 : 30 = 3 : 2. Hence the area of *QOM* is $45\times\frac{3}{3+2}=$ 27 cm^2, which is the same as the area of *QAM*. The area of *MON* is 45 − 27 = 18 cm^2, which is the same as the area of *MBN*. Similarly, the area of *NOP* is $30\times\frac{2}{3+2}=$ 12 cm^2, which is the same as the area of *NCP*. Finally, the area of *POQ* is 30 − 12 = 18 cm^2, which is 18 − 12 = 6 cm^2 more than the area of *PDQ*. The maximum value of the four difference is therefore 6 cm^2.

ANS: 6

15. Right before Carol was born, the age of Eric is equal to the sum of the ages of Alice, Ben and Debra, and the average age of the four was 19. In 2010, the age of Debra was 8 more than the sum of the ages of Ben and Carol, and the average age of the five was 35.2. In 2012, the average age of Ben, Carol, Debra and Eric is 39.5. What is the age of Ben in 2012?

【Solution】

Right before Carol was born, the total age of Alice, Ben, Debra and Eric was 19 × 4 = 76. In 2010, the total age of all five was 35.2 × 5 = 176. It follows that in 2010, the age of Carol was (176 − 76) ÷ 5 = 20, so that she was born in 1990. In 2012, the total age of Ben, Carol, Debra and Eric is 39.5 × 4 = 158. In 2010, the total age of these four was 158 − 2 × 4 = 150. It follows that the age of Alice was 176 − 150 = 26. Note that the age of Eric in 1990 was 76 ÷ 2 = 38. Hence the age of Eric in 2010 was 58, and the total age of Ben, Carol and Debra was 176 − 26 − 58 = 92. It follows that the age of Debra was (92 + 8) ÷ 2 = 50, and the age of Ben was 92 − 50 − 20 = 22. Thus the age of Ben in 2012 is 22 + 2 = 24.

ANS: 24

Taiwan International Mathematics Competition 2012 (TAIMC 2012)

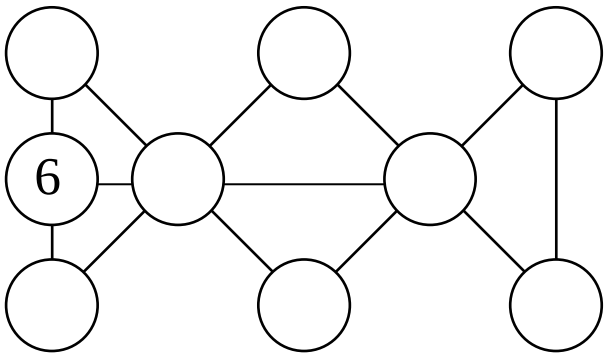
World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

TEAM CONTEST

(with solution and marking scheme)

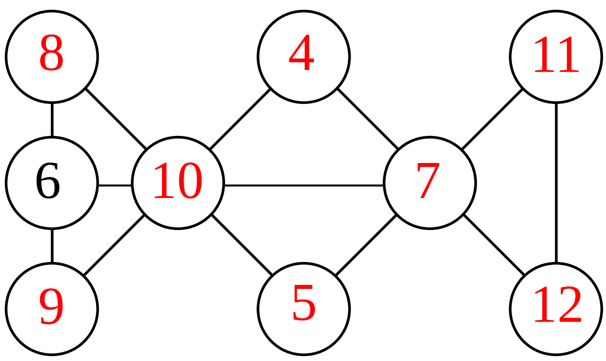
1. Each of the nine circles in the diagram below contains a different positive integer. These integers are consecutive and the sum of numbers in all the circles on each of the seven lines is 23. The number in the circle at the top right corner is less than the number in the circle at the bottom right corner. Eight of the numbers have been erased. Restore them.



【Solution】

Each circle lies on two lines except for the two rimmed circles in the diagram below. The sum of the numbers inside them is 23－6=17. Hence the sum of the nine consecutive number is (7×23－17)÷2=72. The middle number is 72÷9=8, so that the nine numbers are 4, 5, 6, 7, 8, 9, 10, 11 and 12. The line on the right has only two circles. Hence the numbers inside must be 11 and 12, with 11 on top. The remaining numbers in these two lines add up to 12 and 11 respectively, and we must have 11=4+7 since 6 is already used. Now 12=5+7=4+8, so that the number in the rimmed circle on the right must contain 4 or 7. It cannot be 4 as the numbers in the two rimmed circles must add up to 17. Hence 7 is in the rimmed circle on the right and 10 is in the rimmed circle on the left. The placement of the remaining numbers are forced, completing the unique solution shown in the diagram below.

// totally correct [40pt]
// up-side-down or any other answer [0pt]



2. A clay tablet consists of a table of numbers, part of which is shown in the diagram below on the left. The first column consists of consecutive numbers starting from 0. In the first row, each subsequent number is obtained from the preceding one by adding 1. In the second row, each subsequent number is obtained from the preceding one by adding 2. In the third row, each subsequent number is obtained from the preceding one by adding 3, and so on. The tablet falls down and breaks up into pieces, which are swept away except for the two shown in the diagram below on the right in magnified forms, each with a smudged square. What is the sum of the two numbers on these two squares?

0	1	2	3	4	5	
1	3	5	7	9	11	
2	5	8	11	14	17	
3	7	11	15	19	23	
4	9	14	19	24	29	
5	11	17	23	29	35	

?	2012	2023

2012
2683
?

【Solution】

The smudged number in the first piece is 2012 − (2023 − 2012) = 2001. Note that the table is symmetric about the main diagonal. Hence the smudged number in the second piece is 2683 + (2683 − 2012) = 3354. The desired sum is therefore 2001+3354=5355

// "2001" for left blank [+5pt]

// "3354" for right blank [+10pt]

// correct answer [+25pt]

// a proof without correct answer [-5pt]

ANS: 5355

3. In a row of numbers, each is either 2012 or 1. The first number is 2012. There is exactly one 1 between the first 2012 and the second 2012. There are exactly two 1s between the second 2012 and the third 2012. There are exactly three 1s between the third 2012 and the fourth 2012, and so on. What is the sum of the first 2012 numbers in the row?

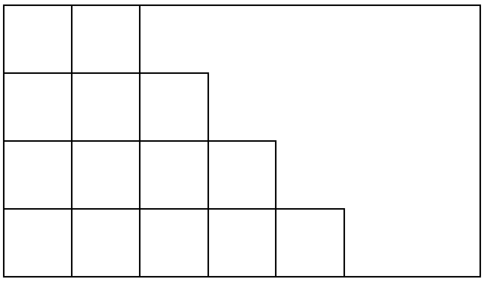
【Solution】

Let us break the row down into a staircase, with 2 numbers in the first, 3 numbers in the second, 4 numbers in the third, and so on, as shown in the diagram below on the left. Let us first calculate the total number of numbers in the first four rows. Make a second copy of the staircase, turn it upside down and put it together with the original copy to form a rectangle, as shown in the diagram below on the right. This rectangle has 3 more columns than rows. Hence it contains 4 × 7 = 28 numbers, which means that the original staircase contains 28 ÷ 2 = 14 numbers. The property that the rectangle contains 3 more columns than rows holds true regardless of the size of the staircase. We want the number of numbers in the rectangle to be as close to 2×2012 = 4024 as possible, without going over. Now 61×64 = 3904 while 62 × 65 = 4030. Hence we take a staircase with 61 rows, containing 3904 ÷ 2 = 1952 numbers. To bring the total up to 2012, we use an incomplete 62nd rows. Thus among the first 2012 numbers in the original row, there are 62 copies of 2012 and 2012−62 = 1950 copies of 1, for a total of 62 × 2012 + 1950 = 126694.

// totally correct [40pt]

// any other answer [0pt]

2012	1			
2012	1	1		
2012	1	1	1	
2012	1	1	1	1



ANS: 126694

4. In a test, one-third of the questions were answered incorrectly by Andrea and 7 questions were answered incorrectly by Barbara. One fifth of the questions were answered incorrectly by both of them. What was the maximum number of questions which were answered correctly by both of them?

【Solution】

The fraction of the questions answered incorrectly only by Andrea was $\frac{1}{3}-\frac{1}{5}=\frac{2}{15}$.

Hence the number of questions must be a multiple of 15. Since answered 7 questions incorrectly, at most 7 questions may be answered incorrectly by both of them, so that the total number of questions is at most $7\div\frac{1}{5}=35$. To get the maximum number of questions answered correctly by both of them, we take the largest possible number of questions, which is 30. Then $30\times\frac{1}{5}=6$ questions were answered incorrectly by both, 7－6 = 1 question answered incorrectly by Barbara only, and $30\times\frac{2}{15}=4$ questions answered incorrectly by Andrea only. The number of questions answered correctly by both of them is therefore 30－6－1－4=19.

// (#all questions) is a multiple of 15 [+10pt]

// (#all questions) has upper bound 35 [+10pt]

// describe the results of the test [+10pt]

// correct answer [+5pt]

// check the case of "(#all questions)=15" [+5pt]

ANS: 19

5. Five different positive integers are multiplied two at a time, yielding ten products. The smallest product is 28, the largest product is 240 and 128 is also one of the products. What is the sum of these five numbers?

【Solution】

Note that 28 is the product of the smallest two numbers while 240 is the product of the largest two numbers. Hence the smallest two numbers are 1 and 28, 2 and 14 or 4 and 7. Note that the second smallest number is no less than 7, so that the second largest number is no less than 9. Hence the largest two numbers are 10 and 24, 12 and 20 or 15 and 16. Note that the second largest number is no greater than 15, so that the second smallest number is no greater than 13. Hence the smallest two numbers are 4 and 7. Now 128 is not divisible by 7, or by any of the possible values for the second largest number, namely 10, 12 and 15. The smallest number 4 cannot be one of its factors as the other factor 32 is greater than any possible value of the largest number. It follows that 128 is the product of the middle and the largest number. The largest number must be 16 as neither 20 or 24 divides 128. Hence the five numbers are 4, 7, 8, 15 and 16, and their sum is 50.

// totally correct [40pt]

// any other answer [0pt]

ANS: 50

6. The diagram below shows a square *MNPQ* inside a rectangle *ABCD* where $AB - BC = 7$ cm. The sides of the rectangle parallel to the sides of the square. If the total area of *ABNM* and *CDQP* is 123 cm^2 and the total area of *ADQM* and *BCPN* is 312 cm^2, what is the area of *MNPQ* in cm^2?

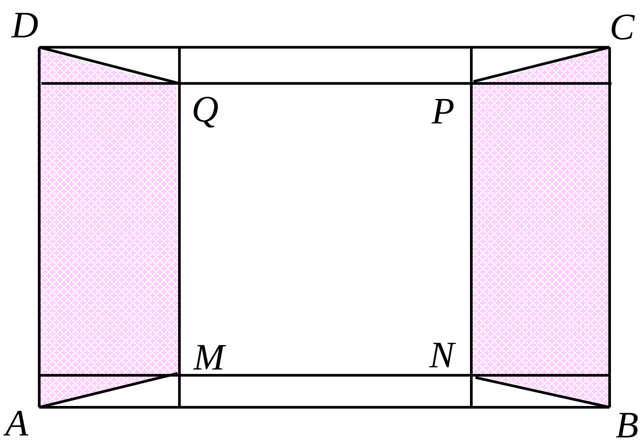
【Solution】

The diagram below is to be regarded as a frame with a square hole in the middle. The difference between the area of the shaded part of the frame and the area of the unshaded part of the frame is 312 − 123 = 189 cm^2. This difference is unchanged if we disregard the eight congruent right triangles at the four corners. The remaining parts consist of two shaded rectangles and two unshaded rectangles, whose heights are all equal to *MN*. Since the difference between their combined widths is 7 cm, we have *MN* = 189 ÷ 7 = 27 cm, so that the area of *MNPQ* is $27^2 = 729$ cm^2.

// move MNPQ to a corner of ABCD [+10pt]

// correct answer [+10pt]

// correct answer with proof [40pt]



ANS: 729 cm^2

7. Two companies have the same number of employees. The first company hires new employees so that its workforce is 11 times its original size. The second company lays off 11 employees. After the change, the number of employees in the first company is a multiple of the number of employees in the second

company. What is the maximum number of employees in each company before the change?

【Solution】

Suppose the number of employees in each company originally is not a multiple of 11. After the change, the number of employees in the second company divides the original number of employees, and must therefore divide the difference, which is 11. Since it cannot be a multiple of 11, it must be equal to 1, so that originally each company has 12 employees. Suppose the number of employees in each company originally is a multiple of 11. Divide each company into 11 branches of equal size. After the change, the number of employees in each branch of the second company divides 11 times the number of employees in each branch originally. Hence it must be a divisor of 11. If it is 1, then originally each branch has 2 employees and each company has 22 employees. If it is 11, then originally each branch has 12 employees and each company has 132 employees. Thus the maximum number is 132.

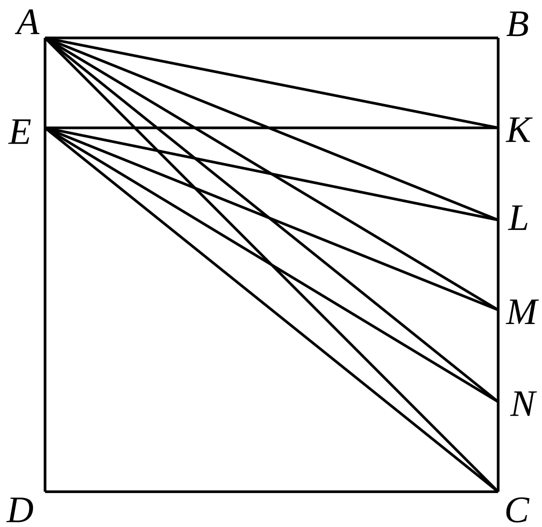
// totally correct [40pt]

// any other answer [0pt]

ANS: 132

8. $ABCD$ is a square. K, L, M and N are points on BC such that $BK = KL = LM = MN = NC$. E is the point on AD such that $AE = BK$. In degrees, what is the measure of

$$\angle AKE + \angle ALE + \angle AME + \angle ANE + \angle ACE\ ?$$



【Solution】

Note that $EABK$, $EAKL$, $EALM$, $EAMN$ and $EANC$ are all parallelograms since each has a pair of equal and parallel opposite sides. It follows that $\angle AKE = \angle KAB$, $\angle ALE = \angle LAK$, $\angle AME = \angle MAL$, $\angle ANE = \angle NAM$ and $\angle ACE = \angle CAN$. Hence $\angle AKE + \angle ALE + \angle AME + \angle ANE + \angle ACE = \angle BAC = 45°$

// correct answer [+10pt]

// correct answer with proof [40pt]

ANS: 45°

9. The numbers 1 and 8 have been put into two squares of a 3×3 table, as shown in the diagram below. The remaining seven squares are to be filled with the numbers 2, 3, 4, 5, 6, 7 and 9, using each exactly once, such that the sum of the

numbers is the same in any of the four 2×2 subtables shaded in the diagram below. Find all possible solutions.

1		
		8

1		
		8

1		
		8

1		
		8

【Solution】

By considering the first two shaded subtables, we see that the number below 1 and the number above 8 must differ by $8 - 1 = 7$. They can only be 9 and 2 respectively, as shown in the diagram below. By considering the last two shaded subtables, we see that the numbers below 9 and 8 must be consecutive. They can only be 3 and 4, 4 and 5, 5 and 6 or 6 and 7, as shown in the diagram below. It is easy to complete the table in the first three cases. In the last case, the difference between the top number and the bottom number in the middle column must be $6 - 1 = 5 = 7 - 2$, but the maximum difference among the remaining numbers, namely 3, 4 and 5, is only 2. Thus there are only three solutions.

// totally correct [40pt]
// miss any solution [0pt]
// any other answer [0pt]

1	7	2
9	6	8
3	5	4

1	6	2
9	7	8
4	3	5

1	7	2
9	4	8
5	3	6

1		2
9		8
6		7

10. At the beginning of each month, an adult red ant gives birth to three baby black ants. An adult black ant eats one baby black ant, gives birth to three baby red ants, and then dies. During the month, baby ants become adult ants, and the cycle continues. If there are 9000000 red ants and 1000000 black ants on Christmas day, what was the difference between the number of red ants and the number of black ants on Christmas day a year ago?

【Solution】

Let us consider the cycle two months at a time. The number of new red ants in the first month is 3 times the original number of black ants. The number of new red ants in the second month is 3 times the number of surviving baby black ants in the first month. This number is 3 times the original number of red ants minus the original number of black ants. Hence in two months, the net gain in the number of red ants is 9 times the original number of red ants. In other words, the number of red ants is 10 times the number two months ago. The number black ants in the first month is 3 times the original number of red ants minus the original number of black ants. The number of baby black ants born at the beginning of the second month is 3 times the original number of red ants and 9 times the original number of black ants. Of these, 3 times the original number of red ants minus the original number of black ants are eaten. Hence the number of black ants is also 10 times the number two months ago. In a year, the number of red ants increases six times, to 1000000 times the original number. Since there are 9000000 red ants and 1000000 black ants on Christmas day, there were 9 red ants and 1 black ant on Christmas day a year ago, and the difference was 8.

// correct answer [+10pt]

// partial score from the following four cases must not exceed [+25pt]

// calculate the situation of “next month” correctly [+10pt]

// calculate the situation of “next two month” correctly [+10pt]

// calculate the situation of “last month” correctly [+10pt]

// calculate the situation of “last two month” correctly [+15pt]

// “10x” every two month [+5pt]

// correct answer with proof [40pt]

// a proof without correct answer [-5pt]

ANS: 8