Solutions to Chapter 10 Exercise Problems

Problem 10.1

Two spur gears have a diametral pitch of 6. Gear 2 has 24 teeth, and gear 3 has 48. The working pressure angle is 20˚, and both gears are standard involutes. Determine the length of the contact line and the contact ratio.

Solution:

From Table 10.1, the addendum for both gears is given by

$$a = \frac{1}{P_d} = \frac{1}{6} = 0.167\text{in} = a_2 = a_3$$

Similarly, the circular pitch for both gears is given by

$$P_c = \frac{\pi}{P_d} = \frac{\pi}{6} = 0.524 \text{ in}$$

and from Eq. (10.13), the base pitch is related to the circular pitch by

$$P_b = P_c\cos\ = 0.524\cos 20° = 0.492$$

From Eq. (10.7), the two pitch radii are given by

$$r_{p2} = \frac{N_2}{2P_d} = \frac{24}{2(6)} = 2$$

and

$$r_{p3} = \frac{N_3}{2P_d} = \frac{48}{2(6)} = 4$$

The length of the line of contact is given by Eq. (10.17) as

$$\begin{aligned}\lambda &= -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}{}^2\sin^2\phi} - r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}{}^2\sin^2\phi}\\ &= -2\sin 20° + \sqrt{0.167^2 + 2(0.167)(2) + 2^2\sin^2 20°} - 4\sin 20°\\ &\quad + \sqrt{0.167^2 + 2(0.167)(4) + 4^2\sin^2 20°} = 0.825\text{in}\end{aligned}$$

From Eq. (10.18), the contact ratio is

$$m_c = \frac{\lambda}{P_b} = \frac{0.825}{0.492} = 1.678$$

Problem 10.2

For the gear pairs given below and meshing at their correct center-to-center distance, determine whether any interference is present and determine the contact ratio for each case. Assume that the addendum is $1/P_d$ in each case, and if any interference is present, assume the interference is removed by cutting off the ends of the gear teeth before determining the contact ratio.

(a) $14\frac{1}{2}°$ involute gears having 30 and 45 teeth
(b) 20° involute gears having 20 teeth and a rack
(c) 25° involute gears having 30 and 60 teeth

Solution:

Let gear 2 be the pinion. Then the condition of no interference in the pinion is as follows (see Fig. 10.6):

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi} \le r_{p2}\sin\phi \qquad (1)$$

From Table 10.4, we know the addendum and the pitch radii:

$$a_2 = a_3 = \frac{1}{P_d}$$

$$r_{p2} = \frac{N_2}{2P_d}$$

and

$$r_{p3} = \frac{N_3}{2P_d}$$

Equation (1) can then be rewritten as:

$$-\frac{N_3}{2P_d}\sin\phi + \sqrt{\left(\frac{1}{P_d}\right)^2 + 2\frac{1}{P_d}\frac{N_3}{2P_d} + \left(\frac{N_3}{2P_d}\right)^2\sin^2\phi} \le \frac{N_2}{2P_d}\sin\phi$$

Simplifying

$$-N_3\sin\phi + \sqrt{4 + 4N_3 + N_3^2\sin^2\phi} \le N_2\sin\phi$$

or

$$(N_2 + N_3)\sin\phi \ge \sqrt{4 + 4N_3 + N_3^2\sin^2\phi}$$

or

$$(N_2 + N_3)^2\sin^2\phi \ge 4 + 4N_3 + N_3^2\sin^2\phi$$

Simplifying further, the condition for no interference becomes

$$1 + N_3 \le \frac{N_2^2 + 2N_2N_3}{4}\sin^2\phi$$

(a) Substitue the values, N_2=30, N_3=45, and ϕ =14.5. Then

$$1 + 45 \le \frac{30^2 + 2(30)(45)}{4}\sin^2 14.5°$$

no interference exists. Now,

$$\lambda = -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2\sin^2\phi} - r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi}$$

$$= -\frac{N_2}{2P_d}\sin\phi + \sqrt{\frac{1}{P_d^2} + \frac{N_2}{P_d^2} + \frac{N_2^2}{4P_d^2}\sin^2\phi} - \frac{N_3}{2P_d}\sin\phi + \sqrt{\frac{1}{P_d^2} + \frac{N_3}{P_d^2} + \frac{N_3^2}{4P_d^2}\sin^2\phi}$$

The contact ratio is given by:

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos\phi} = \frac{-\frac{N_2+N_3}{2P_d}\sin\phi + \sqrt{\frac{1}{P_d^2} + \frac{N_2}{P_d^2} + \frac{N_2^2}{4P_d^2}\sin^2\phi} + \sqrt{\frac{1}{P_d^2} + \frac{N_3}{P_d^2} + \frac{N_3^2}{4P_d^2}\sin^2\phi}}{\frac{\pi}{P_d}\cos\phi}$$

$$= \frac{-\frac{N_2+N_3}{2}\sin\phi + \sqrt{1 + N_2 + \frac{N_2^2}{4}\sin^2\phi} + \sqrt{1 + N_3 + \frac{N_3^2}{4}\sin^2\phi}}{\pi\cos\phi}$$

$$= \frac{-\frac{30+45}{2}\sin 14.5^\circ + \sqrt{1 + 30 + \frac{30^2}{4}\sin^2 14.5^\circ} + \sqrt{1 + 45 + \frac{45^2}{4}\sin^2 14.5^\circ}}{\pi\cos 14.5^\circ} = 2.02$$

(b) From Table 10.2, we know when $\phi = 20^\circ$, $N_{min} = 18 < 20$, so no interference exists.

$$\lambda = -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2\sin^2\phi} + \frac{a}{\sin\phi} = -\frac{N_2}{2P_d}\sin\phi + \frac{1}{P_d}\sqrt{1 + N_2 + \frac{N_2^2}{4}\sin^2\phi} + \frac{1}{P_d\sin\phi}$$

so the contact ratio is given by:

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos\phi} = \frac{-\frac{N_2}{2}\sin\phi + \frac{1}{\sin\phi} + \sqrt{1 + N_2 + \frac{N_2^2}{4}\sin^2\phi}}{\pi\cos\phi}$$

$$= \frac{-\frac{20}{2}\sin 20^\circ + \frac{1}{\sin 20^\circ} + \sqrt{1 + 20 + \frac{20^2}{4}\sin^2 20^\circ}}{\pi\cos 20^\circ} = 1.77$$

(c) From Table 10.2, we know when $\phi = 25$, $N_{min} = 12 < 30$, so no interference exists. Using the equation developed above

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos\phi} = \frac{-\frac{N_2+N_3}{2P_d}\sin\phi + \sqrt{\frac{1}{P_d^2} + \frac{N_2}{P_d^2} + \frac{N_2^2}{4P_d^2}\sin^2\phi} + \sqrt{\frac{1}{P_d^2} + \frac{N_3}{P_d^2} + \frac{N_3^2}{4P_d^2}\sin^2\phi}}{\frac{\pi}{P_d}\cos\phi}$$

$$= \frac{-\frac{N_2 + N_3}{2}\sin\phi + \sqrt{1 + N_2 + \frac{N_2^2}{4}\sin^2\phi} + \sqrt{1 + N_3 + \frac{N_3^2}{4}\sin^2\phi}}{\pi\cos\phi}$$

$$= \frac{-\frac{30 + 60}{2}\sin 25^\circ + \sqrt{1 + 30 + \frac{30^2}{4}\sin^2 25^\circ} + \sqrt{1 + 60 + \frac{60^2}{4}\sin^2 25^\circ}}{\pi\cos 25^\circ} = 1.514$$

Problem 10.3

A 20° involute pinion having 30 teeth is meshing with a 60-tooth internal gear. The addendum of the pinion is $1.25/P_d$, and the addendum of the gear is $1/P_d$. Is there any interference? Determine the contact ratio. If there is interference, assume that the interfering portion is removed by cutting off the ends of the teeth.

Solution:

Let gear 2 be the pinion. Then the condition of no interference in the pinion is as follows (see Fig. 10.6):

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi} \le r_{p2}\sin\phi \qquad (1)$$

From Table 10.4, we know the addendum and the pitch radii:

$$a_2 = a_3 = \frac{1}{P_d}$$

$$r_{p2} = \frac{N_2}{2P_d}$$

and

$$r_{p3} = \frac{N_3}{2P_d}$$

Equation (1) can then be rewritten as:

$$-\frac{N_3}{2P_d}\sin\phi + \sqrt{\left(\frac{1}{P_d}\right)^2 + 2\frac{1}{P_d}\frac{N_3}{2P_d} + \left(\frac{N_3}{2P_d}\right)^2\sin^2\phi} \le \frac{N_2}{2P_d}\sin\phi$$

Simplifying

$$-N_3\sin\phi + \sqrt{4 + 4N_3 + N_3^2\sin^2\phi} \le N_2\sin\phi$$

or

$$(N_2 + N_3)\sin\phi \ge \sqrt{4 + 4N_3 + N_3^2\sin^2\phi}$$

or

$$(N_2 + N_3)^2\sin^2\phi \ge 4 + 4N_3 + N_3^2\sin^2\phi$$

Simplifying further, the condition for no interference becomes

$$1 + N_3 \le \frac{N_2^2 + 2N_2N_3}{4}\sin^2\phi$$

If N_2=30, N_3=60, and ϕ =20, then

$$1+60 \leq \frac{30^2+2(30)(60)}{4}\sin^2 20^\circ$$

and no interference exists. Now,

$$\lambda = -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2\sin^2\phi} - r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi}$$

$$= -\frac{N_2}{2P_d}\sin\phi + \sqrt{\frac{1.25^2}{P_d^2} + \frac{1.25N_2}{P_d^2} + \frac{N_2^2}{4P_d^2}\sin^2\phi} - \frac{N_3}{2P_d}\sin\phi + \sqrt{\frac{1}{P_d^2} + \frac{N_3}{P_d^2} + \frac{N_3^2}{4P_d^2}\sin^2\phi}$$

The contact ratio is given by:

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos\phi} = \frac{-\frac{N_2+N_3}{2P_d}\sin\phi + \sqrt{\frac{1.25^2}{P_d^2} + \frac{1.25N_2}{P_d^2} + \frac{N_2^2}{4P_d^2}\sin^2\phi} + \sqrt{\frac{1}{P_d^2} + \frac{N_3}{P_d^2} + \frac{N_3^2}{4P_d^2}\sin^2\phi}}{\frac{\pi}{P_d}\cos\phi}$$

$$= \frac{-\frac{N_2+N_3}{2}\sin\phi + \sqrt{1.25^2 + 1.25N_2 + \frac{N_2^2}{4}\sin^2\phi} + \sqrt{1 + N_3 + \frac{N_3^2}{4}\sin^2\phi}}{\pi\cos\phi}$$

$$= \frac{-\frac{30+60}{2}\sin 20^\circ + \sqrt{1.25^2 + 1.25(30) + \frac{30^2}{4}\sin^2 20^\circ} + \sqrt{1+60+\frac{60^2}{4}\sin^2 20^\circ}}{\pi\cos 20^\circ} = 1.894$$

Problem 10.4

What is the largest gear that will mesh with a 20° standard full-depth gear of 22 teeth with no interference?

Solution:

From Table 10.2, when $\phi = 20^\circ$, $N_{min} = 18 < 22$. Therefore, there will be no interference with any gear, and the largest gear is a rack.

Problem 10.5

What is the smallest gear that will mesh with a 20° standard full-depth gear of 22 teeth with no interference?

Solution:

Let gear 2 be the pinion. Then the condition of no interference in the pinion is as follows (see Fig. 10.6):

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}{}^2\sin^2\phi} \le r_{p2}\sin\phi \qquad (1)$$

From Table 10.4, we know the addendum and the pitch radii:

$$a_2 = a_3 = \frac{1}{P_d}$$

$$r_{p2} = \frac{N_2}{2P_d}$$

and

$$r_{p3} = \frac{N_3}{2P_d}$$

Equation (1) can then be rewritten as:

$$-\frac{N_3}{2P_d}\sin\phi + \sqrt{\left(\frac{1}{P_d}\right)^2 + 2\frac{1}{P_d}\frac{N_3}{2P_d} + \left(\frac{N_3}{2P_d}\right)^2\sin^2\phi} \le \frac{N_2}{2P_d}\sin\phi$$

Simplifying

$$-N_3\sin\phi + \sqrt{4 + 4N_3 + N_3^2\sin^2\phi} \le N_2\sin\phi$$

or

$$(N_2 + N_3)\sin\phi \ge \sqrt{4 + 4N_3 + N_3^2\sin^2\phi}$$

or

$$(N_2 + N_3)^2\sin^2\phi \ge 4 + 4N_3 + N_3^2\sin^2\phi$$

Simplifying further, the condition for no interference becomes

$$1 + N_3 \le \frac{N_2^2 + 2N_2N_3}{4}\sin^2\phi$$

Substitue the values, N_2=22, N_3=45, and ϕ =20˚. Then

$$1 + 22 \le \frac{N_2^2 + 2N_2(22)}{4}\sin^2 20^\circ$$

or

$$N_2 \ge 13.64$$

so

$$N_{2min} = 14$$

Therefore, the smallest gear has 14 teeth.

Problem 10.6

Assume a gear has a diametral pitch of 6. Determine the addendum, dedendum, and clearance if the pressure angle is 20˚ full depth, 25˚ full depth, 20˚ stub teeth.

Solution:

From Table 10.1, when $\phi = 20^\circ$, and we have full depth,

$$a = \frac{1}{P_d} = \frac{1}{6} = 0.167$$

$$b = \frac{1.25}{P_d} = \frac{1.25}{6} = 0.208$$

$$c = \frac{0.25}{P_d} = \frac{0.25}{6} = 0.0417$$

When $\phi = 25$, full depth,

$$a = \frac{1}{P_d} = \frac{1}{6} = 0.167$$

$$b = \frac{1.25}{P_d} = \frac{1.25}{6} = 0.208$$

$$c = \frac{0.25}{P_d} = \frac{0.25}{6} = 0.0417$$

When $\phi = 20$, stub depth

$$a = \frac{0.8}{P_d} = \frac{0.8}{6} = 0.133$$

$$b = \frac{1}{P_d} = \frac{1}{6} = 0.167$$

$$c = \frac{0.2}{P_d} = \frac{0.2}{6} = 0.0333$$

Problem 10.7

Assume two meshing gears have a diametral pitch of 6 and a 20° pressure angle. The gear has 38 teeth, and the pinion has 24. Determine the design center distance. Now assume that the center distance is increased by 0.01 in. Determine the pressure angle for the new center distance.

Solution:

From Table 10.4, the center distance is given by:

$$C = r_{p2} + r_{p3} = \frac{N_2 + N_3}{2P_d} = \frac{24 + 38}{(2)(6)} = 5.167$$

The new center distance is:

$$C_m = C + e = 5.167 + 0.01 = 5.177$$

The pressure angle for the new center distance is:

$$\phi_m = \cos^{-1}\left(\frac{r_{b2}}{r_{pm2}}\right) = \cos^{-1}\left(\frac{r_{p2}\cos}{C_m \frac{N_2}{N_2 + N_3}}\right) = \cos^{-1}\left(\frac{\frac{24}{(2)(6)}\cos 20°}{(5.177)\frac{24}{24+38}}\right) = 20.312°$$

Problem 10.8

Assume a standard full-depth rack has a diametral pitch of 2. Determine the smallest gear that will mesh with the rack without interference if the pressure angle is (a) 20˚, (b) 25˚.

Solution:

Use formular (10.25),

$$N \geq \frac{2}{\sin^2\phi}$$

(a) When =20˚

$$N \geq \frac{2}{\sin^2 20°} = 17.10$$

so N_{min}=18.

(b) When =25˚

$$N \geq \frac{2}{\sin^2 25°} = 11.20$$

so N_{min}=12.

Problem 10.9

Assume two meshing gears have a diametral pitch of 8 and a 25˚ pressure angle. The gear has 60 teeth, and the pinion has 30 Determine the design center distance. Now assume that the center distance is increased by 0.012 in. Determine the pressure angle for the new center distance.

Solution:

From Table 10.4, the center distance is given by:

$$C = r_{p2} + r_{p3} = \frac{N_2 + N_3}{2P_d} = \frac{30+60}{(2)(8)} = 5.625$$

The new center distance is:

$$C_m = C + e = 5.625 + 0.012 = 5.637$$

The pressure angle for the new center distance is:

$$\phi_m = \cos^{-1}\left(\frac{r_{b2}}{r_{pm2}}\right) = \cos^{-1}\left(\frac{r_{p2}\cos}{C_m\dfrac{N_2}{N_2+N_3}}\right) = \cos^{-1}\left(\frac{\dfrac{30}{(2)(8)}\cos 25°}{(5.637)\dfrac{30}{30+60}}\right) = 25.26°$$

Problem 10.10

Two standard gears have a diametral pitch of 2 and a pressure angle of $14\frac{1}{2}°$. The tooth numbers are 14 and 16. Determine whether interference occurs. If it does, compute the amount that the addendum(s) must be shortened to remove the interference, and the new contact ratio.

Solution:

Let gear 2 be the pinion. Then the condition of no interference in the pinion is as follows (see Fig. 10.6):

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi} \le r_{p2}\sin\phi$$

From Table 10.4, we know the addendum and the pitch radii:

$$a_2 = a_3 = \frac{1}{P_d}$$

$$r_{p2} = \frac{N_2}{2P_d}$$

and

$$r_{p3} = \frac{N_3}{2P_d}$$

The interference equation can then be rewritten as:

$$-\frac{N_3}{2P_d}\sin\phi + \sqrt{\left(\frac{1}{P_d}\right)^2 + 2\frac{1}{P_d}\frac{N_3}{2P_d} + \left(\frac{N_3}{2P_d}\right)^2\sin^2\phi} \le \frac{N_2}{2P_d}\sin\phi$$

Simplifying

$$-N_3\sin\phi + \sqrt{4 + 4N_3 + N_3^2\sin^2\phi} \le N_2\sin\phi$$

or

$$(N_2 + N_3)\sin\phi \ge \sqrt{4 + 4N_3 + N_3^2\sin^2\phi}$$

or

$$(N_2 + N_3)^2\sin^2\phi \ge 4 + 4N_3 + N_3^2\sin^2\phi$$

Simplifying further, the condition for no interference becomes

$$1 + N_3 \le \frac{N_2^2 + 2N_2N_3}{4}\sin^2\phi$$

Substitute the values, N_2=14, N_3=16, and ϕ =14.5. Since

$$1 + 16 \le \frac{14^2 + 2(14)(16)}{4} \sin^2 14.5°$$

Interference exists.

The conditions of no interference here should be:

$$u = -r_{p3} \sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2 \sin^2\phi} \le r_{p2} \sin\phi \tag{1}$$

$$v = -r_{p2} \sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2 \sin^2\phi} \le r_{p3} \sin\phi \tag{2}$$

From Table 10.4, we know the addendum and the pitch radii:

$$r_{p2} = \frac{N_2}{2P_d} = \frac{14}{2(2)} = 3.5$$

and

$$r_{p3} = \frac{N_3}{2P_d} = \frac{16}{2(2)} = 4$$

Substituting values into a modified form of Eqs. (1) and (2) gives,

$$a_3^2 + 2a_3(4) + 4^2 \sin^2 14.5° \le (3.4 + 4)^2 \sin^2 14.5° \Rightarrow a_{3_m} = 0.304$$

$$a_2^2 + 2a_2(3.5) + 3.5^2 \sin^2 14.5° \le (3.4 + 4)^2 \sin^2 14.5° \Rightarrow a_{2_m} = 0.374$$

so the addendums must be shortened by:

$$\Delta a_2 = a_2 - a_{2_m} = 0.5 - 0.374 = 0.126$$

$$\Delta a_3 = a_3 - a_{3_m} = 0.5 - 0.304 = 0.196$$

The contact line becomes:

$$\lambda = u + v = (r_{p2} + r_{p3}) \sin\phi = (3.5 + 4) \sin 14.5° = 1.878$$

The contact ratio is:

$$m_c = \frac{\lambda}{P_b} = \frac{1.878}{\frac{\pi}{2} \cos 14.5°} = 1.235$$

Problem 10.11

Two standard gears have 18 and 32 teeth, respectively. The diametral pitch is 10, and the pinion rotates at 1000 rpm. Determine the following: (a) center distance, (b) pitch diameters, (c) circular pitch, (d) pitch line velocity, (e) angular velocity of the gear.

Solution:

(a) From Table 10.4, we know the center distance:

$$C = \frac{N_2 + N_3}{2P_d} = \frac{18 + 32}{(2)(10)} = 2.5$$

(b) The pitch diameters:

$$d_{p2} = \frac{N_2}{P_d} = \frac{18}{10} = 1.8$$

and

$$d_{p3} = \frac{N_3}{P_d} = \frac{32}{10} = 3.2$$

(c) The circular pitch:

$$P_c = \frac{\pi}{P_d} = \frac{\pi}{10} = 0.314$$

(d) The pitch line velocity:

$$v_p = \omega_2 \cdot r_{p2} = \omega_2 \cdot \frac{d_{p2}}{2} = (1000)(\frac{2\pi}{60})(\frac{1.8}{2}) = 94.25 \text{ in / s}$$

(e) The angular velocity of the gear:

$$\omega_3 = \omega_2 \cdot \frac{N_2}{N_3} = 1000 \cdot \frac{18}{32} = 562.5 \text{ rpm}$$

Problem 10.12

Two standard gears have a diametral pitch of 10 and a velocity ratio of 2.5. The center distance is 3.5 in. Determine the number of teeth on each gear.

Solution:

From Table 10.4, we know the velocity ratio and the center distance:

$$R = \frac{r_{p3}}{r_{p2}} = 2.5 \Rightarrow r_{p3} = 2.5 r_{p2}$$

$$C = r_{p2} + r_{p3} = 3.5$$

so

$$r_{p2} = 1, \qquad r_{p3} = 2.5$$

From Table 10.4, the number of teeth on each gear is:

$$N_2 = 2P_d r_{p2} = (2)(10)(1) = 20$$

and

$$N_3 = 2P_d r_{p3} = (2)(10)(2.5) = 50$$

Problem 10.13

Is it possible to specify arbitrary values for the velocity ratio, center distance, and diametral pitch in a problem such as that given in Problem 10.12? Explain.

Solution:

If we specify arbitrary values, we always can get solutions from the equations in Problem 10.12, but it is possible to get a gear with too few teeth. Then interference will occur between the two gears.

Problem 10.14

Two standard meshing gears are to have 9 and 36 teeth, respectively. They are to be cut with a 20° full-depth cutter with a diametral pitch of 3. (a) Determine the amount that the addendum of the gear is to be shortened to eliminate interference. (b) If the addendum of the pinion is increased the same amount, determine the new contact ratio.

Solution:

The condition of of no interference is as follows:

$$a_3^2 + 2a_3 r_{p3} + r_{p3}^2 \sin^2\phi \le (r_{p2} + r_{p3})^2 \sin^2\phi$$

From Table 10.4, we know the pitch radii:

$$r_{p2} = \frac{N_2}{2P_d} = \frac{9}{(2)(3)} = 1.5$$

and

$$r_{p3} = \frac{N_3}{2P_d} = \frac{36}{(2)(3)} = 6$$

Substitute values for the pitch radii and $\phi = 20$ into the condition of no interference,

$$a_3^2 + 2a_3(6) + 6^2 \sin^2 20° \le (1.5 + 6)\sin 20° \Rightarrow a_{3_m} = 0.194$$

$$a_3 = \frac{1}{P_d} = \frac{1}{3} = 0.333$$

so the gear should be shortened by

$$\Delta a_3 = a_3 - a_{3_m} = 0.333 - 0.194 = 0.139$$

The pinion is increased by the same amount, so

$$\Delta a_2 = 0.139$$

and

$$a_{2_m} = a_2 + \Delta a_2 = 0.333 + 0.139 = 0.472$$

The length of the line of contact is:

$$\lambda = u + v = r_{p2} \sin\phi + \left(-r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2 \sin^2\phi}\right) = \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2 \sin^2\phi}$$

$$= \sqrt{0.472^2 + 2(0.472)(1.5) + 1.5^2 \sin^2 20^\circ} = 1.379$$

From Eq. (10.18), the contact ratio is

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos\phi} = \frac{1.379}{\frac{\pi}{3}\cos 20^\circ} = 1.4$$

Problem 10.15

Two standard meshing gears are to have 13 and 20 teeth, respectively. They are to be cut with a 20° full-depth cutter with a diametral pitch of 2. To reduce interference, the center distance is increased by 0.1 in. Determine the following:

a) Whether the interference is completely eliminated
b) The pitch diameters of both gears
c) The new pressure angle
d) The new contact ratio

Solution

The condition of no interference is as follows:

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2 \sin^2\phi} \le r_{p2}\sin\phi$$

The new center distance is:

$$C_m = C + e = \frac{N_2 + N_3}{2P_d} + e = \frac{13+20}{(2)(2)} + 0.1 = 8.35$$

and the new pitch radii are:

$$r_{pm2} = C_m \cdot \frac{N_2}{N_2 + N_3} = 8.35 \cdot \frac{13}{13+20} = 3.29$$

and

$$r_{pm3} = C_m \cdot \frac{N_3}{N_2 + N_3} = 8.35 \cdot \frac{20}{13+20} = 5.06$$

The pressure angle for the new center distance is:

$$\phi_m = \cos^{-1}\left(\frac{r_{b2}}{r_{pm2}}\right) = \cos^{-1}\left(\frac{r_{p2}\cos\phi}{r_{pm2}}\right) = \cos^{-1}\left(\frac{\frac{13}{(2)(2)}\cos 20°}{3.29}\right) = 21.83°$$

and the addendums are:

$$a_2 = a_3 = \frac{1}{P_d} = \frac{1}{2} = 0.5$$

(a) Substituting the relevant values into the condition of no interference, we have

$$\begin{aligned} u &= -r_{pm3}\sin\phi_m + \sqrt{a_3^2 + 2a_3 r_{pm3} + r_{pm3}{}^2\sin^2\phi_m} \\ &= -5.06\sin 21.83° + \sqrt{0.5^2 + 2(0.5)(5.06) + 5.06^2\sin^2 21.83°} = 1.093 \\ &\le r_{pm2}\sin\phi_m = 3.29\sin 21.83° = 1.223 \end{aligned}$$

So, the interference is completely eliminated.

(b) The pitch diameters of both gears:

$$d_{pm2} = 2r_{pm2} = (2)(3.29) = 6.58$$

and

$$d_{pm3} = 2r_{pm3} = (2)(5.06) = 10.12$$

(c) The new pressure angle is

$$\phi_m = 21.83°$$

(d) The new contact ratio:

$$\begin{aligned} v &= -r_{pm2}\sin\phi_m + \sqrt{a_2^2 + 2a_2 r_{pm2} + r_{pm2}{}^2\sin^2\phi_m} \\ &= -3.29\sin 21.83° + \sqrt{0.5^2 + 2(0.5)(3.29) + 3.29^2\sin^2 21.83°} = 1.021 \end{aligned}$$

The length of the line of contact is:

$$\lambda = u + v = 1.093 + 1.021 = 2.114$$

The contact ratio is:

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{d_{b2}}{N_2}} = \frac{\lambda}{\frac{(2r_{p2}\cos\ )}{N_2}} = \frac{2.114}{\frac{(2)(3.25)\cos 20°}{13}} = 1.432$$

Problem 10.16

Assume that you have a 13-tooth pinion and a 50-tooth gear. What is the smallest (nonstandard) pressure angle that can be used if interference is to be avoided? What is the smallest pressure angle that can be used if only standard pressure angles can be considered?

Solution:

The condition of no interference is as follows:

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2 \sin^2\phi} \le r_{p2}\sin\phi$$

or

$$a_3^2 + 2a_3 r_{p3} + r_{p3}^2 \sin^2\phi \le (r_{p2} + r_{p3})^2 \sin^2\phi$$

or

$$\sin\phi \ge \sqrt{\frac{a_3^2 + 2a_3 r_{p3}}{r_{p2}^2 + 2r_{p2} r_{p3}}} = \sqrt{\frac{1 + N_3}{\frac{N_2^2}{4} + \frac{N_2 N_3}{2}}} = \sqrt{\frac{1 + 50}{\frac{13^2}{4} + \frac{(13)(50)}{2}}} = 0.373$$

so $\phi \ge 21.88°$.

Therefore $\phi_{min} \ge 21.88°$.

If only standard pressure angles can be considered, $\phi_{min} = 25°$

Problem 10.17

A standard, full-depth spur gear has a pressure angle of 25° and an outside diameter of 225 mm. If the gear has 48 teeth, find the module and circular pitch.

Solution:

From Table 10.4, the outside diameter is:

$$d_o = d_p + 2a = N \cdot m + 2m = (N + 2)m$$

or

$$225 = (48 + 2)m \Rightarrow m = 4.5\,\text{mm}$$

Therefore the module is $m = 4.5$mm.

The circular pitch is:

$$P_c = \pi m = \pi(4.5) = 14.15\,\text{mm}$$

Problem 10.18

The pinion of a pair of spur gears has 16 teeth and a pressure angle of 20°. The velocity ratio is to be 3:2, and the module is 6.5 mm. Determine the initial center distance. If the center distance is increased by 3 mm, find the resulting pressure angle.

Solution:

From Table 10.4, the velocity ratio is given by:

$$R = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{3}{2} \Rightarrow N_3 = \frac{3}{2}N_2 = \frac{3}{2}(16) = 24$$

The center distance is given by:

$$C = \frac{N_2 + N_3}{2}m = \frac{16+24}{2}6.5 = 130\,\text{mm}$$

Therefore, the initial center distance is 130mm.

The new center distance is:

$$C_m = C + e = 130 + 3 = 133$$

The pressure angle for the new center distance is:

$$\phi_m = \cos^{-1}\left(\frac{r_{b2}}{r_{pm2}}\right) = \cos^{-1}\left(\frac{r_{p2}\cos\phi}{C_m \frac{N_2}{N_2+N_3}}\right) = \cos^{-1}\left(\frac{\frac{16}{2}(6.5)\cos 20^\circ}{(133)\frac{16}{16+24}}\right) = 23.29^\circ$$

Problem 10.19

Two standard, full-depth spur gears are to have 10 teeth and 35 teeth, respectively. The cutter has a 20˚ pressure angle with a module of 10 mm. Determine the amount by which the addendum of the gear must be reduced to avoid interference. Then determine the length of the new path of contact and the contact ratio.

Solution:

The condition of of no interference is as follows:

$$-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3r_{p3} + r_{p3}^2\sin^2\phi} \le r_{p2}\sin\phi$$

or

$$a_3^2 + 2a_3r_{p3} + r_{p3}^2\sin^2\phi \le (r_{p2} + r_{p3})^2\sin^2\phi$$

From Table 10.4, we know the pitch radii:

$$r_{p2} = \frac{N_2m}{2} = \frac{(10)(10)}{2} = 50$$

and

$$r_{p3} = \frac{N_3m}{2} = \frac{(35)(10)}{2} = 175$$

plug the pitch radii and $\phi = 20^\circ$ into the condition of no interference and simplifying,

$$a_3^2 + 2a_3(175) + 175^2\sin^2 20^\circ \le (50+175)^2\sin^2 20^\circ \Rightarrow a_{3m} = 6.56$$

$$a_3 = m = 10$$

so the gear should be shortened by

$$\Delta a_3 = a_3 - a_{3m} = 10 - 6.56 = 3.44$$

The length of the new path of contact is:

$$\lambda_m = u + v = r_{p3}\sin\phi + \left(-r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}{}^2\sin^2\phi}\right)$$

$$= \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}{}^2\sin^2\phi} = \sqrt{10^2 + 2(10)(50) + 50^2\sin^2 20^\circ} = 37.32$$

From Eq. (10.18), the new contact ratio is

$$m_{c_m} = \frac{\lambda_m}{P_b} = \frac{\lambda_m}{\pi m \cos\phi} = \frac{37.32}{\pi(10)\cos 20^\circ} = 1.264$$

Problem 10.20

A standard, full-depth spur gear tooth has been cut with a 20° hob and has a diametral pitch of 6. The tooth thickness at a radius of 2.1 in is 0.1860 in. Determine the thickness of the gear tooth at the base circle.

Solution:

From Table 10.4, we have the following relationships:

$$t_p = \frac{\pi}{2P_d} \tag{1}$$

$$\cos\beta = \frac{r_p \cos\phi}{r} \tag{2}$$

$$t = 2r\left[\frac{t_p}{2r_p} + \text{inv}\phi - \text{inv}\beta\right] \tag{3}$$

$$r_p = \frac{N}{2P_d} \tag{4}$$

From Eqs. (1), (3), and (4),

$$t = 2r\left[\frac{\frac{\pi}{2P_d}}{2\left(\frac{N}{2P_d}\right)} + \text{inv}\phi - \text{inv}\beta\right] = 2r\left[\frac{\pi}{2N} + \text{inv}\phi - \text{inv}\beta\right] \tag{5}$$

Combining Eqs. (2) and (4),

$$\cos\beta = \frac{\frac{N}{2P_d}\cos\phi}{r} = \frac{N}{2rP_d}\cos\phi \Rightarrow N = \frac{2rP_d\cos\beta}{\cos\phi} \tag{6}$$

Now combining Eqs. (6) and (5),

$$t = 2r\left[\frac{\pi}{2N} + \text{inv}\phi - \text{inv}\beta\right] = 2r\left[\frac{\pi}{2\dfrac{2rP_d\cos\beta}{\cos\phi}} + \text{inv}\phi - \text{inv}\beta\right] \tag{7}$$

or

$$t = 2r\left[\frac{\pi\cos\phi}{4rP_d\cos\beta} + \text{inv}\phi - \text{inv}\beta\right]$$

or

$$2r\left[\frac{\pi\cos\phi}{4rP_d\cos\beta} + \text{inv}\phi - \text{inv}\beta\right] - t = 0 \tag{8}$$

Equation (8) is nonlinear in β; however, it can be solved easily using a nonlinear equation solver. The answer is $\beta = 26.498588°$. Once β is known, we can solve for N from Eq. (6). Then,

$$N = \frac{2rP_d\cos\beta}{\cos\phi} = \frac{2(2.1)(6)\cos(26.498588°)}{\cos(20°)} = 24$$

The base circle radius is given by

$$r_b = r_p\cos\phi = \frac{N}{2P_d}\cos\phi = \frac{24}{2(6)}\cos(20°) = 1.879385$$

From Eq. (7), the thickness of the gear at the base circle is

$$t_b = 2r_b\left[\frac{\pi}{2N} + \text{inv}\phi - \text{inv}\beta_b\right]$$

The value of β at the base circle is zero. Therefore, the tooth thickness at the base circle is

$$t_b = 2r_b\left[\frac{\pi}{2N} + \text{inv}\phi - \text{inv}\beta_b\right] = 2(1.879385)\left[\frac{\pi}{2(24)} + \text{inv}(20°) - 0\right] = 0.31713 \text{ in.}$$

Problem 10.21

A pair of standard, full-depth spur gears has been cut with a 25° hob. The pinion has 31 teeth and the gear 60 teeth. The diametral pitch is 4. Find the velocity ratio, the pitch circle radii, the outside diameters, the center distance, and the contact ratio.

Solution:

From Table 10.4, the velocity ratio is given by:

$$R = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{60}{31} = 1.935$$

The pitch radii:

$$r_{p2} = \frac{N_2}{2P_d} = \frac{31}{(2)(4)} = 3.875$$

and

$$r_{p3} = \frac{N_3}{2P_d} = \frac{60}{(2)(4)} = 7.5$$

The outside diameters are:

$$d_{o2} = d_{p2} + 2a_2 = 2(3.875) + 2(0.25) = 8.25$$

$$d_{o3} = d_{p3} + 2a_3 = 2(7.5) + 2(0.25) = 15.5$$

The center distance is:

$$C = r_{p2} + r_{p3} = 3.875 + 7.5 = 11.375$$

The length of the line of contact is given by Eq. (10.17) as

$$\begin{aligned}\lambda &= -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2\sin^2\phi} - r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi} \\ &= -3.875\sin 25° + \sqrt{0.25^2 + 2(0.25)(3.875) + 3.875^2\sin^2 25°} \\ &\quad - 7.5\sin 25° + \sqrt{0.25^2 + 2(0.25)(7.5) + 7.5^2\sin^2 25°} = 1.079\end{aligned}$$

From Eq. (10.18), the contact ratio is

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos\phi} = \frac{1.079}{\frac{\pi}{4}\cos 25°} = 1.516$$

Problem 10.22

A pair of standard, full-depth spur gears has been cut with a 25° hob. The pinion has 14 teeth and the gear 51 teeth. The diametral pitch is 5. Find the velocity ratio, the pitch circle radii, the outside diameters, the center distance, and the contact ratio.

<u>Solution:</u>

From Table 10.4, the velocity ratio is given by:

$$R = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{51}{14} = 3.643$$

The pitch radii:

$$r_{p2} = \frac{N_2}{2P_d} = \frac{14}{(2)(5)} = 1.4$$

and

$$r_{p3} = \frac{N_3}{2P_d} = \frac{51}{(2)(5)} = 5.1$$

The outside diameters are:

$$d_{o2} = d_{p2} + 2a_2 = 2(1.4) + 2(0.2) = 3.2$$

$$d_{o3} = d_{p3} + 2a_3 = 2(5.1) + 2(0.2) = 10.6$$

The center distance is:

$$C = r_{p2} + r_{p3} = 1.4 + 5.1 = 6.5$$

The length of the line of contact is given by Eq. (10.17) as

$$\lambda = -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2\sin^2\phi} - r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi}$$

$$= -1.4\sin 25° + \sqrt{0.2^2 + 2(0.2)(1.4) + 1.4^2\sin^2 25°} - 5.1\sin 25° + \sqrt{0.2^2 + 2(0.2)(5.1) + 5.1^2\sin^2 25°} = 0.821$$

From Eq. (10.18), the contact ratio is

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos} = \frac{0.821}{\frac{\pi}{5}\cos 25°} = 1.442$$

Problem 10.23

A pair of standard, full-depth spur gears has been cut with a 20˚ hob. The pinion has 27 teeth and the gear 65 teeth. The diametral pitch is 2. Find the velocity ratio, the pitch circle radii, the outside diameters, the center distance, and the contact ratio.

Solution:

From Table 10.4, the velocity ratio is given by:

$$R = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{65}{27} = 2.407$$

The pitch radii:

$$r_{p2} = \frac{N_2}{2P_d} = \frac{27}{(2)(2)} = 6.75$$

and

$$r_{p3} = \frac{N_3}{2P_d} = \frac{65}{(2)(2)} = 16.25$$

The outside diameters are:

$$d_{o2} = d_{p2} + 2a_2 = 2(6.75) + 2(0.5) = 14.5$$

$$d_{o3} = d_{p3} + 2a_3 = 2(16.25) + 2(0.5) = 33.5$$

The center distance is:

$$C = r_{p2} + r_{p3} = 6.75 + 16.25 = 23$$

The length of the line of contact is given by Eq. (10.17) as

$$\lambda = -r_{p2}\sin\phi + \sqrt{a_2^2 + 2a_2 r_{p2} + {r_{p2}}^2\sin^2\phi} - r_{p3}\sin\phi + \sqrt{a_3^2 + 2a_3 r_{p3} + {r_{p3}}^2\sin^2\phi}$$

$$= -6.75\sin 20° + \sqrt{0.5^2 + 2(0.5)(6.75) + 6.75^2\sin^2 20°}$$
$$-16.25\sin 20° + \sqrt{0.5^2 + 2(0.5)(16.25) + 16.25^2\sin^2 20°} = 2.529$$

From Eq. (10.18), the contact ratio is

$$m_c = \frac{\lambda}{P_b} = \frac{\lambda}{\frac{\pi}{P_d}\cos} = \frac{2.529}{\frac{\pi}{2}\cos 20°} = 1.713$$