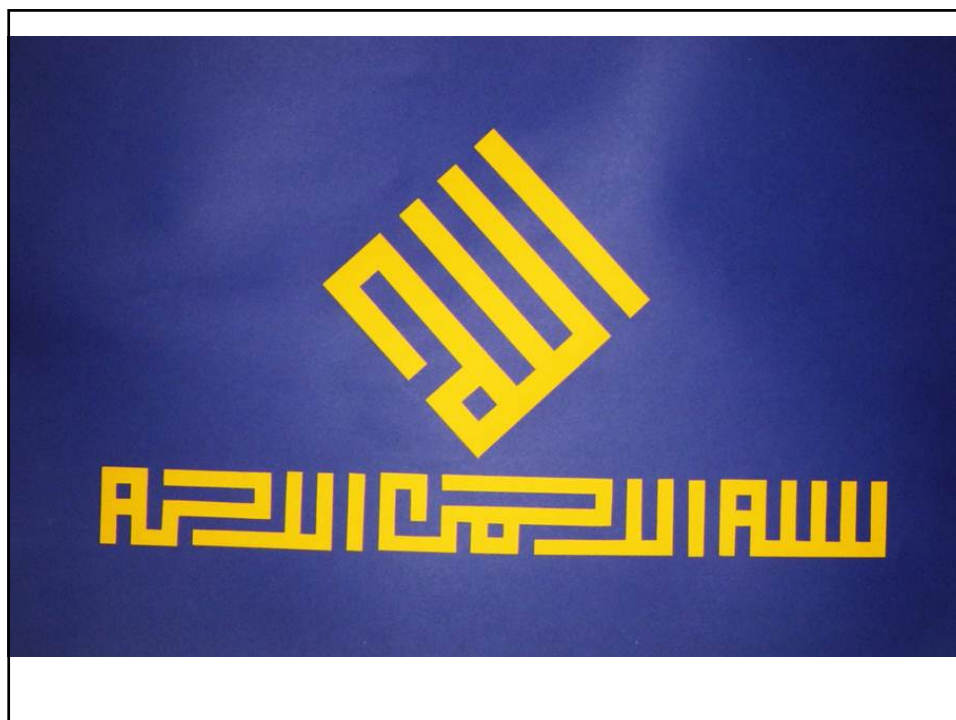




دانشکده فنی و مهندسی
گروه مهندسی عمران

پایداری سازه ها

(Structural Stability)

دکتر مهرداد حجازی

۱۳۸۹

فصل سوم

تیر - ستون ها (Beam-Columns)

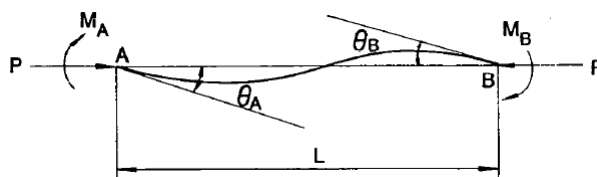


Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran

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VII - معادلات شیب - تغییر مکان (Slope-Deflection Equations)

- در این قسمت، معادلات شیب - تغییر مکان برای یک تیر - ستون (تحت اثر توام خمش و نیروی محوری) بدون اثبات بیان می شود.
- در اولین قسمت تیر - ستون بدون تغییر مکان نسبی دو انتها و فقط تحت ممان های انتهایی در نظر گرفته می شود.



Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran

University of Isfahan

مقادیر S_{ij} و S_{ii} برای مقادیر مختلف kL در جدول بعدی نشان داده شده است و در شکل بعد ترسیم شده است.



Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran

University of Isfahan

Table 3.7 Stability Functions ($kL = \pi\sqrt{P/P_e}$)

kL	P/P_e	compression		tension	
		S_{ii}	S_{ij}	S_{ii}	S_{ij}
0.	0.	4. 0000	2. 0000	4. 0000	2. 0000
0. 0500	0. 0003	3. 9997	2. 0001	4. 0003	1. 9999
0. 1000	0. 0010	3. 9987	2. 0003	4. 0013	1. 9997
0. 1500	0. 0023	3. 9970	2. 0008	4. 0030	1. 9993
0. 2000	0. 0041	3. 9947	2. 0013	4. 0053	1. 9987
0. 2500	0. 0063	3. 9917	2. 0021	4. 0083	1. 9979
0. 3000	0. 0091	3. 9876	2. 0028	4. 0120	1. 9970
0. 3500	0. 0124	3. 9833	2. 0039	4. 0157	1. 9956
0. 4000	0. 0162	3. 9786	2. 0054	4. 0211	1. 9946
0. 4500	0. 0205	3. 9729	2. 0068	4. 0268	1. 9932
0. 5000	0. 0253	3. 9665	2. 0084	4. 0332	1. 9917
0. 5500	0. 0306	3. 9595	2. 0102	4. 0401	1. 9900
0. 6000	0. 0365	3. 9517	2. 0121	4. 0477	1. 9881
0. 6500	0. 0428	3. 9433	2. 0143	4. 0560	1. 9861
0. 7000	0. 0496	3. 9342	2. 0166	4. 0649	1. 9839
0. 7500	0. 0570	3. 9244	2. 0191	4. 0744	1. 9816
0. 8000	0. 0648	3. 9139	2. 0218	4. 0846	1. 9791
0. 8500	0. 0732	3. 9027	2. 0246	4. 0954	1. 9764
0. 9000	0. 0821	3. 8908	2. 0277	4. 1069	1. 9737
0. 9500	0. 0914	3. 8782	2. 0309	4. 1189	1. 9707

Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran

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1. 0000	0. 1013	3. 8649	2. 0344	4. 1316	1. 9677
1. 0500	0. 1117	3. 8508	2. 0380	4. 1449	1. 9645
1. 1000	0. 1226	3. 8360	2. 0419	4. 1588	1. 9611
1. 1500	0. 1340	3. 8205	2. 0460	4. 1734	1. 9577
1. 2000	0. 1459	3. 8043	2. 0502	4. 1885	1. 9541
1. 2500	0. 1583	3. 7873	2. 0547	4. 2042	1. 9503
1. 3000	0. 1712	3. 7695	2. 0594	4. 2205	1. 9465
1. 3500	0. 1847	3. 7510	2. 0644	4. 2374	1. 9425
1. 4000	0. 1986	3. 7317	2. 0695	4. 2549	1. 9384
1. 4500	0. 2130	3. 7116	2. 0749	4. 2729	1. 9342
1. 5000	0. 2280	3. 6907	2. 0806	4. 2916	1. 9299
1. 5500	0. 2434	3. 6690	2. 0865	4. 3107	1. 9255
1. 6000	0. 2594	3. 6466	2. 0926	4. 3305	1. 9210
1. 6500	0. 2758	3. 6233	2. 0990	4. 3508	1. 9163
1. 7000	0. 2928	3. 5991	2. 1057	4. 3716	1. 9116
1. 7500	0. 3103	3. 5741	2. 1127	4. 3929	1. 9068
1. 8000	0. 3283	3. 5483	2. 1199	4. 4148	1. 9019
1. 8500	0. 3468	3. 5216	2. 1275	4. 4373	1. 8969
1. 9000	0. 3658	3. 4940	2. 1353	4. 4602	1. 8919
1. 9500	0. 3853	3. 4655	2. 1434	4. 4836	1. 8867



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2. 0000	0. 4053	3. 4361	2. 1519	4. 5076	1. 8815
2. 0500	0. 4258	3. 4058	2. 1607	4. 5320	1. 8762
2. 1000	0. 4468	3. 3745	2. 1699	4. 5569	1. 8708
2. 1500	0. 4684	3. 3422	2. 1794	4. 5823	1. 8654
2. 2000	0. 4904	3. 3090	2. 1893	4. 6082	1. 8599
2. 2500	0. 5129	3. 2748	2. 1996	4. 6345	1. 8544
2. 3000	0. 5360	3. 2395	2. 2102	4. 6613	1. 8488
2. 3500	0. 5595	3. 2032	2. 2213	4. 6886	1. 8431
2. 4000	0. 5836	3. 1659	2. 2328	4. 7163	1. 8374
2. 4500	0. 6082	3. 1274	2. 2447	4. 7444	1. 8317
2. 5000	0. 6333	3. 0878	2. 2572	4. 7730	1. 8259
2. 5500	0. 6588	3. 0471	2. 2701	4. 8020	1. 8201
2. 6000	0. 6849	3. 0052	2. 2834	4. 8314	1. 8142
2. 6500	0. 7115	2. 9622	2. 2974	4. 8612	1. 8083
2. 7000	0. 7386	2. 9179	2. 3118	4. 8915	1. 8024
2. 7500	0. 7662	2. 8723	2. 3268	4. 9221	1. 7965
2. 8000	0. 7944	2. 8254	2. 3425	4. 9531	1. 7905
2. 8500	0. 8230	2. 7772	2. 3587	4. 9845	1. 7845
2. 9000	0. 8521	2. 7276	2. 3756	5. 0162	1. 7785
2. 9500	0. 8817	2. 6766	2. 3932	5. 0484	1. 7725



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3. 0000	0. 9119	2. 6242	2. 4115	5. 0809	1. 7665
3. 0500	0. 9425	2. 5703	2. 4305	5. 1137	1. 7605
3. 1000	0. 9737	2. 5148	2. 4503	5. 1469	1. 7544
3. 1500	1. 0054	2. 4577	2. 4709	5. 1805	1. 7484
3. 2000	1. 0375	2. 3990	2. 4924	5. 2143	1. 7424
3. 2500	1. 0702	2. 3385	2. 5148	5. 2485	1. 7363
3. 3000	1. 1034	2. 2763	2. 5382	5. 2831	1. 7303
3. 3500	1. 1371	2. 2122	2. 5626	5. 3179	1. 7243
3. 4000	1. 1713	2. 1463	2. 5880	5. 3530	1. 7183
3. 4500	1. 2060	2. 0783	2. 6146	5. 3885	1. 7123
3. 5000	1. 2412	2. 0083	2. 6424	5. 4242	1. 7063
3. 5500	1. 2769	1. 9362	2. 6714	5. 4603	1. 7003
3. 6000	1. 3131	1. 8618	2. 7017	5. 4966	1. 6944
3. 6500	1. 3498	1. 7851	2. 7335	5. 5332	1. 6884
3. 7000	1. 3871	1. 7060	2. 7668	5. 5701	1. 6825
3. 7500	1. 4248	1. 6243	2. 8016	5. 6073	1. 6766
3. 8000	1. 4631	1. 5400	2. 8382	5. 6447	1. 6708
3. 8500	1. 5018	1. 4528	2. 8765	5. 6823	1. 6649
3. 9000	1. 5411	1. 3627	2. 9168	5. 7203	1. 6591
3. 9500	1. 5809	1. 2696	2. 9592	5. 7584	1. 6533



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4. 0000	1. 6211	1. 1731	3. 0037	5. 7968	1. 6476
4. 0500	1. 6619	1. 0733	3. 0507	5. 8355	1. 6419
4. 1000	1. 7032	0. 9698	3. 1001	5. 8744	1. 6362
4. 1500	1. 7450	0. 8624	3. 1523	5. 9135	1. 6305
4. 2000	1. 7873	0. 7510	3. 2074	5. 9528	1. 6249
4. 2500	1. 8301	0. 6353	3. 2656	5. 9923	1. 6193
4. 3000	1. 8734	0. 5149	3. 3273	6. 0321	1. 6138
4. 3500	1. 9172	0. 3897	3. 3926	6. 0720	1. 6083
4. 4000	1. 9616	0. 2592	3. 4619	6. 1122	1. 6028
4. 4500	2. 0064	0. 1231	3. 5356	6. 1526	1. 5974
4. 5000	2. 0518	-0. 0191	3. 6140	6. 1931	1. 5920
4. 5500	2. 0976	-0. 1678	3. 6975	6. 2339	1. 5867
4. 6000	2. 1440	-0. 3234	3. 7866	6. 2748	1. 5814
4. 6500	2. 1908	-0. 4867	3. 8819	6. 3159	1. 5761
4. 7000	2. 2382	-0. 6582	3. 9839	6. 3572	1. 5709
4. 7500	2. 2861	-0. 8387	4. 0934	6. 3987	1. 5658
4. 8000	2. 3344	-1. 0289	4. 2112	6. 4403	1. 5606
4. 8500	2. 3833	-1. 2299	4. 3381	6. 4821	1. 5556
4. 9000	2. 4327	-1. 4427	4. 4751	6. 5241	1. 5505
4. 9500	2. 4826	-1. 6685	4. 6235	6. 5662	1. 5456



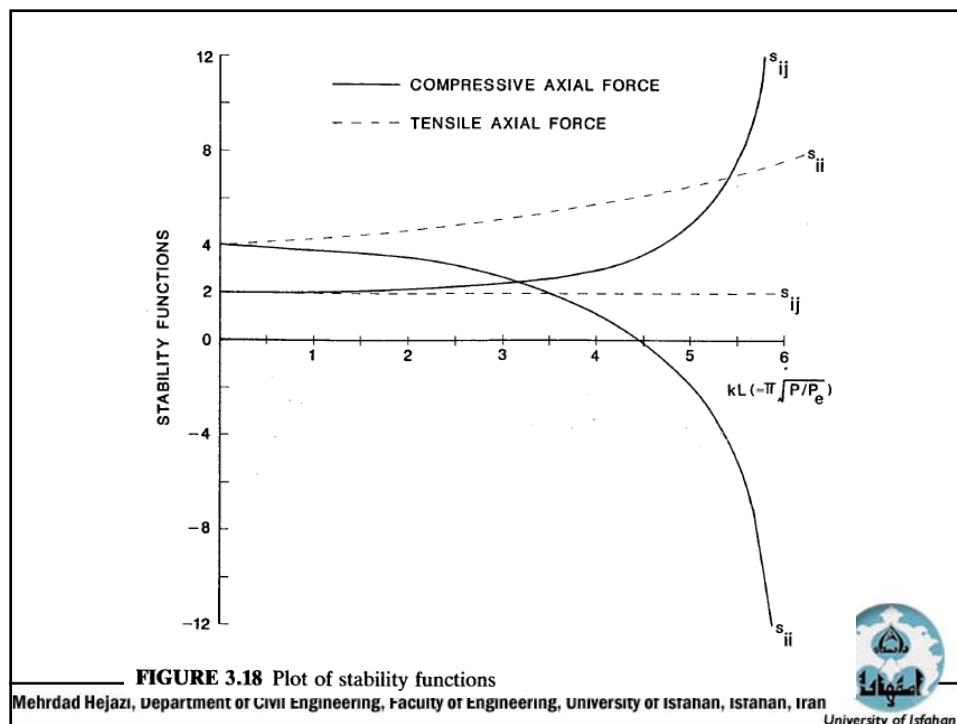
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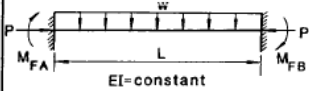
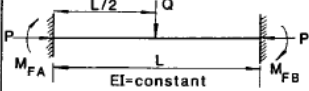
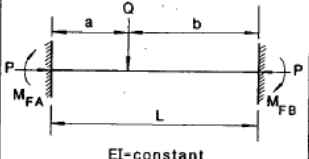
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5. 0000	2. 5330	-1. 9087	4. 7845	6. 6085	1. 5406
5. 0500	2. 5839	-2. 1651	4. 9599	6. 6509	1. 5357
5. 1000	2. 6354	-2. 4394	5. 1514	6. 6934	1. 5309
5. 1500	2. 6873	-2. 7341	5. 3613	6. 7362	1. 5261
5. 2000	2. 7397	-3. 0516	5. 5921	6. 7790	1. 5213
5. 2500	2. 7927	-3. 3953	5. 8470	6. 8220	1. 5166
5. 3000	2. 8461	-3. 7689	6. 1297	6. 8652	1. 5120
5. 3500	2. 9001	-4. 1770	6. 4447	6. 9084	1. 5074
5. 4000	2. 9545	-4. 6254	6. 7977	6. 9518	1. 5028
5. 4500	3. 0095	-5. 1210	7. 1957	6. 9953	1. 4983
5. 5000	3. 0650	-5. 6727	7. 6472	7. 0390	1. 4938
5. 5500	3. 1209	-6. 2916	8. 1635	7. 0827	1. 4894
5. 6000	3. 1774	-6. 9923	8. 7589	7. 1266	1. 4851
5. 6500	3. 2344	-7. 7937	9. 4524	7. 1706	1. 4807
5. 7000	3. 2919	-8. 7215	10. 2693	7. 2147	1. 4765
5. 7500	3. 3499	-9. 8106	11. 2447	7. 2590	1. 4722
5. 8000	3. 4084	-11. 1107	12. 4279	7. 3033	1. 4680
5. 8500	3. 4675	-12. 6943	13. 8915	7. 3477	1. 4639
5. 9000	3. 5270	-14. 6717	15. 7455	7. 3922	1. 4598
5. 9500	3. 5870	-17. 2192	18. 1662	7. 4369	1. 4558
6. 0000	3. 6476	-20. 6379	21. 4544	7. 4816	1. 4518
6. 0500	3. 7086	-25. 4868	26. 1690	7. 5264	1. 4478
6. 1000	3. 7702	-32. 9355	33. 4794	7. 5714	1. 4439
6. 1500	3. 8322	-45. 9092	46. 3106	7. 6164	1. 4401
6. 2000	3. 8948	-74. 3671	74. 6217	7. 6615	1. 4363
6. 2500	3. 9579	-188. 3001	188. 4032	7. 7067	1. 4325

Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran

University of Isfahan



Expressions for Fixed-End Moments ($u = \frac{L}{2} \sqrt{\frac{P}{EI}}$)	
Case	Fixed-End Moments
 $EI = \text{constant}$	$M_{FA} = -M_{FB} = -\frac{wL^2}{12} \left[\frac{3(\tan u - u)}{u^2 \tan u} \right]$
 $EI = \text{constant}$	$M_{FA} = -M_{FB} = -\frac{QL}{8} \left[\frac{2(1 - \cos u)}{u \sin u} \right]$
 $EI = \text{constant}$	$M_{FA} = -\frac{QL}{d} \left[\frac{2ub}{L} \cos 2u - 2u \cos \frac{2ub}{L} - \sin 2u \right. \\ \left. + \sin \frac{2ua}{L} + \sin \frac{2ub}{L} + \frac{2ua}{L} \right]$ $M_{FB} = \frac{QL}{d} \left[\frac{2ua}{L} \cos 2u - 2u \cos \frac{2ua}{L} - \sin 2u \right. \\ \left. + \sin \frac{2ub}{L} + \sin \frac{2ua}{L} + \frac{2ub}{L} \right]$ $d = 2u(2 - 2 \cos 2u - 2u \sin 2u)$

Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran



Beam-Columns

End

Mehrdad Hejazi, Department of Civil Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran

