باسمه تعالی

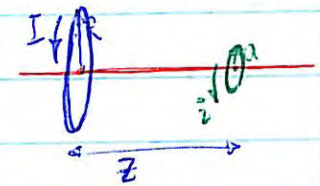
الف) میدان را بر روی محور حلقه بدست آورید.

$$B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}$$

ب) مطابق شکل دو حلقه هم محور را در نظر بگیرید. نیروی وارد بر حلقه‌ی کوچکتر را بیابید.

$$dB_z \times \pi\rho^2 + B_\rho \times 2\pi\rho\, dz = 0 \to B_\rho = -\frac{\rho}{2}\frac{dB_z}{dz} \to B_\rho = -\frac{3\mu_0 R^2 I z\rho}{4(R^2+z^2)^{5/2}}$$

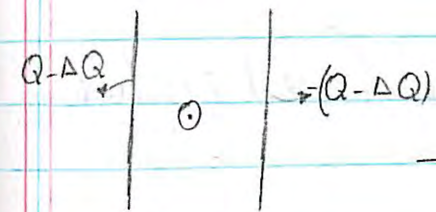
$$d\vec F_z = i(d\vec L \times \vec B_\rho) \to \vec F_z = -\frac{3\mu_0 i \pi z I R^2 a^2}{2(R^2+z^2)^{5/2}}\hat z$$



مثال: یک کره‌ی رسانا بین صفحات یک خازن قرار می‌دهیم. تغییر پتانسیل دو سر خازن چقدر است؟

دو شرایط فیزیکی متفاوت می‌گیریم. Q ثابت / V ثابت

V ثابت



$P = L\Delta Q$, $q = CV \to dq = cdV + Vdc$

$$\to -\Delta Q = V\Delta C \to -\frac{P}{L} = \epsilon_0 L\,\Delta C \to$$

$$-\frac{4\pi\epsilon_0 a^3 E_0}{L} = \epsilon_0 L\Delta C \to \Delta C = -\frac{4\pi\epsilon_0 a^3}{L^2}$$

بسمه تعالی

$$cdV + Vdc = 0 \to \Delta V = -\frac{V\Delta C}{C} \to \Delta V = \left(\frac{4\pi a^3}{LA}\right)V \leftarrow$$ Q ثابت

مثال: حلقه‌ای به شعاع r دارای بار Q است. اگر کمان بسیار کوچکی از آن را برداریم شدت میدان الکتریکی در محل سابق کمان را بدست آورید.

$$m\frac{d\vec v}{dt} = q\vec E \to \vec v_f - \vec v_i = \frac{q\vec E}{m}\tau \to \bar{\vec v} = \frac{\vec v_i+\vec v_f}{2} = \vec v_i + \frac{q\vec E}{2m}\tau$$

$$\to \vec v_D = \frac{q\tau}{2m}\vec E \quad , \quad \bar{\vec v_i} = 0 \quad , \quad \tau = \bar\tau$$

$$m\frac{d\vec v}{dt} = \vec F - \alpha\vec v \quad , \quad t\to\infty: \vec v \to \vec v_D \to \alpha = \frac{2m}{\tau}$$

$$\frac{d\vec v}{dt} + \frac{2}{\tau}\vec v = \frac{\vec F}{m} = \vec f \quad , \quad e^{-\frac{2t}{\tau}}\vec u = \vec v \to$$

$$\frac{d\vec u}{dt} = e^{\frac{2t}{\tau}}\vec f_{(t)} \to \vec v_{(t)} = \vec v_{(0)}e^{-\frac{2t}{\tau}} + \int_0^t \vec f_{(t')}\, e^{\frac{2(t'-t)}{\tau}}\,dt'$$

$$m\frac{d\vec v}{dt} = \vec F + \vec A_{(\vec v)} \quad , \quad \vec v = \vec v_{(0)} : \vec F = \frac{2m\vec v_D}{\tau} + \vec A_{(\vec v_D)} = 0 \to \vec A_{(\vec v_D)} = -\frac{2m\vec v_D}{\tau}$$

برای زمان های دور $\frac{d\vec v}{dt}$ صفر است.

$D^n X + D^{n-1} X a_{n-1} + \cdots + a_0 X = f_{(t)} \to$

معادله مشخصه: $(s^n + a_{n-1} s^{n-1} + \cdots + a_0) e^{st} = 0 \to$ n ریشه مختلط

$\frac{d}{d\theta}(ae^{a\theta}) = ae^{a\theta}$, $e^0 = 1$; $e^{i\theta}$ را طوری تعریف می‌کنیم که داشته باشیم:

$e^{i\theta} = f_{(\theta)} + i g_{(\theta)} \to f_{(\theta)} = \cos\theta$, $g_{(\theta)} = \sin\theta$

$s = b + i\omega$, $\bar{s} = b - i\omega$ اگر s جواب بود $\bar{s}$ نیز جواب است

$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

$(D^n + a_{n-1} D^{n-1} + \cdots + a_0) y = f$ | $[P_{(D)}] y^H = 0$, $y^H = e^{s_1 t} Z_{(t)}$

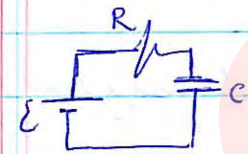
$P_{(D)} = (D - s_1)^{k_1} \cdots (D - s_l)^{k_l}$ | $[P_{(D)}](e^{s_1 t} Z_{(t)}) = 0$

$P_{(s)} = (s - s_1) \cdots (s - s_n)$

$K_1 + K_2 + \cdots + K_l = n$

$\vec{v} = \frac{\tau}{2m} \vec{F} \to \vec{j} = \frac{\tau n q^2}{2m} \vec{E} \to \vec{j} = \sigma \vec{E}$

$\vec{E}_{(r)} = (V_0 - V_1) \vec{e}_{(\vec{r})} \to \vec{j} = \sigma (V_0 - V_1) \vec{e}_{(\vec{r})} \to I = \sigma (V_0 - V_1) \int \vec{e}_{(\vec{r})} ds$



$\frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \cdots + a_0 x = f$, $D = \frac{d}{dt}$

$\to (D^n + a_{n-1} D^{n-1} + \cdots + a_0) x^P = f \to (D^n + a_{n-1} D^{n-1} + \cdots + a_0)(x - x^P) = 0$

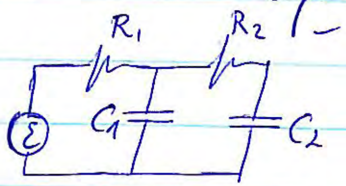
$(D^n + a_{n-1} D^{n-1} + \cdots + a_0) x = f$

$x = x^P + x^H$ ← جواب عمومی معادله همگن

جواب خاص

$\frac{dx}{dt} = f_{(x)}$, $\begin{cases} \forall \varepsilon > 0 : \exists \delta > 0 \quad |x - x_0| < \delta \to |f_{(x)} - f_{(x_0)}| < \varepsilon & \text{: پیوسته} \\ \forall M > 0 : \exists \varepsilon > 0 \quad |x - x_0| < M\varepsilon \to \quad // & \text{لیپ‌شیتز پیوسته} \end{cases}$

اگر $P_{(b)}=0$ در این صورت باید یک t در $\tilde{Q}_{(t)}$ ضرب کنیم و اگر جواب مضاعف معادله بود t^2 ضرب می‌کنیم.



$$\begin{cases} C_1\dfrac{dV_1}{dt}+\dfrac{V_1-\mathcal{E}}{R_1}+\dfrac{V_1-V_2}{R_2}=0 \\ C_2\dfrac{dV_2}{dt}+\dfrac{V_2-V_1}{R_2}=0 \end{cases} \rightarrow \begin{cases} C_1\dfrac{dV_1}{dt}+\dfrac{V_1}{R_1}+\dfrac{V_1-V_2}{R_2}=0 \\ C_2\dfrac{dV_2}{dt}+\dfrac{V_2-V_1}{R_2}=0 \end{cases}$$

$$\rightarrow \underbrace{\begin{pmatrix} C_1D+\frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & C_2D+\frac{1}{R_2} \end{pmatrix}}_{P_{(D)}} \underbrace{\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}}_{\tilde{V}} = \begin{pmatrix} \frac{\mathcal{E}}{R_1} \\ 0 \end{pmatrix} \rightarrow$$

$$[P_{(D)}]\,V=\mathcal{E} \rightarrow [P_{(D)}]\,V^P=\mathcal{E} \quad , \quad [P_{(D)}]\,V^H=0$$

$$V^H_{(t)}=V^H e^{st} \rightarrow [P_{(s)}]\,V^H e^{st}=0 \rightarrow [P_{(s)}]\,V^H=0 \rightarrow$$

$$D(e^{s_1t}Z_{(t)})=S_1e^{s_1t}Z_{(t)}+e^{s_1t}DZ_{(t)}=e^{s_1t}(D+S_1)Z_{(t)}$$

$$D^2(e^{s_1t}Z_{(t)})=D\left(e^{s_1t}\underbrace{(D+S_1)Z_{(t)}}_{w_{(t)}}\right)=e^{s_1t}(D+S_1)(D+S_1)Z_{(t)}$$

$$[P_{(D)}](e^{s_1t}Z_{(t)})=e^{s_1t}P_{(D+S_1)}Z_{(t)}$$

$$P_{(D+S_1)}=D^{K_1}\dots(D+S_1-S_l)^{K_l} \rightarrow$$

$$(D^{K_1}\dots(D+S_1-S_l)^{K_l})Z_{(t)}=0 \rightarrow (\dots D^{K_1})Z_{(t)}=0 \rightarrow$$

$$Z_{(t)}=\{1,t,\dots,t^{K_1-1}\} \rightarrow y^H_{(t)}=\{e^{s_1t},te^{s_1t},t^{K_1-1}e^{s_1t}\}$$

اگر $f_{(t)}$ بصورت $Q_{(t)}e^{bt}$ باشد در این صورت جواب خصوصی می‌شود: $y^P_{(t)}=\tilde{Q}_{(t)}e^{bt}$ چندجمله‌ای از درجه l و $P_{(b)}\neq 0$

$$P_{(D)}[e^{bt}\tilde{Q}_{(t)}]=e^{bt}[P_{(D+b)}]\tilde{Q}_{(t)}=e^{bt}Q_{(t)} \rightarrow$$

$$[P_{(D+b)}]\tilde{Q}=Q$$

$\det[P_{(s_1)}]=0$

$\mathbb{F}$ میدان — $\mathbb{V}$ بردارها

$+$ — $\times$

بخش

$\mathbb{F}$	$\mathbb{V}$
بسته	بسته
جابه‌جایی	جابه‌جایی
شرکت‌پذیر	شرکت‌پذیر
عضو خنثی	عضو خنثی
	عضو وارون

$\mathbb{F}\times\mathbb{V}\to\mathbb{V}$

$a\times v = v\times a \to$ تعریف نشان دادن است

$a\times'(b\times' v) = (a\times b)\times' v$

$(a+b)\times' v = (a\times' v)+(b\times' v)$

$a\times'(u+v) = a\times' u + a\times' v$

$1\times' \underbrace{v}_{\text{بردار}} = v$

$f_1,\dots,f_l$ خطی مستقل‌اند اگر $\sum_i \underbrace{\alpha_i}_{\text{عدد}} f_i = 0 \to$ α_i ها صفر شوند

اگر $e_1,\dots,e_n$ خطی مستقل باشند و هر vای به این مجموعه اضافه کنیم خطی وابسته شود می‌توان ثابت کرد هر برداری را می‌توان برحسب $e_1,\dots,e_n$ نوشت و یکتا نوشت.

اگر برداری داشته باشیم در فضا که هر برداری را بتوانیم برحسب آن بنویسیم تمامی بردارهای فضا خطی وابسته‌اند.

n تعداد بعد فضاست.

$g: \mathbb{V}\to\mathbb{W}$, $\quad e_1,e_2,\dots,e_n \to f_1,f_2,\dots,f_n$

خطی: $g(\alpha u+\beta v)=\alpha g_{(u)}+\beta g_{(v)}$

$g_{(u)} = g\left(\sum_i u_i e_i\right) = \sum_i u_i g_{(e_i)} = \sum_{i,a} u_i g_{ai} f_a =$

$\sum_a [g_{(u)}]_a f_a \to [g_{(u)}]_a = \sum_i g_{ai} u_i$

$W \in img(T)$ $\qquad f_1, \dots, f_k$ $\quad$ پایه‌های $img(T)$

$W = \sum_a W_a f_{(a)} \qquad \exists\, e_a \in V \qquad T(e_a) = f_{(a)}$

$V \supset e_1, \dots, e_k$

$\alpha_1 e_1 + \dots + \alpha_k e_k = 0 \to \alpha_1 T_{e_1} + \dots + \alpha_k T_{e_k} = 0 \to$

$\alpha_1 f_1 + \dots + \alpha_k f_{(k)} = 0$

$A_{(t)} = a\cos(\omega t + \varphi) = Re\left[a e^{i\varphi} e^{-i\omega t}\right] = Re\left(A e^{-i\omega t}\right)$

$V_{(t)} = R I_{(t)} \quad , \quad Re\left(V e^{-i\omega t}\right) = R\, Re\left(\mathcal{I} e^{-i\omega t}\right)$

$\to V = R\mathcal{I}$

یادوری بحث حالات چند قطبی بالاتر داریم:

$d\phi = K \dfrac{(\vec{r} - \vec{r}')\cdot \vec{p}_{(\vec{r}')}}{|\vec{r} - \vec{r}'|^3} \quad , \quad \vec{P}\,\Delta V = \vec{p}$

عدد: عنصرهای ماتریسی g

بردار

$g_{(e_i)} = \sum_a g_{ai} f_{(a)}$

$h: U \to V \qquad g\circ h: U \to W$

$d_1, \dots, d_l$

$(g\circ h)(d_\alpha) = g[h_{(d_\alpha)}] = g\left[\sum_i h_{i\alpha} e_i\right] = \sum_i h_{i\alpha} g_{(e_i)}$

$= \sum_{i,a} h_{i\alpha} g_{ai} f_a = \sum_a (g\circ h)_{a\alpha} f_{(a)}$

$(g\circ h)_{a\alpha} = \sum_i h_{i\alpha} g_{ai} = \sum_i g_{ai} h_{i\alpha}$

$T: V \to W$

$ker_{(T)} = \{u \in V \mid T_u = 0\}$

$img_{(T)} = \{T_u \mid u \in V\}$

$\underbrace{dim}_{\text{بعد}}\left(\underbrace{dom(T)}_{\text{دامنه}}\right) = dim(ker(T)) + dim(img(T))$

$$= \sum_j P_j \partial_i E_j = \sum_j \varepsilon_0 \chi E_j \partial_i E_j = \frac{\varepsilon_0 \chi}{2} \sum_j \partial_i (\vec{E}_j \cdot \vec{E}_j)$$

$$\vec{F} = \frac{\varepsilon_0 \chi}{2} \int \sum_j \partial_i (E_j^2)\, dx\, dy\, dz$$

دی الکتریک

اگر z ~~؟~~ به اندازه‌ی کافی نزدیک به رسانا نباشد، نیروی z و y صفر می‌شوند

و همچنین هیچ لبه‌ای به لبه‌ی دیگر نزدیک نباشد.

$$\mathrm{I} \xrightarrow{\text{بار ثابت}} \mathrm{II} \qquad \mathrm{I} \xrightarrow{\text{پتانسیل ثابت}} \mathrm{III} \qquad \mathrm{II} \xrightarrow{\text{جسم ثابت}} \mathrm{III}$$

$\cancel{\Delta\phi_{\mathrm{I}\to\mathrm{II}} = 0}$ $\quad \Delta Q_{\mathrm{I}\to\mathrm{II}} = 0$

$$\Delta\phi_{\mathrm{I}\to\mathrm{III}} = 0$$

$$\Delta \xi_{\mathrm{II}\to\mathrm{III}} = 0 \,, \quad \Delta Q_{\mathrm{II}\to\mathrm{III}} = \Delta Q_{\mathrm{I}\to\mathrm{III}} \quad \Big| \quad \Delta \xi_{\mathrm{I}\to\mathrm{III}} = \Delta \xi_{\mathrm{I}\to\mathrm{II}}$$

$$\Delta\phi_{\mathrm{II}\to\mathrm{III}} = -\Delta\phi_{\mathrm{I}\to\mathrm{II}}$$

$$\Delta U_{\mathrm{I}\to\mathrm{III}} = \Delta U_{\mathrm{I}\to\mathrm{II}} + \Delta U_{\mathrm{II}\to\mathrm{III}} = \frac{1}{2} Q \cdot \phi_{\mathrm{I}\to\mathrm{II}} +$$

$$\frac{1}{2}\left(\phi \Delta Q_{\mathrm{II}\to\mathrm{III}} + Q \Delta\phi_{\mathrm{II}\to\mathrm{III}}\right) = \frac{1}{2} Q \Delta\phi_{\mathrm{I}\to\mathrm{II}} + Q \Delta\phi_{\mathrm{II}\to\mathrm{III}}$$

$$m\ddot{\vec{r}} = q\vec{E} - m\gamma\dot{\vec{r}} - m\omega_0^2 \vec{r}$$

$$\rightarrow \vec{r}_f = \frac{q\vec{E}_f}{m(\omega_0^2 - \omega^2 - i\gamma\omega)} \rightarrow \vec{p} = \frac{nq^2 \Delta V}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}$$

$$\rightarrow \chi = \frac{nq^2}{m\varepsilon_0(\omega_0^2 - \omega^2 - i\gamma\omega)} \,, \quad \chi = \sum \frac{n_a q_a^2}{m_a \varepsilon_0 (\omega_a^2 - \omega^2 - i\gamma_a\omega)}$$

$$\sum f_a = 1 \,, \quad \frac{n_a}{n} = f_a \rightarrow \chi = \frac{nq^2}{\varepsilon_0 m} \sum_a \frac{f_a}{\omega_a^2 - \omega^2 - i\gamma_a\omega}$$

$\rightarrow \omega = 0 ; \quad \chi = \frac{nq^2}{m\varepsilon_0} \sum \frac{f_a}{\omega_a^2} > 0 \rightarrow$ البته اگر ω_a صفر شود

نمی‌توانیم قطعی بگوییم.

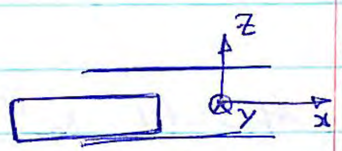
$$\omega_0 = 0 \,, \quad \omega \to 0$$

$$\rightarrow -i\omega\chi = \frac{nq^2}{\varepsilon_0 m} \times \frac{f_0}{\gamma_0} \xrightarrow{\frac{nq\dot{\vec{r}}}{\varepsilon_0}} \vec{J} = \left(\frac{f_0 n q^2}{m\gamma_0}\right)\vec{E}$$

$$-i\omega\chi\vec{E} \Rightarrow \frac{d\vec{r}}{dt}$$

$$\omega \to \infty \,, \quad \chi = -\frac{nq^2}{m\varepsilon_0\omega^2} < 0$$

$$\vec{f} = \vec{P} \cdot \nabla \vec{E} \Rightarrow \sum_j P_j \partial_j E_i = f_i$$



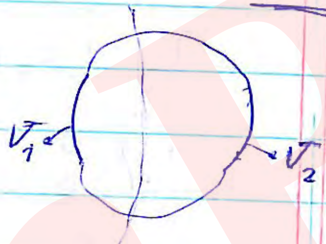
$$\vec{E} = \hat{n}(\vec{E}\cdot\hat{n}) + \hat{t}(\vec{E}\cdot\hat{t}) = \frac{\hat{n}(\vec{E}_0\cdot\hat{n})\varepsilon_0}{\varepsilon} + \hat{t}(\vec{E}_0\cdot\hat{t})$$

$$= \vec{E}_0 + [\hat{n}(\vec{E}_0\cdot\hat{n})](\frac{\varepsilon_0}{\varepsilon} - 1)$$

$$\vec{P} = (\varepsilon-\varepsilon_0)\vec{E} = (\varepsilon-\varepsilon_0)(\vec{E}_0 + \frac{\varepsilon_0-\varepsilon}{\varepsilon}\hat{n}(\vec{E}_0\cdot\hat{n})) \rightarrow$$

$$\tilde{U} = -\frac{1}{2}V\left[(\varepsilon-\varepsilon_0)\vec{E}_0\cdot\vec{E}_0 - \frac{(\varepsilon-\varepsilon_0)^2}{\varepsilon}(\vec{E}_0\cdot\hat{n})^2\right]$$

$$\begin{cases}\vec{\nabla}\cdot(\sigma\vec{E}) = 0 \\ \vec{\nabla}\cdot(\varepsilon\vec{E}) = 0\end{cases} \xrightarrow{\text{اگر } \sigma_{(r)} = \alpha\varepsilon_{(r)}} \phi_1 = \phi_2$$



$$I = \int \sigma\vec{E}\cdot\vec{ds} = \alpha\int\varepsilon\vec{E}\cdot\vec{ds} = \alpha Q \rightarrow RC = \frac{1}{\alpha}$$

$$\begin{cases}\vec{\nabla}\cdot\vec{P} = -\rho_b \\ \vec{P}\cdot\hat{n} = \sigma_b\end{cases} \quad , \quad \vec{P} = \varepsilon_0\chi\vec{E}$$ برای:

$$\begin{cases}\vec{P} = \sum_{n=1}^{\infty}\chi^n\vec{P}_n \\ \vec{E} = \sum_{n=1}^{\infty}\chi^{n-1}\vec{E}_{n-1}\end{cases} \longrightarrow \vec{P}_n = \varepsilon_0\vec{E}_{n-1}$$

$$= -\frac{1}{2}Q\Delta\phi_{I\to II} = -\Delta U_{I\to II}$$

$$F = -\frac{\partial U}{\partial z}\Big|_{\text{بار ثابت}} = \frac{\partial U}{\partial z}\Big|_{\text{پتانسیل ثابت}} \quad , \quad U = \frac{1}{2}\int\rho\phi\, dv$$

$$U = \frac{1}{2}\int\vec{E}\cdot\vec{D}\,dv \Rightarrow \frac{1}{2}\left[\int\vec{E}\cdot\vec{D}\,dv - \int\vec{E}_0\cdot\vec{D}_0\,dv\right] = \tilde{U}$$

$\tilde{U} = U - U_0$ U_0 برای حالتی است که دی الکتریک را برداشته ایم.

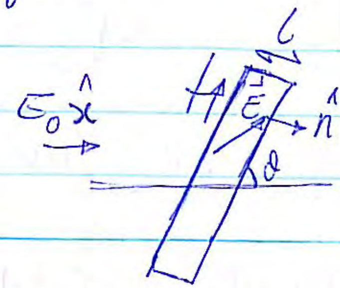
$$\Rightarrow \tilde{U} = \frac{1}{2}\left[\int\phi\,\vec{\nabla}\cdot\vec{D}\,dv - \int\phi_0\,\vec{\nabla}\cdot\vec{D}_0\,dv\right] =$$

$$\frac{1}{2}\left[\int\phi\,\vec{\nabla}\cdot\vec{D}_0\,dv - \int\phi_0\,\vec{\nabla}\cdot\vec{D}\,dv\right] = \frac{1}{2}\left[\int(\vec{E}\cdot\vec{D}_0 - \vec{E}_0\cdot\vec{D})\,dv\right]$$

$$= \frac{1}{2}\int\vec{E}_0\cdot(\varepsilon_0\vec{E} - \vec{D})\,dv = -\frac{1}{2}\int\vec{E}_0\cdot\vec{P}\,dv$$

$$\vec{E}_0\cdot\hat{t} = \vec{E}\cdot\hat{t}$$

$$\varepsilon_0\vec{E}_0\cdot\hat{n} = \varepsilon\vec{E}\cdot\hat{n}$$



میدان ثابت:

$$\begin{cases} m\dfrac{dv_x}{dt} = qBv_y \\ m\dfrac{dv_y}{dt} = -qBv_x \end{cases} \to m\frac{d}{dt}(v_x+iv_y) = -iqB(v_x+iv_y)$$

$$\to (v_x+iv_y)_{(t)} = \underbrace{(v_x+iv_y)_0}_{v_0 e^{i\varphi}}\, e^{-\frac{iqB}{m}t} \to \begin{cases} v_x = v_{(0)}\cos(\omega t+\varphi) \\ v_y = v_0\sin(\varphi-\omega t) \end{cases}$$

$$\vec F_{B(q'\to q)} = \frac{\mu_0}{4\pi}\frac{qq'\,\vec v\times(\vec v'\times(\vec r-\vec r'))}{|\vec r-\vec r'|^3}, \quad \vec F_{e(q'\to q)} = \frac{qq'(\vec r-\vec r')}{4\pi\varepsilon_0|\vec r-\vec r'|^3}$$

$$\to F_B \sim \frac{vv'}{c^2}F_e$$

$$\vec B = \frac{\mu_0 I}{4\pi}\oint\frac{d\vec r'\times(\vec r-\vec r')}{|\vec r-\vec r'|^3} = \frac{\mu_0 I}{4\pi}\vec\nabla\times\oint\frac{d\vec r'}{|\vec r-\vec r'|}$$

$$\int\frac{d\vec r'}{|\vec r-\vec r'|} = \int d\vec r'\left\{\frac{1}{r}+\frac{\vec r\cdot\vec r'}{r^3}+\dots\right\} = \frac{1}{r^3}\oint d\vec r'\,(\vec r\cdot\vec r')+\dots$$

ولی $\vec a\cdot\int d\vec r'(\vec r\cdot\vec r') = \int d\vec r'\cdot[\vec a(\vec r\cdot\vec r')] = \int d\vec s'\cdot\nabla'\times[\vec a(\vec r\cdot\vec r')]$

$$= \int d\vec s'\cdot(\vec r\times\vec a) = \int(d\vec s\times\vec r)\cdot\vec a$$

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الکترودینامیک دوره طلاها

تقارن‌ها:

1) مشاهده: $\sim q$ $\quad \vec F_B = q\vec{\mathcal F}_B$

2) مشاهده: عمود بر سرعت و خطی با آن

$$(F_B)_i = q\sum_j B_{ij}v_j \to \sum_{i,j} v_iv_jB_{ij} = 0$$

$$\to \sum_{i,j}(u_i+v_i)(u_j+v_j)B_{ij} = 0 \Rightarrow \sum_{i,j}(u_iv_j+u_jv_i)B_{ij} = 0$$

$\to \sum_{i,j} u_iv_j(B_{ij}+B_{ji}) = 0 \to$ B یک ماتریس پادمتقارن است

$$B = \begin{pmatrix} 0 & B_3 & -B_2 \\ -B_1 & 0 & B_1 \\ B_2 & -B_3 & 0 \end{pmatrix}, \quad \sum_{i,j} v_iu_jB_{ij} = \sum_{i,j} v_ju_iB_{ji}$$

$$\to \vec F = q\vec v\times\vec B$$

$$\vec F' = q(\vec E'+\vec v'\times\vec B') = q(\vec E+\vec v\times\vec B) \Rightarrow$$

$$\vec B = \vec B', \quad \vec E' = \vec E - \vec v\times\vec B$$

$\frac{\partial}{\partial r_j}(r_i r_k)$

$$\sum_j \int \left(r_k \frac{\partial r_i}{\partial r_j} + r_i \frac{\partial r_k}{\partial r_j}\right) \dot{J}_j \, dv =$$

$$\sum_j \int dv \left[\underbrace{\frac{\partial}{\partial r_j}(r_i r_k \dot{J}_j)}_{\int r_i r_k \dot{J}\cdot ds} - r_i r_k \underbrace{\frac{\partial \dot{J}_j}{\partial r_j}}_{\nabla\cdot\dot{J}}\right] = 0$$

$$\varepsilon_{ijk} = \begin{cases} 1 & 123 \text{ جایگشت زوج} \\ -1 & 123 \text{ جایگشت فرد} \\ 0 & \text{دست کم ۲ شاخص تکراری} \end{cases}$$

$$(\vec{A}\times\vec{B})_i = \sum_{j,k} \varepsilon_{ijk} A_j B_k$$

$$A_i B_j - A_j B_i = \sum_k \varepsilon_{ijk} (\vec{A}\times\vec{B})_k$$

$$I = 0 + \sum_j \varepsilon_{kij} \int \frac{1}{2}(\vec{r}\times \vec{J})_j \, dv = \sum_j \varepsilon_{kij} m_j$$

$$\rightarrow \vec{B} = \vec{\nabla}\times\left\{0 + \frac{\mu_0}{4\pi r^3} \sum_k r_k \sum_j \varepsilon_{ijk} m_j + \dots\right\} \Rightarrow$$

$$\vec{B} = \vec{\nabla}\times\left(\frac{\mu_0}{4\pi r^3} \vec{m}\times\vec{r}\right)$$

$$\rightarrow \int \frac{d\vec{r}\,'}{|\vec{r}-\vec{r}\,'|} = \int \frac{d\vec{s}\times\vec{r}}{r^3} \Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \vec{\nabla}\times\left(\frac{\vec{m}\times\vec{r}}{r^3}\right)$$

$$= \frac{\mu_0}{4\pi}\left\{\vec{m}\,\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) - \vec{m}\cdot\nabla\left(\frac{\vec{r}}{r^3}\right)\right\} = \frac{\mu_0}{4\pi}\left(-\frac{\vec{m}}{r^3} + \frac{3\vec{r}(\vec{m}\cdot\vec{r})}{r^5}\right)$$

$$\rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{3\hat{r}(\hat{r}\cdot\vec{m}) - \vec{m}}{r^3} \quad , \quad \nabla'(\vec{r}\cdot\vec{r}\,') = \vec{r}$$

$$\vec{B}_{(\vec{r})} = \nabla\times\left\{\frac{\mu_0}{4\pi}\int dv' \frac{J_{(\vec{r}\,')}}{|\vec{r}-\vec{r}\,'|}\right\} \quad , \quad \frac{1}{|\vec{r}-\vec{r}\,'|} = \frac{1}{r} + \frac{\vec{r}\cdot\vec{r}\,'}{r^3} + \dots$$

$$\int \dot{J}_{i(\vec{r}\,')} dv' = \int dv' \sum_j \delta_{ij} \dot{J}_{j(\vec{r}\,')} = \int dv' \sum_j \left[\frac{\partial r'_i}{\partial r'_j} \dot{J}_j(\vec{r}\,')\right] =$$

$$\int dv' \sum_j \left\{\underbrace{\frac{\partial}{\partial r'_j}\left[r'_i \dot{J}_{j(\vec{r}\,')}\right]}_{\nabla'\cdot(r'_i \dot{\vec{J}})} - \underbrace{r'_i \frac{\partial \dot{J}_j}{\partial r'_j}}_{r_i \vec{\nabla}\cdot\dot{J}}\right\} = 0$$

اگر جریان محدود باشد صفر $\leftarrow \nabla'\cdot(r'_i \dot{\vec{J}})$

$$\int \sum_k r_k r'_k \dot{J}_{i(\vec{r}\,')} dv' = \sum_k r_k \int r'_k \dot{J}_{i(\vec{r}\,')} dv'$$

$$\int r'_k \dot{J}_{i(\vec{r}\,')} dv = \frac{1}{2}\int (r_k \dot{J}_i + r_i \dot{J}_k) dv + \frac{1}{2}\int (r_k \dot{J}_i - r_i \dot{J}_k) dv = I$$

$$\Rightarrow \int (r_k \dot{J}_i + r_i \dot{J}_k) dv = \sum_j \int (r_k \delta_{ij} + r_i \delta_{jk}) \dot{J}_j =$$

الکترودینامیک دوره طلاها

$$\vec A_{(\vec r)} \neq \frac{\mu_0}{4\pi}\int \frac{[\frac12 \vec r'\times \vec J_{(\vec r')}]\times(\vec r-\vec r')}{|\vec r-\vec r'|^3}\, dv$$

$$\vec B = \vec\nabla\times\vec A \to \vec\nabla\cdot\vec B = 0 \to \vec B = \vec\nabla\times\vec A' \to \vec A' - \vec A = \nabla\Lambda$$

$$\downarrow$$

$$\oint \vec B\cdot d\vec s = 0$$

$$\vec F = \int \vec J\times\vec B\, dv = \int \vec J_{(\vec r)}\times\left[\vec B_0 + (\vec r-\vec r_0)\cdot\nabla\vec B_0 + \cdots\right] dv$$

$$\Rightarrow F_i = \sum_{jk}\varepsilon_{ijk}\int J_j \sum_l r_l \frac{\partial B_k}{\partial r_l}\Big|_0 dv + \cdots$$

$$= \sum_{jkl}\varepsilon_{ijk}\frac{\partial B_k}{\partial r_l}\int dv\,(r_l J_j + \cdots) = \sum_{j,k,l,n}\varepsilon_{ijk}\varepsilon_{ljn}\frac{\partial B_k}{\partial r_l} m_n$$

$$= \sum_{k,l,n}(\delta_{il}\delta_{kn} - \delta_{in}\delta_{kl})\frac{\partial B_k}{\partial r_l} m_n + \cdots = \sum_k \frac{\partial B_k}{\partial r_i} m_k$$

$$\to \vec F = \vec\nabla(\vec m\cdot\vec B)$$

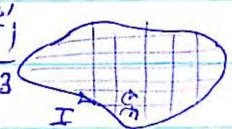
اگر $\vec\nabla\times\vec B = 0$, $\sum_k \frac{\partial B_k}{\partial r_i} m_k = \sum_k \frac{\partial B_i}{\partial r_k} m_k = (\vec m\cdot\nabla)B_i$

$$\vec m = \int \frac12 (\vec r\times\vec J)\, dv = \int I\, d\vec s$$

$\vec J = \rho_e \vec v$, چگالی تکانه $= \rho_m \vec v$ اگر $\frac{q}{m} =$ ثابت

$$\to \vec m = \frac12 \frac{q}{m}\vec L$$

$$\vec A_{(\vec r)} = \frac{\mu_0}{4\pi}\int dv' \frac{\vec J_{(\vec r')}}{|\vec r-\vec r'|} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec s'\times(\vec r-\vec r')}{|\vec r-\vec r'|^3}$$



$$\to \vec B_{(\vec r)} = \frac{\mu_0 I}{4\pi}\int \frac{3(\vec r-\vec r')((\vec r-\vec r')\cdot d\vec s') - [(\vec r-\vec r')\cdot(\vec r-\vec r')]\, d\vec s'}{|\vec r-\vec r'|^5}$$

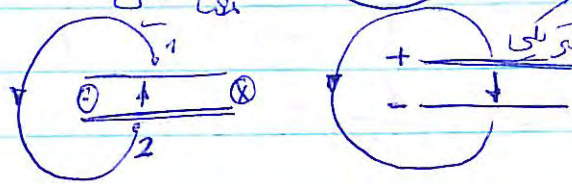
$$\to \vec B_{(\vec r)} = -\nabla\left\{\frac{\mu_0}{4\pi} I \int \frac{(\vec r-\vec r')\cdot d\vec s'}{|\vec r-\vec r'|^3}\right\}$$ اگر r روی سیم نباشد :

$$\to \vec B_{(\vec r)} = +\nabla\left\{\frac{\mu_0}{4\pi} I\Omega\right\} \to \vec B_{(\vec r)} = +\frac{\mu_0 I}{4\pi}\nabla\Omega\ ,$$ Ω : زاویه فضایی

$\int \vec E\cdot d\vec r = 0$ در $\int \vec B\cdot d\vec r = \frac{\mu_0 I}{4\pi}\, 4\pi = \mu_0 I$

$$\int_1^2 \vec B\cdot d\vec r = \phi_{m_1} - \phi_{m_2}$$

نیمه طی   الکتریکی



$$[\det(R)]^2 = 1 \rightarrow \det R = \pm 1$$

ماتریس $MU = \lambda U$ (بردار، عدد) $\rightarrow (M - \lambda 1)U = 0 \rightarrow$

$\det(M - \lambda 1) = 0$ λ : ویژه مقدار u ویژه بردار

چند جمله‌ای از درجه $n = 0$

$(M - \lambda_1 1)(M - \lambda_2 1) \cdots (M - \lambda_n 1) = 0$ تجزیه

$$(R\vec{u}) \cdot (R\vec{v}) = \vec{u} \cdot \vec{v} \rightarrow (R\vec{u}^*) \cdot (R\vec{u}) = \vec{u}^* \cdot \vec{u}$$

فرض : $R\vec{u} = \lambda \vec{u} \rightarrow \overset{\text{حقیقی}}{R}\vec{u}^* = \lambda^* \vec{u}^*$

$$\rightarrow (\lambda^* \vec{u}^*) \cdot (\lambda \vec{u}) = \vec{u}^* \cdot \vec{u} \rightarrow (\lambda^* \lambda - 1)\underbrace{(\vec{u}^* \cdot \vec{u})}_{>0} = 0$$

$$\rightarrow \lambda \lambda^* = 1 \rightarrow |\lambda| = 1$$

$$\det(R - \lambda 1) = 0 \rightarrow (R_{11} - \lambda_1)(R_{22} - \lambda_2)(R_{33} - \lambda_3) = 0$$

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اگر دو جریان (حلقه) داشته باشیم که در یکدیگر رفته‌اند می‌توانیم با فرمول دو قطبی مغناطیسی نیرو را حساب کنیم.

$$\vec{J} \rightarrow T_{\vec{a}} \vec{J} \qquad (T_{\vec{a}} \vec{J})(\vec{r}) = \vec{J}(\vec{r} - \vec{a})$$

$$\vec{B} \rightarrow T_{\vec{a}} \vec{B} \qquad (T_a \vec{B})_{(\vec{r})} = \vec{B}(\vec{r} - \vec{a})$$

$$\vec{J} \rightarrow \tilde{\vec{J}}_{(\vec{r})} = (R(\vec{J}))(R^{-1}(\vec{r})) \quad , \quad (R\vec{u}) \cdot (R\vec{v}) = \vec{u} \cdot \vec{v}$$

$$\sum_{i,j} u_i v_j \delta_{ij} = \sum_{i,j} \left(\sum_k R_{ik} u_k\right)\left(\sum_l R_{jl} v_l\right) \delta_{ij}$$

$$= \sum_{k,l} \delta_{k,l} u_k v_l \qquad \sum_{i,j} R_{ik} R_{jl} \delta_{ij} = \delta_{kl}$$

$$\sum_i (R^{-1})_{ki} R_{il} = \delta_{kl} \rightarrow \sum_i (R^{-1})_{ki} R_{il} = \delta_{kl}$$

$$(R^{-1})_{ki} = R_{ik} \rightarrow \det(R^{-1}) = \det(R)$$

$$\det(AB) = (\det A)(\det B) \rightarrow \det(RR^{-1}) = 1 = \det R \det R^{-1}$$

$(\lambda-\lambda_1)(\lambda-\lambda_2)(\lambda-\lambda_3)=0 \to \lambda^3+a\lambda^2+b\lambda+c=0$

$\to c=\det(R)=-\lambda_1\lambda_2\lambda_3$,

a,b,c حقیقی $\to (\lambda^*)^3+a(\lambda^*)^2+b\lambda^*+c=0 \to$

اگر λ های مختلط داشتیم پس λ^* هم جواب است پس ~~یکی~~

دیگری 1 است. $(\det(R)=1)$

منفی حرف های بالا. $(\det(R)=-1)$

$(R\vec{u})\cdot(R\vec{v})=\vec{u}\cdot\vec{v}$ ، فرض: $\vec{u}$ ویژه بردار ، $\vec{u}\cdot\vec{v}=0$

$\lambda(R\vec{u})\cdot\vec{v}=\vec{u}\cdot\vec{v}=0 \to (R\vec{u})\cdot\vec{v}=0$

فرض: $Re_1=e_1$, e_2 , e_3

$Re_j=\sum_i R_{ij}e_i \to Re_2=\sum_i R_{i2}e_i=\sum_{i=2}^{3}R_{i2}e_i$

ترکیب خطی

$=\sum_{i=2}^{3} R'_{i2}e_i$ $\qquad u,v\in \mathrm{span}(e_2,e_3)$

$R\vec{u}=R'\vec{u}$, $R\vec{v}=R'\vec{v}$

$\vec{u}\cdot\vec{v}=(R\vec{u})\cdot(R\vec{v})=(R'u)\cdot(R'\vec{v})$

$$R=\begin{bmatrix}1&0&0\\0&\multicolumn{2}{c}{\boxed{R'}}\\0&&\end{bmatrix}$$

$$\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}a&c\\b&d\end{pmatrix}=\begin{pmatrix}a^2+b^2&ac+bd\\ac+bd&c^2+d^2\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$a^2+b^2=1$, $c^2+d^2=1$, $ac+bd=0$

$a=\cos\theta \to b=\pm\sin\theta=s\sin\theta \to d=\alpha\cos\theta$, $\alpha=\pm1$

$\to c=-\alpha s\sin\theta \to R'=\begin{pmatrix}\cos\theta & s\sin\theta\\ -\alpha s\sin\theta & \alpha\cos\theta\end{pmatrix} \to \det R'=\alpha$

$\to \alpha=1,\ s=-1:\ \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \qquad \alpha=-1,\ s=1:\ \begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix}$

$$\vec B_{(\vec r)} = \frac{\mu_0}{4\pi}\int \frac{(R\vec J)(R^{-1}\vec r')\times(\vec r-\vec r')}{|\vec r-\vec r'|^3}\,dv'$$

$$R\vec r'' = \vec r' \to \vec B_{(\vec r)} = \frac{\mu_0}{4\pi}\int \frac{(R\vec J)\,\vec r''\times(\vec r - R\vec r'')}{|\vec r - R\vec r''|^3}\,dv''$$

$$\frac{\mu_0}{4\pi}\int \frac{\{R\vec J_{(\vec r'')}\}\times[R(R^{-1}\vec r-\vec r'')]}{[R(R^{-1}\vec r-\vec r'')]^3}\,dv''$$

$$= \frac{\mu_0}{4\pi}(\det R)\,R\int dv''\,\frac{[\vec J_{(\vec r'')}]\times(R^{-1}\vec r-\vec r'')}{|R^{-1}\vec r-\vec r''|^3}$$

$$\to \vec B_{(\vec r)} = (\det R)[(R\vec B)(R^{-1}\vec r)]$$

تقارن: $S_{ij}=S_{ji}$. پادتقارن: $A_{ij}=-A_{ji}$

ترانهاده: $T_{ij}=T^{-1}_{ji}$

$$T_{a\hat z}\vec J = \vec J \longrightarrow T_{a\hat z}\vec B = \vec B \longrightarrow B_{(\vec r - a\hat z)} = B_{(\vec r)}$$

دوران حول z به اندازه α

$$(R\vec B)(R^{-1}\vec r) = \vec B_{(\vec r)} = B_{1(\vec r)}\hat\rho + B_{2(\vec r)}\hat\varphi + B_{3(\vec r)}\hat z$$

$$\to R\{B_1(R^{-1}\vec r)[\hat\rho(R^{-1}\vec r)] + B_2(R^{-1}\vec r)[\hat\varphi(R^{-1}\vec r)]$$

$$+B_3(R^{-1}\vec r)[\hat z(R^{-1}\vec r)]\} = B_1(R^{-1}\vec r)\hat\rho + B_2(R^{-1}\vec r)\hat\varphi + B_3(R^{-1}\vec r)\hat z$$

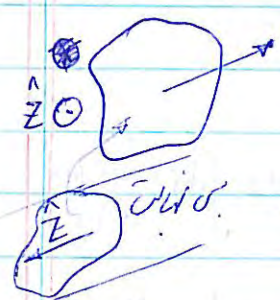
$$R(\hat\rho(R^{-1}\vec r)) = \hat\rho_{(\vec r)}$$

$$(R\vec J)(R^{-1}\vec r) = -\vec J \to (R\vec B)(R^{-1}\vec r) = -\vec B_{(\vec r)} \to B_z = 0$$

R انعکاس نسبت به صفحه عمود بر سیم

$$\to \vec B = B_\rho\hat\varphi$$

تقارن محوری + دارای انعکاس نسبت به هر صفحه شامل z $\Rightarrow J_\varphi = 0$



یک بار جریان و بار دیگر بار روی آن می‌ریزیم طوری که:

سیم بی‌نهایت. $J_{s(x,y)} = \alpha\,\sigma_{(x,y)} \to \vec B = (\mu_0\varepsilon_0\alpha)(\hat z\times\vec E)$

$$\vec A = \frac{\mu_0}{4\pi}\int dv'\,\frac{\vec M_{(\vec r')}\times(\vec r-\vec r')}{|\vec r-\vec r'|^3} = \frac{\mu_0}{4\pi}\int dv'\,\vec M_{(\vec r')}\times\nabla'\left(\frac{1}{|\vec r-\vec r'|}\right) =$$

$$\frac{\mu_0}{4\pi}\int dv'\left\{-\nabla'\left[\frac{\vec M_{(\vec r')}}{|\vec r-\vec r'|}\right] + \frac{(\vec\nabla\times\vec M)_{(\vec r')}}{|\vec r-\vec r'|}\right\}$$

$$\frac{\partial}{\partial t}\Big|_{\vec r} = \frac{\partial}{\partial t} \quad , \quad \frac{\partial}{\partial t'}\Big|_{\vec r'} = \frac{\partial}{\partial t'}$$

$(\vec r, t) \to (\vec r', t')$, $t = t'$, $\vec r' = \vec r + \vec v t$, $\vec r = \vec r' - \vec v t'$

$$\frac{\partial}{\partial t'} = \frac{\partial t}{\partial t'}\frac{\partial}{\partial t} + \sum_i \frac{\partial r_i}{\partial t'}\frac{\partial}{\partial r_i} = \frac{\partial}{\partial t} - \sum_i v_i \frac{\partial}{\partial r_i}$$

$$\frac{\partial}{\partial r'_i} = \underbrace{\frac{\partial t}{\partial r'_i}}_{0}\frac{\partial}{\partial t} + \sum_j \underbrace{\frac{\partial r_j}{\partial r'_i}}_{\delta_{ij}}\frac{\partial}{\partial r_j} = \frac{\partial}{\partial r_i}$$

$$\to \begin{cases} \dfrac{\partial}{\partial t'} = \dfrac{\partial}{\partial t} - \vec v \cdot \vec\nabla \quad , \quad \nabla' = \nabla \\ \vec\nabla' \cdot \left(\vec B'_{(\vec r', t)}\right) = \vec\nabla \cdot \left(\vec B_{(\vec r, t)}\right) = 0 \ \checkmark \end{cases}$$

$$\vec\nabla' \times E' = \vec\nabla' \times (\vec E - \vec v \times \vec B) = \vec\nabla \times \vec E - \vec v(\vec\nabla \cdot \vec B) + (\vec v \cdot \vec\nabla)\vec B$$

$$\to \vec\nabla' \times \vec E' = \cancel{\quad} (\vec v \cdot \vec\nabla)\vec B$$

$$\frac{\partial \vec B}{\partial t} = 0 = \frac{\partial B'}{\partial t'} + (\vec v \cdot \vec\nabla')\vec B' \to \vec\nabla' \times \vec E' = -\frac{\partial \vec B'}{\partial t'} \ \checkmark$$

$$\vec a \cdot \int dv\,(\vec\nabla \times \vec C) = \int dv\, \vec\nabla \cdot (\vec C \times \vec a) = \oint d\vec s \cdot (\vec C \times \vec a) = \vec a \cdot \oint d\vec s \times \vec C$$

$$(\vec\nabla \times \vec C) \cdot \vec a = \sum_{i,j,k} (\varepsilon_{ijk} \partial_i C_j) a_k = \sum_i \partial_i \sum_{j,k} \varepsilon_{ijk} C_j a_k = \sum_i \partial_i (\vec C \times \vec a)_i$$

مثال: $\vec\nabla \cdot (\vec A \times \vec B) = \nabla \cdot (\overset{\downarrow}{\vec A} \times \vec B) + \vec\nabla \cdot (\vec A \times \overset{\downarrow}{\vec B}) = \vec B \cdot (\vec\nabla \times \vec A) - \vec A \cdot (\vec\nabla \times \vec B)$

$$\to \vec A_{b(\vec r)} = \frac{\mu_0}{4\pi}\left[\oint ds' \frac{\vec M_{(\vec r')} \times \hat n'}{|\vec r - \vec r'|} + \int dv' \frac{(\vec\nabla \times \vec M)_{(\vec r')}}{|\vec r - \vec r'|}\right]$$

$$\to \vec J_b = \vec\nabla \times \vec M \qquad \vec J_{bs} = \vec M \times \hat n$$

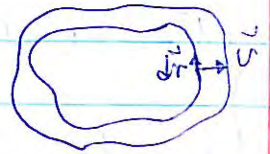
$$\rho_M = -\vec\nabla \cdot \vec M \quad , \quad \sigma_M = \vec M \cdot \hat n$$

$$\Phi_M = \frac{\mu_0}{4\pi}\int dv' \frac{\vec M_{(\vec r')} \cdot (\vec r - \vec r')}{|\vec r - \vec r'|^3} = \frac{\mu_0}{4\pi}\left[\int dv' \frac{(-\vec\nabla \cdot \vec M)_{(\vec r')}}{|\vec r - \vec r'|} + \int ds' \frac{\hat n' \cdot \vec M_{(\vec r')}}{|\vec r - \vec r'|}\right]$$

خطوط $\vec H$ برای سیم‌لوله همان $\vec D$ برای قطبش است.

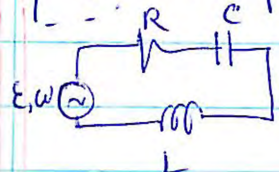
$$\vec B'_{(\vec r', t)} = \vec B_{(\vec r, t)} \quad , \quad \vec E_{(\vec r, t)} = \vec E'_{(\vec r', t)} + \vec v \times \vec B'_{(\vec r', t)}$$

$$\vec\nabla \cdot \vec B = 0 \quad , \quad \vec\nabla \times \vec E = 0 \quad , \quad \frac{\partial \vec E}{\partial t} = 0 \quad , \quad \frac{\partial \vec B}{\partial t} = 0$$



$$-\mathcal{E}+RI=\oint \vec{E}\cdot d\vec{r}=-\frac{d\Psi_M}{dt}=-L\frac{dI}{dt}$$

چون L (خودالقایی مدار) خیلی کوچک است می‌توانیم تقاضای ولتاژ ننویسیم.

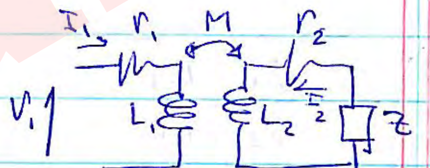


$$\mathcal{I}=\frac{\mathcal{E}}{R-i\omega L+\frac{i}{\omega C}}=\frac{\mathcal{E}}{R+i\left(\frac{1}{\omega C}-L\omega\right)}$$

$$\rightarrow \angle\mathcal{I}=\angle\mathcal{E}+\tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)$$

$$|\mathcal{I}|=\frac{|\mathcal{E}|}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}\rightarrow I_{(t)}=|\mathcal{I}|\cos\left(\omega t-\angle\mathcal{E}-\tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)\right)$$

$$\mathcal{E}=Re(\mathcal{E}e^{i\omega t})$$



$$\vec{\nabla}''\times\vec{E}''=\vec{\nabla}'\times(\vec{E}'-\vec{U}\times\vec{B}')=\vec{\nabla}'\times\vec{E}'+(\vec{U}\cdot\vec{\nabla}')\vec{B}'=$$

$$-\left(\frac{\partial}{\partial t'}-\vec{U}\cdot\vec{\nabla}'\right)\vec{B}'=-\frac{\partial\vec{B}''}{\partial t''}$$

$$\frac{\partial}{\partial t'}-\vec{U}\cdot\nabla'=\frac{\partial}{\partial t''}$$

$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0\quad,\quad\vec{B}=\vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}\times\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)=0\rightarrow\vec{E}+\frac{\partial\vec{A}}{\partial t}=-\vec{\nabla}\phi\rightarrow\vec{E}=-\vec{\nabla}\phi-\frac{\partial\vec{A}}{\partial t}$$

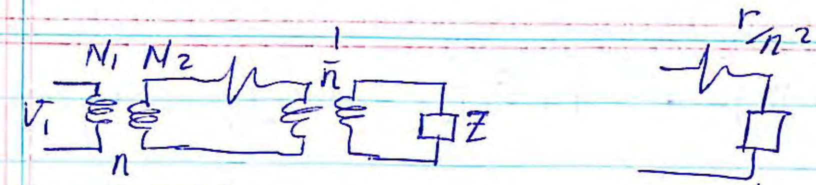
اگر بخواهیم $\vec{E},\vec{B}$ تغییر نکند: $\rightarrow\phi'=\phi-\frac{\partial\lambda}{\partial t}$

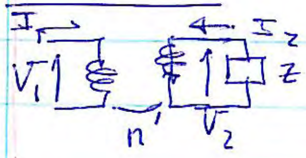
$$\vec{A}'=\vec{A}+\vec{\nabla}\lambda$$

$$emf=\oint_S(\vec{E}+\vec{v}\times\vec{B})\cdot d\vec{r}=-\int_S\frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}+\oint_{\partial S}(\vec{v}\times\vec{B})\cdot d\vec{r}$$

$$=-\left[\int_S\frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}+\int_{\partial S}(\vec{B}\times\vec{v})\cdot d\vec{r}\right]=-\frac{d}{dt}\left(\int\vec{B}\cdot d\vec{s}\right)$$

$$\frac{d}{dt}\int_S\vec{B}\cdot d\vec{s}=\int_S\frac{\partial\vec{B}}{\partial t}\cdot d\vec{s}+\oint_{\partial S}(\vec{v}\times d\vec{r})\cdot\vec{B}$$





$$\begin{cases} V_2 = n' V_1 \; ; \; I_2 = \frac{1}{n'} I_1 \\ \frac{V_2}{I_1} = Z = n'^2 \frac{V_1}{I_1} \rightarrow \frac{V_1}{I_1} = \frac{Z}{n'^2} \end{cases}$$

$$U = \frac{\mu_0 n^2 I^2}{2} lS \rightarrow F = -\frac{1}{2}\mu_0 S K^2$$ اضافی

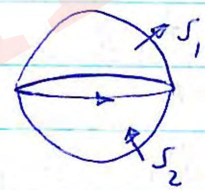
$$F = -\frac{\partial U}{\partial l} \leftarrow I$$ ثابت

$$F_z = -2\pi a I n \int B_{\rho(a)} dz \xrightarrow{\vec{\nabla}\cdot\vec{B}=0} F_z = 2\pi I n \int_0^a \rho d\rho \int \frac{\partial B_z}{\partial z} dz$$

$$B_z = 2\pi I n \int_0^a \rho d\rho \left[B_{z\,\text{بی}} - B_{z\,(\text{انتها})} \right] = -\frac{S}{2}\mu_0 K^2$$

$$\int \vec{j}\cdot d\vec{s} = -\frac{d}{dt}\int \rho dV = -\frac{dQ_{(in)}}{dt}$$

$$\vec{\nabla}\cdot\vec{j} + \frac{\partial \rho}{\partial t} = 0$$



$$I_{S_1} + I_{S_2} = 0 \rightarrow I_{(S)} = 0 \rightarrow \times \rightarrow \vec{\nabla}\times\vec{B} \neq \mu_0 \vec{j}$$ لزوماً

$$\begin{cases} V_1 = (r_1 - i\omega L_1) I_1 - i\omega M I_2 \\ V_2 = (r_2 - i\omega L_2) I_2 - i\omega M I_1 \end{cases} , \quad V_2 = -Z I_2$$

$$\rightarrow \frac{V_1}{V_2} = \frac{\omega^2 M^2 + (r_1 - i\omega L_1)(Z + r_2 - i\omega L_2)}{-i\omega M Z}$$

$$= \frac{\omega^2 (M^2 - L_1 L_2) - i\omega (L_1 Z + L_1 r_2 + L_2 r_1) + r_1 (Z + r_2)}{-i\omega M Z}$$

$$\rightarrow |M^2 - L_1 L_2| \simeq 0 ,$$

$$\frac{|L_1 Z + L_1 r_2 + L_2 r_1|}{L_1 L_2 - M^2} < \omega < \frac{(Z + r_2) r_1}{|L_1 Z + L_1 r_2 + L_2 r_1|} \rightarrow$$

$$\frac{L_1 |Z|}{L_1 L_2 - M^2} < \omega < \frac{r_1}{L_1}$$

$$L_1 r_2 + L_2 r_1 \ll L_1 |Z|$$

$$\rightarrow \frac{V_1}{V_2} = \frac{L_1}{M}$$

$$\rightarrow \frac{d\mathcal{E}_m}{dt} = -\frac{d}{dt}\left\{\int\left(\frac{\varepsilon_0}{2}\vec{E}\cdot\vec{E}+\frac{\vec{B}\cdot\vec{B}}{2\mu_0}\right)d\tau - \int\frac{\vec{E}\times\vec{B}}{\mu_0}\cdot d\vec{s}\right\}$$

$$u_e = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 \quad , \quad \vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$$

$$\frac{d\mathcal{E}_m}{dt} = \int \vec{J}_f\cdot\vec{E}\,d\tau = \int\left(-\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + (\vec{\nabla}\times\vec{H})\cdot\vec{E}\right)d\tau$$

$$= \int\left[-\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\vec{\nabla}\times\vec{E} - \vec{\nabla}\cdot(\vec{E}\times\vec{H})\right]d\tau =$$

$$-\int(\vec{E}\times\vec{H})\cdot d\vec{s} - \int\left(\vec{E}\cdot\frac{\partial\vec{D}}{\partial t}+\vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right)d\tau$$

$-\int(\vec{E}\times\vec{H})\cdot d\vec{s} - \frac{\partial}{\partial t}\int\left(\frac{\vec{E}\cdot\vec{D}}{2}+\frac{\vec{H}\cdot\vec{B}}{2}\right)d\tau$; برای محیط خطی

$\rightarrow u_e = \frac{\vec{E}\cdot\vec{D}}{2}+\frac{\vec{H}\cdot\vec{B}}{2}$ اما نه لزوماً همگن

$$\delta U_e = \int d\tau\,(\vec{E}\cdot\delta\vec{D}+\vec{H}\cdot\delta\vec{B})$$

$$\delta U_e + \delta\mathcal{E}_m = -\delta t\oint \vec{J}_e\cdot d\vec{s}$$

مثال: $\vec{E} = \frac{\vec{J}}{\sigma}$, $B_\varphi = \frac{\mu_0 \rho}{2}\vec{J} \rightarrow \vec{S} = -\frac{\rho J^2}{2\sigma}\hat{\rho}$

$$\vec\nabla\times\vec B = \mu_0(\vec J+\vec J_D) \rightarrow \vec\nabla\cdot\vec J + \vec\nabla\cdot\vec J_D = 0 \rightarrow \vec\nabla\cdot\vec J_D - \varepsilon_0\vec\nabla\cdot\frac{\partial\vec E}{\partial t}=0$$

$\rightarrow \vec J_D = \varepsilon_0\frac{\partial\vec E}{\partial t}$: فرض می‌کنیم - با تجربه می‌خونه

$$\rightarrow \vec\nabla\times\vec B = \mu_0\vec J + \mu_0\varepsilon_0\frac{\partial\vec E}{\partial t}$$

$$\begin{cases}\vec\nabla\cdot\vec D = \rho_f\\ \vec\nabla\times\vec H - \frac{\partial\vec D}{\partial t} = \vec J_f\end{cases} \qquad \begin{cases}\vec\nabla\cdot\vec B = 0\\ \vec\nabla\times\vec E + \frac{\partial\vec B}{\partial t} = 0\end{cases}$$

$$\vec F = q(\vec E+\vec v\times\vec B) \rightarrow dW = q\vec v\cdot\vec E = \rho\vec v\cdot\vec E\,d\tau = \vec J\cdot\vec E\,d\tau$$

$\rightarrow \frac{d\mathcal E_m}{dt} = \int\vec J\cdot\vec E\,d\tau$ توان مکانیکی

$$\frac{d}{dt}(\mathcal E_m+\mathcal E_e) = -\oint\vec J_\varepsilon\cdot d\vec s$$

$$\frac{d\mathcal E_m}{dt} = \int\vec E\cdot\left(\frac{1}{\mu_0}\vec\nabla\times\vec B - \varepsilon_0\frac{\partial\vec E}{\partial t}\right)d\tau = \int\left[\frac{\partial}{\partial t}\left(-\frac{\varepsilon_0}{2}\vec E\cdot\vec E\right)\right]d\tau$$

$$+\int\left[(\vec\nabla\times\vec E)\cdot\frac{\vec B}{\mu_0} - \frac{\vec\nabla\cdot(\vec E\times\vec B)}{\mu_0}\right]dv$$

$$\overbrace{\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}}^{M}\ddot{x} = -\overbrace{\begin{pmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_3+K_2 \end{pmatrix}}^{K} x$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} e^{st} = X e^{st}$$

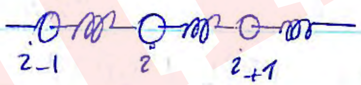
$$M\ddot{x} = -Kx \longrightarrow x(K + MS^2) = 0 \longrightarrow \det(MS^2 + K) = 0$$

$$(m_1 s^2 + K_1 + K_2)(m_2 s^2 + K_2 + K_3) = K_2^2$$

$$R_e[a_{(1)} e^{-i\omega_{(1)}t} X_{(1)}] + R_e[a_{(2)} e^{-i\omega_{(2)}t} X_{(2)}] =$$

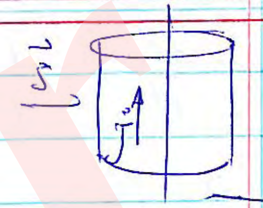
$$= |a_{(1)}| \cos[\omega_{(1)}t - \varphi_{(1)}] \begin{pmatrix} 1 \\ 1 \end{pmatrix} + |a_{(2)}| \cos[\omega_{(2)}t - \varphi_{(2)}] \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$m\ddot{z}_i = K(z_{i+1} + z_{i-1} - 2z_i)$$



$$z_{j(t)} = Z_j e^{st} = Z_j e^{-i\omega t}$$

$$\longrightarrow Z_{i+1} + \left(\frac{m\omega^2}{k} - 2\right) Z_i + Z_{i-1} = 0 \longrightarrow Z_i = u^i$$



$$\begin{cases} \vec\nabla \cdot \vec E = \dfrac{\rho_f}{\varepsilon} \\ \vec\nabla \times \vec B - \mu\varepsilon \dfrac{\partial \vec E}{\partial t} = \mu \vec J_f \end{cases} \longrightarrow \vec\nabla \times (\vec\nabla \times \vec E) + \mu\varepsilon \frac{\partial \vec E}{\partial t} = -\mu \frac{\partial \vec J_f}{\partial t}$$

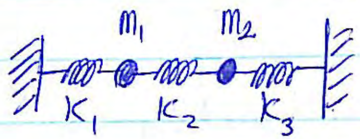
$$\longrightarrow \vec\nabla(\vec\nabla \cdot \vec E) - \vec\nabla^2 \vec E + \mu\varepsilon \frac{\partial^2 \vec E}{\partial t^2} = -\mu \frac{\partial \vec J_f}{\partial t} \longrightarrow$$

$$\left(\vec\nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t^2}\right)\vec E = \frac{\vec\nabla \rho_f}{\varepsilon} + \mu \frac{\partial \vec J_f}{\partial t}$$

بیرون چشمه ، $\left(\vec\nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t^2}\right)\vec E = 0$, $\vec\nabla \cdot \vec E = 0$

$$\left(\nabla^2 - \mu\varepsilon \frac{\partial^2}{\partial t^2}\right)\vec B = -\mu \vec\nabla \times \vec J_f \quad , \quad \vec\nabla \cdot \vec B = 0$$

$$\begin{cases} m_1 \ddot x_1 = -K_1 x_1 + K_2(x_2 - x_1) \\ m_2 \ddot x_2 = K_2(x_1 - x_2) - K_3 x_2 \end{cases}$$



$$M\ddot{z}_{(x,t)} = K\left[z_{(x-\Delta,t)} + z_{(x+\Delta,t)} - 2z_{(x,t)}\right] \longrightarrow$$

$$-M\omega^2 = K\left[e^{-ik\Delta} + e^{ik\Delta} - 2\right] = 2K(\cos k\Delta - 1)$$

$$\longrightarrow \omega^2 = 4\frac{K}{M}\sin^2\left(\frac{k\Delta}{2}\right)$$ رابطه‌ی پاشندگی

$$\longrightarrow 0 \le \omega \le 2\sqrt{\frac{K}{M}}$$ رشته‌ی بی‌نهایت

حرکت واداشته با بسامد ω $\qquad \omega > 2\sqrt{\frac{K}{M}}$

$$\cos(k\Delta) = 1 - \frac{M\omega^2}{2K} = 1 - \frac{2\omega^2}{\omega_c^2} < -1 \quad ; \quad k\Delta = \pi + \zeta$$

$$\cos(\pi+\zeta) = 1 - \frac{2\omega^2}{\omega_c^2} \qquad \cos\zeta = \frac{2\omega^2}{\omega_c^2} - 1 > 1 \to$$

$$\cosh(i\zeta) = \frac{2\omega^2}{\omega_c^2} - 1 \to i\zeta = \pm\cosh^{-1}\left(\frac{2\omega^2}{\omega_c^2} - 1\right)$$

$$k\Delta = \pi \pm i\cosh^{-1}\left(\frac{2\omega^2}{\omega_c^2} - 1\right) \to ikx = \frac{i\pi x}{\Delta} \pm \left((-1)\cosh^{-1}\left(\frac{2\omega^2}{\omega_c^2} - 1\right)\right)\frac{x}{\Delta}$$

$$\Delta k = \pi + i\cosh^{-1}\left(\frac{2\omega^2}{\omega_c^2} - 1\right) \to e^{ikx} = e^{\frac{i\pi x}{\Delta}} e^{-\frac{x}{\Delta}\cosh^{-1}\left(\frac{2\omega^2}{\omega_c^2} - 1\right)}$$

$$\to u + \frac{1}{u} + \left(\frac{m\omega^2}{k} - 2\right) = 0 \to u^2 + \left(\frac{m\omega^2}{k} - 2\right)u + 1 = 0$$

$$\to z_j = A u^j e^{-i\omega t} + B u^{-j} e^{-i\omega t}$$

رشته‌ی بی‌نهایت: $u = e^{i\overset{\text{حقیقی}}{\theta}} \to |u| = 1$

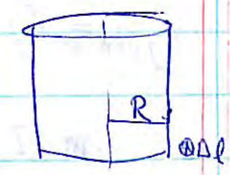
$$\frac{m\omega^2}{k} = 2(1-\cos\theta) \to 0 \le \omega^2 \le \frac{4k}{m} \to 0 \le \omega \le 2\sqrt{\frac{k}{m}}$$

$\omega = 2\sqrt{\frac{k}{m}}\sin\left(\frac{\theta}{2}\right)$ ، جواب خاص ، $z_{\pm\theta, j, t} = e^{ij\theta - i\left(2\sqrt{\frac{k}{m}}\sin\frac{\theta}{2}\right)t}$

θ پیوسته است. یعنی ω پیوسته است

$$\int_0^{2\pi} d\theta \left[A_{(\theta)} z_{+\theta, j, t} + A_{-(\theta)} z_{-\theta, j}\right]$$

$$U = \frac{\pi R^2 L}{2\mu_0}(\mu_0 n I)^2 \to \left(\frac{\partial U}{\partial R}\right)_I = \frac{2\pi R L}{2\mu_0}(\mu_0 n I)^2$$



$$P = \frac{(\mu_0 n I)^2}{2\mu_0} = \frac{B^2}{2\mu_0}$$

$$\frac{B}{2}(nI\Delta z)\Delta l \quad ; \quad \overline{B}(nI\Delta z)\Delta l$$

N تا جواب (ω جمعی) داریم.

$K\Delta \ll 1: \quad \omega^2 = \frac{K}{M}\Delta^2 K^2 \rightarrow \frac{\omega}{k} = \Delta\sqrt{\frac{K}{M}}$

$v_p = \frac{\Delta x}{\Delta t} = \frac{\omega}{k} \longrightarrow$ محیط پاشنده نیست

$M\ddot{\xi} = K\left(\Delta^2 \xi'' + \frac{1}{12}\Delta^2 \xi^{IV} + \dots\right) \rightarrow \rho\ddot{\xi} = F\xi''$

$\rightarrow -\rho\omega^2 = -FK^2 \quad ; \quad K\Delta = F \quad , \quad \frac{M}{\Delta} = \rho$

$f\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)\psi_{(x,t)} = 0 \rightarrow f_{(-i\omega, ik)} = 0$

$\frac{F}{\rho} = v^2 \rightarrow -\ddot{\xi} + v^2\xi'' = 0 \rightarrow (-D_t^2 + v^2 D_x^2)\xi = 0$

$\rightarrow (-D_t + vD_x)(D_t + vD_x)\xi = 0 \longrightarrow D_\eta D_\xi \xi = 0$

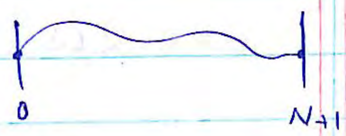
$D_\xi = \frac{\partial t}{\partial \xi}D_t + \frac{\partial x}{\partial \xi}D_x$

$$\text{طول مشخصه} = \frac{\Delta}{\cosh^{-1}\left(\frac{2\omega^2}{\omega_c^2} - 1\right)}$$

تخمین: $\omega^2 \approx \frac{K}{M}\Delta^2 K^2$ ؛ برای بلندکم $\quad v_p = \frac{\omega}{K}$

$\rightarrow v_p = \Delta\sqrt{\frac{K}{M}}$

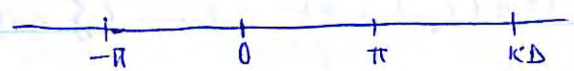
$\xi = e^{-i\omega t}\left(ae^{ikx} + be^{-ikx}\right)$



$x = 0: \quad b = -a \rightarrow \xi = e^{-i\omega t}\sin(kx)$

$\sin kL = 0 \rightarrow K = \frac{n\pi}{L} = \frac{n\pi}{\Delta(N+1)}$

$\sin(kx) = \sin(Kj\Delta) = \sin\left(\frac{jn\pi}{N+1}\right)$



$\sin(j(K\Delta + \pi)) = \sin(j(K\Delta - \pi)) \quad ; \quad K\Delta - \pi = -K'\Delta$

$0 < K\Delta < \pi \Big\} \Rightarrow \sin(-jK'\Delta) = -\sin(jK'\Delta) \rightarrow$

$$n\pi < u_n < (n+\tfrac{1}{2})\pi$$

$$n \gg 1: \quad u_n + n\pi + \delta_n \to \delta_n = \frac{m}{M} \times \frac{1}{\pi n}$$

$$n=0: \quad \frac{m}{M} \ll 1 \to u_0 = \sqrt{\frac{m}{M}} \xrightarrow{\text{دقیق‌تر}} u_0^2 = \frac{m}{M+\frac{m}{3}}$$

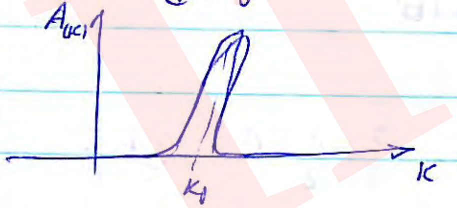
$$\to \omega_0 = \frac{u_0}{L}\sqrt{\frac{F}{\rho}} = \sqrt{\frac{m}{M+\frac{m}{3}}}\sqrt{\frac{K L}{m/L}} \times \frac{1}{L}$$

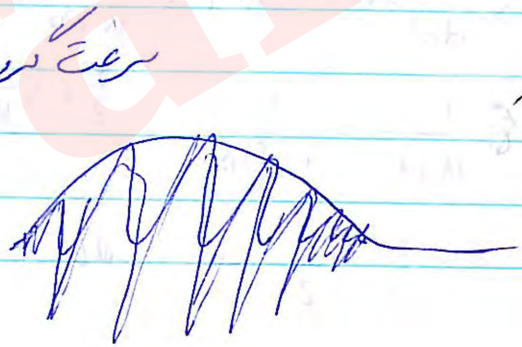
$$\to \omega_0 = \sqrt{\frac{K}{M+\frac{m}{3}}}$$

$$\Psi_{(x,t)} = \int A_{(k)} e^{i(kx-\omega t)} dk = \int A_k e^{i(k_0x-\omega_0 t) + i\left[(k-k_0)x - \frac{d\omega}{dk}\big|_0 (k-k_0)t\right]}$$

$$= e^{i(k_0x-\omega_0 t)} \int A_{(k)} dk\, e^{i(k-k_0)(x-\frac{d\omega}{dk}|_0 t)} = e^{ik_0(x-v_p t)} g_{(x-v_g t)}$$

$$v_g = \frac{d\omega}{dk}\Big|_0$$ سرعت گروه





$$\begin{cases} \frac{\partial t}{\partial \xi} = \alpha \\ \frac{\partial t}{\partial \eta} = -\alpha \end{cases} \quad \begin{cases} \frac{\partial x}{\partial \xi} = \alpha v \\ \frac{\partial x}{\partial \eta} = \alpha v \end{cases} \to \begin{cases} t = \alpha(\xi-\eta) \\ x = \alpha v(\xi+\eta) \end{cases}$$

$$\to \begin{cases} \xi = \frac{x+vt}{2\alpha v} \\ \eta = \frac{x-vt}{2\alpha v} \end{cases} \xrightarrow{\alpha = \frac{1}{2v}} \begin{cases} \xi = x+vt \\ \eta = x-vt \end{cases}$$

$$D_\eta D_\xi y = 0 \to D_\xi y = f_{(\xi)} \to y = f_{(\xi)} + g_{(\eta)}$$

$$\to y = f_{(x+vt)} + g_{(x-vt)}$$

L $\boxed{M}$ ρ, F $\quad M\ddot{y}_{(L,t)} = -F y'_{(L,t)}$

$$\to -M\omega^2 \sin(kL) = FK\cos(kL) \to kL\tan(kL) = \frac{m}{M}$$

$kL = u$

در نتیجه بالایی بیاید تو و پایینی برود بیرون.

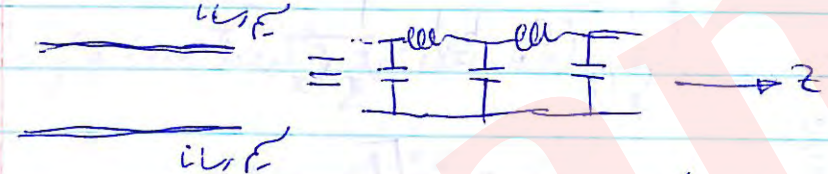
$$\rho \frac{\partial^2 z}{\partial t^2} = F\frac{\partial^2 z}{\partial x^2} - \rho\gamma \frac{\partial z}{\partial t}$$

$z_{(x=0)} = z_{(x=L)} = 0$; $z \sim e^{-i\omega t + ikx}$

$-\rho\omega^2 = -Fk^2 + i\rho\gamma\omega \to \omega^2 = v^2k^2 - i\gamma\omega$

$z \sim e^{-i\omega t}\sin(kx)$ $\quad \sin kl = 0 \to kl = n\pi \to k = \frac{n\pi}{l}$

$\to \omega = -\frac{i\gamma}{2} \pm \sqrt{-\frac{\gamma^2}{4}+v^2k^2} \to z \sim \sin\left(\frac{n\pi x}{l}\right) e^{-\frac{\gamma t}{2}} e^{-i\pm\sqrt{v^2k^2-\frac{\gamma^2}{4}}\,t}$



حالتی را در نظر می‌گیریم که $\frac{\partial B_z}{\partial t} \to B_z = 0 \to (\vec\nabla\times\vec E)_z = 0$

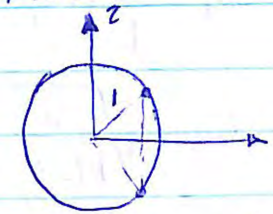
روی صفحه $z=C$ می‌توانیم پتانسیل تعریف کنیم.

$m\ddot z_j = K(z_{j+1}+z_{j-1}-2z_j) - \gamma m \dot z_j$

$\to -m\omega^2 z_j = K(z_{j+1}+z_{j-1}-2z_j) + im\gamma\omega z_j$

$-m\omega^2 = K(u+\frac{1}{u}-2) \to u^2+2b'u+1=0$, $b' = -\cos\theta$

با اضافه کردن $u^2 - \left(2 - \frac{m\omega^2}{k} - \frac{im\gamma\omega}{k}\right)u + 1 = 0$



$\delta b' = \frac{im\gamma\omega^2}{k}$

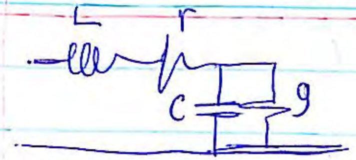
می‌خواهیم ببینیم برای γ کوچک جواب‌ها چه می‌شود. طبیعتاً یکی بیرون و یکی باید داخل باشد همیشه. پس اگر یکی افتاد بیرون همیشه بیرون است.

$\angle \frac{du}{db'}\delta b' = \angle u + \angle\frac{-1}{u+b'} + \frac{\pi}{2}$

$-\frac{\pi}{2} + \angle\frac{1}{u+b'}$

$\frac{1}{u+b'} = \frac{1}{\pm i\sin\theta}$

پایین $\frac{\pi}{2}-\frac{\pi}{2}=0$ $\quad$ بالا $-\frac{\pi}{2}-\frac{\pi}{2}=-\pi$



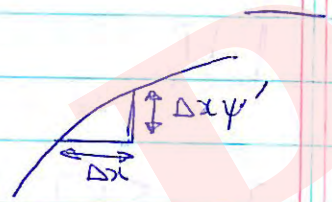
$$-i\omega l + r = -i\omega\left(l - \frac{r}{i\omega}\right)$$

$$-i\omega c + g = -i\omega\left(c - \frac{g}{i\omega}\right)$$

$$\frac{\partial v}{\partial z} = i\omega l I = i\omega l I - rI \to -l\frac{\partial I}{\partial t} - rI$$

$$\frac{\partial I}{\partial z} = (i\omega c - g)v$$

$$\Delta x\sqrt{1+\psi'^2} = \Delta x\left(1 + \frac{\psi'^2}{2}\right) \to$$



$$\delta U = \frac{1}{2}K\left[\Delta x - \Delta x_0 + \frac{1}{2}\psi'^2\Delta x\right]^2$$

$$= \frac{1}{2}K(\Delta x - \Delta x_0)^2 + \frac{1}{2}K(\Delta x - \Delta x_0)\Delta x\,\psi'^2 + \dots \to U = \frac{1}{2}\int F\psi'^2 dx$$

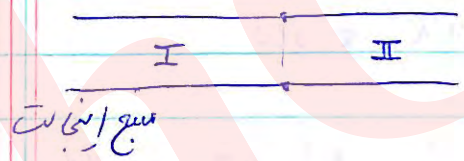
$$P = -F\psi'\dot{\psi} = \frac{F}{v}\dot{\psi}^2 = \sqrt{F\rho}\,\dot{\psi}^2 = Z\dot{\psi}^2$$

$$\frac{F}{\rho} = v^2 \quad , \quad \sqrt{F\rho} = Z$$

$$v_{(z)} = v_{(z+\Delta z)} + l\Delta z\frac{\partial I}{\partial t} \to$$

$$\frac{\partial v}{\partial z} = -l\frac{\partial I}{\partial t} \quad , \quad \frac{\partial I}{\partial z} = -c\frac{\partial v}{\partial t}$$

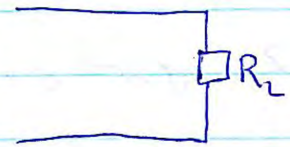
$$\to \frac{\partial^2 v}{\partial z^2} = lc\frac{\partial^2 v}{\partial t^2} \quad , \quad v = \frac{1}{\sqrt{lc}}$$



$$\begin{cases} V_I = a_I e^{ik_I x} + b_I e^{-ik_I x} \\ V_{II} = a_{II} e^{ik_{II} x} \end{cases}$$

$$\begin{cases} V_{I(0)} = V_{II(0)} \\ I_{I(0)} = I_{II(0)} \end{cases} \to \begin{cases} a_I + b_I = a_{II} \\ \frac{1}{V_I l_I}(a_I - b_I) = \frac{1}{V_{II} l_{II}} a_{II} \end{cases}$$

$$\frac{b_I}{a_I} = \frac{1 - \frac{R_I}{R_{II}}}{1 + \frac{R_I}{R_{II}}} \quad , \quad R = \sqrt{\frac{l}{c}} = vl$$



$$\frac{a+b}{\frac{1}{R}(a-b)} = R_L \quad \frac{a+b}{a-b} = \frac{R_L}{R} \quad , \quad \frac{b}{a} = \frac{1 - \frac{R}{R_L}}{1 + \frac{R}{R_L}}$$

$\longrightarrow \; : \; vnC_1\left(T_0 - \lambda \frac{\partial T}{\partial x}\right)$

$\longleftarrow \; : \; vnC_1\left(T_0 + \frac{\partial T}{\partial x}\lambda\right)$

$\to \dot{J} = vnC\lambda \frac{\partial T}{\partial x} \to K = \lambda v C$

$\to \frac{\ell^2 f}{\lambda v} \cdot \frac{\ell}{\lambda} = \frac{\ell}{10^3 D} \to$ اندازه مولکول

$\delta P = \mathcal{P} e^{-i\omega t + i\vec{k}\cdot\vec{r}}$ $\qquad \vec{V} = \frac{\vec{K}}{\omega\rho}\mathcal{P}$

دامنه سرعت

$P_{\text{توان}} = \int \vec{I}\cdot d\vec{s}$

$u_K = \frac{1}{2}\rho\vec{v}\cdot\vec{v}$ $\qquad u_p = \frac{1}{2}\frac{(\delta P)^2}{B}$

در آستانه شنوایی $\quad |\mathcal{P}| \sim 3\times 10^{-5}\ \text{pa} \to 10^{-11}\ \text{m}$

در آستانه دردناکی $\quad$ " $\quad |\mathcal{P}| \sim 30\ \text{pa} \to 10^{-5}\ \text{m}$

$$\begin{cases} \vec\nabla\cdot(\rho\vec v) + \frac{\partial \rho}{\partial t} = 0 \\ \rho\left[\frac{\partial \vec v}{\partial t} + \vec v\cdot\vec\nabla\vec v\right] = -\nabla P \end{cases} \qquad \begin{cases} \vec v_0 = 0 \\ P_0 , \rho_0 \ \text{ثابت} \end{cases}$$

$$\begin{cases} \frac{\partial}{\partial t}\delta\rho + \rho_0 \vec\nabla\cdot(\delta\vec v) = 0 \to \frac{\rho_0}{B}\frac{\partial}{\partial t}(\delta P) + \rho_0\vec\nabla\cdot(\delta\vec v) = 0 \\ \rho_0\frac{\partial}{\partial t}(\delta\vec v) = -\vec\nabla(\delta P) \to \frac{\rho_0}{B}\frac{\partial^2}{\partial t^2}(\delta P) - \nabla^2(\delta P) = 0 \\ \frac{\delta P}{\delta\rho} = \frac{B}{\rho_0} \end{cases}$$

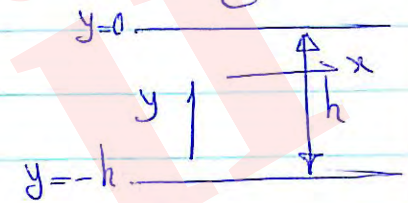
$\vec{J} = -K\vec\nabla T \quad ; \quad \int \vec{J}\cdot d\vec{s} = \int C\frac{\partial T}{\partial t}dv \to$

$K\nabla^2 T = -C\frac{\partial T}{\partial t} \to \frac{C}{\tau} \sim \frac{K}{\ell^2} \to \tau \sim \frac{\ell^2 C}{K}$

زمان لازم برای هم دما شدن

$$\frac{C\ell^2 f}{K}\begin{cases} \ll 1 & \text{همدما} \\ \gg 1 & \text{بی دررو} \end{cases}$$

عمق مایع و ابعاد طولی آن بزرگ است.



$$\psi_{(x,t)} = A\cos(\omega t - kx)$$

$$\begin{cases}\vec{\nabla}\cdot\vec{v} = 0 \\ \vec{\nabla}\times\vec{v} = 0 \to \vec{v} = +\vec{\nabla}\phi\end{cases} \to \nabla^2\phi = 0$$

شرایط مرزی $v_{y(y=-h)} = 0$; $v_{y(y=0)} = -A\omega\sin(\omega t - kx)$

$$\to \left.\frac{\partial\phi}{\partial y}\right|_{y=-h} = 0 \quad ; \quad \left.\frac{\partial\phi}{\partial y}\right|_{y=0} = -A\omega\sin(\omega t - kx)$$

$$\phi = \sin(\omega t - kx)\, f_{(y)} \to -k^2 f_{(y)}\sin(\omega t - kx) + f''_{(y)}\sin(\omega t - kx) = 0$$

$$\to f'' = k^2 f \to f = B\cosh(k(y+h))$$

$$\frac{\partial\phi}{\partial y} = Bk\sin(\omega t - kx)\sinh(k(y+h)) \to B = -\frac{A\omega}{k\sinh(kh)}$$

$$\phi = -\frac{A\omega}{k\sinh(kh)}\sin(\omega t - kx)\cosh(k(h+y))$$

$$\begin{cases} v_x = \dfrac{A\omega}{\sinh(kh)}\cosh(k(h+y))\cos(\omega t - kx) \\ v_y = -\dfrac{A\omega}{\sinh(kh)}\sinh(k(h+y))\sin(\omega t - kx)\end{cases}$$

$$\frac{\bar{T}}{\text{مساحت}} = \frac{1}{2}\rho \times \frac{1}{2}\frac{A^2\omega^2}{\sinh^2(kh)}\int_{-h}^{0} dy\left(\cosh^2\theta + \sinh^2\theta\right)$$

$$= \frac{\rho A^2\omega^2\cosh(kh)}{4k\sinh(kh)} \quad ; \quad \frac{\bar{U}_g}{\text{مساحت}} = \frac{\rho g A^2}{4} = \frac{1}{2}\rho g A^2\langle\cos^2(\omega t - kx)\rangle$$

$$L = \int\sqrt{1+\psi'^2}\,dx = \int\left(\frac{1}{2}\psi'^2 + 1\right)dx \to \frac{\bar{U}_\tau}{\text{مساحت}} = \frac{1}{4}A^2k^2\tau$$

$\bar{U}_g + \bar{U}_\tau = \bar{T} \to \rho g + k^2\tau = \dfrac{\rho\omega^2}{k\tanh(kh)}$ معادلهٔ پاشندگی

برای آب $\tau \ll 0 \Rightarrow \omega^2 = gk\tanh(kh)$ / طول موج از ۲ سانتیمتر >

برای کم‌عمق $kh \ll 1 \Rightarrow \dfrac{\omega}{k} = \sqrt{gh}$

برای عمیق $kh \gg 1$: $\omega^2 = gk$

$$[O(\tfrac{\partial}{\partial t}, \nabla)][R\vec{\psi}_{(t,R^{-1}(\vec r))}] = R(O\vec\psi)(t, R^{-1}(\vec r))$$

$$[O(-i\omega, iR\vec k)](R\vec\psi) = R[O(-i\omega, i\vec k)]\vec\varphi$$

$$[O(-i\omega, iR\vec k)]R = R[O(-i\omega, i\vec k)]$$

اگر $R\vec k = \vec k$, $[O(-i\omega, i\vec k)]R = R[O(-i\omega, i\vec k)]$

$$OR(\vec\psi_\parallel + \vec\psi_\perp) = RO(\vec\psi_\parallel + \vec\psi_\perp) \to$$

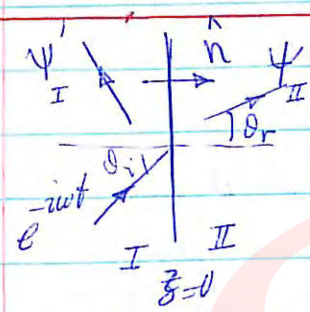
$$O(\vec\psi_\parallel + R\vec\psi_\perp) = RO\vec\psi_\parallel + RO\vec\psi_\perp \to$$ دوران 180

$$O(\vec\psi_\parallel - \vec\psi_\perp) = RO\vec\psi_\parallel + RO\vec\psi_\perp \xrightarrow{\vec\psi_\perp = 0} O\vec\psi_\parallel = RO\vec\psi_\parallel$$

$$\to O\vec\psi_\parallel = \lambda_\parallel \vec\psi_\parallel \equiv Mu = \lambda u$$

دوران 180 : $R(O\vec\psi_\perp) = -O\vec\psi_\perp \to (O\vec\psi_\perp)\cdot\vec k = 0$

$$\to O = \begin{bmatrix} \overset{1\times1}{O_\parallel} & \\ & \underset{2\times2}{O_\perp} \end{bmatrix}$$



$$Ae^{i\vec k_I\cdot\vec\rho} + Be^{i\vec k'_I\cdot\vec\rho} = Ce^{i\vec k_{II}\cdot\vec\rho}$$

$$\to \vec K_{I\parallel} = \vec K'_{I\parallel} = \vec K_{II\parallel} = \vec K_\parallel \to$$

تابش و بازتاب در یک صفحه است .

$$\sin\theta_r = \frac{K_{II\parallel}}{K_{II}} \to \frac{\sin\theta_r}{\sin\theta_i} = \frac{K_I}{K_{II}} = \frac{v_{II}}{v_I}$$

$$\underbrace{[O(\tfrac{\partial}{\partial t}, \nabla)]}_{\text{ماتریس}}\underbrace{\vec\psi_{(t,\vec r)}}_{\text{بردار}} = 0 \to \vec\psi = \vec\varphi e^{-i\omega t + i\vec k\cdot\vec r}$$

$$[O(-i\omega, i\vec k)]\vec\varphi = 0 \to \det[O(-i\omega, i\vec k)] = 0$$

$\tilde{\vec\psi}$: $\tilde{\vec\psi}_{(t,\vec r)} = R\vec\psi_{(t,R^{-1}(\vec r))}$ R: دوران

$$R\vec\varphi e^{-i\omega t + \underbrace{i\vec k\cdot(R^{-1}\vec r)}_{(R\vec k)\cdot\vec r}}$$

$$[O(-i\omega, i\vec k)](R\vec\psi_\perp) + [O(-i\omega, i\vec k)]\vec\psi_\parallel = 0$$

$R\vec k = \vec k$, $\vec\psi = \psi_\parallel + \vec\psi_\perp$

C.C.: Complex Conjugate

$\vec E_{\text{درونی}} = K\frac{3(\vec r\cdot\hat r)\hat r - \vec r}{r^3} \to (\vec E_{\vec r})_i = \sum_j \left[K\frac{3r_ir_j - \vec r\cdot\vec r\,\delta_{ij}}{r^5}\right]\vec r_j$

$\vec E_{\vec r} = M\vec r \to \vec E_{\text{درونی}} = \sum_a M_a\vec r_a = \gamma\left(\sum_a M_a\right)\vec E$

$\sum_i M_{ii} = K\frac{3\vec r\cdot\vec r - \vec r\cdot\vec r(3)}{r^5} = 0 \to \vec E_{\text{درونی}} = 0$

$$\begin{cases}\nabla\times\vec B = \mu\varepsilon(-i\omega\vec E) + \mu\sigma\vec E \\ \nabla\times\vec E - i\omega\vec B = 0\end{cases}$$

$\sigma \gg \varepsilon\omega$ ،μ, ε حقیقی و ثابت

$i\omega\varepsilon \to -\sigma$

$\varepsilon \to -\frac{\sigma}{i\omega}$

$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\frac{i\mu\sigma}{\omega}}}$, $K = \frac{\omega}{v} = \omega\sqrt{\frac{i\mu\sigma}{\omega}} \to K^2 = i\mu\sigma\omega$

$\to -\nabla^2\psi = -\mu\sigma\frac{\partial\psi}{\partial t}$

$\sqrt{i} = \sqrt{e^{i\pi/2}} = e^{\frac{i\pi}{4}} = \frac{1+i}{\sqrt2}$, $\vec K = K\hat K$ ~ حقیقی

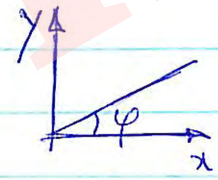
$\vec E = E_{\text{حقیقی}}(\alpha\hat x + \beta\hat y)$; $|\alpha|^2 + |\beta|^2 = 1$

$\frac{\beta}{\alpha} =$ حقیقی $\to$ قطبش خطی $\to \vec E = E(\alpha\hat x + \beta\hat y)\cos\omega t\ \hat z$

$\frac{\beta}{\alpha} = \pm i \to$ قطبش دایره‌ای $\to \vec E = \frac{E}{\sqrt2}(\hat x\cos\omega t \pm \hat y\sin\omega t)\ \hat z$

$\to \mathrm{Re}\left[(\hat x \pm i\hat y)e^{-i\omega t}\right]$

قطبش بیضوی $\vec E = E(\alpha\cos\omega t\,\hat x + |\beta|\hat y\cos(\omega t - \theta))\ \hat z$



$\hat e_{(\varphi)} = \hat x\cos\varphi + \hat y\sin\varphi$

$\hat e_\pm = \frac{\hat x \pm i\hat y}{\sqrt2}$

$\to \hat x = \frac{\hat e_+ + \hat e_-}{\sqrt2}$ $\qquad \hat y = \frac{\hat e_+ - \hat e_-}{i\sqrt2}$

$\to \hat e_{(\varphi)} = \frac{1}{\sqrt2}\left(e^{-i\varphi}\hat e_+ + \hat e_- e^{i\varphi}\right)$

$\frac{1}{\sqrt2}\left[\hat e_+ e^{-i\varphi + ik_+l} + \hat e_- e^{i\varphi + ik_-l}\right] = (\ )\frac{1}{\sqrt2}\left[\hat e_+ e^{-i\varphi + i\frac{k_+ - k_-}{2}l} + \hat e_-\, c.c.\right]$

$$\rho_m = mn = \rho_{m_0} + m(n-n_0) = \rho_{m_0} + \delta\rho_m$$

$$\rho_e = (n-n_0)q = \frac{q}{m}\delta\rho_m$$

n متحرک n_0 ثابت ها

$$\begin{cases} \frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) = 0 \\ \rho_m\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + nq\vec{E} \end{cases} \qquad \begin{cases} \rho_m \frac{dP}{d\rho_m} = B \\ \vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\varepsilon_0} \end{cases}$$

$$\begin{cases} \frac{\partial \delta n}{\partial t} + n_0\vec{\nabla}\cdot\vec{v} = 0 \\ mn_0\frac{\partial \vec{v}}{\partial t} + \vec{\nabla}(\delta P) = n_0 q\vec{E} \end{cases} \qquad \begin{cases} \frac{\delta P}{\delta n} = \frac{B}{n_0} \\ \frac{n_0}{B}\frac{\partial(\delta P)}{\partial t} + n_0\vec{\nabla}\cdot\vec{v} = 0 \end{cases}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{q\,\delta_n}{\varepsilon_0} \qquad mn_0\frac{\partial}{\partial t}\vec{\nabla}\cdot\vec{v} = -\nabla^2(\delta P) + n_0 q\frac{q\,\delta n}{\varepsilon_0}$$

$$\frac{\partial^2}{\partial t^2}(\delta P) + \frac{B}{mn_0}\left[-\nabla^2(\delta P) + \frac{n_0^2 q^2}{\varepsilon_0 B}\delta P\right] = 0 \rightarrow$$

$$\frac{B}{mn_0}\nabla^2(\delta P) = \left(\frac{\partial^2}{\partial t^2} + \frac{n_0 q^2}{m\varepsilon_0}\right)\delta P \rightarrow v^2\nabla^2(\delta P) = \left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\delta P$$

$$\rightarrow v^2 k^2 = \omega^2 - \omega_p^2 \rightarrow v_p = \frac{\sqrt{v^2k^2 + \omega_p^2}}{k}, \quad v_g = \frac{v^2}{v_p}$$

$$k = \sqrt{\mu\sigma\omega}\,\frac{1+i}{\sqrt{2}} = \frac{1+i}{\delta}, \quad \delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

$$(\nabla^2 + k^2)\vec{E} = 0, \quad \vec{E} = \hat{z}E_{(\rho,z)}$$

استوانه

$$\left(\frac{1+i}{\delta}\right)^2 = \frac{2i}{\delta^2} \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial E}{\partial \rho}\right) + \frac{2i}{\delta^2}E = 0, \quad \vec{E} = E_{\rho}\,\hat{z}$$

$$\delta \gg R: \quad \frac{\partial^2 E}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E}{\partial \rho} + \frac{2i}{\delta^2}E = 0,$$

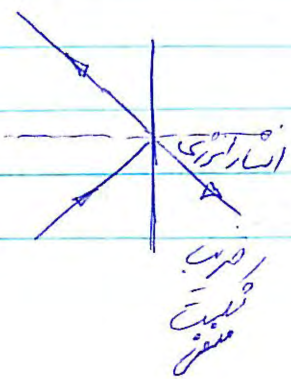
$$E = E_0 + \rho E_1 + \rho^2 E_2 + \dots \rightarrow 2E_2 + \frac{E_1}{\rho} + 2E_2 + \frac{2i}{\delta^2}E_0 = 0$$

$$E_1 = 0, \quad E_2 = -\frac{i}{2\delta^2}E_0, \quad E_3 = 0, \quad E_4 = -\frac{E_0}{16\delta^4}$$

$$\rightarrow E_\rho = E_0\left(1 - \frac{i}{2\delta}\rho^2 - \frac{\rho^4}{16\delta^4}\right) \rightarrow |E|^2 = |E_0|^2\left(1 + \frac{\rho^4}{8\delta^4}\right)$$

$\delta \ll R$: رفتار نمایی است.

$\mu, \varepsilon < 0$



$$i\vec{k}\times\vec{E} - i\omega\vec{B} = 0 \rightarrow \vec{H} = \frac{\vec{k}\times\vec{E}}{\mu\omega}$$

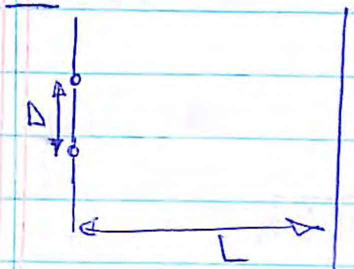
$$\vec{S} = \frac{1}{2}\mathrm{Re}(\vec{E}\times H^*) = \frac{E^2}{2\mu\omega}\vec{k}$$

$$A_{(\theta,\theta_i)} = C e^{i\cdot}\int_{-D/2}^{D/2} dx'\, e^{ik(\sin\theta_i-\sin\theta)x'} = C e^{i\cdot}\,\frac{e^{\frac{ikD}{2}(\sin\theta_i-\sin\theta)} - e^{-\frac{ikD}{2}(\sin\theta_i-\sin\theta)}}{ik(\sin\theta_i-\sin\theta)}$$

$$\rightarrow A = (\ )\,\frac{\sin\left(\frac{kD}{2}(\sin\theta_i-\sin\theta)\right)}{\frac{kD}{2}(\sin\theta_i-\sin\theta)} = (\ )\,\frac{\sin a}{a}$$

$\theta_i = 0$, $\dfrac{\sin\left(\frac{kD}{2}\sin\theta\right)}{\frac{kD}{2}\sin\theta} \rightarrow \dfrac{kD}{2}\sin\theta = \pi$ / نوار تاریک اول

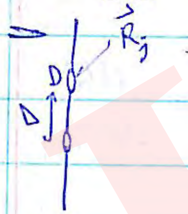
$$\rightarrow \theta \sim \frac{2\pi}{kD} = \frac{\lambda}{D}$$



نقطه‌ای : $e^{i(\vec k_i-\vec k)\cdot\frac{\Delta}{2}\hat x} + e^{-i(\vec k_i-\vec k)\cdot(\frac{\Delta}{2}\hat x)}$

$$= e^{\frac{ikD}{2}(\sin\theta_i-\sin\theta)} + e^{-\frac{ik\Delta}{2}(\sin\theta_i-\sin\theta)}$$

$$= \cos\left[\frac{kD}{2}(\sin\theta_i-\sin\theta)\right] \Rightarrow \frac{k\Delta\,\theta}{2} = n\pi \rightarrow \theta = \frac{n\lambda}{D}$$



$$\int ds' e^{i(\vec k_i-\vec k)\cdot\vec r'} = \sum_j \int_{S_j} ds' e^{i(\vec k_i-\vec k)\cdot\vec r'}$$

$$= \sum_j e^{i(\vec k_i-\vec k)\cdot\vec R_j}\int_{S_j} e^{i(\vec k_i-\vec k)\cdot\vec r''}\, ds''$$

در بسامدهای زیاد : $\varepsilon = \varepsilon_0(1+\chi) = \varepsilon_0\left(1-\frac{\omega_p^2}{\omega^2}\right)$ $\quad \omega \gg \omega_p$

$$k^2 = \varepsilon\mu\omega^2 = \frac{\omega^2}{c^2}\left(1-\frac{\omega_p^2}{\omega^2}\right) \quad , \quad c^2k^2 = \omega^2-\omega_p^2$$

$$\rightarrow v_p = c\,\frac{\sqrt{k^2+\frac{\omega_p^2}{c^2}}}{k} \quad , \quad v_g = \frac{ck}{\sqrt{k^2+\frac{\omega_p^2}{c^2}}}$$

فاز $= \vec k_i\cdot\vec r'$ $\qquad A_{(\vec r)} = (\ )\int ds' \dfrac{e^{i(\vec k_i\cdot\vec r' + k|\vec r-\vec r'|)}}{|\vec r-\vec r'|}$

$$\varphi = \vec k_i\cdot\vec r' + k|\vec r-\vec r'|$$

$$\rightarrow A_{(\vec r)} = (\ )\,\frac{e^{ikr}}{r}\int ds' e^{i[(\vec k_i-\vec k)\cdot\vec r']}$$

$|\vec r-\vec r'| \simeq r - \dfrac{\vec r\cdot\vec r'}{r}$, $\vec r$ مکان مشاهده , $\vec r'$ مکان جسم ; $r \gg r'$

$$\rightarrow \vec k = k\hat r = k\frac{\vec r}{r}$$

فاز $= k(x'\sin\theta_i - x'\sin\theta)$; $\vec k_i = k(\hat z\cos\theta_i + \hat x\sin\theta_i)$

تابش: از تحلیل ابعادی داریم:
$$P \sim \frac{K p^2 \omega^4}{c^3}$$

q تابع زمان است یا بار نقطه‌ای نوسان می‌کند. (تقریباً) $q \wedge\wedge\wedge$

$$\vec{E} \approx K \frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3} e^{ikr} = K e^{ikr} \vec{\mathcal{E}} \quad , \quad \vec{B} \approx 0$$

$$\begin{cases} \vec{\nabla}\times\vec{E} - i\omega\vec{B} = 0 \longrightarrow \vec{\nabla}\times\vec{E} - iKc\vec{B} = 0 \\ \vec{\nabla}\times\vec{B} + \frac{i\omega}{c^2}\vec{E} = 0 \longrightarrow \end{cases}$$

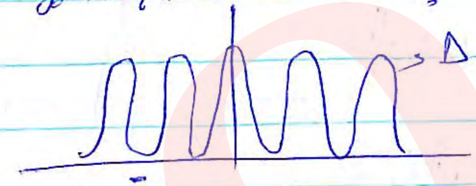
$$\begin{cases} \vec{\nabla}\times(e^{ikr}\vec{\mathcal{E}}) - iK(e^{ikr}\vec{\mathcal{B}}) = 0 \\ \vec{\nabla}\times(e^{ikr}\vec{\mathcal{B}}) + iK(e^{ikr}\vec{\mathcal{E}}) = 0 \end{cases} , \quad \begin{cases} \vec{E} = K e^{ikr}\vec{\mathcal{E}} \\ c\vec{B} = K e^{ikr}\vec{\mathcal{B}} \end{cases}$$

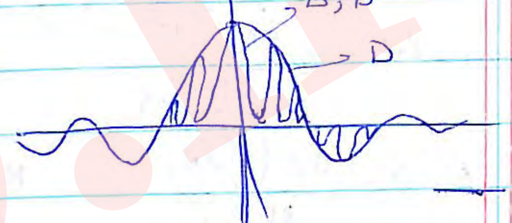
$$\vec{\mathcal{E}}_0 = \frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3} \quad , \quad \vec{\mathcal{B}}_0 = 0$$

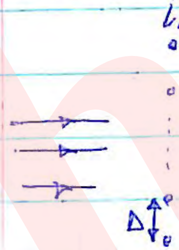
$$\begin{cases} \vec{\nabla}\times\vec{\mathcal{E}} + iK\hat{r}\times\vec{\mathcal{E}} - iK\vec{\mathcal{B}} = 0 \\ \vec{\nabla}\times\vec{\mathcal{B}} + iK\hat{r}\times\vec{\mathcal{B}} + iK\vec{\mathcal{E}} = 0 \end{cases} \qquad \begin{cases} \vec{\nabla}\cdot\vec{\mathcal{E}} + iK\hat{r}\cdot\vec{\mathcal{E}} = 0 \\ \vec{\nabla}\cdot\vec{\mathcal{B}} + iK\hat{r}\cdot\vec{\mathcal{B}} = 0 \end{cases}$$

اگر چشمه‌ها مثل هم باشند:
$$= A_{\theta i} \sum_j e^{i(\vec{K}_i - \vec{K})\cdot\vec{R}_j}$$

$A_{کل} = A_i \times A$ حاصل از چشمه‌های نقطه‌ای







$$\sum_{j=0}^{N-1} e^{iK(-\sin\theta) j\Delta} = \frac{1 - e^{-iKN\Delta\sin\theta}}{1 - e^{-iK\Delta\sin\theta}} = (\ ) \frac{\sin\left(\frac{NK\Delta}{2}\sin\theta\right)}{\sin\left(\frac{K\Delta}{2}\sin\theta\right)}$$

$$\sin\theta_n = \frac{2n\pi}{K\Delta} = \frac{n\lambda}{\Delta}$$

$$\frac{NK\Delta}{2}\sin(\theta_n + \delta\theta_n) = \pi + \frac{NK\Delta}{2}\sin\theta_n \rightarrow \delta\theta_n = \frac{\lambda}{N\Delta\cos\theta_n}$$

$$\lambda \rightarrow \theta_n \quad , \quad \lambda + \Delta\lambda \rightarrow \theta_n + \Delta\theta_n$$

$$\sin(\theta_n + \Delta\theta_n) = \frac{n(\lambda + \Delta\lambda)}{\Delta} \rightarrow \cos\theta_n \Delta\theta_n = \frac{n\Delta\lambda}{\Delta} \rightarrow \Delta\theta_n = \frac{n\Delta\lambda}{\Delta\cos\theta_n}$$

$$\rightarrow \frac{n\Delta\lambda}{\Delta\cos\theta_n} = \frac{\lambda}{N\Delta}\frac{1}{\cos\theta_n} \rightarrow \frac{\Delta\lambda}{\lambda} = \frac{1}{nN}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times H^{*} = \frac{K^2k^4}{2\mu_0 C}\frac{[(\vec{p}\times\vec{r})\times\hat{r}]\times(\vec{p}\times\vec{r})}{r} = \frac{K^2k^4}{2\mu_0 C}\hat{r}\frac{|\vec{p}\times\vec{r}|^2}{r^2}$$

$$\longrightarrow P = \frac{K^2k^4}{2\mu_0 C}\int|\vec{r}\times\hat{r}|^2 d\Omega \Rightarrow P = \frac{Kk^4 c p^2}{3}$$

$$\longrightarrow P = \frac{Kq^2|\vec{R}|^2\omega^4}{3c^3} = \frac{Kq^2}{3c^3}|\vec{a}|^2 = \frac{2Kq^2}{3c^3}\bar{a}^2 \longrightarrow$$ دقیق

برای شتاب سینوسی دقیق، که $|\vec{a}|$ دامنه شتاب

$$P_{(t)} = \frac{2Kq^2}{3c^3}a^2_{(t-\frac{R}{c})}$$ برای غیر نسبیتی (سرعت کم)

$$\frac{dE}{dt} = \frac{Kq^2}{c^3}\left(\frac{Kq^2}{mr^2}\right)^2 = -\frac{Kq^2}{r^2}\frac{dr}{dt} \longrightarrow \frac{dr^3}{dt} = -\frac{(Kq^2)^2}{m^2c^3}$$

$$\longrightarrow \tau = \frac{m^2c^3R_0^3}{(Kq^2)^2} \longrightarrow \tau = 10^{-9}s$$ برای هیدروژن

$$\overbrace{\vec{p}\times\vec{r}}^{B}\ ,\ \overbrace{\vec{p}\ ,\ (\vec{p}\cdot\vec{r})\vec{r}}^{E}\qquad (\vec{p},\vec{r})$$

$$\vec{\nabla}\times\vec{B}_1 + iK\vec{\mathcal{E}}_0 = 0 \qquad ,\ \vec{B}_1 = \alpha K\frac{\vec{p}\times\vec{r}}{r^3}$$

$$\vec{\nabla}\times\vec{B}_1 = \alpha K\vec{\mathcal{E}}_0 \rightarrow \alpha = -i \rightarrow \vec{B}_1 = -iK\frac{\vec{p}\times\vec{r}}{r^3}$$

$$\vec{\nabla}\times\vec{\mathcal{E}}_1 + iK\hat{r}\times\vec{\mathcal{E}}_0 = 0 \rightarrow \vec{\nabla}\times\vec{\mathcal{E}}_1 + iK\frac{\vec{p}\times\hat{r}}{r^3} = 0$$

$$\vec{\nabla}\cdot\vec{\mathcal{E}}_1 + iK\hat{r}\cdot\vec{\mathcal{E}}_0 = 0 \qquad ,\ \vec{\mathcal{E}}_1 = \beta K\frac{\vec{p}}{r^2} + \gamma K\frac{(\vec{p}\cdot\vec{r})\vec{r}}{r^4}$$

$$\vec{\nabla}\times\vec{\mathcal{E}}_1 = \beta K\frac{-2\hat{r}}{r^3}\times\vec{p} + \gamma K\frac{\vec{p}\times\hat{r}}{r^3} \longrightarrow 2\beta + \gamma + i = 0$$

$$\vec{\nabla}\cdot\vec{\mathcal{E}}_1 = \beta K\frac{-2\vec{p}\cdot\hat{r}}{r^3} \qquad \rightarrow \gamma = -3i$$

$$\rightarrow \vec{\mathcal{E}}_1 = -iK\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r} - \vec{p}r^2}{r^5}\right] = -iKr\vec{\mathcal{E}}_0$$

$$\vec{B}_2 = \delta K^2\frac{\vec{p}\times\vec{r}}{r^2} \rightarrow \vec{\nabla}\times\vec{B}_2 = \delta K^2\left[r\frac{3(\vec{p}\cdot\vec{r})\vec{r} - \vec{p}r^2}{r^5} + \hat{r}\times\frac{\vec{p}\times\vec{r}}{r^3}\right]$$

$$\rightarrow \delta = -1 \rightarrow \vec{B}_2 = -K^2\frac{\vec{p}\times\vec{r}}{r^2}\ ,\ \vec{\mathcal{E}}_2 = +K^2\frac{\vec{p} - (\vec{p}\cdot\hat{r})\hat{r}}{r}$$

$$\int \vec{j}\cdot\vec{E}\,dv = IV = \left[\mathrm{Re}\left(v e^{-i\omega t}\right)\right]\left[\mathrm{Re}\left(\mathcal{J} e^{-i\omega t}\right)\right]$$

$$= |v|\cos(\omega t-\hat{v})\,|\mathcal{J}|\cos(\omega t-\hat{\mathcal{J}}) \Rightarrow$$

$$\bar{P} = \frac{1}{2}|\mathcal{J}||v|\left(\cos(2\omega t-\hat{v}-\hat{\mathcal{J}})+\cos(\hat{v}-\hat{\mathcal{J}})\right)$$

$$\rightarrow \bar{P} = \frac{1}{2}|\mathcal{J}||v|\cos(\hat{v}-\hat{\mathcal{J}})$$

$$v = |v|e^{i\hat{v}} \qquad \mathcal{J} = |\mathcal{J}|e^{i\hat{\mathcal{J}}}$$

$$\bar{P} = \frac{1}{2}|\mathcal{J}||v|\,\mathrm{Re}\left(e^{i(\hat{v}-\hat{\mathcal{J}})}\right) = \frac{1}{2}\mathrm{Re}\left(v\mathcal{J}^{*}\right)$$

$$\mathcal{E}(T_1,T_2,\dots,T_n)=T_{11}\,\mathcal{E}(e_1,T_2,\dots,T_n)+T_{21}\,\mathcal{E}(e_2,T_2,\dots,T_n)$$

$$+\dots=T_{11}\,\mathcal{E}(e_1,\tilde{T}_2,\dots,\tilde{T}_n)+T_{21}\,\mathcal{E}(e_2,\tilde{\tilde{T}}_2,\dots,\tilde{\tilde{T}}_n)+\dots$$

که مؤلفه اول صفر

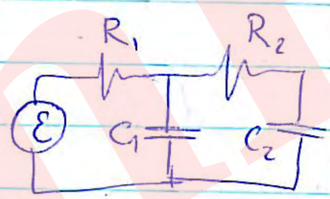
$$\det T=T_{11}\det(\tilde{T}_2,\dots,\tilde{T}_n)+T_{21}(-1)\det(\tilde{\tilde{T}}_2,\dots,\tilde{\tilde{T}}_n)$$

$T_{ij}\,(-1)^{i+j}$ برای تعیین منفی و مثبت

M وارون پذیر نباشد : $\dim[\ker(M)]>0 \to \dim[\mathrm{img}(M)]<n$

$$\to \mathcal{E}(Me_1,Me_2,\dots,Me_n)=0 \to \det M=0$$

$$\det\begin{pmatrix} C_1 s+\frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & C_2 s+\frac{1}{R_2}\end{pmatrix}=0$$



ماتریس معکوس

$$\Delta_{ij}=(-1)^{i+j}\det\tilde{M}_{ij}$$

$$M^{-1}=\frac{\Delta_{ij}}{\det(M)}$$

$$M=\begin{pmatrix}1&2&3\\0&1&-1\\2&0&1\end{pmatrix}\to \Delta=\begin{pmatrix}1&-2&-2\\-2&-5&4\\ \dots&\dots&\dots\end{pmatrix}$$

$$\det(M)=4=(1\times1)+(2\times-2)+(3\times-2)=$$

$$(0\times-2)+(-5\times1)+(4\times-1)$$

$$V=\alpha_1 V_1^H e^{s_1 t}+\alpha_2 V_2^H e^{s_2 t}$$

برای زمان های بزرگ جواب همگن صفر می شود.

$$I_1=C_1\frac{dV_1}{dt}$$

$$\cancel{P=\dots}\quad P=\sum q_a\vec{V}_a\cdot\vec{E}=\sum(\vec{J}\Delta V)\cdot\vec{E}$$

$$\mathcal{E}_{(u,v)} = \mathcal{E}_{(u_1e_1+u_2e_2,\ v_1e_1+v_2e_2)} = (u_1v_2 - u_2v_1)\,\mathcal{E}_{(e_1,e_2)}$$

برای 3 تا هم می‌توان کاری مشابه بالا کرد.

$\mathcal{E}'_{(u,v)}$ را طوری تعریف می‌کنیم که خواص $\mathcal{E}$ را داشته باشد.

$$\mathcal{E}'_{(u,v)} = (u_1v_2 - u_2v_1)\,\mathcal{E}'_{(e_1,e_2)}$$

پس α و β ای حتما می‌توان یافت که: $\alpha\mathcal{E}'_{(u,v)} + \beta\mathcal{E}_{(u,v)} = 0$

$$\mathcal{E}_{(e_1,e_2,e_3)} = -\mathcal{E}_{(e_2,e_1,e_3)} = \mathcal{E}_{(e_2,e_3,e_1)}$$

به تعداد جایگشت‌ها بستگی ندارد.

$$T:\ \mathbb{V} \to \mathbb{V}$$

$$\mathcal{E}(e_1,e_2,\dots,e_n) \neq 0$$

$$\mathcal{E}(Tu_1, Tu_2,\dots,Tu_n) = \tilde{\mathcal{E}}_{(u_1,\dots,u_n)}$$

$$\tilde{\mathcal{E}}_{(\alpha u_1+\alpha' u'_1,\ \dots)} = \mathcal{E}_{(T(\alpha u_1+\alpha' u'_1),\ \dots)} =$$

$$\mathcal{E}_{(\alpha Tu+\alpha' Tu',\ \dots)} \to \tilde{\mathcal{E}}_{(u_1,u_2,\dots,u_n)} = \left[\det(T)\right]\mathcal{E}_{(u_1,u_2,\dots,u_n)}$$

$$\to \det(T) = \frac{\mathcal{E}(Te_1,Te_2,\dots,Te_n)}{\mathcal{E}(e_1,e_2,\dots,e_n)}$$

$$\mathcal{E}(STe_1, STe_2, \dots, STe_n) =$$

$$(\det S)\,\mathcal{E}(Te_1, Te_2, \dots, Te_n) =$$

$$(\det(S))(\det(T))\,\mathcal{E}(e_1,e_2,\dots,e_n)$$

$$\to \det(ST) = \det(S)\det(T)$$

$$\mathrm{mat}(T) = \big((Te_1),(Te_2),\dots,(Te_n)\big) = (T_1,T_2,\dots,T_n)$$

$$\det(T) = \frac{\mathcal{E}(T_1,T_2,\dots,T_n)}{\mathcal{E}(e_1,e_2,\dots,e_n)}$$

$$T_1 = T_{11}e_1 + T_{21}e_2 + \dots + T_{n1}e_n$$

بار پوسته ی داخلی q است   در حالت خاص $\sigma_0 = \delta$ پتانسیل در فضای بین

دو پوسته را بدست آورید ـ ثابت کنید

نیروی خالص وارد بر پوسته ی داخلی را مشخص کنید.

رابطه ی بین پتانسیل در دو پوسته چنین است: $\varphi_{(r,\theta)} = \left(A_0 + \frac{B_0}{r}\right) + \delta\left(A_1 r + \frac{B_1}{r^2}\right)\cos\theta$

فضای

تابستان 87 ـ آزمون 2 ـ سوال

$0 \le x \le 2a : (\rho a + 2\rho a + \sigma_1 + \sigma_2) S = Q = 0$

$\boxed{\sigma_1 + \sigma_2 = -3\rho a}$ $0 < x < a : E_1 = \frac{\sigma_1}{\varepsilon_0} + \frac{\rho x}{\varepsilon_0}$

$a < x < 2a : E_2 = (E_1)_{x=a} + \frac{2\rho x'}{\varepsilon_0} = \left(\frac{\sigma_1 + \rho a}{\varepsilon_0}\right) + \frac{2\rho x'}{\varepsilon_0}$

$\Delta V_1 = V_{1x} - V_0 = -\frac{1}{\varepsilon_0}\int(\sigma_1 + \rho x)\,dx = -\frac{\left(\sigma_1 x + \frac{\rho x^2}{2}\right)}{\varepsilon_0}$

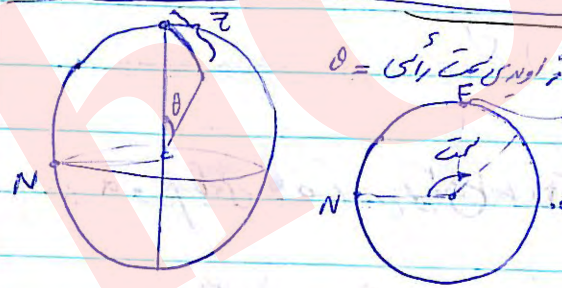
$V_1\big|_{x=a} = -\left(\frac{\sigma_1 a}{\varepsilon_0} + \frac{\rho a^2}{2\varepsilon_0}\right) \to \Delta V_2 = -\int E_2\,dx = -\left(\frac{\sigma_1 + \rho a}{\varepsilon_0}\right)x' - \frac{2\rho\left(\frac{x'^2}{2}\right)}{\varepsilon_0}$

$V_2 = V_1 - \frac{1}{\varepsilon_0}\left((\sigma_1 + \rho a)x' + \frac{\rho}{\varepsilon_0}x'^2\right)$

$x' = a \to V_2 = V_1 - \frac{1}{\varepsilon_0}(\sigma_1 a + \rho a^2 + \rho a^2) \to V_2 = -\frac{1}{\varepsilon_0}\left(\sigma_1 a + \frac{\rho a^2}{2} + \sigma_1 a + 2\rho a^2\right)$

$\to V_2 = -\frac{a}{\varepsilon_0}\left(2\sigma_1 + \frac{5\rho a}{\varepsilon_0}\right) = V_0 \to 2\sigma_1 + \frac{5}{2}\rho a = -\varepsilon_0 \frac{V_0}{a}$

$\sigma_1 = -\frac{1}{2}\left(\frac{\varepsilon_0 V_0}{a} + \frac{5\rho a}{2}\right) \to \boxed{\sigma_1 = -\left(\frac{2\varepsilon_0 V_0 + 5\rho a^2}{4a}\right)}$



$\theta = $ زاویه ی سمت الرأس $= z$

مبدأ طول جغرافیایی گرینویچ است.

$L = m\sqrt{GMa(1-e^2)}$ $E = -\frac{GMm}{2a}$

$\varepsilon(u,v) = $ مساحت جبری

$\varepsilon(u,u) = 0$

$\varepsilon(u,v) = -\varepsilon(v,u)$

$\varepsilon(\alpha u, v) = \alpha\,\varepsilon(u,v)$

مثال: یک فواره می‌تواند ذرات آب را در همه‌ی جهات با سرعت اولیه‌ی v_0 پرتاب

کند. معادله‌ی رویه‌ی بوجود آمده را بدست آورید $y = \frac{-gx^2}{2v_0^2} + \frac{v_0^2}{2g}$

مثال: تعداد سوراخ‌ها در واحد سطح برای یک آب‌پاش تابعی از α است. این تابع را

برحسب α و بصورت تناسبی به گونه‌ای مشخص کنید که در تمامی دور از فواره آبیاری

بصورت یکنواخت انجام شود. $N = n_{(\alpha)} \times 2\pi R^2 \sin\alpha\, d\alpha \sim 2\pi r\, dr$

$$r = \frac{v_0^2 \sin 2\left(\frac{\pi}{2}-\alpha\right)}{g} = \frac{v_0^2 \sin(2\alpha)}{g} \to dr = \frac{v_0^2}{g}\left(2\cos(2\alpha)\, d\alpha\right)$$

$$n_{(\alpha)} \sim \frac{r\, dr}{\sin 2\alpha\, d\alpha} \to n_{(\alpha)} \sim \frac{\sin 4\alpha}{\sin \alpha}$$

مثال:

$$x_1\left(t-\sqrt{\tfrac{2h}{g}}\right) = a\sin\omega\left(t-\sqrt{\tfrac{2h}{g}}\right) \to \dot{x}_1 = a\omega\cos\omega\left(t-\sqrt{\tfrac{2h}{g}}\right)$$

$$\Delta x = \dot{x}_1 \sqrt{\tfrac{2h}{g}} = x_{2t} - x_1\left(t-\sqrt{\tfrac{2h}{g}}\right)$$