Algebra

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

are the solutions of $ax^2 + bx + c = 0$.

Multiplication and Factoring

$u^2 - v^2 = (u + v)(u - v)$

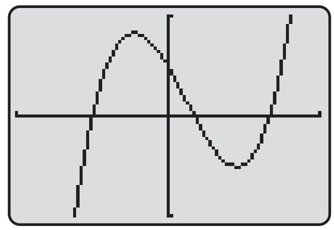
$(u \pm v)^2 = u^2 \pm 2uv + v^2$

$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$

$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$

Equations and Graphs

The solutions of the equation $f(x) = 0$ are the x-intercepts of the graph of $y = f(x)$.



Exponents

$c^r c^s = c^{r+s}$

$\frac{c^r}{c^s} = c^{r-s}$

$(c^r)^s = c^{rs}$

$(cd)^r = c^r d^r$

$\left(\frac{c}{d}\right)^r = \frac{c^r}{d^r} \quad (d \neq 0)$

$c^{-r} = \frac{1}{c^r} \quad (c \neq 0)$

Natural Logarithms

For $v, w > 0$ and any u, k:

$\ln v = u$ means $e^u = v$

$\ln(vw) = \ln v + \ln w$

$\ln\left(\frac{v}{w}\right) = \ln v - \ln w$

$\ln(v^k) = k(\ln v)$

Logarithms to Base b

For $v, w > 0$ and any u, k:

$\log_b v = u$ means $b^u = v$

$\log_b(vw) = \log_b v + \log_b w$

$\log_b\left(\frac{v}{w}\right) = \log_b v - \log_b w$

$\log_b(v^k) = k(\log_b v)$

Special Notation

$\ln v$ means $\log_e v$

$\log v$ means $\log_{10} v$

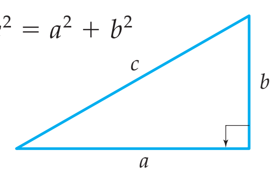
Change of Base Formula

$$\log_b v = \frac{\ln v}{\ln b}$$

Geometry

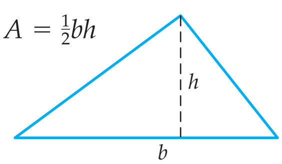
The Pythagorean Theorem

$c^2 = a^2 + b^2$



Area of a Triangle

$A = \frac{1}{2}bh$

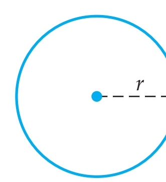


Circles

Diameter $= 2r$

Circumference $= 2\pi r$

Area $= \pi r^2$

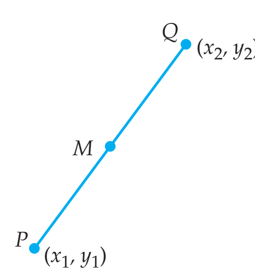


Distance Formula

Length of segment $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Midpoint Formula

Midpoint M of segment $PQ = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



Slope of nonvertical line through (x_1, y_1) and (x_2, y_2)

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Equation of line with slope m through (x_1, y_1)

$$y - y_1 = m(x - x_1)$$

Equation of line with slope m and y-intercept b

$$y = mx + b$$

Trigonometry

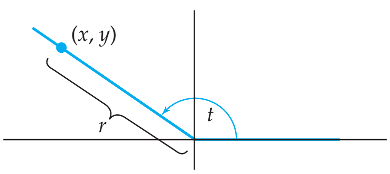
If t is a real number and P is the point where the terminal side of an angle of t radians in standard position meets the unit circle, then

$$\cos t = x\text{-coordinate of } P \qquad \sin t = y\text{-coordinate of } P$$

$$\tan t = \frac{\sin t}{\cos t} \qquad \csc t = \frac{1}{\sin t} \qquad \sec t = \frac{1}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t}$$

Point-in-the-Plane Description

For any real number t and point (x, y) on the terminal side of an angle of t radians in standard position:



$$\sin t = \frac{y}{r} \qquad \cos t = \frac{x}{r} \qquad \tan t = \frac{y}{x} \quad (x \neq 0)$$

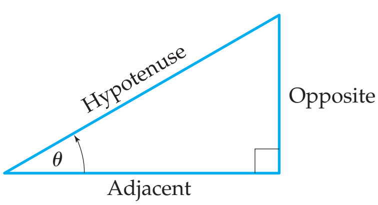
$$\csc t = \frac{r}{y} \quad (y \neq 0) \qquad \sec t = \frac{r}{x} \quad (x \neq 0) \qquad \cot t = \frac{x}{y} \quad (y \neq 0)$$

Periodic Graphs

If $A \neq 0$ and $b > 0$, then each of $f(t) = A\sin(bt + c)$ and $g(t) = A\cos(bt + c)$ has

$$\text{amplitude } |A|, \qquad \text{period } 2\pi/b, \qquad \text{phase shift } -c/b.$$

Right Triangle Trigonometry

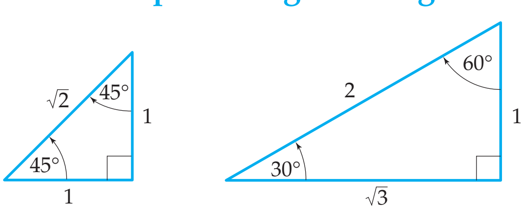


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Special Values

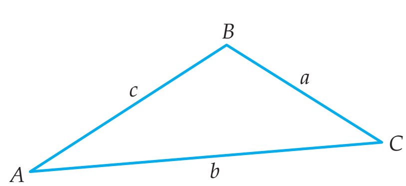
θ Degrees	θ Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	undefined

Special Right Triangles



Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$



Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area Formulas

Herron's Formula: Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a + b + c)$ Area $= \frac{1}{2}ab\sin C$

Trigonometric Identities

Reciprocal Identities

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Negative Angle Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

Periodicity Identities

$$\sin(x + 2\pi) = \sin x \qquad \cos(x + 2\pi) = \cos x$$

$$\csc(x + 2\pi) = \csc x \qquad \sec(x + 2\pi) = \sec x$$

$$\tan(x + \pi) = \tan x \qquad \cot(x + \pi) = \cot x$$

Cofunction Identities

$$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \cos x = \sin\left(\frac{\pi}{2} - x\right)$$

$$\tan x = \cot\left(\frac{\pi}{2} - x\right) \qquad \cot x = \tan\left(\frac{\pi}{2} - x\right)$$

$$\sec x = \csc\left(\frac{\pi}{2} - x\right) \qquad \csc x = \sec\left(\frac{\pi}{2} - x\right)$$

Addition and Subtraction Identities

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Identities

$$\sin 2x = 2 \sin x \cos x \qquad \cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = 2\cos^2 x - 1$$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}} \qquad \tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}} \qquad \tan\frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

Product Identities

$$\sin x \cos y = \frac{1}{2}(\sin(x + y) + \sin(x - y))$$

$$\sin x \sin y = \frac{1}{2}(\cos(x - y) - \cos(x + y))$$

$$\cos x \cos y = \frac{1}{2}(\cos(x + y) + \cos(x - y))$$

$$\cos x \sin y = \frac{1}{2}(\sin(x + y) - \sin(x - y))$$

Factoring Identities

$$\sin x + \sin y = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

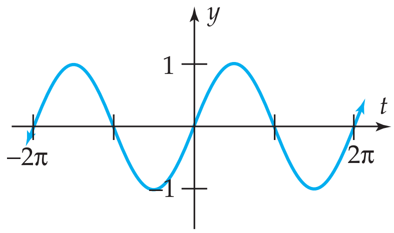
$$\sin x - \sin y = 2\cos\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$

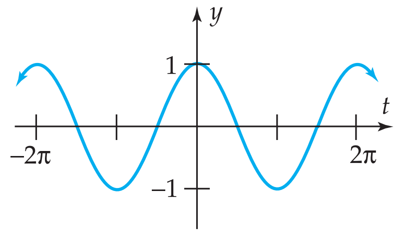
$$\cos x - \cos y = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

Trigonometric Functions

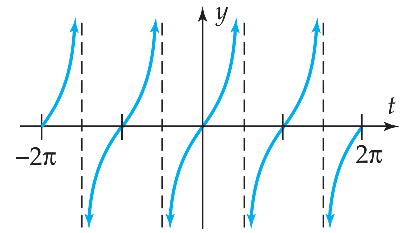
$f(t) = \sin t$



$f(t) = \cos t$



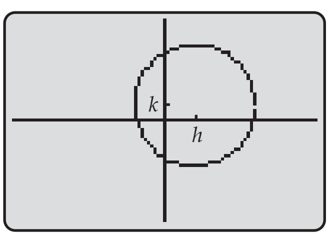
$f(t) = \tan t$



Rectangular and Parametric Equations for Conic Sections

Circles
Center (h, k), radius r

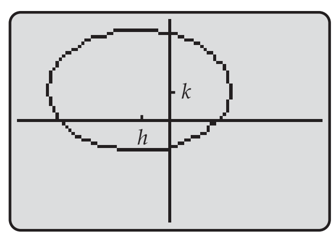
$$(x-h)^2 + (y-k)^2 = r^2$$



$$x = r\cos t + h \qquad y = r\sin t + k \qquad (0 \le t \le 2\pi)$$

Ellipse
Center (h, k)

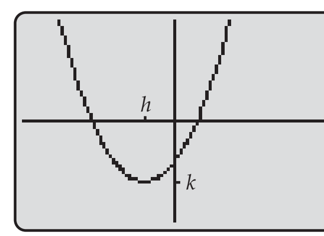
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$$x = a\cos t + h \qquad y = b\sin t + k \qquad (0 \le t \le 2\pi)$$

Parabola
Vertex (h, k)

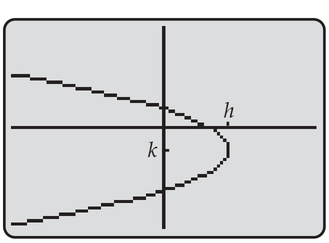
$$(x-h)^2 = 4p(y-k)$$



$$x = t \qquad y = \frac{(t-h)^2}{4p} + k \qquad (t \text{ any real})$$

Parabola
Vertex (h, k)

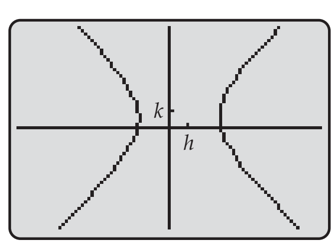
$$(y-k)^2 = 4p(x-h)$$



$$x = \frac{(t-k)^2}{4p} + h \qquad y = t \qquad (t \text{ any real})$$

Hyperbola
Center (h, k)

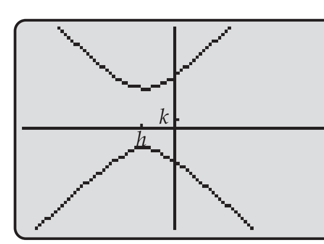
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$x = \frac{a}{\cos t} + h \qquad y = b\tan t + k \qquad (0 \le t \le 2\pi)$$

Hyperbola
Center (h, k)

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



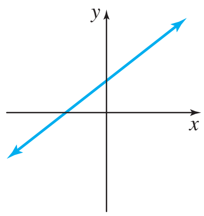
$$x = b\tan t + h \qquad y = \frac{a}{\cos t} + k \qquad (0 \le t \le 2\pi)$$

Catalog of Basic Functions

Linear Functions

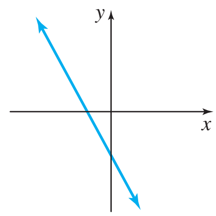
$f(x) = mx + b$

Slope $= m > 0$

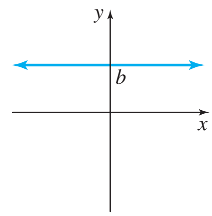


$f(x) = mx + b$

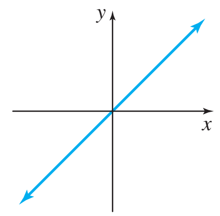
Slope $= m < 0$



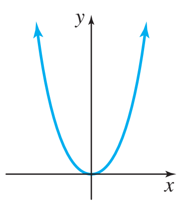
Constant Function $f(x) = b$



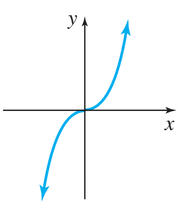
Identity Function $f(x) = x$



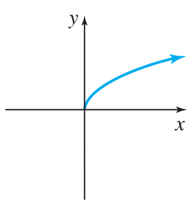
Square Function $f(x) = x^2$



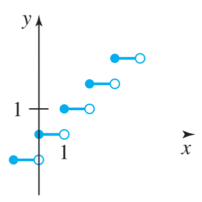
Cube Function $f(x) = x^3$



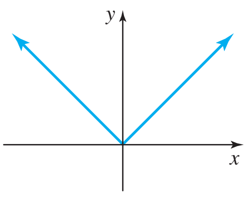
Square Root Function $f(x) = \sqrt{x}$



Greatest Integer Function $f(x) = [x]$

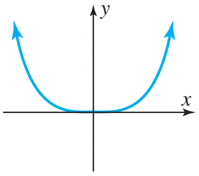


Absolute Value Function $f(x) = |x|$

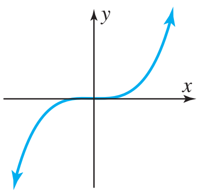


Power Functions

$f(x) = x^n$ (n **even**)

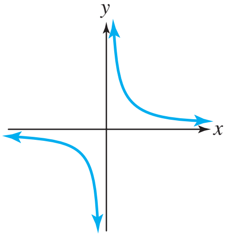


$f(x) = x^n$ (n **odd**)

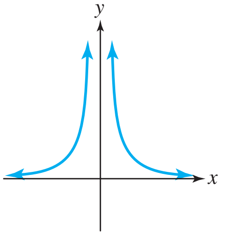


Reciprocal Functions

$f(x) = \dfrac{1}{x}$

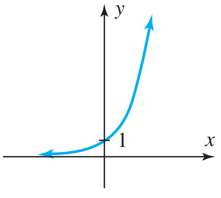


$f(x) = \dfrac{1}{x^2}$

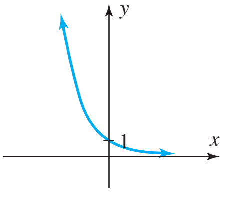


Exponential Functions

$f(x) = b^x$ $(b > 1)$

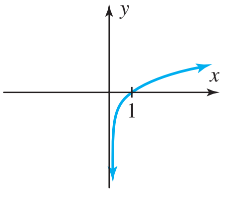


$f(x) = b^x$ $(0 < b < 1)$

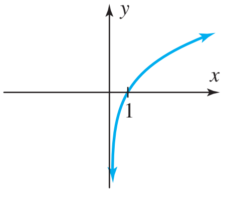


Logarithmic Functions

$f(x) = \log x$



$f(x) = \ln x$



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