Solutions to Chapter 11 Exercise Problems

Problem 11.1

Two helical gears are cut with a spur gear hob that has a diametral pitch of 4 and a pressure angle of 20˚. The pinion has 15 teeth, the gear has 35 teeth, and the helix angle is 30˚. Determine the minim um recommended face width. Using the minimum face width, find the transverse diametral pitch, th e pitch cylinder radii, and the axial, transverse, and total contact ratios.

Solution

The limiting condition for the face width is given by Eq. (11.15)

$$F \geq \frac{1.15 p_n}{\sin\psi}$$

The normal pitch is given by

$$p_n = \frac{\pi}{P_n}$$

Therefore,

$$F \geq \frac{1.15\pi}{P_n \sin\psi}$$

Now

$$\frac{1.15\pi}{P_n \sin\psi} = \frac{1.15\pi}{4\sin 30} = 1.8064$$

Therefore,

$$F \geq 1.8064$$

The properties of the gears in the normal direction will be the same as those of the hob. The transve rse diametral pitch (P_t) for both gears is related to the normal diametral pitch by Eq. (11.5). Based on the diametral pitch of the hob,

$$P_t = P_n \cos\psi = 4\cos 30 = 3.464$$

The pitch cylinder diameters are related to the diametral pitch through Eq. (11.4) Therefore,

$$D_t = \frac{N_t}{P_t}$$

and the pitch cylinder radius for the pinion is

$$r_{2_t} = \frac{N_{2_t}}{2P_t} = \frac{15}{2(3.464)} = 2.165\ in$$

The pitch cylinder radius for the gear is

$$r_{3t} = \frac{N_{3t}}{2P_t} = \frac{35}{2(3.464)} = 5.052\,in$$

The transverse contact ratio is given by Eq. (11.16) as

$$m_{c_t} = \frac{p_t(\lambda_2 + \lambda_3)}{\pi\cos\phi_t}$$

where

$$\lambda_i = -r_{p_i}\sin\phi_t + \sqrt{a_i^2 + 2a_i r_{p_i} + r_{p_i}^2 \sin^2\phi_t}\,,\;\; i = 2, 3$$

The addenda are determined by the hob. Because a standard hob is used, both addenda are given b y

$$a = \frac{1}{P_n} = \frac{1}{4} = 0.25\,inch$$

The transverse circular pitch is given by Eq. (11.3) as

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{3.464} = 0.907$$

and the transverse pressure angle is given by Eq. (11.7) as

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20}{\cos 30}\right) = 22.796^\circ$$

Now,

$$\begin{aligned}\lambda_2 &= -r_{p2}\sin\phi_t + \sqrt{a_2^2 + 2a_2 r_{p2} + r_{p2}^2\sin^2\phi_t}\\ &= -2.165\sin(22.796) + \sqrt{(0.25)^2 + 2(0.25)(2.165) + [2.165\sin(22.796)]^2} = 0.5208\end{aligned}$$

and

$$\begin{aligned}\lambda_3 &= -r_{p3}\sin\phi_t + \sqrt{a_3^2 + 2a_3 r_{p3} + r_{p3}^2\sin^2\phi_t}\\ &= -5.0518\sin(22.796) + \sqrt{(0.25)^2 + 2(0.25)(5.0518) + [5.0518\sin(22.796)]^2} = 0.5764\end{aligned}$$

Therefore, the transverse contact ratio is

$$m_{c_t} = \frac{p_t(\lambda_2 + \lambda_3)}{\pi\cos\phi_t} = \frac{0.907(0.5208 + 0.5764)}{\pi\cos(22.796)} = 1.3123$$

The axial contact ratio is given by Eq. (11.16) as

$$m_{c_a} = \frac{F\tan\psi}{p_t} = \frac{1.8064\tan 30}{0.907} = 1.1500$$

The total contact ratio is

$$m_c = m_{c_t} + m_{c_a} = 2.4623 + 1.1500 = 2.4623$$

Problem 11.2

Two helical gears are cut with the same tooth numbers and with the same cutter as given in Problem 11.1. The helix angle is 30˚. Find the transverse pressure angle, the transverse diametral pitch, and the axial pitch.

Solution

The transverse pressure angle is given by Eq. (11.7) as

$$\tan\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = \tan^{-1}(0.4203) = 22.79^\circ$$

The transverse diametral pitch (P_t) for both gears is related to the normal diametral pitch by Eq. (11. 5). Based on the diametral pitch of the hob,

$$P_t = P_n\cos\psi = 4\cos 30 = 3.464$$

The axial pitch is given by Eq. (11.6). Then

$$p_a = \frac{p_n}{\sin\psi} = \frac{\pi}{P_n\sin\psi} = \frac{\pi}{4\sin 30^\circ} = 1.5708$$

Problem 11.3

Two parallel helical gears are cut with a 20˚ normal pressure angle and a 45˚ helix angle. They have a diametral pitch of 12 in the normal plane and have 10 and 41 teeth, respectively. Find the transver se pressure angle, transverse circular pitch, and transverse diametral pitch. Also determine the mini mum face width, and using that face width, determine the total contact ratio.

Solution

The transverse pressure angle is given by Eq. (11.7) as

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 45^\circ}\right) = \tan^{-1}(0.5147) = 27.24^\circ$$

The transverse diametral pitch (P_t) for both gears is related to the normal diametral pitch by Eq. (11. 5). Based on the diametral pitch of the hob,

$$P_t = P_n\cos\psi = 12\cos 45 = 8.485$$

The transverse circular pitch is given by Eq. (11.3) as

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{8.485} = 0.3702$$

The limiting condition for the face width is given by Eq. (11.15)

$$F \geq \frac{1.15p_n}{\sin\psi}$$

The normal pitch is given by

$$p_n = \frac{\pi}{P_n}$$

Therefore,

$$F \geq \frac{1.15\pi}{P_n \sin\psi}$$

Now

$$\frac{1.15\pi}{P_n \sin\psi} = \frac{1.15\pi}{12 \sin 45} = 0.4258$$

Therefore,

$$F \geq 0.4258$$

The pitch cylinder diameters are related to the diametral pitch through Eq. (11.4) Therefore,

$$D_t = \frac{N_t}{P_t}$$

and the pitch cylinder radius for the pinion is

$$r_{2_t} = \frac{N_{2_t}}{2P_t} = \frac{10}{2(8.485)} = 0.589 \text{ in}$$

The pitch cylinder radius for the gear is

$$r_{3_t} = \frac{N_{3_t}}{2P_t} = \frac{41}{2(8.485)} = 2.416 \text{ in}$$

The transverse contact ratio is given by Eq. (11.16) as

$$m_{c_t} = \frac{p_t(\lambda_2 + \lambda_3)}{\pi \cos\phi_t}$$

where

$$\lambda_i = -r_{p_i} \sin\phi_t + \sqrt{a_i^2 + 2a_i r_{p_i} + r_{p_i}^2 \sin^2\phi_t}\,, \quad i = 2, 3$$

The addenda are determined by the hob. Because a standard hob is used, both addenda are given b y

$$a = \frac{1}{P_n} = \frac{1}{12} = 0.08333 \text{ in}$$

Now,

$$\lambda_2 = -r_{2_t}\sin\phi_t + \sqrt{a_2^2 + 2a_2 r_{2_t} + r_{2_t}^2\sin^2\phi_t}$$
$$= -0.589\sin(27.24) + \sqrt{(0.08333)^2 + 2(0.08333)(0.589) + [0.589\sin(27.24)]^2} = 0.1521$$

and

$$\lambda_3 = -r_{3_t}\sin\phi_t + \sqrt{a_3^2 + 2a_3 r_{3_t} + r_{3_t}^2\sin^2\phi_t}$$
$$= -2.416\sin(27.24) + \sqrt{(0.08333)^2 + 2(0.08333)(2.416) + [2.416\sin(27.24)]^2} = 0.1719$$

Therefore, the transverse contact ratio is

$$m_{c_t} = \frac{p_t(\lambda_2 + \lambda_3)}{\pi\cos\phi_t} = \frac{0.3702(0.1521 + 0.1719)}{\pi\cos(27.24)} = 0.9841$$

The axial contact ratio is given by Eq. (11.16) as

$$m_{c_a} = \frac{F\tan\psi}{p_t} = \frac{0.4258\tan 45}{0.3702} = 1.1500$$

The total contact ratio is

$$m_c = m_{c_t} + m_{c_a} = 0.9841 + 1.1500 = 2.1341$$

Problem 11.4

A helical gear has 18 teeth and a transverse diametral pitch of 6. The face width is 1.5, and the helix angle is 25°. Determine the axial pitch, normal pitch, lead, transverse pitch diameter, and minimum face width.

Solution

The transverse pressure angle is given by Eq. (11.7) as

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20°}{\cos 25°}\right) = \tan^{-1}(0.9063) = 21.88°$$

The transverse diametral pitch (P_t) for the gear is related to the normal diametral pitch by Eq. (11.5) . Based on the transverse diametral pitch

$$P_n = P_t / \cos\psi = 6 / \cos 25 = 6.620$$

The transverse pitch diameter is given by

$$d_t = \frac{N_2}{P_t} = \frac{18}{6} = 3$$

The transverse circular pitch is given by Eq. (11.3) as

$$p_t = \frac{\pi}{P_t} = \frac{\pi}{6} = 0.5236$$

The axial pitch is given by Eq. (11.6). Then

$$p_a = \frac{p_n}{\sin\psi} = \frac{\pi}{P_n \sin\psi} = \frac{\pi}{6.620\sin 25^\circ} = 1.1229$$

The lead is given by Eq. (11.25) as

$$L = N_2 p_a = 18(1.1229) = 20.212 \text{ in}$$

The limiting condition for the face width is given by Eq. (11.15)

$$F \geq \frac{1.15 p_n}{\sin\psi}$$

The normal pitch is given by

$$p_n = \frac{\pi}{P_n}$$

Therefore,

$$F_{\min} \geq \frac{1.15\pi}{P_n \sin\psi}$$

Now

$$\frac{1.15\pi}{P_n \sin\psi} = \frac{1.15\pi}{12\sin 45} = 1.2913$$

Therefore,

$$F_{\min} \geq 1.2913$$

Problem 11.5

Two helical gears have 20 and 34 teeth and a normal diametral pitch of 8. The left-handed pinion h as a helix angle of 40˚ and a rotational speed of 1000 rpm. The gear is also left handed and has a h elix angle of 40˚. Determine the angular velocity of the gear, transverse diametral pitch of each gear, and pitch diameters.

Solution

The angular velocity of the gear is given by Eq. (11.11) as

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2}$$

Then,

$$\omega_3 = \omega_2 \frac{N_2}{N_3} = 1000\frac{34}{20} = 1700 \text{ rpm}$$

The transverse diametral pitch (P_t) for each gear is related to the normal diametral pitch by Eq. (11.5). Then

$$P_t = P_n \cos\psi = 8\cos 40 = 6.128$$

The pitch diameter for the pinion is given by

$$d_2 = \frac{N_2}{P_t} = \frac{20}{6.128} = 3.263 \text{ in}$$

And the pitch diameter for the gear is given by

$$d_3 = \frac{N_3}{P_t} = \frac{34}{6.128} = 5.548 \text{ in}$$

Problem 11.6

Two standard spur gears have a diametral pitch of 10, a pressure angle of 20˚, and a velocity ratio equal to 3.5:1. The center distance is 8.55 in. Two helical gears are to be used to replace the two spur gears such that the center distance and angular velocity ratio remain unchanged. The helical gears are also to be cut with the same hob as that used to cut the spur gears. Determine the helix angle, tooth numbers, and minimum face width for the new gears if the helix angle is kept to a minimum.

Solution

For the helical gears, we need to determine the helix angle and the number of teeth on each gear. We will find the number of teeth and transverse diametral pitch first and then determine the helix angle. The velocity ratio is given by Eq. (11.11) as

$$\frac{\omega_2}{\omega_3} = \frac{r_{p3}}{r_{p2}} = \frac{D_{p3}}{D_{p2}} = \frac{N_3 / P_t}{N_2 / P_t} = \frac{N_3}{N_2}$$

and the center distance is given by Eq. (11.12)

$$C = r_2 + r_3 = \frac{D_2 + D_3}{2}$$

For the spur gear,

$$d_3 = \frac{N_3}{N_2} d_2 = 3.5 d_2$$

and

$$C = 8.55 = \frac{d_2 + d_3}{2} = \frac{d_2 + 3.5 d_2}{2} = \frac{4.5 d_2}{2} = 2.25 d_2$$

Therefore,

$$d_2 = 3.8 \text{ in}$$

and

$$d_3 = 3.5 d_2 = 13.3 \text{ in}$$

Also,

$$N_2 = P_n d_2 = 10(3.8) = 38$$

and

$$N_3 = P_n d_3 = 10(13.3) = 133$$

The pitch radii for the helical gears must be the same as the corresponding radii for the spur gears. However, the tooth numbers can and will be different. The transverse diametral pitch is related to the normal diametral pitch by

$$P_t = P_n \cos\psi \tag{11.5}$$

and to the teeth numbers by

$$P_t = \frac{N_2}{2r_{p2}} = \frac{N_3}{2r_{p3}} \tag{11.19}$$

From Eq. (11.5), it is clear that $P_t < P_n$. Therefore, based on Eq. (11.19), the tooth numbers on the helical gears must be less than those on the spur gears. As a result, when we investigate tooth numbers that satisfy Eq. (11.18), we need only consider values which are lower than the corresponding values for the spur gears. A set of values is

N_2	$N_3 = 3.5N_2$	
38	133	(spur gear)
37	129.5	
36	126	

From the table, the first set of teeth numbers that are integers are $N_2 = 36$ and $N_3 = 126$. For these numbers, the transverse pitch is given by Eq. (11.19) as

$$P_t = \frac{N_2}{2r_{p2}} = \frac{N_3}{2r_{p3}} = \frac{36}{2(1.8)} = \frac{126}{2(6.65)} = 9.474$$

From Eq. (11.5),

$$\psi = \cos^{-1}\left(\frac{P_t}{P_n}\right) = \cos^{-1}\left(\frac{9.474}{10}\right) = 18.672°$$

Notice that this is the lowest helix angle possible (other than 0) if the center distance and velocity ratio are to be maintained. The minimum face width is given by Eq. (11.14):

$$F \geq \frac{1.15p_t}{\tan\psi} \text{ or } F \geq \frac{1.15\pi}{P_t \tan\psi}$$

Therefore,

$$F \geq \left(\frac{1.15\pi}{9.474\tan(18.672)} = 1.128 \text{ in}\right)$$

Problem 11.7

Two standard spur gears have a diametral pitch of 16 and a pressure angle of 20˚. The tooth numbe rs are 36 and 100, and the gears were meshed at a standard center distance. After the gear reducer was designed and tested, the noise of the drive was found to be excessive. Therefore, the decision w as made to replace the spur gears with helical gears. The helix angle chosen was 22˚, and the tooth numbers were to remain unchanged. Determine the change in center distance required.

Solution

The center distance for the spur gears is given by Eq. (11.12)

$$C_c = r_2 + r_3 = \frac{D_2 + D_3}{2} = \frac{N_2 + N_3}{2P_n} = \frac{36+100}{2(16)} = 4.25 \text{ in}$$

For the helical gears,

$$C_h = \frac{d_{t2} + d_{t3}}{2} = \frac{N_2 + N_3}{2P_t} = \frac{N_2 + N_3}{2P_n \cos\psi} = \frac{C_c}{\cos\psi} = \frac{4.25}{\cos 22} = 4.583 \text{ in}$$

The change in center distance is given by

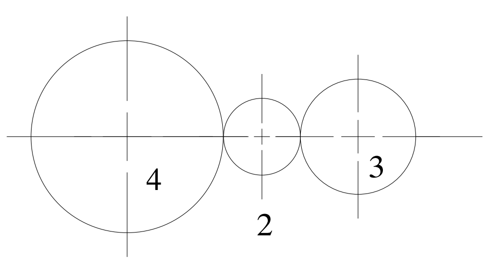
$$\Delta C = C_h - C_c = 4.583 - 4.25 = 0.333 \text{ in}$$

Problem 11.8

A spur gear transmission consists of a pinion that drives two gears. The pinion has 24 teeth and a d iametral pitch of 12. The velocity ratio for the pinion and one gear is 3:2 and for the pinion and the other gear is 5:2. To reduce the noise level, all three gears are to be replaced by helical gears such t hat the center distances and velocity ratios remain the same. The helical gears will be cut with a 16 pitch, 20˚ hob. If the helix angle is kept as low as possible, determine the number of teeth, face widt h, hand, helix angle, and outside diameter for each of the gears.

Solution

Assume that the gear arrangement is as shown in the figure.



For the helical gears, we need to determine the helix angle and the number of teeth on each gear. W e will find the number of teeth and transverse diametral pitch first and then determine the helix angle . For the pinion,

$$d_2 = \frac{N_2}{P_n} = \frac{24}{12} = 2 \text{ in}$$

The velocity ratio for the first gears is given by Eq. (11.11) as

$$\frac{\omega_2}{\omega_3} = \frac{r_{p3}}{r_{p2}} = \frac{d_{t3}}{d_{t2}} = \frac{N_3 / P_t}{N_2 / P_t} = \frac{N_3}{N_2} = \frac{3}{2}$$

Therefore,

$$N_3 = \frac{3}{2}N_2 = \frac{3}{2}24 = 36$$

and

$$d_{t3} = \frac{3}{2}d_{t2} = \frac{3}{2}(2) = 3 \text{ in}$$

For the second gear,

$$\frac{\omega_2}{\omega_4} = \frac{r_{p4}}{r_{p2}} = \frac{d_{t4}}{d_{t2}} = \frac{N_4 / P_t}{N_2 / P_t} = \frac{N_4}{N_2} = \frac{5}{2}$$

Therefore,

$$N_4 = \frac{5}{2}N_2 = \frac{5}{2}24 = 60$$

and

$$d_{t4} = \frac{5}{2}d_{t2} = \frac{5}{2}(2) = 5 \text{ in}$$

The center distances is given by Eq. (11.12). Then

$$C_1 = r_2 + r_3 = \frac{d_{t2} + d_{t3}}{2} = \frac{2+3}{2} = 2.5 \text{ in}$$

and

$$C_2 = r_2 + r_4 = \frac{d_{t2} + d_{t4}}{2} = \frac{2+5}{2} = 3.5 \text{ in}$$

The pitch radii for the helical gears must be the same as the corresponding radii for the spur gears. However, the tooth numbers can and will be different. The transverse diametral pitch is related to t he normal diametral pitch by

$$P_t = P_n \cos\psi \tag{11.5}$$

and to the teeth numbers by

$$P_t = \frac{N_2}{2r_{p2}} = \frac{N_3}{2r_{p3}} \tag{11.19}$$

From Eq. (11.5), it is clear that $P_t < P_n$. Therefore, based on Eq. (11.19), the tooth numbers on the helical gears must be less than those on the spur gears. As a result, when we investigate tooth numb ers that satisfy Eq. (11.18), we need only consider values which are lower than the corresponding va lues for the spur gears. A set of values is

N_2	$N_4 = 1.5N_2$	$N_4 = 2.5N_2$
24	36	60(spur gear)
23	34.5	57.5
22	33	55

From the table, the first set of teeth numbers that are integers for the three gears are N_2 = 22 and N_3 = 33, and N_4 = 55. For these numbers, the transverse pitch is given by Eq. (11.19) as

$$P_t = \frac{N_2}{2r_{p2}} = \frac{N_3}{2r_{p3}} = \frac{N_4}{2r_{p4}} = \frac{22}{2(1)} = \frac{33}{2(1.5)} = \frac{55}{2(2.5)} = 11$$

From Eq. (11.5),

$$\psi = \cos^{-1}\left(\frac{P_t}{P_n}\right) = \cos^{-1}\left(\frac{11}{16}\right) = 46.567^\circ$$

Notice that this is the lowest helix angle possible (other than 0) if the center distance and velocity rat io are to be maintained. The minimum face width is given by Eq. (11.14):

$$F \geq \frac{1.15p_t}{\tan\psi} \text{ or } F \geq \frac{1.15\pi}{P_t \tan\psi}$$

Therefore,

$$F \geq \left(\frac{1.15\pi}{11\tan(46.567)} = 0.311 \text{ in}\right)$$

The blank diameters of the three gears are given by

$$d_{o2} = d_{p2} + 2a_2 = d_{p2} + 2\frac{k}{P_n} = 2 + 2\frac{1}{16} = 2.125 \text{ in},$$

$$d_{o3} = d_{p3} + 2a_3 = d_{p3} + 2\frac{k}{P_n} = 3 + 2\frac{1}{16} = 3.125 \text{ in}$$

and

$$d_{o4} = d_{p4} + 2a_4 = d_{p4} + 2\frac{k}{P_n} = 5 + 2\frac{1}{16} = 5.125 \text{ in}$$

The hand of the gears is arbitrary; however, the pinion will be one hand and the two gears will be th e opposite hand. For example, if the pinion is right handed, gears 3 and 4 will be left handed.

Problem 11.9

A pair of helical gears have a module in the normal plane of 3 mm, a normal presure angle of 20˚, an d a helix angle of 45˚. The gears mesh with parallel shafts and have 30 and 48 teeth. Determine the transverse module, the pitch diameters, the center distance, and the minimum face width.

Solution

The transverse pressure angle is given by Eq. (11.7) as

$$\phi_t = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 45^\circ}\right) = \tan^{-1}(0.5147) = 27.24^\circ$$

The transverse module (m_t) for both gears is related to the normal module by

$$m_t = \frac{d_t}{N} = \frac{m_n}{\cos\psi} = \frac{3}{\cos 45} = 4.243 \text{ mm}$$

The transverse pitch diameters are

$$d_{t2} = N_2 m_t = 30(4.243) = 127.28 \text{ mm}$$

and

$$d_{t3} = N_3 m_t = 48(4.243) = 203.66 \text{ mm}$$

The center distance is given by

$$C = r_2 + r_3 = \frac{d_{t2} + d_{t3}}{2} = \frac{127.28 + 203.66}{2} = 165.47 \text{ mm}$$

The limiting condition for the face width is given by Eq. (11.15)

$$F \geq \left(\frac{1.15 p_n}{\sin\psi} = \frac{1.15\pi m_n}{\sin\psi} = \frac{1.15\pi 3}{\sin 45} = 15.328 \text{ mm}\right)$$

Problem 11.10

Two 20° spur gears have 36 and 90 teeth and a module of 1.5. The spur gears are to be replaced by helical gears such that the center distance and velocity ratio are not changed. The maximum allowe d face width is 12.7 mm, and the hob module is 1.5 mm. Design the helical gear pair that has the s mallest helix angle possible. Determine the numbers of teeth, the face width, the helix angle, and the outside diameters of the gears.

Solution

For the helical gears, we need to determine the helix angle and the number of teeth on each gear. W e will find the number of teeth and transverse diametral pitch first and then determine the helix angle . The velocity ratio is given by Eq. (11.11) as

$$\frac{\omega_2}{\omega_3} = \frac{r_{p3}}{r_{p2}} = \frac{d_{p3}}{d_{p2}} = \frac{N_3 / P_t}{N_2 / P_t} = \frac{N_3}{N_2} = \frac{90}{36} = 2.5 \qquad (1)$$

and the center distance is given by Eq. (11.12)

$$C = r_2 + r_3 = \frac{d_2 + d_3}{2}$$

For the spur gear,

$$d_2 = N_2 m = 36(1.5) = 54 \text{ mm}, \tag{2}$$

$$d_3 = N_3 m = 90(1.5) = 135 \text{ mm}, \tag{3}$$

Assume that full depth gears are used. Then the addendum length is given by

$$a = \frac{1}{P_n} = m_n$$

The outside diameter of the gears are given by

$$d_{o2} = d_2 + 2m_n = 54 + 2(1.5) = 57 \text{ mm}$$

and

$$d_{o3} = d_3 + 2m_n = 135 + 2(1.5) = 138 \text{ mm}$$

The center distance is given by

$$C = \frac{d_2 + d_3}{2} = \frac{54 + 135}{2} = 94.5 \text{ mm} \tag{4}$$

The pitch radii for the helical gears must be the same as the corresponding radii for the spur gears. However, the tooth numbers can and will be different. The transverse module is related to the norm al module by

$$m_t = m_n / \cos\psi \tag{5}$$

and to the teeth numbers by

$$m_t = \frac{d_{t2}}{N_2} = \frac{d_{t3}}{N_3} \tag{6}$$

From Eq. (5), it is clear that $m_t > m_n$. Therefore, based on Eq. (6), the tooth numbers on the helical gears must be less than those on the spur gears. As a result, when we investigate tooth numbers tha t satisfy Eq. (1), we need only consider values which are lower than the corresponding values for th e spur gears. A set of values is

N_2	$N_3 = 2.5N_2$
36	90 (spur gear)
35	87.5
34	85
33	82.5
32	80
31	77.5
30	75

From the table, the first set of teeth numbers that are integers are $N_2 = 34$ and $N_3 = 85$. For these n umbers, the transverse module is given by

$$m_t = \frac{d_{p2}}{N_2} = \frac{d_{p3}}{N_3} = \frac{54}{34} = 1.588 \text{ mm}$$

From Eq. (5), the helix angle is given by

$$\psi = \cos^{-1}\left(\frac{m_n}{m_t}\right) = \cos^{-1}\left(\frac{1.5}{1.588}\right) = 19.19°$$

Notice that this is the lowest helix angle possible (other than 0) if the center distance and velocity rat io are to be maintained. The minimum face width is given by

$$F \geq \frac{1.15\pi m_t}{\tan\psi}$$

Therefore,

$$F \geq \left(\frac{1.15\pi m_t}{\tan\psi} = \frac{1.15\pi 1.588}{\tan(19.19)} = 16.486 \text{ mm}\right)$$

This exceeds the maximum allowed face width of 12.7 mm. Therefore, we must try the next lower v alues for the tooth numbers. These are N_2 = 32 and N_3 = 80. For these numbers, the transverse m odule is given by

$$m_t = \frac{d_{p2}}{N_2} = \frac{d_{p3}}{N_3} = \frac{54}{32} = 1.688 \text{ mm}$$

From Eq. (5), the helix angle is given by

$$\psi = \cos^{-1}\left(\frac{m_n}{m_t}\right) = \cos^{-1}\left(\frac{1.5}{1.688}\right) = 27.266°$$

The minimum face width is given by

$$F \geq \left(\frac{1.15\pi m_t}{\tan\psi} = \frac{1.15\pi 1.688}{\tan(27.266)} = 11.833 \text{ mm}\right)$$

Problem 11.11

Two helical gears are cut with a 20˚ hob with a module of 2. One gear is right handed, has a 30˚ hel ix angle, and has 36 teeth. The second gear is left handed, has a 40˚ helix angle, and has 72 teeth. Determine the shaft angle, the angular velocity ratio, and the center distance.

Solution

The shaft angle is given by Eq. (11.21). Then,

$$\Sigma = \psi_2 \pm \psi_3 = 30° - 40° = -10°$$

The angular velocity of the gear is given by Eq. (11.11) as

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2}$$

Then,

$$\frac{\omega_3}{\omega_2} = \frac{N_2}{N_3} = \frac{36}{72} = 2$$

The pitch diameter for gear 2 is given by

$$d_{t2} = \frac{N_2 m_n}{\cos\psi} = \frac{36(2)}{\cos(30)} = 83.14 \text{ mm}$$

and for gear 3,

$$d_{t2} = \frac{N_2 m_n}{\cos\psi} = \frac{72(2)}{\cos(40)} = 187.98 \text{ mm}$$

The center distance is

$$C = r_2 + r_3 = \frac{d_2 + d_3}{2} = \frac{83.14 + 187.98}{2} = 135.56 \text{ mm}$$

Problem 11.12

Two crossed shafts are connected by helical gears such that the velocity ratio is 3:1, and the shaft an gle is 60˚. The center distance is 10 in, and the normal diametral pitch is 8. The pinion has 35 teeth . Assume that the gears are the same hand and determine the helix angles, pitch diameters, and reco mmended face widths.

Solution

The velocity ratio is given by

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = 3$$

Therefore,

$$N_3 = 3N_2 = 3(35) = 105$$

The pitch diameters for the two gears are given by

$$d_{p2} = \frac{N_2}{P_{t2}} = \frac{N_2}{P_n \cos\psi_2} \tag{1}$$

and

$$d_{p3} = \frac{N_3}{P_{t3}} = \frac{N_3}{P_n \cos\psi_3} \tag{2}$$

The center distance is

$$C = r_2 + r_3 = \frac{d_2 + d_3}{2} = \frac{1}{2P_n}\left[\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3}\right] = 10 \text{ in}$$

Or,

$$\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3} = 10(2)P_n$$

and

$$\frac{35}{\cos\psi_2} + \frac{105}{\cos\psi_3} = 2(10)(8) = 160 \tag{3}$$

Both gears are of the same hand. Therefore, the shaft angle is given by

$$\Sigma = \psi_2 + \psi_3 = 60^\circ \tag{4}$$

To find the helix angles, we must solve Eqs. (3) and (4) simultaneously. First use Eq. (4) to solve f or ψ_2 in terms of ψ_3. Then substitute the result into Eq. (3). This gives

$$\frac{35}{\cos(60 - \psi_3)} + \frac{105}{\cos\psi_3} = 160$$

This equation can be solved iteratively for ψ_3 using MATLAB. The result is

$$\psi_2 = 32.298^\circ$$

and

$$\psi_3 = 27.702^\circ$$

The pitch diameters are given by

$$d_{p2} = \frac{N_2}{P_n\cos\psi_2} = \frac{35}{8\cos(32.298^\circ)} = 5.176 \text{ in}$$

and

$$d_{p3} = \frac{N_2}{P_n\cos\psi_3} = \frac{105}{8\cos(27.702^\circ)} = 14.182 \text{ in}$$

The minimum face width for each gear is given by

$$F \geq \left[\frac{1.15p_n}{\sin\psi} = \frac{1.15\pi}{P_n\sin\psi}\right]$$

Then,

$$F_2 \geq \left[\frac{1.15\pi}{P_n\sin\psi_2} = \frac{1.15\pi}{8\sin(32.298^\circ)} = 0.845 \text{ in}\right]$$

and

$$F_3 \geq \left[\frac{1.15\pi}{P_n\sin\psi_3} = \frac{1.15\pi}{8\sin(27.702^\circ)} = 0.971 \text{ in}\right]$$

The recommended face widths would be 0.85 in and 1.00 in for gears 2 and 3, respectively.

Problem 11.13

Two crossed shafts are connected by helical gears such that the velocity ratio is 3:2, and the shaft an gle is 90˚. The center distance is 5 in. Select a pair of gears that will satisfy the design constraints. What other information might be considered to reduce the number of arbitrary choices for the desi gn?

Solution

To complete the design, we need to select diametral pitch for the hob used to cut the gears and the n umber of teeth on one of the gears. We also need to identify if both gears are the same hand or opp osite hand.

Initially assume that the helix angles on the gears are the same hand. Also select a hob with a diame tral pitch of 12 and assume that the pinion has 30 teeth. If these values do not give helix angles whi ch are approximately equal, we can select other values.

The velocity ratio is given by

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{3}{2} = 1.5$$

Therefore,

$$N_3 = 1.5N_2 = 1.5(30) = 45$$

The pitch diameters for the two gears are given by

$$d_{p2} = \frac{N_2}{P_{t2}} = \frac{N_2}{P_n \cos\psi_2} \tag{1}$$

and

$$d_{p3} = \frac{N_3}{P_{t3}} = \frac{N_3}{P_n \cos\psi_3} \tag{2}$$

The center distance is

$$C = r_2 + r_3 = \frac{d_2 + d_3}{2} = \frac{1}{2P_n}\left[\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3}\right] = 5\text{ in}$$

Or,

$$\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3} = 5(2)P_n$$

and

$$\frac{30}{\cos\psi_2} + \frac{45}{\cos\psi_3} = 2(5)(12) = 120 \tag{3}$$

Assume that both gears are of the same hand. Therefore, the shaft angle is given by

$$\Sigma = \psi_2 + \psi_3 = 90^\circ \tag{4}$$

To find the helix angles, we must solve Eqs. (3) and (4) simultaneously. First use Eq. (4) to solve f or ψ_2 in terms of ψ_3. Then substitute the result into Eq. (3). This gives

$$\frac{30}{\cos(90-\psi_3)}+\frac{45}{\cos\psi_3}=120$$

This equation can be solved iteratively for ψ_3 using MATLAB. The result is

-0.00001 32.149 57.851 2.953 7.047 0.566 0.356

$$\psi_2 = 32.149^\circ$$

and

$$\psi_3 = 57.851^\circ$$

The pitch diameters are given by

$$d_{p2}=\frac{N_2}{P_n\cos\psi_2}=\frac{30}{12\cos(32.149^\circ)}=2.953 \text{ in}$$

and

$$d_{p3}=\frac{N_2}{P_n\cos\psi_3}=\frac{30}{12\cos(57.851^\circ)}=7.047 \text{ in}$$

The minimum face width for each gear is given by

$$F \geq \left[\frac{1.15p_n}{\sin\psi}=\frac{1.15\pi}{P_n\sin\psi}\right]$$

Then,

$$F_2 \geq \left[\frac{1.15\pi}{P_n\sin\psi_2}=\frac{1.15\pi}{12\sin(32.149^\circ)}=0.566 \text{ in}\right]$$

and

$$F_3 \geq \left[\frac{1.15\pi}{P_n\sin\psi_3}=\frac{1.15\pi}{12\sin(57.851^\circ)}=0.356 \text{ in}\right]$$

The recommended face widths would be 0.60 in and 0.40 in for gears 2 and 3, respectively.

To reduce the number of arbitrary choices we could specify limits on the helix angles or on the toot h numbers. Also, we have not considered strength yet. The stress and wear equations for the gear t eeth will add additional constraints.

Problem 11.14

A helical gear with a normal diametral pitch of 8 is to be used to drive a spur gear at a shaft angle of 45˚. The helical gear has 21 teeth, and the velocity ratio is 2:1. Determine the helix angle for the he lical gear and the pitch diameter of both gears.

Solution

The helix angle for the spur gear is 0; therefore, the helix angle for the helical gear must be the same as the shaft angle or 45˚. Both gears must have the same normal diametral pitch. The helical gear i s the pinion (gear 2) and the gear is gear 3.

The velocity ratio is given by

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{3}{2} = 1.5$$

Therefore,

$$N_3 = 2N_2 = 2(21) = 42$$

The pitch diameters for the two gears are given by

$$d_{p2} = \frac{N_2}{P_{t2}} = \frac{N_2}{P_n \cos\psi_2} = \frac{21}{P_n \cos 45} = 3.712 \text{ in} \tag{1}$$

and

$$d_{p3} = \frac{N_3}{P_{t3}} = \frac{N_3}{P_n \cos\psi_3} = \frac{41}{8\cos 0} = 5.125 \text{ in} \tag{2}$$

Problem 11.15

Two crossed helical gears connect shafts making an angle of 45˚. The pinion is right handed, has a helix angle of 20˚, and contains 30 teeth. The gear is also right handed and contains 45 teeth. The t ransverse diametral pitch of the gear is 5. Determine the pitch diameter, the normal pitch, and the le ad for each gear.

Solution

Because both gears are of the same hand, the relationship between the helix angles and shaft angle i s

$$\Sigma = \psi_2 + \psi_3 = 45^\circ$$

Therefore,

$$\psi_3 = 45^\circ - \psi_2 = 45^\circ - 20^\circ = 35^\circ$$

The pitch diameters for the gear is given by

$$d_{p3} = \frac{N_3}{P_{t3}} = \frac{45}{5} = 9 \text{ in}$$

and the normal diametral pitch for both gears is

$$P_n = \frac{P_{t3}}{\cos\psi_3} = \frac{5}{\cos 35^\circ} = 6.104$$

The normal pitch is given by

$$p_n = \frac{\pi}{P_n} = \frac{\pi}{6.104} = 0.515 \text{ in}$$

This is a nonstandard pitch and would require a special hob. The pitch diameter for the pinion is

$$d_{p3} = \frac{N_3}{P_{t3}} = \frac{N_3}{P_n \cos\psi_3}$$

The pitch diameters are given by

$$d_{p2} = \frac{N_2}{P_n \cos\psi_2} = \frac{30}{6.104\cos(20^\circ)} = 5.230 \text{ in}$$

The axial pitches for the two gears are

$$p_{a2} = \frac{p_n}{\sin\psi_2} = \frac{0.515}{\sin(20^\circ)} = 1.505 \text{ in}$$

and

$$p_{a3} = \frac{p_n}{\sin\psi_3} = \frac{0.515}{\sin(35^\circ)} = 0.897 \text{ in}$$

The leads for the two gears are

$$L_2 = N_2 p_{a2} = 30(1.505) = 45.15$$

and

$$L_3 = N_3 p_{a3} = 45(0.897) = 40.365$$

Problem 11.16

The worm of a worm gear set has 2 teeth, and the gear has 58 teeth. The worm axial pitch is 1.25 in , and the pitch diameter is 3 in. The shaft angle is 90˚. Determine the center distance for the two ge ars, the helix angle, and the lead for the worm.

Solution

Because the shaft angle is 90˚, the transverse circular pitch of the gear is equal to the axial pitch of t he worm. Therefore,

$$p_{t3} = 1.25$$

Therefore, the pitch diameter of the gear is

$$d_{t3} = \frac{p_{t3} N_3}{\pi} = \frac{1.25(58)}{\pi} = 23.078 \text{ in}$$

The center distance is

$$C = r_2 + r_3 = \frac{d_2 + d_3}{2} = \frac{3 + 23.078}{2} = 13.039 \text{ in}$$

To determine the helix angles, use the relationships for the shaft angle and angular velocity ratio. B ecause both gears are of the same hand, the relationship between the helix angles and shaft angle is

$$\Sigma = \psi_2 + \psi_3 = 90^\circ \qquad (1)$$

The angular velocity ratio is

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_{t3}\cos\psi_3}{d_{t2}\cos\psi_2} = \frac{d_{t3}\cos\psi_3}{d_{t2}\cos\psi_2}$$

Then,

$$\frac{58}{2} = \frac{23.078\cos\psi_3}{3\cos\psi_2}$$

and

$$\cos\psi_2 = 0.26526\cos\psi_3 \tag{2}$$

Combining Eqs. (1) and (2),

$$\cos(90 - \psi_3) - 0.26526\cos\psi_3 = 0 \tag{3}$$

Equation (3) can be solved iteratively for ψ_3 by using MATLAB. The result is

$$\psi_2 = 75.144^\circ$$

and

$$\psi_3 = 14.856^\circ$$

The lead for the worm is given by

$$L_2 = N_2 p_{a2} = 2(1.25) = 2.5$$

Problem 11.17

The shaft angle between two shafts is 90˚, and the shafts are to be connected through a worm gear s et. The center distance is 3 in, and the velocity ratio is 30:1. Determine a worm and gear that will sa tisfy the design requirements. Specify the number of teeth, lead angle, and pitch diameter for each g ear. Also, determine the face width for the gear.

Solution

This problem has considerable design latitude. We need to satisfy only three equations. One equat ion is for the shaft angle

$$\Sigma = \psi_2 + \psi_3 = 90^\circ$$

The second is for the velocity ratio

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_{t3}\cos\psi_3}{d_{t2}\cos\psi_2} = \frac{d_{t3}\cos\psi_3}{d_{t2}\cos\psi_2}$$

and the third is for the center distance

$$C = r_2 + r_3 = \frac{N_2}{2P_{t2}} + \frac{N_3}{2P_{t3}} = \frac{N_2}{2P_n\cos\psi_2} + \frac{N_3}{2P_n\cos\psi_3} = \frac{1}{2P_n}\left[\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3}\right]$$

We can rewrite the equations in terms of the helix angles, tooth numbers, and normal diametral pitc h. Doing this and substituting for the known variables,

$$\psi_2 + \psi_3 = 90^\circ \tag{1}$$

$$\frac{N_3}{N_2} = \frac{30}{1} \tag{2}$$

$$\frac{1}{2P_n}\left[\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3}\right] = 3 \tag{3}$$

This gives us three equations in five unknowns. Therefore, we must select values for two of the vari ables. Let us choose $N_2 = 2$ and $P_n = 12$. Then from Eq. (2),

$$N_3 = 30N_2 = 30(2) = 60$$

Equations (1) and (2) can then be combined as

$$\frac{1}{2P_n}\left[\frac{N_2}{\cos\psi_2} + \frac{N_3}{\cos\psi_3}\right] = \frac{1}{2(12)}\left[\frac{2}{\cos(90-\psi_3)} + \frac{60}{\cos\psi_3}\right] = 3$$

or

$$\frac{2}{\cos(90-\psi_3)} + \frac{60}{\cos\psi_3} = 3(2)(12) = 72 \tag{4}$$

Equation (4) can be solved iteratively using MATLAB. The result is

$$\psi_2 = 79.504$$

and

$$\psi_3 = 10.496$$

The pitch diameter of the gear is

$$d_{t3} = \frac{p_{t3}N_3}{\pi} = \frac{1.25(58)}{\pi} = 23.078 \text{ in}$$

The center distance is

$$C = r_2 + r_3 = \frac{d_2 + d_3}{2} = \frac{3 + 23.078}{2} = 13.039 \text{ in}$$

To determine the helix angles, use the relationships for the shaft angle and angular velocity ratio. B ecause both gears are of the same hand, the relationship between the helix angles and shaft angle is

$$\Sigma = \psi_2 + \psi_3 = 90^\circ \tag{1}$$

The angular velocity ratio is

$$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_{t3}\cos\psi_3}{d_{t2}\cos\psi_2} = \frac{d_{t3}\cos\psi_3}{d_{t2}\cos\psi_2}$$

Then,

$$\frac{58}{2} = \frac{23.078\cos\psi_3}{3\cos\psi_2}$$

and

$$\cos\psi_2 = 0.26526\cos\psi_3 \tag{2}$$

Combining Eqs. (1) and (2),

$$\cos(90 - \psi_3) - 0.26526\cos\psi_3 = 0 \tag{3}$$

Equation (3) can be solved iteratively for ψ_3 by using MATLAB. The result is

$$\psi_2 = 79.504$$

and

$$\psi_3 = 10.496$$

The transverse pitch diameters are

$$d_{t2} = \frac{N_2}{2P_n\cos\psi_2} = \frac{2}{2(12)\cos(79.504)} = 0.457 \text{ in}$$

and

$$d_{t3} = \frac{N_3}{2P_n\cos\psi_3} = \frac{60}{2(12)\cos(10.496)} = 2.543 \text{ in}$$

The lead for the worm and gear are

$$L_2 = N_2 p_{a2} = \frac{N_2 p_n}{\sin\psi_2} = \frac{N_2\pi}{P_n\sin\psi_2} = \frac{2\pi}{12\sin(79.504)} = 0.532$$

and

$$L_3 = N_3 p_{a3} = \frac{N_3 p_n}{\sin\psi_3} = \frac{N_3\pi}{P_n\sin\psi_3} = \frac{2\pi}{12\sin(10.496)} = 2.874$$

The minimum facewidth for the gear is given by

$$F \ge \left[\frac{1.15p_n}{\sin\psi} = \frac{1.15\pi}{P_n\sin\psi}\right]$$

Then,

$$F_3 \ge \left[\frac{1.15\pi}{P_n\sin\psi_3} = \frac{1.15\pi}{12\sin(10.496^\circ)} = 1.653 \text{ in}\right]$$

For this type of gear, the minimum facewidth is not very meaningful. The worm contacts the gear at one point only, so the entire face is not contacted.

Problem 11.18

A worm with two teeth drives a gear with 50 teeth. The gear has a pitch diameter of 8 in and a helix angle of 20˚. The shaft angle between the two shafts is 80˚. Determine the lead and pitch diameter of the worm.

Solution

The normal diametral pitch for the gears is

$$P_n = \frac{N_3}{2d_{t3}\cos\psi_3} = \frac{50}{2(8)\cos 20^\circ} = 3.326$$

The shaft angles for the two gears are related by

$$\Sigma = \psi_2 + \psi_3 = 80^\circ$$

Therefore,

$$\psi_2 = 80^\circ - \psi_3 = 80^\circ - 20^\circ = 60^\circ$$

The pitch diameter of the worm is then given by

$$d_{t2} = \frac{N_2}{2P_n\cos\psi_2} = \frac{2}{2(3.326)\cos(60)} = 0.601 \text{ in}$$

The lead for the worm is

$$L_2 = N_2 p_{a2} = \frac{N_2 p_n}{\sin\psi_2} = \frac{N_2\pi}{P_n\sin\psi_2} = \frac{2\pi}{3.326\sin(60)} = 2.181$$

Problem 11.19

Two straight-toothed bevel gears mesh with a shaft angle of 90˚ and a diametral pitch of 5. The pini on has 20 teeth, and the gear ratio is 2:1. The addendum and dedendum are the same as for 20˚ stu b teeth. For the gear, determine the pitch radius, cone angle, outside diameter, cone distance, and fac e width.

Solution

The number of teeth on the gear is given by

$$N_3 = N_2\frac{\omega_2}{\omega_3} = 20\frac{2}{1} = 40$$

The pitch cone angle for the gear is given by Eq. (11.27). Then,

$$\gamma_3 = \tan^{-1}\left[\frac{N_3}{N_2}\right] = \tan^{-1}(2) = 63.435^\circ$$

The pitch cone angle for the pinion is given Eq. (11.31). Then,

$$\gamma_2 = \Sigma - \gamma_3 = 90^\circ - 63.435^\circ = 26.565^\circ$$

The pitch radius for the large end of the gear is given by

$$r_3 = \frac{N_3}{2P_n} = \frac{40}{2(5)} = 4$$

The cone distance is given by

$$r_o = \frac{d_2}{2\sin\gamma_2} = \frac{d_3}{2\sin\gamma_3} = \frac{r_3}{\sin\gamma_3} = \frac{4}{\sin(63.435)} = 4.472 \text{ in}$$

The outside diameter at the back of the gear is

$$d_{o3} = d_3 + 2a = d_3 + 2\frac{k}{P_n} = 4 + 2\frac{0.8}{5} = 4.32 \text{ in}$$

The face width limit is given by Eq. (11.43). Then

$$F < [0.3r_o = 0.3(4.472) = 1.342]$$

or

$$F < \left[\frac{10}{P_n} = \frac{10}{5} = 2\right]$$

We should use the smaller value for F or

$$F = 2 \text{ in}$$

Problem 11.20

A pair of straight-toothed bevel gears mesh with a shaft angle of 90˚ and a diametral pitch of 6. The pinion has 18 teeth, and the gear ratio is 2:1. The addendum and dedendum are the same as for 20˚ full-depth spur-gear teeth. Determine the number of teeth on the gear and the pitch diameters of bo th the pinion and gear. Also, for the gear, determine the pitch-cone angle, outside diameter, cone dis tance, and face width.

Solution

The number of teeth on the gear is given by

$$N_3 = N_2\frac{\omega_2}{\omega_3} = 18\frac{2}{1} = 36$$

The pitch cone angle for the gear is given by Eq. (11.27). Then,

$$\gamma_3 = \tan^{-1}\left[\frac{N_3}{N_2}\right] = \tan^{-1}(2) = 63.435^\circ$$

The pitch-cone angle for the pinion is given Eq. (11.31). Then,

$$\gamma_2 = \Sigma - \gamma_3 = 90^\circ - 63.435^\circ = 26.565^\circ$$

The pitch diameter for the large end of the gear is given by

$$d_3 = \frac{N_3}{P_n} = \frac{36}{(6)} = 6 \text{ in}$$

The pitch diameter for the pinion is given by Eq. (11.32). Then

$$d_2 = d_3 \frac{N_2}{N_3} = 6\frac{18}{36} = 3 \text{ in}$$

The cone distance is given by

$$r_o = \frac{d_2}{2\sin\gamma_2} = \frac{d_3}{2\sin\gamma_3} = \frac{r_3}{\sin\gamma_3} = \frac{3}{\sin(63.435)} = 3.354 \text{ in}$$

The outside diameter at the back of the gear is

$$d_{o3} = d_3 + 2a = d_3 + 2\frac{k}{P_n} = 6 + 2\frac{1}{6} = 6.333 \text{ in}$$

The face width limit is given by Eq. (11.43). Then

$$F < [0.3r_o = 0.3(3.354) = 1.006]$$

and

$$F < \left[\frac{10}{P_n} = \frac{10}{6} = 1.666\right]$$

We should use the smaller value for F or

$$F = 1 \text{ in}$$

Problem 11.21

A pair of straight-toothed bevel gears mesh with a shaft angle of 80˚ and a diametral pitch of 7. The pinion has 20 teeth and a pitch cone angle of 40˚. The gear ratio is 3:2. Determine the number of t eeth on the gear and the pitch diameters of both the pinion and gear. Also, determine the equivalent spur gear radii for both the pinion and the gear.

Solution

The number of teeth on the gear is given by

$$N_3 = N_2 \frac{\omega_2}{\omega_3} = 20\frac{3}{2} = 30$$

The pitch cone angle for the gear is given by Eq. (11.31).

$$\gamma_3 = \Sigma - \gamma_2 = 80^\circ - 40^\circ = 40^\circ$$

The pitch diameter for the large end of the pinion is given by

$$d_2 = \frac{N_2}{P_n} = \frac{20}{7} = 2.857 \text{ in}$$

and for the gear,

$$d_3 = \frac{N_3}{P_n} = \frac{30}{7} = 4.286 \text{ in}$$

The equivalent spur gear radius for the pinion is given by Eq. (11.39) as

$$r_{e2} = \frac{d_2}{2\cos\gamma_2} = \frac{2.857}{2\cos(40)} = 1.865 \text{ in}$$

and for the gear,

$$r_{e3} = \frac{d_3}{2\cos\gamma 3} = \frac{4.286}{2\cos(40)} = 2.797 \text{ in}$$

Problem 11.22

A pair of straight-toothed bevel gears mesh with a shaft angle of 45˚ and a module of 5.08. The pin ion has 16 teeth and a pitch cone angle of 20˚. The gear ratio is 3:2. Determine the number of teeth on the gear and the pitch diameters of both the pinion and gear. Also determine the back-cone dist ance and the back-cone angle for the gear.

Solution

The number of teeth on the gear is given by

$$N_3 = N_2 \frac{\omega_2}{\omega_3} = 16\frac{3}{2} = 24$$

The pitch cone angle for the gear is given by Eq. (11.31).

$$\gamma_3 = \Sigma - \gamma_2 = 45^\circ - 20^\circ = 25^\circ$$

The pitch diameter for the large end of the pinion is given by

$$d_2 = N_2 m_n = 16(5.08) = 81.28 \text{ mm}$$

and for the gear,

$$d_3 = N_3 m_n = 24(5.08) = 121.92 \text{ mm}$$

The back-cone radius for the gear is given by Eq. (11.39) as

$$r_{e3} = \frac{d_3}{2\cos\gamma_3} = \frac{121.92}{2\cos(40)} = 79.58 \text{ mm}$$

The back-cone angle is given by

$$\beta = 2\sin^{-1}\left(\frac{r_3}{r_{e3}}\right) = 2\sin^{-1}\left[\frac{121.92}{2(79.58)}\right] = 100^{\circ}$$