$$\lim_{x \to +\infty} (e^x + x)^{\frac{2}{x}}$$

$$y = (e^x + x)^{\frac{2}{x}} \Leftrightarrow \ln(y) = \ln(e^x + x)^{\frac{2}{x}}$$

$$\Leftrightarrow \ln y = \frac{2}{x} \ln(e^x + x)$$

$$\Leftrightarrow \lim_{x \to +\infty} \ln(y) = \lim_{x \to +\infty} \frac{2}{x} \ln(e^x + x)$$

$$\Leftrightarrow \ln(\lim_{x \to +\infty} y) = \lim_{x \to +\infty} \frac{2 \ln(e^x + x)}{x}$$

$$\Leftrightarrow \ln(\lim_{x \to +\infty} y) \underset{=}{\text{Hop}} \lim_{x \to +\infty} \frac{2 \frac{e^x + 1}{e^x + x}}{1}$$

$$\Leftrightarrow \ln(\lim_{x \to +\infty} y) = 2$$

$$\boxed{\begin{array}{l} \ln a = y \\ a = e^y \end{array}}$$

$$\Leftrightarrow \lim_{x \to +\infty} y = e^2$$

$$\int (\cos^4 x - \sin^4 x)\,dx$$

$$= \int (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)\,dx$$

$$= \int (\cos^2 x - \sin^2 x) \times 1 \times dx$$

$$= \int \cos 2x\,dx = \frac{1}{2}\sin 2x + c$$

$$\int \cos^{11} x \sin^3 x\,dx = \int \cos^{11} x \sin^2 x \sin x\,dx$$

$$= \int \cos^{11} x (1 - \cos^2 x) \sin x\,dx$$

$$\begin{cases} u = \cos x \\ du = -\sin x\,dx \longrightarrow -du = \sin x\,dx \end{cases}$$

$$= -\int u^{11}(1 - u^2)\,du = -\frac{u^{12}}{12} + \frac{u^{14}}{14} + c$$

$$= -\frac{1}{12}\cos^{12} x + \frac{1}{14}\cos^{14} x + c$$

$$\int \frac{\sqrt{\tanh x + 1}}{\cosh^2 x}\,dx \qquad \begin{cases} 1+\tanh x = u \\ \dfrac{dx}{\cosh^2 x} = du \end{cases}$$

$$= \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} = \frac{2}{3}(1+\tanh x)\sqrt{1+\tanh x}$$

$$\int \arcsin x\,dx \qquad \begin{cases} u = \arcsin x \\ du = \dfrac{1}{\sqrt{1-x^2}}\,dx \end{cases}$$

$$= uv - \int v\,du \qquad \begin{cases} dv = dx \rightarrow v = x \end{cases}$$

$$= x\sin^{-1}(x) - \int x \times \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= x\sin^{-1}(x) + \sqrt{1-x^2}$$

$$\int \sin\sqrt{x}\, dx \qquad \begin{cases} t := \sqrt{x} \\ t^2 = x \iff 2t\,dt = dx \end{cases}$$

$$= \int 2t \sin t\, dt = \qquad \begin{cases} u = t \rightarrow du = dt \\ dv = \sin t\, dt \\ \Rightarrow v = -\cos t \end{cases}$$

$$= 2\left(uv - \int v\, du\right)$$

$$= 2\left(-t\cos t - \int 1 \times (-\cos t)\, dt\right)$$

$$= 2(-t\cos t + \sin t) + c$$

$$= 2(-\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x}) + c$$

$$\int \sin(\ln(x))\, dx \qquad \begin{cases} u := \sin(\ln(x)) \\ du = \frac{1}{x}\cos x\, dx \end{cases} \qquad \begin{cases} dv = dx \\ \Rightarrow v = x \end{cases}$$

$$\Rightarrow \int \sin(\ln x)\, dx = uv - \int v\, du$$

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$$= x\sin(\ln x) - \boxed{\int \cos(\ln x)\, dx}$$

$$= x\sin(\ln x) - \boxed{\left[x\cos(\ln x) + \int \frac{x}{x} \times \sin(\ln(x))\, dx\right]}$$

$$= x\sin(\ln x) - x\cos(\ln(x)) \cancel{+} - \int \sin(\ln x)\, dx$$

$$\iff 2\int \sin(\ln(x))\, dx = x(\sin(\ln x) - \cos(\ln x))$$

$$\iff \int \sin(\ln(x)) = \frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + c$$

$$\lim_{x \to 0^+} x^{\sin x} = ?$$

$$\left\{ y = x^{\sin x} \iff \ln y = \ln x^{\sin x} \right.$$

$$\iff \ln y = \sin x \ln x$$

$$\lim_{x \to 0^+} \ln(y) = \lim_{x \to 0^+} \sin x \ln x$$

$$\ln(\lim_{x \to 0^+} y) = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{\sin x}}$$

$$\ln(\lim_{x \to 0^+} y) \overset{Hop}{=\!=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}}$$

$$\ln(\lim_{x \to 0^+} y) = \lim_{x \to 0^+} \frac{-\sin x}{x} \cdot \tan x = -1 \times 0 = 0$$

$$\Rightarrow \ln(\lim_{x \to 0^+} y) = 0 \iff \lim_{x \to 0^+} y = e^0 = 1$$

$$\int \frac{\sqrt{\tanh x + 1}}{\cosh^2 x}\, dx \qquad \begin{cases} 1 + \tanh x = u \\ \dfrac{dx}{\cosh^2 x} = du \end{cases}$$

$$= \int \sqrt{u}\, du = \frac{2}{3} u^{3/2} = \frac{2}{3}(1+\tanh x)\sqrt{1+\tanh x}$$

$$\int \arcsin x\, dx \qquad \begin{cases} u = \arcsin x \\ du = \dfrac{1}{\sqrt{1-x^2}}\, dx \end{cases}$$

$$= uv - \int v\, du \qquad \{\, dv = dx \rightarrow v = x$$

$$= x \sin^{-1}(x) - \int x \times \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= x \sin^{-1}(x) - 2\sqrt{1-x^2}$$

$$\int \frac{\cos x - \sin x}{1 + \sin 2x}\, dx$$

$$= \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x \cos x}\, dx \quad \begin{cases} 1 = \cos^2 x + \sin^2 x \\ \sin 2x = 2\sin x \cos x \end{cases}$$

$$= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2}\, dx$$

$$\begin{cases} u = \sin x + \cos x \\ du = (\cos x - \sin x)\, dx \end{cases}$$

$$= \int \frac{du}{u^2} = -\frac{1}{u} + c = -\frac{1}{\sin x + \cos x} + c$$

$$\int \frac{e^x}{1 + e^{2x}}\, dx \qquad \begin{cases} u = e^x \\ du = e^x\, dx \\ e^{2x} = (e^x)^2 = u^2 \end{cases}$$

$$= \int \frac{du}{1 + u^2}$$

$$= \tan^{-1}(u) + c = \tan^{-1}(e^x) + c$$

$$\int \sqrt{\frac{x+1}{x-1}}\,dx = \int \sqrt{\frac{(x+1)(x+1)}{(x-1)(x+1)}}\,dx$$

$$= \int \frac{x+1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx + \int \frac{1}{\sqrt{x^2-1}}\,dx$$

$$= \sqrt{x^2-1} + \cosh^{-1}(x) + c$$

$$\int \frac{x}{\cos^2 x}\,dx = \qquad \begin{cases} u = x \Rightarrow du = dx \\ dv = \frac{1}{\cos^2 x}\,dx \\ \quad \Rightarrow v = \tan x \end{cases}$$

$$uv - \int v\,du = x\tan x - \int \tan x\,dx$$

$$= x\tan x - \int \frac{\sin x}{\cos x}\,dx \qquad \begin{cases} t = \cos t \\ dt = -\sin x\,dx \end{cases}$$

$$= x\tan x + \int \frac{dt}{t}$$

$$= x\tan x + \ln|t| + c = x\tan x + \ln(\cos x) + c$$

$$\int \sqrt{1-\sin x}\,dx = \int \sqrt{(\cos\tfrac{x}{2})^2 + (\sin(\tfrac{x}{2}))^2 - 2\sin\tfrac{x}{2}\cos\tfrac{x}{2}}\,dx$$

$$= \int \sqrt{(\cos x/2 - \sin\tfrac{x}{2})^2}\,dx$$

$$= \int |\cos x/2 - \sin\tfrac{x}{2}|\,dx$$

$$= 2 \times |\sin\tfrac{x}{2} + \cos\tfrac{x}{2}| + c$$