باشگاه المپیاد بعثت
besat olympiad club

پاسخنامه تشریحی

مرحله دوم
المپیاد فیزیک

دوره 38

گروه المپیاد فیزیک بعثت

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۲۴ فروردین ۱۴۰۴

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فشار (0.8 bar)	فشار (1.0 bar)	دما (c°)
11.08	10.99	12
16.07	14.96	17
20.22	20.72	22
26.94	26.77	27

الف)

$$\frac{PV}{T} = C \rightarrow \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \rightarrow \quad V_2 = V_1 \times \frac{T_2}{T_1} \times \frac{P_1}{P_2}$$

$$V_2 = 100 \times \frac{290}{300} \times \frac{1}{0.8} \backsimeq \boxed{120.8\ (m^3)}$$

ب)

$$\Delta U = \frac{5}{2}(P_2V_2 - P_1V_1) \backsimeq \boxed{8.\bar{3} \times 10^5 (J)}$$

پ)

$$\Delta m = m_1 - m_2 = (\beta V_1 - \alpha V_2) = 100\left(26.77 - 15.07 \times \frac{29}{24}\right) \backsimeq \boxed{856.0(\ g)}$$

ت)

$$Q = \Delta m \times L_v = 856 \times 2400 \backsimeq \boxed{2054 \times 10^3 (J)}$$

ث)

$$m = 100 \times 100 \times 10 \times 26.77 = \boxed{2677 \times 10^3 (\ g)}$$

ج)

$$\Delta m = 10^3 \times 856 \times 10^{-3} \backsimeq \boxed{856.0(kg)}$$

چ)

$$\frac{P}{\rho\, T} = C \qquad \rho_2 = \rho_1 \times \frac{T_1}{T_2} \times \frac{P_2}{P_1}$$

$$\rho A u = C \qquad \rho_1 \times 100 \times 100 \times 10 = \rho_2 \times 100 \times h \times 30$$

$$h = \frac{100}{3} \times \frac{\rho_1}{\rho_1} = \frac{100}{3} \times \frac{T_2}{T_1} \times \frac{P_1}{P_2} = \frac{100}{3} \times \frac{29}{30} \times \frac{1}{0.8} \backsimeq \boxed{40.28(m)}$$

ح)

$$Q = 2054 \times 10^6 - 833 \times 10^6 = \rho A u g H = 1.2 \times 10^4 \times 10 \times 10 \times H$$

$$H \backsimeq \boxed{1018\ (\mathrm{m})}$$

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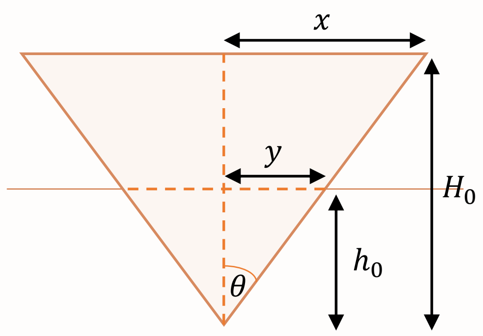
الف)



$$\tan\theta = \frac{x}{H_0} = \frac{y}{h_0}$$

$$h_0 = \sqrt{\frac{\rho_A}{\rho_{Hg}}}\, H_0$$

$$\rho_A H_0 x = \rho_{Hg} h_0 y$$

ب)

$$\Delta h = H_0(1+\alpha_A \Delta T)\sqrt{\frac{\rho_A(1-3\alpha_A\Delta T)}{\rho_{Hg}(1-\beta_{Hg}\Delta T)}} = \frac{H_0}{2}\sqrt{\frac{\rho_A}{\rho_{Hg}}}\,(\beta_{Hg}-\alpha_A)\Delta T$$

پ)

$$\Delta h = \frac{0.2}{2}\times\frac{1}{\sqrt{4}}\times 25\times 16\times 10^{-5} = 2\times 10^{-4}(m)$$

ت)

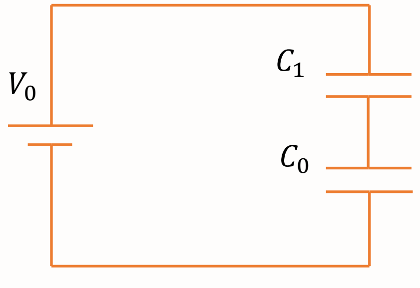
$$\Delta d = \Delta h - \Delta H \quad \rightarrow \quad \Delta d = \frac{H_0}{2}\Delta T\left(\sqrt{\frac{\rho_A}{\rho_{Hg}}}\,(\beta_{Hg}-\alpha_A) - 2\alpha_A\right)$$

ث)

$$C = \frac{A\varepsilon_0}{d} \qquad \frac{dC}{d(d)} = -\frac{A\varepsilon_0}{d^2} = -\frac{C}{d} \qquad \frac{\Delta C}{C} = -\frac{\Delta d}{d}$$

$$\frac{\Delta C}{C} = -\frac{1}{d}\times\frac{H_0}{2}\Delta T\left(\sqrt{\frac{\rho_A}{\rho_{Hg}}}\,(\beta_{Hg}-\alpha_A) - 2\alpha_A\right)$$

ج)



$$V_0 = q\left(\frac{1}{C_0}+\frac{1}{C_1}\right) \qquad V = \frac{q}{C_0} = \frac{V_0 C_1}{C_1 + C_0}$$

$$\Delta V = -\frac{V_0 C_1 \Delta C}{(C_1+C_0)^2} \rightarrow (C_1 = C_0) \rightarrow \Delta V = -\frac{V_0 \Delta C}{4C_0}$$

ح)

$$\rightarrow \Delta V = -\frac{V_0\Delta C}{4C_0} = 0.05\ (V)$$

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الف)

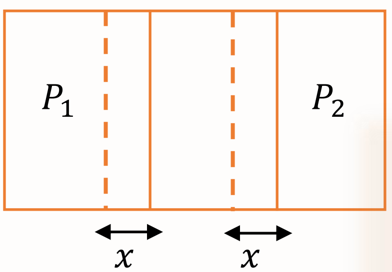
$$Q = nAxe \rightarrow \quad \sigma = nxe$$

$$F = -Ee = -\left(\frac{\sigma}{2\varepsilon_0}\right)e = -\frac{nxe^2}{2\varepsilon_0} = m_e a$$

$$a = -\frac{ne^2}{2m_e\varepsilon_0}x \qquad \omega = \sqrt{\frac{ne^2}{2m_e\varepsilon_0}} \qquad \boxed{f = \frac{1}{2\pi}\sqrt{\frac{ne^2}{2m_e\varepsilon_0}}}$$

ب)

$$\lambda_n = \frac{2L}{n} \qquad \lambda = 3d \qquad PV = C$$



$$P(l - x) = P_0 l \qquad P_0 = \frac{nN_A k_B T}{Al}$$

$$P_2 = P_0(1 + \frac{x}{l}) \qquad \rho = \frac{nN_A m_e}{Al}$$

$$P_1 = P_0(1 - \frac{x}{l})$$

$$\Delta P = -2P_0\left(\frac{x}{l}\right) \rightarrow F = -2P_0 A\left(\frac{x}{l}\right) = \rho Ala \qquad a = -2P_0\left(\frac{x}{\rho l^2}\right)$$

$$\omega = \sqrt{\frac{2P_0}{\rho l^2}} = \sqrt{\frac{2k_BT}{m_e l^2}} \qquad \boxed{\omega = \sqrt{\frac{72k_BT}{m_e\lambda^2}}}$$

پ)

$$\frac{F}{M} = cte \rightarrow \qquad -M{\omega_1}^2 x - M{\omega_1}^2 x = Ma$$

$$\Omega^2 = {\omega_1}^2 + {\omega_2}^2 \qquad \Omega^2 = \frac{ne^2}{2m_e\varepsilon_0} + \frac{72k_BT}{m_e\lambda^2}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{ne^2}{2m_e\varepsilon_0} + \frac{72k_BT}{m_e\lambda^2}}}$$

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الف)

$$\left.\begin{array}{l} \varepsilon = -\dfrac{d\varphi}{dt} = -\dfrac{d\varphi_z}{dz}\dfrac{dz}{dt} = u\varphi' \\ P = Ri^2 = fv \end{array}\right\} \qquad \boxed{f = \frac{u}{R}{\varphi'}^2}$$

ب)

$$\varepsilon = -\frac{d\varphi}{dt} = -\frac{d\varphi_{z-bk}}{dz}\frac{dz}{dt} = u{\varphi'}_{z-bk} \qquad \sum_{k=-N}^{N} P_k = fV \qquad R = \frac{\rho(2\pi r)}{ab}$$

$$f=\frac{abu}{2\rho\pi r}\sum_{k=-N}^{N}{\varphi'_{z-bk}}^2$$

پ)

$$r=z-bk$$

$$b=dr=d(z-bk)$$

$$f=\frac{au}{2\rho\pi r}\sum_{k=-\infty}^{\infty}{\varphi'_r}^2dr$$

ت)

$$\varepsilon_{tot}=u\sum_{k=-N}^{N}\varphi'_{z-bk}$$

$$\frac{u_{cu}}{u_{Al}}=\frac{\rho_{Al}}{\rho_{cu}}$$

ث)

$$P=\frac{u^2ab}{2\pi r\rho(2N+1)}\left(\sum_{k=-N}^{N}\varphi'_{z-bk}\right)^2=fu \xrightarrow[b=\Delta(z-bk)]{b(2N+1)=L}$$

$$f=\frac{ua}{2\pi r\rho L}\left(\sum_{k=-N}^{N}\varphi'_{z-bk}\,\Delta(z-bk)\right)^2$$

ج)

$$r=z-bk$$

$$\lim_{b\to 0}f=\frac{ua}{2\pi r\rho L}\left(\sum_{k=-\infty}^{\infty}\varphi'_r\,dr\right)^2$$

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الف)

انبساط : $\sigma blg\sin\theta+\mu\sigma b(l-x_1)g\cos\theta=\mu\sigma bx_1g\cos\theta$

ب)

انقباض : $\sigma blg\sin\theta-\mu\sigma b(l-x_2)g\cos\theta=-\mu\sigma bx_2g\cos\theta$

پ)

$$\longrightarrow \quad x_1=\frac{l}{2}\frac{\mu+\tan\theta}{\mu} \qquad x_2=\frac{l}{2}\frac{\mu-\tan\theta}{\mu}$$

ت)

$$\left.\begin{aligned}\delta_{C\to H}&=x_1\alpha(T_H-T_C)\\ \delta_{H\to C}&=-x_2\alpha(T_H-T_C)\end{aligned}\right\}\quad \delta=\alpha(T_H-T_C)\left(\frac{l}{2}\frac{\mu+\tan\theta}{\mu}-\frac{l}{2}\frac{\mu-\tan\theta}{\mu}\right)$$

$$=\alpha(T_H-T_C)\frac{l\tan\theta}{\mu}$$

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الف)

ب)

$$N = mR\omega^2$$
$$f_s = mg$$

$$\omega_{min} = \sqrt{\frac{g}{\mu_s R}}$$

پ)

ت)

ث)

$$N = mR\omega^2 - mg\sin\theta$$
$$f_s = mg\cos\theta$$

$$\omega_{min} = \sqrt{\frac{g}{R}\sqrt{1+(\frac{1}{\mu_s})^2}}$$

$$\omega^2 \geq \frac{g}{R}(\sin\theta + \frac{\cos\theta}{\mu_s})$$

ج)

$$\vec{g} = -g\sin\alpha\hat{z} - g\cos\alpha\hat{y}$$
$$f_{s1} = mg\cos\theta\cos\alpha$$
$$N = mR\omega^2 - mg\sin\theta\cos\alpha$$
$$f_{s1} = mg\sin\alpha$$

چ)

$$\omega^2 \geq \frac{g}{R}(\cos\alpha\sin\theta + \frac{1}{\mu_s}\sqrt{\sin\alpha^2 + (\cos\theta\cos\alpha)^2})$$

ح)

$$\frac{d\omega^2}{d\theta} = 0 = \cos\theta\cos\alpha - \frac{2\cos\alpha^2\cos\theta\sin\theta}{2\sqrt{\sin\alpha^2 + (\cos\theta\cos\alpha)^2})}$$

$$\cos\theta = 0 \qquad \theta = \frac{\pi}{2},\frac{3\pi}{2}$$

$$\theta = \frac{\pi}{2} \to \omega^2 = \frac{g}{R}(\cos\alpha + \frac{\sin\alpha}{\mu_s})$$

$$\theta = \frac{3\pi}{2} \to \omega^2 = \frac{g}{R}(-\cos\alpha + \frac{\sin\alpha}{\mu_s})$$

$$\sin\theta = \pm\frac{1}{\cos\alpha\sqrt{1+(\frac{1}{\mu_s})^2}}$$

$$\omega^2 = \frac{g}{R}\sqrt{1+(\frac{1}{\mu_s})^2} \quad , \quad \omega^2 = \frac{g}{R}\frac{(\frac{1}{\mu_s})^2 - 1}{\sqrt{1+(\frac{1}{\mu_s})^2}}$$

$$\cos\alpha\sqrt{1+(\frac{1}{\mu_s})^2} \geq 1 \quad \to \omega_{min} = \sqrt{\frac{g}{R}(\cos\alpha + \frac{\sin\alpha}{\mu_s})}$$

$$\cos\alpha\sqrt{1+(\frac{1}{\mu_s})^2} \leq 1 \quad \to \omega_{min} = \sqrt{\frac{g}{R}\sqrt{1+(\frac{1}{\mu_s})^2}}$$

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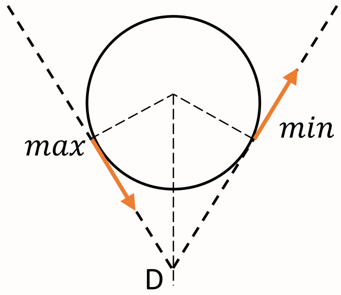
الف)



$cos\theta = 1 \quad f_{max}$

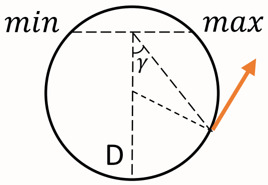
$cos\theta = -1 \quad f_{min}$

$$f_{max} = \frac{f_0}{1-\frac{v}{c}}$$

$$f_{min} = \frac{f_0}{1+\frac{v}{c}}$$

هر دو گیرنده داخل هستند، چون اکسترمم ها در بیرون هستند.

ب)



$$\frac{sin\alpha}{d} = \frac{sin\gamma}{R}$$

$$f = \frac{f_0}{1-\frac{v}{c}\frac{d}{R}sin\gamma}$$

$$f = \frac{f_0}{1-\frac{v}{c}cos\theta} = \frac{f_0}{1-\frac{v}{c}sin\alpha}$$

$$f_{max} = \frac{f_0}{1-\frac{v}{c}\frac{d}{R}}$$

$$f_{min} = \frac{f_0}{1+\frac{v}{c}\frac{d}{R}}$$

با توجه به اینکه اکسترمم های هر دو در یک زمان می باشد، پس هر دو داخل نیستند. زیرا در یک نقطه دو دریافت کننده قرار نمی گیرد.

پ) با توجه به اینکه $\frac{\text{درون}\, f_{min}}{\text{بیرون}\, f_{min}} > 1$

نقطه ۱ بیرون قرار دارد و نقطه ۲ داخل است.

بیرون $\quad f_{min} = \frac{f_0}{1+\frac{v}{c}}$

درون $\quad f_{min} = \frac{f_0}{1+\frac{v}{c}\frac{d}{R}}$

ت) با توجه به نمودار داریم:

$f_0 = 488.75(s^{-1})$

$v = 49.5(\frac{m}{s})$

$\alpha = \beta = \frac{\pi}{3}$

$L = 354.375(m)$

$d_1 = 472.5(m)$

$R = 236.25(m)$

$d_2 = 1181.125(m)$

موفق باشید. با تشکر