تمرینات مروری فصل ۶

۱)

$x^2-4x+3=0\to x=1\ ,\ x=3$

$=\int_1^3 x^2-4x+3dx=\left[\frac{1}{3}x^3-2x^2+3x\right]_1^3=$

$\left[\frac{1}{3}(27)-2(9)+9\right]-\left[\frac{1}{3}-2+3\right]=\frac{4}{3}$

۲)

$y=12x-2x^2 \qquad 12x-2x^2=x^2-6x$

$3x^2-18x=0\Rightarrow 3x(x-6)=0\Rightarrow \begin{matrix}x=0\\ x=6\end{matrix}$

$\int_0^6\left|12x-2x^2-x^2+6x\right|dx=\int_0^6\left|-3x^2+18x\right|dx$

$=\int_0^6\left|-3x^2+18x\right|dx=\left[-x^3+9x^2\right]_0^6=$

$=\left[-216+(9\times 36)\right]=108$

$y=x^2-6x\Rightarrow y'=2x-6\Rightarrow x=3$

$y=12x-2x^2 \qquad x=\frac{-12}{-4}=3$

۳)

$y=x^3 \qquad y=\sqrt[3]{x}=\int_0^1\sqrt[3]{x}-x^3=$

$=\left[\frac{3}{4}x\sqrt[3]{x}-\frac{1}{4}x^4\right]_0^1=\frac{1}{2}$

۴)

$\int_0^{\frac{3\pi}{2}}Sinx+Cosx\,dx+\int_{\frac{3\pi}{4}}^{\pi}-Cosx-Sinx=$

$-Cosx\ \_Sinx\Big]_0^{\frac{3\pi}{4}}+\left(-Sinx+Cosx\right]_{\frac{3\pi}{4}}^{\pi}$

$=\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)-(-1+0)+(0-1)+\sqrt{2}=2\sqrt{2}$

۵)

$S=\int_0^1(-y^3+y^2)dy=\frac{-1}{4}y^4+\frac{1}{3}y^3\Big]_0^1=\frac{1}{12}$

۶)

$y=-x^2+5 \qquad x^2=5-y$

$y=-4 \qquad x=\pm\sqrt{5-y}$

$\int_{-4}^{5}\sqrt{5-y}+\sqrt{5-y}=2\int_{-4}^{5}\sqrt{5-y}=2\int_{-4}^{5}$

$(5-y)^{\frac{1}{2}}dy=-2\int_{-4}^{5}-(5-y)^{\frac{1}{2}}dy=$

$\frac{-4}{3}(5-y)^{\frac{3}{2}}\Big]_{-4}^{5}=\frac{-4}{3}(5-5)+\frac{4}{3}(5+4)^{\frac{3}{2}}$

$=27\times\frac{4}{3}=36$

۷)

$S=\left|\int_0^1 x^3+3x^2+2x-2x^2-4x\,dx\right|$

$\left|\int_0^1 x^3+x^2-2x\,dx\right|=\left[\frac{1}{4}x^4+\frac{1}{3}x^3-x^2\right]_0^1=\frac{5}{12}$

۸) $y=e^{-x^2} \quad x=3 \quad y=0 \quad x=0$

$\int_0^3 e^{-x^2}dx$

۹) $y=\sqrt{(x-2)(x-3)}\ \ ,\ y=0$

۱۰)

$y\geq 0 \quad x=0 \quad x^2+y^2=4$

$\Rightarrow -y^2+4=x^2 \Rightarrow x=\sqrt{-y^2+4}$

$=\int_{-2}^{+2}\sqrt{-y^2+4dy}$

۱۱)

$\int_0^{\sqrt[4]{3}}\left(8^{\frac{2}{4}}-y^{\frac{2}{3}}\right)^{\frac{3}{2}}dy$

۱۲)

$\int_1^3\pi\left(\sqrt{x-1}\right)^2dx=\pi\int_1^3\left(\sqrt{x-1}\right)^2dx$

$=\pi\int_1^3(x-1)dx=\pi\frac{(x-1)^2}{2}\Big]_1^3=\pi\left(\frac{4}{2}-0\right)=2\pi$

۱۳)

$V=\pi\int_1^3\left(4y-y^2-3\right)^2dy=$

$$\pi\int_1^3\left(16y^2+y^4+9-8y^3-24y+6y^2\right)$$

$$\Rightarrow\int_1^3\left(y^4-8y^3+22y^2-24y+9\right)dy=$$

$$\frac{1}{5}y^5-2y^4+\frac{22}{3}y^3-12y^2+9y\Big]_1^3$$

$$=3/6-2/5=1/1$$

۱۴)

$$V=\int_{-(a-h)}^{a-h}\pi\left((\sqrt{a^2-y^2})^2-(a-h)^2\right)dy$$

۱۵)

$$V=\int_0^1\pi\,dx+\int_1^4\pi(4)dx+\int_4^5\pi(9)dx=22\pi$$

۱۶)

$$f(x)=\begin{cases}\dfrac{1}{2} & 0\le x\le 1\\ x^2-2x+2 & 2\le x\le 4\end{cases}$$

$$V=\int_0^1\pi\left(\frac{1}{2}\right)^2dx+\int_2^4\pi\left[\left(x^2-2x+2\right)\right]^2dx$$

$$=\pi\frac{1}{4}x\Big]_0^1$$

$$+\pi\int_2^4\left(x^4+4x^2+4+2\left(-2x^3+2x^2-4x\right)\right)dx$$

$$=\pi\int_2^4\left(x^4-4x^3+8x^2-8x+4\right)dx$$

$$=\pi\frac{1}{4}x\Big]_0^1+\frac{\pi x^5}{5}-x^4+\frac{8x^3}{3}-4x^2+4x\Big]_2^4$$

$$=\frac{\pi}{4}+\left(\pi\left(\frac{4^5}{5}\right)-4^4+\frac{8\times4^3}{3}-4\times16+16\right)-$$

$$\left(\pi\left(\frac{2^5}{5}\right)-2^4+\frac{8\times2^3}{3}-4\times4+8\right)$$

$$=\frac{136}{15}$$

۱۷)

$$\pi\int_0^4x=\pi\left(\frac{1}{2}x^2\right)\Big]_0^4=8\pi$$

۱۸)

$$\int_0^3\pi(x+2)^2=\pi$$

$$\left(\frac{(x+2)^3}{3}\right)\Big]_0^3=\left(\frac{125}{3}-\frac{8}{3}\right)\pi=\frac{117}{3}\pi$$

۱۹)

$$V=\pi\int_{-9}^0(3)^2-\left(\sqrt{-x}\right)^2dx=\int_{-9}^0\pi(9+x)dx$$

$$=\pi\left(9x+\frac{1}{2}x^2\right]_{-9}^0=\frac{81\pi}{2}$$

۲۰)

ج) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}Cos^2x\,\pi=\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1+Cos2x}{2}\Rightarrow$

$$\frac{\pi}{2}\left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}Cos2x\right]=\frac{\pi}{2}(x)\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}+\frac{\pi}{4}Sin2x\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$\Rightarrow\left(\frac{\pi^2}{4}+\frac{\pi^2}{4}\right)+\frac{\pi}{4}\times0+\frac{\pi}{4}\times0=\frac{\pi^2}{2}$$

$$=\frac{\pi}{4}+\frac{\pi}{4}=\frac{2\pi}{4}$$

۲۱)

$$\int_c^d\pi\left(f(y)\right)^2dy$$

۲۲)

$$\pi\int_0^4\left(2-\sqrt{x}\right)^2dx=\pi\int_0^4\left(4+x-4\sqrt{x}\right)dx$$

$$=\pi\left[4x+\frac{1}{2}x^2-\frac{8}{3}x\sqrt{x}\right]_0^4$$

$$=\pi\left[16+8-\frac{64}{3}\right]=\frac{8\pi}{3}$$

۲۳)

$$y=\sqrt{x}\Rightarrow x=y$$

$$V=\pi\int_0^2\left(4-y^2\right)^2dy$$

$$\pi\int_0^2\left(16+y^4-8y^2\right)dy=16y+\frac{1}{5}y^5-\frac{8}{3}y^3\Big]_0^2=49\pi$$

۲۴)

$$f'(x)=\frac{3}{2}\times\frac{1}{3}\times2x\left(x^2+2\right)^{\frac{1}{2}}\Rightarrow f'(x)=x\left(x^2+2\right)^{\frac{1}{2}}$$

$$\to S=\int_{۰}^{۱}\sqrt{۱+x^{۴}+۲x^{۲}}\,dx=\int_{۰}^{۱}\left(x^{۲}+۱\right)dx$$

$$=\left[\frac{۱}{۳}x^{۳}+x\right]_{۰}^{۱}=\frac{۴}{۳}$$

۲۵)

$$y=\frac{x^{۴}}{۴}+\frac{۱}{۸x^{۲}}\quad ,\ ۱\le x\le ۳$$

$$y'=x^{۳}-\frac{۱}{۴x^{۳}}\Rightarrow l=\int_{۱}^{۳}\sqrt{۱+\left(x^{۳}-\frac{۱}{۴x^{۳}}\right)^{۲}}$$

$$=\int_{۱}^{۳}\sqrt{\left(x^{۳}+\frac{۱}{۴x^{۳}}\right)^{۲}}=\int_{۱}^{۳}\left(x^{۳}+\frac{۱}{۴x^{۳}}\right)dx=$$

$$\frac{۱}{۴}x^{۴}-\frac{۱}{۸x^{۲}}\Big]_{۱}^{۳}=\left(\frac{۸۱}{۴}-\frac{۱}{۷۲}\right)-\left(\frac{۱}{۸}\right)$$

۲۶) $f'(x)=۳x^{۲}\Rightarrow S=\int_{۰}^{۱}\sqrt{۱+۹x^{۴}}dx$

۲۷)

$$x^{\frac{۲}{۳}}+y^{\frac{۲}{۳}}=۱\Rightarrow y^{\frac{۲}{۳}}=۱-x^{\frac{۲}{۳}}$$

$$\Rightarrow y=\left(۱-x^{\frac{۲}{۳}}\right)^{\frac{۳}{۲}}\Rightarrow y'=\frac{۳}{۲}\times\frac{۲}{۳}x^{\frac{-۱}{۳}}\left(۱-x^{\frac{۲}{۳}}\right)^{\frac{۱}{۲}}$$

$$=-x^{-\frac{۱}{۳}}\left(۱-x^{\frac{۲}{۳}}\right)^{\frac{۱}{۲}}$$

$$(y')^{۲}=x^{\frac{-۲}{۳}}\left(۱-x^{\frac{۲}{۳}}\right)=x^{-\frac{۲}{۳}}-۱\Rightarrow ۱+(y')^{۲}=x^{-\frac{۲}{۳}}$$

$$L=\int x^{-\frac{۱}{۳}}dx=\frac{۳}{۲}x^{\frac{۲}{۳}}$$

۲۸)

$$y=۲x^{\frac{۳}{۲}}\Rightarrow y'=۳x^{\frac{۱}{۲}}\Rightarrow y'^{۲}=۹x$$

$$L=\int_{۰}^{۱}\sqrt{۱+۹x}\,dx=\frac{۲}{۳\times۹}\left(۱+۹x\right)^{\frac{۳}{۲}}\Big]_{۰}^{۱}$$

$$=\frac{۲}{۲۷}\left((۱۰)^{\frac{۳}{۲}}-۱\right)$$

۲۹)

$$y=\frac{x^{۲}}{۳}+\frac{۱}{۴x}\Rightarrow y'=\frac{۲}{۳}x-\frac{۱}{۴x^{۲}}$$

$$L=\int_{۰}^{\infty}\sqrt{۱+\left(\frac{۲}{۳}x-\frac{۱}{۴x^{۲}}\right)^{۲}}\,dx$$