In The Name Of God

Power Electronics

Diode Rectifiers

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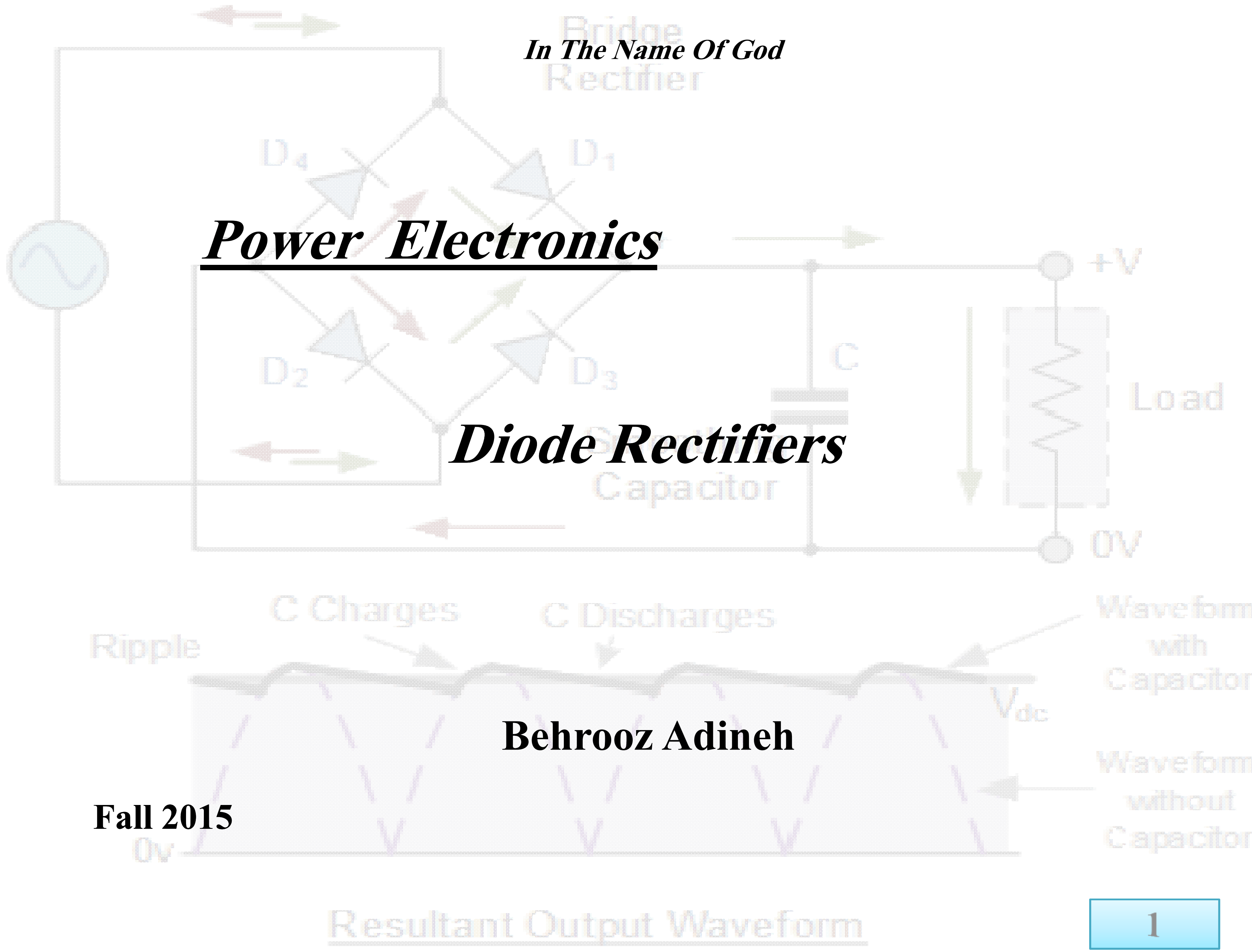
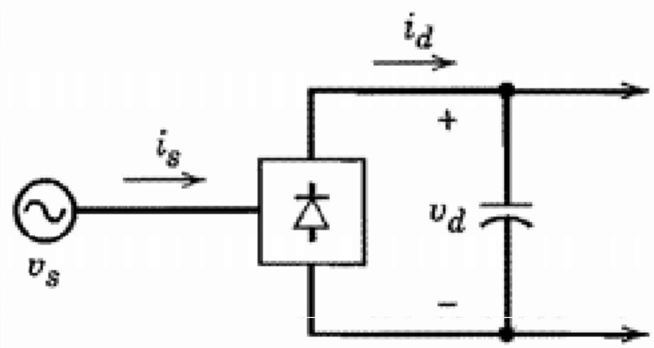
Fall 2015



The dc output voltage of a rectifier should be as ripple free as possible.

Rectification is the process of conversion of alternating input voltage to direct output voltage. As stated before, a rectifier converts ac power to dc. In diode-based rectifiers, the output voltage cannot be controlled.



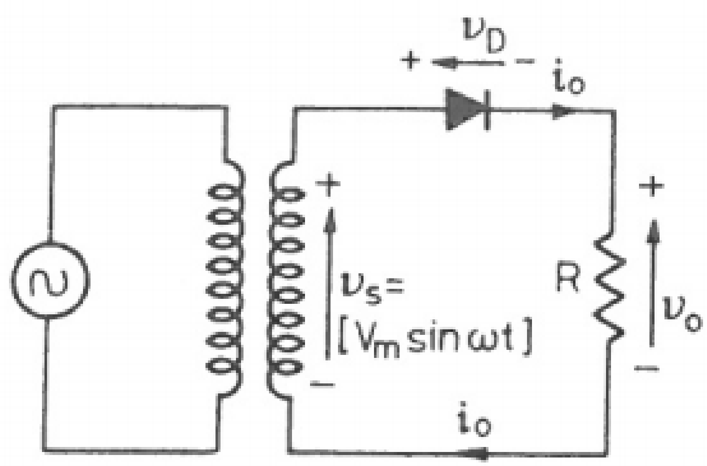
Diodes are extensively used in rectifiers. A *rectifier* is a circuit that converts an ac signal into a unidirectional signal. A rectifier is a type of dc–ac converter. Depending on the type of input supply, the rectifiers are classified into two types: (1) single phase and (2) three phase. For the sake of simplicity the diodes are considered to be ideal. By "ideal" we mean that the reverse recovery time t_{rr} and the forward voltage drop V_D are negligible. That is, $t_{rr} = 0$ and $V_D = 0$.

Single Phase Diode Rectifiers

This is the simplest type of uncontrolled rectifier. It is never used in industrial applications because of its poor performance. Its study is, however, useful in understanding the principle of rectifier operation.

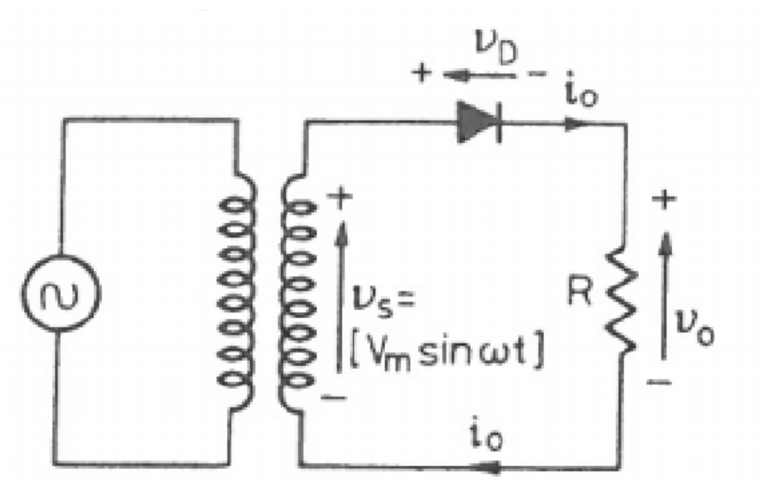
In a single-phase half-wave rectifier, for one cycle of supply voltage, there is one half-cycle of output, or load, voltage. As such, it is also called *single-phase one-pulse rectifier.*

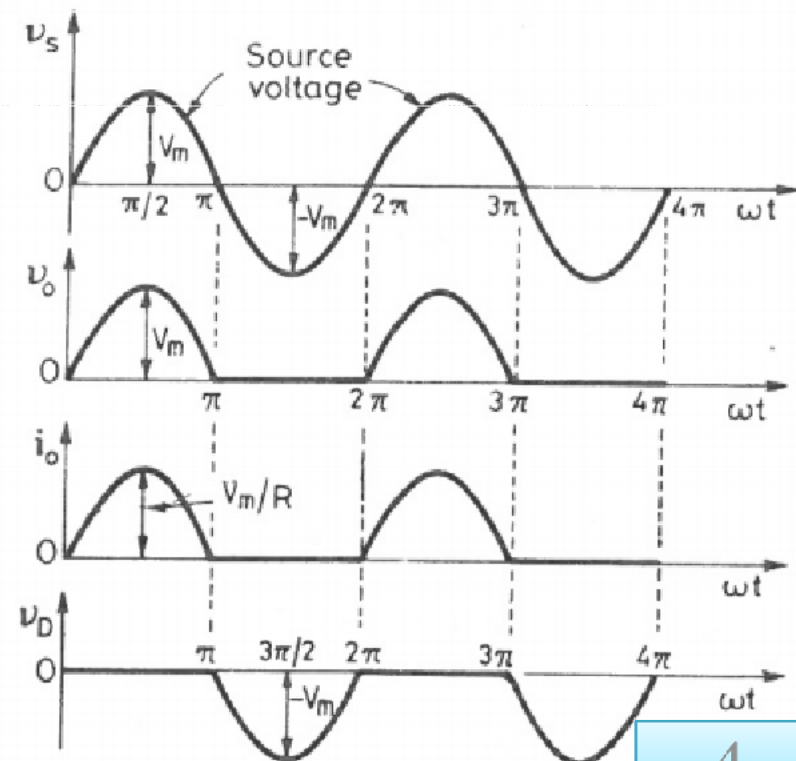
The load on the output side of rectifier may be R, RL or RL with a flywheel diode. These are now discussed briefly.



Resistive Load

During the positive half cycle, diode is forward biased, it therefore conducts from $\omega t = 0°$ to $\omega t = \pi$. During the positive half cycle, output voltage v_0 = source voltage v_s and load current $i_0 = v_0/R$. At $\omega t = \pi$, $v_0 = 0$ and for R load, i_0 is also zero. As soon as v_s tends to become negative after $\omega t = \pi$, diode D is reverse biased, it is therefore turned off and goes into blocking state. Output voltage, as well as output current, are zero from $\omega t = \pi$ to $\omega t = 2\pi$. After $\omega t = 2\pi$, diode is again forward biased and conduction begins.





A rectifier is a power processor that should give a dc output voltage with a minimum amount of harmonic contents. At the same time, it should maintain the input current as sinusoidal as possible and in phase with the input voltage so that the power factor is near unity. The power-processing quality of a rectifier requires the determination of harmonic contents of the input current, the output voltage, and the output current. We can use Fourier series expansions to find the harmonic contents of voltages and currents. There are different types of rectifier circuits and the performances of a rectifier are normally evaluated in terms of the following parameters:

The *average* value of the output (load) voltage, V_{dc}

The average value of the output (load) current, I_{dc}

The output dc power,

$$P_{dc} = V_{dc} I_{dc}$$

The root-mean-square (rms) value of the output voltage, V_{rms}

The rms value of the output current, I_{rms}

The output ac power

$$P_{ac} = V_{rms} I_{rms}$$

The *efficiency* (or *rectification ratio*) of a rectifier, which is a figure of merit and permits us to compare the effectiveness, is defined as

$$\eta = \frac{P_{dc}}{P_{ac}}$$

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The output voltage can be considered as composed of two components: (1) the dc value, and (2) the ac component or ripple.

The *effective* (rms) value of the ac component of output voltage is

$$V_{ac} = \sqrt{V_{rms}^2 - V_{dc}^2}$$

The *form factor*, which is a measure of the shape of output voltage, is

$$FF = \frac{V_{rms}}{V_{dc}}$$

The *ripple factor*, which is a measure of the ripple content, is defined as

$$RF = \frac{V_{ac}}{V_{dc}}$$

Substituting Eq. (3.4) in Eq. (3.6), the ripple factor can be expressed as

$$RF = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1} = \sqrt{FF^2 - 1}$$

The *transformer utilization factor* is defined as

$$TUF = \frac{P_{dc}}{V_s I_s}$$

If ϕ is the angle between the fundamental components of the input current and voltage, ϕ is called the *displacement angle*. The *displacement factor* is defined as

$$\mathrm{DF} = \cos\phi$$

The *harmonic factor* (HF) of the input current is defined as

$$\mathrm{HF} = \left(\frac{I_s^2 - I_{s1}^2}{I_{s1}^2}\right)^{1/2} = \left[\left(\frac{I_s}{I_{s1}}\right)^2 - 1\right]^{1/2}$$

where I_{s1} is the fundamental component of the input current I_s. Both I_{s1} and I_s are expressed here in rms. The input *power factor* (PF) is defined as

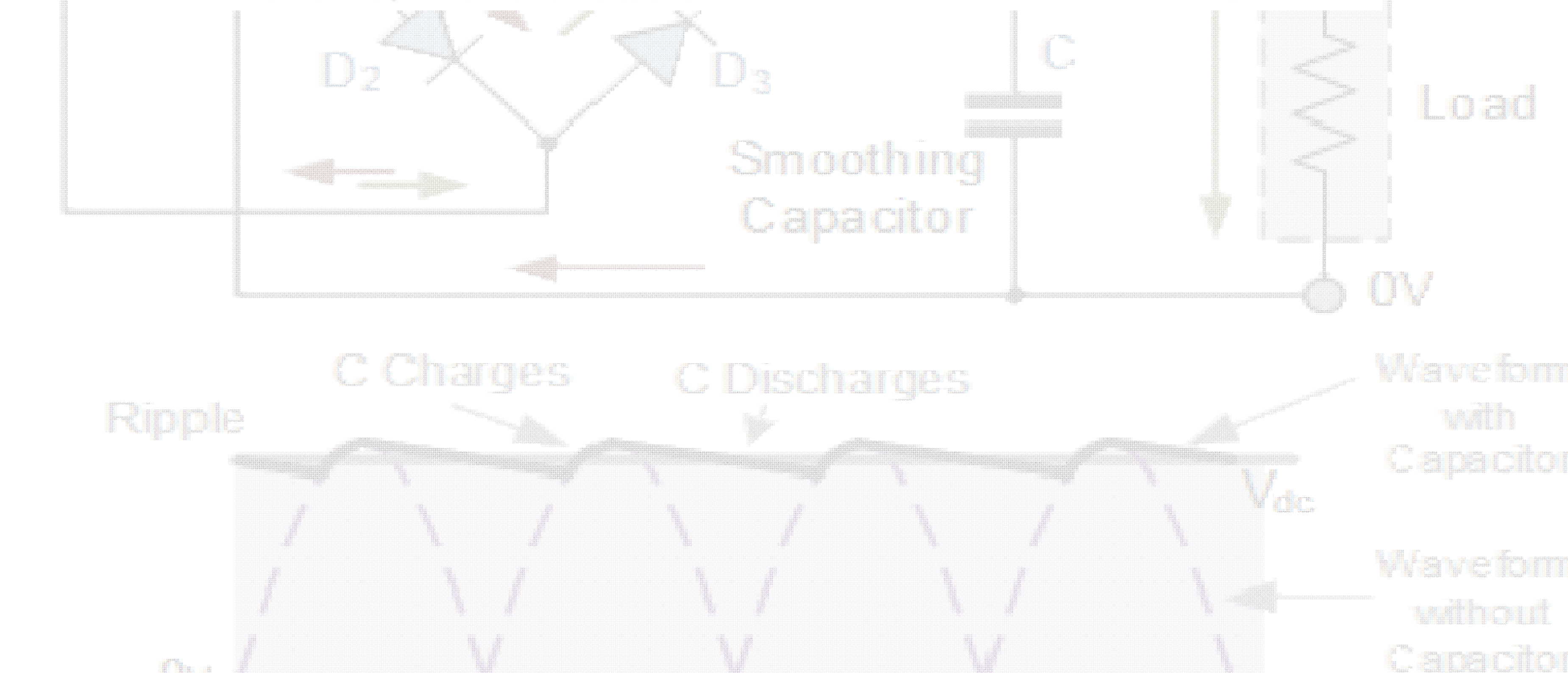
$$\mathrm{PF} = \frac{V_s I_{s1}}{V_s I_s}\cos\phi = \frac{I_{s1}}{I_s}\cos\phi$$

Crest factor (CF), which is a measure of the peak input current $I_{s(\text{peak})}$ as compared with its rms value I_s, is often of interest to specify the peak current ratings of devices and components. CF of the input current is defined by

$$\mathrm{CF} = \frac{I_{s(\text{peak})}}{I_s}$$

Notes

1. HF is a measure of the distortion of a waveform and is also known as *total harmonic distortion* (THD).
2. If the input current i_s is purely sinusoidal, $I_{s1} = I_s$ and the power factor PF equals the displacement factor DF. The displacement angle ϕ becomes the impedance angle $\theta = \tan^{-1}(\omega L/R)$ for an RL load.
3. Displacement factor DF is often known as *displacement power factor* (DPF).
4. An ideal rectifier should have $\eta = 100\%$, $V_{ac} = 0$, RF = 0, TUF = 1, HF = THD = 0, and PF = DPF = 1.



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Resultant Output Waveform

Average value of output (or load) voltage,

$$V_0 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin \omega t \, d(\omega t)\right]$$

$$= \frac{V_m}{2\pi} \left| -\cos \omega t \right|_0^{\pi} = \frac{V_m}{\pi}$$

Rms value of output voltage, $V_{or} = \left[\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t)\right]^{1/2}$

$$= \frac{V_m}{\sqrt{2\pi}}\left[\int_0^{\pi} \frac{1-\cos 2\omega t}{2} \cdot d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{2}$$

Here the subscript '*r*' is used to denote rms value.

Average value of load current,

$$I_0 = \frac{V_0}{R} = \frac{V_m}{\pi R}$$

Rms value of load current, $I_{or} = \frac{V_{or}}{R} = \frac{V_m}{2R}$

Peak value of load, or diode, current

$$= \frac{V_m}{R}$$

Peak inverse voltage, PIV, is an important parameter in the design of rectifier circuits *PIV* is the maximum voltage that appears across the device (here diode) during its blocking state. $\text{PIV} = V_m = \sqrt{2} \cdot V_s = \sqrt{2}$ (rms value of transformer secondary voltage).

Power delivered to resistive load = (*rms* load voltage) (*rms* load current)

$$= V_{or} \cdot I_{or} = \frac{V_m}{2} \cdot \frac{V_m}{2R} = \frac{V_m^2}{4R} = \frac{V_s^2}{2R} = I_{or}^2 R$$

Input power factor

$$= \frac{\text{Power delivered to load}}{\text{Input } VA}$$

$$= \frac{V_{or} \cdot I_{or}}{V_s \cdot I_{or}} = \frac{V_{or}}{V_s} = \frac{\sqrt{2} V_s}{2 V_s} = 0.707 \text{ lag.}$$

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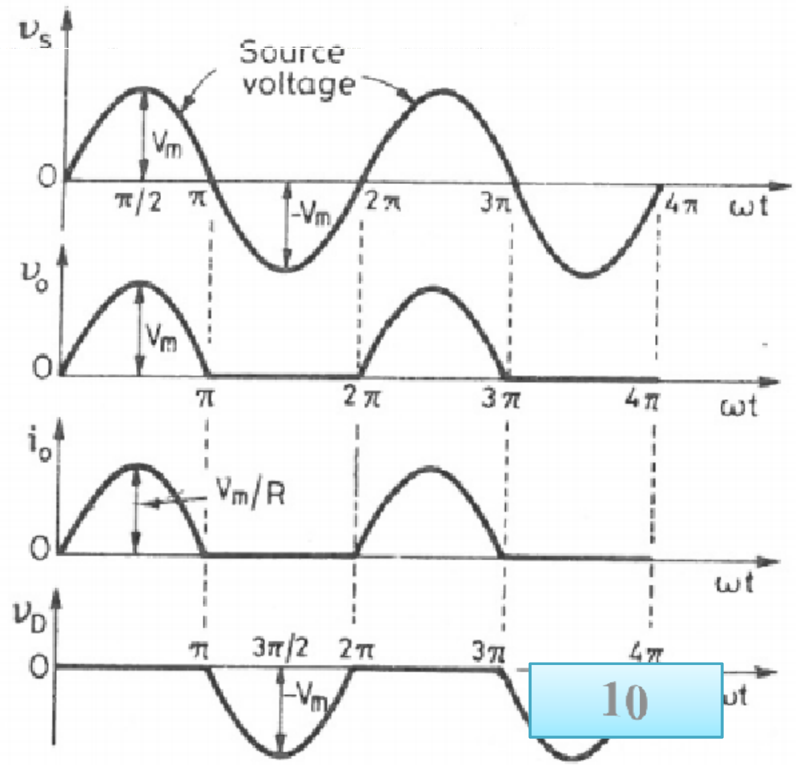

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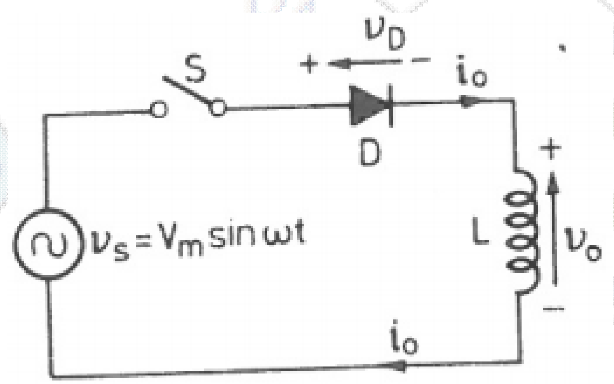
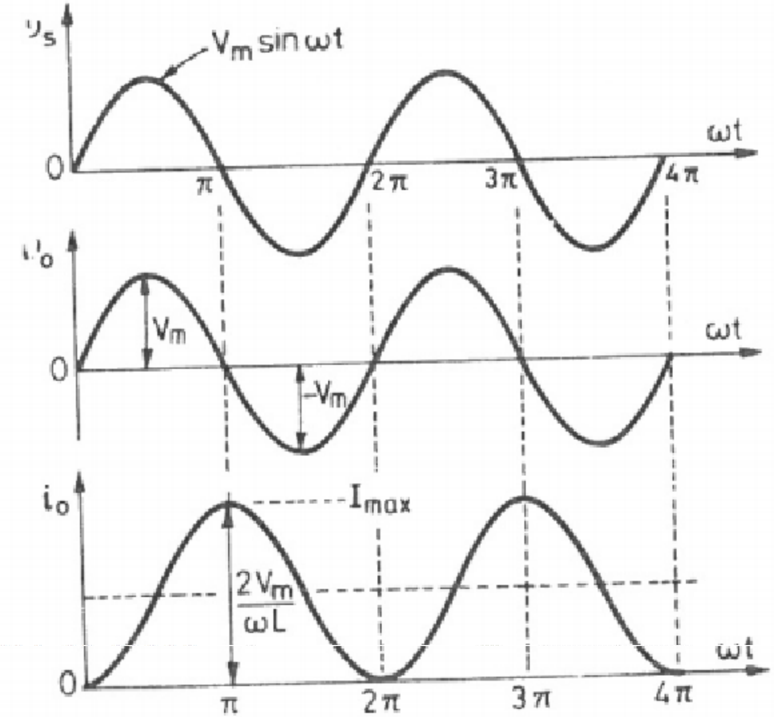
Inductive Load





When switch S is closed at $\omega t = 0$, diode starts conducting. *KVL* for this circuit gives

$$v_s = v_0 = L\frac{di_o}{dt} = V_m \sin \omega t \qquad i_0 = \frac{V_m}{L}\int \sin \omega t \cdot dt = -\frac{V_m}{\omega L}\cos \omega t + A$$

At $\omega t = 0,\ i_0 = 0$

$$0 = -\frac{V_m}{\omega L} + A$$

$$A = V_m / \omega L$$

$$i_0 = \frac{V_m}{\omega L}(1 - \cos \omega t)$$

Output voltage, $$v_0 = L\frac{di_o}{dt} = L\frac{V_m}{\omega L}[\sin \omega t]\,\omega = V_m \sin \omega t = v_s$$

Average value of output voltage, $V_0 = 0$

The output current i_0 consists of dc component and fundamental frequency component of frequency ω.

Peak value of current I_{max} occurs at $\omega t = \pi$ $$I_{max} = \frac{V_m}{\omega L}(1 + 1) = \frac{2V_m}{\omega L}$$

Average value of current, $$I_0 = \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m}{\omega L}(1 - \cos \omega t)\, d(\omega t)$$

$$= \frac{V_m}{\omega L} = \frac{1}{2} I_{max}$$

Rms value of fundamental current, I_{1r} is given by

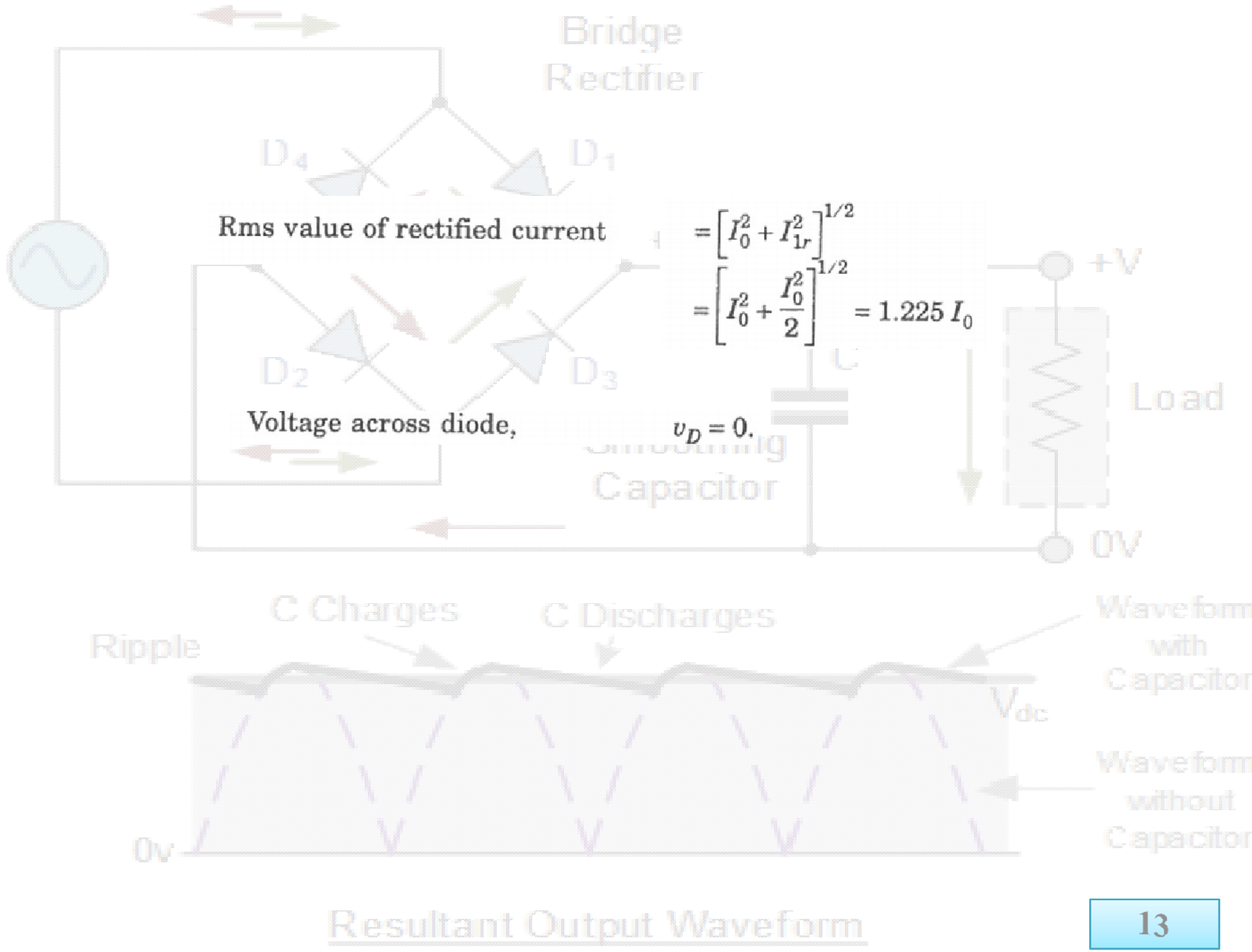
$$I_{1r} = \left[\frac{1}{2\pi}\left(\frac{V_m}{\omega L}\right)^2 \int_0^{2\pi} (\cos \omega t)^2\, d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2}\cdot \omega L} = \frac{V_s}{\omega L} = \frac{I_0}{\sqrt{2}}$$

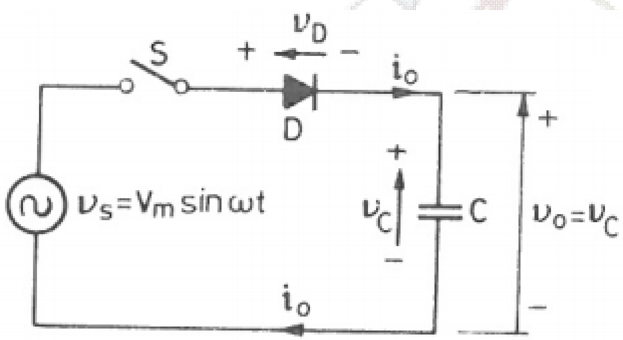
Rms value of rectified current $= \left[I_0^2 + I_{1r}^2\right]^{1/2}$

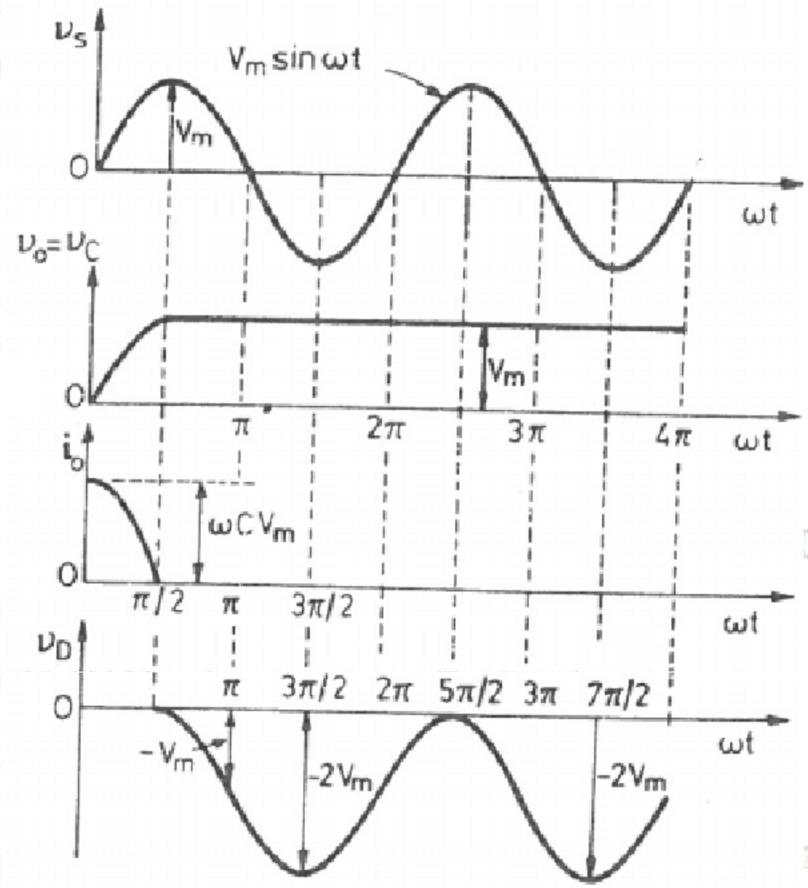
$$= \left[I_0^2 + \frac{I_0^2}{2}\right]^{1/2} = 1.225\, I_0$$

Voltage across diode, $v_D = 0.$



Capacitor Load





when switch S is closed at $\omega t = 0$

$$i_0 = C\frac{dv_s}{dt} = C\frac{d}{dt}(V_s \sin \omega t)$$

$$= \omega C\, V_m \cos \omega t$$

Output voltage, $$v_0 = \frac{1}{C}\int i dt = V_m \sin \omega t = v_s = v_C$$

Capacitor is charged to voltage V_m at $\omega t = \frac{\pi}{2}$ and subsequently this voltage remains constant at V_m.

Capacitor current or load current is maximum at $\omega t = 0$. Its value at $\omega t = 0$ is $\omega C V_m$

The diode conducts for $\frac{\pi}{2\omega}$ seconds only from $\omega t = 0$ to $\omega t = \frac{\pi}{2}$.

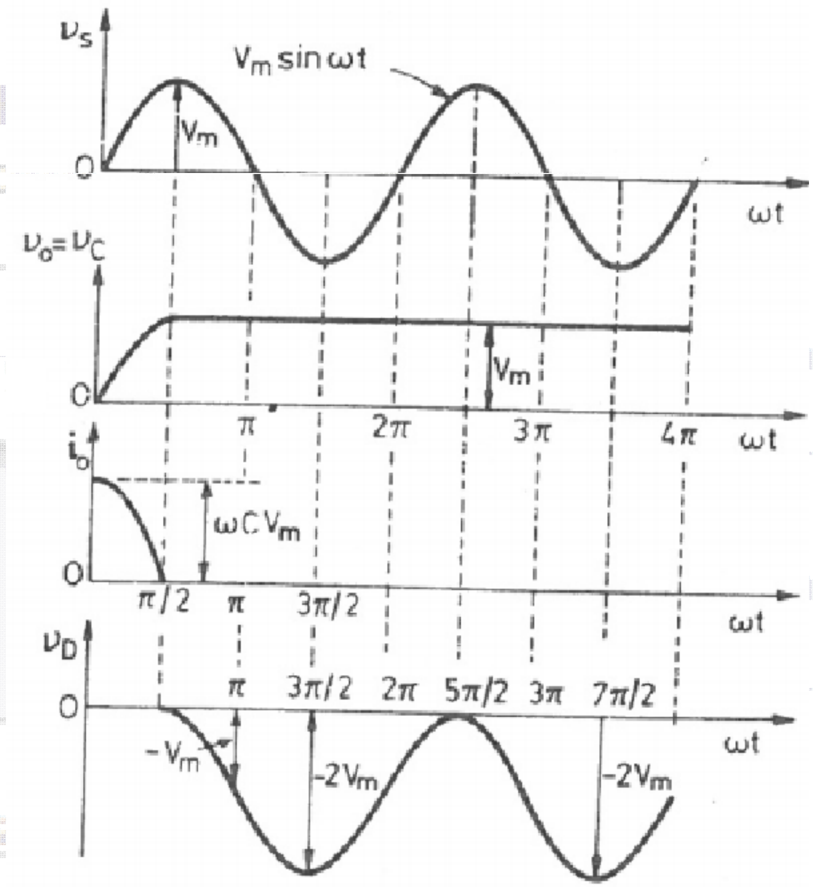
During this interval, diode voltage is, therefore, zero.

After $\omega t = \pi/2$, diode voltage v_D is given by

$$v_D = -v_0 + v_s = -V_m + V_m \sin \omega t$$
$$= V(\sin \omega t - 1)$$

the time origin is redefined at $\omega t = \pi/2$.

At $\omega t = \frac{3\pi}{2}$, $v_D = -2\,V_m$.



Average value of voltage across diode,

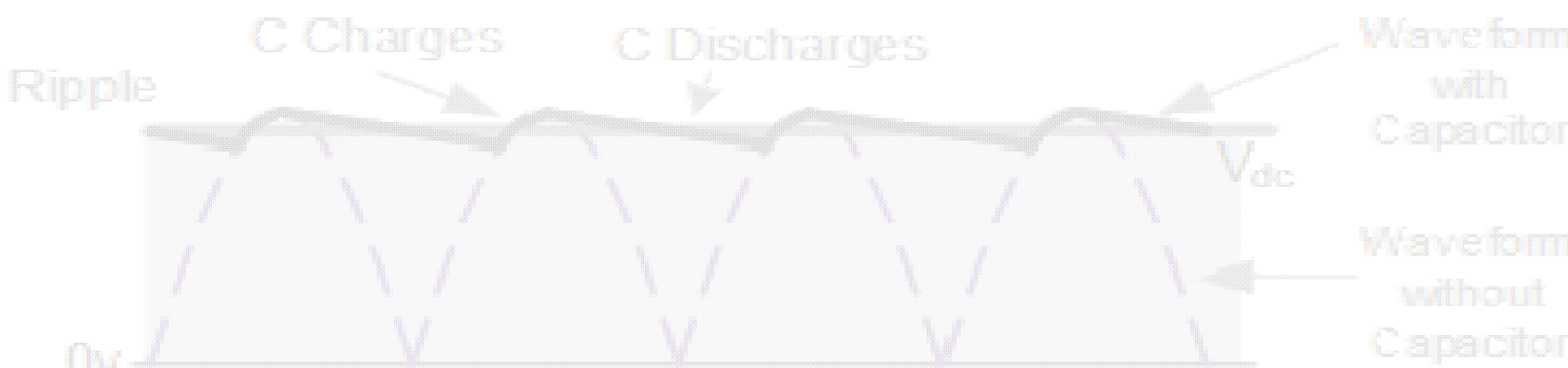
$$V_D = \frac{1}{2\pi}\int_0^{2\pi} V_m\,(\sin\omega t - 1)\,d(\omega t)$$
$$= V_m = \sqrt{2}\,V_s$$

Rms value of fundamental component of voltage across diode,

$$V_{1r} = \left[\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\omega t\,d(\omega t)\right]^{1/2} = \frac{V_m}{\sqrt{2}}$$

Rms value of voltage across diode

$$= \sqrt{V_D^2 + V_{1r}^2} = 1.225\,V_m$$



Resultant Output Waveform

Example

A single-phase 230 V, 1 kW heater is connected across single-phase 230 V, 50 Hz supply through a diode. Calculate the power delivered to the heater element. Find also the peak diode current and input power factor.

Heater resistance, $R = \dfrac{230^2}{1000}\ \Omega$

Rms value of output voltage $V_{or} = \dfrac{\sqrt{2} \times 230}{2}$

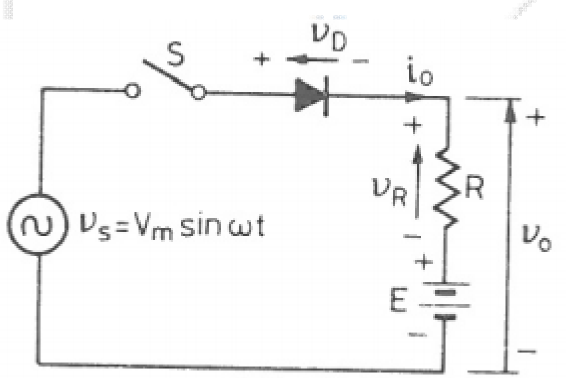
Power absorbed by heater element

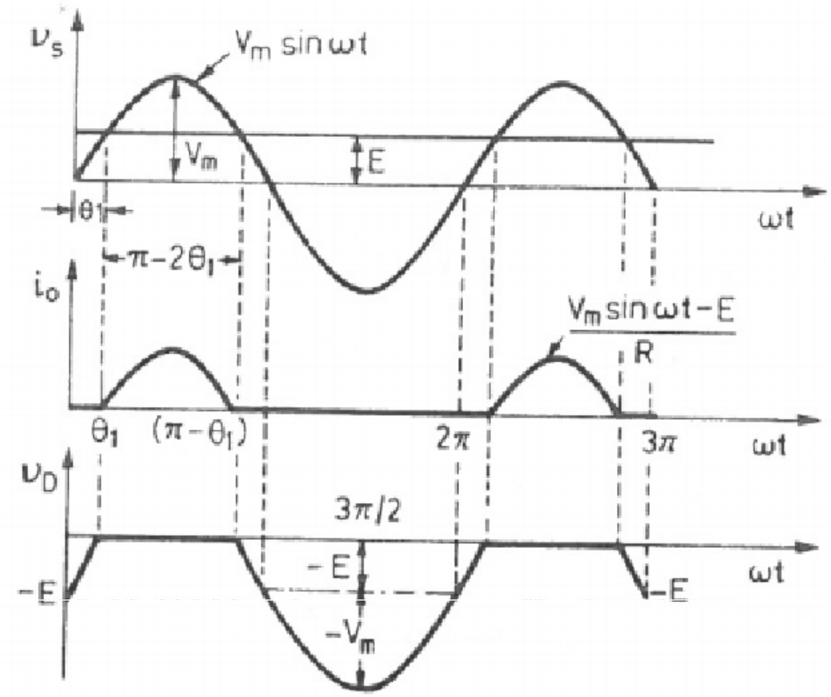
$$= \frac{V_{or}^2}{R} = \frac{2 \times 230^2}{4} \times \frac{1000}{230^2} = 500 \text{ W}$$

Peak value of diode current $\dfrac{\sqrt{2} \times 230}{230^2} \times 1000 = 6.1478 \text{ A}$

Input power factor $= \dfrac{V_{or}}{V_s} = \dfrac{\sqrt{2} \times 230}{2} \times \dfrac{1}{230} = 0.707$ lag.

RE Load





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If the output is connected to a battery, the rectifier can be used as a battery charger.

If the switch S is closed at $\omega t = 0°$ or when $v_o = 0$, then diode would not conduct at $\omega t = 0$ because diode is reverse biased until source voltage v_s equals E. When $V_m \sin \theta_1 = E$, diode D starts conducting and the turn-on angle θ_1 is given by

$$\theta_1 = \sin^{-1}\left(\frac{E}{V_m}\right)$$

The diode now conducts from $\omega t = \theta_1$ to $\omega t = (\pi - \theta_1)$, *i.e.* conduction angle for diode is $(\pi - 2\theta_1)$

the voltage equation for the circuit is

$$V_m \sin \omega t = E + i_0 R$$

$$i_0 = \frac{V_m \sin \omega t - E}{R}$$

Average value of this current is given by

$$I_0 = \frac{1}{2\pi R}\left[\int_{\theta_1}^{\pi-\theta_1} (V_m \sin \omega t - E)\, d(\omega t)\right]$$

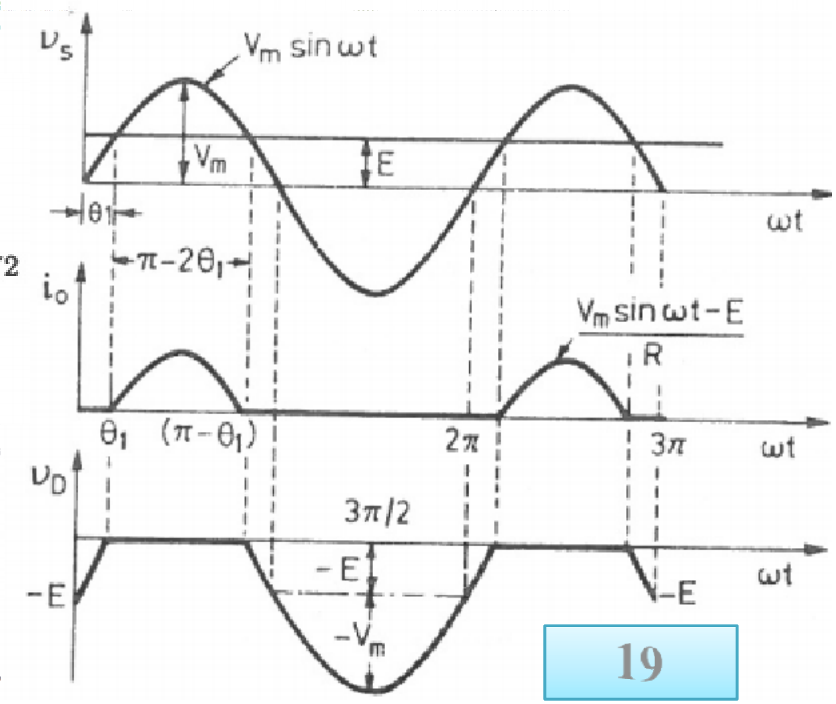
$$= \frac{1}{2\pi R}\,[2\, V_m \cos \theta_1 - E\,(\pi - 2\theta_1)]$$

Rms value of the load current

$$I_{or} = \left[\frac{1}{2\pi}\int_{\theta_1}^{\pi-\theta_1}\left(\frac{V_m \sin \omega t - E}{R}\right)\cdot d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi\theta^2}\int_{\theta_1}^{\pi-\theta_1} (V_m^2 \sin^2 \omega t + E^2 - 2\, V_m\, E \sin \omega t)\, d(\omega t)\right]^{1/2}$$

$$= \left[\frac{1}{2\pi R^2}\left\{(V_s^2 + E^2)\,(\pi - 2\theta_1) + V_s^2 \sin 2\theta_1 - 4\, V_m\, E \cos \theta_1\right\}\right]^{1/2}$$



Power delivered to load,

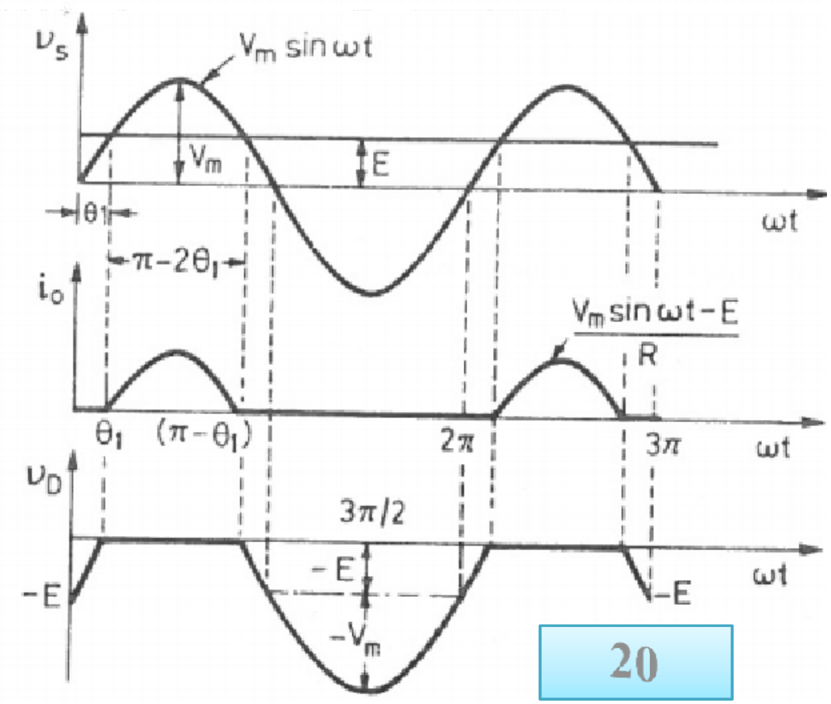
$$P = E\,I_0 + I_{or}^2\,R \ \text{watts}$$

$$\text{Supply pf} = \frac{\text{Power delivered to load}}{\text{(Source voltage) (rms value of source current)}}$$

$$= \frac{E\,I_0 + I_{or}^2\,R}{V_s \cdot I_{or}}$$

at $\omega t = 0°$, $v_D = -E$ and at $\omega t = \theta_1$, $v_D = 0$

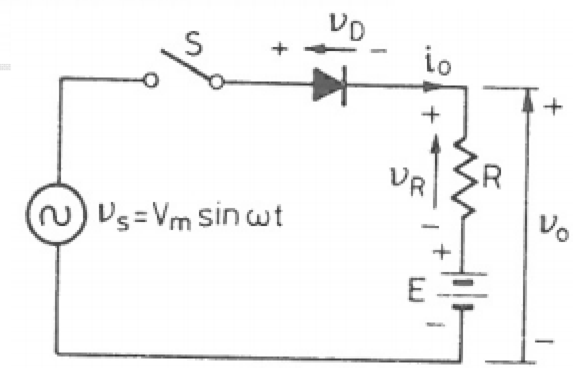
During the period diode conducts, $v_D = 0$. When $\omega t = 3\pi/2$, $v_s = -V_m$ and $v_D = -(V_m + E)$. Thus *PIV* for diode is $(V_m + E)$.



Example

A dc battery of constant emf E is charged through a resistor. For source voltage of 235 V, 50 Hz and for $R = 8\Omega$, $E = 150$ V,

(a) find the value of average charging current,
(b) find the power supplied to battery and that dissipated in the resistor,
(c) calculate the supply pf,
(d) find the charging time in case battery capacity is 1000 Wh and
(e) find rectifier efficiency and PIV of the diode.



Solution : (*a*) The diode will start conducting at an angle θ_1, where

$$\theta_1 = \sin^{-1} \frac{150}{\sqrt{2} \times 230} = 27.466°$$

Average value of charging current, from Eq. (3.38), is

$$I_0 = \frac{1}{2\pi \times 8}\left[2 \cdot \sqrt{2} \times 230 \cos 27.466° - 150\left(\pi - \frac{2 \times 27.466 \times \pi}{180}\right)\right]$$

$$= 4.9676 \text{ A}$$

(*b*) Power delivered to battery

$$= E\,I_0 = 150 \times \times\, 4.9676 = 745.14 \text{ W}$$

Rms value of charging current, from Eq. (3.39), is

$$I_{or} = \left[\frac{1}{2\pi \times 64}\left\{(230^2 + 150^2)\left(\pi - 2 \times 27.466 \times \frac{\pi}{180}\right) + 230^2 \sin 27.466 - 4\sqrt{2} \times 230\ 150 \cos 27.466°\right\}\right] = 9.2955 \text{ A}$$

Power dissipated in resistor

$$= I_{or}^2\, R = (9.2955)^2 \times 8 = 691.25 \text{ W}$$

(*c*) From Eq. (3.41), the supply

$$pf = \frac{745.14 + 691.25}{230 \times 9.2955} = 0.672 \text{ lag}$$

(*d*) (Power delivered to battery) (charging time in hours)

= Battery capacity in Wh.

$\therefore$ Charging time $= \dfrac{1000}{745.14} = 1.342$ h

(*e*) Rectifier efficiency $= \dfrac{\text{Power delivered to battery}}{\text{Total input power}}$

$$= \frac{745.14}{745.14 + 691.25} \times 100 = 51.876\%$$

(*f*) PIV of diode $= V_m + E = \sqrt{2} \times 230 + 150 = 475.22$ V.

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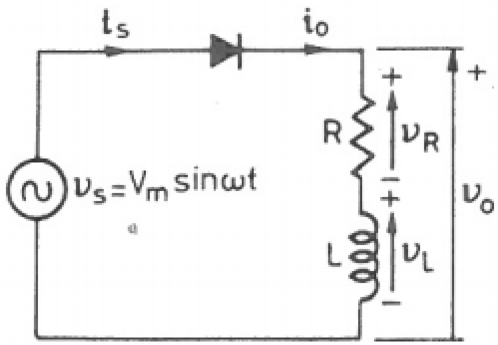
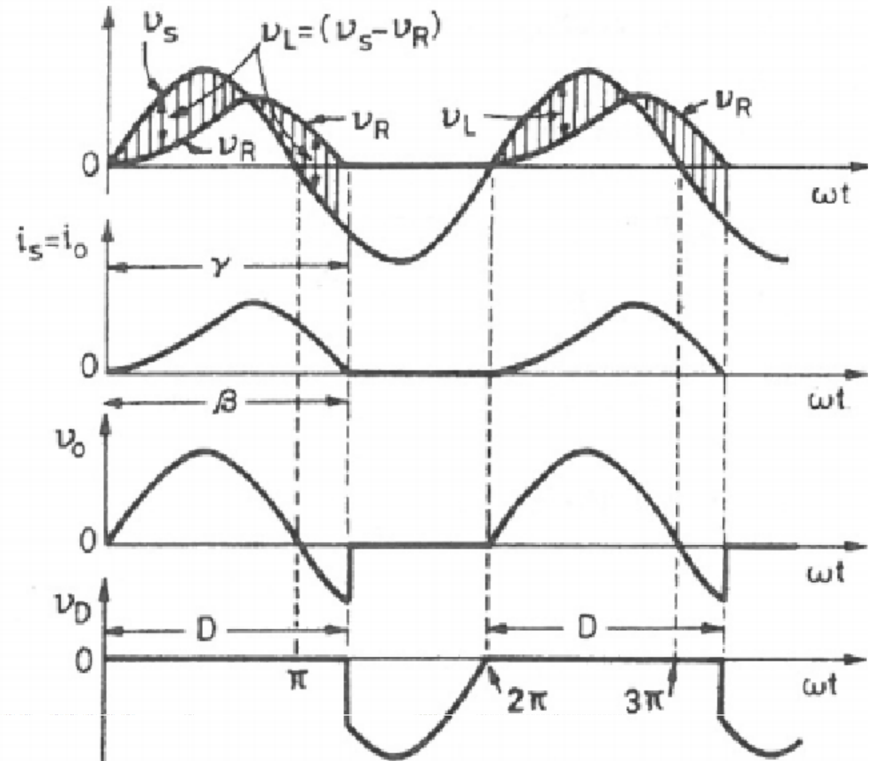
RL Load





Current i_0 continues to flow-even after source voltage v_s has become negative ; this is because of the presence of inductance L in the load circuit. Voltage $v_R = i_0 R$ has the same waveshape as that of i_0. Inductor voltage $v_L = v_s - v_R$ is also shown. The current i_0 flows till the two areas A and B are equal. Area A represents the energy stored by L and area B the energy released by L. It must be noted that average value of voltage v_L across inductor L is zero.

When $i_0 = 0$ at $\omega t = \beta$; $v_L = 0$, $v_R = 0$ and voltage v_s appears as reverse bias across diode D as shown. At β, voltage v_D across diode jumps from zero to $V_m \sin\beta$ where $\beta > \pi$. Here $\beta = \gamma$ is also the conduction angle of the diode.

Average value of output voltage,

$$V_0 = \frac{1}{2\pi}\int_0^{\beta} V_m \sin\omega t \cdot d(\omega t)$$

$$= \frac{V_m}{2\pi}(1 - \cos\beta)$$

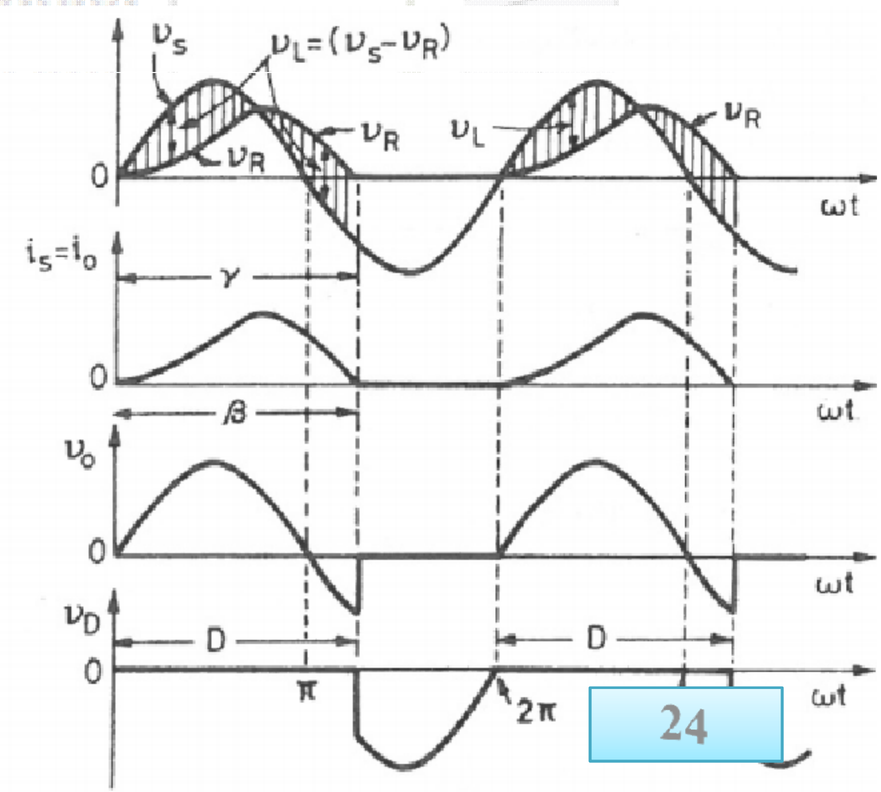
Average value of load or output current

$$I_0 = \frac{V_0}{R} = \frac{V_m}{2\pi R}(1 - \cos\beta)$$

A general expression for output current i_0 for $0 < \omega t < \beta$ can be obtained as under :

When diode is conducting, KVL for the circuit

$$Ri_0 + L\frac{di_0}{dt} = V_m \sin\omega t$$



The load, or output, current i_0 consists of two components, one steady state component i_s and the other transient component i_t. Here i_s is given by

$$i_s = \frac{V_m}{\sqrt{R_t^2\ X^2}} \sin(\omega t - \phi)$$

where $\phi = \tan^{-1}\frac{X}{R}$ and $X = \omega L$. Here ϕ is the angle by which rms current I_s lags V_s.

The transient component i_t can be obtained fiom force-free equation

$$Ri_t + L\frac{di_t}{dt} = 0$$

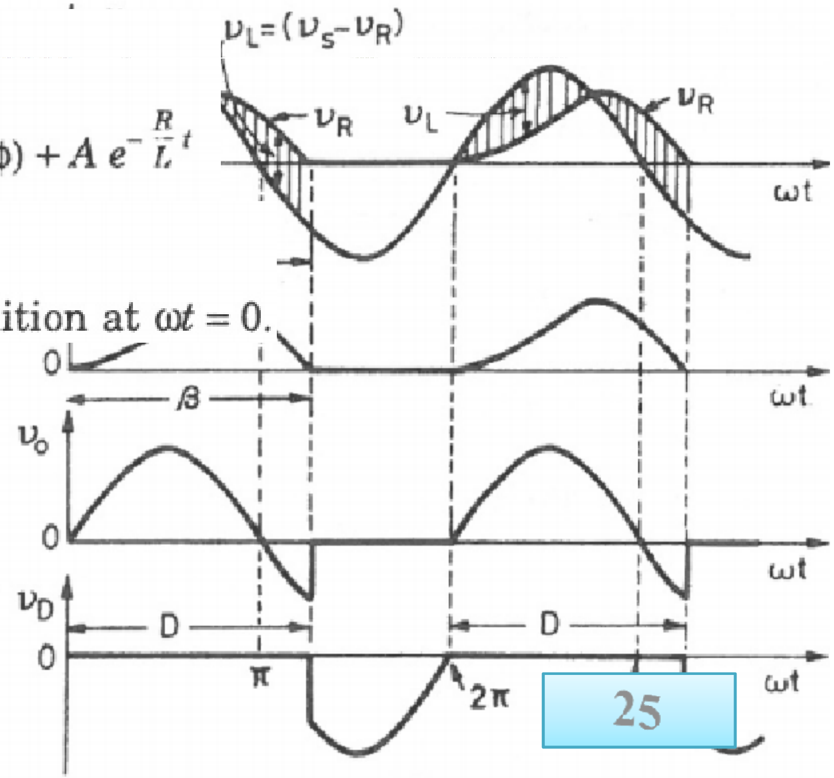
Its solution gives $\qquad i_t = A\, e^{-\frac{R}{L}t}$

Total solution for current i_0 is, therefore, given by

$$i_0 = i_s + i_t = \frac{V_m}{Z}\sin(\omega t - \phi) + A\, e^{-\frac{R}{L}t}$$

where $\qquad Z = \sqrt{R_t^2 + X^2}$

Constant A can be obtained from the boundary condition at $\omega t = 0$.



$$i_0 = \frac{V_m}{Z}\left[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\frac{R}{L}t}\right]$$

for $0 \le \omega t \le \beta$

Resultant Output V

$$i_0 = \frac{V_m}{Z}\left[\sin(\omega t - \phi) + \sin\phi \cdot e^{-\frac{R}{L}t}\right]$$

for $0 \le \omega t \le \beta$

when $\omega t = \beta$, $i_0 = 0$.

$$\sin(\beta - \phi) + \sin\phi \cdot \exp\left[-\frac{R}{\omega L}\beta\right] = 0$$

The solution of this transcendental equation can give the value of extinction angle β.