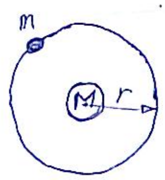
بسمه تعالی

تابستان 84 - دوره ی 18 - امتحان اول

1)



$$t = k \times r^{\alpha} \times M^{\beta} \times G^{\gamma}$$

$$G = \frac{m^3}{kg \cdot s^2} \longrightarrow [t] = m^{\alpha} \times kg^{\beta} \times m^{3\gamma} \times kg^{-\gamma} \times s^{-2\gamma} = s \longrightarrow \begin{cases} \beta = \gamma \\ \gamma = -\frac{1}{2} \\ \alpha = -3\gamma \end{cases} \longrightarrow \begin{cases} \beta = \gamma = -\frac{1}{2} \\ \alpha = \frac{3}{2} \end{cases} \longrightarrow$$

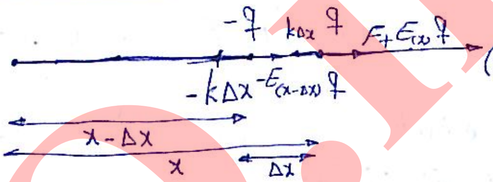
$$t = k \frac{r^{3/2}}{\sqrt{GM}} \longrightarrow T = c \frac{r^{3/2}}{\sqrt{GM}}, \quad t = c' \frac{r^{3/2}}{\sqrt{GM}} \longrightarrow \frac{T}{t} = c'' \longrightarrow$$

t: زمان سقوط

T: زمان تناوب

$$\frac{T_{me}}{t_{me}} = \frac{T_{se}}{t_{se}} \longrightarrow \frac{28}{21} = \frac{365}{t_{se}} \longrightarrow t_{se} = \frac{3 \times 365}{4} \longrightarrow \boxed{t_{se} = 273.75 \text{ day}}$$

2) الف)



$$\begin{cases} F_{(q)} = F + E_{(x)} q - k\Delta x \\ F_{(-q)} = k\Delta x - E_{(x-\Delta x)} q \end{cases}$$

ب)

$$\begin{cases} k\Delta x = F + E_{(x)} q \\ k\Delta x = E_{(x-\Delta x)} q \\ \Delta x = -\Delta a \end{cases} \longrightarrow F = q\left(E_{(x-\Delta x)} - E_{(x)}\right) \longrightarrow \frac{F}{\Delta x} = q\left(\frac{E_{(x)} - E_{(x-\Delta x)}}{-\Delta x}\right) = q\frac{dE}{da} = -qE' \longrightarrow$$

تعریف: $\Delta x = -\Delta a$

$$F = -qE'\Delta x \longrightarrow \Delta x (k + qE') = E_{(x)} q \longrightarrow \Delta x = \frac{qE_{(x)}}{k + qE'_{(x)}} \longrightarrow \boxed{\Delta x = \frac{qE_{(x)}}{k}\left(1 - \frac{qE'}{k}\right)}$$

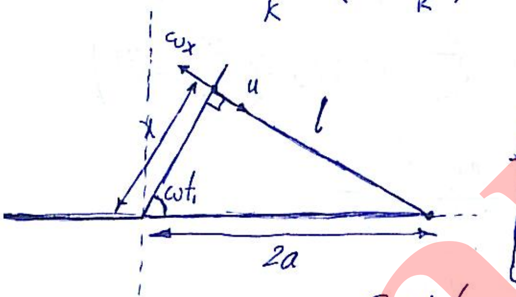
$\boxed{k \gg qE'_{(x)}}$

$qE'_{(x)}$ کوچک است $\longrightarrow \Delta x \to 0$

$$F = -qE'_{(x)} \cdot \frac{qE_{(x)}}{k}\left(1 - \frac{qE'}{k}\right) \longrightarrow \boxed{F = \frac{-q^2 E_{(x)} E'_{(x)}}{k}}$$

ج)

3) الف)



$$\begin{cases} l = 2a \sin(\omega t_1) \\ x = 2a \cos(\omega t_1) \\ T = \frac{\pi}{\omega} + t_1 + \frac{2a \sin(\omega t_1)}{u - 2a\omega \cos(\omega t_1)} \end{cases}, \quad \frac{dT}{dt_1} = 0 \longrightarrow$$

$$1 + 2a\omega \frac{\cos(\omega t_1)\left(u - 2a\omega \cos(\omega t_1)\right) - 2a\omega \sin^2(\omega t_1)}{\left(u - 2a\omega \cos(\omega t_1)\right)^2} = 0 \longrightarrow \left(u - 2a\omega \cos(\omega t_1)\right)^2 = 2a\omega\left(2a\omega - u\cos(\omega t_1)\right)$$

$$\frac{u}{2a\omega} = \alpha$$

$$\longrightarrow \alpha^2 + \cos^2(\omega t_1) - 2\alpha \cos(\omega t_1) = 1 - \alpha \cos(\omega t_1) \longrightarrow \cos^2(\omega t_1) - \alpha \cos(\omega t_1) + (\alpha^2 - 1) = 0 \longrightarrow$$

$$\cos(\omega t_1) = \frac{\alpha \pm \sqrt{\alpha^2 - 4\alpha^2 + 4}}{2} \longrightarrow \cos(\omega t_1) = \frac{\alpha \pm \sqrt{4 - 3\alpha^2}}{2}, \quad \Delta \geq 0 \longrightarrow 3\alpha^2 \leq 4 \longrightarrow \alpha \leq \frac{2}{\sqrt{3}}$$

$$\longrightarrow \cos(\omega t_1) = \frac{\alpha + \sqrt{4 - 3\alpha^2}}{2} \longrightarrow \sin(\omega t_1) = \frac{1}{2}\sqrt{4 - \alpha^2 - 4 + 3\alpha^2 - 2\alpha\sqrt{4 - 3\alpha^2}} \longrightarrow \sin(\omega t_1) = \frac{1}{2}\sqrt{2\alpha\left(\alpha - \sqrt{4 - 3\alpha^2}\right)}$$

$$u_{max} \longrightarrow \frac{\pi}{\omega} + t_1 + \frac{\sin(\omega t_1)}{\omega\left(\alpha - \cos(\omega t_1)\right)} = \frac{2\pi}{\omega} \longrightarrow \cos^{-1}\left(\frac{\alpha + \sqrt{4 - 3\alpha^2}}{2}\right) + \frac{\sqrt{2\alpha\left(\alpha - \sqrt{4 - 3\alpha^2}\right)}}{\alpha - \sqrt{4 - 3\alpha^2}} = \pi \longrightarrow$$

$$\boxed{\cos^{-1}\left(\frac{\alpha + \sqrt{4 - 3\alpha^2}}{2}\right) + \sqrt{\frac{2\alpha}{\alpha - \sqrt{4 - 3\alpha^2}}} = \pi} \longrightarrow \underbrace{\frac{\alpha + \sqrt{4 - 3\alpha^2}}{2}}_{\beta} = -\cos\left(\underbrace{\sqrt{\frac{2\alpha}{\alpha - \sqrt{4 - 3\alpha^2}}}}_{\alpha}\right) \longrightarrow \cos(\alpha) = -\beta, \quad \frac{\pi}{6} \leq \beta < \frac{\pi}{2}$$

$$, \frac{2\pi}{3} \leq \alpha \quad \frac{\pi}{2}$$

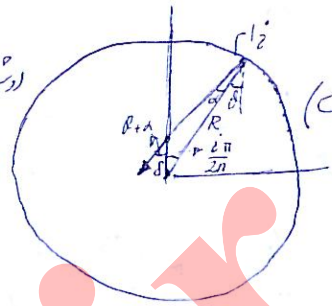
باسمه تعالی

ب)

$$u_{min} \rightarrow \frac{\pi}{\omega} + t_1 + \frac{\sin(\omega t_1)}{\omega(\alpha - \cos(\omega t_1))} = \frac{6\pi}{\omega} \longrightarrow$$

$$\boxed{\cos^{-1}\left(\frac{\alpha + \sqrt{4-5\alpha^2}}{2}\right) + \sqrt{\frac{2\alpha}{\alpha - \sqrt{4-3\alpha^2}}} = 5\pi}$$

4) الف) روش اول: طبق تقارن



$$\theta_i = i \times \frac{2\pi}{4n} = \frac{i\pi}{2n} = \theta \quad , \quad \vec{E}_{(x)} = 0$$

$$E_{y_i} = \frac{kq \cos(\theta + \alpha)}{R^2 + \delta^2 - 2\delta R \cos\theta}$$

$$\delta \cos\theta + \sqrt{R^2 + \delta^2 - 2\delta R\cos\theta}\,\cos\alpha = R \longrightarrow \cos\alpha = \frac{R - \delta\cos\theta}{R\left(1 + \frac{\delta^2}{R^2} - \frac{2\delta}{R}\cos\theta\right)^{1/2}} \rightarrow$$

$$\cos\alpha = \left(1 - \frac{\delta}{R}\cos\theta\right)\left(1 - \frac{\delta^2}{2R^2} + \frac{\delta}{R}\cos\theta + \frac{3}{2}\cdot\frac{\delta^2}{R^2}\cos^2\theta\right) = 1 - \frac{\delta^2}{2R^2} + \frac{\delta}{R}\cos\theta + \frac{3}{2}\frac{\delta^2}{R^2}\cos^2\theta - \frac{\delta}{R}\cos\theta - \frac{\delta^2}{R^2}\cos^2\theta \rightarrow \cos\alpha = 1 - \frac{\delta^2}{2R^2}\sin^2\theta$$

$$\sin\alpha = \frac{\delta \sin\theta}{\sqrt{R^2 + \delta^2 - 2\delta R\cos\theta}} \longrightarrow \sin\alpha = \frac{\delta}{R}\sin\theta\left(1 + \frac{\delta}{R}\cos\theta\right) \longrightarrow \sin\theta = \frac{\delta}{R}\sin\theta + \frac{\delta^2}{R^2}\sin\theta\cos\theta$$

$$\longrightarrow E_{y_i} = \frac{kq}{R^2}\left(1 - \frac{\delta^2}{R^2} + \frac{2\delta}{R}\cos\theta + 4\frac{\delta^2}{R^2}\cos^2\theta\right)\left(\cos\theta - \frac{\delta^2}{2R^2}\sin^2\theta\cos\theta - \frac{\delta}{R}\sin^2\theta - \frac{\delta^2}{R^2}\sin^2\theta\cos\theta\right) \longrightarrow$$

$$E_{y_i} = \frac{kq}{R^2}\left(\cos\theta - \frac{\delta^2}{2R^2}\sin^2\theta\cos\theta - \frac{\delta}{R}\sin^2\theta - \frac{\delta^2}{R^2}\sin^2\theta\cos\theta - \frac{\delta^2}{R^2}\cos\theta + \frac{2\delta}{R}\cos^2\theta - \frac{2\delta^2}{R^2}\sin^2\theta\cos\theta + \frac{4\delta^2}{R^2}\cos^3\theta\right) \longrightarrow$$

$$E_{y_i} = \frac{kq}{R^2}\left(\left(1 - \frac{\delta^2}{R^2}\right)\cos\theta - \frac{7}{2}\frac{\delta^2}{R^2}\,\frac{\cos\theta - \cos\theta\cos(2\theta)}{2} - \frac{\delta}{R}\left(\frac{1 - \cos(2\theta)}{2}\right) + \left(\frac{\delta}{R} + \frac{2\delta^2}{R^2}\cos\theta\right)(1 + \cos(2\theta))\right) \longrightarrow$$

$$E_{y_i} = \frac{kq}{R^2}\left(\left(1 - \frac{\delta^2}{R^2} - \frac{7}{4}\frac{\delta^2}{R^2}\right)\cos\theta + \frac{7}{4}\frac{\delta^2}{R^2}\left(\frac{\cos(3\theta) + \cos\theta}{2}\right) - \frac{\delta}{2R} + \frac{\delta}{R} + \frac{2\delta^2}{R^2}\cos\theta + \frac{3\delta}{2R}\cos(2\theta) + \frac{2\delta^2}{R^2}\left(\frac{\cos\theta + \cos(3\theta)}{2}\right)\right) \longrightarrow$$

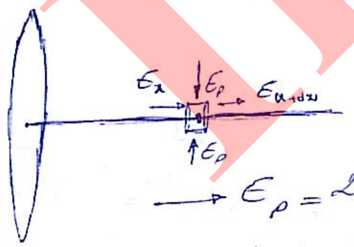
$$E_y = \frac{kq}{R^2}\left(\left(1 - \frac{\delta^2}{R^2} - \frac{7}{4}\frac{\delta^2}{R^2} + \frac{7}{4}\frac{\delta^2}{2R^2} + \frac{2\delta^2}{R^2} + \frac{\delta^2}{R^2}\right)\sum_{i=1}^{4n}\cos\left(\frac{i\pi}{2n}\right) + \frac{\delta}{2R}\cdot 4n + \left(\frac{7}{4}\frac{\delta^2}{R^2} + \frac{2\delta^2}{R^2}\right)\cdot\frac{1}{2}\sum_{i=1}^{4n}\cos\left(\frac{3i\pi}{2n}\right) + \frac{3\delta}{2R}\sum_{i=1}^{4n}\cos\left(\frac{i\pi}{n}\right)\right)$$

$$\sum_{i=1}^{4n}\cos\left(\frac{i\pi}{2n}\right) = \frac{\sin\left(\left(4n + \frac{1}{2}\right)\frac{\pi}{2n}\right)}{2\sin\left(\frac{\pi}{4n}\right)} - \frac{1}{2} = 0 \quad , \quad \sum_{i=1}^{n}\cos(i\theta) = \frac{\sin\left(\left(n + \frac{1}{2}\right)\theta\right)}{2\sin\left(\frac{\theta}{2}\right)} - \frac{1}{2}$$

$$\sum_{i=1}^{4n}\cos\left(\frac{i\pi}{n}\right) = \frac{\sin\left(\left(4n + \frac{1}{2}\right)\frac{\pi}{n}\right)}{2\sin\left(\frac{\pi}{2n}\right)} - \frac{1}{2} = 0 \quad , \quad \sum_{i=1}^{4n}\cos\left(\frac{3i\pi}{2n}\right) = \frac{\sin\left(\left(4n + \frac{1}{2}\right)\frac{3\pi}{2n}\right)}{2\sin\left(\frac{3\pi}{4n}\right)} - \frac{1}{2} = 0$$

$$\longrightarrow E_y = \frac{kq}{R^2}\cdot\frac{2\delta n}{R} \rightarrow \boxed{E_y = \frac{q\delta n}{2\pi\varepsilon_0 R^3}}$$

روش دوم:



$$E_x = \frac{kq}{R^2 + x^2}\cdot\frac{x}{\sqrt{R^2 + x^2}}\cdot 4n \longrightarrow E_x = 4kqn\cdot\frac{x}{(R^2 + x^2)^{3/2}}$$

$$(E_{(x+dx)} - E_x)\cdot\pi\rho^2 = 2\pi\rho\, dx\, E_\rho \rightarrow E_\rho = \frac{\rho}{2}\frac{dE_x}{dx} = \frac{\rho}{2}\cdot 4kqn\cdot\frac{(x^2 + R^2)^{3/2} - \frac{3}{2}\cdot 2x\cdot x\,(R^2 + x^2)^{1/2}}{(R^2 + x^2)^3}$$

$$\longrightarrow E_\rho = 2kq\rho n\cdot\frac{R^2 - 2x^2}{(R^2 + x^2)^{5/2}} \xrightarrow{x=0} E = \frac{2kq\delta n}{R^3} \longrightarrow \boxed{E = \frac{q\delta n}{2\pi\varepsilon_0 R^3}}$$

ب) روش اول:

$$E = \frac{\delta}{2\pi\varepsilon_0 R^3}(qn) \xrightarrow{\frac{4qn}{2\pi R} = \lambda} E = \frac{\delta}{2\pi\varepsilon_0 R^3}\cdot\frac{\pi\lambda R}{2} \longrightarrow \boxed{E = \left(\frac{\lambda}{4\varepsilon_0 R^2}\right)\delta}$$

روش دوم:

$$E_y = \frac{k\lambda}{R}\left(\left(1 + \frac{9}{8}\frac{\delta^2}{R^2}\right)\int_0^{2\pi}\cos\theta\, d\theta + \frac{\delta}{2R}\int_0^{2\pi}d\theta + \left(\frac{15\delta^2}{8R^2}\right)\int_0^{2\pi}\cos(3\theta)\, d\theta + \frac{3\delta}{2R}\int_0^{2\pi}\cos(2\theta)\, d\theta\right) \longrightarrow \boxed{E = \left(\frac{\lambda}{4\varepsilon_0 R^2}\right)\delta}$$

پ)

$$\boxed{E = \left(\frac{N\lambda}{4\varepsilon_0 R^2}\right)\delta}$$

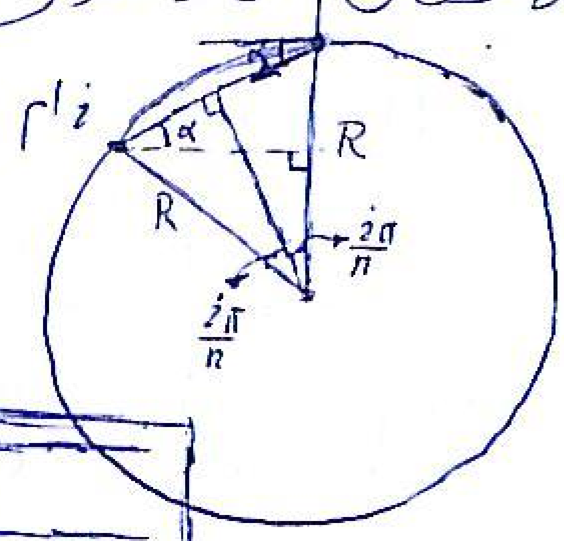
ت)

$$b = \frac{2N\lambda R\, d\theta}{\ldots} \longrightarrow \boxed{b = \frac{N\lambda}{\ldots}}$$

امیرصبحی

امتحان دوم تابستان 84 ـ دوره‌ی 18

با سمت تقارن



۱) الف)

$$\vec F_r = \frac{-Gm^2}{4R^2\sin^2\left(\frac{i\pi}{n}\right)}\cdot \sin\alpha\,\hat r \quad , \quad 0=\vec F_\theta \text{ چون تقارن}$$

$$\alpha + \left(\frac{\pi}{2}-\frac{2i\pi}{n}\right)+\frac{i\pi}{n}=\frac{\pi}{2} \longrightarrow \alpha = \frac{i\pi}{n} \longrightarrow \boxed{\vec F_r = \frac{-Gm^2}{4R^2}\sum_{i=1}^{n-1}\frac{1}{\sin\left(\frac{i\pi}{n}\right)}\,\hat r}$$

ب)

$$-\frac{Gm^2}{4R^2}\sum_{i=1}^{n-1}\frac{1}{\sin\left(\frac{i\pi}{n}\right)} = -R\omega^2 \longrightarrow \omega = \frac{m}{2R}\sqrt{\frac{G}{R}\sum_{i=1}^{n-1}\frac{1}{\sin\left(\frac{i\pi}{n}\right)}} \longrightarrow \boxed{T = \frac{4R\pi}{m}\sqrt{\frac{R}{G\sum_{i=1}^{n-1}\frac{1}{\sin\left(\frac{i\pi}{n}\right)}}}}$$

۲) الف)

$$\vec E_{(r)} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i}\frac{q_i}{|\vec r_i-\vec r|^3}(\vec r_i-\vec r)$$

$$\vec E'_{(r)} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i}\frac{a\,q_i}{|k\vec r_i-\vec r|^3}(k\vec r_i-\vec r) \longrightarrow \vec E'_{(r)} = \frac{a}{4\pi\varepsilon_0 k^2}\sum_{i=1}^{i}\frac{q_i}{\left|\vec r_i-\frac{\vec r}{k}\right|^3}\left(\vec r_i-\frac{\vec r}{k}\right) \longrightarrow \boxed{\vec E'_{(r)} = \frac{a}{k^2}\vec E_{\left(\frac{\vec r}{k}\right)}}$$

ب)

$$\begin{cases} \vec E_{(r)} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_{-\infty}^{+\infty}\frac{\lambda(x)\,dx}{|\vec x-\vec r|^3}(\vec x-\vec r) \longrightarrow \vec E_{\left(\frac{r}{k}\right)} = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\int_{-\infty}^{+\infty}\frac{\lambda(x)\,dx}{\left|\vec x-\frac{\vec r}{k}\right|^3}(\vec x-\vec r) \\ \vec E'_{(r)} = \dfrac{a}{4\pi\varepsilon_0}\displaystyle\int_{-\infty}^{+\infty}\frac{\lambda\left(\frac{x}{k}\right)dx}{|\vec x-\vec r|^3}(\vec x-\vec r) \longrightarrow \vec E'_{(r)} = \dfrac{a}{(4\pi\varepsilon_0)k^2}\displaystyle\int_{-\infty}^{+\infty}\frac{\lambda\left(\frac{x}{k}\right)dx}{\left|\frac{\vec x}{k}-\frac{\vec r}{k}\right|^3}\left(\frac{\vec x}{k}-\frac{\vec r}{k}\right) \end{cases}$$

حال می‌خواهیم المان به چگالیت $\lambda(x)$ در نقطه kx را در نظر بگیریم و چون بازه‌ی انتگرال از $-\infty$ تا $+\infty$ است تغییری در آن ایجاد نمی‌شود

$$\vec E'_{(r)} = \frac{a}{4\pi\varepsilon_0 k}\int_{-\infty}^{+\infty}\frac{\lambda(x)\,dx}{\left|\vec x-\frac{\vec r}{k}\right|^3}\left(\vec x-\frac{\vec r}{k}\right) \longrightarrow \boxed{\vec E' = \frac{a}{k}\vec E_{\left(\frac{\vec r}{k}\right)}}$$

۳) الف)

$$\boxed{F = \frac{Q^2\Delta S}{32\pi^2\varepsilon_0 R^4}}$$

ب)

$$U = -\int F\cdot dr = -\int\frac{4\pi\sigma^2}{2\varepsilon_0}r^2dr = -W \longrightarrow W = \int\frac{2\pi}{\varepsilon_0}\times\frac{Q^2}{16\pi^2r^4}\cdot r^2dr \longrightarrow W = \frac{Q^2}{8\pi\varepsilon_0}\left.-\frac{1}{r}\right|_{R_1}^{R_2} \longrightarrow$$

$$\boxed{W = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

ج)

$$\boxed{U = \frac{Q^2}{8\pi\varepsilon_0 R}}$$

د)

$$U_T = 2T\times 4\pi r^2 + \frac{Q^2}{8\pi\varepsilon_0 r} \xrightarrow{\frac{dU_T}{dr}=0} 16T\pi r_0 = \frac{Q^2}{8\pi\varepsilon_0 r_0^2} \longrightarrow \boxed{r_0 = \sqrt[3]{\frac{Q^2}{128\pi^2\varepsilon_0 T}}}$$

$$U_{T_{min}} = 8\pi T\cdot\frac{Q}{16\pi}\sqrt[3]{\frac{Q}{4\pi\varepsilon_0^2T^2}} + 16T\pi\cdot\frac{Q}{16\pi}\sqrt[3]{\frac{Q}{4\pi\varepsilon_0^2T^2}} \longrightarrow \boxed{U_{T_{min}} = \frac{3QT}{2}\sqrt[3]{\frac{Q}{4\pi\varepsilon_0^2T^2}}}$$

هـ)

$$U'_{T_{min}} = 2\times\frac{3QT^2}{4}\sqrt[3]{\frac{Q}{8\pi\varepsilon_0^2T^2}} \longrightarrow \boxed{U'_{T_{min}} = \frac{U_{T_{min}}}{\sqrt[3]{2}}}$$

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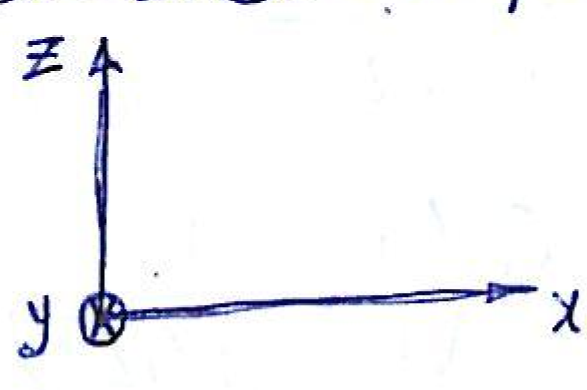
۱) الف)

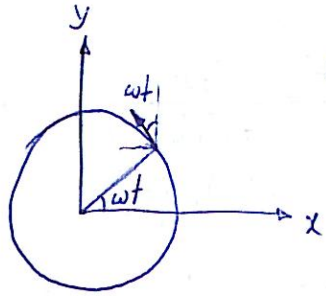


$$\begin{cases} V_y = \omega R \\ V_z = u \\ V_x = 0 \end{cases} \longrightarrow \begin{cases} y = \omega R t \\ z = ut \\ x = R \end{cases} \longrightarrow \begin{cases} z = \left(\frac{u}{\omega R}\right)y \\ x = R \end{cases}$$

بسمه تعالی

1) ب

$$\begin{cases} z = u(T-t) \longrightarrow T-t = \frac{z}{u} \longrightarrow t = T-\frac{z}{u} \\ y = R\sin(\omega t) + \omega R\cos(\omega t)(T-t) \\ x = R\cos(\omega t) - \omega R\sin(\omega t)(T-t) \end{cases} \longrightarrow x^2+y^2 = R^2+\omega^2R^2(T-t)^2 \longrightarrow \boxed{\frac{x^2+y^2}{R^2} = \frac{u^2+\omega^2z^2}{u^2}}$$



$$\boxed{\begin{cases} y = R\left(\sin\left(\left(T-\frac{z}{u}\right)\omega\right) + \frac{\omega}{u} z\cos\left(\omega\left(T-\frac{z}{u}\right)\right)\right) \\ x = R\left[\cos\left(\omega\left(T-\frac{z}{u}\right)\right) - \frac{\omega}{u} z\sin\left(\omega\left(T-\frac{z}{u}\right)\right)\right] \end{cases}} \longrightarrow \cancel{y = R\sqrt{1+\frac{\omega^2z^2}{u^2}}\sin\left(\omega T-\frac{\omega z}{u}\right)}$$

2) ب

$$\frac{y}{x} = \tan\varphi = \frac{\sin\left(\omega\left(T-\frac{z}{u}\right)\right) + \left(\frac{\omega z}{u}\right)\cos\left(\omega\left(T-\frac{z}{u}\right)\right)}{\cos\left(\omega\left(T-\frac{z}{u}\right)\right) - \left(\frac{\omega z}{u}\right)\sin\left(\omega\left(T-\frac{z}{u}\right)\right)} = \frac{\tan\left(\omega\left(T-\frac{z}{u}\right)\right)+\tan\alpha}{1-\tan\alpha\tan\left(\omega\left(T-\frac{z}{u}\right)\right)} = \tan\left(\alpha+\omega\left(T-\frac{z}{u}\right)\right)$$

$$\varphi = \alpha+\omega\left(T-\frac{z}{u}\right) \longrightarrow \boxed{\varphi = \tan^{-1}\left(\frac{\omega z}{u}\right) + \omega\left(T-\frac{z}{u}\right)} \xrightarrow{\frac{\omega z}{u}\to\infty} \boxed{\varphi = \left(\frac{\pi}{2}+\omega T\right) - \frac{\omega z}{u}}$$

3) ب

T ثابت: $\Delta\varphi = -\frac{\omega\Delta z}{u} \longrightarrow \boxed{d = \frac{2\pi u}{\omega}}$

2) الف ، ب)

$$\begin{cases} \varphi_1 = \frac{kq_b}{b} + \frac{kQ}{c} + \frac{kq_a}{r} \\ \varphi_2 = \frac{kq_b}{b} + \frac{k(Q+q_a)}{r} \end{cases} \longrightarrow \begin{cases} \varphi_{(a)} = 0 = \frac{q_b}{b} + \frac{Q}{c} + \frac{q_a}{a} \\ \varphi_{(b)} = 0 = q_b + Q + q_a \end{cases} \longrightarrow \begin{cases} q_b = -b\left(\frac{Q}{c}+\frac{q_a}{a}\right) \\ q_b = -(Q+q_a) \end{cases} \longrightarrow$$

$$\frac{Q+q_a}{b} = \frac{Q}{c} + \frac{q_a}{a} \longrightarrow q_a = \frac{\frac{1}{c}-\frac{1}{b}}{\frac{1}{b}-\frac{1}{a}}Q \longrightarrow \boxed{q_a = -\frac{a(b-c)}{c(b-a)}Q} \longrightarrow \boxed{q_b = -\frac{b(c-a)}{c(b-a)}Q}$$

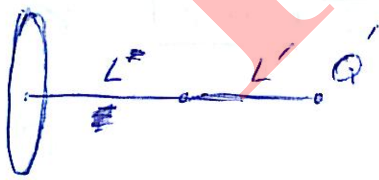
$$A_1 = k\left(\frac{Q}{c} - \frac{c-a}{(b-a)c}Q\right) \longrightarrow \boxed{A_1 = \frac{kQ(b-c)}{c(b-a)}}, \quad \boxed{B_1 = -\frac{ka(b-c)}{c(b-a)}Q}$$

$$\boxed{A_2 = -\frac{k(c-a)}{c(b-a)}Q} \qquad B_2 = k(Q+q_a) = -kq_b \longrightarrow \boxed{B_2 = \frac{kb(c-a)}{c(b-a)}Q}$$

ج و د)

$\varphi_{(a)} = \varphi_{(c)} = 0 \longrightarrow$ معادلات مستقل است $\longrightarrow$ (د) = (ج) = (ب) جواب

3) الف



$$d\varphi = \frac{k\,dq}{\sqrt{r^2+L^2}} = \frac{k\times 2\pi r\,dr\,\sigma}{\sqrt{r^2+L^2}} = k\pi\sigma\left(\frac{d(r^2+L^2)}{\sqrt{r^2+L^2}}\right) \longrightarrow$$

$$\varphi = \frac{kQ'}{L'} + \frac{2k\pi\,Q}{\pi R^2}\left(\sqrt{R^2+L^2}-L\right) \longrightarrow \boxed{\varphi = \frac{kQ'}{L'} + \frac{2kQ}{R^2}\left(\sqrt{R^2+L^2}-L\right)}$$

ب)

$$dE = \frac{k\times 2\pi r\,dr\,\sigma}{r^2+L^2}\cdot\frac{L}{\sqrt{r^2+L^2}} \longrightarrow \int dE = k\sigma\pi L\int_0^R \frac{d(r^2+L^2)}{(r^2+L^2)^{3/2}} = \frac{2k\pi LQ}{\pi R^2}\left(\frac{1}{L}-\frac{1}{\sqrt{R^2+L^2}}\right) \longrightarrow$$

$$E = \frac{2kLQ}{R^2}\left(\frac{1}{L}-\frac{1}{\sqrt{R^2+L^2}}\right) - \frac{kQ'}{L'^2} \xrightarrow{E=0} \boxed{L' = \sqrt{\frac{R^2Q'\sqrt{R^2+L^2}}{2Q\left(\sqrt{R^2+L^2}-L\right)}}}$$

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3) ج) روش اول:

$$V_{(z)} = \frac{2kQ}{R^2}\left(\sqrt{R^2+(z+L)^2} - (z+L)\right) + \frac{kQ'}{L'-z} \to$$

$$V'_{(z)} = \frac{2kQ}{R^2}\left(\frac{2(z+L)}{2\sqrt{R^2+(z+L)^2}} - 1\right) + \frac{kQ'}{(L'-z)^2} \to V''_{(z)} = \frac{2kQ}{R^2}\cdot\frac{\sqrt{R^2+(z+L)^2} - \frac{(z+L)^2}{\sqrt{R^2+(z+L)^2}}}{R^2+(z+L)^2} + \frac{2kQ'}{(L'-z)^3} \to$$

$$\boxed{V''_{(z)} = \frac{2kQ}{\left(R^2+(z+L)^2\right)^{3/2}} + \frac{2kQ'}{(L'-z)^3}} \to \boxed{V''_{(0)} = \frac{2kQ}{(R^2+L^2)^{3/2}} + \frac{2kQ'}{L'^3}}$$

روش دوم:

$$V''_{(z)} = -E'_{(z)} \to V''_{(z)} = \left(\frac{2kQ}{R^2}\left(\frac{z+L}{\sqrt{R^2+(z+L)^2}} - 1\right) + \frac{kQ'}{(L'-z)^2}\right)' \to \boxed{V''_{(0)} = \frac{2kQ}{(R^2+L^2)^{3/2}} + \frac{2kQ'}{L'^3}}$$

د)

$$V = a_0 + a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2 + a_7xy + a_8yz + a_9xz$$

$x = y = 0 = z \to V = V_0 \to \boxed{a_0 = V_0}$ $\quad V_{(x)} = V_{(-x)},\ V_{(y)} = V_{(-y)} \to \boxed{a_1 = a_2 = a_7 = a_8 = a_9 = 0}$

تقارن x,y $\to \boxed{a_4 = a_5}$, $\nabla^2 V = 0 \to a_4 + a_5 + a_6 = 0 \to \boxed{a_4 = a_5 = -\frac{a_6}{2}} \to$

$$V = a_0 + a_3z + a_6\left(-\frac{x^2+y^2}{2} + z^2\right) \to \vec{E} = -\vec{\nabla}V = -\left(-a_6(x\hat{i}+y\hat{j}) + (a_3 + 2a_6z)\hat{k}\right) \xrightarrow{\vec{E}_{(0,0,0)}=0} \boxed{a_3 = 0} \to$$

$$V = V_0 + a_6\left(z^2 - \frac{x^2+y^2}{2}\right) \to V''_{(0,0,0)} = 2a_6 \to \boxed{a_6 = k\left(\frac{Q}{(R^2+L^2)^{3/2}} + \frac{Q'}{L'^3}\right)} \to$$

$$\boxed{V = k\left(\frac{Q'}{L'} + \frac{2Q}{R^2}\left(\sqrt{R^2+L^2} - L\right) + \left(\frac{Q}{(R^2+L^2)^{3/2}} + \frac{Q'}{L'^3}\right)\left(z^2 - \frac{x^2+y^2}{2}\right)\right)}$$ یا $$\boxed{V = V_0 + a_6\left(z^2 - \frac{x^2+y^2}{2}\right)}$$

~~ه) بار دره و ترس و بار نقطه‌ای [illegible] هم علامت و مثبت اند. پس اگر ذره از محور z خارج شود [illegible] محور z [illegible] وارد می کند و آن را [illegible]~~

امتحان دوم - تابستان 84 - دوره ی 18

2) ب)

$$\vec{E}_{(r)} = \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{\lambda_{(x)}\,dx\,(\vec{x}-\vec{r})}{|\vec{x}-\vec{r}|^3}$$

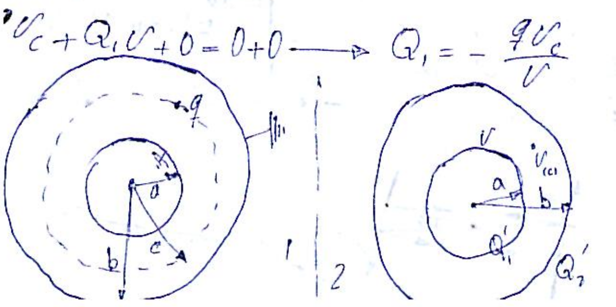
$$\vec{E}'_{(r)} = \frac{a}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{\lambda(\frac{x}{k})\,dx\,(\vec{x}-\vec{r})}{|\vec{x}-\vec{r}|^3} = \frac{a}{4\pi\varepsilon_0 k}\int_{-\infty}^{+\infty}\frac{\lambda(\frac{x}{k})\,d(\frac{x}{k})\,(\frac{\vec{x}}{k}-\frac{\vec{r}}{k})}{|\frac{\vec{x}}{k}-\frac{\vec{r}}{k}|^3} = \frac{a}{4\pi\varepsilon_0 k}\int_{-\infty}^{+\infty}\frac{\lambda_{(u)}\,du\,(\vec{u}-\frac{\vec{r}}{k})}{|\vec{u}-\frac{\vec{r}}{k}|^3}, \quad u = \frac{x}{k} \to$$

$$\boxed{\vec{E}'_{(r)} = \frac{a}{k}\vec{E}_{(\frac{\vec{r}}{k})}}$$

امتحان سوم -

2) ج) روش دوم: گرین

$$qV_c + Q_1V + 0 = 0 + 0 \to Q_1 = -\frac{qV_c}{V}$$



$$\varphi = A + \frac{B}{r} \to \begin{cases} A + \frac{B}{a} = V \\ A + \frac{B}{b} = 0 \end{cases} \to A = \frac{-aV}{b-a}, \quad B = \frac{abV}{b-a} \to$$

$$V_c = \frac{-aV}{b-a} + \frac{aVb}{c(b-a)} = \frac{aV}{b-a}\left(\frac{b}{c} - 1\right) \to \boxed{Q_1 = -\frac{qa(b-c)}{c(b-a)}}$$

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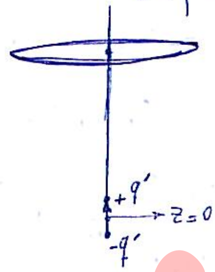
3) ه) $V = V_0 + a_8\left(z^2 - \frac{r^2}{2}\right) \longrightarrow V = V_0 + a_8 z^2\left(1 - \frac{\tan^2\theta}{2}\right) \longrightarrow \frac{\partial^2 V}{\partial z^2} = a_8(2 - \tan^2\theta) \longrightarrow 2 - \tan^2\theta > 0 \longrightarrow \boxed{-\sqrt{2} < \tan\theta < \sqrt{2}}$

2) الف)



$$\vec{F} = kqq'\left(\frac{-(z+\delta/2)}{\left(R^2+(z+\frac{\delta}{2})^2\right)^{3/2}} + \frac{z-\frac{\delta}{2}}{\left(R^2+(z-\frac{\delta}{2})^2\right)^{3/2}}\right)\hat{z} + kqq'\left(\frac{-1}{\left(R^2+(z+\frac{\delta}{2})^2\right)^{3/2}} + \frac{1}{\left(R^2+(z-\frac{\delta}{2})^2\right)^{3/2}}\right)R\hat{r}$$

$$\frac{1}{(R^2+\delta z+z^2)^{3/2}} = \frac{1}{(R^2+z^2)^{3/2}}\left(1 - \frac{3\delta z}{2(R^2+z^2)}\right) \longrightarrow$$

$$F_z = \frac{kqq'}{(R^2+z^2)^{3/2}}\left(z - \frac{\delta}{2} + \frac{3\delta z^2}{2(R^2+z^2)} - z - \frac{\delta}{2} + \frac{3\delta z^2}{2(R^2+z^2)}\right) \longrightarrow \boxed{F_z = \frac{kq\vec{P}(z^2-2R^2)}{(R^2+z^2)^{5/2}}}$$

$$F_r = \frac{kqq'R}{(R^2+z^2)^{3/2}}\left(1 + \frac{3\delta z}{2(R^2+z^2)} - 1 + \frac{3\delta z}{2(R^2+z^2)}\right) \longrightarrow \boxed{F_r = \frac{3kqRzP}{(R^2+z^2)^{5/2}}}$$

ب) $2T\sin\left(\frac{d\theta}{2}\right) = F \rightarrow 2\pi T = F \rightarrow 2\pi Y\frac{\Delta a}{a} = \frac{3kqazP}{(a^2+z^2)^{5/2}} \longrightarrow \boxed{\Delta a_{(1)} = \frac{3kqa^2zP}{2\pi Y(a^2+z^2)^{5/2}}}$

ج) $\Delta a^{(2)} = \frac{3kqPz}{2\pi Y}\left(\frac{(a+\Delta a_{(1)}, a)}{(a^2+z^2)^{5/2}\left(1+\frac{2\Delta a_{(1)}a}{a^2+z^2}\right)^{5/2}}\right) = \frac{3kqPza}{2\pi Y(a^2+z^2)^{5/2}}\left(a + 2\Delta a_{(1)} - \frac{5a^2\Delta a_{(1)}}{a^2+z^2}\right) \longrightarrow$

$$\Delta a_{(2)} = \frac{3kqPza}{2\pi Y(a^2+z^2)^{5/2}} \times \frac{(2z^2-3a^2)}{a^2+z^2} \times \frac{3kqazP}{2\pi Y(a^2+z^2)^{5/2}}\,a \longrightarrow \Delta a_{(2)} = \left(\frac{3kqazP}{2\pi Y}\right)^2\left(\frac{2z^2-3a^2}{(a^2+z^2)^6}\right)a \rightarrow \boxed{\Delta a_{(2)} = \left(\frac{3kqPaz}{2\pi Y(a^2+z^2)^3}\right)^2(2z^2-3a^2)a}$$

3) الف) $F = -\frac{dU}{dr} = +\frac{d(\vec{P}\cdot d\vec{E})}{dr} = \lambda\frac{d(E^2)}{dr} \rightarrow F = \lambda\left(\frac{V}{\ln(\frac{b}{a})}\right)^2 \times \frac{-2}{r^3} \rightarrow \boxed{F = -\left(\frac{2\lambda V^2}{\ln^2(\frac{b}{a})}\right)\frac{1}{r^3}}$

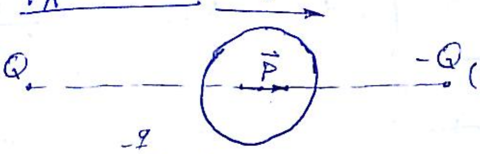
$$E \times 2\pi r L = \frac{Q}{\varepsilon_0} \rightarrow E = \left(\frac{Q}{2\pi L\varepsilon_0}\right)\frac{1}{r} \rightarrow V = \frac{Q}{2\pi L\varepsilon_0}\ln\left(\frac{b}{a}\right) \rightarrow E = \left(\frac{V}{\ln(\frac{b}{a})}\right)\frac{1}{r}$$

ب) $U - U_0 + (k - k_0) = 0 \longrightarrow \lambda(E^2 - E_0^2) = \frac{1}{2}m(\dot{r}^2 - \dot{r}_0^2) \longrightarrow \dot{r}^2 = \frac{2\lambda}{m}\left(\frac{V}{\ln(\frac{b}{a})}\right)^2\left(\frac{1}{r^2} - \frac{1}{b^2}\right) \rightarrow \int_b^a \frac{r\,dr}{\sqrt{b^2-r^2}} = -\left(\frac{V}{b\ln(\frac{b}{a})}\right)\sqrt{\frac{2\lambda}{m}}\int_0^t dt$

$$\longrightarrow -\frac{1}{2}\times 2\times\sqrt{b^2-a^2} = -\left(\frac{V}{b\ln(\frac{b}{a})}\right)\sqrt{\frac{2\lambda}{m}}\frac{L}{V_0} \longrightarrow \boxed{L_{min} = \frac{bV_0\ln(\frac{b}{a})\sqrt{b^2-a^2}}{V}\sqrt{\frac{2\lambda}{m}}} \rightarrow \sqrt{\frac{m}{\lambda}},\ 2\ \text{اضافی}$$

$$P = 2\pi\varepsilon_0 R \times E_0 \rightarrow \boxed{\lambda = 2\pi\varepsilon_0 R}$$

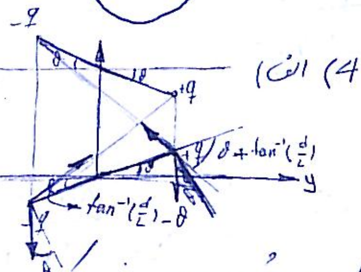
ج)



$$\sum\tau = \frac{q^2 L/2}{4\pi\varepsilon_0}\left(\frac{\sin(\theta+\tan^{-1}(\frac{d}{L}))}{L^2+d^2} - \frac{\cos\theta}{(d-L\theta)^2} + \frac{\cos\theta}{(d+L\theta)^2} - \frac{\sin(\tan^{-1}(\frac{d}{L})-\theta)}{L^2+d^2}\right) \longrightarrow$$

4) الف)

$$\sum\tau = \frac{q^2L/2}{4\pi\varepsilon_0}\left(\frac{\theta d + L\theta - d + L\theta}{(L^2+d^2)^{3/2}} + \frac{-4L\theta d}{d^4}\right) = \frac{2L^2\theta q^2}{2\times 4\pi\varepsilon_0}\left(\frac{1}{(L^2+d^2)^{3/2}} - \frac{2}{d^3}\right) \rightarrow \sum\tau < 0 \rightarrow$$

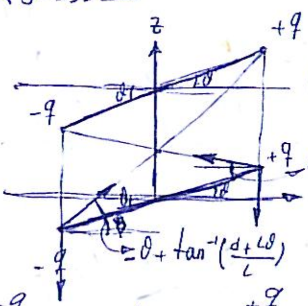


تعادل پایدار $\longrightarrow 2kq^2L^2\theta\left(\frac{2}{d^3} - \frac{1}{(d^2+L^2)^{3/2}}\right) + \frac{mL^2}{2}\ddot{\theta} = 0 \longrightarrow \boxed{\omega = \sqrt{\frac{4kq^2}{m}\left(\frac{2}{d^3} - \frac{1}{(d^2+L^2)^{3/2}}\right)}}$

~~جواب تشریحی: دو نیروی که بارهای هم نام به هم وارد می‌کنند با هم و دو نیرویی که بارهای غیر همنام به هم وارد می‌کنند با هم تعادل ...~~

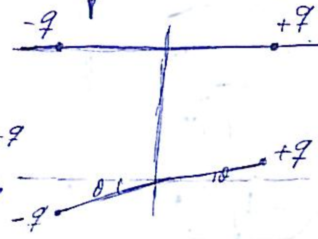
~~$\frac{kq^2}{d-L\theta} > \frac{kq^2}{d+L\theta}$ گشتاور دادستانی~~ ~~$\frac{kq^2}{L^2+d^2}$ گشتاور دادستانی > گشتاور منفی~~ ~~چون θ در راستای ... است~~

ب) $\sum\tau = kq^2\frac{L}{2}\left(\frac{\sin(\theta+\tan^{-1}(\frac{d-L\theta}{L}))}{L^2+(d-L\theta)^2} - \frac{\sin(-\theta+\tan^{-1}(\frac{d+L\theta}{L}))}{L^2+(d+L\theta)^2}\right) = kq^2\frac{L}{2}\left(\frac{\theta L + d - L\theta}{(L^2+(d-L\theta)^2)^{3/2}} - \frac{-L\theta+d+L\theta}{(L^2+(d+L\theta)^2)^{3/2}}\right)$



$$= \frac{kq^2L}{2(L^2+d^2)^{3/2}}\left(d + \frac{3Ld^2\theta}{L^2+d^2} - d + \frac{3Ld^2\theta}{L^2+d^2}\right) = \frac{2kq^2L^2\theta}{2(L^2+d^2)^{3/2}}\left(\frac{3d^2}{L^2+d^2}\right) \longrightarrow \sum\tau > 0 \longrightarrow \boxed{\text{تعادل ناپایدار}}$$

ج)



همان طور که در شکل زیر معلوم است گشتاورها θ را افزایش می‌دهند و همچنین میله‌ی بالایی در جهت $\hat{\theta}$ می‌چرخانند و با افزایش θ نیز گشتاورها بیشتر می‌شوند. پس تعادل هر دو میله اندازه‌ی $\boxed{\text{ناپایدار}}$ است

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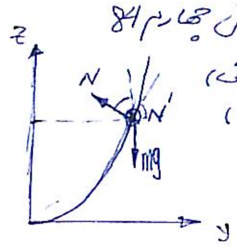
5) الف)

ب)



$$\begin{cases} N\cos\alpha - mg = m\ddot{z} \\ -N\sin\alpha = m(\ddot{r}-\omega^2 r) \\ N' = m(2\omega\dot{r}) \\ \tan\alpha = \frac{dz}{dr} = 2k\alpha r^{2k-1} \end{cases} \longrightarrow \tan\alpha = \frac{\omega^2 r - \ddot{r}}{g+\ddot{z}} = \frac{dz}{dr} \longrightarrow \omega^2 r\,dr - \ddot{r}\,dr = g\,dz + \ddot{z}\,dz \longrightarrow$$

$$\omega^2 r\,dr - \dot{r}\,d\dot{r} = g\,dz + \dot{z}\,d\dot{z} \longrightarrow \frac{\dot{r}^2}{2} + \frac{\dot{z}^2}{2} + gz - \frac{\omega^2 r^2}{2} = E' \longrightarrow$$

$$\frac{\dot{r}^2}{2}\left(1+4k^2\alpha^2 r^{4k-2}\right) + \alpha g r^{2k} - \frac{\omega^2 r^2}{2} = E' \longrightarrow$$

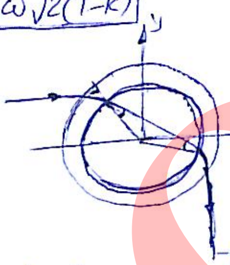
$$\boxed{f_{(r)} = \frac{1+4k^2\alpha^2 r^{4k-2}}{2}} \quad , \quad \boxed{K_{(r)} = \alpha g r^{2k} - \frac{\omega^2 r^2}{2}}$$

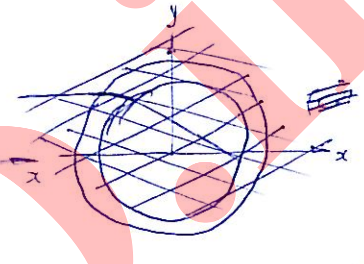
ج)

$$K_{(r)} = \frac{U_{eff}}{m} \longrightarrow \frac{\partial U_{eff}}{m\,\partial r} = 2k\alpha g r^{2k-1} - \omega^2 r \longrightarrow \frac{\partial^2 U_{eff}}{m\,\partial r^2} = 2k(2k-1)\alpha g r^{2k-2} - \omega^2 \Big\} \longrightarrow \boxed{r_0^{2k-2} = \frac{\omega^2}{2k\alpha g}} \longrightarrow \boxed{r_0 = \left(\frac{\omega^2}{2k\alpha g}\right)^{\frac{1}{2k-2}}}$$

$$\frac{\partial^2 U_{eff}}{m\,\partial r^2} = 2\omega^2(k-1) \longrightarrow \begin{cases} k>1 \longrightarrow \text{تعادل پایدار} \longrightarrow \boxed{\Omega = \omega\sqrt{2(k-1)}} \\ k<1 \longrightarrow \text{تعادل ناپایدار} \longrightarrow \boxed{\tau = \frac{1}{\omega\sqrt{2(1-k)}}} \end{cases}$$

6) الف)





$$(R+\delta R)u_0\sin\theta = R\,v_\theta \longrightarrow \boxed{v_\theta = \left(1+\frac{\delta R}{R}\right)u_0\sin\theta}$$

$$E = \frac{Q}{2\pi r\ell\varepsilon_0} \longrightarrow V_0 = \frac{Q}{2\pi\varepsilon_0\ell}\ln\left(\frac{R+\delta R}{R}\right) \longrightarrow \vec{E} = \left(\frac{V_0}{\ln(1+\frac{\delta R}{R})}\right)\frac{1}{r}\hat{r} \longrightarrow$$

$$\vec{F}_{(r)} = -\left(\frac{eV_0}{\ln(1+\frac{\delta R}{R})}\right)\frac{\hat{r}}{r} \longrightarrow \Delta U = -eV_0\frac{\ln\left(\frac{R+r}{R}\right)}{\ln\left(1+\frac{\delta R}{R}\right)}$$

$$\cancel{\frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}} \quad \frac{1}{2}m(v_\theta^2+v_r^2) + U = \frac{1}{2}mu_0^2 + U_0 \longrightarrow \left(1+\frac{\delta R}{R}\right)^2 u_0^2\sin^2\theta + v_r^2 = u_0^2 + \frac{2eV_0}{m} \longrightarrow \boxed{v_r = \mp\sqrt{u_0^2\left(1-\left(1+\frac{\delta R}{R}\right)^2\sin^2\theta_0\right) + \frac{2eV_0}{m}}}$$

$$\longrightarrow \boxed{\vec{u}_1^{(1)} = \mp\sqrt{u_0^2\cos^2\theta_0 + \frac{2eV_0}{m}\left(1-u_0^2\sin^2\theta_0\frac{\delta R}{R}\right)}\,\hat{r} + \left(1+\frac{\delta R}{R}\right)u_0\sin\theta\,\hat{\theta}} \longrightarrow \boxed{\vec{u}_1^{(0)} = -\sqrt{u_0^2\cos^2\theta + \frac{2eV_0}{m}}\,\hat{r} + u_0\sin\theta\,\hat{\theta}}$$

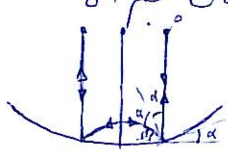
ب)

$$u_{1\theta} = 0 \longrightarrow \sin\theta = 0 \longrightarrow \boxed{\theta = 0}$$

پ) θ ورود بدلیل تشکیل مثلث متساوی الساقین با θ خروج برابر است. بدلیل عدم وجود میدان الکتریکی داخل استوانه سرعت ورود با سرعت خروج برابر است. پس ~~...~~ خروج بار از استوانه مانند ورود آن است؛ با این تفاوت که برعکس مسیر آن است. پس $\boxed{u_2 = u_0}$

امتحان پنجم 84

1)



$$\frac{2V_0^2\sin\theta\cos\theta}{g} = 2x \longrightarrow \cancel{2y_0 x\sin(4\alpha) = 2x} \longrightarrow \cancel{2}(y_0-y)\sin(4\alpha) = \cancel{2x} \longrightarrow \left(y_0 y - \frac{y^2}{2}\right)4 = \frac{x^2}{2} \longrightarrow \cancel{x^2+4(y-y_0)^2}$$

$$x^2 = -4(y^2-2y_0 y) = -4(y-y_0)^2 + 4y_0^2 \longrightarrow (y-y_0)^2 + \frac{x^2}{4} = y_0^2 \longrightarrow \boxed{y'^2 + \left(\frac{x}{2}\right)^2 = y_0^2}$$ مبدأ در y_0 باشد معادله بیضی.

2) الف)

$$\frac{1}{2}m\left(R^2-\frac{\ell^2}{4}\right)\dot{\theta}^2 + \frac{1}{2}\times\frac{1}{12}m\ell^2\times\dot{\theta}^2 - mg\sqrt{R^2-\frac{\ell^2}{4}}\cos\theta = -mg\sqrt{R^2-\frac{\ell^2}{4}}\cos\theta_0 \longrightarrow$$

$$\frac{\dot{\theta}^2}{2}\left(R^2-\frac{\ell^2}{6}\right) = \frac{g\sqrt{R^2-\ell^2/4}}{2}(\theta_0^2-\theta^2) \longrightarrow \boxed{\dot{\theta} = \mp\sqrt{\frac{g\sqrt{R^2-\ell^2/4}}{R^2-\ell^2/6}}\sqrt{\theta_0^2-\theta^2}} \quad \text{یا} \quad \boxed{\dot{\theta} = \mp\sqrt{\frac{2g\sqrt{R^2-\ell^2/4}}{R^2-\ell^2/6}}\sqrt{\cos\theta-\cos\theta_0}}$$

ب)

$$\vec{v} = +\sqrt{R^2-\frac{\ell^2}{4}}\,\dot{\theta}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right) \longrightarrow$$

$$\boxed{\vec{v} = \left(R^2-\frac{\ell^2}{4}\right)\sqrt{\frac{g}{R^2-\ell^2/6}}\sqrt{\theta_0^2-\theta^2}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)}$$

$$\boxed{\vec{v} = \left(R^2-\frac{\ell^2}{4}\right)\sqrt{\frac{2g}{R^2-\ell^2/6}}\sqrt{\cos\theta-\cos\theta_0}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)}$$

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$$\ddot{\theta}\left(R^2-\frac{\ell^2}{6}\right) + g\sqrt{R^2-\frac{\ell^2}{4}}\,\theta = 0 \longrightarrow \boxed{\omega = \sqrt{\frac{g\sqrt{R^2-\ell^2/4}}{R^2-\ell^2/6}}}$$