به نام یگانه مغناطیس جهان

And God Said:

$\nabla.E = \frac{\rho}{\epsilon_0}$	(1)	Gauss' law
$\nabla.B = 0$	(2)	Magnetic monopoles
$\nabla \times E = -\frac{\partial B}{\partial t}$	(3)	Faraday's law
$\nabla \times H = J + \frac{\partial D}{\partial t}$	(4)	Ampere-Maxwell law

and there was LIGHT !

وقف عام

نمونه سوالات مهم المپیاد فیزیک مباحث:
الکتریسیته و الکترومغناطیس؛ مناسب مرحله دوم
کشوری، دوره تابستانه (مرحله نهایی المپیاد
کشوری)، دوره انتخابی تیم، مرحله جهانی

((گلچین سوالات سطح نیمه دشوار
و دشوار مرحله های ایالتی و کشوری ژاپن))

Problem 1

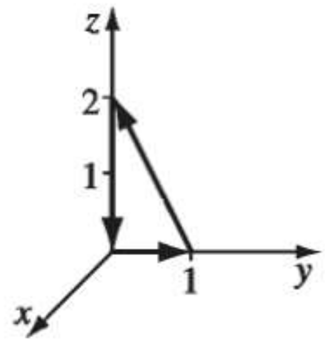
Find the transformation matrix that converts the components of a vector in spherical polar coordinates into its components in circular cylindrical coordinates. Then find the matrix of the inverse transformation.

Problem 2

a) Compute the line integral of

$$\mathbf{v} = \mathrm{r}\cos^2(\theta)\hat{\mathbf{r}} - \mathrm{r}\cos(\theta)\sin(\theta)\,\hat{\boldsymbol{\theta}} + 3\mathrm{r}\hat{\boldsymbol{\varphi}}$$

for the path shown in the following figure:



b) Compute the integral $\int_S (\nabla \times \vec{\mathbf{v}})\, ds$, then test the **Stokes' Theorem** for this function.

Problem 3

Let $\mathbf{A}(\mathbf{r}) = \mathbf{c}\exp(\mathrm{i}\mathbf{k}\,.\,\mathbf{r})$ where $\mathbf{c}$ is constant. Show that, in every case, the replacement $\nabla \to i\mathbf{k}$ produces the correct answer for $\nabla \cdot \mathbf{A}$, $\nabla \times \mathbf{A}$, $\nabla \times (\nabla \times \mathbf{A})$, $\nabla(\nabla \cdot \mathbf{A})$, and $\nabla^2\mathbf{A}$. (**Note:** For $\nabla \times (\nabla \times \mathbf{A})$, $\nabla(\nabla \cdot \mathbf{A})$, and $\nabla^2\mathbf{A}$, use vector identities)

Problem 4

Show that:

a) $\int_V (\nabla \mathrm{T})\, dv = \oint_{\partial V} \mathrm{T} d\vec{s}$

b) $\int_V (\nabla \times \vec{\mathrm{v}})\, dv = -\oint_{\partial V} \vec{\mathrm{v}} \times d\vec{s}$

c) $\int_V [\mathrm{T}\nabla^2\mathrm{U} + (\nabla T).(\nabla \mathrm{U})]\, dv = \oint_{\partial V} (\mathrm{T}\nabla\mathrm{U}) d\vec{s}$

d) $\int_S \nabla\mathrm{T} \times d\vec{s} = -\oint_{\partial S} \mathrm{T} d\vec{l}$

e) $\oint_{\partial S} (\vec{a}\,.\vec{r}) d\vec{l} = -\vec{a} \times \int_S d\vec{s}$ (Where $\vec{a}$ is a constant vector and $\vec{r}$ the position vector)

Problem 5

a) Compute the Laplacian of

$$f_a(r) = -\frac{1}{4\pi}\frac{1}{\sqrt{r^2 + a^2}}$$

b) Using part (a) show that

$$-\nabla^2 \frac{1}{4\pi r} = \delta(\vec{\boldsymbol{r}})$$

Problem 1

A two-dimensional disk of radius R carries a uniform charge per unit area $\sigma > 0$.

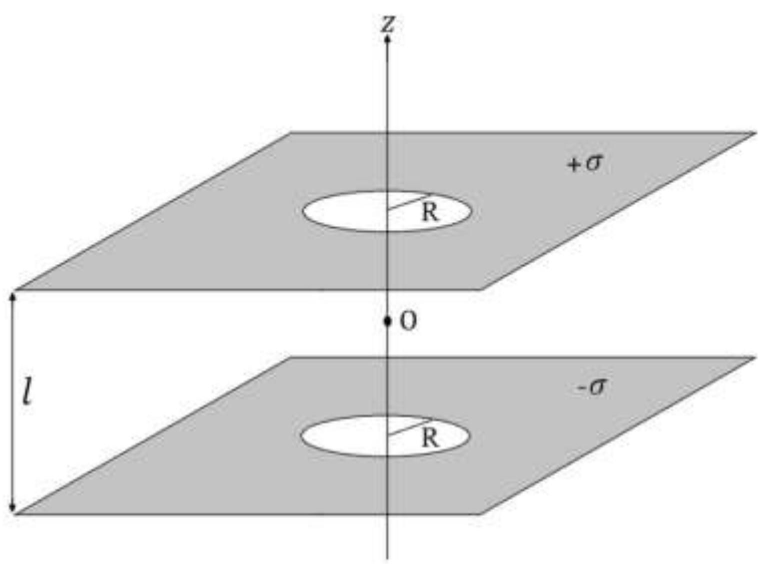
a) Calculate the potential at any point on the rim of the disk. (**This part is independent of others**)

b) Calculate the potential and electrical field at any point on the symmetry axis of the disk.

c) The figure below shows two infinite surfaces are held a distance l apart, that carry uniform surface charge densities $+\sigma$ (top) and $-\sigma$ (bottom). A disk of radius R is hollowed out from each of two infinite surfaces. Use part **(b)** and calculate the electrical field at the origin O (The distance between O and each surface is $/2$).

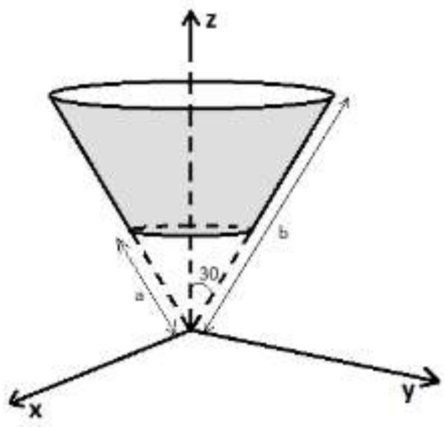
d) Sketch the electric field pattern everywhere in x-z plane.

e) **(optional-bonus)** Find the radial component of electric field near the z-axis in $z \gg l$. (**Hint:** Draw a small cylinder through z-axis in $z \gg l$ and use **Gauss's Law** to find a relation between $E_z(z)$ and $E_r(z, r)$.)



Problem 2

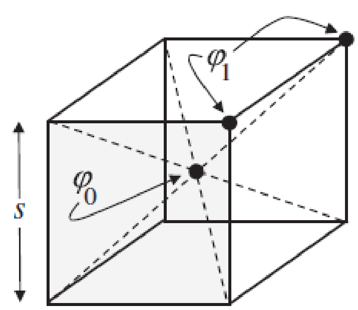
A truncated cone, as shown in the figure, has a nonuniform surface charge density $\sigma = 2z$. Find the potential at the origin.



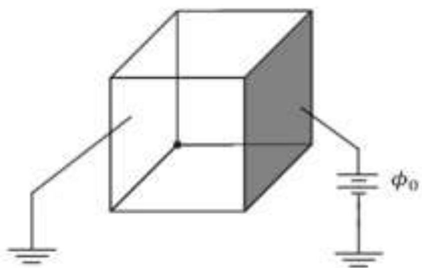
$$\begin{cases} a < r < b \\ 0 < \varphi < 2\pi \\ \theta = 30° \end{cases}$$

Problem 3

a) The figure below shows a cube filled uniformly with charge. Determine the ratio φ_0/φ_1 of the potential at the center of the cube to the potential at the corner of the cube.



b) The cube in following figure has 5 grounded sides. The sixth side (shaded side) is isolated from other sides and connected to potential ϕ_0. What is the potential at the center of the cube and why?



Problem 4

Assume that charge is distributed on the z=0 plane with a surface density

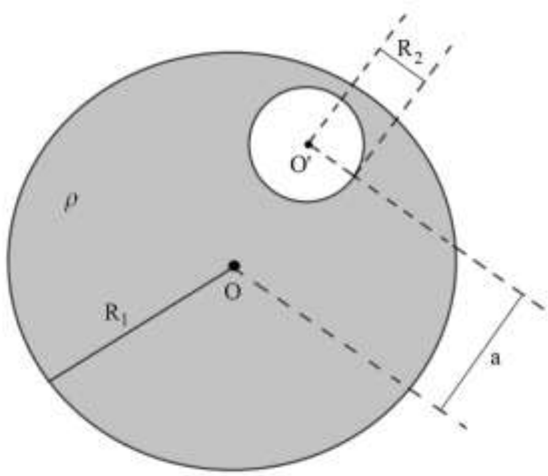
$$\sigma(\rho) = \frac{-qd}{2\pi(\rho^2 + s^2)^{\frac{3}{2}}}$$

a) Find the total charge Q on the plane.
b) Show that the potential $\varphi(z)$ produced by $\sigma(\rho)$ on the z-axis is identical to the potential produced by a point with charge Q on the axis at z=-s.

Problem 5

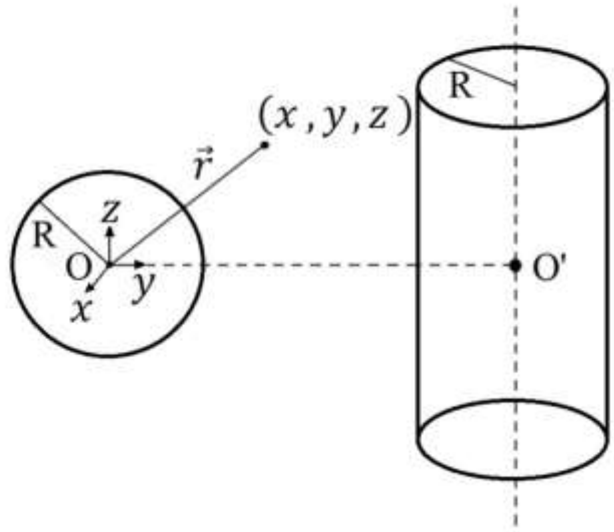
A spherical cavity, of radius R_2, is hollowed out from the interior of a sphere of radius R_1 that carries a uniform volume charge density ρ. The distance between the centers of two spheres is a.

a) Find the electrical field at the center of the hollow sphere (at point O').
b) Find the potential at that point.



Problem 6

Consider a solid sphere of radius R and uniform volume charge density ρ and a long (infinite) cylinder of radius R and uniform volume charge density ρ.



The axis of cylinder is parallel to z-axis and $\overrightarrow{OO'} = 4R\,\hat{y}$. If $\vec{r}$ is the position vector of any point (x, y, z) in space with respect to origin O, find the electrical field vector in terms of x, y, z and other parameters at any point:

a) Inside the sphere

b) Outside the sphere and cylinder

c) Inside the cylinder

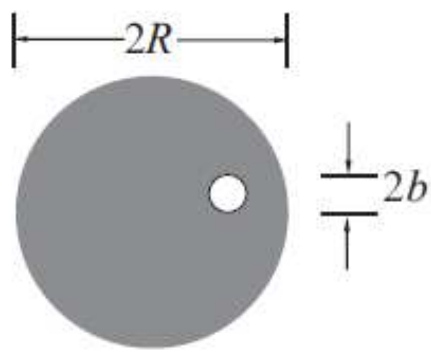
And also compute the:

d) $V_O - V_{O'}$

e) $V_A - V_B$ (Where A is a point on the surface of sphere $(R, 0{,}0)$ and B is a point on the lateral surface of cylinder $(R, 4R, 0)$.)

*** Problem 1**

The figure below shows a circular hole of radius b bored through a spherical shell with radius R and uniform surface charge density σ. Find the electric field at the center of the hole.



*** Problem 2**

Evaluate the relevant part of the integral $U_E = \frac{1}{2}\int d^3r\, \rho(\boldsymbol{r})\varphi(\boldsymbol{r})$ to find the interaction energy V_E between two identical insulating spheres, each with radius R and charge Q distributed uniformly over their surfaces. The center-to-center separation between the spheres is $d > 2R$. Do *not* assume that $d \gg R$.

Problem 3

A large space that defined by region $-a < z < a$, carries a volume charge density:

$$\rho_v = \frac{\rho_0}{a}|z - 0.5a|$$

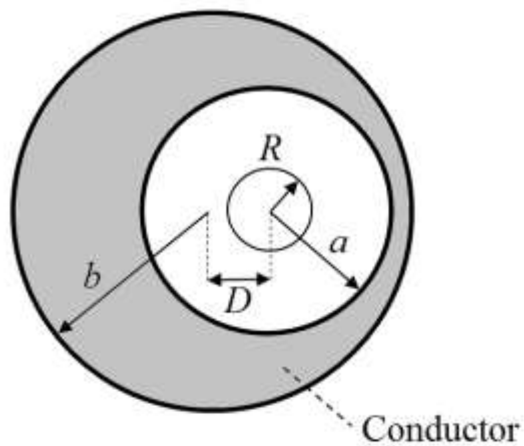
Find the electric field everywhere in all space.

*** Problem 4**

An uncharged conducting layer is placed between two spheres of radii a and b. The distance between the centers of these spheres is D. A sphere of radius R that carries a uniform volume charge density ρ is concentric with the interior sphere.

a) Find the potential everywhere. (For four regions: 1) inside the charged sphere, 2) inside the cavity and outside the charged sphere, 3) inside the conductor and 4) outside the conductor)

b) Find the surface charge density induced on interior and exterior surface of conductor.

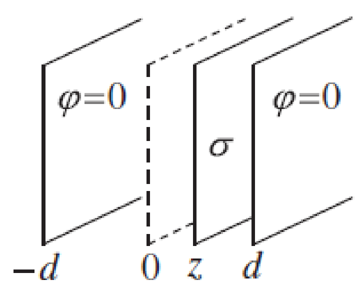


*** Problem 5**

Two infinite conducting planes are held at zero potential at $z = -d$ and $z = d$. An infinite sheet with uniform charge per unit area σ is interposed between them at an arbitrary point.

(a) Find the charge density induced on each grounded plane and the potential at the position of the sheet of charge.

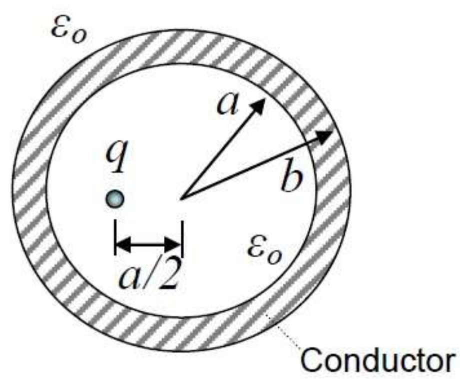
(b) Find the force per unit area which acts on the sheet of charge.



Problem 6

Consider a spherical shell conductor as shown in following figure. First, electrical charge Q given to the shell and then the point charge q placed inside the shell at the point shown in figure. After the electrostatic balance,

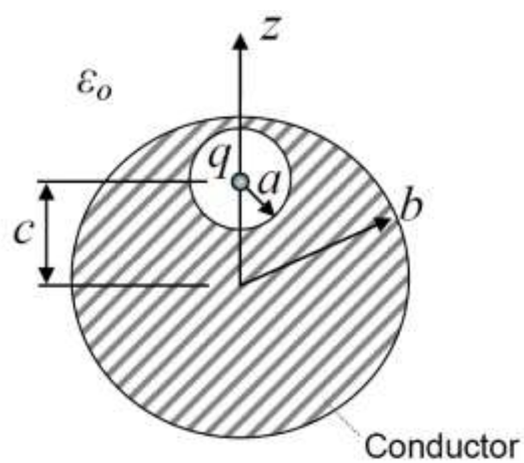
a) Find the electric field in region $r \geq b$.
b) What is the electric potential of the shell in this case?
c) Find the electric potential at the center of spherical shell.



Problem 7

The figure below shows a spherical conductor of radius b centered at the origin that has a spherical cavity of radius a carved out of it. A point charge q placed at the center of spherical cavity. In addition, the electrical charge Q is given to this spherical conductor. After the electrostatic balance,

a) Calculate $\phi(0,0,z)$, the electric potential at any point on the z-axis.
b) Plot the $\phi(0,0,z)$ in terms of z.



Problem 8

A model hydrogen atom is composed of a point nucleus with charge $+|e|$ and an electron charge distribution

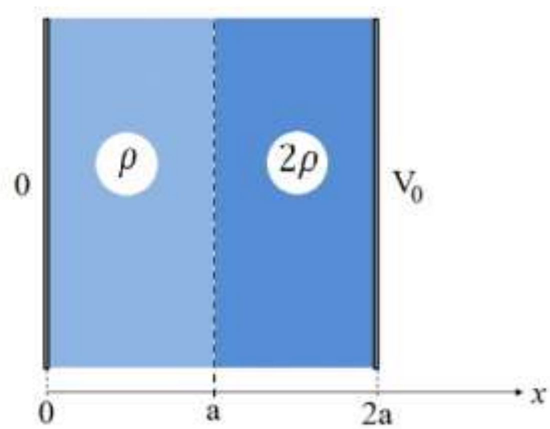
$$\rho_{-}(r) = -\frac{|e|}{\pi a^2 r}\exp(-\frac{2r}{a})$$

Show that the ionization energy (the energy to remove the electronic charge and disperse it to infinity) of this atom is

$$I = \frac{3}{8}\frac{e^2}{\pi\epsilon_0 a}$$

*** Problem 9**

Two infinite conducting planes are held at zero potential at $x = 0$, and potential V_0 at $x = 2a$. Region $0 < x < a$ and $a < x < 2a$ are filled by uniform volume charge density ρ and 2ρ, respectively.



a) Find the electric field $E(x)$ in different areas within interval $-\infty < x < +\infty$.
b) Find the electric potential $V(x)$ in different areas within interval $-\infty < x < +\infty$.
c) Find the charge density induced on both side of each infinite conducting plane.
d) (**optional**) Plot the electric field $E(x)$.
e) (**optional**) Plot the electric potential $V(x)$.

*** Problem 10**

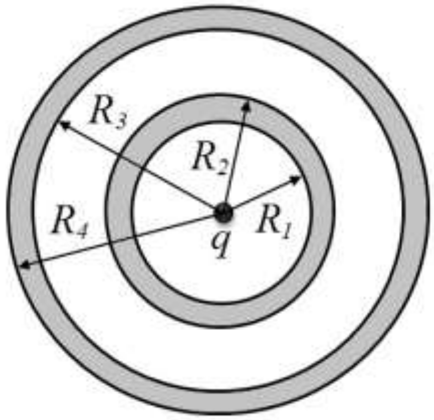
The point charge q is placed at the center of two concentric conducting shells. If we give electric charge Q_1 to inner shell and electric charge Q_2 to outer shell,

a) Find the charge distribution on the both surfaces of each spherical shell.

b) Find the electrical field within regions $r < R_1$, $R_1 < r < R_2$, $R_2 < r < R_3$, $R_3 < r < R_4$ and $r > R_4$.

c) What is the electric potential difference between two spherical shells?

d) If we connect two spherical shells to each other by a conducting wire, what will be the final charge on each of two spherical shells?



Problem 1

Let the space between two concentric spheres with radii a and $R \geq a$ be filled uniformly with charge.

a) Calculate the total energy U_E in terms of total charge Q and the variable $x{=}a/R$.

b) Minimize U_E with respect to x (keeping the total charge Q constant).

Problem 2

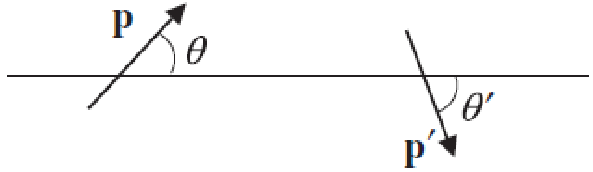
Find the electric dipole moment of

a) a ring with charge per unit length $\lambda = \lambda_0 \cos\phi$ where ϕ is the angular variable in cylindrical coordinates.

b) a sphere with charge per unit areas $\sigma = \sigma_0 \cos\theta$ where θ is the polar angle measured from the positive z-axis.

*** Problem 3**

Two coplanar dipoles are oriented as shown in the figure below.



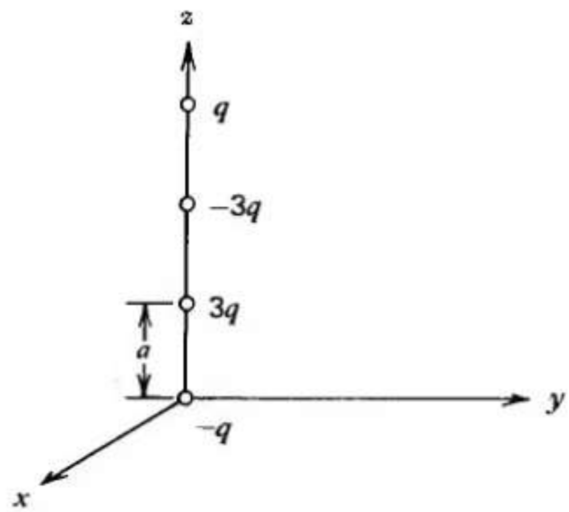
Find the equilibrium value of the angle θ' if the angle θ is fixed.

Problem 4

Show that the potential at large distances from a linear octupole shown in the figure is

$$\frac{6qa^3 P_3(\cos\theta)}{4\pi\varepsilon_0 r^4}$$

where $P_3(\cos\theta) = \frac{1}{2}(5cos^3\theta - 3cos\theta)$.



*** Problem 5**

In a medium with the constant electrical field $\vec{E} = 20\,\hat{z}$, one puts a metal sphere of radius $a = 0.5\, m$, which is connected to a battery with potential $V_0 = 10\, V$. If the function of potential around the sphere is given by:

$$V(r,\theta) = k_1 + \left(k_2 r\; + \frac{k_3}{r^2}\right)\cos(\theta), \qquad (r \geq a)$$

a) Find the coefficients k_1, k_2 and k_3.

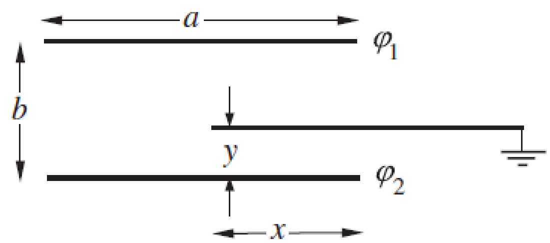
b) Find the amount of electric charge accumulated within the area $0 < \theta < \frac{\pi}{2}$ and $0 < \varphi < \pi$.

Problem 6

How much work does it take to make a spherical shell of inner radius a and outer radius $b = 2a$, which bears the volume charge density $\rho = 2\left(1 - \frac{r}{a}\right)$?

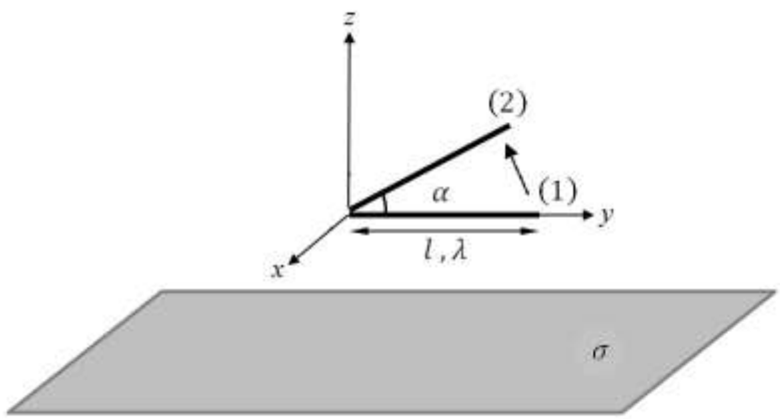
* Problem 7

A grounded metal plate is partially inserted into a parallel-plate capacitor with potential difference $\varphi_2 - \varphi_1 > 0$ as shown in the diagram below. Find the elements of the capacitance matrix. Assume that all plates extend a distance d in the direction perpendicular to the paper. Ignore fringing fields.



* Problem 8

A very large suface carries a uniform surface charge density σ. A thin bar with a length of l uniformly charged with λ, placed in parallel at the top the surface. How much work does it take to rotate the bar around its end ? (with rotation angle α)



* Problem 9

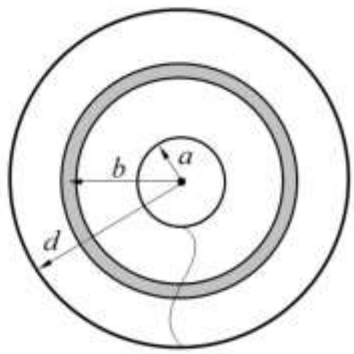
A spherical conducting shell with radius b is concentric with and encloses a conducting ball with radius a. Compute the capacitance $C = Q/\Delta\phi$ when

a) the shell is grounded and the ball has charge Q.
b) the ball is grounded and the shell has charge Q.

* Problem 10

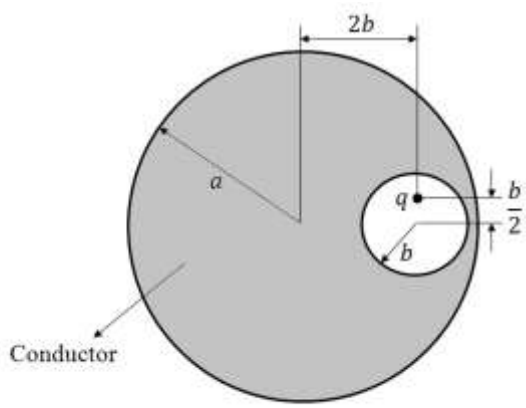
A capacitor consists of three concentric spherical shells of radii a, b and d ($a < b < d$) as shown in figure below. One connects the outer and inner shells to each other by a thin, insulated wire, through a small hole within the middle shell. (pass up the effects of the hole)

- **a)** Find the capacitance of the system.
- **b)** If the charge Q_B is given to the middle shell, find the charge distribution on this spherical shell.



Problem 11

Following figure shows an isolated conducting sphere of radius a with an spherical cavity of radius b. The point charge q is located at the distance of $\frac{b}{2}$ form the center of cavity. What is the potential at the center of cavity?



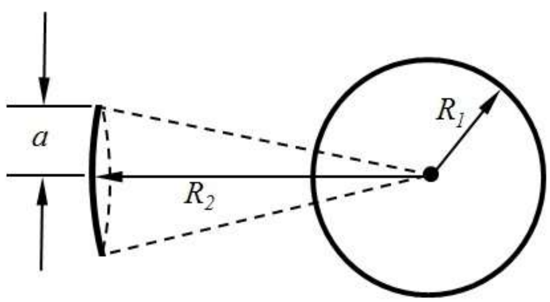
* Problem 12

For a system of N conductors which bear charges $Q_1, Q_2, \ldots$ and Q_N, one writes:

$$V_i = \sum_{j=1}^{N} P_{ij}\, Q_j$$

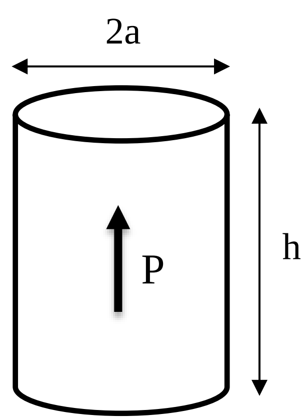
Where V_i are the potential of the ith conductor and Q_j are the charge of the jth conductor. P_{ij} are the *coefficients of potential* that are related to geometrical properties of system and independent of the charges of the conductors. One can show that $P_{ij} = P_{ji}$.

Now consider the figure below, where an skull-cap is located at the distance of R_2 (there was an sphere of radius R_2, which this scull-cap is taken from that) from an sphere of radius R_1. The distance from the edge of the scull-cap to its axis is a. According to given information, if the scull-cap carries the uniform charge per unit area σ, what is the potential of the sphere of radius R_1? (Use the method of *coefficients of potential*)



*** Problem 1**

A cylinder with radius a and height h is polarized uniformly parallel to its axis. Find the electric field and potential on the axis of cylinder.



Problem 2

A sphere of radius R carries a polarization $\boldsymbol{P}(\boldsymbol{r}) = k\boldsymbol{r}$ where k is a constant and $\boldsymbol{r}$ is the vector from the center.

a) Calculate the bound charges σ_b and ρ_b.

b) Find the electric field inside and outside the sphere.

*** Problem 3**

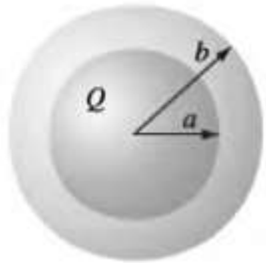
The polarization in all of space has the form:

$$\mathbf{P} = \begin{cases} P\hat{r} & r > R \\ 0 & r < R \end{cases}$$

where P and R are constants. Find the polarization charge density and the electric field everywhere.

Problem 4

A spherical conductor, of radius a carries a charge Q. It is surrounded by linear dielectric material of susceptibility χ_e out to radius b. Find the energy of this configuration.



*** Problem 5**

The space between the plates of a parallel-plate capacitor is filled with dielectric material whose dielectric constant varies linearly from 1 at the bottom plate (x = 0) to 2 at the top plate (x = d). The capacitor is connected to a battery of voltage V. Find all the bound charge, and check that the total is zero.

*** Problem 6**

For a configuration of charges and currents confined within a volume V, show that

$$\int_V \mathbf{J} dv = \frac{d\mathbf{p}}{dt}$$

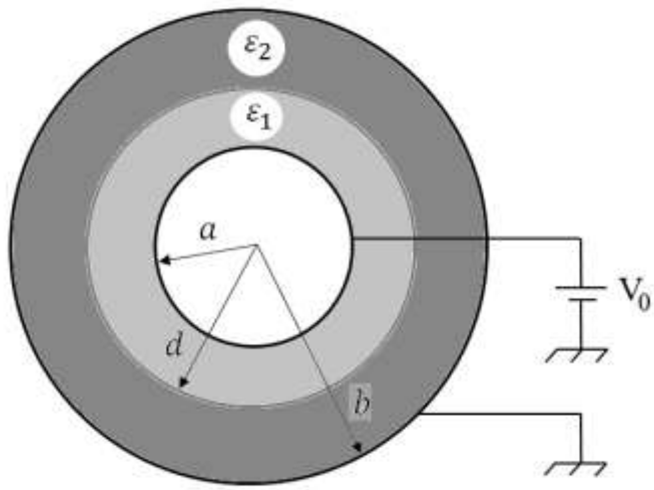
where **p** is the total dipole moment.

*** Problem 7**

Figure below shows two concentric conducting spherical shells of radii a and b. The space between the conducting shells is filled with two different dielectric materials whose dielectric constants within interval $a < r < d$ is $\varepsilon_1 = 6\varepsilon_0$, and within interval $d < r < b$ is $\varepsilon_2 = \varepsilon_0(1 + \frac{1}{r^2})$. The potential of inner shell is $V_0 = 120\,V$ and outer shell is zero. ($a = 0.58\,m, b = 1.7m$ and $d = 1m$)

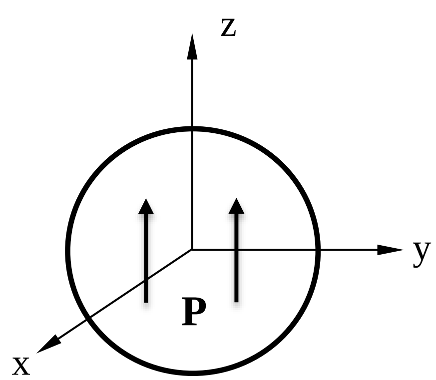
a) Find the function of potential within $a < r < d$ and $d < r < b$.
b) Find the bound surface charges over the surfaces of dielectrics (σ_b).
c) Find the bound volume charges within the dielectrics (ρ_b).

d) (Optional) Find the total bound surface charges (Q_{σ_b}) and total bound volume charges (Q_{ρ_b}), and then show $Q_{\rho_b} = |Q_{\sigma_b}|$ (Because of the approximations that may be used in calculation, one might reach to $Q_{\rho_b} \approx |Q_{\sigma_b}|$)



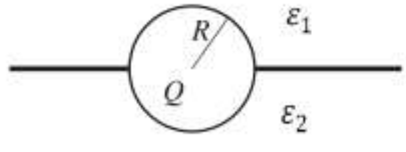
Problem 1

A sphere of radius R which its center is located at the origin, carries a polarization $\mathbf{P} = P_0\,\hat{z}$. Find the electric potential inside and outside the sphere. (**Hint:** It is easier to use the potential of dipole instead of bound charges; the potential of a dipole is $V = (1/4\pi\varepsilon_0)[(\mathbf{p}\,.\,\mathbf{R})/R^3]$, divide the volume of the sphere to small sections and then sum the potentials of sections.)



Problem 2

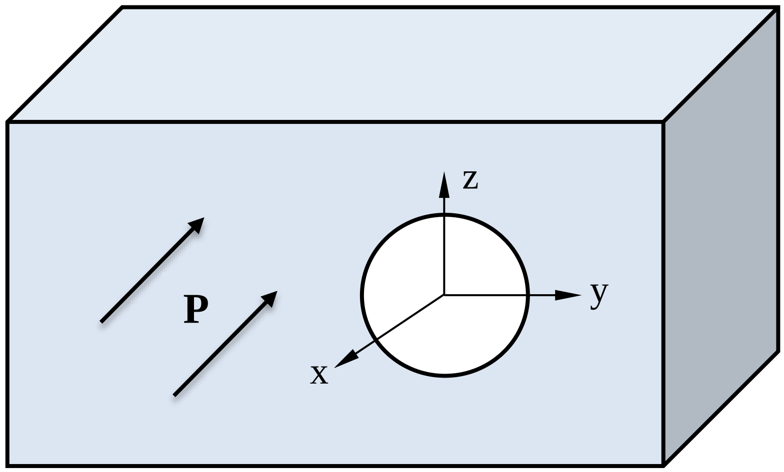
A conducting sphere with radius R and charge Q sits at the origin of coordinates. The space outside the sphere above the z = 0 plane has dielectric constant ε_1. The space outside the sphere below the z = 0 plane has dielectric constant ε_2.



a) Find the potential everywhere outside the conductor.
b) Find the distributions of free charge and polarization charge wherever they may be.

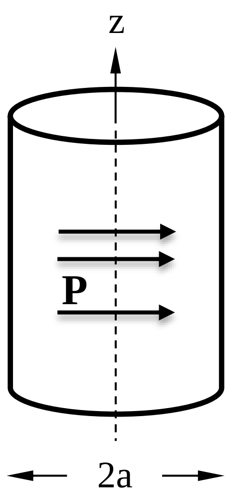
*** Problem 3**

Figure below shows a large dielectric which is polarized with $\mathbf{P_0} = P_0\,(\sin(\theta_0)\ \hat{\mathbf{y}} + \cos(\theta_0)\,\hat{\mathbf{z}}\,)$, where P_0 is a constant. A sphere of radius a is hollowed out from the dielectric. If this cavity has no effect on the polarization of dielectric, find the electric field at the center of spherical cavity.



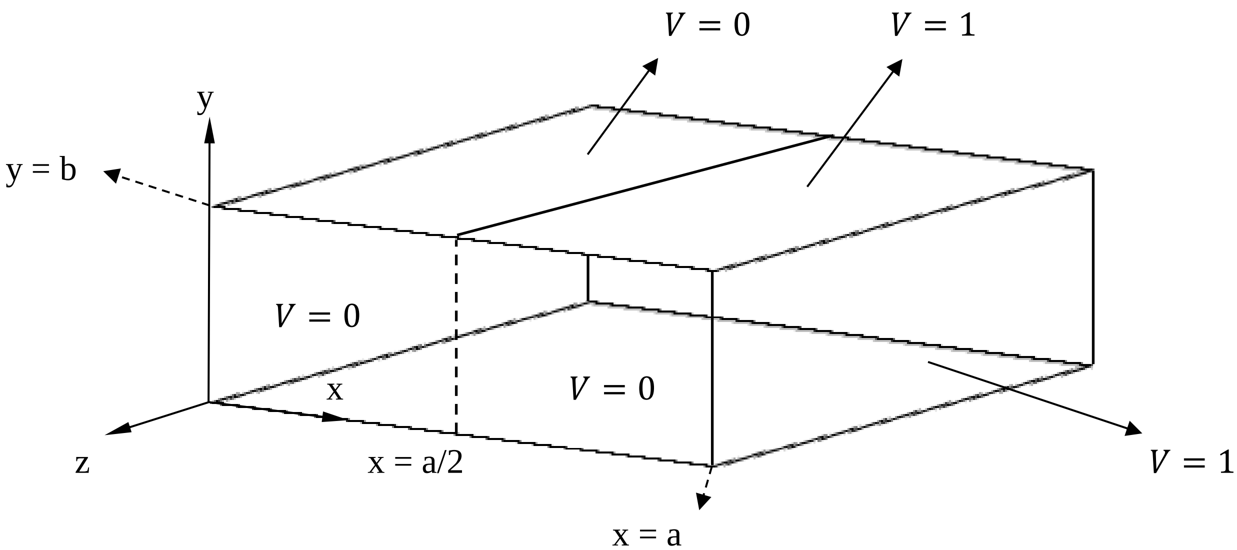
*** Problem 4**

An infinite cylinder with radius a is polarized uniformly perpendicular to its axis ($\mathbf{P} = P_0\,\hat{\mathbf{x}}$). Find the electric field and potential, inside and outside the cylinder. (Use the hint of problem1)



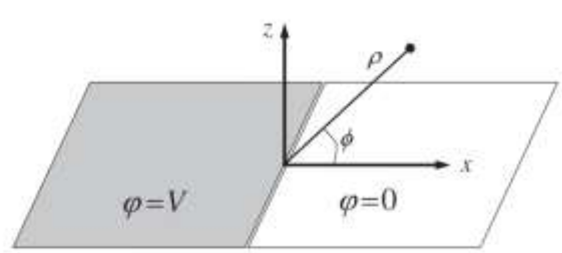
Problem 5

Consider following figure. Four conductor plates at $x = 0$, $x = a$, $y = 0$ and $y = b$ constitute an infinite tunnel through z-axis (A tiny gap at $x = 0$, $x = a$, $y = 0$ and $y = b$ prevents electrical contact between the plates.). The potential of each plate is shown in figure. Find the potential function inside the tunnel.



*** Problem 6**

The $x > 0$ half of a conducting plane at $z = 0$ is held at zero potential. The $x < 0$ half of the plane is held at potential V . A tiny gap at $x = 0$ prevents electrical contact between the two halves.



a) Explain why the $z > 0$ potential $\phi(\rho,\varphi)$ in plane polar coordinates cannot depend on the radial variable ρ.
b) Find the electrostatic potential in the $z > 0$ half-space.
c) Find the electric field in the $z > 0$ half-space, and make a quantitative sketch of the electric field lines.

*** Problem 9**

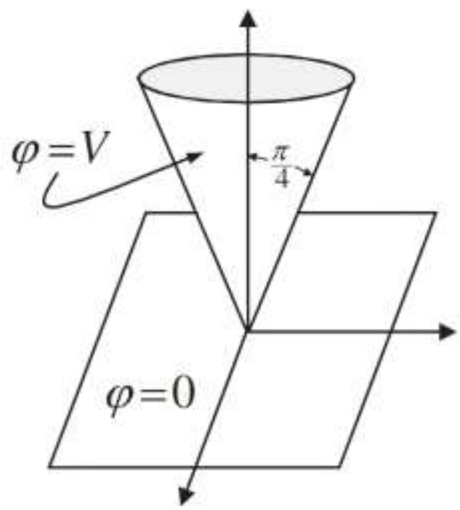
a) Consider two conical conducting surfaces in $\theta = \alpha$ and $\theta = \beta$ which are connected to potential V_0 and 0, respectively. (The radius of both cones is R.)

a-1) Explain why $\phi(r, \theta, \varphi) = \phi(\theta)$ in the space between two conical conductors.

a-2) Find the function of potential within $\alpha < \theta < \beta$ (in the region between two cones).

a-3) Find the capacitance between two conical conducting surfaces.

b) A capacitor is formed by the infinite grounded plane $z = 0$ and an infinite, solid, conducting cone with interior angle $\pi/4$ held at potential V. A tiny insulating spot at the cone vertex (the origin of coordinates) isolates the two conductors. Find the potential between the plates.



c) (**Optional**) In the spherical coordinates, the space between $(\frac{\pi}{3} < \theta < \frac{\pi}{2})$ and $(\frac{\pi}{2} < \theta < \frac{2\pi}{3})$ is filled with two different dielectric materials whose dielectric constants are $\varepsilon_1 = 3\varepsilon_0$ and $\varepsilon_2 = 5\varepsilon_0$, respectively. The surface $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$ are connected to potential $V_0 = 120\ V$ and 0, respectively.

c-1) Find the electric filed and potential within two dielectrics.

c-2) Find the capacitance per unit between two surfaces $\theta = \frac{\pi}{3}$ and $\theta = \frac{2\pi}{3}$.

Problem 7

There is a metal sphere of radius a in the free space. One puts the sphere in an infinite space whose dielectric constant is $\varepsilon = \varepsilon_0(1 + \frac{k}{r^2})$. The capacitance of the sphere in this case is twice the free space capacitance ($C = 2C_0$), find the constant k.

* Problem 8

The space between two coaxial cylinders of radii a and b is filled with a dielectric material whose dielectric constant is $\varepsilon_r = 1 + \frac{1}{R}$. The potential of inner shell is $V_0 = 100\,V$ and outer shell is zero. ($a = 0.5\,m, b = 3m$, but you can write your answer in terms of a, b and V_0).

a) Find the function of potential between two cylinders.
b) Find the capacitance between two conductors.
c) Find the bound charges within ($a < r < b$) and on the surfaces of dielectric.

Problem 10

Consider three concentric spherical conducting shell of radii a, b and c ($a < b < c$). The inner and outer sphere are connected to ground and a surface charge ρ_s is distributed on the middle sphere. The space $a < r < b$ is filled with $\varepsilon_1 = 3\varepsilon_0$ and the space $b < r < c$ is filled with $\varepsilon_2 = 5\varepsilon_0$.

a) Find the electric field and potential between spheres.
b) Find the capacitance of the system.
c) Find the bound surface charges on the inner and outer spheres.

And God Said:

"

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\oint_C \mathbf{E} \cdot d\ell + \frac{d\Phi_M}{dt} = 0$$

$$\oint_C \mathbf{B} \cdot d\ell = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

"

and there was LIGHT !