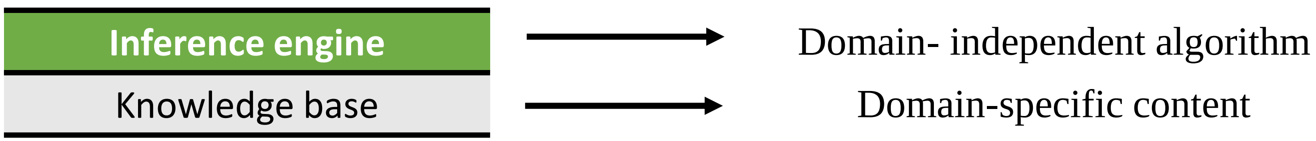
Logic and propositional logic

Chapter 7, Part 1

Outline

- **Knowledge-based agents**
- **Wumpus world**
- **Logic in general**

Knowledge bases



- Knowledge base (*KB*) = set of sentences in a formal language
- Declarative approach to building an agent (or other system):

  Tell it what it needs to know

  Then it can ***ask*** itself what to do - answers should follow from the ***KB***

- Agents can be viewed at the *knowledge level*
  - *i.e., what they know, regardless of how implemented*
- Or at the *implementation level*
  - *i.e., data structure in KB and algorithm that manipulate them*

A simple knowledge-based agent

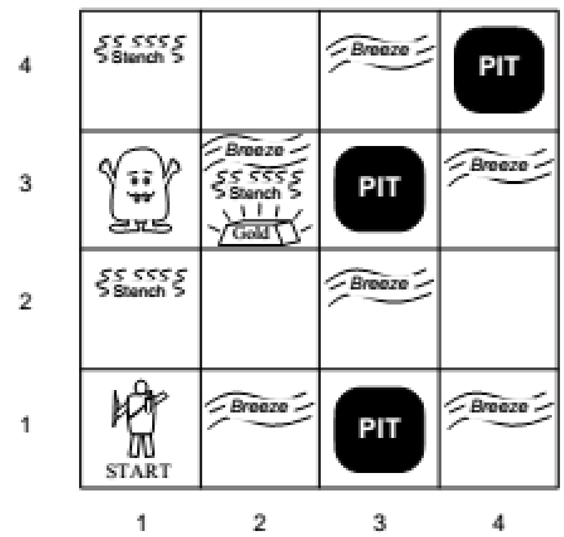
```
function KB-AGENT( percept) returns an action
   static: KB, a knowledge base
           t, a counter, initially 0, indicating time

   TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
   action ← ASK(KB, MAKE-ACTION-QUERY(t))
   TELL(KB, MAKE-ACTION-SENTENCE(action, t))
   t ← t + 1
   return action
```

- The agent must be able to:
  - Represent states, actions,etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

Wumpus World PEAS description

- **Performance measure**
  - gold+1000,death-1000
  - -1per step,-10 for using the arrow
- **Environment**
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square
- **Actuators :** Left turn,Right turn,Forward,Grab,Release,Shoot
- **Sensors :** Breeze,Glitter,Smell



Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

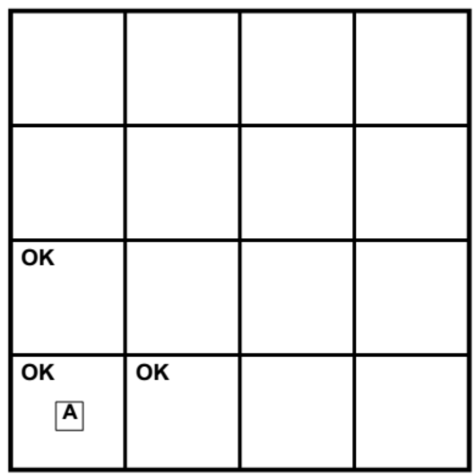
Static?? Yes—Wumpus and Pits do not move

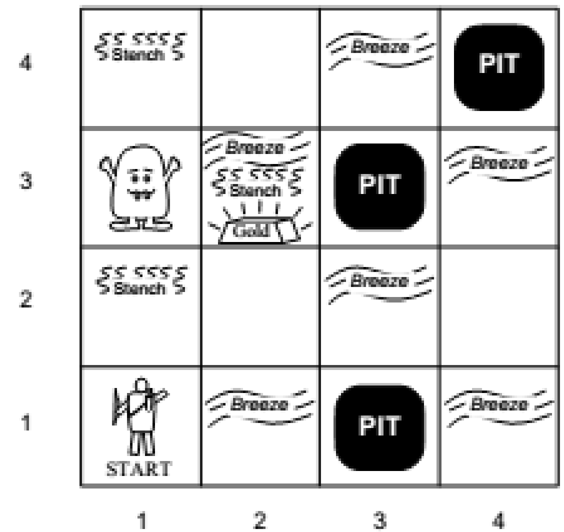
Discrete?? Yes

Single-agent??Yes—Wumpus is essentially a natural feature

Exploring a wumpus world

- A = Agent
- B = Breeze
- G = Glitter, gold
- OK = Safe square
- P = Pit
- S = Stench
- V = visited
- W= Wumpus





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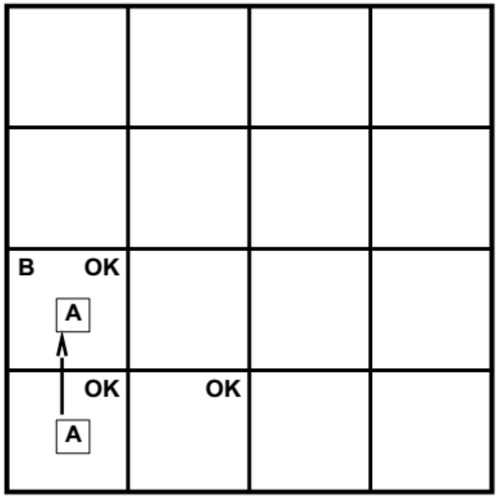

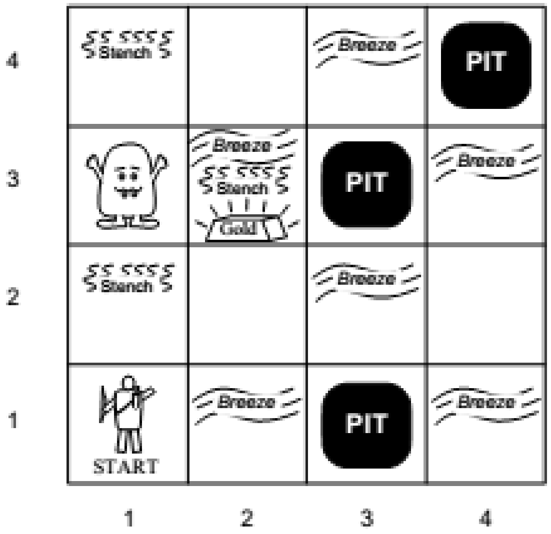

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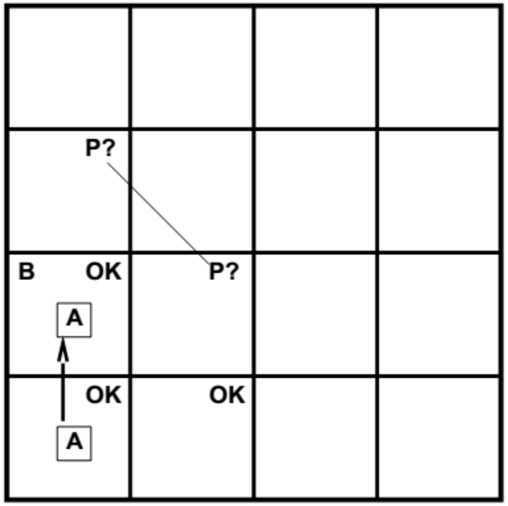

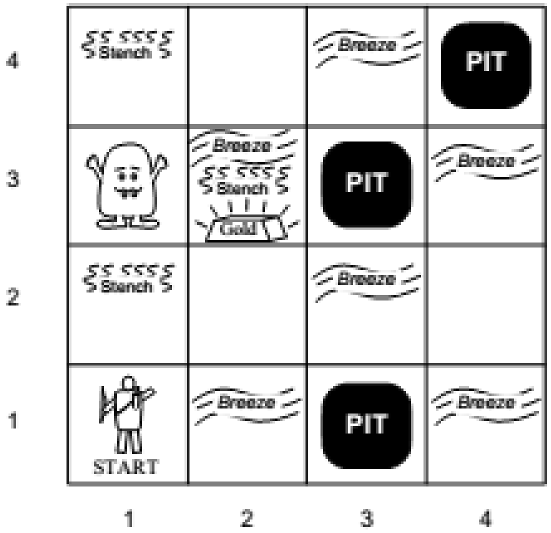

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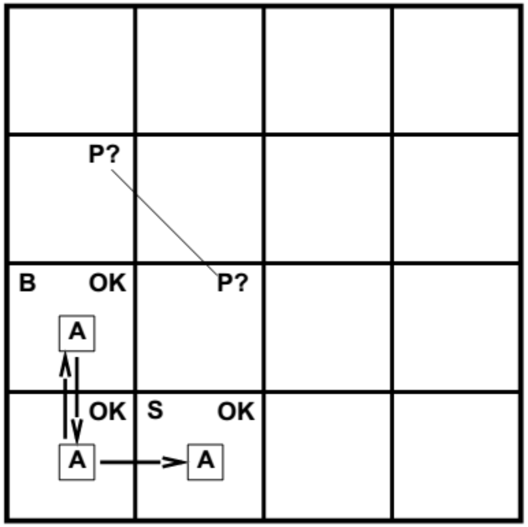

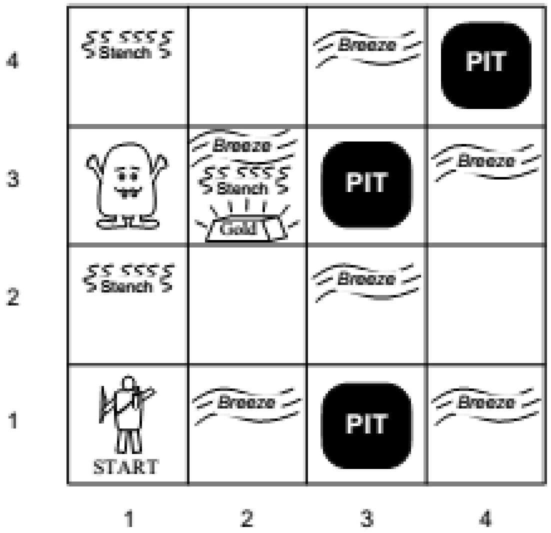

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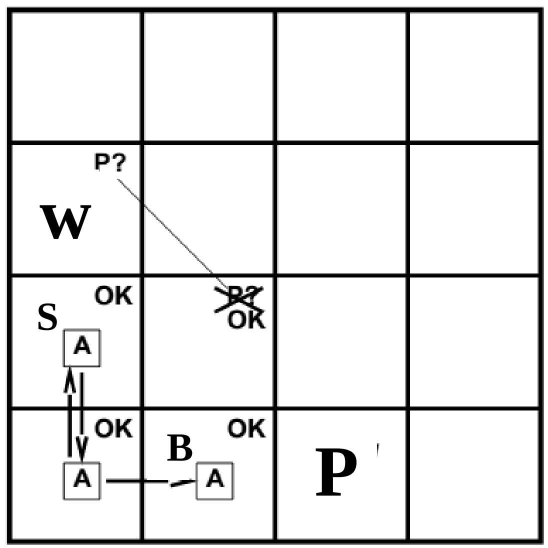

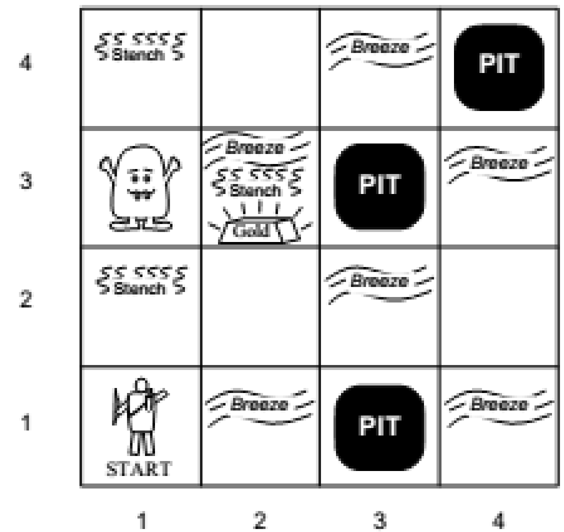

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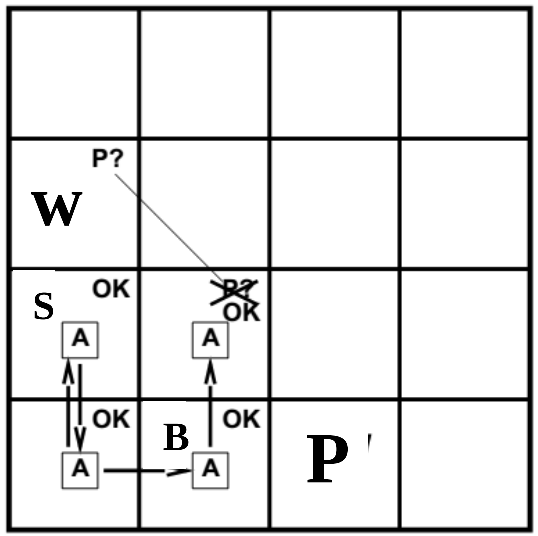

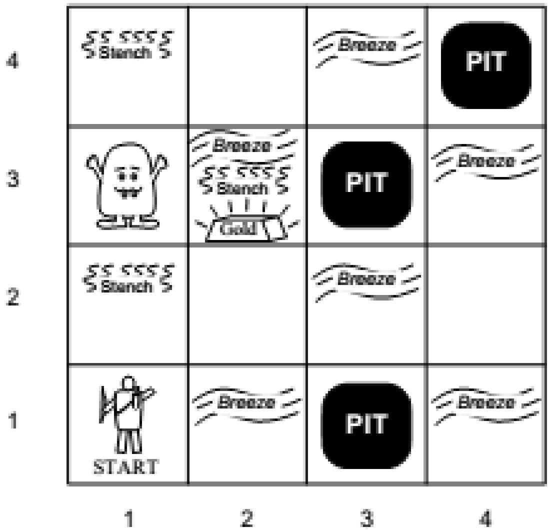

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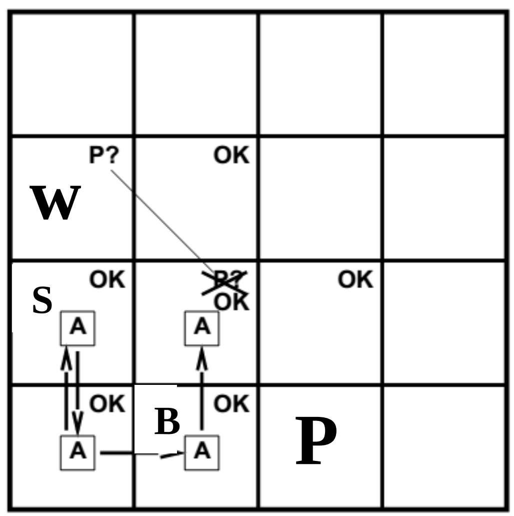

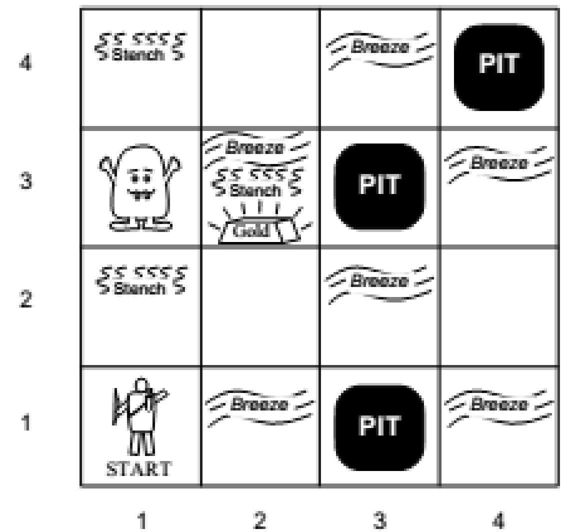

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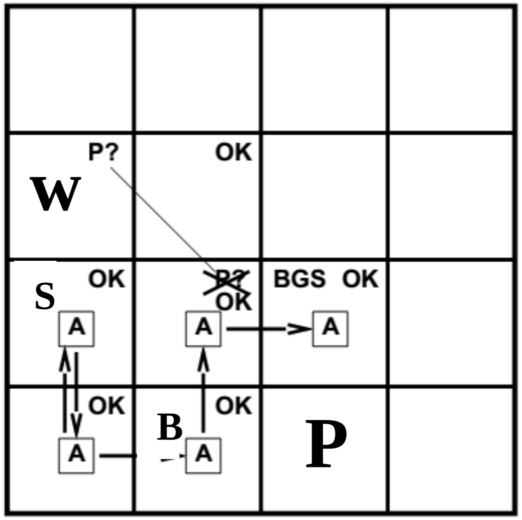
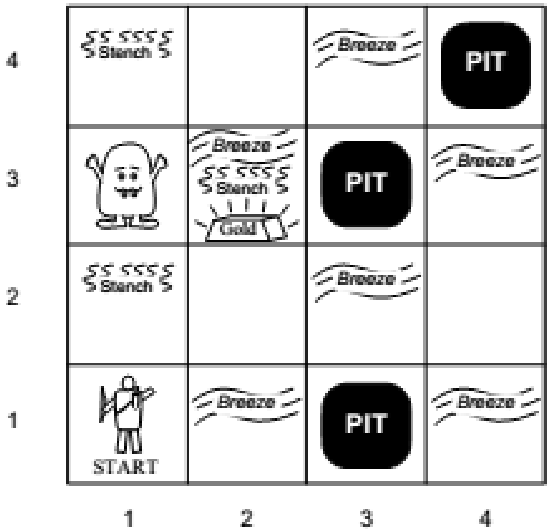




Logic in general

- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the“meaning”of sentences;
  - i.e.,define truth of a sentence in a world
  - E.g.,the language of arithmetic
    $x+2 \geq y$ is a sentence; $x2+y>$ is not a sentence
    $x+2 \geq y$ is true iff the number $x+2$ is no less than the number y
    $x+2 \geq y$ is true in a world where $x=7, y=1$
    $x+2 \geq y$ is false in a world where $x=0, y=6$

Propositional logic: Syntax

- Atomic sentence:
  - A *proposition symbol* representing a true or false statement
- Negation:
  - If P is a sentence, $\neg$ P is a sentence
- Conjunction:
  - If P and Q are sentences, $P \wedge Q$ is a sentence
- Disjunction:
  - If P and Q are sentences, $P \vee Q$ is a sentence
- Implication:
  - If P and Q are sentences, sentences, $P \Rightarrow Q$ is a sentence
- Biconditional:
  - If P and Q are sentences, $P \Leftrightarrow Q$ is a sentence
- **$\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$** are called *logical connectives*

Propositional logic: Semantics

- A **model** specifies the true/false status of each proposition symbol in the knowledge base
  - E.g., **P** is true, **Q** is true, **R** is false
  - With three symbols, there are 8 possible models, and they can enumerated exhaustively
- Rules for evaluating truth with respect to a model:

$\neg P$	is true	iff	P	is false			
$P \wedge Q$	is true	iff	P	is true	and	Q	is true
$P \vee Q$	is true	iff	P	is true	or	Q	is true
$P \Rightarrow Q$	is true	iff	P	is false	or	Q	is true
$P \Leftrightarrow Q$	is true	iff	$P \Rightarrow Q$	is true	and	$Q \Rightarrow P$	is true

Truth table

- A **truth table** specifies the truth value of composite sentence for each possible assignments of truth values to its atoms

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

Logical equivalent

- Two sentences are **logically equivalent** iff true in same models:

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutatively of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutatively of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg (\neg \alpha) \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg \alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg (\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$	DeMorgan
$\neg (\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta)$	DeMorgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Validity and satisfiability

- A sentence is **valid** if it is true in **all** models,
  - e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$
- A sentence is **satisfiable** if it is true in **some** model
  - e.g., $A \vee B$, C
- A sentence is **unsatisfiable** if it is true in **no** models
  - e.g., $A \wedge \neg A$

Entailment

- Entailment means that one thing follows from another:

$$KB \models \alpha$$

- Knowledgebase KB entails sentence α

  if and only if

α is true in all worlds where KB is true

- E.g., $x = 0$ entails $x * y = 0$

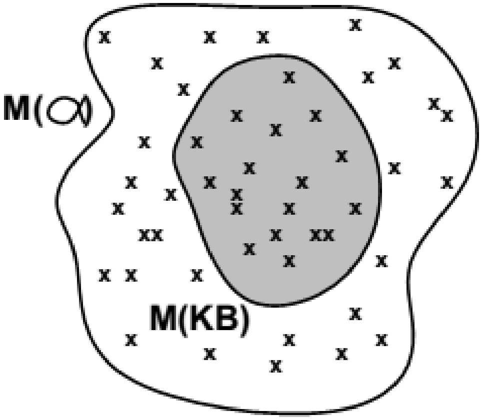
- $KB \models \alpha$ iff $(KB \Rightarrow \alpha)$ is valid
- $KB \models \alpha$ iff $(KB \land \neg \alpha)$ is unsatisfiable

models

- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say m is a **model** of a sentence α if α is true in m

  $M(\alpha)$ is the set of all models of α

  Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$



Entailment in the wumpus world

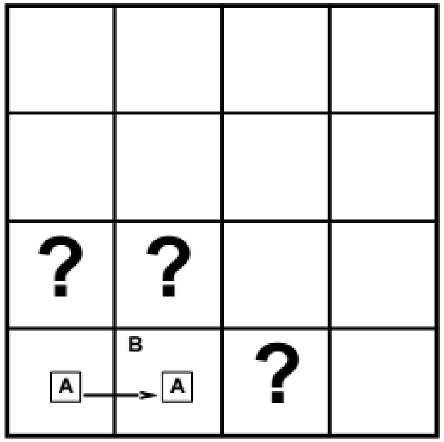
- Situation after detecting nothing in [1,1],

  moving right , breeze in[2,1]

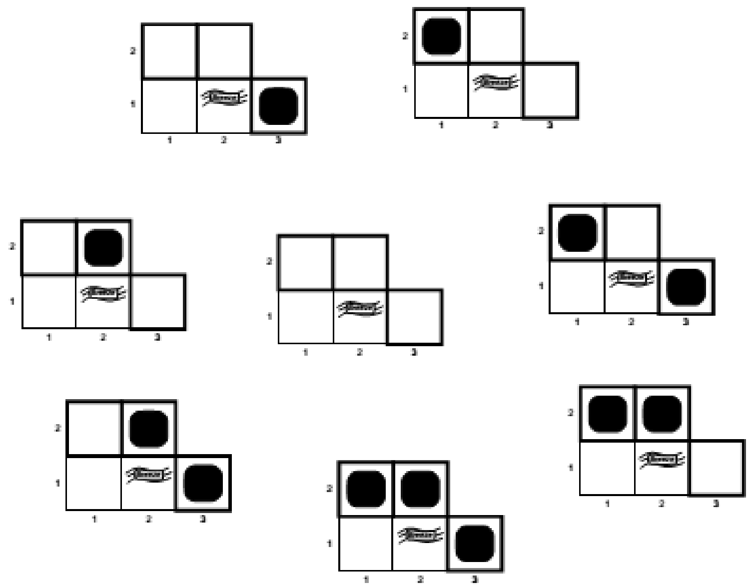
  Consider possible models for ?s

  assuming only pits

  3 Boolean choices ⇒ 8 possible models

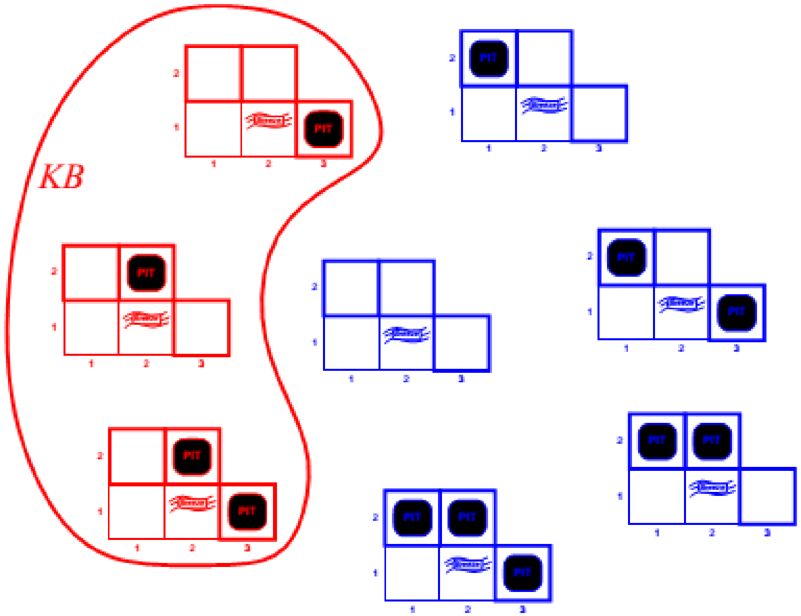


Wumpus models



Wumpus models

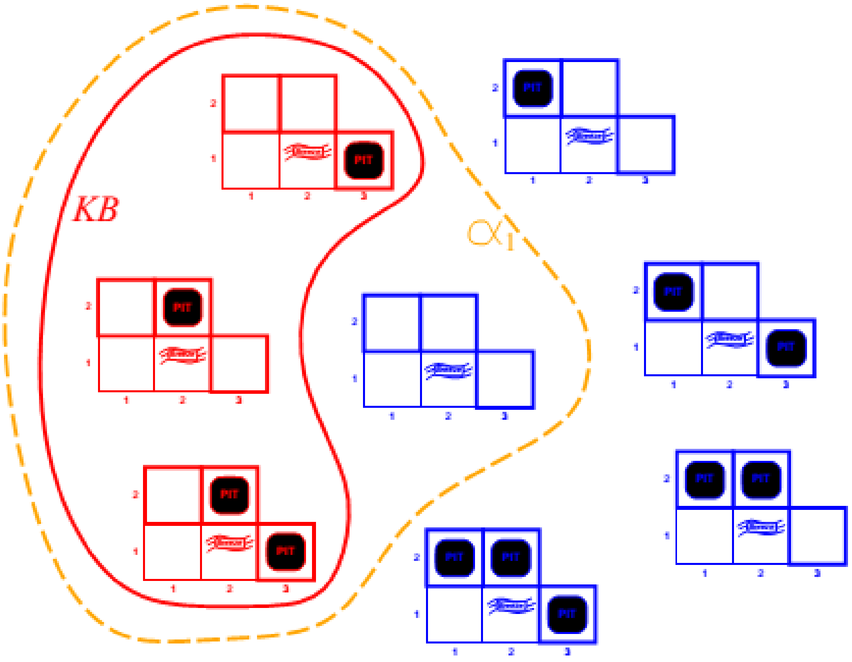
$\alpha 1$= [2,1] is safe”???



Wumpus models

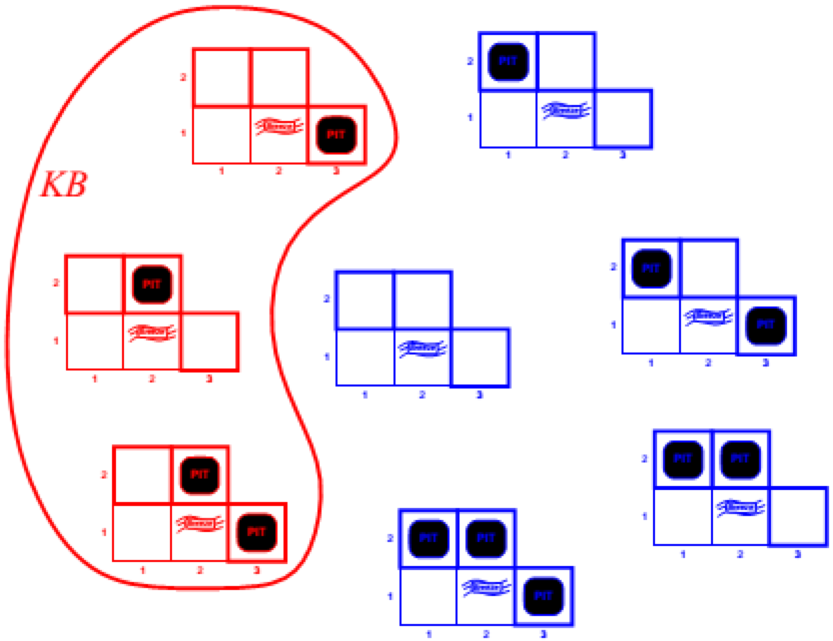
$\alpha 1$= [2,1] is safe"

KB|=$\alpha 1$, proved by model checking



Wumpus models

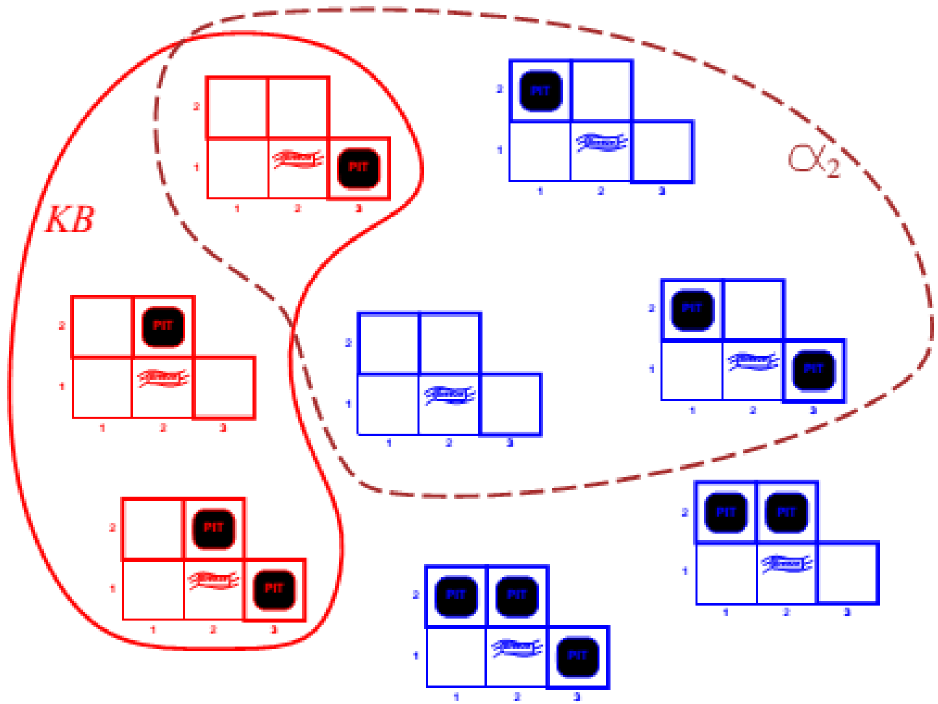
α1= [2,2] is safe???????



Wumpus models

$\alpha 2$= [2,2] is safe

$KB \not\models \alpha 2$



axioms

- An **axiom** is just a sentence asserted to be true about the domain.
- E.g.,"Pits cause breezes in adjacent squares"

$$B1,1 \Leftrightarrow (P1,2 \vee P2,1)$$

$$B2,1 \Leftrightarrow (P1,1 \vee P2,2 \vee P3,1)$$

- I.e., "A square is breezy if and only if there is an adjacent pit"
- We need axiom sets for every time step!

inference

- $KB \vdash_i \alpha$ means sentence α can be derived from *KB* by procedure *i*
  Consequences of *KB* are a haystack ; α is a needle.
- Entailment = needle in haystack;
- Inference = finding it
- Soundness: i is sound if
  - When ever $KB \vdash_i \alpha$,it is also true that KB|=α
- Completeness: i is complete if
  - whenever KB|=α,it is also true that $KB \vdash_i \alpha$

End chapter 7, part1