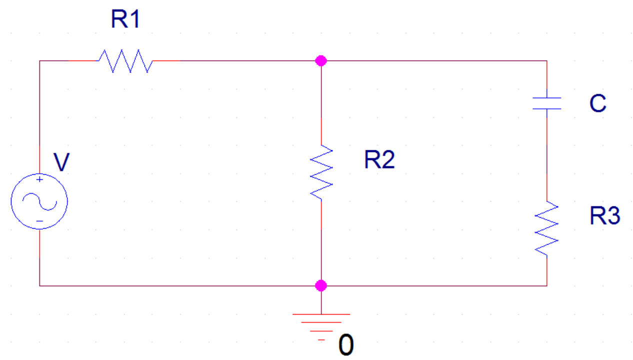
**تحلیل مدار شامل مقاومت و خازن :**




$$V_C = A\sin(\omega t)$$

$$i_3 = CA\,\omega\cos(\omega t)$$

$$V_{R_3} = R_3 i_3$$

$$V_{R_2} = V_C + V_{R_3}$$

$$i_2 = \frac{V_{R_2}}{R_2} = \frac{V_{C_1} + V_{R_3}}{R_2} \quad , \quad i_1 = i_2 + i_3 \quad , \quad V_{R_1} = R_1 i_1 = R_1(i_2 + i_3)$$

$$V = V_{R_1} + V_{R_2}$$

از همه ی تساوی های بالا نتیجه می شود :

$$V = R_1\left(\frac{A\sin(\omega t) + R_3 CA\,\omega\cos(\omega t)}{R_2} + CA\,\omega\cos(\omega t)\right) + A\sin(\omega t) + R_3 CA\,\omega\cos(\omega t)$$

$$\frac{V}{A} = \left(\frac{R_1}{R_2} + 1\right)\sin(\omega t) + \left(\frac{R_1 R_3}{R_2} + R_1 + R_3\right)C\omega\cos(\omega t)$$

$$= \sqrt{\left(\frac{R_1}{R_2} + 1\right)^2 + \left(\left(\frac{R_1 R_3}{R_2} + R_1 + R_3\right)C\omega\right)^2}\ \sin\left(\omega t + \tan^{-1}\frac{\left(\frac{R_1 R_3}{R_2} + R_1 + R_3\right)C\omega}{\left(\frac{R_1}{R_2} + 1\right)}\right)$$

$$V_{in} = D\sin(\omega t + \Delta\varphi)$$

$$D = A\sqrt{\left(\frac{R_1}{R_2} + 1\right)^2 + \left(\left(\frac{R_1 R_3}{R_2} + R_1 + R_3\right)C\omega\right)^2}$$

$$\Delta\varphi = \tan^{-1}\frac{\left(\frac{R_1 R_3}{R_2} + R_1 + R_3\right)C\omega}{\left(\frac{R_1}{R_2} + 1\right)} = \tan^{-1}\left(\frac{(R_1 R_3 + R_1 R_2 + R_2 R_3)}{(R_1 + R_2)}C\omega\right)$$

$$V_{R_2} = E\ \sin\left(\omega t + \Delta\varphi'\right)$$

$$E = A\sqrt{1 + (R_3 C\omega)^2}$$

$$\Delta\varphi' = \tan^{-1}(R_3 C\omega)$$

$$\frac{V_{R_2}}{V_{in}} = \frac{E\ \sin(\omega t + \Delta\varphi')}{D\sin(\omega t + \Delta\varphi)}$$

$$\left|\frac{V_{R_2}}{V_{in}}\right| = \frac{E}{D} = \sqrt{\frac{1\ +\ (R_3C\omega)^2}{\left(\frac{R_1}{R_2}+1\right)^2\ +\ \left(\left(\frac{R_1R_3}{R_2}+R_1+R_3\right)C\omega\right)^2}}$$

$$\omega \to 0:\ \left|\frac{V_{R_2}}{V_{in}}\right| = \frac{R_2}{R_1+R_2}$$

$$\omega \to \infty:\ \left|\frac{V_{R_2}}{V_{in}}\right| = \sqrt{\frac{(R_3)^2}{\left(\frac{R_1R_3}{R_2}+R_1+R_3\right)^2}} = \frac{R_3}{\frac{R_1R_3}{R_2}+R_1+R_3} = \frac{R_2R_3}{R_1R_3+R_1R_2+R_2R_3} =$$

$$\frac{\frac{R_2R_3}{R_2+R_3}}{R_1+\frac{R_2R_3}{R_2+R_3}} = \frac{R_2||R_3}{R_1+R_2||R_3}$$

$$\Delta\varphi' - \Delta\varphi = \tan^{-1}(R_3C\omega) -\ \tan^{-1}\left(\frac{(R_1R_3+R_1R_2+R_2R_3)}{(R_1+R_2)}C\omega\right)$$

از دو طرف تانژانت می گیریم:

$$\tan\left(\Delta\varphi - \Delta\varphi'\right) = \frac{\tan(\Delta\varphi) - \tan\left(\Delta\varphi'\right)}{1+\tan(\Delta\varphi)\tan(\Delta\varphi')}$$

$$= \frac{R_3C\omega - \frac{\left(R_1R_3+R_1R_2+R_2R_3\right)}{(R_1+R_2)}C\omega}{1+\frac{\left(R_1R_3+R_1R_2+R_2R_3\right)}{(R_1+R_2)}R_3C^2\omega^2}$$

$$\Delta\varphi - \Delta\varphi' = \tan^{-1}\left(\frac{-R_1R_2}{(R_1+R_2)+(R_1R_3+R_1R_2+R_2R_3)R_3C^2\omega^2}C\omega\right)$$

$$\omega \to 0:\ \Delta\varphi - \Delta\varphi' = 0$$

$$\omega \to \infty:\ \Delta\varphi - \Delta\varphi' = 0$$

$$R_1 = R_2 = R_3 = 1\Omega \quad , \quad C = 1F$$

در شکل زیر نمودار $20 * \log(\left|\frac{V_{R_2}}{V_{in}}\right|)$ بر حسب $\log(\omega)$ به کمک نرم افزار متمتیکا رسم شده است.

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In[55]:= ParametricPlot[{Log10[t], 20*Log10[Sqrt[(1+t*t)/(4+9*t*t)]]}, {t, -20, 7}]
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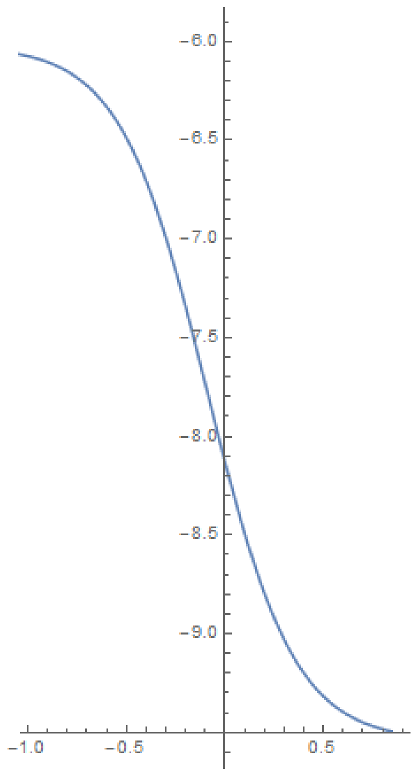
Out[55]=




در شکل زیر نمودار $\Delta\varphi - \Delta\varphi'$ بر حسب $\log(\omega)$ به کمک نرم افزار متمتیکا رسم شده است.

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In[64]:= ParametricPlot[{Log10[t], ArcTan[-t/(2+3*t*t)]}, {t, -20, 7}]
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Out[64]=