الگوریتم ۱ الگوریتم GMRES (m)

Require: x_0, A, b, tol, m

1: $\beta = \|r_0\|_2, r_0 = b - Ax_0, v_1 = r_0/\beta$
2: initial vector v_1 and Arnoldi process to obtain orthogonal basic V.
3: calculate y_m that minimize $\|\beta e_1 - \bar{H}_m y\|_2$. ▷ least square problem
4: **if** $\|\beta e_1 - \bar{H}_m y\|_2 < tol$ **then**
5: $\quad x_m = x_0 + V_m y_m$
6: **else**
7: $\quad x_0 = x_m$, **go to** 1 ▷ Restart
8: **end if**

الگوریتم ۲ حل $A_d u + N(u) + B^T p = f$ و $Bu = g$

Require: Initialize *tolerance*, u and p

1: Linearize $N(u)$ using u from the previous step using a Picard or Newton linearization scheme to create the matrix F and the right-hand side $\bar{f}$
2: Solve

$$\begin{bmatrix} F & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} F & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta p \end{bmatrix} + \begin{bmatrix} f \\ g \end{bmatrix}$$

3: update

$$\begin{bmatrix} u \\ p \end{bmatrix} = \begin{bmatrix} u \\ p \end{bmatrix} + \begin{bmatrix} \delta u \\ \delta p \end{bmatrix}$$

4: **if** $\left\| \begin{bmatrix} F & B^T \\ B & 0 \end{bmatrix} - \begin{bmatrix} f \\ g \end{bmatrix} \right\|_2 < tolerance \left\| \begin{bmatrix} f \\ g \end{bmatrix} \right\|$ **then**
5: $\quad$ Convergence
6: **else**
7: $\quad$ **go to** 2
8: **end if**

الگوریتم ۳ الگوریتم GMRES (m)

Require: $r_0 = b - Ax_0$ and $\beta = \|r_0\|_2$ and $v_1 = r_0/\beta$

1: **for** $j = 1, \cdots, m$ **do**
2: $\quad w_j = Av_j$ ▷ using Modified Gram-Schmidt
3: $\quad$ **for** $i = 1, \cdots, m$ **do**
4: $\quad\quad h_{ij} = \langle w_j, v_i \rangle$
5: $\quad\quad w_j = w_j - h_{ij} v_i$
6: $\quad$ **end for**
7: $\quad h_{j+1,j} = \|w_j\|_2$
8: $\quad$ **if** $h_{j+1,j} = 0$ **then**
9: $\quad\quad m = j$ **go to** 13 ▷ breakdown
10: $\quad$ **end if**
11: $\quad v_{j+1} = w_j / h_{j+1,j}$
12: **end for**
13: Define Hessenberg matrix $\bar{H}_m = h_{ij}$ that is $(m+1) \times m$
14: Compute y_m the minimizer $\|\beta e_1 - \bar{H}_m y\|_2$ and $x = x_0 + V_m y_m$