پاسخ ۴-۵

قسمت د)

$$\frac{dy}{dx}=2+y \longrightarrow \frac{dy}{dx}-y-2=0 \qquad 0\leq x \leq 10$$

$$y(0)=y(10)=0 \text{ و } N_1(x)=x^2(10-x)^2$$

$$y^*=C_1x^2(10-x)^2$$

$$\frac{dy^*}{dx}=C_1\left((2x)(10-x)^2+2(-1)(10-x)x^2\right)=(4x^3-60x^2+200x)C_1$$

$$R(x;C_1)=(4x^3-60x^2+200x)C_1-(100x^2-20x^3+x^4)C_1-2$$

$$R(x;C_1)=(-x^4+24x^3-160x^2+200x)C_1-2$$

$$\int_0^{10}N_1(x)R(x)dx=\int_0^{10}(x^2(10-x^2))((-x^4+24x^3-160x^2+200x)C_1-2)dx$$

$$=\int_0^{10}(100x^2-20x^3+x^4)((-x^4+24x^3-160x^2+200x)C_1-2)dx$$

$$\Rightarrow\int_0^{10}(-100x^6+2400x^5-16000x^4+20000x^3+20x^7-480x^6+3200x^5-4000x^4-x^8+24x^7-160x^6+200x^5)C_1dx$$

$$=\int_0^{10}200x^2-40x^3+2x^4dx=200\frac{x^3}{3}+\left(-40\frac{x^4}{4}\right)+2\frac{x^5}{5}\Big]_0^{10}=\frac{200(10)^3}{3}-40\frac{(10)^4}{4}+2\frac{(10)^5}{5}=\frac{20000}{3}$$

$$\int_0^{10}C_1(-740x^6+5800x^5-5600x^4+20000x^3+47x^7-x^8)dx$$

$$=\left(-740\frac{x^7}{7}+5800\frac{x^6}{6}-5600\frac{x^5}{5}+20000\frac{x^4}{4}+47\frac{x^8}{8}-\frac{x^9}{9}\right)C_1\Big]_0^{10}=\frac{20000}{3}\Rightarrow C_1=2.058\times10^{-5}$$

$$\Rightarrow y^*=2.058\times10^{-5}x^2(10-x)^2$$

پاسخ ۴-۵
قسمت هـ)

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=x \quad , \ 0\leqslant x\leqslant 1 \quad , \ y(0)=y(1)=0 \ , \ N_1(x)=x(x-1)^2$$

$$y^*=C_1x(x-1)^2$$

$$\frac{dy^*}{dx}=C_1\left((x-1)^2+2x(x-1)\right)=C_1(x^2-2x+1+2x^2-2x)=C_1(3x^2-4x+1)$$

$$\frac{d^2y^*}{dx^2}=C_1(6x-4)$$

$$C_1(6x-4)-3C_1(3x^2-4x+1)+C_1x(x^2-2x+1)-x=0$$

$$R(x;C_1)=C_1(6x-4)-3C_1(3x^2-4x+1)$$

$$R(x;C_1)=C_1(6x-4-9x^2+12x-3+x^3-2x^2+x)-x=0$$

$$\int_0^1 x(x-1)^2\left((x^3-11x^2+19x-7)-x\right)dx=0 \Rightarrow \underbrace{\int_0^1 x(x-1)^2(x^3-11x^2+19x-7)\,dx}_{2}+\underbrace{\int x(x-1)^2(-x)\,dx}_{1}$$

$$1 \rightarrow \int_0^1 (x^4-2x^3+x^2)\,dx=\frac{x^5}{5}-2\frac{x^4}{4}+\frac{x^3}{3}\Big]_0^1=\frac{1}{5}-\frac{2}{4}+\frac{1}{3}=\frac{1}{30}=0.033$$

$$2 \rightarrow \int_0^1 (x^6-11x^5+19x^4-7x^3-2x^5+22x^4-38x^3+14x^2+x^4-11x^3+19x^2-7x)C_1\,dx=0.033$$

$$=\int_0^1 (x^6-13x^5+42x^4-56x^3+33x^2-7x)C_1\,dx=\left(\frac{1}{7}-\frac{13}{6}+\frac{42}{5}-\frac{56}{4}+\frac{33}{3}-\frac{7}{2}\right)C_1$$

$$\Rightarrow \frac{x^7}{7}-13\frac{x^6}{6}+42\frac{x^5}{5}-56\frac{x^4}{4}+33\frac{x^3}{3}-7\frac{x^2}{2}\Big]_0^1 C_1$$

$$\Rightarrow -\frac{13}{105}C_1=\frac{1}{30}\Rightarrow C_1=-\frac{7}{26}\simeq -0.27$$

$$\Rightarrow y^*=-0.27\,x(x-1)^2$$

$$y''-3y'+y=x \rightarrow \lambda^2-3\lambda+1=0 \rightarrow \Delta=9-4=5 \rightarrow \lambda=\frac{3\pm\sqrt{5}}{2} \begin{cases}\lambda_1=2.618\\ \lambda_2=0.382\end{cases}$$

$$\rightarrow y=C_1e^{\lambda_1}+C_2e^{\lambda_2}=C_1e^{2.618}+C_2e^{0.382}=$$ جواب عمومی

$$y_p=x^2e^{2x}x=x^3e^{2x}$$ جواب خصوصی

$$y=C_1e^{2.618x}+C_2e^{0.382x}+x^3e^{2x}$$

$$y(0)=0\rightarrow C_1+C_2=0$$

$$y(1)=0\rightarrow C_1e^{2.618}+C_2e^{0.382}+e^2=0$$

$$\Rightarrow \begin{cases}C_1=-0.603\\ C_2=0.603\end{cases} \Rightarrow y=-0.603e^{2.618x}+0.603e^{0.382x}+x^3e^{2x}$$

پاسخ ۷-۵

قسمت ب)

$$K_x A\frac{d^2T}{dx^2}-hPT=hPT_a \rightarrow K_x A\frac{d^2T}{dx^2}-hPT_a-hPT=0$$

$$\int_{x_1}^{x_2}\left(K_x A\frac{d^2T}{dx^2}-hPT_a-hPT\right)N_i(x)A\,dx=0$$

$$=\left[K_x A\left(\underbrace{A\int_{x_1}^{x_2}\frac{d^2T}{dx^2}N_i(x)\,dx}_{1}-hPT_a\int_{x_1}^{x_2}N_i(x)\,dx-hP\int_{x_1}^{x_2}T\,N_i(x)\,dx\right)\right]=0$$

با انتگرال‌گیری جزء به جزء

$$1\rightarrow A\int_{x_1}^{x_2}\frac{d^2T}{dx^2}N_i(x)\,dx=AN_i(x)\frac{dT}{dx}\Big]_{x_1}^{x_2}-A\int_{x_1}^{x_2}\frac{dN_i(x)}{dx}\frac{dT}{dx}\,dx$$

$$\begin{cases}N_i(x)=u\rightarrow du=\dfrac{dN_i(x)}{dx}\\[2mm] \dfrac{d^2T}{dx^2}dx=dv\rightarrow v=\dfrac{dT}{dx}\end{cases}$$

$$\Rightarrow K_xA^2N_i(x)\frac{dT}{dx}\Big]_{x_1}^{x_2}-K_xA^2\int_{x_1}^{x_2}\frac{dN_i(x)}{dx}\frac{dT}{dx}\,dx-hPT_a\int_{x_1}^{x_2}N_i(x)\,dx-hP\int_{x_1}^{x_2}TN_i(x)\,dx=0$$

$$\Rightarrow K_xA^2\int_{x_1}^{x_2}\frac{dN_i(x)}{dx}\frac{dT}{dx}\,dx=K_xA^2N_i(x)\frac{dT}{dx}\Big]_{x_1}^{x_2}-hPT_a\int_{x_1}^{x_2}N_i(x)\,dx-hP\int_{x_1}^{x_2}TN_i(x)\,dx=0$$

$$T=T_1N_1(x)+T_2N_2(x)$$

$$N_1(x)=1-\frac{x}{L}$$

$$N_2(x)=\frac{x}{L}$$

$$i=1\rightarrow \underbrace{K_xA^2N_1(x)\frac{d}{dx}\left(N_1(x)T_1+N_2(x)T_2\right)\Big]_{x_1}^{x_2}}_{1}-\underbrace{hPT_a\int_{x_1}^{x_2}N_1(x)\,dx}_{2}-\underbrace{hP\int_{x_1}^{x_2}\left(T_1N_1(x)+T_2N_2(x)\right)N_1(x)\,dx}_{3}=$$

$$1\rightarrow K_xA^2N_1(x)\frac{d}{dx}\left(N_1(x)T_1+N_2(x)T_2\right)\Big]_{x_1}^{x_2}=K_xA^2\left(1-\frac{x}{L}\right)\left(-\frac{1}{L}T_1+\frac{1}{L}T_2\right)\Big]_{x_1}^{x_2}$$

$$=K_xA^2\left(-\left(-\frac{1}{L}T_1+\frac{1}{L}T_2\right)\right)=\frac{K_xA^2}{L}(T_1-T_2)$$

$$2\rightarrow hPT_a\int_{x_1}^{x_2}N_1(x)\,dx=hPT_a\int_{x_1}^{x_2}\left(1-\frac{x}{L}\right)dx=hPT_a\left(x-\frac{x^2}{2L}\right)\Big]_{x_1}^{x_2}$$

$$= hPT_a\left(L - \frac{L^2}{2L}\right) = \frac{hPT_aL}{2}$$

$$③ \rightarrow hP\int_{x_1}^{x_2}(T_1N_1(x)+T_2N_2(x))N_1(x)\,dx \Longrightarrow hP\int_{x_1}^{x_2}\left(T_1\left(1-\frac{x}{L}\right)+T_2\left(\frac{x}{L}\right)\right)\left(1-\frac{x}{L}\right)dx$$

$$= hP\int_{x_1}^{x_2}\left(T_1\left(1-\frac{x}{L}\right)^2+T_2\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)\right)dx = hP\int_{x_1}^{x_2}\left(T_1\left(1-\frac{2x}{L}+\frac{x^2}{L^2}\right)+T_2\left(\frac{x}{L}-\frac{x^2}{L^2}\right)\right)dx$$

$$= hP\left(T_1\left(x-\frac{2x^2}{2L}+\frac{x^3}{3L^2}\right)+T_2\left(\frac{x^2}{2L}-\frac{x^3}{3L^2}\right)\right)\Bigg]_{x_1}^{x_2} = hP\left(T_1\left(L-L+\frac{L}{3}\right)+T_2\left(\frac{L^2}{2L}-\frac{L^3}{3L^2}\right)\right)$$

$$= hP\left(T_1\frac{L}{3}+T_2\frac{L}{6}\right) = hPL\left(\frac{T_1}{3}+\frac{T_2}{6}\right) = \frac{hPL}{18}(6T_1+3T_2)$$

$$\Rightarrow \frac{K_xA^2}{L}(T_1-T_2) - \frac{hPT_aL}{2} - \frac{hPL}{18}(6T_1+3T_2)$$ بازای $i=1$ به این عبارت رسیدیم

$$i=2 \rightarrow \underbrace{K_xA^2N_2(x)\frac{dT}{dx}\Bigg]_{x_1}^{x_2}}_{①} - \underbrace{hPT_a\int_{x_1}^{x_2}N_i(x)\,dx}_{②} - \underbrace{hP\int_{x_1}^{x_2}TN_i(x)\,dx}_{③} = 0$$

$$① \rightarrow K_xA^2\left(\frac{x}{L}\right)\left(-\frac{1}{L}T_1+\frac{1}{L}T_2\right)\Bigg]_{x_1}^{x_2} = K_xA^2\frac{1}{L}(T_2-T_1) = \frac{K_xA^2}{L}(T_2-T_1)$$

$$② \rightarrow hPT_a\int_{x_1}^{x_2}\left(\frac{x}{L}\right)dx = hPT_a\frac{x^2}{2L}\Bigg]_{x_1}^{x_2} = \frac{hPT_aL}{2}$$

$$③ \rightarrow hP\int_{x_1}^{x_2}T\left(\frac{x}{L}\right)dx = hP\int_{x_1}^{x_2}\left(\left(1-\frac{x}{L}\right)T_1+\left(\frac{x}{L}\right)T_2\right)\left(\frac{x}{L}\right)dx$$

$$= hP\int_{x_1}^{x_2}\left(\left(\frac{x}{L}-\frac{x^2}{L^2}\right)T_1+\frac{x^2}{L^2}T_2\right)dx = hP\left(\left(\frac{x^2}{2L}-\frac{x^3}{3L^2}\right)T_1+\frac{x^3}{3L^2}T_2\right)\Bigg]_{x_1}^{x_2}$$

$$= hP\left(\frac{L}{6}T_1+\frac{L}{3}T_2\right) = hPL\left(\frac{T_1}{6}+\frac{T_2}{3}\right)$$

$$\Rightarrow \frac{K_xA^2}{L}(T_2-T_1) - \frac{hPT_aL}{2} - hPL\left(\frac{T_1}{6}+\frac{T_2}{3}\right)$$ بازای $i=2$ به این عبارت رسیدیم

تا اینجا سمت راست معادله را به ازای $i=1$ و $i=2$ بدست آوردیم.

حال سمت چپ معادله را بازای $i=1$ و $i=2$ بدست می آوریم.

$$K_x A^2 \int_{x_1}^{x_2} \frac{dN_i(x)}{dx}\frac{dT}{dx}\, dx$$

$$i=1 \to K_x A^2 \int_{x_1}^{x_2} -\frac{1}{L}\frac{d}{dx}\left(T_1N_1(x)+T_2N_2(x)\right)dx$$

$$= -\frac{K_xA^2}{L}\left[\int_{x_1}^{x_2} T_1\left(1-\frac{x}{L}\right)dx + \int_{x_1}^{x_2} T_2\left(\frac{x}{L}\right)dx\right] = -\frac{K_xA^2}{L}\left(\left(T_1x - \frac{T_1x^2}{2L}\right) + \frac{T_2x^2}{2L}\right)\Bigg]_{x_1}^{x_2}$$

$$= -\frac{K_xA^2}{L}\left(T_1L - \frac{T_1L}{2} + \frac{T_2L}{2}\right) = -\frac{K_xA^2}{L}\left(T_1 \cancel{L}/2 + T_2 \cancel{L}/2\right) = -\frac{K_xA^2}{2}(T_1+T_2)$$

$$i=2 \to K_xA^2\int_{x_1}^{x_2} \frac{1}{L}\frac{d}{dx}\left(T_1N_1(x)+T_2N_2(x)\right)dx = \frac{K_xA^2}{2}(T_1+T_2)$$

$$\Rightarrow -\frac{K_xA^2}{2}(T_1+T_2) = \frac{K_xA^2}{L}(T_1-T_2) - \frac{hPT_aL}{2} - \frac{hPL}{18}(6T_1+3T_2)$$

$$K_xA^2\left(\frac{T_1+T_2}{2}+\frac{T_1-T_2}{L}\right) = \frac{hPL}{2}T_a - \frac{hPL}{18}(6T_1+3T_2)$$

$$\frac{K_xA^2}{2}T_1 + \frac{K_xA^2}{L}T_1 + \frac{hPL}{3}T_1 + \frac{K_xA^2}{2}T_2 - \frac{K_xA^2}{L}T_2 + \frac{hPL}{6}T_2 = \frac{hPL}{2}T_a$$

$$T_1\left(\frac{K_xA^2}{2}+\frac{K_xA^2}{L}+\frac{hPL}{3}\right) + T_2\left(\frac{K_xA^2}{2}-\frac{K_xA^2}{L}+\frac{hPL}{6}\right) = \frac{hPL}{2}T_a$$

$$\frac{K_xA^2}{2}(T_1+T_2) = \frac{K_xA^2}{L}(T_2-T_1) - \frac{hPT_aL}{2} - hPL\left(\frac{T_1}{6}+\frac{T_2}{3}\right)$$

$$\left(\frac{K_xA^2}{2}+\frac{K_xA^2}{L}+\frac{hPL}{6}\right)T_1 + \left(\frac{K_xA^2}{2}-\frac{K_xA^2}{L}+\frac{hPL}{3}\right)T_2 = -\frac{hPLT_a}{2}$$

$$\Rightarrow \begin{bmatrix} \frac{K_xA^2}{2}+\frac{K_xA^2}{L}+\frac{hPL}{3} & \frac{K_xA^2}{2}-\frac{K_xA^2}{L}+\frac{hPL}{6} \\ \frac{K_xA^2}{2}+\frac{K_xA^2}{L}+\frac{hPL}{6} & \frac{K_xA^2}{2}-\frac{K_xA^2}{L}+\frac{hPL}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} \frac{hPL}{2}T_a \\ -\frac{hPL}{2}T_a \end{bmatrix}$$

پاسخ ۱۲-۵

$$\frac{E\,d^2u}{dx^2}=0 \quad \text{و} \quad A=A_0\left(1-\frac{x}{2L}\right)$$

با انتگرال گیری جزء به جزء داریم:

$$\int_0^L N_i\left(E\frac{d^2u}{dx^2}\right)A_0\left(1-\frac{x}{2L}\right)dx = A_0E\int_0^L \underbrace{N_i\left(1-\frac{x}{2L}\right)}_{u}\underbrace{\frac{d^2u}{dx^2}dx}_{dv}$$

$$du=\left(\frac{dN_i(x)}{dx}\left(1-\frac{x}{2L}\right)-\frac{N_i(x)}{2L}\right)dx$$

$$\frac{d^2u}{dx^2}dx=dv \rightarrow v=\frac{du}{dx}$$

$$=N_i(x)\left(1-\frac{x}{2L}\right)\frac{du}{dx}\Big]_0^L-\int_0^L\left(\frac{dN_i(x)}{dx}\left(1-\frac{x}{2L}\right)-\frac{N_i(x)}{2L}\right)\frac{du}{dx}dx$$

$$=N_i(x)\left(1-\frac{x}{2L}\right)\frac{du}{dx}\Big]_0^L-\int_0^L\frac{dN_i(x)}{dx}\frac{du}{dx}-\frac{x\,dN_i(x)}{2L\,dx}\frac{du}{dx}-\frac{N_i(x)}{2L}\frac{du}{dx}dx$$

$$=N_i(x)\left(1-\frac{x}{2L}\right)\frac{du}{dx}\Big]_0^L-\underbrace{\int_0^L\frac{dN_i(x)}{dx}\frac{du}{dx}dx}_{1}-\underbrace{\int_0^L\frac{x\,dN_i(x)}{2L\,dx}\frac{du}{dx}dx}_{2}-\underbrace{\int_0^L\frac{N_i(x)}{2L}\frac{du}{dx}dx}_{3}$$

$$1\rightarrow\begin{cases}\int_0^L\frac{dN_i(x)}{dx}\left(\frac{u_1}{x_2-x_1}\right)(-1)\,dx=\frac{u_1}{-x_2+x_1}\int_0^L\frac{dN_i(x)}{dx}dx\\[2ex] \int_0^L\frac{dN_i(x)}{dx}\left(\frac{u_2}{x_2-x_1}\right)(1)\,dx=\frac{u_2}{x_2-x_1}\int_0^L\frac{dN_i(x)}{dx}dx\end{cases}\left(\frac{u_2}{x_2-x_1}-\frac{u_1}{x_2-x_1}\right)\int_0^L\frac{dN_i(x)}{dx}dx$$

$$=\frac{u_2-u_1}{x_2-x_1}\int_0^L\frac{dN_i(x)}{dx}dx=\frac{u_2-u_1}{x_2-x_1}\int_0^L dN_i(x)=\frac{u_2-u_1}{x_2-x_1}N_i(x)\Big]_0^L\begin{cases}\xrightarrow{i=1}\frac{u_2-u_1}{x_2-x_1}\left(\frac{x_2-x}{x_2-x_1}\right)\Big]_0^L=\frac{u_1-u_2}{L}\\[2ex]\xrightarrow{i=2}\frac{u_2-u_1}{x_2-x_1}\left(\frac{x-x_1}{x_2-x_1}\right)\Big]_0^L=\frac{u_2-u_1}{L}\end{cases}$$

$$2\rightarrow\begin{cases}\frac{1}{2L}\int_0^L\frac{dN_i(x)}{dx}\left(\frac{x\,u_1}{x_2-x_1}\right)(-1)\,dx=\frac{-u_1}{2L(x_2-x_1)}\int_0^L x\frac{dN_i(x)}{dx}dx=\frac{-u_1}{2L(x_2-x_1)}\int_0^L x\,dN_i(x)\\[2ex]\frac{1}{2L}\int_0^L\frac{dN_i(x)}{dx}\left(\frac{x\,u_2}{x_2-x_1}\right)(1)\,dx=\frac{u_2}{2L(x_2-x_1)}\int_0^L x\frac{dN_i(x)}{dx}dx=\frac{u_2}{2L(x_2-x_1)}\int_0^L x\,dN_i(x)\end{cases}\frac{u_2-u_1}{2L(x_2-x_1)}\int_0^L x\,dN_i(x)$$

$$=\frac{u_2-u_1}{2L(x_2-x_1)}\int_0^L x\,dN_i(x)\begin{cases}\xrightarrow{i=1}\frac{u_2-u_1}{2L(x_2-x_1)}\int_0^L x\left(\frac{-1}{x_2-x_1}\right)dx=\frac{u_2-u_1}{2L(x_2-x_1)^2}\int_0^L -x\,dx=\frac{u_1-u_2}{2L(x_2-x_1)^2}\;\frac{x^2}{2}\Big]_0^L=\frac{u_1-u_2}{4L}\\[2ex]\xrightarrow{i=2}\frac{u_2-u_1}{2L(x_2-x_1)}\int_0^L x\left(\frac{1}{x_2-x_1}\right)dx=\frac{u_2-u_1}{2L(x_2-x_1)^2}\int_0^L x\,dx=\frac{u_2-u_1}{2L(x_2-x_1)^2}\;\frac{x^2}{2}\Big]_0^L=\frac{u_2-u_1}{4L}\end{cases}$$

$$(3) \rightarrow \begin{cases} \frac{1}{2L}\int_0^L N_i(x)\frac{du}{dx}dx = \frac{1}{2L}\int_0^L N_i(x)\left(\frac{u_1}{x_2-x_1}\right)(-1)dx = \frac{-u_1}{2L(x_2-x_1)}\int_0^L N_i(x)\,dx \\ \frac{1}{2L}\int_0^L N_i(x)\frac{du}{dx}dx = \frac{1}{2L}\int_0^L N_i(x)\left(\frac{u_2}{x_2-x_1}\right)(1)dx = \frac{u_2}{2L(x_2-x_1)}\int_0^L N_i(x)\,dx \end{cases} \frac{u_2-u_1}{2L(x_2-x_1)}\int_0^L N_i(x)dx$$

$$= \frac{u_2-u_1}{2L(x_2-x_1)}\int_0^L N_i(x)dx \begin{cases} \xrightarrow{i=1} \frac{u_2-u_1}{2L(x_2-x_1)^2}\int_0^L (x_2-x)\,dx = \frac{u_2-u_1}{2L(x_2-x_1)^2}\left(x_2x-\frac{x^2}{2}\right)\Big]_0^L = \frac{u_2-u_1}{4L} \\ \xrightarrow{i=2} \frac{u_2-u_1}{2L(x_2-x_1)^2}\int_0^L (x-x_1)\,dx = \frac{u_2-u_1}{2L(x_2-x_1)^2}\left(\frac{x^2}{2}-x_1x\right)\Big]_0^L = \frac{u_1-u_2}{4L} \end{cases}$$

$$\Rightarrow A_0E(1+2+3) = A_0E\,N_i(x)\left(1-\frac{x}{2L}\right)\underbrace{\frac{du}{dx}}_{\varepsilon}\Big]_0^L$$

$$\xRightarrow{i=1} A_0E\left(\frac{u_1-u_2}{L}+\frac{u_1-u_2}{4L}+\frac{u_2-u_1}{4L}\right) = A_0E\left(\frac{x_2-x_1}{x_2-x_1}\right)\left(1-\frac{x}{2L}\right)\frac{du}{dx}\Big]_0^L$$

$$\Rightarrow A_0E\left(\frac{u_1-u_2}{L}\right) = -A_0E\frac{du}{dx}\Big]_0^L = -A_0\sigma = -F \qquad (4)$$

$$\xRightarrow{i=2} A_0E\left(\frac{u_2-u_1}{L}+\frac{u_2-u_1}{4L}+\frac{u_1-u_2}{4L}\right) = A_0E\left(\frac{x-x_1}{x_2-x_1}\right)\left(1-\frac{x}{2L}\right)\frac{du}{dx}\Big]_0^L$$

$$\Rightarrow A_0E\left(\frac{u_2-u_1}{L}\right) = A_0E\left(\frac{1}{2}\right)\frac{du}{dx}\Big]_0^L = \frac{1}{2}A_0\sigma = \frac{1}{2}F \qquad (5)$$

$$\xRightarrow{(4),(5)} \frac{A_0E}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -F \\ \frac{1}{2}F \end{bmatrix}$$

پاسخ ۱۳-۵

$$\int_{x_1}^{x_2} N_{i(x)}\left[\frac{d^2}{dx^2}(EI_z)\frac{d^2v}{dx^2} - q_{(x)}\right]dx = 0$$

$$\underbrace{\int_{x_1}^{x_2} N_{i(x)}\frac{d^2}{dx^2}\left(EI_z\frac{d^2v}{dx^2}\right)dx}_{1} - \underbrace{\int_{x_1}^{x_2} N_{i(x)}\, q_{(x)}\, dx}_{2} = 0$$

$$u = N_{i(x)} \to du = \frac{dN_{i(x)}}{dx}$$

$$\frac{d^2}{dx^2}\left(EI_z\frac{d^2v}{dx^2}\right)dx = dv \to v = \frac{d}{dx}\left(EI_z\frac{d^2v}{dx^2}\right)$$

$$uv - \int v\,du \to \underbrace{N_{i(x)}\frac{d}{dx}\left(EI_z\frac{d^2v}{dx^2}\right)}_{3}\Bigg]_{x_1}^{x_2} - \underbrace{\int_{x_1}^{x_2}\frac{dN_{i(x)}}{dx}\frac{d}{dx}\left(EI_z\frac{d^2v}{dx^2}\right)dx}_{4}$$

$$3 \to EN_{i(x)}\frac{d}{dx}\left(I_z\frac{d^2v}{dx^2}\right)\Bigg]_{x_1}^{x_2} = EN_{i(x)}\left(\frac{dI_z}{dx}\frac{d^2v}{dx^2} + I_z\frac{d^3v}{dx^3}\right)\Bigg]_{x_1}^{x_2}$$

$$= EN_{i(x)}\frac{dI_z}{dx}\frac{d^2v}{dx^2}\Bigg]_{x_1}^{x_2} + EN_{i(x)}I_z\frac{d^3v}{dx^3}\Bigg]_{x_1}^{x_2}$$

$$4 \to E\int_{x_1}^{x_2}\underbrace{\frac{dN_{i(x)}}{dx}}_{u}\,\underbrace{\frac{d}{dx}\left(I_z\frac{d^2v}{dx^2}\right)dx}_{dv}$$

$$\left.\begin{aligned} du &= \frac{d^2N_{i(x)}}{dx} \\ v &= I_z\frac{d^2v}{dx^2}\end{aligned}\right\} uv - \int v\,du = E\frac{dN_{i(x)}}{dx}I_z\frac{d^2v}{dx^2}\Bigg]_{x_1}^{x_2} - E\int_{x_1}^{x_2} I_z\frac{d^2v}{dx^2}\frac{d^2N_{i(x)}}{dx^2}dx$$

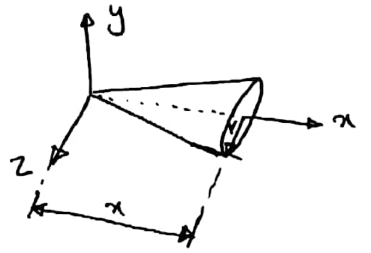
$$EN_{i(x)}\frac{dI_z}{dx}\frac{d^2v}{dx^2}\Bigg]_{x_1}^{x_2} + \underbrace{EN_{i(x)}I_z\frac{d^3v}{dx^3}}_{N_{i(x)}V_{(x)}}\Bigg]_{x_1}^{x_2} - \underbrace{E\frac{dN_{i(x)}}{dx}I_z\frac{d^2v}{dx^2}}_{\frac{dN_{i(x)}}{dx}M_{(x)}}\Bigg]_{x_1}^{x_2} + E\int_{x_1}^{x_2} I_z\frac{d^2v}{dx^2}\frac{d^2N_{i(x)}}{dx^2}dx - \int_{x_1}^{x_2} N_{i(x)}q_{(x)}dx$$

$$E\int_{x_1}^{x_2} I_z\frac{d^2v}{dx^2}\frac{d^2N_{i(x)}}{dx^2}dx = -EN_{i(x)}\frac{dI_z}{dx}\frac{d^2v}{dx^2}\Bigg]_{x_1}^{x_2} + N_{i(x)}V_{(x)}\Bigg]_{x_1}^{x_2} + \frac{dN_{i(x)}}{dx}M_{(x)}\Bigg]_{x_1}^{x_2} + \int_{x_1}^{x_2} N_{i(x)}\,q_{(x)}\,dx$$

$$q_{(x)} = q$$

$$N_1(x) = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right) \text{ ، } N_2(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2} \text{ ، } N_3(x) = \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \text{ ، } N_4(x) = \frac{x^3}{L^2} - \frac{x^2}{L}$$

$$I_z = \frac{3}{5} m\left(\frac{1}{4}r^2 + x^2\right) \longrightarrow \frac{dI_z}{dx} = \frac{3}{5} m(2x) = \frac{6mx}{5}$$



$$i=1 \longrightarrow -E\left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right)\frac{6}{5}mx\,\frac{M(x)}{EI_z}\Bigg]_{x_1}^{x_2}$$

$$= -\left(1 - \frac{3L^2}{L^2} + \frac{2L^3}{L^3}\right)\frac{6}{5}mL\,\frac{M(L)}{\frac{3}{5}m\left(\frac{1}{4}r^2 + L^2\right)} = 0$$

$$EI_z\frac{d^2v}{dx^2} = M(x) \longrightarrow \frac{d^2v}{dx^2} = \frac{M(x)}{EI_z}$$

$$N_1(x)\,V(x)\Big]_{x_1}^{x_2} = \left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right)V(x)\Big]_{x_1}^{x_2} = -V(x=0)$$

$$\frac{dN_1(x)}{dx}M(x)\Big]_{x_1}^{x_2} = \left(-\frac{6x}{L^2} + \frac{6x^2}{L^3}\right)M(x)\Big]_{x_1}^{x_2} = 0$$

$$\int_{x_1}^{x_2} N_1(x)\,q\,dx = q\int_{x_1}^{x_2}\left(1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3}\right)dx = q\left(x - \frac{3x^3}{3L^2} + \frac{2x^4}{4L^3}\right)\Bigg]_{x_1=0}^{x_2=L} = q\left(L - L + \tfrac{1}{2}L\right) = \frac{qL}{2}$$

$$\Rightarrow F_1 = \frac{qL}{2} - V(x=0) = \frac{qL}{2} - V_1$$

$$N_2(x) = x - \frac{2x^2}{L} + \frac{x^3}{L^2} \Rightarrow -EN_2(x)\frac{dI_z}{dx}\frac{d^2v}{dx^2}\Bigg]_{x_1}^{x_2} = -E\left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)\frac{6}{5}mx\,\frac{M(x)}{EI_z}\Bigg]_{x_1}^{x_2} = 0$$

$$N_2(x)\,V(x)\Big]_{x_1}^{x_2} = \left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)V(x)\Big]_{x_1}^{x_2} = 0$$

$$\frac{dN_2(x)}{dx}M(x)\Big]_{x_1}^{x_2} = \left(1 - \frac{4x}{L} + \frac{3x^2}{L^2}\right)M(x)\Big]_{x_1}^{x_2} = -M(x=0)$$

$$\int_{x_1}^{x_2} N_2(x)\,q\,dx = q\int_{x_1}^{x_2}\left(x - \frac{2x^2}{L} + \frac{x^3}{L^2}\right)dx = q\left(\frac{x^2}{2} - \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right)\Bigg]_{x_1}^{x_2} = q\left(\frac{L^2}{2} - \frac{2}{3}L^2 + \frac{L^2}{4}\right) = \frac{qL^2}{12}$$

$$F_2 = \frac{qL^2}{12} - M(x=0)$$

$$N_3(x)=\frac{3x^2}{L^2}-\frac{2x^3}{L^3}$$

$$-EN_3(x)\,\frac{dI_z}{dx}\,\frac{d^2v}{dx^2}\Big]_{x_1}^{x_2}=-E\left(\frac{3x^2}{L^2}-\frac{2x^3}{L^3}\right)\left(\frac{6}{5}mx\right)\frac{M(x)}{EI_z}\Big]_{x_1}^{x_2}=(1)\left(\frac{6}{5}mL\right)\frac{M(x=L)}{\frac{3}{5}m\left(\frac{1}{4}r^2+L^2\right)}=\frac{8L}{r^2+4L^2}M(x=L)$$

$$N_3(x)\,v(x)\Big]_{x_1}^{x_2}=\left(\frac{3x^2}{L^2}-\frac{2x^3}{L^3}\right)v(x)\Big]_{x_1}^{x_2}=v(x=L)$$

$$\frac{dN_3(x)}{dx}M(x)\Big]_{x_1}^{x_2}=\left(\frac{6x}{L^2}-\frac{6x^2}{L^3}\right)M(x)\Big]_{x_1}^{x_2}=0$$

$$q\int_{x_1}^{x_2}\left(\frac{3x^2}{L^2}-\frac{2x^3}{L^3}\right)dx=q\left(\frac{3x^3}{3L^2}-\frac{2x^4}{4L^3}\right)\Big]_{x_1}^{x_2}=q\left(L-\frac{1}{2}L\right)=\frac{1}{2}qL$$

$$\Rightarrow F_3=\frac{1}{2}qL+v(x=L)-\frac{8L}{r^2+4L^2}M(x=L)$$

$$N_4(x)=\frac{x^3}{L^2}-\frac{x^2}{L}$$

$$-EN_4(x)\,\frac{dI_z}{dx}\,\frac{d^2v}{dx^2}\Big]_{x_1}^{x_2}=-E\left(\frac{x^3}{L^2}-\frac{x^2}{L}\right)\left(\frac{6}{5}mx\right)\frac{M(x)}{EI_z}\Big]_{x_1}^{x_2}=0$$

$$N_4(x)\,v(x)\Big]_{x_1}^{x_2}=\left(\frac{x^3}{L^2}-\frac{x^2}{L}\right)v(x)\Big]_{x_1}^{x_2}=0$$

$$\frac{dN_4(x)}{dx}M(x)\Big]_{x_1}^{x_2}=\left(\frac{3x^2}{L^2}-\frac{2x}{L}\right)M(x)\Big]_{x_1}^{x_2}=M(x=L)$$

$$q\int_{x_1}^{x_2}\left(\frac{x^3}{L^2}-\frac{x^2}{L}\right)dx=q\left(\frac{x^4}{4L^2}-\frac{x^3}{3L}\right)\Big]_{x_1}^{x_2}=q\left(-\frac{L^2}{12}\right)=-\frac{1}{12}qL^2$$

$$\Rightarrow F_4=-\frac{1}{12}qL^2+M(x=L)$$

پاسخ ۱۸-۵

$$\frac{dy}{dx}+y=3 \qquad 0\leq x\leq 1$$

$$y_{(x)}=3+Ce^{-x} \quad , \quad y_{(0)}=0$$

$$\left.\begin{array}{l} y^*=C_0+C_1x \\ y^*_{(0)}=0 \longrightarrow C_0=0 \end{array}\right\} y^*=C_1x \longrightarrow \frac{dy}{dx}=C_1$$

$$R(x;C_1)=0 \Rightarrow C_1+C_1x-3=0 \Rightarrow C_1(1+x)=3 \Rightarrow C_1=\frac{3}{1+x}$$

$$\Rightarrow y^*=\frac{3}{1+x}x=\frac{3x}{1+x}$$

$$y_{(x)}=3+Ce^{-x} \; , \; y_{(0)}=0 \Rightarrow 3+Ce^{0}=0 \Rightarrow C=-3 \Rightarrow y_{(x)}=3-3e^{-x}$$

$$\frac{dy}{dx}=3-y \longrightarrow \frac{dy}{3-y}=dx \longrightarrow -\ln 3-y=x+C \longrightarrow e^{-\ln 3-y}=e^{x+C} \longrightarrow e^{\ln\frac{1}{3-y}}=e^{x+C}$$

$$\longrightarrow \frac{1}{3-y}=e^xe^C \xrightarrow{e^C=c'} \frac{1}{3-y}=c'e^x \Rightarrow (3-y)(c'e^x)=1 \longrightarrow 3c'e^x-yc'e^x=1$$

$$\longrightarrow 3c'e^x-1=yc'e^x \longrightarrow y=\frac{3c'e^x-1}{c'e^x}=3-\frac{1}{c'e^x}=3-\frac{1}{c'}e^{-x} \xrightarrow{\frac{-1}{c'}=c''} y=3+c''e^{-x}$$

$$\xrightarrow{c''=-3} y=3-3e^{-x}$$

با تقریب خوبی می توان گفت دو معادله نمودارهای بسیار نزدیکی بهم داشته و تقریباً دو جواب یکسان اند.

اما یک مرحله دیگر ادامه می دهیم:

$$y^*_{(x)}=C_0+C_1x+C_2x^2$$

$$y^*_{(0)}=0 \Rightarrow C_0=0 \Rightarrow y^*_{(x)}=C_1x+C_2x^2$$

$$\frac{dy^*}{dx}=C_1+2C_1x$$

اما یک معادله داریم و دو مجهول

اگر از معادله اصلی کمک بگیریم:

$$R(x;C_1)=0 \Rightarrow C_1+2C_2x+C_1x+C_2x^2-3=0$$

$$y_{(x)}=3-3e^{-x} \longrightarrow \begin{cases} y(1)=1{,}8963 \longrightarrow 2C_1+3C_2=4{,}8963 \\ y(\frac{1}{2})=1{,}1804 \longrightarrow 6C_1+5C_2=14{,}7216 \end{cases} \Rightarrow \begin{cases} C_1=3{,}2148 \\ C_2=-0{,}5111 \end{cases}$$

$$\longrightarrow y^*=3{,}2148x-0{,}5111x^2$$

بعد از رسم نمودار معادله روبه رو بازهم متوجه می شویم که تقریباً جواب ها یکسان اند.
(منحنی ها برهم منطبق اند)