A Comprehensive Branch Model for Transformers

Figure 1. shows the basic equivalent circuit of transformer in respect to the complex current (I_i, I_i', I_j), complex voltages (V_i, V_i', V_j), complex tap ratio (t) and admittance y.

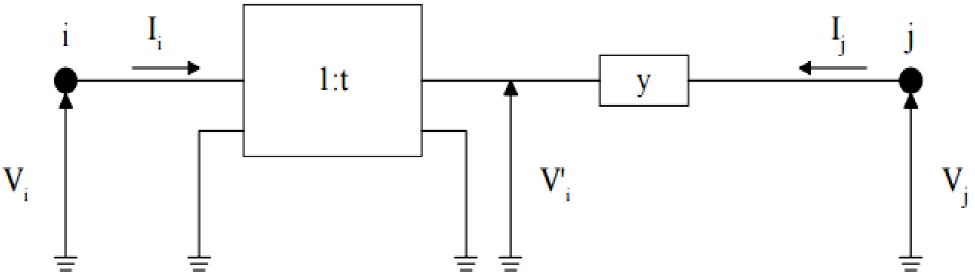

Figure 1. Transformer equivalent circuit

The voltage and current ratio can be then defined as follows:

$$V_i : V_i' = 1 : t \text{ and } I_i : I_i' = t^* : 1 \text{ due to } V_i^* I_i = V_i'^* I_i' \qquad \text{(T1)}$$

where:

- $*$ -refers to conjugate complex number
- V_i -is the complex voltage at the *i* end of the line *i-j*,
- V_i' -is the complex voltage behind the ideal transformer,
- V_j -is the complex voltage at the *j* end of the line *i-j*,
- I_i -is the complex current at the *i* end of the line *i-j*,
- I_i' -is the complex current behind the ideal transformer,
- I_j -is the complex current at the *j* end of the line *i-j*,
- t -refers to the complex tap ratio of the transformer.

The transformer equivalent circuit shown in Fig. 1. can be transformed to an equivalent π circuit using the following equations:

$$I_i = t^* I_i' = t^*(V_i' - V_j)y = t^*(tV_i - V_j)y = t^2 V_i y - t^* V_j y,$$

$$I_j = (V_j - V_i')y = (V_j - tV_i)y = -tV_i y + V_j y,$$

or in a matrix form:

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} t^2 y & -t^* y \\ -ty & y \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{T2}$$

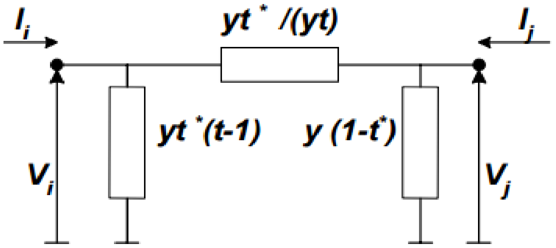

Figure 2. Comprehensive branch model for8 transformers

Based on equation (T2) a comprehensive branch model is shown in Fig. 2. It should be noted that only phase shifter transformer has $y_{ij} \neq y_{ji}$, while for all others types of transformer $t^* = t$ and consequently $y_{ij} = y_{ji}$. Besides, this branch model assumes that the transformer admittance is behind the off nominal side of transformer. Some other branch models are given in [1].

The complex line flow from node *i* to node *j* can be formulated as:

$$S_{ij} = V_i^* \{yt^*(t-1)V_i + (V_i - V_j)yt^*\} = V_i^2 t^2 y - V_i^2 t^* y + V_i^2 t^* y - V_i^* V_j t^* y,$$

$$S_{ij} = V_i^2 t^2 y - V_i^* V_j t^* y, \tag{T3}$$

Using polar coordinates the voltages, tap ratio and admittance can be written as follows:

- $V_i = V_i e^{j\theta_i}, V_i = \|V_i\|, \theta_i = \angle V_i,$ (T4)
- $V_j = V_j e^{j\theta_j}, V_j = \|V_j\|, \theta_j = \angle V_j,$ (T5)

- $\boldsymbol{t} = \mathrm{t}e^{j\theta}, \mathrm{t} = \|\boldsymbol{t}\|, \theta = \angle \boldsymbol{t}$, (T6)
- $\boldsymbol{y} = \mathrm{y}e^{\boldsymbol{j}\psi} = \mathrm{g} + \boldsymbol{j}\mathrm{b}, \psi = \arctan\frac{\mathrm{b}}{\mathrm{g}}$. (T7)

Substituting the complex variables with the polar coordinates given in equations (T4-T7), equation (T3) can be rewritten as:

$$\boldsymbol{S}_{ij} = V_i^2 t^2 (g + \boldsymbol{j}b) - V_i V_j t y e^{-\boldsymbol{j}(\theta_i - \theta_j - \psi + \theta)}), \tag{T8}$$

در رابطه با سوال ۸۷ بررسی

سلام لطفا بدید مهندس عرب بررسی کنه ببینه بازم با افزایش تپ توان راکتیو کاهش پیدا میکنه ممنون

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