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Stochastic programming analysis and solutions to schedule overcrowded operating rooms in China

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ABSTRACT

As a result of the growing demand for health services, China's large city hospitals have become markedly overstretched, resulting in delicate and complex operating room scheduling problems. While the operating rooms are struggling to meet demand, they face idle times because of (human) resources being pulled away for other urgent demands, and cancellations for economic and health reasons. In this research we analyze the resulting stochastic operating room scheduling problems, and the improvements attainable by scheduled cancellations to accommodate the large demand while avoiding the negative consequences of excessive overtime work. We present a three-stage recourse model which formalizes the scheduled cancellations and is anticipative to further uncertainty. We develop a solution method for this three-stage model which relies on the sample average approximation and the L-shaped method. The method exploits the structure of optimal solutions to speed up the optimization. Scheduled cancellations can significantly and substantially improve the operating room schedule when the costs of cancellations are close to the costs of overtime work. Moreover, the proposed methods illustrate how the adverse impact of cancellations (by patients) for economic and health reasons can be largely controlled. The (human) resource unavailability however is shown to cause a more than proportional loss of solution value for the surgery scheduling problems occurring in China's large city hospitals, even when applying the proposed solution techniques, and requires different management measures.

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1. Introduction

In the first decade of the present millennium, China's GDP has grown at an average rate of more than 10% [35]. These economic developments have gone hand in hand with social and demographic developments. The urban population grew from 452 million to 721 million [45], the public transportation system improved considerably, and health insurance coverage grew from below 30% around the turn of the millennium to over 95% in 2011 [34]. These changes have driven an enormous growth in demand for health services, and in health expenditures of which 71% are accounted for by hospitals [2]. As a result of these developments, particularly the demand for services at the large (level 3) hospitals in big cities increased [38]. Despite a tenfold growth in government spending on health [28] and a growth in the number of hospitals by more than 40% since the year 2000 [36], the increase in health service

capacity has not been able to cope with the rising demand. The level 3 hospitals in big cities have become markedly overstretched [41]. These phenomena are concretely illustrated by the 2013 data provided for the purpose of the analysis presented in this manuscript by Shanghai General Hospital, where the actual average surgical workload exceeded the daily capacity by as much as 20%, and average operating room opening hours are almost 14 h daily.

Because a referral system is lacking, an important part of the increased demand directly reaches the hospitals in the form of ever higher numbers of outpatients, which tend to pull away physicians and other staff from wards and operating rooms. The number of outpatient visits to hospitals has grown from 2.12 billion per year to 3.45 billion per year in the first decade of the new millennium [29]. The increase in outpatient services may cause physicians to be late for operating room shifts or to be called away during operating room shifts, causing idle time at the operating room. (From the complete operating room data for the year 2013, we estimate that idle time at Shanghai General Hospital is around 17%.) In the same decade, the number of inpatient visits in China has more than doubled from 53 million to 133 million annually. Meara et al. [33] recently conservatively estimated the annually

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foremost in appointment scheduling for outpatients. For instance LaGanga and Lawrence [24] and Berg et al. [4] use overbooking to hedge against patient no-show and present simulation results showing a significant improvement in access and provider productivity, while increasing both patient wait times and provider overtime.

With regard to surgery scheduling, May et al. [32] conclude from a literature review that 'it remains to be seen if the existing results and observations regarding manufacturing replanning and rescheduling would extend to surgery' (where rescheduling refers to the possibility to adjust the initial schedule during execution). Much of the literature on surgical scheduling optimizes the sequence and schedule for a fixed pool of patients while taking the stochastic nature of several problem parameters, especially surgery duration, into account. Mancilla and Storer [31], Denton et al. [12] and Berg et al. [4] simultaneously consider patient waiting time, resource idle time, and overtime. Xiao et al. [47] propose an adaptive scheduling approach for a problem that is closely related to the one considered in this paper, yet without considering cancellation. Stepaniak et al. [39] present a simulation study on cancellation, which they refer to as 'patient rejection'. Formal scheduling models which explicitly include cancellation, as is particularly relevant for overcrowded hospitals, appear to have received little or no attention in operating room scheduling so far.

The scheduling process we adopt matches a multiple stage stochastic programming approach. Standard two-stage stochastic programs with linear or convex functions are often solved using the L-shaped method or Bender's decomposition [44,6,7]. However, our recourse decision (scheduled cancellations) is still anticipative to further uncertainty, namely the second shift surgery durations, unavailability and cancellations. As such, the decision problem can be viewed as a three-stage recourse model [5,6]. Solving the scheduling problem is further complicated because the recourse function is integer. Laporte and Louveaux [26] propose modified L-shaped decomposition with adjusted optimal cuts for two stage stochastic program with integer recourse. Angulo et al. [1] alternately generate optimal cuts of the linear sub-problem and the integer sub-problem, which improves the practical convergence (see also [15,8]). We follow a sample average approximation approach (SAA) which uses this framework. Moreover, we prove and exploit a specific relationship between the first-stage realization and the optimal number of scheduled cancellations to speed up the computation of integer cuts. We use Jensen's inequality [17] to upper bound the minus second (and third) stage cost, a technique that was proposed by Batun et al. [3].

We now review studies on the occurrence and cause of surgery cancellations. Cancellation of surgery is a common phenomenon globally and appears to be more frequent in developing countries. For instance, Kumar and Gandhi [23] (India) report that 17.6% of scheduled surgeries are canceled on the day of surgery. Several authors, e.g., Kumar and Gandhi [23], Kolawole and Bolaji [22] (Nigeria), Chiu et al. [10] (China), Chalya et al. [9] (Tanzania), analyze causes of cancellation, citing variations and prolonged durations of previous surgeries as a prime source. A Daily Briefing [11] report discusses a case study in the USA in which 6.7% of scheduled surgeries in 2009 are canceled, one-third of which was due to hospital related causes, such as poor scheduling. In addition, Yoon et al. [49] (Korea), Hussain and Khan [16] (Pakistan), Perroca et al. [37] (Brazil) and Fernando et al. [14] (UK) explore cancellations. The latter authors point at the management role to address the inefficiencies that cancellations may cause. The Lancet Commission on Global Surgery posits that management might be even more important in settings in which maximal use of the few available resources is a practical necessity to advance on meeting the unmet global need of 143 million surgeries yearly [33].

Various authors report cancellation rates of between 10% and

15% for Chinese hospitals. Jiang et al. [18] report that 12.88% of children's elective surgeries are canceled in Hunan children's hospital in 2010 due to emergent infection (70.30%), inappropriate preoperative preparation (15.12%), poor scheduling and other factors (14.58%). Jie et al. [19] take a statistical analysis on Guangdong General Hospital, which is a large general hospital, and show that the cancellation rate is at 11.2%. Causes for cancellations are patients' illnesses (65.97%), lack of preoperative preparations (14.03%), economic reasons and risk concerns (10.99%), and accidents (9.01%). (Economic reasons refer to the patients inability to pay.) Li et al. [27] study cancellation at Zunyi Medical College, and report as main causes of cancellation: upper respiratory tract infection (18.39%), high blood pressure (12.86%), lack of preoperative preparation (11.79%), and economic concerns (9.64%). Xiang et al. [46] report a cancellation rate of 5.1% caused by recent changes in health conditions (55.8%), patients' determination changes (23.1%), and poor scheduling. Zhang et al. [50] report a 2010 case study and find that the cancellation rate is 13.9%, due to illnesses (68.7%), exogenous cancellations (20.3%), and preoperative preparations (7.7%). The reader may refer to Xu et al. [48] for related work. Next to scheduling related reasons, several of these authors mention the length of schedules and workload as reasons for *scheduled cancellations*.

Briefly reflecting on these causes of cancellations, we notice that they are mostly attributed to emergent infection, illness, recent changes in health condition and the like. It is not uncommon that these conditions relate to hospital acquired infections, which are preventable. Procedures for hospitalization and infection prevention may reduce the prevalence of these cancellations. Another important source of cancellation stems from the high out-of-pocket (co-)payments patients have difficulty to effectuate. Improvements in health insurance coverage, as currently in progression, may reduce the number of these economically driven cancellations. In our computational experiments we explore scenarios in which exogenous cancellations are less frequent.

3. The model

3.1. Problem description and notation

For the single operating room scheduling problem under consideration, we denote by \hat{t} the regular working time. For example, in Shanghai General Hospital, \hat{t} equals 570 min (9.5 h). An initial schedule is made at least one day ahead. This initial schedule specifies a sequence for the patients and expected starting times of their surgeries. The patients to be scheduled are selected from a given set $I = \{1, 2, \dots, n_p\}$. The reward of performing surgery on patient $i \in I$ equals r_i . This reward can be interpreted strictly financially, in which case it corresponds to the associated hospital revenue [13], or can be defined more broadly to incorporate for instance also the benefits for the patients (see also [47]). Notice that in the latter case, the corresponding values may not be readily available from hospital information systems. Scheduled cancellation of surgery for patient $i \in I$ leads to a penalty of c_i , which can in turn be a financial penalty incurred by the insurer, including wasted pre-operative costs, and more generally including patient inconveniences and losses of health.

Each patient $i \in I$ has an associated surgical time distribution, which will be denoted by ξ_i' . We assume that the surgery times for different patients are independent. We also include a probability of exogenous cancellation, which will be denoted by p_i for all $i \in I$. There are no rewards for exogenously canceled surgeries and they do not take time except for a constant t_d switching time. For each patient $i \in I$, selecting patient i thus consumes ξ_i' time units of

operating theater capacity with probability $1 - p_i$, and t_d time units of capacity with probability p_i . We let ξ_i represent this compound random variable which equals ξ'_i with probability $1 - p_i$ and t_d with probability p_i .

In many practical contexts, a number of patients may have the same characteristics (from the perspective of scheduling), because they have to undergo the same procedure. To accommodate this, if for patients i and i' we have that $r_i = r_{i'}$, $c_i = c_{i'}$, $p_i = p_{i'}$, and that ξ'_i and $\xi'_{i'}$ are identically distributed, then we will say that patients i and i' belong to the same surgery class. This will be denoted by $i \sim i'$. Let it be noted however that ξ'_i and $\xi'_{i'}$ will still be independent. More explicitly, while patients may share characteristics, the surgery time distributions ξ'_i and $\xi'_{i'}$ of each pair of patients i and i' are independent, even if ξ'_i and $\xi'_{i'}$ are identically distributed.

As outlined in the introduction, unavailability of surgical resources (staff and/or facilities) is another important source of uncertainty which reduces the effective time available for surgery in the operating room. We thus introduce η_1 and η_2 , which represent the total length of such interruptions in the first and second shift, respectively.

In practice, decision making regarding scheduled cancellations may for instance take place daily at a fixed moment in time (see e.g. [39] for example set this moment at 2 PM). We adopt a different approach, which guarantees a first shift of patients that their surgeries will be scheduled, and allows to inform a second shift of scheduled patients that they will either receive final confirmation or notification of cancellation after the first shift is completed. We consider this approach to be more patient centered as it eliminates uncertainty for the first shift patients and provides clarity to all others after this first shift has been completed. To this purpose, we set the moment of decision making on scheduled cancellations upon completion of half of the scheduled patients (rounded down in case of an odd number of patients). The time of completion of the first shift therefore forms the recourse moment in the proposed multi-stage stochastic programming approach. The second stage thus entails to decide on possible scheduled cancellations of surgeries for patients scheduled in the second shift. After this recourse moment, the second shift surgery durations are revealed and final costs are incurred, making the problem a three-stage recourse model [6].

Following current practice, we assume that scheduled cancellations always regard the last patients in the sequence implied by the surgical schedule, working backwards through the sequence if more than one scheduled patient is canceled. To model the scheduled cancellations we introduce *positions*. All patients scheduled in the first shift are considered to be in position $j=0$, because their order is inconsequential from the viewpoint of our model. For the second shift, we introduce positions $j \in \{1, \dots, \kappa\}$, that are to be filled sequentially, starting from position 1. We later comment on how to set κ . The set of all positions will be denoted by $\{0, 1, \dots, \kappa\}$; this includes the first and second shifts.

We introduce binary decision variables x_{ij} , $i \in I$, $j \in \{0, 1, \dots, \kappa\}$, where x_{ij} equals 1 if patient i is scheduled in the j th position, and 0 otherwise. For convenience, let $\mathbf{x} = \{x_{ij} | i \in I, j \in \{0, 1, \dots, \kappa\}\}$. By interpretation, $\sum_{i=1}^{n_p} x_{i0}$ represents the number of patients scheduled for the first shift. Second shift slots $j \in \{1, \dots, \kappa\}$ may contain at most a single patient. To balance the patient numbers between the shifts as described above, we use the restriction $\sum_{i=1}^{n_p} x_{i0} = \lfloor \sum_{i=1}^{n_p} \sum_{j=0}^{\kappa} x_{ij} / 2 \rfloor$, where $\lfloor x \rfloor$ is the largest integer no greater than x . We thus need no more than $\lfloor n_p / 2 \rfloor + 1$ second shift positions, and may set $\kappa = \lfloor n_p / 2 \rfloor + 1$ accordingly.

To specify the three-stage recourse model with (integer) recourse, we create i.i.d. copies s_j of each random variable ξ_i , which will represent the surgery times in the first shift. Variables s_i and

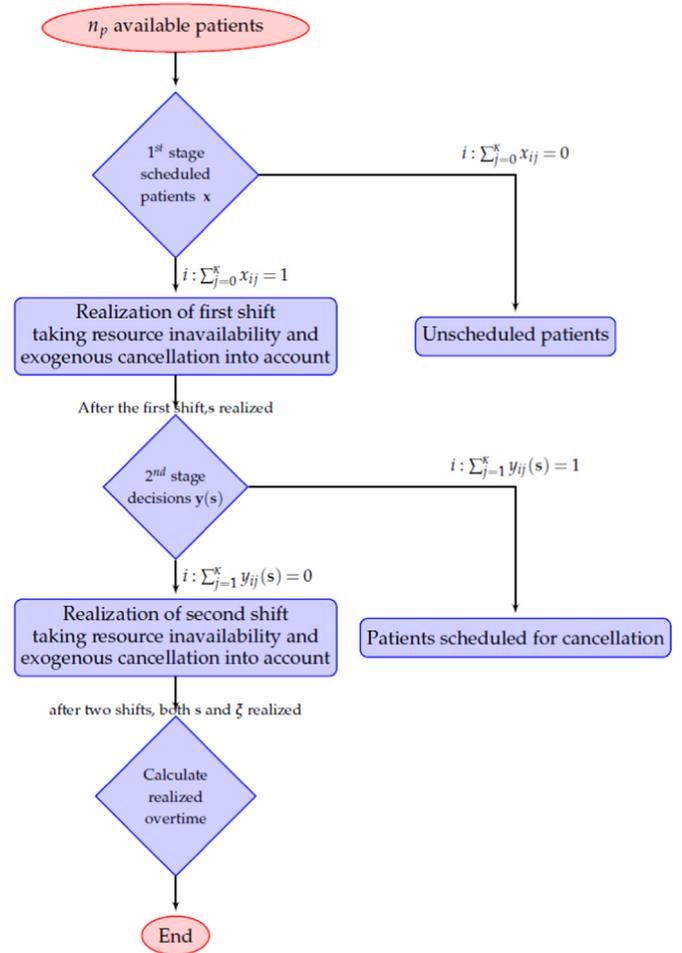


Fig. 1. A chart showing the flow of the patients in the various decision stages in our scheduling problem.

ξ_i follow the same distribution but are independent. We then denote the first shift of the schedule by $\mathbf{s} = (s_1, s_2, \dots, s_{n_p}, \eta_1)$, and the second shift by $\xi = (\xi_1, \dots, \xi_{n_p}, \eta_2)$. We set the rewards for patients corresponding to exogenous cancellations to zero. Thus, reward loss due to exogenous cancellations can be modeled as $\sum_{i=1}^{n_p} I_0(s_i) r_i x_{i0}$, and the indicator function $I_0(s_i) = 1$ if $s_i = t_d$, and 0 otherwise. Next consider the binary decision variables $y_{ij}(\mathbf{s})$, $i \in I$, $j \in \{1, \dots, \kappa\}$, which depend on the outcome of \mathbf{s} . We let $y_{ij}(\mathbf{s}) = 1$ if treatment of patient i in slot j is canceled under scenario \mathbf{s} , and $y_{ij}(\mathbf{s}) = 0$ otherwise. For convenience, let $\mathbf{y} = \{y_{ij} | i \in I, j \in \{1, \dots, \kappa\}\}$. Scheduled cancellation of patient i results in a penalty r_i . Moreover, scheduled cancellations require zero time. The total amount of time that schedule (\mathbf{x}, \mathbf{y}) takes is therefore $\sum_{i=1}^{n_p} s_i x_{i0} + \eta_1 + \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} [x_{ij} - y_{ij}(\mathbf{s})] \xi_i + \eta_2$. The loss of reward in the second stage due to exogenous cancellation is $\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} [x_{ij} - y_{ij}(\mathbf{s})] r_i I_0(\xi_i)$.

We assume that overtime work incurs a cost, which may include financial costs such as salary, employee dissatisfaction, and patient safety risks, which increase with the duration of overtime (see also Section 1). We therefore model the overtime cost function to be piecewise linear and convex, as illustrated in the example in Fig. 2. In the example overtime starts after 570 min and overtime cost per time unit becomes more expensive per time unit after 120 min of overtime.

formulation nevertheless allows to deduce some structural properties.

Lemma 1. *The second stage decision problems $Q(\mathbf{x}, \mathbf{s})$ and $\tilde{Q}(\mathbf{x}, \mathbf{s})$ are equivalent for any $\mathbf{x} \in X$ and realization of \mathbf{s} .*

All proofs of lemmas and propositions are provided in [Appendix A](#). This lemma yields the following equivalent formulation of (1), which will be analyzed in the next section:

$$\max_{\mathbf{x} \in X} \sum_{i=1}^{n_p} \sum_{j=0}^k r_i \cdot x_{ij} - E_s \tilde{Q}(\mathbf{x}, \mathbf{s}) \quad (15)$$

4. Analytical Insights

4.1. Structural properties of the second stage problem

In this section we develop a relation between the capacity used by the first shift and the cancellations in the second shift for a fixed schedule $\mathbf{x} \in X$. Firstly, we introduce some notations:

$$\hat{S} = \sum_{i=1}^{n_p} x_{i0} s_i + \eta_1 \quad (16)$$

$$R(\mathbf{s}) = \sum_{i=1}^{n_p} I_0(s_i) r_i x_{i0} \quad (17)$$

$$g(\hat{S}, k_\xi) = \sum_{i=1}^{n_p} \sum_{j=k_\xi+1}^k c_i x_{ij} + E_\xi \left[D \left[\hat{S} + \sum_{i=1}^{n_p} \sum_{j=1}^{k_\xi} \xi_i x_{ij} + \eta_2 \right] + \sum_{i=1}^{n_p} \sum_{j=1}^{k_\xi} I_0(\xi_i) r_i x_{ij} \right] \quad (18)$$

$$f(\hat{S}) = \min_{0 \leq k_\xi \leq k_{\max}} g(\hat{S}, k_\xi) \quad (19)$$

$$k_\xi^* = \max \{ \arg \min_{0 \leq k_\xi \leq k_{\max}} g(\hat{S}, k_\xi) \} \quad (20)$$

Thus, we let \hat{S} denote the total realized time of the first shift for a given first stage solution \mathbf{x} and we let $g(\hat{S}, k_\xi)$ be the corresponding second stage cost (excluding $R(\mathbf{s})$) when k_ξ patients are kept in the second shift. By k_ξ^* , we denote the optimal number of patients to keep (not scheduled for cancellation), i.e., the index minimizing $g(\hat{S}, k_\xi)$, choosing the largest possible index in case of a tie. The associated minimum cost is denoted by $f(\hat{S})$.

Proposition 1. *Let $\mathbf{x} \in X$ be given, and conditioned on \hat{S} , then $g(\hat{S}, k_\xi)$ is a supermodular function.*

With Proposition 1 at hand, we can then prove that:

Proposition 2. *Let $\mathbf{x} \in X$ be given, and consider two realizations of the total time of the first shift: \hat{S}_1 and \hat{S}_2 with $\hat{S}_1 \leq \hat{S}_2$, then $k_{\hat{S}_1}^* \geq k_{\hat{S}_2}^*$.*

Since $Q(\mathbf{x}, \mathbf{s})$ and $\tilde{Q}(\mathbf{x}, \mathbf{s})$ are equivalent by Lemma 1, the intuitive practical interpretation of this result is that the number of scheduled cancellations increases with the length of realization of the first shift. The result will be used in the L-shaped method to accelerate the solution of the [integer subproblem](#).

We now rewrite the second stage cost function (18) conditioned on \hat{S} as follows: $F(\hat{S}, \mathbf{y}) = \sum_{i=1}^{n_p} \sum_{j=1}^k c_i \cdot y_{ij} + E_\xi [D[\hat{S} + \sum_{i=1}^{n_p} \sum_{j=1}^k \xi_i (x_{ij} - y_{ij}) + \eta_2] + \sum_{i=1}^{n_p} \sum_{j=1}^k I_0(\xi_i) r_i (x_{ij} - y_{ij})]$. Because the L-shaped method requires convexity, the following result

is helpful to solve the relaxed model with continuous recourse:

Lemma 2. *Let $\mathbf{x} \in \bar{X}$ be given and \hat{S} be defined by (16), then $F(\hat{S}, \mathbf{y})$ is convex in $\mathbf{y} \in \bar{Y}(\mathbf{x})$.*

Observing that $R(\mathbf{s})$ is independent of \mathbf{y} , we therefore also have that the second stage objective function is convex in $\mathbf{y} \in \bar{Y}(\mathbf{x})$. The convexity of the second stage objective function in $\mathbf{y} \in \bar{Y}(\mathbf{x})$ will be used in the L-shaped method in [Section 5.3](#) to approximately evaluate the original subproblem with integer recourse.

We conclude this section by a general convexity result for the minimum cost function of the continuous relaxation of the second stage problem, which is further used in [Section 4.2](#).

Proposition 3. *Let $\mathbf{x} \in X$ be given and \hat{S} be defined by (16), then $f(\hat{S}) = \min_{\mathbf{y} \in \bar{Y}} F(\hat{S}, \mathbf{y})$ and $f(\hat{S})$ is convex in \hat{S} . Besides, $\sum_{i=1}^{n_p} I_0(s_i) r_i x_{i0}$ is also convex in \mathbf{s} .*

4.2. Convexity of the second stage problem

We now proceed to derive optimality cuts for the integral master problem and its continuous relaxation on the basis of Jensen's inequality. By [Proposition 3](#) and [Lemma 1](#), we can apply Jensen's inequality [17] to obtain

$$E(f(\hat{S}) + R(\mathbf{s})) \geq f(E(\hat{S})) + R(E(\mathbf{s})) \quad (21)$$

By definition, $\min_{\mathbf{y} \in \bar{Y}} F(\hat{S}, \mathbf{y}) \geq f(\hat{S})$ for $\forall \hat{S}$. Now, by taking expectation on both sides and using inequality (21), we can further derive that

$$E(\min_{\mathbf{y} \in \bar{Y}} F(\hat{S}, \mathbf{y}) + R(\mathbf{s})) \geq f(E(\hat{S})) + R(E(\mathbf{s})) \quad (22)$$

We will use inequalities (21) and (22) to strengthen our L-shaped algorithm by formulating valid inequalities for continuous and integral master problems, cf. [Batun et al. \[3\]](#).

5. Solution methods

As our research questions require to compare the optimal solutions of various models and parameter settings, we now set out to describe solution techniques designed to present near to optimal solutions. More specifically we present a solution method based on SAA in [Section 5.2](#). Because of the stochasticity still involved after the second stage, we require many samples to accurately represent the stochastic nature of the problem, which makes the SAA approach non-standard and computationally challenging. We use the theoretical results derived in [Section 4](#) to reduce the computation times required to solve the SAA in [Section 5.3](#). In [Appendix B.2](#), the resulting formulation is strengthened using Jensen's inequalities.

5.1. Linearizing the objective function

In order to formulate the SAA as a MIP, we linearize the objective function by writing the overtime cost function as follows:

$$D(x) = \min_{\phi_v} \sum_{v=0}^q \tau_v \phi_v$$

$$\text{s.t.} \quad \sum_{v=0}^q \phi_v = x$$

$$\phi_v \in [0, l_v]$$

Note that each piecewise linear convex function on $[0, \infty)$ with $q + 1$ breakpoints can be written in this fashion. Here, the length of interval $v \in \{0, \dots, q\}$ is l_v , and its slope is τ_v . The slopes should

satisfy $\tau_u \geq \tau_v$ for $u \geq v$.

5.2. SAA formulation

For the SAA, we use \hat{n} independent samples of \mathbf{s} , for which we will use the index $n \in \{1, \dots, \hat{n}\}$, and \hat{m} independent samples of ξ , for which we will use the index $m \in \{1, \dots, \hat{m}\}$. Denote the first shift surgery time for patient i for sample n by s_{in} , and the time lost due to resource unavailability by η_{1n} . Denote the second shift surgery time and time lost due to resource unavailability for sample m by ξ_{im} and η_{2m} , respectively. Solving the problem consists in finding first stage decisions $\mathbf{x} = \{x_{ij}|i \in I, j \in \{0, 1, \dots, \kappa\}\}$, and for each sample $n \in \{1, \dots, \hat{n}\}$ a second stage decision $\mathbf{y}(n) = \{y_{ij}(n)|i \in I, j \in \{0, 1, \dots, \kappa\}\}$, such that each $\mathbf{y}(n) \in Y(\mathbf{x})$. Here, $\mathbf{y}(n)$ is short for $\mathbf{y}(\mathbf{s}_n)$.

We now formulate the associated sample average approximation (SAA) for (1):

$$\max_{\mathbf{x} \in X} \sum_{i=1}^{n_p} \sum_{j=0}^{\kappa} r_i x_{ij} - \frac{1}{\hat{n}} \sum_{n=1}^{\hat{n}} Q(\mathbf{x}, n) \tag{23}$$

where

$$Q(\mathbf{x}, n) = \min_{\mathbf{y}(n), l} \left[l + \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} c_i y_{ij}(n) + \frac{1}{\hat{m}} \sum_{m=1}^{\hat{m}} \left[\left(\sum_{v=0}^q \tau_v \phi_v(n, m) \right) - \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} l_0(\xi_{mi}) r_i y_{ij}(n) \right] \right] \tag{24}$$

$$\text{s.t. } l \geq \sum_{i=1}^{n_p} l_0(s_{in}) r_i x_{i0} + \frac{1}{\hat{m}} \sum_{m=1}^{\hat{m}} \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} l_0(\xi_{mi}) r_i x_{ij} \tag{25}$$

$$\sum_{v=0}^q \phi_v(n, m) = \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_{mi}(x_{ij} - y_{ij}(n)) + \eta_{2m} + \sum_{i=1}^{n_p} s_{in} x_{i0} + \eta_{1n}, \tag{26}$$

$$\forall n \in \{1, \dots, \hat{n}\}, m \in \{1, \dots, \hat{m}\}$$

$$\phi_v(n, m) \in [0, l_v], \quad \forall n \in \{1, \dots, \hat{n}\}, m \in \{1, \dots, \hat{m}\}, \tag{27}$$

$$v \in \{0, \dots, q\}$$

$$\mathbf{y}(n) \in Y(\mathbf{x}), \quad \forall n \in \{1, \dots, \hat{n}\} \tag{28}$$

where l is introduced to simplify the formulation. Note that by definition, the set inclusions $\mathbf{x} \in X$ and $\mathbf{y}(n) \in Y(\mathbf{x})$ can be expressed using linear inequalities and binary variables. For example, $\mathbf{y}(n) \in Y(\mathbf{x})$ can be expressed using (9)–(11), with $y_{ij}(n)$ taking the place of y_{ij} . We let $Q(\mathbf{x}) = \frac{\sum_{n=1}^{\hat{n}} Q(\mathbf{x}, n)}{\hat{n}}$.

For the L-shaped method introduced in Section 5.3 we will also use the continuous recourse relaxation $Q_{LP}(\mathbf{x}, n)$ of $Q(\mathbf{x}, n)$, which is obtained by relaxing (28) to

$$\mathbf{y}(n) \in \bar{Y}(\mathbf{x}). \tag{29}$$

$$\text{We let } Q_{LP}(\mathbf{x}) = \frac{\sum_{n=1}^{\hat{n}} Q_{LP}(\mathbf{x}, n)}{\hat{n}}.$$

5.3. Application of L-shaped method

The L-shaped method iteratively generates feasibility and optimality cuts. For the problem under consideration, only optimality cuts are needed. Denote the set of generated optimality cuts by Θ .

Each optimality cut provides a lower bound to the second stage cost. That is, for every $\mathbf{x} \in X$ and $(\mathbf{v}_k, \rho_k) \in \Theta$ we have that $Q(\mathbf{x}) \geq \mathbf{v}_k^T \mathbf{x} + \rho_k$ and $-Q(\mathbf{x}) \leq -\mathbf{v}_k^T \mathbf{x} - \rho_k$ [26] (here \mathbf{v}_k^T is the transpose of \mathbf{v}_k).

$$\max_{\mathbf{x}} \sum_{i=1}^{n_p} \sum_{j=0}^{\kappa} r_i x_{ij} + \theta \tag{30}$$

$$\text{s.t. } \theta \leq -\mathbf{v}_k^T \mathbf{x} - \rho_k, \quad \forall (\mathbf{v}_k, \rho_k) \in \Theta \tag{31}$$

$$\mathbf{x} \in X \tag{32}$$

Notice that θ bounds the minus of the second stage cost, i.e., θ bounds $-Q(\mathbf{x})$. We will also refer to the *relaxed master problem*, in which (32) is replaced by $\mathbf{x} \in \bar{X}$. In order to strengthen both the master problem and the relaxed master problem, Jensen's inequality is added in the form of an additional constraint involving θ (cf. Appendix B.2).

In the course of our algorithm, we will generate two types of cuts. For the first type, which will be referred to as continuous recourse optimality cuts, we note that for every $\mathbf{x}_l \in \bar{X}$, we can use Benders' decomposition [6] to obtain a cut (\mathbf{v}, ρ) such that $\mathbf{v}^T \mathbf{x}_l + \rho = Q_{LP}(\mathbf{x}_l)$. That is, at \mathbf{x}_l the cut is tight for the continuous recourse relaxation.

For the second type, which will be referred to as integer optimality cuts, note that for every $\mathbf{x}_l \in X$ we may compute $Q(\mathbf{x}_l)$ by solving the integer second-stage problems. We can then generate a cut (\mathbf{v}, ρ) that represents the inequality $\theta \leq -(Q(\mathbf{x}_l) - l_0)(\sum_{(i,j) \in S(\mathbf{x}_l)} x_{ij} - \sum_{(i,j) \notin S(\mathbf{x}_l)} x_{ij} - |S(\mathbf{x}_l)|) - Q(\mathbf{x}_l)$. Here $S(\mathbf{x}_l) = \{(i, j)|x_{ij} = 1\}$. The constant l_0 is a lower bound of $Q(\mathbf{x})$ over $\mathbf{x} \in X$ [1]. We can set $l_0 = 0$ in our case.

To efficiently compute $Q(\mathbf{x})$, we apply the submodularity result derived in Section 4.1. More precisely, the procedure can be described as follows:

- (a) Let Π be a set containing information on cancellations, and initially $\Pi = \emptyset$, \mathbf{x} is a given first stage solution, $n = 1$;
- (b) If $n = 1$, calculate its first stage realization $\hat{\delta}_n$ by (16), get its objective value $Q(\mathbf{x}, n)$ and cancellation decision $\mathbf{y}(n)$ by integer subproblem (24)–(28), and meanwhile store a triple $(\hat{\delta}_n, \hat{\delta}_n, \mathbf{y}(n))$ into Π , here $\hat{\delta}_n$ acts as both a lower bound (LB) and an upper bound (UB) of first stage realization values that lead to cancellation decision $\mathbf{y}(n)$, $n = n + 1$;
- (c) If $n \leq \hat{n}$, calculate first stage realization $\hat{\delta}_n$ by (16),
 1. if $\hat{\delta}_n$ falls in $[LB^\pi, UB^\pi]$ of any triple π in Π , then we directly get its optimal cancellation decision the same as \mathbf{y}^π , evaluate its objective value $Q(\mathbf{x}, n)$;
 2. otherwise, calculate its $\mathbf{y}(n)$ by integer subproblem (24)–(28) and get its objective value $Q(\mathbf{x}, n)$. If the newly calculated $\mathbf{y}(n)$ equals \mathbf{y}^π in any triple π in Π , we update its $UB^\pi = \max\{UB^\pi, \hat{\delta}_n\}$, $LB^\pi = \min\{LB^\pi, \hat{\delta}_n\}$, otherwise add triple $(\hat{\delta}_n, \hat{\delta}_n, \mathbf{y}(n))$ to Π , let $n = n + 1$ and then go to step (c).

Our overall L-shaped algorithm follows the same general structure as the algorithm described in Angulo et al. [1] and can be found in Appendix B.1.

6. Computational results

In this section, we will apply the methods and algorithms developed in this paper to solve instances derived from Shanghai General Hospital data, and analyze how reductions of resource

unavailability and exogenous cancellation can alleviate the problems caused by overcrowding. Moreover, we present comparative analysis on the ESC model which allows scheduled cancellations and the ECO model which does not. To this end, we employ the methods developed in previous sections to obtain lower and upper bounds on the performance of these models, cf. Appendix D. Before discussing the results in Section 6.3, we consider the setup of the experiments in this section.

We consider two scheduling models:

1. Exogenous Cancellations Only (ECO): Patients are scheduled a day ahead, and processed accordingly. (Exogenous cancellations still occur.)
2. Exogenous and Scheduled Cancellations (ESC): Cancellation of surgeries (in reverse order of the scheduled sequence) is allowed after the completion of the first shift (as introduced in Section 3).

The ECO model is obtained by imposing $y_{ij}(s) = 0$ for all i, j in the ESC model.

Surgery time distribution: To apply the methods developed in this paper, we fit surgery time distributions to surgery data collected between October 2013 and October 2014 at Shanghai General Hospital. For practical and statistical reasons, we consider instances containing the six surgery classes with highest volumes over this period. Our tests revealed that the log-normal distribution fits the data well, as is confirmed by the QQ-plots depicted in Appendix C. The corresponding parameters are given in Table 1. Note that the flexibility of the SAA approach can easily deal with the log-normal distribution that is difficult to handle analytically.

The base case: Having estimated these surgery time distributions, we now first construct a basic problem instance, referred to as base case, and consider variations for the purpose of sensitivity analysis. For the base case, we set surgical time distributions for six patient classes based on Table 1. To account for surgery specific set-up times, we add 5 min to the surgery durations, which is close to the median reported setup time. We assume that 3 patients are available for each of the six classes, so $n_p = 18$. On the basis of the evidence reported in Section 2, we set the probability of exogenous cancellations to 15%. Following personal communication and data analysis regarding the time between surgeries which exceeds the regular setup time, we estimate the time lost per exogenous cancellation to be 15 min. Adding 5 min of normal setup time reserved for the next patient, this gives 20 min in total to prepare the next patient in case of exogenous cancellation. Resource unavailability is also derived from Shanghai General Hospital data. We estimate the average daily resource unavailability to equal 2 h, which we divide evenly over the shifts. Specifically, we set resource unavailability for both first and second shift as i.i.d log-

Table 1

The mean (m) and standard deviation (s) (in minutes) of the log-normal distribution with parameters (μ, σ) fitted to data for various surgery classes, surgery classes are sorted in increasing order of mean.

Departments	Index	Number of observations	Log-normal parameters		Mean and std deviation	
			μ	σ	m	s
Obstetrical	1	2949	4.02	.41	60.75	25.75
Gynecology	2	5368	4.11	.88	90.14	97.62
Orthopedic	3	2236	4.70	.59	130.86	84.70
General	4	4003	4.85	.59	152.13	98.91
Thoracic	5	1303	4.98	.52	165.67	91.82
Neurosurgical	6	1234	5.06	.68	197.67	150.42

Table 2

Intervals and slopes of the overtime cost function and associated terminology.

Terminology	Regular time	Regular overtime	Excessive overtime
Interval (min)	[0,570]	(570,690]	(690, ∞)
Slope	0	1.5	2.0

normal distributions with parameters $\mu = 4$ and $\sigma = 0.5$, and thus a mean of approximately 62 min. Table 2 gives the intervals on which the overtime cost function is linear, as well as the slopes for those intervals. Overtime costs are thus only incurred after regular working hours, which has a duration of 9.5 h, and additional costs for excessive overtime are occurred after 11.5 h. Lacking specific financial data, as well as data on health benefits from surgery, we normalize the reward r for each of the surgeries to equal the expected surgical duration m' , where m' equals m plus the five minutes' preparation time. The penalty associated with scheduled cancellation is set to 1.05 times the reward in the base case.

6.1. Results

Section 6.1.1 investigates the performance of the developed solution methods for the base case and three variations. It also presents the comparative analysis between ESC and ECO in terms of optimal solution values. Section 6.1.2 investigates the impact of decreasing resource unavailability and reducing exogenous cancellations as means to alleviate the problems caused by overcrowding.

6.1.1. Comparative analysis

We consider four cases in order to compare the performances of the ESC and ECO policies. The three variations of the base case are obtained by varying the rewards and penalties of the surgery classes. Note that overtime costs, rewards, and cancellation costs should be understood relative to each other: the cost coefficients measure the relative importance of achieving the various conflicting objectives. The final objective is referred to as yield. Table 3 lists the variations and the base case. Remember that m' is the average surgical time including preparation time, which is set at $m' = m + 5$. For the resulting cases, we determine the yields obtained by our algorithms for ESC and ECO, as well as associated upper bounds. The results are summarized in Table 4. The table shows that, with one mild exception, our algorithm consistently finds solutions that are within 1% from the corresponding upper bound. In view of the stochasticity involved in the third stage of the three-stage recourse model, after the scheduled cancellations are decided, we consider this performance satisfactory.

Table 5 compares the solution values obtained for ESC and ECO and provides insight on the benefits of allowing scheduled cancellations. Naturally, these benefits depend on the cancellation cost. The benefit of scheduled cancellations is as much as 11.23% in the base case, and then reduces as scheduled cancellations become

Table 3

The base case and three variations for computing the reward r and cancellation penalty c from the mean m' and standard deviation s of the surgery time plus preparation time.

Case	Reward (r)	Penalty (c)
Base case	m'	$1.05 m'$
Case a	m'	$1.2 m'$
Case b	$m' + 0.5 s$	$1.05(m' + 0.5 s)$
Case c	$m' + 0.5 s$	$1.2(m' + 0.5 s)$

Table 4

The yield obtained by the ESC and ECO scheduling policies using the algorithms developed in this paper, as well as associated upper bounds and optimality gaps.

Policy	Statistic	Base case	Case A	Case B	Case C
ESC	Yield	397.02 ± 0.51	376.29 ± 0.69	609.22 ± 0.69	589.21 ± 0.83
	Upper bound	400.61 ± 0.65	381.15 ± 0.91	614.25 ± 2.12	594.10 ± 1.88
	Gap (%)	(0.90 ± 0.20)	(1.27 ± 0.42)	(0.82 ± 0.46)	(0.82 ± 0.45)
ECO	Yield	356.95 ± 0.62	356.95 ± 0.62	585.47 ± 0.93	585.47 ± 0.93
	Upper bound	359.86 ± 0.49	359.86 ± 0.49	585.47 ± 0.93	585.47 ± 0.93
	Gap (%)	(0.81 ± 0.31)	(0.81 ± 0.31)	(0 ± 0.32)	(0 ± 0.32)

Table 5

The improvement of ESC over ECO for each of four cases, as well as the ratio between the costs of cancelling a surgery versus the cost of performing the surgery in (excessive) overtime.

Statistic	Base case	Case A	Case B	Case C
ESC vs ECO ((ESC-ECO)/ECO, in %)	(11.23 ± 0.32)	(5.42 ± 0.37)	(4.06 ± 0.28)	(0.64 ± 0.30)
Cost ratio of cancellation vs regular overtime	1.05:1.275	1.20:1.275	1.41:1.275	1.61:1.275
Cost ratio of cancellation vs excessive overtime	1.05:1.70	1.20:1.70	1.41:1.70	1.61:1.70

penalized heavier. In comparison to a weak and simple upper bound which assumes that there is a revenue of 1 for every expected non-idle unit of regular operating room time and no cost of cancellation or overtime work, ESC closes around 50% of the gap between this bound and the solution value for ECO. A similar result holds for case B. ESC closes less than 25% of this gap for the cases A and C.

To allow the reader to appreciate the effects of increasing the costs of scheduled cancellations, we tabulate the cost ratio between performing a surgery in overtime and cancelling the surgery, as well as the ratio between performing a surgery in excessive overtime and cancelling the surgery. (Note that the cancellation decision is nontrivial even though these ratios are known: At the moment of deciding on scheduled cancellations there is considerable uncertainty regarding the starting times of second shift surgeries.) These ratios vary case by case. They also depend to a limited extent

on the surgery class, but relative variation is less than 8.86% over the six surgery classes. Table 5 gives the average ratio over the six surgery classes for each case. The calculated ratios account for the probability of exogenous cancellation in the expected surgery durations. Overtime is associated with increased risks of complications and medical errors, as well as dissatisfaction among scarce staff. By consequence, hospitals may prefer scheduled cancellations and delays of the corresponding patients to the next day to performing the surgery in overtime. The *base case* assumes that it is $\frac{1.275}{1.05}$ times more desirable to cancel a surgery than to perform it in regular overtime. For the base case, the yield improvement of ESC over ECO is $(11.23 \pm 0.32)\%$, which shows that there is considerable value in allowing scheduled cancellations, even if the cancellation decision is to be taken already after completing the first shift of at most half of the scheduled patients. Cases A and B represent cases where scheduled cancellations are only $\frac{1.275}{1.20}$ and $\frac{1.275}{1.41}$ times more desirable than performing the corresponding surgeries in regular overtime, while excessive overtime is still much more undesirable relative to scheduled cancellation. In that case, the value of allowing cancellations reduces to $(5.42 \pm 0.37)\%$ and $(4.06 \pm 0.28)\%$, respectively. In Case C, the penalty for scheduled cancellation is so high that the recourse offers little improvement opportunity. It is therefore not surprising that the value of scheduled cancellations is very limited in case C at $(0.64 \pm 0.30)\%$.

6.1.2. Insights

In this section, reward and penalty cost are fixed to the base case, and we explore the impact of reducing exogenous cancellations and resource unavailability to the ESC and ECO policies as means to alleviate hospital overcrowding problems. We vary the rate of exogenous cancellations to be 0%, 5% and 15%. The latter is based on existing evidence reported in the scientific literature (cf. Section 2). The 5% appears to be a lower bound among the values reported in the scientific literature. The 0% scenario merely gives insight in the overall potential of eliminating exogenous cancellations altogether. The results are shown in Fig. 3. The figure shows that exogenous cancellation has a significant adverse impact on performance: For the ESC policy, yields increase by 5% as exogenous cancellation rate decreases from 15% to 0%. For the ECO policy, this increase is 8%. These results show that ESC can better control the adverse impact of cancellations than ECO. For all tested values of exogenous cancellation rate, the ESC policy significantly outperforms the ECO policy by about 9%, which underlines the potential value of scheduled cancellations in dealing with uncertainties, even if they will be reduced in the future.

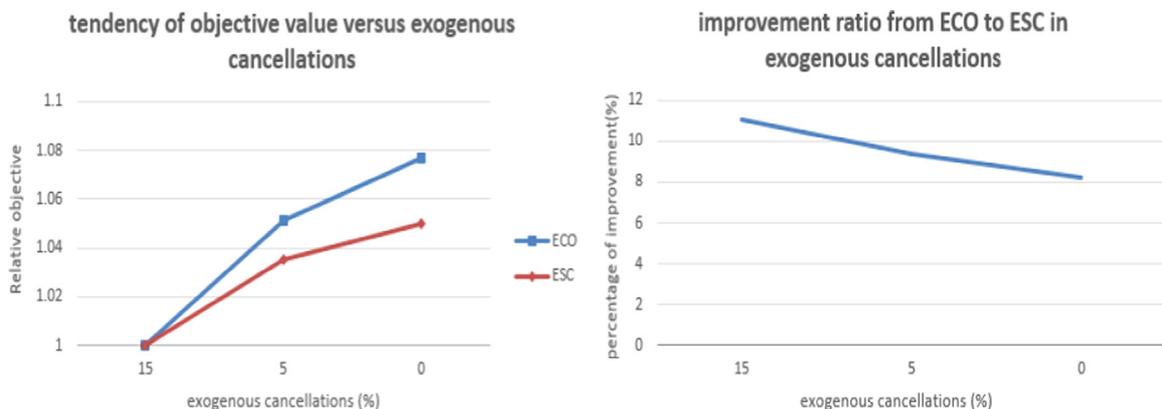


Fig. 3. The impact of the exogenous cancellations. The left figure shows the relative improvement in objective as the exogenous cancellation rate is reduced, where the improvement is measured with respect to the base case of 15% exogenous cancellation rate for both ECO and ESC. The right figure shows the relative improvement of ESC over ECO as the exogenous cancellation rate is reduced.

With these solution methods at hand, we have further analyzed exogenous cancellation and resource unavailability. As the latter may be in the order of 20% of regular opening hours, it is clear that improving unavailability holds great potential to alleviate the problems caused by overcrowding. Our results reveal that – while scheduled cancellations can limit the negative impact of resource unavailability – the overall impact on the solution values is more than proportional to the unavailability and exceeds 25% in the presented instances. A practical implication is therefore that hospitals and patients can greatly benefit from better management and control of the operational deployment of (human) resources to reduce their unavailability.

Although evidence indicates that exogenous cancellation may apply to as much as 15% of scheduled surgeries, it poses fewer difficulties for operating room scheduling and utilization than resource availability. This holds particularly true for the ESC model as its optimal solution value does not improve beyond 5%, even when exogenous cancellation is reduced by the full 15%. The impact for the model without scheduled cancellations is larger, confirming the potential of scheduled cancellations. From a practical operating room management perspective, these results imply that reduction of exogenous cancellations is worth considering after implementation of scheduled cancellations and reducing human resource unavailability. Especially so as the causes of scheduled cancellations are beyond the control of operating room management. As exogenous cancellations often follow from financial barriers and worsening of health status, reducing exogenous cancellations remains of urgency and importance.

While our analysis relies on data from a single hospital, Shanghai General Hospital, we believe that the model, solution methods, and analyses are likely to have relevance for the many other level 3 large city hospitals in China, which are presently overcrowded and face further demand increases. Similar problems occur in other developing countries as well. Our research presents first theoretical advancements on the resulting operating room scheduling problems as well as practical improvement suggestions. At the same time, it is clear that it has limitations and poses new research questions. For example, models which set the recourse moment at a fixed moment in time, or divide the shifts based on minutes workload rather than numbers of patients are worthy of further study. Moreover, one may consider the problem of determining the optimal moment in time, workload minutes, or relative patient number after which to end the first shift. We therefore hope that our research motivates other researchers to advance the work on the presently under-researched urgent operations management problems occurring in the operating rooms of China and other – mostly developing – countries, serving the far majority of the global population.

Appendix A. Proofs of lemmas and theorems

Lemma 1. The second stage decision problems $Q(\mathbf{x}, \mathbf{s})$ and $\tilde{Q}(\mathbf{x}, \mathbf{s})$ are equivalent for any $\mathbf{x} \in X$ and realization of \mathbf{s} .

Proof. Let $y_{ij}(\mathbf{s})$ and $k_{\mathbf{s}}$ are, respectively, the optimal solution to $Q(\mathbf{x}, \mathbf{s})$ and $\tilde{Q}(\mathbf{x}, \mathbf{s})$. Then $y_{ij}(\mathbf{s}) \in Y(\mathbf{x})$ and $k_{\mathbf{s}} \in Z$.

• Let k satisfy $\sum_{i=1}^{n_p} \sum_{j=1}^k x_{ij} = k_{\mathbf{s}}$, next we equivalently transform $k_{\mathbf{s}}$ into a solution $y_{ij}^k(\mathbf{s})$:

$$y_{ij}^k(\mathbf{s}) = \begin{cases} 0, & \forall i = 1, \dots, n_p, j = 1, \dots, k, \\ x_{ij}, & \text{otherwise} \end{cases} \quad (\text{A.1})$$

Obviously, $y_{ij}^k(\mathbf{s})$ is a feasible solution to $Q(\mathbf{x}, \mathbf{s})$, and $\tilde{Q}(\mathbf{x}, \mathbf{s}) \geq Q(\mathbf{x}, \mathbf{s})$.

• Let

$$k_{\mathbf{s}}^y = \sum_{i=1}^{n_p} \sum_{j=1}^k (x_{ij} - y_{ij}(\mathbf{s})) \quad (\text{A.2})$$

then $k_{\mathbf{s}}^y \in Z$ and $\tilde{Q}(\mathbf{x}, \mathbf{s}) \leq Q(\mathbf{x}, \mathbf{s})$.

Summarizing above, we can conclude that $Q(\mathbf{x}, \mathbf{s})$ and $\tilde{Q}(\mathbf{x}, \mathbf{s})$ are equivalent for any $\mathbf{x} \in X$ and realization of \mathbf{s} .□

Proposition 1. Let $\mathbf{x} \in X$ be given, and condition on \hat{s} , then $g(\hat{s}, k_{\hat{s}})$ is a supermodular function.

Proof. To prove that $g(\hat{s}, k_{\hat{s}})$ is supermodular in $(\hat{s}, k_{\hat{s}})$, we should prove that $\forall \hat{s}_1 \geq \hat{s}_2, k_{\hat{s}_1} \leq k_{\hat{s}_2}$,

$$g(\hat{s}_1, k_{\hat{s}_2}) + g(\hat{s}_2, k_{\hat{s}_1}) \geq g(\hat{s}_1, k_{\hat{s}_1}) + g(\hat{s}_2, k_{\hat{s}_2}) \quad (\text{A.3})$$

Expanding their expressions and merge similar items, inequality (A.3) is equivalent to the following:

$$\begin{aligned} E_{\xi} D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{k_{\hat{s}_2}} x_{ij} \xi_i + \hat{s}_1 + \eta_2 \right) + E_{\xi} D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{k_{\hat{s}_1}} x_{ij} \xi_i + \hat{s}_2 + \eta_2 \right) \\ \geq E_{\xi} D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{k_{\hat{s}_1}} x_{ij} \xi_i + \hat{s}_1 + \eta_2 \right) + E_{\xi} D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{k_{\hat{s}_2}} x_{ij} \xi_i + \hat{s}_2 + \eta_2 \right) \end{aligned} \quad (\text{A.4})$$

The convexity of function $D(x)$ in x justifies inequality (A.4) and the proof is done.□

Proposition 2. Let $\mathbf{x} \in X$ be given, and consider two realizations of the total time for the first shift: \hat{s}_1 and \hat{s}_2 with $\hat{s}_1 \leq \hat{s}_2$, then $k_{\hat{s}_1}^* \geq k_{\hat{s}_2}^*$.

Proof. As Proposition 1 showed, $g(\hat{s}, k_{\hat{s}})$ is supermodular in vector $(\hat{s}, k_{\hat{s}})$, by introducing $t = -\hat{s}$, we can get submodular function $g(t, k_{\hat{s}})$, and applying the property of submodular function [43], we can get that $k_{\hat{s}}^*$ increases in t , i.e., $k_{\hat{s}}^*$ decreases in \hat{s} .□

Lemma 2. Let $\mathbf{x} \in \bar{X}$ be given and \hat{s} be defined by (17), then $F(\hat{s}, \mathbf{y})$ is convex in $\mathbf{y} \in \bar{Y}(\mathbf{x})$.

Proof. We will prove that $\forall \mathbf{y}_1 = (y_{ij}^1)_{n_p \times \kappa}, \mathbf{y}_2 = (y_{ij}^2)_{n_p \times \kappa}$ and $\lambda \geq 0$,

$$F(\hat{s}, \lambda \mathbf{y}_1 + (1 - \lambda) \mathbf{y}_2) \leq \lambda F(\hat{s}, \mathbf{y}_1) + (1 - \lambda) F(\hat{s}, \mathbf{y}_2)$$

remark that $\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} I_0(\xi_i) r_i (x_{ij} - y_{ij})$ is linear in \mathbf{y} and make no difference in the convexity, the above inequality holds if

$$\begin{aligned} E_{\xi} D \left[\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i [\lambda (x_{ij} - y_{ij}^1) + (1 - \lambda) (x_{ij} - y_{ij}^2)] + \hat{s} + \eta_2 \right] \\ \leq \lambda E_{\xi} D \left[\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i (x_{ij} - y_{ij}^1) + \hat{s} + \eta_2 \right] \\ + (1 - \lambda) E_{\xi} D \left[\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i (x_{ij} - y_{ij}^2) + \hat{s} + \eta_2 \right] \end{aligned}$$

Since $D(x)$ is convex in x , the second inequality holds for any $\lambda \in [0, 1]$ and $\mathbf{y}_1, \mathbf{y}_2$, and the proposition is true.□

Proposition 3. Let $\mathbf{x} \in X$ be given and \hat{s} be defined by (17), then $f(\hat{s}) = \min_{\mathbf{y} \in \bar{Y}} F(\hat{s}, \mathbf{y})$ and $f(\hat{s})$ is convex in \hat{s} . Besides, $R(\mathbf{s})$ defined in $\sum_{i=1}^{n_p} I_0(s_i) r_i x_{i0}$ is also convex in \mathbf{s} .

Proof. We can easily get $f(\hat{s}) = \min_{\mathbf{y} \in \bar{Y}} F(\hat{s}, \mathbf{y})$ by Lemma 1. Next we will prove that $\forall \hat{s}_1, \hat{s}_2, \hat{s}_1 \geq \hat{s}_2$ and $\lambda \geq 0$,

$$f(\lambda \hat{s}_1 + (1 - \lambda) \hat{s}_2) \leq \lambda f(\hat{s}_1) + (1 - \lambda) f(\hat{s}_2) \quad (\text{A.5})$$

Let $\mathbf{y}_1 \in \bar{Y}$ and $\mathbf{y}_2 \in \bar{Y}$ be, respectively, the optimal solution to $f(\hat{s}_1)$

and $f(\hat{s}_2)$, then $\lambda \mathbf{y}_1 + (1 - \lambda) \mathbf{y}_2$ is a feasible solution to $f(\lambda \hat{s}_1 + (1 - \lambda) \hat{s}_2)$ and

$$f(\lambda \hat{s}_1 + (1 - \lambda) \hat{s}_2) \leq F(\lambda \hat{s}_1 + (1 - \lambda) \hat{s}_2, \lambda \mathbf{y}_1 + (1 - \lambda) \mathbf{y}_2)$$

What's more,

$$\begin{aligned} & \lambda f(\hat{s}_1) + (1 - \lambda) f(\hat{s}_2) - F(\lambda \hat{s}_1 + (1 - \lambda) \hat{s}_2, \lambda \mathbf{y}_1 + (1 - \lambda) \mathbf{y}_2) \\ &= \lambda E_{\xi} [D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - y_{ij}^1) + \hat{s}_1 + \eta_2 \right) + \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} I_0(\xi_i) r_i(x_{ij} - y_{ij}^1)] \\ & \quad + (1 - \lambda) E_{\xi} [D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - y_{ij}^2) + \hat{s}_2 + \eta_2 \right) \\ & \quad + \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} I_0(\xi_i) r_i(x_{ij} - y_{ij}^2)] \\ & \quad - E_{\xi} [D \left(\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - \lambda y_{ij}^1 - (1 - \lambda) y_{ij}^2) + \lambda \hat{s}_1 + (1 - \lambda) \hat{s}_2 + \eta_2 \right) \\ & \quad + \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} I_0(\xi_i) r_i(x_{ij} - \lambda y_{ij}^1 - (1 - \lambda) y_{ij}^2)] \\ &= \lambda E_{\xi} D \left[\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - y_{ij}^1) + \hat{s}_1 + \eta_2 \right] \\ & \quad + (1 - \lambda) E_{\xi} D \left[\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - y_{ij}^2) + \hat{s}_2 + \eta_2 \right] \\ & \quad - E_{\xi} D \left[\lambda \left(\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - y_{ij}^1) + \hat{s}_1 + \eta_2 \right) \right. \\ & \quad \left. + (1 - \lambda) \left(\sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_i(x_{ij} - y_{ij}^2) + \hat{s}_2 + \eta_2 \right) \right] \geq 0 \end{aligned}$$

Since $D(x)$ is convex in x , the last inequality holds and inequality (A.5) is true. Moreover, $I_0(\xi_i)$ is convex in \mathbf{s} , which can directly derive that $R(\mathbf{s})$ is also convex in \mathbf{s} . \square

Appendix B. Application of L-shaped method

B.1. Algorithm of L-shaped method

Our overall L-shaped algorithm follows the same general structure as the algorithm described in Angulo et al. [1]. Based on the above analysis, it can be described as follows:

Algorithm 1.

- Step 0 Initiate $\theta = \emptyset$. Throughout, Θ will be used for the master problem.
- Step 1 Optimize the integral master problem to obtain an optimal solution $\mathbf{x} \in X$ and corresponding objective value z and θ . If $\frac{\theta - (-Q_{LP}(\mathbf{x}))}{(-Q_{LP}(\mathbf{x}))} > \epsilon$, add the corresponding optimality cut (\mathbf{v}, ρ) to Θ and go to Step 1, otherwise go to Step 2.
- Step 2 If $\frac{\theta - (-Q(\mathbf{x}))}{(-Q(\mathbf{x}))} > \epsilon$, add the corresponding integer optimality cut to Θ , and go to Step 1. Otherwise, if $\frac{\theta - (-Q(\mathbf{x}))}{(-Q(\mathbf{x}))} \leq \epsilon$, terminate, designating \mathbf{x} as the ϵ -optimal solution.

B.2. Upper bound by Jensen's inequality

The L-shaped method from the previous section can be

enhanced by adding Jensen's inequality as a constraint to both the integral and relaxed master problem. By the results obtained in Section 4.2, the second stage costs can be bounded from below if all first shift surgeries take on their expected value (for the SAA approach, this translated to replacing the expected value by the sample mean). To explicitly give the constraints, let $\bar{s}_i = \frac{\sum_{n=1}^{\hat{n}} (s_{in})}{\hat{n}}$,

and $\bar{\eta}_1 = \frac{\sum_{n=1}^{\hat{n}} \eta_{1n}}{\hat{n}}$, and let $\bar{\mathbf{y}} = \{\bar{y}_{ij} | i \in I, j \in \{1, \dots, \kappa\}\}$ denote the second stage decisions if all first-stage random variables take on their expected value, in which case $I_0(\bar{s}_i) = 0$. Then,

$$\theta \leq - \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} c_i \bar{y}_{ij} - \frac{1}{\hat{m}} \sum_{m=1}^{\hat{m}} \left[\sum_{v=0}^q \tau_v \bar{\phi}_v(m) + \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} I_0(\xi_{im}) r_i(x_{ij} - \bar{y}_{ij}) \right] \quad (B.1)$$

$$\sum_{v=0}^q \bar{\phi}_v(m) = \sum_{i=1}^{n_p} \sum_{j=1}^{\kappa} \xi_{im}(x_{ij} - \bar{y}_{ij}) + \eta_{2m} + \sum_{i=1}^{n_p} \bar{s}_i x_{i0} + \bar{\eta}_1 \quad (B.2)$$

$$\mathbf{y} \in \bar{Y}(\mathbf{x}) \quad (B.3)$$

$$\bar{\phi}_v(m) \in [0, l_v], \quad m \in \{1, \dots, \hat{m}\}, \quad v \in \{0, \dots, q\} \quad (B.4)$$

These constraints extend the results in Batun et al. [3] for our problem.

Appendix C. QQ-plots of surgery time distribution

The log-normal distribution fits the surgery time data quite well, as is confirmed by the QQ-plots depicted in Fig. C1.

Appendix D. Obtaining performance bounds

The general method for obtaining upper and lower bound estimates from the SAA of two-stage stochastic programs has been discussed in Mak et al. [30] and Kleywegt et al. [21]. Let us recall, however, that the ESC scheduling problem is a three-stage SP. Upper and lower bounds are therefore obtained from the SAA (23)–(28) as follows. (Recall that we are maximizing.) The SAA objective averages all combinations of \hat{n} first shift samples and \hat{m} second shift samples, which equals a total of $\hat{n} \times \hat{m}$ combinations. Because the problem is three-stage, it requires relatively many samples to sufficiently accurately represent the randomness. In our numerical experiments we use $\hat{n} = \hat{m} = 500$, for a total of 250,000 combinations. The target accuracy ϵ of Algorithm 1 is set at 0.5% when running time is shorter than 24 h, and is increased to 2% when this running time bound is exceeded. An upper bound estimate is obtained by averaging the upper bound on the optimal objective value for 10 collections of $\hat{n} \times \hat{m}$ samples. (Thus, the our upper bound becomes weaker as ϵ increases.)

To obtain a lower bound estimate, we select a solution $\mathbf{x}' \in X$ that optimizes the SAA for a 500×500 sample. We then fix the schedule to this \mathbf{x}' , and solve the SAA for a single first shift sample ($\hat{n} = 1$), while setting $\hat{m} = 2000$. This yields a single appropriate cancellation decision for that first shift realization. The outcome for the first shift sample with that cancellation decision is evaluated using a new, independent set of 2000 second shift realizations. This yields an unbiased lower bound estimate. A reliable lower bound estimate with associated standard deviation is obtained by averaging the result of this procedure for 2000 replications, i.e., (1) generate new first shift and second shift

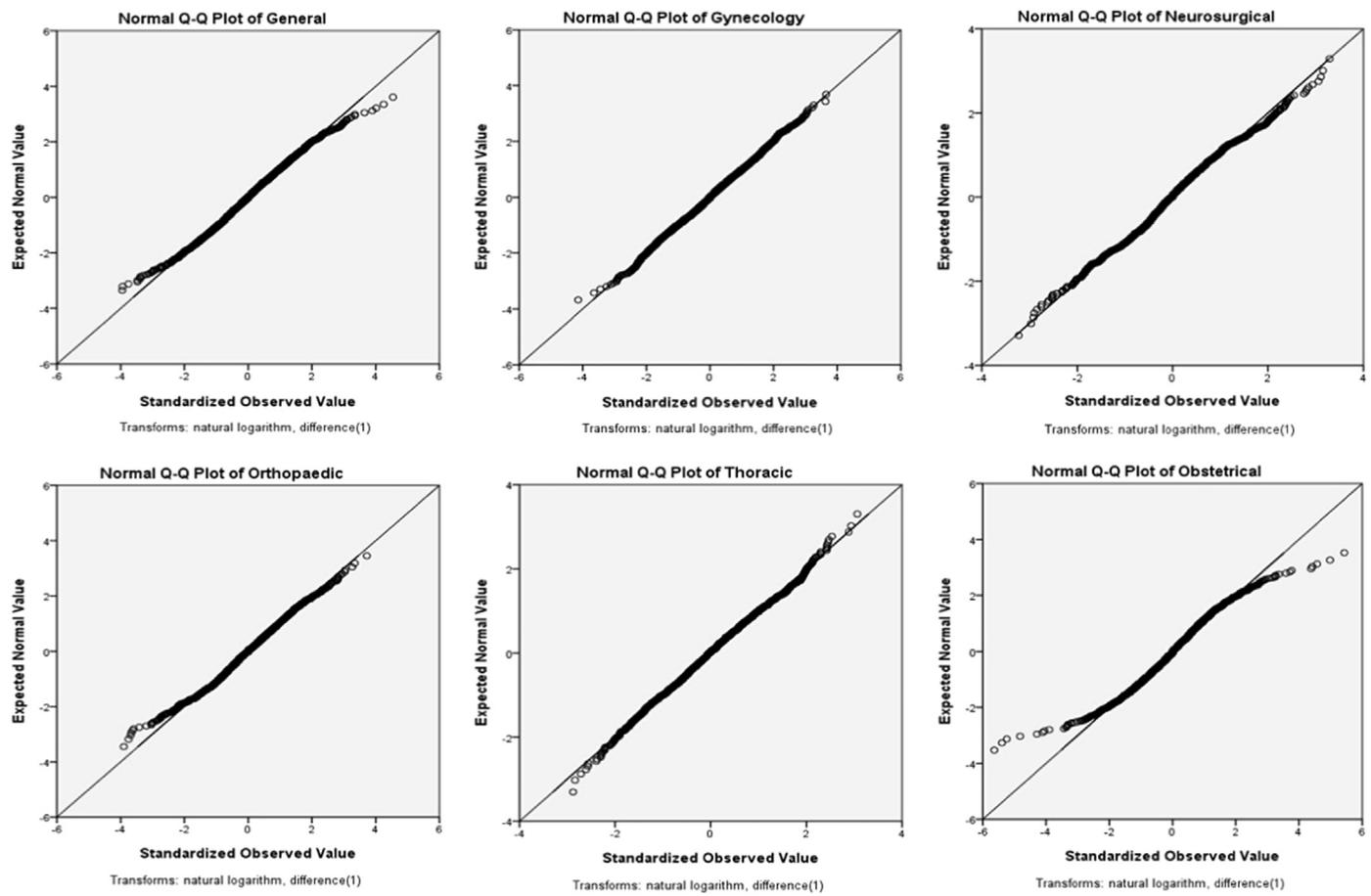


Fig. C1. QQ plots of the data versus the fitted lognormal distribution, for various surgery classes.

samples, (2) determine an appropriate cancellation decision, and (3) evaluate the outcome of the first shift sample and cancellation decision with a new second shift sample.

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