۱- تفاضل پیشرو روش اول:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{f_{i,j+1} - f_{i,j}}{\Delta y}\right) + O(\Delta y) = \frac{1}{\Delta y}\left(\frac{\partial f}{\partial x}|_{i,j+1} - \frac{\partial f}{\partial x}|_{i,j}\right) + O(\Delta y) =$$

$$= \frac{1}{\Delta y}\left(\frac{f_{i+1,j+1} - f_{i,j+1}}{\Delta x} - \frac{f_{i+1,j} - f_{i,j}}{\Delta x}\right) + O(\Delta x, \Delta y) = \frac{f_{i+1,j+1} - f_{i,j+1} - f_{i+1,j} + f_{i,j}}{\Delta y \Delta x} + O(\Delta x, \Delta y)$$

۲- تفاضل پیشرو روش دوم:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{f_{i+1,j} - f_{i,j}}{\Delta x}\right) + O(\Delta x) = \frac{1}{\Delta x}\left(\frac{\partial f}{\partial y}|_{i+1,j} - \frac{\partial f}{\partial y}|_{i,j}\right) + O(\Delta x) =$$

$$= \frac{1}{\Delta x}\left(\frac{f_{i+1,j+1} - f_{i+1,j}}{\Delta y} - \frac{f_{i,j+1} - f_{i,j}}{\Delta y}\right) + O(\Delta x, \Delta y) = \frac{f_{i+1,j+1} - f_{i+1,j} - f_{i,j+1} + f_{i,j}}{\Delta y \Delta x} + O(\Delta x, \Delta y)$$

۳- تفاضل پسرو روش اول:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{f_{i,j} - f_{i,j-1}}{\Delta y}\right) + O(\Delta y) = \frac{1}{\Delta y}\left(\frac{\partial f}{\partial x}|_{i,j} - \frac{\partial f}{\partial x}|_{i,j-1}\right) + O(\Delta y) =$$

$$= \frac{1}{\Delta y}\left(\frac{f_{i,j} - f_{i-1,j}}{\Delta x} - \frac{f_{i,j-1} - f_{i-1,j-1}}{\Delta x}\right) + O(\Delta x, \Delta y) = \frac{f_{i,j} - f_{i-1,j} - f_{i,j-1} + f_{i-1,j-1}}{\Delta y \Delta x} + O(\Delta x, \Delta y)$$

۴- تفاضل پسرو روش دوم:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{f_{i,j} - f_{i-1,j}}{\Delta x}\right) + O(\Delta x) = \frac{1}{\Delta x}\left(\frac{\partial f}{\partial y}|_{i,j} - \frac{\partial f}{\partial y}|_{i-1,j}\right) + O(\Delta x) =$$

$$= \frac{1}{\Delta x}\left(\frac{f_{i,j} - f_{i,j-1}}{\Delta y} - \frac{f_{i-1,j} - f_{i-1,j-1}}{\Delta y}\right) + O(\Delta x, \Delta y) = \frac{f_{i,j} - f_{i,j-1} - f_{i-1,j} + f_{i-1,j-1}}{\Delta y \Delta x} + O(\Delta x, \Delta y)$$

۵- تفاضل سنترال روش اول:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y}\right) + O(\Delta y^2) = \frac{1}{2\Delta y}\left(\frac{\partial f}{\partial x}|_{i,j+1} - \frac{\partial f}{\partial x}|_{i,j-1}\right) + O(\Delta y^2) =$$

$$= \frac{1}{2\Delta y}\left(\frac{f_{i+1,j+1} - f_{i-1,j+1}}{2\Delta x} - \frac{f_{i+1,j-1} - f_{i-1,j-1}}{2\Delta x}\right) + O(\Delta x^2, \Delta y^2)$$

$$= \frac{f_{i+1,j+1} - f_{i-1,j+1} - f_{i+1,j-1} + f_{i-1,j-1}}{4\Delta y \Delta x} + O(\Delta x^2, \Delta y^2)$$

۶- تفاضل سنترال روش دوم:

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(\frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x}\right) + O(\Delta x^2) = \frac{1}{2\Delta x}\left(\frac{\partial f}{\partial y}|_{i+1,j} - \frac{\partial f}{\partial y}|_{i-1,j}\right) + O(\Delta x^2) =$$

$$= \frac{1}{2\Delta x}\left(\frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta y} - \frac{f_{i-1,j+1} - f_{i-1,j-1}}{2\Delta y}\right) + O(\Delta x^2, \Delta y^2)$$

$$= \frac{f_{i+1,j+1} - f_{i+1,j-1} - f_{i-1,j+1} + f_{i-1,j-1}}{4\Delta y \Delta x} + O(\Delta x^2, \Delta y^2)$$