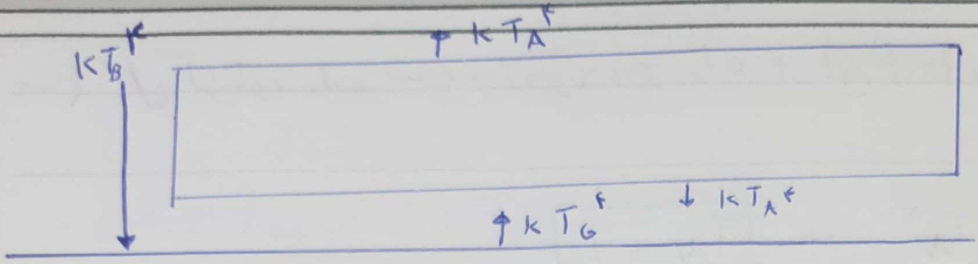
پاسخ سوال ۱

الف)



سیستم در تعادل ⟸ گرمای داده شده به زمین و جو باید خارج شود ⟸

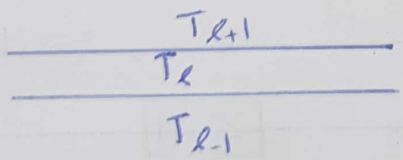
$$S_s = kT_A^4 = kT_B^4$$

$$\boxed{T_A = T_B}$$

گرمای داده شده به جو باید خارج شود از آن ⟸

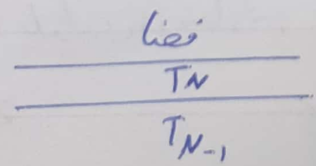
$$\left.\begin{aligned} kT_A^4 + kT_A^4 &= kT_G^4 \\ T_A &= T_B \end{aligned}\right\} \Rightarrow \boxed{T_G = 2^{\frac{1}{4}} T_B}$$

ب)



گرمای داده شده به لایه ℓ باید از آن خارج شود ⟸

$$2kT_\ell^4 = kT_{\ell-1}^4 + kT_{\ell+1}^4 \rightsquigarrow \boxed{2T_\ell^4 = T_{\ell-1}^4 + T_{\ell+1}^4}$$



گرمای داده شده به لایه N باید از آن خارج شود

$$\Rightarrow 2 \times kT_N^4 = kT_{N-1}^4 \qquad \boxed{2^{\frac{1}{4}} T_N = T_{N-1}}$$

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پ) کل گرمای داده شده به زمین و جو باید از آن ۴ خارج شوند (مانند قسمت الف)

$$\Rightarrow kT_B^4 = kT_N^4 \rightsquigarrow \boxed{T_N = T_B}$$

بنابراین می‌شود فهمید همواره دمای آخرین لایه T_B است.

ت)

$$2T_\ell^4 = T_{\ell-1}^4 + T_{\ell+1}^4$$

برای حل این معادله سه روش معرفی می‌کنم

روش یک:

$$T_\ell^4 = x_\ell \rightsquigarrow 2x_\ell = x_{\ell-1} + x_{\ell+1}$$

حدس می‌زنم که جواب معادله بازگشتی $Az^\ell = x_\ell$ است

جایگذاری در معادله:

$$2Az^\ell = Az^{\ell-1} + Az^{\ell+1}$$

$$\rightsquigarrow z^2 + 1 - 2z \qquad (z-1)^2 = 0 \qquad \boxed{z=1}$$

ریشه مضاعف

$$\Leftarrow \quad x_\ell = (A + B\ell)z^\ell$$

جایگذاری در معادله:

$$2(A+B\ell)z^\ell = (A + B(\ell-1))z^{\ell-1} + (A+B(\ell+1))z^{\ell+1}$$

$z=1$

$$2A + 2B\ell = 2A + B(\ell-1) + B(\ell+1) \quad \checkmark$$

پس جواب $\boxed{x_\ell = A + B\ell}$ است

پاسخ سوال

روش دوم: فرض می‌کنیم x_0 و x_1 را داریم سپس x_2 را برحسب آن‌ها بدست می‌آوریم

بعد x_3 را برحسب x_0 و x_1 بدست می‌آوریم. بعد x_4 و ...

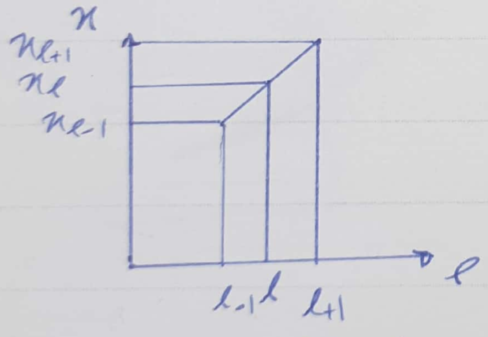
می‌بینیم که x_ℓ خطی است پس جواب کلی آن $A+B\ell$ است

روش سوم:

$$2x_\ell = x_{\ell+1} + x_{\ell-1}$$

$$x_\ell = \frac{x_{\ell+1} + x_{\ell-1}}{2}$$

این یعنی x_ℓ میانگین $x_{\ell+1}$ و $x_{\ell-1}$ است که نتیجه می‌دهد x_ℓ و $x_{\ell+1}$ و $x_{\ell-1}$ روی یک خط اند



پس می‌توان فهمید تمام x_ℓ ها روی یک خط اند پس $x_\ell = A + B\ell$

اما برای بدست آوردن A و B نیاز به دو شرایط مرزی داریم

$T_B = T_N$ $\qquad$ $2^{1/4} T_N = T_{N-1} = 2^{1/4} T_B$

$x_N = A + BN = T_B^4$ $\qquad$ $x_{N-1} = A + B(N-1) = 2T_B^4$

حل ۲ معادله ۲ مجهول $\Rightarrow$ $B = -T_B^4$ $\qquad$ $A = T_B^4(1+N)$

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$$\Longrightarrow \varkappa_\ell = T_\ell^4 = T_B^4(1+N) - T_B^4 \ell$$

$$\boxed{T_\ell = T_B\,(1+N-\ell)^{\frac{1}{4}}}$$

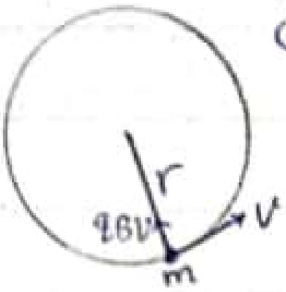
$$T_{\ell=0} = \boxed{T_E = T_B\,(1+N)^{\frac{1}{4}}}$$ زمین = لایه صفر

پاسخ سوال ۳

آ) معادله‌ی نیرو : $\frac{mV^2}{r} = qVB \Rightarrow \frac{V}{r} = \frac{qB}{m}$

$\tau =$ دوره‌ی تناوب $= \frac{2\pi r}{V}$

$\Rightarrow \boxed{\tau = \frac{2\pi m}{qB}}$



ب) کار انجام شده بر روی بار = تغییرات انرژی جنبشی $\Rightarrow (KS)(q\frac{V_0}{S}) = \frac{m}{2}V_K^2$

$\Rightarrow \boxed{V_K = \sqrt{\frac{2KqV_0}{m}}}$

پ) $\frac{mV_K^2}{r_K} = qV_KB \Rightarrow V_K \times \frac{m}{qB} = r_K = \boxed{\sqrt{\frac{2KmV_0}{qB^2}}}$

ت) t شامل دو قسمت است. t_{in} مدتی است که درون استوانه‌ها بوده و t_{out} مدتی است که درون شکاف بوده :

$t_{in} = (K-1)\frac{\tau}{2} = (K-1)\times\frac{\pi m}{qB}$

برای محاسبه‌ی t_{out} مسیرشکاف‌ها را به هم می‌چسبانیم:

$\underbrace{\bullet \qquad\qquad\qquad \bullet}_{KS}$

$KS = (\frac{V_0 q}{mS}) \times \frac{t_{out}^2}{2} \Rightarrow t_{out} = S\sqrt{\frac{2Km}{qV_0}} \longrightarrow \boxed{t = \frac{\pi m}{qB}(K-1) + S\sqrt{\frac{2mK}{qV_0}}}$

ث) $l = KS + \sum_{i=1}^{K-1}\pi r_i = KS + \pi\sum_{i=1}^{K-1}\sqrt{\frac{2mV_0}{qB^2}}\sqrt{i} \simeq KS + \frac{\pi}{B}\sqrt{\frac{2mV_0}{q}}\left(\frac{2}{3}\sqrt{K^3} - \frac{\sqrt{K}}{2}\right)$

$\Rightarrow \boxed{l \simeq KS + \frac{\pi}{B}\sqrt{\frac{2mV_0K}{q}}\left(\frac{2}{3}K - \frac{1}{2}\right)}$

ج) $\frac{T}{2} = S_{min}\sqrt{\frac{2Km}{qV_0}} \Rightarrow S_{min} = \sqrt{\frac{qV_0}{2Km}} \times \frac{\pi m}{qB} = \boxed{\frac{\pi}{B}\sqrt{\frac{mV_0}{2Kq}}}$

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$V_0 = ۵۰ (KV)$ ، $\frac{mV_K^۲}{۲} = ۲۵ (MeV)$ ، $q = e$ بار الکترون ، $\frac{m}{q} = ۱٫۰۴ \times ۱۰^{-۸} \left(\frac{Kg}{C}\right)$ ، $B = ۱٫۰۴ (T)$ (۵

$S = ۲ (mm)$

$$\Longrightarrow \frac{mV_K^۲}{۲} = K q V_0 = ۲۵ \times ۱۰^۶ \times e \,(V) = K \times e \times \left(۵ \times ۱۰^۴ (V)\right)$$

$$\Longrightarrow \boxed{K = ۵۰۰} \longrightarrow t = (K-۱) \times \frac{\pi m}{qB} + S\sqrt{\frac{۲mK}{qV_0}} = ۴۹۹ \times \pi \times ۱۰^{-۸} (S) + ۲ \times ۱۰^{-۳} \sqrt{\frac{۱۰^۳ \times ۱٫۰۴ \times ۱۰^{-۸}}{۵ \times ۱۰^۴}} (S)$$

$$= ۴۹۹ \pi \times ۱۰^{-۸} (S) + ۲ \times ۱۰^{-۳} \sqrt{۲٫۰۸ \times ۱۰^{-۱۰}} (S) \Longrightarrow \boxed{t \approx ۱٫۵۷ \times ۱۰^{-۵} (S)}$$

$$l \approx KS + \frac{\pi}{B}\sqrt{\frac{۲m}{q} V_0 K}\left(\frac{۲}{۳}K - \frac{۱}{۲}\right) \approx \boxed{۳٫۱۸ (m)}$$

پاسخ سوال ۵

آ)

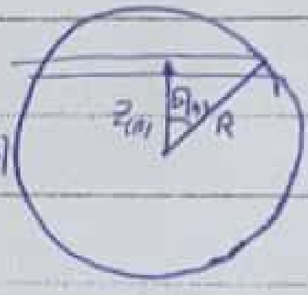
طبق تقارن $E_{(r)} = 0$ است

$$|\vec{E}| = E_z = \frac{\frac{q}{N}}{4\pi\epsilon_0 (r^2+z^2)} \cdot \frac{z \times N}{\sqrt{r^2+z^2}} \Rightarrow \boxed{E = \frac{qz}{4\pi\epsilon_0 (r^2+z^2)^{3/2}}}$$

$$N \to \infty \Rightarrow E = \frac{qz}{4\pi\epsilon_0 (r^2+z^2)^{3/2}}$$

تمام بارها از محور به یک فاصله اند:

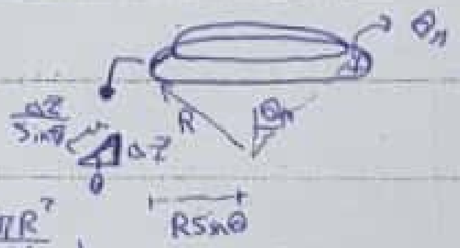
$$\varphi = \frac{N\left(\frac{q}{N}\right)}{4\pi\epsilon_0 \sqrt{r^2+z^2}} = \boxed{\frac{q}{4\pi\epsilon_0 \sqrt{r^2+z^2}} = \varphi_{1n}}$$



$$z_{(n)} = R - \Delta z\left(-\frac{1}{2}+n\right)$$

$$\cos(\theta_n) = \frac{z_n}{R} = \frac{R - \frac{2R}{N}\left(n-\frac{1}{2}\right)}{R} \qquad \Delta z = \frac{2R}{N}$$

$$\Rightarrow \cos(\theta_n) = 1 - \frac{2n}{N} + \frac{1}{N} \Rightarrow \boxed{\cos(\theta_n) = 1 - \frac{(2n-1)}{N}}$$



$$S_{(n)} = 2\pi(R\sin\theta_n)\left(\frac{\Delta z}{\sin\theta_n}\right) = 2\pi R \Delta z$$

$$\Rightarrow \boxed{S_n = \frac{4\pi R^2}{N}} \Rightarrow Q_{(n)} = Q\,\frac{S_n}{S_{\text{کل}}} = Q\left(\frac{\frac{4\pi R^2}{N}}{\frac{4\pi R^2}{1}}\right)$$

$$\Rightarrow \boxed{Q_{(n)} = \frac{Q}{N}}$$

پاسخ سوال ③

$$\Rightarrow E = \sum_{n=1}^{N} \frac{\left(\frac{q}{N}\right)\left(d - R\cos\theta_n\right)}{4\pi\epsilon_0\left((R\sin\theta_n)^2 + (d - R\cos\theta_n)^2\right)^{3/2}}\,\hat{z}$$ ⇐ به صورت جمع N حلقه

$$\Rightarrow \boxed{E = \sum_{n=1}^{N} \frac{\frac{Q}{N}\left(d - R\cos\theta_n\right)\hat{z}}{4\pi\epsilon_0\left(R^2 + d^2 - 2Rd\cos\theta_n\right)^{3/2}}} \quad , \quad \boxed{V = \sum_{n=1}^{N} \frac{\frac{Q}{N}}{4\pi\epsilon_0\left(R^2 + d^2 - 2Rd\cos\theta\right)^{1/2}}}$$

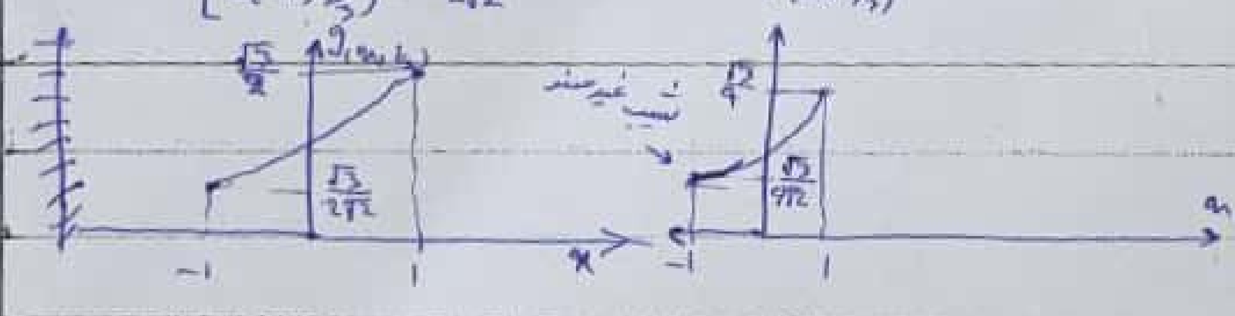
$$\therefore E = \frac{Q}{4\pi\epsilon_0 N d^2} \sum_{n=1}^{N} \overbrace{\frac{1 - \alpha x_n}{\left(1 + x_n^2 - 2x_n\alpha\right)^{3/2}}}^{f_{(\alpha, x)}}$$

$$\therefore V = \frac{Q}{4\pi\epsilon_0 N d} \sum_{n=1}^{N} \underbrace{\frac{1}{\left(1 + x_n^2 - 2\alpha x_n\right)^{1/2}}}_{g_{(x,\alpha)}}$$

$$\alpha = \tfrac{1}{3} \Rightarrow f_{(x,\frac{1}{3})} = \frac{1 - \frac{x}{3}}{\left(1 + x^2 - \frac{2}{3}x\right)^{3/2}} \quad , \quad g_{(x,\frac{1}{3})} = \frac{1}{\left(1 + x^2 - \frac{2}{3}x\right)^{1/2}}$$

$$x \Big|_{-1}^{1} \qquad x = 1 \Rightarrow \begin{cases} f_{(1,\frac{1}{3})} = \dfrac{\frac{2}{3}}{\frac{8}{(3)^{3/2}}} = \dfrac{\sqrt{3}}{4} \\ g_{(1,\frac{1}{3})} = \cancel{\dfrac{\sqrt{3}}{\sqrt{8}}}\ \dfrac{\sqrt{3}}{2} \end{cases}$$

$$x = -1 \begin{cases} f_{(-1,\frac{1}{3})} = \dfrac{\frac{4}{3}}{\left(\frac{8}{3}\right)^{3/2}} = \dfrac{\sqrt{3}}{4\sqrt{2}} \\ g_{(-1,\frac{1}{3})} = \dfrac{\sqrt{3}}{2\sqrt{2}} \end{cases}$$



پاسخ سوال ۲

الف)

۲	$N>۴$	۴	۳	N
۱	۱	۱	$\frac{۳}{۲}$	۹
$۲\sqrt{۳}$	$۲\tan(\frac{۲\pi}{N})$	∞	∞	۴

پاسخ سوال

ب)

$$y = D\tan(2\omega t) \to \boxed{v = 2\omega D\sec^2(2\omega t)}$$

پ)

$$t - t_m = \frac{\frac{d}{\cos(2\omega t_m)}}{c} = \frac{d}{c\cos(2\omega t_m)} \to \boxed{t = t_m + \frac{d}{c\cos^2(2\omega t_m)}}$$

$$v = \frac{dy}{dt_m}\left(\frac{1}{\frac{dt}{dt_m}}\right) = \left(\frac{2\omega D}{\cos^2(2\omega t_m)}\right)\left(\frac{1}{1 + \frac{2\omega D}{c}\frac{\sin(2\omega t_m)}{\cos^2(2\omega t_m)}}\right)$$

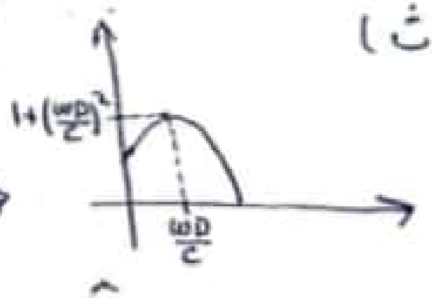
ت)

$$\boxed{v = \frac{2\omega D c}{c\cos^2(2\omega t_m) + 2\omega D\sin(2\omega t_m)}}$$

ث)

$$z = \frac{2\omega D}{v} = \cos^2(2\omega t_m) + \frac{2\omega D}{c}\sin(2\omega t_m)$$

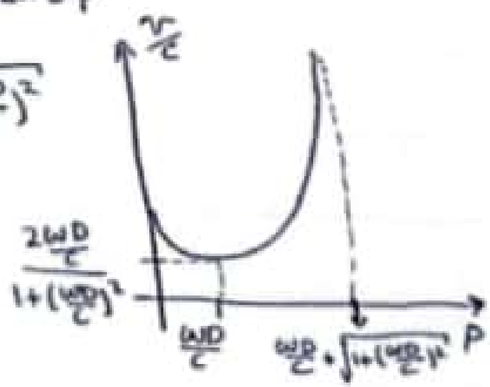
$$P = \sin(2\omega t_m) \to z = \frac{2\omega D}{c}P + 1 - P^2 \to$$



ج)

$$\frac{v}{c} = \frac{2\omega D}{c - c\sin^2(2\omega t_m) + 2\omega D\sin(2\omega t_m)} = \frac{2\omega D}{c - cP^2 + 2\omega D P}$$

بی‌نهایت : $c - cP^2 + 2\omega D P = 0 \to P = \frac{\omega D}{c} \pm \sqrt{1 + \left(\frac{\omega D}{c}\right)^2}$



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(ه

$$P^2 - \frac{2\omega D}{c} P - 1 = 2\frac{\omega D}{c}$$

$$P_1' = \frac{\omega D}{c} - \sqrt{\left(\frac{\omega D}{c}\right)^2 + \left(1 - 2\frac{\omega D}{c}\right)}$$

$$P_2' = \frac{\omega D}{c} + \sqrt{\left(\frac{\omega D}{c}\right)^2 + \left(1 - 2\frac{\omega D}{c}\right)}$$

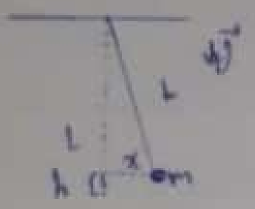
$\sin(2\theta)$ باید بین $\frac{\omega D}{c} + \sqrt{1+\left(\frac{\omega D}{c}\right)^2}$ و یا بین $\frac{\omega D}{c} - \sqrt{1+\left(\frac{\omega D}{c}\right)^2}$ و $\frac{2\omega D}{c} - 1$ باشد :

$$\frac{dt}{dt_m} T_0 = T = \left(1 + \frac{2\omega D \sin(2\theta)}{c \cos^2(2\theta)}\right) T_0$$

$$\boxed{T \geq T_0}$$

پاسخ سوال 5

آ)



$$V = mgh$$

$$h = L - \sqrt{L^2 - x^2} = L\left(1 - \sqrt{1 - \frac{x^2}{L^2}}\right) \simeq L\left(1 - 1 + \frac{x^2}{2L^2}\right) = \frac{x^2}{2L}$$

$$\to U_{(x)} = \boxed{\frac{mgx^2}{2L}} \qquad U_{(x)} = \frac{1}{2}kx^2 \Rightarrow k = \boxed{\frac{mg}{L}}$$

ب)

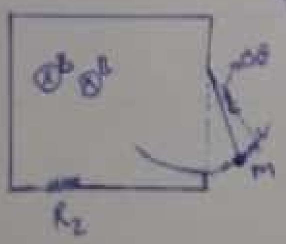
$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \to \dot{E} = mav + kxv = 0 \Rightarrow ma + kx = 0 \to \frac{a}{x} = -\frac{k}{m} = \boxed{-\frac{g}{L}}$$

پ) فرض می‌کنیم $0<A$

$$x = A\cos(\omega t + \varphi) \to v = -A\omega\sin(\omega t + \varphi) \to a = -A\omega^2\cos(\omega t + \varphi)$$

$$\Rightarrow \frac{a}{x} = -\omega^2 = -\frac{g}{L} \Rightarrow \omega = \boxed{\sqrt{\frac{g}{L}}} \qquad t=0: v=0,\ x>0 \Rightarrow \boxed{\varphi = 0}$$

ت)



$$\dot{\varphi} = -\mathcal{E}$$

$$\varphi = BA,\ B = \text{const} \Rightarrow \dot{\varphi} = B\dot{A}$$

$$\text{در } \Delta t:\ \Delta S = \frac{L \times L\Delta\theta}{2} = \frac{L^2\Delta\theta}{2}, \quad \Delta\theta = \dot{\theta}\,\Delta t = \frac{v}{L}\Delta t \quad \Rightarrow \Delta S = \frac{vL}{2}\Delta t \Rightarrow \dot{S} = \frac{vL}{2}$$

$$\Rightarrow \mathcal{E} = \boxed{-\frac{BvL}{2}}$$

ث)

$$P = I^2R = \frac{\mathcal{E}^2}{R} = \frac{\mathcal{E}^2}{R_1 + R_2} = \frac{B^2v^2L^2}{4(R_1 + R_2)}$$

$$E = K + U = \frac{1}{2}mv^2 + \frac{mgx^2}{2L} \to \dot{E} = mva + \frac{mgvx}{L} = -P = -\frac{B^2v^2L^2}{4(R_1 + R_2)}$$

$$\to \boxed{ma + \frac{mg}{L}x + \frac{B^2L^2}{4(R_1 + R_2)}v = 0} \qquad \boxed{k = \frac{mg}{L},\ b = \frac{B^2L^2}{4(R_1 + R_2)}}$$

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ج)

$$x = Ae^{-\gamma t}\cos(\omega' t+\beta) \Rightarrow v = -Ae^{-\gamma t}\left(\gamma\cos(\omega' t+\beta)+\omega'\sin(\omega' t+\beta)\right)$$

$$\Rightarrow a = Ae^{-\gamma t}\left(\gamma\omega'\sin(\omega' t+\beta)-\omega'^2\cos(\omega' t+\beta)\right)+Ae^{-\gamma t}\left(\gamma^2\cos(\omega' t+\beta)+\gamma\omega'\sin(\omega' t+\beta)\right)$$

$$ma+bv+Kx=0 \Rightarrow K\cos(\omega' t+\beta)+\left(-b\gamma\cos(\omega' t+\beta)-b\omega'\sin(\omega' t+\beta)\right)+2m\gamma\omega'\sin(\omega' t+\beta)+m\gamma^2\cos(\omega' t+\beta) - m\omega'^2 \ldots = 0$$

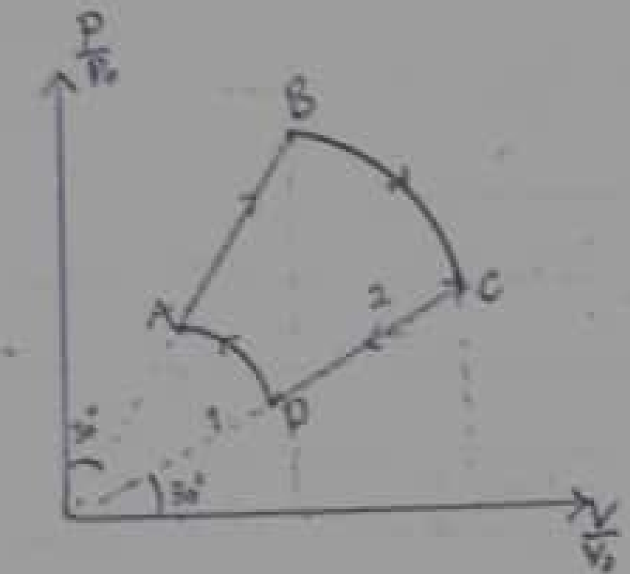
$$\Rightarrow \begin{cases} -b\omega'+2m\gamma\omega'=0 \Rightarrow \boxed{\gamma=\dfrac{b}{2m}} \\ K-b\gamma+m\gamma^2-m\omega'^2=0 \Rightarrow \omega'^2=\gamma^2+\dfrac{K}{m}-\dfrac{b\gamma}{m} \end{cases} \Rightarrow \omega'^2=\frac{b^2}{4m^2}+\frac{K}{m}-\frac{b^2}{2m^2}=\frac{K}{m}-\frac{b^2}{4m^2}$$

$$\Rightarrow \boxed{\omega'=\sqrt{\frac{K}{m}-\frac{b^2}{4m^2}}}$$

$$\boxed{\gamma=\frac{B^2L^2}{8m(R_1+R_2)}} \qquad \boxed{\omega'=\sqrt{\frac{g}{L}-\frac{B^4L^4}{64m^2(R_1+R_2)^2}}}$$

پاسخ سوال 6

فرمول ها:



$$PV = nRT$$
$$U = \frac{3}{2} nRT$$
$$\Delta U = W + Q$$
$$dW = -PdV$$

الف)

$A: \quad V_A = \frac{V_0}{2} \qquad P_A = \frac{\sqrt{3}}{2} P_0 \qquad T_A = \frac{\sqrt{3}}{4} \frac{P_0 V_0}{nR} = \frac{\sqrt{3}}{4} T_0$

$B: \quad V_B = V_0 \qquad P_B = \sqrt{3}\, P_0 \qquad T_B = \sqrt{3} \frac{P_0 V_0}{nR} = \sqrt{3}\, T_0$

$C: \quad V_C = \frac{\sqrt{3}}{2} V_0 \qquad P_C = \frac{P_0}{2} \qquad T_C = \frac{\sqrt{3}}{4} \frac{P_0 V_0}{nR} = \frac{\sqrt{3}}{4} T_0$

$D: \quad V_D = \sqrt{3}\, V_0 \qquad P_D = P_0 \qquad T_D = \sqrt{3} \frac{P_0 V_0}{nR} = \sqrt{3}\, T_0$

ب)

$W = -$ مساحت زیر نمودار PV $= -P_0 V_0$ (مساحت زیر نمودار)

$W_{A\to B}: \; -P_0 V_0 \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8} \right) = \boxed{-P_0 V_0 \frac{3\sqrt{3}}{8}} \rightarrow$ منفی

$W_{B\to C}: \; -P_0 V_0 \left(\frac{\pi}{3} + \cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{\sqrt{3}}{2}} \right) = \boxed{-P_0 V_0 \frac{\pi}{3}} \rightarrow$ منفی

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نام خانوادگی:
مبحث:

پاسخ سوال | ادامه 6

$W_{C\to D}: P_0V_0\left(\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{8}\right) = \boxed{P_0V_0\,\frac{3\sqrt{3}}{8}} \to$ مثبت

$W_{D\to A}: P_0V_0\left(\frac{\pi}{12}+\cancel{\frac{\sqrt{3}}{16}}-\cancel{\frac{\sqrt{3}}{16}}\right) = \boxed{P_0V_0\,\frac{\pi}{12}} \to$ مثبت

$W_{\text{کل}} = \sum W = P_0V_0\left(\frac{\pi}{12}-\frac{\pi}{3}\right) = -P_0V_0\,\frac{\pi}{4}$

$Q = \Delta U - W \to Q_{A\to B} = \frac{3}{2}P_0V_0\left(\sqrt{3}-\frac{\sqrt{3}}{\cancel{4}}\right) + \frac{3\sqrt{3}}{\cancel{8}}P_0V_0 = \boxed{\frac{3\sqrt{3}}{2}P_0V_0}$ مثبت

$Q_{B\to C}: \frac{3}{2}P_0V_0\left(\sqrt{3}-\sqrt{3}\right) + \frac{\pi}{3}P_0V_0 = \boxed{\frac{\pi}{3}P_0V_0} \to$ مثبت

$Q_{C\to D} = \frac{3}{2}P_0V_0\left(\frac{\sqrt{3}}{\cancel{4}}-\sqrt{3}\right) - \frac{3\sqrt{3}}{\cancel{8}}P_0V_0 = \boxed{\frac{-3\sqrt{3}}{2}P_0V_0} \to$ منفی

$Q_{D\to A} = \frac{3}{2}V_0P_0\left(\frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4}\right) - \frac{\pi}{12}P_0V_0 = \boxed{\frac{-\pi}{12}P_0V_0} \to$ منفی

$Q = U - W \Rightarrow dQ = dU - dW = 0$ $\Leftarrow$ در مرز باید $dq = 0$

$\to dU = dW \to d\left(\frac{3}{2}PV\right) = -PdV = \frac{3}{2}(PdV + VdP)$

$\mapsto \frac{5}{2}PdV = -\frac{3}{2}VdP \Rightarrow -\frac{5}{3}\,\frac{P}{V} = \frac{dP}{dV}$

$\frac{dP}{dV} =$ شیب نمودار $= -\cot\theta \quad , \quad \frac{P}{V} = \tan\theta \Rightarrow \boxed{\tan^2\theta = \frac{3}{5}}$

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$$\tan\theta = \sqrt{\frac{3}{5}} \Rightarrow \theta = \tan^{-1}\left(\sqrt{\frac{3}{5}}\right)$$

$$\rightarrow BC: \; V = 2V_0 \cos\left(\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)$$

$$P = 2P_0 \sin\left(\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)$$

$$T = T_0 \sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)$$

$$DA: \; V = V_0 \cos\left(\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)$$

$$P = P_0 \sin\left(\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)$$

$$T = \frac{T_0}{2} \sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right);$$

$$Q = \Delta U - W = \frac{3}{2}P_0V_0\left(\sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right) - \frac{\sqrt{3}}{4}\right)$$

$A \rightarrow Q=0$ (DC)

$$+ P_0V_0\left(4\pi \cdot \frac{\frac{\pi}{3} - \tan^{-1}\left(\sqrt{\frac{3}{5}}\right)}{2\pi} + \frac{\sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)}{2} - \frac{\sqrt{3}}{8}\right)$$

$$= \boxed{P_0V_0\left(\frac{5}{2}\sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right) - \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{3} - \tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)\right)}$$

$$Q = \frac{3}{2}P_0V_0\left(\frac{1}{2}\sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right) - \frac{\sqrt{3}}{4}\right) + P_0V_0\left(\frac{1}{2}\left(\frac{\pi}{3} - \tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)\right.$$

$A \rightarrow Q=0$ (DA)

$$\left. + \frac{\sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)}{4} - \frac{\sqrt{3}}{8}\right)$$

$$= \boxed{P_0V_0\left(\sin\left(2\tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right) - \frac{\sqrt{3}}{2} + \frac{1}{2}\left(\frac{\pi}{3} - \tan^{-1}\left(\sqrt{\frac{3}{5}}\right)\right)\right)}$$

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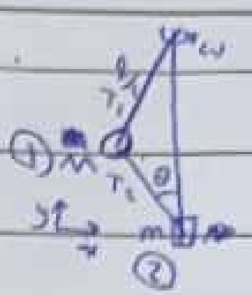
پاسخ سوال ۶

$\ddot{x}_i = \frac{d^2 x_i}{dt^2}$



$M\ddot{y}_1 = (T_1 - T_2)\cos\theta - Mg$ (I)

$M\ddot{x}_1 = (T_1 + T_2)\sin\theta - M(\frac{l}{2}\sin\theta)\omega^2$ (II)   در حالت تعادل $(\ddot{x}_1, \ddot{y}_1, \ddot{y}_2 = 0)$

$m\ddot{y}_2 = T_2\cos\theta - mg$ (III)

(III) $T_2\cos\theta = mg$

(I) $Mg = (T_1 - T_2)\cos\theta$ $\Rightarrow T_1\cos\theta = Mg + T_2\cos\theta$ $\Rightarrow T_1\cos\theta = (M+m)g$ (IIII)

(II) $T_1 + T_2 = \frac{Ml\omega^2}{2} \xrightarrow[\text{III}]{\text{IIII}} \frac{Ml\omega^2}{2} = \frac{g(M+2m)}{\cos\theta} \Rightarrow \cos\theta = \frac{2g(M+2m)}{Ml\omega^2} < 1$

$\Rightarrow \omega_m^2 > \frac{2g(M+2m)}{Ml} \Rightarrow \boxed{\omega_m^2 = \frac{2g(M+2m)}{Ml}}$

$\omega = 2\omega_m \Rightarrow \cos\theta = \frac{1}{4} \times \frac{2g(M+2m)}{Ml\omega_m^2} \Rightarrow \boxed{\cos\theta = \frac{1}{4}} \Rightarrow \sin\theta = \sqrt{1 - \frac{1}{16}} = \frac{\sqrt{15}}{4}$

$T_1 = \frac{(m+M)g}{\cos\theta} \Rightarrow \boxed{T_1 = 4(m+M)g}$, $T_2 = \frac{mg}{\cos\theta} \Rightarrow \boxed{T_2 = 4mg}$

$r = \frac{l}{2}\sin\theta \Rightarrow r = \frac{l}{2} \times \sqrt{\frac{15}{4}} \Rightarrow \boxed{r = \frac{l\sqrt{15}}{8}}$

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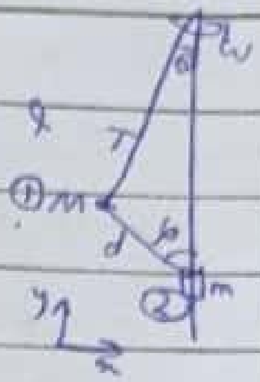
$$M\ddot{y}_1 = T(\cos\theta - \cos\phi) - Mg \quad (1)$$

$$M\ddot{x}_1 = T(\sin\theta + \sin\phi) - M(d\sin\phi)\omega^2 \quad (2)$$

$$m\ddot{y}_2 = mg - T\cos\phi \quad (3)$$

در تعادل شتاب ها برابر صفر اند



$$r = d\sin\phi = (\ell - d)\sin\theta \qquad u = \frac{d}{\ell}$$

$$\Rightarrow \sin\phi = \left(\frac{\ell}{d} - 1\right)\sin\theta = \left(\frac{1}{u} - 1\right)\sin\theta = \left(\frac{-u+1}{u}\right)\sin\theta \Rightarrow \sin\phi = \frac{1-u}{u}\sin\theta$$

$$M = 2m \qquad \Rightarrow \cos^2\phi = 1 - \left(\frac{1-u}{u}\right)^2\sin^2\theta$$

$$(1),(3) \Rightarrow T\cos\theta = Mg + T\cos\phi \Rightarrow T\cos\theta = (M+m)g = 3mg$$

$$\left.\begin{array}{l} T\cos\theta = 3mg \\ T\cos\phi = mg \leftarrow (3) \end{array}\right\} \Rightarrow \frac{\cos\theta}{\cos\phi} = 3$$

$$\cos^2\theta = 9\cos^2\phi = 9\left(1 - \left(\frac{1-u}{u}\right)^2\sin^2\theta\right) = 9\left(1 - \left(\frac{1-u}{u}\right)^2 + \left(\frac{1-u}{u}\right)^2\cos^2\theta\right) \to \cos^2\theta$$

$$\Rightarrow \boxed{\cos^2\theta = \frac{9\left(1 - \left(\frac{1-u}{u}\right)^2\right)}{1 - 9\left(\frac{1-u}{u}\right)^2}}$$

$$(2) \Rightarrow \frac{T}{Md}\cdot\frac{\sin\phi + \sin\theta}{\sin\phi} = \omega^2, \quad T = \frac{3mg}{\cos\theta}, \quad M = 2m, \quad \sin\phi = \left(\frac{1}{u} - 1\right)\sin\theta$$

$$\Rightarrow \omega^2 = \frac{\left(\frac{3mg}{\cos\theta}\right)}{2md}\cdot\frac{\frac{1}{u}\sin\theta}{\frac{1-u}{u}\sin\theta} = \frac{3}{2}\cdot\frac{g}{(1-u)\cos\theta\, d}, \quad d = \ell u$$

$$\Rightarrow \boxed{\omega^2 = \frac{\frac{g}{\ell u}}{2(1-u)}\sqrt{\frac{1 - 9\left(\frac{1-u}{u}\right)^2}{1 - \left(\frac{1-u}{u}\right)^2}}} \Rightarrow \boxed{\omega^2 = \frac{g}{2u\ell(1-u)}\sqrt{\frac{u^2 - 9(1-u)^2}{u^2 - (1-u)^2}}}$$

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$$\Rightarrow u=\frac{1}{4} \Rightarrow \cos^2\theta = \frac{9(1-(9))}{1-81} = \frac{9(\cancel{1-9})}{(\cancel{1-9})(1+9)} = \frac{9}{10}$$

$$\Rightarrow \boxed{\cos\theta = \frac{3}{\sqrt{10}} \quad \cos\phi = \frac{1}{\sqrt{10}}} \Rightarrow \omega^2 = \frac{2g}{9\left(\frac{3}{4}\right)}\sqrt{10} = \boxed{\frac{8\sqrt{10}\, g}{3l} = \omega^2}$$

$$r = d\sin\phi = 9u\sin\phi \qquad \sin\phi = \frac{1}{\sqrt{10}}\left(\sqrt{10-1}\right) = \frac{3}{\sqrt{10}}$$

$$\Rightarrow \boxed{r = \frac{3l}{4\sqrt{10}}}$$

$$\frac{\omega^2 l}{g} = \frac{1}{2} \times \frac{1}{u(1-u)}\sqrt{\frac{u^2-9(1-u)^2}{u^2-(1-u)^2}} \qquad \begin{aligned} &u^2-(1-u)^2 = 2u-1 \\ &u^2-9(1-u)^2 = 18u-9-8u^2 \end{aligned}$$

$$\frac{d\frac{\omega^2 l}{g}}{du} = 0 \Rightarrow \frac{u(1-u)\sqrt{u^2-(1-u)^2}\,(9-8u)}{\sqrt{u^2-9(1-u)^2}} = \sqrt{u^2-9(1-u)^2}\left((1-2u)\sqrt{2u-1} + \frac{u(1-u)}{\sqrt{2u-1}}\right)$$

$$\Rightarrow u(1-u)\sqrt{2u-1}\,(9-8u) = \left(u^2-9(1-u)^2\right)\left(\frac{u(1-u)-1-4u^2+4u}{\sqrt{2u-1}}\right)$$

$$\Rightarrow u(1-u)(2u-1)(9-8u) = \left(u^2-9(1-u)^2\right)\left(5u-5u^2-1\right) = \left(18u-9-8u^2\right)\left(5u-5u^2-1\right)$$

$$u^4(16)+u^3(\overbrace{-18-8}^{-26}-16+16)+u^2(9+\underbrace{18+8}_{35}) = u^4(40)+u^3(\underbrace{-90-40}_{-130})+u^2(\underbrace{90+8+45}_{98+45})+u(-18-45)+9$$

$$24u^4 - 104u^3 + u^2(108) - 63u + 9 = 0$$

$$\boxed{u^4 - \frac{13}{3}u^3 + \frac{27}{6}u^2 - \frac{21}{8}u + \frac{3}{8} = 0}$$

پاسخ سوال

المپیاد فیزیک انرژی اتمی بهار ۱۴۰۱ ت