Nonlinear Control of Flyback type DC to DC Converters: An Indirect Backstepping Approach

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*Abstract***— Flyback type DC to DC converters are basically a transformer–isolated version of the standard buck-boost converter. In this work, a nonlinear control methodology for the flyback type DC to DC converter is proposed. The proposed method uses a backstepping approach to regulate the corresponding current required to set the output voltage to its desired value as opposed to directly regulating the output voltage. Simulation results are presented to illustrate the effectiveness of the proposed method.**

Keywords— **DC to DC converters, Flyback Converter, Nonlinear Control, Backstepping.**

INTRODUCTION

In recent years, control research on power electronic devices has gained popularity. DC to DC converters are one of the simplest form of power electronic devices. They are widely used in computers, industrial electronic systems, battery operating portable equipments and uninterruptible power supplies. As their name indicates they are used in the conversion of one DC voltage level to another, by storing the input energy temporarily and then releasing that energy to the output at a different voltage. The two basic switched DC to DC converter topologies, refered as the Buck (Step-down) and the Boost (Step-up) are widely known. The other converters such as Buck-Boost, Flyback, forward, Cuk, Half Bridge, Sepic converters, are derived from these two [1]. From a control engineers' perspective a flyback converter, like all other switched DC power converters, is a non-linear, timevarying system that require high-performance control techniques, therefore constitutes an interesting case study. The traditional pulse width modulation (PWM) based controllers used in these systems are based on small signal analysis of DC converters, therefore, are only suited for specific conditions. Under large load variations, PWM type controllers cannot achieve gratifying success. This work focuses on a special type of DC to DC converter mostly known as the flyback converter where the input and output circuits are isolated through a transformer.

One way of providing a robust solution to the control problem of a DC to DC converters, is the use of nonlinear variable-structure like controllers [2]. Due to their switching nature, variable structure controllers are well fitted for switched power circuits especially DC to DC converters [3]. Additionally variable–structure like controllers, like the sliding mode controller, have the advantage of guaranteeing a large extent the stability against parametric uncertainties and load variations.. Sliding mode controller are described in many studies the contribution of the DC switching power converters [3, 4, 5, 6].

Another way of approaching the control problem for a DC to DC converter is applying nonlinear controller techniques as passivity based and/or backstepping approaches. Though these approaches are not as popular their variable structure counterparts, they have been proven effective by many researches [7].

In this paper we propose a nonlinear control methodology for the control of flyback type DC to DC converter. The proposed method uses a backstepping approach to regulate the corresponding current required to set the output voltage to its desired value as opposed to directly regulating the output voltage. Lyapunov type arguments are used to prove the stability of system states and boundedness of closed loop signals. Simulation results are presented to illustrate the effectiveness of the proposed method.

The rest of the work is organized as follows: Section 2 states the dynamical model and the control problem for the DC to DC converter under consideration. The error system development and control design are presented in Section 3. Simulation studies are presented in Section 4. Section 5 contains some concluding remarks.

DYNAMICAL MODEL OF THE SYSTEM

A flyback converter is a specific type of Buck-Boost converter where the input and output circuits are isolated through an ideal transformer. A typical flyback converter circuitry is presented in Fig.1. As shown in the figure the input circuitry of the converter is composed of a DC voltage supply, a MOSFET or an IGBT transistor used as a switching device and primer side of the transformer while the output circuit is composed of a diode switch and an RC circuit.

Fig.1 A Flyback type DC/DC converter circuitry

From a simple analysis of the circuitry, one can conclude that when the switch S, on the input side of the converter is closed (i.e. transistor is ON position) current passing through the primary winding of the transformer will increase rapidly. The energy stored on the primary side of the transformer during this time interval will then induce the secondary part of the transformer, turning the diode ON and charging the capacitor on the output part of the circuitry when the switch S is open (i.e. transistor is OFF position). Therefore the energy required for the output voltage is supplied from the capacitor when the switch is ON and from the tranformator when the switch is OFF. For control design purposes[7], the main objective is to adjust the duty cycle of the transistor, therefore the diode, online so that the output voltage, $V_o(t)$, is kept at a desired value even when the load resistance, R , is varying.

The differential equations describing a flyback converter, as the one given in Fig. 1, can be obtained to have the following form. [8]:

$$
\dot{x}_1 = -\frac{L_M}{L_1 L_2 - L_M^2} (1 - u) x_3 + \frac{L_2}{L_1 L_2 - L_M^2} E u
$$
\n
$$
\dot{x}_2 = \frac{L_1}{L_1 L_2 - L_M^2} (1 - u) x_3 - \frac{L_M}{L_1 L_2 - L_M^2} E u
$$
\n
$$
\dot{x}_3 = \frac{1}{C} (1 - u) x_2 - \frac{1}{RC} x_3
$$
\n(1)

where x_1, x_2, x_3 are the used to express the current passing through the primary winding of the transformer on the input side, current passing through the secondary winding of the transformer on the output side and the output voltage measured at the resistor terminals respectively. E is the DC voltage supply, L_1 is the inductance of the primary side of the transformer, L_2 is the inductance of the secondary side while L_M is the mutual inductance between the two windings,. $u(t)$ is the switching function for the input transistor and $(1 - u)$ is the switching function for the diode D. Applying the transformation

$$
u = \begin{cases} 1, t_k < t \le t_k + \mu(x(t_k))T \\ 0, t_k + \mu(x(t_k))T \le t < t_k + T = t_{k+1} \end{cases}
$$
 (2)

the continuous state average model for the pulse width modulated (PWM) regulated system of (1) can be obtained to have the following form [8].

$$
\dot{z}_1 = -\frac{L_M}{L_1 L_2 - L_M^2} (1 - \mu) z_3 + \frac{L_2}{L_1 L_2 - L_M^2} E \mu
$$
\n
$$
\dot{z}_2 = \frac{L_1}{L_1 L_2 - L_M^2} (1 - \mu) z_3 - \frac{L_M}{L_1 L_2 - L_M^2} E \mu
$$
\n
$$
\dot{z}_3 = \frac{1}{C} (1 - \mu) z_2 - \frac{1}{RC} z_3
$$
\n(3)

where the discrete control input signal $u(t)$ is substituted by the continuous, but limited, duty ratio function $\mu(t)$.

The control objective is now to regulate the transformed state $z_3(t)$ at a desired contstant output voltage level which we will refer as v_d via duty ratio function $\mu(t)$.

ERROR SYSTEM DEVELOPMENT AND CONTROLLER DESIGN

To quantify the control objective we define the output error of the converter, $e_0(t) \in \mathbb{R}$, and the indirect error signal $e_i(t) \in \mathbb{R}$, as follows

$$
e_0 = z_3 - v_d
$$

\n
$$
e_i = z_2 - z_{2d}
$$
\n(4)

where v_d , is the desired output voltage and z_{2d} is the necessary current passing through the secondary winding of the transformer that ensures the desired voltage is achieved at the output when the overall system is at steady state. To calculate the value of z_{2d} , we need to establish a relationship between the average output voltage and the average current passing through the secondary winding of the transformer at the desired equilibrium point. Assuming that a constant duty ratio $\mu = \mu_*$ is achieved at desired equilibrium state, the coresponding state equilibrium values for the average output voltage, denoted by $(z_3)_*$ and the averaged current passing through the secondary winding , denoted by (z_2) can be obtained from (3) as

$$
(z_2)_* = \frac{L_M E \mu_*}{R L_1 (1 - \mu_*)^2} \text{ and } (z_3)_* = \frac{L_M E \mu_*}{L_1 (1 - \mu_*)} \tag{5}
$$

From (5) it is straightforward to show that in order to ensure $(z_3)_* = z_{3d} = v_d$ the corresponding value for $(z_2)_* = z_{2d}$ have to be in the following form

$$
z_{2d} = \frac{v_d (L_M E + v_d L_1)}{R L_M E} \tag{6}
$$

Similar to that of the Buck-Boost converter, stabilization of the flyback converter using only the output voltage is not achievable. To overcome this; researches have proposed indirect controllers where the regulation around the desired point is achieved indirectly by stabilizing the average current input $z_2(t)$ around the corresponding

equilibrium value z_{2d} [9]. Taking the time derivative of $e_i(t)$ defined in (4) and inserting for $\dot{z}_2(t)$ from (3) we obtain

$$
\dot{e}_i = \frac{L_1}{L_1 L_2 - L_M^2} (1 - \mu) z_3 - \frac{L_M}{L_1 L_2 - L_M^2} \mu E \tag{7}
$$

where the fact that $\dot{z}_{2d} = 0$ has been utilized. To facilitate the backstepping design we added and subtracted $(1 \mu$) α term to the right hand side of (7) to obtain

$$
\dot{e}_i = (1 - \mu)\eta - \frac{L_M}{L_1 L_2 - L_M^2} \mu E + (1 - \mu)\alpha \tag{8}
$$

with the auxiliary term $\eta(t)$ defined as

$$
\eta \triangleq \frac{L_1}{L_1 L_2 - L_M^2} z_3 - \alpha \tag{9}
$$

and $\alpha(t)$ is the auxiliary control signal yet to be designed. Based on the structure of (9) and the subsequent stability analysis the auxiliary control input signal $\alpha(t)$ is designed as follows:

$$
\alpha \triangleq \frac{1}{(1-\mu)} \left[-k_i e_i + \frac{L_M}{L_1 L_2 - L_M^2} \mu E \right] \tag{10}
$$

where k_i is a positive scalar control gain. After substituting (10) into (8), the closed-loop error dynamics for $e_i(t)$ is obtained to have the following form

$$
\dot{e}_i = (1 - \mu)\eta - k_i e_i. \tag{11}
$$

The backstepping type control design also requires the dynamics of the auxiliary signal $\eta(t)$. To this end, we take the time derivative of (9) and insert for the time derivative of $\alpha(t)$ designed in (10) to obtain

$$
\dot{\eta} = \frac{L_1}{L_1 L_2 - L_M^2} \left[-\frac{1}{C} (1 - \mu) z_2 - \frac{1}{RC} z_3 \right] \n- \frac{\mu}{(1 - \mu)^2} \left[-k_i e_i + \frac{L_M}{L_1 L_2 - L_M^2} E \right] \n+ k_i \left[\frac{L_1}{L_1 L_2 - L_M^2} (1 - \mu) z_3 - \frac{\mu}{(1 - \mu)} \frac{L_M}{L_1 L_2 - L_M^2} E \right]
$$
\n(12)

Based on the structure of (11) and (12) and the subsequent stability analysis, the dynamically generated duty ratio function is designed as

$$
\dot{\mu} = \frac{(1 - \mu)^2}{\left[-k_i e_i + \frac{L_M}{L_1 L_2 - L_M^2} E \right]} \left[k_{\eta} \eta + (1 - \mu) e_i + \Delta \right] \tag{13}
$$

where the auxiliary control function Δ used in (13) is defined as

$$
\Delta = \frac{L_1}{L_1 L_2 - L_M^2} \left[-\frac{1}{C} (1 - \mu) z_2 - \frac{1}{RC} z_3 \right]
$$

+ $k_i \left[\frac{L_1}{L_1 L_2 - L_M^2} z_3 - \frac{\mu}{(1 - \mu)} \frac{L_M}{L_1 L_2 - L_M^2} E \right]$ (14)

and k_n is a positive control gain.

Remark : It is obvious from (13) that the proposed controller has a singularity at $k_i e_i = \frac{L_M}{L_1 L_2 - L_M^2} E$. This *drawback of the controller can either be avoided by design (i.e. selection of the induction values of the transformer) or artificially keeping the value of the denominator of (13)away from zero.*

Inserting (13) and (14) back in (12) the closed loop dynamics for the auxiliary signal $\eta(t)$ is obtained to have the following form

$$
\dot{\eta} = -k_{\eta}\eta - (1 - \mu)e_i \tag{15}
$$

We are now ready to state the following Theorem

Theorem : *The nonlinear controller proposed in (13), (14) with the auxiliary control input signal of (10) ensures that the current tracking error signal* $e_i(t)$ *defined in* (4) , exponentially converges to zero in the sense that $e_i(t)$ $and \eta(t)$ are bounded by an exponentially decaying *envelope. Which indirectly ensures that the output voltage is also regulated around the desired value.*

Proof: We start our proof by introducing the following non-negative function of the form

$$
V = \frac{1}{2}e_i^2 + \frac{1}{2}\eta^2\tag{16}
$$

After taking the time derivative of (16) along with (11) and (15) and cancelling common term we obtain

$$
\dot{V} = -k_i e_i^2 - k_\eta \eta^2 \tag{17}
$$

From the structure of (16) and (17) we can use standard Lyapunov arguments to show that $e_i(t)$, $\eta(t) \in \mathcal{L}_{\infty}$ (are bounded). Then following standard signal chasing arguments we can conclude that all signals in the closedloop error system remain bounded when the singularity defined in *Remark 1* is avoided. Moreover the structure of (16) and (17) enable us to conclude that the bound on $e_i(t)$, $\eta(t)$ exponentially decays to zero. Since the convergence of the current tracking error is achieved we also can guarantee that the output voltage is also regulated around the desired set point.

SIMULATION RESULTS

The controller proposed in (13) and (10) was simulated on the flyback converter given in fig.1 with the input voltage set to E= 48V, the inductance of the transformer selected as

$$
L_1 = 500 \,\mu H, L_2 = 500 \,\mu H, L_M = 420 \,\mu H \,, \tag{18}
$$

The output capacitance value used in the simulations was

$$
C = 2400 \,\mu\text{F} \tag{19}
$$

and the PWM switching freqeuncy was set to 50kHz. As shown in Figure 2 the controller attempts to indirectly regulate the output voltage. Backstepping controller output is $\mu(t)$ and by integration along time the control signal $\mu(t)$ was obtained. Two sets of simulations for desired output voltages 5V and 10 V are performed. Other set of simulation for desired output voltages 5V and $R = 5$ ohm are performed.

Fig.2 A Block Diagram of the Controller Implementation

 On the first set of simulations the output resistance R was set to 2Ω , and the output current and voltage values are recorded. The control gains defined in (10) and (13) were selected as follows

$$
k_i = 0.000003, \ k_{\eta} = 19313000 \tag{24}
$$

For the second set of simulations the desired output voltage is set to 10 V and the controller gains were selected as

$$
k_i = 0.000003, k_{\eta} = 2610000 \tag{25}
$$

For the third set of simulations the desired output voltage is set to 5 V, load 5 ohm and the controller gains were selected as

$$
k_i = 0.000003, k_{\eta} = 3400000 \tag{26}
$$

The results of the simulations are presented in Figures 3,4 and 5.

Fig.3 The output voltage and current when the desired output voltage is set to 5 V and $R = 2\Omega$

Fig.4 The output voltage and current when the desired output voltage is set to 10 V and $R = 2\Omega$

Fig.5 The output voltage and current when the desired output voltage is set to 5 *V* and $R = 5Ω$

RESULTS AND DISCUSSION

In this paper a nonlinear controller for a special type of DC to DC converter known as the Flyback type converter has been presented. Simulation motivated duty ratio synthesizer was derived for the indirect output voltage stabilization of dc to dc power converter of the Flyback type converter. The proposed controller is based on backstepping approach and ensures that the current tracking error is driven to zero which indirectly enforces the output voltage to be regulated around the desired set point for varies value of output load. Simulation results are presented to illustrate the effectiveness of the proposed method.

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