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$$\left. \begin{aligned} U_e &= \frac{JG}{rL} \theta^r \\ U_F &= -W = -T\theta \end{aligned} \right\} \Pi = U_e + U_F = \frac{JG}{rL} (\theta_r - \theta_1)^2 - T_1 \theta_1 - T_r \theta_r$$

پاسخ ۲-۱۵

$$\Rightarrow \begin{cases} \frac{\partial \Pi}{\partial \theta_1} = 0 \Rightarrow \frac{JG}{rL} (2(-1)(\theta_r - \theta_1)) - T_1 = 0 \\ \frac{\partial \Pi}{\partial \theta_r} = 0 \Rightarrow \frac{JG}{rL} (2(1)(\theta_r - \theta_1)) - T_r = 0 \end{cases} \Rightarrow \begin{cases} \frac{JG}{L} (\theta_1 - \theta_r) = T_1 \\ \frac{JG}{L} (\theta_r - \theta_1) = T_r \end{cases}$$

$$\xrightarrow{\text{نویسند به صورت ماتریسی}} \frac{JG}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_r \end{bmatrix} = \begin{bmatrix} T_1 \\ T_r \end{bmatrix}$$

$$u(x) = N_1(x) u_1 + N_r(x) u_r$$

$$u(x=0) = u_1$$

$$N_1(0) = 1$$

$$N_1(x) = \cos\left(\frac{\pi x}{rL}\right)$$

$$u(x=L) = u_r$$

$$N_r(0) = 0$$

$$N_r(x) = \sin\left(\frac{\pi x}{rL}\right)$$

$$N_1(L) = 0$$

$$N_r(L) = 1$$



پاسخ ۲-۱۶

$$u(x=0) = N_1(0) u_1 + N_r(0) u_r = u_1 \Rightarrow N_1(0) u_1 = u_1 \Rightarrow N_1(0) = 1$$

$$u(x=L) = u_r \Rightarrow N_1(L) u_1 + N_r(L) u_r = u_r \Rightarrow N_r(L) u_r = u_r \Rightarrow N_r(L) = 1$$

$$u_1 = u_r \Rightarrow u = N_1(x) u + N_r(x) u \Rightarrow N_1(x) + N_r(x) = 1 \Rightarrow \cos\left(\frac{\pi x}{rL}\right) + \sin\left(\frac{\pi x}{rL}\right) = 1 \Rightarrow \begin{cases} x=0 \\ x=L \end{cases}$$

$$N_1'(x) = -\frac{\pi}{rL} \sin\left(\frac{\pi x}{rL}\right)$$

$$N_r'(x) = \frac{\pi}{rL} \cos\left(\frac{\pi x}{rL}\right)$$

$$\epsilon_x = \frac{du}{dx} = \frac{-\frac{\pi}{rL} u_1 \sin\left(\frac{\pi x}{rL}\right) + \frac{\pi}{rL} u_r \cos\left(\frac{\pi x}{rL}\right)}{1} = \frac{\pi}{rL} \left(-u_1 \sin\left(\frac{\pi x}{rL}\right) + u_r \cos\left(\frac{\pi x}{rL}\right) \right)$$

$$\epsilon_x \Big|_{x=0} = \frac{\pi}{rL} (u_r) = \frac{\pi}{rL} u_r$$

$$\epsilon_x \Big|_{x=L} = \frac{\pi}{rL} (-u_1) = -\frac{\pi}{rL} u_1$$

چون مقدار کرنش تغییر کرده بنابراین توابع درونیاب معرفی کرده مناسب نیستند.