۱) اتحادهای مهم ...

① $(a \pm b)^2 = a^2 \pm 2ab + b^2$

② $(a+b)^2 + (a-b)^2 = 2a^2 + 2b^2$ $\quad (a+b)^2 - (a-b)^2 = 4ab$

③ $(a+b)(a-b) = a^2 - b^2$

④ $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

⑤ $(x+a)(x+b) = x^2 + (a+b)x + ab$

⑥ $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

⑦ $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

⑧ $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - ac - bc)$

۲) اتحادهای مثلثاتی ۲

① $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$ $\quad \cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ $\quad \tan\alpha = \frac{1}{\cot\alpha}$

② $\sin^2\alpha + \cos^2\alpha = 1$ $\quad 1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$ $\quad 1 + \cot^2\alpha = \frac{1}{\sin^2\alpha}$

③ $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

④ $\cos(\alpha \pm \beta) = \cos\alpha.\cos\beta \mp \sin\alpha.\sin\beta$ $\quad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

⑤ $\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \cos^2\alpha - \sin^2\alpha$

⑥ $\sin^2\alpha = \frac{1 - \cos 2\alpha}{2}$ $\quad \cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$

⑦ $(\sin\alpha \pm \cos\alpha)^2 = 1 \pm \sin 2\alpha$

⑧ $\tan\alpha + \cot\alpha = \frac{2}{\sin 2\alpha}$ $\quad \tan\alpha - \cot\alpha = -2\cot 2\alpha$

⑨ $\sin 2\alpha = \frac{2\tan\alpha}{1 + \tan^2\alpha}$ $\quad \cos 2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$ $\quad \frac{2\tan\alpha}{1 - \tan^2\alpha} = \tan 2\alpha$

(۱۱) $\sin^4\alpha + \cos^4\alpha = 1 - 2\sin^2\alpha\cdot\cos^2\alpha = 1 - \frac{1}{2}\sin^2 2\alpha$

(۱۲) $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\cdot\tan\beta}$ $\qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

(۱۳) $\cot(\alpha+\beta) = \frac{\cot\alpha\cdot\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$ $\qquad \cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha}$

(۱۴) $\sin\alpha \pm \cos\alpha = \sqrt{2}\sin(\alpha \pm \frac{\pi}{4})$ $\qquad \cos\alpha \mp \sin\alpha = \sqrt{2}\cos(\alpha \pm \frac{\pi}{4})$

(۱۵) $\sin 2\alpha = 2\sin\alpha - 4\sin^3\alpha$ $\qquad \cos 2\alpha = 4\cos^3\alpha - 3\cos\alpha$

(۱۶) $\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cdot\cos\left(\frac{\alpha \mp \beta}{2}\right)$

(۱۷) $\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\cos\left(\frac{\alpha-\beta}{2}\right)$

(۱۸) $\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)$

(۱۹) $\tan\alpha \pm \tan\beta = \frac{\sin(\alpha \pm \beta)}{\cos\alpha\cdot\cos\beta}$ $\qquad \cot\alpha \pm \cot\beta = \frac{\sin(\beta \pm \alpha)}{\sin\alpha\cdot\sin\beta}$

(۲۰) $\sin\alpha\cdot\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$

(۲۱) $\sin\alpha\cdot\sin\beta = \frac{-1}{2}\left(\cos(\alpha+\beta) - \cos(\alpha-\beta)\right)$

(۲۲) $\cos\alpha\cdot\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$

۳) روابط مشتق

مشتق تابع	تابع	مشتق تابع	تابع
		$y'=0$	$y=a$
$y'=au'$	$y=au$	$y'=a$	$y=ax$
$y'=anu'u^{n-1}$	$y=au^n$	$y'=anx^{n-1}$	$y=ax^n$
$y'=u'+v'$	$y=u+v$	$y'=1$	$y=a+x$
$y'=u'vw+v'uw+w'uv$	$y=uvw$		
$y'=\frac{u'v-v'u}{v^2}$	$y=\frac{u}{v}$		
$y'=\frac{u'}{2\sqrt{u}}$	$y=\sqrt{u}$	$y'=\frac{1}{2\sqrt{x}}$	$y=\sqrt{x}$
$y'=\frac{mu'}{n\sqrt[n]{u^{n-m}}}$	$y=\sqrt[n]{u^m}$	$y'=\frac{m}{n\sqrt[n]{x^{n-m}}}$	$y=\sqrt[n]{x^m}$
$y'=u'a^u\ln a$	$y=a^u$	$y'=a^x\ln a$	$y=a^x$
$y'=u'e^u$	$y=e^u$	$y'=e^x$	$y=e^x$
$y'=\frac{u'}{u}$	$y=\ln u$	$y'=\frac{1}{x}$	$y=\ln x$
$y'=\frac{u'}{u\ln a}$	$y=\log_a u$	$y'=\frac{1}{x\ln a}$	$y=\log_a x$
$y'=n.a.u'.\cos u.\sin^{n-1}u$	$y=a\sin^n u$	$y'=\cos x$	$y=\sin x$
$y'=-n.a.u'.\sin u.\cos^{n-1}u$	$y=a\cos^n u$	$y'=-\sin x$	$y=\cos x$
$y'=n.a.u'\left(1+\tan^2 u\right).\tan^{n-1}u$	$y=a\tan^n u$	$1+\tan^2 x$	$y=\tan x$
$y'=-n.a.u'\left(1+\cot^2 u\right).\cot^{n-1}u$	$y=a\cot^n u$	$y'=-\left(1+\cot^2 x\right)$	$y=\cot x$
$y'=\frac{u'}{\sqrt{1-u^2}}$	$y=\sin^{-1}u$	$y'=\frac{1}{\sqrt{1-x^2}}$	$y=\sin^{-1}x$
$y=\frac{-u'}{\sqrt{1-u^2}}$	$y=\cos^{-1}u$	$y'=\frac{-1}{\sqrt{1-x^2}}$	$y=\cos^{-1}x$
$y'=\frac{u'}{1+u^2}$	$y=\tan^{-1}u$	$y'=\frac{1}{1+x^2}$	$y=\tan^{-1}x$
$y'=\frac{-u}{1+u^2}$	$y=\cot^{-1}u$	$y'=\frac{-1}{1+x^2}$	$y=\cot^{-1}x$
$y'=\frac{\begin{vmatrix}a & b\\ c & d\end{vmatrix}}{(cx+d)^2}$	$y=\frac{ax+b}{cx+d}$	$y'=\frac{-a}{x^2}$	$y=\frac{a}{x}$
$y'=\frac{-u'an}{x^{n+1}}$	$y=\frac{a}{u^n}$	$y'=\frac{-an}{x^{n+1}}$	$y=\frac{a}{x^n}$

فرمولهای انتگرال

① $\int 1\,dx = \int dx = x + c$

② $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq 1) \qquad \int x^{-1} dx = \ln x$

③ $\int k a^{nx}\,dx = \frac{k a^{nx}}{n \ln a} + c$

④ $\int e^{ax} = \frac{e^{ax}}{a} + c$

⑤ $\int k \sin ax\,dx = -\frac{k}{a} \cos ax + c$

⑥ $\int k \cos ax\,dx = \frac{k}{a} \sin ax + c$

⑦ $\int k \tan ax\,dx = \frac{-k}{a} \ln|\cos ax| + c$

⑧ $\int k \cot ax\,dx = \frac{k}{a} \ln|\sin ax| + c$

⑨ $\int (1 + \tan^2 ax)\,dx = \frac{1}{a} \tan ax + c$

⑩ $\int (1 + \cot^2 ax)\,dx = \frac{-1}{a} \cot ax + c$

⑪ $\int \frac{du}{u} = \ln|u| + c$

⑫ $\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b} \sin^{-1} \frac{bx}{a} + c = \frac{-1}{b} \cos^{-1} \frac{bx}{a} + c$

⑬ $\int \frac{dx}{a^2 + b^2x^2} = \frac{1}{ab} \tan^{-1} \frac{bx}{a} + c = \frac{-1}{b} \cot^{-1} \frac{bx}{a} + c$

* موفق باشید *

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