Example

Farmer Bill has 300 acres and plans to plant wheat and soybeans. Each acre of wheat costs $275 to plant, maintain and harvest while each acre of soybeans costs $140 to plant, maintain and harvest. The farmer has a crop loan of $60,000 available to cover costs. Each acre of wheat will yield 120 bu. of wheat, while each acre of soybeans will yield 30 bu. of soybeans. Farmer Bill has contracted to sell the wheat and soybeans for $3.00 per bu. and $6.00 per bu., respectively. However, the farmer must store both the wheat and soybeans for several months after harvest in his storage facility which has a maximum capacity of 24,000 bu. Farmer Bill wants to know how many acres of each crop to plant in order to maximize profit. Formulate a linear programming model for this problem.

- Setting up a table, we have

	Yield/acre (bushel)	Cost/acre ($)	Profit/acre ($)
Wheat	120	275	????
Soy Beans	30	140	????
Capacity	24,000	60,000	

Profit/acre for Wheat:

Cost: ($275/acre)/(120 bu/acre) = $2.29

Profit/bu = $3 - $2.29 = $0.7083/bu

Profit/acre = $0.7083/bu (120 bu/acre) = $85

Profit/acre for Soybeans: $40

- Decision Variables

Let Z = Profit for 300 acres of crops

Let X_1 = number of acres of wheat to plant

Let X_2 = number of acres of soybeans to plant

Objective Function

$$\text{Max } Z = \$85X_1 + \$40X_2$$

- Constraints

$120X_1 + 30X_2 \leq 24,000$ (yield -storage capacity)

$\$275X_1 + \$140X_2 \leq \$60,000$ (costs -loan amount)

$X_1 + X_2 \leq 300$ (acres –size of farm)

$X_1, X_2 \geq 0$

Graphical Solution to LP Models

- Two approaches:
  - Graphically (enumeration)
  - Mathematically (Simplex Method; Excel)

- Constraints define the feasible region
- What is the feasible region?

  set of points (or values) that the decision variables can have and simultaneously satisfy all of the constraints in the LP model.

Graphical Solution (Cont.)

- Two variables... we can graph the FR and locate the optimal solution/point
- Must first plot the constraints
- Recall that the lines are of the form

$$aX_1 + bX_2 = c$$

where a, b, and c are constants.

- Let’s look at the Farmer Bill Example...

Graphical Solution: Farmer Bill Example

- Recall the model

  Max $Z = \$85X_1 + \$40X_2$

  subject to

  $120X_1 + 30X_2 \leq 24,000$ (yield)

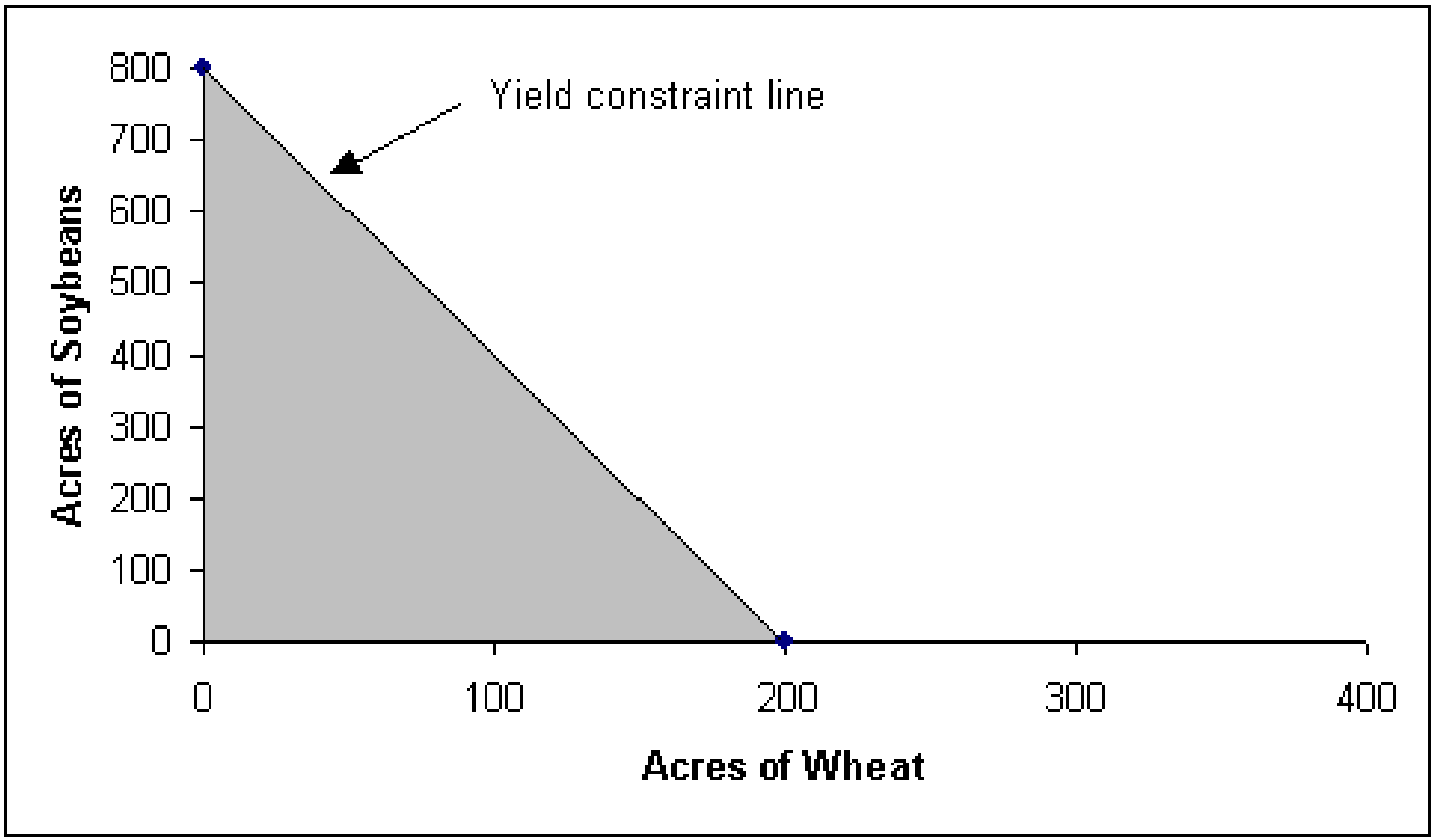
  $\$275X_1 + \$140X_2 \leq \$60,000$ (costs)

  $X_1 + X_2 \leq 300$ (acres)

  $X_1, X_2 \geq 0$

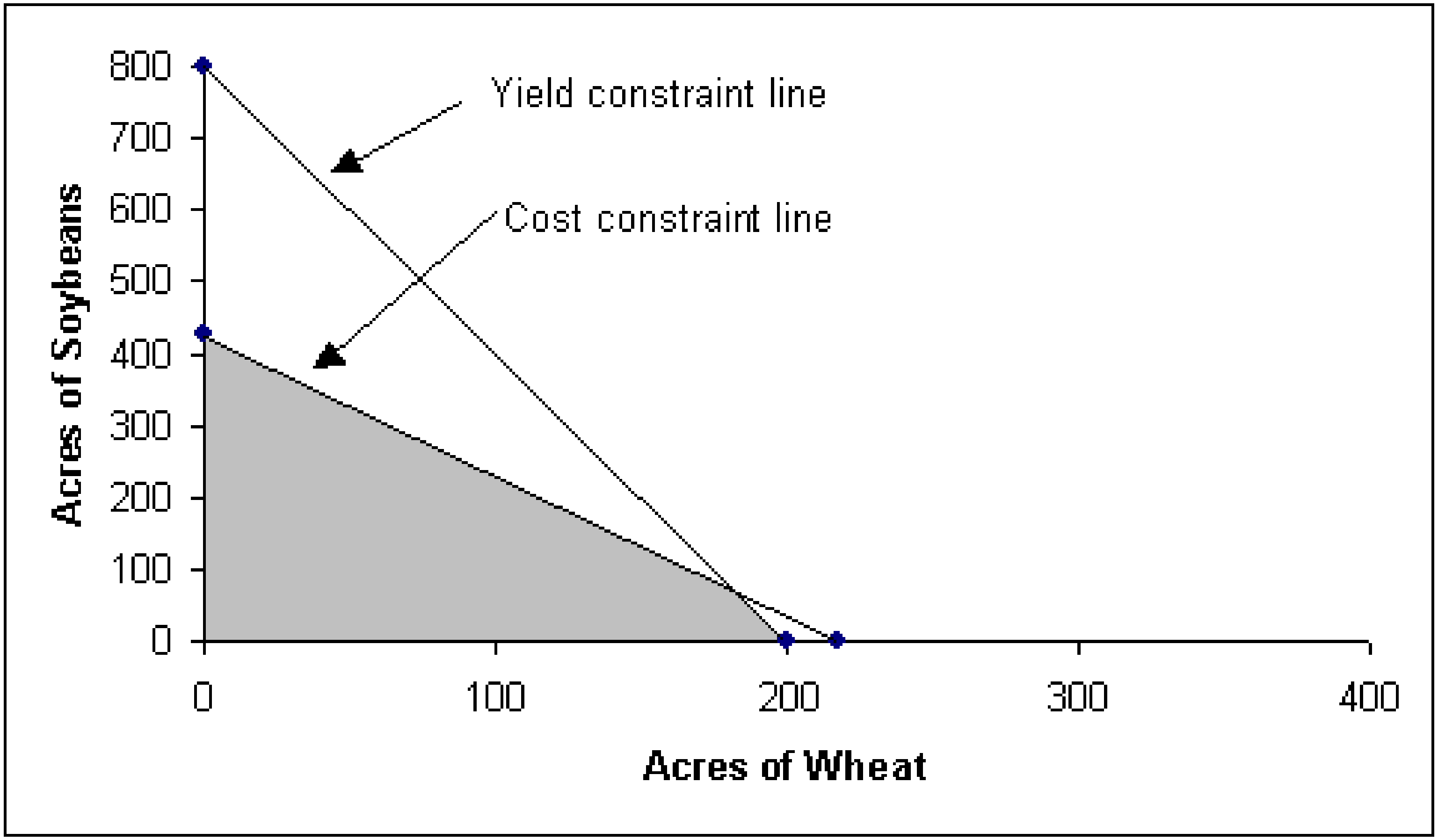
Graphical Solution: Farmer Bill Example

Now we can plot the yield constraint:



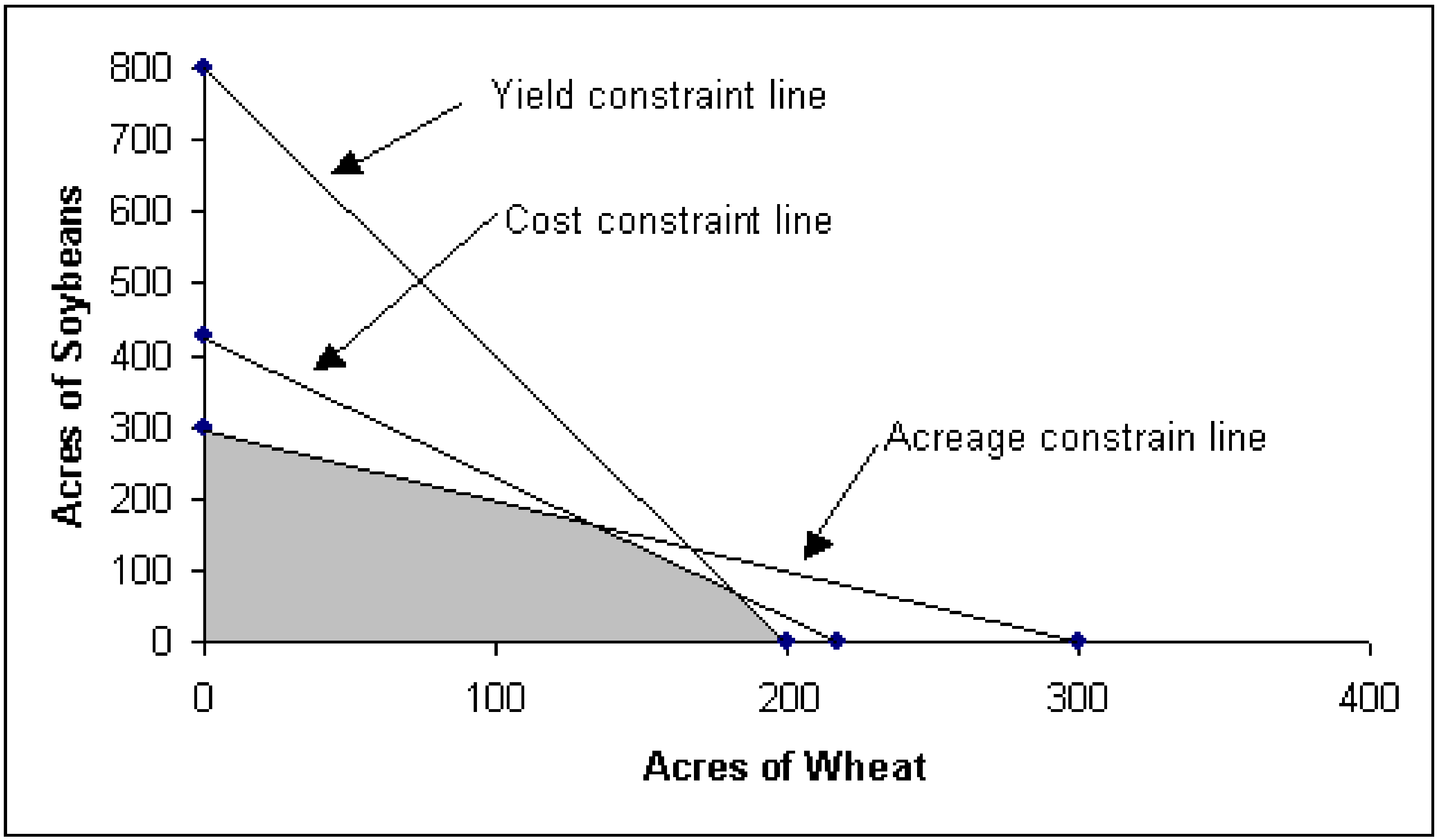
Graphical Solution: Farmer Bill Example

Now we can plot the cost constraint line along with the yield.

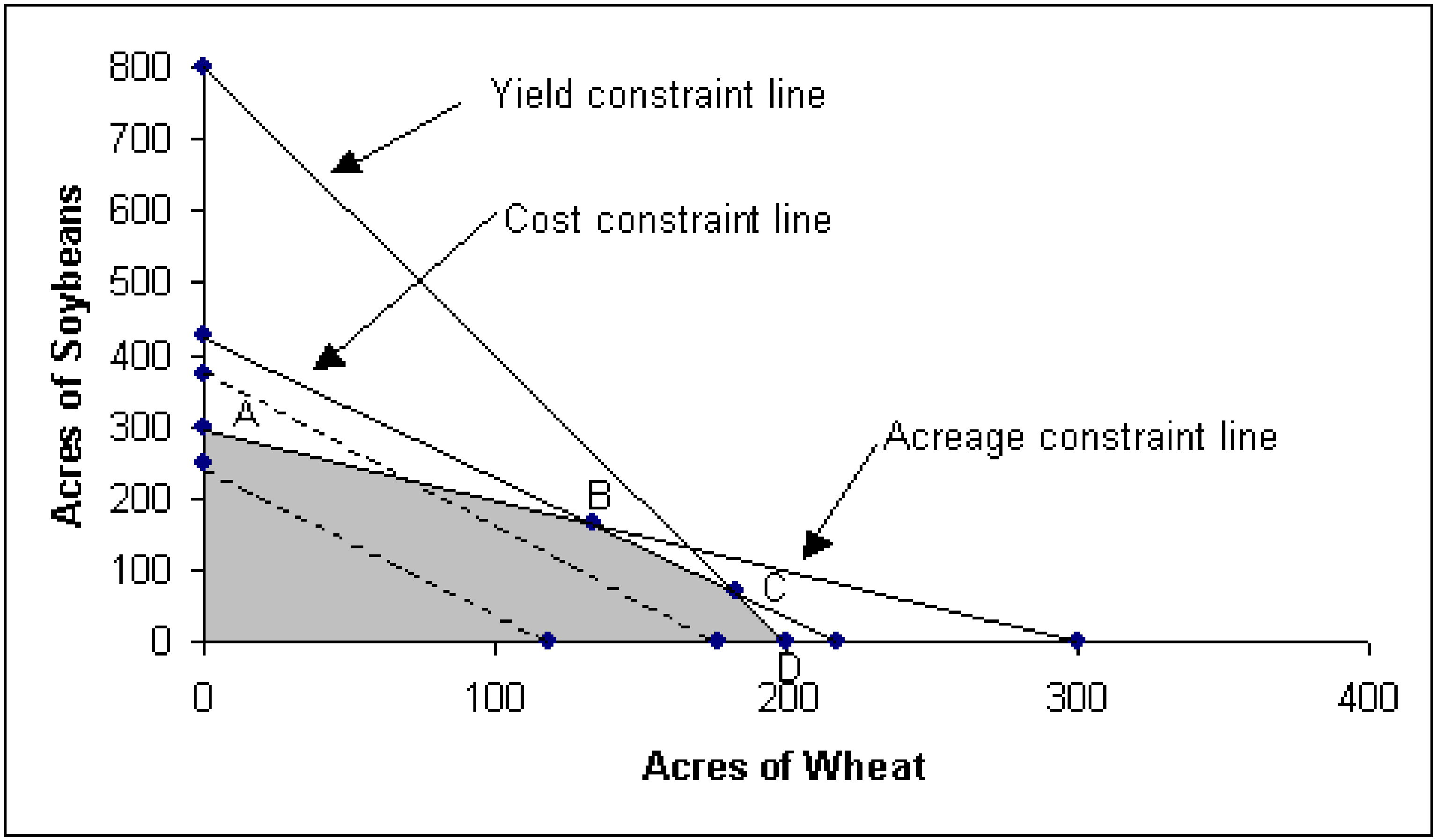


Graphical Solution: Farmer Bill Example

Plotting all constraints...



Graphical Solution: Farmer Bill Example
Why Boundary Points → Solution (Cont.)



Graphical Solution: Farmer Bill

- Level curves make process rely on "visual inspection."
- Another alternative: Enumeration of corner points

	Corner Point	Obj. Function Value (Z)
A	(0, 300)	$12,000.00
B	(133.33, 166.67)	$17,999.85
C	(182.5, 70.18)	$18,315.79
D	(200, 0)	$17,000.00