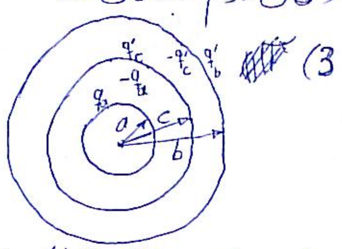
بسمه تعالی

امتحان دوم تابستان 88

3)



$$\int \vec{E}\cdot ds = \frac{q_{in}}{\varepsilon_0} \to \boxed{q_{in_a}=0} \to \boxed{q_a = q_{out_a}}$$

$$\begin{cases} \frac{kq_a}{a}+\frac{kq_b}{b}+\frac{kQ}{c}=\varphi_a \to q_a = \left(\frac{\varphi_a}{k}-\frac{Q}{c}\right)a - \frac{q_b a}{b} \\ \frac{kq_a}{b}+\frac{kQ}{b}+\frac{kq_b}{b}=\varphi_b \to q_a+q_b = \left(\frac{\varphi_b}{k}-\frac{Q}{b}\right)b \end{cases} \to \frac{a\varphi_a}{k}-\frac{aQ}{c}-\frac{aq_b}{b} = \frac{b\varphi_b}{k} - q_b - Q \to$$

$$q_b\left(\frac{b-a}{b}\right) = \left(\frac{a-c}{c}\right)Q + \frac{b\varphi_b - a\varphi_a}{k} \to \boxed{q_b = \left(\frac{b}{b-a}\right)\left(\frac{b\varphi_b-a\varphi_a}{k}+\frac{Q(a-c)}{c}\right)} \to q_a = \frac{a\varphi_a}{k}-\frac{aQ}{c}-\frac{a}{b-a}\left(\frac{b\varphi_b-a\varphi_a}{k}+\frac{aQ}{c}-Q\right)\to$$

$$q_a = \frac{a\varphi_a}{k}-\frac{aQ}{c}-\frac{ab\varphi_b}{(b-a)k}+\frac{a^2\varphi_a}{k(b-a)}-\frac{a^2Q}{c(b-a)}+\frac{aQ}{b-a}\to \boxed{q_a = \frac{ab(\varphi_a-\varphi_b)}{k(b-a)}+\frac{a(c-b)}{c(b-a)}Q}\to$$

الف)

$$E=\begin{cases} 0 & r<a \\ \frac{k(q_a+Q)}{r^2} & c<r<b \\ \frac{kq_a}{r^2} & a<r<c \\ \frac{k(q_a+q_b+Q)}{r^2} & b<r \end{cases} \longrightarrow E=\begin{cases} 0 & r<a \\ \left(b(\varphi_a-\varphi_b)+\frac{k(c-b)}{c}Q\right)\times\frac{a}{(b-a)r^2} & a<r<c \\ \frac{b}{(b-a)r^2}\left(a(\varphi_a-\varphi_b)+\frac{(c-a)k}{c}Q\right) & c<r<b \\ \frac{b\varphi_b}{r^2} & b<r \end{cases}$$

ب)

$$\boxed{q_{in_a}=0},\ \boxed{q_{in_c}=-q_a},\ \boxed{q_{out_c}=-q_{in_b}=Q+q_a},\ q_{out_b}=Q+q_a+q_b \to \boxed{q_{out_b}=\frac{b\varphi_b}{k}}$$

ج)

$$V=\begin{cases} \varphi_a & 0<r<a \\ k\left(\frac{q_a}{r}+\frac{q_b}{b}+\frac{q_c}{c}\right) & a<r<c \\ k\left(\frac{q_a+q_c}{r}+\frac{q_b}{b}\right) & c<r<b \\ k\left(\frac{q_a+q_b+q_c}{r}\right) & b<r \end{cases} \longrightarrow V_{(r)}=\begin{cases} \varphi_a & r<a \\ \frac{kQ}{c}+\left(\frac{1}{b-a}\right)\left(\frac{ab(\varphi_a-\varphi_b)}{r}+\frac{k(c-b)a}{rc}Q+(b\varphi_b-a\varphi_a)+\frac{k(a-c)}{c}Q\right) & a<r<c \\ \left(\frac{ab(\varphi_a-\varphi_b)}{r(b-a)}+\frac{k(ac-ab+bc-ac)}{c(b-a)r}Q\right)+\frac{1}{(b-a)}\left(b\varphi_b-a\varphi_a+\frac{k(a-c)Q}{c}\right) & c<r<b \\ \frac{b\varphi_b}{r} & b<r \end{cases}$$

$$\to V_{(r)}=\begin{cases} \varphi_a & r\le a \\ \frac{kQ}{c}+\frac{1}{(b-a)}\left(\frac{ab(\varphi_a-\varphi_b)}{r}+\frac{k(c-b)a}{rc}Q+(b\varphi_b-a\varphi_a)+\frac{k(a-c)}{c}Q\right) & a\le r<c \\ \frac{1}{(b-a)}\left(\frac{ab(\varphi_a-\varphi_b)}{r}+(b\varphi_b-a\varphi_a)+\frac{kb(c-a)}{cr}Q+\frac{k(a-c)}{c}Q\right) & c\le r\le b \\ \frac{b\varphi_b}{r} & b<r \end{cases}$$

د)

$$V_c=\frac{1}{(b-a)}\left(\frac{a\varphi_a(b-c)+b\varphi_b(c-a)}{c}+\frac{k(c-a)}{c}Q\left(\frac{b-c}{c}\right)\right)\to \boxed{V_c=\frac{\left(a\varphi_a(b-c)+b\varphi_b(c-a)\right)c+k(c-a)(b-c)Q}{c^2(b-a)}}$$

4) الف)

$$g+\epsilon y = \ddot{y}_{(0)}+\epsilon\ddot{y}_{(1)} = -g+\epsilon y_{(0)}\to$$

$$y_{(0)} = -\frac{gt^2}{2}+v_0\sin\theta\, t$$

$$\ddot{y}_{(1)} = -\frac{gt^2}{2}+v_0\sin\theta\, t \to \dot{y}_{(1)} = -\frac{gt^3}{6}+v_0\sin\theta\frac{t^2}{2}\to y_{(1)} = -\frac{gt^4}{24}+\frac{v_0\sin\theta\, t^3}{6} \to \boxed{y_{(t)} = t\left(-\frac{gt}{2}+v_0\sin\theta\right)+\frac{\epsilon t^3}{6}\left(-\frac{gt}{4}+v_0\sin\theta\right)}$$

باسمه تعالی

$$t=\frac{x}{v_0\cos\theta} \longrightarrow \boxed{y_{(x)}=\left(-\frac{gx^2}{2v_0^2\cos^2\theta}+x\tan\theta\right)+\frac{\epsilon x^3}{6v_0^3\cos^3\theta}\left(-\frac{gx}{4v_0\cos\theta}+v_0\sin\theta\right)}$$

چون در راستای x شتاب نداریم پس سرعت در راستای x ثابت است و با پرتابه‌ی عادی تفاوتی ندارد. فرض می‌کنیم که نقطه‌ی پرتاب تا اوج تابع $f_{(x)}$ پیموده شود. اندکی پس از نقطه‌ی اوج معادلات همان معادلات قبل از نقطه‌ی اوج است و شرایط اولیه‌ی سرعت هم یکسان است. پس می‌توانیم تصور کنیم که فیلم را برعکس می‌کنیم. پس مسیر متقارن است.

ب)

$$y_{(x)}=0 \longrightarrow -\frac{g(R_{(0)}+\epsilon R_{(1)})^2}{2v_0^2\cos^2\theta}+(R_0+\epsilon R_{(1)})\tan\theta+\frac{\epsilon R_{(0)}^3}{6v_0^3\cos^3\theta}\left(-\frac{gR_{(0)}}{4v_0\cos\theta}+v_0\sin\theta\right)=0 \longrightarrow$$

$$-\frac{g}{2v_0^2\cos^2\theta}\left(R_{(0)}^2+2\epsilon R_{(0)}R_{(1)}\right)+R_0\tan\theta+\epsilon R_{(1)}\tan\theta-\frac{g\epsilon R_{(0)}^4}{24v_0^4\cos^4\theta}+\frac{\epsilon R_{(0)}^3\sin\theta}{6v_0^2\cos^3\theta}=0 \longrightarrow$$

$$\begin{cases} -\frac{gR_{(0)}^2}{2v_0^2\cos^2\theta}+R_{(0)}\frac{\sin\theta}{\cos\theta}=0 \longrightarrow \boxed{R_{(0)}=\frac{2v_0^2\sin\theta\cos\theta}{g}} \\ -\frac{gR_{(0)}R_{(1)}}{v_0^2\cos^2\theta}+R_{(1)}\tan\theta-\frac{gR_{(0)}^4}{24v_0^4\cos^4\theta}+\frac{R_{(0)}^3\sin\theta}{6v_0^2\cos^3\theta}=0 \longrightarrow \end{cases}$$

$$-\frac{g}{v_0^2\cos^2\theta}\times\frac{2v_0^2\sin\theta\cos\theta}{g}\times R_{(1)}+R_{(1)}\frac{\sin\theta}{\cos\theta}-\frac{g}{24v_0^4\cos^4\theta}\times\frac{16v_0^8\sin^4\theta\cos^4\theta}{g^4}+\frac{\sin\theta}{6v_0^2\cos^3\theta}\times\frac{8v_0^6\sin^3\theta\cos^3\theta}{g^3}=0 \longrightarrow$$

$$-R_{(1)}\times\frac{\sin\theta}{\cos\theta}-\frac{2v_0^4\sin^4\theta}{3g^3}+\frac{4v_0^4\sin^4\theta}{3g^3}=0 \longrightarrow R_{(1)}=\frac{\cos\theta}{\sin\theta}\times\frac{2v_0^4\sin^4\theta}{3g^3} \longrightarrow \boxed{R_{(1)}=\frac{2v_0^4\sin^3\theta\cos\theta}{3g^3}} \longrightarrow$$

$$R=\frac{2v_0^2\sin\theta\cos\theta}{g}+\frac{2\epsilon v_0^4\sin^3\theta\cos\theta}{3g^3} \longrightarrow \boxed{R=\frac{2v_0^2\sin\theta\cos\theta}{g}\left(1+\frac{\epsilon v_0^2\sin^2\theta}{3g^2}\right)}$$

پ) روش اول:

$$y_{(t)}=0 \longrightarrow -\frac{g(t_{(0)}+\epsilon t_{(1)})^2}{2}+v_0\sin\theta\,(t_{(0)}+\epsilon t_{(1)})+\frac{\epsilon t_{(0)}^3}{6}\left(-\frac{g\,t_{(0)}}{4}+v_0\sin\theta\right) \longrightarrow$$

$$\begin{cases} -\frac{gt_{(0)}^2}{2}+v_0t_{(0)}\sin\theta=0 \longrightarrow \boxed{t_{(0)}=\frac{2v_0\sin\theta}{g}} \\ -gt_{(0)}t_{(1)}+v_0t_{(1)}\sin\theta-\frac{gt_{(0)}^4}{24}+\frac{v_0t_{(0)}^3\sin\theta}{6}=0 \longrightarrow -v_0t_{(1)}\sin\theta-\frac{g}{24}\times\frac{16v_0^4\sin^4\theta}{g^4}+\frac{v_0\sin\theta}{6}\times\frac{8v_0^3\sin^3\theta}{g^3}=0 \longrightarrow \end{cases}$$

$$t_{(1)}=\frac{v_0^3\sin^3\theta}{g^3}\left(-\frac{2}{3}+\frac{4}{3}\right) \longrightarrow \boxed{t_{(1)}=\frac{2v_0^3\sin^3\theta}{3g^3}} \longrightarrow \boxed{t_{(1)}=\frac{2v_0^3}{3g^3}} \longrightarrow \boxed{t=\frac{2v_0\sin\theta}{g}\left(1+\frac{\epsilon v_0^2\sin^2\theta}{3g^2}\right)}$$

روش دوم:

$$R=v_0\cos\theta\,t \longrightarrow R_{(1)}=v_0\cos\theta\,t_{(1)} \longrightarrow \boxed{t_{(1)}=\frac{2v_0^3\sin^3\theta}{3g^3}} \text{ یا } R=v_0\cos\theta\,t \longrightarrow \boxed{t=\frac{2v_0\sin\theta}{g}\left(1+\frac{\epsilon v_0^2\sin^2\theta}{3g^2}\right)}$$

ت) طبق تقارن مسیر می‌توان گفت سرعت بازگشت $v=v_0\sin\theta$ یا همان $v=v_0$ است.
به طور دیگر می‌توان این طور گفت که نیروی کندکننده یک نیروی پایستار است؛ پس مستقل از مسیر است؛ پس کار آن صفر است. پس سرعت بازگشت همان سرعت رفت است.

ث)

$$R=\frac{2v_0^2}{g}\left(\sin\theta\cos\theta+\frac{\epsilon v_0^2}{3g^2}\sin^3\theta\cos\theta\right) \xrightarrow{\frac{dR}{d\theta}=0} \cos^2\theta-\sin^2\theta+\frac{\epsilon v_0^2}{3g^2}\left(3\sin^2\theta\cos^2\theta-\sin^4\theta\right)=0$$

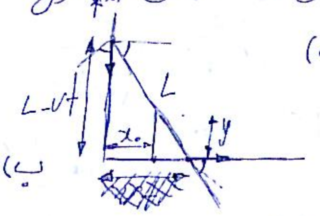
$$\longrightarrow \cos(2\theta_{(0)}+2\epsilon\theta_{(1)})+\frac{\epsilon v_0^2\sin^2(\theta_{(0)})}{3g^2}\left(3\cos^2\theta_{(0)}-\sin^2\theta_{(0)}\right)=0 \longrightarrow$$

$$\begin{cases} \cos(2\theta_{(0)})=0 \longrightarrow \boxed{\theta_{(0)}=\frac{\pi}{4}} \\ -2\theta_{(1)}\sin(2\theta_{(0)})+\frac{v_0^2\cos^2\theta_{(0)}\sin^2(\theta_{(0)})}{g^2}-\frac{v_0^2\sin^4\theta_{(0)}}{3g^2}=0 \longrightarrow -2\theta_{(1)}=\frac{v_0^2}{2g^2}\left(\frac{1}{2}-\frac{1}{6}\right) \longrightarrow \boxed{\theta=\frac{\pi}{4}+\frac{\epsilon v_0^2}{12g^2}} \end{cases}$$

بسمه تعالی

تابستان 88 - آزمون اول

2) الف)



$v\sin\theta = v'\cos\theta \to v' = v\tan\theta \to \boxed{v' = \frac{(L-vt)v}{\sqrt{vt(2L-vt)}}}$

ب)

$\frac{y}{L-vt} = \frac{\sqrt{vt(2L-vt)} - x_0}{\sqrt{vt(2L-vt)}} \to \boxed{y = \left(1 - \frac{x_0}{\sqrt{vt(2L-vt)}}\right)(L-vt)} \to$

$\dot y = \left(-v - x_0 v\,\frac{-v\sqrt{vt(2L-vt)} - \frac{(L-vt)(2L-2vt)v}{2\sqrt{vt(2L-vt)}}}{vt(2L-vt)}\right) \to \dot y = \left(-v + \frac{2Lvt - v^2t^2 + L^2 + v^2t^2 - 2Lvt}{(vt(2L-vt))^{3/2}}\, v x_0\right) \to$

$\boxed{\dot y = \left(\frac{L^2 x_0}{(vt(2L-vt))^{3/2}} - 1\right) v}$

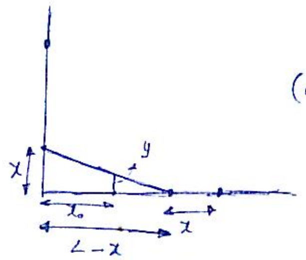
$\dot y = 0 \to \sqrt{vt(2L-vt)} = (L^2x_0)^{1/3} \to 2Lvt - v^2t^2 = (L^2x_0)^{2/3} \to L^2 - (L-vt)^2 = (L^2x_0)^{2/3} \to (L-vt)^2 = L^2 - (L^2x_0)^{2/3}$

ب)

$\to vt = L - L\sqrt{1-\left(\frac{x_0^2}{L^2}\right)^{1/3}} \to t = \frac{L\left(1-\sqrt{1-(L^{-2}x_0^2)^{1/3}}\right)}{v} \to f(x) = \left(1 - \frac{x_0}{(L^2x_0)^{1/3}}\right)\left(1 - \left(\frac{x_0^2}{L^2}\right)^{1/3}\right)^{1/2} L \to$

$\cancel{f(x) = \left(1-\sqrt[3]{\frac{x}{L}}\right)\left(1-\sqrt[3]{Lx^2}\right)L}$ $\quad \boxed{f(x) = L\left(1-\left(\frac{x_0}{L}\right)^{2/3}\right)^{3/2}}$

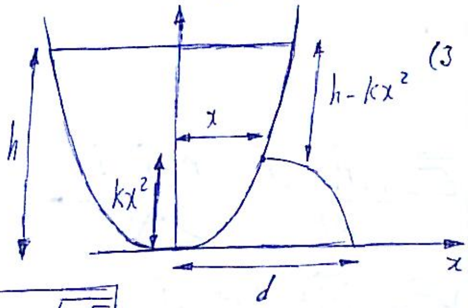
پ)



$\frac{y}{x} = \left(1 - \frac{x_0}{L-x}\right) \to \frac{dy}{dx} = 1 - x_0\frac{(L-x)+x}{(L-x)^2} = 1 - \frac{Lx_0}{(L-x)^2} = 0 \to L - x = \sqrt{Lx_0} \to x = L - \sqrt{Lx_0} \to$

$\boxed{f(x) = (\sqrt L - \sqrt{x_0})\left(1-\sqrt{\frac{x_0}{L}}\right)\sqrt L} \to \boxed{f(x) = (\sqrt L - \sqrt x)^2}$

3)



$d = x + \sqrt{2g(h-kx^2)}\, t \to d = x + 2x\sqrt{k(h-kx^2)} \to$

$t = \sqrt{\frac{2kx^2}{g}}$

$1 + 2\sqrt k\left(\sqrt{h-kx^2} + \frac{-2kx^2}{2\sqrt{h-kx^2}}\right) = 0 \to \frac{h-2kx^2}{\sqrt{h-kx^2}} = -\frac{1}{2\sqrt k} \to$

$4k(h-2kx^2)^2 = h - kx^2 \to kh - k^2x^2 = 4(kh-2k^2x^2)^2 \to$

$\frac18 - k^2x^2 = 4\left(\frac1{64} + 4k^4x^4 - \frac{k^2x^2}{2}\right) \to 16k^4x^4 + \frac1{16} - k^2x^2 - \frac18 = 0 \to$

$16k^4x^4 - k^2x^2 - \frac1{16} = 0 \to k^2x^2 = \frac{1\pm\sqrt{1+4}}{32} \to kx = \frac14\sqrt{\frac{1+\sqrt5}{2}} \to \boxed{x = 2h\sqrt{\frac{1+\sqrt5}{2}}} \to$

$d = x\left(1 + 2\sqrt{kh - k^2x^2}\right) \to d = 2h\sqrt{\frac{1+\sqrt5}{2}}\left(1 + 2\sqrt{\frac18 - \frac{1+\sqrt5}{32}}\right) \to \boxed{d = 2h\sqrt{\frac{1+\sqrt5}{2}}\left(1+\frac12\sqrt{\frac{3-\sqrt5}{2}}\right)}$

$4kh^2 + 16k^3x^4 - 16k^2hx^2 = h - kx^2 \to$

نقض

$16k^3x^4 + (-16k^2h + k)x^2 + (4kh^2 - h) = 0 \to \Delta \ge 0 \to k^2 + 256k^4h^2 - 32k^3h \ge 64k^3(4kh^2 - h) \to$

$1 + 256k^2h^2 - 32kh \ge 256k^2h^2 - 64kh \to -32kh \le 1 \to$ بدیهی $\to$ $\boxed{\text{هیچ شرطی روی h وجود ندارد}}$

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2) الف)

$q' = -\frac{a}{\sqrt{R^2+z^2}}\, q \to \boxed{q' = -\frac{2\pi a\lambda R}{\sqrt{R^2+z^2}}} \xrightarrow{r = \frac{a^2R}{R^2+z^2}} \lambda' = -\frac{a\lambda R}{\sqrt{R^2+z^2}} \times \frac{R^2+z^2}{a^2R} \to \boxed{\lambda' = -\frac{\sqrt{R^2+z^2}}{a}\lambda}$

$\frac{r}{R} = \frac{a^2}{R^2+z^2} \to \boxed{r = \frac{a^2R}{R^2+z^2}}$ مرتبه اول

ب)

$\pi\rho^2 E_z = 2\pi\rho E_\rho dz \to E_\rho^{(1)} = -\frac{\rho}{2}\frac{dE_z}{dz}$, $\vec\nabla\times\vec E = 0 \to \frac{\partial E_\rho}{\partial z} = \frac{\partial E_z}{\partial \rho} \to E_z^{(2)} = -\frac12\int\rho\left(\frac{\partial^2 E_z}{\partial z^2}\right)d\rho = E_z + c \to$

$\boxed{E_z^{(2)} = E_z^{(1)} - \frac{\rho^2}{4}E''_{z}} \to E_z'' = 0 \to F_z^{(1)} = q'E_z \to$

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$$F_z^{(1)} = kq'Q\left(\frac{-z}{(z^2+R^2)^{3/2}} + \frac{z-x}{(R^2+(z-x)^2)^{3/2}}\right) = -\frac{k\times\lambda\cdot 2\pi R}{(R^2+z^2)^{3/2}}\times -\frac{2\pi\lambda aR}{\sqrt{R^2+z^2}}\times\left(-z+(z-x)\left(1+\frac{3}{2}\times\frac{2xz}{z^2+R^2}\right)\right)=0$$

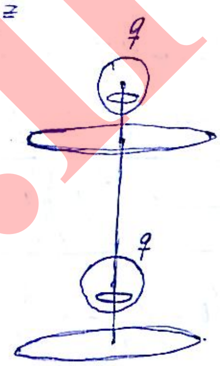
$$F_z^{(3)} = -\frac{kq'\times\lambda\times 2\pi Rz}{(R^2+z^2)^{3/2}} + \left(E_{(z-x)}^{(1)} - \frac{r^2}{2}E''^{(1)}_{(z-x)}\right)q' = q'\left(E_{(z-x)} - E_z\right) = q'\times\frac{k\lambda\times 2\pi R}{(R^2+z^2)^{3/2}}\left(\frac{z-x}{\left(1+\frac{x^2-2zx}{R^2+z^2}\right)^{3/2}} - z\right) = \frac{k\lambda\times 2\pi Rq'}{(R^2+z^2)^{3/2}}\left((z-x)\left(1+\frac{3zx}{R^2+z^2}\right)-z\right)$$

$$F_{(z)}^{(2)} = \frac{k\lambda\times 2\pi R}{(R^2+z^2)^{3/2}}\times -\frac{2\pi a\lambda R}{\sqrt{R^2+z^2}}\times\frac{a^2 z}{R^2+z^2}\times\left(\frac{3z^2}{R^2+z^2}-1\right) \longrightarrow \cancel{F_{(z)} = kaz} \quad \boxed{F_{(z)} = -kaz\left(\frac{2\pi\lambda Ra}{(R^2+z^2)^2}\right)^2(2z^2-R^2)}$$

$$E_z = \frac{2k\lambda\pi Rz}{(z^2+R^2)^{3/2}} \longrightarrow E'_{(z)} = \frac{\lambda R}{2\varepsilon_0}\cdot\frac{(z^2+R^2)^{3/2} - \frac{3(z^2+R^2)^{1/2}}{2}\times 2z}{(z^2+R^2)^3} = \frac{\lambda R}{2\varepsilon_0}\times\frac{R^2-2z^2}{(z^2+R^2)^{5/2}} \longrightarrow E''_{(z)} = \frac{\lambda R}{2\varepsilon_0}\times\frac{-4z(z^2+R^2)^{5/2} - 5(R^2-2z^2)(z^2+R^2)^{3/2}z}{(z^2+R^2)^5}$$

$$\longrightarrow E''_{(z)} = -\frac{\lambda R}{2\varepsilon_0}\times\frac{4z^2+4R^2+5R^2-10z^2}{(z^2+R^2)^{7/2}}z \longrightarrow E''_{(z)} = -\frac{\lambda R}{2\varepsilon_0}\cdot\frac{9R^2-6z^2}{(z^2+R^2)^{7/2}}z \longrightarrow E''_{(z)} = -\frac{3\lambda R}{2\varepsilon_0}\cdot\frac{3R^2-2z^2}{(z^2+R^2)^{7/2}}z$$

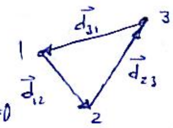
(ب



$$\vec{E} = \frac{\rho_1\vec{r}_1+\rho_2\vec{r}_2}{3\varepsilon_0} \xrightarrow{\vec{r}_2=\vec{r}_1-\vec{d}} \vec{E} = \frac{(\rho_1+\rho_2)\vec{r}_1-\rho_2\vec{d}}{3\varepsilon_0} \longrightarrow \begin{cases}\rho_1+\rho_2=0\\ \vec{E} = -\frac{\rho_2\vec{d}}{3\varepsilon_0} = \frac{\rho_1\vec{d}}{3\varepsilon_0}\end{cases}$$ (a (3

$$\vec{E} = \frac{\rho_1\vec{r}_1+\rho_2\vec{r}_2+\rho_3\vec{r}_3}{3\varepsilon_0} \longrightarrow \vec{E} = \frac{(\rho_1+\rho_2+\rho_3)\vec{r}_2+(\rho_3\vec{d}_{32}+\rho_1\vec{d}_{12})}{3\varepsilon_0} \longrightarrow \boxed{\rho_1+\rho_2+\rho_3=0}$$ (b

$$\vec{r}_1 = \vec{d}_{12}+\vec{r}_2 \quad , \quad \vec{r}_3 = \vec{d}_{32}+\vec{r}_2$$



$$\vec{d}_{23}+\vec{d}_{31}+\vec{d}_{12}=0 \longrightarrow \vec{d}_{32}=\vec{d}_{12}+\vec{d}_{31} \longrightarrow \vec{E} = \frac{(\rho_3+\rho_1)\vec{d}_{12}+\rho_3\vec{d}_{31}}{3\varepsilon_0} \xrightarrow{\vec{E}\cdot\vec{d}_{13}=0} -\rho_3(d_{13})^2+(\rho_1+\rho_3)(\vec{d}_{13}\cdot\vec{d}_{12})=0$$ (c

$$\longrightarrow \boxed{\frac{\rho_1}{\rho_3} = \frac{(d_{13})^2-(\vec{d}_{13}\cdot\vec{d}_{12})}{\vec{d}_{13}\cdot\vec{d}_{12}}} \qquad \vec{E} = \frac{(\rho_1+\rho_2+\rho_3)\vec{r}_1+(\rho_2\vec{d}_{21}+\rho_3\vec{d}_{31})}{3\varepsilon_0} \longrightarrow \rho_2(\vec{d}_{12}\cdot\vec{d}_{13})+\rho_3(d_{13})^2=0 \longrightarrow \boxed{\frac{\rho_2}{\rho_3} = -\frac{(d_{13})^2}{\vec{d}_{12}\cdot\vec{d}_{13}}}$$

$$\longrightarrow \boxed{\frac{\rho_1}{\rho_2} = \frac{(d_{13})^2-(\vec{d}_{13}\cdot\vec{d}_{12})}{(d_{13})^2}} \longrightarrow \boxed{\frac{\rho_1}{\rho_3} = \frac{\vec{d}_{23}\cdot\vec{d}_{13}}{\vec{d}_{13}\cdot\vec{d}_{12}}} \ , \ \boxed{\frac{\rho_1}{\rho_2} = \frac{\vec{d}_{23}\cdot\vec{d}_{13}}{(d_{13})^2}}$$

$$\vec{E} = \frac{\sum_{i=1}^{N}\rho_i\vec{r}_i}{3\varepsilon_0} = \frac{\rho_1\vec{r}_1+\rho_2\vec{r}_2+\cdots+\rho_N\vec{r}_N}{3\varepsilon_0} \longrightarrow \vec{E} = \frac{\vec{r}_1\sum_{i=1}^{N}\rho_i+\sum_{i=2}^{N}\rho_i\vec{d}_{i1}}{3\varepsilon_0} \longrightarrow \begin{cases}\boxed{\sum_{i=1}^{N}\rho_i=0}\\ \boxed{\vec{E} = \frac{\sum_{i=2}^{N}\rho_i\vec{d}_{i1}}{3\varepsilon_0}}\end{cases}$$ (d

$$\vec{r}_2 = \vec{d}_{21}+\vec{r}_1 \ , \ \vec{r}_3 = \vec{d}_{31}+\vec{r}_1 \ , \ \ldots \ , \ \vec{r}_N = \vec{d}_{N1}+\vec{r}_1$$

$$B_1 = A\lambda^\alpha\times\mu_0^\beta\times\varepsilon_0^\gamma\times e^\zeta\times h^\theta \longrightarrow \frac{N}{A\times m} = m^\alpha\times\frac{N^\beta}{A^{2\beta}}\times\frac{A^{2\gamma}\times s^{2\gamma}}{m^{2\gamma}\times N^\gamma}\times A^\zeta\times s^\zeta\times N^\theta\times m^\theta\times s^\theta \longrightarrow$$ (a (4

$$1=\beta+\theta-\gamma \qquad -2=-2\beta+\theta+4\gamma+\zeta-2\theta \longrightarrow \boxed{\zeta=4\beta-1}$$

$$-1=2\gamma-2\beta+\zeta \qquad \longrightarrow -1=-\theta+2\gamma \rightarrow \theta=2\gamma+1 \rightarrow \boxed{\theta=1-2\beta}$$

$$-2=\alpha+\beta-2\gamma-\gamma+2\theta \qquad \gamma+1=1-\beta \rightarrow \boxed{\gamma=-\beta} \qquad \alpha-4\beta-2-3\beta-\beta \longrightarrow \boxed{\alpha=-2}$$

$$\longrightarrow \begin{cases} B_1 = A\left(\frac{h}{\lambda^2 e}\right)\left(\frac{e^4\mu_0}{h^2\varepsilon_0}\right)^\beta \\ B_2 = B\left(\frac{h}{\lambda^2 e}\right)\left(\frac{e^4\mu_0}{h^2\varepsilon_0}\right)^\beta \end{cases}$$

بسمه تعالی

امتحان سوم 88

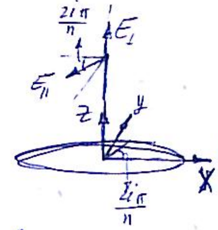
b) $\xi > \lambda \to \boxed{Sn \text{ ! ابررسانای نوع}}$ ، $\boxed{B_2 = \frac{h}{e\xi^2}}$ ، $\boxed{B_1 = \frac{h}{e\lambda^2}}$

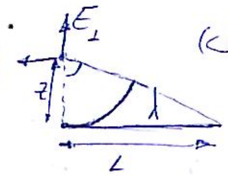
c) $\boxed{\begin{cases} \lambda > \xi & \text{نوع } 2 \\ \xi > \lambda & \text{نوع } ! \end{cases}}$

d)

امتحان چهارم 88

1) الف)





$$\vec{E}_i = (\vec{E}_\perp)\hat z + \left(-E_\parallel \cos\left(\frac{2i\pi}{n}\right)\right)\hat x + \left(-E_\parallel \sin\left(\frac{2i\pi}{n}\right)\right)\hat y \to$$

$$\vec E = (nE_\perp)\hat z - E_\parallel\left(\left(\frac{\sin\left((n+\frac12)\times\frac{2\pi}{n}\right)}{2\sin\left(\frac{\pi}{n}\right)} - \frac12\right)\hat x + \left(\frac{\sin\left((\frac{n+1}{2})\times\frac{2\pi}{n}\right)\sin(\pi)}{\sin\left(\frac{\pi}{n}\right)}\right)\hat y\right) \to \vec E = nE_\perp \hat z$$

$$dE_\perp = \frac{k\lambda z\, d\theta}{z^2}\cos\theta \to E_\perp = \frac{k\lambda}{z}\times\frac{L}{\sqrt{L^2+z^2}} \to \boxed{\vec E = \frac{nk\lambda L}{z\sqrt{z^2+L^2}}\hat z}$$

با انتگرال هم می‌شد

ب)

$$\vec E_r^{(1)} = -\frac{\vec r}{2}\frac{\partial E_z}{\partial z} \to \vec E_r = -\frac{nk\lambda L}{2}\vec r\times\left(-\frac{1}{z^2\sqrt{z^2+L^2}} - \frac{1}{(z^2+L^2)^{3/2}}\right) \to \boxed{\vec E_r = \frac{nk\lambda L(2z^2+L^2)}{2z^2(z^2+L^2)^{3/2}}\vec r}$$

ج)

$$\vec\nabla\cdot\vec E = 0 \to \frac{\partial E_\rho}{\partial z} = \frac{\partial E_z}{\partial \rho} \to E_z^{(2)} = E_z^{(1)} - \frac{r^2}{4}E''^{(1)}_{(z)}$$

$$E''^{(1)}_{(z)} = -\left(\frac{-2}{z^3\sqrt{z^2+L^2}} - \frac{1}{z(z^2+L^2)^{3/2}} - \frac{3z}{(z^2+L^2)^{5/2}}\right)\times nk\lambda L \to E_z^{(2)} = nk\lambda L\left(\frac{1}{z\sqrt{L^2+z^2}} - \frac{r^2(4z^4+2L^4+5z^2L^2)}{z^3(z^2+L^2)^{5/2}}\right)$$

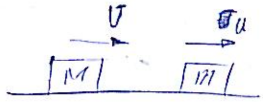
$$= E''^{(1)}_{(z)} = \frac{nk\lambda L}{\sqrt{z^2+L^2}}\left(\frac{2z^4+2L^4+4z^2L^2+z^4+z^2L^2+3z^4}{z^3(z^2+L^2)^2}\right) \to E''^{(1)}_{(z)} = \left(\frac{4z^4+2L^4+5z^2L^2}{z^3(z^2+L^2)^{5/2}}\right)nk\lambda L$$

$$\to \boxed{E_z^{(2)} = \frac{nk\lambda L}{z\sqrt{z^2+L^2}}\left(1 - \frac{r^2(4z^4+2L^4+5z^2L^2)}{z^2(z^2+L^2)^2}\right)}$$

د)

$$\sigma\times 2\pi r\, dr = n\lambda\, dr \to \boxed{\sigma = \frac{n\lambda}{2\pi r}}$$

2) الف)



$$\begin{cases} MV_n + mU_n = Mv_n + mu_n \\ \varepsilon = \frac{v_n - u_n}{U_n - V_n} \to U_n = V_n + \frac{-u_n+v_n}{\varepsilon} \end{cases} \to V_n(M+m) = \frac{mu_n}{\varepsilon} - \frac{mv_n}{\varepsilon} + Mv_n + mu_n \to \boxed{V_n = \frac{m(1+\varepsilon)u_n + (M\varepsilon - m)v_n}{\varepsilon(M+m)}}$$

$$\to \boxed{U_n = \frac{M(1+\varepsilon)v_n + (m\varepsilon - M)u_n}{\varepsilon(M+m)}} \quad , \quad 0 = -\frac{F}{2M}T_n^2 + (u_n - v_n)T_n \to \boxed{T_n = \frac{2M}{F}(u_n - v_n)}$$

ج)

$$V_{n+1} = v_n + \frac{F}{M}T_n \to v_{n+1} = 2u_n - v_n \to m(1+\varepsilon)u_{n+1} + (M\varepsilon - m)v_{n+1} = \varepsilon(M+m)(2u_n - v_n)$$

$$U_{n+1} = u_n \to M(1+\varepsilon)v_{n+1} + (m\varepsilon - M)u_{n+1} = \varepsilon(M+m)u_n \to v_{n+1} = -\frac{(m\varepsilon - M)}{M(1+\varepsilon)}u_{n+1} + \frac{\varepsilon(M+m)}{M(1+\varepsilon)}u_n \to$$

$$m(1+\varepsilon)u_{n+1} + \frac{\varepsilon(M+m)(M\varepsilon - m)}{M(1+\varepsilon)}u_n - \frac{(M\varepsilon - m)(m\varepsilon - M)}{M(1+\varepsilon)}u_{n+1} = 2\varepsilon(M+m)u_n + \frac{\varepsilon(m\varepsilon - M)(M+m)}{M(1+\varepsilon)}u_n - \frac{\varepsilon^2(M+m)^2}{M(1+\varepsilon)}u_{n-1} \to$$

$$\left(Mm(1+\varepsilon^2+2\varepsilon) - (Mm\varepsilon^2 - M^2\varepsilon - m^2\varepsilon + mM)\right)u_{n+1} + \varepsilon(M+m)\left((M-m)(1+\varepsilon) - 2M(1+\varepsilon)\right)u_n + \varepsilon^2(M+m)^2u_{n-1} = 0 \to$$

$$(M+m)^2u_{n+1} - (1+\varepsilon)(M+m)^2u_n + \varepsilon(M+m)^2u_{n-1} = 0 \to u_{n+1} - (1+\varepsilon)u_n + \varepsilon u_{n-1} = 0 \to (y-\varepsilon)(y-1) = 0 \to$$

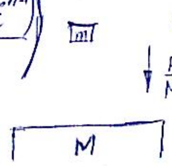
$$u_n = A_1\varepsilon^n + A_2 \to v_{n+1} = A_2 + \frac{A_1\varepsilon^{n+1}}{M(1+\varepsilon)}(\cancel{\varepsilon}M + m - \varepsilon^2 m + M\cancel{\varepsilon}) \to v_{n+1} = A_2 + A_1\varepsilon^{n+1}\left(\frac{2M + m(1-\varepsilon^2)}{M(1+\varepsilon)}\right)$$

$$\begin{cases} M(1+\varepsilon)v_1 + (m\varepsilon - M)u_1 = 0 \\ \sqrt{\frac{2FL}{M}} = \frac{m(1+\varepsilon)u_1 + (M\varepsilon - m)v_1}{\varepsilon(M+m)} \end{cases} \to \begin{cases} (M+m)\varepsilon A_2 + A_1\varepsilon(2M + m(1-\varepsilon^2)) + A_1\varepsilon(m\varepsilon - M) = 0 \to A_1\varepsilon(M+m) + A_2\varepsilon(M+m) = 0 \to A_1 = -A_2 \\ \sqrt{\frac{2FL}{M}} = A_2 + \frac{A_1\varepsilon m(1+\varepsilon) + A_1(M\varepsilon - m)\times\left(\frac{2M + m(1-\varepsilon^2)}{M(1+\varepsilon)}\right)\varepsilon}{\varepsilon(M+m)} \end{cases} \to$$

$$\sqrt{\frac{2FL}{M}} = \frac{A_1}{M+m}\left(\frac{(-M - m + m + m\varepsilon)M(1+\varepsilon) + (M\varepsilon - m)\left((M - m\varepsilon) + (M+m)\right)}{M(1+\varepsilon)}\right) = \frac{A_1\left((m\varepsilon - M)(M + M\varepsilon - M\varepsilon + m) + (M+m)(M\varepsilon - m)\right)}{(M+m)M(1+\varepsilon)}$$

$$= \frac{A_1(M+m)(\varepsilon - 1)}{M(1+\varepsilon)} \to A_1 = \frac{M(1+\varepsilon)}{(M+m)(\varepsilon - 1)}\sqrt{\frac{2FL}{M}} \to \boxed{u_n = \frac{M(1+\varepsilon)(\varepsilon^n - 1)}{(M+m)(\varepsilon - 1)}\sqrt{\frac{2FL}{M}}} \to \boxed{v_n = \frac{M(1+\varepsilon)}{(M+m)(\varepsilon-1)}\sqrt{\frac{2FL}{M}}\left(\left(\frac{2M + m(1-\varepsilon)}{M(1+\varepsilon)}\right)\varepsilon^n - 1\right)}$$

ب)



$$\boxed{u_n - v_n = \varepsilon^n\sqrt{\frac{2FL}{M}}} \quad , \quad T_n = 2\times\frac{M}{F}\times\varepsilon^n\sqrt{\frac{2FL}{M}} \to t_n = 2\sqrt{\frac{2LM}{F}}\times\frac{\varepsilon(1-\varepsilon^{n-1})}{1-\varepsilon} + \sqrt{\frac{2LM}{F}} = \sqrt{\frac{2LM}{F}}\left(1 + \frac{2\varepsilon(1-\varepsilon^{n-1})}{1-\varepsilon}\right)$$

$$F\times t_n = (M+m)v_{cm} \to \boxed{v_{cm} = \frac{\sqrt{2LMF}}{M+m}\left(1 + \frac{2\varepsilon(1-\varepsilon^{n-1})}{1-\varepsilon}\right)} \to \boxed{v_{cm} = \frac{\sqrt{2LMF}}{M+m}\left(\frac{1+\varepsilon-2\varepsilon^n}{1-\varepsilon}\right)}$$

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ج) روش دوم:

$$\begin{cases} u_n - v_n = \varepsilon^n\sqrt{\frac{2FL}{M}} \\ m u_n + M v_n = \sqrt{\frac{2FLM}{1-\varepsilon}}\,(1+\varepsilon-2\varepsilon^n)\end{cases} \longrightarrow v_n, u_n$$ بدست می آید

د) $$t_n = \sqrt{\frac{2LM}{F}}\left(\frac{1+\varepsilon-2\varepsilon^n}{1-\varepsilon}\right), \quad n\to\infty \Rightarrow \varepsilon^n\to 0 \to \boxed{T' = \left(\frac{1+\varepsilon}{1-\varepsilon}\right)\sqrt{\frac{2LM}{F}}}$$

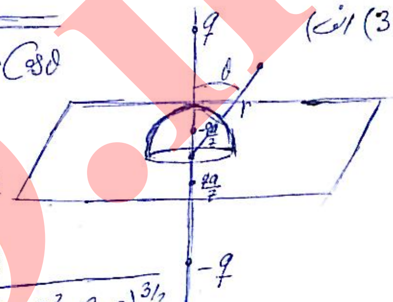
$$x_{n_m} = u_n T_n \to x_m = \sum_{n=1}^{n-1} u_n T_n = \frac{4LM(1+\varepsilon)}{(M+m)(1-\varepsilon)}\sum_{n=1}^{n-1}(\varepsilon^n - \varepsilon^{2n}) = \frac{4LM(1+\varepsilon)}{(M+m)(1-\varepsilon)}\left(\frac{\varepsilon(1-\varepsilon^{n-1})}{1-\varepsilon} - \frac{\varepsilon^2(1-\varepsilon^{2n-1})}{1-\varepsilon^2}\right)\to$$

$$x_{T'} = \frac{4LM(1+\varepsilon)\varepsilon}{(M+m)(1-\varepsilon)^2(1+\varepsilon)}\left((1+\varepsilon)-\varepsilon\right) \to \boxed{x_{T'} = \frac{4LM\varepsilon}{(M+m)(1-\varepsilon^2)}}$$

$$F(L+x_{T'}) = Q + \frac12(M+m)v^2_{cm_{T'}} \to F\left(L+\frac{4LM\varepsilon}{(M+m)(1-\varepsilon^2)}\right) - \frac{2LMF(1+\varepsilon)^2}{2(M+m)(1-\varepsilon)^2} = Q \to \boxed{Q = FL - \frac{MLF(1+\varepsilon^3+7\varepsilon^2)}{(M+m)(1-\varepsilon)^2(1+\varepsilon)}}$$

3) الف)

$$V = \frac{kq}{\sqrt{r^2+z^2-2rz\cos\theta}} - \frac{kq}{\sqrt{r^2+z^2+2rz\cos\theta}} - \frac{kqa}{z\sqrt{\frac{a^4}{z^2}+r^2-\frac{2a^2}{z}r\cos\theta}} + \frac{kqa}{z\sqrt{\frac{a^4}{z^2}+r^2+\frac{2a^2}{z}r\cos\theta}}$$



$$\to \vec E = kq\left(-\frac{2r-2z\cos\theta}{2(r^2+z^2-2rz\cos\theta)^{3/2}} + \frac{2r+2z\cos\theta}{2(r^2+z^2+2rz\cos\theta)^{3/2}}\right)\hat r + \frac{kqa}{z}\left(\frac{2r-\frac{2a^2}{z}\cos\theta}{2\left(\frac{a^4}{z^2}+r^2-\frac{2a^2}{z}r\cos\theta\right)^{3/2}}\right.$$

$$\left.-\frac{2r+\frac{2a^2}{z}\cos\theta}{2\left(\frac{a^4}{z^2}+r^2+\frac{2a^2}{z}r\cos\theta\right)^{3/2}}\right)\hat r + \frac{kq}{r}\left(-\frac{2rz\sin\theta}{2(r^2+z^2-2rz\cos\theta)^{3/2}} - \frac{2rz\sin\theta}{2(r^2+z^2+2rz\cos\theta)^{3/2}} + \frac{2a^3 r\sin\theta}{2z^2\left(\frac{a^4}{z^2}+r^2-\frac{2a^2}{z}r\cos\theta\right)^{3/2}}\right.$$

$$\left.+\frac{2a^3r\sin\theta}{2z^2\left(\frac{a^4}{z^2}+r^2+\frac{2a^2}{z}r\cos\theta\right)^{3/2}}\right)\hat\theta$$

ب) $$\sigma = \frac{q}{4\pi}\left(\frac{-1}{(a+z)^2} - \frac{1}{(z-a)^2} - \frac{a}{z\left(a-\frac{a^2}{z}\right)^2} + \frac{a}{z\left(a+\frac{a^2}{z}\right)^2}\right)\to$$

$$\sigma = -\frac{q}{4\pi}\left(\frac{z-a}{a(z+a)^2} - \frac{z+a}{a(a-z)^2}\right) = \frac{-q\left((z-a)^3-(z+a)^3\right)}{4\pi a(z+a)^2(z-a)^2}\to \boxed{\sigma = \frac{(a^2+3z^2)q}{2\pi(z^2-a^2)^2}}$$

ج) $$\frac{\sigma}{\varepsilon_0} = \frac{dQ}{2\pi a^2\sin\theta\, d\theta\,\varepsilon_0} = \frac{q}{4\pi\varepsilon_0}\left(\frac{-z\cos\theta+a}{(a^2+z^2-2az\cos\theta)^{3/2}} - \frac{z\cos\theta+a}{(a^2+z^2+2az\cos\theta)^{3/2}} + \frac{\left(\frac{a}{z}\cos\theta-1\right)}{az\left(\frac{a^2}{z^2}+1-\frac{2a}{z}\cos\theta\right)^{3/2}} + \frac{\frac{a}{z}\cos\theta+1}{az\left(\frac{a^2}{z^2}+1+\frac{2a}{z}\cos\theta\right)^{3/2}}\right)$$

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{-z\cos\theta+a+z\cos\theta-\frac{z^2}{a}}{(a^2+z^2-2az\cos\theta)^{3/2}} + \frac{z\cos\theta+\frac{z^2}{a}-z\cos\theta-a}{(a^2+z^2-2az\cos\theta)^{3/2}}\right) = \frac{q(z^2-a^2)}{4\pi a\varepsilon_0}\left(\frac{1}{(a^2+z^2-2az\cos\theta)^{3/2}}+\frac{1}{(a^2+z^2+2az\cos\theta)^{3/2}}\right)$$

$$dQ = \frac{-qa(z^2-a^2)}{2}\left(\int_0^\theta \frac{d\cos\theta}{(a^2+z^2-2az\cos\theta)^{3/2}} + \int_0^\theta\frac{d\cos\theta}{(a^2+z^2+2az\cos\theta)^{3/2}}\right) = \frac{q(z^2-a^2)}{4z}\left(\int_0^{\pi/2}\frac{d(a^2+z^2-2az\cos\theta)}{(a^2+z^2-2az\cos\theta)^{3/2}} - \int_0^{\pi/2}\frac{d(a^2+z^2+2az\cos\theta)}{(a^2+z^2+2az\cos\theta)^{3/2}}\right)$$

$$\to Q = -\frac{q(z^2-a^2)}{2z}\left(-\frac{1}{\sqrt{a^2+z^2}}+\frac{1}{z-a}-\frac{1}{\sqrt{a^2+z^2}}+\frac{1}{z+a}\right)\to \boxed{Q = q\left(\frac{z^2-R^2}{z\sqrt{z^2+R^2}}-1\right)}$$

د) $U = \frac12\sum q_i V_i$ پتانسیل صفحه صفر است

$$U = \frac{qV}{2} = \frac{kq^2}{2}\left(\frac{-1}{z+a} - \frac{a}{z\left(z-\frac{a^2}{z}\right)} + \frac{a}{z\left(z+\frac{a^2}{z}\right)}\right)\to U = \frac{kq^2\left(a^4-z^4-a(z^2+a^2)(z+a)+a(z+a)(z^2-a^2)\right)}{2(z^4-a^4)(z+a)}$$

$$= \frac{kq^2\left(a^4-z^4-az^3-a^2z^2-a^3z-a^4+az^3-a^3z+a^2z^2-a^4\right)}{2(z^4-a^4)(z+a)}\to \boxed{U = -\frac{kq^2(a^4+z^4+2a^3z)}{2(z^4-a^4)(z+a)}}$$

5) الف) $$\frac{m\,d(r^2\dot\theta)}{dt} = -br\dot\theta\to\boxed{J = J_0 e^{-\frac{bt}{m}}}$$

ب) $$-b\dot r + F_{(r)} = m(\ddot r - r\dot\theta^2)\to\boxed{F_{(r)} = -\frac{mv^2}{R}}, \quad \boxed{\dot r = \frac{v^2}{R}+\frac{F_{(R)}}{m}}$$

ج) $$\frac{d\theta}{dt} = \frac{J_0 e^{-\frac{bt}{m}}}{r^2}\to\int_0^{2\pi} r^2\, d\theta = -\frac{J_0 m}{b}\left(e^{-\frac{bT}{m}}-1\right)\to\boxed{\overline{r^2} = \frac{2\pi/mJ_0}{b}\left(1-e^{-\frac{bT}{m}}\right)}$$

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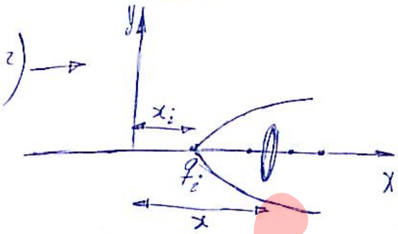
5) الف)

$$\frac{q_1}{\varepsilon_0}\times\frac{2\pi r_1^2(1-\cos\alpha)}{4\pi r_1^2}=\frac{q_2}{\varepsilon_0}\times\frac{2\pi r_2^2(1-\cos\beta)}{4\pi r_2^2}\longrightarrow\frac{q_1}{q_2}\sin^2\left(\frac{\alpha}{2}\right)=\sin^2\left(\frac{\beta}{2}\right)\longrightarrow\boxed{\sin\left(\frac{\beta}{2}\right)=\sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)}$$

ب)

$$d\varphi_i=\frac{kq_i(x-x_i)}{\left((x-x_i)^2+y^2\right)^{3/2}}\times 2\pi y\,dy\longrightarrow d\varphi=k\pi\sum_{i=1}^{N}\frac{(x-x_i)q_i}{\left((x-x_i)^2+y^2\right)^{3/2}}d\left((x-x_i)^2+y^2\right)\longrightarrow$$



پ)

$$\varphi=k\pi\sum_{i=1}^{N}\frac{q_i(x-x_i)}{-\frac{1}{2}}\Bigg|_0^y\frac{1}{\sqrt{y^2+(x-x_i)^2}}\longrightarrow$$

$$\varphi=-2k\pi\sum_{i=1}^{N}q_i(x-x_i)\left(\frac{1}{\sqrt{y^2+(x-x_i)^2}}-\frac{1}{x-x_i}\right)\longrightarrow\varphi=2k\pi\sum_{i=1}^{N}q_i\left(1-\frac{x-x_i}{\sqrt{(x-x_i)^2+y^2}}\right)=\frac{q_j(1-\cos\alpha_j)}{2\varepsilon_0}$$

$$\longrightarrow\sum_{i=1}^{N}q_i\left(1-\frac{x-x_i}{\sqrt{(x-x_i)^2+y^2}}\right)=q_j(1-\cos\alpha_j')\longrightarrow$$ معادله‌ی خط میدانی که از بار q_j با زاویه‌ی α_j جدا شده است

$$\boxed{\cancel{f_{(x,y)}=\sum_{i=1}^{N}q_i\left(1-\frac{1}{\sqrt{1+\left(\frac{y}{x-x_i}\right)^2}}\right)}}\longrightarrow$$ اگر تنها دو بار q_1, q_2 باشد: $$\boxed{\cancel{f_{(x,y)}=(q_1-q_2)-\frac{q_1}{\sqrt{1+\left(\frac{y}{x-x_1}\right)^2}}+\frac{q_2}{\sqrt{1+\left(\frac{y}{x}\right)^2}}}}\Longrightarrow$$

$$\cancel{\frac{q}{2}-q_1\cos\alpha}=\frac{q_2}{\sqrt{1+\left(\frac{y}{x}\right)^2}}-\frac{q_1}{\sqrt{1+\left(\frac{y}{x-x_1}\right)^2}}\qquad\boxed{\sum_{i=1}^{N}q_i\frac{x-x_i}{\sqrt{(x-x_i)^2+y^2}}=C}$$

$$\frac{q_1(x-x_1)}{\sqrt{(x-x_1)^2+y^2}}-\frac{q_2(x-x_2)}{\sqrt{(x_2-x)^2+y^2}}=C\longrightarrow\frac{q_1\epsilon}{\epsilon(1+\tan^2\alpha)^{1/2}}-\frac{q_2(x_1-x_2+\epsilon)}{\left((x_1-x_2+\epsilon)^2+\epsilon^2\tan^2\alpha\right)^{1/2}}=\frac{q_1(x_2-x_1-\epsilon)}{\left((x_2-x_1-\epsilon)^2+\epsilon^2\tan^2\beta\right)^{1/2}}-\frac{q_2(-\epsilon)}{\epsilon(1+\tan^2\beta)^{1/2}}\longrightarrow$$

$$q_1\cos\alpha+q_2=q_1+q_2\cos\beta\longrightarrow q_1\sin^2\left(\frac{\alpha}{2}\right)=q_2\sin^2\left(\frac{\beta}{2}\right)\longrightarrow\boxed{\sin\left(\frac{\beta}{2}\right)=\sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)}$$

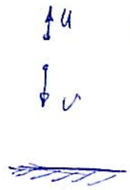
امتحان چهارم 88

4) a)

$$QE\left(\frac{2h_0}{v}\right)=2mv\longrightarrow\boxed{h_0=\frac{mv^2}{QE}}$$

$$\cancel{T=\frac{h}{v}+\delta t\longrightarrow T=\frac{h}{v}\left(1+\epsilon\left(1-\frac{m}{M}\right)\right)}\qquad \mathcal{E}=\frac{1}{2}mv^2+QEh\longrightarrow\dot{\mathcal{E}}=mv\dot{v}+QE\dot{h}$$

b)



$$\frac{d\mathcal{E}}{dt}=-\frac{\frac{1}{2}mv^2(2\epsilon)}{\frac{2h}{v}}\longrightarrow\dot{\mathcal{E}}=-\frac{mv^3\epsilon}{2h}\longrightarrow mv\dot{v}+QE\dot{h}=-\frac{mv^3\epsilon}{2h}\longrightarrow,\ h=\frac{2mv\dot{v}}{QE}\longrightarrow$$

$$\longrightarrow\dot{h}=\frac{2}{QE}\left(-\frac{mv^3}{2h}\epsilon-QE\dot{h}\right)\longrightarrow3\dot{h}=-\frac{mv^3\epsilon}{QEh}\longrightarrow\boxed{\dot{h}=-\frac{\epsilon}{3}\sqrt{\frac{QE}{m}}\,h^{1/2}}$$

c)

$$\longrightarrow\int\frac{dh}{\sqrt{h}}=-\frac{\epsilon}{3}\sqrt{\frac{QE}{m}}\int dt\longrightarrow2\left(\sqrt{h}-\sqrt{h_0}\right)=-\frac{\epsilon}{3}\sqrt{\frac{QE}{m}}\,t\longrightarrow\boxed{h=\left(\sqrt{h_0}-\frac{\epsilon}{6}\sqrt{\frac{QE}{m}}\,t\right)^2}\Longrightarrow\boxed{T=\frac{6}{\epsilon}\sqrt{\frac{mh_0}{QE}}}$$

d)

$$F=\frac{mv^2}{h},\ k=\frac{1}{2}mv^2\longrightarrow dk=-F\,dh\longrightarrow\frac{1}{2}m\,d(v^2)+\frac{mv^2dh}{h}=0\longrightarrow\frac{dv}{v}+\frac{dh}{h}=0\longrightarrow hv=\text{ثابت}\longrightarrow$$

$$3h_0v=h_0\times\sqrt{\frac{QEh_0}{m}}\longrightarrow\boxed{v=\frac{1}{3}\sqrt{\frac{QEh_0}{m}}}$$

امتحان سوم 88) 1) الف)

$$\vec{dl}=Rd\varphi\,\hat{\varphi}+(Rd\varphi)\tan\theta\,\hat{k}\longrightarrow\Delta h=\frac{Rd\varphi}{\cos\theta},\qquad\vec{v}=v_0\hat{k}+\omega R\hat{\varphi}$$

$$\vec{dF}=-\frac{\xi_1}{\Delta h}(\vec{v}\cdot\vec{dl})\vec{dl}-\xi_2\left(\Delta h\vec{v}-\frac{(\vec{v}\cdot\vec{dl})\vec{dl}}{\Delta h}\right)\longrightarrow\vec{dF}=-\frac{\xi_1\cos\theta}{Rd\varphi}(\omega R^2d\varphi)Rd\varphi(\hat{\varphi}+\tan\theta\hat{k})-\xi_2\left(\frac{Rd\varphi}{\cos\theta}R\omega\hat{\varphi}-\frac{\omega R^2d\varphi}{\frac{Rd\varphi}{\cos\theta}}\right.$$

$$\left.(Rd\varphi(\hat{\varphi}+\tan\theta\hat{k})\right)\longrightarrow\vec{dF}=\omega R^2d\varphi\left(-\xi_1\cos\theta(\hat{\varphi}+\tan\theta\hat{k})-\xi_2\left(\frac{\hat{\varphi}}{\cos\theta}-\cos\theta(\hat{\varphi}+\tan\theta\hat{k})\right)\right)\longrightarrow$$

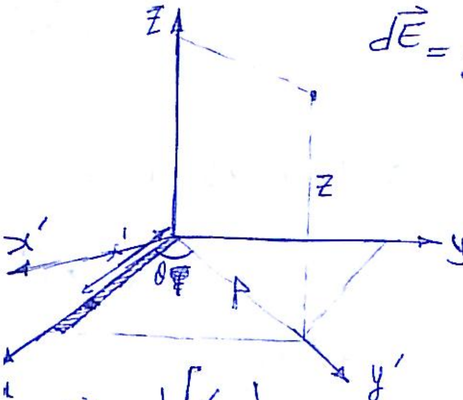
ب)

$$\vec{F}=\omega R^2\left(-\xi_1\sin\theta\hat{k}\frac{L\cos\theta}{R}+\xi_2\sin\theta\times\frac{L\cos\theta}{R}\right)\longrightarrow\boxed{\vec{F}=L\omega R\cos\theta\sin\theta(\xi_2-\xi_1)\hat{n}_1}\longrightarrow\boxed{\vec{F}=L\omega R\sin\theta\cos\theta(\xi_1-\xi_2)\hat{n}_2}$$

$$\vec{dF}=-\frac{\xi_1\cos\theta}{Rd\varphi}\left(R^2d\varphi\,\omega+v_0Rd\varphi\tan\theta\right)(Rd\varphi)(\hat{\varphi}+\tan\theta\hat{k})\longrightarrow-\xi_2\left(\frac{Rd\varphi}{\cos\theta}(R\omega\hat{\varphi}+v_0\hat{k})-(Rd\varphi)(R\omega+v_0\tan\theta)\frac{\cos\theta}{Rd\varphi}(Rd\varphi)(\hat{\varphi}+\tan\theta\hat{k})\right)\longrightarrow$$

باسمه تعالی

امتحان چهارم 88 (1



$$d\vec E = \frac{K\lambda dx'(-x'\hat x + \rho\cos\theta\hat x + \rho\sin\theta\hat y + z\hat z)}{((\rho\cos\theta - x')^2 + (\rho^2\sin^2\theta + z^2))^{3/2}} \to$$

$$\vec E = K\lambda\left[-\frac12\int\frac{d(z^2+\rho^2\sin^2\theta+(\rho\cos\theta-x')^2)}{((\rho\cos\theta-x')^2+(\rho^2\sin^2\theta+z^2))^{3/2}}\hat x + (\rho\sin\theta\hat y + z\hat z)\int\frac{d(x'-x)}{((\rho\cos\theta-x')^2+(\rho^2\sin^2\theta+z^2))^{3/2}}\right]$$

$$\to \vec E = K\lambda\left[\left(\frac{1}{\sqrt{\rho^2\sin^2\theta+z^2+(\rho\cos\theta-l)^2}} - \frac{1}{\sqrt{\rho^2+z^2}}\right)\hat x + \frac{(\rho\sin\theta\hat y+z\hat z)}{\rho^2\sin^2\theta+z^2}\left(\frac{l-\rho\cos\theta}{(z^2+\rho^2\sin^2\theta+(l-\rho\cos\theta)^2)^{1/2}} + \frac{\rho\cos\theta}{\sqrt{z^2+\rho^2}}\right)\right]$$

$$\xrightarrow[\text{اول}]{\text{تقریب}} \vec E_{(1)} = K\lambda\left[\left(\frac{1}{\sqrt{z^2+l^2}}\left(1+\frac{\rho l\cos\theta}{z^2+l^2}\right)-\frac1z\right)\hat x + \frac{(\rho\sin\theta\hat y+z\hat z)}{z^2}\left(\frac{(l-\rho\cos\theta)}{\sqrt{z^2+l^2}}\left(1+\frac{\rho l\cos\theta}{z^2+l^2}\right)+\frac{\rho\cos\theta}{z}\right)\right] \to$$

$$\vec E_{(1)} = K\lambda\left[\left(\frac{1}{\sqrt{z^2+l^2}}\left(1+\frac{\rho l\cos\theta}{z^2+l^2}\right)-\frac1z\right)(\sin\theta\hat x'+\cos\theta\hat y') + \frac1z\left(\frac{l-\rho\cos\theta}{\sqrt{z^2+l^2}}+\frac{\rho l^2\cos\theta}{(z^2+l^2)^{3/2}}+\frac{\rho\cos\theta}{z}\right)\hat z\right.$$

$$\left.+\frac{\rho l\sin\theta}{z^2\sqrt{z^2+l^2}}(\sin\theta\hat y'-\cos\theta\hat x')\right] \to \vec E_{(1)} = K\lambda\left[\left(\left(\frac{1}{\sqrt{z^2+l^2}}-\frac1z\right)\sin\theta - \frac{\rho l^3\sin(2\theta)}{2z^2(z^2+l^2)^{3/2}}\right)\hat x'\right.$$

$$\left.+\frac1z\left(\frac{l}{\sqrt{z^2+l^2}} - \frac{\rho z^2\cos\theta}{(z^2+l^2)^{3/2}}+\frac{\rho\cos\theta}{z}\right)\hat z + \left(\frac{\rho l(z^2+\frac{l^2}{2})}{z^2(z^2+l^2)^{3/2}} + \left(\frac{1}{\sqrt{z^2+l^2}}-\frac1z\right)\cos\theta - \frac{\rho l^3\cos(2\theta)}{2z^2(z^2+l^2)^{3/2}}\right)\hat y'\right]$$

$$\to \vec E = \sum_{i=1}^N \vec E_i = K\lambda\left[\left(\left(\frac{1}{\sqrt{z^2+l^2}}-\frac1z\right)\sum_{i=1}^N\sin\left(\theta+\frac{2(i-1)\pi}{N}\right) - \frac{\rho l^3}{2z^2(z^2+l^2)^{3/2}}\sum_{i=1}^N\sin\left(2\theta+\frac{4(i-1)\pi}{N}\right)\right)\hat x'\right.$$

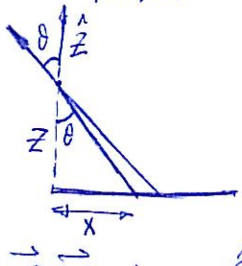
$$+\frac1z\left(\frac{Nl}{\sqrt{z^2+l^2}}+\left(\frac{\rho}{z}-\frac{\rho z^2}{(z^2+l^2)^{3/2}}\right)\sum_{i=1}^N\cos\left(\theta+\frac{2(i-1)\pi}{N}\right)\right)\hat z + \left(\frac{\rho l(z^2+\frac{l^2}{2})N}{z^2(z^2+l^2)^{3/2}}+\left(\frac{1}{\sqrt{z^2+l^2}}-\frac1z\right)\sum_{i=1}^N\cos\left(\theta+\frac{2(i-1)\pi}{N}\right)\right.$$

$$\left.\left.-\frac{\rho l^3}{2z^2(z^2+l^2)^{3/2}}\sum_{i=1}^N\cos\left(2\theta+\frac{4(i-1)\pi}{N}\right)\right)\hat y'\right]$$

$$\sum_{i=1}^N\sin\left(\theta+\frac{2(i-1)\pi}{N}\right) = \sin\left(\theta-\frac{2\pi}{N}\right)\sum_{i=1}^N\cos\left(\frac{2i\pi}{N}\right)+\cos\left(\theta-\frac{2\pi}{N}\right)\sum_{i=1}^N\sin\left(\frac{2i\pi}{N}\right)=0 \to \sum_{i=1}^N\cos\left(\theta+\frac{2(i-1)\pi}{N}\right)=0$$

$$\sum_{i=1}^N\cos\left(2\theta+\frac{4(i-1)\pi}{N}\right) = \cos\left(2\theta-\frac{4\pi}{N}\right)\sum_{i=1}^N\cos\left(\frac{4i\pi}{N}\right)-\sin\left(2\theta-\frac{4\pi}{N}\right)\sum_{i=1}^N\sin\left(\frac{4i\pi}{N}\right)=0\to\sum_{i=1}^N\sin\left(2\theta+\frac{4(i-1)\pi}{N}\right)=0$$

$$\to \vec E^{(1)}_{(\rho,\phi,z)} = K\lambda\left[\frac{Nl}{z\sqrt{l^2+z^2}}\hat z+\frac{Nl(z^2+\frac{l^2}{2})}{z^2(z^2+l^2)^{3/2}}\rho\hat\rho\right] \to \boxed{\vec E^{(1)}_{(\rho,\phi,z)} = \frac{NK\lambda l}{z^2(z^2+l^2)^{3/2}}\left(z(z^2+l^2)\hat z+\left(z^2+\frac{l^2}{2}\right)\rho\hat\rho\right)}$$ (ب



$$dE_z = \frac{NK\lambda\, dx\, z}{(z^2+x^2)^{3/2}} \to E_z = \frac{NK\lambda z}{z^2}\times\frac{l}{\sqrt{z^2+l^2}} \to \boxed{E_z=\frac{NK\lambda l}{z\sqrt{z^2+l^2}}}$$ (الف

$$\nabla\times\vec E = 0 \to \frac{\partial E_z}{\partial\rho} = \frac{\partial E_\rho}{\partial z}\to E_z = E_z^{(0)}+\int E_\rho^{(1)}d\rho$$ (ج

باسمه تعالی

فاینال 88) سؤال iii] a)

$$\vec{E}_p^{(1)} = -\frac{P}{2}\frac{\partial E_z^{(0)}}{\partial z} \longrightarrow \boxed{E_z^{(2)} = E_z^{(0)} - \frac{E_z''}{4}P^2}$$

b) و c)

$$\boxed{V_1 = Kq_1\left(\frac{1}{R_2} - \frac{1}{2H}\right)} \quad , \quad \boxed{Q_2 = -q_1}$$

$$\cancel{U_{\text{برهم‌کنش}} = U_T - U_{11}} \longrightarrow \boxed{\cancel{U_{\text{برهم‌کنش}} = 0}} \longrightarrow \boxed{\cancel{\vec{F}_{\text{برهم‌کنش}} = 0}}$$

$$\cancel{U_T = \frac{1}{2}q_1 Kq_1\left(\frac{1}{R} - \frac{1}{2H}\right)} - \cancel{U_T = \frac{1}{2}q_1V_1 - \frac{1}{2}q_1V_1 + U_{q_1\,\text{نقطه‌ای}}} \quad , \quad \cancel{U_{11} = \frac{1}{2}qV - \frac{1}{2}qV + U_{q_1\,\text{نقطه‌ای}}}$$

$$U_{\text{برهم‌کنش}} = U_T - U_{11} \longrightarrow \boxed{U_{\text{برهم‌کنش}} = -\frac{Kq_1^2}{4H}} \longrightarrow \boxed{F = \frac{Kq_1^2}{4H^2}}$$

$$U_T = \frac{1}{2}\sum q_i V_i = \frac{1}{2}q_1\left(V_1 + \int_{\text{داخل کره کوچک}} \vec{E}\cdot d\vec{r}\right) \qquad U_{11} = \frac{1}{2}q_1\left(\frac{Kq_1}{R_2} + \int_{\text{داخل کره کوچک}} \vec{E}\cdot d\vec{r}\right)$$

باسمه تعالی

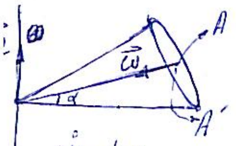
$$\vec F = -\xi_1 \cos\theta (R\omega + v_1 \tan\theta)\, l \cos\theta \frac{\sin\theta}{\cos\theta}\hat k - \xi_2\left(\frac{L\cos\theta}{\cos\theta} v_1 \hat k - L\cos\theta(\omega R + v_1\tan\theta)\cos\theta\frac{\sin\theta}{\cos\theta}\hat k\right) \longrightarrow$$

$$\vec F = L\omega R \sin\theta\cos\theta(\xi_2-\xi_1)\hat k + v_1 L\left(-\xi_1\sin^2\theta + \xi_2(-1+\sin^2\theta)\right)\hat k \longrightarrow \boxed{\vec F = L\omega R\frac{\sin(2\theta)}{2}(\xi_2-\xi_1)\hat k - v_1 L(\xi_1\sin^2\theta + \xi_2\cos^2\theta)\hat k}$$

$$\longrightarrow \boxed{v_c = \frac{\omega R \sin(2\theta)(\xi_1-\xi_2)}{\xi_1\sin^2\theta + \xi_2\cos^2\theta}}$$

امتحان دوم 89

3) الف)



$$v_{A'} = h\cos\alpha\,\omega - R\cos\alpha\,\dot\psi = 0 \longrightarrow \dot\psi = \frac{h}{R}\omega \longrightarrow \boxed{\dot\psi = \frac{\omega}{\tan\alpha}}$$

$$\boxed{\vec r_A = R\cos\alpha\,\hat k + h\cos\alpha\left(\cos(\omega t)\hat i + \sin(\omega t)\hat j\right)}$$

ب) غلط

$$\dot\psi = k\omega \longrightarrow \frac{1}{\tan\alpha} \in Q \longrightarrow \boxed{\tan\alpha \in Q}$$

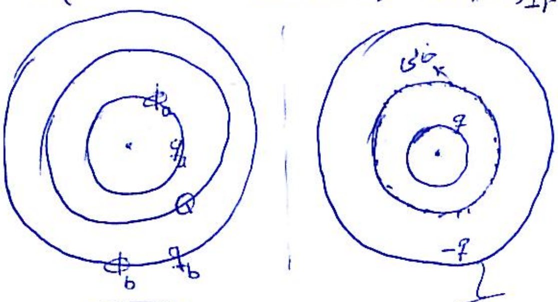
ج)

$$\vec r_B = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} r - R(1-\cos\psi)\sin\alpha \\ -R\sin\psi \\ R(1-\cos\psi)\cos\alpha \end{pmatrix} \longrightarrow \boxed{\begin{aligned}\vec r_B = &\left((r - R(1-\cos\psi)\sin\alpha)\cos\varphi + R\sin\psi\sin\varphi\right)\hat i + \\ &\left((r - R(1-\cos\psi)\sin\alpha)\sin\varphi - R\sin\psi\cos\varphi\right)\hat j + R(1-\cos\psi)\cos\alpha\,\hat k\end{aligned}}$$

د)

درست :

$$\vec V = (\vec\Omega + \vec\omega)\times\vec r \xrightarrow{v_{A'}=0} (\vec\omega\times\vec\Omega)_{\perp \hat r} = 0 \longrightarrow \Omega = \omega\sin\alpha \longrightarrow \omega = \frac{\Omega}{\sin\alpha}$$



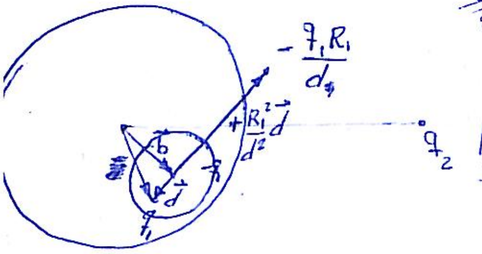
امتحان دوم 88 سؤال 3) روش دوم : زرین

$$\phi_a q + \phi_b(-q) = Kq\left(\frac{1}{a}-\frac{1}{b}\right)q_a + Kq\left(\frac{1}{c}-\frac{1}{b}\right)Q \longrightarrow$$

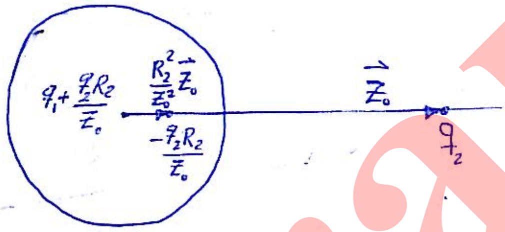
$$\boxed{q_a = \frac{4\pi\varepsilon_0(\phi_a-\phi_b) - Q\left(\frac{1}{c}-\frac{1}{b}\right)}{\frac{1}{a}-\frac{1}{b}}} \longrightarrow$$ به همین ترتیب q_b در می‌آید.

روش سوم : خازن

i (1 a) $\boxed{\vec E_{in2out1} = 0}$,



$$\boxed{\vec E_{in_1} = \frac{Kq_1(\vec r - \vec b - \vec d)}{|\vec r - \vec b - \vec d|^3} - \frac{Kq_1R_1\left(\vec r - \vec b - \frac{R_1^2}{d^2}\vec d\right)}{d_1\left|\vec r - \vec b - \frac{R_1^2}{d^2}\vec d\right|^3}}$$



$$\boxed{\vec E_{out2} = \frac{K\left(q_1 + \frac{q_2R_2}{Z_0}\right)\vec r}{r^3} + \frac{Kq_2(\vec r - \vec Z_0)}{|\vec r - \vec Z_0|^3} - \frac{Kq_2R_2\left(\vec r - \frac{R_2^2}{Z_0^2}\vec Z_0\right)}{Z_0\left|\vec r - \frac{R_2^2}{Z_0^2}\vec Z_0\right|^3}}$$

(b

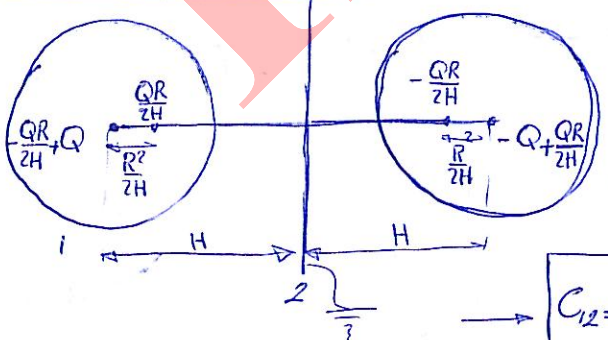
$$\vec F_1 = \frac{-Kq_1R_1\left(\frac{R_1^2}{d^2}+1\right)\vec d}{d_1\left(\frac{R_1^2}{d^2}+1\right)^3 d^3} \longrightarrow \boxed{\vec F_1 = -\frac{Kq_1R_1\vec d}{(R_1^2+d^2)^2}}$$

$$= \frac{Kq_2\left(q_1 + \frac{q_2R_2}{Z_0}\right)\vec Z_0}{Z_0^3} - \frac{Kq_2^2R_2\vec Z_0}{(Z_0^2-R_2^2)^2} \quad (b)$$

(c)

$$\boxed{\frac{q_1Z_0 + q_2R_2}{Z_0^4} = \frac{q_2R_2}{(Z_0^2-R_2^2)^2}}$$

ii] (a



$$\begin{cases} Q_1 = C_{11}v_1 \\ Q_2 = C_{12}v_1 \end{cases} \longrightarrow \begin{cases} Q = KQ\left(\frac{1}{R}-\frac{1}{2H}\right)C_{11} \\ -Q = KQ\left(\frac{1}{R}-\frac{1}{2H}\right)C_{12} \end{cases}$$

(b

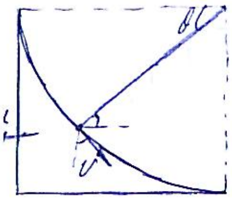
$$v_1 = \frac{K\left(Q - \frac{QR}{2H}\right)}{R} - \frac{KQR/2H}{2H - R - \frac{R^2}{2H}} + \frac{KQ\,R/2H}{2H-R} = KQ\left(\frac{1}{R}-\frac{1}{2H}\right)$$

$$\longrightarrow \boxed{C_{12} = -C_{11} = -\frac{R}{K}\left(1+\frac{R}{2H}\right)}$$

$$\longrightarrow \boxed{Q_2 = -\frac{Rv}{K}\left(1+\frac{R}{2H}\right)}$$

بسمه تعالی

تابستان 88



5) الف)

$$\begin{cases}\frac{1}{2}m|\vec{v}+\vec{u}|^2+\frac{1}{2}Mu^2+mag(1-\sin\theta)=mag \rightarrow \frac{1}{2}(M+m)u^2+\frac{1}{2}mv(v-2u\sin\theta)=mga\sin\theta \\ m(v\sin\theta-u)=Mu \rightarrow u=\frac{mv\sin\theta}{M+m}\end{cases}$$

$$\rightarrow 2mag\sin\theta=\frac{m^2v^2\sin^2\theta}{M+m}+mv\left(v-\frac{2mv\sin^2\theta}{M+m}\right)\rightarrow 2mga\sin\theta=\frac{-m^2\sin^2\theta+(M+m)m}{M+m}v^2$$

$$\rightarrow v^2=\frac{2ag(M+m)\sin\theta}{M+m\cos^2\theta}\rightarrow a_n=\frac{v^2}{a}\rightarrow \boxed{a_n=\frac{2(M+m)g\sin\theta}{M+m\cos^2\theta}}$$

ب) اشتباه کردیم

$$\cancel{2v\dot{v}=2ag(M+m)\times\frac{(M+m\cos^2\theta)\cos\theta+2m\sin^2\theta\cos\theta}{(M+m\cos^2\theta)^2}\dot{\theta}=2\times\frac{2ag(M+m)\sin\theta}{(M+m\cos^2\theta)}\times\dot{v}\rightarrow a_T=}$$

ب)

$$2v\dot{v}=2ag(M+m)\times\frac{(M+m\cos^2\theta)\cos\theta+2m\sin^2\theta\cos\theta}{(M+m\cos^2\theta)^2}\times\frac{v}{a}\rightarrow\boxed{a_T=\frac{\cos\theta(M+m)\left((M+m)+m\sin^2\theta\right)}{(M+m\cos^2\theta)^2}g}$$

پ)

$$u=\frac{mv\sin\theta}{M+m}\rightarrow a_x=\frac{m}{M+m}(v\cos\theta\dot{\theta}+a_T\sin\theta)=\frac{m}{M+m}\left(\frac{2g(M+m)\sin\theta\cos\theta}{M+m\cos^2\theta}+\frac{(M+m)\left((M+m)+m\sin^2\theta\right)g\sin\theta\cos\theta}{(M+m\cos^2\theta)^2}\right)$$

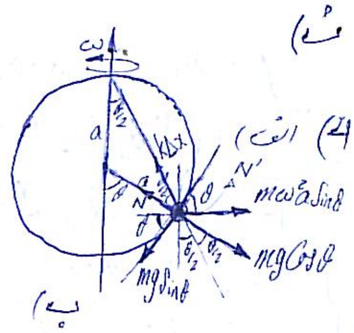
$$a_x=\frac{m}{M+m}\times\frac{g(M+m)\sin\theta\cos\theta}{M+m\cos^2\theta}\left(\frac{2M+2m\cos^2\theta+M+m\sin^2\theta}{M+m\cos^2\theta}\right)\rightarrow\boxed{a_x=\frac{\left(3M+m(1+\cos^2\theta)\right)mg\sin\theta\cos\theta}{(M+m\cos^2\theta)^2}}$$

ت)

$$m(g\sin\theta+a_n)=N\rightarrow N=mg\sin\theta\left(\frac{M+m\cos^2\theta+2M+2m}{M+m\cos^2\theta}\right)\rightarrow N=\frac{(3M+m(2+\cos^2\theta))mg\sin\theta}{M+m\cos^2\theta}\rightarrow\boxed{F_y=\frac{(3M+m(2+\cos^2\theta))mg\sin\theta}{M+m\cos^2\theta}}$$

ث)

$$u=\frac{mv}{M+m}\rightarrow u=\frac{m}{M+m}\times\sqrt{\frac{2ag(M+m)}{m}}\rightarrow\boxed{u=\sqrt{\frac{2mag}{M+m}}}$$



4) الف)

$$k\left(2a\cos\left(\tfrac{\theta}{2}\right)-a\right)\sin\left(\tfrac{\theta}{2}\right)+m\omega^2a\sin\theta\cos\theta-mg\sin\theta=ma\ddot{\theta}\rightarrow$$

$$\boxed{\ddot{\theta}=\frac{k}{m}\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)\sin\left(\tfrac{\theta}{2}\right)+\omega^2\sin\theta\cos\theta-\frac{g}{a}\sin\theta}$$

ب)

$$-N+mg\cos\theta+m\omega^2a\sin^2\theta-ka\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)\cos\left(\tfrac{\theta}{2}\right)=-a\dot{\theta}^2m\rightarrow$$

$$\boxed{N=mg\cos\theta+m\omega^2a\sin^2\theta+ma\dot{\theta}^2-ka\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)\cos\left(\tfrac{\theta}{2}\right)}$$

~~پ)~~

$$\cancel{E=\text{ثابت}\rightarrow\frac{1}{2}ma^2\dot{\theta}^2+\frac{1}{2}ka^2\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)^2-2mga\cos^2\left(\tfrac{\theta}{2}\right)=\text{ثابت}}\rightarrow\boxed{V_{eff(\theta)}=\frac{1}{2}ka^2\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)^2-2mga\cos^2\left(\tfrac{\theta}{2}\right)}$$

~~ت)~~

$$\cancel{\frac{\partial V_{eff}}{\partial\theta}=0\rightarrow ka^2\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)\times\sin\left(\tfrac{\theta}{2}\right)+2mga\cos\left(\tfrac{\theta}{2}\right)\sin\left(\tfrac{\theta}{2}\right)+\frac{1}{2}m\omega^2a^2\sin\left(\tfrac{\theta}{2}\right)\cos\left(\tfrac{\theta}{2}\right)=0}$$

یا V_{eff} هم می توان مستقیم

پ)

$$\ddot{\theta}=0\rightarrow\boxed{\frac{k}{m}\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)\sin\left(\tfrac{\theta}{2}\right)+\omega^2\sin\theta\cos\theta-\frac{g}{a}\sin\theta=0}$$

پ)

$$E-W_f=\text{ثابت}\rightarrow\frac{1}{2}ma^2\dot{\theta}^2+\frac{1}{2}ka^2\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)^2+\frac{1}{2}ma^2\omega^2\sin^2\theta-\int N'a\,d\varphi-2mag\cos^2\left(\tfrac{\theta}{2}\right)=\text{ثابت}$$

$$dW_f=N'd\varphi\,a\ ,\ N'=m(2\dot{r}\sin\theta\dot{\varphi}+r\sin\theta\ddot{\varphi}+2r\cos\theta\dot{\theta}\dot{\varphi})\rightarrow N'=2ma\cos\theta\dot{\theta}\omega$$

$$\rightarrow W_f=2ma^2\omega\int\sin\theta\cos\theta\cdot\frac{d\theta}{dt}\times\omega dt\Rightarrow W_f=2ma^2\omega^2\times\frac{1}{2}\Delta(\sin^2\theta)\rightarrow W_f=m\omega^2a^2\Delta(\sin^2\theta)=m\omega^2a^2\sin^2\theta+c'$$

$$\rightarrow V_{eff(\theta)}=\frac{1}{2}ka^2\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)^2+\frac{1}{2}m\omega^2a^2\sin^2\theta-m\omega^2a^2\sin^2\theta-2mga\cos^2\left(\tfrac{\theta}{2}\right)\rightarrow$$

$$\boxed{V_{eff(\theta)}=\frac{1}{2}ka^2\left(2\cos\left(\tfrac{\theta}{2}\right)-1\right)^2-\frac{1}{2}m\omega^2a^2\sin^2\theta-2mga\cos^2\left(\tfrac{\theta}{2}\right)}$$

ث)

$$\theta=\theta_0+\theta_{(1)}\rightarrow\frac{k}{m}\left(2\cos\left(\tfrac{\theta_0}{2}\right)-\theta_{(1)}\sin\left(\tfrac{\theta_0}{2}\right)-1\right)\left(\sin\left(\tfrac{\theta_0}{2}\right)+\theta_{(1)}\frac{\cos\left(\frac{\theta_0}{2}\right)}{2}\right)+\omega^2(\sin\theta_0+\theta_{(1)}\cos\theta_0)(\cos\theta_0-\theta_{(1)}\sin\theta_0)-\frac{g}{a}(\sin\theta_0+\theta_{(1)}\cos\theta_0)$$

$$=\ddot{\theta}_{(1)}$$

بسمه تعالی

$$\to\begin{cases}\frac{k}{m}\left(2\cos(\frac{\theta_0}{2})-1\right)\sin(\frac{\theta_0}{2})+\omega^2\sin\theta_0\cos\theta_0-\frac{g}{a}\sin\theta_0=0\to\frac{ak}{mg}\left(2\cos(\frac{\theta_0}{2})-1\right)=2\cos(\frac{\theta_0}{2})\to\boxed{\cos(\frac{\theta_0}{2})=\frac{ak}{2(mg+ak)}}\\ \frac{k}{m}\left(-\frac{\theta_{(1)}}{2}\cos(\frac{\theta_0}{2})-\theta_{(1)}\sin^2(\frac{\theta_0}{2})+\theta_{(1)}\cos^2(\frac{\theta_0}{2})\right)+\omega^2\left(\theta_{(1)}\cos^2(\theta_0)-\theta_{(1)}\sin^2\theta_0\right)-\frac{g}{a}\theta_{(1)}\cos\theta_0-\ddot{\theta}_{(1)}=0\to\end{cases}$$

$$\ddot{\theta}_{(1)}+\left(\frac{g}{a}\cos\theta_0+\frac{k}{2m}\cos(\frac{\theta_0}{2})-\cos(\theta_0)\times\frac{k}{m}\right)\theta_{(1)}=0\xrightarrow{\frac{mg}{ak}=\alpha}\ddot{\theta}_{(1)}+\frac{k}{m}\left((\alpha-1)\cos\theta_0+\frac{1}{2}\cos(\frac{\theta_0}{2})\right)\theta_{(1)}=0$$

$$\cos(\frac{\theta_0}{2})=\frac{1}{2(1-\alpha)}\to\cos\theta_0=\frac{1}{2(1-\alpha)^2}-1\to\ddot{\theta}_{(1)}+\frac{k}{m}\left(\frac{1}{2(\alpha-1)}-(\alpha-1)-\frac{1}{4(\alpha-1)}\right)\theta_{(1)}=0\to\ddot{\theta}_{(1)}+\frac{k}{m}\left(\frac{1}{4(\alpha-1)}-(\alpha-1)\right)\theta_{(1)}=0$$

$\to$ اگر : $\left.\begin{matrix}(\alpha-1)<\frac{1}{2}\\ -(\alpha-1)<-\frac{1}{2}\end{matrix}\right\}$ ← هارمونیک ← تعادل پایدار ← $\boxed{\Omega=\sqrt{\frac{k}{m}\left(\frac{ak}{4(mg-ak)}-\frac{mg-ak}{ak}\right)}}$, $\begin{cases}1<\frac{mg}{ak}<\frac{3}{2}\\ \frac{mg}{ak}<\frac{1}{2}\end{cases}$ شرط تعادل پایدار

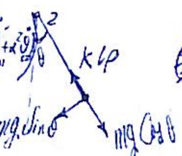
$$\sin(\frac{\theta_0}{2})=0\to\theta_0=0\to\ddot{\theta}_{(1)}+\left(\frac{g}{a}+\frac{k}{2m}-\frac{k}{m}\right)\theta_{(1)}=0\to\boxed{\Omega=\sqrt{\frac{g}{a}-\frac{k}{2m}}}$$, $\frac{mg}{ak}>\frac{1}{2}$ شرط پایداری

$$\sin(\theta_0)=0\to\theta_0=0\to\ddot{\theta}_{(1)}+(-\omega^2+\frac{g}{a})\theta_{(1)}=0\to\boxed{\Omega=\sqrt{\frac{g}{a}-\omega^2}}$$, $\frac{g}{a}>\omega^2$ (ج

$$\cos\theta_0=\frac{g}{a\omega^2}\to\ddot{\theta}_{(1)}+\left(\frac{g^2}{a^2\omega^2}+\omega^2-\frac{g^2}{a^2\omega^2}\right)\theta_{(1)}=0\to\boxed{\Omega=\omega}$$

(د

$$\begin{cases}mg\cos\theta-kl\rho=m(l\ddot{\rho}-l(1+\rho)\dot{\theta}^2)\\ -g\sin\theta=2l\dot{\rho}\dot{\theta}+l(\rho+1)\ddot{\theta}\end{cases}\to\begin{cases}\alpha\cos(\theta_0'+\alpha\theta_{(1)}'+\alpha^2\theta_{(2)}')-(\alpha\rho_{(1)}'+\alpha^2\rho_{(2)}')=\frac{m}{k}\left(\alpha\ddot{\rho}_{(1)}'+\alpha^2\ddot{\rho}_{(2)}'-(1+\alpha\rho_{(1)}'+\alpha^2\rho_{(2)}')(\dot{\theta}_0'+\alpha\dot{\theta}_{(1)}'+\alpha^2\dot{\theta}_{(2)}')^2\right)\\ -\omega^2\sin(\theta_{(0)}'+\alpha\theta_{(1)}'+\alpha^2\theta_{(2)}')=2(\alpha\dot{\rho}_{(1)}'+\alpha^2\dot{\rho}_{(2)}')(\dot{\theta}_{(0)}'+\alpha\dot{\theta}_{(1)}'+\alpha^2\dot{\theta}_{(2)}')+(1+\alpha\rho_{(1)}'+\alpha^2\rho_{(2)}')(\ddot{\theta}_{(0)}'+\alpha\ddot{\theta}_{(1)}'+\alpha^2\ddot{\theta}_{(2)}')\end{cases}$$



$$\to\begin{cases}\alpha\cos\theta_{(0)}-\alpha\theta_{(1)}\sin\theta_{(0)}-\rho_{(1)}-\rho_{(2)}=\frac{m}{k}\left(\ddot{\rho}_{(1)}+\ddot{\rho}_{(2)}-\rho_{(2)}\dot{\theta}_{(0)}^2-\rho_{(1)}(\dot{\theta}_{(0)}^2+2\dot{\theta}_{(0)}\dot{\theta}_{(1)})-(\dot{\theta}_{(0)}^2+\dot{\theta}_{(1)}^2+2\dot{\theta}_{(0)}\dot{\theta}_{(1)}+2\dot{\theta}_{(0)}\dot{\theta}_{(2)})\right)\\ -\omega^2\left(\sin\theta_{(0)}(1-\frac{\theta_{(1)}^2}{2})+\cos\theta_{(0)}(\theta_{(1)}+\theta_{(2)})\right)=2\left(\dot{\rho}_{(1)}\dot{\theta}_{(0)}+\dot{\rho}_{(1)}\dot{\theta}_{(1)}+\dot{\rho}_{(2)}\dot{\theta}_{(0)}\right)+\left(\ddot{\theta}_{(0)}+\ddot{\theta}_{(1)}+\ddot{\theta}_{(2)}+\rho_{(1)}\ddot{\theta}_{(0)}+\rho_{(1)}\ddot{\theta}_{(1)}+\rho_{2}\ddot{\theta}_{(0)}\right)\end{cases}\to$$

$$\boxed{-\omega^2\sin\theta_{(0)}=\ddot{\theta}_{(0)}}\to\boxed{2\omega^2\cos\theta_{(0)}=\dot{\theta}_{(0)}^2}\to\theta_{(0)}=F_{(t)}$$

(a

$$\begin{cases}\alpha\cos\theta_{(0)}-\rho_{(1)}=\frac{m}{k}(\ddot{\rho}_{(1)}-\rho_{(1)}\dot{\theta}_{(0)}^2-2\dot{\theta}_{(0)}\dot{\theta}_{(1)})-\frac{m}{k}\dot{\theta}_{(0)}^2\to\frac{m}{k}\ddot{\rho}_{(1)}+\rho_{(1)}=3\alpha\cos(F)\to\boxed{\rho_{(1)}=3\alpha\cos(F)}\\ -\omega^2\theta_{(1)}\cos\theta_{(0)}=2\dot{\theta}_{(0)}\dot{\rho}_{(1)}+\ddot{\theta}_{(1)}+\rho_{(1)}\ddot{\theta}_{(0)}\to-\omega^2\theta_{(1)}\cos\theta_{(0)}=2\times3\alpha\sin(F)\times-2\omega^2\cos F+\ddot{\theta}_{(1)}-3\alpha\omega^2\sin F\cos F\to\end{cases}$$

(b

$$\to15\omega^2\alpha\sin F\cos F=\ddot{\theta}_{(1)}+\omega^2\theta_{(1)}\cos F$$

(ai (1

$$\vec{E}=\sum\frac{kq_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}\to\boxed{\vec{E}=kq_1\left(\frac{\vec{r}}{r^3}+\frac{(x+\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{3}})\hat{x}+(y-\frac{d}{\sqrt{6}})\hat{y}+(z-\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{2}})\hat{z}}{\left[(x+\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{3}})^2+(y-\frac{d}{\sqrt{6}})^2+(z-\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{2}})^2\right]^{3/2}}-\frac{(x+\frac{b}{\sqrt{2}})\hat{x}+\vec{y}+(z-\frac{b}{\sqrt{2}})\hat{z}}{\left(r^2+b^2+\sqrt{2}b(x-z)\right)^{3/2}}\right)+\frac{kq_2(\vec{r}-z_0\hat{z})}{(r^2+z_0^2-2\vec{r}\cdot(z_0\hat{z}))^{3/2}}}$$

(bi

$$\vec{E}_{q_1}=kq_1\left(-\frac{(-\frac{d}{\sqrt{3}})\hat{x}+(\frac{d}{\sqrt{6}})\hat{y}-(\frac{d}{\sqrt{2}})\hat{z}}{d^3}+\frac{(-\frac{b}{\sqrt{2}}-\frac{d}{\sqrt{3}})\hat{x}+(\frac{d}{\sqrt{6}})\hat{y}+(\frac{b}{\sqrt{2}}-\frac{d}{\sqrt{2}})\hat{z}}{\left[\frac{b^2}{2}+\frac{d^2}{3}+\sqrt{\frac{2}{3}}bd+\frac{d^2}{6}+\frac{b^2}{2}+\frac{d^2}{2}-bd\right]^{3/2}}\right)+kq_2\frac{(-\frac{b}{\sqrt{2}}-\frac{d}{\sqrt{3}})\hat{x}+(\frac{d}{\sqrt{6}})\hat{y}+(\frac{b}{\sqrt{2}}-\frac{d}{\sqrt{2}}-z_0)\hat{z}}{\left[\frac{b^2}{2}+\frac{d^2}{3}+\sqrt{\frac{2}{3}}bd+\frac{d^2}{6}+\frac{b^2}{2}+\frac{d^2}{2}-bd+z_0(z_0+\sqrt{2}(d-b))\right]^{3/2}}$$

$$\to\boxed{\vec{F}_{q_1}=kq_1^2\left(\frac{\frac{1}{\sqrt{3}}\hat{x}-\frac{1}{\sqrt{6}}\hat{y}+\frac{1}{\sqrt{2}}\hat{z}}{d^2}+\frac{-(\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{3}})\hat{x}+(\frac{d}{\sqrt{6}})\hat{y}+(\frac{b-d}{\sqrt{2}})\hat{z}}{\left(b^2+d^2+bd(\sqrt{\frac{2}{3}}-1)\right)^{3/2}}\right)+kq_1q_2\frac{-(\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{3}})\hat{x}+(\frac{d}{\sqrt{6}})\hat{y}+(\frac{b-d}{\sqrt{2}}-z_0)\hat{z}}{\left[b^2+d^2+bd(\sqrt{\frac{2}{3}}-1)+z_0(z_0+\sqrt{2}(d-b))\right]^{3/2}}}$$

$$\boxed{\vec{F}_{q_2}=kq_1q_2\frac{(\frac{b}{\sqrt{2}}+\frac{d}{\sqrt{3}})\hat{x}-(\frac{d}{\sqrt{6}})\hat{y}+(\frac{d-b}{\sqrt{2}}+z_0)\hat{z}}{\left[b^2+d^2+bd(\sqrt{\frac{2}{3}}-1)+z_0(z_0+\sqrt{2}(d-b))\right]^{3/2}}+\left(\frac{kq_1q_2}{z_0^2}\right)\hat{z}-\frac{\frac{b}{\sqrt{2}}\hat{x}+(z_0-\frac{b}{\sqrt{2}})\hat{z}}{(z_0^2+b^2-\sqrt{2}bz_0)^{3/2}}kq_1q_2}$$

بسمه تعالی

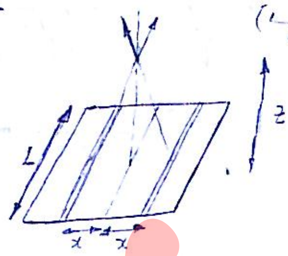
نهائی 88

7) الف)

$$dE=\frac{k\lambda R d\theta \cos\theta}{R^2}\rightarrow E=\frac{2k}{S}\times\frac{q}{L}\times\frac{L}{2\sqrt{\frac{L^2}{4}+S^2}}\rightarrow \boxed{E=\frac{kq}{S\sqrt{S^2+\frac{L^2}{4}}}}\quad \text{يا}\ \boxed{E=\frac{2k\lambda}{S}\sin\theta}$$

ب)

$$dE=2\times\frac{2k\lambda}{\sqrt{x^2+z^2}}\times\frac{L}{2\sqrt{x^2+z^2+\frac{L^2}{4}}}\times\frac{z}{\sqrt{x^2+z^2}}\rightarrow E=2k\sigma Lz\int_0^{L/2}\frac{dx}{(x^2+z^2)\sqrt{x^2+z^2+\frac{L^2}{4}}}\xrightarrow[\frac{x}{z}=\tan\varphi]{1+\frac{L^2}{4z^2}=C^2}$$



$$E=\frac{2k\sigma Lz}{z^2}\int_0^{\varphi}\frac{d\varphi}{\sqrt{C^2+\tan^2\varphi}}=\frac{2k\sigma L}{z}\int_0\frac{d(\sin\varphi)}{\sqrt{C^2-(C^2-1)\sin^2\varphi}}=\frac{2k\sigma L}{z\sqrt{\frac{L^2}{4z^2}}}\int\frac{d(\sin\varphi)}{\sqrt{\frac{C^2}{C^2-1}-\sin^2\varphi}}=$$

$$4k\sigma\left|\sin^{-1}\left(\frac{\sin\varphi}{\sqrt{1+\frac{L^2}{4z^2}}}\times\left(\frac{4z^2}{L^2}\right)^{-1/2}\right)\right|_{\varphi_0}=4k\sigma\sin^{-1}\left(\frac{L}{\sqrt{4z^2+L^2}}\times\frac{L}{\sqrt{L^2+4z^2}}\right)\rightarrow\boxed{E=\frac{\sigma}{\pi\varepsilon_0}\sin^{-1}\left(\frac{L^2}{L^2+4z^2}\right)}$$

ج) دو

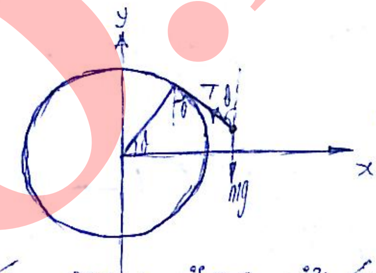
$$E=\frac{\sigma}{\pi\varepsilon_0}\beta\ ,\ \sin\beta=\frac{L^2}{L^2+4z^2}\rightarrow \sin\beta_0+\beta_{(1)}\cos\beta_0=1-\frac{4z^2}{L^2}\rightarrow\boxed{\beta_0=\frac{\pi}{2}}$$

$$\sin(\beta_0+\beta_{(1)}+\beta_{(2)})=1-\frac{4z^2}{L^2}\rightarrow\cos(\beta_{(1)}+\beta_{(2)})=1-\frac{4z^2}{L^2}=\cos\beta_{(1)}\cos\beta_{(2)}-\sin\beta_{(1)}\sin\beta_{(2)}\rightarrow\left(1-\frac{\beta_{(1)}^2}{2}\right)=1-\frac{4z^2}{L^2}\rightarrow\boxed{\beta_{(1)}=2\sqrt{2}\,\frac{z}{L}}$$

$$\rightarrow E=\frac{\sigma}{\pi\varepsilon_0}\left(\frac{\pi}{2}+2\sqrt{2}\,\frac{z}{L}\right)\rightarrow\boxed{E_{(0)}=\frac{\sigma}{2\varepsilon_0}}\ ,\ \boxed{E_{(1)}=\frac{2\sqrt{2}\sigma}{\pi\varepsilon_0}\cdot\frac{z}{L}}$$

8) الف)

$$\boxed{x=R(\cos\theta+\theta\sin\theta)}\qquad\boxed{y=R(\sin\theta-\theta\cos\theta)}$$



ب)

$$\begin{cases}\dot{x}=R(\dot\theta\sin\theta+\theta\dot\theta\cos\theta-\dot\theta\sin\theta)\\ \dot{y}=R(\dot\theta\cos\theta+\theta\dot\theta\sin\theta-\dot\theta\cos\theta)\end{cases}\rightarrow\begin{cases}\ddot{x}=R(\dot\theta^2\cos\theta+\theta\ddot\theta\cos\theta-\theta\dot\theta^2\sin\theta)\\ \ddot{y}=R(\dot\theta^2\sin\theta+\ddot\theta\theta\sin\theta+\theta\dot\theta^2\cos\theta)\end{cases}$$

ج) و د)

$$\begin{cases}T\cos\theta-mg=m\ddot{y}\\ -T\sin\theta=m\ddot{x}\end{cases}\rightarrow-\frac{\sin\theta}{\cos\theta}=\frac{\ddot{x}}{g+\ddot{y}}\rightarrow-\frac{g}{R}\sin\theta-\dot\theta^2\sin^2\theta-\theta\ddot\theta\sin^2\theta-\cancel{\theta\dot\theta^2\sin\theta\cos\theta}=\dot\theta^2\cos^2\theta+\theta\ddot\theta\cos^2\theta-\cancel{\theta\dot\theta^2\sin\theta\cos\theta}$$

$$\rightarrow+\frac{g}{R}\sin\theta+\dot\theta^2+\theta\ddot\theta=0\rightarrow\left(\frac{2}{\theta}\right)(\dot\theta^2)+\frac{d(\dot\theta^2)}{d\theta}=-\frac{2g}{R}\frac{\sin\theta}{\theta}\rightarrow\dot\theta^2=e^{-\int_{\theta_0}^{\theta}\frac{2}{\theta}d\theta}\left(\int_{\theta_0}^{\theta}e^{\int\frac{2}{\theta}d\theta}\times-\frac{2g}{R}\cdot\frac{\sin\theta}{\theta}\,d\theta+C\right)$$

$$\dot\theta^2=e^{\ln\left(\frac{\theta_0^2}{\theta^2}\right)}\left(-\frac{2g}{R}\int_{\theta_0}^{\theta}e^{\ln\left(\frac{\theta^2}{\theta_0^2}\right)}\times\frac{\sin\theta}{\theta}\,d\theta+C\right)\rightarrow\dot\theta^2=\frac{\theta_0^2}{\theta^2}\left(\frac{2g}{R\theta_0^2}\int_{\theta_0}^{\theta}\theta\,d\cos\theta+C\right)$$

$$\int_{\theta_0}^{\theta}\theta\,d\cos\theta=\Big|_{\theta_0}^{\theta}\theta\cos\theta-\int_{\theta_0}^{\theta}\cos\theta\,d\theta=\theta\cos\theta-\theta_0\cos\theta_0-\sin\theta+\sin\theta_0\rightarrow\dot\theta^2=\frac{1}{\theta^2}\left(\frac{2g}{R}(\theta\cos\theta-\sin\theta)+\left(\frac{2g}{R}(\sin\theta_0-\theta\cos\theta_0)+C\theta_0^2\right)\right)\rightarrow$$

$$\dot\theta^2=\frac{2g}{R\theta^2}(\theta\cos\theta-\sin\theta)+\frac{v^2}{R^2}\rightarrow\boxed{\dot\theta=\sqrt{\frac{2g}{R\theta^2}(\theta\cos\theta-\sin\theta)+\frac{v^2}{R\theta}}}$$

ه)

$$\theta=\theta_{(0)}+\lambda\theta_{(1)}\rightarrow(\theta_0+\lambda\theta_{(1)})(\dot\theta_{(0)}+\lambda\dot\theta_{(1)})=\frac{v}{R}\left(1+\lambda(\theta_{(0)}\cos\theta_{(0)}-\sin\theta_0)\right)\rightarrow\theta_{(0)}\dot\theta_{(0)}+\lambda\left(\theta_0\dot\theta_{(1)}+\dot\theta_{(0)}\theta_{(1)}\right)=\frac{v}{R}+\frac{v\lambda}{R}(\theta_0\cos\theta_0-\sin\theta_0)$$

$$\rightarrow\begin{cases}\theta_{(0)}\frac{d\theta_{(0)}}{dt}=\frac{v}{R}\rightarrow\boxed{\theta_{(0)}=\sqrt{\frac{2vt}{R}}}\\ \theta_{(0)}\dot\theta_{(1)}+\dot\theta_{(0)}\theta_{(1)}=\frac{v}{R}(\theta_{(0)}\cos\theta_{(0)}-\sin\theta_{(0)})\end{cases}\rightarrow\sqrt{\frac{2vt}{R}}\,\dot\theta_{(1)}+\sqrt{\frac{v}{2Rt}}\,\theta_{(1)}=\frac{v}{R}\cdot(\theta_{(0)}\cos\theta_{(0)}-\sin\theta_{(0)})\rightarrow$$

$$\dot\theta_{(1)}+\frac{\theta_{(1)}}{2t}=\frac{v}{R}\left(\cos\theta_{(0)}-\frac{\sin\theta_{(0)}}{\theta_{(0)}}\right)\rightarrow\sqrt{t}\,\dot\theta_{(1)}+\frac{\theta_{(1)}}{2\sqrt{t}}=\frac{v}{R}\left(\sqrt{t}\cos\theta_{(0)}-\sqrt{\frac{R}{2v}}\sin\theta_{(0)}\right)=(\sqrt{t}\,\theta_{(1)})^{\cdot}$$

$$\int\sqrt{a}\cos(\sqrt{a})\,da=2\int a\,d(\sin\sqrt{a})=2\left(a\sin\sqrt{a}-\int\sin\sqrt{a}\,da\right)=2\left(a\sin\sqrt{a}-2\sin\sqrt{a}+2\sqrt{a}\cos\sqrt{a}\right)$$

$$\int\sin\sqrt{a}\,da=2\int u\sin u\,du=-2\int u\,d\cos u\ ,\ \boxed{u=\sqrt{a}}\rightarrow-2\int\cos^{-1}(v)\,dv\ ,\ \boxed{v=\cos u}\rightarrow-2\left(v\cos^{-1}(v)+\int\frac{v}{\sqrt{1-v^2}}dv\right)=-2\left(v\cos^{-1}(v)-\frac{1}{2}\int\frac{d(1-v^2)}{\sqrt{1-v^2}}\right)$$

$$=2\left(\sqrt{1-v^2}-v\cos^{-1}(v)\right)=2\left(\sin\sqrt{a}-\sqrt{a}\cos\sqrt{a}\right)$$

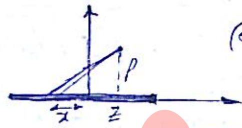
باسمه تعالی

$$\sqrt{t}\,\theta_{(1)} = \frac{v}{R}\left(\int \sqrt{t}\cos\sqrt{\frac{2vt}{R}}\,dt - \int \sin\sqrt{\frac{2vt}{R}}\,dt\right) = \frac{v}{R}\left(\frac{R}{2v}\sqrt{\frac{R}{2v}}\int \sqrt{\frac{2vt}{R}}\cos\sqrt{\frac{2vt}{R}}\, d\left(\frac{2vt}{R}\right) - \frac{R}{2v}\int \sin\sqrt{\frac{2vt}{R}}\, d\left(\frac{2vt}{R}\right)\right) \longrightarrow$$

$$\sqrt{t}\,\theta_{(1)} = \frac{1}{2}\left(\sqrt{\frac{R}{2v}}\times 2\left(\frac{2vt}{R}\sin\sqrt{\frac{2vt}{R}} - 2\sin\sqrt{\frac{2vt}{R}} + 2\sqrt{\frac{2vt}{R}}\cos\sqrt{\frac{2vt}{R}}\right)\right) \longrightarrow$$

$$\theta = \sqrt{\frac{2vt}{R}} + \frac{2gR}{v^2}\sqrt{\frac{R}{2vt}}\left(\frac{\cancel{2}vt}{R}\sin\sqrt{\frac{2vt}{R}} - \sin\sqrt{\frac{2vt}{R}} + \sqrt{\frac{2vt}{R}}\cos\sqrt{\frac{2vt}{R}}\right) \longrightarrow \boxed{\theta = \sqrt{\frac{2vt}{R}} + \frac{2gR}{v^2}\sqrt{\frac{R}{2vt}}\left[\left(\frac{vt}{R}-1\right)\sin\sqrt{\frac{2vt}{R}} + \sqrt{\frac{2vt}{R}}\cos\sqrt{\frac{2vt}{R}}\right]}$$

10) الف)



$$dv = \frac{k\lambda\,dx}{\sqrt{\rho^2+(z-x)^2}} \longrightarrow v = \frac{kq}{l}\int_{-l/2}^{l/2}\frac{d(x-z)}{\sqrt{\rho^2+(x-z)^2}} \longrightarrow v = \frac{kq}{l}\ln\left(\frac{(\frac{l}{2}-z)+\sqrt{\rho^2+(\frac{l}{2}-z)^2}}{-\frac{l}{2}-z+\sqrt{\rho^2+(\frac{l}{2}+z)^2}}\right)\quad \boxed{e^{\frac{lv}{kq}} = c}$$

$$\left(\frac{l}{2}-z\right)+\sqrt{\rho^2+\left(\frac{l}{2}-z\right)^2} = -c\left(\frac{l}{2}+z\right)+c\sqrt{\rho^2+\left(\frac{l}{2}+z\right)^2} \longrightarrow c^2\left(\frac{l}{2}+z\right)^2+\left(\frac{l}{2}-z\right)^2 = c^2\left(\rho^2+\left(\frac{l}{2}+z\right)^2\right)+\left(\rho^2+\left(\frac{l}{2}-z\right)^2\right)-2c\sqrt{\left(\rho^2+\left(\frac{l}{2}-z\right)^2\right)\left(\rho^2+\left(\frac{l}{2}+z\right)^2\right)}$$

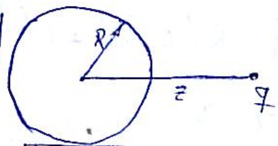
$$-2c\left(\frac{l^2}{4}-z^2\right) \longrightarrow -2c\left(\frac{l^2}{4}-z^2\right)+\rho^2(c^2+1) = 2c\sqrt{\left(\rho^2+\left(\frac{l}{2}+z\right)^2\right)\left(\rho^2+\left(\frac{l}{2}-z\right)^2\right)} \longrightarrow$$

$$4c^2\left(\frac{l^2}{4}-z^2\right)^2+\rho^4(c^4+1+2c^2)-4c(c^2+1)\rho^2\left(\frac{l^2}{4}-z^2\right) = 4c^2\left(\rho^4+\left(\frac{l^2}{4}-z^2\right)^2+2\rho^2\left(\frac{l^2}{4}+z^2\right)\right) \longrightarrow$$

$$\rho^2(c^2-1)^2+4c\left((c^2+1-2c)z^2-(c^2+1+2c)\frac{l^2}{4}\right) = 0 \longrightarrow \rho^2(c^2-1)^2+4c(c-1)^2z^2 = c(c+1)^2l^2 \longrightarrow$$

$$\frac{(c-1)^2(c+1)^2}{c(c+1)^2l^2}\rho^2+\frac{4c(c-1)^2}{c(c+1)^2l^2}z^2 = 1 \longrightarrow \boxed{\left(\frac{\rho}{\frac{l\sqrt{c}}{c-1}}\right)^2+\left(\frac{z}{\frac{l(1+c)}{2(c-1)}}\right)^2 = 1} \longrightarrow$$ بیضی وار دوار است.

ب)



$$\begin{cases} V_1 = P_{11}q_1+P_{12}q_2 \\ V_2 = P_{21}q_1+P_{22}q_2 \end{cases} \Bigg|\ q_2 = 0 \longrightarrow \begin{cases} V_1 = P_{11}q_1 = \frac{kq_1}{R} \\ V_2 = P_{21}q_1 = \frac{kq_1}{z} \end{cases} \longrightarrow \begin{cases} \boxed{P_{11} = \frac{k}{R}} \\ \boxed{P_{12} = P_{21} = \frac{k}{z}} \end{cases} \longrightarrow Q_1 = -\frac{P_{12}}{P_{11}}q \longrightarrow \boxed{Q_1 = -\frac{R}{z}q}$$

الف)

$$\cancel{(a-f)+(a+f)}\ 2\sqrt{f^2+b^2} = (a-f)+(a+f) \longrightarrow f = \pm\sqrt{a^2-b^2} \Rightarrow f^2 = \frac{l^2}{(c-1)^2}\left(\frac{(1+c)^2}{4}-c\right) = \frac{l^2(c-1)^2}{4(c-1)^2} \longrightarrow \boxed{f = \pm\frac{l}{2}}$$

ب) جای بیضی‌وار یک میله قرار می‌دهیم با گاوس نشان می‌دهیم q داخل میله با q روی سطح بیضی‌وار برابر است.

$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{\text{بیضی}}}{\varepsilon_0} = \frac{q_{\text{میله}}}{\varepsilon_0} \longrightarrow q_{\text{بیضی}} = q_{\text{میله}}$$

$$\begin{cases} V_1 = P_{11}q_1+P_{12}q_2 \\ V_2 = P_{21}q_1+P_{22}q_2 \end{cases} \Bigg|\ q_2 = 0 \longrightarrow \begin{cases} V_1 = P_{11}q_1 \Rightarrow \\ V_2 = P_{21}q_1 \Rightarrow \end{cases} \begin{cases} P_{11} = \frac{k}{l}\ln\left(\frac{\frac{l}{2}+a+(\frac{l}{2}+a)}{a-\frac{l}{2}+(a-\frac{l}{2})}\right) \\ P_{21} = P_{12} = \frac{k}{l}\ln\left(\frac{z+\frac{l}{2}+\sqrt{(x^2+y^2)+(z+\frac{l}{2})^2}}{z-\frac{l}{2}+\sqrt{(x^2+y^2)+(z-\frac{l}{2})^2}}\right) \end{cases} \longrightarrow l = 2\sqrt{a^2-b^2} \longrightarrow$$

$$v = -\frac{kq}{l}\int_{-l/2}^{l/2}\frac{d(z-x)}{\sqrt{\rho^2+(z-x)^2}} \longrightarrow v = \frac{kq}{l}\ln\left(\frac{z+\frac{l}{2}+\sqrt{\rho^2+(z+\frac{l}{2})^2}}{z-\frac{l}{2}+\sqrt{\rho^2+(z-\frac{l}{2})^2}}\right)$$

$$\longrightarrow \begin{cases} \boxed{P_{11} = \frac{k}{2\sqrt{a^2-b^2}}\ln\left(\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}\right)} \\ \boxed{P_{21} = P_{12} = \frac{k}{2\sqrt{a^2-b^2}}\ln\left(\frac{z+\sqrt{a^2-b^2}+\sqrt{(x^2+y^2)+(z+\sqrt{a^2-b^2})^2}}{z-\sqrt{a^2-b^2}+\sqrt{(x^2+y^2)+(z-\sqrt{a^2-b^2})^2}}\right)} \end{cases}$$

$$V_1 = 0 \longrightarrow Q_1 = -\frac{P_{12}}{P_{11}}q \longrightarrow$$

$$\boxed{Q_1 = -q\log_{\left(\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}\right)}\left(\frac{z+\sqrt{a^2-b^2}+\sqrt{(x^2+y^2)+(z+\sqrt{a^2-b^2})^2}}{z-\sqrt{a^2-b^2}+\sqrt{(x^2+y^2)+(z-\sqrt{a^2-b^2})^2}}\right)}$$