CORRECTIONS TO SOLUTIONS MANUAL

In the new edition, some chapter problems have been reordered and equations and figure references have changed. The solutions manual is based on the preview edition and therefore must be corrected to apply to the new edition. Below is a list reflecting those changes.

The “NEW” column contains the problem numbers in the new edition. If that problem was originally under another number in the preview edition, that number will be listed in the “PREVIEW” column on the same line. In addition, if a reference used in that problem has changed, that change will be noted under the problem number in quotes. Chapters and problems not listed are unchanged.

For example:

NEW	PREVIEW
4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”

The above means that problem 4.18 in the new edition was problem 4.5 in the preview edition. To find its solution, look up problem 4.5 in the solutions manual. Also, the problem 4.5 solution referred to “Fig. 4.35” and “Fig. 4.36” and should now be “Fig. 4.38” and “Fig. 4.39,” respectively.

CHAPTER 3

NEW	PREVIEW
3.1	3.8
3.2	3.9
3.3	3.11
3.4	3.12
3.5	3.13
3.6	3.14
3.7	3.15
“From 3.6”	“From 3.14”
3.8	3.16
3.9	3.17
3.10	3.18
3.11	3.19
3.12	3.20
3.13	3.21
3.14	3.22
3.15	3.1

3.16	3.2
3.17	3.2’
3.18	3.3
3.19	3.4
3.20	3.5
3.21	3.6
3.22	3.7
3.23	3.10
3.24	3.23
3.25	3.24
3.26	3.25
3.27	3.26
3.28	3.27
3.29	3.28

CHAPTER 4

NEW	PREVIEW
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4.1	4.12
4.2	4.13
4.3	4.14
4.4	4.15
4.5	4.16
4.6	4.17
4.7	4.18
“p. 4.6”	“p. 4.17”
4.8	4.19
4.9	4.20
4.10	4.21
4.11	4.22
4.12	4.23
4.13	4.24
“p. 4.9”	“p. 4.20”
4.14	4.1
“(4.52)”	“(4.51)”
“(4.53)”	“(4.52)”
4.15	4.2
4.16	4.3
4.17	4.4
4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”
4.19	4.6
“Fig 4.39(c)”	“Fig 4.36(c)”

4.20	4.7
4.21	4.8
4.22	4.9
4.23	4.10
4.24	4.11
4.25	4.25
4.26	4.26
“p. 4.9”	“p. 4.20”

CHAPTER 5

NEW	PREVIEW
-----	---------
5.1	5.16
5.2	5.17
5.3	5.18
5.4	5.19
5.5	5.20
5.6	5.21
5.7	5.22
5.8	5.23
5.9	5.1
5.10	5.2
5.11	5.3
5.12	5.4
5.13	5.5
5.14	5.6
5.15	5.7
5.16	5.8
5.17	5.9
5.18	5.10
“Similar to 5.18(a)”	“Similar to 5.10(a)”
5.19	5.11
5.20	5.12
5.21	5.13
5.22	5.14
5.23	5.15

CHAPTER 6

NEW	PREVIEW
-----	---------
6.1	6.7
6.2	6.8

6.3	6.9
“from eq(6.23)”	“from eq(6.20)”
6.4	6.10
6.5	6.11
“eq (6.52)”	“eq (6.49)”
6.6	6.1
6.7	6.2
6.8	6.3
6.9	6.4
6.10	6.5
6.11	6.6
6.13	6.13
“eq (6.56)”	“eq (6.53)”
“problem 3”	“problem 9”
6.16	6.16
“to (6.23) & (6.80)”	“to (6.20) & (6.76)”
6.17	6.17
“equation (6.23)”	“equation (6.20)”

CHAPTER 7

NEW	PREVIEW
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7.2	7.2
“eqn. (7.59)”	“eqn. (7.57)”
7.17	7.17
“eqn. (7.59)”	“eqn. (7.57)
7.19	7.19
“eqns 7.66 and 7.67”	“eqns 7.60 and 7.61”
7.21	7.21
“eqn. 7.66”	“eqn. 7.60”
7.22	7.22
“eqns 7.70 and 7.71”	“eqns. 7.64 and 7.65”
7.23	7.23
“eqn. 7.71”	“eqn. 7.65”
7.24	7.24
“eqn 7.79”	“eqn 7.73”

CHAPTER 8

NEW	PREVIEW
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8.1	8.5
8.2	8.6

8.3	8.7
8.4	8.8
8.5	8.9
8.6	8.10
8.7	8.11
8.8	8.1
8.9	8.2
8.10	8.3
8.11	8.4
8.13	8.13
“problem 8.5”	“problem 8.9”

CHAPTER 13

NEW	PREVIEW
3.17	3.17
“Eq. (3.123)”	“Eq. (3.119)”

CHAPTER 14 - New Chapter, “Oscillators”

CHAPTER 15 - New Chapter, “Phase-Locked Loops”

CHAPTER 16 - Was Chapter 14 in Preview Ed.

Change all chapter references in solutions manual from 14 to 16.

CHAPTER 17 - Was Chapter 15 in Preview Ed.

Change all chapter references in solutions manual from 15 to 17.

CHAPTER 18 - Was Chapter 16 in Preview Ed.

NEW	PREVIEW
18.3	16.3
“Fig. 18.12(c)”	“Fig. 16.13(c)”
18.8	16.8
“Fig. 18.33(a,b,c,d)”	“Fig. 16.34(a,b,c,d)”

Also, change all chapter references from 16 to 18.

Chapter 14
Oscillators

14.1 Open-Loop Transfer Function:

$$H(s) = \frac{-(g_m R_D)^2}{(1+\frac{s}{\omega_0})^2}, \quad \omega_0 = \frac{1}{R_D C_L}$$

The gain drops to unity at $\frac{g_m R_D}{(1+\frac{\omega_u^2}{\omega_0^2})^{1/2}} = 1$, which for $g_m R_D \gg 1$, yields, $\omega_u \gg \omega_0$ and $\omega_u \cong \omega_0 \cdot g_m R_D = \frac{g_m}{C_L}$. The phase changes from $-180°$ at $\omega \approx 0$ to $-2\tan^{-1}\frac{\omega_u}{\omega_0} - 180°$ at ω_u; i.e., the phase change at ω_u is $-2\tan^{-1}(g_m R_D)$ and the phase margin is equal to $180° - 2\tan^{-1}(g_m R_D)$.

14.2 (a) $g_m R_D \geq 2 \Rightarrow R_D \geq 400\,\Omega$.

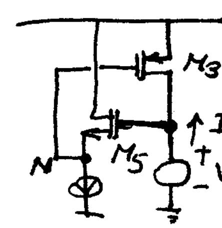
(b) $\begin{cases} \omega_{osc} = \sqrt{3}\,\omega_0 = \sqrt{3}/(R_D C_L) \\ \text{Total Gain} = (g_m R_D)^3 = 16 \Rightarrow R_D = 504\,\Omega \end{cases} \Rightarrow C_L = 0.547\,pF$

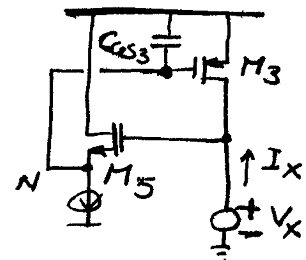
14.3 Each stage must provide a small-signal gain of 2. That is, $g_{m1} R_1 = 2$. With small swings, each transistor carries half of the tail current. For square-law devices, therefore, we have

$$g_{m1} R_1 = 2 = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\, R_1 = 2 \Rightarrow$$

$$I_{SS} \geq \frac{4}{\mu_n C_{ox} \frac{W}{L} R_1^2}$$

14.4 Neglecting body effect of M_5, we have $V_N \approx V_X$. Thus, the gate and drain of M_3 experience equal voltage variations. That is, M_3 operates as a diode-connected device, providing an impedance of $1/g_{m3}$.





14.5 $\dfrac{V_N}{V_X} = \dfrac{\dfrac{1}{C_{GS3}s}}{\dfrac{1}{C_{GS3}s} + \dfrac{1}{g_{m5}}}$ $(\gamma = \lambda = 0)$

$$= \frac{g_{m5}}{g_{m5} + C_{GS3}s} \Rightarrow \frac{I_X}{V_X} = \frac{g_{m3}g_{m5}}{g_{m5} + C_{GS3}s}$$

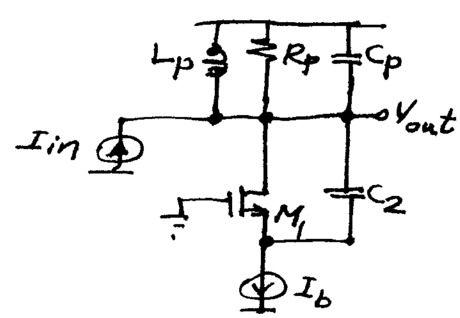
$\Rightarrow \dfrac{V_X}{I_X} = \dfrac{1}{g_{m3}} + \dfrac{C_{GS3}}{g_{m3}g_{m5}}s$ $\Rightarrow$ The impedance is always inductive.

14.6 To avoid latchup, $g_m R_S < 1 \Rightarrow R_S < \dfrac{1}{g_m}$.

14.7 The drain currents saturate near I_{SS} and 0 for a short while, creating a "squarish" waveform. The output voltages are the result of injecting the currents into the tanks. Since the tanks provide suppression at higher harmonics, V_X and V_Y are filtered versions of I_{D1} and I_{D2}.

14.8 For the circuit to oscillate, the loop gain must exceed unity: $g_m R_P > 1 \Rightarrow g_m > 1/R_P$. For square-law devices, $\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} > \frac{1}{R_P}$. Thus, $I_{SS} > \dfrac{1}{\mu_n C_{ox}\frac{W}{L} R_P^2}$.

For M_1 and M_2 not to enter the triode region, the maximum value of V_X and the minimum value of V_Y must differ by no more than V_{TH}. That is, the peak-to-peak swing at X or Y must be less than V_{TH}. Since the peak-to-peak swing is $\approx I_{SS}R_P$, we must have $I_{SS}R_P < V_{TH}$.



14.9 Since the total current flowing thru M_1 and C_2 is equal to I_b, a constant value.

Thus, $\dfrac{V_{out}}{I_{in}} = (L_P s) \,||\, R_P \,||\, \dfrac{1}{C_P s}$.

14.10 Replace R_p with $R_p \| \frac{1}{C_p s} = \frac{R_p}{R_p C_p s + 1}$ in Eq. (14.40). The denominator then reduces to:

$$R_p C_1 C_2 L_p s^3 + (C_1 + C_2) L_p R_p C_p s^3 + (C_1+C_2) L_p s^2 + [g_m L_p R_p C_p s + g_m L_p + R_p (C_1 + C_2)] s + g_m R_p$$

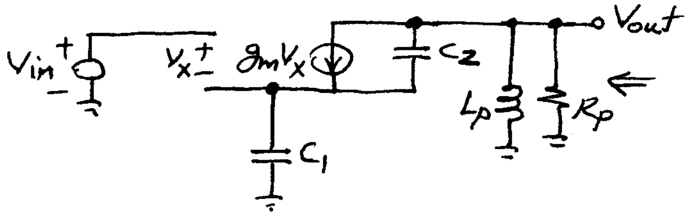
Grouping the imaginary terms and equating their sum to zero, we have

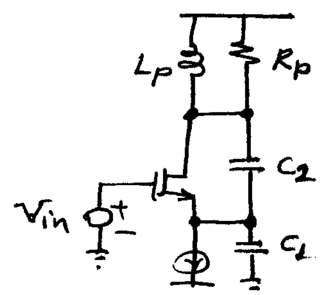
$$-R_p L_p \omega^3 [C_1 C_2 + (C_1 + C_2) C_p] + [g_m L_p + R_p (C_1 + C_2)] \omega = 0$$

Assuming $g_m L_p \ll R_p (C_1 + C_2)$, we obtain

$$\omega^2 = \frac{1}{L_p \left(\frac{C_1 C_2}{C_1 + C_2} + C_p \right)}.$$

14.11





The current thru $R_p \| (L_p s)$ is equal to $V_{out}(\frac{1}{R_p} + \frac{1}{L_p s})$. The negative of this current flows thru C_1, generating a voltage $-V_{out}(\frac{1}{R_p} + \frac{1}{L_p s})\frac{1}{C_1 s}$ across it. Thus, $V_X = V_{in} + V_{out}(\frac{1}{R_p} + \frac{1}{L_p s})\frac{1}{C_1 s}$. Also, the current thru C_2 is equal to $[V_{out} + V_{out}(\frac{1}{R_p} + \frac{1}{L_p s})\frac{1}{C_1 s}] C_2 s$.

Adding $g_m V_X$ and the current thru C_2 and equating the result to $-V_{out}(\frac{1}{R_p} + \frac{1}{L_p s})$, we have

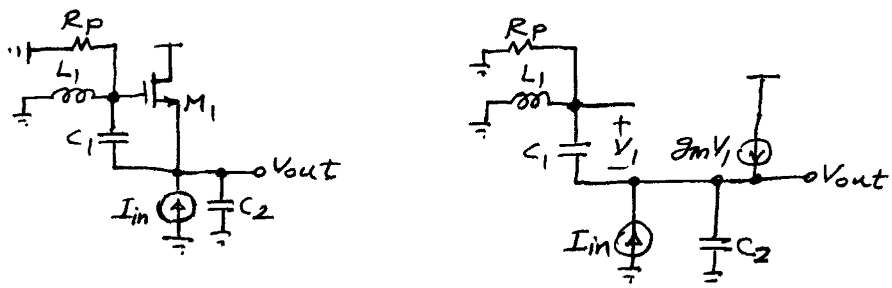
$$\left[V_{in} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] g_{m1} + \left[V_{out} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] C_2 s = -V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right).$$

It follows that

$$\frac{V_{out}}{V_{in}} = \frac{-g_m L_p R_p C_1 s^2}{R_p L_p C_2 C_1 s^3 + L_p (C_1 + C_2) s^2 + [g_{m1} L_p + R_p (C_1 + C_2)] s + g_{m1} R_p}.$$

Note that the denomintor is the same as in Eq. (14.40).

14.12



$$V_1 = -(I_{in} - V_{out}C_2 s + g_m V_1)/C_1 s \Rightarrow V_1(1+g_m/C_1 s) = \frac{-I_{in}+V_{out}C_2 s}{C_1 s}$$

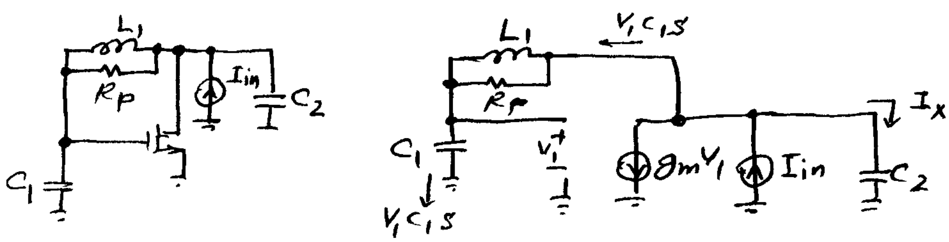
$$\Rightarrow V_1 = \frac{-I_{in}+V_{out}C_2 s}{g_m + C_1 s}$$

Writing a KVL, we have $-V_1 C_1 s \dfrac{R_p L_1 s}{R_p + L_1 s} = V_1 + V_{out}$.

It follows that

$$V_{out} = -\frac{I_{in}+V_{out}C_2 s}{g_m + C_1 s}\left[1+\frac{C_1 s R_p L_1 s}{R_p+L_1 s}\right].$$

Simplifying and calculating the denominator of V_{out}/I_{in}, we have $R_pL_1C_1C_2s^3 + L_1(C_1+C_2)s^2 + [R_p(C_1+C_2)+g_mL_1]s + g_mR_p$, which is the same as Eq. (14.40). Thus, the oscillation conditions are the same as those of Colpitts oscillator.



We can consider V_1 as the output because for oscillation to begin the gain from I_{in} to V_1 must be infinite as well. First, assume $R_p = \infty$:

$$I_X = +V_1 C_1 s \left(L_1 s + \frac{1}{C_1 s}\right) C_2 s = -g_m V_1 + I_{in} - V_1 C_1 s$$

$$\Rightarrow V_1\left[C_1C_2 s^2 \left(L_1 s + \frac{1}{C_1 s}\right) + g_m + C_1 s\right] = I_{in}$$

Now, include R_p: $V_1\left[C_1C_2s^2\left(\dfrac{R_pL_1s}{R_p+L_1s} + \dfrac{1}{C_1s}\right) + g_m + C_1 s\right] = I_{in}$

$$\Rightarrow V_1\left[\frac{\cancel{C_1}C_2s^2(R_pC_1L_1s^2 + R_p + L_1 s) + (g_m + C_1 s)\cancel{(C_1 s)}(R_p+L_1 s)}{C_1 s(R_p+L_1 s)}\right] = I_{in}$$

(C_1s is factored from numerator & denominator.)

$\Rightarrow$ denominator of V_1/I_{in} is

$$R_pC_1C_2L_1s^3 + R_pC_2 s + L_1C_2 s^2 + g_mR_p + g_mL_1 s + C_1R_p s + C_1L_1 s^2$$

$$= R_pC_1C_2L_1s^3 + L_1(C_1+C_2)s^2 + [R_p(C_1+C_2) + g_mL_1]s + g_mR_p,$$

the same as that in Eq. (14.40).

14.13 $I_T = 1\,mA,\ (\frac{W}{L})_{1,2} = 50/0.5$

(a) For a three-stage ring, the minimum gain per stage at low freqs must be 2. Thus, $g_{m1,2} R_{1,2} = 2$ (when no current flows thru M_3 and M_4). $\Rightarrow R_{1,2} = 2/g_{m1,2}$. ($g_{m1,2} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_{1,2} I_T}$.)

(b) $g_{m3,4} R = 0.5$ with $I_{D3,4} = 0.5\,mA$.

$$g_{m3,4} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_{3,4} I_T} = \underbrace{g_{m1,2}}_{=\frac{2}{R}} \sqrt{\frac{(W/L)_{3,4}}{(W/L)_{1,2}}}$$

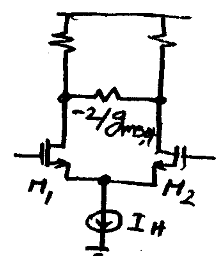
$$\Rightarrow \frac{2}{R} \sqrt{\frac{(W/L)_{3,4}}{(W/L)_{1,2}}}\, R = 0.5$$

$$\Rightarrow (W/L)_{3,4} = 0.25^2 (W/L)_{1,2}.$$

(c) The voltage gain must be equal to 2 with a diff pair tail current of I_H while M_3 and M_4 carry all of I_T.

$$|A_V| = g_{m1,2} (R_{1,2} \| \frac{-1}{g_{m3,4}})$$

$$= g_{m1,2} \frac{R_{1,2}}{1 - g_{m3,4} R_{1,2}}$$



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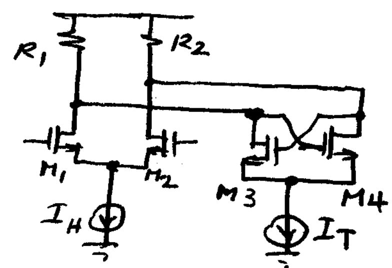


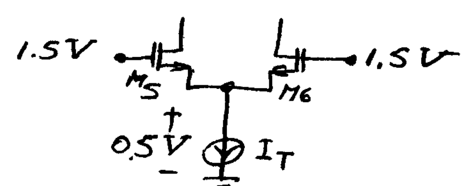
If $g_{m3,4} R_{1,2} < 1$ (to avoid latch-up), then

$$g_{m1,2} R_{1,2} > 2(1 - g_{m3,4} R_{1,2})$$

$$\Rightarrow \sqrt{2 \frac{I_H}{2} \mu_n C_{ox} (\frac{W}{L})_{1,2}}\, R_{1,2} > 2\left(1 - \sqrt{2 \frac{I_T}{2} \mu_n C_{ox} (\frac{W}{L})_{3,4}}\, R_{1,2}\right)$$

Thus, I_H can be determined.

(d) Neglecting body effect for simplicity, we have



$$\frac{I_T}{2} = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})_{5,6} (V_{GS5,6} - V_{TH5,6})^2$$

$$\Rightarrow (\frac{W}{L})_{5,6} = \frac{I_T}{\mu_n C_{ox} (V_{GS5} - V_{TH5,6})^2} \quad \text{and } V_{GS5,6} + 0.5\,V = 1.5\,V.$$

14.14 If each inductor contributes a cap of C_1, then

$$f_{osc,min} = \frac{1}{2\pi\sqrt{L\,(C_0+C_1)}}\ ,\quad f_{osc,max} = \frac{1}{2\pi\sqrt{L\,(0.62C_0+C_1)}}$$

Thus, the tuning range is given by $\dfrac{f_{osc,max}}{f_{osc,min}} = \sqrt{\dfrac{C_0+C_1}{0.62C_0+C_1}}$,

which is less than 27%. For example, if $C_1 = 0.2\,C_0$, then,

$f_{osc,max}/f_{osc,min} \cong 1.21$.

14.15 (a) $L_P = 5\ nH$, $C_X = 0.5\ pF$ $\quad f_{osc} = 1\ GHz = \dfrac{1}{2\pi\sqrt{5nH \times (C_X + C_D)}}$

$\Rightarrow C_D = 4.566\ pF$.

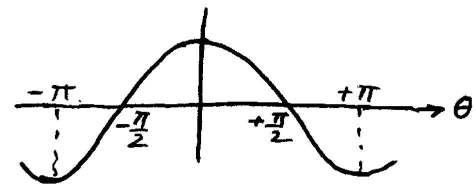
(b) $Q = \dfrac{L\omega}{R_P} = 4 \Rightarrow R_P = 125.7\ \Omega \Rightarrow$

With a 1-mA tail current, the peak-to-peak swing on each side is approximately equal to 126 mV.

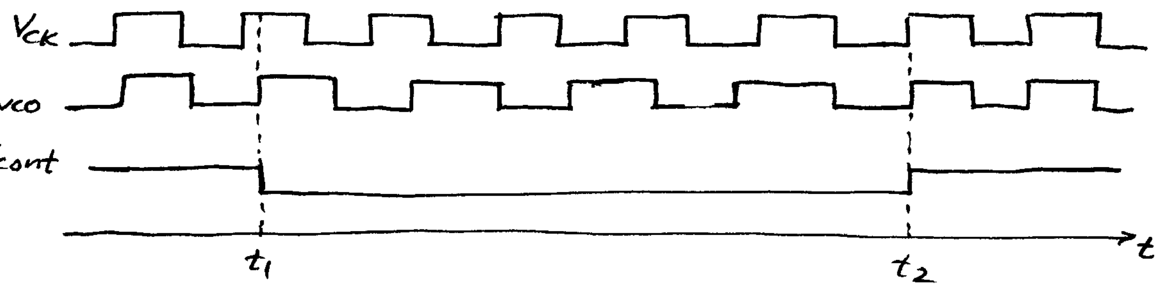
Chapter 15
Phase-Locked Loops

15.1 With two signals $V_1 \cos\omega t$ and $V_2 \cos(\omega t+\theta)$, the product is $V_{out} = \frac{1}{2}V_1V_2\left[\cos(2\omega t+\theta) + \cos\theta\right]$. If the high-freq. component is filtered out, $\overline{V_{out}} \propto \cos\theta$.

· The phase detector is linear only for a small neighborhood around $\theta = \pm\frac{\pi}{2}$.



15.2



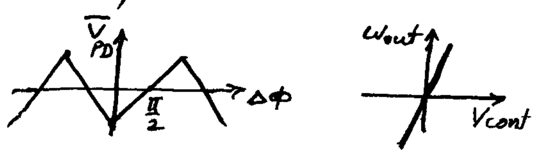
The difference between the two frequencies is integrated between t_1 and t_2 to accumulate a difference of ϕ_0:

$$(f_H - f_L)(t_2 - t_1) = \frac{\phi_0}{2\pi}$$

$$\Rightarrow \quad t_2 - t_1 = \frac{\phi_0}{2\pi(f_H - f_L)}$$

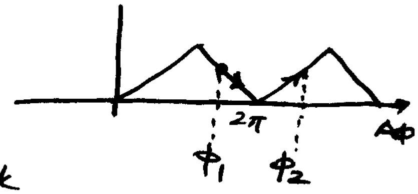
15.3 The VCO still requires a dc voltage that defines the frequency of operation. A high-pass filter would not provide the dc component.

15.4 The loop must lock such that the phase difference is away from zero because the PD gain drops to zero at $\Delta\phi=0$. With a large loop gain, the PD output settles around half of its full scale. This point can be better seen in a fully-differential implementation:

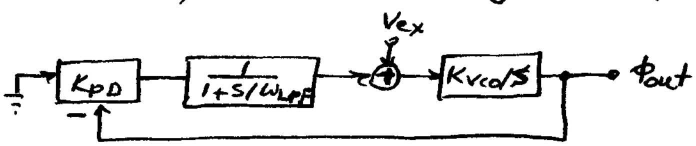


$\Rightarrow V_{cont}$ is small $\Rightarrow \Delta\phi \simeq 90°$.

15.5 Suppose the loop begins with $\Delta\phi = \phi_1$. If the feedback is positive, the loop accumulates so much phase to drive the PD toward ϕ_2, where the feedback is negative and the loop can settle.



15.6 Note: ϕ_{ex} should be changed to V_{ex}.



$$\left(-\phi_{out}\cdot K_{PD}\cdot\frac{1}{1+\frac{s}{\omega_{LPF}}}+V_{ex}\right)\frac{K_{VCO}}{s}=\phi_{out}$$

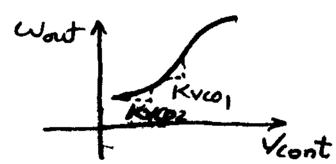
$$\Rightarrow \phi_{out}\left(1+\frac{K_{PD}K_{VCO}}{s(1+\frac{s}{\omega_{LPF}})}\right)=V_{ex}\frac{K_{VCO}}{s}\Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}}=\frac{K_{VCO}}{s+\frac{K_{PD}K_{VCO}}{1+\frac{s}{\omega_{LPF}}}}=\frac{K_{VCO}(1+\frac{s}{\omega_{LPF}})}{\frac{s^2}{\omega_{LPF}}+s+K_{PD}K_{VCO}}$$

15.7 $\zeta=\frac{1}{2}\sqrt{\frac{\omega_{LPF}}{K_{PD}K_{VCO}}}$ $\quad\sqrt{\frac{K_{VCO1}}{K_{VCO2}}}=1.5$

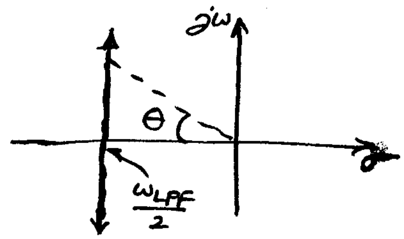
$\Rightarrow \frac{K_{VCO1}}{K_{VCO2}}=2.25$

The slope can vary by a factor of 2.25.



15.8 $\tan\varphi=\frac{Im(pole)}{-Re(pole)}=\frac{\sqrt{1-\zeta^2}}{\zeta}$

This is indeed as if $\zeta=\cos\varphi$ and $\sqrt{1-\zeta^2}=\sin\varphi$.



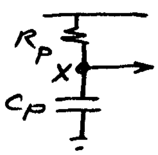
15.9 $K_{VCO}=100\ MHz/V$, $K_{PD}=1\ V/rad$, $\omega_{LPF}=2\pi(1\ MHz)$

$\Rightarrow \xi=\frac{1}{2}\sqrt{\frac{1\ MHz}{(1\ V/rad)(100\ MHz/V)}}=0.05$ $\quad\frac{\omega_n}{2\pi}=\sqrt{(1MHz)(1V/rad)(100MHz/V)}$ $=10\ MHz$.

The loop is heavily underdamped. $\tau=318\ ns$

Step response $\simeq[1-e^{-t/318\,ns}\sin(2\pi\times10\ MHz\times t+\theta)]\,u(t)$, $\theta\cong90°$.

15.10

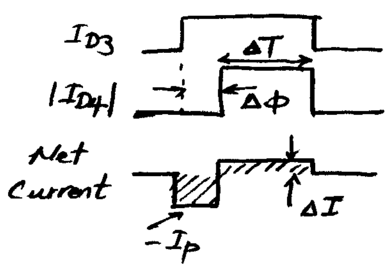


If the control voltage is sensed at node X, then R_P appears in series with the current sources in the charge pump, failing to provide a zero.

15.11 From (15.40), $\frac{I_{out}}{\Delta\phi}(s) = \frac{I_P}{2\pi}$. Since I_{out} is multiplied by the series combination of R_P and C_P:

$$\frac{V_{out}}{\Delta\phi}(s) = \frac{I_P}{2\pi}\left(R_P + \frac{1}{C_P s}\right).$$

15.12



$\Delta\phi$ must be such that the net current is zero. If the current mismatch equals ΔI and the width of $|I_{D4}|$ pulses is ΔT, then

$$\left(\frac{\Delta\phi}{2\pi} \cdot T_P\right) I_P = \Delta T \cdot \Delta I, \text{ where } T_P \text{ is the period.}$$

$$\Rightarrow \Delta\phi = 2\pi \frac{\Delta T}{T_P} \cdot \frac{\Delta I}{I_P}$$

15.13 $\omega_{out} = \omega_0 + K_{VCO} V_{cont}$, $V_{cont} = V_m \cos\omega_m t$. The VCO output is

$$V_{out} = V_0 \cos\left[\int \omega_{out}\, dt\right] = V_0 \cos\left[\omega_0 t + K_{VCO} V_m \int \cos\omega_m t\, dt\right]$$

$$= V_0 \cos\omega_0 t \cos\left(K_{VCO}\frac{V_m}{\omega_m}\sin\omega_m t\right) - V_0 \sin\omega_0 t \sin\left(K_{VCO}\frac{V_m}{\omega_m}\sin\omega_m t\right).$$

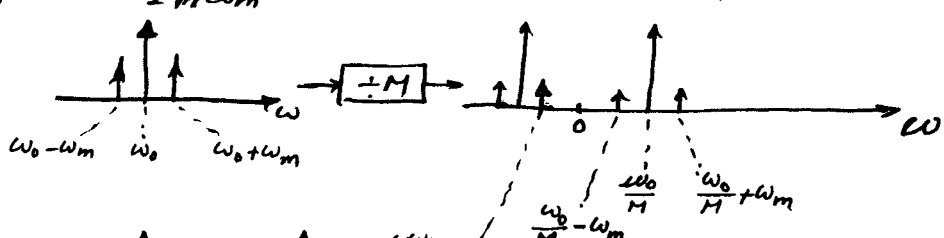
For small V_m, $V_{out}(t) \approx V_0 \cos\omega_0 t - \frac{K_{VCO} V_m V_0}{2\omega_m}\left[\cos(\omega_0 - \omega_m)t - \cos(\omega_0 + \omega_m)t\right]$.

The divider output is expressed as

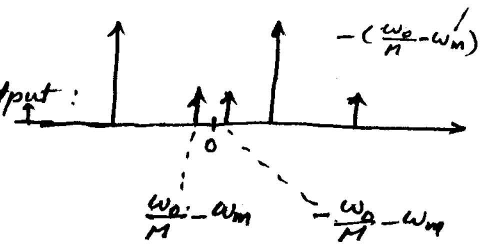
$$V_{out,M} = V_0 \cos\left[\frac{\omega_0 t}{M} + \frac{K_{VCO} V_m}{M}\int \cos\omega_m t\, dt\right]$$

$$\approx V_0 \cos\frac{\omega_0}{M}t - \frac{K_{VCO} V_m V_0}{2M\omega_m}\left[\cos\left(\frac{\omega_0}{M} - \omega_m\right)t - \cos\left(\frac{\omega_0}{M} + \omega_m\right)t\right].$$

If $\frac{\omega_0}{M} > \omega_m$,



If $\frac{\omega_0}{M} > \omega_m$, output:



(aliasing)

15.14 $\quad S_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1} \qquad \xi \propto \sqrt{I_p K_{vco}}$

$\omega_n \propto \sqrt{I_p K_{vco}}$

As $I_p K_{vco}$ starts from small values, $S_{1,2}$ are complex:

$Re\{S_{1,2}\} = -\xi\omega_n \quad Im\{S_{1,2}\} = \pm\omega_n\sqrt{1-\xi^2}$.

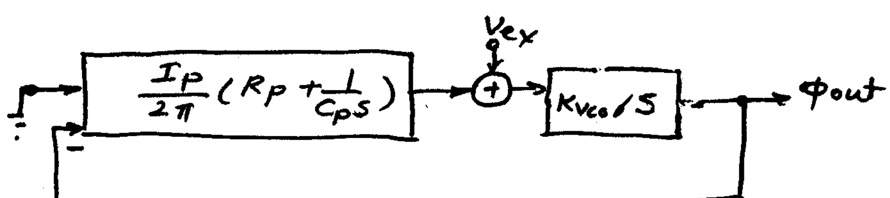
Noting that $\omega_n = \dfrac{2\xi}{R_pC_p}$, we can write $\omega_n^2 - \dfrac{2\xi\omega_n}{R_pC_p} = 0$

Adding $\left(\dfrac{1}{R_pC_p}\right)^2$ to both sides and subtracting and adding $-\xi^2\omega_n^2$, we obtain $\left(-\xi\omega_n + \dfrac{1}{R_pC_p}\right)^2 + \omega_n^2(1-\xi^2) = \left(\dfrac{1}{R_pC_p}\right)^2$, which is a circle centered at $-\dfrac{1}{R_pC_p}$ with a radius equal to $\dfrac{1}{R_pC_p}$.

For $\xi \geq 1$, the poles become real and move away from each other: $-\xi\omega_n + \omega_n\sqrt{\xi^2-1}$ and $-\xi\omega_n - \omega_n\sqrt{\xi^2-1}$. If $\xi \to \infty$, then $-\xi\omega_n + \omega_n\sqrt{\xi^2-1} = \omega_n(-\xi + \sqrt{\xi^2-1}) = \omega_n\xi\left(-1+\sqrt{1-\dfrac{1}{\xi^2}}\right)$

$\approx \omega_n\xi\left(-1+\left(1-\dfrac{1}{2\xi^2}\right)\right) \approx \dfrac{-\omega_n}{2\xi} = \dfrac{-1}{R_pC_p}$.

15.15 $\quad$ Note: ϕ_{ex} should be changed to V_{ex}.



$$\left[-\phi_{out}\cdot\frac{I_p}{2\pi}\left(\frac{R_pC_pS+1}{C_pS}\right) + V_{ex}\right]\frac{K_{vco}}{S} = \phi_{out}$$

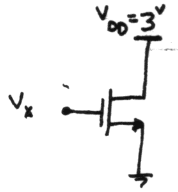
$$\Rightarrow \phi_{out}\left[1 + \frac{I_pK_{vco}(R_pC_pS+1)}{2\pi C_pS^2}\right] = V_{ex}\,\frac{K_{vco}}{S} \Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}} = \frac{K_{vco}(2\pi C_pS^2)}{2\pi C_pS^2 + I_pK_{vco}R_pC_pS + 1}$$

15.16 When the VCO frequency is far from the input frequency, the PFD operates as a frequency detector, comparing the VCO and input frequencies. Thus, the VCO transfer function must relate the output frequency to the control voltage: $\Delta\omega_{out} = K_{VCO}\,\Delta V_{cont}$ $\Rightarrow$ the order of the system falls by one (compared to when the VCO phase is of interest: K_{VCO}/s.)

Chapter 2

2.1) a) NMOS :

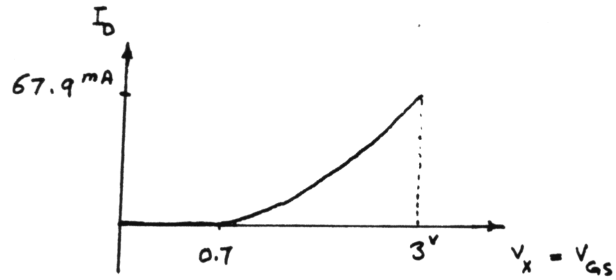


for $V_x < V_{TH}\ (=0.7)$ device is off , $I_D \simeq 0$

for $V_x \geqslant 0.7$

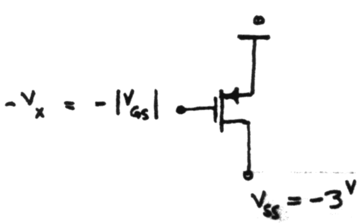
$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L_{eff}} (V_x - 0.7)^2 (1 + \lambda \cdot 3^V) \quad (L_{eff} = 0.5^{\mu} - 2L_D)$$

$$I_D = 12.8 \left(\frac{mA}{V^2}\right) \cdot (V_x - 0.7)^2$$



b) PMOS :

Solution is the same

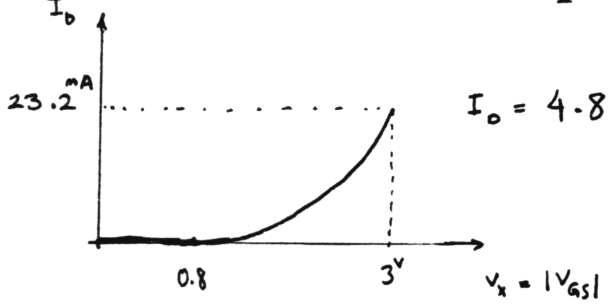


for $|V_{GS}| < V_{TH}\ (= 0.8)$ $I_D \simeq 0$

for $|V_{GS}| \geqslant 0.8$

$$I_D = \frac{1}{2}\mu_p C_{ox} \frac{W}{L_{eff}} (V_x - 0.8)^2 (1 + \lambda \cdot 3^V)$$

$$I_D = 4.8 \left(\frac{mA}{V^2}\right) \cdot (V_x - 0.8)^2$$



2.2) a) N mos

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66 \frac{mA}{V}$$ (Neglecting L_D)

$$r_o = \frac{1}{\lambda I_D} = 20^{K\Omega}$$

Intrinsic gain $= g_m r_o = 7.33 \frac{V}{V}$

b) PMOS

$$g_m = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} = 1.96 \frac{mA}{V}$$

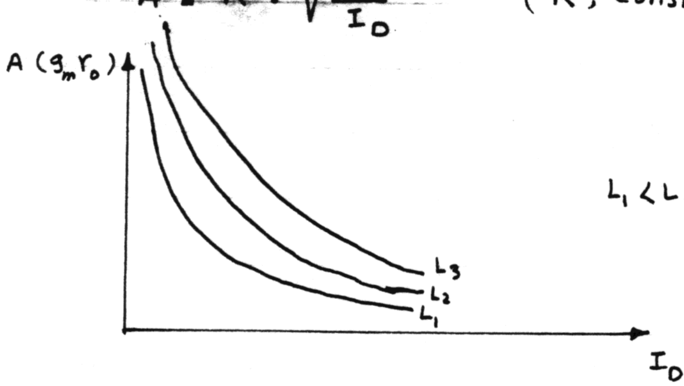
$$r_o = \frac{1}{\lambda I} = \frac{1}{0.2 \cdot 0.5^{mA}} = 10^{K\Omega}$$

$$g_m r_o = 19.6 \frac{V}{V}$$

2.3) $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ $\quad r_o = \frac{1}{\lambda I_D}$ $\quad$ Assume $\lambda = \frac{\alpha}{L}$

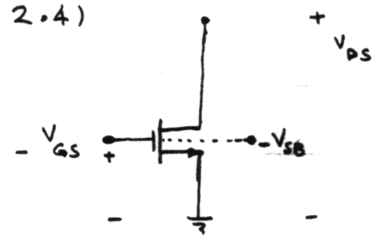
$$A = g_m r_o = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} \cdot \frac{L}{\alpha I_D}$$

$$A = K \cdot \sqrt{\frac{WL}{I_D}}$$ (K; Constant)



$L_1 < L_2 < L_3$

2.4)



I_D versus V_{GS} : (for NMOS)

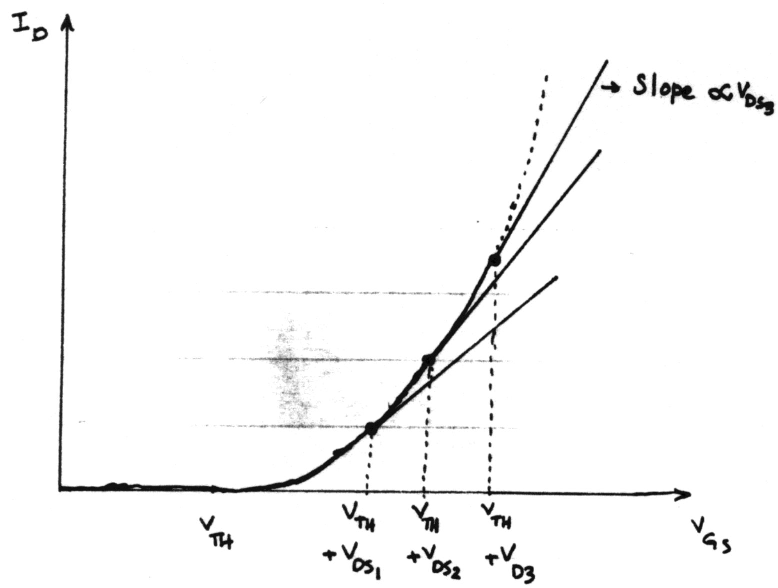
I) for $V_{GS} < V_{TH}$, $I_D \approx 0$

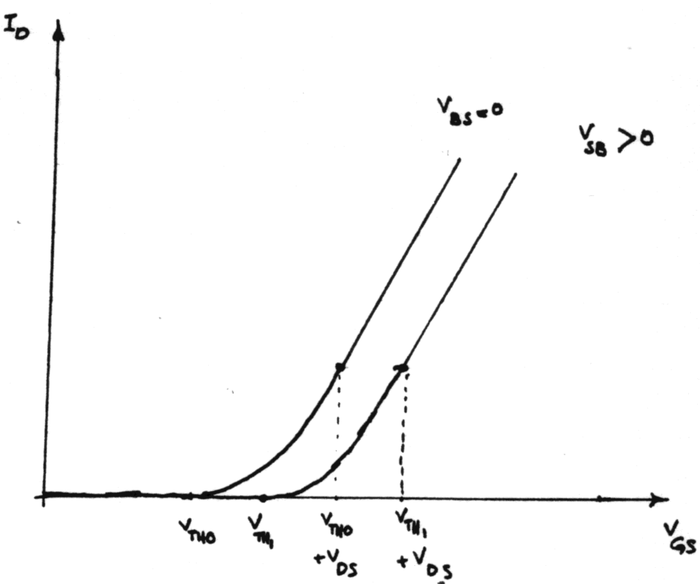
II) for $V_{TH} < V_{GS} < V_{TH} + V_{DS}$ $\Rightarrow$ Device is in the Saturation region

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

III) for $V_{GS} > V_{TH} + V_{DS}$ $\Rightarrow$ Device Operates in the triode region

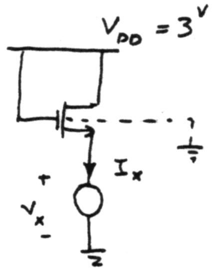
$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$





Changing V_{SB} just shifts the curve to the right for $V_{SB} > 0$ or to the left for $V_{SB} < 0$

2.5) a)



$\lambda = 0.1$, $\gamma = 0.45$, $2\phi_F = 0.9$, $V_{TH0} = 0.7$

$V_{GS} = 3 - V_x$, $V_{DS} = 3 - V_x$, $V_{SB} = V_x$

$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

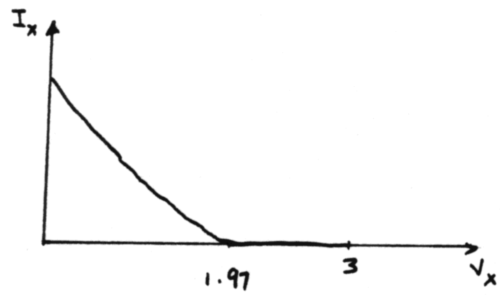
So, $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right)\right)^2 (1 + \lambda(3 - V_x))$

The above equation is valid for

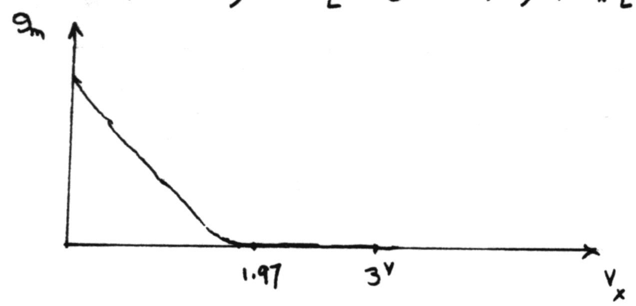
$3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right) > 0$, i.e. $V_x < 1.97^V$

So, $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2.727 - V_x - 0.45\sqrt{0.9 + V_x}\right)^2 (1.3 - 0.1 V_x)$

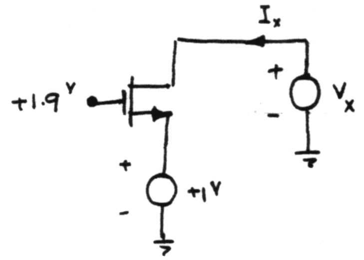
and $I_x = 0$ for $1.97 < V_x$



$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_x}$



2.5) b)



$\lambda = \gamma = 0 \qquad V_{TH} = 0.7$

for $0 < V_x < 1$, S and D exchang their roles.

$V_{GS} = 1.9 - V_x \qquad V_{DS} = 1 - V_x \quad , V_{OD} = 1.2 - V_x$

$I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[(1.2 - V_x) \times 2 \times (1 - V_x) - (1 - V_x)^2 \right]$

$I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (1 - V_x)(1.4 - V_x)$.

$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS} = \mu_n C_{ox} \frac{W}{L} (1 - V_x)$ (absolute Value)

The above equations are valid for $V_x < 1$

Then the direction of current is reversed.

$V_{GS} = 1.9 - 1 = 0.9 \qquad V_{DS} = V_x - 1 \quad , V_{OD} = 0.9 - 0.7 = 0.2$

for $V_x < 1.2$, device operates in the triode region.

$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \times 0.2 \times (V_x - 1) - (V_x - 1)^2 \right]$

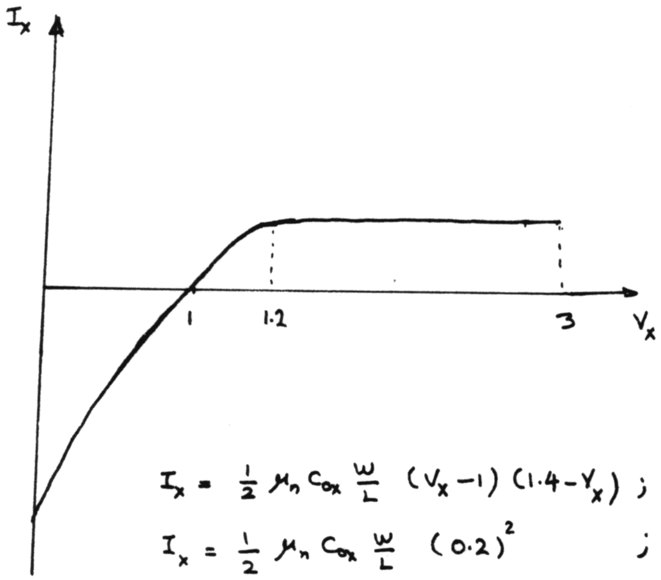
$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - 1)$

for $V_x > 1.2$, Device goes into Saturation region

2.5) b Cont

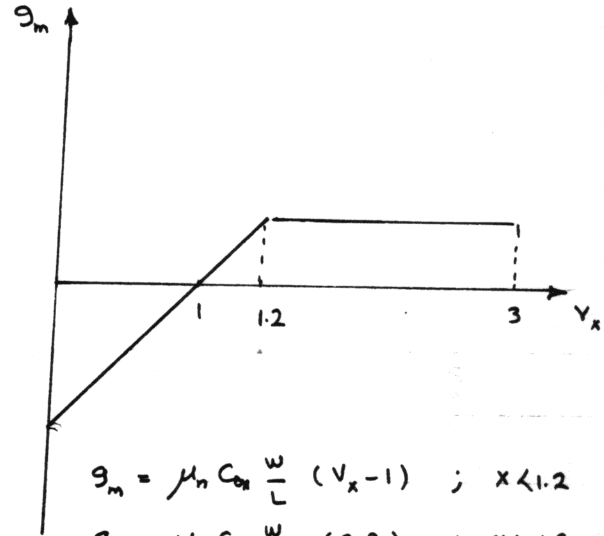
$$\text{So,} \quad I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2 ,$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$$



$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_x - 1)(1.4 - V_x) \; ; \; x < 1.2$

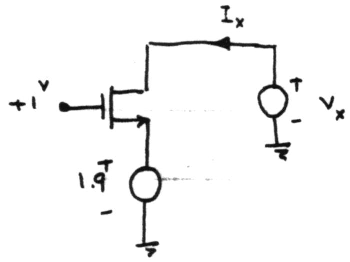
$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2 \; ; \; x > 1.2$



$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - 1) \; ; \; x < 1.2$

$g_m = \mu_n C_{ox} \frac{W}{L} (0.2) \; ; \; x > 1.2$

2.5) c



$\lambda = \gamma = 0$ $V_{TH} = 0.7$

S and D exchange their roles.

$V_{GS} = 1 - V_x$ $V_{DS} = 1.9 - V_x$ $V_{OD} = V_{GS} - V_{TH} = 0.3 - V_x$

Device is in Saturation region, So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$

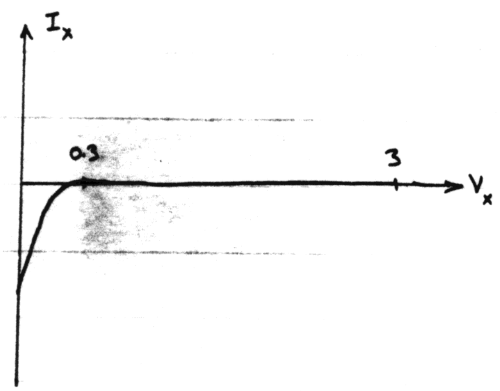
Device turns off when $V_x = 0.3$ and never turns on again.

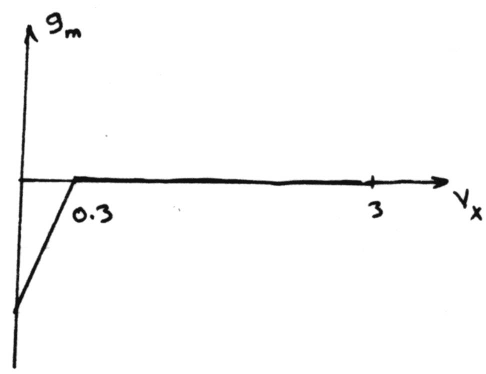
So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$; $x < 0.3$

$I_x = 0$; other wise

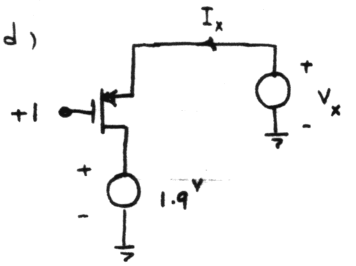
Then $g_m = -\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)$; $x < 0.3$

$g_m = 0$; Other wise





2.5) d)



$V_{TH} = -0.8$ $\quad \gamma = 0$

D and S exchang their roles.

$V_{GS} = -0.9 \qquad V_{DS} = V_x - 1.9$

for $V_x < 1.8$:

$$I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

Device remains in the saturation region until

$V_x = 1.9 - 0.1 = 1.8$, then device goes into the triode

region.

for $1.8 < V_x < 1.9$:

$$I_x = -\mu_p C_{ox}\frac{W}{L}\left[(-0.1)(V_x - 1.9) - \frac{1}{2}(V_x - 1.9)^2\right]$$

$$g_m = +\mu_p C_{ox}\frac{W}{L}(V_x - 1.9)$$

for $V_x > 1.9$:

S and D exchange their roles again, when $V_x = 1.9$

for $V_x > 1.9$, Device operates in the triode region.

$V_{GS} = 1 - V_x \quad , \quad V_{DS} = 1.9 - V_x$

$$I_x = +\mu_p C_{ox}\frac{W}{L}\left[(1.8 - V_x)(1.9 - V_x) - \frac{1}{2}(1.9 - V_x)^2\right]$$

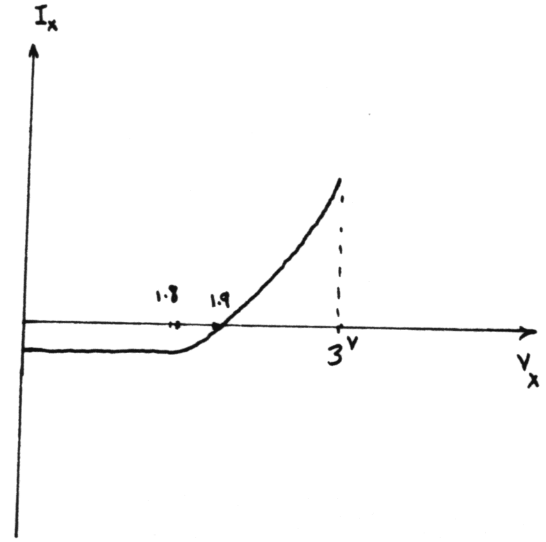
$$g_m = -\mu_p C_{ox}\frac{W}{L}(1.9 - V_x)$$

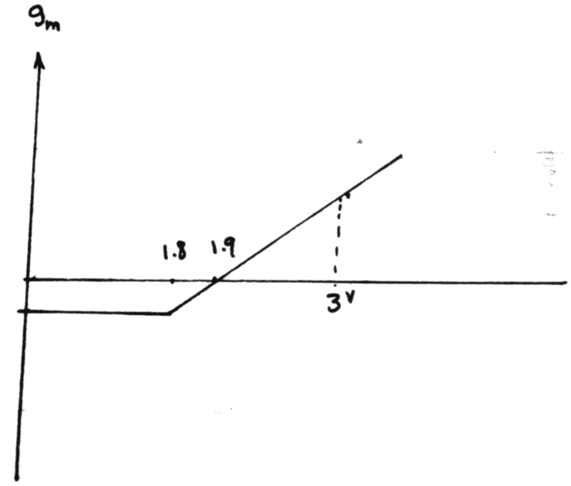
2.5)d So, $0 < V_x < 1.8$ $\quad I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

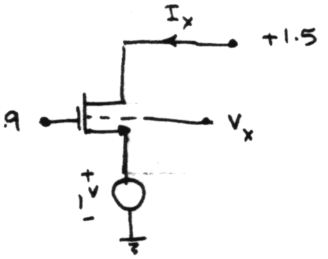
$1.8 < V_x < 3$ $\quad I_x = +\mu_p C_{ox}\frac{W}{L}\times\frac{1}{2}(V_x-1.9)(V_x-1.7)$

$$g_m = \mu_p C_{ox}\frac{W}{L}(V_x-1.9)$$





2.5) e)



$V_{TH0} = 0.7$ $\quad \gamma = 0.45$ $\quad 2\phi_F = 0.9$ $\quad , \lambda = 0$

$V_{SB} = 1 - V_x$

$V_{TH} = 0.7 + 0.45(\sqrt{0.9 + 1 - V_x} - \sqrt{0.9})$

$V_{GS} = 0.9$ $\qquad V_{DS} = 0.5$

for $V_x = 0$, $V_{TH} = 0.893$ So device is in Saturation region.

So $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45(\sqrt{1.9 - V_x} - \sqrt{0.9})\right)^2$

$g_m = \mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45(\sqrt{1.9 - V_x} - \sqrt{0.9})\right)$

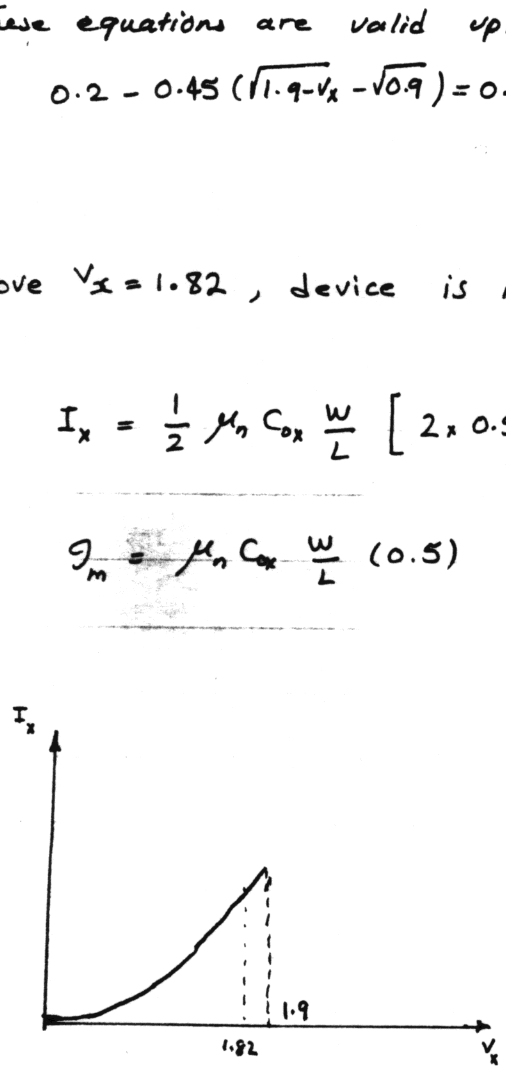
These equations are valid upto the edge of triode region, i.e.

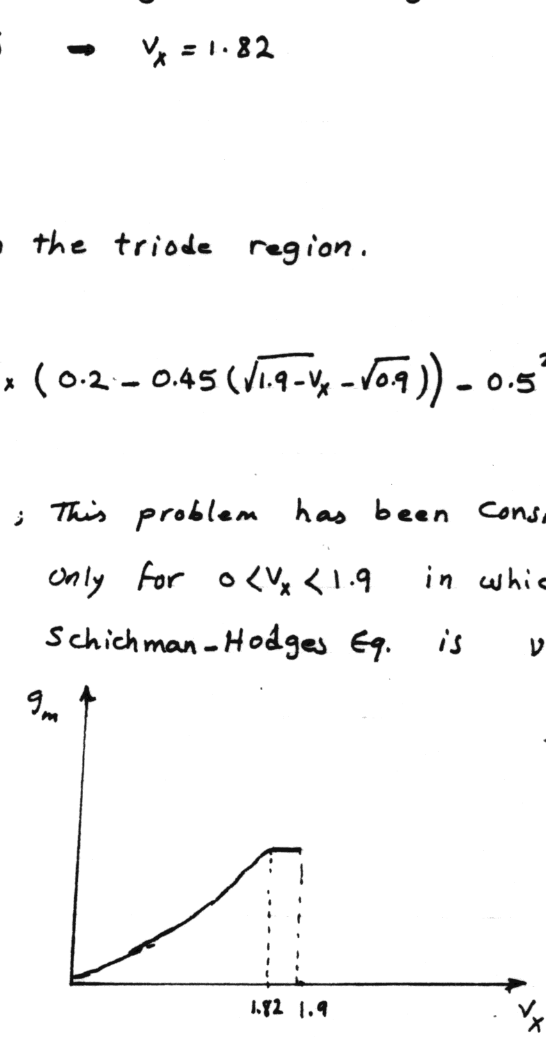
$0.2 - 0.45(\sqrt{1.9 - V_x} - \sqrt{0.9}) = 0.5 \rightarrow V_x = 1.82$

Above $V_x = 1.82$, device is in the triode region.

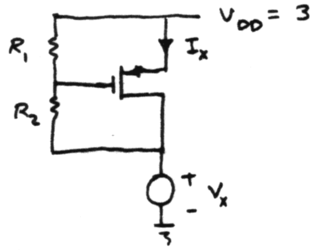
$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2 \times 0.5 \times \left(0.2 - 0.45(\sqrt{1.9 - V_x} - \sqrt{0.9})\right) - 0.5^2\right]$

$g_m = \mu_n C_{ox}\frac{W}{L}(0.5)$; This problem has been considered only for $0 < V_x < 1.9$ in which Schichman-Hodges Eq. is valid for V_{TH}.





2.6) a)



$\gamma = 0$

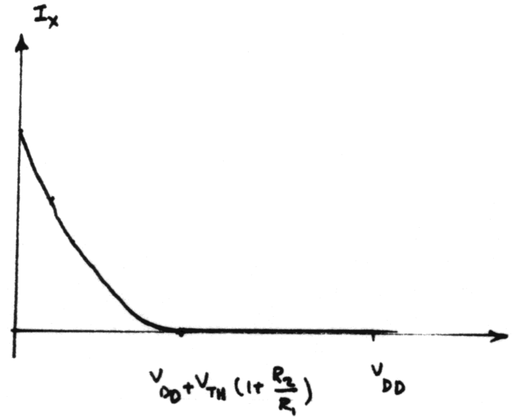
$$V_{SG} = (V_{DD} - V_X)\frac{R_1}{R_1 + R_2} \qquad V_{SD} = V_{DD} - V_X$$

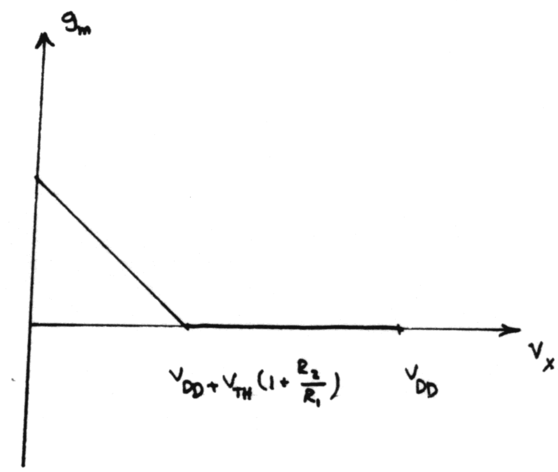
for $V_{SG} > |V_{TH}|$ Device is in the Saturation region (Device is off; otherwise)

$$(V_{DD} - V_X)\frac{R_1}{R_1+R_2} > -V_{TH}$$

$$V_X < V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) \Rightarrow I_X = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{DD} - V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]^2$$

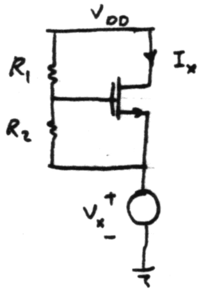
$$g_m = \mu_p C_{ox}\frac{W}{L}\left[(V_{DD} - V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]$$





If $V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ (e.g. for small value of R_1), device never turns on!

2.6) b)



$\gamma = 0$

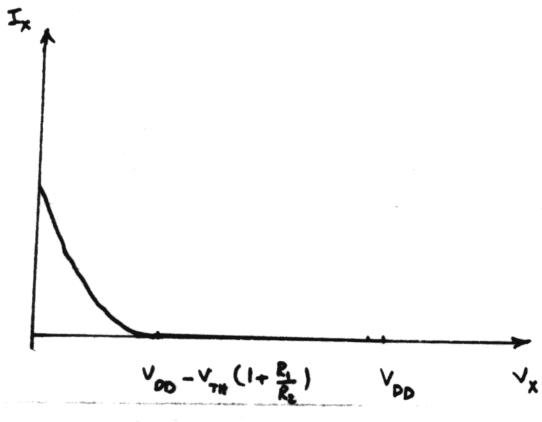
$$V_{GS} = (V_{DD} - V_x)\frac{R_2}{R_1 + R_2} \qquad V_{DS} = V_{DD} - V_x$$

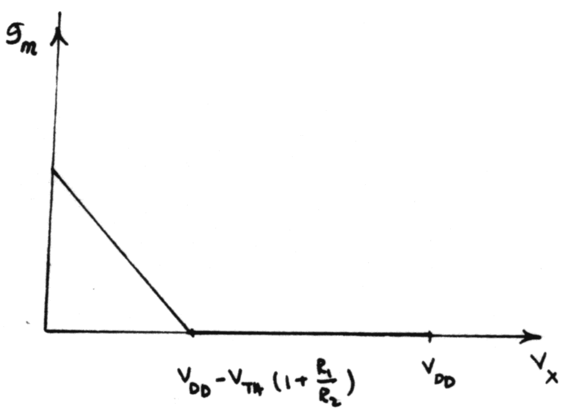
for $V_{GS} > V_{TH}$, Device is in the saturation region and

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_x)\frac{R_2}{R_1 + R_2} - V_{TH}\right]^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_x)\frac{R_2}{R_1 + R_2} - V_{TH}\right]$$

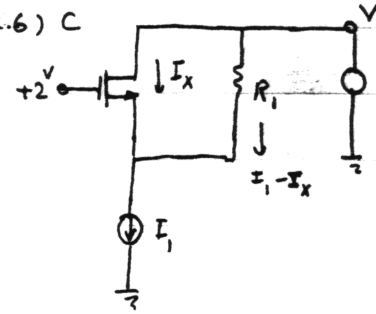
for $V_x < V_{DD} - V_{TH}\left(1 + \frac{R_1}{R_2}\right)$ (i.e. $V_{GS} > V_{TH}$)





If $V_{DD} - V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ device doesn't turn on.

2.6) C



I_x and $I_R = I_1 - I_x$ have the same polarity

So, $0 \leqslant I_x \leqslant I_1$

for $0 < V_x < 2 - V_{TH}$ (1.3) Device is in the triode.

$$V_{GS} = 2 - V_x + R_1 (I_1 - I_x) \quad , \quad V_{DS} = R_1 (I_1 - I_x)$$

$$I_x = I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) - V_{DS} \right] V_{DS}$$

$$\Rightarrow (*) \; I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_x) + 2(2 - V_{TH} - V_x) \right] \left\{ R_1 (I_1 - I_x) \right\}$$

The above equation peresents $I_x - V_x$ characteristics in this region.

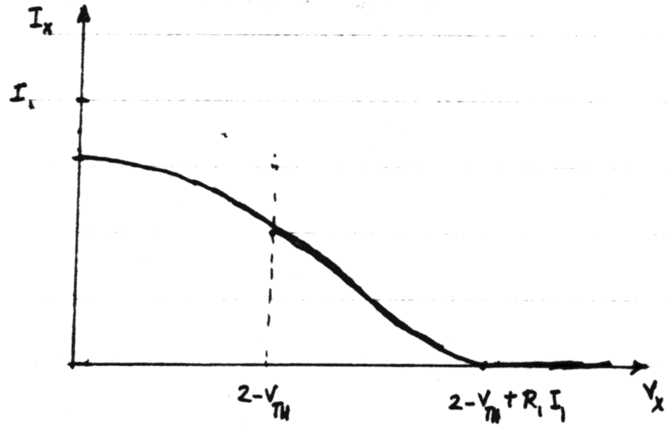
In this region $g_m = \mu_n C_{ox} V_{DS} = \mu_n C_{ox} R_1 (I_1 - I_x)$

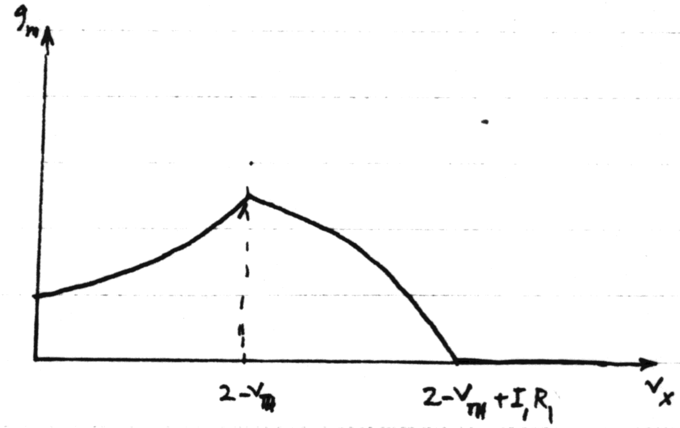
Then device enters the Saturation region; $V_{GS} = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]^2$$

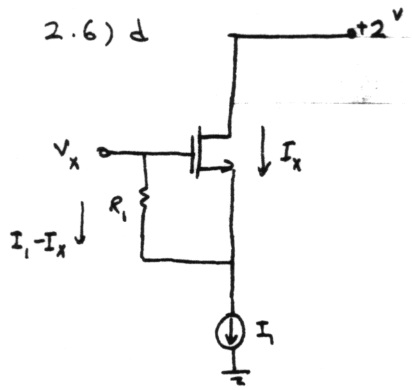
$$g_m = \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]$$

Then device turns off when $V_x = 2 - V_{TH} + R_1 I_1$





2.6) d



Assumption : $R_1 I_1 > V_{TH}$

for $0 < V_X < 2 + V_{TH}$: Device is in the saturation region

$$V_{GS} = R_1 (I_1 - I_X)$$

$$I_D = I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_X) - V_{TH} \right]^2$$

I_X is a constant that can be derived by solving the above equation.

Then device enters the triode region for $V_X > 2 + V_{TH}$

In this case $V_{GS} = R_1 (I_1 - I_X)$ $\quad V_{DS} = 2 - \left[V_X - R_1 (I_1 - I_X) \right] = 2 - V_X + R_1 (I_1 - I_X)$

$$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \left[R (I_1 - I_X) - V_{TH} \right] - 2 + V_X - R_1 (I_1 - I_X) \right] \times (2 - V_X + R_1 (I_1 - I_X))$$

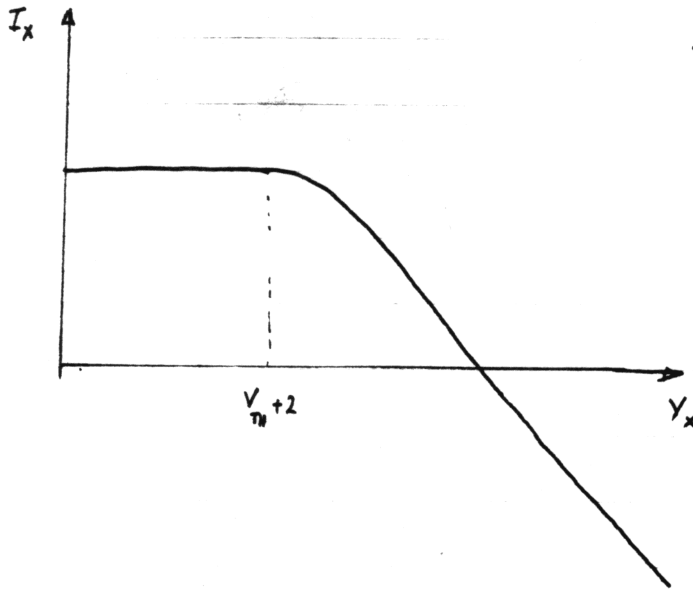
$$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\left(R_1 (I_1 - I_X) - V_{TH} \right) + (V_X - 2 - V_{TH}) \right] \left[\left(R_1 (I_1 - I_X) - V_{TH} \right) - (V_X - 2 - V_{TH}) \right]$$

(*) $$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\left(R_1 (I_1 - I_X) - V_{TH} \right)^2 - (V_X - 2 - V_{TH})^2 \right]$$

The second term shows that I_X decreases when we increase V_X

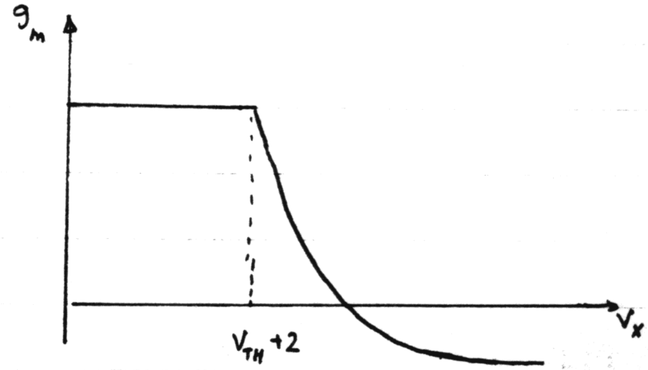
The polarity of I_X changes for higher V_X (Device still is in triode)

(*) presents $I_X - V_X$ relationship in this region.

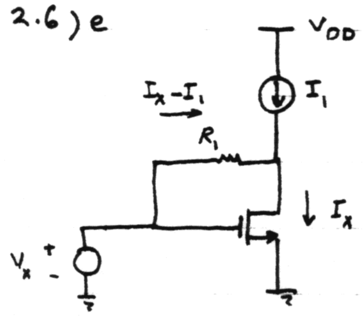


$g_m = \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_X) - V_{TH} \right]$; $V_X < 2 + V_{TH}$

$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS} = \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_X) + 2 - V_X \right]$ $V_X > 2 + V_{TH}$



2.6) e



for $0 < V_X < V_{TH}$ Device is off $I_X = 0$ $g_m = 0$

Then device turns on (in the saturation region)

$$I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_X - V_{TH})^2$$

Transistor is in the saturation until $V_{GD} = R_1 (I_X - I_1) = V_{TH}$, Then device enters the triode region. (when $I_X = I_1 + \frac{V_{TH}}{R_1}$, i.e. $V_X = V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox} \frac{W}{L}}}$)

So, $V_{TH} < V_X < V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox} \frac{W}{L}}}$ $I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_X - V_{TH})^2$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_X - V_{TH})$$

2.6) e Cont.

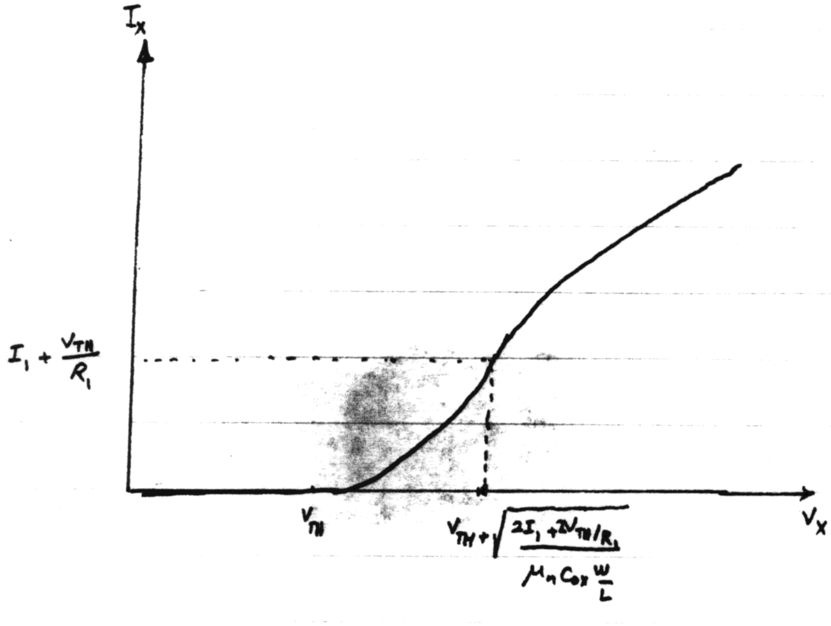
Then device enters the triode region. $V_{GS} = V_X$ $V_{DS} = V_X - R_1(I_X - I_1)$

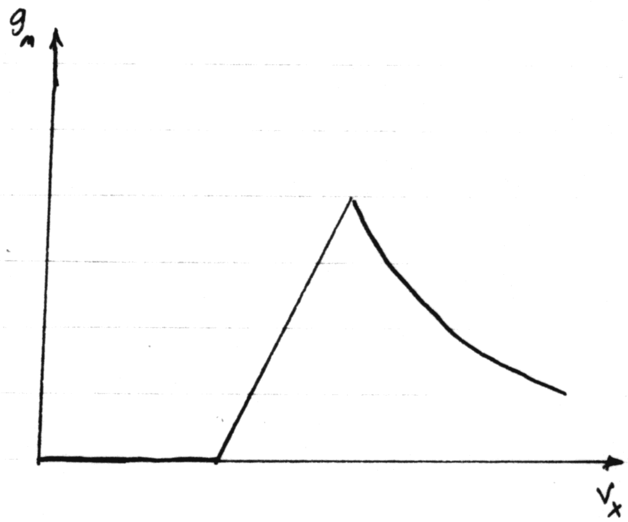
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH}) - V_{DS}\right]V_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_X - V_{TH}) - V_X + R_1(I_X - I_1)\right] \times$$
$$(V_X - R_1(I_X - I_1))$$

$$(*)\quad I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_X + R_1(I_X - I_1) - 2V_{TH}\right)\left(V_X - R_1(I_X - I_1)\right)$$

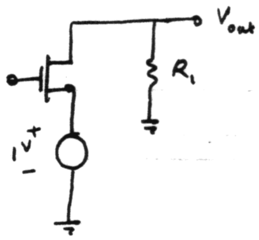
The above equation presents $I_X - V_X$ relationship in triode region.

In this region, $g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}(V_X - R_1(I_X - I_1))$





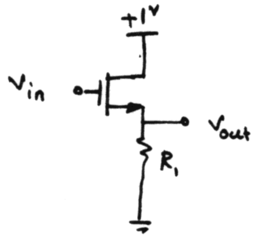
2.7) a



$\lambda = \gamma = 0$ $\qquad V_{TH} = 0.7$

Drain and Source exchange their roles.

$\Rightarrow$



for $0 < V_{in} < 0.7$ $\qquad$ device is off $\qquad V_{out} = 0$

for $0.7 < V_{in} < 1.7$ $\qquad$ device is in the saturation region

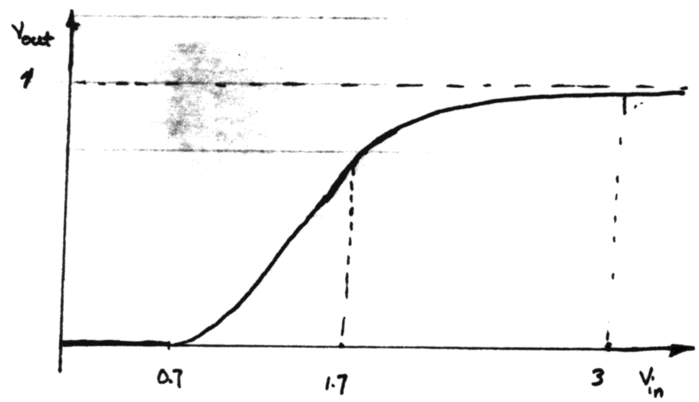
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in} - V_{out} - 0.7\right)^2 \Rightarrow \text{Input - Output relationship}$$

for $1.7 < V_{in} < 3$ $\qquad$ device is in the triode region

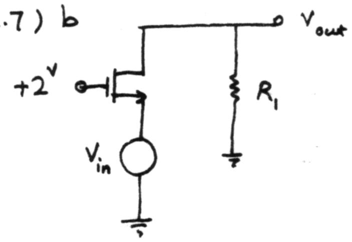
$V_{GS} = V_{in} - V_{out}$ $\qquad V_{DS} = 1 - V_{out}$

$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(V_{in} - V_{out} - 0.7\right)\left(1 - V_{out}\right) - \left(1 - V_{out}\right)^2\right]$$

$\Rightarrow$ Input - output relationship

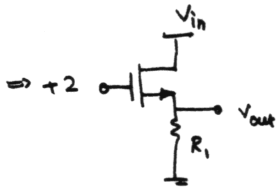


2.7) b



$\gamma = \lambda = 0 \qquad V_{TH} = 0.7$

Drain and source exchange their roles!



for $0 < V_{in} < 1.3$ device is in triode

$V_{GS} = 2 - V_{out} \qquad V_{DS} = V_{in} - V_{out}$

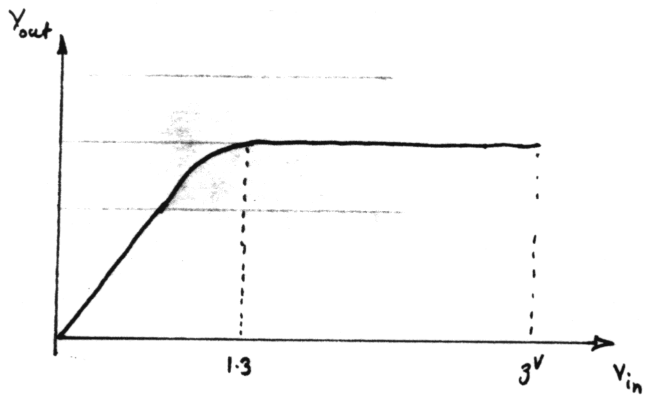
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(2 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right]$$

Input output relationship is presented by the above equation.

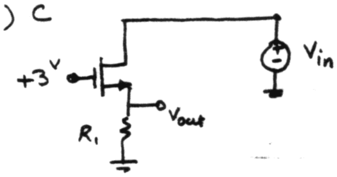
for $1.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2 - V_{out} - 0.7)^2$$

V_{out} doesn't depend on V_{in} and it is constant for $V_{in} > 1.3$



2.7) c



$\gamma = \lambda = 0 \qquad V_{TH} = 0.7$

for $0 < V_{in} < 2.3$ device is in triode

$$V_{GS} = 3 - V_{out} \qquad V_{DS} = V_{in} - V_{out}$$

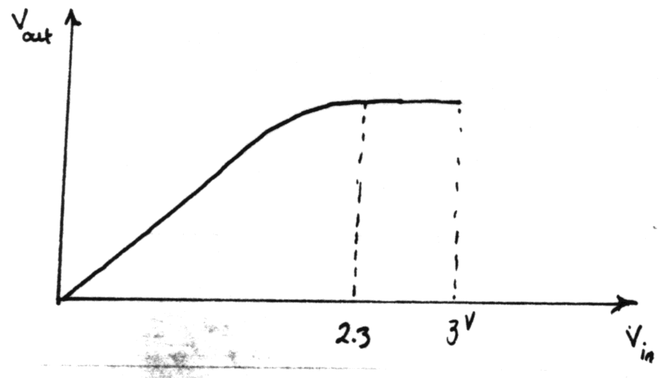
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right]$$

Input-output relationship is presented by the above equation.

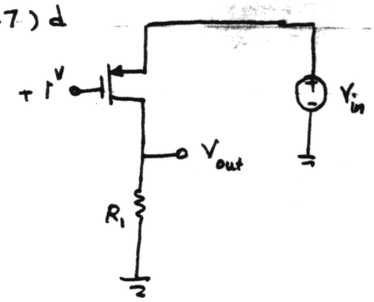
for $2.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3 - V_{out} - 0.7)^2$$

V_{out} is constant for $V_{in} > 2.3$ (It doesn't depend on V_{in})



2.7) d



$|V_{TH}| = 0.8 \qquad \gamma = \lambda = 0$

for $0 < V_{in} < 1.8$ device is off $\Rightarrow V_{out} = 0$

Then device tunns on (in sat.) and V_{out} goes up

Until $V_{out} = 1.8$, then device enters the triode region for $V_{in} > 1.8$ and $V_{out} < 1.8$

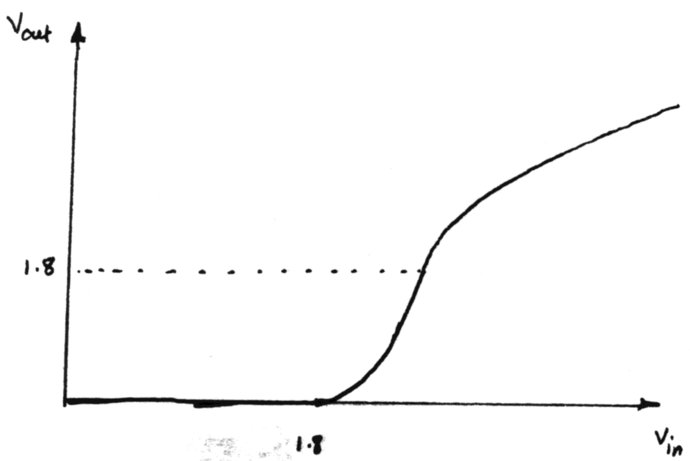
$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{in} - 1.8)^2 \Rightarrow V_{out} = \frac{1}{2}\mu_p C_{ox} R_1 \frac{W}{L}(V_{in} - 1.8)^2$$

This is good for $1.8 < V_{in} < 1.8 + \sqrt{\dfrac{2 \times 1.8^{V}}{\mu_p C_{ox}\frac{W}{L}R_1}}$

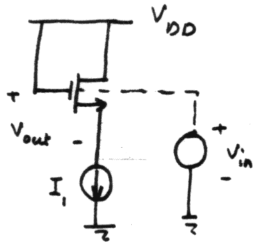
for $V_{in} > 1.8 + \sqrt{\dfrac{2 \times 1.8}{\mu_p C_{ox}\frac{W}{L}R_1}}$

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2(V_{in} - 1.8)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right].$$

Input - output relationship is presented by the above equation.



2.8) a



$V_S = V_{DD} - V_{out}$ $\quad V_B = V_{in}$, $V_{SB} = V_{DD} - V_{out} - V_{in}$

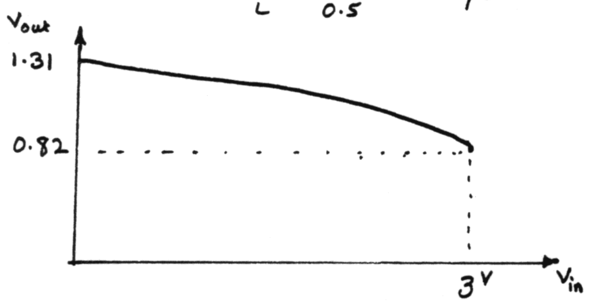
$$I_D = I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{out} - V_{TH})^2$$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right)$$

$$\Rightarrow I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{out} - V_{TH0} - \gamma\left(\sqrt{2\varphi_F + V_{DD} - V_{out} - V_{in}} - \sqrt{2\varphi_F}\right)\right)^2$$

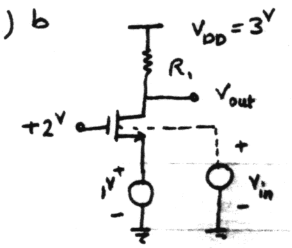
for each V_{in}, the above equation should be solved to obtain V_{out}

for $\frac{W}{L} = \frac{50}{0.5}$ $\quad I_1 = 1^{mA}$



Assumption: $2\varphi_F + V_{DD} - V_{out} - V_{in} > 0$

2.8) b



$V_{SB} = 1 - V_{in}$ $\quad V_{gs} = 1$

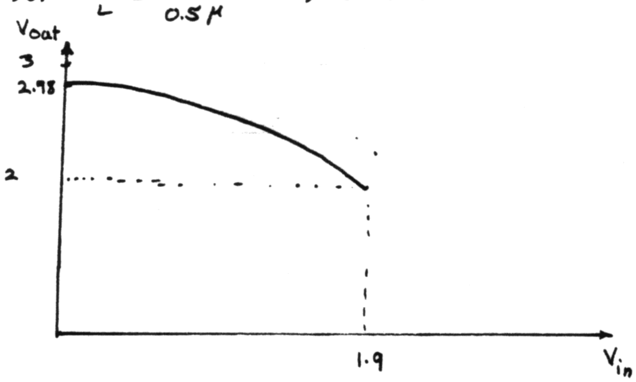
$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right)$$

$$V_{TH} = 0.7 + 0.45\left(\sqrt{1.9 - V_{in}} - \sqrt{0.9}\right)$$

Assumption: V_{in} varies from 0 to 1.9 and R_1 is small enough to guarantee that the device remains in the saturation region.

$$V_{out} = 3 - R_1 \cdot \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(0.3 - 0.45\left(\sqrt{1.9 - V_{in}} - \sqrt{0.9}\right)\right)^2$$

for $\frac{W}{L} = \frac{50\,\mu}{0.5\,\mu}$, $R = 0.2\,K$



2.8) c Drain and Source exchange their roles , ∴ $V_{THO} = 0.7$ $\gamma = 0.45$ $2\phi_F = 0.9$

Assumption : $V_{SB} > -2\phi_F$ $(V_{out} - V_{in} > -2\phi_F)$ ⇒ Device is in the Saturation.

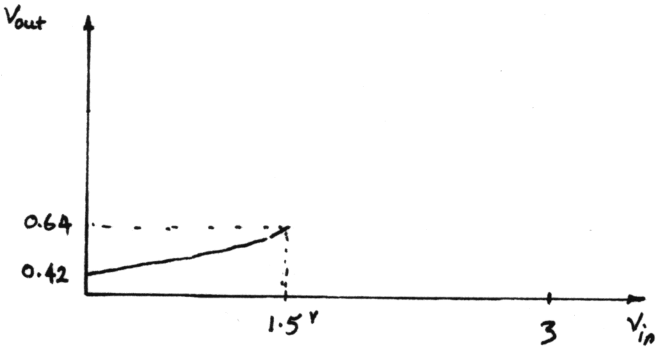
$$V_{TH} = 0.7 + 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right) \qquad V_{GS} = 2 - V_{out}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_{out} - 0.7 - 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right)\right)^2$$

$$I_D = \frac{V_{out}}{R_1}$$

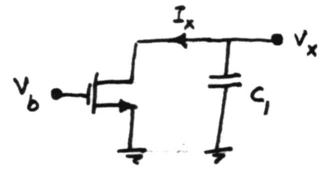
$$(*) \quad \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_{out} - 0.7 - 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right)\right)^2$$

Input – output relationship is presented by the above equation.



$\frac{W}{L} = \frac{50}{0.5}$ $R = 100\,\Omega$

2.9) a



$\gamma = \lambda = 0$ $\quad V_{TH} = 0.7$

Assume $V_b > V_{TH}$

for $V_b - 0.7 < V_x < 3$ device is in saturation

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{TH})^2$$

$$V_x = -\frac{1}{C_1}\int I_x\,dt + 3^V = 3 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{TH})^2 t$$

Then device goes into triode, for $0 < V_x < V_b - 0.7$

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_b - 0.7)V_x - V_x^2\right] = -\frac{dV_x}{dt} \times C_1$$

$$\Rightarrow \quad -dt\,\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha} = \frac{dV_x}{V_x\left[2(V_b - 0.7) - V_x\right]}$$

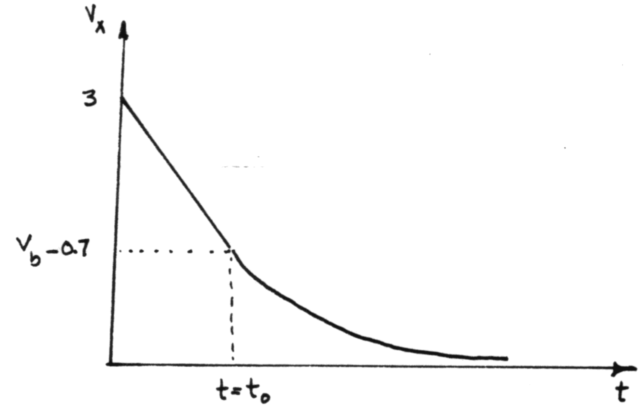
$$-\alpha\,dt = \left[\frac{1}{V_x} + \frac{1}{2(V_b - 0.7) - V_x}\right] \times \frac{1}{2(V_b - 0.7)}$$

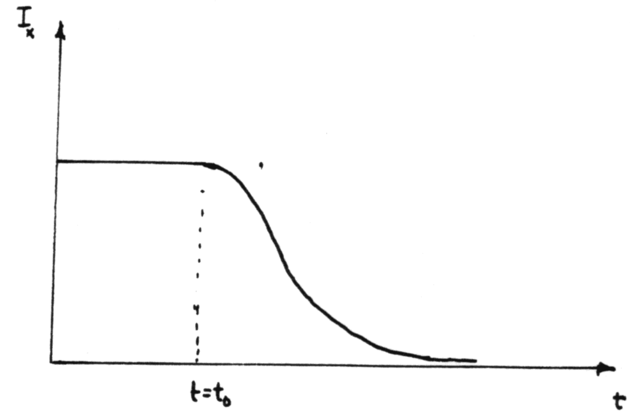
$$\Rightarrow \quad -\alpha(t - t_0) = \left[\ln\frac{V_x}{2(V_b - 0.7) - V_x}\right]\frac{1}{2(V_b - 0.7)} \qquad @\,t = t_0,\ V_x = V_b - 0.7$$

$$\Rightarrow \quad \frac{2(V_b - 0.7) - V_x}{V_x} = e^{2\alpha(V_b - 0.7)(t - t_0)}$$

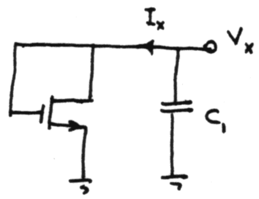
$$\Rightarrow \quad V_x = \frac{2(V_b - 0.7)}{1 + e^{2\alpha(V_b - 0.7)(t - t_0)}}$$

$$I_x = -C_1\frac{dV_x}{dt} = \frac{4\alpha C_1 (V_b - 0.7)^2 e^{2\alpha(V_b - 0.7)(t - t_0)}}{\left(1 + e^{2\alpha(V_b - 0.7)(t - t_0)}\right)^2}$$





2.9) b



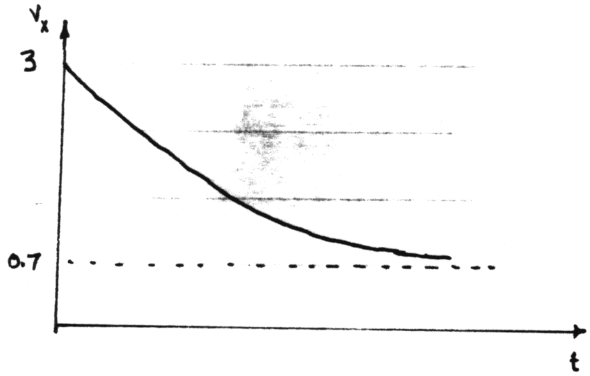
Device is always in the saturation region.

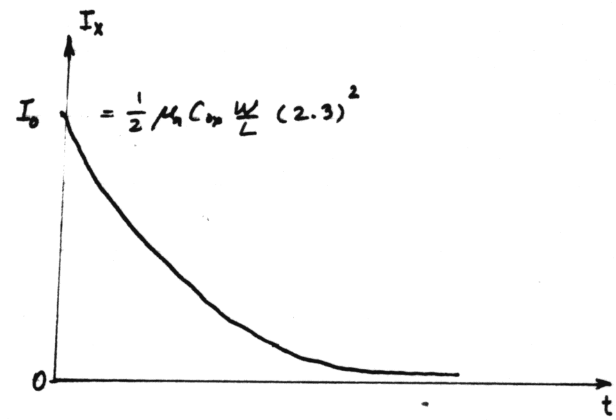
$$I_x = -C_1 \frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_x - 0.7)^2$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} \frac{1}{C_1}}_{\alpha} dt = -\frac{dV_x}{(V_x-0.7)^2} \qquad \Rightarrow \alpha t = \frac{1}{V_x - 0.7} + K$$

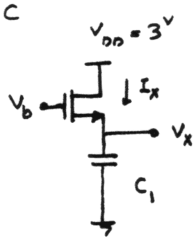
$$@\ t=0\ ,\ V_x = 3 \Rightarrow \alpha t = \frac{1}{V_x-0.7} - \frac{1}{2.3} \Rightarrow V_x = 0.7 + \frac{1}{\alpha t + 1/2.3}$$

$$I_x = -C_1 \frac{dV_x}{dt} = \frac{\alpha C_1}{\left(\alpha t + \frac{1}{2.3}\right)^2}$$



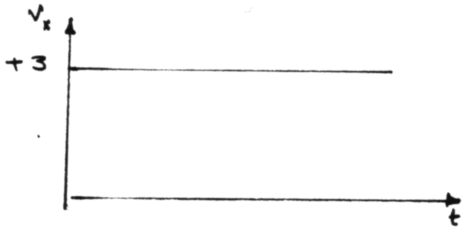


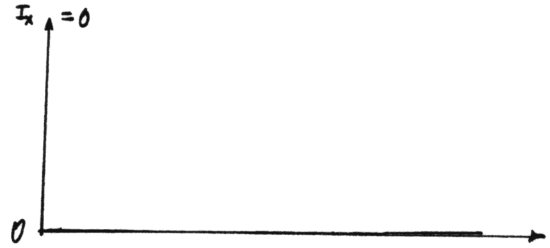
2.9) c



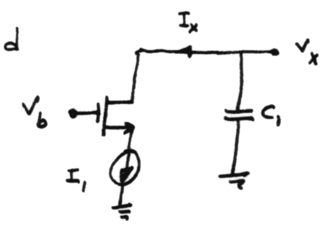
@ $t=0$ $\quad V_x = 3 \;,\; V_{DD} = 3^V \Rightarrow V_{DS} = 0 \Rightarrow I_x = 0$

And the Circuit remains in this state





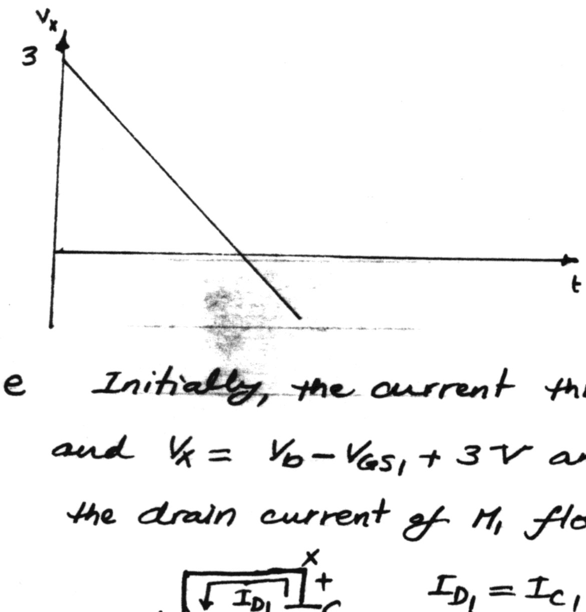
2.9) d

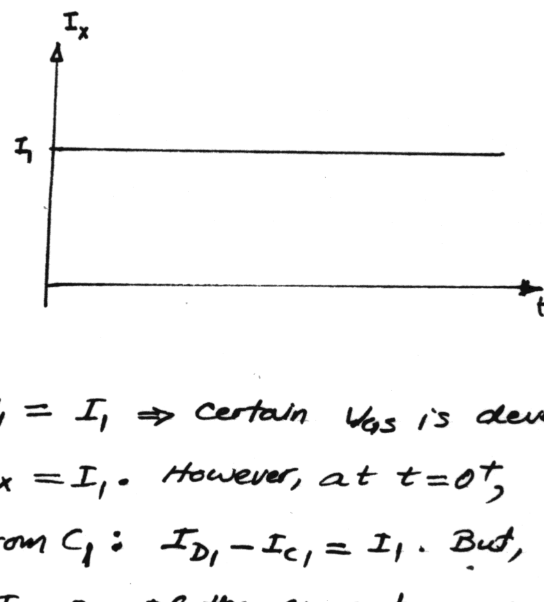


$I_x = I_1$

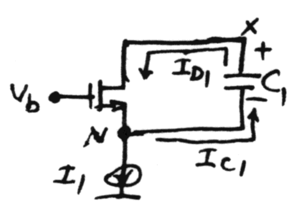
$$-C_1 \frac{dV_x}{dt} = I_1 \Rightarrow V_x = 3 - \frac{I_1}{C} t$$

Infact these Equations are valid until I_1 is no longer an ideal current Source.





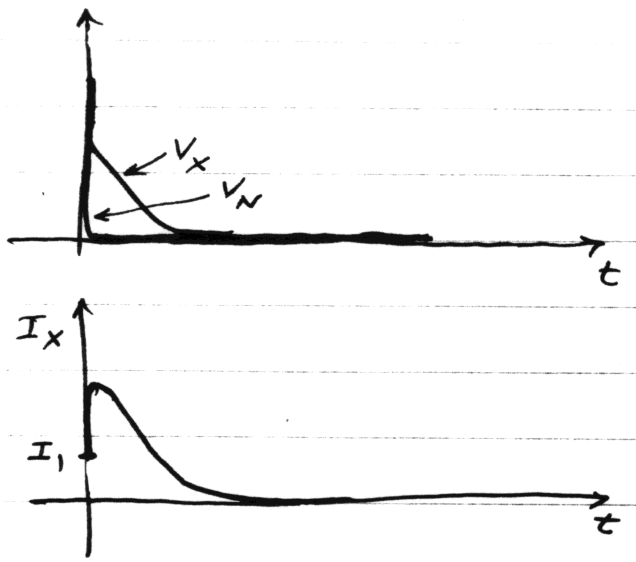
2.9) e Initially, the current thru $M_1 = I_1 \Rightarrow$ certain V_{GS} is developed and $V_X = V_b - V_{GS_1} + 3V$ and $I_X = I_1$. However, at $t = 0^+$, the drain current of M_1 flows from C_1: $I_{D_1} - I_{C_1} = I_1$. But, $I_{D_1} = I_{C_1} \Rightarrow I_1 = 0$. If the current source is ideal, V_N jumps to $-\infty$ (actually about 0.6V below 0, where the S-B diode turns on.)



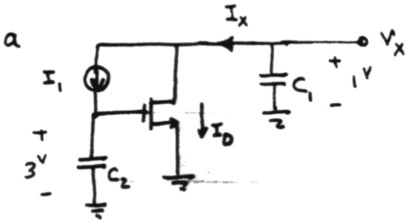
If I_1 is not ideal, V_N jumps to zero and C_1 discharges

2.9) e (cnt'd)

through M_1:



2.10) a



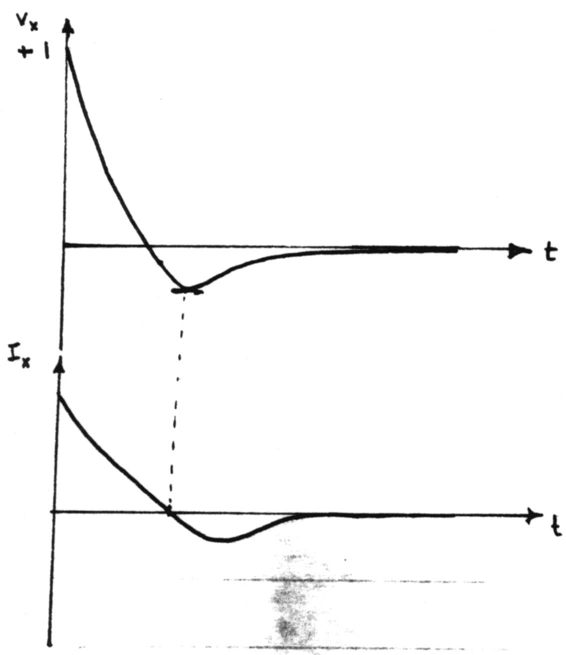
$V_G = 3 + \frac{I_1 t}{C}$

This circuit settles at $t=\infty$, when $V_G = \infty$

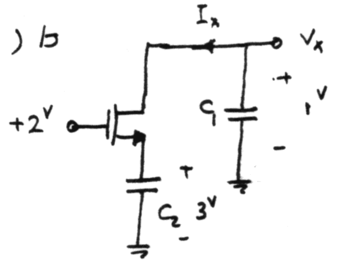
$I_D = -I_1$, $V_{DS} = 0$ (Actually, Drain and Source exchange their roles after a specific time at which $I_x = I_1$ and afterward V_x becomes negative) However, transistor always operates in the triode region.

$$I_x = I_1 + \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[2\left(3 + \frac{I_1}{C_2} t - 0.7\right) V_x - V_x^2 \right] = -C_1 \frac{dV_x}{dt}$$

The values of V_x can be obtained by numerical methods



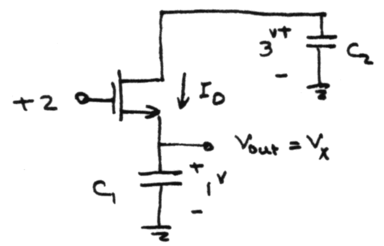
2.10) b



Drain and source exchange their roles.

$(\gamma = \lambda = 0)$ $\quad V_{TH} = 0.7$

⇒



$\int I_D\, dt = q \qquad V_x = 1 + \frac{q}{C_1}\ , \quad V_D = V_{C_2} = 3 - \frac{q}{C_2}$

V_x goes up until transistor turns off when $V_x = 1.3$

Assumption: Transistor is in saturation.

This assumption is correct if: $\quad V_D = 3 - \frac{q}{C_2} > 1.3 \quad (2 - 0.7)$

$$V_x(\infty) = 1 + \frac{q(\infty)}{C_1} = 1.3 \qquad V_D(\infty) = 3 - \frac{q(\infty)}{C_2} = 3 - 0.3\frac{C_1}{C_2} > 1.3$$

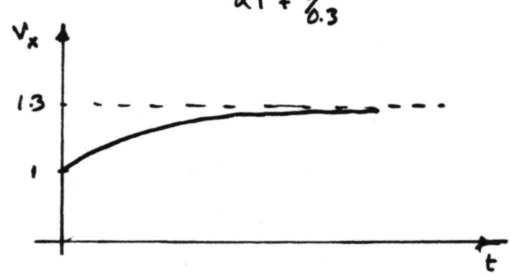
$$0.3\frac{C_1}{C_2} < 1.7 \qquad C_1 < 5.67\, C_2$$

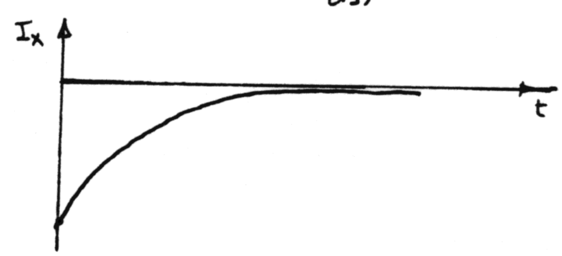
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - 1 - \frac{q}{C_1} - 0.7\right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{1}{C_1}}_{\alpha} dt = \frac{dq/C_1}{(0.3 - q/C_1)^2} \Rightarrow \alpha t = \frac{1}{0.3 - q/C_1} + K \quad (t=0, q=0)$$

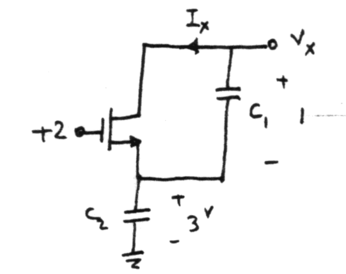
$$\Rightarrow \alpha t = \frac{1}{0.3 - q/C_1} - \frac{1}{0.3} \Rightarrow \frac{q}{C_1} = 0.3 - \frac{1}{\alpha t + \frac{1}{0.3}} \qquad V_x = 1 + \frac{q}{C_1}$$

$$\Rightarrow V_x = 1.3 - \frac{1}{\alpha t + 1/0.3} \qquad I_x = -C_1\frac{dV_x}{dt} = \frac{-\alpha C_1}{(\alpha t + 1/0.3)^2}$$





2.10) c

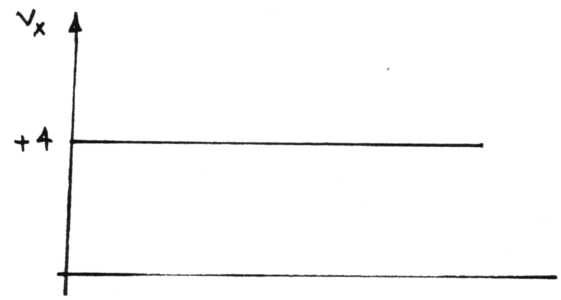


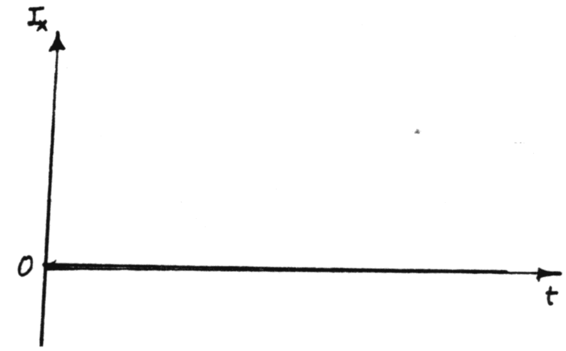
At $t=0$ $\quad V_G = 2 \quad V_S = 3 \quad V_D = 4$

Device is off and doesn't turn on.

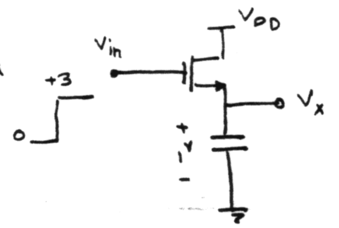
The Circuit remains in this state.

So, $V_X = 4 \quad I_X = 0$





2.11) a



$\gamma = \lambda = 0$ $\quad V_{TH} = 0.7$

At $t = 0^+$, device turns on (in sat) and starts charging the capacitor, until device turns off

when; $V_x = V_{in} - V_{TH} = 3 - 0.7 = 2.3$

$$I_c = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3 - V_x)^2 \quad ; V_{GS} = 3 - V_x - 0.7$$

$$I_c = C_1 \frac{dV_x}{dt}$$

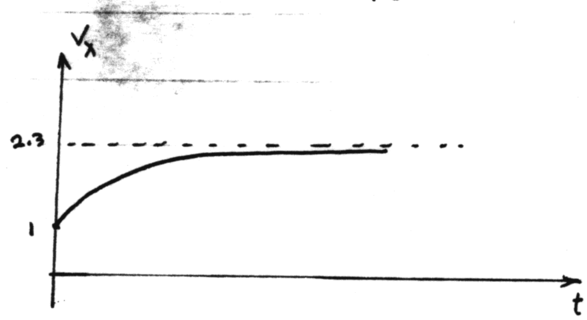
$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L} \times \frac{1}{C_1}}_{\alpha}(2.3 - V_x)^2 = \frac{dV_x}{dt}$$

$$\Rightarrow \alpha\, dt = \frac{dV_x}{(2.3 - V_x)^2} \quad \Rightarrow \alpha t + K_0 = \frac{1}{2.3 - V_x}$$

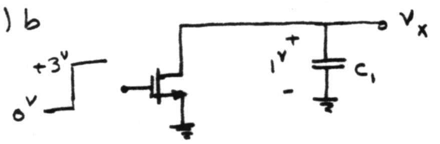
$(t = 0, V_x = 1)$ $\quad \alpha \times 0 + K_0 = \frac{1}{2.3 - 1} \quad \Rightarrow K_0 = 1/1.3$

$$\Rightarrow \frac{1}{1.3} + \alpha t = \frac{1}{2.3 - V_x} \quad \Rightarrow 2.3 - V_x = \frac{1}{\alpha t + 1/1.3}$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/1.3}$$



2.11) b



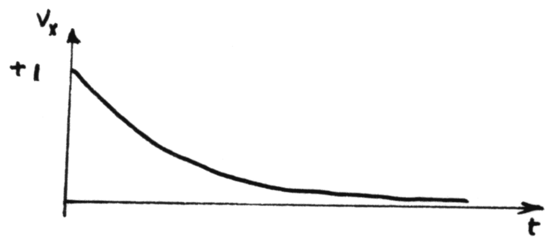
Transistor turns on at $t=0$, and discharges C_1 until $V_X = 0$, (device always operats in triode)

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_X - V_X^2\right] = -C_1\frac{dV_X}{dt}$$

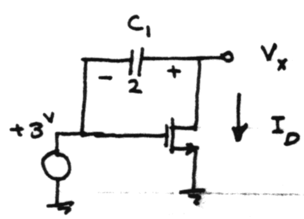
$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha}\left[4.6V_X - V_X^2\right] = -\frac{dV_X}{dt} \Rightarrow -\alpha\,dt = \frac{dV_X}{V_X(4.6-V_X)}$$

$$\Rightarrow -\alpha t = \left(\frac{1}{V_X} + \frac{1}{4.6-V_X}\right)\frac{1}{4.6} + K \quad , \; @t=0 \;, V_X = 1$$

$$\frac{1}{3.6}e^{-\alpha t} = \frac{V_X}{4.6-V_X} \longrightarrow V_X = \frac{4.6}{1+3.6\,e^{4.6\alpha t}}$$



2.11) C



At $t=0^+$, $V_X = 5$, device is in Saturation region

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-0.7)^2$, V_X decreases until

$V_X = 2.3$ at $t=t_0$, then device enters triode region

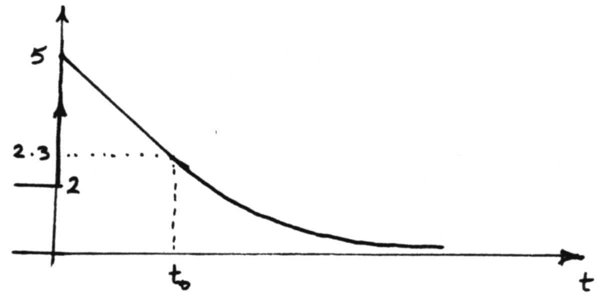
for $t < t_0$ $(V_X > 2.3)$ $\quad V_X = 5 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3)^2\, t/C_1$

for $t > t_0$ $\quad I_D = -C_1\frac{dV_X}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_X - V_X^2\right]$

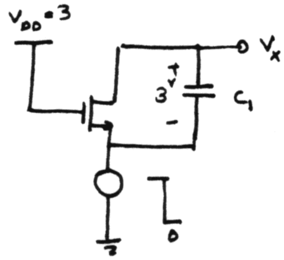
$$\Rightarrow \frac{dV_X}{V_X(4.6-V_X)} = \underbrace{-\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt$$

2.11) c, Cont. $\quad -\alpha(t-t_0) = \left[\ln \frac{V_x}{4.6-V_x}\right] \cdot \frac{1}{4.6}$ $\quad t=t_0, \; V_x = 2.3$

$$\Rightarrow V_x = \frac{4.6}{1+e^{4.6\alpha(t-t_0)}}$$



2.11) d



At $t=0^+$, $V_x = 3$ device is in saturation

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-0.7)^2$, V_x decreases until

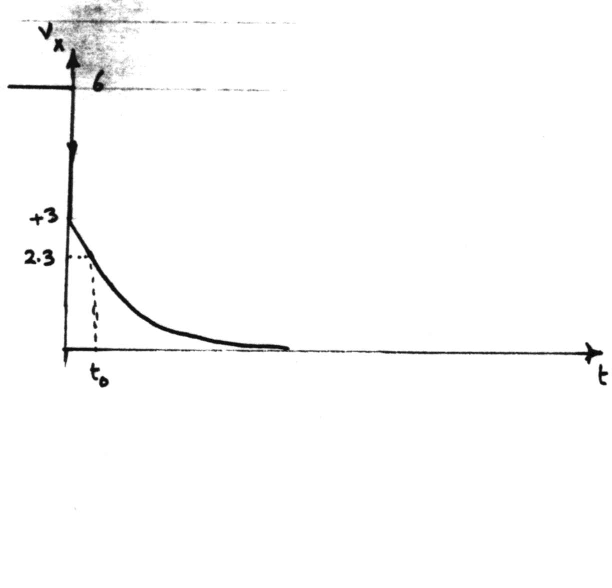
$V_x = 2.3$ at $t=t_0$, then device enters triode region.

for $t<t_0$ $\quad V_x = 3 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3)^2\frac{t}{C_1}$; $\quad 2.3 < V_x < 3$

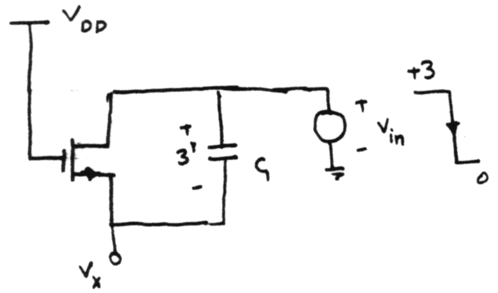
for $t>t_0$ $\quad I_D = -C_1\frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_x - V_x^2\right]$

$$\frac{dV_x}{V_x(4.6-V_x)} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{1}{C_1}dt \qquad ,(t=t_0, \; V_x = 2.3)$$

$$-\alpha(t-t_0) = \left[\ln\frac{V_x}{4.6-V_x}\right]\frac{1}{4.6} \Rightarrow V_x = \frac{4.6}{1+e^{4.6\alpha(t-t_0)}}$$

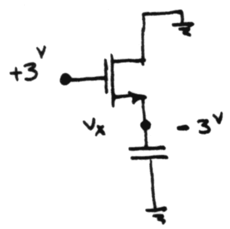


2.12) a)



Device is in the triode region.

$t \geq 0^+$



$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\,(2.3 - V_x)(-V_x) - V_x^2\right]$$

$$\begin{cases} V_{GS} = 3 - V_x \\ V_{DS} = -V_x \end{cases}$$

$$I_D = C_1 \frac{dV_x}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L} \times \frac{1}{C_1}}_{\alpha}\left[V_x^2 - 4.6\,V_x\right] = \frac{dV_x}{dt}$$

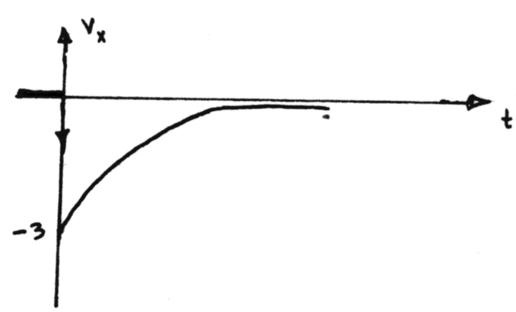
$$\Rightarrow \alpha\, dt = \frac{dV_x}{V_x^2 - 4.6\,V_x} = dV_x\left(\frac{1}{V_x - 4.6} + \frac{-1}{V_x}\right) \times \frac{1}{4.6}$$

$$\Rightarrow 4.6\,\alpha t + K_0 = \ln\left(\frac{V_x - 4.6}{V_x}\right) \qquad ; \qquad V_x(0^+) = -3$$

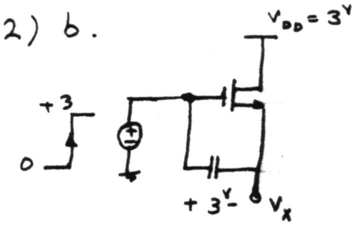
$$\Rightarrow K_0 = \ln\frac{7.6}{3} \qquad \Rightarrow \frac{V_x - 4.6}{V_x} = \frac{7.6}{3}e^{4.6\alpha t}$$

$$\Rightarrow \frac{4.6}{V_x} = 1 - \frac{7.6}{3}e^{4.6\alpha t}$$

$$\Rightarrow V_x = \frac{-4.6}{\frac{7.6}{3}e^{4.6\alpha t} - 1}$$

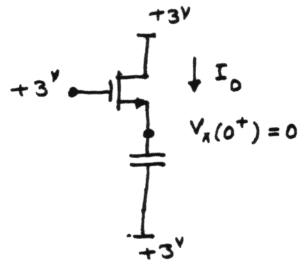


2.12) b.



Device is in saturation region

$t = 0^+$

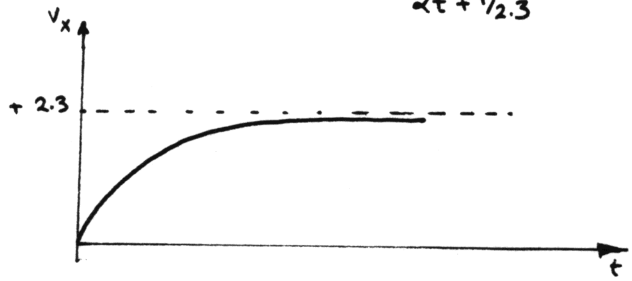


$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\,(3 - V_x - 0.7)^2 = C_1\frac{dV_x}{dt}$$

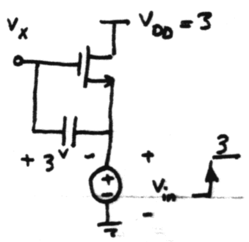
$$\frac{dV_x}{(2.3 - V_x)^2} = \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt$$

$$\Rightarrow \frac{1}{2.3 - V_x} = \alpha t + K \qquad (t = 0,\ V_x = 0) \Rightarrow \frac{1}{2.3 - V_x} - \frac{1}{2.3} = \alpha t$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/2.3}$$

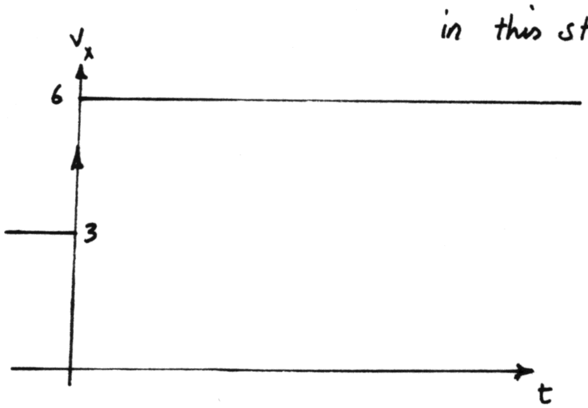


2.12) c

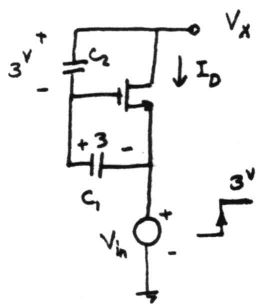


At $t = 0^+$ $V_D = 3$ $V_S = 3$ $V_G = 6$

So, $V_{DS} = 0$ and $I_C = I_D = 0$ And circuit remains in this state. $V_x(0^-) = 3$, $V_x(t) = 6$



2.12) d



Assume that the device remains in the saturation region until it turns off when $V_{gs} = 0.7$

$$V_{c_1} = V_{gs} = 3 - \frac{1}{C_1}\int I_D\,dt \qquad V_{c_2} = V_{dg} = 3 - \frac{1}{C_2}\int I_D\,dt$$

This assumption is correct if $V_{dg} > -0.7$ when $V_{gs} = 0.7$

$$\int I_D\,dt = q(t) \qquad V_{gs} = 3 - \frac{q}{C_1} = 0.7 \Rightarrow \frac{q}{C_1} = 2.3 \qquad V_{dg} = 3 - \frac{q}{C_2} > -0.7$$

$$\Rightarrow \frac{q}{C_2} < 3.7 \qquad 2.3\,\frac{C_1}{C_2} < 3.7 \qquad \Rightarrow C_1 < 1.61\,C_2$$

With this assumption,

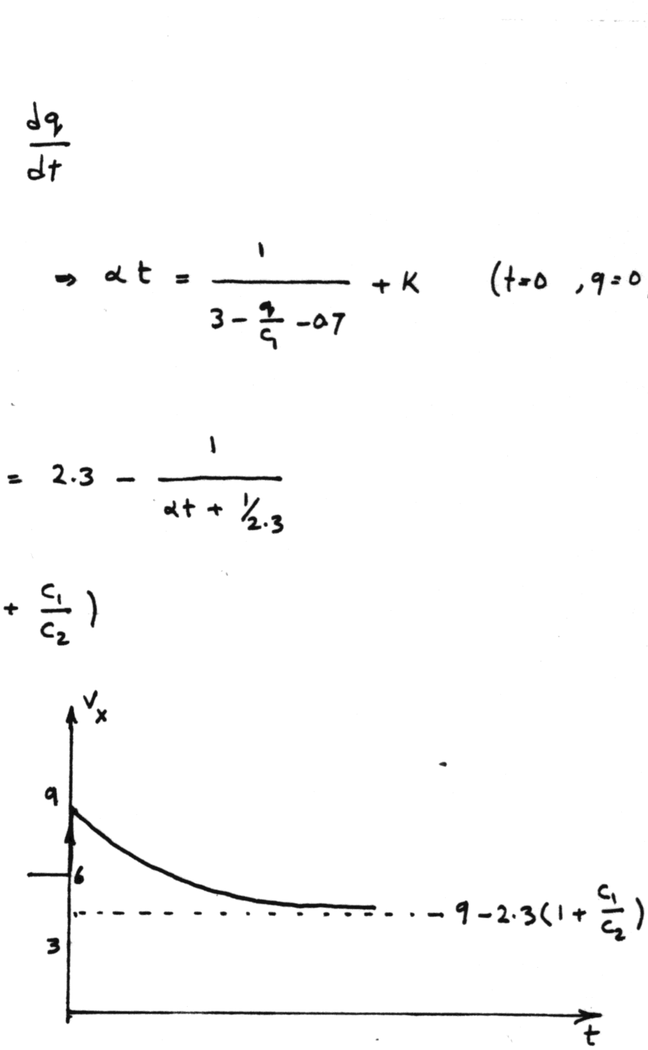
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - \frac{q}{C_1} - 0.7\right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt = \frac{dq/C_1}{\left(3 - \frac{q}{C_1} - 0.7\right)^2} \Rightarrow \alpha t = \frac{1}{3 - \frac{q}{C_1} - 0.7} + K \qquad (t=0,\ q=0)$$

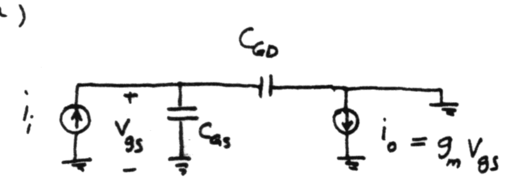
$$\Rightarrow \alpha t = \frac{1}{2.3 - \frac{q}{C_1}} - \frac{1}{2.3} \qquad \Rightarrow \frac{q}{C_1} = 2.3 - \frac{1}{\alpha t + 1/2.3}$$

$$V_x = 3 + 3 - \frac{q}{C_1} + 3 - \frac{q}{C_2} = 9 - \frac{q}{C_1}\left(1 + \frac{C_1}{C_2}\right)$$

$$V_x(t) = 9 - \left(1 + \frac{C_1}{C_2}\right)\frac{2.3\,\alpha t}{\alpha t + 1/2.3}$$



2.13) a)



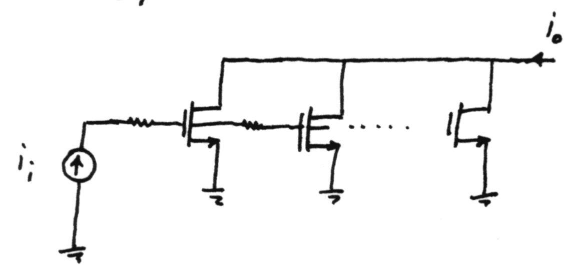
$$i_i = (C_{GS} + C_{GD})\, s\, V_{gs}$$
$$i_o = g_m V_{gs}$$

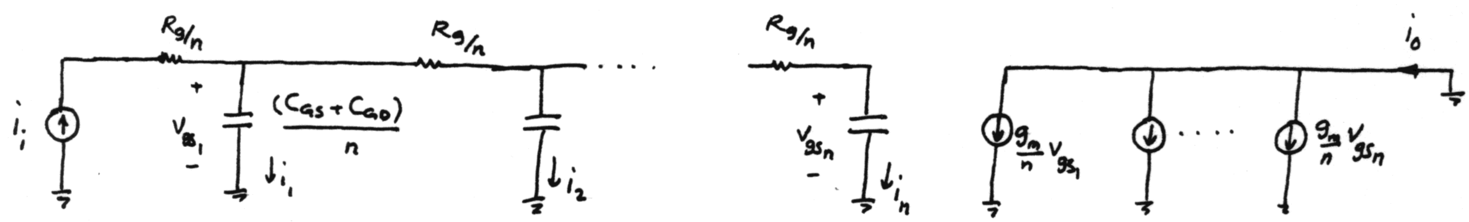
$$\beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{GS} + C_{GD})s} \; ; \quad |\beta| = 1 \Rightarrow \frac{g_m}{(C_{GS} + C_{GD})\omega_T} = 1$$

$$\Rightarrow \omega_T = \frac{g_m}{(C_{GS} + C_{GD})} \rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$

Approximation : $g_m v_{gs}$ is the output current.

b)





$$i_K = \frac{1}{n} (C_{GS} + C_{GD})\, s\, V_{gsK} \qquad K = 1 \cdots n$$

$$(*) \quad i_i = i_1 + i_2 + \cdots + i_n = \frac{1}{n} (C_{GS} + C_{GD})\, s\, (V_{gs_1} + V_{gs2} + \cdots + V_{gsn})$$

$$(**) \quad i_o = \frac{g_m}{n} V_{gs1} + \cdots + \frac{g_m}{n} V_{gsn} = \frac{g_m}{n} (V_{gs1} + V_{gs2} + \cdots + V_{gsn})$$

$$(*), (**) \Rightarrow \beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{GD} + C_{GS})s} \; ; \quad |\beta| = 1 \Rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$

c) $$f_T = \frac{g_m}{2\pi\,(C_{GS} + C_{GD})}$$

$$g_m = \mu\, C_{ox}\, \frac{W}{L}\, (V_{GS} - V_{TH})$$

$$C_{GS} + C_{GD} \simeq C_{ox}\, WL$$

$$\Longrightarrow \quad f_T = \frac{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}{2\pi\, C_{ox}\, WL} \simeq \frac{\mu}{2\pi}\, \frac{(V_{GS} - V_{TH})}{L^2}$$

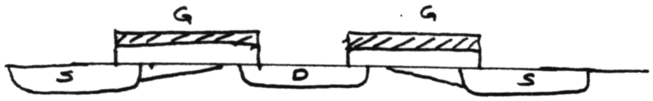
2.14)

$$f_T = \frac{g_m}{2\pi\,(C_{GS} + C_{GD})} \quad ; \quad g_m = \frac{I_D}{\xi V_T}$$

In the subthreshold $C_{GS} = C_{GD} = W\, C_{ov}$ (Fig 2.?.33)

So, $$f_T = \frac{I_D / \xi V_T}{4\pi\, W C_{ov}} = \frac{I_D}{4\pi \xi V_T\, W L_D C_{ox}}$$

2.15)



$\left\{ \begin{array}{l} C_{j0} = 0.56\,e{-3}\ F/m^2 \\ m_j = 0.6 \end{array} \right.$ $\quad C_{jsw0} = 0.35\,e{-11}\ F/m \quad m_{jsw} = 0.2$

$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{2\phi_F}\right)^m}$$

$$C_{DB} = \frac{W}{2} E C_j + 2\left(\frac{W}{2} + E\right) C_{jsw}$$

$$C_{SB} = 2\left[\frac{W}{2} E C_j' + 2\left(\frac{W}{2} + E\right) C_{jsw}'\right]$$

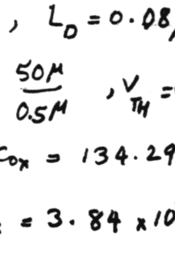
$$C_{GD} = 2\left(\frac{W}{2} C_{ov}\right) \qquad C_{ov} = L_D C_{ox}$$

$$C_{GS} = \frac{2WLC_{ox}}{3} + W C_{ov}$$

$$C_{GB} = (WLC_{ox})C_d / (WLC_{ox} + C_d) \;;\; C_d = WL\sqrt{q\epsilon_{si} N_{sub}/2\phi_F}$$

$W = 50\mu \qquad L = 0.5\mu \qquad E = 1.5\mu$

3



$$I = \frac{1}{2}\mu_n C_{ox}\frac{W}{L - 2L_D}(V_{GS} - V_{TH})^2 \;,\; 1^{mA} = \frac{1}{2} \times 0.13429 \times \frac{50}{0.5 - 0.16}(V_{GS} - 0.7)^2$$

$$V_{GS} = 1.0182 \qquad g_m = \frac{2I_D}{V_{GS} - V_{TH}} = 6.285\ mA/V \;,\; V_{DB} = 1.0182$$

$\lambda = 0 \;,\; L_D = 0.08\,\mu m$

$\frac{W}{L} = \frac{50\mu}{0.5\mu} \;,\; V_{TH} = 0.7$

$\mu_n C_{ox} = 134.29\ \mu A/V^2$

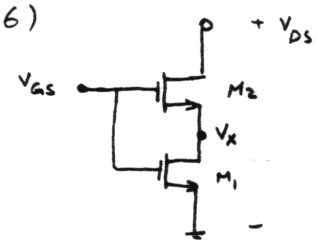
$C_{ox} = 3.84 \times 10^{-3}\ F/m^2$

$C_{GD} = 15.4\ fF \qquad C_{GS} = 79.36\ fF$

$C_{SB} = 42.4\ fF \qquad C_{DB} = 13.5\ fF$

$$f_T = \frac{g_m}{2\pi(C_{GD} + C_{GS})} = 10.6\ GHz$$

2.16)



CASE I , M_1 : Triode M_2 : Triode

$$V_{OD1} = V_{GS} - V_{TH} \qquad V_{OD2} = V_{GS} - V_X - V_{TH}$$

$$V_{DS1} = V_X \qquad V_{DS2} = V_{DS} - V_X$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] \qquad (*)$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH}-V_X)(V_{DS}-V_X) - (V_{DS}-V_X)^2\right]$$

$$I_{D1} = I_{D2} \Rightarrow 2(V_{GS}-V_{TH})V_X - V_X^2 = 2(V_{GS}-V_{TH})V_{DS} + 2V_X^2 - 2V_X(V_{GS}-V_{TH}) - \cancel{2V_X V_{DS}} - V_{DS}^2 - V_X^2 + \cancel{2V_X V_{DS}}$$

$$\Rightarrow 2\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] = 2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2 \qquad (**)$$

$$(*),(**) \Rightarrow I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} \times \frac{1}{2}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right] \quad \left(\frac{W}{2L} \text{ in Triode}\right)$$

CASE II , M_1 : Triode , M_2 : Sat

$$I_{D1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] \qquad (*)$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_X-V_{TH})^2$$

$$I_{D1} = I_{D2} \Rightarrow V_X^2 - 2V_X(V_{GS}-V_{TH}) + (V_{GS}-V_{TH})^2 = 2(V_{GS}-V_{TH})V_X - V_X^2$$

$$\Rightarrow (V_{GS}-V_{TH})^2 = 2\left[2(V_{GS}-V_{TH})V_X - V_X^2\right] \qquad (**)$$

2.16) Cont. $(*), (**) \Rightarrow I_{D_1} = I_{D_2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \times \frac{1}{2} (V_{GS} - V_{TH})^2 \quad \left(\frac{W}{2L} \text{ in Sat}\right)$

Note That M_1 is always in triode, because V_{OD2} is always positive

i.e. $V_{GS2} - V_{TH} > 0 \Rightarrow V_{GS} - V_X - V_{TH} > 0 \Rightarrow V_{GS} - V_{TH} > V_X$

$\Rightarrow V_{GS_1} - V_{TH} > V_{DS_1} \Rightarrow M_1$ is in the triode region.

Saturation – triode transition edge of M_2:

We show that the transition point the saturation and triode region of the equivalent transitor is the same as that of M_2.

$V_{OD2} = V_{GS} - V_X - V_{TH}$ $\quad V_{DS2} = V_{DS} - V_X$

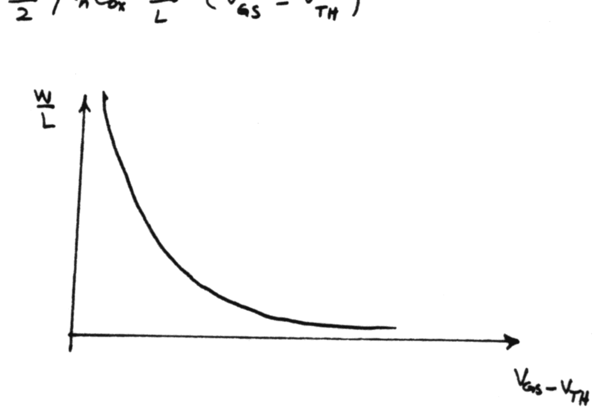
for $V_{OD2} > V_{DS2}$; M_2 is in the triode region, i.e. $V_{GS} - V_{TH} > V_{DS}$

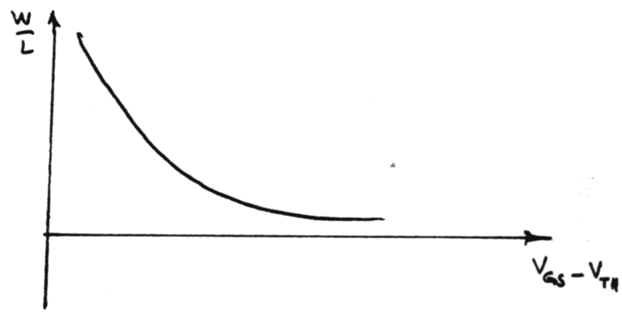
It means that when M_2 is in the saturation, then the equivalent transistor is in the saturation, and vice versa.

2.17)

In Saturation region, $I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$

$$\Rightarrow \frac{W}{L} = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

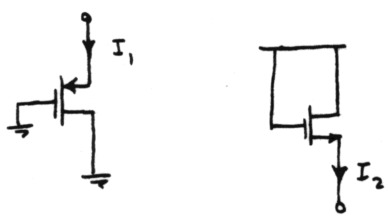




$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})}$$

2.18)



These structures cannot operate as current sources, because their currents strongly depend on source voltages, but an ideal current source should provide a constant current, independent of its voltage.

2.19) From Eq. (2.1) we know that $V_{TH} = \varphi_{MS} + 2\varphi_F + \frac{Q_{dep}}{C_{ox}}$, where

φ_{MS} and φ_F are constant values, so any changes in V_{TH}

come from the third term, in fact $\Delta V_{TH} = \frac{\Delta Q_{dep}}{C_{ox}}$ and

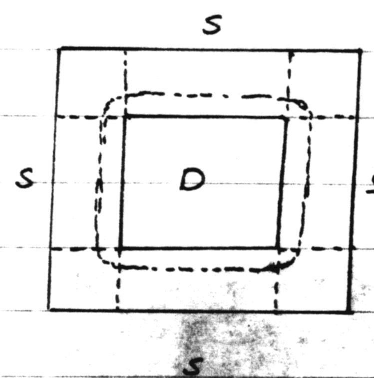
From Eq (2.22), we have $\Delta V_{TH} = \gamma\left(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right)$ (in fact,

this is definition of γ). from pn junction theory we know

that Q_{dep} is proportional to $\sqrt{N_{sub}}$, so γ is directly

proportional to $\sqrt{N_{sub}}$ and inversely proportional to C_{ox}.

2.20)



This structure operates as a traditional device does, in fact if we neglect edges we have four Mosfets in parallel, where the aspect ratio of each is $\frac{W}{L}$.
So the overal aspect ratio is almost $\frac{4W}{L}$

Drain junction capacitance: $C_{DB} = W^2 C_J + 4W C_{JSW}$

Drain junction capacitance of devices shown in fig 2.32 a, b for the aspect ratio of $\frac{4W}{L}$

$$C_{DB(a)} = 4WE\,C_J + (8W + 2E)\,C_{jsw}$$

$$C_{DB(b)} = 2WE\,C_J + (4W + 2E)\,C_{jsw}$$

The value of side wall capacitance in the ring structure is less than that in folded and traditional structures, but the bottom capacitance of ring structure is higher than that of the other two structures. (for $W > 4E$)

2.21) We first check the terminals of the device with a multimeter in order to find BS or BD junctions. There are 12 experiments in total of which two lead to conduction and remaining ones show no conduction. If we find one of those two conductions then we are done. Finding B and S (or D), we need to do one other experiment between B (Cathode of junction) and one of the two remaining terminals; In case of no Connection, the terminal under test is G, otherwise it is D (or S). In worst case with a maximum of 8 experiments, each terminal can be specified. It is as follows:

Assume, the two selected terminals do not Conduct in both directions and this is the case for the other two terminals. Upto this point, four experiments have been done while not yet encountering any Conduction. It is clear that one group Consists of

G and B and the other Comprises from D and S, Because at least one Conduction should be observed if B were in the same group with one of the source or Drain. In the next step, we pick up one terminal from each group to undergo the conductivity test. Assume, no Conduction happens in either direction (Worst case). It means that we had chosen G from (G B) group. Thusfor, we have done six experiments. We change both of terminals and now we have chosen B for sure. and in worst case, we will find a connection in 8th experiment. Now, we know B and S (D), Bulk's groupmate is Gate and Source's (Drain's) groupmate is Drain (Source).

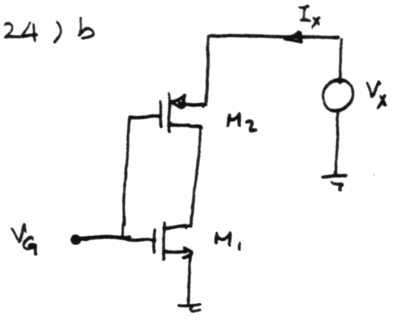
2.22) If we don't know the type of device, In eight experiment we cannot distinguish between B and S (D) and we should perform another experiment, which is exchanging one of

2.22) Cont. terminals with its groupmate. If we still had the Conduction then the exchanged terminal and its groupmate are Source and Drain, otherwise the exchanged terminal is Bulk.

2.23) a) NO, Because in DC model equations of MOSFET, we always have the product of $\mu_n C_{ox}$ and $\frac{W}{L}$.

b) NO, Because we cannot obtain as many independent equations as the unknown quantities. But if the difference between the aspect ratios is known, then $\mu_n C_{ox}$ and both $\frac{W}{L}$, are attainable.

2.24) b



CASE I : $V_G < V_{THN} \Rightarrow M_1$: off $\quad I_x = 0$

$g_m = 0$

CASE II : $V_G > V_{THN}$

for $\quad 0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0 \quad (M_2 : \text{off}) \quad g_m = \dfrac{\partial I_x}{\partial V_G} = 0$

Then M_2 turns on (in sat), M_1 still is in triode region

$$I_x = \frac{1}{2} \mu_p Cox \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2$$

This is correct until M_1, goes into saturation, when

$$\frac{1}{2} \mu_p Cox \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2 = \frac{1}{2} \mu_n Cox \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$$

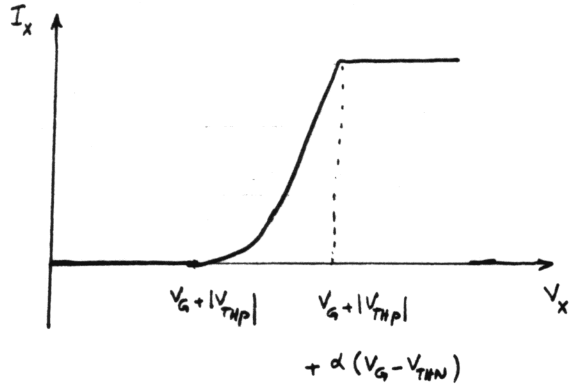
i.e. $$V_X = V_G + |V_{THP}| + \underbrace{\sqrt{\frac{\mu_n \, (W/L)_n}{\mu_p \, (W/L)_p}}}_{\alpha} (V_G - V_{THN})$$

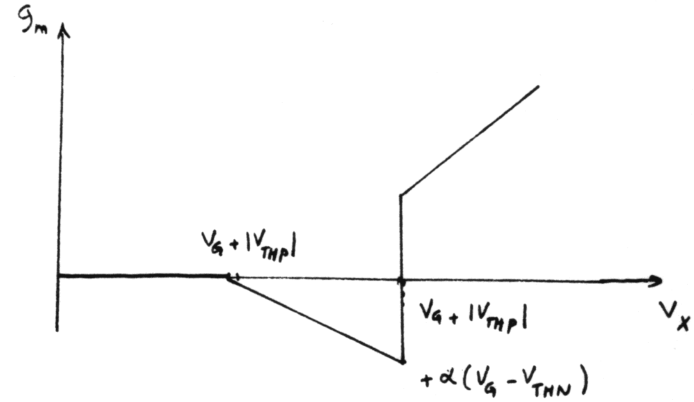
And afterward, M_2 goes into triode region and $I_x = \frac{1}{2} \mu_n Cox \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$

So, $0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0 \qquad g_m = \dfrac{\partial I_x}{\partial V_G} = 0$

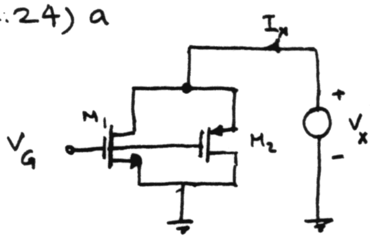
$V_G + |V_{THP}| < V_X < V_G + |V_{THP}| + \alpha(V_G - V_{THN}) \qquad I_x = \frac{1}{2} \mu_p Cox \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2 \quad g_m = \mu_p Cox \left(\frac{W}{L}\right)_p (V_G + |V_{THP}| - V_X,$

$V_G + |V_{THP}| + \alpha(V_G - V_{THN}) < V_X \qquad I_x = \frac{1}{2} \mu_n Cox \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2 \quad g_m = \mu_n Cox \left(\frac{W}{L}\right)_N (V_G - V_{THN})$





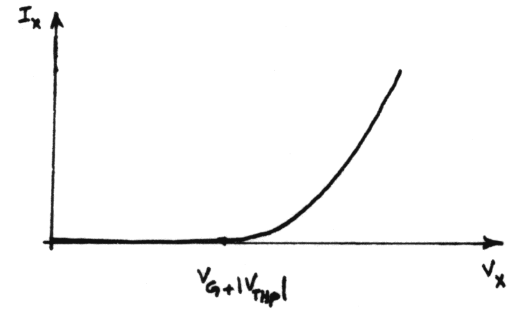
2.24) a



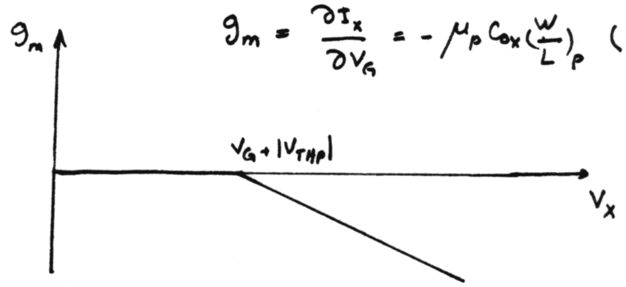
CASE I : $V_G < V_{THN}$ M_1 : off

for $0 < V_x < V_G + |V_{THP}|$ $I_x = 0$, $g_m = \dfrac{\partial I_x}{\partial V_G} = 0$

for $V_G + |V_{THP}| < V_x$ $\Rightarrow I_x = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$



$g_m = \dfrac{\partial I_x}{\partial V_G} = -\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)$



CASE II : $V_G > V_{THN}$

for $0 < V_x < V_G - V_{THN}$ (M_2 : off M_1 : triode)

$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n \left[2(V_G - V_{THN})V_x - V_x^2\right]$ $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_x$

for $V_G - V_{THN} < V_x < V_G + |V_{THP}|$ (M_2 : off M_1 : Sat)

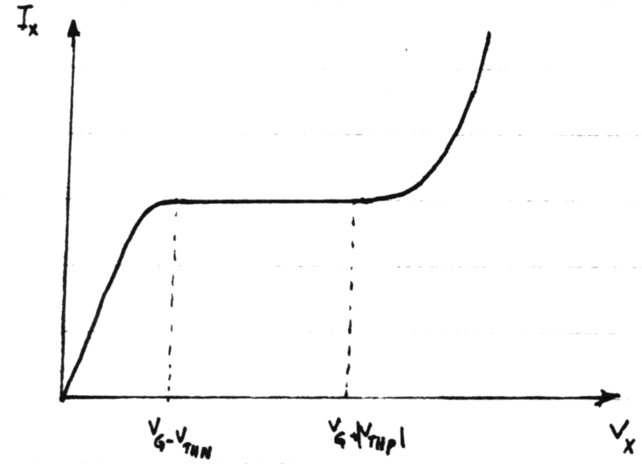
$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN})^2$ $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN})$

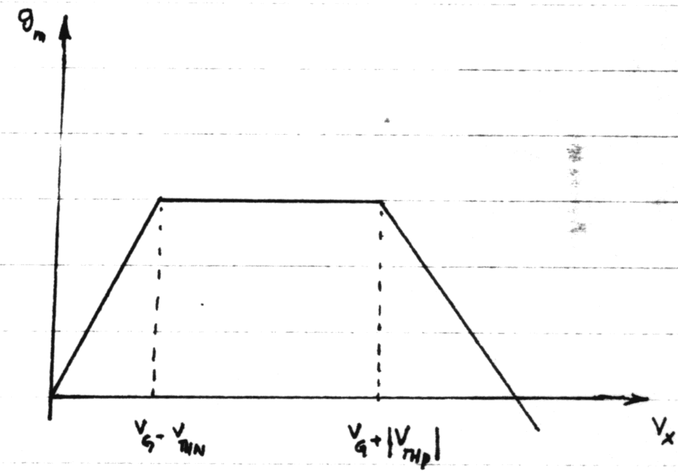
for $V_G + |V_{THP}| < V_x$ (M_2 : Sat M_1 : Sat)

2.24) a Cont.

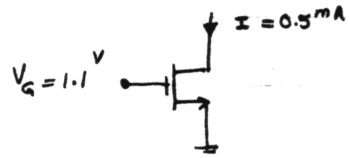
$$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN})^2 + \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$$

$$g_m = \frac{\partial I_x}{\partial V_G} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN}) - \mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)$$





2.25) $V_{TH} = 0.7$ $\lambda = 0.1$ (for $L = 0.5^{\mu}$)



for $L = 0.5^{\mu}$ $\lambda = 0.1$ $\rightarrow r_o = \frac{1}{\lambda I_D} = 20^{K\Omega}$

$$V_{OD} = V_{GS} - V_{TH} = 0.4 \Rightarrow V_{GS} = 1.1^{V}$$

Calculating W, $I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2$

$$0.5^{mA} = \frac{1}{2} \times 0.1343 \frac{mA}{V^2} \times \frac{W}{0.5^{\mu} - 0.16^{\mu}} \times (0.4)^2$$

$\frac{W}{L_{eff}} \simeq 47$ $\Rightarrow$ $W = 15.82\ \mu M$

$$C_{gs} = \frac{2}{3} W L C_{ox} + W C_{ov} \simeq 25\ fF$$

$$C_{GD} = W C_{ov} = 4.85\ fF$$

$$C_{DB} = \frac{W}{2} E C_j + 2\left(\frac{W}{2} + E\right) C_{jsw} \quad (@\ V_D = 0.4) = 10.7\ fF$$

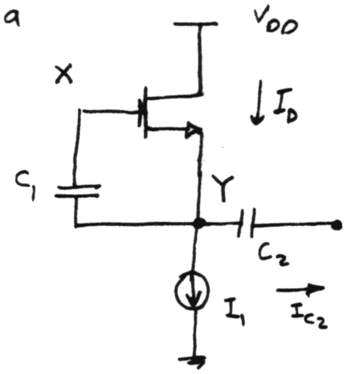
(for folded structure)

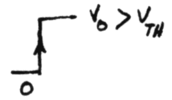
$$\left(C_j = \frac{C_{jo}}{\left(1 + \frac{V_{DB}}{2\varphi_F}\right)^{mj}} = 0.449 \times 10^{-3} \frac{F}{M^2} \quad , \quad C_{jsw} = \frac{C_{jswo}}{\left(1 + \frac{V_{DB}}{2\varphi_F}\right)^{mjsw}} = 0.325 \times 10^{-11} F/M \right)$$

$C_{ox} = 3.84 \times 10^{-3}\ F/m$ $C_{jo} = 0.56 \times 10^{-3}$ $m_j = 0.6$

$C_{jsw_o} = 0.35 \times 10^{-11}$ $m_{jsw_o} = 0.2$

2.26) a





Before applying the pulse

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}}$$

After Applying the Pulse

$$X(0^+) = V_{DD} + V_0$$

$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + V_0$$

for $t > 0$
$$\begin{cases} X(t) = V_{DD} + \alpha(t) \\ Y(t) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + \alpha(t) \end{cases}$$

$\alpha(0^+) = V_0$, Device is in triode

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right] = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - \left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - \alpha(t)\right)\right]\left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - \alpha(t)\right)$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}} - (\alpha(t) - V_{TH})^2\right] = I_1 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(\alpha(t) - V_{TH})^2$$

$$I_{C_2} = I_D - I_1 = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(\alpha(t) - V_{TH})^2 = C_2 \frac{dV_{C_2}}{dt} = C_2\frac{d\alpha(t)}{dt}$$

$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_2}}_{K} dt = \frac{-d\alpha}{(\alpha - V_{TH})^2} \Rightarrow Kt = \frac{1}{\alpha - V_{TH}} - \frac{1}{V_0 - V_{TH}}$$

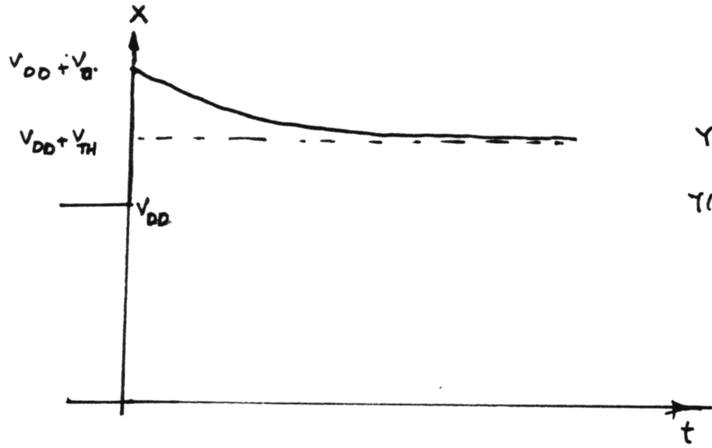
$$\Rightarrow \alpha(t) = V_{TH} + \frac{1}{Kt + \frac{1}{V_0 - V_{TH}}}$$

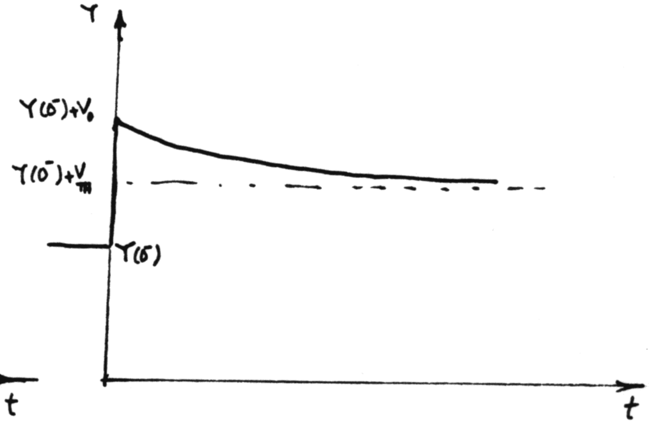
$$\alpha(\infty) = V_{TH}$$

2.26) a Cont.

$$X(\infty) = V_{DD} + V_{TH}$$

$$Y(\infty) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH} = V_{DD} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$





2.26) b

Before applying the pulse.

$X(0^-) = V_{DD}$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$

After applying the pulse

$X(0^+) = V_{DD} - V_{TH}$

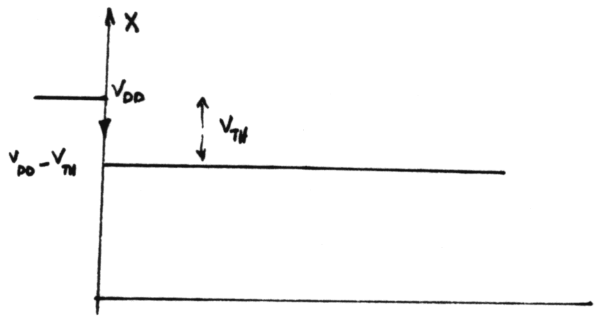
$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - V_{TH}$$

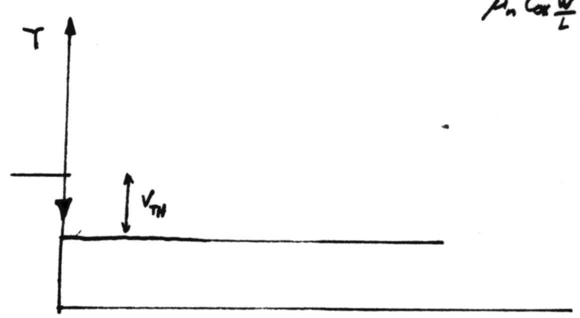
After applying the pulse, device remains in the saturation region, and its current doesn't chang, So, $I_{C_1} = I_{C_2} = 0$

Therefore, the circuit keeps its state.

$X(t) = X(0^+) = V_{DD} - V_{TH}$

$$Y(t) = Y(0^+) = V_{DD} - 2V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$





2.27)

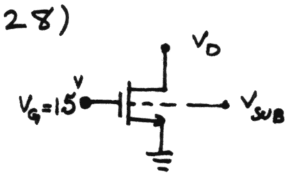
$$I_D = I_o \exp \frac{V_{GS}}{\xi V_T}$$

$$\frac{I_{D2}}{I_{D1}} = \exp \frac{V_{GS2} - V_{GS1}}{\xi V_T} \qquad \frac{I_{D2}}{I_1} = 10 \quad \Rightarrow \quad \Delta V_{GS} = \xi V_T \ln 10$$

$$\Delta V_{GS} = 1.5 \times \ln 10 \times 26^{mV} = 89.8\ mV$$

$$g_m = \frac{I_D}{\xi V_T} = \frac{10^{\mu A}}{1.5 \times 26^{mV}} = 0.26\ {}^{mA}/_{V}$$

2.28)



a) If we decrease V_D below zero, Source and drain exchange their roles and device operates in the triode region.

b) If we increase V_B, V_{TH} decreases, because

$$\Delta V_{TH} = \gamma \left(\sqrt{2\varphi_F - V_B} - \sqrt{2\varphi_F} \right) \text{ is negative.}$$

Therefore, I_D increases.