جدولی از انتگرالها

چند صورت مقدماتی

۱. $\int du = u + C$

۲. $\int a\,du = au + C$

۳. $\int [f(u) + g(u)]\,du = \int f(u)\,du + \int g(u)\,du$

۴. $(n \neq -1)$ $\int u^n du = \frac{u^{n+1}}{n+1} + C$

۵. $\int \frac{du}{u} = \ln|u| + C$

صورگویای شامل $a+bu$

۶. $\int \frac{u\,du}{a+bu} = \frac{1}{b^2}[a+bu-a\ln|a+bu|] + C$

۷. $\int \frac{u^2 du}{a+bu} = \frac{1}{b^3}\left[\frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2\ln|a+bu|\right] + C$

۸. $\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right] + C$

۹. $\int \frac{u^2 du}{(a+bu)^2} = \frac{1}{b^3}\left[a+bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right] + C$

۱۰. $\int \frac{u\,du}{(a+bu)^3} = \frac{1}{b^2}\left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu}\right] + C$

۱۱. $\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

۱۲. $\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

۱۳. $\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

صور شامل $\sqrt{a+bu}$

۱۴. $\int u\sqrt{a+bu}\,du = \frac{2}{15b^3}(3bu-2a)(a+bu)^{3/2} + C$

۱۵. $$\int u^2\sqrt{a+bu}\,du=\frac{2}{105b^3}(15b^2u^2-12abu+8a^2)(a+bu)^{3/2}+C$$

۱۶. $$\int u^n\sqrt{a+bu}\,du=\frac{2u^n(a+bu)^{3/2}}{b(2n+3)}-\frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$$

۱۷. $$\int\frac{u\,du}{\sqrt{a+bu}}=\frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$$

۱۸. $$\int\frac{u^2du}{\sqrt{a+bu}}=\frac{2}{15\,b^3}(3b^2u^2-4abu+8a^2)\sqrt{a+bu}+C$$

۱۹. $$\int\frac{u^n du}{\sqrt{a+bu}}=\frac{2u^n\sqrt{a+bu}}{b(2n+1)}-\frac{2an}{b(2n+1)}\int\frac{u^{n-1}\,du}{\sqrt{a+bu}}$$

۲۰. $$\int\frac{du}{u\sqrt{a+bu}}=\begin{cases}\frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right|+C & a>0\\ \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}}+C & a<0\end{cases}$$

۲۱. $$\int\frac{du}{u^n\sqrt{a+bu}}=-\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}}-\frac{b(2n-3)}{2a(n-1)}\int\frac{du}{u^{n-1}\sqrt{a+bu}}$$

۲۲. $$\int\frac{\sqrt{a+bu}\,du}{u}=2\sqrt{a+bu}+a\int\frac{du}{u\sqrt{a+bu}}$$

۲۳. $$\int\frac{\sqrt{a+bu}du}{u^n}=-\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}}-\frac{b(2n-5)}{2a(n-1)}\int\frac{\sqrt{a+bu}\,du}{u^{n-1}}$$

صور شامل $a^2\pm u^2$

۲۴. $$\int\frac{du}{a^2+u^2}=\frac{1}{a}\tan^{-1}\frac{u}{a}+C$$

۲۵. $$\int\frac{du}{a^2-u^2}=\frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right|+C=\begin{cases}\frac{1}{a}\tanh^{-1}\frac{u}{a}+C & |u|<a\\ \frac{1}{a}\coth^{-1}\frac{u}{a}+C & |u|>a\end{cases}$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C = \begin{cases} -\frac{1}{a}\tanh^{-1}\frac{u}{a} + C & |u|<a \\ -\frac{1}{a}\coth^{-1}\frac{u}{a} + C & |u|>a \end{cases} \quad .۲۶$$

صور شامل $\sqrt{u^2 \pm a^2}$

در فرمولهای ۲۷ تا ۳۸ ما می‌توانیم تبدیلات زیر را انجام دهیم

$$\sinh^{-1}\frac{u}{a} \text{ به } \ln(u+\sqrt{u^2+a^2}) \qquad \cosh^{-1}\frac{u}{a} \text{ به } \ln|u+\sqrt{u^2-a^2}|$$

$$\sinh^{-1}\frac{a}{u} \text{ به } \ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right|$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln|u+\sqrt{u^2 \pm a^2}| + C \quad .۲۷$$

$$\int \sqrt{u^2 \pm a^2}\, du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln|u+\sqrt{u^2 \pm a^2}| + C \quad .۲۸$$

$$\int u^2\sqrt{u^2 \pm a^2}\, du = \frac{u}{8}(2u^2 \pm a^2)\sqrt{u^2 \pm a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2 \pm a^2}| + C \quad .۲۹$$

$$\int \frac{\sqrt{u^2+a^2}\, du}{u} = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C \quad .۳۰$$

$$\int \frac{\sqrt{u^2-a^2}\, du}{u} = \sqrt{u^2-a^2} - a\sec^{-1}\left|\frac{u}{a}\right| + C \quad .۳۱$$

$$\int \frac{\sqrt{u^2 \pm a^2}\, du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln|u+\sqrt{u^2 \pm a^2}| + C \quad .۳۲$$

$$\int \frac{u^2\, du}{\sqrt{u^2 \pm a^2}} = \frac{u}{2}\sqrt{u^2 \pm a^2} - \frac{\pm a^2}{2}\ln|u+\sqrt{u^2 \pm a^2}| + C \quad .۳۳$$

$$\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C \quad .۳۴$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \quad .۳۵$$

$$\int \frac{du}{u^2\sqrt{u^2 \pm a^2}} = -\frac{\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C \quad .۳۶$$

$$\int (u^2 \pm a^2)^{3/2} du = \frac{u}{8}(2u^2 \pm 5a^2)\sqrt{u^2 \pm a^2} \quad .۳۷$$

$$+ \frac{3a^4}{8} \ln |u + \sqrt{u^2 \pm a^2}| + C$$

$$\int \frac{du}{(u^2 \pm a^2)^{3/2}} = \frac{u}{\pm a^2\sqrt{u^2 \pm a^2}} + C \quad .۳۸$$

صور شامل $\sqrt{a^2 - u^2}$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C \quad .۳۹$$

$$\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C \quad .۴۰$$

$$\int u^2\sqrt{a^2 - u^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C \quad .۴۱$$

$$\int \frac{\sqrt{a^2 - u^2}\, du}{u} = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C \quad .۴۲$$

$$= \sqrt{a^2 - u^2} - a\cosh^{-1}\frac{a}{u} + C$$

$$\int \frac{\sqrt{a^2 - u^2}\, du}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \sin^{-1}\frac{u}{a} + C \quad .۴۳$$

$$\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C \quad .۴۴$$

$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C = -\frac{1}{a}\cosh^{-1}\frac{a}{u} + C \quad .۴۵$$

$$\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C \quad .۴۶$$

$$\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2}+\frac{3a^4}{8}\sin^{-1}\frac{u}{a}+C \quad .47$$

$$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}}+C \quad .48$$

صور شامل $2au-u^2$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\cos^{-1}\left(1-\frac{u}{a}\right)+C \quad .49$$

$$\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} \quad .50$$
$$+\frac{a^3}{2}\cos^{-1}\left(1-\frac{u}{a}\right)+C$$

$$\int \frac{\sqrt{2au-u^2}\,du}{u} = \sqrt{2au-u^2}+a\cos^{-1}\left(1-\frac{u}{a}\right)+C \quad .51$$

$$\int \frac{\sqrt{2au-u^2}\,du}{u^2} = -\frac{2\sqrt{2au-u^2}}{u}-\cos^{-1}\left(1-\frac{u}{a}\right)+C \quad .52$$

$$\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(1-\frac{u}{a}\right)+C \quad .53$$

$$\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2}+a\cos^{-1}\left(1-\frac{u}{a}\right)+C \quad .54$$

$$\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2}+\frac{3a^2}{2}\cos^{-1}\left(1-\frac{u}{a}\right)+C \quad .55$$

$$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au}+C \quad .56$$

$$\int \frac{du}{(2au-u^2)^{3/2}} = \frac{u-a}{a^2\sqrt{2au-u^2}}+C \quad .57$$

$$\int \frac{u\,du}{(2au-u^2)^{3/2}} = \frac{u}{a\sqrt{2au-u^2}}+C \quad .58$$

۵۹. $\int \sin u\,du = -\cos u + C$

۶۰. $\int \cos u\,du = \sin u + C$

۶۱. $\int \tan u\,du = \ln|\sec u| + C$

۶۲. $\int \cot u\,du = \ln|\sin u| + C$

۶۳. $\int \sec u\,du = \ln\left|\sec u + \tan u\right| + C = \ln\left|\tan\left(\frac{۱}{۲}\pi + \frac{۱}{۲}u\right)\right| + C$

۶۴. $\int \csc u\,du = \ln|\csc u - \cot u| + C = \ln|\tan\frac{۱}{۲}u| + C$

۶۵. $\int \sec^۲ u\,du = \tan u + C$

۶۶. $\int \csc^۲ u\,du = -\cot u + C$

۶۷. $\int \sec u \tan u\,du = \sec u + C$

۶۸. $\int \csc u \cot u\,du = -\csc u + C$

۶۹. $\int \sin^۲ u\,du = \frac{۱}{۲}u - \frac{۱}{۴}\sin ۲u + C$

۷۰. $\int \cos^۲ u\,du = \frac{۱}{۲}u + \frac{۱}{۴}\sin ۲u + C$

۷۱. $\int \tan^۲ u\,du = \tan u - u + C$

۷۲. $\int \cot^۲ u\,du = -\cot u - u + C$

۷۳. $\int \sin^n u\,du = -\frac{۱}{n}\sin^{n-۱}u\cos u + \frac{n-۱}{n}\int \sin^{n-۲}u\,du$

۷۴. $\int \cos^n u\,du = \frac{۱}{n}\cos^{n-۱}u\sin u + \frac{n-۱}{n}\int \cos^{n-۲}u\,du$

۷۵. $\int \tan^n u\,du = \frac{۱}{n-۱}\tan^{n-۱}u - \int \tan^{n-۲}u\,du$

۷۶. $\int \cot^n u\,du = -\frac{۱}{n-۱}\cot^{n-۱}u - \int \cot^{n-۲}u\,du$

۷۷. $\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du$

۷۸. $\int \csc^n u\,du = -\frac{1}{n-1}\csc^{n-2}u\cot u + \frac{n-2}{n-1}\int \csc^{n-2}u\,du$

۷۹. $\int \sin mu \sin nu\,du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

۸۰. $\int \cos mu \cos nu\,du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

۸۱. $\int \sin mu \cos nu\,du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

۸۲. $\int u \sin u\,du = \sin u - u\cos u + C$

۸۳. $\int u\cos u\,du = \cos u + u\sin u + C$

۸۴. $\int u^2 \sin u\,du = 2u\sin u + (2-u^2)\cos u + C$

۸۵. $\int u^2\cos u\,du = 2u\cos u + (u^2-2)\sin u + C$

۸۶. $\int u^n \sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$

۸۷. $\int u^n \cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$

۸۸. $\int \sin^m u\cos^n u\,du = -\frac{\sin^{m-1}u\cos^{n+1}u}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2}u\cos^n u\,du$

$$= \frac{\sin^{m+1}u\cos^{n-1}u}{m+n} + \frac{n-1}{m+n}\int \sin^m u\cos^{n-2}u\,du$$

صور شامل توابع مثلثاتی معکوس

۸۹. $\int \sin^{-1}u\,du = u\sin^{-1}u + \sqrt{1-u^2} + C$

۹۰. $\int \cos^{-1}u\,du = u\cos^{-1}u - \sqrt{1-u^2} + C$

۹۱. $\int \tan^{-1}u\,du = u\tan^{-1}u - \ln\sqrt{1+u^2} + C$

۹۲. $$\int \cot^{-1} u\,du = u\cot^{-1} u + \ln\sqrt{1+u^2} + C$$

۹۳. $$\int \sec^{-1} u\,du = u\sec^{-1} u - \ln|u+\sqrt{u^2-1}| + C$$
$$= u\sec^{-1} u - \cosh^{-1} u + C$$

۹۴. $$\int \csc^{-1} u\,du = u\csc^{-1} u + \ln|u+\sqrt{u^2-1}| + C$$
$$= u\csc^{-1} u + \cosh^{-1} u + C$$

صور شامل توابع نمایی و لگاریتمی

۹۵. $$\int e^u du = e^u + C$$

۹۶. $$\int a^u du = \frac{a^u}{\ln a} + C$$

۹۷. $$\int ue^u du = e^u(u-1) + C$$

۹۸. $$\int u^n e^u du = u^n e^u - n\int u^{n-1} e^u du$$

۹۹. $$\int u^n a^u du = \frac{u^n a^u}{\ln a} - \frac{n}{\ln a}\int u^{n-1} a^u du + C$$

۱۰۰. $$\int \frac{e^u du}{u^n} = \frac{e^u}{(n-1)u^{n-1}} + \frac{1}{n-1}\int \frac{e^u du}{u^{n-1}}$$

۱۰۱. $$\int \frac{a^u du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1}\int \frac{a^u du}{u^{n-1}}$$

۱۰۲. $$\int \ln u\,du = u\ln u - u + C$$

۱۰۳. $$\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$$

۱۰۴. $$\int \frac{du}{u\ln u} = \ln|\ln u| + C$$

۱۰۵. $$\int e^{au}\sin nu\,du = \frac{e^{au}}{a^2+n^2}(a\sin nu - n\cos nu) + C$$

$$\int e^{au}\cos nu\,du=\frac{e^{au}}{a^2+n^2}(a\cos nu+n\sin nu)+C \quad .۱۰۶$$

صور شامل توابع هیپربولیك

$$\int \sinh u\,du=\cosh u+C \quad .۱۰۷$$

$$\int \cosh u\,du=\sinh u+C \quad .۱۰۸$$

$$\int \tanh u\,du=\ln|\cosh u|+C \quad .۱۰۹$$

$$\int \coth u\,du=\ln|\sinh u|+C \quad .۱۱۰$$

$$\int \operatorname{sech} u\,du=\tan^{-1}(\sinh u)+C \quad .۱۱۱$$

$$\int \operatorname{csch} u\,du=\ln\left|\tanh\tfrac{1}{2}u\right|+C \quad .۱۱۲$$

$$\int \operatorname{sech}^2 u\,du=\tanh u+C \quad .۱۱۳$$

$$\int \operatorname{csch}^2 u\,du=-\coth u+C \quad .۱۱۴$$

$$\int \operatorname{sech} u\tanh u\,du=-\operatorname{sech} u+C \quad .۱۱۵$$

$$\int \operatorname{csch} u\coth u\,du=-\operatorname{csch} u+C \quad .۱۱۶$$

$$\int \sinh^2 u\,du=\tfrac{1}{4}\sinh 2u-\tfrac{1}{2}u+C \quad .۱۱۷$$

$$\int \cosh^2 u\,du=\tfrac{1}{4}\sinh 2u+\tfrac{1}{2}u+C \quad .۱۱۸$$

$$\int \tanh^2 u\,du=u-\tanh u+C \quad .۱۱۹$$

$$\int \coth^2 u\,du=u-\coth u+C \quad .۱۲۰$$

$$\int u\sinh u\,du=u\cosh u-\sinh u+C \quad .۱۲۱$$

$$\int u \cosh u \, du = u \sinh u - \cosh u + C \quad \text{۱۲۲.}$$

$$\int e^{au} \sinh nu \, du = \frac{e^{au}}{a^2 - n^2} (a \sinh nu - n \cosh nu) + C \quad \text{۱۲۳.}$$

$$\int e^{au} \cosh nu \, du = \frac{e^{au}}{a^2 - n^2} (a \cosh nu - n \sinh nu) + C \quad \text{۱۲۴.}$$