

# Reliability-based smart-maintenance model for power system generators

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**Abstract:** In order to provide a reliable service and supply the demand most of the time, all generators in a power grid should be subjected to an effective maintenance plan. The smarter the maintenance performed could result in a better performance of the system. However, a challenge is to minimise maintenance costs that do not compromise the benefits. Considering these facts, this study presents a reliability-based smart-maintenance approach of generators to compute the net-maximum economic benefit. The approach is derived from Kijima model type I to characterise the impact of maintenance over the component's virtual age, and Markov chains to model the component's lifetime. To achieve a more realistic model, generators' failure and repair rates are considered time-dependent variables. Then, the optimum preventive maintenance schedule is obtained by using an advanced algorithm named accelerated quantum particle swarm optimisation in combination with sequential Monte Carlo simulation. The effectiveness of the approach is investigated through a case study with four different scenarios: (i) no preventive maintenance plan, (ii) yearly periodic preventive maintenance, (iii) reliability-centred maintenance and (iv) smart maintenance. The results suggest that the approach is convenient for power system generators and delivers a significant knowledge contribution in the area of maintenance.

## Nomenclature

Some of the symbols and notations used throughout this manuscript are defined below for quick reference. Others are defined following their first appearances, as appropriate.

$V_n$	virtual age after the $n$ th maintenance is executed
$X_n$	time elapse from the $n - 1$ th failure to the $n$ th failure
$q$	degree of maintenance
$\delta$	maintenance factor
$T_U$	guarantee time of the component
$T_W$	end lifetime of the component
$\lambda$	failure rate
$\mu$	repair rate
$\phi$	degradation rate
$\alpha$	failure rate scale parameter
$\beta$	failure rate power parameter
$\omega$	failure rate location parameter
$a$	failure rate displacement parameter
$\sigma$	repair rate scale parameter
$\gamma$	repair rate power parameter
$b$	repair rate displacement parameter
$l$	auxiliary variable

## 1 Introduction

Power systems require adaptive and dynamic maintenance of their components to achieve a higher level of reliability. Such maintenance strategies are vital to limit failures and minimise downtime of the components [1]. The maintenance can be broadly categorised into two basic schemes: (i) preventive maintenance (PM), which is carried out at predetermined intervals or according to prescribed criteria, aimed at reducing the failure risk or performance degradation of the equipment [2]; (ii) corrective maintenance (CM), which is carried out after failure detection and is aimed at restoring an asset to a condition in which it can perform its intended function [2].

Periodic preventive maintenance (PPM) is a typical strategy used in many power system components. This strategy proposes routine maintenance scheme given by the manufacturer's

specification. Even though PPM provides reliable operation of the components, several studies [3, 4] proved that it may not necessarily lead to the optimum benefits. This is because PPM does not consider the composite system operation, consequently, the number of maintenances is high and so the maintenance cost. For this reason, new maintenance approaches have been developed in the last two decades [5–7].

Among the advanced maintenance strategies, literature presents the reliability-centred maintenance (RCM) [7] as one of the most popular strategy. RCM proposes a maintenance schedule optimisation problem, of which the objective is to minimise the maintenance cost while keeping an adequate reliability of the system [8]. The literature reports variations of RCM by incorporating different optimisation techniques such as genetic algorithm [9], particle swarm optimisation [10] and annealing optimisation [11]. Although these studies propose advanced models to guarantee the availability of the components, the main drawback lies in the reliability model of the components. In this context, literature presents the reliability model of the component as Markov process between the state 'operating' and 'not in service' [12]. This idea brings two main drawbacks. In the first instance, component's failure and repair rates are modelled using exponential distribution [12], underestimating the ageing and bringing in less accuracy in reliability assessment. Although recent studies in [7, 8] propose Weibull distribution into RCM to model the ageing effect, deeper analysis on these reveals that the assessment of the degradation of the component is not captured in detail. In the second instance, the alternating renewal process does not consider the inclusion of PM into the model and considers only CM.

The concept of maintenance has reached a new paradigm of smart maintenance (SM). Several researchers proposed this vision based on some applications with the inclusion of smart inspections, smart devices, smart services and asset management. In this regard, Falahati *et al.* [13] propose a SM scheme with the implementation of smart devices to monitor the operation of transformers and circuit breakers in a small-scale distribution system. This study includes a mathematical framework that allows quantifying the impact of system monitoring on system reliability analytically. Subsequently, the study is extended with the incorporation of a novel index called 'monitoring degree' [14]. The vision of SM with

the implementation of smart inspection and smart services are proposed in [15]. This approach is based on eddy current transducer carried out by an inspection robot to record the status of transmission lines. The approach provides effective benefits for the planning of a SM schedule. SM theoretical framework based on management assets is presented in [16]. Such study exhibits a SM decision support system that uses corporate big data analytics, which is applied to transformers and circuit breakers. A comprehensive vision of SM based on management assets is presented in [17]. In this work, the authors propose an approach to deal with the investment required for optimal mid- and long-term decision making of distribution companies in the presence of reward-penalty.

At the power generation level, recent investigations present different SM approaches based on generators maintenance scheduling problem. However, such approaches provide limited transparency of mathematical frameworks that can effectively capture the impact of long-term PM scheduling on the degradation (due to ageing) of the generators. For instance, Balaji *et al.* [18] propose a SM approach using conventional lambda iteration in combination with differential evolution algorithm to minimise the overall generator operation cost while satisfying power system operational constraints. The main drawback with [18] is the lack of consideration of the reliability parameters of the generators, such as the failure and repair rates. Thus, sudden disruptions due to random generator failures are not considered into the analysis, which brings uncertainties into the results. The same issue appears also in [19], with the difference being that the authors propose a discrete integer Cuckoo search algorithm. In [20, 21], the authors propose a vision of SM using stochastic mixed-integer polynomial programming to obtain a proactive PM plan that maximises system reliability. Although the given approaches show significant computational gains, they need the recorded failures scenarios that sometimes are not available in the published literature. The authors of [22, 23] expose a SM schedule for generators based on reliability theory and greedy heuristic local search algorithm, respectively. Both studies seek to maximise the probability of no power generating units in the power system failing during the

scheduling window. Even though the generators reliability model is well presented in [22, 23], their main deficiency is the use of time-independent failure and repair rates that produce inaccuracies for real-world applications. In [24], a vision of a SM for real hydropower systems is found. In this study, the authors suggest a mixed-integer programming model that considers the time windows of the maintenance activities with non-linearities and disjunctions of the hydroelectric production functions to bring an effective maintenance schedule. The gap in this study is that it focuses on short- and mid-term PM planning. In order to get an insight of each SM approach, Table 1 presents a qualitative comparison of the presented literature.

While the implementation of the ageing effect is limited in almost all the available literature, there are few works [26–30] that consider such effect. Nevertheless, these studies are limited to short- and mid- terms PM schedules. Furthermore, these studies do not consider the bathtub curve and half-arch shape to model a more realistic behaviour of the failure and repair rates, respectively. Therefore, the reliability models in these studies carry less accuracies to depict a real impact of PM schedule on system reliability. To overcome these limitations, this paper proposes an innovative SM scheme of generators to enhance the reliability of a power system. The model is based on the following stages: firstly, the Kijima model type I (KMI) is adopted to create a link between component's virtual age; secondly, by the employment of Markov chain, the component's reliability model is obtained. The SM formulates a maintenance schedule optimisation problem which is solved using an advanced algorithm proposed in this paper as accelerated quantum particle swarm optimisation (AQPSO). The main contributions of this paper are: (i) a more realistic and accurate ageing reliability model, which considers the obsolescence state of the power generators; (ii) a novel AQPSO-based algorithm for optimal long-term PM planning of power generators; (iii) a novel mathematical formulation that describes the relationship between generator's lifetime, virtual age, degradation and transition rates; and (iv) an advanced SM algorithm for optimal power system generation adequacy assessment.

**Table 1** Recent SM visions for power system generators

Reference	Failure ( $\lambda$ ) and repair ( $\mu$ ) rate behaviour	Consider degradation due to ageing	States per component	Years of analysis	Technique employed
Balaji <i>et al.</i> [18]	not applied	no	not applied	one	lambda iteration approach and differential evolution
Lakshminarayanan and Kaur [19]	not applied	no	two	one	discrete integer cuckoo search
Basçiftci <i>et al.</i> [20]	$\lambda$ : not specified $\mu$ : not specified	no	not specified	one	stochastic mixed-integer programming and sample average approximation
Jo <i>et al.</i> [21]	$\lambda$ : not specified $\mu$ : not specified	no	not specified	one	mixed-integer polynomial programming
Eygelaar <i>et al.</i> [22]	$\lambda$ : constant $\mu$ : constant	no	two	one	reliability theory
Hosseini <i>et al.</i> [23]	$\lambda$ : constant $\mu$ : constant	no	not specified	one	greedy heuristic local search algorithm
Rodríguez <i>et al.</i> [24]	not applied	no	not applied	one	mixed-integer programming
Azadeh <i>et al.</i> [25]	$\lambda$ : Weibull variation $\mu$ : constant	yes	two	one	Markovian discrete event and Monte Carlo
Yildirim <i>et al.</i> [26]	$\lambda$ : Weibull $\mu$ : Weibull	yes	two	one	mixed-integer optimisation and Monte Carlo
Mo and Sansavini [27]	$\lambda$ : Weibull variation $\mu$ : constant	yes	two	twenty	linear programming and Monte Carlo
Hoseyni <i>et al.</i> [28]	$\lambda$ : Weibull variation $\mu$ : not specified	yes	three	thirty	condition-based probabilistic safety assessment
Selvi <i>et al.</i> [29]	$\lambda$ : Weibull variation $\mu$ : not specified	yes	two	ten	genetic algorithm and Monte Carlo
proposed approach in this paper	$\lambda$ : bathtub curve $\mu$ : half-arch shape	yes	three	fifty	AQPSO and sequential Monte Carlo

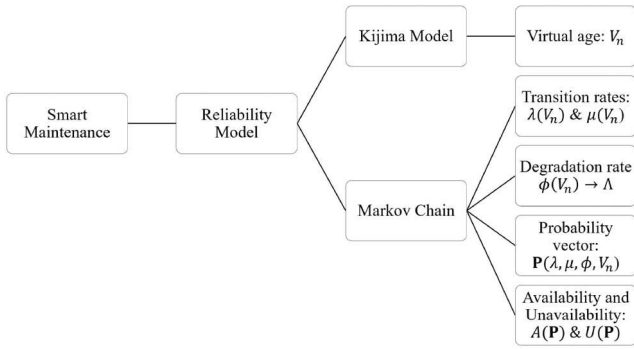


Fig. 1 Reliability concepts for SM

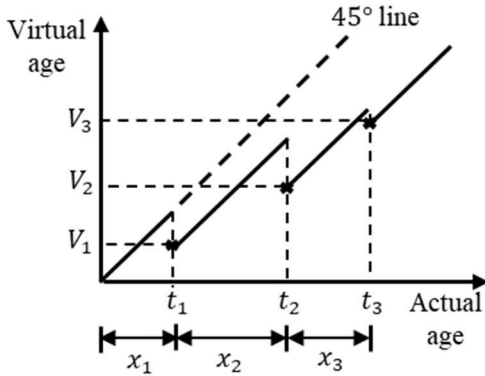


Fig. 2 Virtual age as a function of the actual age

The rest of the paper is organised as follows. Section 2 presents the reliability concepts required for SM; Section 3 presents the mathematical formulations to describe the AQPSO; Section 4 describes the problem formulation; Section 5 presents an algorithm to determine the optimum PM schedule based on the SM scheme; Section 6 presents a case study; the results are analysed in Section 7. Finally, Section 9 gives the conclusion.

## 2 Smart maintenance mathematical framework

SM is an advanced strategy that takes the reliability features of the components to set an effective maintenance plan as input. Fig. 1 presents the reliability concepts needed to formulate the SM mathematical framework.

### 2.1 Virtual age, actual age and maintenance

Maintenance action causes a rejuvenation, which is defined by its virtual age. The KMI [31] is employed to model the relationship between the virtual age before and after the  $n$ th maintenance action. Maintenance influences the virtual age of a component, by an amount proportional to the time elapsed from the  $n-1$ th failure to the  $n$ th failure. Then, the KMI can be described mathematically as [12]

$$V_n = \begin{cases} V_{n-1} + q_{CM}X_n, & \text{if CM is performed} \\ V_{n-1} + q_{PM}X_n, & \text{if PM is performed} \end{cases} \quad (1)$$

KMI can also be represented in terms of the actual age  $t$ . This fact can be appreciated in Fig. 2, in which the relationship between the actual and virtual age is given. Then, the virtual age in terms of the actual age  $t_n$  when the  $n$ th failure occurs is as follows:

$$V_n = \begin{cases} q_{CM}(X_1 + \dots + X_n) = q_{CM}t_n, & \text{if CM is performed} \\ q_{PM}(X_1 + \dots + X_n) = q_{PM}t_n, & \text{if PM is performed} \end{cases} \quad (2)$$

The parameter  $q$  is the degree of efficacy defined as perfect, imperfect and minimal. The term ‘perfect’ ( $q = 0$ ) refers to the restoration of the component to operate as a new, the ‘imperfect’ ( $0 < q < 1$ ) implies a restoration of the component to operate in

between good (similar to new life) and worst (similar to end of life) and in the case where the maintenance is developed with limited effort, is called minimal ( $q = 1$ ) [32].

To simplify KMI, maintenance factor  $\delta$  is incorporated into the model. This factor depends on the type of maintenance and can take only two values: (i) one, if CM is performed; (ii) zero, if PM is performed. Then, (2) can be rewritten as

$$V_n = q_{CM}^{\delta_m} q_{PM}^{1-\delta_m} t_n, \quad \delta_m = \begin{cases} 1, & \text{if CM is performed} \\ 0, & \text{if PM is performed} \end{cases} \quad (3)$$

### 2.2 Markov chain, transition rates and lifetime

A Markov chain is an illustration that contains the possible states of a component, which are connected by transitions rates. This point becomes relevant since the lifetime of the component follows a Markovian process, which is defined by its failure and repair rate.

Regarding the generators’ failure rate, it is common to use the bathtub curve to describe it. Bathtub curve is divided into four stages [33]. The first stage is the infant mortality, which is defined by the policy of the manufacturer and it corresponds to the period of guarantee. In this stage, the component is in a state denominated by ‘operation good as new’. Whenever a failure arises, the component goes to a state characterised by the ‘policy of replacement’. This process is recurrent, and the component goes to the next stage only if no failure event occurs during the guarantee period ends ( $0 \leq t < T_U$ ). The next stage is the useful life ( $T_U \leq t < T_V$ ), in which the component is subjected to the ‘normal operation’ and ‘not in service’ states. The state ‘normal operation’ indicates that the component is operating under rated conditions, while the ‘not in service’ state is driven by any failure that interrupts the operation of the component. In this stage, the failure rate takes a constant value. The next stage is the wear out ( $T_V \leq t < T_W$ ), in which the component presents a high occurrence of failure. This is because the ageing effects become so intense that produces gradual increments on the failure rate of the component. The last stage ( $t \geq T_W$ ) of the curve is the end lifetime, which contains the ‘obsolescence’ state. This stage is the most critical since the component is totally degraded. The failure rate takes extremely high values. Consequently, component replacement is required.

To include maintenance action into the proposed model, the component’s failure rate is treated as a function of the virtual age. Since time  $t$  is the cumulative operation time of the system, the virtual age can be written as [34]

$$V(t) = V_n + (t - t_n) \quad (4)$$

Then, by using Weibull distribution in combination with Gumbel distribution [35], the bathtub curve mathematical model can incorporate maintenance action as given in the following equation [33]:

$$\lambda(t) = \begin{cases} \lambda_I, & 0 \leq V(t) < T_U \\ \lambda_C, & T_U \leq V(t) < T_W \\ \infty, & V(t) \geq T_W \end{cases} \quad (5)$$

where  $\lambda_I = \alpha_I e^{-\beta_I V(t)}$  and  $\lambda_C = \alpha_C + \alpha_C e^{(V(t) - \omega)/\alpha_C}$ .

Regarding the repair rate of the component, it presents the same four stages as in the bathtub curve, but with different behaviour. During the infant mortality, the repairs or restoration of the component is the responsibility of the manufacturer, which is given by  $\mu_I$ . In the useful life, the repair rate decreases at a lower rate, while in the wear out stage the repair rate decreases at a faster rate. This is attributed to the severity of the failures that increase the difficulty to repair the component as time passes by. When the component reaches the end lifetime stage, the component cannot be repaired, and the repair rate goes to zero. Thus, the repair rate can be mathematically written as follows [33]:

$$\mu(t) = \begin{cases} \mu_I, & 0 \leq V(t) < T_U \\ \mu_C, & T_U \leq V(t) < T_W \\ 0, & V(t) \geq T_W \end{cases} \quad (6)$$

where  $\mu_C = b - \sigma e^{\gamma V(t)}$ .

Fig. 3 shows the behaviour of the failure and repair rates divided by stage and states as previously described. It can be observed that although Fig. 3 shows five states, the model can be reduced to three states by starting the analysis at time  $T_U$ . This is attributed to the fact that the manufacturer must ensure the correct operation of the component to perform during the guarantee period (infant mortality). Therefore, the states '0' and 'R' can be neglected, since they do not correspond to the customer. Concerning the replacement action, this is modelled as a forced decision that automatically occurs once the obsolescence state is reached. Hence, the replacement is considered as a non-stochastic process and for this reason it does not appear in the Markov chain process presented in Fig. 3.

Therefore, the relationship between the component's virtual age, transitions rates and lifetime is given by the stochastic matrix of transition states  $\mathbf{H}$  [36]. This is a square matrix that associates row and column with a specific state. For the model given in Fig. 3,  $\mathbf{H}$  is a three by three matrix, where the first row and column associate the state 'Normal operation'; second row and column associate the state 'Not in service'; and third row and column associate the state 'Obsolescence'.

The diagonal terms of the matrix denoted by  $h_{ii}$  are obtained by performing the negative sum of the transition rates that leave the state  $i$ , while the rest of terms  $h_{ij}$  are given by the transition rate that goes from state  $i$  to state  $j$ . Then, the  $\mathbf{H}$  matrix for the proposed model is as follows:

$$\mathbf{H} = \begin{bmatrix} -\lambda_C & \lambda_C & 0 \\ \mu_C & -\mu_C - \phi_W & \phi_W \\ 0 & 0 & 0 \end{bmatrix} \quad (7)$$

### 2.3 Absorbing state and degradation function

An absorbing state is defined as the state that, once entered, cannot be left. For example, the obsolescence presented in Fig. 3 represents an absorbing state since there is no transition rate that allows going out of this state. The expected time to get into an absorbing state is given by the end lifetime of the component, which can be obtained using the formulation given in the following equation [36]:

$$T_W = \frac{\lambda_C + \phi_W + \mu_C}{\lambda_C \phi_W} \quad (8)$$

Solving for  $\phi_W$

$$\phi_W = \frac{\lambda_C + \mu_C}{T_W \lambda_C - 1} \quad (9)$$

Then, the degradation of the component is quantified using the degradation rate and lifetime of the component. The expression that described the degradation is given by [33]

$$\Lambda = \frac{1}{T_W \phi_W} \quad (10)$$

subject to

$$\lim_{t \rightarrow T_W} \Lambda = \infty \quad (11)$$

The constraint presented in (11) is to hold the obsolescence state [33].

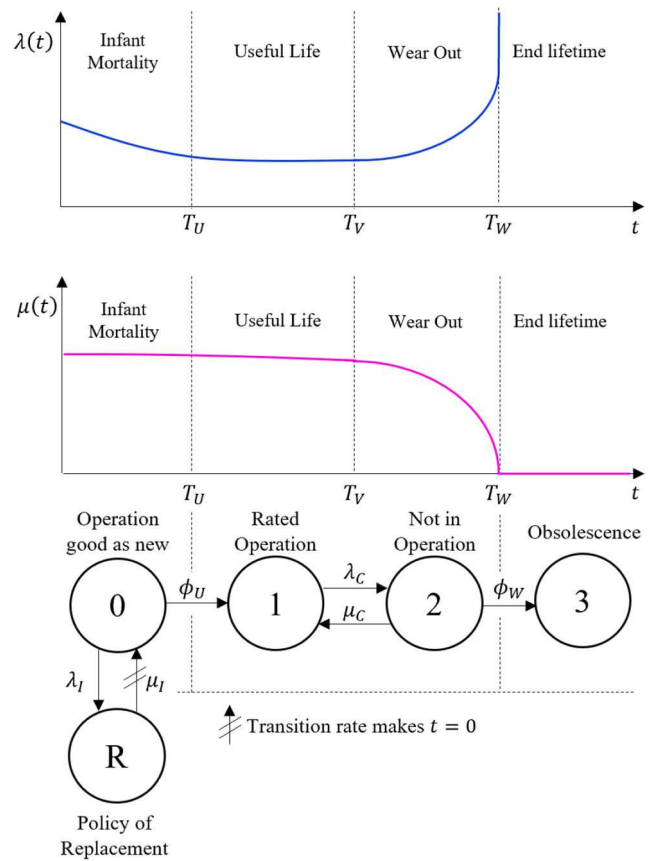


Fig. 3 Failure and repair rates for SM scheme

### 2.4 Function of the probability vector of states

The main goal of this section is to determine an expression that can capture the probability of being in the states given in Fig. 3 as a function of the failure, repair and degradation rate of the generators.

For every state  $s$ , there is a probability function  $P_s$  defined by its transposed transition matrix  $\mathbf{H}^T$ . Mathematically, this is formulated as [36]

$$[P_1(t) \ P_2(t) \ \dots \ P_z(t)]^T = \sum_{s=1}^z K_s \mathbf{v}_s e^{\chi_s t} \quad (12)$$

where  $z$  is the total number of states,  $\mathbf{v}$  and  $\chi$  are the eigenvectors and eigenvalues of  $\mathbf{H}^T$ , and  $K$  is given by the initial conditions;  $T$  is an operator that transpose the matrix. By employing this criterion to (7), the probability vector of states is

$$[P_1(t) \ P_2(t) \ P_3(t)]^T = \sum_{s=1}^3 K_s \mathbf{v}_s e^{\chi_s t} \quad (13)$$

where

$$\begin{aligned} l &= \sqrt{-4\lambda_C \phi_W + (\lambda_C + \mu_C + \phi_W)^2} \\ \chi_1 &= 0; \chi_2 = \frac{-\lambda_C - \mu_C - \phi_W - l}{2}; \\ \chi_3 &= \frac{-\lambda_C - \phi_W - \mu_C + l}{2} \end{aligned} \quad (14)$$

(see (15))

To get the  $K_s$  values, an initial condition is needed. In fact, it is known that at  $V(t) = T_U$  the component is in state '1' ( $P_1|_{V(t)=T_U} = 1; P_2|_{V(t)=T_U} = 0, P_3|_{V(t)=T_U} = 0$ ), therefore from (12)

$$[1 \ 0 \ 0]^T = K_1 \mathbf{v}_1 e^{\chi_1 V(t)} + K_2 \mathbf{v}_2 e^{\chi_2 V(t)} + K_3 \mathbf{v}_3 e^{\chi_3 V(t)} \quad (16)$$

$$\begin{aligned} \mathbf{v}_1 &= [0 \ 0 \ 1]^T; \\ \mathbf{v}_2 &= \begin{bmatrix} \frac{(\lambda_C - \mu_C - \phi_W + l)(\lambda_C + \mu_C + \phi_W + l)}{4\lambda_C\phi_W} & \frac{-\lambda_C - \mu_C - \phi_W - l}{2\phi_W} & 1 \end{bmatrix}^T; \\ \mathbf{v}_3 &= \begin{bmatrix} \frac{(\lambda_C - \mu_C - \phi_W - l)(\lambda_C + \mu_C + \phi_W - l)}{4\lambda_C\phi_W} & \frac{-\lambda_C - \mu_C - \phi_W + l}{2\phi_W} & 1 \end{bmatrix}^T; \end{aligned} \quad (15)$$

Replacing (14) and (15) in (16) and solving for  $K_1$ ,  $K_2$  and  $K_3$

$$\begin{aligned} K_1 &= 1; \\ K_2 &= \frac{(\lambda_C + \mu_C + \phi_W - l)}{2d} e^{0.5T_U(\lambda_C + \mu_C + \phi_W + l)}; \\ K_3 &= \frac{(-\lambda_C - \mu_C - \phi_W - l)}{2d} e^{0.5T_U(\lambda_C + \mu_C + \phi_W - l)}; \end{aligned} \quad (17)$$

Once  $\chi$ ,  $\mathbf{v}$  and  $K$  are quantified in terms of the failure, repair and degradation rates, the probability function of each state can be determined by replacing (14), (15) and (17) in (13).

### 2.5 Reliability model: availability and unavailability

The reliability model of any component is defined by its availability and unavailability. The availability is the sum probabilities of the states defined in the set  $\Theta$  that keep the component in operation. On the other pole, the unavailability is the sum probabilities of the states defined in the set  $\Omega$  that keep the component non-operating. They can be estimated using (18) and (19), respectively

$$A(t) = \sum_{s \in \Theta} P_s(t) \quad (18)$$

$$U(t) = \sum_{s \in \Omega} P_s(t) \quad (19)$$

It is notable that for the model proposed

$$\Theta = \{1\}; \quad \Omega = \{2, 3\} \quad (20)$$

Therefore

$$A(t) = P_1(t) \quad (21)$$

$$U(t) = P_2(t) + P_3(t) \quad (22)$$

At this point, the component's reliability model for SM scheme is described. The next step is to define a method that allows getting the maximum net benefit due to optimum PM scheduling. This is described in the section below.

## 3 Optimisation technique: AQPSO

Metaheuristics approaches define different scenarios to describe the motion of a particle. For instance, [37] traditional PSO presents particles with characteristics of classical physics, such as inertia, speed, acceleration and so on. Hence, the motion of the particles in this scenario is governed by the laws of dynamics and kinematics. Another example is given in [38], which proposes magnetic particles and its motion is described using electromagnetism theory. AQPSO proposes a scenario, where a unidimensional particle lies in a quantum delta potential well. The motion of the particle is driven by quantum mechanics concepts.

AQPSO follows the process described in Fig. 4. AQPSO starts defining the initial population of the particles  $SS$  and total number of iterations  $It$ . The position of the particle ( $x$ ) represents a solution candidate to the optimisation problem; thus, it can be used to evaluate the objective function. Each quantum particle  $\ell$  presents two specific attributes, which are related to memory and communication. The memory attribute refers to the ability to save the best position of the particle by comparing its actual position

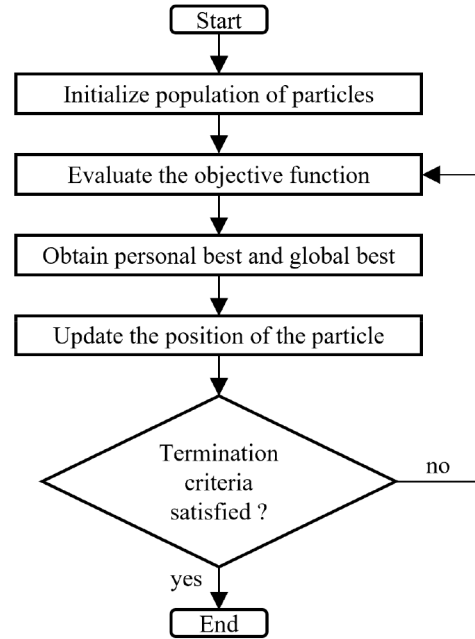


Fig. 4 AQPSO general process

with the position after the motion. The memory attribute is known as 'personal best' and denoted by  $q_\ell$ . The communication attribute refers to the ability to save the particle with the best position among the swarm. The communication attribute is known as 'global best' and is denoted by  $g$ . The personal best and global best are used to define the local attraction between particles. Clerc and Kennedy [37] conducted a trajectory analysis of a particle and demonstrated that this attraction mainly depends the terms  $q_\ell$  and  $g$ . Mathematically, the local attraction of the particle at search step  $k$  is defined as follows:

$$\begin{aligned} D_\ell(k) &= \varphi(k)q_\ell(k) + (1 - \varphi(k))g(k) \\ \varphi(k) &= d_1u_1/(d_1u_1 + d_2u_2) \end{aligned} \quad (23)$$

where  $u$  is a uniformly distributed random number, and  $d$  is a constant of acceleration coefficient which values are between zero and two [37]. The expression given in (23) is important because is needed in the formulation of mathematical expression that describes the motion of the quantum particle.

The process continues with the position update of every particle. The new positions represent an evolution (enhancement) of the actual solutions. The evolution is achieved based on the particle motion mathematical formulations, which is given as follows [39]:

$$\begin{aligned} x_\ell(k+1) &= \\ &\begin{cases} D_\ell(k) + \alpha \left| x_\ell(k) - \frac{1}{SS} \sum_{\ell=1}^{SS} q_\ell(k) \right| \ln\left(\frac{1}{u}\right), & \text{if } \rho \geq 0.5 \\ D_\ell(k) - \alpha \left| x_\ell(k) - \frac{1}{SS} \sum_{\ell=1}^{SS} q_\ell(k) \right| \ln\left(\frac{1}{u}\right), & \text{if } \rho < 0.5 \end{cases} \end{aligned} \quad (24)$$

where  $\rho$  is the best observation given by the cardinality of the observer sets as presented in [39].

The last step of AQPSO is to verify the termination criterion using the total number of iterations  $It$ , and convergence tolerance value  $e$ . The process finishes if one of the conditions given in (25) is satisfied

$$\text{Convergence criteria: } \begin{cases} k = It \\ |g(k) - g(k-1)| \leq e \end{cases} \quad (25)$$

## 4 Cost-benefit problem formulation

This section presents the cost-benefit analysis from the point of view of reliability. The SM model is applied to power generators.

### 4.1 Generation adequacy net benefit

The annual reliability index of the power system under consideration is the loss of energy expectation (LOEE) given in MWh/year. This index is calculated using sequential Monte Carlo simulation (SMCS). SMCS starts by generating random number for each 1 h time slot sampling during the time analysis  $T_y$ . This is executed for each unit generation in the power system, such that if the generated number is greater than the unavailability, the component goes to the 'normal operation' state, otherwise, the component goes to 'not in service' state. The next step is to calculate the margin generation by taking the difference between the available hourly power generation and the hourly demand, as presented in Fig. 5. The sum of the negative margin at experiment  $ex$  determines the energy not supplied  $ENS_{ex}$ .

The value  $ENS_{ex}$  is saved, and one experiment is completed. The process is repeated  $NE$  times. Finally, the LOEE is calculated by [12]

$$\text{LOEE} = \frac{1}{NE} \sum_{ex=1}^{NE} ENS_{ex} \quad (26)$$

Then, the benefit is estimated as follows:

$$\text{Benefit} = K_E \left( \sum_{t=1}^{T_y} E(t) - \text{LOEE} \right) \quad (27)$$

where  $E$  corresponds to the energy served at hour  $t$  and  $K_E$  is the price per unit energy.

On the other hand, maintenance cost is the price paid for the actions taken to preserve or restore a good or a product to an operational state which depends on its type. In the case of CM action, the cost is related to the repair or substitution of the failed part in the component. In case of PM action, the cost is related to the material needed to perform inspection and prevent any failure.

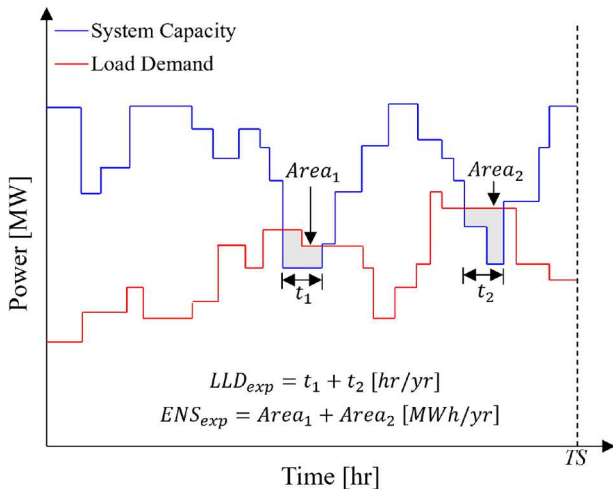


Fig. 5 Generation-demand margin model

Hence, the total maintenance cost in the interval  $(0, T_s]$  can be expressed as [40]

$$\text{Cost}_{\text{maintenance}} = \sum_{e=1}^{NC} (\text{Cost}_{\text{PM},e} M_{\text{PM},e} + \text{Cost}_{\text{CM},e} M_{\text{CM},e}) \quad (28)$$

where  $NC$  is the total number of components in the system,  $\text{Cost}_{\text{PM},e}$  is the price of performing one PM on component  $eth$ ,  $\text{Cost}_{\text{CM},e}$  is the price of performing one CM on component  $eth$ ,  $M_{\text{PM},e}$  is the total number of PM performed on component  $eth$  during time interval  $(0, T_s]$  and  $M_{\text{CM},e}$  is the total number of CM performed on component  $eth$  during time interval  $(0, T_s]$ .

If the time analysis is greater than the component's lifetime, an extra cost will appear. This cost is due to component's renewal labelled as  $\text{Cost}_{\text{renew},e}$ . In addition, there is a cost involved with the operation of the generator defined as  $\text{Cost}_{\text{op},e}$ . Therefore, the total cost is

$$\text{Cost}_{\text{total}} = \text{Cost}_{\text{maintenance}} + \sum_{e=1}^{NC} \text{Cost}_{\text{renew},e} + \sum_{e=1}^{NC} \text{Cost}_{\text{op},e} \quad (29)$$

Finally, the net benefit is defined as given in the following equation:

$$NB = \text{Benefit} - \text{Cost}_{\text{total}} \quad (30)$$

### 4.2 Optimisation problem

The main goal is to maximise the net benefit by obtaining the optimum PM scheduling. Hence, the optimisation problem can be defined as follows:

$$\text{maximise}(NB) \quad (31)$$

Subject to

$$M_{\text{PM},e}, M_{\text{CM},e} \in \mathbb{N} \quad (32)$$

$$(t_{\text{PM},e} \in \mathbb{N}) \leq T_y \quad (33)$$

$$\exists f \in F \Rightarrow \exists \text{CM}: t < T_W \quad (34)$$

The restrictions shown in (32) indicate that the number of CM and PM must be positive integers. Restriction (33) indicates that the time to perform the PM must be an integer in the interval  $(0, T_y]$ . The last restriction (34) states that in case that the failure  $f$  (element of the set of failures events  $F$ ) is detected, a CM will immediately take place, as long as, component's end lifetime is not reached.

## 5 Algorithm proposed for optimum PM schedule

The proposed algorithm is based on AQPSO and it takes as input the generator reliability data, specifically their failure and repair rates of the generators and the load demand profile during the time of analysis. Next, the maximum number of iterations  $It$  and the swarm size  $SS$  is defined. The particle  $x_\ell$  represents a PM schedule. The generators subject to PM are saved in  $x_\ell \cdot gPM$ , while the time to perform the PM is saved in  $x_\ell \cdot t_{\text{PM}}$ . The particle contains a discrete number such that  $x_\ell \cdot gPM = 0$  or  $x_\ell \cdot gPM = 1$  ('0' indicates do not perform maintenance, while '1' indicates the opposite). Since the analysis is for  $k = 0$  then  $q_\ell(0) = x_\ell(0)$ .

The next step is to add the information of particle to the power system, then the generation adequacy assessment takes place and the net benefit for each particle is determined.  $g(0)$  is obtained based on the particle that has the maximum net benefit value. At this point, the first iteration takes place, and the particles start their motion. The attraction parameter  $D_\ell(k)$  is obtained using (23). Then, the new position of the particle is computed using (24). The updated position of the particle represents a new PM schedule that may lead to a better net benefit. Then, generation adequacy is

computed to obtain a new net benefit value, which is saved in  $x_\ell \cdot NB(k)$ .

Next, the particles compete between them to determine the best PM schedule. If  $x_\ell \cdot NB(k) > x_\ell \cdot NB(k-1)$  then the particle updates its position. The 'personal best' is also updated and saved in  $q_\ell$ . This is followed by a second comparison in which the 'global best' is considered. For this purpose, it is required to find the best  $x_\ell(k)$  among the swarm such that it brings the maximum savings. The best particle is saved in the variable  $g'$ . If the  $g' \cdot NB > g \cdot NB$  then the  $x_\ell(k)$  becomes the 'global best', otherwise the 'global best' is not replaced. In case that  $x_\ell \cdot NB(k) \leq x_\ell \cdot NB(k)$  then the process continues with the next particle and one iteration is finished. Then, the process is repeated  $It$  times or the convergence criteria presented in is satisfied. Consider for the convergence criteria a  $|x_\ell \cdot EENS(k) - x_\ell \cdot EENS(k-1)| < 10^{-6}$ . Finally, the

outcome is the best PM schedule. For more details regarding the process of the determination of the effective PM schedule, Fig. 6 is presented.

## 6 Case study

The study incorporates the Roy Billinton Test System (RBTS) [12]. Four scenarios are evaluated: (i) no PM (NPM); (ii) yearly periodic PM (PPM); (iii) RCM using PSO; and (iv) SM. The study is conducted for the next 50 years. To simplify the analysis, the assumptions are: (i) generators reliability and cost data is as given in Tables 2 and 3, respectively; (ii) the failure and repair rates of each generator follow the behaviour described in Fig. 7; (iii) energy price is  $K_E = 0.082$  [£/kWh] with a yearly increment of 3%; and (iv) yearly load profile is as shown in Fig. 8 with a yearly increment of 0.5%; (v) all costs present a yearly increment of 2%; (vi) the operation and repair actions are carried out according to the manufacturer's guide (Fig. 7).

## 7 Results and discussion

### 7.1 Smart maintenance schedule

The PM reduces the occurrence of failures of the generators; then, the greater the number of PM performed, the more reliable the component becomes. Nevertheless, there must be a balance between the maintenance cost and its benefit and over this statement the theory of SM is formulated. By using the algorithm described in Section 5, the PM schedule that maximises the net benefit is obtained. As a result, Figs. 9 and 10 show the optimum PM scheduling applied to hydro and thermal UG, respectively.

To understand these figures, a symbol is given for every UG. Then, the time when the action takes place is given by the interception point formed from the figure axis, that is, the month is given by the x-axis and the year is given by its y-axis. For instance, the first PM to execute for H5 (blue circle) is in June of the second year. It is to be noted that most of the PM starts after the second year. In the subsequent years, an average of one maintenance per year is recommended. Then, between three to four years (on average) before the UG reaches its end lifetime, it is recommended not to perform PM.

A relevant fact that occurs during the optimisation process is that SM defines a hierarchical level for each UG based on its failure rate, capacity and lifetime. For instance, UGs with the higher failure rate, lower lifetime and lower capacity require more maintenance than the ones with lower failure rate, higher lifetime and higher capacity, as presented in Figs. 9 and 10.

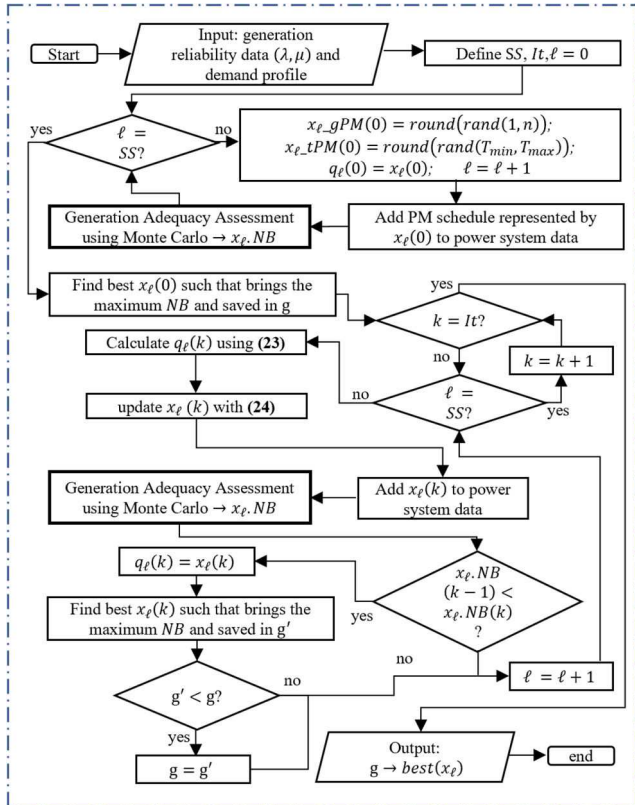


Fig. 6 SM flowchart to maximise the net benefit of generators

Table 2 Generators reliability data

Unit generation, H: hydro T: thermal		H5	T10	H20	T20	H40	T40
No. of units		2	1	4	1	1	2
size, MW		5	10	20	20	40	40
$T_w$ , yr		25	20	30	25	40	35
$\lambda_C$ [1/year]	$\alpha_C$	2.0	4.0	2.4	5.0	3.0	6.0
	$\alpha_C$	0.50	0.40	0.85	0.40	0.88	0.78
	$\omega$	25	20	30	25	40	35
$\mu_C$ [1/year]	$b$	198	196	158	195	147	194
	$\sigma$	0.20	0.10	0.10	0.25	0.30	0.30
	$\gamma$	0.28	0.25	0.25	0.27	0.15	0.19

Table 3 Cost of the unit generations

UG	H5	T10	H10	T20	H40	T40
acquisition cost, M£	40	40	80	60	160	80
operation cost, k£/year	12.5	600	50	680	100	790
PM cost, k£	25	100	60	200	120	220
CM cost, k£	156	625	375	1250	750	1375

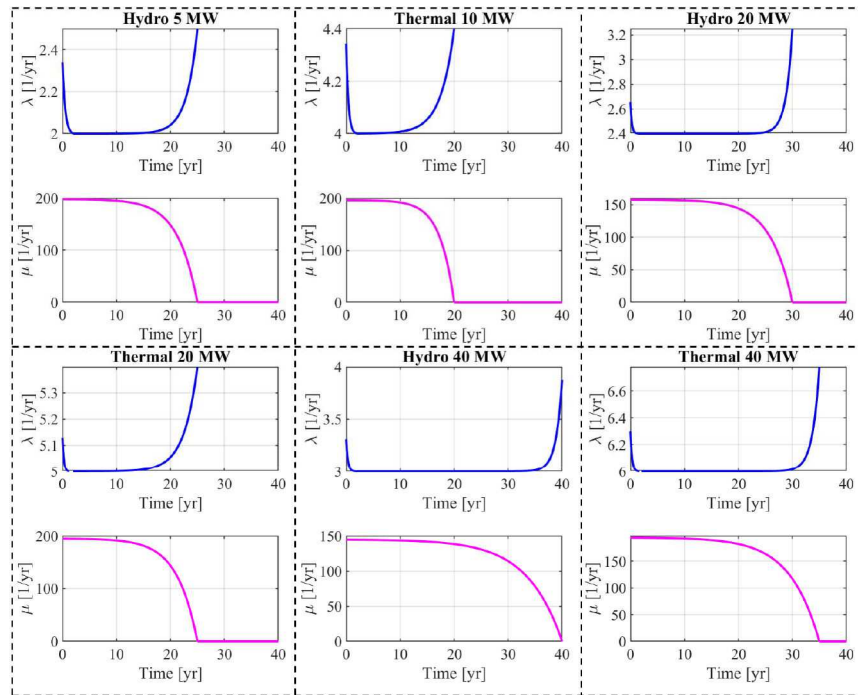


Fig. 7 Failure and repair rate of each unit generation

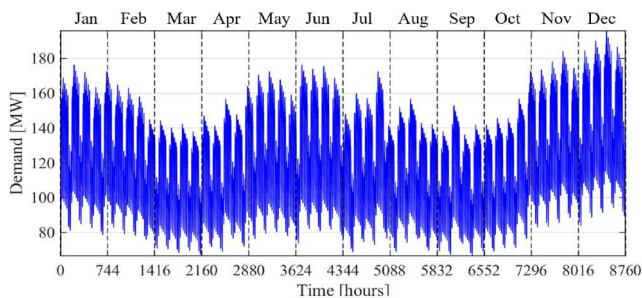


Fig. 8 Yearly load profile

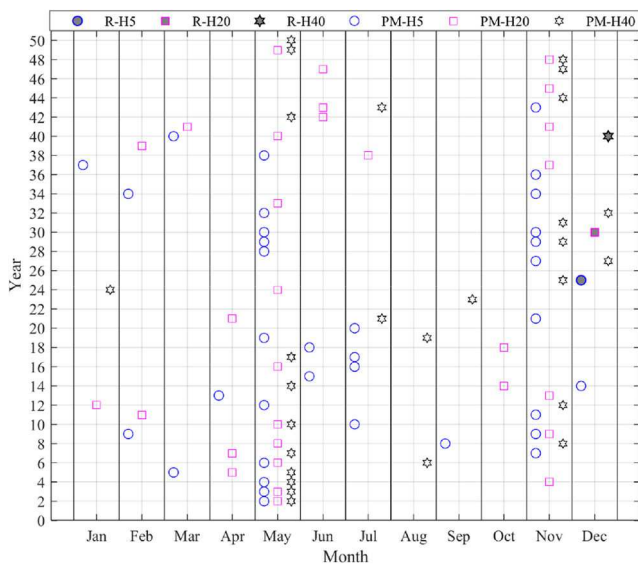


Fig. 9 SM plan for hydro unit generation

## 7.2 Unit generation degradation

As presented in Section 2.3, the degradation of each generator can be obtained using the data given in Table 2 in expression (10). Fig. 11 shows the degradation function for each generator. The figure depicts that there is a gradual increment on the degradation magnitude. The generators reach the obsolescence state when the

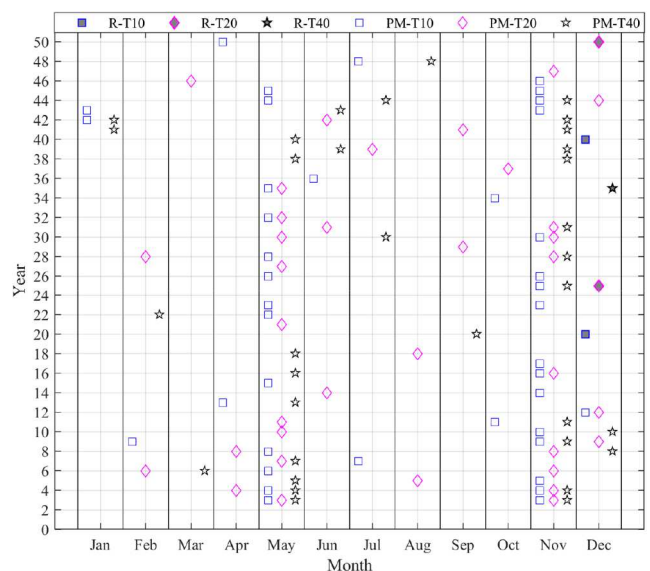


Fig. 10 SM plan for thermal unit generation

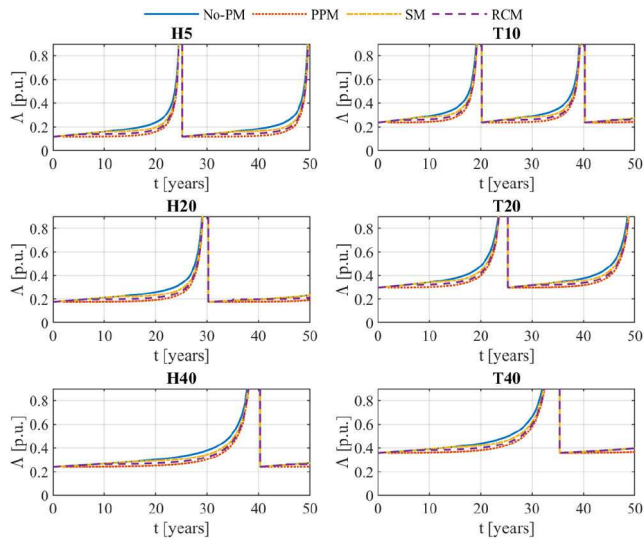
degradation presents an extremely high value, as established in (11).

The maintenance strategy with the lowest degradation belongs to PPM, followed by RCM and SM. This fact is associated with the number of PMs, as presented in Table 4. The more PM executed, the slower the degradation process; this circumstance has repercussion over the reliability of the UG, as is discussed in next section.

## 7.3 Unit generation reliability model

Using the failure and repair rates of the component, probability vector of states is determined for each unit generation by following the process described in Section 2.4, the probability function for each generator is obtained. Then, the reliability model of the component is determined using (21). Fig. 12 presents the availability as a function of time. It can be observed in Fig. 12 that as time passes the probability of being in a 'normal operation' (availability) decreases. At some point, the obsolescence becomes 100% which indicates that the component has reached the end of





**Fig. 11** Degradation of each unit generation under different maintenance strategies

**Table 4** Summary of the maintenances

Unit generation	Number of PM within 50 years			
	NPM	PPM	RCM	SM
H5	0	50	30	26
T10	0	50	35	30
H20	0	50	40	36
T20	0	50	42	39
H40	0	50	45	42
T40	0	50	46	45

its lifetime, then a replacement takes place, and, finally the process starts again.

When no PM is considered (No-PM), the UG's availability decreases faster than the other scenarios. This fact is reasonable since as exhibited in Section 7.2, scenario No-PM presents the highest values of degradation. Concerning PPM, RCM and SM, some UGs have slightly different values of availability than others. Nevertheless, for all UG, at some point, the availability curve of the SM tends to be under the RCM followed by PPM availability curve. Therefore, PPM is the most reliable strategy. However, PPM underestimates the optimal time to perform PM that maximises the net benefit.

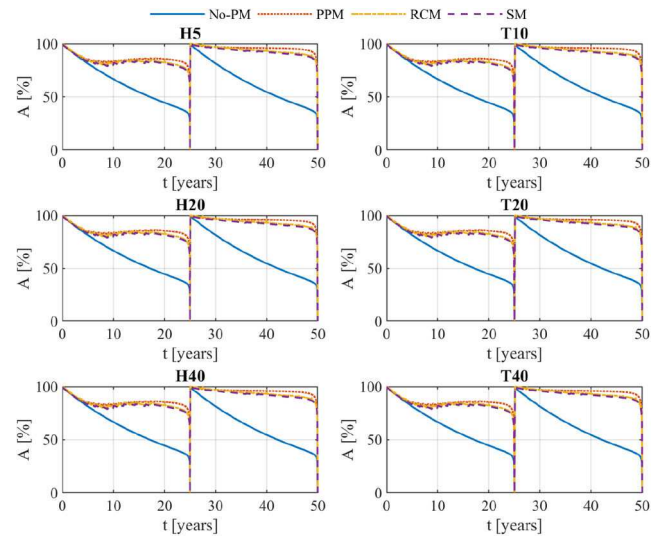
#### 7.4 Net benefit

There are several variables involved in the net benefit mathematical formulation, nevertheless, it mainly depends on the number of PMs executed and LOEE index. Fig. 13 shows the yearly LOEE discrete value for every scenario. In all scenarios, there is a tendency of gradual increments of LOEE due to ageing, and at some points, the values decrease considerably attributed to a UG replacement.

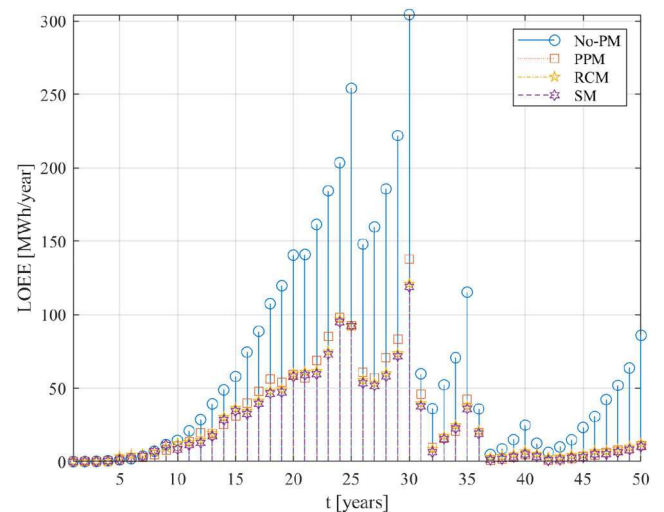
The No-PM scenario shows the highest LOEE, while SM scenario provides the lowest LOEE in most of the time, followed by the RCM and PPM. Consequently, SM presents the lowest value of energy not supplied and brings the highest net benefit since there is an inverse relationship between the benefit and LOEE index, as shown in (27). The net benefit for every scenario is presented in Table 5.

### 8 Computational performance

In order to show the computational efficacy of the AQPSO, the same generation scheduling maintenance problem is solved using traditional particle swarm optimisation with 50 particles and 100 iterations (PSO), genetic algorithm with 50 genes and 100 generations (GA), and simulated annealing with 50 neighbours and



**Fig. 12** Availability of each unit generation under different maintenance strategies



**Fig. 13** LOEE per year

**Table 5** Net benefit after 50 years

Scenario	No-PM	PPM	RCM	SM
net benefit [£] × 10 <sup>6</sup>	0.2456	1.475	1.789	1.989

100 steps (SA). A computer with a RAM of 16.0 GB and processor of Intel Core i7-6700 of 3.40 GHz is used to run the algorithms.

In addition to the power system of 6 bus (RBTS) [12] used in Section 6, the IEEE 73-bus reliability test system (RTS) [41] is also employed. This is introduced to compare the computational efficacy under two different power system sizes that is with small-scale (RBTS) and large-scale (RTS).

The computational efficacy in terms of convergence is presented in Fig. 14. The results depict that for the small-scale power system (RBTS), there is not much difference (<3 iterations) in the convergence between optimisation techniques. However, for the large-scale power system (RTS), there is a notable difference in computational performance. In percentage terms, AQPSO convergence exceeds in iterations by 36.6, 22.5, 19.7% to PSO, GA and SA, respectively. Therefore, the optimisation technique with the highest convergence performance is given by AQPSO.

The time simulation is given in Fig. 15. The results reveal that for the small-scale power system (RBTS), the simulation time difference among the optimisation techniques does not exceed 0.2%. In contrast, the simulation time difference for large-scale power system is very significant. The AQPSO presents the lowest simulation time with 85.2 s, followed by SA with 95.2 s, GA with

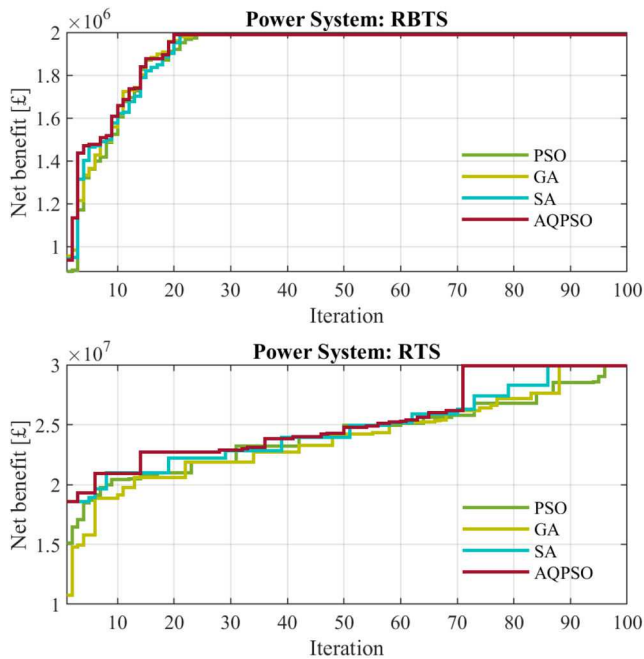


Fig. 14 Convergence behaviour

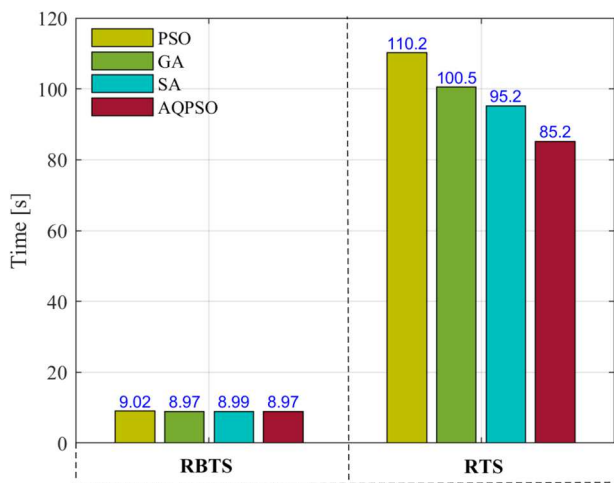


Fig. 15 Simulation time per experiment

100.5 s and PSO with 110.2 s. Thus, AQPSO prove to be the most outperformed optimisation technique (among PSO, GA and SA) due to its inherited properties (convergence iteration and simulation time).

## 9 Conclusion

This paper proposes an innovative approach to SM of generators with the aim of maximising net reliability benefits through the improvement of generators' operating life. The approach incorporates KMI and Markov chain to establish the relationship between the component's lifetime, virtual age and transition rates. The main engine of the model is based on the AQPSO algorithm, which determines the optimum PM schedules of generators.

The case study validated the effectiveness of the approach. The results argue that the greater number of PMs could potentially lead to a considerable level of the improvement in the reliability of power supply. Nevertheless, it is vital to perform PM at the benefit horizons to avoid superseding the cost over benefit.

AQPSO also outperforms PSO, GA and SA in terms of the number of iterations required and simulation time, maintaining the accuracy and robustness. Thus, AQPSO could be considered as one of the most effective technique to solve the SM problem. The effectiveness increases with the size of the power system.

The proposed approach is beneficial for designing SM schedules of generators with the aim of improving the reliability

performances in the long run. Although the paper employs the SM scheme for generators, the model can be extended for other power system components, providing a range of opportunities for the operational planning of modern power systems.

## 10 Acknowledgments

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