Instructor's Solution Manual
for

Fundamentals of Physics, 6/E
by Halliday, Resnick, and Walker

James B. Whitenton
Southern Polytechnic State University

Preface

This booklet includes the solutions relevant to the EXERCISES & PROBLEMS sections of the 6^{th} edition of **Fundamentals of Physics**, by Halliday, Resnick, and Walker. We also include solutions to problems in the Problems Supplement. We have not included solutions or discussions which pertain to the QUESTIONS sections.

I am very grateful for helpful input from J. Richard Christman, Meighan Dillon, Barbara Moore, and Jearl Walker regarding the development of this document.

Chapter 1

1. The metric prefixes (micro, pico, nano, ...) are given for ready reference on the inside front cover of the textbook (see also Table 1-2).

   (a) Since $1\,\text{km} = 1 \times 10^3\,\text{m}$ and $1\,\text{m} = 1 \times 10^6\,\mu\text{m}$,

   $$1\,\text{km} = 10^3\,\text{m} = (10^3\,\text{m})(10^6\,\mu\text{m/m}) = 10^9\,\mu\text{m}\,.$$

   The given measurement is 1.0 km (two significant figures), which implies our result should be written as $1.0 \times 10^9\ \mu\text{m}$.

   (b) We calculate the number of microns in 1 centimeter. Since $1\,\text{cm} = 10^{-2}\,\text{m}$,

   $$1\,\text{cm} = 10^{-2}\,\text{m} = (10^{-2}\,\text{m})(10^6\,\mu\text{m/m}) = 10^4\,\mu\text{m}\,.$$

   We conclude that the fraction of one centimeter equal to $1.0\,\mu\text{m}$ is 1.0×10^{-4}.

   (c) Since $1\,\text{yd} = (3\,\text{ft})(0.3048\,\text{m/ft}) = 0.9144\,\text{m}$,

   $$1.0\,\text{yd} = (0.91\,\text{m})(10^6\,\mu\text{m/m}) = 9.1 \times 10^5\,\mu\text{m}\,.$$

2. The customer expects $20 \times 7056\ \text{in}^3$ and receives $20 \times 5826\ \text{in}^3$, the difference being 24600 cubic inches, or

   $$\left(24600\ \text{in}^3\right)\left(\frac{2.54\ \text{cm}}{1\ \text{inch}}\right)^3\left(\frac{1\ \text{L}}{1000\ \text{cm}^3}\right) = 403\ \text{L}$$

   where Appendix D has been used (see also Sample Problem 1-2).

3. Using the given conversion factors, we find

   (a) the distance d *in rods* to be

   $$d = 4.0\,\text{furlongs} = \frac{(4.0\,\text{furlongs})(201.168\,\text{m/furlong})}{5.0292\,\text{m/rod}} = 160\,\text{rods}\,,$$

   (b) and that distance *in chains* to be

   $$d = \frac{(4.0\,\text{furlongs})(201.168\,\text{m/furlong})}{20.117\,\text{m/chain}} = 40\,\text{chains}\ .$$

4. (a) Recalling that 2.54 cm equals 1 inch (exactly), we obtain

   $$(0.80\ \text{cm})\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{6\ \text{picas}}{1\ \text{inch}}\right)\left(\frac{12\ \text{points}}{1\ \text{pica}}\right) \approx 23\ \text{points}\ ,$$

   (b) and

   $$(0.80\ \text{cm})\left(\frac{1\ \text{inch}}{2.54\ \text{cm}}\right)\left(\frac{6\ \text{picas}}{1\ \text{inch}}\right) \approx 1.9\ \text{picas}\ .$$

5. Various geometric formulas are given in Appendix E.

   (a) Substituting
   $$R = \left(6.37 \times 10^6\,\mathrm{m}\right)\left(10^{-3}\,\mathrm{km/m}\right) = 6.37 \times 10^3\,\mathrm{km}$$
   into *circumference* $= 2\pi R$, we obtain 4.00×10^4 km.

   (b) The surface area of Earth is
   $$4\pi R^2 = 4\pi\left(6.37 \times 10^3\,\mathrm{km}\right)^2 = 5.10 \times 10^8\ \mathrm{km}^2\ .$$

   (c) The volume of Earth is
   $$\frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(6.37 \times 10^3\,\mathrm{km}\right)^3 = 1.08 \times 10^{12}\ \mathrm{km}^3\ .$$

6. (a) Using the fact that the area A of a rectangle is width×length, we find
   $$\begin{aligned} A_{\text{total}} &= (3.00\text{ acre}) + (25.0\,\text{perch})(4.00\,\text{perch}) \\ &= (3.00\,\text{acre})\left(\frac{(40\,\text{perch})(4\,\text{perch})}{1\text{ acre}}\right) + 100\,\text{perch}^2 \\ &= 580\text{ perch}^2\ . \end{aligned}$$
   We multiply this by the perch^2 → rood conversion factor ($1\,\text{rood}/40\,\text{perch}^2$) to obtain the answer: $A_{\text{total}} = 14.5$ roods.

   (b) We convert our intermediate result in part (a):
   $$A_{\text{total}} = (580\text{ perch}^2)\left(\frac{16.5\text{ ft}}{1\text{ perch}}\right)^2 = 1.58 \times 10^5\ \text{ft}^2\ .$$
   Now, we use the feet → meters conversion given in Appendix D to obtain
   $$A_{\text{total}} = \left(1.58 \times 10^5\ \text{ft}^2\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)^2 = 1.47 \times 10^4\ \text{m}^2\ .$$

7. The volume of ice is given by the product of the semicircular surface area and the thickness. The semicircle area is $A = \pi r^2/2$, where r is the radius. Therefore, the volume is
   $$V = \frac{\pi}{2}r^2 z$$
   where z is the ice thickness. Since there are 10^3 m in 1 km and 10^2 cm in 1 m, we have
   $$r = (2000\,\text{km})\left(\frac{10^3\,\text{m}}{1\,\text{km}}\right)\left(\frac{10^2\,\text{cm}}{1\,\text{m}}\right) = 2000 \times 10^5\ \text{cm}\ .$$
   In these units, the thickness becomes
   $$z = (3000\,\text{m})\left(\frac{10^2\,\text{cm}}{1\,\text{m}}\right) = 3000 \times 10^2\,\text{cm}\ .$$
   Therefore,
   $$V = \frac{\pi}{2}\left(2000 \times 10^5\,\text{cm}\right)^2\left(3000 \times 10^2\,\text{cm}\right) = 1.9 \times 10^{22}\,\text{cm}^3\ .$$

8. The total volume V of the real house is that of a triangular prism (of height $h = 3.0$ m and base area $A = 20 \times 12 = 240$ m^2) in addition to a rectangular box (height $h' = 6.0$ m and same base). Therefore,
   $$V = \frac{1}{2}hA + h'A = \left(\frac{h}{2} + h'\right)A = 1800\ \text{m}^3\ .$$

(a) Each dimension is reduced by a factor of 1/12, and we find

$$V_{\text{doll}} = (1800\ \text{m}^3)\left(\frac{1}{12}\right)^3 \approx 1.0\ \text{m}^3\ .$$

(b) In this case, each dimension (relative to the real house) is reduced by a factor of 1/144. Therefore,

$$V_{\text{miniature}} = (1800\ \text{m}^3)\left(\frac{1}{144}\right)^3 \approx 6.0 \times 10^{-4}\ \text{m}^3\ .$$

9. We use the conversion factors found in Appendix D.

$$1\,\text{acre}\cdot\text{ft} = (43,560\,\text{ft}^2)\cdot\text{ft} = 43,560\,\text{ft}^3\,.$$

Since 2 in. = (1/6) ft, the volume of water that fell during the storm is

$$V = (26\,\text{km}^2)(1/6\,\text{ft}) = (26\,\text{km}^2)(3281\,\text{ft/km})^2(1/6\,\text{ft}) = 4.66 \times 10^7\,\text{ft}^3\,.$$

Thus,

$$V = \frac{4.66 \times 10^7\,\text{ft}^3}{4.3560 \times 10^4\,\text{ft}^3/\text{acre}\cdot\text{ft}} = 1.1 \times 10^3\,\text{acre}\cdot\text{ft}\,.$$

10. The metric prefixes (micro (μ), pico, nano, ...) are given for ready reference on the inside front cover of the textbook (also, Table 1-2).

$$\begin{aligned} 1\,\mu\text{century} &= (10^{-6}\,\text{century})\left(\frac{100\,\text{y}}{1\,\text{century}}\right)\left(\frac{365\,\text{day}}{1\,\text{y}}\right)\left(\frac{24\,\text{h}}{1\,\text{day}}\right)\left(\frac{60\,\text{min}}{1\,\text{h}}\right) \\ &= 52.6\,\text{min}\ . \end{aligned}$$

The percent difference is therefore

$$\frac{52.6\,\text{min} - 50\,\text{min}}{50\,\text{min}} = 5.2\%\,.$$

11. We use the conversion factors given in Appendix D and the definitions of the SI prefixes given in Table 1-2 (also listed on the inside front cover of the textbook). Here, "ns" represents the nanosecond unit, "ps" represents the picosecond unit, and so on.

(a) $1\,\text{m} = 3.281\,\text{ft}$ and $1\,\text{s} = 10^9\,\text{ns}$. Thus,

$$3.0 \times 10^8\,\text{m/s} = \left(\frac{3.0 \times 10^8\,\text{m}}{\text{s}}\right)\left(\frac{3.281\,\text{ft}}{\text{m}}\right)\left(\frac{\text{s}}{10^9\,\text{ns}}\right) = 0.98\,\text{ft/ns}\,.$$

(b) Using $1\,\text{m} = 10^3\,\text{mm}$ and $1\,\text{s} = 10^{12}\,\text{ps}$, we find

$$\begin{aligned} 3.0 \times 10^8\,\text{m/s} &= \left(\frac{3.0 \times 10^8\,\text{m}}{\text{s}}\right)\left(\frac{10^3\,\text{mm}}{\text{m}}\right)\left(\frac{\text{s}}{10^{12}\,\text{ps}}\right) \\ &= 0.30\ \text{mm/ps}\ . \end{aligned}$$

12. The number of seconds in a year is 3.156×10^7. This is listed in Appendix D and results from the product

$$(365.25\,\text{day/y})(24\,\text{h/day})(60\,\text{min/h})(60\,\text{s/min})\ .$$

(a) The number of shakes in a second is 10^8; therefore, there are indeed more shakes per second than there are seconds per year.

(b) Denoting the age of the universe as 1 u-day (or 86400 u-sec), then the time during which humans have existed is given by

$$\frac{10^6}{10^{10}} = 10^{-4} \text{ u-day} ,$$

which we may also express as

$$\left(10^{-4} \text{ u-day}\right)\left(\frac{86400 \text{ u-sec}}{1 \text{ u-day}}\right) = 8.6 \text{ u-sec} .$$

13. None of the clocks advance by exactly 24 h in a 24-h period but this is not the most important criterion for judging their quality for measuring time intervals. What is important is that the clock advance by the same amount in each 24-h period. The clock reading can then easily be adjusted to give the correct interval. If the clock reading jumps around from one 24-h period to another, it cannot be corrected since it would impossible to tell what the correction should be. The following gives the corrections (in seconds) that must be applied to the reading on each clock for each 24-h period. The entries were determined by subtracting the clock reading at the end of the interval from the clock reading at the beginning.

CLOCK	Sun. -Mon.	Mon. -Tues.	Tues. -Wed.	Wed. -Thurs.	Thurs. -Fri.	Fri. -Sat
A	−16	−16	−15	−17	−15	−15
B	−3	+5	−10	+5	+6	−7
C	−58	−58	−58	−58	−58	−58
D	+67	+67	+67	+67	+67	+67
E	+70	+55	+2	+20	+10	+10

Clocks C and D are both good timekeepers in the sense that each is consistent in its daily drift (relative to WWF time); thus, C and D are easily made "perfect" with simple and predictable corrections. The correction for clock C is less than the correction for clock D, so we judge clock C to be the best and clock D to be the next best. The correction that must be applied to clock A is in the range from 15 s to 17s. For clock B it is the range from −5 s to +10 s, for clock E it is in the range from −70 s to −2 s. After C and D, A has the smallest range of correction, B has the next smallest range, and E has the greatest range. From best the worst, the ranking of the clocks is C, D, A, B, E.

14. The time on any of these clocks is a straight-line function of that on another, with slopes $\neq 1$ and y-intercepts $\neq 0$. From the data in the figure we deduce

$$\begin{aligned} t_C &= \frac{2}{7}t_B + \frac{594}{7} \\ t_B &= \frac{33}{40}t_A - \frac{662}{5} . \end{aligned}$$

These are used in obtaining the following results.

(a) We find

$$t'_B - t_B = \frac{33}{40}\left(t'_A - t_A\right) = 495 \text{ s}$$

when $t'_A - t_A = 600$ s.

(b) We obtain

$$t'_C - t_C = \frac{2}{7}\left(t'_B - t_B\right) = \frac{2}{7}(495) = 141 \text{ s} .$$

(c) Clock B reads $t_B = (33/40)(400) - (662/5) \approx 198$ s when clock A reads $t_A = 400$ s.

(d) From $t_C = 15 = (2/7)t_B + (594/7)$, we get $t_B \approx -245$ s.

15. We convert meters to astronomical units, and seconds to minutes, using

$$\begin{aligned} 1000\,\text{m} &= 1\,\text{km} \\ 1\,\text{AU} &= 1.50\times 10^8\,\text{km} \\ 60\,\text{s} &= 1\,\text{min}\;. \end{aligned}$$

Thus, 3.0×10^8 m/s becomes

$$\left(\frac{3.0\times 10^8\,\text{m}}{\text{s}}\right)\left(\frac{1\,\text{km}}{1000\,\text{m}}\right)\left(\frac{\text{AU}}{1.50\times 10^8\,\text{km}}\right)\left(\frac{60\,\text{s}}{\text{min}}\right) = 0.12\,\text{AU/min}\,.$$

16. Since a change of longitude equal to 360° corresponds to a 24 hour change, then one expects to change longitude by $360^\circ/24 = 15^\circ$ before resetting one's watch by 1.0 h.

17. The last day of the 20 centuries is longer than the first day by

$$(20\,\text{century})(0.001\,\text{s/century}) = 0.02\,\text{s}\,.$$

The average day during the 20 centuries is $(0 + 0.02)/2 = 0.01$ s longer than the first day. Since the increase occurs uniformly, the cumulative effect T is

$$\begin{aligned} T &= \text{(average increase in length of a day)(number of days)} \\ &= \left(\frac{0.01\,\text{s}}{\text{day}}\right)\left(\frac{365.25\,\text{day}}{\text{y}}\right)(2000\,\text{y}) \\ &= 7305\text{ s} \end{aligned}$$

or roughly two hours.

18. We denote the pulsar rotation rate f (for frequency).

$$f = \frac{1\text{ rotation}}{1.55780644887275\times 10^{-3}\,\text{s}}$$

(a) Multiplying f by the time-interval $t = 7.00$ days (which is equivalent to 604800 s, if we ignore *significant figure* considerations for a moment), we obtain the number of rotations:

$$N = \left(\frac{1\,\text{rotation}}{1.55780644887275\times 10^{-3}\,\text{s}}\right)(604800\,\text{s}) = 388238218.4$$

which should now be rounded to 3.88×10^8 rotations since the time-interval was specified in the problem to three significant figures.

(b) We note that the problem specifies the *exact* number of pulsar revolutions (one million). In this case, our unknown is t, and an equation similar to the one we set up in part (a) takes the form

$$\begin{aligned} N &= ft \\ 1\times 10^6 &= \left(\frac{1\text{ rotation}}{1.55780644887275\times 10^{-3}\,\text{s}}\right)t \end{aligned}$$

which yields the result $t = 1557.80644887275$ s (though students who do this calculation on their calculator might not obtain those last several digits).

(c) Careful reading of the problem shows that the time-uncertainty *per revolution* is $\pm 3\times 10^{-17}$ s. We therefore expect that as a result of one million revolutions, the uncertainty should be $(\pm 3\times 10^{-17})(1\times 10^6) = \pm 3\times 10^{-11}$ s.

19. If M_E is the mass of Earth, m is the average mass of an atom in Earth, and N is the number of atoms, then $M_E = Nm$ or $N = M_E/m$. We convert mass m to kilograms using Appendix D ($1\,\text{u} = 1.661\times10^{-27}\,\text{kg}$). Thus,

$$N = \frac{M_E}{m} = \frac{5.98\times10^{24}\,\text{kg}}{(40\,\text{u})(1.661\times10^{-27}\,\text{kg/u})} = 9.0\times10^{49}\ .$$

20. To organize the calculation, we introduce the notion of density (which the students have probably seen in other courses):

$$\rho = \frac{m}{V}\ .$$

   (a) We take the volume of the leaf to be its area A multiplied by its thickness z. With density $\rho = 19.32\ \text{g/cm}^3$ and mass $m = 27.63$ g, the volume of the leaf is found to be

$$V = \frac{m}{\rho} = 1.430\ \text{cm}^3\ .$$

   We convert the volume to SI units:

$$(1.430\ \text{cm}^3)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 = 1.430\times10^{-6}\ \text{m}^3\ .$$

   And since $V = A\,z$ where $z = 1\times10^{-6}$ m (metric prefixes can be found in Table 1-2), we obtain

$$A = \frac{1.430\times10^{-6}\,\text{m}^3}{1\times10^{-6}\,\text{m}} = 1.430\ \text{m}^2\ .$$

   (b) The volume of a cylinder of length ℓ is $V = A\ell$ where the cross-section area is that of a circle: $A = \pi r^2$. Therefore, with $r = 2.500\times10^{-6}$ m and $V = 1.430\times10^{-6}\,\text{m}^3$, we obtain

$$\ell = \frac{V}{\pi r^2} = 7.284\times10^4\ \text{m}\ .$$

21. We introduce the notion of density (which the students have probably seen in other courses):

$$\rho = \frac{m}{V}$$

and convert to SI units: $1\,\text{g} = 1\times10^{-3}\,\text{kg}$.

   (a) For volume conversion, we find $1\,\text{cm}^3 = (1\times10^{-2}\,\text{m})^3 = 1\times10^{-6}\,\text{m}^3$. Thus, the density in kg/m^3 is

$$1\,\text{g/cm}^3 = \left(\frac{1\,\text{g}}{\text{cm}^3}\right)\left(\frac{10^{-3}\,\text{kg}}{\text{g}}\right)\left(\frac{\text{cm}^3}{10^{-6}\,\text{m}^3}\right) = 1\times10^3\ \text{kg/m}^3\ .$$

   Thus, the mass of a cubic meter of water is 1000 kg.

   (b) We divide the mass of the water by the time taken to drain it. The mass is found from $M = \rho V$ (the product of the volume of water and its density):

$$M = (5700\,\text{m}^3)(1\times10^3\,\text{kg/m}^3) = 5.70\times10^6\ \text{kg}\ .$$

   The time is $t = (10\,\text{h})(3600\,\text{s/h}) = 3.6\times10^4\,\text{s}$, so the *mass flow rate* R is

$$R = \frac{M}{t} = \frac{5.70\times10^6\,\text{kg}}{3.6\times10^4\,\text{s}} = 158\ \text{kg/s}\ .$$

22. The volume of the water that fell is

$$\begin{aligned} V &= (26\,\text{km}^2)(2.0\,\text{in.}) \\ &= (26\,\text{km}^2)\left(\frac{1000\,\text{m}}{1\,\text{km}}\right)^2 (2.0\,\text{in.})\left(\frac{0.0254\,\text{m}}{1\,\text{in.}}\right) \\ &= (26\times 10^6\,\text{m}^2)(0.0508\,\text{m}) \\ &= 1.3\times 10^6\ \text{m}^3\ . \end{aligned}$$

We write the mass-per-unit-volume (density) of the water as:

$$\rho = \frac{m}{V} = 1\times 10^3\ \text{kg/m}^3\ .$$

The mass of the water that fell is therefore given by $m = \rho V$:

$$\begin{aligned} m &= \left(1\times 10^3\,\text{kg/m}^3\right)\left(1.3\times 10^6\,\text{m}^3\right) \\ &= 1.3\times 10^9\ \text{kg}\ . \end{aligned}$$

23. We introduce the notion of density (which the students have probably seen in other courses):

$$\rho = \frac{m}{V}$$

and convert to SI units: $1000\,\text{g} = 1\,\text{kg}$, and $100\,\text{cm} = 1$ m.

(a) The density ρ of a sample of iron is therefore

$$\rho = \left(7.87\,\text{g/cm}^3\right)\left(\frac{1\,\text{kg}}{1000\,\text{g}}\right)\left(\frac{100\,\text{cm}}{1\,\text{m}}\right)^3$$

which yields $\rho = 7870$ kg/m^3. If we ignore the empty spaces between the close-packed spheres, then the density of an individual iron atom will be the same as the density of any iron sample. That is, if M is the mass and V is the volume of an atom, then

$$V = \frac{M}{\rho} = \frac{9.27\times 10^{-26}\,\text{kg}}{7.87\times 10^3\,\text{kg/m}^3} = 1.18\times 10^{-29}\ \text{m}^3\ .$$

(b) We set $V = 4\pi R^3/3$, where R is the radius of an atom (Appendix E contains several geometry formulas). Solving for R, we find

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3(1.18\times 10^{-29}\,\text{m}^3)}{4\pi}\right)^{1/3} = 1.41\times 10^{-10}\,\text{m}\,.$$

The center-to-center distance between atoms is twice the radius, or 2.82×10^{-10} m.

24. The metric prefixes (micro (μ), pico, nano, ...) are given for ready reference on the inside front cover of the textbook (see also Table 1-2). The surface area A of each grain of sand of radius $r = 50\,\mu\text{m} = 50\times 10^{-6}$ m is given by $A = 4\pi(50\times 10^{-6})^2 = 3.14\times 10^{-8}$ m^2 (Appendix E contains a variety of geometry formulas). We introduce the notion of density (which the students have probably seen in other courses):

$$\rho = \frac{m}{V}$$

so that the mass can be found from $m = \rho V$, where $\rho = 2600$ kg/m^3. Thus, using $V = 4\pi r^3/3$, the mass of each grain is

$$m = \left(\frac{4\pi\left(50\times 10^{-6}\,\text{m}\right)^3}{3}\right)\left(2600\,\frac{\text{kg}}{\text{m}^3}\right) = 1.36\times 10^{-9}\ \text{kg}\ .$$

We observe that (because a cube has six equal faces) the indicated surface area is 6 m^2. The number of spheres (the grains of sand) N which have a total surface area of 6 m^2 is given by

$$N = \frac{6\,\mathrm{m}^2}{3.14 \times 10^{-8}\,\mathrm{m}^2} = 1.91 \times 10^8 \ .$$

Therefore, the total mass M is given by

$$M = Nm = \left(1.91 \times 10^8\right)\left(1.36 \times 10^{-9}\ \mathrm{kg}\right) = 0.260\,\mathrm{kg} \ .$$

25. From the Figure we see that, regarding differences in positions Δx, 212 S is equivalent to 258 W and 180 S is equivalent to 156 Z. Whether or not the origin of the Zelda path coincides with the origins of the other paths is immaterial to consideration of Δx.

(a)

$$\Delta x = (50.0\ \mathrm{S})\left(\frac{258\ \mathrm{W}}{212\ \mathrm{S}}\right) = 60.8\ \mathrm{W}$$

(b)

$$\Delta x = (50.0\ \mathrm{S})\left(\frac{156\ \mathrm{Z}}{180\ \mathrm{S}}\right) = 43.3\ \mathrm{Z}$$

26. The first two conversions are easy enough that a *formal* conversion is not especially called for, but in the interest of *practice makes perfect* we go ahead and proceed formally:

(a)

$$(11\ \text{tuffet})\left(\frac{2\ \text{peck}}{1\ \text{tuffet}}\right) = 22\ \text{peck}$$

(b)

$$(11\ \text{tuffet})\left(\frac{0.50\ \text{bushel}}{1\ \text{tuffet}}\right) = 5.5\ \text{bushel}$$

(c)

$$(5.5\ \text{bushel})\left(\frac{36.3687\ \mathrm{L}}{1\ \text{bushel}}\right) \approx 200\ \mathrm{L}$$

27. We make the assumption that the clouds are directly overhead, so that Figure 1-3 (and the calculations that accompany it) apply. Following the steps in Sample Problem 1-4, we have

$$\frac{\theta}{360^\circ} = \frac{t}{24\ \mathrm{h}}$$

which, for $t = 38$ min $= 38/60$ h yields $\theta = 9.5^\circ$. We obtain the altitude h from the relation

$$d^2 = r^2 \tan^2\theta = 2rh$$

which is discussed in that Sample Problem, where $r = 6.37 \times 10^6$ m is the radius of the earth. Therefore, $h = 8.9 \times 10^4$ m.

28. In the simplest approach, we set up a ratio for the total increase in *horizontal depth* x (where $\Delta x = 0.05$ m is the increase in horizontal depth per step)

$$x = N_{\rm steps}\Delta x = \left(\frac{4.57}{0.19}\right)(0.05) = 1.2 \text{ m} .$$

However, we can approach this more carefully by noting that if there are $N = 4.57/.19 \approx 24$ rises then under normal circumstances we would expect $N - 1 = 23$ runs (horizontal pieces) in that staircase. This would yield $(23)(0.05) = 1.15$ m, which – to two significant figures – agrees with our first result.

29. Abbreviating wapentake as "wp" and assuming a hide to be 110 acres, we set up the ratio 25 wp/11 barn along with appropriate conversion factors:

$$\frac{(25\,\text{wp})\left(\frac{100\,\text{hide}}{1\,\text{wp}}\right)\left(\frac{110\,\text{acre}}{1\,\text{hide}}\right)\left(\frac{4047\,\text{m}^2}{1\,\text{acre}}\right)}{(11\,\text{barn})\left(\frac{1\times10^{-28}\,\text{m}^2}{1\,\text{barn}}\right)} \approx 1\times 10^{36} .$$

30. It is straightforward to compute how many seconds in a year (about 3×10^7). Now, if we estimate roughly one breath per second (or every two seconds, or three seconds – it won't affect the result) then to within an order of magnitude, a person takes 10^7 breaths in a year.

31. A day is equivalent to 86400 seconds and a meter is equivalent to a million micrometers, so

$$\frac{(3.7\,\text{m})(10^6\,\mu\text{m/m})}{(14\,\text{day})(86400\,\text{s/day})} = 3.1\ \mu\text{m/s} .$$

32. The mass in kilograms is

$$(28.9\,\text{piculs})\left(\frac{100\,\text{gin}}{1\,\text{picul}}\right)\left(\frac{16\,\text{tahil}}{1\,\text{gin}}\right)\left(\frac{10\,\text{chee}}{1\,\text{tahil}}\right)\left(\frac{10\,\text{hoon}}{1\,\text{chee}}\right)\left(\frac{0.3779\,\text{g}}{1\,\text{hoon}}\right)$$

which yields 1.747×10^6 g or roughly 1750 kg.

33. (a) In atomic mass units, the mass of one molecule is $16 + 1 + 1 = 18$ u. Using Eq. 1-9, we find

$$(18\,\text{u})\left(\frac{1.6605402\times10^{-27}\,\text{kg}}{1\,\text{u}}\right) = 3.0\times10^{-26}\ \text{kg} .$$

(b) We divide the total mass by the mass of each molecule and obtain the (approximate) number of water molecules:

$$\frac{1.4\times10^{21}}{3.0\times10^{-26}} \approx 5\times10^{46} .$$

34. (a) We find the volume in cubic centimeters

$$(193\,\text{gal})\left(\frac{231\,\text{in}^3}{1\,\text{gal}}\right)\left(\frac{2.54\,\text{cm}}{1\,\text{in}}\right)^3 = 7.31\times10^5\ \text{cm}^3$$

and subtract this from 1×10^6 cm^3 to obtain 2.69×10^5 cm^3. The conversion gal $\rightarrow$ in^3 is given in Appendix D (immediately below the table of Volume conversions).

(b) The volume found in part (a) is converted (by dividing by $(100\,\text{cm/m})^3$) to 0.731 m^3, which corresponds to a mass of

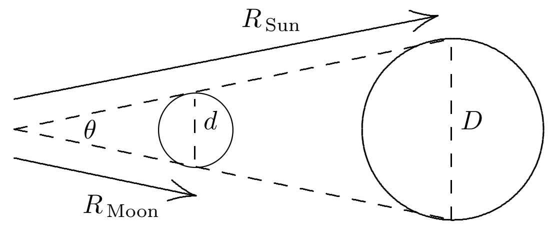
$$\left(1000\,\text{kg/m}^3\right)\left(0.731\,\text{m}^2\right) = 731\ \text{kg}$$

using the density given in the problem statement. At a rate of 0.0018 kg/min, this can be filled in

$$\frac{731\,\text{kg}}{0.0018\,\text{kg/min}} = 4.06\times10^5\ \text{min}$$

which we convert to 0.77 y, by dividing by the number of minutes in a year (365 days)(24 h/day)(60 min/h).

35. (a) When θ is measured in radians, it is equal to the arclength divided by the radius. For very large radius circles and small values of θ, such as we deal with in this problem, the arcs may be approximated as straight lines – which for our purposes correspond to the diameters d and D of the Moon and Sun, respectively. Thus,



$$\theta = \frac{d}{R_{\text{Moon}}} = \frac{D}{R_{\text{Sun}}} \implies \frac{R_{\text{Sun}}}{R_{\text{Moon}}} = \frac{D}{d}$$

which yields $D/d = 400$.

(b) Various geometric formulas are given in Appendix E. Using r_s and r_m for the radius of the Sun and Moon, respectively (noting that their ratio is the same as D/d), then the Sun's volume divided by that of the Moon is

$$\frac{\frac{4}{3}\pi r_s^3}{\frac{4}{3}\pi r_m^3} = \left(\frac{r_s}{r_m}\right)^3 = 400^3 = 6.4 \times 10^7 \ .$$

(c) The angle should turn out to be roughly 0.009 rad (or about half a degree). Putting this into the equation above, we get

$$d = \theta R_{\text{Moon}} = (0.009)\left(3.8 \times 10^5\right) \approx 3.4 \times 10^3 \text{ km} \ .$$

36. (a) For the minimum (43 cm) case, 9 cubit converts as follows:

$$(9\,\text{cubit})\left(\frac{0.43\,\text{m}}{1\,\text{cubit}}\right) = 3.9 \text{ m} \ .$$

And for the maximum (43 cm) case we obtain

$$(9\,\text{cubit})\left(\frac{0.53 \text{ m}}{1\,\text{cubit}}\right) = 4.8 \text{ m} \ .$$

(b) Similarly, with $0.43\,\text{m} \to 430\,\text{mm}$ and $0.53\,\text{m} \to 530\,\text{mm}$, we find 3.9×10^3 mm and 4.8×10^3 mm, respectively.

(c) We can convert length and diameter first and then compute the volume, or first compute the volume and then convert. We proceed using the latter approach (where d is diameter and ℓ is length).

$$\begin{aligned} V_{\text{cylinder, min}} &= \frac{\pi}{4}\,\ell\, d^2 = 28 \text{ cubit}^3 \\ &= \left(28\,\text{cubit}^3\right)\left(\frac{0.43\,\text{m}}{1\,\text{cubit}}\right)^3 \\ &= 2.2 \text{ m}^3 \ . \end{aligned}$$

Similarly, with 0.43 m replaced by 0.53 m, we obtain $V_{\text{cylinder, max}} = 4.2\,\text{m}^3$.

37. (a) Squaring the relation $1\,\text{ken} = 1.97\,\text{m}$, and setting up the ratio, we obtain

$$\frac{1\,\text{ken}^2}{1\,\text{m}^2} = \frac{1.97^2\,\text{m}^2}{1\,\text{m}^2} = 3.88 \ .$$

(b) Similarly, we find

$$\frac{1\,\text{ken}^3}{1\,\text{m}^3} = \frac{1.97^3\,\text{m}^3}{1\,\text{m}^3} = 7.65 \ .$$

(c) The volume of a cylinder is the circular area of its base multiplied by its height. Thus,

$$\pi r^2 h = \pi(3.00)^2(5.50) = 155.5 \text{ ken}^3 \ .$$

(d) If we multiply this by the result of part (b), we determine the volume in cubic meters: $(155.5)(7.65) = 1.19 \times 10^3 \text{ m}^3$.

38. Although we can look up the distance from Cleveland to Los Angeles, we can just as well (for an order of magnitude calculation) assume it's some relatively small fraction of the circumference of Earth – which suggests that (again, for an order of magnitude calculation) we can estimate the distance to be roughly r, where $r \approx 6 \times 10^6$ m is the radius of Earth. If we take each toilet paper sheet to be roughly 10 cm (0.1 m) then the number of sheets needed is roughly $6 \times 10^6/0.1 = 6 \times 10^7 \approx 10^8$.

39. Using the (exact) conversion $2.54\,\text{cm} = 1\,\text{in.}$ we find that 1 ft = (12)(2.54)/100 = 0.3048 m (which also can be found in Appendix D). The volume of a cord of wood is $8 \times 4 \times 4 = 128 \text{ ft}^3$, which we convert (multiplying by 0.3048^3) to 3.6 m^3. Therefore, one cubic meter of wood corresponds to $1/3.6 \approx 0.3$ cord.

40. (a) When θ is measured in radians, it is equal to the arclength s divided by the radius R. For a very large radius circle and small value of θ, such as we deal with in Fig. 1-9, the arc may be approximated as the straight line-segment of length 1 AU. First, we convert $\theta = 1$ arcsecond to radians:

$$(1\,\text{arcsecond})\left(\frac{1\,\text{arcminute}}{60\,\text{arcsecond}}\right)\left(\frac{1^\circ}{60\,\text{arcminute}}\right)\left(\frac{2\pi\,\text{radian}}{360^\circ}\right)$$

which yields $\theta = 4.85 \times 10^{-6}$ rad. Therefore, one parsec is

$$R_\text{o} = \frac{s}{\theta} = \frac{1\text{ AU}}{4.85 \times 10^{-6}} = 2.06\ \times 10^5\text{ AU} \ .$$

Now we use this to convert $R = 1$ AU to parsecs:

$$R = (1\text{ AU})\left(\frac{1\text{ pc}}{2.06\ \times 10^5\text{ AU}}\right) = 4.9 \times 10^{-6}\,\text{pc} \ .$$

(b) Also, since it is straightforward to figure the number of seconds in a year (about 3.16×10^7 s), and (for constant speeds) distance = speed×time, we have

$$1\,\text{ly} = (186{,}000\,\text{mi/s})\left(3.16 \times 10^7\,\text{s}\right) 5.9 \times 10^{12}\ \text{mi}$$

which we convert to AU by dividing by 92.6×10^6 (given in the problem statement), obtaining 6.3×10^4 AU. Inverting, the result is $1\,\text{AU} = 1/6.3 \times 10^4 = 1.6 \times 10^{-5}$ ly.

(c) As found in the previous part, $1\,\text{ly} = 5.9 \times 10^{12}\,\text{mi}$.

(d) We now know what one parsec is in AU (denoted above as R_o), and we also know how many miles are in an AU. Thus, one parsec is equivalent to

$$\left(92.9 \times 10^6\,\text{mi/AU}\right)\left(2.06\ \times 10^5\,\text{AU}\right) = 1.9 \times 10^{13}\ \text{mi} \ .$$

41. We reduce the stock amount to British teaspoons:

$$\begin{aligned} 1\text{ breakfastcup} &= 2 \times 8 \times 2 \times 2 = 64\text{ teaspoons} \\ 1\text{ teacup} &= 8 \times 2 \times 2 = 32\text{ teaspoons} \\ 6\text{ tablespoons} &= 6 \times 2 \times 2 = 24\text{ teaspoons} \\ 1\text{ dessertspoon} &= 2\text{ teaspoons} \end{aligned}$$

which totals to 122 teaspoons – which corresponds (since liquid measure is being used) to 122 U.S. teaspoons. Since each U.S cup is 48 teaspoons, then upon dividing $122/48 \approx 2.54$, we find this amount corresponds to two-and-a-half U.S. cups plus a remainder of precisely 2 teaspoons. For the nettle tops, one-half quart is still one-half quart. For the rice, one British tablespoon is 4 British teaspoons which (since dry-goods measure is being used) corresponds to 2 U.S. teaspoons. Finally, a British saltspoon is $\frac{1}{2}$ British teaspoon which corresponds (since dry-goods measure is again being used) to 1 U.S. teaspoon.

42. (a) *Mega*phone.
 (b) *micro*phone.
 (c) *deka*card ("deck of cards").
 (d) *Giga*low ("gigalo").
 (e) *tera*bull ("terrible").
 (f) *deci*mate.
 (g) *centi*pede.
 (h) *nano*nannette. ("No No Nanette").
 (i) *pico*boo ("peek-a-boo").
 (j) *atto*boy ("at-a-boy").
 (k) Two *hecto*withit ("to heck with it").
 (l) Two *kilo*mockingbird ("to kill a mockingbird").

43. The volume removed in one year is

$$V = \left(75 \times 10^4 \text{ m}^2\right)(26 \text{ m}) \approx 2 \times 10^7 \text{ m}^3$$

which we convert to cubic kilometers:

$$V = \left(2 \times 10^7 \text{ m}^3\right)\left(\frac{1 \text{ km}}{1000 \text{ m}}\right)^3 = 0.020 \text{ km}^3 \ .$$

44. (a) Using Appendix D, we have $1 \text{ ft} = 0.3048 \text{ m}$, $1 \text{ gal} = 231 \text{ in.}^3$, and $1 \text{ in.}^3 = 1.639 \times 10^{-2}$ L. From the latter two items, we find that $1 \text{ gal} = 3.79$ L. Thus, the quantity $460 \text{ft}^2/\text{gal}$ becomes

$$\left(\frac{460 \text{ ft}^2}{\text{gal}}\right)\left(\frac{1 \text{ m}}{3.28 \text{ ft}}\right)^2\left(\frac{1 \text{ gal}}{3.79 \text{ L}}\right) = 11.3 \text{ m}^2/\text{L} \ .$$

 (b) Also, since 1 m^3 is equivalent to 1000 L, our result from part (a) becomes

$$\left(\frac{11.3 \text{ m}^2}{\text{L}}\right)\left(\frac{1000 \text{ L}}{1 \text{ m}^3}\right) = 1.13 \times 10^4 \text{ m}^{-1} \ .$$

 (c) The inverse of the original quantity is $(460 \text{ ft}^2/\text{gal})^{-1} = 2.17 \times 10^{-3} \text{ gal}/\text{ft}^2$, which is the volume of the paint (in gallons) needed to cover a square foot of area. From this, we could also figure the paint thickness (it turns out to be about a tenth of a millimeter, as one sees by taking the reciprocal of the answer in part (b)).

Chapter 2

1. Assuming the horizontal velocity of the ball is constant, the horizontal displacement is

$$\Delta x = v\Delta t$$

where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. Converting v to meters per second, we have $160\,\text{km/h} = 44.4\,\text{m/s}$. Thus

$$\Delta t = \frac{\Delta x}{v} = \frac{18.4\,\text{m}}{44.4\,\text{m/s}} = 0.414\,\text{s}\,.$$

The velocity-unit conversion implemented above can be figured "from basics" (1000 m = 1 km, 3600 s = 1 h) or found in Appendix D.

2. Converting to SI units, we use Eq. 2-3 with d for distance.

$$\begin{aligned} s_{\text{avg}} &= \frac{d}{t} \\ (110.6\ \text{km/h})\left(\frac{1000\,\text{m/km}}{3600\ \text{s/h}}\right) &= \frac{200.0\,\text{m}}{t} \end{aligned}$$

which yields $t = 6.510$ s. We converted the speed km/h $\rightarrow$ m/s by converting each unit (km $\rightarrow$ m, h $\rightarrow$ s) individually. But we mention that the "one-step" conversion can be found in Appendix D ($1\,\text{km/h} = 0.2778\,\text{m/s}$).

3. We use Eq. 2-2 and Eq. 2-3. During a time t_c when the velocity remains a positive constant, speed is equivalent to velocity, and distance is equivalent to displacement, with $\Delta x = v\,t_c$.

   (a) During the first part of the motion, the displacement is $\Delta x_1 = 40$ km and the time interval is

   $$t_1 = \frac{(40\,\text{km})}{(30\,\text{km/h})} = 1.33\,\text{h}\ .$$

   During the second part the displacement is $\Delta x_2 = 40\,\text{km}$ and the time interval is

   $$t_2 = \frac{(40\,\text{km})}{(60\,\text{km/h})} = 0.67\,\text{h}\,.$$

   Both displacements are in the same direction, so the total displacement is $\Delta x = \Delta x_1 + \Delta x_2 = 40\,\text{km} + 40\,\text{km} = 80\,\text{km}$. The total time for the trip is $t = t_1 + t_2 = 2.00\,\text{h}$. Consequently, the average velocity is

   $$v_{\text{avg}} = \frac{(80\,\text{km})}{(2.0\,\text{h})} = 40\,\text{km/h}\ .$$

   (b) In this example, the numerical result for the average speed is the same as the average velocity 40 km/h.

(c) In the interest of saving space, we briefly describe the graph (with kilometers and hours understood): two contiguous line segments, the first having a slope of 30 and connecting the origin to $(t_1, x_1) = (1.33, 40)$ and the second having a slope of 60 and connecting (t_1, x_1) to $(t, x) = (2.00, 80)$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (t, x).

4. If the plane (with velocity v) maintains its present course, and if the terrain continues its upward slope of 4.3°, then the plane will strike the ground after traveling

$$\Delta x = \frac{h}{\tan\theta} = \frac{35\,\text{m}}{\tan 4.3^\circ} = 465.5 \text{ m} \approx 0.465 \text{ km} .$$

This corresponds to a time of flight found from Eq. 2-2 (with $v = v_{\text{avg}}$ since it is constant)

$$t = \frac{\Delta x}{v} = \frac{0.465\,\text{km}}{1300\,\text{km/h}} = 0.000358 \text{ h} \approx 1.3 \text{ s} .$$

This, then, estimates the time available to the pilot to make his correction.

5. (a) Denoting the travel time and distance from San Antonio to Houston as T and D, respectively, the average speed is

$$s_{\text{avg 1}} = \frac{D}{T} = \frac{(55\,\text{km/h})\frac{T}{2} + (90\,\text{km/h})\frac{T}{2}}{T} = 72.5 \text{ km/h}$$

which should be rounded to 73 km/h.

(b) Using the fact that time = distance/speed while the speed is constant, we find

$$s_{\text{avg 2}} = \frac{D}{T} = \frac{D}{\frac{D/2}{55\,\text{km/h}} + \frac{D/2}{90\,\text{km/h}}} = 68.3 \text{ km/h}$$

which should be rounded to 68 km/h.

(c) The total distance traveled ($2D$) must not be confused with the net displacement (zero). We obtain for the two-way trip

$$s_{\text{avg}} = \frac{2D}{\frac{D}{72.5\,\text{km/h}} + \frac{D}{68.3\,\text{km/h}}} = 70 \text{ km/h} .$$

(d) Since the net displacement vanishes, the average velocity for the trip in its entirety is zero.

(e) In asking for a *sketch*, the problem is allowing the student to arbitrarily set the distance D (the intent is *not* to make the student go to an Atlas to look it up); the student can just as easily arbitrarily set T instead of D, as will be clear in the following discussion. In the interest of saving space, we briefly describe the graph (with kilometers-per-hour understood for the slopes): two contiguous line segments, the first having a slope of 55 and connecting the origin to $(t_1, x_1) = (T/2, 55T/2)$ and the second having a slope of 90 and connecting (t_1, x_1) to (T, D) where $D = (55 + 90)T/2$. The average velocity, from the graphical point of view, is the slope of a line drawn from the origin to (T, D).

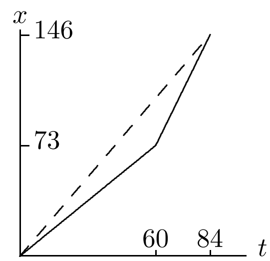
6. (a) Using the fact that time = distance/velocity while the velocity is constant, we find

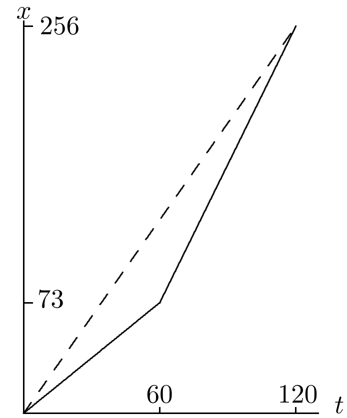
$$v_{\text{avg}} = \frac{73.2\,\text{m} + 73.2\,\text{m}}{\frac{73.2\,\text{m}}{1.22\,\text{m/s}} + \frac{73.2\,\text{m}}{3.05\,\text{m}}} = 1.74\,\text{m/s} .$$

(b) Using the fact that distance $= vt$ while the velocity v is constant, we find

$$v_{\text{avg}} = \frac{(1.22\,\text{m/s})(60\,\text{s}) + (3.05\,\text{m/s})(60\,\text{s})}{120\,\text{s}} = 2.14\,\text{m/s} .$$

(c) The graphs are shown below (with meters and seconds understood). The first consists of two (solid) line segments, the first having a slope of 1.22 and the second having a slope of 3.05. The slope of the dashed line represents the average velocity (in both graphs). The second graph also consists of two (solid) line segments, having the same slopes as before – the main difference (compared to the first graph) being that the stage involving higher-speed motion lasts much longer.





7. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write $x = (3\,\text{m/s})t - (4\,\text{m/s}^2)t^2 + (1\,\text{m/s}^3)t^3$. We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously.

   (a) Plugging in $t = 1$ s yields $x = 0$. With $t = 2$ s we get $x = -2$ m. Similarly, $t = 3$ s yields $x = 0$ and $t = 4$ s yields $x = 12$ m. For later reference, we also note that the position at $t = 0$ is $x = 0$.

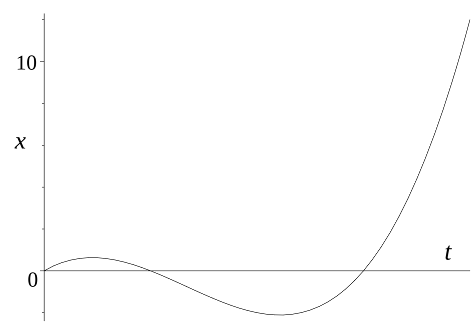
   (b) The position at $t = 0$ is subtracted from the position at $t = 4$ s to find the displacement $\Delta x = 12$ m.

   (c) The position at $t = 2$ s is subtracted from the position at $t = 4$ s to give the displacement $\Delta x = 14$ m. Eq. 2-2, then, leads to

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{14}{2} = 7 \text{ m/s} .$$

   (d) The horizontal axis is $0 \le t \le 4$ with SI units understood.

Not shown is a straight line drawn from the point at $(t, x) = (2, -2)$ to the highest point shown (at $t = 4$ s) which would represent the answer for part (c).



8. Recognizing that the gap between the trains is closing at a constant rate of 60 km/h, the total time which elapses before they crash is $t = (60\,\text{km})/(60\,\text{km/h}) = 1.0$ h. During this time, the bird travels a distance of $x = vt = (60\,\text{km/h})(1.0\,\text{h}) = 60$ km.

9. Converting to seconds, the running times are $t_1 = 147.95$ s and $t_2 = 148.15$ s, respectively. If the runners were equally fast, then

$$s_{\text{avg }1} = s_{\text{avg }2} \implies \frac{L_1}{t_1} = \frac{L_2}{t_2} .$$

From this we obtain

$$L_2 - L_1 = \left(\frac{148.15}{147.95} - 1\right) L_1 \approx 1.35 \text{ m}$$

where we set $L_1 \approx 1000$ m in the last step. Thus, if L_1 and L_2 are no different than about 1.35 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 than 1.4 m then runner 2 is actually the faster.

10. The velocity (both magnitude and sign) is determined by the slope of the x versus t curve, in accordance with Eq. 2-4.

(a) The armadillo is to the left of the coordinate origin on the axis between $t = 2.0$ s and $t = 4.0$ s.

(b) The velocity is negative between $t = 0$ and $t = 3.0$ s.

(c) The velocity is positive between $t = 3.0$ s and $t = 7.0$ s.

(d) The velocity is zero at the graph minimum (at $t = 3.0$ s).

11. We use Eq. 2-4.

(a) The velocity of the particle is

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(4 - 12t + 3t^2\right) = -12 + 6t .$$

Thus, at $t = 1$ s, the velocity is $v = (-12 + (6)(1)) = -6$ m/s.

(b) Since $v < 0$, it is moving in the negative x direction at $t = 1$ s.

(c) At $t = 1$ s, the *speed* is $|v| = 6$ m/s.

(d) For $0 < t < 2$ s, $|v|$ decreases until it vanishes. For $2 < t < 3$ s, $|v|$ increases from zero to the value it had in part (c). Then, $|v|$ is larger than that value for $t > 3$ s.

(e) Yes, since v smoothly changes from negative values (consider the $t = 1$ result) to positive (note that as $t \to +\infty$, we have $v \to +\infty$). One can check that $v = 0$ when $t = 2$ s.

(f) No. In fact, from $v = -12 + 6t$, we know that $v > 0$ for $t > 2$ s.

12. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds.

(a) We plug into the given equation for x for $t = 2.00$ s and $t = 3.00$ s and obtain $x_2 = 21.75$ cm and $x_3 = 50.25$ cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{50.25\,\text{cm} - 21.75\,\text{cm}}{3.00\,\text{s} - 2.00\,\text{s}}$$

which yields $v_{\text{avg}} = 28.5$ cm/s.

(b) The instantaneous velocity is $v = \frac{dx}{dt} = 4.5t^2$, which yields $v = (4.5)(2.00)^2 = 18.0$ cm/s at time $t = 2.00$ s.

(c) At $t = 3.00$ s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ cm/s.

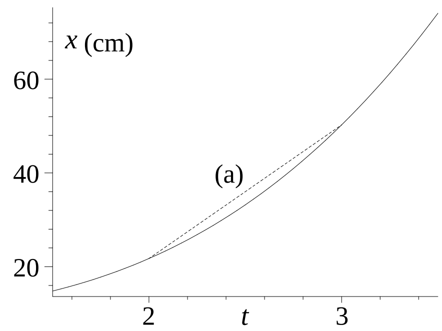
(d) At $t = 2.50$ s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ cm/s.

(e) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm). Therefore,

$$x_m = 9.75 + 1.5t_m^3 \implies t_m = 2.596$$

in seconds. Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ cm/s.

(f) The answer to part (a) is given by the slope of the straight line between $t = 2$ and $t = 3$ in this x-vs-t plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.

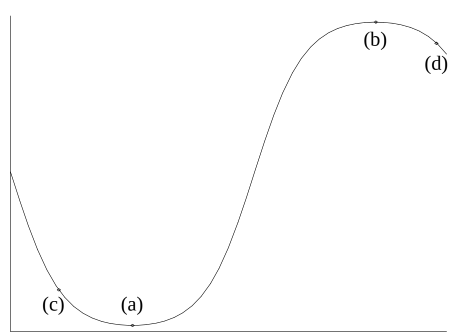


13. Since $v = \frac{dx}{dt}$ (Eq. 2-4), then $\Delta x = \int v\,dt$, which corresponds to the area under the v vs t graph. Dividing the total area A into rectangular (base×height) and triangular ($\frac{1}{2}$base×height) areas, we have

$$\begin{aligned} A &= A_{0<t<2} + A_{2<t<10} + A_{10<t<12} + A_{12<t<16} \\ &= \frac{1}{2}(2)(8) + (8)(8) + \left((2)(4) + \frac{1}{2}(2)(4)\right) + (4)(4) \end{aligned}$$

with SI units understood. In this way, we obtain $\Delta x = 100$ m.

14. From Eq. 2-4 and Eq. 2-9, we note that the sign of the velocity is the sign of the slope in an x-vs-t plot, and the sign of the acceleration corresponds to whether such a curve is concave up or concave down. In the interest of saving space, we indicate sample points for parts (a)-(d) in a single figure; this means that all points are not at $t = 1$ s (which we feel is an acceptable modification of the problem – since the datum $t = 1$ s is not used).



Any change from zero to non-zero values of $\vec{v}$ represents increasing $|\vec{v}|$ (speed). Also, $\vec{v} \parallel \vec{a}$ implies that the particle is going faster. Thus, points (a), (b) and (d) involve increasing speed.

15. We appeal to Eq. 2-4 and Eq. 2-9.

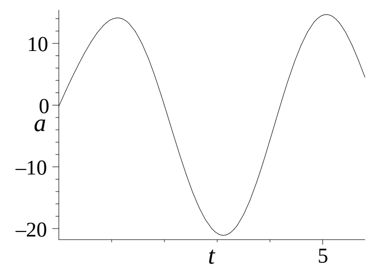
(a) This is v^2 – that is, the velocity squared.

(b) This is the acceleration a.

(c) The SI units for these quantities are $(\text{m/s})^2 = \text{m}^2/\text{s}^2$ and m/s^2, respectively.

16. Eq. 2-9 indicates that acceleration is the slope of the v-vs-t graph.

Based on this, we show here a sketch of the acceleration (in m/s^2) as a function of time. The values along the acceleration axis should not be taken too seriously.



17. We represent its initial direction of motion as the $+x$ direction, so that $v_0 = +18$ m/s and $v = -30$ m/s (when $t = 2.4$ s). Using Eq. 2-7 (or Eq. 2-11, suitably interpreted) we find

$$a_{\text{avg}} = \frac{(-30) - (+18)}{2.4} = -20 \text{ m/s}^2$$

which indicates that the average acceleration has magnitude 20 m/s^2 and is in the opposite direction to the particle's initial velocity.

18. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during $5\,\text{min} \le t \le 10\,\text{min}$ is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$.

(a) Here, the entire interval considered is $\Delta t = 8 - 2 = 6$ min which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3$ min$=$ 180 s. His position at $t = 2$ min is $x = 0$ and his position at $t = 8$ min is $x = v\Delta t' = (2.2)(180) = 396$ m. Therefore,

$$v_{\text{avg}} = \frac{396\,\text{m} - 0}{360\,\text{s}} = 1.10 \text{ m/s} .$$

(b) The man is at rest at $t = 2$ min and has velocity $v = +2.2$ m/s at $t = 8$ min. Thus, keeping the answer to 3 significant figures,

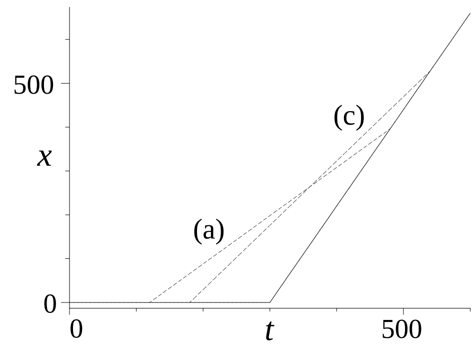
$$a_{\text{avg}} = \frac{2.2\,\text{m/s} - 0}{360\,\text{s}} = 0.00611 \text{ m/s}^2 .$$

(c) Now, the entire interval considered is $\Delta t = 9 - 3 = 6$ min (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4$ min$=$ 240 s). His position at $t = 3$ min is $x = 0$ and his position at $t = 9$ min is $x = v\Delta t' = (2.2)(240) = 528$ m. Therefore,

$$v_{\text{avg}} = \frac{528\,\text{m} - 0}{360\,\text{s}} = 1.47 \text{ m/s} .$$

(d) The man is at rest at $t = 3$ min and has velocity $v = +2.2$ m/s at $t = 9$ min. Consequently, $a_{\text{avg}} = 2.2/360 = 0.00611$ m/s^2 just as in part (b).

(e) The horizontal line near the bottom of this x-vs-t graph represents the man standing at $x = 0$ for $0 \leq t < 300$ s and the linearly rising line for $300 \leq t \leq 600$ s represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results.



The graph of v-vs-t is not shown here, but would consist of two horizontal "steps" (one at $v = 0$ for $0 \leq t < 300$ s and the next at $v = 2.2$ m/s for $300 \leq t \leq 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connected the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{avg}).

19. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. Thus, $x(t) = 50t + 10t^2$ with SI units (meters and seconds) understood.

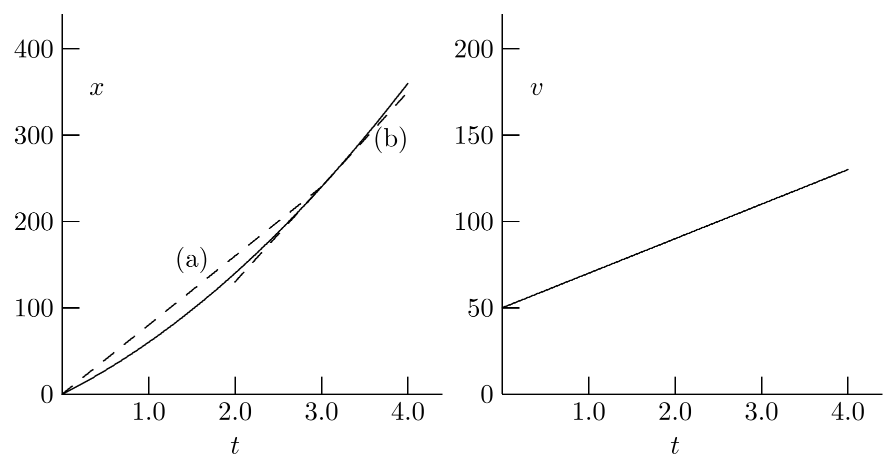
(a) The average velocity during the first 3 s is given by

$$v_{\text{avg}} = \frac{x(3) - x(0)}{\Delta t} = \frac{(50)(3) + (10)(3)^2 - 0}{3} = 80 \text{ m/s} .$$

(b) The instantaneous velocity at time t is given by $v = dx/dt = 50 + 20t$, in SI units. At $t = 3.0$ s, $v = 50 + (20)(3.0) = 110$ m/s.

(c) The instantaneous acceleration at time t is given by $a = dv/dt = 20 \text{ m/s}^2$. It is constant, so the acceleration at any time is $20 \, \text{m/s}^2$.

(d) and (e) The graphs below show the coordinate x and velocity v as functions of time, with SI units understood. The dotted line marked (a) in the first graph runs from $t = 0$, $x = 0$ to $t = 3.0$ s, $x = 240$ m. Its slope is the average velocity during the first 3 s of motion. The dotted line marked (b) is tangent to the x curve at $t = 3.0$ s. Its slope is the instantaneous velocity at $t = 3.0$ s.



20. Using the general property $\frac{d}{dx} \exp(bx) = b \exp(bx)$, we write

$$v = \frac{dx}{dt} = \left(\frac{d\,(19t)}{dt} \right) \cdot e^{-t} + (19t) \cdot \left(\frac{d\,e^{-t}}{dt} \right) .$$

If a concern develops about the appearance of an argument of the exponential ($-t$) apparently having units, then an explicit factor of $1/T$ where $T = 1$ second can be inserted and carried through the computation (which does not change our answer). The result of this differentiation is

$$v = 16(1-t)e^{-t}$$

with t and v in SI units (s and m/s, respectively). We see that this function is zero when $t = 1$ s. Now that we know *when* it stops, we find out *where* it stops by plugging our result $t = 1$ into the given function $x = 16te^{-t}$ with x in meters. Therefore, we find $x = 5.9$ m.

21. In this solution, we make use of the notation $x(t)$ for the value of x at a particular t. The notations $v(t)$ and $a(t)$ have similar meanings.

    (a) Since the unit of ct^2 is that of length, the unit of c must be that of length/time^2, or m/s^2 in the SI system. Since bt^3 has a unit of length, b must have a unit of length/time^3, or m/s^3.

    (b) When the particle reaches its maximum (or its minimum) coordinate its velocity is zero. Since the velocity is given by $v = dx/dt = 2ct - 3bt^2$, $v = 0$ occurs for $t = 0$ and for

    $$t = \frac{2c}{3b} = \frac{2(3.0\,\mathrm{m/s^2})}{3(2.0\,\mathrm{m/s^3})} = 1.0\,\mathrm{s}\,.$$

    For $t = 0$, $x = x_0 = 0$ and for $t = 1.0\,\mathrm{s}, x = 1.0\,\mathrm{m} > x_0$. Since we seek the maximum, we reject the first root ($t = 0$) and accept the second ($t = 1\,\mathrm{s}$).

    (c) In the first 4 s the particle moves from the origin to $x = 1.0\,\mathrm{m}$, turns around, and goes back to

    $$x(4\,\mathrm{s}) = (3.0\,\mathrm{m/s^2})(4.0\,\mathrm{s})^2 - (2.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^3 = -80\,\mathrm{m}\ .$$

    The total path length it travels is $1.0\,\mathrm{m} + 1.0\,\mathrm{m} + 80\,\mathrm{m} = 82\,\mathrm{m}$.

    (d) Its displacement is given by $\Delta x = x_2 - x_1$, where $x_1 = 0$ and $x_2 = -80\,\mathrm{m}$. Thus, $\Delta x = -80\,\mathrm{m}$.

    (e) The velocity is given by $v = 2ct - 3bt^2 = (6.0\,\mathrm{m/s^2})t - (6.0\,\mathrm{m/s^3})t^2$. Thus

    $$\begin{aligned} v(1\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(1.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(1.0\,\mathrm{s})^2 = 0 \\ v(2\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(2.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(2.0\,\mathrm{s})^2 = -12\,\mathrm{m/s} \\ v(3\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(3.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(3.0\,\mathrm{s})^2 = -36.0\,\mathrm{m/s} \\ v(4\,\mathrm{s}) &= (6.0\,\mathrm{m/s^2})(4.0\,\mathrm{s}) - (6.0\,\mathrm{m/s^3})(4.0\,\mathrm{s})^2 = -72\,\mathrm{m/s}\ . \end{aligned}$$

    (f) The acceleration is given by $a = dv/dt = 2c - 6b = 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})t$. Thus

    $$\begin{aligned} a(1\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(1.0\,\mathrm{s}) = -6.0\,\mathrm{m/s^2} \\ a(2\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(2.0\,\mathrm{s}) = -18\,\mathrm{m/s^2} \\ a(3\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(3.0\,\mathrm{s}) = -30\,\mathrm{m/s^2} \\ a(4\,\mathrm{s}) &= 6.0\,\mathrm{m/s^2} - (12.0\,\mathrm{m/s^3})(4.0\,\mathrm{s}) = -42\,\mathrm{m/s^2}\ . \end{aligned}$$

22. For the automobile $\Delta v = 55 - 25 = 30\,\mathrm{km/h}$, which we convert to SI units:

    $$a = \frac{\Delta v}{\Delta t} = \frac{(30\,\mathrm{km/h})\left(\frac{1000\,\mathrm{m/km}}{3600\,\mathrm{s/h}}\right)}{(0.50\,\mathrm{min})(60\,\mathrm{s/min})} = 0.28\ \mathrm{m/s^2}\ .$$

    The change of velocity for the bicycle, for the same time, is identical to that of the car, so its acceleration is also 0.28 m/s^2.

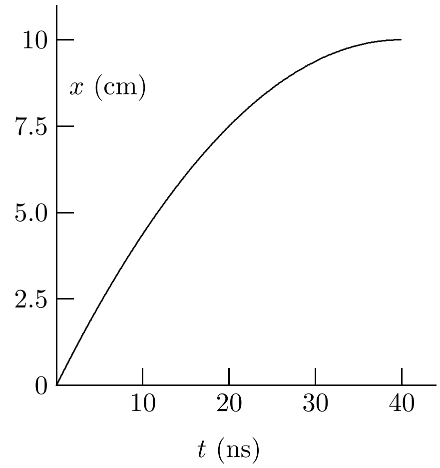
23. The constant-acceleration condition permits the use of Table 2-1.

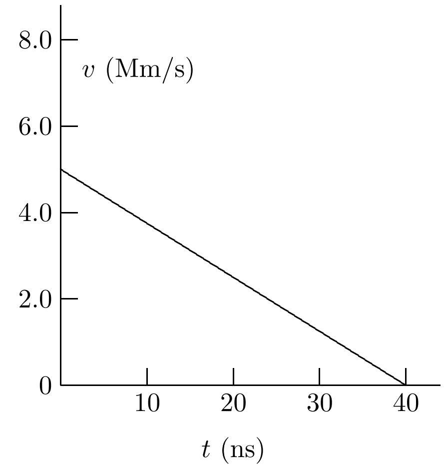
(a) Setting $v = 0$ and $x_0 = 0$ in $v^2 = v_0^2 + 2a(x - x_0)$, we find

$$x = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left(\frac{5.00 \times 10^6}{-1.25 \times 10^{14}}\right) = 0.100 \text{ m} .$$

Since the muon is slowing, the initial velocity and the acceleration must have opposite signs.

(b) Below are the time-plots of the position x and velocity v of the muon from the moment it enters the field to the time it stops. The computation in part (a) made no reference to t, so that other equations from Table 2-1 (such as $v = v_0 + at$ and $x = v_0 t + \frac{1}{2}at^2$) are used in making these plots.





24. The time required is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7). First, we convert the velocity change to SI units:

$$\Delta v = (100\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 27.8 \text{ m/s} .$$

Thus, $\Delta t = \Delta v/a = 27.8/50 = 0.556$ s.

25. We use $v = v_0 + at$, with $t = 0$ as the instant when the velocity equals +9.6 m/s.

(a) Since we wish to calculate the velocity for a time *before* $t = 0$, we set $t = -2.5$ s. Thus, Eq. 2-11 gives

$$v = (9.6\,\text{m/s}) + \left(3.2\,\text{m/s}^2\right)(-2.5\,\text{s}) = 1.6 \text{ m/s} .$$

(b) Now, $t = +2.5$ s and we find

$$v = (9.6\,\text{m/s}) + \left(3.2\,\text{m/s}^2\right)(2.5\,\text{s}) = 18 \text{ m/s} .$$

26. The bullet starts at rest ($v_0 = 0$) and after traveling the length of the barrel ($\Delta x = 1.2$ m) emerges with the given velocity ($v = 640$ m/s), where the direction of motion is the positive direction. Turning to the constant acceleration equations in Table 2-1, we use

$$\Delta x = \frac{1}{2}(v_0 + v)\,t \quad .$$

Thus, we find $t = 0.00375$ s (about 3.8 ms).

27. The constant acceleration stated in the problem permits the use of the equations in Table 2-1.

(a) We solve $v = v_0 + at$ for the time:

$$t = \frac{v - v_0}{a} = \frac{\frac{1}{10}\left(3.0 \times 10^8\,\text{m/s}\right)}{9.8\,\text{m/s}^2} = 3.1 \times 10^6 \text{ s}$$

which is equivalent to 1.2 months.

(b) We evaluate $x = x_0 + v_0 t + \frac{1}{2}at^2$, with $x_0 = 0$. The result is

$$x = \frac{1}{2}\left(9.8\,\text{m/s}^2\right)\left(3.1\times10^6\,\text{s}\right)^2 = 4.7\times10^{13}\,\text{m} .$$

28. From Table 2-1, $v^2 - v_0^2 = 2a\Delta x$ is used to solve for a. Its minimum value is

$$a_{\text{min}} = \frac{v^2 - v_0^2}{2\Delta x_{\text{max}}} = \frac{(360\,\text{km/h})^2}{2(1.80\,\text{km})} = 36000\ \text{km/h}^2$$

which converts to 2.78 m/s^2.

29. Assuming constant acceleration permits the use of the equations in Table 2-1. We solve $v^2 = v_0^2 + 2a(x - x_0)$ with $x_0 = 0$ and $x = 0.010$ m. Thus,

$$a = \frac{v^2 - v_0^2}{2x} = \frac{\left(5.7\times10^5\right)^2 - \left(1.5\times10^5\right)^2}{2(0.01)} = 1.62\times10^{15}\ \text{m/s}^2 .$$

30. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \frac{\Delta v}{\Delta t} = \frac{(1020\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)}{1.4\,\text{s}} = 202.4\ \text{m/s}^2 .$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = \frac{202.4}{9.8}\,g = 21g .$$

31. We choose the positive direction to be that of the initial velocity of the car (implying that $a < 0$ since it is slowing down). We assume the acceleration is constant and use Table 2-1.

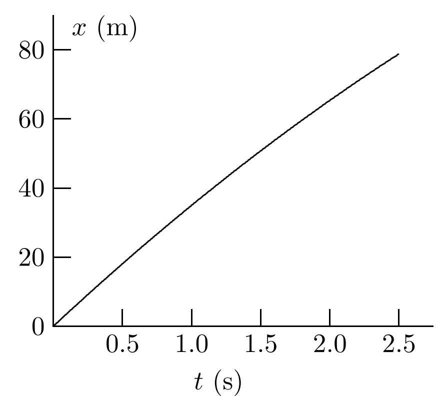
(a) Substituting $v_0 = 137\,\text{km/h} = 38.1\,\text{m/s}$, $v = 90\,\text{km/h} = 25\,\text{m/s}$, and $a = -5.2\,\text{m/s}^2$ into $v = v_0 + at$, we obtain

$$t = \frac{25\,\text{m/s} - 38\,\text{m/s}}{-5.2\,\text{m/s}^2} = 2.5\ \text{s} .$$

(b) We take the car to be at $x = 0$ when the brakes are applied (at time $t = 0$). Thus, the coordinate of the car as a function of time is given by

$$x = (38)t + \frac{1}{2}(-5.2)t^2$$

in SI units. This function is plotted from $t = 0$ to $t = 2.5$ s on the graph to the right. We have not shown the v-vs-t graph here; it is a descending straight line from v_0 to v.



32. From the figure, we see that $x_0 = -2.0$ m. From Table 2-1, we can apply $x - x_0 = v_0 t + \frac{1}{2}at^2$ with $t = 1.0$ s, and then again with $t = 2.0$ s. This yields two equations for the two unknowns, v_0 and a. SI units are understood.

$$\begin{aligned} 0.0 - (-2.0) &= v_0\,(1.0) + \frac{1}{2}a(1.0)^2 \\ 6.0 - (-2.0) &= v_0\,(2.0) + \frac{1}{2}a(2.0)^2 . \end{aligned}$$

Solving these simultaneous equations yields the results $v_0 = 0.0$ and $a = 4.0$ m/s^2. The fact that the answer is positive tells us that the acceleration vector points in the $+x$ direction.

33. The problem statement (see part (a)) indicates that $a =$constant, which allows us to use Table 2-1.

    (a) We take $x_0 = 0$, and solve $x = v_0 t + \frac{1}{2}at^2$ (Eq. 2-15) for the acceleration: $a = 2(x - v_0 t)/t^2$. Substituting $x = 24.0\,\text{m}$, $v_0 = 56.0\,\text{km/h} = 15.55\,\text{m/s}$ and $t = 2.00\,\text{s}$, we find

    $$a = \frac{2\,(24.0\,\text{m} - (15.55\,\text{m/s})(2.00\,\text{s}))}{(2.00\,\text{s})^2} = -3.56\,\text{m/s}^2 \ .$$

    The negative sign indicates that the acceleration is opposite to the direction of motion of the car. The car is slowing down.

    (b) We evaluate $v = v_0 + at$ as follows:

    $$v = 15.55\,\text{m/s} - \left(3.56\,\text{m/s}^2\right)(2.00\,\text{s}) = 8.43\ \text{m/s}$$

    which is equivalent to 30.3 km/h.

34. We take the moment of applying brakes to be $t = 0$. The deceleration is constant so that Table 2-1 can be used. Our primed variables (such as $v'_\text{o} = 72$ km/h $= 20$ m/s) refer to one train (moving in the $+x$ direction and located at the origin when $t = 0$) and unprimed variables refer to the other (moving in the $-x$ direction and located at $x_0 = +950$ m when $t = 0$). We note that the acceleration vector of the unprimed train points in the *positive* direction, even though the train is slowing down; its initial velocity is $v_\text{o} = -144$ km/h $= -40$ m/s. Since the primed train has the lower initial speed, it should stop sooner than the other train would (were it not for the collision). Using Eq 2-16, it should stop (meaning $v' = 0$) at

    $$x' = \frac{(v')^2 - (v'_\text{o})^2}{2a'} = \frac{0 - 20^2}{-2} = 200\ \text{m} \ .$$

    The speed of the other train, when it reaches that location, is

    $$v = \sqrt{v_\text{o}^2 + 2a\Delta x} = \sqrt{(-40)^2 + 2(1.0)(200 - 950)} = \sqrt{100} = 10\ \text{m/s}$$

    using Eq 2-16 again. Specifically, its velocity at that moment would be -10 m/s since it is still traveling in the $-x$ direction when it crashes. If the computation of v had failed (meaning that a negative number would have been inside the square root) then we would have looked at the possibility that there was no collision and examined how far apart they finally were. A concern that can be brought up is whether the primed train collides before it comes to rest; this can be studied by computing the time it stops (Eq. 2-11 yields $t = 20$ s) and seeing where the unprimed train is at that moment (Eq. 2-18 yields $x = 350$ m, still a good distance away from contact).

35. The acceleration is constant and we may use the equations in Table 2-1.

    (a) Taking the first point as coordinate origin and time to be zero when the car is there, we apply Eq. 2-17 (with SI units understood):

    $$x = \frac{1}{2}(v + v_0)\,t = \frac{1}{2}(15 + v_0)(6) \ .$$

    With $x = 60.0$ m (which takes the direction of motion as the $+x$ direction) we solve for the initial velocity: $v_0 = 5.00$ m/s.

    (b) Substituting $v = 15$ m/s, $v_0 = 5$ m/s and $t = 6$ s into $a = (v - v_0)/t$ (Eq. 2-11), we find $a = 1.67$ m/s^2.

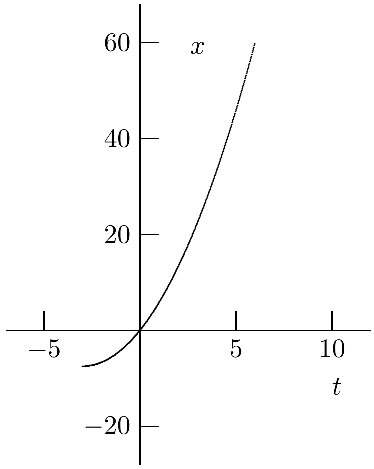
    (c) Substituting $v = 0$ in $v^2 = v_0^2 + 2ax$ and solving for x, we obtain

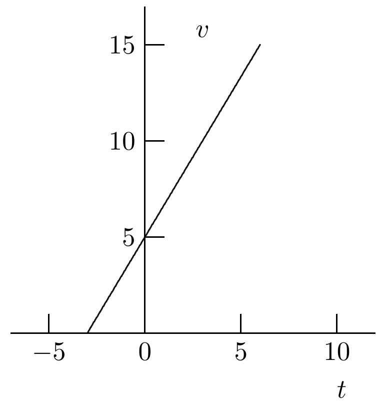
    $$x = -\frac{v_0^2}{2a} = -\frac{5^2}{2(1.67)} = -7.50\ \text{m} \ .$$

(d) The graphs require computing the time when $v = 0$, in which case, we use $v = v_0 + at' = 0$. Thus,

$$t' = \frac{-v_0}{a} = \frac{-5}{1.67} = -3.0 \text{ s}$$

indicates the moment the car was at rest. SI units are assumed.





36. We denote the required time as t, assuming the light turns green when the clock reads zero. By this time, the distances traveled by the two vehicles must be the same.

(a) Denoting the acceleration of the automobile as a and the (constant) speed of the truck as v then

$$\Delta x = \left(\frac{1}{2}at^2\right)_{\text{car}} = (vt)_{\text{truck}}$$

which leads to

$$t = \frac{2v}{a} = \frac{2(9.5)}{2.2} = 8.6 \text{ s} .$$

Therefore,

$$\Delta x = vt = (9.5)(8.6) = 82 \text{ m} .$$

(b) The speed of the car at that moment is

$$v_{\text{car}} = at = (2.2)(8.6) = 19 \text{ m/s} .$$

37. We denote t_r as the reaction time and t_b as the braking time. The motion during t_r is of the constant-velocity (call it v_0) type. Then the position of the car is given by

$$x = v_0 t_r + v_0 t_b + \frac{1}{2}at_b^2$$

where v_0 is the initial velocity and a is the acceleration (which we expect to be negative-valued since we are taking the velocity in the positive direction and we know the car is decelerating). *After* the brakes are applied the velocity of the car is given by $v = v_0 + at_b$. Using this equation, with $v = 0$, we eliminate t_b from the first equation and obtain

$$x = v_0 t_r - \frac{v_0^2}{a} + \frac{1}{2}\frac{v_0^2}{a} = v_0 t_r - \frac{1}{2}\frac{v_0^2}{a} .$$

We write this equation for each of the initial velocities:

$$x_1 = v_{01} t_r - \frac{1}{2}\frac{v_{01}^2}{a}$$

and

$$x_2 = v_{02} t_r - \frac{1}{2} \frac{v_{02}^2}{a} \ .$$

Solving these equations simultaneously for t_r and a we get

$$t_r = \frac{v_{02}^2 x_1 - v_{01}^2 x_2}{v_{01} v_{02} (v_{02} - v_{01})}$$

and

$$a = -\frac{1}{2} \frac{v_{02} v_{01}^2 - v_{01} v_{02}^2}{v_{02} x_1 - v_{01} x_2} \ .$$

Substituting $x_1 = 56.7\,\text{m}$, $v_{01} = 80.5\,\text{km/h} = 22.4\,\text{m/s}$, $x_2 = 24.4$ m and $v_{02} = 48.3\,\text{km/h} = 13.4$ m/s, we find

$$t_r = \frac{13.4^2(56.7) - 22.4^2(24.4)}{(22.4)(13.4)(13.4 - 22.4)} = 0.74 \text{ s}$$

and

$$a = -\frac{1}{2} \frac{(13.4)22.4^2 - (22.4)13.4^2}{(13.4)(56.7) - (22.4)(24.4)} = -6.2 \text{ m/s}^2 \ .$$

The *magnitude* of the deceleration is therefore 6.2 m/s^2. Although rounded off values are displayed in the above substitutions, what we have input into our calculators are the "exact" values (such as $v_{02} = \frac{161}{12}$ m/s).

38. In this solution we elect to wait until the last step to convert to SI units. Constant acceleration is indicated, so use of Table 2-1 is permitted. We start with Eq. 2-17 and denote the train's initial velocity as v_t and the locomotive's velocity as v_ℓ (which is also the final velocity of the train, if the rear-end collision is barely avoided). We note that the distance Δx consists of the original gap between them D as well as the forward distance traveled during this time by the locomotive $v_\ell t$. Therefore,

$$\frac{v_t + v_\ell}{2} = \frac{\Delta x}{t} = \frac{D + v_\ell t}{t} = \frac{D}{t} + v_\ell \ .$$

We now use Eq. 2-11 to eliminate time from the equation. Thus,

$$\frac{v_t + v_\ell}{2} = \frac{D}{(v_\ell - v_t)/a} + v_\ell$$

leads to

$$a = \left(\frac{v_t + v_\ell}{2} - v_\ell\right)\left(\frac{v_\ell - v_t}{D}\right) = -\frac{1}{2D}(v_\ell - v_t)^2 \ .$$

Hence,

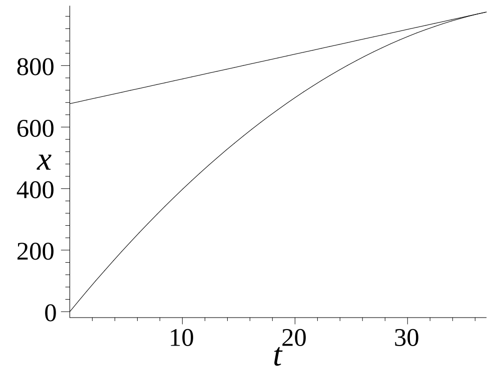
$$a = -\frac{1}{2(0.676\,\text{km})}\left(29\,\frac{\text{km}}{\text{h}} - 161\,\frac{\text{km}}{\text{h}}\right)^2 = -12888 \text{ km/h}^2$$

which we convert as follows:

$$a = \left(-12888\,\text{km/h}^2\right)\left(\frac{1000\,\text{m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600\,\text{s}}\right)^2 = -0.994 \text{ m/s}^2$$

so that its *magnitude* is 0.994 m/s^2. A graph is shown below for the case where a collision is just avoided (x along the vertical axis is in meters and t along the horizontal axis is in seconds). The top (straight) line shows the motion of the locomotive and the bottom curve shows the motion of the passenger train.

The other case (where the collision is not quite avoided) would be similar except that the slope of the bottom curve would be greater than that of the top line at the point where they meet.



39. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22$ m/s^2 and $a_2 = -1.22$ m/s^2. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ m/s.

   (a) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x \quad \Longrightarrow \quad \Delta x = \frac{5.08^2}{2(1.22)}$$

   which yields $\Delta x = 10.59 \approx 10.6$ m.

   (b) Using Eq. 2-11, we have

$$t_1 = \frac{v - v_0}{a_1} = \frac{5.08}{1.22} = 4.17 \text{ s} .$$

   The deceleration time t_2 turns out to be the same so that $t_1 + t_2 = 8.33$ s. The distances traveled during t_1 and t_2 are the same so that they total to $2(10.59) = 21.18$ m. This implies that for a distance of $190 - 21.18 = 168.82$ m, the elevator is traveling at constant velocity. This time of constant velocity motion is

$$t_3 = \frac{168.82\,\text{m}}{5.08\,\text{m/s}} = 33.21 \text{ s} .$$

   Therefore, the total time is $8.33 + 33.21 \approx 41.5$ s.

40. Neglect of air resistance justifies setting $a = -g = -9.8$ m/s^2 (where *down* is our $-y$ direction) for the duration of the fall. This is constant acceleration motion, and we may use Table 2-1 (with Δy replacing Δx).

   (a) Using Eq. 2-16 and taking the negative root (since the final velocity is downward), we have

$$v = -\sqrt{v_0^2 - 2g\Delta y} = -\sqrt{0 - 2(9.8)(-1700)} = -183$$

   in SI units. Its magnitude is therefore 183 m/s.

   (b) No, but it is hard to make a convincing case without more analysis. We estimate the mass of a raindrop to be about a gram or less, so that its mass and speed (from part (a)) would be less than that of a typical bullet, which is good news. But the fact that one is dealing with *many* raindrops leads us to suspect that this scenario poses an unhealthy situation. If we factor in air resistance, the final speed is smaller, of course, and we return to the relatively healthy situation with which we are familiar.

41. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Starting the clock at the moment the wrench is dropped ($v_o = 0$), then $v^2 = v_o^2 - 2g\Delta y$ leads to

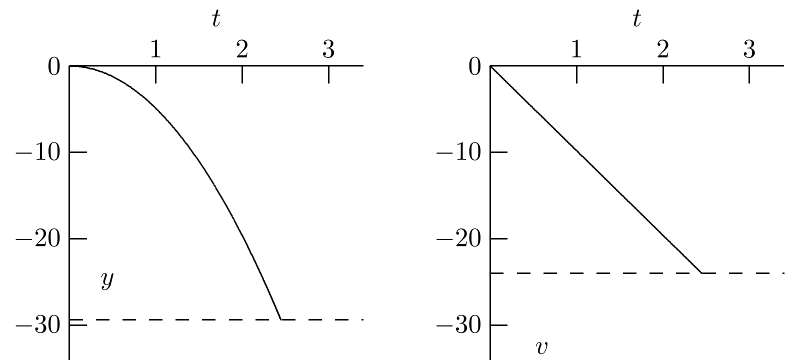
$$\Delta y = -\frac{(-24)^2}{2(9.8)} = -29.4 \text{ m}$$

so that it fell through a height of 29.4 m.

(b) Solving $v = v_0 - gt$ for time, we find:

$$t = \frac{v_0 - v}{g} = \frac{0 - (-24)}{9.8} = 2.45 \text{ s} .$$

(c) SI units are used in the graphs, and the initial position is taken as the coordinate origin. In the interest of saving space, we do not show the acceleration graph, which is a horizontal line at $-9.8\,\text{m/s}^2$.



42. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the fall. This is constant acceleration motion, which justifies the use of Table 2-1 (with Δy replacing Δx).

(a) Noting that $\Delta y = y - y_0 = -30$ m, we apply Eq. 2-15 and the quadratic formula (Appendix E) to compute t:

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \implies t = \frac{v_0 \pm \sqrt{v_0^2 - 2g\Delta y}}{g}$$

which (with $v_0 = -12$ m/s since it is downward) leads, upon choosing the positive root (so that $t > 0$), to the result:

$$t = \frac{-12 + \sqrt{(-12)^2 - 2(9.8)(-30)}}{9.8} = 1.54 \text{ s} .$$

(b) Enough information is now known that any of the equations in Table 2-1 can be used to obtain v; however, the one equation that does *not* use our result from part (a) is Eq. 2-16:

$$v = \sqrt{v_0^2 - 2g\Delta y} = 27.1 \text{ m/s}$$

where the positive root has been chosen in order to give *speed* (which is the magnitude of the velocity vector).

43. We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g = -9.8$ m/s^2 (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is $a =$ constant motion.

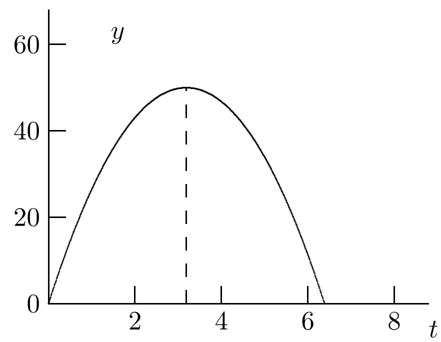
(a) At the highest point the velocity of the ball vanishes. Taking $y_0 = 0$, we set $v = 0$ in $v^2 = v_0^2 - 2gy$, and solve for the initial velocity: $v_0 = \sqrt{2gy}$. Since $y = 50$ m we find $v_0 = 31$ m/s.

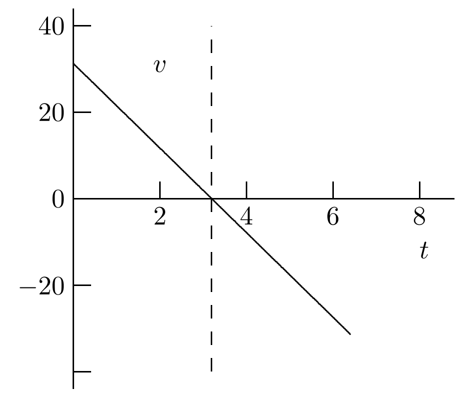
(b) It will be in the air from the time it leaves the ground until the time it returns to the ground ($y = 0$). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time $t > 0$) we have

$$y = v_0 t - \frac{1}{2} g t^2 \implies t = \frac{2v_0}{g}$$

which (using our result from part (a)) produces $t = 6.4$ s. It is possible to obtain this without using part (a)'s result; one can find the time just for the rise (from ground to highest point) from Eq. 2-16 and then double it.

(c) SI units are understood in the x and v graphs shown. In the interest of saving space, we do not show the graph of a, which is a horizontal line at $-9.8\,\mathrm{m/s^2}$.





44. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8$ m/s^2 (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have $a_2 = +25g = 245$ m/s^2.

(a) The time of fall is given by Eq. 2-15 with $v_0 = 0$ and $y = 0$. Thus,

$$t = \sqrt{\frac{2y_0}{g}} = \sqrt{\frac{2(145)}{9.8}} = 5.44 \text{ s} .$$

(b) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other equations can be used but they would use the result from part (a)).

$$v = -\sqrt{v_0^2 - 2g\,(y - y_0)} = -\sqrt{2gy_0} = -53.3 \text{ m/s}$$

where the negative root is chosen since this is a downward velocity.

(c) For the catching process, the answer to part (b) plays the role of an *initial* velocity ($v_0 = -53.3$ m/s) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2 = \frac{v^2 - v_0^2}{2a_2} = \frac{-(-53.3)^2}{2(245)} = -5.80 \text{ m}$$

where the negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

45. Taking the $+y$ direction *downward* and $y_0 = 0$, we have $y = v_0 t + \frac{1}{2} g t^2$ which (with $v_0 = 0$) yields $t = \sqrt{2y/g}$.

(a) For this part of the motion, $y = 50$ m so that

$$t = \sqrt{\frac{2(50)}{9.8}} = 3.2 \text{ s} .$$

(b) For this next part of the motion, we note that the total displacement is $y = 100$ m. Therefore, the total time is

$$t = \sqrt{\frac{2(100)}{9.8}} = 4.5 \text{ s} .$$

The difference between this and the answer to part (a) is the time required to fall through that second 50 m distance: $4.5 - 3.2 = 1.3$ s.

46. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to $y = 0$.

(a) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 - 2g\,(y - y_0)} = \sqrt{v_0^2 + 2gh} .$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity).

(b) We use the quadratic formula to solve Eq. 2-15 for t, with v_0 replaced with $-v_0$,

$$\Delta y = -v_0 t - \frac{1}{2} g t^2 \implies t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}$$

where the positive root is chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 + 2gh} - v_0}{g} .$$

(c) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation) .

(d) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \implies t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root again chosen to yield $t > 0$. With $y = 0$ and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 + 2gh} + v_0}{g} .$$

47. We neglect air resistance, which justifies setting $a = -g = -9.8 \text{ m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis.

(a) Using $y = v_0 t - \frac{1}{2} g t^2$, with $y = 0.544$ m and $t = 0.200$ s, we find

$$v_0 = \frac{y + \frac{1}{2} g t^2}{t} = \frac{0.544 + \frac{1}{2}(9.8)(0.200)^2}{0.200} = 3.70 \text{ m/s} .$$

(b) The velocity at $y = 0.544$ m is

$$v = v_0 - gt = 3.70 - (9.8)(0.200) = 1.74 \text{ m/s} .$$

(c) Using $v^2 = v_0^2 - 2gy$ (with different values for y and v than before), we solve for the value of y corresponding to maximum height (where $v = 0$).

$$y = \frac{v_0^2}{2g} = \frac{3.7^2}{2(9.8)} = 0.698 \text{ m} .$$

Thus, the armadillo goes $0.698 - 0.544 = 0.154$ m higher.

48. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis. The total time of fall can be computed from Eq. 2-15 (using the quadratic formula).

$$\Delta y = v_0 t - \frac{1}{2}gt^2 \implies t = \frac{v_0 + \sqrt{v_0^2 - 2g\Delta y}}{g}$$

with the positive root chosen. With $y = 0$, $v_0 = 0$ and $y_0 = h = 60$ m, we obtain

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}} = 3.5 \text{ s} .$$

Thus, "1.2 s earlier" means we are examining where the rock is at $t = 2.3$ s:

$$y - h = v_0(2.3) - \frac{1}{2}g(2.3)^2 \implies y = 34 \text{ m}$$

where we again use the fact that $h = 60$ m and $v_0 = 0$.

49. The speed of the boat is constant, given by $v_b = d/t$. Here, d is the distance of the boat from the bridge when the key is dropped (12 m) and t is the time the key takes in falling. To calculate t, we put the origin of the coordinate system at the point where the key is dropped and take the y axis to be positive in the *downward* direction. Taking the time to be zero at the instant the key is dropped, we compute the time t when $y = 45$ m. Since the initial velocity of the key is zero, the coordinate of the key is given by $y = \frac{1}{2}gt^2$. Thus

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(45\,\text{m})}{9.8\,\text{m/s}^2}} = 3.03 \text{ s} .$$

Therefore, the speed of the boat is

$$v_b = \frac{12\,\text{m}}{3.03\,\text{s}} = 4.0 \text{ m/s} .$$

50. With $+y$ upward, we have $y_0 = 36.6$ m and $y = 12.2$ m. Therefore, using Eq. 2-18 (the last equation in Table 2-1), we find

$$y - y_0 = vt + \frac{1}{2}gt^2 \implies v = -22 \text{ m/s}$$

at $t = 2.00$ s. The term *speed* refers to the magnitude of the velocity vector, so the answer is $|v| = 22.0$ m/s.

51. We first find the velocity of the ball just before it hits the ground. During contact with the ground its average acceleration is given by

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

where Δv is the change in its velocity during contact with the ground and $\Delta t = 20.0 \times 10^{-3}$ s is the duration of contact. Now, to find the velocity just *before* contact, we put the origin at the point where the ball is dropped (and take $+y$ upward) and take $t = 0$ to be when it is dropped. The ball strikes the ground at $y = -15.0$ m. Its velocity there is found from Eq. 2-16: $v^2 = -2gy$. Therefore,

$$v = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-15.0)} = -17.1 \text{ m/s}$$

where the negative sign is chosen since the ball is traveling downward at the moment of contact. Consequently, the average acceleration during contact with the ground is

$$a_{\text{avg}} = \frac{0 - (-17.1)}{20.0 \times 10^{-3}} = 857 \text{ m/s}^2 .$$

The fact that the result is positive indicates that this acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

52. The y axis is arranged so that ground level is $y = 0$ and $+y$ is upward.

(a) At the point where its fuel gets exhausted, the rocket has reached a height of

$$y' = \frac{1}{2}at^2 = \frac{(4.00)(6.00)^2}{2} = 72.0 \text{ m} .$$

From Eq. 2-11, the speed of the rocket (which had started at rest) at this instant is

$$v' = at = (4.00)(6.00) = 24.0 \text{ m/s} .$$

The additional height Δy_1 the rocket can attain (beyond y') is given by Eq. 2-16 with vanishing final speed: $0 = v'^{\,2} - 2g\Delta y_1$. This gives

$$\Delta y_1 = \frac{v'^{\,2}}{2g} = \frac{(24.0)^2}{2(9.8)} = 29.4 \text{ m} .$$

Recalling our value for y', the total height the rocket attains is seen to be $72.0 + 29.4 = 101$ m.

(b) The time of free-fall flight (from y' until it returns to $y = 0$) after the fuel gets exhausted is found from Eq. 2-15:

$$-y' = v't - \frac{1}{2}gt^2 \implies -72.0 = (24.0)t - \frac{9.80}{2}t^2 .$$

Solving for t (using the quadratic formula) we obtain $t = 7.00$ s. Recalling the upward acceleration time used in part (a), we see the total time of flight is $7.00 + 6.00 = 13.0$ s.

53. The average acceleration during contact with the floor is given by $a_{\text{avg}} = (v_2 - v_1)/\Delta t$, where v_1 is its velocity just before striking the floor, v_2 is its velocity just as it leaves the floor, and Δt is the duration of contact with the floor (12×10^{-3} s). Taking the y axis to be positively upward and placing the origin at the point where the ball is dropped, we first find the velocity just before striking the floor, using $v_1^2 = v_0^2 - 2gy$. With $v_0 = 0$ and $y = -4.00$ m, the result is

$$v_1 = -\sqrt{-2gy} = -\sqrt{-2(9.8)(-4.00)} = -8.85 \text{ m/s}$$

where the negative root is chosen because the ball is traveling downward. To find the velocity just after hitting the floor (as it ascends without air friction to a height of 2.00 m), we use $v^2 = v_2^2 - 2g(y - y_0)$ with $v = 0$, $y = -2.00$ m (it ends up two meters *below* its initial drop height), and $y_0 = -4.00$ m. Therefore,

$$v_2 = \sqrt{2g(y - y_0)} = \sqrt{2(9.8)(-2.00 + 4.00)} = 6.26 \text{ m/s} .$$

Consequently, the average acceleration is

$$a_{\text{avg}} = \frac{v_2 - v_1}{\Delta t} = \frac{6.26 + 8.85}{12.0 \times 10^{-3}} = 1.26 \times 10^3 \text{ m/s}^2 .$$

The positive nature of the result indicates that the acceleration vector points upward. In a later chapter, this will be directly related to the magnitude and direction of the force exerted by the ground on the ball during the collision.

54. The height reached by the player is $y = 0.76$ m (where we have taken the origin of the y axis at the floor and $+y$ to be upward).

(a) The initial velocity v_0 of the player is

$$v_0 = \sqrt{2gy} = \sqrt{2(9.8)(0.76)} = 3.86 \text{ m/s} .$$

This is a consequence of Eq. 2-16 where velocity v vanishes. As the player reaches $y_1 = 0.76 - 0.15 = 0.61$ m, his speed v_1 satisfies $v_0^2 - v_1^2 = 2gy_1$, which yields

$$v_1 = \sqrt{v_0^2 - 2gy_1} = \sqrt{(3.86)^2 - 2(9.80)(0.61)} = 1.71 \text{ m/s} .$$

The time t_1 that the player spends *ascending* in the top $\Delta y_1 = 0.15$ m of the jump can now be found from Eq. 2-17:

$$\Delta y_1 = \frac{1}{2}(v_1 + v)\, t_1 \implies t_1 = \frac{2(0.15)}{1.71 + 0} = 0.175 \text{ s}$$

which means that the total time spend in that top 15 cm (both ascending and descending) is $2(0.17) = 0.35$ s $= 350$ ms.

(b) The time t_2 when the player reaches a height of 0.15 m is found from Eq. 2-15:

$$0.15 = v_0 t_2 - \frac{1}{2} g t_2^2 = (3.86) t_2 - \frac{9.8}{2} t_2^2 ,$$

which yields (using the quadratic formula, taking the smaller of the two positive roots) $t_2 = 0.041$ s $= 41$ ms, which implies that the total time spend in that bottom 15 cm (both ascending and descending) is $2(41) = 82$ ms.

55. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to the origin of the y axis. The time drop 1 leaves the nozzle is taken as $t = 0$ and its time of landing on the floor t_1 can be computed from Eq. 2-15, with $v_0 = 0$ and $y_1 = -2.00$ m.

$$y_1 = -\frac{1}{2} g t_1^2 \implies t_1 = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-2.00)}{9.8}} = 0.639 \text{ s} .$$

At that moment,the fourth drop begins to fall, and from the regularity of the dripping we conclude that drop 2 leaves the nozzle at $t = 0.639/3 = 0.213$ s and drop 3 leaves the nozzle at $t = 2(0.213) = 0.426$ s. Therefore, the time in free fall (up to the moment drop 1 lands) for drop 2 is $t_2 = t_1 - 0.213 = 0.426$ s and the time in free fall (up to the moment drop 1 lands) for drop 3 is $t_3 = t_1 - 0.426 = 0.213$ s. Their positions at that moment are

$$\begin{aligned} y_2 &= -\frac{1}{2} g t_2^2 = -\frac{1}{2}(9.8)(0.426)^2 = -0.889 \text{ m} \\ y_3 &= -\frac{1}{2} g t_3^2 = -\frac{1}{2}(9.8)(0.213)^2 = -0.222 \text{ m} , \end{aligned}$$

respectively. Thus, drop 2 is 89 cm below the nozzle and drop 3 is 22 cm below the nozzle when drop 1 strikes the floor.

56. The graph shows $y = 25$ m to be the highest point (where the speed momentarily vanishes). The neglect of "air friction" (or whatever passes for that on the distant planet) is certainly reasonable due to the symmetry of the graph.

   (a) To find the acceleration due to gravity g_p on that planet, we use Eq. 2-15 (with $+y$ up)

$$y - y_0 = vt + \frac{1}{2}g_p t^2 \implies 25 - 0 = (0)(2.5) + \frac{1}{2}g_p(2.5)^2$$

   so that $g_p = 8.0$ m/s^2.

   (b) That same (max) point on the graph can be used to find the initial velocity.

$$y - y_0 = \frac{1}{2}(v_0 + v)\,t \implies 25 - 0 = \frac{1}{2}(v_0 + 0)\,(2.5)$$

   Therefore, $v_0 = 20$ m/s.

57. Taking $+y$ to be upward and placing the origin at the point from which the objects are dropped, then the location of diamond 1 is given by $y_1 = -\frac{1}{2}gt^2$ and the location of diamond 2 is given by $y_2 = -\frac{1}{2}g(t-1)^2$. We are starting the clock when the first object is dropped. We want the time for which $y_2 - y_1 = 10$ m. Therefore,

$$-\frac{1}{2}g(t-1)^2 + \frac{1}{2}gt^2 = 10 \implies t = (10/g) + 0.5 = 1.5 \text{ s} .$$

58. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. When something is thrown straight up and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-25), the time of flight t is half of its time of ascent t_a, which is given by Eq. 2-18 with $\Delta y = H$ and $v = 0$ (indicating the maximum point).

$$H = vt_a + \frac{1}{2}gt_a^2 \implies t_a = \sqrt{\frac{2H}{g}}$$

   Writing these in terms of the total time in the air $t = 2t_a$ we have

$$H = \frac{1}{8}gt^2 \implies t = 2\sqrt{\frac{2H}{g}} .$$

   We consider two throws, one to height H_1 for total time t_1 and another to height H_2 for total time t_2, and we set up a ratio:

$$\frac{H_2}{H_1} = \frac{\frac{1}{8}gt_2^2}{\frac{1}{8}gt_1^2} = \left(\frac{t_2}{t_1}\right)^2$$

   from which we conclude that if $t_2 = 2t_1$ (as is required by the problem) then $H_2 = 2^2 H_1 = 4H_1$.

59. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. We placing the coordinate origin on the ground. We note that the initial velocity of the package is the same as the velocity of the balloon, $v_0 = +12$ m/s and that its initial coordinate is $y_0 = +80$ m.

   (a) We solve $y = y_0 + v_0 t - \frac{1}{2}gt^2$ for time, with $y = 0$, using the quadratic formula (choosing the positive root to yield a positive value for t).

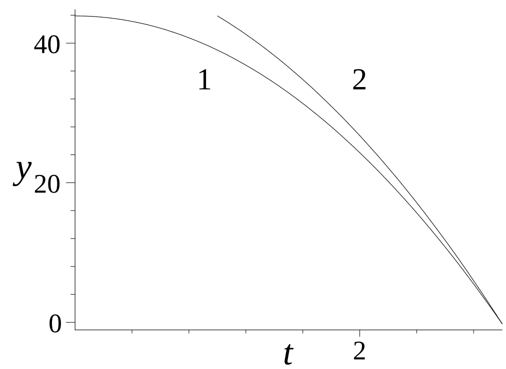
$$t = \frac{v_0 + \sqrt{v_0^2 + 2gy_0}}{g} = \frac{12 + \sqrt{12^2 + 2(9.8)(80)}}{9.8} = 5.4 \text{ s}$$

(b) If we wish to avoid using the result from part (a), we could use Eq. 2-16, but if that is not a concern, then a variety of formulas from Table 2-1 can be used. For instance, Eq. 2-11 leads to $v = v_0 - gt = 12 - (9.8)(5.4) = -41$ m/s. Its final *speed* is 41 m/s.

60. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Eq. 2-15 (with Δy replacing Δx) because this is constant acceleration motion. We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial downward velocity $-v_0$, so that v_0 is being used for the initial *speed*). SI units are used throughout.

$$\begin{aligned}\Delta y' &= 0(t) - \frac{1}{2}gt^2 \\ \Delta y &= (-v_0)(t-1) - \frac{1}{2}g(t-1)^2\end{aligned}$$

Since the problem indicates $\Delta y' = \Delta y = -43.9$ m, we solve the first equation for t (finding $t = 2.99$ s) and use this result to solve the second equation for the initial speed of the second stone:



$$-43.9 = (-v_0)(1.99) - \frac{1}{2}(9.8)(1.99)^2$$

which leads to $v_0 = 12.3$ m/s.

61. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with Δy replacing Δx) because the ball has constant acceleration motion. We use primed variables (except t) with the constant-velocity elevator (so $v' = 20$ m/s), and unprimed variables with the ball (with initial velocity $v_0 = v' + 10 = 30$ m/s, relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height y (relative to the ground) with $v^2 = v_0^2 - 2g(y - y_o)$, where the highest point is characterized by $v = 0$. Thus,

$$y = y_o + \frac{v_0^2}{2g} = 76 \text{ m}$$

where $y_o = y'_o + 2 = 30$ m (where $y'_o = 28$ m is given in the problem) and $v_0 = 30$ m/s relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and "fixes" the coordinate origin to it); in this case, one describes the elevator motion with $y' = y'_o + v't$ and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0e}t - \frac{1}{2}gt^2 \implies t = \frac{v_{0e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where $v_{0e} = 20$ m/s is the initial velocity of the ball relative to the elevator and $\Delta y_e = -2.0$ m is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for t ; the result is $t = 4.2$ s.

62. We neglect air resistance, which justifies setting $a = -g = -9.8$ m/s^2 (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_o = 0$).

   (a) We apply Eq. 2-16 to both measurements, with SI units understood.

$$\begin{aligned} v_B^2 = v_0^2 - 2gy_B \quad &\Longrightarrow \quad \left(\frac{1}{2}v\right)^2 + 2g(y_A + 3) = v_0^2 \\ v_A^2 = v_0^2 - 2gy_A \quad &\Longrightarrow \quad v^2 + 2gy_A = v_0^2 \end{aligned}$$

   We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 2g(3) = v^2 + 2gy_A \quad \Longrightarrow \quad 2g(3) = \frac{3}{4}v^2$$

   which yields $v = \sqrt{2g(4)} = 8.85$ m/s.

   (b) An object moving upward at A with speed $v = 8.85$ m/s will reach a maximum height $y - y_A = v^2/2g = 4.00$ m above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B.

63. The object, once it is dropped ($v_0 = 0$) is in free-fall ($a = -g = -9.8$ m/s^2 if we take *down* as the $-y$ direction), and we use Eq. 2-15 repeatedly.

   (a) The (positive) distance D from the lower dot to the mark corresponding to a certain reaction time t is given by $\Delta y = -D = -\frac{1}{2}gt^2$, or $D = gt^2/2$. Thus for $t_1 = 50.0\,\text{ms}$

$$D_1 = \frac{(9.8\,\text{m/s}^2)(50.0 \times 10^{-3}\,\text{s})^2}{2} = 0.0123\,\text{m} = 1.23\,\text{cm}\ .$$

   (b) For $t_2 = 100\,\text{ms}$

$$D_2 = \frac{(9.8\,\text{m/s}^2)(100 \times 10^{-3}\,\text{s})^2}{2} = 0.049\,\text{m} = 4D_1\ ;$$

   for $t_3 = 150\,\text{ms}$

$$D_3 = \frac{(9.8\,\text{m/s}^2)(150 \times 10^{-3}\,\text{s})^2}{2} = 0.11\,\text{m} = 9D_1\ ;$$

   for $t_4 = 200\,\text{ms}$

$$D_4 = \frac{(9.8\,\text{m/s}^2)(200 \times 10^{-3}\,\text{s})^2}{2} = 0.196\,\text{m} = 16D_1\ ;$$

   and for $t_4 = 250\,\text{ms}$

$$D_5 = \frac{(9.8\,\text{m/s}^2)(250 \times 10^{-3}\,\text{s})^2}{2} = 0.306\,\text{m} = 25D_1\ .$$

64. During free fall, we ignore the air resistance and set $a = -g = -9.8$ m/s^2 where we are choosing *down* to be the $-y$ direction. The initial velocity is zero so that Eq. 2-15 becomes $\Delta y = -\frac{1}{2}gt^2$ where Δy represents the *negative* of the distance d she has fallen. Thus, we can write the equation as $d = \frac{1}{2}gt^2$ for simplicity.

(a) The time t_1 during which the parachutist is in free fall is (using Eq. 2-15) given by

$$d_1 = 50\,\text{m} = \frac{1}{2}gt_1^2 = \frac{1}{2}\left(9.80\,\text{m/s}^2\right)t_1^2$$

which yields $t_1 = 3.2\,\text{s}$. The *speed* of the parachutist just before he opens the parachute is given by the positive root $v_1^2 = 2gd_1$, or

$$v_1 = \sqrt{2gh_1} = \sqrt{(2)(9.80\,\text{m/s}^2)(50\,\text{m})} = 31\,\text{m/s}\,.$$

If the final speed is v_2, then the time interval t_2 between the opening of the parachute and the arrival of the parachutist at the ground level is

$$t_2 = \frac{v_1 - v_2}{a} = \frac{31\,\text{m/s} - 3.0\,\text{m/s}}{2\,\text{m/s}^2} = 14\,\text{s}\ .$$

This is a result of Eq. 2-11 where *speeds* are used instead of the (negative-valued) velocities (so that final-velocity minus initial-velocity turns out to equal initial-speed minus final-speed); we also note that the acceleration vector for this part of the motion is positive since it points upward (opposite to the direction of motion – which makes it a deceleration). The total time of flight is therefore $t_1 + t_2 = 17\,\text{s}$.

(b) The distance through which the parachutist falls after the parachute is opened is given by

$$d = \frac{v_1^2 - v_2^2}{2a} = \frac{(31\,\text{m/s})^2 - (3.0\,\text{m/s})^2}{(2)(2.0\,\text{m/s}^2)} \approx 240\,\text{m}\,.$$

In the computation, we have used Eq. 2-16 with both sides multiplied by -1 (which changes the negative-valued Δy into the positive d on the left-hand side, and switches the order of v_1 and v_2 on the right-hand side). Thus the fall begins at a height of $h = 50 + d \approx 290\,\text{m}$.

65. The time t the pot spends passing in front of the window of length $L = 2.0$ m is $0.25\,\text{s}$ each way. We use v for its velocity as it passes the top of the window (going up). Then, with $a = -g = -9.8\ \text{m/s}^2$ (taking *down* to be the $-y$ direction), Eq. 2-18 yields

$$L = vt - \frac{1}{2}gt^2 \quad \Longrightarrow \quad v = \frac{L}{t} - \frac{1}{2}gt\ .$$

The distance H the pot goes above the top of the window is therefore (using Eq. 2-16 with the *final velocity* being zero to indicate the highest point)

$$H = \frac{v^2}{2g} = \frac{(L/t - gt/2)^2}{2g} = \frac{(2.00/0.25 - (9.80)(0.25)/2)^2}{(2)\,(9.80)} = 2.34\,\text{m}\ .$$

66. The time being considered is 6 years and roughly 235 days, which is approximately $\Delta t = 2.1 \times 10^7$ s. Using Eq. 2-3, we find the average speed to be

$$\frac{30600 \times 10^3\,\text{m}}{2.1 \times 10^7\,\text{s}} = 0.15\ \text{m/s}\ .$$

67. We assume constant velocity motion and use Eq. 2-2 (with $v_{\text{avg}} = v > 0$). Therefore,

$$\Delta x = v\Delta t = \left(303\,\frac{\text{km}}{\text{h}}\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)\left(100 \times 10^{-3}\,\text{s}\right) = 8.4\,\text{m}\ .$$

68. For each rate, we use distance $d = vt$ and convert to SI using 0.0254 cm = 1 inch (from which we derive the factors appearing in the computations below).

(a) The total distance d comes from summing

$$\begin{aligned}
d_1 &= \left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.762\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 7.62\,\text{m} \\
d_2 &= \left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.381\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 3.81\,\text{m} \\
d_3 &= \left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.914\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 13.72\,\text{m} \\
d_4 &= \left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.457\,\text{m/step}}{60\,\text{s/min}}\right)(5\,\text{s}) = 6.86\,\text{m}
\end{aligned}$$

so that $d = d_1 + d_2 + d_3 + d_4 = 32$ m.

(b) Average velocity is computed using Eq. 2-2: $v_{\text{avg}} = 32/20 = 1.6$ m/s, where we have used the fact that the total time is 20 s.

(c) The total time t comes from summing

$$\begin{aligned}
t_1 &= \frac{8\,\text{m}}{\left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.762\,\text{m/step}}{60\,\text{s/min}}\right)} = 5.25\,\text{s} \\
t_2 &= \frac{8\,\text{m}}{\left(120\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.381\,\text{m/step}}{60\,\text{s/min}}\right)} = 10.5\,\text{s} \\
t_3 &= \frac{8\,\text{m}}{\left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.914\,\text{m/step}}{60\,\text{s/min}}\right)} = 2.92\,\text{s} \\
t_4 &= \frac{8\,\text{m}}{\left(180\,\frac{\text{steps}}{\text{min}}\right)\left(\frac{0.457\,\text{m/step}}{60\,\text{s/min}}\right)} = 5.83\,\text{s}
\end{aligned}$$

so that $t = t_1 + t_2 + t_3 + t_4 = 24.5$ s.

(d) Average velocity is computed using Eq. 2-2: $v_{\text{avg}} = 32/24.5 = 1.3$ m/s, where we have used the fact that the total distance is $4(8) = 32$ m.

69. The statement that the stoneflies have "constant speed along a straight path" means we are dealing with constant velocity motion (Eq. 2-2 with v_{avg} replaced with v_{s} or v_{ns}, as the case may be).

(a) We set up the ratio and simplify (using d for the common distance).

$$\frac{v_{\text{s}}}{v_{\text{ns}}} = \frac{d/t_{\text{s}}}{d/t_{\text{ns}}} = \frac{t_{\text{ns}}}{t_{\text{s}}} = \frac{25.0}{7.1} = 3.52$$

(b) We examine Δt and simplify until we are left with an expression having numbers and no variables other than v_{s}. Distances are understood to be in meters.

$$\begin{aligned}
t_{\text{ns}} - t_{\text{s}} &= \frac{2}{v_{\text{ns}}} - \frac{2}{v_{\text{s}}} \\
&= \frac{2}{(v_{\text{s}}/3.52)} - \frac{2}{v_{\text{s}}} \\
&= \frac{2}{v_{\text{s}}}(3.52 - 1) \\
&\approx \frac{5}{v_{\text{s}}}
\end{aligned}$$

70. We orient $+x$ along the direction of motion (so a will be negative-valued, since it is a deceleration), and we use Eq. 2-7 with $a_{\text{avg}} = -3400g = -3400(9.8) = -3.33 \times 10^4$ m/s^2 and $v = 0$ (since the recorder finally comes to a stop).

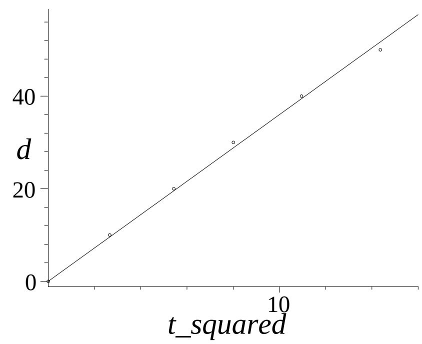
$$a_{\text{avg}} = \frac{v - v_0}{\Delta t} \implies v_0 = \left(3.33 \times 10^4\,\text{m/s}^2\right)\left(6.5 \times 10^{-3}\,\text{s}\right)$$

which leads to $v_0 = 217$ m/s.

71. (a) It is the intent of this problem to treat the $v_0 = 0$ condition rigidly. In other words, we are not fitting the distance to just any second-degree polynomial in t; rather, we requiring $d = At^2$ (which meets the condition that d and its derivative is zero when $t = 0$). If we perform a leastsquares fit with this expression, we obtain $A = 3.587$ (SI units understood). We return to this discussion in part (c). Our expectation based on Eq. 2-15, assuming no error in starting the clock at the moment the acceleration begins, is $d = \frac{1}{2}at^2$ (since he started at the coordinate origin, the location of which presumably is something we can be fairly certain about).

(b) The graph (d on the vertical axis, SI units understood) is shown.

The horizontal axis is t^2 (as indicated by the problem statement) so that we have a straight line instead of a parabola.



(c) Comparing our two expressions for d, we see the parameter A in our fit should correspond to $\frac{1}{2}a$, so $a = 2(3.587) \approx 7.2\,\text{m/s}^2$. Now, other approaches might be considered (trying to fit the data with $d = Ct^2 + B$ for instance, which leads to $a = 2C = 7.0\,\text{m/s}^2$ and $B \neq 0$), and it might be useful to have the class discuss the assumptions made in each approach.

72. (a) We estimate $x \approx 2$ m at $t = 0.5$ s, and $x \approx 12$ m at $t = 4.5$ s. Hence, using the definition of average velocity Eq. 2-2, we find

$$v_{\text{avg}} = \frac{12 - 2}{4.5 - 0.5} = 2.5 \text{ m/s} \quad .$$

(b) In the region $4.0 \leq t \leq 5.0$, the graph depicts a straight line, so its slope represents the instantaneous velocity for any point in that interval. Its slope is the average velocity between $t = 4.0$ s and $t = 5.0$ s:

$$v_{\text{avg}} = \frac{16.0 - 8.0}{5.0 - 4.0} = 8.0 \text{ m/s} \quad .$$

Thus, the instantaneous velocity at $t = 4.5$ s is 8.0 m/s. (Note: similar reasoning leads to a value needed in the next part: the slope of the $0 \leq t \leq 1$ region indicates that the instantaneous velocity at $t = 0.5$ s is 4.0 m/s.)

(c) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{8.0 - 4.0}{4.5 - 0.5} = 1.0 \text{ m/s}^2 \quad .$$

(d) The instantaneous acceleration is the instantaneous rate-of-change of the velocity, and the constant x vs. t slope in the interval $4.0 \leq t \leq 5.0$ indicates that the velocity is constant during that interval. Therefore, $a = 0$ at $t = 4.5$ s.

73. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = 6.0t^2 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = 12t$$

with SI units understood. This expressions are used in the parts that follow.

(a) Using the definition of average velocity, Eq. 2-2, we find

$$v_{\text{avg}} = \frac{x(2) - x(1)}{2.0 - 1.0} = \frac{2(2)^3 - 2(1)^3}{1.0} = 14 \text{ m/s} \ .$$

(b) The average acceleration is defined by Eq. 2-7:

$$a_{\text{avg}} = \frac{v(2) - v(1)}{2.0 - 1.0} = \frac{6(2)^2 - 6(1)^2}{1.0} = 18 \text{ m/s}^2 \ .$$

(c) The value of $v(t)$ when $t = 1.0$ s is $v(1) = 6(1)^2 = 6.0$ m/s.

(d) The value of $a(t)$ when $t = 1.0$ s is $a(1) = 12(1) = 12$ m/s^2.

(e) The value of $v(t)$ when $t = 2.0$ s is $v(2) = 6(2)^2 = 24$ m/s.

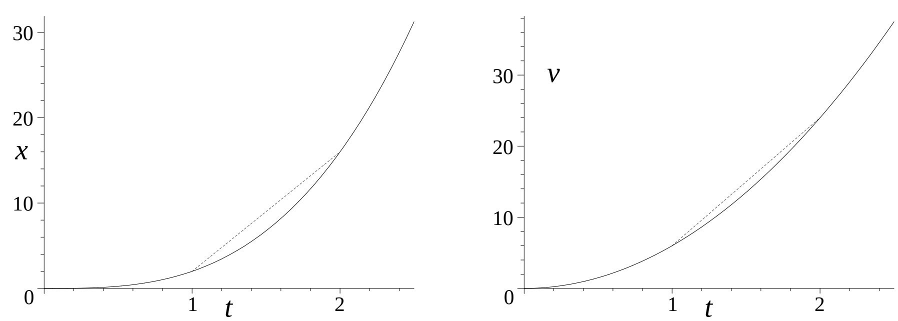
(f) The value of $a(t)$ when $t = 2.0$ s is $a(2) = 12(2) = 24$ m/s^2.

(g) We don't expect average values of a quantity, say, heights of trees, to equal any particular height for a specific tree, but we are sometimes surprised at the different kinds of averaging that can be performed. Now, the acceleration is a linear function (of time) so its average as defined by Eq. 2-7 is, not surprisingly, equal to the arithmetic average of its $a(1)$ and $a(2)$ values. The velocity is not a linear function so the result of part (a) is not equal to the arithmetic average of parts (c) and (e) (although it is fairly close). This reminds us that the calculus-based definition of the average a function (equivalent to Eq. 2-2 for v_{avg}) is not the same as the simple idea of an arithmetic average of two numbers; in other words,

$$\frac{1}{t' - t} \int_t^{t'} f(\tau)\, d\tau \neq \frac{f(t') + f(t)}{2}$$

except in very special cases (like with linear functions).

(h) The graphs are shown below, $x(t)$ on the left and $v(t)$ on the right. SI units are understood. We do not show the tangent lines (representing instantaneous slope values) at $t = 1$ and $t = 2$, but we do show line segments representing the average quantities computed in parts (a) and (b).



74. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). We denote the 1.00 s duration mentioned in the problem as $t - t'$ where t is the value of time when it lands and t' is one second prior to that. The corresponding distance is $y - y' = 0.50h$, where y denotes the location of the ground. In these terms, y is the same as h, so we have $h - y' = 0.50h$ or $0.50h = y'$.

(a) We find t' and t from Eq. 2-15 (with $v_0 = 0$):

$$\begin{aligned} y' &= \frac{1}{2}gt'^2 \implies t' = \sqrt{\frac{2y'}{g}} \\ y &= \frac{1}{2}gt^2 \implies t = \sqrt{\frac{2y}{g}} \; . \end{aligned}$$

Plugging in $y = h$ and $y' = 0.50h$, and dividing these two equations, we obtain

$$\frac{t'}{t} = \sqrt{\frac{2(0.50h)/g}{2h/g}} = \sqrt{0.50} \; .$$

Letting $t' = t - 1.00$ (SI units understood) and cross-multiplying, we find

$$t - 1.00 = t\sqrt{0.50} \implies t = \frac{1.00}{1 - \sqrt{0.50}}$$

which yields $t = 3.41$ s.

(b) Plugging this result into $y = \frac{1}{2}gt^2$ we find $h = 57$ m.

(c) In our approach, we did not use the quadratic formula, but we did "choose a root" when we assumed (in the last calculation in part (a)) that $\sqrt{0.50} = +2.236$ instead of -2.236. If we had instead let $\sqrt{0.50} = -2.236$ then our answer for t would have been roughly 0.6 s which would imply that $t' = t - 1$ would equal a negative number (indicating a time *before* it was dropped) which certainly does not fit with the physical situation described in the problem.

75. (a) Let the height of the diving board be h. We choose *down* as the $+y$ direction and set the coordinate origin at the point where it was dropped (which is when we start the clock). Thus, $y = h$ designates the location where the ball strikes the water. Let the depth of the lake be D, and the total time for the ball to descend be T. The speed of the ball as it reaches the surface of the lake is then $v = \sqrt{2gh}$ (from Eq. 2-16), and the time for the ball to fall from the board to the lake surface is $t_1 = \sqrt{2h/g}$ (from Eq. 2-15). Now, the time it spends descending in the lake (at constant velocity v) is

$$t_2 = \frac{D}{v} = \frac{D}{\sqrt{2gh}} \; .$$

Thus, $T = t_1 + t_2 = \sqrt{\frac{2h}{g}} + \frac{D}{\sqrt{2gh}}$, which gives

$$D = T\sqrt{2gh} - 2h = (4.80)\sqrt{(2)(9.80)(5.20)} - (2)(5.20) = 38.1 \text{ m} \; .$$

(b) Using Eq. 2-2, the average velocity is

$$v_{\text{avg}} = \frac{D + h}{T} = \frac{38.1 + 5.20}{4.80} = 9.02 \text{ m/s}$$

where (recalling our coordinate choices) the positive sign means that the ball is going downward (if, however, upwards had been chosen as the positive direction, then this answer would turn out negative-valued).

(c) We find v_0 from $\Delta y = v_0 t + \frac{1}{2}gt^2$ with $t = T$ and $\Delta y = h + D$. Thus,

$$v_0 = \frac{h + D}{T} - \frac{gT}{2} = \frac{5.20 + 38.1}{4.80} - \frac{(9.8)(4.80)}{2} = -14.5 \text{ m/s}$$

where (recalling our coordinate choices) the negative sign means that the ball is being thrown upward.

76. The time Δt is $2(60) + 41 = 161$ min and the displacement $\Delta x = 370$ cm. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{370}{161} = 2.3 \text{ cm/min} .$$

77. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ and find the latter two quantities by differentiating:

$$v(t) = \frac{dx(t)}{t} = -15t^2 + 20 \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -30t$$

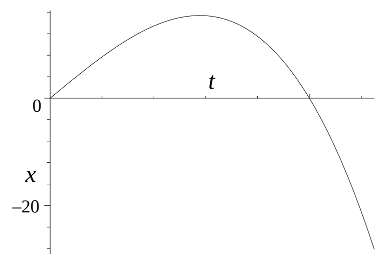
with SI units understood. This expressions are used in the parts that follow.

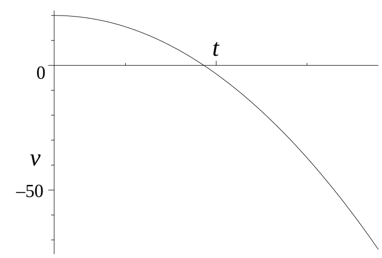
(a) From $0 = -15t^2 + 20$, we see that the only positive value of t for which the particle is (momentarily) stopped is $t = \sqrt{20/15} = 1.2$ s.

(b) From $0 = -30t$, we find $a(0) = 0$ (that is, it vanishes at $t = 0$).

(c) It is clear that $a(t) = -30t$ is negative for $t > 0$ and positive for $t < 0$.

(d) We show the two of the graphs below (the third graph, $a(t)$, which is a straight line through the origin with slope $= -30$ is omitted in the interest of saving space). SI units are understood.





78. (a) It follows from Eq. 2-8 that $v - v_0 = \int a\,dt$, which has the geometric interpretation of being the area under the graph. Thus, with $v_0 = 2.0$ m/s and that area amounting to 3.0 m/s (adding that of a triangle to that of a square, over the interval $0 \le t \le 2$ s), we find $v = 2.0 + 3.0 = 5.0$ m/s (which we will denote as v_2 in the next part). The information given that $x_0 = 4.0$ m is not used in this solution.

(b) During $2 < t \le 4$ s, the graph of a is a straight line with slope 1.0 m/s^3. Extrapolating, we see that the intercept of this line with the a axis is zero. Thus, with SI units understood,

$$v = v_2 + \int_{2.0}^{t} a\ d\tau = 5.0 + \int_{2.0}^{t} (1.0)\tau\, d\tau = 5.0 + \frac{(1.0)t^2 - (1.0)(2.0)^2}{2}$$

which yields $v = 3.0 + 0.50t^2$ in m/s.

79. We assume the train accelerates from rest ($v_0 = 0$ and $x_0 = 0$) at $a_1 = +1.34 \text{ m/s}^2$ until the midway point and then decelerates at $a_2 = -1.34 \text{ m/s}^2$ until it comes to a stop ($v_2 = 0$) at the next station. The velocity at the midpoint is v_1 which occurs at $x_1 = 806/2 = 403$ m.

(a) Eq. 2-16 leads to

$$v_1^2 = v_0^2 + 2a_1x_1 \implies v_1 = \sqrt{2(1.34)(403)}$$

which yields $v_1 = 32.9$ m/s.

(b) The time t_1 for the accelerating stage is (using Eq. 2-15)

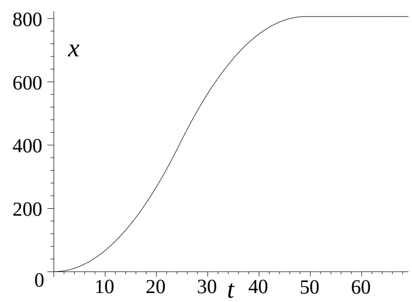
$$x_1 = v_0t_1 + \frac{1}{2}a_1t_1^2 \implies t_1 = \sqrt{\frac{2(403)}{1.34}}$$

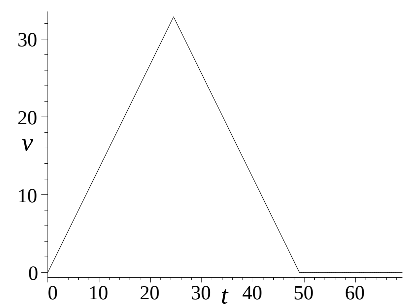
which yields $t_1 = 24.53$ s. Since the time interval for the decelerating stage turns out to be the same, we double this result and obtain $t = 49.1$ s for the travel time between stations.

(c) With a "dead time" of 20 s, we have $T = t + 20 = 69.1$ s for the total time between start-ups. Thus, Eq. 2-2 gives

$$v_{\text{avg}} = \frac{806\,\text{m}}{69.1\,\text{s}} = 11.7 \text{ m/s} .$$

(d) We show the two of the graphs below. The third graph, $a(t)$, is not shown to save space; it consists of three horizontal "steps" – one at 1.34 during $0 < t < 24.53$ and the next at -1.34 during $24.53 < t < 49.1$ and the last at zero during the "dead time" $49.1 < t < 69.1$). SI units are understood.





80. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D up the hill is, of course, the same as the distance down the hill, and since the speed is constant (during each stage of the motion) we have speed$= D/t$. Thus, the average speed is

$$\frac{D_{\text{up}} + D_{\text{down}}}{t_{\text{up}} + t_{\text{down}}} = \frac{2D}{\frac{D}{v_{\text{up}}} + \frac{D}{v_{\text{down}}}}$$

which, after canceling D and plugging in $v_{\text{up}} = 40$ km/h and $v_{\text{down}} = 60$ km/h, yields 48 km/h for the average speed.

81. During T_r the velocity v_0 is constant (in the direction we choose as $+x$) and obeys $v_0 = D_r/T_r$ where we note that in SI units the velocity is $v_0 = 200(1000/3600) = 55.6$ m/s. During T_b the acceleration is opposite to the direction of v_0 (hence, for us, $a < 0$) until the car is stopped ($v = 0$).

(a) Using Eq. 2-16 (with $\Delta x_b = 170$ m) we find

$$v^2 = v_0^2 + 2a\Delta x_b \implies a = -\frac{v_0^2}{2\Delta x_b}$$

which yields $|a| = 9.08$ m/s^2.

(b) We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{9.08}{9.8}\right) 9 = 0.926g .$$

(c) We use Eq. 2-17 to obtain the braking time:

$$\Delta x_b = \frac{1}{2}(v_0 + v)\,T_b \implies T_b = \frac{2(170)}{55.6} = 6.12 \text{ s} .$$

(d) We express our result for T_b as a multiple of the reaction time T_r by setting up a ratio:

$$T_b = \left(\frac{6.12}{400 \times 10^{-3}}\right) T_r = 15.3T_r \quad .$$

(e) We are only asked what the *increase* in distance D is, due to $\Delta T_r = 0.100$ s, so we simply have

$$\Delta D = v_0 \Delta T_r = (55.6)(0.100) = 5.56 \text{ m} .$$

82. We take $+x$ in the direction of motion. We use subscripts 1 and 2 for the data. Thus, $v_1 = +30$ m/s, $v_2 = +50$ m/s and $x_2 - x_1 = +160$ m.

(a) Using these subscripts, Eq. 2-16 leads to

$$a = \frac{v_2^2 - v_1^2}{2\,(x_2 - x_1)} = \frac{50^2 - 30^2}{2(160)} = 5.0 \text{ m/s}^2 .$$

(b) We find the time interval corresponding to the displacement $x_2 - x_1$ using Eq. 2-17:

$$t_2 - t_1 = \frac{2\,(x_2 - x_1)}{v_1 + v_2} = \frac{2(160)}{30 + 50} = 4.0 \text{ s} .$$

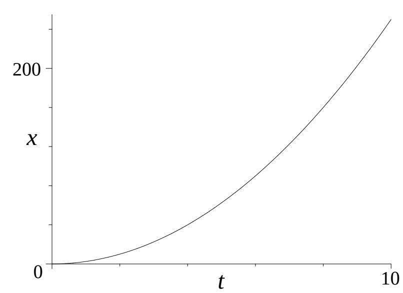
(c) Since the train is at rest ($v_0 = 0$) when the clock starts, we find the value of t_1 from Eq. 2-11:

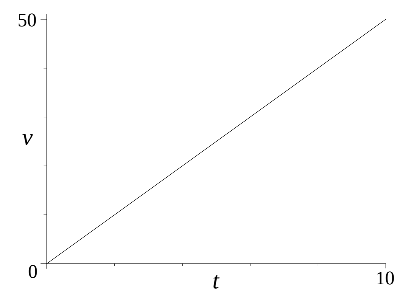
$$v_1 = v_0 + a t_1 \implies t_1 = \frac{30}{5.0} = 6.0 \text{ s} .$$

(d) The coordinate origin is taken to be the location at which the train was initially at rest (so $x_0 = 0$). Thus, we are asked to find the value of x_1. Although any of several equations could be used, we choose Eq. 2-17:

$$x_1 = \frac{1}{2}\,(v_0 + v_1)\,t_1 = \frac{1}{2}(30)(6.0) = 90 \text{ m} .$$

(e) The graphs are shown below, with SI units assumed.





83. Direction of $+x$ is implicit in the problem statement. The initial position (when the clock starts) is $x_0 = 0$ (where $v_0 = 0$), the end of the speeding-up motion occurs at $x_1 = 1100/2 = 550$ m, and the subway comes to a halt ($v_2 = 0$) at $x_2 = 1100$ m.

(a) Using Eq. 2-15, the subway reaches x_1 at

$$t_1 = \sqrt{\frac{2x_1}{a_1}} = \sqrt{\frac{2(550)}{1.2}} = 30.3 \text{ s} .$$

The time interval $t_2 - t_1$ turns out to be the same value (most easily seen using Eq. 2-18 so the total time is $t_2 = 2(30.3) = 60.6$ s.

(b) Its maximum speed occurs at t_1 and equals

$$v_1 = v_0 + a_1 t_1 = 36.3 \text{ m/s} .$$

(c) The graphs are not shown here, in the interest of saving space. They are very similar to those shown in the solution for problem 79, above.

84. We note that the running time for Bill Rodgers is $\Delta t_R = 2(3600) + 10(60) = 7800$ s. We also note that the magnitude of the average velocity (Eq. 2-2) and Eq. 2-3 (for average speed) agree in this exercise (which is not usually the case).

   (a) Denoting the Lewis' average velocity as v_L (similarly for Rodgers), we find

$$v_L = \frac{100\,\text{m}}{10\,\text{s}} = 10 \text{ m/s} \quad v_R = \frac{42000\,\text{m}}{7800\,\text{s}} = 5.4 \text{ m/s} .$$

   (b) If Lewis continued at this rate, he would covered $D = 42000$ m in

$$\Delta t_L = \frac{D}{v_L} = \frac{42000}{10} = 4200 \text{ s}$$

   which is equivalent to 1 h and 10 min.

85. We choose *down* as the $+y$ direction and use the equations of Table 2-1 (replacing x with y) with $a = +g$, $v_0 = 0$ and $y_0 = 0$. We use subscript 2 for the elevator reaching the ground and 1 for the halfway point.

   (a) Eq. 2-16, $v_2^2 = v_0^2 + 2a(y_2 - y_0)$, leads to

$$v_2 = \sqrt{2gy_2} = \sqrt{2(9.8)(120)} = 48.5 \text{ m/s} .$$

   (b) The time at which it strikes the ground is (using Eq. 2-15)

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2(120)}{9.8}} = 4.95 \text{ s} .$$

   (c) Now Eq. 2-16, in the form $v_1^2 = v_0^2 + 2a(y_1 - y_0)$, leads to

$$v_1 = \sqrt{2gy_1} = \sqrt{2(9.8)(60)} = 34.2 \text{ m/s} .$$

   (d) The time at which it reaches the halfway point is (using Eq. 2-15)

$$t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(60)}{9.8}} = 3.50 \text{ s} .$$

86. To find the "launch" velocity of the rock, we apply Eq. 2-11 to the maximum height (where the speed is momentarily zero)

$$v = v_0 - gt \implies 0 = v_0 - (9.8)(2.5)$$

so that $v_0 = 24.5$ m/s (with $+y$ up). Now we use Eq. 2-15 to find the height of the tower (taking $y_0 = 0$ at the ground level)

$$y - y_0 = v_0 t + \frac{1}{2}at^2 \implies y - 0 = (24.5)(1.5) - \frac{1}{2}(9.8)(1.5)^2 \quad .$$

Thus, we obtain $y = 26$ m.

87. We take the direction of motion as $+x$, so $a = -5.18$ m/s^2, and we use SI units, so $v_0 = 55(1000/3600) = 15.28$ m/s.

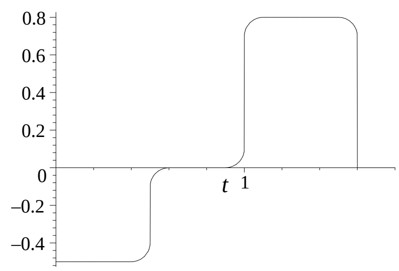
   (a) The velocity is constant during the reaction time T, so the distance traveled during it is $d_r = v_0T - (15.28)(0.75) = 11.46$ m. We use Eq. 2-16 (with $v = 0$) to find the distance d_b traveled during braking:

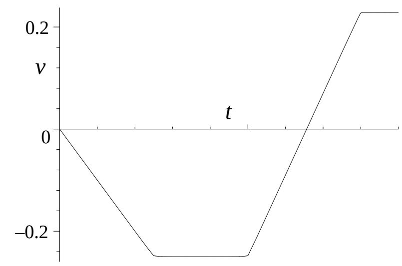
$$v^2 = v_0^2 + 2ad_b \implies d_b = -\frac{15.28^2}{2(-5.18)}$$

   which yields $d_b = 22.53$ m. Thus, the total distance is $d_r + d_b = 34.0$ m, which means that the driver *is* able to stop in time. And if the driver were to continue at v_0, the car would enter the intersection in $t = (40\,\text{m})/(15.28\,\text{m/s}) = 2.6$ s which is (barely) enough time to enter the intersection before the light turns, which many people would consider an acceptable situation.

(b) In this case, the total distance to stop (found in part (a) to be 34 m) is greater than the distance to the intersection, so the driver cannot stop without the front end of the car being a couple of meters into the intersection. And the time to reach it at constant speed is $32/15.28 = 2.1$ s, which is too long (the light turns in 1.8 s). The driver is caught between a rock and a hard place.

88. We assume $v_0 = 0$ and integrate the acceleration to find the velocity. In the graphs below (the first is the acceleration, like Fig. 2-35 but with some numbers we adopted, and the second is the velocity) we modeled the curve in the textbook with straight lines and circular arcs for the rounded corners, and literally integrated it. The intent of the textbook was not, however, to go through such an involved procedure, and one should be able to obtain a close approximation to the shape of the velocity graph below (the one on the right) just by applying the idea that constant nonzero acceleration means a linearly changing velocity.





89. We take the direction of motion as $+x$, take $x_0 = 0$ and use SI units, so $v = 1600(1000/3600) = 444$ m/s.

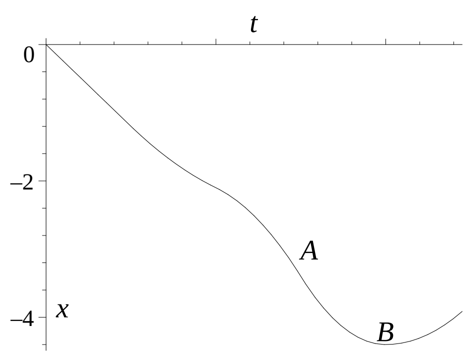
(a) Eq. 2-11 gives $444 = a(1.8)$ or $a = 247\ \text{m/s}^2$. We express this as a multiple of g by setting up a ratio:

$$a = \left(\frac{247}{9.8}\right) g = 25g\ .$$

(b) Eq. 2-17 readily yields

$$x = \frac{1}{2}(v_0 + v)\, t = \frac{1}{2}(444)(1.8) = 400\ \text{m}\ .$$

90. The graph is shown below. We assumed each interval described in the problem was one time unit long. A marks where the curve is steepest and B is where it is least steep (where it, in fact, has zero slope).



91. We use the functional notation $x(t)$, $v(t)$ and $a(t)$ in this solution, where the latter two quantities are obtained by differentiation:

$$v(t) = \frac{dx(t)}{dt} = -12t \quad \text{and} \quad a(t) = \frac{dv(t)}{dt} = -12$$

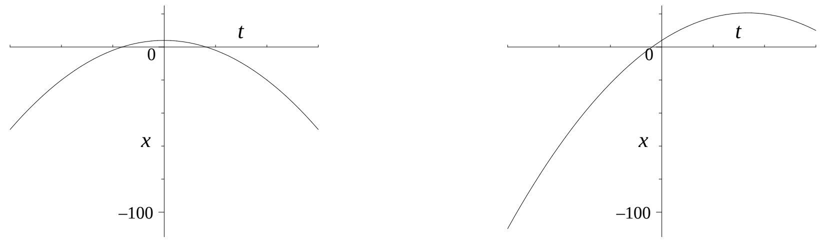
with SI units understood.

(a) From $v(t) = 0$ we find it is (momentarily) at rest at $t = 0$.

(b) We obtain $x(0) = 4.0$ m

(c) Requiring $x(t) = 0$ in the expression $x(t) = 4.0 - 6.0t^2$ leads to $t = \pm 0.82$ s for the times when the particle can be found passing through the origin.

(d) We show both the asked-for graph (on the left) as well as the "shifted" graph which is relevant to part (e). In both cases, the time axis is given by $-3 \le t \le 3$ (SI units understood).



(e) We arrived at the graph on the right (shown above) by adding $20t$ to the $x(t)$ expression.

(f) Examining where the slopes of the graphs become zero, it is clear that the shift causes the $v = 0$ point to correspond to a larger value of x (the top of the second curve shown in part (d) is higher than that of the first).

92. (a) The slope of the graph (at a point) represents the velocity there, and the up-or-down concavity of the curve there indicates the $\pm$ sign of the acceleration. Thus, during AB we have $v > 0$ and $a = 0$ (since it is a straight line). During BC, we still have $v > 0$ but there is some curvature and a downward concavity is indicated (so $a < 0$). The segment CD is horizontal, implying the particle remains at the same position for some time; thus, $v = a = 0$ during CD. Clearly, the slope is negative during DE (so $v < 0$) but whether or not the graph is curved is less clear; we believe it is, with an upward concavity ($a > 0$).

(b) The key word is "obviously." Since it seems plausible to us that the curved portions can be "fit" with parabolic arcs (indications of constant acceleration by Eq. 2-15), then our answer is "no."

(c) Neither signs of slopes nor the sign of the concavity depends on a global shift in one axis or another (or, for that matter, on rescalings of the axes themselves) so the answer again is "no."

93. (a) The slope of the graph (at a point) represents the velocity there, and the up-or-down concavity of the curve there indicates the $\pm$ sign of the acceleration. Thus, during AB we have positive slope ($v > 0$) and $a < 0$ (since it is concave downward). The segment BC is horizontal, implying the particle remains at the same position for some time; thus, $v = a = 0$ during BC. During CD we have $v > 0$ and $a > 0$ (since it is concave upward). Clearly, the slope is positive during DE (so $v > 0$) but whether or not the graph is curved is less clear; we believe it is not, so $a = 0$.

(b) The key word is "obviously." Since it seems plausible to us that the curved portions can be "fit" with parabolic arcs (indications of constant acceleration by Eq. 2-15), then our answer is "no."

(c) Neither signs of slopes nor the sign of the concavity depends on a global shift in one axis or another (or, for that matter, on rescalings of the axes themselves) so the answer again is "no."

94. This problem consists of two parts: part 1 with constant acceleration (so that the equations in Table 2-1 apply), $v_0 = 0$, $v = 11.0$ m/s, $x = 12.0$ m, and $x_0 = 0$ (adopting the starting line as the coordinate origin); and, part 2 with constant velocity (so that $x - x_0 = vt$ applies) with $v = 11.0$ m/s, $x_0 = 12.0$, and $x = 100.0$ m.

(a) We obtain the time for part 1 from Eq. 2-17

$$x - x_0 = \frac{1}{2}(v_0 + v)\,t_1 \implies 12.0 - 0 = \frac{1}{2}(0 + 11.0)t_1$$

so that $t_1 = 2.2$ s, and we find the time for part 2 simply from $88.0 = (11.0)t_2 \;\; \rightarrow t_2 = 8.0$ s. Therefore, the total time is $t_1 + t_2 = 10.2$ s.

(b) Here, the total time is required to be 10.0 s, and we are to locate the point x_p where the runner switches from accelerating to proceeding at constant speed. The equations for parts 1 and 2, used above, therefore become

$$\begin{aligned} x_p - 0 &= \frac{1}{2}(0 + 11.0)t_1 \\ 100.0 - x_p &= (11.0)(10.0 - t_1) \end{aligned}$$

where in the latter equation, we use the fact that $t_2 = 10.0 - t_1$. Solving the equations for the two unknowns, we find that $t_1 = 1.8$ s and $x_p = 10.0$ m.

95. We take $+x$ in the direction of motion, so $v_0 = +24.6$ m/s and $a = -4.92$ m/s^2. We also take $x_0 = 0$.

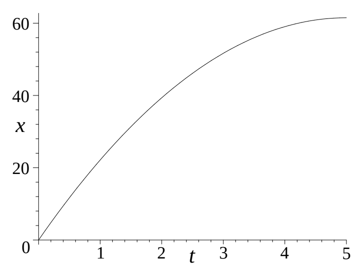
(a) The time to come to a halt is found using Eq. 2-11:

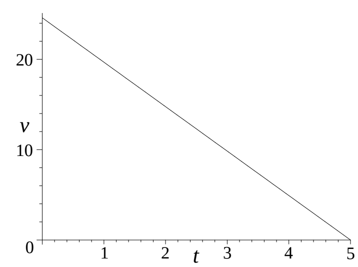
$$0 = v_0 + at \implies t = -\frac{24.6}{-4.92} = 5.00 \text{ s} .$$

(b) Although several of the equations in Table 2-1 will yield the result, we choose Eq. 2-16 (since it does not depend on our answer to part (a)).

$$0 = v_0^2 + 2ax \implies x = -\frac{24.6^2}{2(-4.92)} = 61.5 \text{ m} .$$

(c) Using these results, we plot $v_0 t + \frac{1}{2}at^2$ (the x graph, shown below, on the left) and $v_0 + at$ (the v graph, below right) over $0 \le t \le 5$ s, with SI units understood.





96. We take $+x$ in the direction of motion, so

$$v = (60\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = +16.7 \text{ m/s}$$

and $a > 0$. The location where it starts from rest ($v_0 = 0$) is taken to be $x_0 = 0$.

(a) Eq. 2-7 gives $a_{\text{avg}} = (v - v_0)/t$ where $t = 5.4$ s and the velocities are given above. Thus, $a_{\text{avg}} = 3.1$ m/s^2.

(b) The assumption that $a =$constant permits the use of Table 2-1. From that list, we choose Eq. 2-17:

$$x = \frac{1}{2}\,(v_0 + v)\,t = \frac{1}{2}(16.7)(5.4) = 45 \text{ m} .$$

(c) We use Eq. 2-15, now with $x = 250$ m:

$$x = \frac{1}{2}at^2 \implies t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(250)}{3.1}}$$

which yields $t = 13$ s.

97. Converting to SI units, we have $v = 3400(1000/3600) = 944$ m/s (presumed constant) and $\Delta t = 0.10$ s. Thus, $\Delta x = v\Delta t = 94$ m.

98. The (ideal) driving time before the change was $t = \Delta x/v$, and after the change it is $t' = \Delta x/v'$. The time saved by the change is therefore

$$t - t' = \Delta x\left(\frac{1}{v} - \frac{1}{v'}\right) = \Delta x\left(\frac{1}{55} - \frac{1}{65}\right) = \Delta x(0.0028\,\mathrm{h/mi})$$

which becomes, converting $\Delta x = 700/1.61 = 435$ mi (using a conversion found on the inside front cover of the textbook), $t - t' = (435)(0.0028) = 1.2$ h. This is equivalent to 1 h and 13 min.

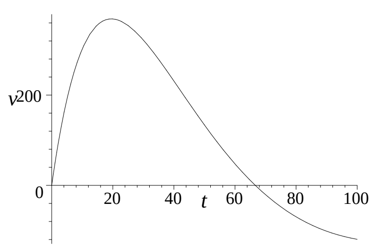
99. (a) With the understanding that these are good to three significant figures, we write the function (in SI units) as

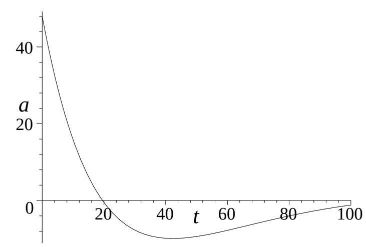
$$x(t) = -32 + 24t^2e^{-0.03t}$$

and find the velocity and acceleration functions by differentiating (calculus is reviewed Appendix E). We find

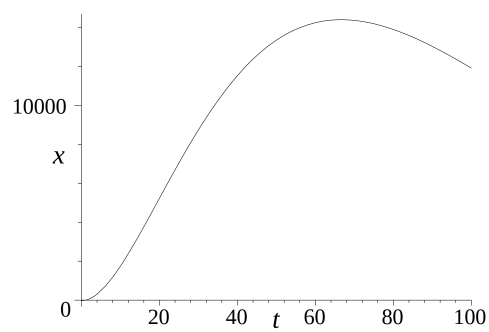
$$v(t) = 24t(2 - 0.03t)e^{-0.03t} \quad \text{and} \quad a(t) = 24\left(2 - 0.12t + 0.0009t^2\right)e^{-0.03t}\ .$$

(b) The $v(t)$ and $a(t)$ graphs are shown below (SI units understood). The time axis in both cases runs from $t = 0$ to $t = 100$ s. We include the $x(t)$ graph in the next part, accompanying our discussion of its root (which is, as suggested by the graph, a small positive value of t).





(c) We seek to find a positive value of t for which $24t^2e^{-0.03t} = 32$. We turn to the calculator or to a computer for its (numerical) solution. In this case, we ignore the roots outside the $0 \le t \le 100$ range (such as $t = -1.14$ s and $t = 387.77$ s) and choose $t = 1.175$ s as our answer. All of these are rounded-off values. We find $v = 53.5$ m/s and $a = 43.1$ m/s^2 at this time.



(d) It is much easier to find when $24t(2 - 0.03t)e^{-0.03t} = 0$ since the roots are clearly $t_1 = 0$ and $t_2 = 2/0.03 = 66.7$ s. We find $x(t_1) = -32.0$ m and $a(t_1) = 48.0$ m/s^2 at the first root, and we find $x(t_2) = 1.44 \times 10^4$ m and $a(t_2) = -6.50$ m/s^2 at the second root.

100. We take $+x$ in the direction of motion, so $v_0 = +30$ m/s, $v_1 = +15$ m/s and $a < 0$. The acceleration is found from Eq. 2-11: $a = (v_1 - v_0)/t_1$ where $t_1 = 3.0$ s. This gives $a = -5.0$ m/s^2. The displacement (which in this situation is the same as the distance traveled) to the point it stops ($v_2 = 0$) is, using Eq. 2-16,

$$v_2^2 = v_0^2 + 2a\Delta x \implies \Delta x = -\frac{30^2}{2(-5)} = 90 \text{ m} .$$

101. We choose the direction of motion as the positive direction. We work with the kilometer and hour units, so we write $\Delta x = 0.088$ km.

(a) Eq. 2-16 leads to

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{65^2 - 85^2}{2(0.088)}$$

which yields $a = -1.7 \times 10^4$ km/h^2.

(b) In this case, we obtain

$$a = \frac{60^2 - 80^2}{2(0.088)} = -1.6 \times 10^4 \text{ km/h}^2 .$$

(c) In this final situation, we find

$$a = \frac{40^2 - 50^2}{2(0.088)} = -5.1 \times 10^3 \text{ km/h}^2 .$$

102. Let the vertical distances between Jim's and Clara's feet and the jump-off level be H_J and H_C, respectively. At the instant this photo was taken, Clara has fallen for a time T_C, while Jim has fallen for T_J. Thus (using Eq. 2-15 with $v_0 = 0$) we have

$$H_J = \frac{1}{2}gT_J^2 \quad \text{and} \quad H_C = \frac{1}{2}gT_C^2 .$$

Measuring directly from the photo, we get $H_J \approx 3.6$ m and $H_C \approx 6.3$ m, which yields $T_J \approx 0.86$ s and $T_C \approx 1.13$ s. Jim's waiting time is therefore $T_C - T_J \approx 0.3$ s.

103. We choose *down* as the $+y$ direction and place the coordinate origin at the top of the building (which has height H). During its fall, the ball passes (with velocity v_1) the top of the window (which is at y_1) at time t_1, and passes the bottom (which is at y_2) at time t_2. We are told $y_2 - y_1 = 1.20$ m and $t_2 - t_1 = 0.125$ s. Using Eq. 2-15 we have

$$y_2 - y_1 = v_1 (t_2 - t_1) + \frac{1}{2}g (t_2 - t_1)^2$$

which immediately yields

$$v_1 = \frac{1.20 - \frac{1}{2}(9.8)(0.125)^2}{0.125} = 8.99 \text{ m/s} .$$

From this, Eq. 2-16 (with $v_0 = 0$) reveals the value of y_1:

$$v_1^2 = 2gy_1 \implies y_1 = \frac{8.99^2}{2(9.8)} = 4.12 \text{ m} .$$

It reaches the ground ($y_3 = H$) at t_3. Because of the symmetry expressed in the problem ("upward flight is a reverse of the fall") we know that $t_3 - t_2 = 2.00/2 = 1.00$ s. And this means $t_3 - t_1 = 1.00 + 0.125 = 1.125$ s. Now Eq. 2-15 produces

$$\begin{aligned} y_3 - y_1 &= v_1 (t_3 - t_1) + \frac{1}{2}g (t_3 - t_1)^2 \\ y_3 - 4.12 &= (8.99)(1.125) + \frac{1}{2}(9.8)(1.125)^2 \end{aligned}$$

which yields $y_3 = H = 20.4$ m.

104. (a) Using the fact that the area of a triangle is $\frac{1}{2}$(base)(height) (and the fact that the integral corresponds to area under the curve) we find, from $t = 0$ through $t = 5$ s, the integral of v with respect to t is 15 m. Since we are told that $x_0 = 0$ then we conclude that $x = 15$ m when $t = 5.0$ s.

(b) We see directly from the graph that $v = 2.0$ m/s when $t = 5.0$ s.

(c) Since $a = \frac{dv}{dt}$ = slope of the graph, we find that the acceleration during the interval $4 < t < 6$ is uniformly equal to $-2.0\,\text{m/s}^2$.

(d) Thinking of $x(t)$ in terms of accumulated area (on the graph), we note that $x(1) = 1$ m; using this and the value found in part (a), Eq. 2-2 produces

$$v_{\text{avg}} = \frac{x(5) - x(1)}{5 - 1} = \frac{15 - 1}{4} = 3.5 \text{ m/s} .$$

(e) From Eq. 2-7 and the values $v(t)$ we read directly from the graph, we find

$$a_{\text{avg}} = \frac{v(5) - v(1)}{5 - 1} = \frac{2 - 2}{4} = 0.$$

105. (First problem of **Cluster 1**)
The two parts of this problem are as follows. Part 1 (motion from A to B) consists of constant acceleration (so Table 2-1 applies) and involves the data $v_0 = 0$, $v = 10.0$ m/s, $x_0 = 0$ and $x = 40.0$ m (taking point A as the coordinate origin and orienting the positive x axis towards B and C). Part 2 (from B to C) consists of constant velocity motion (so the simple equation $\frac{\Delta x}{\Delta t} = v$ applies) with $v = 10.0$ m/s and $\Delta t = 10.0$ s.

(a) Eq. 2-16 is an efficient way of finding the part 1 acceleration:

$$v^2 = v_0^2 + 2a\,(x - x_0) \quad \Longrightarrow \quad (10.0)^2 = 0 + 2a(40.0)$$

from which we obtain $a = 1.25$ m/s^2.

(b) Using Eq. 2-17 avoids using the result from part (a) and finds the time readily.

$$x - x_0 = \frac{1}{2}\,(v_0 + v)\,t \quad \Longrightarrow \quad 40.0 - 0 = \frac{1}{2}(0 + 10.0)t$$

This leads to $t = 8.00$ s, for part 1.

(c) We find the distance traveled in part 2 with $\Delta x = v\Delta t = (10.0)(10.0) = 100$ m.

(d) The average velocity is defined by Eq. 2-2

$$v_{\text{avg}} = \frac{x_C - x_A}{t_C - t_A} = \frac{140 - 0}{18 - 0} = 7.78 \text{ m/s} .$$

106. (Second problem of **Cluster 1**)
The two parts of this problem are as follows. Part 1 (motion from A to B) consists of constant acceleration (so Table 2-1 applies) and involves the data $v_0 = 20.0$ m/s, $v = 30.0$ m/s, $x_0 = 0$ and $t_1 = 10.0$ s (taking point A as the coordinate origin, orienting the positive x axis towards B and C, and starting the clock when it passes point A). Part 2 (from B to C) also involves uniformly accelerated motion but now with the data $v_0 = 30.0$ m/s, $v = 15.0$ m/s, and $\Delta x = x - x_0 = 150$ m.

(a) The distance for part 1 is given by

$$x - x_0 = \frac{1}{2}\,(v_0 + v)\,t_1 = \frac{1}{2}(20.0 + 30.0)(10.0)$$

which yields $x = 250$ m.

(b) The time t_2 for part 2 is found from the same formula as in part (a).

$$x - x_0 = \frac{1}{2}(v_0 + v)\, t_2 \quad \Longrightarrow \quad 150 = \frac{1}{2}(30.0 + 15.0)t_2 \ .$$

This results in $t_2 = 6.67$ s.

(c) The definition of average velocity is given by Eq. 2-2:

$$v_{\rm avg} = \frac{x_C - x_A}{t_C - t_A} = \frac{400 - 0}{16.7} = 24.0 \text{ m/s} \ .$$

(d) The definition of average acceleration is given by Eq. 2-7:

$$a_{\rm avg} = \frac{v_C - v_A}{t_C - t_A} = \frac{15.0 - 20.0}{16.7} = -0.30 \text{ m/s}^2 \ .$$

107. (Third problem of **Cluster 1**)
The problem consists of two parts (A to B at constant velocity, then B to C with constant acceleration). The constant velocity in part 1 is 20 m/s (taking the positive direction in the direction of motion) and $t_1 = 5.0$ s. In part 2, we have $v_0 = 20$ m/s, $v = 0$, and $t_2 = 10$ s.

(a) We find the distance in part 1 from $x - x_0 = vt_1$, so we obtain $x = 100$ m (taking A to be at the origin). And the position at the end of part 2 is then found using Eq. 2-17.

$$x = x_0 + \frac{1}{2}(v_0 + v)\, t_2 = 100 + \frac{1}{2}(20 + 0)(10) = 200 \text{ m} \ .$$

(b) The acceleration in part (a) can be found using Eq. 2-11.

$$v = v_0 + at_2 \quad \Longrightarrow \quad 0 = 20 + a(10) \ .$$

Thus, we find $a = -2.0$ m/s^2. The negative sign indicates that the acceleration vector points opposite to the chosen positive direction (the direction of motion), which is what we expect of a deceleration.

108. (Fourth problem of **Cluster 1**)
The part 1 motion in this problem is simply that of constant velocity, so $x_B - x_0 = v_1 t_1$ applies with $t_1 = 5.00$ s and $x_0 = x_A = 0$ if we choose point A as the coordinate origin. Next, the part 2 motion consists of constant acceleration (so the equations of Table 2-1, such as Eq. 2-17, apply) with $x_0 = x_B$ (an unknown), $v_0 = v_B$ (also unknown, but equal to the v_1 above), $x_C = 300$ m, $v_C = 10.0$ m/s, and $t_2 = 20.0$ s. The equations describing parts 1 and 2, respectively, are therefore

$$\begin{aligned} x_B - x_A = v_1 t_1 \quad &\Longrightarrow \quad x_B = v_1(5.00) \\ x_C - x_B = \frac{1}{2}(v_B + v_C)\, t_2 \quad &\Longrightarrow \quad 300 - x_B = \frac{1}{2}(v_B + 10.0)\,(20.0) \end{aligned}$$

(a) We use the fact that $v_A = v_1 = v_B$ in solving this set of simultaneous equations. Adding equations, we obtain the result $v_1 = 13.3$ m/s.

(b) In order to find the acceleration, we use our result from part (a) as the initial velocity in Eq. 2-14 (applied to the part 2 motion):

$$v = v_0 + at_2 \quad \Longrightarrow \quad 10.0 = 13.3 + a(20.0)$$

Thus, $a = -0.167$ m/s^2.

109. (Fifth problem of **Cluster 1**)

The problem consists of two parts, where part 1 (A to B) involves constant velocity motion for $t_1 = 5.00$ s and part 2 (B to C) involves uniformly accelerated motion. Assuming the coordinate origin is at point A and the positive axis is directed towards B and C, then we have $x_C = 250$ m, $a_2 = -0.500$ m/s^2, and $v_C = 0$.

(a) We set up the uniform velocity equation for part 1 ($\Delta x = vt$) and Eq. 2-16 for part 2 ($v^2 = v_0^2 + 2a\Delta x$) as a simultaneous set of equations to be solved:

$$\begin{aligned} x_B - 0 &= v_1(5.00) \\ 0^2 &= v_B^2 + 2(-0.500)\,(250 - x_B)\ . \end{aligned}$$

Bearing in mind that $v_A = v_1 = v_B$, we can solve the equations by, for instance, substituting the first into the second – eliminating x_B and leading to a quadratic equation for v_1:

$$v_1^2 + 5v_1 - 150 = 0\ .$$

The positive root gives us $v_1 = 13.5$ m/s.

(b) We obtain the duration t_2 of part 2 from Eq. 2-11:

$$v = v_0 + at_2 \quad\Longrightarrow\quad 0 = 13.5 + (-0.500)t_2$$

which yields the value $t_2 = 27.0$ s. Therefore, the total time is $t_1 + t_2 = 32.0$ s.

110. (Sixth problem of **Cluster 1**)

Both part 1 and part 2 of this problem involve uniformly accelerated motion, but at different rates a_1 and a_2. We take the coordinate origin at point A and direct the positive axis towards B and C. In these terms, we are given $x_A = 0$, $x_C = 1300$ m, $v_A = 0$, and $v_C = 50$ m/s. Further, the time-duration for each part is given: $t_1 = 20$ s and $t_2 = 40$ s.

(a) We have enough information to apply Eq. 2-17 ($\Delta x = \frac{1}{2}(v_0 + v)t$) to parts 1 and 2 and solve the simultaneous set:

$$\begin{aligned} x_B - x_A = \frac{1}{2}\,(v_A + v_B)\,t_1 &\quad\Longrightarrow\quad x_B = \frac{1}{2}\,v_B\,(20) \\ x_C - x_B = \frac{1}{2}\,(v_B + v_C)\,t_2 &\quad\Longrightarrow\quad 1300 - x_B = \frac{1}{2}\,(v_B + 50)\,(40) \end{aligned}$$

Adding equations, we find $v_B = 10$ m/s.

(b) The other unknown in the above set of equations is now easily found by plugging the result for v_B back in: $x_B = 100$ m.

(c) We can find a_1 a variety of ways, using the just-obtained results. We note that Eq. 2-11 is especially easy to use.

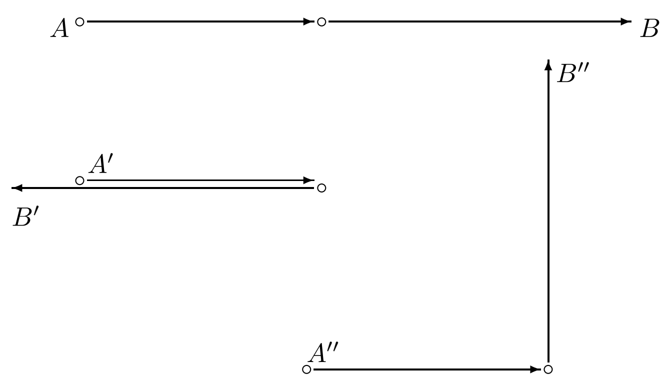
$$v = v_0 + a_1t_1 \quad\Longrightarrow\quad 10 = 0 + a_1(20)$$

This leads to $a_1 = 0.50$ m/s^2.

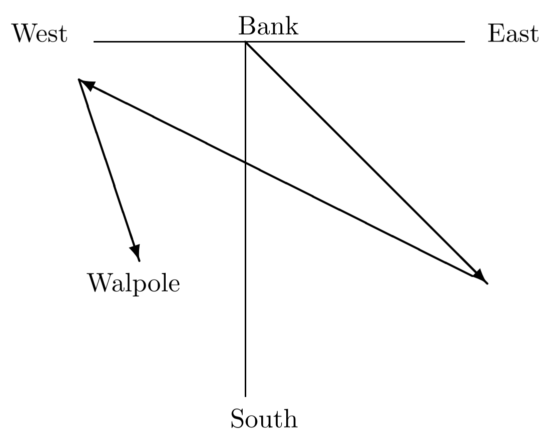
(d) To find a_2 we proceed as just as we did in part (c), so that Eq. 2-11 for part 2 becomes $50 = 10 + a_2(40)$. Therefore, the acceleration is $a_2 = 1.0\,\text{m/s}^2$.

Chapter 3

1. The vectors should be parallel to achieve a resultant 7 m long (the unprimed case shown below), antiparallel (in opposite directions) to achieve a resultant 1 m long (primed case shown), and perpendicular to achieve a resultant $\sqrt{3^2 + 4^2} = 5$ m long (the double-primed case shown). In each sketch, the vectors are shown in a "head-to-tail" sketch but the resultant is not shown. The resultant would be a straight line drawn from beginning to end; the beginning is indicated by A (with or without primes, as the case may be) and the end is indicated by B.



2. A sketch of the displacements is shown. The resultant (not shown) would be a straight line from start (Bank) to finish (Walpole). With a careful drawing, one should find that the resultant vector has length 29.5 km at 35° west of south.



3. The x component of $\vec{a}$ is given by $a_x = 7.3\cos 250° = -2.5$ and the y component is given by $a_y = 7.3\sin 250° = -6.9$. In considering the variety of ways to compute these, we note that the vector is 70° below the $-x$ axis, so the components could also have been found from $a_x = -7.3\cos 70°$ and $a_y = -7.3\sin 70°$. In a similar vein, we note that the vector is 20° from the $-y$ axis, so one could use $a_x = -7.3\sin 20°$ and $a_y = -7.3\cos 20°$ to achieve the same results.

4. The angle described by a full circle is $360° = 2\pi$ rad, which is the basis of our conversion factor. Thus,

$$(20.0°)\,\frac{2\pi\,\text{rad}}{360°} = 0.349 \text{ rad}$$

and (similarly) $50.0° = 0.873$ rad and $100° = 1.75$ rad. Also,

$$(0.330\,\text{rad})\,\frac{360°}{2\pi\,\text{rad}} = 18.9°$$

and (similarly) $2.10\,\text{rad} = 120°$ and $7.70\,\text{rad} = 441°$.

5. The textbook's approach to this sort of problem is through the use of Eq. 3-6, and is illustrated in Sample Problem 3-3. However, most modern graphical calculators can produce the results quite efficiently using rectangular $\leftrightarrow$ polar "shortcuts."

   (a) $\sqrt{(-25)^2 + 40^2} = 47.2$ m

   (b) Recalling that $\tan(\theta) = \tan(\theta + 180°)$, we note that the two possibilities for $\tan^{-1}(40/-25)$ are $-58°$ and $122°$. Noting that the vector is in the third quadrant (by the signs of its x and y components) we see that $122°$ is the correct answer. The graphical calculator "shortcuts" mentioned above are designed to correctly choose the right possibility.

6. The x component of $\vec{r}$ is given by $15\cos 30° = 13$ m and the y component is given by $15\sin 30° = 7.5$ m.

7. The point P is displaced vertically by $2R$, where R is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or πR. Since $R = 0.450$ m, the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the x axis is horizontal and the y axis is vertical, the vector displacement (in meters) is $\vec{r} = (1.414\,\hat{\imath} + 0.900\,\hat{\jmath})$. The displacement has a magnitude of

$$|\vec{r}| = \sqrt{(\pi R)^2 + (2R)^2} = R\sqrt{\pi^2 + 4} = 1.68 \text{ m}$$

and an angle of

$$\tan^{-1}\left(\frac{2R}{\pi R}\right) = \tan^{-1}\left(\frac{2}{\pi}\right) = 32.5°$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

8. Although we think of this as a three-dimensional movement, it is rendered effectively two-dimensional by referring measurements to its well-defined plane of the fault.

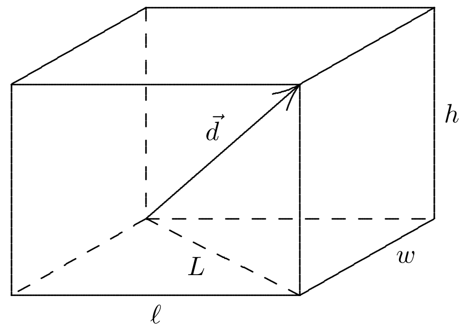
   (a) The magnitude of the net displacement is

$$|\vec{AB}| = \sqrt{|AD|^2 + |AC|^2} = \sqrt{17^2 + 22^2} = 27.8 \text{ m} .$$

   (b) The magnitude of the vertical component of $\vec{AB}$ is $|AD|\sin 52.0° = 13.4$ m.

9. The length unit meter is understood throughout the calculation.

   (a) We compute the distance from one corner to the diametrically opposite corner: $d = \sqrt{3.00^2 + 3.70^2 + 4.30^2} = 6.42$.

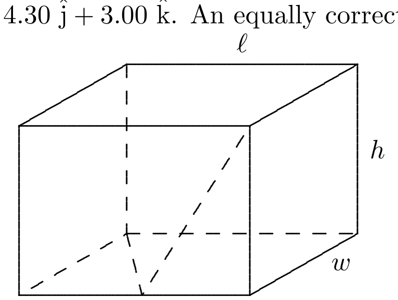


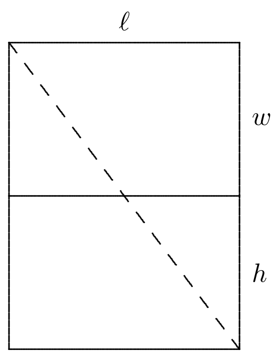
(b) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement.

(c) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h$.

(d) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector.

(e) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is $w = 3.70$, the y component of the displacement is 4.30, and the z component is 3.00. Thus $\vec{d} = 3.70\,\hat{\imath} + 4.30\,\hat{\jmath} + 3.00\,\hat{k}$. An equally correct answer is gotten by interchanging the length, width, and height.



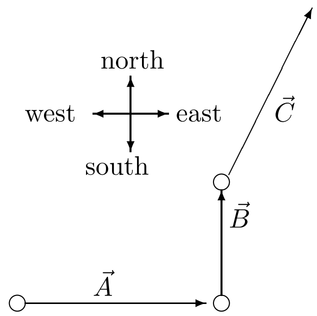


(f) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is

$$L_{\text{min}} = \sqrt{(w+h)^2 + \ell^2} = \sqrt{(3.70 + 3.00)^2 + 4.30^2} = 7.96 \text{ m} .$$

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result

of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\imath}$ direction ($+x$ direction) and *north* as the $\hat{\jmath}$ direction ($+y$ direction). All distances are understood to be in kilometers. We note that the angle between $\vec{C}$ and the x axis is $60°$. Thus,



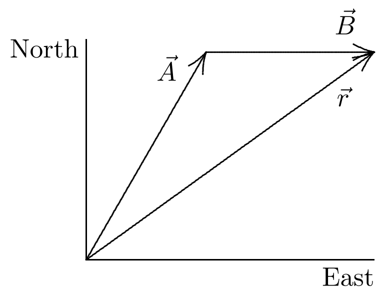
$$\begin{aligned}\vec{A} &= 50\ \hat{\imath}\\ \vec{B} &= 30\ \hat{\jmath}\\ \vec{C} &= 25\ \cos(60°)\ \hat{\imath} + 25\ \sin(60°)\ \hat{\jmath}\\ \vec{r} = \vec{A}+\vec{B}+\vec{C} &= 62.50\ \hat{\imath} + 51.65\ \hat{\jmath}\end{aligned}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{62.50^2 + 51.65^2} \approx 81\ \text{km}\ .$$

and its angle (counterclockwise from $+x$ axis) is $\tan^{-1}(51.65/62.50) \approx 40°$, which is to say that it points $40°$ *north of east*. Although the resultant $\vec{r}$ is shown in our sketch, it would be a direct line from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

11. The diagram shows the displacement vectors for the two segments of her walk, labeled $\vec{A}$ and $\vec{B}$, and the total ("final") displacement vector, labeled $\vec{r}$. We take east to be the $+x$ direction and north to be the $+y$ direction. We observe that the angle between $\vec{A}$ and the x axis is $60°$. Where the units are not explicitly shown, the distances are understood to be in meters. Thus, the components of $\vec{A}$ are $A_x = 250\cos 60° = 125$ and $A_y = 250\sin 30° = 216.5$. The components of $\vec{B}$ are $B_x = 175$ and $B_y = 0$. The components of the total displacement are $r_x = A_x + B_x = 125 + 175 = 300$ and $r_y = A_y + B_y = 216.5 + 0 = 216.5$.



   (a) The magnitude of the resultant displacement is

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2} = \sqrt{300^2 + 216.5^2} = 370\ \text{m}\ .$$

   (b) The angle the resultant displacement makes with the $+x$ axis is

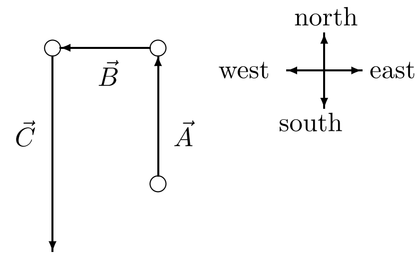
$$\tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{216.5}{300}\right) = 36°\ .$$

   (c) The total *distance* walked is $d = 250 + 175 = 425$ m.

   (d) The total distance walked is greater than the magnitude of the resultant displacement. The diagram shows why: $\vec{A}$ and $\vec{B}$ are not collinear.

12. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result

of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\imath}$ direction ($+x$ direction) and *north* as the $\hat{\jmath}$ direction ($+y$ direction). All distances are understood to be in kilometers. Therefore,



$$\begin{aligned}\vec{A} &= 3.1\,\hat{\jmath}\\ \vec{B} &= -2.4\,\hat{\imath}\\ \vec{C} &= -5.2\,\hat{\jmath}\\ \vec{r}=\vec{A}+\vec{B}+\vec{C} &= -2.1\,\hat{\imath}-2.4\,\hat{\jmath}\end{aligned}$$

which means that its magnitude is

$$|\vec{r}| = \sqrt{(-2.1)^2+(-2.4)^2} \approx 3.2 \text{ km} .$$

and the two possibilities for its angle are

$$\tan^{-1}\left(\frac{-2.4}{-2.1}\right) = 41^\circ, \quad \text{or} \quad 221^\circ .$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar $\leftrightarrow$ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the $+x$ axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

13. We write $\vec{r} = \vec{a} + \vec{b}$. When not explicitly displayed, the units here are assumed to be meters. Then $r_x = a_x + b_x = 4.0 - 13 = -9.0$ and $r_y = a_y + b_y = 3.0 + 7.0 = 10$. Thus $\vec{r} = (-9.0\,\text{m})\,\hat{\imath} + (10\,\text{m})\,\hat{\jmath}$. The magnitude of the resultant is

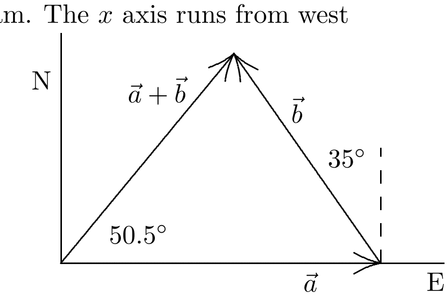
$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(-9.0)^2 + (10)^2} = 13 \text{ m} .$$

The angle between the resultant and the $+x$ axis is given by $\tan^{-1}(r_y/r_x) = \tan^{-1} 10/(-9.0)$ which is either -48° or 132°. Since the x component of the resultant is negative and the y component is positive, characteristic of the second quadrant, we find the angle is 132° (measured counterclockwise from $+x$ axis).

14. The x, y and z components (with meters understood) of $\vec{r}$ are

$$\begin{aligned} r_x = c_x + d_x &= 7.4 + 4.4 = 12\\ r_y = c_y + d_y &= -3.8 - 2.0 = -5.8\\ r_z = c_z + d_z &= -6.1 + 3.3 = -2.8 .\end{aligned}$$

15. The vectors are shown on the diagram. The x axis runs from west to east and the y axis run from south to north. Then $a_x = 5.0\,\text{m}$, $a_y = 0$, $b_x = -(4.0\,\text{m})\sin 35^\circ = -2.29\,\text{m}$, and $b_y = (4.0\,\text{m})\cos 35^\circ = 3.28\,\text{m}$.



(a) Let $\vec{c} = \vec{a} + \vec{b}$. Then $c_x = a_x + b_x = 5.0\,\text{m} - 2.29\,\text{m} = 2.71\,\text{m}$ and $c_y = a_y + b_y = 0 + 3.28\,\text{m} = 3.28\,\text{m}$. The magnitude of c is

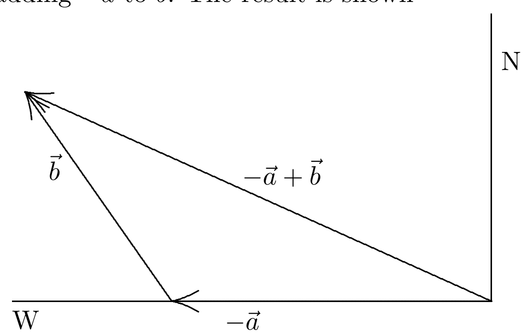
$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.71\,\text{m})^2 + (3.28\,\text{m})^2} = 4.3\ \text{m}\ .$$

(b) The angle θ that $\vec{c} = \vec{a} + \vec{b}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\frac{c_y}{c_x} = \tan^{-1}\frac{3.28\,\text{m}}{2.71\,\text{m}} = 50.4^\circ\ .$$

The second possibility ($\theta = 50.4^\circ + 180^\circ = 126^\circ$) is rejected because it would point in a direction opposite to $\vec{c}$.

(c) The vector $\vec{b} - \vec{a}$ is found by adding $-\vec{a}$ to $\vec{b}$. The result is shown on the diagram to the right. Let $\vec{c} = \vec{b} - \vec{a}$. Then $c_x = b_x - a_x = -2.29\,\text{m} - 5.0\,\text{m} = -7.29\,\text{m}$ and $c_y = b_y - a_y = 3.28\,\text{m}$. The magnitude of $\vec{c}$ is $c = \sqrt{c_x^2 + c_y^2} = 8.0\,\text{m}$.



(d) The tangent of the angle θ that $\vec{c}$ makes with the $+x$ axis (east) is

$$\tan\theta = \frac{c_y}{c_x} = \frac{3.28\,\text{m}}{-7.29\,\text{m}} = -4.50, .$$

There are two solutions: -24.2° and 155.8°. As the diagram shows, the second solution is correct. The vector $\vec{c} = -\vec{a} + \vec{b}$ is 24° north of west.

16. All distances in this solution are understood to be in meters.

(a) $\vec{a} + \vec{b} = (3.0\,\hat{\imath} + 4.0\,\hat{\jmath}) + (5.0\,\hat{\imath} - 2.0\,\hat{\jmath}) = 8.0\,\hat{\imath} + 2.0\,\hat{\jmath}$.

(b) The magnitude of $\vec{a} + \vec{b}$ is

$$|\vec{a} + \vec{b}| = \sqrt{8.0^2 + 2.0^2} = 8.2\ \text{m}\ .$$

(c) The angle between this vector and the $+x$ axis is $\tan^{-1}(2.0/8.0) = 14^\circ$.

(d) $\vec{b} - \vec{a} = (5.0\,\hat{\imath} - 2.0\,\hat{\jmath}) - (3.0\,\hat{\imath} + 4.0\,\hat{\jmath}) = 2.0\,\hat{\imath} - 6.0\,\hat{\jmath}$.

(e) The magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$|\vec{b} - \vec{a}| = \sqrt{2.0^2 + (-6.0)^2} = 6.3\ \text{m}\ .$$

(f) The angle between this vector and the $+x$ axis is $\tan^{-1}(-6.0/2.0) = -72^\circ$. The vector is 72° *clockwise* from the axis defined by $\hat{\imath}$.

17. All distances in this solution are understood to be in meters.

(a) $\vec{a} + \vec{b} = (4.0 + (-1.0))\,\hat{\imath} + ((-3.0) + 1.0)\,\hat{\jmath} + (1.0 + 4.0)\,\hat{k} = 3.0\,\hat{\imath} - 2.0\,\hat{\jmath} + 5.0\,\hat{k}$.

(b) $\vec{a} - \vec{b} = (4.0 - (-1.0))\,\hat{\imath} + ((-3.0) - 1.0)\,\hat{\jmath} + (1.0 - 4.0)\,\hat{k} = 5.0\,\hat{\imath} - 4.0\,\hat{\jmath} - 3.0\,\hat{k}$.

(c) The requirement $\vec{a} - \vec{b} + \vec{c} = 0$ leads to $\vec{c} = \vec{b} - \vec{a}$, which we note is the opposite of what we found in part (b). Thus, $\vec{c} = -5.0\,\hat{\imath} + 4.0\,\hat{\jmath} + 3.0\,\hat{k}$.

18. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6).

   (a) The magnitude of $\vec{a}$ is $\sqrt{4^2 + (-3)^2} = 5.0$ m.

   (b) The angle between $\vec{a}$ and the $+x$ axis is $\tan^{-1}(-3/4) = -37°$. The vector is 37° *clockwise* from the axis defined by $\hat{\imath}$.

   (c) The magnitude of $\vec{b}$ is $\sqrt{6^2 + 8^2} = 10$ m.

   (d) The angle between $\vec{b}$ and the $+x$ axis is $\tan^{-1}(8/6) = 53°$.

   (e) $\vec{a} + \vec{b} = (4 + 6)\,\hat{\imath} + ((-3) + 8)\,\hat{\jmath} = 10\,\hat{\imath} + 5\,\hat{\jmath}$, with the unit meter understood. The magnitude of this vector is $\sqrt{10^2 + 5^2} = 11$ m; we rounding to two significant figures in our results.

   (f) The angle between the vector described in part (e) and the $+x$ axis is $\tan^{-1}(5/10) = 27°$.

   (g) $\vec{b} - \vec{a} = (6 - 4)\,\hat{\imath} + (8 - (-3))\,\hat{\jmath} = 2\,\hat{\imath} + 11\,\hat{\jmath}$, with the unit meter understood. The magnitude of this vector is $\sqrt{2^2 + 11^2} = 11$ m, which is, interestingly, the same result as in part (e) (exactly, not just to 2 significant figures) (this curious coincidence is made possible by the fact that $\vec{a} \perp \vec{b}$).

   (h) The angle between the vector described in part (g) and the $+x$ axis is $\tan^{-1}(11/2) = 80°$.

   (i) $\vec{a} - \vec{b} = (4 - 6)\,\hat{\imath} + ((-3) - 8)\,\hat{\jmath} = -2\,\hat{\imath} - 11\,\hat{\jmath}$, with the unit meter understood. The magnitude of this vector is $\sqrt{(-2)^2 + (-11)^2} = 11$ m.

   (j) The two possibilities presented by a simple calculation for the angle between the vector described in part (i) and the $+x$ direction are $\tan^{-1}(11/2) = 80°$, and $180° + 80° = 260°$. The latter possibility is the correct answer (see part (k) for a further observation related to this result).

   (k) Since $\vec{a} - \vec{b} = (-1)(\vec{b} - \vec{a})$, they point in opposite (antiparallel) directions; the angle between them is 180°.

19. Many of the operations are done efficiently on most modern graphical calculators using their built-in vector manipulation and rectangular ↔ polar "shortcuts." In this solution, we employ the "traditional" methods (such as Eq. 3-6). Where the length unit is not displayed, the unit meter should be understood.

   (a) Using unit-vector notation,

$$\begin{aligned}
\vec{a} &= 50\cos(30°)\,\hat{\imath} + 50\sin(30°)\,\hat{\jmath} \\
\vec{b} &= 50\cos(195°)\,\hat{\imath} + 50\sin(195°)\,\hat{\jmath} \\
\vec{c} &= 50\cos(315°)\,\hat{\imath} + 50\sin(315°)\,\hat{\jmath} \\
\vec{a} + \vec{b} + \vec{c} &= 30.4\,\hat{\imath} - 23.3\,\hat{\jmath}\ .
\end{aligned}$$

   The magnitude of this result is $\sqrt{30.4^2 + (-23.3)^2} = 38$ m.

   (b) The two possibilities presented by a simple calculation for the angle between the vector described in part (a) and the $+x$ direction are $\tan^{-1}(-23.2/30.4) = -37.5°$, and $180° + (-37.5°) = 142.5°$. The former possibility is the correct answer since the vector is in the fourth quadrant (indicated by the signs of its components). Thus, the angle is $-37.5°$, which is to say that it is roughly 38° *clockwise* from the $+x$ axis. This is equivalent to 322.5° counterclockwise from $+x$.

   (c) We find $\vec{a} - \vec{b} + \vec{c} = (43.3 - (-48.3) + 35.4)\,\hat{\imath} - (25 - (-12.9) + (-35.4))\,\hat{\jmath} = 127\,\hat{\imath} + 2.6\,\hat{\jmath}$ in unit-vector notation. The magnitude of this result is $\sqrt{127^2 + 2.6^2} \approx 1.3 \times 10^2$ m.

   (d) The angle between the vector described in part (c) and the $+x$ axis is $\tan^{-1}(2.6/127) \approx 1°$.

   (e) Using unit-vector notation, $\vec{d}$ is given by

$$\begin{aligned}
\vec{d} &= \vec{a} + \vec{b} - \vec{c} \\
&= -40.4\,\hat{\imath} + 47.4\,\hat{\jmath}
\end{aligned}$$

   which has a magnitude of $\sqrt{(-40.4)^2 + 47.4^2} = 62$ m.

(f) The two possibilities presented by a simple calculation for the angle between the vector described in part (e) and the $+x$ axis are $\tan^{-1}(47.4/(-40.4)) = -50°$, and $180° + (-50°) = 130°$. We choose the latter possibility as the correct one since it indicates that $\vec{d}$ is in the second quadrant (indicated by the signs of its components).

20. Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Where the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\begin{aligned}\vec{E} &= 3.73\,\hat{\imath} + 4.70\,\hat{\jmath}\\ \vec{F} &= 1.29\,\hat{\imath} - 4.83\,\hat{\jmath}\\ \vec{G} &= 1.45\,\hat{\imath} + 3.73\,\hat{\jmath}\\ \vec{H} &= -5.20\,\hat{\imath} + 3.00\,\hat{\jmath}\\ \vec{E}+\vec{F}+\vec{G}+\vec{H} &= 1.28\,\hat{\imath} + 6.60\,\hat{\jmath}\;.\end{aligned}$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{1.28^2 + 6.60^2} = 6.72$ m. Its angle measured counterclockwise from the $+x$ axis is $\tan^{-1}(6.6/1.28) = 79° = 1.38$ rad.

21. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}$, $\vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the $+x$ axis is 135° and apply Eq. 3-5 and Eq. 3-6 where appropriate.

(a) The x component of $\vec{r}$ is $10\cos 30° + 10\cos 135° = 1.59$ m.

(b) The y component of $\vec{r}$ is $10\sin 30° + 10\sin 135° = 12.1$ m.

(c) The magnitude of $\vec{r}$ is $\sqrt{1.59^2 + 12.1^2} = 12.2$ m.

(d) The angle between $\vec{r}$ and the $+x$ direction is $\tan^{-1}(12.1/1.59) = 82.5°$.

22. If we wish to use Eq. 3-5 in an unmodified fashion, we should note that the angle between $\vec{C}$ and the $+x$ axis is $180° + 20° = 200°$.

(a) The x component of $\vec{B}$ is given by $C_x - A_x = 15\cos 200° - 12\cos 40° = -23.3$ m, and the y component of $\vec{B}$ is given by $C_y - A_y = 15\sin 200° - 12\sin 40° = -12.8$ m. Consequently, its magnitude is $\sqrt{(-23.3)^2 + (-12.8)^2} = 26.6$ m.

(b) The two possibilities presented by a simple calculation for the angle between $\vec{B}$ and the $+x$ axis are $\tan^{-1}((-12.8)/(-23.3)) = 28.9°$, and $180° + 28.9° = 209°$. We choose the latter possibility as the correct one since it indicates that $\vec{B}$ is in the third quadrant (indicated by the signs of its components). We note, too, that the answer can be equivalently stated as $-151°$.

23. We consider $\vec{A}$ with (x, y) components given by $(A\cos\alpha, A\sin\alpha)$. Similarly, $\vec{B} \to (B\cos\beta, B\sin\beta)$. The angle (measured from the $+x$ direction) for their vector sum must have a slope given by

$$\tan\theta = \frac{A\sin\alpha + B\sin\beta}{A\cos\alpha + B\cos\beta}\;.$$

The problem requires that we now consider the orthogonal direction, where $\tan\theta + 90° = -\cot\theta$. If this (the negative reciprocal of the above expression) is to equal the slope for their vector *difference*, then we must have

$$-\frac{A\cos\alpha + B\cos\beta}{A\sin\alpha + B\sin\beta} = \frac{A\sin\alpha - B\sin\beta}{A\cos\alpha - B\cos\beta}\;.$$

Multiplying both sides by $A\sin\alpha + B\sin\beta$ and doing the same with $A\cos\alpha - B\cos\beta$ yields

$$A^2\,\cos^2\alpha - B^2\,\cos^2\beta = A^2\,\sin^2\alpha - B^2\,\sin^2\beta\ .$$

Rearranging, using the $\cos^2\phi + \sin^2\phi = 1$ identity, we obtain

$$A^2 = B^2 \quad\Longrightarrow\quad A = B\ .$$

In a *later* section, the scalar (dot) product of vectors is presented and this result can be revisited with a more compact derivation.

24. If we wish to use Eq. 3-5 directly, we should note that the angles for $\vec{Q}$,$\vec{R}$ and $\vec{S}$ are 100°, 250° and 310°, respectively, if they are measured counterclockwise from the $+x$ axis.

    (a) Using unit-vector notation, with the unit meter understood, we have

$$\begin{aligned}
\vec{P} &= 10\cos(25^\circ)\,\hat{\imath} + 10\sin(25^\circ)\,\hat{\jmath}\\
\vec{Q} &= 12\cos(100^\circ)\,\hat{\imath} + 12\sin(100^\circ)\,\hat{\jmath}\\
\vec{R} &= 8\cos(250^\circ)\,\hat{\imath} + 8\sin(250^\circ)\,\hat{\jmath}\\
\vec{S} &= 9\cos(310^\circ)\,\hat{\imath} + 9\sin(310^\circ)\,\hat{\jmath}\\
\vec{P}+\vec{Q}+\vec{R}+\vec{S} &= 10.0\,\hat{\imath} + 1.6\,\hat{\jmath}
\end{aligned}$$

    (b) The magnitude of the vector sum is $\sqrt{10^2 + 1.6^2} = 10.2$ m and its angle is $\tan^{-1}(1.6/10) \approx 9.2^\circ$ measured counterclockwise from the $+x$ axis. The appearance of this solution would be quite different using the vector manipulation capabilities of most modern graphical calculators, although the principle would be basically the same.

25. Without loss of generality, we assume $\vec{a}$ points along the $+x$ axis, and that $\vec{b}$ is at θ measured counterclockwise from $\vec{a}$. We wish to verify that

$$r^2 = a^2 + b^2 + 2ab\cos\theta$$

    where $a = |\vec{a}| = a_x$ (we'll call it a for simplicity) and $b = |\vec{b}| = \sqrt{b_x^2 + b_y^2}$. Since $\vec{r} = \vec{a} + \vec{b}$ then $r = |\vec{r}| = \sqrt{(a + b_x)^2 + b_y^2}$. Thus, the above expression becomes

$$\begin{aligned}
\left(\sqrt{(a+b_x)^2 + b_y^2}\right)^2 &= a^2 + \left(\sqrt{b_x^2 + b_y^2}\right)^2 + 2ab\cos\theta\\
a^2 + 2ab_x + b_x^2 + b_y^2 &= a^2 + b_x^2 + b_y^2 + 2ab\cos\theta
\end{aligned}$$

    which makes a valid equality since (the last term) $2ab\cos\theta$ is indeed the same as $2ab_x$ (on the left-hand side). In a *later* section, the scalar (dot) product of vectors is presented and this result can be revisited with a somewhat different-looking derivation.

26. The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $1.69\,\hat{\imath} + 3.63\,\hat{\jmath}$ and $-2.87\,\hat{\imath} + 4.10\,\hat{\jmath}$, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

    (a) Adding corresponding components, we obtain $\vec{R} = -3.18\,\hat{\imath} + 4.72\,\hat{\jmath}$.

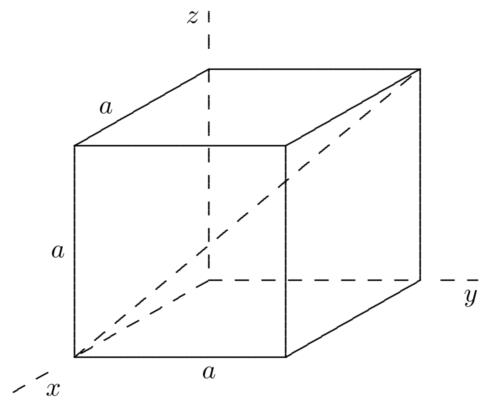
    (b) and (c) Converting this result to polar coordinates (using Eq. 3-6 or functions on a vector-capable calculator), we obtain

$$(-3.18, 4.72) \quad\longrightarrow\quad (5.69\ \angle\ 124^\circ)$$

    which tells us the magnitude is 5.69 m and the angle (measured counterclockwise from $+x$ axis) is 124°.

27. (a) There are 4 such lines, one from each of the corners on the lower face to the diametrically opposite corner on the upper face. One is shown on the diagram. Using an xyz coordinate system as shown (with the origin at the back lower left corner) The position vector of the "starting point" of the diagonal shown is $a\,\hat{\imath}$ and the position vector of the ending point is $a\,\hat{\jmath} + a\,\hat{k}$, so the vector along the line is the difference $a\,\hat{\jmath} + a\,\hat{k} - a\,\hat{\imath}$.

The point diametrically opposite the origin has position vector $a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$ and this is the vector along the diagonal. Another corner of the bottom face is at $a\,\hat{\imath} + a\,\hat{\jmath}$ and the diametrically opposite corner is at $a\,\hat{k}$, so another cube diagonal is $a\,\hat{k} - a\,\hat{\imath} - a\,\hat{\jmath}$. The fourth diagonal runs from $a\,\hat{\jmath}$ to $a\,\hat{\imath} + a\,\hat{k}$, so the vector along the diagonal is $a\,\hat{\imath} + a\,\hat{k} - a\,\hat{\jmath}$.



(b) Consider the vector from the back lower left corner to the front upper right corner. It is $a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$. We may think of it as the sum of the vector $a\,\hat{\imath}$ parallel to the x axis and the vector $a\,\hat{\jmath} + a\,\hat{k}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $\sqrt{a^2 + a^2} = a\sqrt{2}$ and the magnitude of the parallel component is a, $\tan\theta = \left(a\sqrt{2}\right)/a = \sqrt{2}$. Thus $\theta = 54.7°$. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.

(c) The length of any of the diagonals is given by $\sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$.

28. Reference to Figure 3-18 (and the accompanying material in that section) is helpful. If we convert $\vec{B}$ to the magnitude-angle notation (as $\vec{A}$ already is) we have $\vec{B} = (14.4\ \angle\ 33.7°)$ (appropriate notation especially if we are using a vector capable calculator in polar mode). Where the length unit is not displayed in the solution, the unit meter should be understood. In the magnitude-angle notation, rotating the axis by $+20°$ amounts to subtracting that angle from the angles previously specified. Thus, $\vec{A} = (12.0\ \angle 40.0°)'$ and $\vec{B} = (14.4\ \angle\ 13.7°)'$, where the 'prime' notation indicates that the description is in terms of the new coordinates. Converting these results to (x, y) representations, we obtain

$$\begin{aligned} \vec{A} &= 9.19\,\hat{\imath}' + 7.71\,\hat{\jmath}' \\ \vec{B} &= 14.0\,\hat{\imath}' + 3.41\,\hat{\jmath}' \end{aligned}$$

with the unit meter understood, as already mentioned.

29. We apply Eq. 3-20 and Eq. 3-27.

(a) The scalar (dot) product of the two vectors is $\vec{a} \cdot \vec{b} = ab\cos\phi = (10)(6.0)\cos 60° = 30$.

(b) The magnitude of the vector (cross) product of the two vectors is $|\vec{a} \times \vec{b}| = ab\sin\phi = (10)(6.0)\sin 60° = 52$.

30. We consider all possible products and then simplify using relations such as $\hat{\imath} \cdot \hat{k} = 0$ and $\hat{\imath} \cdot \hat{\imath} = 1$. Thus,

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \left(a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}\right) \cdot \left(b_x\,\hat{\imath} + b_y\,\hat{\jmath} + b_z\,\hat{k}\right) \\ &= a_x b_x\,\hat{\imath} \cdot \hat{\imath} + a_x b_y\,\hat{\imath} \cdot \hat{\jmath} + a_x b_z\,\hat{\imath} \cdot \hat{k} + a_y b_x\,\hat{\jmath} \cdot \hat{\imath} + a_y b_j\,\hat{\jmath} \cdot \hat{\jmath} + \cdots \\ &= a_x b_x(1) + a_x b_y(0) + a_x b_z(0) + a_y b_x(0) + a_y b_j(1) + \cdots \end{aligned}$$

which is seen to reduce to the desired result (one might wish to show this in two dimensions before tackling the additional tedium of working with these three-component vectors).

31. Since $ab\cos\phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos\phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab} \; .$$

The magnitudes of the vectors given in the problem are

$$\begin{aligned} a = |\vec{a}| &= \sqrt{(3.0)^2 + (3.0)^2 + (3.0)^2} = 5.2 \\ b = |\vec{b}| &= \sqrt{(2.0)^2 + (1.0)^2 + (3.0)^2} = 3.7 \; . \end{aligned}$$

The angle between them is found from

$$\cos\phi = \frac{(3.0)(2.0) + (3.0)(1.0) + (3.0)(3.0)}{(5.2)(3.7)} = 0.926 \; .$$

The angle is $\phi = 22°$.

32. We consider all possible products and then simplify using relations such as $\hat{\imath} \times \hat{\imath} = 0$ and the important fundamental products

$$\begin{aligned} \hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} &= \hat{k} \\ \hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} &= \hat{\imath} \\ \hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} &= \hat{\jmath} \; . \end{aligned}$$

Thus,

$$\begin{aligned} \vec{a} \times \vec{b} &= \left(a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k}\right) \times \left(b_x\,\hat{\imath} + b_y\,\hat{\jmath} + b_z\,\hat{k}\right) \\ &= a_x b_x\,\hat{\imath} \times \hat{\imath} + a_x b_y\,\hat{\imath} \times \hat{\jmath} + a_x b_z\,\hat{\imath} \times \hat{k} + a_y b_x\,\hat{\jmath} \times \hat{\imath} + a_y b_j\,\hat{\jmath} \times \hat{\jmath} + \cdots \\ &= a_x b_x(0) + a_x b_y(\hat{k}) + a_x b_z(-\hat{\jmath}) + a_y b_x(-\hat{k}) + a_y b_j(0) + \cdots \end{aligned}$$

which is seen to simplify to the desired result.

33. The area of a triangle is half the product of its base and altitude. The base is the side formed by vector $\vec{a}$. Then the altitude is $b\sin\phi$ and the area is $A = \frac{1}{2}ab\sin\phi = \frac{1}{2}|\vec{a} \times \vec{b}|$.

34. Applying Eq. 3-23, $\vec{F} = q\vec{v} \times \vec{B}$ (where q is a scalar) becomes

$$F_x\,\hat{\imath} + F_y\,\hat{\jmath} + F_z\,\hat{k} = q\,(v_y B_z - v_z B_y)\,\hat{\imath} + q\,(v_z B_x - v_x B_z)\,\hat{\jmath} + q\,(v_x B_y - v_y B_x)\,\hat{k}$$

which – plugging in values – leads to three equalities:

$$\begin{aligned} 4.0 &= 2\,(4.0B_z - 6.0B_y) \\ -20 &= 2\,(6.0B_x - 2.0B_z) \\ 12 &= 2\,(2.0B_y - 4.0B_x) \end{aligned}$$

Since we are told that $B_x = B_y$, the third equation leads to $B_y = -3.0$. Inserting this value into the first equation, we find $B_z = -4.0$. Thus, our answer is

$$\vec{B} = -3.0\,\hat{\imath} - 3.0\,\hat{\jmath} - 4.0\,\hat{k} \; .$$

35. Both proofs shown below utilize the fact that the vector (cross) product of $\vec{a}$ and $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. This is mentioned in the book, and is fundamental to its discussion of the right-hand rule.

(a) $(\vec{b} \times \vec{a})$ is a vector that is perpendicular to $\vec{a}$, so the scalar product of $\vec{a}$ with this vector is zero. This can also be verified by using Eq. 3-30, and then (with suitable notation changes) Eq. 3-23.

(b) Let $\vec{c} = \vec{b} \times \vec{a}$. Then the magnitude of $\vec{c}$ is $c = ab\sin\phi$. Since $\vec{c}$ is perpendicular to $\vec{a}$ the magnitude of $\vec{a} \times \vec{c}$ is ac. The magnitude of $\vec{a} \times (\vec{b} \times \vec{a})$ is consequently $|\vec{a} \times (\vec{b} \times \vec{a})| = ac = a^2 b \sin\phi$. This too can be verified by repeated application of Eq. 3-30, although it must be admitted that this is much less intimidating if one is using a math software package such as MAPLE or Mathematica.

36. If a vector capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method. Eq. 3-30 leads to

$$2\vec{A} \times \vec{B} = 2\left(2\,\hat{\imath} + 3\,\hat{\jmath} - 4\,\hat{k}\right) \times \left(-3\,\hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k}\right) = 44\,\hat{\imath} + 16\,\hat{\jmath} + 34\,\hat{k}\ .$$

We now apply Eq. 3-23 to evaluate $3\vec{C} \cdot \left(2\vec{A} \times \vec{B}\right)$:

$$3\left(7\,\hat{\imath} - 8\,\hat{\jmath}\right) \cdot \left(44\,\hat{\imath} + 16\,\hat{\jmath} + 34\,\hat{k}\right) = 3\left((7)(44) + (-8)(16) + (0)(34)\right) = 540\ .$$

37. From the figure, we note that $\vec{c} \perp \vec{b}$, which implies that the angle between $\vec{c}$ and the $+x$ axis is 120°.

(a) Direct application of Eq. 3-5 yields the answers for this and the next few parts. $a_x = a\cos 0° = a = 3.00$ m.

(b) $a_y = a\sin 0° = 0$.

(c) $b_x = b\cos 30° = (4.00\,\text{m})\cos 30° = 3.46$ m.

(d) $b_y = b\sin 30° = (4.00\,\text{m})\sin 30° = 2.00$ m.

(e) $c_x = c\cos 120° = (10.0\,\text{m})\cos 120° = -5.00$ m.

(f) $c_y = c\sin 30° = (10.0\,\text{m})\sin 120° = 8.66$ m.

(g) In terms of components (first x and then y), we must have

$$\begin{aligned} -5.00\,\text{m} &= p(3.00\,\text{m}) + q(3.46\,\text{m}) \\ 8.66\,\text{m} &= p(0) + q(2.00\,\text{m})\ . \end{aligned}$$

Solving these equations, we find $p = -6.67$

(h) and $q = 4.33$ (note that it's easiest to solve for q first). The numbers p and q have no units.

38. We apply Eq. 3-20 with Eq. 3-23. Where the length unit is not displayed, the unit meter is understood.

(a) We first note that $a = |\vec{a}| = \sqrt{3.2^2 + 1.6^2} = 3.58$ m and $b = |\vec{b}| = \sqrt{0.5^2 + 4.5^2} = 4.53$ m. Now,

$$\begin{aligned} \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y &= ab\cos\phi \\ (3.2)(0.5) + (1.6)(4.5) &= (3.58)(4.53)\cos\phi \end{aligned}$$

which leads to $\phi = 57°$ (the inverse cosine is double-valued as is the inverse tangent, but we know this is the right solution since both vectors are in the same quadrant).

(b) Since the angle (measured from $+x$) for $\vec{a}$ is $\tan^{-1}(1.6/3.2) = 26.6°$, we know the angle for $\vec{c}$ is $26.6° - 90° = -63.4°$ (the other possibility, $26.6° + 90°$ would lead to a $c_x < 0$). Therefore, $c_x = c\cos -63.4° = (5.0)(0.45) = 2.2$ m.

(c) Also, $c_y = c\sin -63.4° = (5.0)(-0.89) = -4.5$ m.

(d) And we know the angle for $\vec{d}$ to be $26.6° + 90° = 116.6°$, which leads to $d_x = d\cos 116.6° = (5.0)(-0.45) = -2.2$ m.

(e) Finally, $d_y = d\sin 116.6° = (5.0)(0.89) = 4.5$ m.

39. The solution to problem 27 showed that each diagonal has a length given by $a\sqrt{3}$, where a is the length of a cube edge. Vectors along two diagonals are $\vec{b} = a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$ and $\vec{c} = -a\,\hat{\imath} + a\,\hat{\jmath} + a\,\hat{k}$. Using Eq. 3-20 with Eq. 3-23, we find the angle between them:

$$\cos\phi = \frac{b_x c_x + b_y c_y + b_z c_z}{bc} = \frac{-a^2 + a^2 + a^2}{3a^2} = \frac{1}{3}\,.$$

The angle is $\phi = \cos^{-1}(1/3) = 70.5°$.

40. (a) The vector equation $\vec{r} = \vec{a} - \vec{b} - \vec{v}$ is computed as follows: $(5.0 - (-2.0) + 4.0)\hat{\imath} + (4.0 - 2.0 + 3.0)\hat{\jmath} + ((-6.0) - 3.0 + 2.0)\hat{k}$. This leads to $\vec{r} = 11\,\hat{\imath} + 5.0\,\hat{\jmath} - 7.0\,\hat{k}$.

(b) We find the angle from $+z$ by "dotting" (taking the scalar product) $\vec{r}$ with $\hat{k}$. Noting that $r = |\vec{r}| = \sqrt{11^2 + 5^2 + (-7)^2} = 14$, Eq. 3-20 with Eq. 3-23 leads to

$$\vec{r}\cdot\hat{k} = -7.0 = (14)(1)\cos\phi \implies \phi = 120°\ .$$

(c) To find the component of a vector in a certain direction, it is efficient to "dot" it (take the scalar product of it) with a unit-vector in that direction. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{-2\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}}{\sqrt{(-2)^2 + 2^2 + 3^2}}\ .$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(5)(-2) + (4)(2) + (-6)(3)}{\sqrt{(-2)^2 + 2^2 + 3^2}} = -4.9\ .$$

(d) One approach (if we all we require is the magnitude) is to use the vector cross product, as the problem suggests; another (which supplies more information) is to subtract the result in part (c) (multiplied by $\hat{b}$) from $\vec{a}$. We briefly illustrate both methods. We note that if $a\cos\theta$ (where θ is the angle between $\vec{a}$ and $\vec{b}$) gives a_b (the component along $\hat{b}$) then we expect $a\sin\theta$ to yield the orthogonal component:

$$a\sin\theta = \frac{|\vec{a}\times\vec{b}|}{b} = 7.3$$

(alternatively, one might compute θ form part (c) and proceed more directly). The second method proceeds as follows:

$$\begin{aligned} \vec{a} - a_b\,\hat{b} &= (5.0 - 2.35)\hat{\imath} + (4.0 - (-2.35))\hat{\jmath} + ((-6.0) - (-3.53))\hat{k} \\ &= 2.65\,\hat{\imath} + 6.35\,\hat{\jmath} - 2.47\,\hat{k} \end{aligned}$$

This describes the perpendicular part of $\vec{a}$ completely. To find the magnitude of this part, we compute

$$\sqrt{2.65^2 + 6.35^2 + (-2.47)^2} = 7.3$$

which agrees with the first method.

41. The volume of a parallelepiped is equal to the product of its altitude and the area of its base. Take the base to be the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$. Its area is $bc\sin\phi$, where ϕ is the angle between $\vec{b}$ and $\vec{c}$. This is just the magnitude of the vector (cross) product $\vec{b}\times\vec{c}$. The altitude of the parallelepiped is $a\cos\theta$, where θ is the angle between $\vec{a}$ and the normal to the plane of $\vec{b}$ and $\vec{c}$. Since $\vec{b}\times\vec{c}$ is normal to that plane, θ is the angle between $\vec{a}$ and $\vec{b}\times\vec{c}$. Thus, the volume of the parallelepiped is $V = a|\vec{b}\times\vec{c}|\cos\theta = \vec{a}\cdot(\vec{b}\times\vec{c})$.

42. We apply Eq. 3-30 and Eq. 3-23.

(a) $\vec{a}\times\vec{b} = (a_x b_y - a_y b_x)\,\hat{k}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $((3.0)(4.0) - (5.0)(2.0))\,\hat{k} = 2.0\,\hat{k}$.

(b) $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y$ yields $(3)(2) + (5)(4) = 26$.

(c) $\vec{a}+\vec{b} = (3+2)\,\hat{\imath} + (5+4)\,\hat{\jmath}$, so that $\left(\vec{a}+\vec{b}\right)\cdot\vec{b} = (5)(2) + (9)(4) = 46$.

(d) Several approaches are available. In this solution, we will construct a $\hat{b}$ unit-vector and "dot" it (take the scalar product of it) with $\vec{a}$. In this case, we make the desired unit-vector by

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\,\hat{\imath} + 4\,\hat{\jmath}}{\sqrt{2^2+4^2}} \ .$$

We therefore obtain

$$a_b = \vec{a}\cdot\hat{b} = \frac{(3)(2)+(5)(4)}{\sqrt{2^2+4^2}} = 5.8 \ .$$

43. We apply Eq. 3-30 and Eq.3-23. If a vector capable calculator is used, this makes a good exercise for getting familiar with those features. Here we briefly sketch the method.

(a) We note that $\vec{b}\times\vec{c} = -8\,\hat{\imath} + 5\,\hat{\jmath} + 6\,\hat{k}$. Thus, $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right) = (3)(-8) + (3)(5) + (-2)(6) = -21$.

(b) We note that $\vec{b}+\vec{c} = 1\,\hat{\imath} - 2\,\hat{\jmath} + 3\,\hat{k}$. Thus, $\vec{a}\cdot\left(\vec{b}+\vec{c}\right) = (3)(1) + (3)(-2) + (-2)(3) = -9$.

(c) Finally, $\vec{a}\times\left(\vec{b}+\vec{c}\right) = ((3)(3)-(-2)(-2))\,\hat{\imath} + ((-2)(1)-(3)(3))\,\hat{\jmath} + ((3)(-2)-(3)(1))\,\hat{k} = 5\,\hat{\imath} - 11\,\hat{\jmath} - 9\,\hat{k}$.

44. The components of $\vec{a}$ are $a_x = 0$, $a_y = 3.20\cos 63^\circ = 1.45$, and $a_z = 3.20\sin 63^\circ = 2.85$. The components of $\vec{b}$ are $b_x = 1.40\cos 48^\circ = 0.937$, $b_y = 0$, and $b_z = 1.40\sin 48^\circ = 1.04$.

(a) The scalar (dot) product is therefore

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = (0)(0.937) + (1.45)(0) + (2.85)(1.04) = 2.97 \ .$$

(b) The vector (cross) product is

$$\begin{aligned}
\vec{a}\times\vec{b} &= (a_y b_z - a_z b_y)\,\hat{\imath} + (a_z b_x - a_x b_z)\,\hat{\jmath} + (a_x b_y - a_y b_x)\,\hat{k} \\
&= ((1.45)(1.04) - 0))\,\hat{\imath} + ((2.85)(0.937) - 0))\,\hat{\jmath} + (0 - (1.45)(0.94))\,\hat{k} \\
&= 1.51\,\hat{\imath} + 2.67\hat{\jmath} - 1.36\,\hat{k} \ .
\end{aligned}$$

(c) The angle θ between $\vec{a}$ and $\vec{b}$ is given by

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{ab}\right) = \cos^{-1}\left(\frac{2.96}{(3.30)(1.40)}\right) = 48^\circ \ .$$

45. We observe that $|\hat{\imath}\times\hat{\imath}| = |\hat{\imath}|\,|\hat{\imath}|\sin 0^\circ$ vanishes because $\sin 0^\circ = 0$. Similarly, $\hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0$. When the unit vectors are perpendicular, we have to do a little more work to show the cross product results. First, the magnitude of the vector $\hat{\imath}\times\hat{\jmath}$ is

$$\left|\hat{\imath}\times\hat{\jmath}\right| = |\hat{\imath}|\left|\hat{\jmath}\right|\sin 90^\circ$$

which equals 1 because $\sin 90^\circ = 1$ and these are all units vectors (each has magnitude equal to 1). This is consistent with the claim that $\hat{\imath}\times\hat{\jmath} = \hat{k}$ since the magnitude of $\hat{k}$ is certainly 1. Now, we use the right-hand rule to show that $\hat{\imath}\times\hat{\jmath}$ is in the positive z direction. Thus $\hat{\imath}\times\hat{\jmath}$ has the same magnitude and direction as $\hat{k}$, so it is equal to $\hat{k}$. Similarly, $\hat{k}\times\hat{\imath} = \hat{\jmath}$ and $\hat{\jmath}\times\hat{k} = \hat{\imath}$. If, however, the coordinate system is left-handed, we replace $\hat{k}\to-\hat{k}$ in the work we have shown above and get

$$\hat{\imath}\times\hat{\imath} = \hat{\jmath}\times\hat{\jmath} = \hat{k}\times\hat{k} = 0 \ .$$

just as before. But the relations that are different are

$$\hat{\imath}\times\hat{\jmath} = -\hat{k} \quad \hat{k}\times\hat{\imath} = -\hat{\jmath} \quad \hat{\jmath}\times\hat{k} = -\hat{\imath} \ .$$

46. (a) By the right-hand rule, $\vec{A} \times \vec{B}$ points upward if $\vec{A}$ points north and $\vec{B}$ points west. If $\vec{A}$ and $\vec{B}$ have magnitude$=1$ then, by Eq. 3-27, the result also has magnitude equal to 1.
    (b) Since $\cos 90° = 0$, the scalar dot product between perpendicular vectors is zero. Thus, $\vec{A} \cdot \vec{B} = 0$ is $\vec{A}$ points down and $\vec{B}$ points south.
    (c) By the right-hand rule, $\vec{A} \times \vec{B}$ points south if $\vec{A}$ points east and $\vec{B}$ points up. If $\vec{A}$ and $\vec{B}$ have unit magnitude then, by Eq. 3-27, the result also has unit magnitude.
    (d) Since $\cos 0° = 1$, then $\vec{A} \cdot \vec{B} = AB$ (where A is the magnitude of $\vec{A}$ and B is the magnitude of $\vec{B}$). If, additionally, we have $A = B = 1$, then the result is 1.
    (e) Since $\sin 0° = 0$, $\vec{A} \times \vec{B} = 0$ if both $\vec{A}$ and $\vec{B}$ point south.

47. Let A denote the magnitude of $\vec{A}$; similarly for the other vectors. The vector equation is $\vec{A} + \vec{B} = \vec{C}$ where $B = 8.0$ m and $C = 2A$. We are also told that the angle (measured in the 'standard' sense) for $\vec{A}$ is $0°$ and the angle for $\vec{C}$ is $90°$, which makes this a right triangle (when drawn in a "head-to-tail" fashion) where B is the size of the hypotenuse. Using the Pythagorean theorem,

$$B = \sqrt{A^2 + C^2} \implies 8.0 = \sqrt{A^2 + 4A^2}$$

which leads to $A = 8/\sqrt{5} = 3.6$ m.

48. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones are counterclockwise from $+x$). Thus, vector $\vec{d_1}$ has magnitude $d_1 = 4$ (with the unit meter and three significant figures assumed) and direction $\theta_1 = 225°$. Also, $\vec{d_2}$ has magnitude $d_2 = 5$ and direction $\theta_2 = 0°$, and vector $\vec{d_3}$ has magnitude $d_3 = 6$ and direction $\theta_3 = 60°$.

    (a) The x-component of $\vec{d_1}$ is $d_1 \cos\theta_1 = -2.83$ m.
    (b) The y-component of $\vec{d_1}$ is $d_1 \sin\theta_1 = -2.83$ m.
    (c) The x-component of $\vec{d_2}$ is $d_2 \cos\theta_2 = 5.00$ m.
    (d) The y-component of $\vec{d_2}$ is $d_2 \sin\theta_2 = 0$.
    (e) The x-component of $\vec{d_3}$ is $d_3 \cos\theta_3 = 3.00$ m.
    (f) The y-component of $\vec{d_3}$ is $d_3 \sin\theta_3 = 5.20$ m.
    (g) The sum of x-components is $-2.83 + 5.00 + 3.00 = 5.17$ m.
    (h) The sum of y-components is $-2.83 + 0 + 5.20 = 2.37$ m.
    (i) The magnitude of the resultant displacement is $\sqrt{5.17^2 + 2.37^2} = 5.69$ m.
    (j) And its angle is $\theta = \tan^{-1}(2.37/5.17) = 24.6°$ which (recalling our coordinate choices) means it points at about $25°$ north of east.
    (k) and (ℓ) This new displacement (the direct line home) when vectorially added to the previous (net) displacement must give zero. Thus, the new displacement is the negative, or opposite, of the previous (net) displacement. That is, it has the same magnitude (5.69 m) but points in the opposite direction ($25°$ south of west).

49. Reading carefully, we see that the (x, y) specifications for each "dart" are to be interpreted as $(\Delta x, \Delta y)$ descriptions of the corresponding displacement vectors. We combine the different parts of this problem into a single exposition. Thus, along the x axis, we have (with the centimeter unit understood)

$$30.0 + b_x - 20.0 - 80.0 = -140.0 \quad ,$$

and along y axis we have

$$40.0 - 70.0 + c_y - 70.0 = -20.0 \quad .$$

Hence, we find $b_x = -70.0$ cm and $c_y = 80.0$ cm. And we convert the final location $(-140, -20)$ into polar coordinates and obtain $(141 \angle -172°)$, an operation quickly done using a vector capable calculator in polar mode. Thus, the ant is 141 cm from where it started at an angle of $-172°$, which means $172°$ clockwise from the $+x$ axis or $188°$ counterclockwise from the $+x$ axis.

50. We find the components and then add them (as scalars, not vectors). With $d = 3.40$ km and $\theta = 35.0°$ we find $d\cos\theta + d\sin\theta = 4.74$ km.

51. We choose $+x$ east and $+y$ north and measure all angles in the "standard" way (positive ones counterclockwise from $+x$, negative ones clockwise). Thus, vector $\vec{d}_1$ has magnitude $d_1 = 3.66$ (with the unit meter and three significant figures assumed) and direction $\theta_1 = 90°$. Also, $\vec{d}_2$ has magnitude $d_2 = 1.83$ and direction $\theta_2 = -45°$, and vector $\vec{d}_3$ has magnitude $d_3 = 0.91$ and direction $\theta_3 = -135°$. We add the x and y components, respectively:

$$x: \quad d_1\cos\theta_1 + d_2\cos\theta_2 + d_3\cos\theta_3 = 0.651 \text{ m}$$
$$y: \quad d_1\sin\theta_1 + d_2\sin\theta_2 + d_3\sin\theta_3 = 1.723 \text{ m} .$$

(a) The magnitude of the direct displacement (the vector sum $\vec{d}_1+\vec{d}_2+\vec{d}_3$) is $\sqrt{0.651^2 + 1.723^2} = 1.84$ m.

(b) The angle (understood in the sense described above) is $\tan^{-1}(1.723/0.651) = 69°$. That is, the first putt must aim in the direction 69° north of east.

52. (a) We write $\vec{b} = b\,\hat{\text{j}}$ where $b > 0$. We are asked to consider

$$\frac{\vec{b}}{d} = \left(\frac{b}{d}\right)\hat{\text{j}}$$

in the case $d > 0$. Since the coefficient of $\hat{\text{j}}$ is positive, then the vector points in the $+y$ direction.

(b) If, however, $d < 0$, then the coefficient is negative and the vector points in the $-y$ direction.

(c) Since $\cos 90° = 0$, then $\vec{a}\cdot\vec{b} = 0$, using Eq. 3-20.

(d) Since $\vec{b}/d$ is along the y axis, then (by the same reasoning as in the previous part) $\vec{a}\cdot(\vec{b}/d) = 0$.

(e) By the right-hand rule, $\vec{a}\times\vec{b}$ points in the $+z$ direction.

(f) By the same rule, $\vec{b}\times\vec{a}$ points in the $-z$ direction. We note that $\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$ is true in this case and quite generally.

(g) Since $\sin 90° = 1$, Eq. 3-27 gives $|\vec{a}\times\vec{b}| = ab$ where a is the magnitude of $\vec{a}$. Also, $|\vec{a}\times\vec{b}| = |\vec{b}\times\vec{a}|$ so both results have the same magnitude.

(h) and (i) With $d > 0$, we find that $\vec{a}\times(\vec{b}/d)$ has magnitude ab/d and is pointed in the $+z$ direction.

53. (a) With $a = 17.0$ m and $\theta = 56.0°$ we find $a_x = a\cos\theta = 9.51$ m.

(b) And $a_y = a\sin\theta = 14.1$ m.

(c) The angle relative to the new coordinate system is $\theta' = 56 - 18 = 38°$. Thus, $a'_x = a\cos\theta' = 13.4$ m.

(d) And $a'_y = a\sin\theta' = 10.5$ m.

54. Since $\cos 0° = 1$ and $\sin 0° = 0$, these follows immediately from Eq. 3-20 and Eq. 3-27.

55. (a) The magnitude of the vector $\vec{a} = 4.0\vec{d}$ is $(4.0)(2.5) = 10$ m.

(b) The direction of the vector $\vec{a} = 4.0\vec{d}$ is the same as the direction of $\vec{d}$ (north).

(c) The magnitude of the vector $\vec{c} = -3.0\vec{d}$ is $(3.0)(2.5) = 7.5$ m.

(d) The direction of the vector $\vec{c} = -3.0\vec{d}$ is the opposite of the direction of $\vec{d}$. Thus, the direction of $\vec{c}$ is south.

56. The vector sum of the displacements $\vec{d}_{\text{storm}}$ and $\vec{d}_{\text{new}}$ must give the same result as its originally intended displacement $\vec{d}_{\text{o}} = 120\,\hat{\text{j}}$ where east is $\hat{\text{i}}$, north is $\hat{\text{j}}$, and the assumed length unit is km. Thus, we write

$$\vec{d}_{\text{storm}} = 100\,\hat{\text{i}} \quad \text{and} \quad \vec{d}_{\text{new}} = A\,\hat{\text{i}} + B\,\hat{\text{j}} .$$

(a) The equation $\vec{d}_{\text{storm}} + \vec{d}_{\text{new}} = \vec{d}_{\text{o}}$ readily yields $A = -100$ km and $B = 120$ km. The magnitude of $\vec{d}_{\text{new}}$ is therefore $\sqrt{A^2 + B^2} = 156$ km.

(b) And its direction is $\tan^{-1}(B/A) = -50.2°$ or $180° + (-50.2°) = 129.8°$. We choose the latter value since it indicates a vector pointing in the second quadrant, which is what we expect here. The answer can be phrased several equivalent ways: 129.8° counterclockwise from east, or 39.8° west from north, or 50.2° north from west.

57. (a) The height is $h = d \sin\theta$, where $d = 12.5$ m and $\theta = 20.0°$. Therefore, $h = 4.28$ m.

(b) The horizontal distance is $d\cos\theta = 11.7$ m.

58. (a) We orient $\hat{\imath}$ eastward, $\hat{\jmath}$ northward, and $\hat{k}$ upward. The displacement in meters is consequently $1000\,\hat{\imath} + 2000\,\hat{\jmath} - 500\,\hat{k}$.

(b) The net displacement is zero since his final position matches his initial position.

59. (a) If we add the equations, we obtain $2\vec{a} = 6\vec{c}$, which leads to $\vec{a} = 3\vec{c} = 9\,\hat{\imath} + 12\,\hat{\jmath}$.

(b) Plugging this result back in, we find $\vec{b} = \vec{c} = 3\,\hat{\imath} + 4\,\hat{\jmath}$.

60. (First problem in **Cluster 1**)
The given angle $\theta = 130°$ is assumed to be measured counterclockwise from the $+x$ axis. Angles (if positive) in our results follow the same convention (but if negative are clockwise from $+x$).

(a) With $A = 4.00$, the x-component of $\vec{A}$ is $A\cos\theta = -2.57$.

(b) The y-component of $\vec{A}$ is $A\sin\theta = 3.06$.

(c) Adding $\vec{A}$ and $\vec{B}$ produces a vector we call R with components $R_x = -6.43$ and $R_y = -1.54$. Using Eq. 3-6 (or special functions on a calculator) we present this in magnitude-angle notation: $\vec{R} = (6.61\ \angle -167°)$.

(d) From the discussion in the previous part, it is clear that $\vec{R} = -6.43\,\hat{\imath} - 1.54\,\hat{\jmath}$.

(e) The vector $\vec{C}$ is the difference of $\vec{A}$ and $\vec{B}$. In unit-vector notation, this becomes

$$\vec{C} = \vec{A} - \vec{B} = \left(-2.57\,\hat{\imath} - 3.06\,\hat{\jmath}\right) - \left(-3.86\,\hat{\imath} - 4.60\,\hat{\jmath}\right)$$

which yields $\vec{C} = 1.29\,\hat{\imath} + 7.66\,\hat{\jmath}$.

(f) Using Eq. 3-6 (or special functions on a calculator) we present this in magnitude-angle notation: $\vec{C} = (7.77\ \angle\ 80.5°)$.

(g) We note that $\vec{C}$ is the "constant" in all six pictures. Remembering that the negative of a vector simply reverses it, then we see that in form or another, all six pictures express the relation $\vec{C} = \vec{A} - \vec{B}$.

61. (Second problem in **Cluster 1**)

(a) The dot (scalar) product of $3\vec{A}$ and $\vec{B}$ is found using Eq. 3-23:

$$3\vec{A}\cdot\vec{B} = 3\,(4.00\cos 130°)\,(-3.86) + 3\,(4.00\sin 130°)\,(-4.60) = -12.5\ .$$

(b) We call the result $\vec{D}$ and combine the scalars ($(3)(4) = 12$). Thus, $\vec{D} = (4\vec{A}) \times (3\vec{B})$ becomes, using Eq. 3-30,

$$12\vec{A}\times\vec{B} = 12\,((4.00\cos 130°)\,(-4.60) - (4.00\sin 130°)\,(-3.86))\,\hat{k}$$

which yields $\vec{D} = 284\,\hat{k}$.

(c) Since $\vec{D}$ has magnitude 284 and points in the $+z$ direction, it has radial coordinate 284 and angle-measured-from-z-axis equal to 0°. The angle measured in the xy plane does not have a well-defined value (since this vector does not have a component in that plane)

(d) Since $\vec{A}$ is in the xy plane, then it is clear that $\vec{A} \perp \vec{D}$. The angle between them is 90°.

(e) Calling this new result $\vec{G}$ we have

$$\vec{G} = (4.00\cos 130°)\,\hat{\imath} + (4.00\sin 130°)\,\hat{\jmath} + (3.00)\hat{k}$$

which yields $\vec{G} = -2.57\,\hat{\imath} + 3.06\hat{\jmath} + 3.00\,\hat{k}$.

(f) It is straightforward using a vector-capable calculator to convert the above into spherical coordinates. We, however, proceed "the hard way", using the notation in Fig. 3-44 (where θ is in the xy plane and ϕ is measured from the z axis):

$$\begin{aligned} |\vec{G}| = r &= \sqrt{(-2.57)^2 + 3.06^2 + 3.00^2} = 5.00 \\ \phi &= \tan^{-1}(4.00/3.00) = 53.1° \\ \theta &= 130° \quad \text{given in problem 60} \ . \end{aligned}$$

62. (Third problem in **Cluster 1**)

(a) Looking at the xy plane in Fig. 3-44, it is clear that the angle to $\vec{A}$ (which is the vector lying *in* the plane, not the one rising out of it, which we called $\vec{G}$ in the previous problem) measured counterclockwise from the $-y$ axis is $90° + 130° = 220°$. Had we measured this *clockwise* we would obtain (in absolute value) $360° - 220° = 140°$.

(b) We found in part (b) of the previous problem that $\vec{A}\times\vec{B}$ points along the z axis, so it is perpendicular to the $-y$ direction.

(c) Let $\vec{u} = -\hat{\jmath}$ represent the $-y$ direction, and $\vec{w} = 3\,\hat{k}$ is the vector being added to $\vec{B}$ in this problem. The vector being examined in this problem (we'll call it $\vec{Q}$) is, using Eq. 3-30 (or a vector-capable calculator),

$$\vec{Q} = \vec{A} \times \left(\vec{B} + \vec{w}\right) = 9.19\,\hat{\imath} + 7.71\,\hat{\jmath} + 23.7\,\hat{k}$$

and is clearly in the first octant (since all components are positive); using Pythagorean theorem, its magnitude is $Q = 26.52$. From Eq. 3-23, we immediately find $\vec{u} \cdot \vec{Q} = -7.71$. Since $\vec{u}$ has unit magnitude, Eq. 3-20 leads to

$$\cos^{-1}\left(\frac{\vec{u}\cdot\vec{Q}}{Q}\right) = \cos^{-1}\left(\frac{-7.71}{26.52}\right)$$

which yields a choice of angles 107° or −107°. Since we have already observed that $\vec{Q}$ is in the first octant, the the angle measured counterclockwise (as observed by someone high up on the $+z$ axis) from the $-y$ axis to $\vec{Q}$ is 107°.

Chapter 4

1. Where the length unit is not specified (in this solution), the unit meter should be understood.

   (a) The position vector, according to Eq. 4-1, is $\vec{r} = -5.0\,\hat{\imath} + 8.0\,\hat{\jmath}$ (in meters).

   (b) The magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = 9.4$ m.

   (c) Many calculators have polar $\leftrightarrow$ rectangular conversion capabilities which make this computation more efficient than what is shown below. Noting that the vector lies in the xy plane, we are using Eq. 3-6:

   $$\tan^{-1}\left(\frac{8.0}{-5.0}\right) = -58^\circ \quad \text{or} \quad 122^\circ$$

   where we choose the latter possibility (122° measured counterclockwise from the $+x$ direction) since the signs of the components imply the vector is in the second quadrant.

   (d) In the interest of saving space, we omit the sketch. The vector is 32° counterclockwise from the $+y$ direction, where the $+y$ direction is assumed to be (as is standard) +90° counterclockwise from $+x$, and the $+z$ direction would therefore be "out of the paper."

   (e) The displacement is $\Delta\vec{r} = \vec{r}\,' - \vec{r}$ where $\vec{r}$ is given in part (a) and $\vec{r}\,' = 3.0\,\hat{\imath}$. Therefore, $\Delta\vec{r} = 8.0\,\hat{\imath} - 8.0\,\hat{\jmath}$ (in meters).

   (f) The magnitude of the displacement is $|\Delta\vec{r}| = \sqrt{8^2 + (-8)^2} = 11$ m.

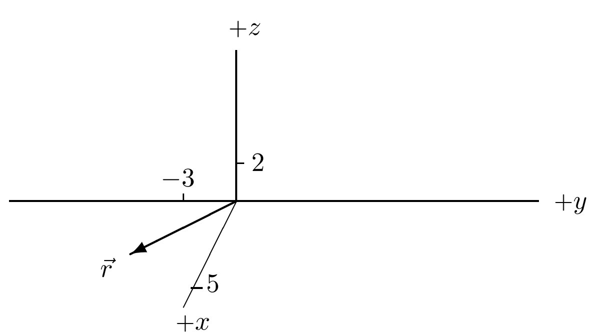
   (g) The angle for the displacement, using Eq. 3-6, is found from

   $$\tan^{-1}\left(\frac{8.0}{-8.0}\right) = -45^\circ \quad \text{or} \quad 135^\circ$$

   where we choose the former possibility (−45°, which means 45° measured clockwise from $+x$, or 315° counterclockwise from $+x$) since the signs of the components imply the vector is in the fourth quadrant.

2. (a) The magnitude of $\vec{r}$ is $\sqrt{5.0^2 + (-3.0)^2 + 2.0^2} = 6.2$ m.

   (b) A sketch is shown. The coordinate values are in meters.



3. Where the unit is not specified, the unit meter is understood. We use Eq. 4-2 and Eq. 4-3.

   (a) With the initial position vector as $\vec{r}_1$ and the later vector as $\vec{r}_2$, Eq. 4-3 yields

   $$\Delta r = ((-2) - 5)\,\hat{\imath} + (6 - (-6))\,\hat{\jmath} + (2 - 2)\,\hat{k} = -7.0\,\hat{\imath} + 12\,\hat{\jmath}$$

   for the displacement vector in unit-vector notation (in meters).

   (b) Since there is no z component (that is, the coefficient of $\hat{k}$ is zero), the displacement vector is in the xy plane.

4. We use a coordinate system with $+x$ eastward and $+y$ upward. We note that 123° is the angle between the initial position and later position vectors, so that the angle from $+x$ to the later position vector is $40° + 123° = 163°$. In unit-vector notation, the position vectors are

$$\begin{aligned}\vec{r}_1 &= 360\cos(40°)\,\hat{\imath} + 360\sin(40°)\,\hat{\jmath} = 276\,\hat{\imath} + 231\,\hat{\jmath}\\ \vec{r}_2 &= 790\cos(163°)\,\hat{\imath} + 790\sin(163°)\,\hat{\jmath} = -755\,\hat{\imath} + 231\,\hat{\jmath}\end{aligned}$$

respectively (in meters). Consequently, we plug into Eq. 4-3

$$\Delta r = ((-755) - 276)\,\hat{\imath} + (231 - 231)\,\hat{\jmath}$$

and find the displacement vector is horizontal (westward) with a length of 1.03 km. If unit-vector notation is not a priority in this problem, then the computation can be approached in a variety of ways – particularly in view of the fact that a number of vector capable calculators are on the market which reduce this problem to a very few keystrokes (using magnitude-angle notation throughout).

5. The average velocity is given by Eq. 4-8. The total displacement $\Delta\vec{r}$ is the sum of three displacements, each result of a (constant) velocity during a given time. We use a coordinate system with $+x$ East and $+y$ North. In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{40\,\text{min}}{60\,\text{min/h}}\right)\hat{\imath} = 40\,\hat{\imath}$$

in kilometers. The second displacement has a magnitude of $60\,\frac{\text{km}}{\text{h}} \cdot \frac{20\,\text{min}}{60\,\text{min/h}} = 20$ km, and its direction is 40° north of east. Therefore,

$$\Delta\vec{r}_2 = 20\cos(40°)\,\hat{\imath} + 20\sin(40°)\,\hat{\jmath} = 15.3\,\hat{\imath} + 12.9\,\hat{\jmath}$$

in kilometers. And the third displacement is

$$\Delta\vec{r}_3 = -\left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{50\,\text{min}}{60\,\text{min/h}}\right)\hat{\imath} = -50\,\hat{\imath}$$

in kilometers. The total displacement is

$$\begin{aligned}\Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3\\ &= 40\,\hat{\imath} + 15.3\,\hat{\imath} + 12.9\,\hat{\jmath} - 50\,\hat{\imath}\\ &= 5.3\,\hat{\imath} + 12.9\,\hat{\jmath}\quad(\text{km})\ .\end{aligned}$$

The time for the trip is $40 + 20 + 50 = 110$ min, which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \left(\frac{5.3\,\text{km}}{1.83\,\text{h}}\right)\hat{\imath} + \left(\frac{12.9\,\text{km}}{1.83\,\text{h}}\right)\hat{\jmath} = 2.90\,\hat{\imath} + 7.01\,\hat{\jmath}$$

in kilometers-per-hour. If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of magnitude $\sqrt{2.9^2 + 7.01^2} = 7.59$ km/h, which is inclined 67.5° north of east (or, 22.5° east of north). If unit-vector notation is not a priority in this problem, then the computation can be approached in a variety of ways – particularly in view of the fact that a number of vector capable calculators are on the market which reduce this problem to a very few keystrokes (using magnitude-angle notation throughout).

6. Using Eq. 4-3 and Eq. 4-8, we have

$$\begin{aligned}\vec{v}_{\text{avg}} &= \frac{(-2.0\,\hat{\imath} + 8.0\,\hat{\jmath} - 2.0\,\hat{\mathrm{k}}) - (5.0\,\hat{\imath} - 6.0\,\hat{\jmath} + 2.0\,\hat{\mathrm{k}})}{10} \\ &= -0.70\,\hat{\imath} + 1.40\,\hat{\jmath} - 0.40\,\hat{\mathrm{k}}\end{aligned}$$

in meters-per-second.

7. To emphasize the fact that the velocity is a function of time, we adopt the notation $v(t)$ for $\frac{dx}{dt}$.

(a) Eq. 4-10 leads to

$$v(t) = \frac{d}{dt}\left(3.00t\,\hat{\imath} - 4.00t^2\,\hat{\jmath} + 2.00\,\hat{\mathrm{k}}\right) = 3.00\,\hat{\imath} - 8.00t\,\hat{\jmath}$$

in meters-per-second.

(b) Evaluating this result at $t = 2$ s produces $\vec{v} = 3.0\,\hat{\imath} - 16.0\,\hat{\jmath}$ m/s.

(c) The speed at $t = 2$ s is $v = |\vec{v}| = \sqrt{3^2 + (-16)^2} = 16.3$ m/s.

(d) And the angle of $\vec{v}$ at that moment is one of the possibilities

$$\tan^{-1}\left(\frac{-16}{3}\right) = -79.4^\circ \quad \text{or} \quad 101^\circ$$

where we choose the first possibility (79.4° measured clockwise from the $+x$ direction, or 281° counterclockwise from $+x$) since the signs of the components imply the vector is in the fourth quadrant.

8. On the one hand, we could perform the vector addition of the displacements with a vector capable calculator in polar mode $((75 \angle 37^\circ) + (65 \angle -90^\circ) = (63 \angle -18^\circ))$, but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with $+x$ East and $+y$ North. Lengths are in kilometers and times are in hours.

(a) We perform the vector addition of individual displacements to find the net displacement of the camel.

$$\begin{aligned}\Delta\vec{r}_1 &= 75\cos(37^\circ)\,\hat{\imath} + 75\sin(37^\circ)\,\hat{\jmath} \\ \Delta\vec{r}_2 &= -65\,\hat{\jmath} \\ \Delta\vec{r}_1 + \Delta\vec{r}_2 &= 60\,\hat{\imath} - 20\,\hat{\jmath} \quad \text{km} .\end{aligned}$$

If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length $\sqrt{60^2 + (-20)^2} = 63$ km, which is directed at 18° south of east.

(b) We use the result from part (a) in Eq. 4-8 along with the fact that $\Delta t = 90$ h. In unit vector notation, we obtain

$$\vec{v}_{\text{avg}} = \frac{60\,\hat{\imath} - 20\,\hat{\jmath}}{90} = 0.66\,\hat{\imath} - 0.22\,\hat{\jmath}$$

in kilometers-per-hour. This result in magnitude-angle notation is $\vec{v}_{\text{avg}} = 0.70$ km/h at 18° south of east.

(c) Average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain $140/90 = 1.56$ km/h.

(d) The net displacement is required to be the 90 km East from A to B. The displacement from the resting place to B is denoted $\vec{r}_3$. Thus, we must have (in kilometers)

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 = 90\,\hat{\imath}$$

which produces $\vec{r}_3 = 30\,\hat{\imath} + 20\,\hat{\jmath}$ in unit-vector notation, or $(36 \angle\ 33^\circ)$ in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{\rm avg}| = \frac{36 \text{ km}}{120 - 90 \text{ h}} = 1.2 \text{ km/h}$$

and the direction of this vector is the same as $\vec{r}_3$ (that is, 33° north of east).

9. We apply Eq. 4-10 and Eq. 4-16.

   (a) Taking the derivative of the position vector with respect to time, we have

   $$\vec{v} = \frac{d}{dt}\left(\hat{\imath} + 4t^2\,\hat{\jmath} + t\,\hat{k}\right) = 8t\,\hat{\jmath} + \hat{k}$$

   in SI units (m/s).

   (b) Taking another derivative with respect to time leads to

   $$\vec{a} = \frac{d}{dt}\left(8t\,\hat{\jmath} + \hat{k}\right) = 8\,\hat{\jmath}$$

   in SI units (m/s^2).

10. We use Eq. 4-15 with $\vec{v}_1$ designating the initial velocity and $\vec{v}_2$ designating the later one.

   (a) The average acceleration during the $\Delta t = 4$ s interval is

   $$\vec{a}_{\rm avg} = \frac{\left(-2\,\hat{\imath} - 2\,\hat{\jmath} + 5\,\hat{k}\right) - \left(4\,\hat{\imath} - 22\,\hat{\jmath} + 3\,\hat{k}\right)}{4} = -1.5\,\hat{\imath} + 0.5\,\hat{k}$$

   in SI units (m/s^2).

   (b) The magnitude of $\vec{a}_{\rm avg}$ is $\sqrt{(-1.5)^2 + 0.5^2} = 1.6$ m/s^2. Its angle in the xz plane (measured from the $+x$ axis) is one of these possibilities:

   $$\tan^{-1}\left(\frac{0.5}{-1.5}\right) = -18^\circ \quad \text{or} \quad 162^\circ$$

   where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

11. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

   (a) Plugging into the given expression, we obtain

   $$\vec{r}\,\Big|_{t=2} = (2(8) - 5(2))\,\hat{\imath} + (6 - 7(16))\,\hat{\jmath} = 6.00\,\hat{\imath} - 106\,\hat{\jmath}$$

   in meters.

   (b) Taking the derivative of the given expression produces

   $$\vec{v}(t) = \left(6.00t^2 - 5.00\right)\,\hat{\imath} + 28.0t^3\,\hat{\jmath}$$

   where we have written $v(t)$ to emphasize its dependence on time. This becomes, at $t = 2.00$ s, $\vec{v} = 19.0\,\hat{\imath} - 224\,\hat{\jmath}$ m/s.

   (c) Differentiating the $\vec{v}(t)$ found above, with respect to t produces $12.0t\,\hat{\imath} - 84.0t^2\,\hat{\jmath}$, which yields $\vec{a} = 24.0\,\hat{\imath} - 336\,\hat{\jmath}$ m/s^2 at $t = 2.00$ s.

(d) The angle of $\vec{v}$, measured from $+x$, is either

$$\tan^{-1}\left(\frac{-224}{19.0}\right) = -85.2^\circ \quad \text{or} \quad 94.8^\circ$$

where we settle on the first choice (-85.2°, which is equivalent to 275°) since the signs of its components imply that it is in the fourth quadrant.

12. Noting that $\vec{v}_2 = 0$, then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{0 - (6.30\,\hat{\imath} - 8.42\,\hat{\jmath})}{3} = -2.1\,\hat{\imath} + 2.8\,\hat{\jmath}$$

in SI units (m/s^2).

13. Constant acceleration in both directions (x and y) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time t is given by $\vec{v} = \vec{v}_0 + \vec{a}t$, where $\vec{v}_0$ is the initial velocity and $\vec{a}$ is the (constant) acceleration. The x component is $v_x = v_{0x} + a_x t = 3.00 - 1.00t$, and the y component is $v_y = v_{0y} + a_y t = -0.500t$ since $v_{0y} = 0$. When the particle reaches its maximum x coordinate at $t = t_m$, we must have $v_x = 0$. Therefore, $3.00 - 1.00t_m = 0$ or $t_m = 3.00$ s. The y component of the velocity at this time is $v_y = 0 - 0.500(3.00) = -1.50$ m/s; this is the only nonzero component of $\vec{v}$ at t_m.

(b) Since it started at the origin, the coordinates of the particle at any time t are given by $\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$. At $t = t_m$ this becomes

$$(3.00\,\hat{\imath})(3.00) + \frac{1}{2}(-1.00\,\hat{\imath} - 0.50\,\hat{\jmath})(3.00)^2 = 4.50\,\hat{\imath} - 2.25\,\hat{\jmath}$$

in meters.

14. (a) Using Eq. 4-16, the acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left((6.0t - 4.0t^2)\hat{\imath} + 8.0\,\hat{\jmath}\right) = (6.0 - 8.0t)\hat{\imath}$$

in SI units. Specifically, we find the acceleration vector at $t = 3.0$ s to be $(6.0 - 8.0(3.0))\hat{\imath} = -18\,\hat{\imath}$ m/s^2.

(b) The equation is $\vec{a} = (6.0 - 8.0t)\hat{\imath} = 0$; we find $t = 0.75$ s.

(c) Since the y component of the velocity, $v_y = 8.0$ m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have $v = |\vec{v}| = \sqrt{(6.0t - 4.0t^2)^2 + (8.0)^2} = 10$ in SI units (m/s). We solve for t as follows:

$$\begin{array}{rcl} \text{squaring} \quad (6t - 4t^2)^2 + 64 & = & 100 \\ \text{rearranging} \quad (6t - 4t^2)^2 & = & 36 \\ \text{taking square root} \quad 6t - 4t^2 & = & \pm 6 \\ \text{rearranging} \quad 4t^2 - 6t \pm 6 & = & 0 \\ \text{using quadratic formula} \quad t & = & \dfrac{6 \pm \sqrt{36 - 4(4)(\pm 6)}}{2(8)} \end{array}$$

where the requirement of a real positive result leads to the unique answer: $t = 2.2$ s.

15. Since the x and y components of the acceleration are constants, then we can use Table 2-1 for the motion along both axes. This can be handled individually (for Δx and Δy) or together with the unit-vector notation (for Δr). Where units are not shown, SI units are to be understood.

(a) Since $\vec{r}_0 = 0$, the position vector of the particle is (adapting Eq. 2-15)

$$\begin{aligned}\vec{r} &= \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \\ &= (8.0\,\hat{\jmath})t + \frac{1}{2}(4.0\,\hat{\imath} + 2.0\,\hat{\jmath})t^2 \\ &= (2.0t^2)\,\hat{\imath} + (8.0t + 1.0t^2)\,\hat{\jmath}\ .\end{aligned}$$

Therefore, we find when $x = 29$ m, by solving $2.0t^2 = 29$, which leads to $t = 3.8$ s. The y coordinate at that time is $y = 8.0(3.8) + 1.0(3.8)^2 = 45$ m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v} = \vec{v}_0 + \vec{a}t\ .$$

Thus, at $t = 3.8$ s, the velocity is

$$\vec{v} = 8.0\,\hat{\jmath} + (4.0\,\hat{\imath} + 2.0\,\hat{\jmath})(3.8) = 15.2\,\hat{\imath} + 15.6\,\hat{\jmath}$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{15.2^2 + 15.6^2} = 22\ \text{m/s}\ .$$

16. The acceleration is constant so that use of Table 2-1 (for both the x and y motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles A and B requires two things. First, the y motion of B must satisfy (using Eq. 2-15 and noting that θ is measured from the y axis)

$$y = \frac{1}{2}a_y t^2 \quad\Longrightarrow\quad 30 = \frac{1}{2}\left(0.40\cos\theta\right)t^2\ .$$

Second, the x motions of A and B must coincide:

$$vt = \frac{1}{2}a_x t^2 \quad\Longrightarrow\quad 3.0t = \frac{1}{2}\left(0.40\sin\theta\right)t^2\ .$$

We eliminate a factor of t in the last relationship and formally solve for time:

$$t = \frac{3}{0.2\sin\theta}\ .$$

This is then plugged into the previous equation to produce

$$30 = \frac{1}{2}\left(0.40\cos\theta\right)\left(\frac{3}{0.2\sin\theta}\right)^2$$

which, with the use of $\sin^2\theta = 1 - \cos^2\theta$, simplifies to

$$30 = \frac{9}{0.2}\,\frac{\cos\theta}{1-\cos^2\theta} \quad\Longrightarrow\quad 1 - \cos^2\theta = \frac{9}{(0.2)(30)}\,\cos\theta\ .$$

We use the quadratic formula (choosing the positive root) to solve for $\cos\theta$:

$$\cos\theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1)(-1)}}{2} = \frac{1}{2}$$

which yields

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ\ .$$

17. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

    (a) With the origin at the firing point, the y coordinate of the bullet is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -0.019$ m indicates where the bullet hits the target, then

    $$t = \sqrt{\frac{2(0.019)}{9.8}} = 6.2 \times 10^{-2} \text{ s} .$$

    (b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since $x = 30$ m is the horizontal position of the target, we have $x = v_0 t$. Thus,

    $$v_0 = \frac{x}{t} = \frac{30}{6.3 \times 10^{-2}} = 4.8 \times 10^2 \text{ m/s} .$$

18. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

    (a) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$. If t is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

    $$t = \sqrt{\frac{2(1.20)}{9.8}} = 0.495 \text{ s} .$$

    (b) The initial (horizontal) velocity of the ball is $\vec{v} = v_0\,\hat{\imath}$. Since $x = 1.52$ m is the horizontal position of its impact point with the floor, we have $x = v_0 t$. Thus,

    $$v_0 = \frac{x}{t} = \frac{1.52}{0.495} = 3.07 \text{ m/s} .$$

19. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161$ km/h. Converting to SI units, this is $v_0 = 44.7$ m/s.

    (a) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -\frac{1}{2}gt^2$, and the x coordinate is given by $x = v_0 t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x = 18.3/2$ m, then $t = (18.3/2)/44.7 = 0.205$ s.

    (b) And the time to travel the next $18.3/2$ m must also be 0.205 s. It can be useful to write the horizontal equation as $\Delta x = v_0 \Delta t$ in order that this result can be seen more clearly.

    (c) From $y = -\frac{1}{2}gt^2$, we see that the ball has reached the height of $-\frac{1}{2}(9.8)(0.205)^2 = -0.205$ m at the moment the ball is halfway to the batter.

    (d) The ball's height when it reaches the batter is $-\frac{1}{2}(9.8)(0.409)^2 = -0.820$ m, which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

20. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 10$ m/s.

    (a) With the origin at the initial point (where the dart leaves the thrower's hand), the y coordinate of the dart is given by $y = -\frac{1}{2}gt^2$, so that with $y = -PQ$ we have $PQ = \frac{1}{2}(9.8)(0.19)^2 = 0.18$ m.

    (b) From $x = v_0 t$ we obtain $x = (10)(0.19) = 1.9$ m.

21. Since this problem involves constant downward acceleration of magnitude a, similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute a for g. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 1.0 \times 10^9$ cm/s.

   (a) If ℓ is the length of a plate and t is the time an electron is between the plates, then $\ell = v_0 t$, where v_0 is the initial speed. Thus

   $$t = \frac{\ell}{v_0} = \frac{2.0\,\text{cm}}{1.0 \times 10^9\,\text{cm/s}} = 2.0 \times 10^{-9}\ \text{s}\ .$$

   (b) The vertical displacement of the electron is

   $$y = -\frac{1}{2}at^2 = -\frac{1}{2}\left(1.0 \times 10^{17}\ \text{cm/s}^2\right)\left(2.0 \times 10^{-9}\,\text{s}\right)^2 = -0.20\ \text{cm}\ .$$

   (c) and (d) The x component of velocity does not change: $v_x = v_0 = 1.0 \times 10^9\,\text{cm/s}$, and the y component is

   $$v_y = a_y t = \left(1.0 \times 10^{17}\ \text{cm/s}^2\right)\left(2.0 \times 10^{-9}\,\text{s}\right) = 2.0 \times 10^8\,\text{cm/s}\ .$$

22. We use Eq. 4-26

   $$R_{\text{max}} = \left(\frac{v_0^2}{g}\sin 2\theta_0\right)_{\text{max}} = \frac{v_0^2}{g} = \frac{(9.5\,\text{m/s})^2}{9.80\,\text{m/s}^2} = 9.21\ \text{m}$$

   to compare with Powell's long jump; the difference from R_{max} is only $\Delta R = 9.21 - 8.95 = 0.26$ m.

23. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is throwing point (the stone's initial position). The x component of its initial velocity is given by $v_{0x} = v_0 \cos\theta_0$ and the y component is given by $v_{0y} = v_0 \sin\theta_0$, where $v_0 = 20$ m/s is the initial speed and $\theta_0 = 40.0°$ is the launch angle.

   (a) At $t = 1.10\,\text{s}$, its x coordinate is

   $$x = v_0 t\cos\theta_0 = (20.0\,\text{m/s})(1.10\,\text{s})\cos 40.0° = 16.9\,\text{m}$$

   (b) Its y coordinate at that instant is

   $$y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = (20.0\,\text{m/s})(1.10\,\text{s})\sin 40° - \frac{1}{2}(9.80\,\text{m/s}^2)(1.10\,\text{s})^2 = 8.21\,\text{m}\,.$$

   (c) At $t' = 1.80\,\text{s}$, its x coordinate is

   $$x = (20.0\,\text{m/s})(1.80\,\text{s})\cos 40.0° = 27.6\,\text{m}$$

   (d) Its y coordinate at t' is

   $$y = (20.0\,\text{m/s})(1.80\,\text{s})\sin 40° - \frac{1}{2}\left(9.80\,\text{m/s}^2\right)(1.80\,\text{s})^2 = 7.26\ \text{m}\ .$$

   (e) and (f) The stone hits the ground earlier than $t = 5.0\,\text{s}$. To find the time when it hits the ground solve $y = v_0 t\sin\theta_0 - \frac{1}{2}gt^2 = 0$ for t. We find

   $$t = \frac{2v_0}{g}\sin\theta_0 = \frac{2(20.0\,\text{m/s})}{9.8\,\text{m/s}^2}\sin 40° = 2.62\ \text{s}\ .$$

   Its x coordinate on landing is

   $$x = v_0 t\cos\theta_0 = (20.0\,\text{m/s})(2.62\,\text{s})\cos 40° = 40.2\ \text{m}$$

   (or Eq. 4-26 can be used). Assuming it stays where it lands, its coordinates at $t = 5.00\,\text{s}$ are $x = 40.2\,\text{m}$ and $y = 0$.

24. In this projectile motion problem, we have $v_0 = v_x =$ constant, and what is plotted is $v = \sqrt{v_x^2 + v_y^2}$. We infer from the plot that at $t = 2.5$ s, the ball reaches its maximum height, where $v_y = 0$. Therefore, we infer from the graph that $v_x = 19$ m/s.

   (a) During $t = 5$ s, the horizontal motion is $x - x_0 = v_x t = 95$ m.

   (b) Since $\sqrt{19^2 + v_{0y}^2} = 31$ m/s (the first point on the graph), we find $v_{0y} = 24.5$ m/s. Thus, with $t = 2.5$ s, we can use $y_{\max} - y_0 = v_{0y}t - \frac{1}{2}gt^2$ or $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\max} - y_0)$, or $y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0y}\right)t$ to solve. Here we will use the latter:

$$y_{\max} - y_0 = \frac{1}{2}\left(v_y + v_{0y}\right)t \implies y_{\max} = \frac{1}{2}(0 + 24.5)(2.5) = 31 \text{ m}$$

   where we have taken $y_0 = 0$ as the ground level.

25. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of §4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 t \cos\theta_0$ and $0 = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$. Eliminating t leads to $2v_0^2 \sin\theta_0 \cos\theta_0 - gd = 0$. Using $\sin\theta_0 \cos\theta_0 = \frac{1}{2}\sin(2\theta_0)$, we obtain

$$v_0^2 \sin(2\theta_0) = gd \implies \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.8)(45.7)}{460^2}$$

which yields $\sin(2\theta_0) = 2.12 \times 10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = 45.7 \tan 0.0606° = 0.0484 \,\text{m} = 4.84 \text{ cm} .$$

26. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units, $g = 32$ ft/s^2.

   (a) Using $x - x_0 = v_x t$ we obtain $v_x = (40 \text{ ft})/(1.25 \text{ s}) = 32$ ft/s. And $y - y_0 = 0 = v_{0y}t - \frac{1}{2}gt^2$ yields $v_{0y} = \frac{1}{2}(32)(1.25) = 20$ ft/s. Thus, the initial speed is

$$v_0 = |\vec{v}_0| = \sqrt{32^2 + 20^2} = 38 \text{ ft/s} .$$

   (b) Since $v_y = 0$ at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as $v_x = 32$ ft/s.

   (c) We can infer from the figure (or compute from $v_y = 0 = v_{0y} - gt$) that the time to reach the top is 0.625 s. With this, we can use $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to obtain 9.3 ft (where $y_0 = 3$ ft has been used). An alternative approach is to use $v_y^2 = v_{0y}^2 - 2g(y - y_0)$.

27. Taking the y axis to be upward and placing the origin at the firing point, the y coordinate is given by $y = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$ and the y component of the velocity is given by $v_y = v_0 \sin\theta_0 - gt$. The maximum height occurs when $v_y = 0$. Thus, $t = (v_0/g)\sin\theta_0$ and

$$y = v_0 \left(\frac{v_0}{g}\right) \sin\theta_0 \sin\theta_0 - \frac{1}{2}\frac{g(v_0 \sin\theta_0)^2}{g^2} = \frac{(v_0 \sin\theta_0)^2}{2g} .$$

28. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is $v_x = v_0 \cos 40.0°$, the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0}{25.0 \cos 40.0°} = 1.15 \text{ s} .$$

(a) The vertical distance is

$$\begin{aligned}\Delta y &= (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \\ &= (25.0\sin 40.0^\circ)(1.15) - \frac{1}{2}(9.8)(1.15)^2 = 12.0 \text{ m} .\end{aligned}$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value: $v_x = v_0 \cos 40.0^\circ = 19.2$ m/s, while the vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin\theta_0 - gt = 25.0\sin 40.0^\circ - (9.8)(1.15) = 4.80 \text{ m/s} .$$

(c) Since $v_y > 0$ when the ball hits the wall, it has not reached the highest point yet.

29. We designate the given velocity $\vec{v} = 7.6\,\hat{\imath} + 6.1\,\hat{\jmath}$ (SI units understood) as $\vec{v}_1$ – as opposed to the velocity when it reaches the max height $\vec{v}_2$ or the velocity when it returns to the ground $\vec{v}_3$ – and take $\vec{v}_0$ as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial y velocity, that is how we will proceed. Using Eq. 2-16, we have

$$\begin{aligned}v_{1\,y}^2 &= v_{0\,y}^2 - 2g\Delta y \\ 6.1^2 &= v_{0\,y}^2 - 2(9.8)(9.1)\end{aligned}$$

which yields $v_{0\,y} = 14.7$ m/s. Knowing that $v_{2\,y}$ must equal 0, we use Eq. 2-16 again but now with $\Delta y = h$ for the maximum height:

$$\begin{aligned}v_{2\,y}^2 &= v_{0\,y}^2 - 2gh \\ 0 &= 14.7^2 - 2(9.8)h\end{aligned}$$

which yields $h = 11$ m.

(b) Recalling the derivation of Eq. 4-26, but using $v_{0\,y}$ for $v_0 \sin\theta_0$ and $v_{0\,x}$ for $v_0\cos\theta_0$, we have

$$\begin{aligned}0 &= v_{0\,y}t - \frac{1}{2}gt^2 \\ R &= v_{0\,x}t\end{aligned}$$

which leads to $R = \frac{2\,v_{0\,x}\,v_{0\,y}}{g}$. Noting that $v_{0\,x} = v_{1\,x} = 7.6$ m/s, we plug in values and obtain $R = 2(7.6)(14.7)/9.8 = 23$ m.

(c) Since $v_{3\,x} = v_{1\,x} = 7.6$ m/s and $v_{3\,y} = -v_{0\,y} = -14.7$ m/s, we have

$$v_3 = \sqrt{v_{3\,x}^2 + v_{3\,y}^2} = \sqrt{(-14.7)^2 + 7.6^2} = 17 \text{ m/s} .$$

(d) The angle (measured from horizontal) for $\vec{v}_3$ is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7}{7.6}\right) = -63^\circ \quad \text{or} \quad 117^\circ$$

where we settle on the first choice (-63°, which is equivalent to 297°) since the signs of its components imply that it is in the fourth quadrant.

30. We apply Eq. 4-21, Eq. 4-22 and Eq. 4-23.

(a) From $\Delta x = v_{0x}t$, we find $v_{0x} = 40/2 = 20$ m/s.

(b) From $\Delta y = v_{0y}t - \frac{1}{2}gt^2$, we find $v_{0y} = (53 + \frac{1}{2}(9.8)(2)^2)/2 = 36$ m/s.

(c) From $v_y = v_{0y} - gt'$ with $v_y = 0$ as the condition for maximum height, we obtain $t' = 36/9.8 = 3.7$ s. During that time the x-motion is constant, so $x' - x_0 = (20)(3.7) = 74$ m.

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the the initial position for the football as it begins projectile motion in the sense of §4-5), and we let θ_0 be the angle of its initial velocity measured from the $+x$ axis.

(a) $x = 46\,\text{m}$ and $y = -1.5\,\text{m}$ are the coordinates for the landing point; it lands at time $t = 4.5\,\text{s}$. Since $x = v_{0x}t$,

$$v_{0\,x} = \frac{x}{t} = \frac{46\,\text{m}}{4.5\,\text{s}} = 10.2 \text{ m/s} .$$

Since $y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0\,y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5\,\text{m}) + \frac{1}{2}(9.8\,\text{m/s}^2)(4.5\,\text{s})^2}{4.5\,\text{s}} = 21.7 \text{ m/s} .$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0\,x}^2 + v_{0\,y}^2} = \sqrt{(10.2\,\text{m/s})^2 + (21.7\,\text{m/s})^2} = 24 \text{ m/s} .$$

(b) The initial angle satisfies $\tan\theta_0 = v_{0\,y}/v_{0\,x}$. Thus, $\theta_0 = \tan^{-1}(21.7/10.2) = 64.8°$.

32. The initial velocity has no vertical component – only an x component equal to $+2.00$ m/s. Also, $y_0 = +10.0$ m if the water surface is established as $y = 0$.

(a) $x - x_0 = v_x t$ readily yields $x - x_0 = 1.60$ m.

(b) Using $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$, we obtain $y = 6.86$ m when $t = 0.800$ s.

(c) With t unknown and $y = 0$, the equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ leads to $t = \sqrt{2(10)/9.8} = 1.43$ s. During this time, the x-displacement of the diver is $x - x_0 = (2.00 \text{ m/s})(1.43 \text{ s}) = 2.86$ m.

33. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -30°$ since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release: $v_0 = 290$ km/h, which we convert to SI units: $(290)(1000/3600) = 80.6$ m/s.

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos\theta_0)\, t \quad\Longrightarrow\quad t = \frac{700}{(80.6)\cos -30°} = 10.0 \text{ s} .$$

(b) And we use Eq. 4-22 to solve for the initial height y_0:

$$\begin{aligned} y - y_0 &= (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \\ 0 - y_0 &= (-40.3)(10.0) - \frac{1}{2}(9.8)(10.0)^2 \end{aligned}$$

which yields $y_0 = 897$ m.

34. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to $v_{0\,x}$). In this notation, we have

$$v_0 = 5v .$$

Next, we observe $v_0 \cos\theta_0 = v_{0\,x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \implies \theta_0 = \cos^{-1}\frac{1}{5} = 78^\circ .$$

35. We denote h as the height of a step and w as the width. To hit step n, the ball must fall a distance nh and travel horizontally a distance between $(n-1)w$ and nw. We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the y axis to be positive in the upward direction. The coordinates of the ball at time t are given by $x = v_{0\,x}t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0\,y} = 0$). We equate y to $-nh$ and solve for the time to reach the level of step n:

$$t = \sqrt{\frac{2nh}{g}} .$$

The x coordinate then is

$$x = v_{0\,x}\sqrt{\frac{2nh}{g}} = (1.52\,\text{m/s})\sqrt{\frac{2n(0.203\,\text{m})}{9.8\,\text{m/s}^2}} = (0.309\,\text{m})\sqrt{n} .$$

The method is to try values of n until we find one for which x/w is less than n but greater than $n-1$. For $n = 1$, $x = 0.309\,\text{m}$ and $x/w = 1.52$, which is greater than n. For $n = 2$, $x = 0.437\,\text{m}$ and $x/w = 2.15$, which is also greater than n. For $n = 3$, $x = 0.535\,\text{m}$ and $x/w = 2.64$. Now, this is less than n and greater than $n-1$, so the ball hits the third step.

36. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With $\Delta y = 0$, we have

$$\Delta y = (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 \implies t = \frac{(19.5)\sin 45^\circ}{\frac{1}{2}(9.8)} = 2.81 \text{ s} .$$

Then Eq. 4-21 yields $\Delta x = (v_0\cos\theta_0)\,t = 38.3$ m. Thus, using Eq. 4-8 and SI units, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{38.3\,\hat{\imath} - 5\,\hat{\imath}}{2.81} = -5.8\,\hat{\imath}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

37. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write $\theta_0 = -37^\circ$ for the angle measured from $+x$, since the angle given in the problem is measured from the $-y$ direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 4-22 to find v_0 (SI units are understood).

$$\begin{aligned} y - y_0 &= (v_0\sin\theta_0)\,t - \frac{1}{2}gt^2 \\ 0 - 730 &= v_0\sin(-37^\circ)\,(5.00) - \frac{1}{2}(9.8)(5.00)^2 \end{aligned}$$

which yields $v_0 = 202$ m/s.

(b) The horizontal distance traveled is $x = v_0 t\cos\theta_0 = (202)(5.00)\cos -37.0^\circ = 806$ m.

(c) The x component of the velocity (just before impact) is $v_x = v_0\cos\theta_0 = (202)\cos -37.0^\circ = 161$ m/s.

(d) The y component of the velocity (just before impact) is $v_y = v_0\sin\theta_0 - gt = (202)\sin(-37^\circ) - (9.80)(5.00) = -171$ m/s.

38. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that $(x_0, y_0) = (0, 3.0)$ m, and $v_x > 0$ (note that $v_{0y} = 0$).

    (a) To (barely) clear the net, we have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \implies 3.0 - 2.24 = 0 - \frac{1}{2}(9.8)t^2$$

    which gives $t = 0.39$ s for the time it is passing over the net. This is plugged into the x-equation to yield the (minimum) initial velocity $v_x = (8.0 \text{ m})/(0.39 \text{ s}) = 20.3$ m/s.

    (b) We require $y = 0$ and find t from $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$. This value ($t = \sqrt{2(3.0)/9.8} = 0.78$ s) is plugged into the x-equation to yield the (maximum) initial velocity $v_x = (17.0 \text{ m})/(0.78 \text{ s}) = 21.7$ m/s.

39. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26.

    (a) We want to know how high the ball is from the ground when it is at $x = 97.5$ m, which requires knowing the initial velocity. Using the range information and $\theta_0 = 45°$, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{g\,R}{\sin 2\theta_0}} = \sqrt{\frac{(9.8)(107)}{1}} = 32.4 \text{ m/s} .$$

    Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{97.5}{(32.4)\cos 45°} = 4.26 \text{ s} .$$

    At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 = 9.88 \text{ m}$$

    which implies it does indeed clear the 7.32 m high fence.

    (b) At $t = 4.26$ s, the center of the ball is $9.88 - 7.32 = 2.56$ m above the fence.

40. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

    (a) We want to know how high the ball is above the court when it is at $x = 12$ m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{12}{(23.6)\cos 0°} = 0.508 \text{ s} .$$

    At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin\theta_0)\,t - \frac{1}{2}gt^2 = 1.103 \text{ m}$$

    which implies it does indeed clear the 0.90 m high fence.

    (b) At $t = 0.508$ s, the center of the ball is $1.103 - 0.90 = 0.20$ m above the net.

    (c) Repeating the computation in part (a) with $\theta_0 = -5°$ results in $t = 0.510$ s and $y = 0.04$ m, which clearly indicates that it cannot clear the net.

(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at $t = 0.510$ s is $0.90 - 0.04 = 0.86$ m.

41. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. Where units are not displayed, SI units are understood. We use x and y to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s) θ_0 so that $y = 3.44$ m when $x = 50$ m. Writing the kinematic equations for projectile motion: $x = v_0 t \cos\theta_0$ and $y = v_0 t \sin\theta_0 - \frac{1}{2}gt^2$, we see the first equation gives $t = x/v_0 \cos\theta_0$, and when this is substituted into the second the result is

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0} .$$

One may solve this by trial and error: systematically trying values of θ_0 until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution:
Using the trigonometric identity $1/\cos^2\theta_0 = 1 + \tan^2\theta_0$, we obtain

$$\frac{1}{2}\frac{gx^2}{v_0^2}\tan^2\theta_0 - x\tan\theta_0 + y + \frac{1}{2}\frac{gx^2}{v_0^2} = 0$$

which is a second-order equation for $\tan\theta_0$. To simplify writing the solution, we denote $c = \frac{1}{2}gx^2/v_0^2 = \frac{1}{2}(9.80)(50)^2/(25)^2 = 19.6$ m. Then the second-order equation becomes $c\tan^2\theta_0 - x\tan\theta_0 + y + c = 0$. Using the quadratic formula, we obtain its solution(s).

$$\begin{aligned}\tan\theta_0 &= \frac{x \pm \sqrt{x^2 + 4(y+c)c}}{2c} \\ &= \frac{50 \pm \sqrt{50^2 - 4(3.44+19.6)(19.6)}}{2(19.6)} .\end{aligned}$$

The two solutions are given by $\tan\theta_0 = 1.95$ and $\tan\theta_0 = 0.605$. The corresponding (first-quadrant) angles are $\theta_0 = 63°$ and $\theta_0 = 31°$. If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

42. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10\,\text{m/s})^2}{25\,\text{m}} = 4.0\,\text{m/s}^2 .$$

43. We apply Eq. 4-33 to solve for speed v and Eq. 4-32 to find acceleration a.

(a) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is $6.37 \times 10^6\,\text{m} + 640 \times 10^3\,\text{m} = 7.01 \times 10^6$ m. Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01 \times 10^6\,\text{m})}{(98.0\,\text{min})(60\,\text{s/min})} = 7.49 \times 10^3\,\text{m/s} .$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49 \times 10^3\,\text{m/s})^2}{7.01 \times 10^6\,\text{m}} = 8.00\,\text{m/s}^2 .$$

44. We note that the period of revolution is $(1200\,\text{rev/min})^{-1} = 8.3 \times 10^{-4}$ min which becomes, in SI units, $T = 0.050$ s.

(a) The circumference is $c = 2\pi r = 2\pi(0.15) = 0.94$ m.

(b) The speed is $v = c/T = (0.94)/(0.050) = 19$ m/s. This is equivalent to using Eq. 4-33.

(c) The magnitude of the acceleration is $a = v^2/r = 19^2/0.15 = 2.4 \times 10^3$ m/s^2.

(d) As noted above, $T = 50$ ms.

45. We apply Eq. 4-32 to solve for speed v and Eq. 4-33 to find the period T.

(a) We obtain
$$v = \sqrt{ra} = \sqrt{(5.0\,\text{m})(7.0)(9.8\,\text{m/s}^2)} = 19 \text{ m/s} .$$

(b) The time to go around once (the period) is $T = 2\pi r/v = 1.7$ s. Therefore, in one minute ($t = 60$ s), the astronaut executes
$$\frac{t}{T} = \frac{60}{1.7} = 35$$
revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of $7g$ when the radius is 5.0 m.

(c) As noted above, $T = 1.7$ s.

46. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (towards the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences $\vec{a} = 1.83$ m/s^2 east, then the center of the circle is *east* of this location. And the distance is $r = v^2/a = (3.66^2)/(1.83) = 7.32$ m. Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(b) We see the distance is the same, but now the direction of $\vec{a}$ experienced by the passenger is *south* – indicating that the center of the merry-go-round is south of him. Therefore, relative to the center, the passenger at that moment located 7.32 m toward the north.

47. The radius of Earth may be found in Appendix C.

(a) The speed of a person at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s): $v = 2\pi(6.37 \times 10^6\,\text{m})/(8.64 \times 10^4\,\text{s}) = 463\,\text{m/s}$. The magnitude of the acceleration is given by
$$a = \frac{v^2}{R} = \frac{(463\,\text{m/s})^2}{6.37 \times 10^6\,\text{m}} = 0.034 \text{ m/s}^2 .$$

(b) If T is the period, then $v = 2\pi R/T$ is the speed and $a = v^2/R = 4\pi^2R^2/T^2R = 4\pi^2R/T^2$ is the magnitude of the acceleration. Thus
$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37 \times 10^6\,\text{m}}{9.8\,\text{m/s}^2}} = 5.1 \times 10^3\,\text{s} = 84 \text{ min} .$$

48. Eq. 4-32 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r.

(a) The minimum turning radius of the train is given by
$$r_{\text{min}} = \frac{v^2}{a_{\text{max}}} = \frac{(216\,\text{km/h})^2}{(0.050)(9.8\,\text{m/s}^2)} = 7.3 \times 10^3\,\text{m} .$$

(b) The speed of the train must be reduced to no more than
$$v = \sqrt{a_{\text{max}}\, r} = \sqrt{0.050(9.8)(1.00 \times 10^3)} = 22 \text{ m/s}$$
which is roughly 80 km/h.

49. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by $a = v^2/R$, where R is the radius of the wheel, and v is the speed of the passenger. Since the passenger goes a distance $2\pi R$ for each revolution, his speed is

$$v = \frac{2\pi(15\,\text{m})}{12\,\text{s}} = 7.85\,\text{m/s}$$

and his centripetal acceleration is

$$a = \frac{(7.85\,\text{m/s})^2}{15\,\text{m}} = 4.1\,\text{m/s}^2 \ .$$

When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(c) At the lowest point, the centripetal acceleration vector points up, toward the center of the orbit. It has the same magnitude as in part (b).

50. We apply Eq. 4-33 to solve for speed v and Eq. 4-32 to find centripetal acceleration a.

(a) $v = 2\pi r/T = 2\pi(20\,\text{km})/1.0\,\text{s} = 1.3 \times 10^5\,\text{km/s}$.

(b)

$$a = \frac{v^2}{r} = \frac{(126\,\text{km/s})^2}{20\,\text{km}} = 7.9 \times 10^5\,\text{m/s}^2 \ .$$

(c) Clearly, both v and a will increase if T is reduced.

51. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the $+y$ direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by $x = v_0 t$ and $y = -\frac{1}{2}gt^2$ (since $v_{0\,y} = 0$). It hits the ground at $x = 10\,\text{m}$ and $y = -2.0\,\text{m}$. Formally solving the second equation for the time, we obtain $t = \sqrt{-2y/g}$, which we substitute into the first equation:

$$v_0 = x\sqrt{-\frac{g}{2y}} = (10\,\text{m})\sqrt{-\frac{9.8\,\text{m/s}^2}{2(-2.0\,\text{m})}} = 15.7\ \text{m/s} \ .$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7\,\text{m/s})^2}{1.5\,\text{m}} = 160\ \text{m/s}^2 \ .$$

52. We write our magnitude-angle results in the form $(R \angle \theta)$ with SI units for the magnitude understood (m for distances, m/s for speeds, m/s^2 for accelerations). All angles θ are measured counterclockwise from $+x$, but we will occasionally refer to angles ϕ which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see r in the figure). We note that the speed of the particle is $v = 2\pi r/T$ where $r = 3.00$ m and $T = 20.0$ s; thus, $v = 0.942$ m/s. The particle is moving counterclockwise in Fig. 4-37.

(a) At $t = 5.00$ s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00}{20.0} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}\,(360°) = 90°$$

measured from vertical (as explained above). Referring to Fig. 4-37, we see that this position (which is the "3 o'clock" position on the circle) corresponds to $x = 3.00$ m and $y = 3.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (4.24 \angle 45°)$. Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of x and y relative to coordinate origin can be gotten from the angle ϕ from the relations $x = r \sin\phi$ and $y = r - r\cos\phi$. Of course, $R = \sqrt{x^2 + y^2}$ and θ comes from choosing the appropriate possibility from $\tan^{-1}(y/x)$ (or by using particular functions of vector capable calculators).

(b) At $t = 7.50$ s, the particle has traveled a fraction of $7.50/20.0 = 3/8$ of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at $\phi = 3/8\,(360°) = 135°$ measured from vertical in the manner discussed above. Referring to Fig. 4-37, we compute that this position corresponds to $x = 3.00 \sin 135° = 2.12$ m and $y = 3.00 - 3.00 \cos 135° = 5.12$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (5.54 \angle 67.5°)$.

(c) At $t = 10.0$ s, the particle has traveled a fraction of $10.0/20.0 = 1/2$ of a revolution around the circle. Relative to the circle-center, the particle is at $\phi = 180°$ measured from vertical (see explanation, above). Referring to Fig. 4-37, we see that this position corresponds to $x = 0$ and $y = 6.00$ m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as $(R \angle \theta) = (6.00 \angle 90.0°)$.

(d) We subtract the position vector in part (a) from the position vector in part (c): $(6.00 \angle 90.0°) - (4.24 \angle 45°) = (4.24 \angle 135°)$ using magnitude-angle notation (convenient when using vector capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta\vec{R} = (0 - 3)\,\hat{\imath} + (6 - 3)\,\hat{\jmath} = -3\,\hat{\imath} + 3\,\hat{\jmath}$$

which leads to $|\Delta\vec{R}| = 4.24$ m and $\theta = 135°$.

(e) From Eq. 4-8, we have

$$\vec{v}_{\rm avg} = \frac{\Delta\vec{R}}{\Delta t} \quad \text{where} \quad \Delta t = 5.00 \text{ s}$$

which produces $-0.6\,\hat{\imath} + 0.6\,\hat{\jmath}$ m/s in unit-vector notation or $(0.849 \angle 135°)$ in magnitude-angle notation.

(f) The speed has already been noted ($v = 0.942$ m/s), but its direction is best seen by referring again to Fig. 4-37. The velocity vector is tangent to the circle at its "3 o'clock position" (see part (a)), which means $\vec{v}$ is vertical. Thus, our result is $(0.942 \angle 90°)$.

(g) Again, the speed has been noted above ($v = 0.942$ m/s), but its direction is best seen by referring to Fig. 4-37. The velocity vector is tangent to the circle at its "12 o'clock position" (see part (c)), which means $\vec{v}$ is horizontal. Thus, our result is $(0.942 \angle 180°)$.

(h) The acceleration has magnitude $v^2/r = 0.296$ m/s^2, and at this instant (see part (a)) it is horizontal (towards the center of the circle). Thus, our result is $(0.296 \angle 180°)$.

(i) Again, $a = v^2/r = 0.296$ m/s^2, but at this instant (see part (c)) it is vertical (towards the center of the circle). Thus, our result is $(0.296 \angle 270°)$.

53. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so 20 km/h $\rightarrow$ 5.6 m/s, 30 km/h $\rightarrow$ 8.3 m/s, and 45 km/h $\rightarrow$ 12.5 m/s. We choose east as the $+\,\hat{\imath}$ direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-42)

$$\vec{v}_{\rm c\,t} = \vec{v}_{\rm c\,g} - \vec{v}_{\rm t\,g} = 12.5\,\hat{\imath} - (-5.6\,\hat{\imath}) = 18.1\,\hat{\imath} \text{ m/s}$$

relative to the truck. The (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{\rm avg} = \frac{18.1\,\hat{\imath} - (-8.3\,\hat{\imath})}{2.0} = 13\,\hat{\imath} \text{ m/s}^2 \ .$$

(b) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-42)

$$\vec{v}_{0\,\mathrm{c\,g}} = \vec{v}_{0\,\mathrm{c\,t}} + \vec{v}_{0\,\mathrm{t\,g}} = (-8.3\,\hat{\imath}) + (-5.6\,\hat{\imath}) = -13.9\,\hat{\imath}\ \mathrm{m/s}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{\mathrm{avg}} = \frac{12.5\,\hat{\imath} - (-13.9\,\hat{\imath})}{2.0} = 13\,\hat{\imath}\ \mathrm{m/s^2}$$

identical to the result of part (a).

54. We choose upstream as the $+\hat{\imath}$ direction, and use Eq. 4-42.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{\mathrm{b\,g}} = \vec{v}_{\mathrm{b\,w}} + \vec{v}_{\mathrm{w\,g}} = (14\,\mathrm{km/h})\,\hat{\imath} + (-9\,\mathrm{km/h})\,\hat{\imath} = (5\,\mathrm{km/h})\,\hat{\imath}$$

(b) And we use the subscript c for the child.

$$\vec{v}_{\mathrm{c\,g}} = \vec{v}_{\mathrm{c\,b}} + \vec{v}_{\mathrm{b\,g}} = (-6\,\mathrm{km/h})\,\hat{\imath} + (5\,\mathrm{km/h})\,\hat{\imath} = (-1\,\mathrm{km/h})\,\hat{\imath}$$

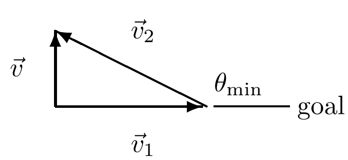
55. When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is $v_p = (15\,\mathrm{m})/(90\,\mathrm{s}) = 0.167\,\mathrm{m/s}$. The escalator moves at $v_e = (15\,\mathrm{m})/(60\,\mathrm{s}) = 0.250\,\mathrm{m/s}$. The speed of the person walking up the moving escalator is $v = v_p + v_e = 0.167\,\mathrm{m/s} + 0.250\,\mathrm{m/s} = 0.417\,\mathrm{m/s}$ and the time taken to move the length of the escalator is

$$t = \ell/v = (15\,\mathrm{m})/(0.417\,\mathrm{m/s}) = 36\,\mathrm{s}\ .$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = (\ell/90) + (\ell/60) = 0.0278\ell$. The time taken is $t = \ell/v = \ell/0.0278\ell = 36\,\mathrm{s}$ and is independent of ℓ.

56. We denote the velocity of the player with $\vec{v}_1$ and the relative velocity between the player and the ball be $\vec{v}_2$. Then the velocity $\vec{v}$ of the ball relative to the field is given by $\vec{v} = \vec{v}_1 + \vec{v}_2$. The smallest angle θ_{min} corresponds to the case when $\vec{v} \perp \vec{v}_1$. Hence,

$$\begin{aligned}\theta_{\mathrm{min}} &= 180^\circ - \cos^{-1}\left(\frac{|\vec{v}_1|}{|\vec{v}_2|}\right) \\ &= 180^\circ - \cos^{-1}\left(\frac{4.0\,\mathrm{m/s}}{6.0\,\mathrm{m/s}}\right) \\ &\approx 130^\circ\ .\end{aligned}$$



57. Relative to the car the velocity of the snowflakes has a vertical component of $8.0\,\mathrm{m/s}$ and a horizontal component of $50\,\mathrm{km/h} = 13.9\,\mathrm{m/s}$. The angle θ from the vertical is found from

$$\tan\theta = v_h/v_v = (13.9\,\mathrm{m/s})/(8.0\,\mathrm{m/s}) = 1.74$$

which yields $\theta = 60^\circ$.

58. We denote the police and the motorist with subscripts p and m, respectively. The coordinate system is indicated in Fig. 4-38.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m\,p} = \vec{v}_m - \vec{v}_p = -60\,\hat{\jmath} - (-80\,\hat{\imath}) = 80\,\hat{\imath} - 60\,\hat{\jmath}\ (\text{km/h})\ .$$

(b) $\vec{v}_{m\,p}$ does happen to be along the line of sight. Referring to Fig. 4-38, we find the vector pointing from car to another is $\vec{r} = 800\,\hat{\imath} - 600\,\hat{\jmath}$ m (from M to P). Since the ratio of components in $\vec{r}$ is the same as in $\vec{v}_{m\,p}$, they must point the same direction.

(c) No, they remain unchanged.

59. Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop is $v_h = 30\,\text{m/s}$, the same as the speed of the train. If v_v is the vertical component of the velocity and θ is the angle between the direction of motion and the vertical, then $\tan\theta = v_h/v_v$. Thus $v_v = v_h/\tan\theta = (30\,\text{m/s})/\tan 70° = 10.9\,\text{m/s}$. The speed of a raindrop is $v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30\,\text{m/s})^2 + (10.9\,\text{m/s})^2} = 32\,\text{m/s}$.

60. Here, the subscript W refers to the water. Our coordinates are chosen with $+x$ being *east* and $+y$ being *north*. In these terms, the angle specifying *east* would be 0° and the angle specifying *south* would be −90° or 270°. Where the length unit is not displayed, km is to be understood.

(a) We have $\vec{v}_{A\,W} = \vec{v}_{A\,B} + \vec{v}_{B\,W}$, so that $\vec{v}_{A\,B} = (22\ \angle - 90°) - (40\ \angle\ 37°) = (56\ \angle - 125°)$ in the magnitude-angle notation (conveniently done with a vector capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{A\,B} = -32\,\hat{\imath} - 46\,\hat{\jmath}\ \ \text{km/h}\ .$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward ($\vec{r} - \vec{r}_0 = \int \vec{v}\,dt$)

$$\vec{r} = (2.5 - 32t)\,\hat{\imath} + (4.0 - 46t)\,\hat{\jmath}$$

with lengths in kilometers and time in hours.

(c) The magnitude of this $\vec{r}$ is

$$r = \sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}$$

We minimize this by taking a derivative and requiring it to equal zero – which leaves us with an equation for t

$$\frac{dr}{dt} = \frac{1}{2}\frac{6286t - 528}{\sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}} = 0$$

which yields $t = 0.084$ h.

(d) Plugging this value of t back into the expression for the distance between the ships (r), we obtain $r = 0.2$ km. Of course, the calculator offers more digits ($r = 0.225...$), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

61. The velocity vector (relative to the shore) for ships A and B are given by

$$\vec{v}_A = -\,(v_A \cos 45°)\ \hat{\imath} + (v_A \sin 45°)\ \hat{\jmath}$$

and

$$\vec{v}_B = -\,(v_B \sin 40°)\ \hat{\imath} - (v_B \cos 40°)\ \hat{\jmath}$$

respectively (where $v_A = 24$ knots and $v_B = 28$ knots). We are taking East as $+\,\hat{\imath}$ and North as $\hat{\jmath}$.

(a) Their relative velocity is

$$\vec{v}_{A\,B} = \vec{v}_A - \vec{v}_B = (v_B \sin 40^\circ - v_A \cos 45^\circ)\,\hat{\imath} + (v_B \cos 40^\circ + v_A \sin 45^\circ)\,\hat{\jmath}$$

the magnitude of which is $|\vec{v}_{A\,B}| = \sqrt{1.0^2 + 38.4^2} \approx 38$ knots. The angle θ which $\vec{v}_{A\,B}$ makes with North is given by

$$\theta = \tan^{-1}\left(\frac{v_{AB,x}}{v_{AB,y}}\right) = \tan^{-1}\left(\frac{1.0}{38.4}\right) = 1.5^\circ$$

which is to say that $\vec{v}_{A\,B}$ points 1.5° east of north.

(b) Since they started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{A\,B}|}{|\vec{v}_{A\,B}|} = \frac{160}{38} = 4.2 \text{ h} .$$

(c) The velocity $\vec{v}_{A\,B}$ does not change with time in this problem, and $\vec{r}_{A\,B}$ is in the same direction as $\vec{v}_{A\,B}$ since they started at the same time. Reversing the points of view, we have $\vec{v}_{A\,B} = -\vec{v}_{B\,A}$ so that $\vec{r}_{A\,B} = -\vec{r}_{B\,A}$ (i.e., they are 180° opposite to each other). Hence, we conclude that B stays at a bearing of 1.5° west of south relative to A during the journey (neglecting the curvature of Earth).

62. The (box)car has velocity $\vec{v}_{c\,g} = v_1\,\hat{\imath}$ relative to the ground, and the bullet has velocity

$$\vec{v}_{0\,b\,g} = v_2 \cos\theta\,\hat{\imath} + v_2 \sin\theta\,\hat{\jmath}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is $\vec{v}_{b\,g} = 0.8v_2 \cos\theta\,\hat{\imath} + 0.8v_2 \sin\theta\,\hat{\jmath}$ (due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with v_3 unspecified) $\vec{v}_{b\,c} = v_3\,\hat{\jmath}$. Now, Eq. 4-42 provides the condition

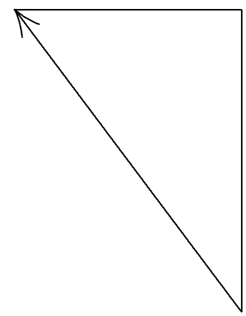
$$\begin{aligned} \vec{v}_{b\,g} &= \vec{v}_{b\,c} + \vec{v}_{c\,g} \\ 0.8v_2 \cos\theta\,\hat{\imath} + 0.8v_2 \sin\theta\,\hat{\jmath} &= v_3\,\hat{\jmath} + v_1\,\hat{\imath} \end{aligned}$$

so that equating x components allows us to find θ. If one wished to find v_3 one could also equate the y components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the x components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\left(\frac{85\left(\frac{1000}{3600}\right)}{0.8(650)}\right)$$

which yields 87° for the direction of $\vec{v}_{b\,g}$ (measured from $\hat{\imath}$, which is the direction of motion of the car). The problem asks, "from what direction was it fired?" – which means the answer is not 87° but rather its supplement 93° (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at 87° measured counterclockwise from the $+x$ direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at -93° (that is, 93° measured clockwise from $+x$).

63. We construct a right triangle starting from the clearing on the south



bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance $(82\,\text{m}) + (1.1\,\text{m/s})t$, where the t-dependent contribution is the distance that the river will carry the boat downstream during time t.

The hypotenuse of this right triangle (the arrow in our sketch) also depends on t and on the boat's speed (relative to the water), and we set it equal to the Pythagorean "sum" of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for t

$$46724 + 180.4t - 14.8t^2 = 0 \; .$$

We solve this and find a positive value: $t = 62.6$ s. The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured "west of north") is then given by

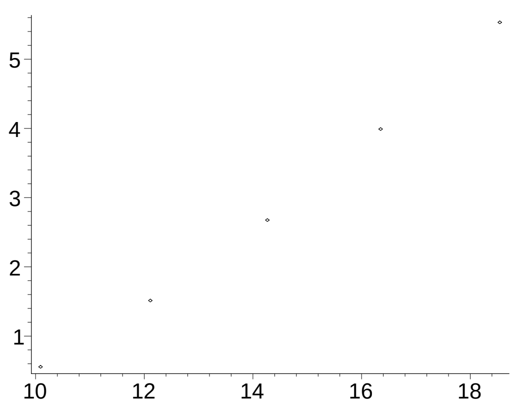
$$\theta = \tan^{-1}\left(\frac{82 + 1.1t}{200}\right) = \tan^{-1}\left(\frac{151}{200}\right) = 37^\circ \; .$$

64. (a) We compute the coordinate pairs (x, y) from $x = v_0 \cos\theta t$ and $x = v_0 \sin\theta t - \frac{1}{2}gt^2$ for $t = 20$ s and the speeds and angles given in the problem. We obtain (in kilometers)

$$(x_A\,,\,y_A) = (10.1, 0.56) \qquad (x_B\,,\,y_B) = (12.1, 1.51)$$
$$(x_C\,,\,y_C) = (14.3, 2.68) \qquad (x_D\,,\,y_D) = (16.4, 3.99)$$

and $(x_E\,,\,y_E) = (18.5, 5.53)$ which we plot in the next part.

(b) The vertical (y) and horizontal (x) axes are in kilometers. The graph does not start at the origin. The curve to "fit" the data is not shown, but is easily imagined (forming the "curtain of death").

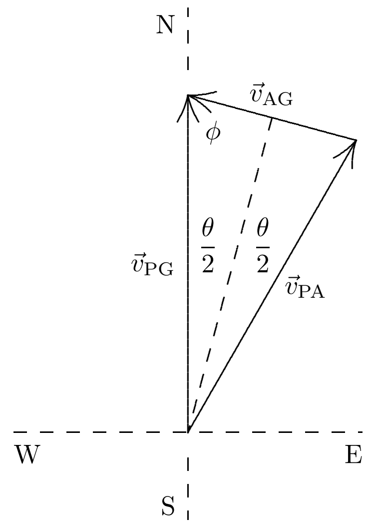


65. We denote $\vec{v}_{\text{PG}}$ as the velocity of the plane relative to the ground, $\vec{v}_{\text{AG}}$ as the velocity of the air relative to the ground, and $\vec{v}_{\text{PA}}$ be the velocity of the plane relative to the air.

(a) The vector diagram is shown below. $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since the magnitudes v_{PG} and v_{PA} are equal the triangle is isosceles, with two sides of equal length. Consider either of the right triangles formed when the bisector of θ is drawn (the dashed line). It bisects $\vec{v}_{AG}$, so

$$\sin(\theta/2) = \frac{v_{AG}}{2v_{PG}} = \frac{70.0\,\text{mi/h}}{2(135\,\text{mi/h})}$$

which leads to $\theta = 30.1°$. Now $\vec{v}_{AG}$ makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction 15° north of west. Thus, it is blowing *from* 75° east of south.



(b) The plane is headed along $\vec{v}_{PA}$, in the direction 30° east of north. There is another solution, with the plane headed 30° west of north and the wind blowing 15° north of east (that is, from 75° west of south).

66. (a) The ball must increase in height by $\Delta y = 0.193$ m and cover a horizontal distance $\Delta x = 0.910$ m during a very short time $t_0 = 1.65 \times 10^{-2}$ s. The statement that the "initial curvature of the ball's path can be neglected" is essentially the same as saying the average velocity for $0 \le t \le t_0$ may be taken equal to the instantaneous initial velocity $\vec{v}_0$. Thus, using Eq. 4-8 to figure its two components, we have

$$\tan\theta_0 = \frac{v_{0y}}{v_{0x}} = \frac{\frac{\Delta y}{t_0}}{\frac{\Delta x}{t_0}} = \frac{\Delta y}{\Delta x}$$

so that $\theta_0 = \tan^{-1}(0.193/0.910) = 12°$.

(b) The magnitude of $\vec{v}_0$ is

$$\sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(\frac{\Delta x}{t_0}\right)^2 + \left(\frac{\Delta y}{t_0}\right)^2} = \frac{\sqrt{\Delta x^2 + \Delta y^2}}{t_0}$$

which yields $v_0 = 56.4$ m/s.

(c) The range is given by Eq. 4-26:

$$R = \frac{v_0^2}{g}\sin 2\theta_0 = 132 \text{ m} .$$

(d) Partly because of its dimpled surface (but other air-flow related effects are important here) the golf ball travels farther than one would expect based on the simple projectile-motion analysis done here.

67. (a) Since the performer returns to the original level, Eq. 4-26 applies. With $R = 4.0$ m and $\theta_0 = 30°$, the initial speed (for the projectile motion) is consequently

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = 6.7 \text{ m/s} .$$

This is, of course, the final speed v for the Air Ramp's acceleration process (for which the initial speed is taken to be zero) Then, for that process, Eq. 2-11 leads to

$$a = \frac{v}{t} = \frac{6.7}{0.25} = 27 \text{ m/s}^2 .$$

We express this as a multiple of g by setting up a ratio: $a = (27/9.8)g = 2.7g$.

(b) Repeating the above steps for $R = 12$ m, $t = 0.29$ s and $\theta_0 = 45°$ gives $a = 3.8g$.

68. The initial position vector $\vec{r}_o$ satisfies $\vec{r} - \vec{r}_o = \Delta\vec{r}$, which results in

$$\vec{r}_o = \vec{r} - \Delta\vec{r} = (3\,\hat{j} - 4\,\hat{k}) - (2\,\hat{i} - 3\,\hat{j} + 6\,\hat{k}) = -2.0\,\hat{i} + 6.0\,\hat{j} - 10\,\hat{k}$$

where the understood unit is meters.

69. We adopt a coordinate system with $\hat{i}$ pointed east and $\hat{j}$ pointed north; the coordinate origin is the flagpole. With SI units understood, we "translate" the given information into unit-vector notation as follows:

$$\begin{aligned} \vec{r}_o &= 40\,\hat{i} \quad \text{and} \quad \vec{v}_o = -10\,\hat{j} \\ \vec{r} &= 40\,\hat{j} \quad \text{and} \quad \vec{v} = 10\,\hat{i}\ . \end{aligned}$$

(a) Using Eq. 4-2, the displacement $\Delta\vec{r}$ is

$$\vec{r} - \vec{r}_o = (56.6\ \angle\ 135°)$$

where we have expressed the result in magnitude-angle notation. The displacement has magnitude $40\sqrt{2} = 56.6$ m and points due Northwest.

(b) Eq. 4-8 shows that $\vec{v}_{avg}$ points in the same direction as $\Delta\vec{r}$, and that its magnitude is simply the magnitude of the displacement divided by the time ($\Delta t = 30$ s). Thus, the average velocity has magnitude $56.6/30 = 1.89$ m/s and points due Northwest.

(c) Using Eq. 4-15, we have

$$\vec{a}_{avg} = \frac{\vec{v} - \vec{v}_o}{\Delta t} = 0.333\,\hat{i} + 0.333\,\hat{j}$$

in SI units. The magnitude of the average acceleration vector is therefore $0.333\sqrt{2} = 0.471\ \text{m/s}^2$, and it points due Northeast.

70. The velocity of Larry is v_1 and that of Curly is v_2. Also, we denote the length of the corridor by L. Now, Larry's time of passage is $t_1 = 150$ s (which must equal L/v_1), and Curly's time of passage is $t_2 = 70$ s (which must equal L/v_2). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1/L + v_2/L} = \frac{1}{\frac{1}{150} + \frac{1}{70}} = 48\ \text{s}\ .$$

71. We choose horizontal x and vertical y axes such that both components of $\vec{v}_0$ are positive. Positive angles are counterclockwise from $+x$ and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos\theta_0\,\hat{i} + (v_0 \sin\theta_0 - gt)\,\hat{j}\ .$$

(a) With $v_0 = 30$ m/s and $\theta_0 = 60°$, we obtain $\vec{v} = 15\,\hat{i} + \,\hat{j}$ in m/s, for $t = 2.0$ s. Converting to magnitude-angle notation, this is $\vec{v} = (16\ \angle\ 23°)$ with the magnitude in m/s.

(b) Now with $t = 5.0$ s, we find $\vec{v} = (27\ \angle -57°)$.

72. (a) The helicopter's speed is $v' = 6.2$ m/s. From the discussions in §4-9 we see that the speed of the package is $v_0 = 12 - v' = 5.8$ m/s, relative to the ground.

(b) Letting $+x$ be in the direction of $\vec{v}_0$ for the package and $+y$ be downward, we have (for the motion of the package)

$$\Delta x = v_0 t \quad \text{and} \quad \Delta y = \frac{1}{2} g t^2$$

where $\Delta y = 9.5$ m. From these, we find $t = 1.39$ s and $\Delta x = 8.08$ m for the package, while $\Delta x'$ (for the helicopter, which is moving in the opposite direction) is $-v'\,t = -8.63$ m. Thus, the horizontal separation between them is $8.08 - (-8.63) = 16.7$ m.

(c) The components of $\vec{v}$ at the moment of impact are $(v_x, v_y) = (5.8, 13.6)$ in SI units. The vertical component has been computed using Eq. 2-11. The angle (which is below horizontal) for this vector is $\tan^{-1}(13.6/5.8) = 67°$.

73. (a) By symmetry, $y = H$ occurs at $x = R/2$ (taking the coordinate origin to be at the launch point). Substituting this into Eq. 4-25 gives

$$H = \frac{R}{2}\tan\theta_0 - \frac{gR^2/4}{2v_0^2\cos^2\theta_0}$$

which leads immediately to

$$\frac{H}{R} = \frac{1}{2}\left(\tan\theta_0 - \frac{gR}{4v_0^2\cos^2\theta_0}\right) .$$

In the far right term, we substitute from Eq. 4-26 for the range:

$$\frac{H}{R} = \frac{1}{2}\left(\tan\theta_0 - \frac{g\left(v_0^2\sin(2\theta_0)/g\right)}{4v_0^2\cos^2\theta_0}\right)$$

which, upon setting $\sin 2\theta_0 = 2\sin\theta_0\cos\theta_0$ and simplifying that last term, yields

$$\frac{H}{R} = \frac{1}{2}\left(\tan\theta_0 - \frac{\sin\theta_0}{2\cos\theta_0}\right)$$

which clearly leads to the relation we wish to prove.

(b) Setting $H/R = 1$ in that relation, we have $\theta_0 = \tan^{-1}(4) = 76°$.

74. (a) The tangent of the angle ϕ is found from the ratio of y to x coordinates of the highest point (taking the coordinate origin to be at the launch point). Using the same notation as in problem 73, we have

$$\phi = \tan^{-1}\left(\frac{H}{\frac{1}{2}R}\right)\tan^{-1}\left(2\frac{H}{R}\right) .$$

Substituting $H/R = \frac{1}{4}\tan\theta_0$ from problem 73, we obtain the relation

$$\tan^{-1}\left(\frac{1}{2}\tan\theta_0\right) .$$

(b) Since $\tan 45° = 1$, then $\phi = \tan^{-1}\left(\frac{1}{2}\right) = 27°$.

75. The initial velocity has magnitude v_0 and because it is horizontal, it is equal to v_x the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0 \quad \text{where} \quad v_y = \sqrt{2gh}$$

where we use Eq. 2-16 with Δx replaced with the $h = 20$ m to obtain that second equality. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to $gh = 4v_0^2$ and therefore to $v_0 = \sqrt{(9.8)(20)}/2 = 7.0$ m/s.

76. (a) The magnitude of the displacement vector $\Delta\vec{r}$ is given by

$$|\Delta\vec{r}| = \sqrt{21.5^2 + 9.7^2 + 2.88^2} = 23.8 \text{ km} .$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8}{3.50} = 6.79 \text{ km/h} .$$

(b) The angle θ in question is given by

$$\theta = \tan^{-1}\left(\frac{2.88}{\sqrt{21.5^2 + 9.7^2}}\right) = 6.96^\circ \ .$$

77. With no acceleration in the x direction yet a constant acceleration of 1.4 m/s^2 in the y direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.0t)\hat{\imath} + \left(\frac{1}{2}(1.4)t^2\right)\hat{\jmath}$$

and $\vec{v}$ is its derivative with respect to t.

(a) At $t = 3.0$ s, therefore, $\vec{v} = 6.0\hat{\imath} + 4.2\hat{\jmath}$ m/s.

(b) At $t = 3.0$ s, the position is $\vec{r} = 18\,\hat{\imath} + 6.3\,\hat{\jmath}$ m.

78. We choose a coordinate system with origin at the clock center and $+x$ rightward (towards the "3:00" position) and $+y$ upward (towards "12:00").

(a) In unit-vector notation, we have (in centimeters) $\vec{r}_1 = 10\,\hat{\imath}$ and $\vec{r}_2 = -10\,\hat{\jmath}$. Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = -10\,\hat{\imath} - 10\,\hat{\jmath} \ \longrightarrow\ (14\ \angle - 135^\circ)$$

where we have switched to magnitude-angle notation in the last step.

(b) In this case, $\vec{r}_1 = -10\,\hat{\jmath}$ and $\vec{r}_2 = 10\,\hat{\jmath}$, and $\Delta\vec{r} = 20\,\hat{\jmath}$ cm.

(c) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

79. We let g_p denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some "inferred" points – such as the max height point at $x = 12.5$ m and $t = 1.25$ s) can be analyzed profitably; for future reference, we label (with subscripts) the first ($(x_0, y_0) = (0, 2)$ at $t_0 = 0$) and last ("final") points ($(x_f, y_f) = (25, 2)$ at $t_f = 2.5$), with lengths in meters and time in seconds.

(a) The x-component of the initial velocity is found from $x_f - x_0 = v_{0x}t_f$. Therefore, $v_{0x} = 25/2.5 =$ 10 m/s. And we try to obtain the y-component from $y_f - y_0 = 0 = v_{0y}t_f - \frac{1}{2}g_p t_f^2$. This gives us $v_{0y} = 1.25g_p$, and we see we need another equation (by analyzing another point, say, the next-to-last one) $y - y_0 = v_{0y}t - \frac{1}{2}g_p t^2$ with $y = 6$ and $t = 2$; this produces our second equation $v_{0y} = 2 + g_p$. Simultaneous solution of these two equations produces results for v_{0y} and g_p (relevant to part (b)). Thus, our complete answer for the initial velocity is $\vec{v} = 10\hat{\imath} + 10\hat{\jmath}$ m/s.

(b) As a by-product of the part (a) computations, we have $g_p = 8.0\,\text{m/s}^2$.

(c) Solving for t_g (the time to reach the ground) in $y_g = 0 = y_0 + v_{0y}t_g - \frac{1}{2}g_p t_g^2$ leads to a positive answer: $t_g = 2.7$ s.

(d) With $g = 9.8\,\text{m/s}^2$, the method employed in part (c) would produce the quadratic equation $-4.9t_g^2 + 10t_g + 2 = 0$ and then the positive result $t_g = 2.2$ s.

80. At maximum height, the y-component of a projectile's velocity vanishes, so the given 10 m/s is the (constant) x-component of velocity.

(a) Using v_{0y} to denote the y-velocity 1.0 s before reaching the maximum height, then (with $v_y = 0$) the equation $v_y = v_{0y} - gt$ leads to $v_{0y} = 9.8$ m/s. The magnitude of the velocity vector at that moment (also known as the *speed*) is therefore

$$\sqrt{v_x^2 + {v_{0y}}^2} = \sqrt{10^2 + 9.8^2} = 14 \text{ m/s} \ .$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using $v_y = v_{0y} - gt$ again but now "starting the clock" at the highest point so that $v_{0y} = 0$ (and $t = 1.0$ s). This leads to $v_y = -9.8$ m/s and ultimately to $\sqrt{10^2 + (-18)^2} = 14$ m/s.

(c) With v_{0y} denoting the y-component of velocity one second before the top of the trajectory – as in part (a) – then we have $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$ where $t = 1.0$ s. This yields $y_0 = -4.9$ m. Alternatively, Eq. 2-18 could have been used, with $v_y = 0$ to the same end. The x_0 value more simply results from $x = 0 = x_0 + (10\,\text{m/s})(1.0\,\text{s})$. Thus, the coordinates (in meters) of the projectile one second before reaching maximum height is $(-10, -4.9)$.

(d) It is clear from symmetry that the coordinate one second after the maximum height is reached is $(10, -4.9)$ (in meters). But this can be verified by considering $t = 0$ at the top and using $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $y_0 = v_{0y} = 0$ and $t = 1$ s. And by using $x - x_0 = (10\,\text{m/s})(1.0\,\text{s})$ where $x_0 = 0$. Thus, $x = 10$ m and $y = -4.9$ m is obtained.

81. With $g_B = 9.8128$ m/s^2 and $g_M = 9.7999$ m/s^2, we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B}\left(\frac{g_B}{g_M} - 1\right)$$

which becomes

$$R_M - R_B = R_B\left(\frac{9.8128}{9.7999} - 1\right)$$

and yields (upon substituting $R_B = 8.09$ m) $R_M - R_B = 0.01$ m.

82. (a) Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields $v_0 = 255.5 \approx 2.6 \times 10^2$ m/s for $x = 9400$ m, $y = -3300$ m, and $\theta_0 = 35°$.

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0\cos\theta_0} = \frac{9400}{255.5\cos 35°} = 45 \text{ s} .$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

83. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for $y = h$:

$$h = y_0 + v_0\sin\theta_0 - \frac{1}{2}gt^2$$

which yields $h = 51.8$ m for $y_0 = 0$, $v_0 = 42$ m/s, $\theta_0 = 60°$ and $t = 5.5$ s.

(b) The horizontal motion is steady, so $v_x = v_{0x} = v_0\cos\theta_0$, but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0\cos\theta_0)^2 + (v_0\sin\theta_0 - gt)^2} = 27 \text{ m/s} .$$

(c) We use Eq. 4-24 with $v_y = 0$ and $y = H$:

$$H = \frac{(v_0\sin\theta_0)^2}{2g} = 67.5 \text{ m} .$$

84. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x\tan\theta_0 - \frac{gx^2}{2\left(v_0\cos\theta_0\right)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0 \ .$$

Thus, with $v_0 = 3 \times 10^6$ m/s and $x = 1$ m, we obtain $y = -5.4 \times 10^{-13}$ m which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that $|y|$ decreases as v_0 is reduced.

85. (a) Using the same coordinate system assumed in Eq. 4-21, we obtain the time of flight

$$t = \frac{\Delta x}{v_0\cos\theta_0} = \frac{20}{15\cos 35^\circ} = 1.63 \text{ s} \ .$$

(b) At that moment, its height of above the ground (taking $y_0 = 0$) is

$$y = \left(v_0\sin\theta_0\right)t - \frac{1}{2}gt^2 = 1.02 \text{ m} \ .$$

Thus, the ball is 18 cm below the center of the circle; since the circle radius is 15 cm, we see that it misses it altogether.

(c) The horizontal component of velocity (at $t = 1.63$ s) is the same as initially:

$$v_x = v_{0x} = v_0\cos\theta_0 = 15\cos 35^\circ = 12.3 \text{ m/s} \ .$$

The vertical component is given by Eq. 4-23:

$$v_y = v_0\sin\theta_0 - gt = 15\sin 35^\circ - (9.8)(1.63) = -7.3 \text{ m/s} \ .$$

Thus, the magnitude of its speed at impact is $\sqrt{v_x^2 + v_y^2} = 14.3$ m/s.

(d) As we saw in the previous part, the sign of v_y is negative, implying that it is now heading down (after reaching its max height).

86. (a) From Eq. 4-22 (with $\theta_0 = 0$), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45)}{9.8}} = 3.03 \text{ s} \ .$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250)(3.03) = 758 \text{ m} \ .$$

(c) And from Eq. 4-23, we find

$$|v_y| = gt = (9.80)(3.03) = 29.7 \text{ m/s} \ .$$

87. Using the same coordinate system assumed in Eq. 4-25, we find x for the elevated cannon from

$$y = x\tan\theta_0 - \frac{gx^2}{2\left(v_0\cos\theta_0\right)^2} \quad \text{where } y = -30 \text{ m.}$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0\cos\theta_0 \left(\frac{v_0\sin\theta_0 + \sqrt{\left(v_0\sin\theta_0\right)^2 - 2gy}}{g}\right)$$

which yields $x = 715$ m for $v_0 = 82$ m/s (from Sample Problem 4-7) and $\theta_0 = 45^\circ$. This is 29 m longer than the 686 m found in that Sample Problem. The "9" in 29 m is not reliable, considering the low level of precision in the given data.

88. (a) With $r = 0.15$ m and $a = 3.0 \times 10614$ m/s^2, Eq. 4-32 gives

$$v = \sqrt{ra} = 6.7 \times 10^6 \text{ m/s} .$$

(b) The period is given by Eq. 4-33:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7} \text{ s} .$$

89. The type of acceleration involved in steady-speed circular motion is the centripetal acceleration $a = v^2/r$ which is at each moment directed towards the center of the circle. The radius of the circle is $r = 12^2/3 = 48$ m. Thus, the car is at the present moment 48 m west of the center of its circular path; this is equally true in part (a) and part (b).

90. (a) With $v = c/10 = 3 \times 10^7$ m/s and $a = 20g = 196$ m/s^2, Eq. 4-32 gives

$$r = \frac{v^2}{a} = 4.6 \times 10^{12} \text{ m} .$$

(b) The period is given by Eq. 4-33:

$$T = \frac{2\pi r}{v} = 9.6 \times 10^5 \text{ s} .$$

Thus, the time to make a quarter-turn is $T/4 = 2.4 \times 10^5$ s or about 2.8 days.

91. (a) Using the same coordinate system assumed in Eq. 4-21 and Eq. 4-22 (so that $\theta_0 = -20.0°$), we use $v_0 = 15.0$ m/s and find the horizontal displacement of the ball at $t = 2.30$ s:

$$\Delta x = (v_0 \cos\theta_0)\, t = 32.4 \text{ m} .$$

(b) And we find the vertical displacement:

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2} g t^2 = -37.7 \text{ m} .$$

92. This is a classic problem involving two-dimensional relative motion; see §4-9. The steps in Sample Problem 4-11 in the textbook are similar to those used here. We align our coordinates so that *east* corresponds to $+x$ and *north* corresponds to $+y$. We write the vector addition equation as $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$. We have $\vec{v}_{WG} = (2.0 \angle 0°)$ in the magnitude-angle notation (with the unit m/s understood), or $\vec{v}_{WG} = 2.0\,\hat{\imath}$ in unit-vector notation. We also have $\vec{v}_{BW} = (8.0 \angle 120°)$ where we have been careful to phrase the angle in the 'standard' way (measured counterclockwise from the $+x$ axis), or $\vec{v}_{BW} = -4.0\,\hat{\imath} + 6.9\,\hat{\jmath}$.

(a) We can solve the vector addition equation for $\vec{v}_{BG}$:

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG} = (2.0 \angle 0°) + (8.0 \angle 120°) = (7.2 \angle 106°)$$

which is very efficiently done using a vector capable calculator in polar mode. Thus $|\vec{v}_{BG}| = 7.2$ m/s, and its direction is 16° west of north, or 74° north of west.

(b) The velocity is constant, and we apply $y - y_0 = v_y t$ in a reference frame. Thus, in the *ground* reference frame, we have $200 = 7.2 \sin(106°)t \;\rightarrow\; t = 29$ s. Note: if a student obtains "28 s", then the student has probably neglected to take the y component properly (a common mistake).

93. The topic of relative motion (with constant velocity motion) in a two-dimensional setting is covered in §4-9. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being $\vec{v}_{PG}$ (east), another leg being $\vec{v}_{AG}$ (magnitude = 20, direction = south), and the hypotenuse being $\vec{v}_{PA}$ (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$|\vec{v}_{PA}| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \implies 70 = \sqrt{|\vec{v}_{PG}|^2 + 20^2}$$

which is easily solved for the ground speed: $|\vec{v}_{PG}| = 67$ km/h.

94. Our coordinate system has $\hat{\text{i}}$ pointed east and $\hat{\text{j}}$ pointed north. All distances are in kilometers, times in hours, and speeds in km/h. The first displacement is $\vec{r}_{AB} = 483\,\hat{\text{i}}$ and the second is $\vec{r}_{BC} = -966\,\hat{\text{j}}$.

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = 483\,\hat{\text{i}} - 966\,\hat{\text{j}} \longrightarrow (1080 \angle -63.4^\circ)$$

where we have expressed the result in magnitude-angle notation in the last step. We observe that the angle can be alternatively expressed as 63.4° south of east, or 26.6° east of south.

(b) Dividing the magnitude of $\vec{r}_{AC}$ by the total time (2.25 h) gives the magnitude of $\vec{v}_{\text{avg}}$ and its direction is the same as in part (a). Thus, $\vec{v}_{\text{avg}} = (480 \angle -63.4^\circ)$ in magnitude-angle notation (with km/h understood).

(c) Assuming the AB trip was a straight one, and similarly for the BC trip, then $|\vec{r}_{AB}|$ is the distance traveled during the AB trip, and $|\vec{r}_{BC}|$ is the distance traveled during the BC trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 + 966}{2.25} = 644 \text{ km/h} .$$

95. We take the initial (x, y) specification to be $(0.000, 0.762)$ m, and the positive x direction to be towards the "green monster." The components of the initial velocity are $(33.53 \angle 55^\circ) \rightarrow (19.23, 27.47)$ m/s.

(a) With $t = 5.00$ s, we have $x = x_0 + v_x t = 96.2$ m.

(b) At that time, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 15.59$ m, which is 4.31 m above the wall.

(c) The moment in question is specified by $t = 4.50$ s. At that time, $x - x_0 = (19.23)(4.5) = 86.5$ m, and $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 = 25.1$ m.

96. The displacement of the one-way trip is the same as the displacement, which has magnitude $D = 4350$ km for the flight (we are in a frame of reference that rotates with the earth). The velocity of the flight relative to the earth is

$$\vec{v}_{fe} = \vec{va} + \vec{ae}$$

where $\vec{ae}$ is the velocity of the (eastward) jet stream (with magnitude $v > 0$), and $\vec{ae}$ is the velocity of the plane relative to the air (with magnitude $u = 966$ m/s). And the magnitudes of the eastward flight velocity (relative to earth) and of the westward flight velocity (primed) are, respectively,

$$|\vec{v}_{fe}| = \frac{D}{t} \quad \text{and} \quad \left|\vec{v}\,'_{fe}\right| = \frac{D}{t'} .$$

The time difference (5/6 of an hour) is therefore

$$\begin{aligned} t' - t &= \frac{D}{\left|\vec{v}\,'_{fe}\right|} - \frac{D}{|\vec{v}_{fe}|} \\ \Delta t &= \frac{D}{u - v} - \frac{D}{u + v} . \end{aligned}$$

Using the quadratic formula to solve for v, we obtain

$$v = \frac{-D + \sqrt{D^2 + u^2(\Delta t)^2}}{\Delta t} = 89 \text{ km/h} .$$

97. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2\,(x\tan\theta_0 - y)}}$$

which yields $v_0 = 23$ ft/s for $g = 32$ ft/s^2, $x = 13$ ft, $y = 3$ ft and $\theta_0 = 55°$.

98. We establish coordinates with $\hat{\imath}$ pointing to the far side of the river (perpendicular to the current) and $\hat{\jmath}$ pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is $|\vec{v}_{bw}| = u = 6.4$ km/h. Its angle, relative to the x axis is θ. With km and h as the understood units, the velocity of the water (relative to the ground) is $\vec{v}_{wg} = 3.2\hat{\jmath}$.

   (a) To reach a point "directly opposite" means that the velocity of her boat relative to ground must be $\vec{bg} = v\,\hat{\imath}$ where $v > 0$ is unknown. Thus, all $\hat{\jmath}$ components must cancel in the vector sum

$$\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$$

   which means the $u\sin\theta = -3.2$, so $\theta = \sin^{-1}(-3.2/6.4) = -30°$.

   (b) Using the result from part (a), we find $v = u\cos\theta = 5.5$ km/h. Thus, traveling a distance of $\ell = 6.4$ km requires a time of $6.4/5.5 = 1.15$ h or 69 min.

   (c) If her motion is completely along the y axis (as the problem implies) then with $v_w = 3.2$ km/h (the water speed) we have

$$t_{\text{total}} = \frac{D}{u + v_w} + \frac{D}{u - v_w} = 1.33 \text{ h}$$

   where $D = 3.2$ km. This is equivalent to 80 min.

   (d) Since

$$\frac{D}{u + v_w} + \frac{D}{u - v_w} = \frac{D}{u - v_w} + \frac{D}{u + v_w}$$

   the answer is the same as in the previous part.

   (e) The case of general θ leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = u\cos\theta\,\hat{\imath} + (u\sin\theta + v_w)\,\hat{\jmath}$$

   where the x component of $\vec{v}_{bg}$ must equal ell/t. Thus,

$$t = \frac{\ell}{u\cos\theta}$$

   which can be minimized using $dt/d\theta = 0$ (though, of course, an easier way is to appeal to either physical or mathematical intuition – concluding that the shortest-time path should have $\theta = 0$). Then $t = 6.4/6.4 = 1.0$ h, or 60 min.

99. With $v_0 = 30$ m/s and $R = 20$ m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218\ .$$

Because $\sin(\phi) = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.6° \quad \text{and} \quad 167.4°\ .$$

Therefore, the two possible launch angles that will hit the target (in the absence of air friction and related effects) are $\theta_0 = 6.3°$ and $\theta_0 = 83.7°$. An alternative approach to this problem in terms of Eq. 4-25 (with $y = 0$ and $1/\cos^2 = 1 + \tan^2$) is possible – and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

100. (a) The time available before the train arrives at the impact spot is

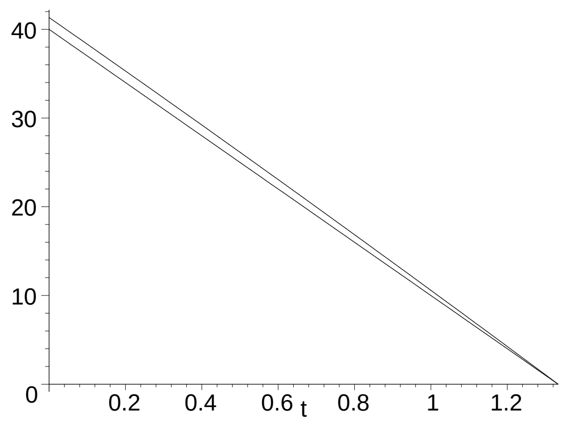
$$t_{\rm train} = \frac{40\,\text{m}}{30\,\text{m/s}} = 1.33 \text{ s}$$

(the train does not reduce its speed). We interpret the phrase "distance between the car and the center of the crossing" to refer to the distance from the front bumper of the car to that point. In which case, the car needs to travel a total distance of $\Delta x = 40 + 5 + 1.5 = 46.5$ m in order for its rear bumper and the edge of the train not to collide (the distance from the center of the train to either edge of the train is 1.5 m). With a starting velocity of $v_0 = 30$ m/s and an acceleration of $a = 1.5$ m/s^2, Eq. 2-15 leads to

$$\Delta x = v_0 t + \frac{1}{2}at^2 \implies t = \frac{-v_0 \pm \sqrt{v_0^2 + 2a\Delta x}}{a}$$

which yields (upon taking the positive root) a time $t_{\rm car} = 1.49$ s needed for the car to make it. Recalling our result for $t_{\rm train}$ we see the car doesn't have enough time available to make it across.

(b) The difference is $t_{\rm car} - t_{\rm train} = 0.16$ s. We note that at $t = t_{\rm train}$ the front bumper of the car is $v_0 t + \frac{1}{2}at^2 = 41.33$ m from where it started, which means it is 1.33 m past the center of the track (but the edge of the track is 1.5 m from the center). If the car was coming from the south, then the point P on the car impacted by the southern-most corner of the front of the train is 2.83 m behind the front bumper (or 2.17 m in front of the rear bumper). The motion of P is what is plotted below (the top graph – looking like a line instead of a parabola because the final speed of the car is not much different than its initial speed). Since the position of the train is on an entirely different axis than that of the car, we plot the distance (in meters) from P to "south" rail of the tracks (the top curve shown), and the distance of the "south" front corner of the train to the line-of-motion of the car (the bottom line shown).



101. (a) With $v_0 = 6.3$ m/s and $R = 0.40$ m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.0988 \; .$$

Because $\sin(\phi) = \sin(180° - \phi)$, there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.0988) = 5.7° \quad \text{and} \quad 174.3° \; .$$

Therefore, the two possible launch angles that will hit the target (in the absence of air friction and related effects) are $\theta_0 = 2.8°$ and $\theta_0 = 87.1°$. But the juggler is trying to achieve a visual effect by having a relatively high trajectory for the balls, so $\theta_0 = 87.1°$ is the result he should choose.

(b) We do not show the graph here. It would be very much like the higher parabola shown in Fig. 4-51.

(c) , (d) and (e) The problem requests that the student work with his graphs, here, but we – for doublechecking purposes – use Eq. 4-26 to calculate $R - 0.40$ m for $\theta_0 - 87.1° = -2°, -1°, 1°$, and $2°$. We obtain the respective values (in meters) $0.28, 0.14, -0.14$, and -0.28.

102. (First problem in **Cluster 1**)
Using the coordinate system employed in §4-5 and §4-6, we have $v_{0x} = v_x > 0$ and $v_{0y} = 0$. Also, $y_0 = h > 0$, $x_0 = 0$, $y = 0$ (when it hits the ground at $t = 3.00$)), and $x = 150$, with lengths in meters and time in seconds.

   (a) The equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ becomes $-h = -\frac{1}{2}(9.8)(3.00)^2$, so that $h = 44.1$ m.

   (b) The equation $v_y = v_{0y} - gt$ gives the y-component of the "final" velocity as $v_y = -(9.8)(3.00) = 29.4$ m/s. The x-component of velocity (which is constant) is computed from $v_x = (x - x_0)/t = 150/3.00 = 50.0$ m/s. Therefore,

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{50^2 + 29.4^2} = 58.0 \text{ m/s} .$$

103. (Second problem in **Cluster 1**)
Using the coordinate system employed in §4-5 and §4-6, we have $v_{0x} = v_0 \cos 30° > 0$ and $v_{0y} = v_0 \sin 30° > 0$. Also, $y_0 = 0$ (corresponding to the dashed line in the figure), $x_0 = 0$, $y = h > 0$ (where it lands at $t = 3.00$), and $x = 100$, with lengths in meters and time in seconds.

   (a) The x-equation determines v_0

$$x - x_0 = v_0 \cos(30)t \implies 100 = v_0(0.866)(3.00)$$

   which leads to $v_0 = 38.5$ m/s. The y-equation $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ becomes $h = (38.5)(\sin 30)(3.00) - \frac{1}{2}(9.8)(3.00)^2 = 13.6$ m.

   (b) As a byproduct of part (a)'s computation, we found $v_0 = 38.5$ m/s.

   (c) Although a somewhat easier method will be found in the energy chapter (especially Chapter 8), we will find the "final" velocity components with the methods of §4-6. We have $v_x = v_{0x} = 38.5 \cos 30 = 33.3$ m/s. And $v_y = v_{0y} - gt = 38.5 \sin 30 - (9.8)(3.00) = -10.2$ m/s. Therefore,

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(33.3)^2 + (-10.2)^2} = 34.8 \text{ m/s} .$$

104. (Third problem in **Cluster 1**)
Following the hint, we have the time-reversed problem with the ball thrown from the roof, towards the left, at 60° measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take $+x$ as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

   (a) With $y_0 = 20.0$, and $y = 0$ at $t = 4.00$, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to $v_0 = 16.9$ m/s. This plugs into the x-equation (with $x_0 = 0$ and $x = d$) to produce $d = (16.9 \cos 60°)(4.00) = 33.7$ m.

   (b) Although a somewhat easier method will be found in the energy chapter (especially Chapter 8), we will find the "final" velocity components with the methods of §4-6. Note that we're still working the time-reversed problem; this "final" $\vec{v}$ is actually the velocity with which it was thrown. We have $v_x = v_{0x} = 16.9 \cos 60° = 8.43$ m/s. And $v_y = v_{0y} - gt = 16.9 \sin 60° - (9.8)(4.00) = -24.6$ m/s. We convert from rectangular components to polar (that is, magnitude-angle) representation:

$$\vec{v} = (8.43, -24.6) \longrightarrow (26.0 \angle -71.1°) .$$

   and we now interpret our result ("undoing" the time reversal) as an initial velocity of magnitude 26 m/s with angle (up from rightward) of 71°.

105. (Fourth problem in **Cluster 1**)
Following the hint, we have the time-reversed problem with the ball thrown from the ground, towards the right, at 60° measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with $+x$ as *rightward* and with positive angles measured counterclockwise. Lengths are in meters and time is in seconds.

(a) The x-equation (with $x_0 = 0$ and $x = 25.0$) leads to $25 = (v_0 \cos 60°)(1.50)$, so that $v_0 = 33.3$ m/s. And with $y_0 = 0$, and $y = h > 0$ at $t = 1.50$, we have $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ where $v_{0y} = v_0 \sin 60°$. This leads to $h = 32.3$ m.

(b) Although a somewhat easier method will be found in the energy chapter (especially Chapter 8), we will find the "final" velocity components with the methods of §4-6. Note that we're still working the time-reversed problem; this "final" $\vec{v}$ is actually the velocity with which it was thrown. We have $v_x = v_{0x} = 33.3 \cos 60° = 16.7$ m/s. And $v_y = v_{0y} - gt = 33.3 \sin 60° - (9.8)(1.50) = 14.2$ m/s. We convert from rectangular to polar in terms of the magnitude-angle notation:

$$\vec{v} = (16.7, 14.2) \longrightarrow (21.9 \angle\ 40.4°)\ .$$

We now interpret this result ("undoing" the time reversal) as an initial velocity (from the edge of the building) of magnitude 22 m/s with angle (down from leftward) of 40°.

106. (Fifth problem in **Cluster 1**)

Let $y_0 = 1.0$ m at $x_0 = 0$ when the ball is hit. Let $y_1 = h$ (the height of the wall) and x_1 describe the point where it first rises above the wall one second after being hit; similarly, $y_2 = h$ and x_2 describe the point where it passes back down behind the wall four seconds later. And $y_f = 1.0$ m at $x_f = R$ is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that v_x is constant, we have $x_2 - x_1 = 50.0 = v_{1x}(4.00)$, which leads to $v_{1x} = 12.5$ m/s. Thus, applied to the full six seconds of motion: $x_f - x_0 = R = v_x(6.00) = 75.0$ m.

(b) We apply $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ to the motion above the wall.

$$y_2 - y_1 = 0 = v_{1y}(4.00) - \frac{1}{2}g(4.00)^2$$

leads to $v_{1y} = 19.6$ m/s. One second earlier, using $v_{1y} = v_{0y} - g(1.00)$, we find $v_{0y} = 29.4$ m/s. We convert from (x, y) to magnitude-angle (polar) representation:

$$\vec{v}_0 = (16.7, 14.2) \longrightarrow (31.9 \angle\ 66.9°)\ .$$

We interpret this result as a velocity of magnitude 32 m/s, with angle (up from rightward) of 67°.

(c) During the first 1.00 s of motion, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ yields $h = 1.0 + (29.4)(1.00) - \frac{1}{2}(9.8)(1.00)^2 = 25.5$ m.

107. (First problem in **Cluster 2**)

(a) Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and $v_y = 0$ at the target, we obtain $v_{0y} = \sqrt{2(9.8)(5.00)} = 9.90$ m/s. Since $v_0 \sin\theta_0 = v_{0y}$, with $v_0 = 12$ m/s, we find $\theta_0 = 55.6°$.

(b) Now, $v_y = v_{0y} - gt$ gives $t = 9.90/9.8 = 1.01$ s. Thus, $\Delta x = (v_0 \cos\theta_0)t = 6.85$ m.

(c) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0 \cos\theta_0 = 6.78$ m/s.

108. (Second problem in **Cluster 2**)

(a) The magnitudes of the components are equal at point A, but in terms of the coordinate system usually employed in projectile motion problems, we have $v_x > 0$ and $v_y = -v_x$. The problem gives v_0 which is related to its components by $v_0^2 = v_{0x}^2 + v_{0y}^2$ which suggests that we look at the pair of equations

$$\begin{aligned} v_y^2 &= v_{0y}^2 - 2g\Delta y \\ v_x^2 &= v_{0x}^2 \end{aligned}$$

which we can add to obtain $2v_x^2 = v_0^2 - 2g\Delta y$ (this is closely related to the type of reasoning that will be employed in some Chapter 8 problems). Therefore, we find $v_x = -v_y = 6.53$ m/s. Therefore, $\Delta y = v_y t + \frac{1}{2}gt^2$ (Eq. 2-16) can be used to find t.

$$3.00 = (-6.53)t + \frac{1}{2}(9.8)t^2 \implies t = 1.69 \text{ or } -0.36$$

from the quadratic formula or or with a polynomial solver available with some calculators. We choose the positive root: $t = 1.69$ s. Finally, we obtain

$$\Delta x = v_x t = 11.1 \text{ m} \ .$$

(b) The speed is $v = \sqrt{v_x^2 + v_y^2} = 9.23$ m/s.

109. (Third problem in **Cluster 2**)

(a) Eq. 4-25, which assumes $(x_0, y_0) = (0, 0)$, gives

$$y = 5.00 = (\tan\theta_0)x - \frac{gx^2}{2\,(v_0\cos\theta_0)^2}$$

where $x = 30.0$ (lengths are in meters and time is in seconds). Using the trig identity suggested in the problem and letting u stand for $\tan\theta_0$, we have a second-degree equation for u (its two roots leading to the values $\theta_{0\,\mathrm{min}}$, and $\theta_{0\,\mathrm{max}}$) parameterized by the initial speed v_0.

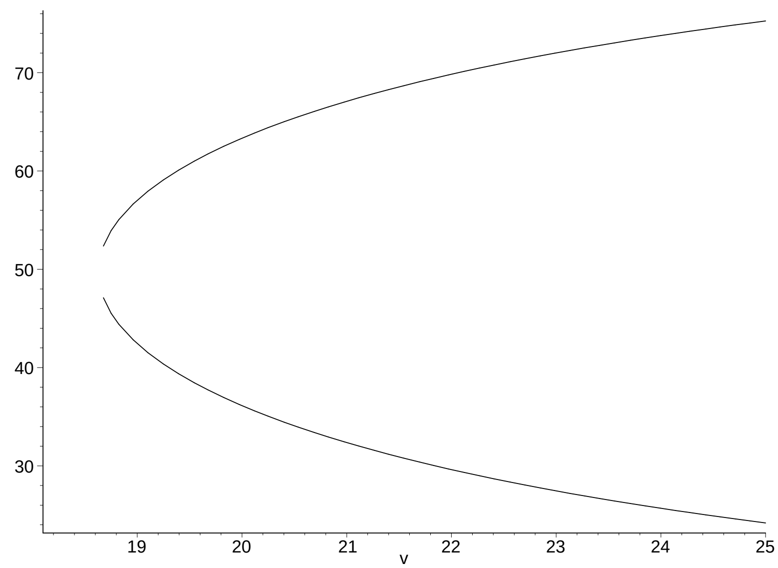
$$\frac{4410}{v_0^2}\,u^2 - 30.0u + \left(\frac{4410}{v_0^2} + 5.00\right) = 0$$

where numerical simplifications have already been made. To see these steps written with the *variables* x, y, v_0 and g made explicit, see the solution to problem 111, below. Now, we solve for u using the quadratic formula, and then find the angles:

$$\theta_0 = \tan^{-1}\left(\frac{1}{294}\,v_0^2 \pm \frac{1}{294}\sqrt{v_0^4 - 98\,v_0^2 - 86436}\right)$$

where the plus is chosen for $\theta_{0\mathrm{max}}$ and the negative is chosen for $\theta_{0\mathrm{min}}$.

(b) These angles are plotted (in degrees) versus v_0 (in m/s) as follows. There are no (real) solutions of the above equations for $18.0 \le v_0 \le 18.6$ m/s (this is further discussed in the next problem).



110. (Fourth problem in **Cluster 2**)
Following the hint in the problem (regarding *analytic* solution), we equate the square root expression, above, to zero:

$$\sqrt{v_0^4 - 98\,v_0^2 - 86436} = 0 \quad\Longrightarrow\quad v_0 = 18.6 \text{ m/s} \;.$$

That solution can be obtained either with the quadratic formula (by writing the equation, first, in terms of $w = v_0^2$) or with a polynomial solver built into many calculators; in the latter approach, this is straightforwardly handled as a fourth degree polynomial. Note that the other root ($v_0 = 15.8$ m/s) is dismissed since we are finding where the *real* solutions for angle disappear as one decreases the initial speed from roughly 20 m/s. In case this problem was assigned without assigning Problem 109 first, then this (the choice of root) might be a confusing point. Plugging $v_0 = 18.6$ m/s into

$$\theta_0 = \tan^{-1}\left(\frac{1}{294}\,v_0^2 \pm \frac{1}{294}\sqrt{v_0^4 - 98\,v_0^2 - 86436}\right)$$

(which is unambiguous since the square root factor is zero) provides the launch angle: $\theta_0 = 49.7°$ in this "critical" case.

111. (Fifth problem in **Cluster 2**)

(a) This builds directly on the solutions of the previous two problems. If we return to the solution of problem 109 without plugging in the data for x, y, and g, we obtain the following expression for the θ_0 roots.

$$\theta_0 = \tan^{-1}\left(\frac{v_0^2}{gx}\left(1 \pm \sqrt{1 - \frac{g}{v_0^2}\left(2y + \frac{gx^2}{v_0^2}\right)}\right)\right)$$

And for the "critical case" of maximum distance for a given launch-speed, we set the square root expression to zero (as in the previous problem) and solve for $x_{\max}$.

$$x_{\max} = \frac{v_0^2}{g}\sqrt{1 - \frac{2gy}{v_0^2}}$$

which one might wish to check for the "straight-up" case (where $x = 0$, and the familiar result $y_{\rm max} = \frac{1}{2}v_0^2/g$ is obtained) and for the "range" case (where $y = 0$ and this then agrees with Eq. 4-26 where $\theta_0 = 45°$). In the problem at hand, we have $y = 5.00$ m, and $v_0 = 15.0$ m/s. This leads to $x_{\rm max} = 17.2$ m.

(b) When the square root term vanishes, the expression for θ_0 becomes

$$\theta_0 = \tan^{-1}\left(\frac{v_0^2}{gx}\right) = 53.1°$$

using $x = x_{\rm max}$ from part (a).

Chapter 5

1. We apply Newton's second law (specifically, Eq. 5-2).

   (a) We find the x component of the force is

   $$F_x = ma_x = ma\cos 20^\circ = (1.00\,\text{kg})(2.00\,\text{m/s}^2)\cos 20^\circ = 1.88\ \text{N}\ .$$

   (b) The y component of the force is

   $$F_y = ma_y = ma\sin 20^\circ = (1.0\,\text{kg})(2.00\,\text{m/s}^2)\sin 20^\circ = 0.684\ \text{N}\ .$$

   (c) In unit-vector notation, the force vector (in Newtons) is

   $$\vec{F} = F_x\,\hat{\imath} + F_y\,\hat{\jmath} = 1.88\,\hat{\imath} + 0.684\,\hat{\jmath}\ .$$

2. We apply Newton's second law (Eq. 5-1 or, equivalently, Eq. 5-2). The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2\right)/m$.

   (a) In the first case

   $$\vec{F}_1 + \vec{F}_2 = \left((3.0\,\text{N})\hat{\imath} + (4.0\,\text{N})\hat{\jmath}\right) + \left((-3.0\,\text{N})\hat{\imath} + (-4.0\,\text{N})\hat{\jmath}\right) = 0$$

   so $\vec{a} = 0$.

   (b) In the second case, the acceleration $\vec{a}$ equals

   $$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\,\text{N})\,\hat{\imath} + (4.0\,\text{N})\hat{\jmath}\right) + \left((-3.0\,\text{N})\,\hat{\imath} + (4.0\,\text{N})\hat{\jmath}\right)}{2.0\,\text{kg}} = 4.0\,\hat{\jmath}\ \text{m/s}^2\ .$$

   (c) In this final situation, $\vec{a}$ is

   $$\frac{\vec{F}_1 + \vec{F}_2}{m} = \frac{\left((3.0\,\text{N})\,\hat{\imath} + (4.0\,\text{N})\hat{\jmath}\right) + \left((3.0\,\text{N})\hat{\imath} + (-4.0\,\text{N})\hat{\jmath}\right)}{2.0\,\text{kg}} = 3.0\,\hat{\imath}\ \text{m/s}^2\ .$$

3. We are only concerned with horizontal forces in this problem (gravity plays no direct role). We take East as the $+x$ direction and North as $+y$. This calculation is efficiently implemented on a vector capable calculator, using magnitude-angle notation (with SI units understood).

   $$\vec{a} = \frac{\vec{F}}{m} = \frac{(9.0\ \angle\ 0^\circ) + (8.0\ \angle\ 118^\circ)}{3.0} = (2.9\ \angle\ 53^\circ)$$

   Therefore, the acceleration has a magnitude of 2.9 m/s^2.

4. Since $\vec{v} =$ constant, we have $\vec{a} = 0$, which implies

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = m\vec{a} = 0 \ .$$

Thus, the other force must be

$$\vec{F}_2 = -\vec{F}_1 = -2\hat{\imath} + 6\hat{\jmath} \ \text{N} \ .$$

5. Since the velocity of the particle does not change, it undergoes no acceleration and must therefore be subject to zero net force. Therefore,

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0 \ .$$

Thus, the third force $\vec{F}_3$ is given by

$$\begin{aligned}\vec{F}_3 &= -\vec{F}_1 - \vec{F}_2 \\ &= -\left(2\,\hat{\imath} + 3\,\hat{\jmath} - 2\,\hat{k}\right) - \left(-5\,\hat{\imath} + 8\,\hat{\jmath} - 2\,\hat{k}\right) \\ &= 3\,\hat{\imath} - 11\,\hat{\jmath} + 4\,\hat{k}\end{aligned}$$

in Newtons. The specific value of the velocity is not used in the computation.

6. The net force applied on the chopping block is $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$, where the vector addition is done using unit-vector notation. The acceleration of the block is given by $\vec{a} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right)/m$.

   (a) The forces exerted by the three astronauts can be expressed in unit-vector notation as follows:

$$\begin{aligned}\vec{F}_1 &= 32(\cos 30^\circ\,\hat{\imath} + \sin 30^\circ\,\hat{\jmath}) \\ &= 27.7\,\hat{\imath} + 16\,\hat{\jmath} \\ \vec{F}_2 &= 55(\cos 0^\circ\,\hat{\imath} + \sin 0^\circ\,\hat{\jmath}) \\ &= 55\,\hat{\imath}\end{aligned}$$

   in Newtons, and

$$\vec{F}_3 = 41\left(\cos(-60^\circ)\hat{\imath} + \sin(-60^\circ)\hat{\jmath}\right) = 20.5\,\hat{\imath} - 35.5\,\hat{\jmath}$$

   in Newtons. The resultant acceleration of the asteroid of mass $m = 120$ kg is therefore

$$\begin{aligned}\vec{a} &= \frac{(27.7\,\hat{\imath} + 16\,\hat{\jmath}) + (55\,\hat{\imath}) + (20.5\,\hat{\imath} - 35.5\,\hat{\jmath})}{120} \\ &= 0.86\,\hat{\imath} - 0.16\,\hat{\jmath} \quad \text{m/s}^2 \ .\end{aligned}$$

   (b) The magnitude of the acceleration vector is

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{0.86^2 + (-0.16)^2} = 0.88 \ \text{m/s}^2 \ .$$

   (c) The vector $\vec{a}$ makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-0.16}{0.86}\right) = -11^\circ \ .$$

7. We denote the two forces $\vec{F}_1$ and $\vec{F}_2$. According to Newton's second law, $\vec{F}_1 + \vec{F}_2 = m\vec{a}$, so $\vec{F}_2 = m\vec{a} - \vec{F}_1$.

   (a) In unit vector notation $\vec{F}_1 = (20.0\,\text{N})\hat{\imath}$ and

$$\vec{a} = -(12\sin 30^\circ \ \text{m/s}^2)\hat{\imath} - (12\cos 30^\circ \ \text{m/s}^2)\hat{\jmath} = -(6.0\,\text{m/s}^2)\hat{\imath} - (10.4\,\text{m/s}^2)\hat{\jmath} \ .$$

   Therefore,

$$\begin{aligned}\vec{F}_2 &= (2.0\,\text{kg})\left(-6.0\,\text{m/s}^2\right)\hat{\imath} + (2.0\,\text{kg})\left(-10.4\,\text{m/s}^2\right)\hat{\jmath} - (20.0\,\text{N})\hat{\imath} \\ &= (-32\,\text{N})\hat{\imath} - (21\,\text{N})\hat{\jmath} \ .\end{aligned}$$

(b) The magnitude of $\vec{F}_2$ is

$$\left|\vec{F}_2\right| = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-32)^2 + (-21)^2} = 38 \text{ N} .$$

(c) The angle that $\vec{F}_2$ makes with the positive x axis is found from $\tan\theta = F_{2y}/F_{2x} = 21/32 = 0.656$. Consequently, the angle is either $33°$ or $33° + 180° = 213°$. Since both the x and y components are negative, the correct result is $213°$.

8. The goal is to arrive at the least magnitude of $\vec{F}_{\text{net}}$, and as long as the magnitudes of $\vec{F}_2$ and $\vec{F}_3$ are (in total) less than or equal to $|\vec{F}_1|$ then we should orient them opposite to the direction of $\vec{F}_1$ (which is the $+x$ direction).

(a) We orient both $\vec{F}_2$ and $\vec{F}_3$ in the $-x$ direction. Then, the magnitude of the net force is $50-30-20 = 0$, resulting in zero acceleration for the tire.

(b) We again orient $\vec{F}_2$ and $\vec{F}_3$ in the negative x direction. We obtain an acceleration along the $+x$ axis with magnitude

$$a = \frac{F_1 - F_2 - F_3}{m} = \frac{50\,\text{N} - 30\,\text{N} - 10\,\text{N}}{12\,\text{kg}} = 0.83 \text{ m/s}^2 .$$

(c) In this case, the forces $\vec{F}_2$ and $\vec{F}_3$ are collectively strong enough to have y components (one positive and one negative) which cancel each other and still have enough x contributions (in the $-x$ direction) to cancel $\vec{F}_1$. Since $|\vec{F}_2| = |\vec{F}_3|$, we see that the angle above the $-x$ axis to one of them should equal the angle below the $-x$ axis to the other one (we denote this angle θ). We require

$$\begin{aligned} -50\,\text{N} &= \vec{F}_{2x} + \vec{F}_{3x} \\ &= -(30\,\text{N})\cos\theta - (30\,\text{N})\cos\theta \end{aligned}$$

which leads to

$$\theta = \cos^{-1}\left(\frac{50\,\text{N}}{60\,\text{N}}\right) = 34° .$$

9. In all three cases the scale is not accelerating, which means that the two cords exert forces of equal magnitude on it. The scale reads the magnitude of either of these forces. In each case the tension force of the cord attached to the salami must be the same in magnitude as the weight of the salami because the salami is not accelerating. Thus the scale reading is mg, where m is the mass of the salami. Its value is $(11.0\,\text{kg})(9.8\,\text{m/s}^2) = 108\,\text{N}$.

10. Three vertical forces are acting on the block: the earth pulls down on the block with gravitational force 3.0 N; a spring pulls up on the block with elastic force 1.0 N; and, the surface pushes up on the block with normal force N. There is no acceleration, so

$$\sum F_y = 0 = N + (1.0 \text{ N}) + (-3.0 \text{ N})$$

yields $N = 2.0$ N. By Newton's third law, the force exerted by the block on the surface has that same magnitude but opposite direction: 2.0 N down.

11. We apply Eq. 5-12.

(a) The mass is $m = W/g = (22\,\text{N})/(9.8\,\text{m/s}^2) = 2.2\,\text{kg}$. At a place where $g = 4.9\,\text{m/s}^2$, the mass is still 2.2 kg but the gravitational force is $F_g = mg = (2.2\,\text{kg})(4.9\,\text{m/s}^2) = 11\,\text{N}$.

(b) As noted, $m = 2.2$ kg.

(c) At a place where $g = 0$ the gravitational force is zero.

(d) The mass is still 2.2 kg.

12. We use $W_p = mg_p$, where W_p is the weight of an object of mass m on the surface of a certain planet p, and g_p is the acceleration of gravity on that planet.

   (a) The weight of the space ranger on Earth is $W_e = mg_e$ which we compute to be $(75\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 7.4 \times 10^2\,\text{N}$.

   (b) The weight of the space ranger on Mars is $W_m = mg_m$ which we compute to be $(75\,\text{kg})(3.8\,\text{m/s}^2) = 2.9 \times 10^2\,\text{N}$.

   (c) The weight of the space ranger in interplanetary space is zero, where the effects of gravity are negligible.

   (d) The mass of the space ranger remains the same (75 kg) at all the locations.

13. According to Newton's second law, the magnitude of the force is given by $F = ma$, where a is the magnitude of the acceleration of the neutron. We use kinematics (Table 2-1) to find the acceleration that brings the neutron to rest in a distance d. Assuming the acceleration is constant, then $v^2 = v_0^2 + 2ad$ produces the value of a:

$$a = \frac{(v^2 - v_0^2)}{2d} = \frac{-(1.4 \times 10^7\,\text{m/s})^2}{2(1.0 \times 10^{-14}\,\text{m})} = -9.8 \times 10^{27}\,\text{m/s}^2 .$$

The magnitude of the force is consequently

$$F = m|a| = (1.67 \times 10^{-27}\,\text{kg})(9.8 \times 10^{27}\,\text{m/s}^2) = 16\text{ N} .$$

14. The child-backpack is in static equilibrium while he waits, so Newton's second law applies with $\sum \vec{F} = 0$. Since students sometimes confuse this with Newton's third law, we phrase our results carefully.

   (a) The magnitude of the normal force $\vec{N}$ exerted upward by the sidewalk is equal, in this situation, to the total weight of the child-backpack, as a result of $\sum \vec{F} = 0$. Thus, $\vec{N} = (33.5\text{ kg})(9.8\text{ m/s}^2) = 328\text{ N}$ and is directed up; this is $\vec{F}_{sc}$ – the force of the sidewalk exerted up on the child's feet. By Newton's third law, the force exerted down (at the child's feet) on the sidewalk is $\vec{F}_{cs} = 328\text{ N}$ downward.

   (b) Except for an entirely negligible gravitation attraction between the child and the concrete, there is no force exerted on the sidewalk by the child when the child is not in contact with it.

   (c) Earth pulls gravitationally on the child, and the child pulls equally in the opposite direction on Earth. This force is the previously computed weight $(29.0)(9.8) = 284$ N. The gravitational force on Earth exerted by the child is 284 N up. But the contact force exerted by the child on the sidewalk (hence, on Earth) is (see part (a)) 328 N downward. Thus, the *net* force exerted by the child on Earth is zero.

   (d) Here the answer is simply the gravitational interaction: 284 N up.

15. We note that the free-body diagram is shown in Fig. 5-18 of the text.

   (a) Since the acceleration of the block is zero, the components of the Newton's second law equation yield $T - mg\sin\theta = 0$ and $N - mg\cos\theta = 0$. Solving the first equation for the tension in the string, we find

$$T = mg\sin\theta = (8.5\,\text{kg})(9.8\,\text{m/s}^2)\sin 30^\circ = 42\text{ N} .$$

   (b) We solve the second equation in part (a) for the normal force N:

$$N = mg\cos\theta = (8.5\,\text{kg})(9.8\,\text{m/s}^2)\cos 30^\circ = 72\text{ N} .$$

(c) When the string is cut, it no longer exerts a force on the block and the block accelerates. The x component of the second law becomes $-mg\sin\theta = ma$, so the acceleration becomes

$$a = -g\sin\theta = -9.8\sin 30^\circ = -4.9$$

in SI units. The negative sign indicates the acceleration is down the plane. The magnitude of the acceleration is 4.9 m/s^2.

16. An excellent analysis of the accelerating elevator is given in Sample Problem 5-8 in the textbook.

(a) From Newton's second law

$$N - mg = ma \quad \text{where} \;\; a = a_{\text{max}} = 2.0 \text{ m/s}^2$$

we obtain $N = 590$ N upward, for $m = 50$ kg.

(b) Again, we use Newton's second law

$$N - mg = ma \quad \text{where} \;\; a = a_{\text{max}} = -3.0 \text{ m/s}^2 \; .$$

Now, we obtain $N = 340$ N upward.

(c) Returning to part (a), we use Newton's third law, and conclude that the force exerted by the passenger on the floor is $\vec{F}_{PF} = 590$ N downward.

17. (a) The acceleration is

$$a = \frac{F}{m} = \frac{20\,\text{N}}{900\,\text{kg}} = 0.022 \text{ m/s}^2 \; .$$

(b) The distance traveled in 1 day ($= 86400\,\text{s}$) is

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(0.0222\,\text{m/s}^2\right)(86400\,\text{s})^2 = 8.3 \times 10^7 \text{ m} \; .$$

(c) The speed it will be traveling is given by

$$v = at = (0.0222\,\text{m/s}^2)(86400\,\text{s}) = 1.9 \times 10^3 \text{ m/s} \; .$$

18. Some assumptions (not so much for realism but rather in the interest of using the given information efficiently) are needed in this calculation: we assume the fishing line and the path of the salmon are horizontal. Thus, the weight of the fish contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 8.7$ kg). Our $+x$ axis is in the direction of the salmon's velocity (away from the fisherman), so that its acceleration ("deceleration") is negative-valued and the force of tension is in the $-x$ direction: $\vec{T} = -T$. We use Eq. 2-16 and SI units (noting that $v = 0$).

$$v^2 = v_0^2 + 2a\Delta x \quad \Longrightarrow \quad a = -\frac{v_0^2}{2\Delta x} = -\frac{2.8^2}{2(0.11)}$$

which yields $a = -36$ m/s^2. Assuming there are no significant horizontal forces other than the tension, Eq. 5-1 leads to

$$\vec{T} = m\vec{a} \quad \Longrightarrow \quad -T = (8.7\,\text{kg})\left(-36\,\text{m/s}^2\right)$$

which results in $T = 3.1 \times 10^2$ N.

19. In terms of magnitudes, Newton's second law is $F = ma$, where F represents $|\vec{F}_{\text{net}}|$, a represents $|\vec{a}|$ (which it does not always do; note the use of a in the previous solution), and m is the (always positive) mass. The magnitude of the acceleration can be found using constant acceleration kinematics (Table 2-1). Solving $v = v_0 + at$ for the case where it starts from rest, we have $a = v/t$ (which we interpret in terms of magnitudes, making specification of coordinate directions unnecessary). The velocity is $v = (1600\,\text{km/h})(1000\,\text{m/km})/(3600\,\text{s/h}) = 444\,\text{m/s}$, so

$$F = (500\,\text{kg})\frac{444\,\text{m/s}}{1.8\,\text{s}} = 1.2 \times 10^5\,\text{N} \; .$$

20. The stopping force $\vec{F}$ and the path of the car are horizontal. Thus, the weight of the car contributes only (via Eq. 5-12) to information about its mass ($m = W/g = 1327$ kg). Our $+x$ axis is in the direction of the car's velocity, so that its acceleration ("deceleration") is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F$.

    (a) We use Eq. 2-16 and SI units (noting that $v = 0$ and $v_0 = 40(1000/3600) = 11.1$ m/s).

    $$v^2 = v_0^2 + 2a\Delta x \implies a = -\frac{v_0^2}{2\Delta x} = -\frac{11.1^2}{2(15)}$$

    which yields $a = -4.12$ m/s^2. Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

    $$\vec{F} = m\vec{a} \implies -F = (1327\,\text{kg})\left(-4.12\,\text{m/s}^2\right)$$

    which results in $F = 5.5 \times 10^3$ N.

    (b) Eq. 2-11 readily yields $t = -v_0/a = 2.7$ s.

    (c) Keeping F the same means keeping a the same, in which case (since $v = 0$) Eq. 2-16 expresses a direct proportionality between Δx and v_0^2. Therefore, doubling v_0 means quadrupling Δx. That is, the new over the old stopping distances is a factor of 4.0.

    (d) Eq. 2-11 illustrates a direct proportionality between t and v_0 so that doubling one means doubling the other. That is, the new time of stopping is a factor of 2.0 greater than the one found in part (c).

21. The acceleration of the electron is vertical and for all practical purposes the only force acting on it is the electric force. The force of gravity is negligible. We take the $+x$ axis to be in the direction of the initial velocity and the $+y$ axis to be in the direction of the electrical force, and place the origin at the initial position of the electron. Since the force and acceleration are constant, we use the equations from Table 2-1: $x = v_0 t$ and

    $$y = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)t^2 .$$

    The time taken by the electron to travel a distance x (= 30 mm) horizontally is $t = x/v_0$ and its deflection in the direction of the force is

    $$y = \frac{1}{2}\frac{F}{m}\left(\frac{x}{v_0}\right)^2 = \frac{1}{2}\left(\frac{4.5 \times 10^{-16}}{9.11 \times 10^{-31}}\right)\left(\frac{30 \times 10^{-3}}{1.2 \times 10^7}\right)^2 = 1.5 \times 10^{-3}\ \text{m} .$$

22. The stopping force $\vec{F}$ and the path of the passenger are horizontal. Our $+x$ axis is in the direction of the passenger's motion, so that the passenger's acceleration ("deceleration") is negative-valued and the stopping force is in the $-x$ direction: $\vec{F} = -F$. We use Eq. 2-16 and SI units (noting that $v = 0$ and $v_0 = 53(1000/3600) = 14.7$ m/s).

    $$v^2 = v_0^2 + 2a\Delta x \implies a = -\frac{v_0^2}{2\Delta x} = -\frac{14.7^2}{2(0.65)}$$

    which yields $a = -167$ m/s^2. Assuming there are no significant horizontal forces other than the stopping force, Eq. 5-1 leads to

    $$\vec{F} = m\vec{a} \implies -F = (41\,\text{kg})\left(-167\,\text{m/s}^2\right)$$

    which results in $F = 6.8 \times 10^3$ N.

23. We note that The rope is 22° from vertical – and therefore 68° from horizontal.

(a) With $T = 760$ N, then its components are

$$\vec{T} = T\cos 68^\circ\,\hat{\imath} + T\sin 68^\circ\,\hat{\jmath} = 285\,\hat{\imath} + 705\,\hat{\jmath}$$

understood to be in newtons.

(b) No longer in contact with the cliff, the only other force on Tarzan is due to earth's gravity (his weight). Thus,

$$\vec{F}_{\rm net} = \vec{T} + \vec{W} = 285\,\hat{\imath} + 705\,\hat{\jmath} - 820\,\hat{\jmath} = 285\,\hat{\imath} - 115\,\hat{\jmath}$$

again understood to be in newtons.

(c) In a manner that is efficiently implemented on a vector capable calculator, we convert from rectangular (x, y) components to magnitude-angle notation:

$$\vec{F}_{\rm net} = (285, -115) \;\longrightarrow\; (307\;\angle - 22^\circ)$$

so that the net force has a magnitude of 307 N.

(d) The angle (see part (c)) has been found to be 22° below horizontal (away from cliff)

(e) Since $\vec{a} = \vec{F}_{\rm net}\,/\,m$ where $m = W/g = 84$ kg, we obtain $\vec{a} = 3.67\ {\rm m/s^2}$

(f) Eq. 5-1 requires that $\vec{a} \parallel \vec{F}_{\rm net}$ so that it is also directed at 22° below horizontal (away from cliff).

24. The analysis of coordinates and forces (the free-body diagram) is exactly as in the textbook in Sample Problem 5-7 (see Fig. 5-18(b) and (c)).

(a) Constant velocity implies zero acceleration, so the "uphill" force must equal (in magnitude) the "downhill" force: $T = mg\sin\theta$. Thus, with $m = 50$ kg and $\theta = 8.0^\circ$, the tension in the rope equals 68 N.

(b) With an uphill acceleration of 0.10 m/s^2, Newton's second law (applied to the x axis shown in Fig. 5-18(b)) yields

$$T - mg\sin\theta = ma \quad\Longrightarrow\quad T - (50)(9.8)\sin 8.0^\circ = (50)(0.10)$$

which leads to $T = 73$ N.

25. (a) Since friction is negligible the force of the girl is the only horizontal force on the sled. The vertical forces (the force of gravity and the normal force of the ice) sum to zero. The acceleration of the sled is

$$a_s = \frac{F}{m_s} = \frac{5.2\,\rm N}{8.4\,\rm kg} = 0.62\ {\rm m/s^2}\ .$$

(b) According to Newton's third law, the force of the sled on the girl is also 5.2 N. Her acceleration is

$$a_g = \frac{F}{m_g} = \frac{5.2\,\rm N}{40\,\rm kg} = 0.13\ {\rm m/s^2}\ .$$

(c) The accelerations of the sled and girl are in opposite directions. Assuming the girl starts at the origin and moves in the $+x$ direction, her coordinate is given by $x_g = \frac{1}{2}a_g t^2$. The sled starts at $x_0 = 1.5$ m and moves in the $-x$ direction. Its coordinate is given by $x_s = x_0 - \frac{1}{2}a_s t^2$. They meet when

$$\begin{aligned} x_g &= x_s \\ \frac{1}{2}a_g t^2 &= x_0 - \frac{1}{2}a_s t^2\ . \end{aligned}$$

This occurs at time

$$t = \sqrt{\frac{2x_0}{a_g + a_s}}\ .$$

By then, the girl has gone the distance

$$x_g = \frac{1}{2}a_g t^2 = \frac{x_0 a_g}{a_g + a_s} = \frac{(15)(0.13)}{0.13 + 0.62} = 2.6 \text{ m} .$$

26. We assume the direction of motion is $+x$ and assume the refrigerator starts from rest (so that the speed being discussed is the velocity v which results from the process). The only force along the x axis is the x component of the applied force $\vec{F}$.

    (a) Since $v_0 = 0$, the combination of Eq. 2-11 and Eq. 5-2 leads simply to

    $$F_x = m\left(\frac{v}{t}\right) \implies v_i = \left(\frac{F\cos\theta_i}{m}\right)t$$

    for $i = 1$ or 2 (where we denote $\theta_1 = 0$ and $\theta_2 = \theta$ for the two cases). Hence, we see that the ratio v_2 over v_1 is equal to $\cos\theta$.

    (b) Since $v_0 = 0$, the combination of Eq. 2-16 and Eq. 5-2 leads to

    $$F_x = m\left(\frac{v^2}{2\Delta x}\right) \implies v_i = \sqrt{2\left(\frac{F\cos\theta_i}{m}\right)\Delta x}$$

    for $i = 1$ or 2 (again, $\theta_1 = 0$ and $\theta_2 = \theta$ is used for the two cases). In this scenario, we see that the ratio v_2 over v_1 is equal to $\sqrt{\cos\theta}$.

27. We choose up as the $+y$ direction, so $\vec{a} = -3.00 \text{ m/s}^2\,\hat{\text{j}}$ (which, without the unit-vector, we denote as a since this is a 1-dimensional problem in which Table 2-1 applies). From Eq. 5-12, we obtain the firefighter's mass: $m = W/g = 72.7$ kg.

    (a) We denote the force exerted by the pole on the firefighter $\vec{F}_{\text{f p}} = F\,\hat{\text{j}}$ and apply Eq. 5-1 (using SI units).

    $$\begin{aligned} \vec{F}_{\text{net}} &= m\vec{a} \\ F - F_g &= ma \\ F - 712 &= (72.7)(-3.00) \end{aligned}$$

    which yields $F = 494$ N. The fact that the result is positive means $\vec{F}_{\text{f p}}$ points up.

    (b) Newton's third law indicates $\vec{F}_{\text{f p}} = -\vec{F}_{\text{p f}}$, which leads to the conclusion that $\vec{F}_{\text{p f}} = 494$ N down.

28. The coordinate choices are made in the problem statement.

    (a) We write the velocity of the armadillo as $\vec{v} = v_x\,\hat{\text{i}} + v_y\,\hat{\text{j}}$. Since there is no net force exerted on it in the x direction, the x component of the velocity of the armadillo is a constant: $v_x = 5.0$ m/s. In the y direction at $t = 3.0$ s, we have (using Eq. 2-11 with $v_{0\,y} = 0$)

    $$v_y = v_{0\,y} + a_y t = v_{0\,y} + \left(\frac{F_y}{m}\right)t = \left(\frac{17}{12}\right)(3.0) = 4.3$$

    in SI units. Thus

    $$\vec{v} = 5.0\,\hat{\text{i}} + 4.3\,\hat{\text{j}} \quad \text{m/s} .$$

    (b) We write the position vector of the armadillo as $\vec{r} = r_x\,\hat{\text{i}} + r_y\,\hat{\text{j}}$. At $t = 3.0$ s we have $r_x = (5.0)(3.0) = 15$ and (using Eq. 2-15 with $v_{0\,y} = 0$)

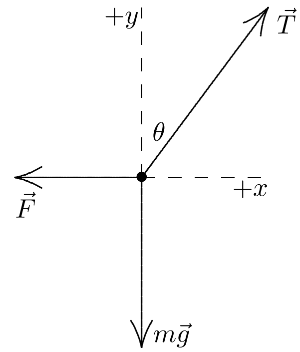
    $$r_y = v_{0\,y}\,t + \frac{1}{2}a_y\,t^2 = \frac{1}{2}\left(\frac{F_y}{m}\right)t^2 = \frac{1}{2}\left(\frac{17}{12}\right)(3.0)^2 = 6.4$$

    in SI units. The position vector at $t = 3.0$ s is therefore

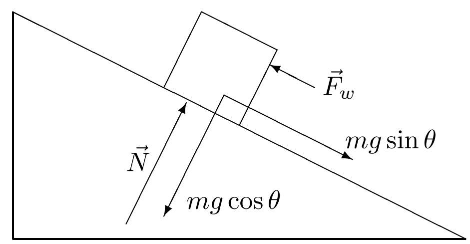
    $$\vec{r} = 15\,\hat{\text{i}} + 6.4\,\hat{\text{j}} \quad \text{m} .$$

29. The solutions to parts (a) and (b) have been combined here. The free-body diagram is shown below, with the tension of the string $\vec{T}$, the force of gravity $m\vec{g}$, and the force of the air $\vec{F}$. Our coordinate system is shown. The x component of the net force is $T\sin\theta - F$ and the y component is $T\cos\theta - mg$, where $\theta = 37°$.

Since the sphere is motionless the net force on it is zero. We answer the questions in the reverse order. Solving $T\cos\theta - mg = 0$ for the tension, we obtain $T = mg/\cos\theta = (3.0 \times 10^{-4})(9.8)/\cos 37° = 3.7 \times 10^{-3}$ N. Solving $T\sin\theta - F = 0$ for the force of the air: $F = T\sin\theta = (3.7\times 10^{-3})\sin 37° = 2.2\times 10^{-3}$ N.



30. We label the 40 kg skier "m" which is represented as a block in the figure shown. The force of the wind is denoted $\vec{F}_w$ and might be either "uphill" or "downhill" (it is shown uphill in our sketch). The incline angle θ is 10°. The $+x$ direction is downhill.



(a) Constant velocity implies zero acceleration; thus, application of Newton's second law along the x axis leads to

$$mg\sin\theta - F_w = 0 \quad .$$

This yields $F_w = 68$ N (uphill).

(b) Given our coordinate choice, we have $a = +1.0$ m/s^2. Newton's second law

$$mg\sin\theta - F_w = ma$$

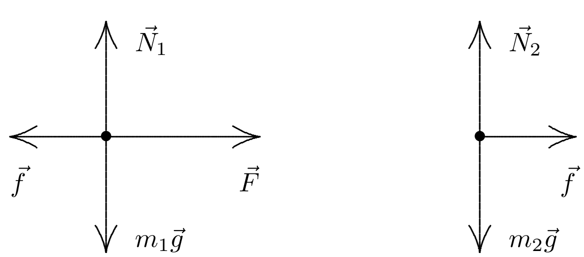
now leads to $F_w = 28$ N (uphill).

(c) Continuing with the forces as shown in our figure, the equation

$$mg\sin\theta - F_w = ma$$

will lead to $F_w = -12$ N when $a = +2.0$ m/s^2. This simply tells us that the wind is opposite to the direction shown in our sketch; in other words, $\vec{F}_w = 12$ N *downhill.*

31. The free-body diagrams for part (a) are shown below. $\vec{F}$ is the applied force and $\vec{f}$ is the force exerted by block 1 on block 2. We note that $\vec{F}$ is applied directly to block 1 and that block 2 exerts the force $-\vec{f}$ on block 1 (taking Newton's third law into account).



(a) Newton's second law for block 1 is $F - f = m_1 a$, where a is the acceleration. The second law for block 2 is $f = m_2 a$. Since the blocks move together they have the same acceleration and the same symbol is used in both equations. From the second equation we obtain the expression $a = f/m_2$, which we substitute into the first equation to get $F - f = m_1 f/m_2$. Therefore,

$$f = \frac{Fm_2}{m_1 + m_2} = \frac{(3.2\,\mathrm{N})(1.2\,\mathrm{kg})}{2.3\,\mathrm{kg} + 1.2\,\mathrm{kg}} = 1.1\,\mathrm{N}\ .$$

(b) If $\vec{F}$ is applied to block 2 instead of block 1 (and in the opposite direction), the force of contact between the blocks is

$$f = \frac{Fm_1}{m_1 + m_2} = \frac{(3.2\,\mathrm{N})(2.3\,\mathrm{kg})}{2.3\,\mathrm{kg} + 1.2\,\mathrm{kg}} = 2.1\,\mathrm{N}\ .$$

(c) We note that the acceleration of the blocks is the same in the two cases. In part (a), the force f is the only horizontal force on the block of mass m_2 and in part (b) f is the only horizontal force on the block with $m_1 > m_2$. Since $f = m_2 a$ in part (a) and $f = m_1 a$ in part (b), then for the accelerations to be the same, f must be larger in part (b).

32. The additional "apparent weight" experienced during upward acceleration is well treated in Sample Problem 5-8. The discussion in the textbook surrounding Eq. 5-13 is also relevant to this.

(a) When $\vec{F}_{\mathrm{net}} = 3F - mg = 0$, we have

$$F = \frac{1}{3}mg = \frac{1}{3}(1400\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right) = 4.6 \times 10^3\ \mathrm{N}$$

for the force exerted by each bolt on the engine.

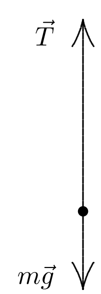
(b) The force on each bolt now satisfies $3F - mg = ma$, which yields

$$F = \frac{1}{3}m(g + a) = \frac{1}{3}(1400)(9.8 + 2.6) = 5.8 \times 10^3\ \mathrm{N}\ .$$

33. The free-body diagram is shown below. $\vec{T}$ is the tension of the cable and $m\vec{g}$ is the force of gravity. If the upward direction is positive, then Newton's second law is $T - mg = ma$, where a is the acceleration.

Thus, the tension is $T = m(g + a)$. We use constant acceleration kinematics (Table 2-1) to find the acceleration (where $v = 0$ is the final velocity, $v_0 = -12\,\mathrm{m/s}$ is the initial velocity, and $y = -42\,\mathrm{m}$ is the coordinate at the stopping point). Consequently, $v^2 = v_0^2 + 2ay$ leads to $a = -v_0^2/2y = -(-12)^2/2(-42) = 1.71\,\mathrm{m/s^2}$. We now return to calculate the tension:

$$\begin{aligned} T &= m(g + a) \\ &= (1600\,\mathrm{kg})(9.8\,\mathrm{m/s^2} + 1.71\,\mathrm{m/s^2}) \\ &= 1.8 \times 10^4\ \mathrm{N}\ . \end{aligned}$$



34. First, we consider all the penguins (1 through 4, counting left to right) as one system, to which we apply Newton's second law:

$$\begin{aligned} F_{\mathrm{net}} &= (m_1 + m_2 + m_3 + m_4)a \\ 222\,\mathrm{N} &= (20\,\mathrm{kg} + 15\,\mathrm{kg} + m_3 + 12\,\mathrm{kg})a\ . \end{aligned}$$

Second, we consider penguins 3 and 4 as one system, for which we have

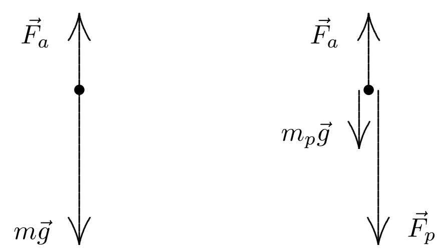
$$\begin{aligned} F'_{\text{net}} &= (m_3 + m_4)a \\ 111\,\text{N} &= (m_3 + 12\,\text{kg})a \; . \end{aligned}$$

We solve these two equations for m_3 to obtain $m_3 = 23\,\text{kg}$. The solution step can be made a little easier, though, by noting that the net force on penguins 1 and 2 is also 111 N and applying Newton's law to them as a single system to solve first for a.

35. We take the down to be the $+y$ direction.

    (a) The first diagram (below) is the free-body diagram for the person and parachute, considered as a single object with a mass of $80\,\text{kg} + 5\,\text{kg} = 85\,\text{kg}$. $\vec{F}_a$ is the force of the air on the parachute and $m\vec{g}$ is the force of gravity. Application of Newton's second law produces $mg - F_a = ma$, where a is the acceleration. Solving for F_a we find

    $$F_a = m(g - a) = (85\,\text{kg})(9.8\,\text{m/s}^2 - 2.5\,\text{m/s}^2) = 620\ \text{N} \; .$$



    (b) The second diagram (above) is the free-body diagram for the parachute alone. $\vec{F}_a$ is the force of the air, $m_p\vec{g}$ is the force of gravity, and $\vec{F}_p$ is the force of the person. Now, Newton's second law leads to $m_pg + F_p - F_a = m_pa$. Solving for F_p, we obtain

    $$F_p = m_p(a - g) + F_a = (5.0)(2.5 - 9.8) + 620 = 580\ \text{N} \; .$$

36. We apply Newton's second law first to the three blocks as a single system and then to the individual blocks. The $+x$ direction is to the right in Fig. 5-37.

    (a) With $m_{\text{sys}} = m_1 + m_2 + m_3 = 67.0$ kg, we apply Eq. 5-2 to the x motion of the system – in which case, there is only one force $\vec{T}_3 = +T_3\,\hat{\imath}$.

    $$\begin{aligned} T_3 &= m_{\text{sys}}\,a \\ 65.0\,\text{N} &= (67.0\,\text{kg})a \end{aligned}$$

    which yields $a = 0.970\ \text{m/s}^2$ for the system (and for each of the blocks individually).

    (b) Applying Eq. 5-2 to block 1, we find

    $$T_1 = m_1a = (12.0\,\text{kg})\left(0.970\ \text{m/s}^2\right) = 11.6\ \text{N} \; .$$

    (c) In order to find T_2, we can either analyze the forces on block 3 or we can treat blocks 1 and 2 as a system and examine its forces. We choose the latter.

    $$T_2 = (m_1 + m_2)\,a = (12.0 + 24.0)(0.970) = 34.9\ \text{N} \; .$$

37. We use the notation g as the acceleration due to gravity near the surface of Callisto, m as the mass of the landing craft, a as the acceleration of the landing craft, and F as the rocket thrust. We take down to be the positive direction. Thus, Newton's second law takes the form $mg - F = ma$. If the thrust is F_1 ($= 3260\,\text{N}$), then the acceleration is zero, so $mg - F_1 = 0$. If the thrust is F_2 ($= 2200\,\text{N}$), then the acceleration is a_2 ($= 0.39\,\text{m/s}^2$), so $mg - F_2 = ma_2$.

(a) The first equation gives the weight of the landing craft: $mg = F_1 = 3260\,\text{N}$.

(b) The second equation gives the mass:

$$m = \frac{mg - F_2}{a_2} = \frac{3260\,\text{N} - 2200\,\text{N}}{0.39\,\text{m/s}^2} = 2.7 \times 10^3 \text{ kg} .$$

(c) The weight divided by the mass gives the acceleration due to gravity: $g = (3260\,\text{N})/(2.7 \times 10^3\,\text{kg}) = 1.2\,\text{m/s}^2$.

38. Although the full specification of $\vec{F}_{\text{net}} = m\vec{a}$ in this situation involves both x and y axes, only the x-application is needed to find what this particular problem asks for. We note that $a_y = 0$ so that there is no ambiguity denoting a_x simply as a. We choose $+x$ to the right and $+y$ up, in Fig. 5-38. We also note that the x component of the rope's tension (acting on the crate) is $T_x = +450 \cos 38° = 355$ N, and the resistive force (pointing in the $-x$ direction) has magnitude $f = 125$ N.

(a) Newton's second law leads to

$$T_x - f = ma \implies a = \frac{355 - 125}{310} = 0.74 \text{ m/s}^2 .$$

(b) In this case, we use Eq. 5-12 to find the mass: $m = W/g = 31.6$ kg. Now, Newton's second law leads to

$$T_x - f = ma \implies a = \frac{355 - 125}{31.6} = 7.3 \text{ m/s}^2 .$$

39. The force diagrams in Fig. 5-18 are helpful to refer to. In adapting Fig. 5-18(b) to this problem, the normal force $\vec{N}$ and the tension $\vec{T}$ should be labeled $F_{\text{m,r}_y}$ and $F_{\text{m,r}_x}$, respectively, and thought of as the y and x components of the force $\vec{F}_{\text{m,r}}$ exerted by the motorcycle on the rider. We adopt the coordinates used in Fig. 5-18 and note that they are not the usual horizontal and vertical axes.

(a) Since the net force equals ma, then the magnitude of the net force on the rider is $(60.0\,\text{kg})(3.0\,\text{m/s}^2) = 1.8 \times 10^2$ N.

(b) We apply Newton's second law to the x axis:

$$F_{\text{m,r}_x} - mg \sin\theta = ma$$

where $m = 60.0$ kg, $a = 3.0$ m/s^2, and $\theta = 10°$. Thus, $F_{\text{m,r}_x} = 282$ N. Applying it to the y axis (where there is no acceleration), we have

$$F_{\text{m,r}_y} - mg \cos\theta = 0$$

which produces $F_{\text{m,r}_y} = 579$ N. Using the Pythagorean theorem, we find

$$\sqrt{F_{\text{m,r}_x}^2 + F_{\text{m,r}_y}^2} = 644 \text{ N} .$$

Now, the magnitude of the force exerted on the rider by the motorcycle is the same magnitude of force exerted by the rider on the motorcycle, so the answer is 6.4×10^2 N.

40. Referring to Fig. 5-10(c) is helpful. In this case, viewing the man-rope-sandbag as a system means that we should be careful to choose a consistent positive direction of motion (though there are other ways to proceed – say, starting with individual application of Newton's law to each mass). We take *down* as positive for the man's motion and *up* as positive for the sandbag's motion and, without ambiguity, denote their acceleration as a. The net force on the system is the difference between the weight of the man and that of the sandbag. The system mass is $m_{\text{sys}} = 85 + 65 = 150$ kg. Thus, Eq. 5-1 leads to

$$(85)(9.8) - (65)(9.8) = m_{\text{sys}}\, a$$

which yields $a = 1.3$ m/s^2. Since the system starts from rest, Eq. 2-16 determines the speed (after traveling $\Delta y = 10$ m) as follows:

$$v = \sqrt{2a\Delta y} = \sqrt{2(1.3)(10)} = 5.1 \text{ m/s} .$$

41. (a) The links are numbered from bottom to top. The forces on the bottom link are the force of gravity $m\vec{g}$, downward, and the force $\vec{F}_{2\text{on}1}$ of link 2, upward. Take the positive direction to be upward. Then Newton's second law for this link is $F_{2\text{on}1} - mg = ma$. Thus $F_{2\text{on}1} = m(a+g) = (0.100\,\text{kg})(2.50\,\text{m/s}^2 + 9.8\,\text{m/s}^2) = 1.23\,\text{N}$.

    (b) The forces on the second link are the force of gravity $m\vec{g}$, downward, the force $\vec{F}_{1\text{on}2}$ of link 1, downward, and the force $\vec{F}_{3\text{on}2}$ of link 3, upward. According to Newton's third law $\vec{F}_{1\text{on}2}$ has the same magnitude as $\vec{F}_{2\text{on}1}$. Newton's second law for the second link is $F_{3\text{on}2} - F_{1\text{on}2} - mg = ma$, so $F_{3\text{on}2} = m(a+g) + F_{1\text{on}2} = (0.100\,\text{kg})(2.50\,\text{m/s}^2 + 9.8\,\text{m/s}^2) + 1.23\,\text{N} = 2.46\,\text{N}$.

    (c) Newton's second for link 3 is $F_{4\text{on}3} - F_{2\text{on}3} - mg = ma$, so $F_{4\text{on}3} = m(a+g) + F_{2\text{on}3} = (0.100\,\text{N})(2.50\,\text{m/s}^2 + 9.8\,\text{m/s}^2) + 2.46\,\text{N} = 3.69\,\text{N}$, where Newton's third law implies $F_{2\text{on}3} = F_{3\text{on}2}$ (since these are magnitudes of the force vectors).

    (d) Newton's second law for link 4 is $F_{5\text{on}4} - F_{3\text{on}4} - mg = ma$, so $F_{5\text{on}4} = m(a+g) + F_{3\text{on}4} = (0.100\,\text{kg})(2.50\,\text{m/s}^2 + 9.8\,\text{m/s}^2) + 3.69\,\text{N} = 4.92\,\text{N}$, where Newton's third law implies $F_{3\text{on}4} = F_{4\text{on}3}$.

    (e) Newton's second law for the top link is $F - F_{4\text{on}5} - mg = ma$, so $F = m(a+g) + F_{4\text{on}5} = (0.100\,\text{kg})(2.50\,\text{m/s}^2 + 9.8\,\text{m/s}^2) + 4.92\,\text{N} = 6.15\,\text{N}$, where $F_{4\text{on}5} = F_{5\text{on}4}$ by Newton's third law.

    (f) Each link has the same mass and the same acceleration, so the same net force acts on each of them: $F_{\text{net}} = ma = (0.100\,\text{kg})(2.50\,\text{m/s}^2) = 0.25\,\text{N}$.

42. The mass of the jet is $m = W/g = 2.36 \times 10^4$ kg. Its acceleration is found from Eq. 2-16:

$$v^2 = v_0^2 + 2a\Delta x \implies a = \frac{85^2}{2(90)} = 40 \text{ m/s}^2 .$$

Thus, Newton's second law provides the needed force F from the catapult.

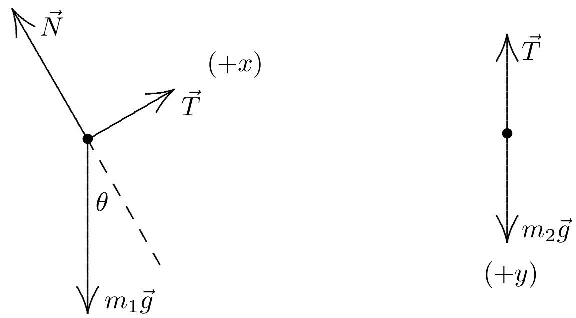
$$F + F_{\text{thrust}} = ma \implies F = \left(2.36 \times 10^4\right)(40) - 107 \times 10^3$$

which yields $F = 8.4 \times 10^5$ N.

43. The free-body diagram for each block is shown below. T is the tension in the cord and $\theta = 30°$ is the angle of the incline. For block 1, we take the $+x$ direction to be up the incline and the $+y$ direction to be in the direction of the normal force $\vec{N}$ that the plane exerts on the block. For block 2, we take the $+y$ direction to be down. In this way, the accelerations of the two blocks can be represented by the same symbol a, without ambiguity. Applying Newton's second law to the x and y axes for block 1 and to the y axis of block 2, we obtain

$$\begin{aligned} T - m_1 g \sin\theta &= m_1 a \\ N - m_1 g \cos\theta &= 0 \\ m_2 g - T &= m_2 a \end{aligned}$$

respectively. The first and third of these equations provide a simultaneous set for obtaining values of a and T. The second equation is not needed in this problem, since the normal force is neither asked for nor is it needed as part of some further computation (such as can occur in formulas for friction).



(a) We add the first and third equations above: $m_2g - m_1g\sin\theta = m_1a + m_2a$. Consequently, we find

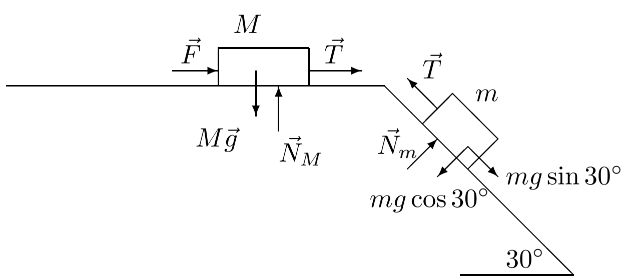
$$a = \frac{(m_2 - m_1\sin\theta)g}{m_1 + m_2} = \frac{(2.30\,\text{kg}) - 3.70\sin 30.0°)\,(9.8)}{3.70 + 2.30} = 0.735\ \text{m/s}^2\ .$$

(b) The result for a is positive, indicating that the acceleration of block 1 is indeed up the incline and that the acceleration of block 2 is vertically down.

(c) The tension in the cord is

$$T = m_1a + m_1g\sin\theta = (3.70)(0.735) + (3.70)(9.8)\sin 30° = 20.8\ \text{N}\ .$$

44. For convenience, we have labeled the 2.0 kg mass m and the 3.0 kg mass M. The $+x$ direction for m is "downhill" and the $+x$ direction for M is rightward; thus, they accelerate with the same sign.



(a) We apply Newton's second law to each block's x axis:

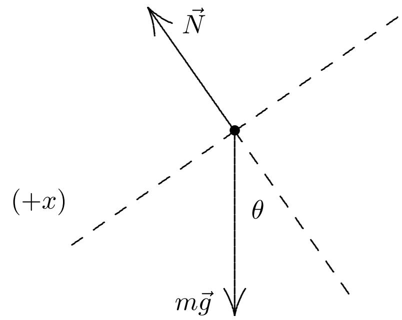
$$\begin{aligned} mg\sin 30° - T &= ma \\ F + T &= Ma \end{aligned}$$

Adding the two equations allows us to solve for the acceleration. With $F = 2.3$ N, we have $a = 1.8\ \text{m/s}^2$. We plug back in to find the tension $T = 3.1$ N.

(b) We consider the "critical" case where the F has reached the *max* value, causing the tension to vanish. The first of the equations in part (a) shows that $a = g\sin 30°$ in this case; thus, $a = 4.9\ \text{m/s}^2$. This implies (along with $T = 0$ in the second equation in part (a)) that $F = (3.0)(4.9) = 14.7$ N in the critical case.

45. The free-body diagram is shown below. $\vec{N}$ is the normal force of the

plane on the block and $m\vec{g}$ is the force of gravity on the block. We take the $+x$ direction to be down the incline, in the direction of the acceleration, and the $+y$ direction to be in the direction of the normal force exerted by the incline on the block. The x component of Newton's second law is then $mg\sin\theta = ma$; thus, the acceleration is $a = g\sin\theta$.



(a) Placing the origin at the bottom of the plane, the kinematic equations (Table 2-1) for motion along the x axis which we will use are $v^2 = v_0^2 + 2ax$ and $v = v_0 + at$. The block momentarily stops at its highest point, where $v = 0$; according to the second equation, this occurs at time $t = -v_0/a$. The position where it stops is

$$\begin{aligned} x &= -\frac{1}{2}\frac{v_0^2}{a} \\ &= -\frac{1}{2}\left(\frac{(-3.50\,\mathrm{m/s})^2}{(9.8\,\mathrm{m/s^2})\sin 32.0^\circ}\right) \\ &= -1.18 \text{ m} . \end{aligned}$$

(b) The time is

$$t = -\frac{v_0}{a} = -\frac{v_0}{g\sin\theta} = -\frac{-3.50\,\mathrm{m/s}}{(9.8\,\mathrm{m/s^2})\sin 32.0^\circ} = 0.674 \text{ s} .$$

(c) That the return-speed is identical to the initial speed is to be expected since there are no dissipative forces in this problem. In order to prove this, one approach is to set $x = 0$ and solve $x = v_0 t + \frac{1}{2}at^2$ for the total time (up and back down) t. The result is

$$t = -\frac{2v_0}{a} = -\frac{2v_0}{g\sin\theta} = -\frac{2(-3.50\,\mathrm{m/s})}{(9.8\,\mathrm{m/s^2})\sin 32.0^\circ} = 1.35\,\mathrm{s} .$$

The velocity when it returns is therefore

$$v = v_0 + at = v_0 + gt\sin\theta = -3.50 + (9.8)(1.35)\sin 32^\circ = 3.50 \text{ m/s} .$$

46. We write the length unit light-month as c·month in this solution.

(a) The magnitude of the required acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{(0.10)(3.0\times 10^8\,\mathrm{m/s})}{(3.0\,\mathrm{days})(86400\,\mathrm{s/day})} = 1.2\times 10^2 \text{ m/s}^2 .$$

(b) The acceleration in terms of g is

$$a = \left(\frac{a}{g}\right) g = \left(\frac{1.2\times 10^2\,\mathrm{m/s^2}}{9.8\,\mathrm{m/s^2}}\right) g = 12g .$$

(c) The force needed is

$$F = ma = \left(1.20\times 10^6\right)\left(1.2\times 10^2\right) = 1.4\times 10^8 \text{ N} .$$

(d) The spaceship will travel a distance $d = 0.1$ c·month during one month. The time it takes for the spaceship to travel at constant speed for 5.0 light-months is

$$t = \frac{d}{v} = \frac{5.0\,\mathrm{c}\cdot\text{months}}{0.1c} = 50 \text{ months}$$

which is about 4.2 years.

47. We take $+y$ to be up for both the monkey and the package.

(a) The force the monkey pulls downward on the rope has magnitude F. According to Newton's third law, the rope pulls upward on the monkey with a force of the same magnitude, so Newton's second law for forces acting on the monkey leads to $F - m_m g = m_m a_m$, where m_m is the mass of the monkey and a_m is its acceleration. Since the rope is massless $F = T$ is the tension in the rope. The rope pulls upward on the package with a force of magnitude F, so Newton's second law for the package is $F + N - m_p g = m_p a_p$, where m_p is the mass of the package, a_p is its acceleration, and N is the normal force exerted by the ground on it. Now, if F is the minimum force required to lift the package, then $N = 0$ and $a_p = 0$. According to the second law equation for the package, this means $F = m_p g$. Substituting $m_p g$ for F in the equation for the monkey, we solve for a_m:

$$a_m = \frac{F - m_m g}{m_m} = \frac{(m_p - m_m)\,g}{m_m} = \frac{(15 - 10)(9.8)}{10} = 4.9 \text{ m/s}^2 \ .$$

(b) As discussed, Newton's second law leads to $F - m_p g = m_p a_p$ for the package and $F - m_m g = m_m a_m$ for the monkey. If the acceleration of the package is downward, then the acceleration of the monkey is upward, so $a_m = -a_p$. Solving the first equation for F

$$F = m_p(g + a_p) = m_p(g - a_m)$$

and substituting this result into the second equation, we solve for a_m:

$$a_m = \frac{(m_p - m_m)\,g}{m_p + m_m} = \frac{(15 - 10)(9.8)}{15 + 10} = 2.0 \text{ m/s}^2 \ .$$

(c) The result is positive, indicating that the acceleration of the monkey is upward.

(d) Solving the second law equation for the package, we obtain

$$F = m_p\,(g - a_m) = (15)(9.8 - 2.0) = 120 \text{ N} \ .$$

48. The direction of motion (the direction of the barge's acceleration) is $+\hat{\imath}$, and $+\hat{\jmath}$ is chosen so that the pull $\vec{F}_{\mathrm{h}}$ from the horse is in the first quadrant. The components of the unknown force of the water are denoted simply F_x and F_y .

(a) Newton's second law applied to the barge, in the x and y directions, leads to

$$\begin{aligned}(7900\,\mathrm{N})\cos 18^\circ + F_x &= ma \\ (7900\,\mathrm{N})\sin 18^\circ + F_y &= 0\end{aligned}$$

respectively. Plugging in $a = 0.12$ m/s^2 and $m = 9500$ kg, we obtain $F_x = 6.4 \times 10^3$ N and $F_y = -2.4 \times 10^3$ N. The magnitude of the force of the water is therefore

$$F_{\text{water}} = \sqrt{F_x^2 + F_y^2} = 6.8 \times 10^3 \text{ N} \ .$$

(b) Its angle measured from $+\hat{\imath}$ is either

$$\tan^{-1}\left(\frac{F_y}{F_x}\right) = -21^\circ \quad \text{or} \quad 159^\circ.$$

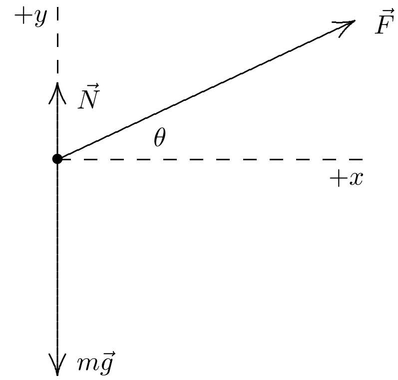
The signs of the components indicate the former is correct, so $\vec{F}_{\text{water}}$ is at 21° measured clockwise from the line of motion.

49. The force diagram (not to scale) for the block is shown below. $\vec{N}$ is the normal force exerted by the floor and $m\vec{g}$ is the force of gravity.

(a) The x component of Newton's second law is $F\cos\theta = ma$, where m is the mass of block and a is the x component of its acceleration. We obtain

$$a = \frac{F\cos\theta}{m} = \frac{(12.0\,\text{N})\cos 25.0^\circ}{5.00\,\text{kg}} = 2.18\,\text{m/s}^2 .$$

This is its acceleration provided it remains in contact with the floor. Assuming it does, we find the value of N (and if N is positive, then the assumption is true but if N is negative then the block leaves the floor). The y component of Newton's second law becomes $N + F\sin\theta - mg = 0$, so $N = mg - F\sin\theta = (5.00)(9.8) - (12.0)\sin 25.0^\circ = 43.9\,\text{N}$. Hence the block remains on the floor and its acceleration is $a = 2.18\,\text{m/s}^2$.



(b) If F is the minimum force for which the block leaves the floor, then $N = 0$ and the y component of the acceleration vanishes. The y component of the second law becomes $F\sin\theta - mg = 0$, so

$$F = \frac{mg}{\sin\theta} = \frac{(5.00)(9.8)}{\sin 25.0^\circ} = 116\ \text{N} .$$

(c) The acceleration is still in the x direction and is still given by the equation developed in part (a):

$$a = \frac{F\cos\theta}{m} = \frac{116\cos 25^\circ}{5.00} = 21.0\ \text{m/s}^2 .$$

50. The motion of the man-and-chair is positive if upward.

(a) When the man is grasping the rope, pulling with a force equal to the tension T in the rope, the total upward force on the man-and-chair due its two contact points with the rope is $2T$. Thus, Newton's second law leads to

$$2T - mg = ma$$

so that when $a = 0$, the tension is $T = 466$ N.

(b) When $a = +1.3\ \text{m/s}^2$ the equation in part (a) predicts that the tension will be $T = 527$ N.

(c) When the man is not holding the rope (instead, the co-worker attached to the ground is pulling on the rope with a force equal to the tension T in it), there is only one contact point between the rope and the man-and-chair, and Newton's second law now leads to

$$T - mg = ma$$

so that when $a = 0$, the tension is $T = 931$ N.

(d) When $a = +1.3\ \text{m/s}^2$ the equation in part (c) predicts that the tension will be $T = 1.05 \times 10^3$ N.

(e) The rope comes into contact (pulling down in each case) at the left edge and the right edge of the pulley, producing a total downward force of magnitude $2T$ on the ceiling. Thus, in part (a) this gives $2T = 931$ N.

(f) In part (b) the downward force on the ceiling has magnitude $2T = 1.05 \times 10^3$ N.

(g) In part (c) the downward force on the ceiling has magnitude $2T = 1.86 \times 10^3$ N.

(h) In part (d) the downward force on the ceiling has magnitude $2T = 2.11 \times 10^3$ N.

51. (a) A small segment of the rope has mass and is pulled down by the gravitational force of the Earth. Equilibrium is reached because neighboring portions of the rope pull up sufficiently on it. Since tension is a force *along* the rope, at least one of the neighboring portions must slope up away from the segment we are considering. Then, the tension has an upward component which means the rope sags.

(b) The only force acting with a horizontal component is the applied force $\vec{F}$. Treating the block and rope as a single object, we write Newton's second law for it: $F = (M + m)a$, where a is the acceleration and the positive direction is taken to be to the right. The acceleration is given by $a = F/(M + m)$.

(c) The force of the rope F_r is the only force with a horizontal component acting on the block. Then Newton's second law for the block gives

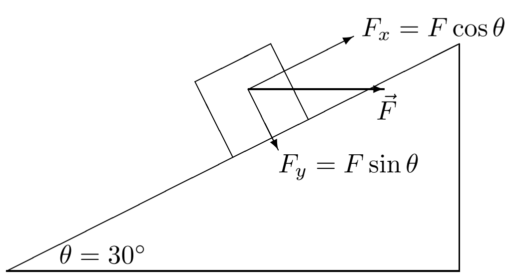
$$F_r = Ma = \frac{MF}{M + m}$$

where the expression found above for a has been used.

(d) Treating the block and half the rope as a single object, with mass $M + \frac{1}{2}m$, where the horizontal force on it is the tension T_m at the midpoint of the rope, we use Newton's second law:

$$T_m = (M + \frac{1}{2}m)a = \frac{(M + \frac{1}{2}m)F}{(M + m)} = \frac{(2M + m)F}{2(M + m)} .$$

52. The coordinate system we wish to use is shown in Fig. 5-18(c) in the textbook, so we resolve this horizontal force into appropriate components.



(a) Referring to Fig. 5-18 in the textbook, we see that Newton's second law applied to the x axis produces

$$F\cos\theta - mg\sin\theta = ma .$$

For $a = 0$, this yields $F = 566$ N.

(b) Applying Newton's second law to the y axis (where there is no acceleration), we have

$$N - F\sin\theta - mg\cos\theta = 0$$

which yields the normal force $N = 1.13 \times 10^3$ N.

53. The forces on the balloon are the force of gravity $m\vec{g}$ (down) and the force of the air $\vec{F}_a$ (up). We take the $+y$ to be up, and use a to mean the *magnitude* of the acceleration (which is not its usual use in this chapter). When the mass is M (before the ballast is thrown out) the acceleration is downward and Newton's second law is $F_a - Mg = -Ma$. After the ballast is thrown out, the mass is $M - m$ (where m is the mass of the ballast) and the acceleration is upward. Newton's second law leads to

$F_a - (M-m)g = (M-m)a$. The earlier equation gives $F_a = M(g-a)$, and this plugs into the new equation to give

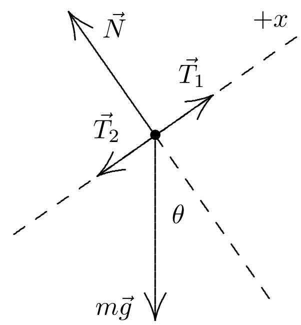
$$M(g-a) - (M-m)g = (M-m)a \implies m = \frac{2Ma}{g+a} .$$

54. The free-body diagram is shown below. Newton's second law for the mass m for the x direction leads to

$$T_1 - T_2 - mg\sin\theta = ma$$

which gives the difference in the tension in the pull cable:

$$\begin{aligned} T_1 - T_2 &= m(g\sin\theta + a) \\ &= (2800)\,(9.8\sin 35^\circ + 0.81) \\ &= 1.8 \times 10^4 \text{ N} . \end{aligned}$$



55. (a) The mass of the elevator is $m = 27800/9.8 = 2837$ kg and (with $+y$ upward) the acceleration is $a = +1.22$ m/s^2. Newton's second law leads to

$$T - mg = ma \implies T = m(g+a)$$

which yields $T = 3.13 \times 10^4$ N for the tension.

(b) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is upward). Thus (with $+y$ upward) the acceleration is now $a = -1.22$ m/s^2, so that the tension $T = m(g+a)$ turns out to be $T = 2.43 \times 10^4$ N in this case.

56. (a) The term "deceleration" means the acceleration vector is in the direction opposite to the velocity vector (which the problem tells us is downward). Thus (with $+y$ upward) the acceleration is $a = +2.4$ m/s^2. Newton's second law leads to

$$T - mg = ma \implies m = \frac{T}{g+a}$$

which yields $m = 7.3$ kg for the mass.

(b) Repeating the above computation (now to solve for the tension) with $a = +2.4$ m/s^2 will, of course, leads us right back to $T = 89$ N. Since the direction of the velocity did not enter our computation, this is to be expected.

57. The mass of the bundle is $m = 449/9.8 = 45.8$ kg and we choose $+y$ upward.

(a) Newton's second law, applied to the bundle, leads to

$$T - mg = ma \implies a = \frac{387 - 449}{45.8}$$

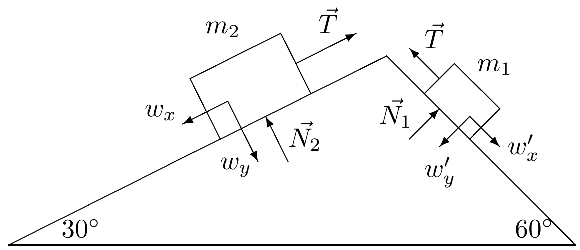
which yields $a = -1.35$ m/s^2 for the acceleration. The minus sign in the result indicates the acceleration vector points down. Any downward acceleration of magnitude greater than this is also acceptable (since that would lead to even smaller values of tension).

(b) We use Eq. 2-16 (with Δx replaced by $\Delta y = -6.1$ m). We assume $v_0 = 0$.

$$|v| = \sqrt{2a\Delta y} = \sqrt{2(-1.35)(-6.1)} = 4.1 \text{ m/s} .$$

For downward accelerations greater than 1.35 m/s^2, the speeds at impact will be larger than 4.1 m/s.

58. For convenience, we have labeled the 2.0 kg box m_1 and the 3.0 kg box m_2 – and their weights w' and w, respectively. The $+x$ axis is "downhill" for m_1 and "uphill" for m_2 (so they both accelerate with the same sign).



We apply Newton's second law to each box's x axis:

$$\begin{aligned} m_1 g \sin 60° - T &= m_1 a \\ T - m_2 g \sin 30° &= m_2 a \end{aligned}$$

Adding the two equations allows us to solve for the acceleration $a = 0.45$ m/s^2. This value is plugged back into either of the two equations to yield the tension $T = 16$ N.

59. (a) There are six legs, and the vertical component of the tension force in each leg is $T \sin\theta$ where $\theta = 40°$. For vertical equilibrium (zero acceleration in the y direction) then Newton's second law leads to

$$6T\sin\theta = mg \implies T = \frac{mg}{6\sin\theta}$$

which (expressed as a multiple of the bug's weight mg) gives roughly $0.26mg$ for the tension.

(b) The angle θ is measured from horizontal, so as the insect "straightens out the legs" θ will increase (getting closer to 90°), which causes $\sin\theta$ to increase (getting closer to 1) and consequently (since $\sin\theta$ is in the denominator) causes T to decrease.

60. (a) Choosing the direction of motion as $+x$, Eq. 2-11 gives

$$a = \frac{88.5\,\text{km/h} - 0}{6.0\,\text{s}} = 15 \text{ km/h/s} .$$

Converting to SI, this is $a = 4.1$ m/s^2.

(b) With mass $m = 2000/9.8 = 204$ kg, Newton's second law gives $\vec{F} = m\vec{a} = 836$ N in the $+x$ direction.

61. (a) Intuition readily leads to the conclusion (that the heavier block should be the hanging one, for largest acceleration). The force that "drives" the system into motion is the weight of the hanging block (gravity acting on the block on the table has no effect on the dynamics, so long as we ignore friction).

(b) In Sample Problem 5-5 (where it was assumed the m is the hanging block) Eq. 5-21 gave the acceleration. Now that we have switched $m \leftrightarrow M$ (so that now M is the hanging block) our new version of Eq. 5-21 is

$$a = \frac{M}{m+M}\,g = 6.5 \text{ m/s}^2 .$$

(c) Switching $m \leftrightarrow M$ has no effect on Eq. 5-22, which yields

$$T = \frac{mM}{m+M}\,g = 13 \text{ N} .$$

62. Making separate free-body diagrams for the helicopter and the truck, one finds there are two forces on the truck ($\vec{T}$ upward, caused by the tension, which we'll think of as that of a single cable, and $m\vec{g}$ downward, where $m = 4500$ kg) and three forces on the helicopter ($\vec{T}$ downward, $\vec{F}_{\text{lift}}$ upward, and $M\vec{g}$ downward, where $M = 15000$ kg). With $+y$ upward, then $a = +1.4$ m/s^2 for both the helicopter and the truck.

(a) Newton's law applied to the helicopter and truck separately gives

$$\begin{aligned} F_{\text{lift}} - T - Mg &= Ma \\ T - mg &= ma \end{aligned}$$

which we add together to obtain

$$F_{\text{lift}} - (M + m)g = (M + m)a \ .$$

From this equation, we find $F_{\text{lift}} = 2.2 \times 10^5$ N.

(b) From the truck equation $T - mg = ma$ we obtain $T = 5.0 \times 10^4$ N.

63. (a) With SI units understood, the net force is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 = (3.0 + (-2.0))\,\hat{\imath} + (4.0 + (-6.0))\,\hat{\jmath}$$

which yields $\vec{F}_{\text{net}} = 1.0\,\hat{\imath} - 2.0\,\hat{\jmath}$ in Newtons.

(b) Using magnitude-angle notation (especially convenient on a vector-capable calculator), the answer to part (a) becomes
$\vec{F}_{\text{net}} = (2.2\,\text{N} \angle - 63°)$.

(c) Since $\vec{F}_{\text{net}}$ is equal to $\vec{a}$ multiplied by a positive scalar (which cannot affect the direction of the vector it multiplies), then the acceleration has the same angle as the net force. The magnitude of $\vec{a}$ comes from dividing the magnitude of $\vec{F}_{\text{net}}$ by the mass ($m = 1.0$ kg). Thus, in magnitude-angle notation, the answer is $\vec{a} = (2.2\,\text{m/s}^2 \angle - 63°)$.

64. We take rightwards as the $+x$ direction. Thus, $\vec{F}_1 = 20\,\hat{\imath}$ in Newtons. In each case, we use Newton's second law $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ where $m = 2.0$ kg.

(a) If $\vec{a} = +10\,\hat{\imath}$ in SI units, then the equation above gives $\vec{F}_2 = 0$.

(b) If $\vec{a} = +20\,\hat{\imath}$ m/s^2, then that equation gives $\vec{F}_2 = 20\,\hat{\imath}$ N.

(c) If $\vec{a} = 0$, then the equation gives $\vec{F}_2 = -20\,\hat{\imath}$ N.

(d) If $\vec{a} = -10\,\hat{\imath}$ m/s^2, the equation gives $\vec{F}_2 = -40\,\hat{\imath}$ N.

(e) If $\vec{a} = -20\,\hat{\imath}$ m/s^2, the equation gives $\vec{F}_2 = -60\,\hat{\imath}$ N.

65. (a) Since the performer's weight is $(52)(9.8) = 510$ N, the rope breaks.

(b) Setting $T = 425$ N in Newton's second law (with $+y$ upward) leads to

$$T - mg = ma \quad \Longrightarrow \quad a = \frac{T}{m} - g$$

which yields $|a| = 1.6$ m/s^2.

66. The mass of the pilot is $m = 735/9.8 = 75$ kg. Denoting the upward force exerted by the spaceship (his seat, presumably) on the pilot as $\vec{F}$ and choosing upward the $+y$ direction, then Newton's second law leads to

$$F - mg_{\text{moon}} = ma \quad \Longrightarrow \quad F = (75)(1.6 + 1.0) = 195 \text{ N} \ .$$

67. With SI units understood, the net force on the box is

$$\vec{F}_{\text{net}} = (3.0 + 14\cos 30^\circ - 11)\,\hat{\imath} + (14\sin 30^\circ + 5.0 - 17)\,\hat{\jmath}$$

which yields $\vec{F}_{\text{net}} = 4.1\,\hat{\imath} - 5.0\,\hat{\jmath}$ in Newtons.

(a) Newton's second law applied to the $m = 4.0$ kg box leads to

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = 1.0\,\hat{\imath} - 1.3\,\hat{\jmath}\ \text{m/s}^2\ .$$

(b) The magnitude of $\vec{a}$ is $\sqrt{1.0^2 + (-1.3)^2} = 1.6$ m/s^2. Its angle is $\tan^{-1}(-1.3/1.0) = -50^\circ$ (that is, 50° measured clockwise from the rightward axis).

68. The net force is in the y direction, so the unknown force must have an x component that cancels the $(8.0\,\text{N})\hat{\imath}$ value of the known force, and it must also have enough y component to give the 3.0 kg object an acceleration of $(3.0\,\text{m/s}^2)\hat{\jmath}$. Thus, the magnitude of the unknown force is

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(-8.0)^2 + 9.0^2} = 12\ \text{N}\ .$$

69. We are only concerned with horizontal forces in this problem (gravity plays no direct role). Thus, $\sum \vec{F} = m\vec{a}$ reduces to $\vec{F}_{\text{avg}} = m\vec{a}$, and we see that the magnitude of the force is ma, where $m = 0.20$ kg and

$$a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

and the direction of the force is the same as that of $\vec{a}$. We take *east* as the $+x$ direction and *north* as $+y$. The acceleration is the *average* acceleration in the sense of Eq. 4-15.

(a) We find the (average) acceleration to be

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{\Delta t} = \frac{(-5.0\,\hat{\imath}) - (2.0\,\hat{\imath})}{0.50} = -14\,\hat{\imath}\ \text{m/s}^2\ .$$

Thus, the magnitude of the force is $(0.20\,\text{kg})(14\,\text{m/s}^2) = 2.8\,\text{N}$ and its direction is $-\,\hat{\imath}$ which means *west* in this context.

(b) A computation similar to the one in part (a) yields the (average) acceleration with two components, which can be expressed various ways:

$$\vec{a} = -4.0\,\hat{\imath} - 10.0\,\hat{\jmath}\ \rightarrow\ (-4.0, -10.0)\ \rightarrow\ (10.8\ \angle\ -112^\circ)$$

Therefore, the magnitude of the force is $(0.20\ \text{kg})(10.8\ \text{m/s}^2) = 2.2$ N and its direction is 112° clockwise from east – which means it is 22° west of south, stated more conventionally.

70. The "certain force" is denoted F is assumed to be the net force on the object when it gives m_1 an acceleration $a_1 = 12$ m/s^2 and when it gives m_2 an acceleration $a_2 = 3.3$ m/s^2. Thus, we substitute $m_1 = F/a_1$ and $m_2 = F/a_2$ in appropriate places during the following manipulations.

(a) Now we seek the acceleration a of an object of mass $m_2 - m_1$ when F is the net force on it. Thus,

$$a = \frac{F}{m_2 - m_1} = \frac{F}{(F/a_2) - (F/a_1)} = \frac{a_1 a_2}{a_1 - a_2}$$

which yields $a = 4.6$ m/s^2.

(b) Similarly for an object of mass $m_2 + m_1$:

$$a = \frac{F}{m_2 + m_1} = \frac{F}{(F/a_2) + (F/a_1)} = \frac{a_1 a_2}{a_1 + a_2}$$

which yields $a = 2.6$ m/s^2.

71. We mention that the textbook treats this particular arrangement of blocks and pulleys in extensive detail in Sample Problem 5-5. Using the usual coordinate system (*right* = $+x$ and *up* = $+y$) for both blocks has the important consequence that for the 3.0 kg block to have a positive acceleration ($a > 0$), block M must have a negative acceleration of the same magnitude ($-a$). Thus, applying Newton's second law to the two blocks, we have

$$\begin{aligned} T &= (3.0 \text{ kg})\left(1.0 \text{ m/s}^2\right) && \text{along } x \text{ axis} \\ T - Mg &= M\left(-1.0 \text{ m/s}^2\right) && \text{along } y \text{ axis .} \end{aligned}$$

(a) The first equation yields the tension $T = 3.0$ N.

(b) The second equation yields the mass $M = 3.0/8.8 = 0.34$ kg.

72. We take $+x$ uphill for the $m = 1.0$ kg box and $+x$ rightward for the $M = 3.0$ kg box (so the accelerations of the two boxes have the same magnitude and the same sign). The uphill force on m is F and the downhill forces on it are T and $mg\sin\theta$, where $\theta = 37°$. The only horizontal force on M is the rightward-pointed tension. Applying Newton's second law to each box, we find

$$\begin{aligned} F - T - mg\sin\theta &= ma \\ T &= Ma \end{aligned}$$

which are added to obtain $F - mg\sin\theta = (m + M)a$. This yields the acceleration

$$a = \frac{12 - (1.0)(9.8)\sin 37°}{1.0 + 3.0} = 1.53 \text{ m/s}^2 \ .$$

Thus, the tension is $T = Ma = (3.0)(1.53) = 4.6$ N.

73. (a) With $v_0 = 0$, Eq. 2-16 leads to

$$a = \frac{v^2}{2\Delta x} = \frac{\left(6.0 \times 10^6 \text{ m/s}\right)^2}{2(0.015 \text{ m})}$$

which yields 1.2×10^{15} m/s^2 for the acceleration. The force responsible for producing this acceleration is

$$F = ma = \left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.2 \times 10^{15} \text{ m/s}^2\right) = 1.1 \times 10^{-15} \text{ N} \ .$$

(b) The weight is $mg = 8.9 \times 10^{-30}$ N, many orders of magnitude smaller than the result of part (a). As a result, gravity plays a negligible role in most atomic and subatomic processes.

74. We denote the thrust as T and choose $+y$ upward. Newton's second law leads to

$$T - Mg = Ma \implies a = \frac{2.6 \times 10^5}{1.3 \times 10^4} - 9.8$$

which yields $a = 10$ m/s^2.

75. (a) The reaction force to $\vec{F}_{MW} = 180$ N west is, by Newton's third law, $\vec{F}_{WM} = 180$ N east.

(b) Applying $\vec{F} = m\vec{a}$ to the woman gives an acceleration $a = 180/45 = 4.0\,\text{m/s}^2$, directed west.

(c) Applying $\vec{F} = m\vec{a}$ to the man gives an acceleration $a = 180/90 = 2.0\,\text{m/s}^2$, directed east.

76. We note that $mg = (15)(9.8) = 147$ N.

(a) The penguin's weight is $W = 147$ N.

(b) The normal force exerted upward on the penguin by the scale is equal to the gravitational pull W on the penguin because the penguin is not accelerating. So, by Newton's second law, $N = W = 147$ N.

(c) The reading on the scale, by Newton's third law, is the reaction force to that found in part (b). Its magnitude is therefore the same: 147 N.

77. Sample Problem 5-8 has a good treatment of the forces in an elevator. We apply Newton's second law (with $+y$ up)

$$N - mg = ma$$

where $m = 100$ kg and a must be estimated from the graph (it is the instantaneous slope at the various moments).

(a) At $t = 1.8$ s, we estimate the slope to be $+1.0\,\mathrm{m/s}^2$. Thus, Newton's law yields $N \approx 1100$ N (up).

(b) At $t = 4.4$ s, the slope is zero, so $N = 980$ N (up).

(c) At $t = 6.8$ s, we estimate the slope to be $-1.7\,\mathrm{m/s}^2$. Thus, Newton's law yields $N = 810$ N (up).

78. From the reading when the elevator was at rest, we know the mass of the object is $m = 65/9.8 = 6.6$ kg. We choose $+y$ upward and note there are two forces on the object: mg downward and T upward (in the cord that connects it to the balance; T is the reading on the scale by Newton's third law).

(a) "Upward at constant speed" means constant velocity, which means no acceleration. Thus, the situation is just as it was at rest: $T = 65$ N.

(b) The term "deceleration" is used when the acceleration vector points in the direction opposite to the velocity vector. We're told the velocity is upward, so the acceleration vector points downward ($a = -2.4$ m/s^2). Newton's second law gives

$$T - mg = ma \implies T = (6.6)(9.8 - 2.4) = 49 \text{ N} .$$

79. Since $(x_0, y_0) = (0, 0)$ and $\vec{v}_0 = 6.0\,\hat{\imath}$, we have from Eq. 2-15

$$\begin{aligned} x &= (6.0)t + \frac{1}{2}a_x t^2 \\ y &= \frac{1}{2}a_y t^2 . \end{aligned}$$

These equations express uniform acceleration along each axis; the x axis points east and the y axis presumably points north (the assumption is that the figure shown in the problem is a view *from above*). Lengths are in meters, time is in seconds, and force is in newtons.

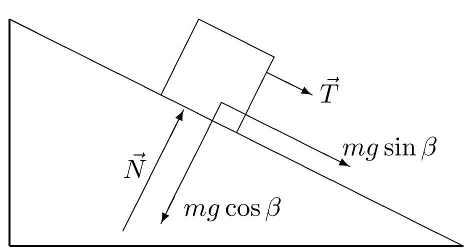
Examination of any non-zero (x, y) point will suffice, though it is certainly a good idea to check results by examining more than one. Here we will look at the $t = 4.0$ s point, at $(8.0, 8.0)$. The x equation becomes $8.0 = (6.0)(4.0) + \frac{1}{2}a_x(4.0)^2$. Therefore, $a_x = -2.0$ m/s^2. The y equation becomes $8.0 = \frac{1}{2}a_y(4.0)^2$. Thus, $a_y = 1.0$ m/s^2. The force, then, is

$$\vec{F} = m\vec{a} = -24\,\hat{\imath} + 12\,\hat{\jmath} \longrightarrow (27 \angle 153^\circ)$$

where the vector has been expressed in unit-vector and then magnitude-angle notation. Thus, the force has magnitude 27 N and is directed 63° west of north (or, equivalently, 27° north of west).

80. We label the 1.0 kg mass m and label the 2.0 kg mass M. We first analyze the forces on m.

The $+x$ direction



is "downhill"
(parallel to $\vec{T}$).

With the acceleration
($5.5\,\mathrm{m/s^2}$) in the positive x direction for m, then Newton's second law, applied to the x axis, becomes

$$T + mg\sin\beta = m(5.5\,\mathrm{m/s^2})$$

But for M, using the more familiar vertical y axis (with *up* as the positive direction), we have the acceleration in the negative direction:

$$F + T - Mg = M(-5.5\,\mathrm{m/s^2})$$

where the tension comes in as an upward force (the cord can pull, not push).

(a) From the equation for M, with $F = 6.0\,\mathrm{N}$, we find the tension $T = 2.6\,\mathrm{N}$.

(b) From the equation for m, using the result from part (a), we obtain the angle $\beta = 17°$.

81. (a) The bottom cord is only supporting a mass of 4.5 kg against gravity, so its tension is $(4.5)(9.8) = 44$ N.

(b) The top cord is supporting a total mass of 8.0 kg against gravity, so the tension there is $(8.0)(9.8) = 78$ N.

(c) In the second picture, the lowest cord supports a mass of 5.5 kg against gravity and consequently has a tension of $(5.5)(9.8) = 54$ N.

(d) The top cord, we are told, has tension 199 N which supports a total of $199/9.8 = 20.3$ kg, 10.3 of which is accounted for in the figure. Thus, the unknown mass in the middle must be $20.3 - 10.3 = 10.0$ kg, and the tension in the cord above it must be enough to support $10.0 + 5.5 = 15.5$ kg, so $T = (15.5)(9.8) = 152$ N. Another way to analyze this is to examine the forces on the 4.8 kg piece; one of the downward forces on it is this T.

82. The mass of the automobile is $17000/9.8 = 1735$ kg, so the net force has magnitude $F = (1735)(3.66) = 6.35 \times 10^2$ N.

83. (First problem in **Cluster 1**)

(a) Using the coordinate system and force resolution shown in the textbook Figure 5-18(c), we apply Newton's second law along the x axis

$$-mg\sin\theta = ma$$

where $\theta = 30.0°$. Thus, $a = -4.9\,\mathrm{m/s^2}$. The magnitude of the acceleration, then, is $4.9\,\mathrm{m/s^2}$.

(b) Applying Newton's second law along the y axis (where there is no acceleration), we have

$$N - mg\cos\theta = 0 \ .$$

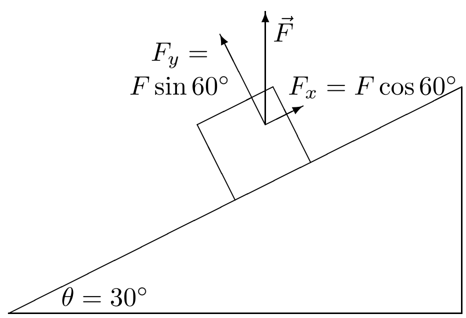
Thus, with $m = 10.0$ kg, we obtain $N = 84.9$ N.

84. (Second problem in **Cluster 1**)

(a) Newton's second law applied to the x axis yields $F - mg\sin\theta = ma$. Thus, with $F = 40.0$ N, we find $a = -0.90\ \mathrm{m/s^2}$. The interpretation is that the magnitude of the acceleration is $0.90\ \mathrm{m/s^2}$ and its direction is downhill.

(b) Substituting $F = 60.0$ N into $F - mg\sin\theta = ma$, we find $a = 1.1\ \mathrm{m/s^2}$. Thus, the acceleration is $1.1\ \mathrm{m/s^2}$ uphill.

85. (Third problem in **Cluster 1**)
The coordinate system we wish to use is shown in Figure 5-18(c) in the textbook, so we resolve this vertical force into appropriate components.



(a) Assuming the block is not pulled entirely off the incline, Newton's second law applied to the x axis yields

$$F_x - mg \sin\theta = ma \ .$$

This leads to $a = -1.9\,\mathrm{m/s^2}$, which we interpret as a acceleration of $1.9\,\mathrm{m/s^2}$ directed *downhill*.

(b) The assumption stated in part (a) implies there is no acceleration in the y direction. Newton's second law along the y axis gives

$$N + F_y - mg \cos\theta = 0 \ .$$

Therefore, $N = 32.9$ N. We note that a negative value of N would have been a sure sign that our assumption was incorrect.

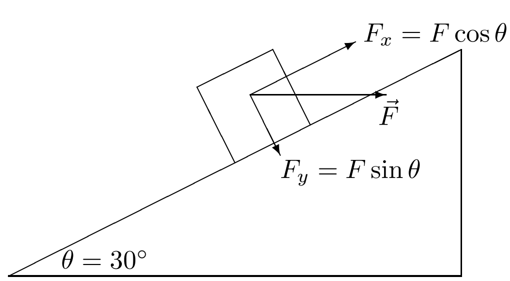
(c) The equation in part (a) can be used to solve for the equilibrium ($a = 0$) value of F:

$$F \cos 60° = mg \sin 30° = 49 \text{ N} \ .$$

Therefore, $F = 98$ N.

(d) There are three forces acting on the block: $\vec{N}, \vec{F}$, and $m\vec{g}$. Equilibrium generally suggests that the "vector triangle" formed by three such vectors closes on itself. In this case, however, two sides of that "triangle" are vertical! $\vec{F}$ is *up* and $m\vec{g}$ is *down*! The insight behind this "squashed triangle" is that $\vec{N}$ (the only vector that is not vertical) has zero magnitude. Thus, the block is not "bearing down" on the incline surface. In fact, in this circumstance, the incline is not needed at all for support; the value $F = 98.0$ N is just what is needed to hold the block (which weighs 98.0 N) aloft.

86. (Fourth problem in **Cluster 1**)
The coordinate system we wish to use is shown in Fig. 5-18 in the textbook, so we resolve this horizontal force into appropriate components.



(a) We apply Newton's second law to the x axis:

$$F_x - mg \sin\theta = ma$$

This yields $a = -1.44\,\mathrm{m/s^2}$, which is interpreted as an acceleration of $1.44\,\mathrm{m/s^2}$ downhill.

(b) Applying Newton's second law to the y axis (where there is no acceleration), we have

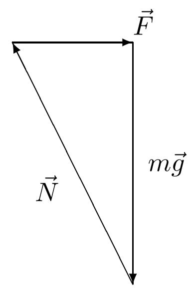
$$N - F_y - mg\cos\theta = 0 \ .$$

This yields the normal force $N = 105$ N.

(c) When we set $a = 0$ in the part (a) equation, we obtain

$$F\cos 30^\circ = mg\sin 30^\circ \ .$$

Therefore, $F = 56.6$ N. Alternatively, we can use a "vector triangle" approach, referred to in the previous problem solution. We form a closed triangle.



We note that the angle between the weight vector and the normal force is θ. Thus, we see $mg\tan\theta = F$, which gives $F = 56.6$ N.

Chapter 6

1. We do not consider the possibility that the bureau might tip, and treat this as a purely horizontal motion problem (with the person's push $\vec{F}$ in the $+x$ direction). Applying Newton's second law to the x and y axes, we obtain

$$\begin{aligned} F - f_{s,\text{max}} &= ma \\ N - mg &= 0 \end{aligned}$$

respectively. The second equation yields the normal force $N = mg$, whereupon the maximum static friction is found to be (from Eq. 6-1) $f_{s,\text{max}} = \mu_s mg$. Thus, the first equation becomes

$$F - \mu_s mg = ma = 0$$

where we have set $a = 0$ to be consistent with the fact that the static friction is still (just barely) able to prevent the bureau from moving.

   (a) With $\mu_s = 0.45$ and $m = 45$ kg, the equation above leads to $F = 198$ N. To bring the bureau into a state of motion, the person should push with any force greater than this value. Rounding to two significant figures, we can therefore say the minimum required push is $F = 2.0 \times 10^2$ N.

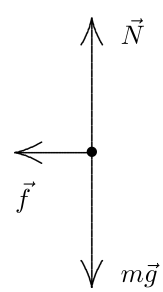
   (b) Replacing $m = 45$ kg with $m = 28$ kg, the reasoning above leads to roughly $F = 1.2 \times 10^2$ N.

2. An excellent discussion and equation development related to this problem is given in Sample Problem 6-3. We merely quote (and apply) their main result (Eq. 6-13)

$$\theta = \tan^{-1} \mu_s = \tan^{-1} 0.04 \approx 2^\circ \ .$$

3. The free-body diagram for the player is shown below. $\vec{N}$ is the normal force of the ground on the player, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. The force of friction is related to the normal force by $f = \mu_k N$. We use Newton's second law applied to the vertical axis to find the normal force. The vertical component of the acceleration is zero, so we obtain $N - mg = 0$; thus, $N = mg$. Consequently,

$$\begin{aligned} \mu_k &= \frac{f}{N} \\ &= \frac{470\,\text{N}}{(79\,\text{kg})\left(9.8\,\text{m/s}^2\right)} \\ &= 0.61 \ . \end{aligned}$$



4. To maintain the stone's motion, a horizontal force (in the $+x$ direction) is needed that cancels the retarding effect due to kinetic friction. Applying Newtons' second to the x and y axes, we obtain

$$\begin{aligned} F - f_k &= ma \\ N - mg &= 0 \end{aligned}$$

respectively. The second equation yields the normal force $N = mg$, so that (using Eq. 6-2) the kinetic friction becomes $f_k = \mu_k mg$. Thus, the first equation becomes

$$F - \mu_k mg = ma = 0$$

where we have set $a = 0$ to be consistent with the idea that the horizontal velocity of the stone should remain constant. With $m = 20$ kg and $\mu_k = 0.80$, we find $F = 1.6 \times 10^2$ N.

5. We denote $\vec{F}$ as the horizontal force of the person exerted on the crate (in the $+x$ direction), $\vec{f_k}$ is the force of kinetic friction (in the $-x$ direction), $\vec{N}$ is the vertical normal force exerted by the floor (in the $+y$ direction), and $m\vec{g}$ is the force of gravity. The magnitude of the force of friction is given by $f_k = \mu_k N$ (Eq. 6-2). Applying Newtons' second to the x and y axes, we obtain

$$\begin{aligned} F - f_k &= ma \\ N - mg &= 0 \end{aligned}$$

respectively.

(a) The second equation yields the normal force $N = mg$, so that the friction is

$$f_k = \mu_k mg = (0.35)(55\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 1.9 \times 10^2 \text{ N} .$$

(b) The first equation becomes

$$F - \mu_k mg = ma$$

which (with $F = 220$ N) we solve to find

$$a = \frac{F}{m} - \mu_k g = 0.56 \text{ m/s}^2 .$$

6. An excellent discussion and equation development related to this problem is given in Sample Problem 6-3. We merely quote (and apply) their main result (Eq. 6-13)

$$\theta = \tan^{-1} \mu_s = \tan^{-1} 0.5 = 27^\circ$$

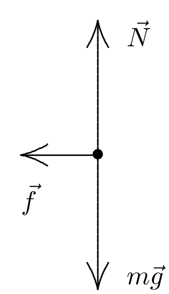
which implies that the angle through which the slope should be *reduced* is $\phi = 45^\circ - 27^\circ \approx 20^\circ$.

7. The free-body diagram for the puck is shown below. $\vec{N}$ is the normal force of the ice on the puck, $\vec{f}$ is the force of friction (in the $-x$ direction), and $m\vec{g}$ is the force of gravity.

(a) The horizontal component of Newton's second law gives $-f = ma$, and constant acceleration kinematics (Table 2-1) can be used to find the acceleration.

Since the final velocity is zero, $v^2 = v_0^2 + 2ax$ leads to $a = -v_0^2/2x$. This is substituted into the Newton's law equation to obtain

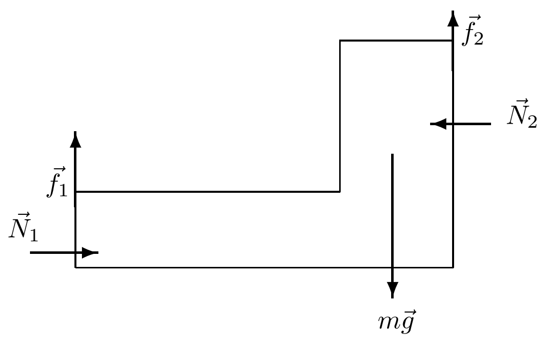
$$\begin{aligned} f &= \frac{mv_0^2}{2x} \\ &= \frac{(0.110\,\text{kg})(6.0\,\text{m/s})^2}{2(15\,\text{m})} \\ &= 0.13 \text{ N} . \end{aligned}$$



(b) The vertical component of Newton's second law gives $N - mg = 0$, so $N = mg$ which implies (using Eq. 6-2) $f = \mu_k mg$. We solve for the coefficient:

$$\mu_k = \frac{f}{mg} = \frac{0.13\,\text{N}}{(0.110\,\text{kg})\left(9.8\,\text{m/s}^2\right)} = 0.12\ .$$

8. (a) The free-body diagram for the person (shown as an L-shaped block) is shown below. The force that she exerts on the rock slabs is not directly shown (since the diagram should only show forces exerted on her), but it is related by Newton's third law) to the normal forces $\vec{N}_1$ and $\vec{N}_2$ exerted horizontally by the slabs onto her shoes and back, respectively. We will show in part (b) that $N_1 = N_2$ so that we there is no ambiguity in saying that the magnitude of her push is N_2. The total upward force due to (maximum) static friction is $\vec{f} = \vec{f}_1 + \vec{f}_2$ where (using Eq. 6-1) $f_1 = \mu_{s\,1} N_1$ and $f_2 = \mu_{s\,2} N_2$. The problem gives the values $\mu_{s\,1} = 1.2$ and $\mu_{s\,2} = 0.8$.



(b) We apply Newton's second law to the x and y axes (with $+x$ rightward and $+y$ upward and there is no acceleration in either direction).

$$\begin{aligned} N_1 - N_2 &= 0 \\ f_1 + f_2 - mg &= 0 \end{aligned}$$

The first equation tells us that the normal forces are equal $N_1 = N_2 = N$. Consequently, from Eq. 6-1

$$\begin{aligned} f_1 &= \mu_{s\,1} N \\ f_2 &= \mu_{s\,2} N \end{aligned}$$

we conclude that

$$f_1 = \left(\frac{\mu_{s\,1}}{\mu_{s\,2}}\right) f_2\ .$$

Therefore, $f_1 + f_2 - mg = 0$ leads to

$$\left(\frac{\mu_{s\,1}}{\mu_{s\,2}} + 1\right) f_2 = mg$$

which (with $m = 49$ kg) yields $f_2 = 192$ N. From this we find $N = f_2/\mu_{s\,2} = 240$ N. This is equal to the magnitude of the push exerted by the rock climber.

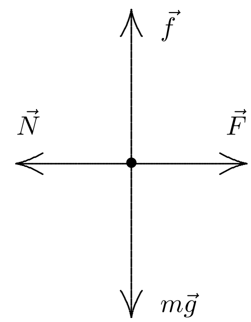
(c) From the above calculation, we find $f_1 = \mu_{s\,1} N = 288$ N which amounts to a fraction

$$\frac{f_1}{W} = \frac{288}{(49)(9.8)} = 0.60$$

or 60% of her weight.

9. (a) The free-body diagram for the block is shown below. $\vec{F}$ is the applied force, $\vec{N}$ is the normal force of the wall on the block, $\vec{f}$ is the force of friction, and $m\vec{g}$ is the force of gravity. To determine if the block falls, we find the magnitude f of the force of friction required to hold it without accelerating and also find the normal force of the wall on the block.

We compare f and $\mu_s N$. If $f < \mu_s N$, the block does not slide on the wall but if $f > \mu_s N$, the block does slide. The horizontal component of Newton's second law is $F - N = 0$, so $N = F = 12\,\text{N}$ and $\mu_s N = (0.60)(12\,\text{N}) = 7.2\,\text{N}$. The vertical component is $f - mg = 0$, so $f = mg = 5.0\,\text{N}$. Since $f < \mu_s N$ the block does not slide.



(b) Since the block does not move $f = 5.0\,\text{N}$ and $N = 12\,\text{N}$. The force of the wall on the block is

$$\vec{F}_w = -N\,\hat{\imath} + f\,\hat{\jmath} = -(12\,\text{N})\,\hat{\imath} + (5.0\,\text{N})\,\hat{\jmath}$$

where the axes are as shown on Fig. 6-21 of the text.

10. In addition to the forces already shown in Fig. 6-22, a free-body diagram would include an upward normal force $\vec{N}$ exerted by the floor on the block, a downward $m\vec{g}$ representing the gravitational pull exerted by Earth, and an assumed-leftward $\vec{f}$ for the kinetic or static friction. We choose $+x$ rightwards and $+y$ upwards. We apply Newton's second law to these axes:

$$\begin{aligned} (6.0\,\text{N}) - f &= ma \\ P + N - mg &= 0 \end{aligned}$$

where $m = 2.5$ kg is the mass of the block.

(a) In this case, $P = 8.0$ N leads to $N = (2.5)(9.8) - 8.0$ so that the normal force is $N = 16.5$ N. Using Eq. 6-1, this implies $f_{s,\max} = \mu_s N = 6.6$ N, which is larger than the 6.0 N rightward force – so the block (which was initially at rest) does not move. Putting $a = 0$ into the first of our equations above yields a static friction force of $f = P = 6.0$ N. Since its value is positive, then our assumption for the direction of $\vec{f}$ (leftward) is correct.

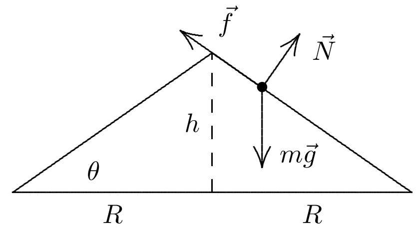
(b) In this case, $P = 10$ N leads to $N = (2.5)(9.8) - 10$ so that the normal force is $N = 14.5$ N. Using Eq. 6-1, this implies $f_{s,\max} = \mu_s N = 5.8$ N, which is less than the 6.0 N rightward force – so the block does move. Hence, we are dealing not with static but with kinetic friction, which Eq. 6-2 reveals to be $f_k = \mu_k N = 3.6$ N. Again, its value is positive, so our assumption for the direction of $\vec{f}$ (leftward) is correct.

(c) In this last case, $P = 12$ N leads to $N = 12.5$ N and thus to $f_{s,\max} = \mu_s N = 5.0$ N, which (as expected) is less than the 6.0 N rightward force – so the block moves. The kinetic friction force, then, is $f_k = \mu_k N = 3.1$ N. Once again, its value is positive, so our assumption for the direction of $\vec{f}$ (leftward) is correct.

11. A cross section of the cone of sand is shown below. To pile the most sand without extending the radius, sand is added to make the height h as great as possible. Eventually, however, the sides become so steep that sand at the surface begins to slide. The goal is to find the greatest height (corresponding to greatest slope) for which the sand does not slide. A grain of sand is shown on the diagram and the forces on it are labeled. $\vec{N}$ is the normal force of the surface, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of (static) friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal

force. We assume the grain does not slide, so its acceleration is zero. Then the x component of Newton's second law is $mg\sin\theta - f = 0$ and the y component is $N - mg\cos\theta = 0$.

The first equation gives $f = mg\sin\theta$ and the second gives $N = mg\cos\theta$. If the grain does not slide, the condition $f < \mu_s N$ must hold. This means $mg\sin\theta < \mu_s mg\cos\theta$ or $\tan\theta < \mu_s$. The surface of the cone has the greatest slope (and the height of the cone is the greatest) if $\tan\theta = \mu_s$.



Since R and h are two sides of a right triangle, $h = R\tan\theta$. Replacing $\tan\theta$ with μ_s we obtain $h = \mu_s R$. We substitute this into the volume equation $V = \pi R^2 h/3$ to obtain the result $V = \pi\mu_s R^3/3$.

12. We denote the magnitude of 110 N force exerted by the worker on the crate as F. The magnitude of the static frictional force can vary between zero and $f_{s,\,\max} = \mu_s N$.

(a) In this case, application of Newton's second law in the vertical direction yields $N = mg$. Thus,

$$\begin{aligned} f_{s,\,\max} &= \mu_s N = \mu_s mg \\ &= (0.37)(35\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 126.9\ \text{N} \end{aligned}$$

which is greater than F. The block, which is initially at rest, stays at rest. This implies, by applying Newton's second law to the horizontal direction, that the magnitude of the frictional force exerted on the crate is $f_s = F = 110\,\text{N}$.

(b) As calculated in part (a), $f_{s,\,\max} = 1.3 \times 10^2\,\text{N}$.

(c) As remarked above, the crate does not move (since $F < f_{s,\,\max}$).

(d) Denoting the upward force exerted by the second worker as F_2, then application of Newton's second law in the vertical direction yields $N = mg - F_2$. Therefore, in this case, $f_{s,\,\max} = \mu_s N = \mu_s(mg - F_2)$. In order to move the crate, F must satisfy $F > f_{s,\,\max} = \mu_s(mg - F_2)$, i.e.,

$$110\,\text{N} > (0.37)\left((35\,\text{kg})\left(9.8\,\text{m/s}^2\right) - F_2\right).$$

The minimum value of F_2 that satisfies this inequality is a value slightly bigger than 45.7 N, so we express our answer as $F_{2,\,\min} = 46\,\text{N}$.

(e) In this final case, moving the crate requires a greater horizontal push from the worker than static friction (as computed in part (a)) can resist. Thus, Newton's law in the horizontal direction leads to

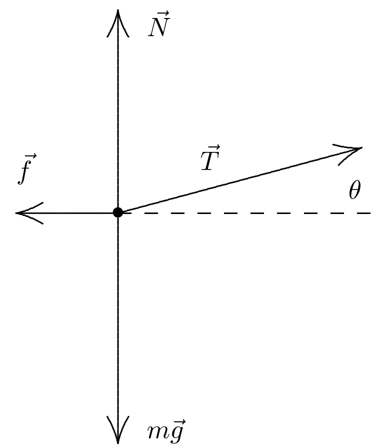
$$\begin{aligned} F + F_2 &> f_{s,\,\max} \\ 110\,\text{N} + F_2 &> 126.9\,\text{N} \end{aligned}$$

which leads (after appropriate rounding) to $F_{2,\,\min} = 17\,\text{N}$.

13. (a) The free-body diagram for the crate is shown below. $\vec{T}$ is the tension force of the rope on the crate, $\vec{N}$ is the normal force of the floor on the crate, $m\vec{g}$ is the force of gravity, and $\vec{f}$ is the force of friction. We take the $+x$ direction to be horizontal to the right and the $+y$ direction to be up. We assume the crate is motionless. The x component of Newton's second law leads to $T\cos\theta - f = 0$ and the y component becomes $T\sin\theta + N - mg = 0$, where $\theta = 15°$ is the angle between the rope and the horizontal.

The first equation gives $f = T\cos\theta$ and the second gives $N = mg - T\sin\theta$. If the crate is to remain at rest, f must be less than $\mu_s N$, or $T\cos\theta < \mu_s(mg - T\sin\theta)$. When the tension force is sufficient to just start the crate moving, we must have $T\cos\theta = \mu_s(mg - T\sin\theta)$. We solve for the tension:

$$\begin{aligned} T &= \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \\ &= \frac{(0.50)(68)(9.8)}{\cos 15^\circ + 0.50 \sin 15^\circ} \\ &= 304 \approx 300 \text{ N} . \end{aligned}$$



(b) The second law equations for the moving crate are $T\cos\theta - f = ma$ and $N + T\sin\theta - mg = 0$. Now $f = \mu_k N$. The second equation gives $N = mg - T\sin\theta$, as before, so $f = \mu_k(mg - T\sin\theta)$. This expression is substituted for f in the first equation to obtain $T\cos\theta - \mu_k(mg - T\sin\theta) = ma$, so the acceleration is

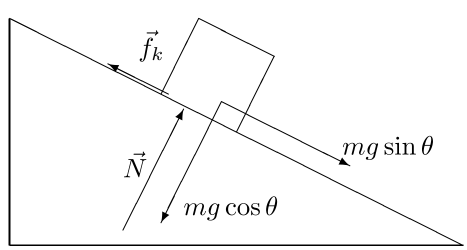
$$a = \frac{T(\cos\theta + \mu_k \sin\theta)}{m} - \mu_k g$$

which we evaluate:

$$a = \frac{(304\,\text{N})(\cos 15^\circ + 0.35 \sin 15^\circ)}{68\,\text{kg}} - (0.35)(9.8\,\text{m/s}^2) = 1.3\,\text{m/s}^2 .$$

14. We first analyze the forces on the pig of mass m. The incline angle is θ.

The $+x$ direction is "downhill."



Application of Newton's second law to the x and y axes leads to

$$\begin{aligned} mg\sin\theta - f_k &= ma \\ N - mg\cos\theta &= 0 . \end{aligned}$$

Solving these along with Eq. 6-2 ($f_k = \mu_k N$) produces the following result for the pig's downhill acceleration:

$$a = g(\sin\theta - \mu_k \cos\theta) .$$

To compute the time to slide from rest through a downhill distance ℓ, we use Eq. 2-15:

$$\ell = v_0 t + \frac{1}{2}at^2 \implies t = \sqrt{\frac{2\ell}{a}} .$$

We denote the frictionless ($\mu_k = 0$) case with a prime and set up a ratio:

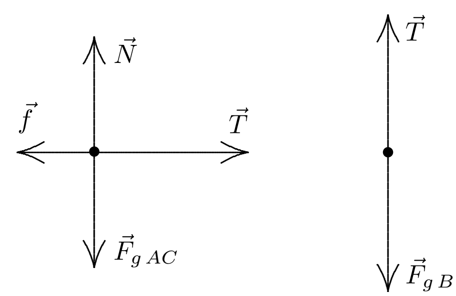
$$\frac{t}{t'} = \frac{\sqrt{2\ell/a}}{\sqrt{2\ell/a'}} = \sqrt{\frac{a'}{a}}$$

which leads us to conclude that if $t/t' = 2$ then $a' = 4a$. Putting in what we found out above about the accelerations, we have

$$g\sin\theta = 4g\left(\sin\theta - \mu_k\cos\theta\right) \ .$$

Using $\theta = 35°$, we obtain $\mu_k = 0.53$.

15. (a) Free-body diagrams for the blocks A and C, considered as a single object, and for the block B are shown below. T is the magnitude of the tension force of the rope, N is the magnitude of the normal force of the table on block A, f is the magnitude of the force of friction, W_{AC} is the combined weight of blocks A and C (the magnitude of force $\vec{F}_{g\,AC}$ shown in the figure), and W_B is the weight of block B (the magnitude of force $\vec{F}_{g\,B}$ shown). Assume the blocks are not moving. For the blocks on the table we take the x axis to be to the right and the y axis to be upward. The x component of Newton's second law is then $T - f = 0$ and the y component is $N - W_{AC} = 0$. For block B take the downward direction to be positive. Then Newton's second law for that block is $W_B - T = 0$. The third equation gives $T = W_B$ and the first gives $f = T = W_B$. The second equation gives $N = W_{AC}$. If sliding is not to occur, f must be less than $\mu_s N$, or $W_B < \mu_s W_{AC}$. The smallest that W_{AC} can be with the blocks still at rest is $W_{AC} = W_B/\mu_s = (22\,\text{N})/(0.20) = 110\,\text{N}$. Since the weight of block A is $44\,\text{N}$, the least weight for C is $110 - 44 = 66\,\text{N}$.



(b) The second law equations become $T - f = (W_A/g)a$, $N - W_A = 0$, and $W_B - T = (W_B/g)a$. In addition, $f = \mu_k N$. The second equation gives $N = W_A$, so $f = \mu_k W_A$. The third gives $T = W_B - (W_B/g)a$. Substituting these two expressions into the first equation, we obtain $W_B - (W_B/g)a - \mu_k W_A = (W_A/g)a$. Therefore,

$$a = \frac{g(W_B - \mu_k W_A)}{W_A + W_B} = \frac{(9.8\,\text{m/s}^2)\,(22\,\text{N} - (0.15)(44\,\text{N}))}{44\,\text{N} + 22\,\text{N}} = 2.3\,\text{m/s}^2 \ .$$

16. We choose $+x$ horizontally rightwards and $+y$ upwards and observe that the 15 N force has components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$.

(a) We apply Newton's second law to the y axis:

$$N - F\sin\theta - mg = 0 \implies N = (15)\sin 40° + (3.5)(9.8) = 44$$

in SI units. With $\mu_k = 0.25$, Eq. 6-2 leads to $f_k = 11$ N.

(b) We apply Newton's second law to the x axis:

$$F\cos\theta - f_k = ma \implies a = \frac{(15)\cos 40° - 11}{3.5} = 0.14$$

in SI units (m/s^2). Since the result is positive-valued, then the block is accelerating in the $+x$ (rightward) direction.

17. (a) Although details in Fig. 6-27 might suggest otherwise, we assume (as the problem states) that only static friction holds block B in place. An excellent discussion and equation development related to this topic is given in Sample Problem 6-3. We merely quote (and apply) their main result (Eq. 6-13) for the maximum angle for which static friction applies (in the absence of additional forces such as the $\vec{F}$ of part (b) of this problem).

$$\theta_{\text{max}} = \tan^{-1} \mu_s = \tan^{-1} 0.63 \approx 32^\circ \ .$$

This is greater than the dip angle in the problem, so the block does not slide.

(b) We analyze forces in a manner similar to that shown in Sample Problem 6-3, but with the addition of a downhill force F.

$$\begin{aligned} F + mg\sin\theta - f_{s,\text{max}} &= ma = 0 \\ N - mg\cos\theta &= 0 \ . \end{aligned}$$

Along with Eq. 6-1 ($f_{s,\text{max}} = \mu_s N$) we have enough information to solve for F. With $\theta = 24^\circ$ and $m = 1.8 \times 10^7$ kg, we find

$$F = mg\,(\mu_s \cos\theta - \sin\theta) = 3.0 \times 10^7 \text{ N} \ .$$

18. We use coordinates and weight-components as indicated in Fig. 5-18 (see Sample Problem 5-7 from the previous chapter).

(a) In this situation, we take $\vec{f_s}$ to point uphill and to be equal to its maximum value, in which case $f_{s,\text{max}} = \mu_s N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$\begin{aligned} F_{\min 1} - mg\sin\theta + f_{s,\text{max}} &= ma = 0 \\ N - mg\cos\theta &= 0 \end{aligned}$$

which (with $\theta = 20^\circ$) leads to

$$F_{\min 1} = mg\,(\sin\theta - \mu_s \cos\theta) = 8.6 \text{ N} \ .$$

(b) Now we take $\vec{f_s}$ to point downhill and to be equal to its maximum value, in which case $f_{s,\text{max}} = \mu_s N$ applies, where $\mu_s = 0.25$. Applying Newton's second law to the block of mass $m = W/g = 8.2$ kg, in the x and y directions, produces

$$\begin{aligned} F_{\min 2} - mg\sin\theta - f_{s,\text{max}} &= ma = 0 \\ N - mg\cos\theta &= 0 \end{aligned}$$

which (with $\theta = 20^\circ$) leads to

$$F_{\min 2} = mg\,(\sin\theta + \mu_s \cos\theta) = 46 \text{ N} \ .$$

A value slightly larger than the "exact" result of this calculation is required to make it accelerate up hill, but since we quote our results here to two significant figures, 46 N is a "good enough" answer.

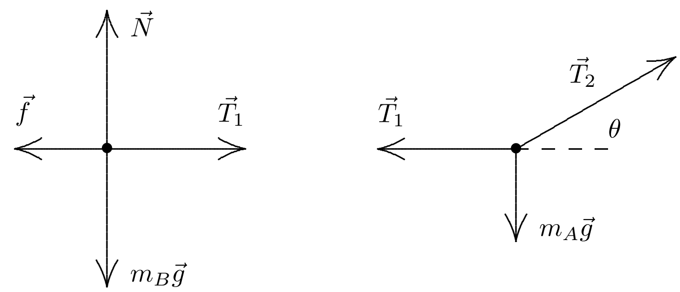
(c) Finally, we are dealing with kinetic friction (pointing downhill), so that

$$\begin{aligned} F - mg\sin\theta - f_k &= ma = 0 \\ N - mg\cos\theta &= 0 \end{aligned}$$

along with $f_k = \mu_k N$ (where $\mu_k = 0.15$) brings us to

$$F = mg\,(\sin\theta + \mu_k \cos\theta) = 39 \text{ N} \ .$$

19. The free-body diagrams for block B and for the knot just above block A are shown below. $\vec{T}_1$ is the tension force of the rope pulling on block B or pulling on the knot (as the case may be), $\vec{T}_2$ is the tension force exerted by the second rope (at angle $\theta = 30°$) on the knot, $\vec{f}$ is the force of static friction exerted by the horizontal surface on block B, $\vec{N}$ is normal force exerted by the surface on block B, W_A is the weight of block A (W_A is the magnitude of $m_A\vec{g}$), and W_B is the weight of block B ($W_B = 711$ N is the magnitude of $m_B\vec{g}$).



For each object we take $+x$ horizontally rightward and $+y$ upward. Applying Newton's second law in the x and y directions for block B and then doing the same for the knot results in four equations:

$$\begin{aligned} T_1 - f_{s,\max} &= 0 \\ N - W_B &= 0 \\ T_2\cos\theta - T_1 &= 0 \\ T_2\sin\theta - W_A &= 0 \end{aligned}$$

where we assume the static friction to be at its maximum value (permitting us to use Eq. 6-1). Solving these equations with $\mu_s = 0.25$, we obtain $W_A = 103 \approx 100$ N.

20. If the block is sliding then we compute the kinetic friction from Eq. 6-2; if it is not sliding, then we determine the extent of static friction from applying Newton's law, with zero acceleration, to the x axis (which is parallel to the incline surface). The question of whether or not it is sliding is therefore crucial, and depends on the maximum static friction force, as calculated from Eq. 6-1. The forces are resolved in the incline plane coordinate system in Figure 6-5 in the textbook. The acceleration, if there is any, is along the x axis, and we are taking uphill as $+x$. The net force along the y axis, then, is certainly zero, which provides the following relationship:

$$\sum \vec{F}_y = 0 \quad \Longrightarrow \quad N = W\cos\theta$$

where $W = 45$ N is the weight of the block, and $\theta = 15°$ is the incline angle. Thus, $N = 43.5$ N, which implies that the maximum static friction force should be $f_{s,\max} = (0.50)(43.5) = 21.7$ N.

(a) For $\vec{P} = 5.0$ N downhill, Newton's second law, applied to the x axis becomes

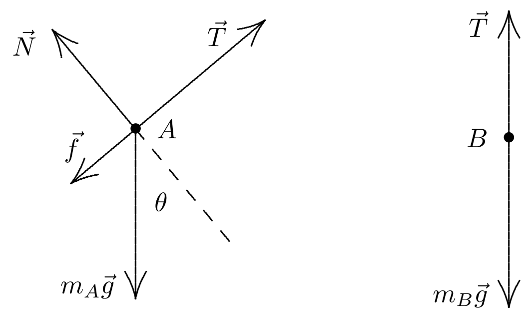
$$f - P - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g} \ .$$

Here we are assuming $\vec{f}$ is pointing uphill, as shown in Figure 6-5, and if it turns out that it points downhill (which *is* a possibility), then the result for f_s will be negative. If $f = f_s$ then $a = 0$, we obtain $f_s = 17$ N, which is clearly allowed since it is less than $f_{s,\max}$.

(b) For $\vec{P} = 8.0$ N downhill, we obtain (from the same equation) $f_s = 20$ N, which is still allowed since it is less than $f_{s,\max}$.

(c) But for $\vec{P} = 15$ N downhill, we obtain (from the same equation) $f_s = 27$ N, which is not allowed since it is larger than $f_{s,\ \max}$. Thus, we conclude that it is the kinetic friction, not the static friction, that is relevant in this case. We compute the result $f_k = (0.34)(43.5) = 15$ N. Here, as in the other parts of this problem, the friction is directed uphill.

21. First, we check to see if the bodies start to move. We assume they remain at rest and compute the force of (static) friction which holds them there, and compare its magnitude with the maximum value $\mu_s N$. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on body A, N is the magnitude of the normal force of the plane on body A, $m_A\vec{g}$ is the force of gravity on body A (with magnitude $W_A = 102$ N), and $m_B\vec{g}$ is the force of gravity on body B (with magnitude $W_B = 32$ N). $\theta = 40°$ is the angle of incline. We are not told the direction of $\vec{f}$ but we assume it is downhill. If we obtain a negative result for f, then we know the force is actually up the plane.



(a) For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force. The x and y components of Newton's second law become

$$\begin{aligned} T - f - W_A \sin\theta &= 0 \\ N - W_A \cos\theta &= 0 \;. \end{aligned}$$

Taking the positive direction to be *downward* for body B, Newton's second law leads to

$$W_B - T = 0 \;.$$

Solving these three equations leads to

$$f = W_B - W_A \sin\theta = 32 - 102 \sin 40° = -34 \text{ N}$$

(indicating that the force of friction is *uphill*) and to

$$N = W_A \cos\theta = 102 \cos 40° = 78 \text{ N}$$

which means that $f_{s,\max} = \mu_s N = (0.56)(78) = 44$ N. Since the magnitude f of the force of friction that holds the bodies motionless is less than $f_{s,\max}$ the bodies remain at rest. The acceleration is zero.

(b) Since A is moving up the incline, the force of friction is downhill with magnitude $f_k = \mu_k N$. Newton's second law, using the same coordinates as in part (a), leads to

$$\begin{aligned} T - f_k - W_A \sin\theta &= m_A\, a \\ N - W_A \cos\theta &= 0 \\ W_B - T &= m_B\, a \end{aligned}$$

for the two bodies. We solve for the acceleration:

$$\begin{aligned} a &= \frac{W_B - W_A \sin\theta - \mu_k W_A \cos\theta}{m_B + m_A} \\ &= \frac{32\,\text{N} - (102\,\text{N}) \sin 40° - (0.25)(102\,\text{N}) \cos 40°}{(32\,\text{N} + 102\,\text{N}) \,/\, \left(9.8\,\text{m/s}^2\right)} \\ &= -3.9 \text{ m/s}^2 \;. \end{aligned}$$

The acceleration is down the plane, which is to say (since the initial velocity was uphill) that the objects are slowing down. We note that $m = W/g$ has been used to calculate the masses in the calculation above.

(c) Now body A is initially moving down the plane, so the force of friction is uphill with magnitude $f_k = \mu_k N$. The force equations become

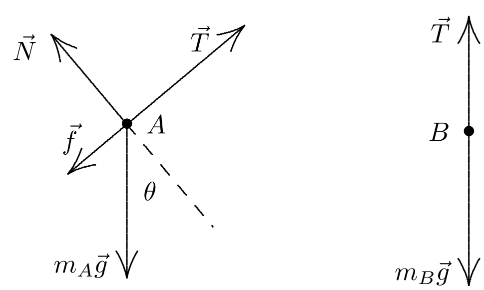
$$\begin{aligned} T + f_k - W_A \sin\theta &= m_A\, a \\ N - W_A \cos\theta &= 0 \\ W_B - T &= m_B\, a \end{aligned}$$

which we solve to obtain

$$\begin{aligned} a &= \frac{W_B - W_A \sin\theta + \mu_k W_A \cos\theta}{m_B + m_A} \\ &= \frac{32\,\mathrm{N} - (102\,\mathrm{N})\sin 40° + (0.25)(102\,\mathrm{N})\cos 40°}{(32\,\mathrm{N} + 102\,\mathrm{N}) \,/\, \left(9.8\,\mathrm{m/s^2}\right)} \\ &= -1.0\ \mathrm{m/s^2}\ . \end{aligned}$$

The acceleration is again downhill the plane. In this case, the objects are speeding up.

22. The free-body diagrams are shown below. T is the magnitude of the tension force of the string, f is the magnitude of the force of friction on block A, N is the magnitude of the normal force of the plane on block A, $m_A\vec{g}$ is the force of gravity on body A (where $m_A = 10$ kg), and $m_B\vec{g}$ is the force of gravity on block B. $\theta = 30°$ is the angle of incline. For A we take the $+x$ to be uphill and $+y$ to be in the direction of the normal force; the positive direction is chosen *downward* for block B.



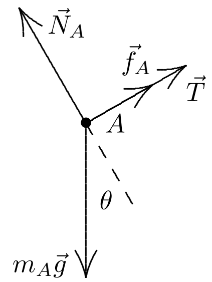
Since A is moving down the incline, the force of friction is uphill with magnitude $f_k = \mu_k N$ (where $\mu_k = 0.20$). Newton's second law leads to

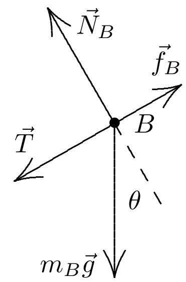
$$\begin{aligned} T - f_k + m_A g \sin\theta &= m_A\, a = 0 \\ N - m_A g \cos\theta &= 0 \\ m_B g - T &= m_B\, a = 0 \end{aligned}$$

for the two bodies (where $a = 0$ is a consequence of the velocity being constant). We solve these for the mass of block B.

$$m_B = m_A\,(\sin\theta - \mu_k \cos\theta) = 3.3\ \mathrm{kg}\ .$$

23. The free-body diagrams for the two blocks are shown below. T is the magnitude of the tension force of the string, $\vec{N}_A$ is the normal force on block A (the leading block), $\vec{N}_B$ is the the normal force on block B, $\vec{f}_A$ is kinetic friction force on block A, $\vec{f}_B$ is kinetic friction force on block B. Also, m_A is the mass of block A (where $m_A = W_A/g$ and $W_A = 3.6$ N), and m_B is the mass of block B (where $m_B = W_B/g$ and $W_B = 7.2$ N). The angle of the incline is $\theta = 30°$.





For each block we take $+x$ downhill (which is toward the lower-left in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first block A and next block B, we arrive at four equations:

$$\begin{aligned} W_A \sin\theta - f_A - T &= m_A\, a \\ N_A - W_A \cos\theta &= 0 \\ W_B \sin\theta - f_B + T &= m_B\, a \\ N_B - W_B \cos\theta &= 0\ . \end{aligned}$$

which, when combined with Eq. 6-2 ($f_A = \mu_{k\,A} N_A$ where $\mu_{k\,A} = 0.10$ and $f_B = \mu_{k\,B} N_B$ where $\mu_{k\,B} = 0.20$), fully describe the dynamics of the system so long as the blocks have the same acceleration and $T > 0$.

(a) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_{k\,A} W_A + \mu_{k\,B} W_B}{W_A + W_B}\right)\cos\theta\right) = 3.5\ \text{m/s}^2\ .$$

(b) We solve the above equations for the tension and obtain

$$T = \left(\frac{W_A\, W_B}{W_A + W_B}\right)(\mu_{k\,B} - \mu_{k\,A})\cos\theta = 0.21\ \text{N}\ .$$

Simply returning the value for a found in part (a) into one of the above equations is certainly fine, and probably easier than solving for T algebraically as we have done, but the algebraic form does illustrate the $\mu_{k\,B} - \mu_{k\,A}$ factor which aids in the understanding of the next part.

(c) Reversing the blocks is equivalent to switching the labels (so A is now the block of weight 7.2 N and $\mu_{k\,A}$ is now the 0.20 value). We see from our algebraic result in part (b) that this gives a negative value for T, which is impossible. We conclude that the above set of four equations are not valid in this circumstance (specifically, a for one block is not equal to a for the other block). The blocks move independently of each other.

24. Treating the two boxes as a single system of total mass $1.0 + 3.0 = 4.0$ kg, subject to a total (leftward) friction of magnitude $2.0 + 4.0 = 6.0$ N, we apply Newton's second law (with $+x$ rightward):

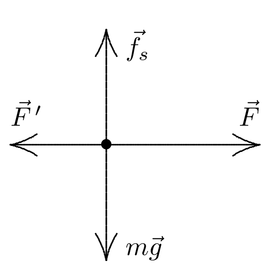
$$\begin{aligned} F - f_{\text{total}} &= m_{\text{total}}\, a \\ 12.0 - 6.0 &= (4.0)a \end{aligned}$$

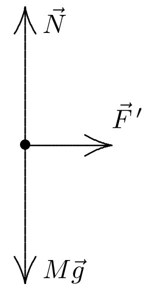
which yields the acceleration $a = 1.5\ \text{m/s}^2$. We have treated F as if it were known to the nearest tenth of a Newton so that our acceleration is "good" to two significant figures. Turning our attention to the larger box (the Wheaties box of mass 3.0 kg) we apply Newton's second law to find the contact force F' exerted by the smaller box on it.

$$\begin{aligned} F' - f_{\text{W}} &= m_{\text{W}}\, a \\ F' - 4.0 &= (3.0)(1.5) \end{aligned}$$

This yields the contact force $F' = 8.5$ N.

25. The free-body diagrams for the two blocks, treated individually, are shown below (first m and then M). F' is the contact force between the two blocks, and the static friction force $\vec{f}_s$ is at its maximum value (so Eq. 6-1 leads to $f_s = f_{s,\max} = \mu_s F'$ where $\mu_s = 0.38$).





Treating the two blocks together as a single system (sliding across a frictionless floor), we apply Newton's second law (with $+x$ rightward) to find an expression for the acceleration.

$$F = m_{\text{total}}\, a \quad \Longrightarrow a = \frac{F}{m+M}$$

This is equivalent to having analyzed the two blocks individually and then combined their equations. Now, when we analyze the small block individually, we apply Newton's second law to the x and y axes, substitute in the above expression for a, and use Eq. 6-1.

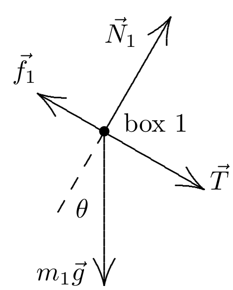
$$\begin{aligned} F - F' = ma \quad &\Longrightarrow \quad F' = F - m\left(\frac{F}{m+M}\right) \\ f_s - mg = 0 \quad &\Longrightarrow \quad \mu_s F' - mg = 0 \end{aligned}$$

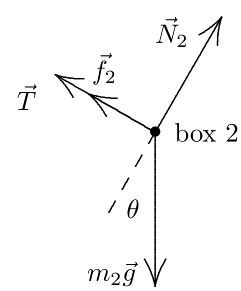
These expressions are combined (to eliminate F') and we arrive at

$$F = \frac{mg}{\mu_s\left(1 - \frac{m}{m+M}\right)}$$

which we find to be $F = 4.9 \times 10^2$ N.

26. The free-body diagrams for the two boxes are shown below. T is the magnitude of the force in the rod (when $T > 0$ the rod is said to be in tension and when $T < 0$ the rod is under compression), $\vec{N}_2$ is the normal force on box 2 (the uncle box), $\vec{N}_1$ is the the normal force on the aunt box (box 1), $\vec{f}_1$ is kinetic friction force on the aunt box, and $\vec{f}_2$ is kinetic friction force on the uncle box. Also, $m_1 = 1.65$ kg is the mass of the aunt box and $m_2 = 3.30$ kg is the mass of the uncle box (which is a lot of ants!).





For each block we take $+x$ downhill (which is toward the lower-right in these diagrams) and $+y$ in the direction of the normal force. Applying Newton's second law to the x and y directions of first box 2 and next box 1, we arrive at four equations:

$$\begin{aligned} m_2 g \sin\theta - f_2 - T &= m_2\, a \\ N_2 - m_2 g \cos\theta &= 0 \\ m_1 g \sin\theta - f_1 + T &= m_1\, a \\ N_1 - m_1 g \cos\theta &= 0\ . \end{aligned}$$

which, when combined with Eq. 6-2 ($f_1 = \mu_1 N_1$ where $\mu_1 = 0.226$ and $f_2 = \mu_2 N_2$ where $\mu_2 = 0.113$), fully describe the dynamics of the system.

(a) We solve the above equations for the tension and obtain

$$T = \left(\frac{m_2\, m_1\, g}{m_2 + m_1}\right)(\mu_1 - \mu_2)\cos\theta = 1.05 \text{ N} .$$

(b) These equations lead to an acceleration equal to

$$a = g\left(\sin\theta - \left(\frac{\mu_2 m_2 + \mu_1 m_1}{m_2 + m_1}\right)\cos\theta\right) = 3.62 \text{ m/s}^2 .$$

(c) Reversing the blocks is equivalent to switching the labels. We see from our algebraic result in part (a) that this gives a negative value for T (equal in magnitude to the result we got before). Thus, the situation is as it was before except that the rod is now in a state of compression.

27. The free-body diagrams for the slab and block are shown below. $\vec{F}$ is the 100 N force applied to the block, $\vec{N}_s$ is the normal force of the floor on the slab, N_b is the magnitude of the normal force between the slab and the block, $\vec{f}$ is the force of friction between the slab and the block, m_s is the mass of the slab, and m_b is the mass of the block. For both objects, we take the $+x$ direction to be to the left and the $+y$ direction to be up.

Applying Newton's second law for the x and y axes for (first) the slab and (second) the block results in four equations:

$$\begin{aligned} f &= m_s a_s \\ N_s - N_b - m_s g &= 0 \\ F - f &= m_b a_b \\ N_b - m_b g &= 0 \end{aligned}$$

from which we note that the maximum possible static friction magnitude would be

$$\mu_s N_b = \mu_s m_b g = (0.60)(10\,\text{kg})(9.8\,\text{m/s}^2) = 59 \text{ N} .$$

We check to see if the block slides on the slab. Assuming it does not, then $a_s = a_b$ (which we denote simply as a) and we solve for f:

$$f = \frac{m_s F}{m_s + m_b} = \frac{(40\,\text{kg})(100\,\text{N})}{40\,\text{kg} + 10\,\text{kg}} = 80 \text{ N}$$

which is greater than $f_{s,\max}$ so that we conclude the block is sliding across the slab (their accelerations are different).

(a) Using $f = \mu_k N_b$ the above equations yield

$$a_b = \frac{F - \mu_k m_b g}{m_b} = \frac{100\,\text{N} - (0.40)(10\,\text{kg})(9.8\,\text{m/s}^2)}{10\,\text{kg}} = 6.1 \text{ m/s}^2 .$$

The result is positive which means (recalling our choice of $+x$ direction) that it accelerates leftward.

(b) We also obtain

$$a_s = \frac{\mu_k m_b g}{m_s} = \frac{(0.40)(10\,\text{kg})(9.8\,\text{m/s}^2)}{40\,\text{kg}} = 0.98\ \text{m/s}^2\ .$$

As mentioned above, this means it accelerates to the left.

28. We may treat all 25 cars as a single object of mass $m = 25 \times 5.0 \times 10^4$ kg and (when the speed is 30 km/h = 8.3 m/s) subject to a friction force equal to $f = 25 \times 250 \times 8.3 = 5.2 \times 10^4$ N.

(a) Along the level track, this object experiences a "forward" force T exerted by the locomotive, so that Newton's second law leads to

$$T - f = ma \implies T = 5.2 \times 10^4 + \left(1.25 \times 10^6\right)(0.20)$$

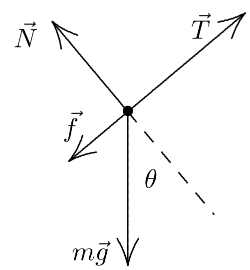
which yields $T = 3.0 \times 10^5$ N.

(b) The free-body diagram is shown below, with θ as the angle of the incline. The $+x$ direction (which is the only direction to which we will be applying Newton's second law) is uphill (to the upper right in our sketch).

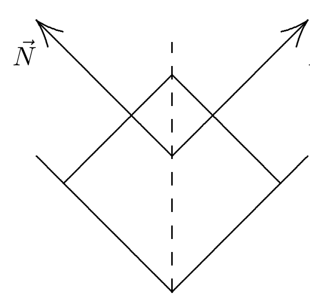
Thus, we obtain

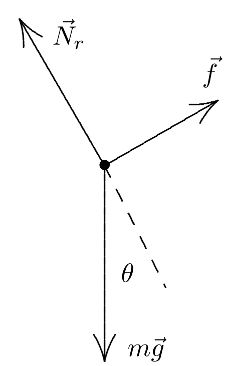
$$T - f - mg\sin\theta = ma$$

where we set $a = 0$ (implied by the problem statement) and solve for the angle. We obtain $\theta = 1.2°$.



29. Each side of the trough exerts a normal force on the crate. The first diagram shows the view looking in toward a cross section. The net force is along the dashed line. Since each of the normal forces makes an angle of 45° with the dashed line, the magnitude of the resultant normal force is given by $N_r = 2N\cos 45° = \sqrt{2}N$. The second diagram is the free-body diagram for the crate (from a "side" view, similar to that shown in the first picture in Fig. 6-36). The force of gravity has magnitude mg, where m is the mass of the crate, and the magnitude of the force of friction is denoted by f. We take the $+x$ direction to be down the incline and $+y$ to be in the direction of $\vec{N}_r$. Then the x component of Newton's second law is $mg\sin\theta - f = ma$ and the y component is $N_r - mg\cos\theta = 0$. Since the crate is moving, each side of the trough exerts a force of kinetic friction, so the total frictional force has magnitude $f = 2\mu_k N = 2\mu_k N_r/\sqrt{2} = \sqrt{2}\mu_k N_r$. Combining this expression with $N_r = mg\cos\theta$ and substituting into the x component equation, we obtain $mg\sin\theta - \sqrt{2}mg\cos\theta = ma$. Therefore $a = g(\sin\theta - \sqrt{2}\mu_k\cos\theta)$.





30. Fig. 6-4 in the textbook shows a similar situation (using ϕ for the unknown angle) along with a free-body diagram. We use the same coordinate system as in that figure.

(a) Thus, Newton's second law leads to

$$\begin{aligned} T\cos\phi - f &= ma & \text{along } x \text{ axis} \\ T\sin\phi + N - mg &= 0 & \text{along } y \text{ axis} \end{aligned}$$

Setting $a = 0$ and $f = f_{s,\max} = \mu_s N$, we solve for the mass of the box-and-sand (as a function of angle):

$$m = \frac{T}{g}\left(\sin\phi + \frac{\cos\phi}{\mu_s}\right)$$

which we will solve with calculus techniques (to find the angle ϕ_m corresponding to the maximum mass that can be pulled).

$$\frac{dm}{dt} = \frac{T}{g}\left(\cos\phi_m - \frac{\sin\phi_m}{\mu_s}\right) = 0$$

This leads to $\tan\phi_m = \mu_s$ which (for $\mu_s = 0.35$) yields $\phi_m = 19°$.

(b) Plugging our value for ϕ_m into the equation we found for the mass of the box-and-sand yields $m = 340$ kg. This corresponds to a weight of $mg = 3.3 \times 10^3$ N.

31. We denote the magnitude of the frictional force αv, where $\alpha = 70\,\mathrm{N\cdot s/m}$. We take the direction of the boat's motion to be positive. Newton's second law gives

$$-\alpha v = m\,\frac{dv}{dt}\ .$$

Thus,

$$\int_{v_0}^{v} \frac{dv}{v} = -\frac{\alpha}{m}\int_0^t dt$$

where v_0 is the velocity at time zero and v is the velocity at time t. The integrals are evaluated with the result

$$\ln\frac{v}{v_0} = -\frac{\alpha t}{m}\ .$$

We take $v = v_0/2$ and solve for time:

$$t = \frac{m}{\alpha}\ln 2 = \frac{1000\,\mathrm{kg}}{70\,\mathrm{N\cdot s/m}}\ln 2 = 9.9\ \mathrm{s}\ .$$

32. In the solution to exercise 4, we found that the force provided by the wind needed to equal $F = 157$ N (where that last figure is not "significant").

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed V along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$V = \sqrt{\frac{2F}{C\rho A}} = \sqrt{\frac{2(157)}{(0.80)(1.21)(0.040)}}$$

which yields $V = 90$ m/s which converts to $V = 3.2 \times 10^2$ km/h.

(b) Doubling our previous result, we find the reported speed to be 6.5×10^2 km/h, which is not reasonable for a terrestrial storm. (A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.)

33. We use Eq. 6-14, $D = \frac{1}{2}C\rho A v^2$, where ρ is the air density, A is the cross-sectional area of the missile, v is the speed of the missile, and C is the drag coefficient. The area is given by $A = \pi R^2$, where $R = 0.265\,\mathrm{m}$ is the radius of the missile. Thus

$$D = \frac{1}{2}(0.75)(1.2\,\mathrm{kg/m^3})\pi(0.265\,\mathrm{m})^2(250\,\mathrm{m/s})^2 = 6.2 \times 10^3\ \mathrm{N}\ .$$

34. Using Eq. 6-16, we solve for the area

$$A = \frac{2\,m\,g}{C\,\rho\,v_t^2}$$

which illustrates the inverse proportionality between the area and the speed-squared. Thus, when we set up a ratio of areas – of the slower case to the faster case – we obtain

$$\frac{A_{\text{slow}}}{A_{\text{fast}}} = \left(\frac{310\,\text{km/h}}{160\,\text{km/h}}\right)^2 = 3.75\ .$$

35. For the passenger jet $D_j = \frac{1}{2}C\rho_1 A v_j^2$, and for the prop-driven transport $D_t = \frac{1}{2}C\rho_2 A v_t^2$, where ρ_1 and ρ_2 represent the air density at 10 km and 5.0 km, respectively. Thus the ratio in question is

$$\frac{D_j}{D_t} = \frac{\rho_1 v_j^2}{\rho_2 v_t^2} = \frac{(0.38\,\text{kg/m}^3)(1000\,\text{km/h})^2}{(0.67\,\text{kg/m}^3)(500\,\text{km/h})^2} = 2.3\ .$$

36. With $v = 96.6$ km/h $= 26.8$ m/s, Eq. 6-17 readily yields

$$a = \frac{v^2}{R} = \frac{26.8^2}{7.6} = 94.7\ \text{m/s}^2$$

which we express as a multiple of g:

$$a = \left(\frac{a}{g}\right) g = \left(\frac{94.7}{9.8}\right) g = 9.7g\ .$$

37. The magnitude of the acceleration of the car as it rounds the curve is given by v^2/R, where v is the speed of the car and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If N is the normal force of the road on the car and m is the mass of the car, the vertical component of Newton's second law leads to $N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s N = \mu_s mg$. If the car does not slip, $f \le \mu_s mg$. This means

$$\frac{v^2}{R} \le \mu_s g \quad \Longrightarrow \quad v \le \sqrt{\mu_s R g}\ .$$

Consequently, the maximum speed with which the car can round the curve without slipping is

$$v_{\max} = \sqrt{\mu_s R g} = \sqrt{(0.60)(30.5)(9.8)} = 13\ \text{m/s}\ .$$

38. We will start by assuming that the normal force (on the car from the rail) points up. Note that gravity points down, and the y axis is chosen positive upwards. Also, the direction to the center of the circle (the direction of centripetal acceleration) is down. Thus, Newton's second law leads to

$$N - mg = m\left(-\frac{v^2}{r}\right)\ .$$

(a) When $v = 11$ m/s, we obtain $N = 3.7 \times 10^3$ N. The fact that this answer is positive means that $\vec{N}$ does indeed point upward as we had assumed.

(b) When $v = 14$ m/s, we obtain $N = -1.3 \times 10^3$ N. The fact that this answer is negative means that $\vec{N}$ points opposite to what we had assumed. Thus, the magnitude of $\vec{N}$ is 1.3 kN and its direction is *down*.

39. The magnitude of the acceleration of the cyclist as it rounds the curve is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve. Since the road is horizontal, only the frictional force of the road on the tires makes this acceleration possible. The horizontal component of Newton's second law is $f = mv^2/R$. If N is the normal force of the road on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $N = mg$. Thus, using Eq. 6-1, the maximum value of static friction is $f_{s,\max} = \mu_s N = \mu_s mg$. If the bicycle does not slip, $f \leq \mu_s mg$. This means
$$\frac{v^2}{R} \leq \mu_s g \implies R \geq \frac{v^2}{\mu_s\, g} .$$
Consequently, the minimum radius with which a cyclist moving at 29 km/h = 8.1 m/s can round the curve without slipping is
$$R_{\min} = \frac{v^2}{\mu_s\, g} = \frac{8.1^2}{(0.32)(9.8)} = 21 \text{ m} .$$

40. The situation is somewhat similar to that shown in the "loop-the-loop" example done in the textbook (see Figure 6-10) except that, instead of a downward normal force, we are dealing with the force of the boom $\vec{F}_B$ on the car – which is capable of pointing any direction. We will assume it to be upward as we apply Newton's second law to the car (of total weight 5000 N):
$$F_B - W = ma \text{ where } m = \frac{W}{g} \ , \text{ and } a = -\frac{v^2}{r}$$
Note that the centripetal acceleration is downward (our choice for negative direction) for a body at the top of its circular trajectory.
   (a) If $r = 10$ m and $v = 5.0$ m/s, we obtain $F_B = 3.7 \times 10^3$ N = 3.7 kN (up).
   (b) If $r = 10$ m and $v = 12$ m/s, we obtain $F_B = -2.3 \times 10^3$ N = -2.3 kN where the minus sign indicates that $\vec{F}_B$ points downward.

41. For the puck to remain at rest the magnitude of the tension force T of the cord must equal the gravitational force Mg on the cylinder. The tension force supplies the centripetal force that keeps the puck in its circular orbit, so $T = mv^2/r$. Thus $Mg = mv^2/r$. We solve for the speed: $v = \sqrt{Mgr/m}$.

42. The magnitude of the acceleration of the cyclist as it moves along the horizontal circular path is given by v^2/R, where v is the speed of the cyclist and R is the radius of the curve.
   (a) The horizontal component of Newton's second law is $f = mv^2/R$, where f is the static friction exerted horizontally by the ground on the tires. Thus,
$$f = \frac{(85.0)(9.00)^2}{25.0} = 275 \text{ N} .$$
   (b) If N is the vertical force of the ground on the bicycle and m is the mass of the bicycle and rider, the vertical component of Newton's second law leads to $N = mg = 833$ N. The magnitude of the force exerted by the ground on the bicycle is therefore
$$\sqrt{f^2 + N^2} = \sqrt{275^2 + 833^2} = 877 \text{ N} .$$

43. (a) At the top (the highest point in the circular motion) the seat pushes up on the student with a force of magnitude $N = 556\,\text{N}$. Earth pulls down with a force of magnitude $W = 667\,\text{N}$. The seat is pushing up with a force that is smaller than the student's weight, and we say the student experiences a decrease in his "apparent weight" at the highest point.
   (b) When the student is at the highest point, the net force toward the center of the circular orbit is $W - F_t$ (note that we are choosing downward as the positive direction). According to Newton's second law, this must equal mv^2/R, where v is the speed of the student and R is the radius of the orbit. Thus
$$mv^2/R = W - N = 667\,\text{N} - 556\,\text{N} = 111 \text{ N} .$$

(c) Now N is the magnitude of the upward force exerted by the seat when the student is at the lowest point. The net force toward the center of the circle is $F_b - W = mv^2/R$ (note that we are now choosing upward as the positive direction). The Ferris wheel is "steadily rotating" so the value mv^2/R is the same as in part (a). Thus,

$$N = \frac{mv^2}{R} + W = 111\,\text{N} + 667\,\text{N} = 778\ \text{N} .$$

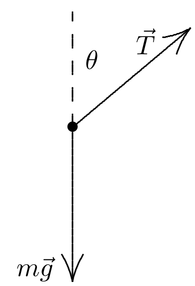
(d) If the speed is doubled, mv^2/R increases by a factor of 4, to 444 N. Therefore, at the highest point we have $W - N = mv^2/R$, which leads to

$$N = 667\,\text{N} - 444\,\text{N} = 223\ \text{N} .$$

Similarly, the normal force at the lowest point is now found to be $N = 667 + 444 \approx 1.1$ kN.

44. The free-body diagram (for the hand straps of mass m) is the view that a passenger might see if she was looking forward and the streetcar was curving towards the right (so $\vec{a}$ points rightwards in the figure) We note that $|\vec{a}| = v^2/R$ where $v = 16$ km/h $= 4.4$ m/s.

Applying Newton's law to the axes of the problem ($+x$ is rightward and $+y$ is upward) we obtain



$$\begin{aligned} T\sin\theta &= m\frac{v^2}{R} \\ T\cos\theta &= mg\ . \end{aligned}$$

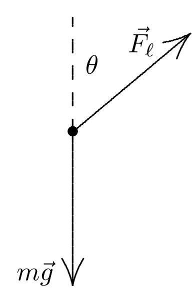
We solve these equations for the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)$$

which yields $\theta = 12°$.

45. The free-body diagram (for the airplane of mass m) is shown below. We note that $\vec{F_\ell}$ is the force of aerodynamic lift and $\vec{a}$ points rightwards in the figure. We also note that $|\vec{a}| = v^2/R$ where $v = 480$ km/h $= 133$ m/s.

Applying Newton's law to the axes of the problem ($+x$ rightward and $+y$ upward) we obtain



$$\begin{aligned} \vec{F_\ell}\ \sin\theta &= m\frac{v^2}{R} \\ \vec{F_\ell}\ \cos\theta &= mg \end{aligned}$$

where $\theta = 40°$. Eliminating mass from these equations leads to

$$\tan\theta = \frac{v^2}{gR}$$

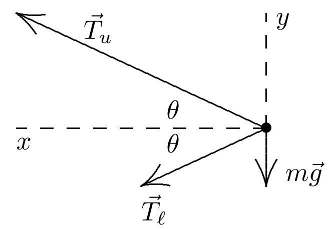
which yields $R = v^2/g\tan\theta = 2.2 \times 10^3$ m.

46. (a) The upward force exerted by the car on the passenger is equal to the downward force of gravity ($W = 500$ N) on the passenger. So the *net* force does not have a vertical contribution; it only has the contribution from the horizontal force (which is necessary for maintaining the circular motion). Thus $\left|\vec{F}_{\text{net}}\right| = F = 210$ N.

(b) Using Eq. 6-18, we have

$$v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(210)(470)}{51.0}} = 44.0 \text{ m/s} .$$

47. (a) The free-body diagram for the ball is shown below. $\vec{T}_u$ is the tension exerted by the upper string on the ball, $\vec{T}_\ell$ is the tension force of the lower string, and m is the mass of the ball. Note that the tension in the upper string is greater than the tension in the lower string. It must balance the downward pull of gravity and the force of the lower string.



(b) We take the $+x$ direction to be leftward (toward the center of the circular orbit) and $+y$ upward. Since the magnitude of the acceleration is $a = v^2/R$, the x component of Newton's second law is

$$T_u \cos\theta + T_\ell \cos\theta = \frac{mv^2}{R} ,$$

where v is the speed of the ball and R is the radius of its orbit. The y component is

$$T_u \sin\theta - T_\ell \sin\theta - mg = 0 .$$

The second equation gives the tension in the lower string: $T_\ell = T_u - mg/\sin\theta$. Since the triangle is equilateral $\theta = 30°$. Thus

$$T_\ell = 35 - \frac{(1.34)(9.8)}{\sin 30°} = 8.74 \text{ N} .$$

(c) The net force is leftward ("radially inward") and has magnitude

$$F_{\text{net}} = (T_u + T_\ell)\cos\theta = (35 + 8.74)\cos 30° = 37.9 \text{ N} .$$

(d) The radius of the path is $[(1.70\,\text{m})/2)]\tan 30° = 1.47\,\text{m}$. Using $F_{\text{net}} = mv^2/R$, we find that the speed of the ball is

$$v = \sqrt{\frac{RF_{\text{net}}}{m}} = \sqrt{\frac{(1.47\,\text{m})(37.9\,\text{N})}{1.34\,\text{kg}}} = 6.45 \text{ m/s} .$$

48. In the solution to exercise 4, we found that the force provided by the wind needed to equal $F = \mu_k mg$. In this situation, we have a much smaller value of μ_k (0.10) and a much larger mass (one hundred stones and the layer of ice). The layer of ice has a mass of

$$m_{\text{ice}} = \left(917\,\text{kg/m}^3\right)(400\,\text{m} \times 500\,\text{m} \times 0.0040\,\text{m})$$

which yields $m_{\text{ice}} = 7.34 \times 10^5$ kg. This added to the mass of the hundred stones (at 20 kg each) comes to $m = 7.36 \times 10^5$ kg.

(a) Setting $F = D$ (for Drag force) we use Eq. 6-14 to find the wind speed v along the ground (which actually is relative to the moving stone, but we assume the stone is moving slowly enough that this does not invalidate the result):

$$v = \sqrt{\frac{\mu_k mg}{4C_{\text{ice}}\rho A_{\text{ice}}}} = \sqrt{\frac{(0.10)\,(7.36 \times 10^5)\,(9.8)}{4(0.002)(1.21)(400 \times 500)}}$$

which yields $v = 19$ m/s which converts to $v = 69$ km/h.

(b) and (c) Doubling our previous result, we find the reported speed to be 139 km/h, which is a reasonable for a storm winds. (A category 5 hurricane has speeds on the order of 2.6×10^2 m/s.)

49. (a) The distance traveled by the coin in 3.14 s is $3(2\pi r) = 6\pi(0.050) = 0.94$ m. Thus, its speed is $v = 0.94/3.14 = 0.30$ m/s.

(b) The acceleration vector (at any instant) is horizontal and points from the coin towards the center of the turntable. This centripetal acceleration is given by Eq. 6-17:

$$a = \frac{v^2}{r} = \frac{0.30^2}{0.050} = 1.8 \text{ m/s}^2 .$$

(c) The only horizontal force acting on the coin is static friction f_s and must be large enough to supply the acceleration of part (b) for the $m = 0.0020$ kg coin. Using Newton's second law,

$$f_s = ma = (0.0020)(1.8) = 3.6 \times 10^{-3} \text{ N}$$

which must point in the same direction as the acceleration (towards the center of the turntable).

(d) We note that the normal force exerted upward on the coin by the turntable must equal the coin's weight (since there is no vertical acceleration in the problem). We also note that if we repeat the computations in parts (a) and (b) for $r' = 0.10$ m, then we obtain $v' = 0.60$ m/s and $a' = 3.6$ m/s^2. Now, if friction is at its maximum at $r = r'$, then, by Eq. 6-1, we obtain

$$\mu_s = \frac{f_{s,\max}}{mg} = \frac{ma'}{mg} = 0.37 .$$

50. (a) The angle made by the cord with the vertical axis is given by $\theta = \cos^{-1}(18/30) = 53°$. This means the radius of the plane's circular path is $r = 30 \sin\theta = 24$ m (we also could have arrived at this using the Pythagorean theorem). The speed of the plane is

$$v = \frac{4.4(2\pi r)}{1\,\text{min}} = \frac{8.8\pi(24\,\text{m})}{60\,\text{s}}$$

which yields $v = 11$ m/s. Eq. 6-17 then gives the acceleration (which at any instant is horizontally directed from the plane to the center of its circular path)

$$a = \frac{v^2}{r} = \frac{11^2}{24} = 5.1 \text{ m/s}^2 .$$

(b) The only horizontal force on the airplane is that component of tension, so Newton's second law gives

$$T \sin\theta = \frac{mv^2}{r} \quad \Longrightarrow \quad T = \frac{(0.75)(11)^2}{24 \sin 53°}$$

which yields $T = 4.8$ N.

(c) The net vertical force on the airplane is zero (since its only acceleration is horizontal), so

$$F_{\text{lift}} = T\cos\theta + mg = 4.8\cos 53° + (0.75)(9.8) = 10 \text{ N} .$$

51. (a) The centripetal force is given by Eq. 6-18:

$$F = \frac{mv^2}{R} = \frac{(1)(465)^2}{6.4 \times 10^6} = 0.034 \text{ N} .$$

(b) Calling downward (towards the center of Earth) the positive direction, Newton's second law leads to

$$mg - T = ma$$

where $mg = 9.80$ N and $ma = 0.034$ N, calculated in part (a). Thus, the tension in the cord by which the body hangs from the balance is $T = 9.80 - 0.03 = 9.77$ N. Thus, this is the reading for a standard kilogram mass, of the scale at the equator of the spinning Earth.

52. There is no acceleration, so the (upward) static friction forces (there are four of them, one for each thumb and one for each set of opposing fingers) equals the magnitude of the (downward) pull of gravity. Using Eq. 6-1, we have

$$4\mu_s N = mg = (79\,\text{kg})\left(9.8\,\text{m/s}^2\right)$$

which, with $\mu_s = 0.70$, yields $N = 2.8 \times 10^2$ N.

53. (a) From Table 6-1 and Eq. 6-16, we have

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \implies C\rho A = 2\frac{mg}{v_t^2}$$

where $v_t = 60$ m/s. We estimate the pilot's mass at about $m = 70$ kg. Now, we convert $v = 1300(1000/3600) \approx 360$ m/s and plug into Eq. 6-14:

$$D = \frac{1}{2}C\rho A v^2 = \frac{1}{2}\left(2\frac{mg}{v_t^2}\right)v^2 = mg\left(\frac{v}{v_t}\right)^2$$

which yields $D = (690)(360/60)^2 \approx 2 \times 10^4$ N.

(b) We assume the mass of the ejection seat is roughly equal to the mass of the pilot. Thus, Newton's second law (in the horizontal direction) applied to this system of mass $2m$ gives the magnitude of acceleration:

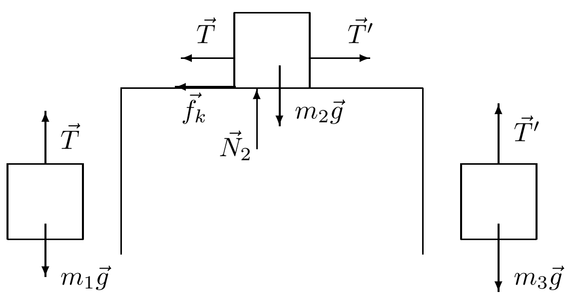
$$|a| = \frac{D}{2m} = \frac{g}{2}\left(\frac{v}{v_t}\right)^2 = 18g\ .$$

54. Although the object in question is a sphere, the area A in Eq. 6-16 is the cross sectional area presented by the object as it moves through the air (the cross section is perpendicular to $\vec{v}$). Thus, A is that of a circle: $A = \pi R^2$. We also note that 16 lb equates to an SI weight of 71 N. Thus,

$$v_t = \sqrt{\frac{2F_g}{C\rho\pi R^2}} \implies R = \frac{1}{145}\sqrt{\frac{2(71)}{(0.49)(1.2)\pi}}$$

which yields a diameter of $2R = 0.12$ m.

55. In the following sketch, T and T' are the tensions in the left and right strings, respectively. Also, $m_1 = M = 2.0$ kg, $m_2 = 2M = 4.0$ kg, and $m_3 = 2M = 4.0$ kg. Since it does, in fact, slide (presumably rightward), the type of friction that is acting upon m_2 is *kinetic* friction.



We use the familiar axes with $+x$ rightward and $+y$ upward for each block. This has the consequence that m_1 and m_2 accelerate with the same sign, but the acceleration of m_3 has the opposite sign. We take this into account as we apply Newton's second law to the three blocks.

$$\begin{aligned} T - m_1 g &= m_1(+a) \\ T' - T - f_k &= m_2(+a) \\ T' - m_3 g &= m_3(-a) \end{aligned}$$

Adding the first two equations, and subtracting the last, we obtain

$$(m_3 - m_1)\,g - f_k = (m_1 + m_2 + m_3)\,a$$

or (using M as in the problem statement)

$$Mg - f_k = 5Ma\ .$$

With $a = 1.5\ \text{m/s}^2$, we find $f_k = 4.6$ N.

56. (a) The component of the weight along the incline (with downhill understood as the positive direction) is $mg\sin\theta$ where $m = 630$ kg and $\theta = 10.2°$. With $f = 62.0$ N, Newton's second law leads to

$$mg\sin\theta - f = ma$$

which yields $a = 1.64\ \text{m/s}^2$. Using Eq. 2-15, we have

$$80.0\ \text{m}\ =\ \left(6.20\ \frac{\text{m}}{\text{s}}\right)t + \frac{1}{2}\left(1.64\ \frac{\text{m}}{\text{s}^2}\right)t^2\ .$$

This is solved using the quadratic formula. The positive root is $t = 6.80$ s.

(b) Running through the calculation of part (a) with $f = 42.0$ N instead of $f = 62$ N results in $t = 6.76$ s.

57. We convert to SI units: $v = 94(1000/3600) = 26$ m/s. Eq. 6-18 yields

$$F = \frac{mv^2}{R} = \frac{(85)(26)^2}{220} = 263\ \text{N}$$

for the horizontal force exerted on the passenger by the seat. But the seat also exerts an upward force equal to $mg = 833$ N. The magnitude of force is therefore $\sqrt{263^2 + 833^2} = 874$ N.

58. (a) Comparing the $t = 2.0$ s photo with the $t = 0$ photo, we see that the distance traveled by the box is

$$d = \sqrt{4.0^2 + 2.0^2} = 4.5\ \text{m}\ .$$

Thus (from Table 2-1, with *downhill* positive) $d = v_0t + \frac{1}{2}at^2$, we obtain $a = 2.2\ \text{m/s}^2$; note that the boxes are assumed to start from rest.

(b) For the axis along the incline surface, we have

$$mg\sin\theta - f_k = ma\ .$$

We compute mass m from the weight $m = 240/9.8 = 24$ kg, and θ is figured from the absolute value of the slope of the graph: $\theta = \tan^{-1} 2.5/5.0 = 27°$. Therefore, we find $f_k = 53$ N.

59. (a) If the skier covers a distance L during time t with zero initial speed and a constant acceleration a, then $L = at^2/2$, which gives the acceleration a_1 for the first (old) pair of skis:

$$a_1 = \frac{2L}{t_1^2} = \frac{2(200\,\text{m})}{(61\,\text{s})^2} = 0.11\,\text{m/s}^2$$

and the acceleration a_2 for the second (new) pair:

$$a_2 = \frac{2L}{t_2^2} = \frac{2(200\,\text{m})}{(42\,\text{s})^2} = 0.23\,\text{m/s}^2\ .$$

(b) The net force along the slope acting on the skier of mass m is

$$F_{\text{net}} = mg\sin\theta - f_k = mg(\sin\theta - \mu_k\cos\theta) = ma$$

which we solve for μ_{k1} for the first pair of skis:

$$\mu_{k1} = \tan\theta - \frac{a_1}{g\cos\theta} = \tan 3.0^\circ - \frac{0.11}{9.8\cos 3.0^\circ} = 0.041$$

and μ_{k2} for the second pair:

$$\mu_{k2} = \tan\theta - \frac{a_2}{g\cos\theta} = \tan 3.0^\circ - \frac{0.23}{9.8\cos 3.0^\circ} = 0.029 .$$

60. (a) The box doesn't move until $t = 2.8$ s, which is when the applied force $\vec{F}$ reaches a magnitude of $F = (1.8)(2.8) = 5.0$ N, implying therefore that $f_{s,\,\text{max}} = 5.0$ N. Analysis of the vertical forces on the block leads to the observation that the normal force magnitude equals the weight $N = mg = 15$ N. Thus, $\mu_s = f_{s,\,\text{max}}/N = 0.34$.

(b) We apply Newton's second law to the horizontal x axis (positive in the direction of motion).

$$F - f_k = ma \implies 1.8t - f_k = (1.5)(1.2t - 2.4)$$

Thus, we find $f_k = 3.6$ N. Therefore, $\mu_k = f_k/N = 0.24$.

61. In both cases (highest point and lowest point), the normal force (on the child from the seat) points up, gravity points down, and the y axis is chosen positive upwards. At the high point, the direction to the center of the circle (the direction of centripetal acceleration) is down, and at the low point that direction is up.

(a) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$N - mg = m\left(-\frac{v^2}{R}\right) .$$

With $m = 26$ kg, $v = 5.5$ m/s and $R = 12$ m, this yields $N = 189$ N which we round off to $N \approx 190$ N.

(b) Now, Newton's second law leads to

$$N - mg = m\left(\frac{v^2}{r}\right)$$

which yields $N = 320$ N. As already mentioned, the direction of $\vec{N}$ is *up* in both cases.

62. The mass of the car is $m = 10700/9.8 = 1.09 \times 10^3$ kg. We choose "inward" (horizontally towards the center of the circular path) as the positive direction.

(a) With $v = 13.4$ m/s and $R = 61$ m, Newton's second law (using Eq. 6-18) leads to

$$f_s = \frac{mv^2}{R} = 3.21 \times 10^3 \text{ N} .$$

(b) Noting that $N = mg$ in this situation, the maximum possible static friction is found to be

$$f_{s,\max} = \mu_s mg = (0.35)(10700) = 3.75 \times 10^3 \text{ N}$$

using Eq. 6-1. We see that the static friction found in part (a) is less than this, so the car rolls (no skidding) and successfully negotiates the curve.

63. (a) The distance traveled in one revolution is $2\pi R = 2\pi(4.6) = 29$ m. The (constant) speed is consequently $v = 29/30 = 0.96$ m/s.

(b) Newton's second law (using Eq. 6-17 for the magnitude of the acceleration) leads to

$$f_s = m\left(\frac{v^2}{R}\right) = m(0.20)$$

in SI units. Noting that $N = mg$ in this situation, the maximum possible static friction is $f_{s,\max} = \mu_s mg$ using Eq. 6-1. Equating this with $f_s = m(0.20)$ we find the mass m cancels and we obtain $\mu_s = 0.20/9.8 = 0.021$.

64. At the top of the hill the vertical forces on the car are the upward normal force exerted by the ground and the downward pull of gravity. Designating $+y$ downward, we have

$$mg - N = \frac{mv^2}{R}$$

from Newton's second law. To find the greatest speed without leaving the hill, we set $N = 0$ and solve for v:

$$v = \sqrt{gR} = \sqrt{(9.8)(250)} = 49.5 \text{ m/s}$$

which converts to $49.5(3600/1000) = 178$ km/h.

65. For simplicity, we denote the 70° angle as θ and the magnitude of the push (80 N) as P. The vertical forces on the block are the downward normal force exerted on it by the ceiling, the downward pull of gravity (of magnitude mg) and the vertical component of $\vec{P}$ (which is upward with magnitude $P\sin\theta$). Since there is no acceleration in the vertical direction, we must have

$$N = P\sin\theta - mg$$

in which case the leftward-pointed kinetic friction has magnitude

$$f_k = \mu_k\left(P\sin\theta - mg\right) .$$

Choosing $+x$ rightward, Newton's second law leads to

$$P\cos\theta - f_k = ma \implies a = \frac{P\cos\theta - \mu_k\left(P\sin\theta - mg\right)}{m}$$

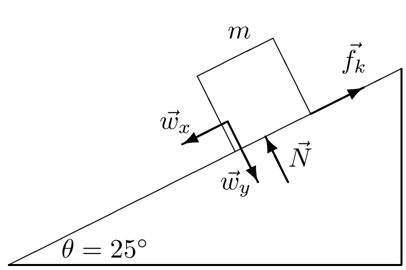
which yields $a = 3.4$ m/s^2 when $\mu_k = 0.40$ and $m = 5.0$ kg.

66. Probably the most appropriate picture in the textbook to represent the situation in this problem is in the previous chapter: Fig. 5-9. We adopt the familiar axes with $+x$ rightward and $+y$ upward, and refer to the 85 N horizontal push of the worker as P (and assume it to be rightward). Applying Newton's second law to the x axis and y axis, respectively, produces

$$\begin{aligned} P - f_k &= ma \\ N - mg &= 0 \ . \end{aligned}$$

Using $v^2 = v_0^2 + 2a\Delta x$ we find $a = 0.36$ m/s^2. Consequently, we obtain $f_k = 71$ N and $N = 392$ N. Therefore, $\mu_k = f_k/N = 0.18$.

67. In the figure below, $m = 140/9.8 = 14.3$ kg is the mass of the child. We use $\vec{w}_x$ and $\vec{w}_y$ as the components of the gravitational pull of Earth on the block; their magnitudes are $w_x = mg\sin\theta$ and $w_y = mg\cos\theta$.



(a) With the x axis directed up along the incline (so that $a = -0.86\ \text{m/s}^2$), Newton's second law leads to

$$f_k - 140 \sin 25^\circ = m\,(-0.86)$$

which yields $f_k = 47$ N. We also apply Newton's second law to the y axis (perpendicular to the incline surface), where the acceleration-component is zero:

$$N - 140 \cos 25^\circ = 0 \implies N = 127\ \text{N}\ .$$

Therefore, $\mu_k = f_k/N = 0.37$.

(b) Returning to our first equation in part (a), we see that if the downhill component of the weight force were insufficient to overcome static friction, the child would not slide at all. Therefore, we require $140 \sin 25^\circ > f_{s,\,\text{max}} = \mu_s N$, which leads to $\tan 25^\circ = 0.47 > \mu_s$. The minimum value of μ_s equals μ_k and is more subtle; reference to §6-1 is recommended. If μ_k exceeded μ_s then when static friction were overcome (as the incline is raised) then it should start to move – which is impossible if f_k is large enough to cause deceleration! The bounds on μ_s are therefore given by $\tan 25^\circ > \mu_s > \mu_k$.

68. (a) The intuitive conclusion, that the tension is greatest at the bottom of the swing, is certainly supported by application of Newton's second law there:

$$T - mg = \frac{mv^2}{R} \implies T = m\left(g + \frac{v^2}{R}\right)$$

where Eq. 6-18 has been used. Increasing the speed eventually leads to the tension at the bottom of the circle reaching that breaking value of 40 N.

(b) Solving the above equation for the speed, we find

$$v = \sqrt{R\left(\frac{T}{m} - g\right)} = \sqrt{(0.91)\left(\frac{40}{0.37} - 9.8\right)}$$

which yields $v = 9.5$ m/s.

69. (a) We denote the apparent weight of the crew member of mass m on the spaceship as $W_a = 300\,\text{N}$, his weight on Earth as $W_e = mg = 600\,\text{N}$, and the radius of the spaceship as $R = 500\,\text{m}$. Since $mv_s^2/R = W_a$, we get

$$v_s = \sqrt{\frac{W_a R}{m}} = \sqrt{\left(\frac{W_a}{W_e}\right) gR}$$

where we substituted $m = W_e/g$. Thus,

$$v_s = \sqrt{\left(\frac{300\,\text{N}}{600\,\text{N}}\right)(9.8\,\text{m/s}^2)\,(500\,\text{m})} = 49.5\ \text{m/s}\ .$$

(b) For any object of mass m on the spaceship $W_a = mv^2/R \propto v^2$, where v is the speed of the circular motion of the object relative to the center of the circle. In the previous case $v = v_s = 49.5\,\text{m/s}$, and in the present case $v = 10\,\text{m/s} + 49.5\,\text{m/s} = 59.5\,\text{m/s} \equiv v'$. Thus the apparent weight of the running crew member is

$$W'_a = W_a\left(\frac{v'}{v}\right)^2 = (300\,\text{N})\left(\frac{59.5\,\text{m/s}}{49.5\,\text{m/s}}\right)^2 = 4.3 \times 10^2\ \text{N}\ .$$

70. We refer the reader to Sample Problem 6-11, and use the result Eq. 6-29:

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

with $v = 60(1000/3600) = 17$ m/s and $R = 200$ m. The banking angle is therefore $\theta = 8.1°$. Now we consider a vehicle taking this banked curve at $v' = 40(1000/3600) = 11$ m/s. Its (horizontal) acceleration is $a' = v'^2/R$, which has components parallel the incline and perpendicular to it.

$$a_{\|} = a' \cos\theta = \frac{v'^2 \cos\theta}{R} \quad \text{and} \quad a_{\perp} = a' \sin\theta = \frac{v'^2 \sin\theta}{R}$$

These enter Newton's second law as follows (choosing downhill as the $+x$ direction and away-from-incline as $+y$):

$$mg\sin\theta - f_s = ma_{\|} \quad \text{and} \quad N - mg\cos\theta = ma_{\perp}$$

and we are led to

$$\frac{f_s}{N} = \frac{mg\sin\theta - mv'^2\cos\theta/R}{mg\cos\theta + mv'^2\sin\theta/R} .$$

We cancel the mass and plug in, obtaining $f_s/N = 0.078$. The problem implies we should set $f_s = f_{s,\max}$ so that, by Eq. 6-1, we have $\mu_s = 0.078$.

71. (a) The force which provides the horizontal acceleration v^2/R necessary for the circular motion of radius $R = 0.25$ m is $T\sin\theta$, where T in the tension in the $L = 1.2$ m string and θ is the angle of the string measured from vertical. The other component of tension must equal the bob's weight so that there is no vertical acceleration: $T\cos\theta = mg$. Combining these observations leads to

$$\frac{v^2}{R} = g\tan\theta \quad \text{where} \quad \sin\theta = \frac{R}{L}$$

so that $\theta = \sin^{-1}(0.25/1.2) = 12°$ and $v = \sqrt{gR\tan\theta} = 0.72\,\text{m/s}$. It should be mentioned that Sample Problem 6-11 discusses the conical pendulum.

(b) Thus, $a = v^2/R = 2.1$ m/s^2.

(c) The tension is

$$T = \frac{mg}{\cos\theta} = \frac{(0.050)(9.8)}{\cos 12°} = 0.50 \text{ N} .$$

72. (a) Our $+x$ direction is horizontal and is chosen (as we also do with $+y$) so that the components of the 100 N force $\vec{F}$ are non-negative. Thus, $F_x = F\cos\theta = 100$ N, which the textbook denotes F_h in this problem.

(b) Since there is no vertical acceleration, application of Newton's second law in the y direction gives

$$N + F_y = mg \quad \Longrightarrow N = mg - F\sin\theta$$

where $m = 25$ kg. This yields $N = 245$ N in this case ($\theta = 0°$).

(c) Now, $F_x = F_h = F\cos\theta = 86.6$ N for $\theta = 30°$.

(d) And $N = mg - F\sin\theta = 195$ N.

(e) We find $F_x = F_h = F\cos\theta = 50$ N for $\theta = 60°$.

(f) And $N = mg - F\sin\theta = 158$ N.

(g) The condition for the chair to slide is

$$F_x > f_{s,\max} = \mu_s N \quad \text{where} \quad \mu_s = 0.42 .$$

For $\theta = 0°$, we have

$$F_x = 100\,\text{N} < f_{s,\max} = (0.42)(245) = 103\,\text{N}$$

so the crate remains at rest.

(h) For $\theta = 30.0°$, we find

$$F_x = 86.6\,\mathrm{N} > f_{s,\,\max} = (0.42)(195) = 81.9\,\mathrm{N}$$

so the crate slides.

(i) For $\theta = 60°$, we get

$$F_x = 50.0\,\mathrm{N} < f_{s,\,\max} = (0.42)(158) = 66.4\,\mathrm{N}$$

which means the crate must remain at rest.

73. We note that $N = mg$ in this situation, so $f_k = \mu_k mg = (0.32)(220) = 70.4\,\mathrm{N}$ and $f_{s,\max} = \mu_s mg = (0.41)(220) = 90.2\,\mathrm{N}$.

(a) The person needs to push at least as hard as the static friction maximum if he hopes to start it moving. Denoting his force as P, this means a value of P slightly larger than 90.2 N is sufficient. Rounding to two figures, we obtain $P = 90$ N.

(b) Constant velocity (zero acceleration) implies the push equals the kinetic friction, so $P = 70$ N.

(c) Applying Newton's second law, we have

$$P - f_k = ma \quad \Longrightarrow \quad a = \frac{\mu_s mg - \mu_k mg}{m}$$

which simplifies to $a = g(\mu_s - \mu_k) = 0.88\ \mathrm{m/s^2}$.

74. Except for replacing f_s with f_k, Fig. 6-5 in the textbook is appropriate. With that figure in mind, we choose uphill as the $+x$ direction. Applying Newton's second law to the x axis, we have

$$f_k - W\sin\theta = ma \quad \text{where} \quad m = \frac{W}{g}\ ,$$

and where $W = 40$ N, $a = +0.80\ \mathrm{m/s}^2$ and $\theta = 25°$. Thus, we find $f_k = 20$ N. Along the y axis, we have

$$\sum \vec{F}_y = 0 \quad \Longrightarrow \quad N = W\cos\theta$$

so that $\mu_k = f_k/N = 0.56$.

75. We use the familiar horizontal and vertical axes for x and y directions, with rightward and upward positive, respectively. The rope is assumed massless so that the force exerted by the child $\vec{F}$ is identical to the tension uniformly through the rope. The x and y components of $\vec{F}$ are $F\cos\theta$ and $F\sin\theta$, respectively. The static friction force points leftward.

(a) Newton's Law applied to the y axis, where there is presumed to be no acceleration, leads to

$$N + F\sin\theta - mg = 0$$

which implies that the maximum static friction is $\mu_s(mg - F\sin\theta)$. If $f_s = f_{s,\,\max}$ is assumed, then Newton's second law applied to the x axis (which also has $a = 0$ even though it is "verging" on moving) yields

$$\begin{aligned} F\cos\theta - f_s &= ma\ , \quad \text{or} \\ F\cos\theta - \mu_s(mg - F\sin\theta) &= 0 \end{aligned}$$

which we solve, for $\theta = 42°$ and $\mu_s = 0.42$, to obtain $F = 74$ N.

(b) Solving the above equation algebraically for F, with W denoting the weight, we obtain

$$F = \frac{\mu_s W}{\cos\theta + \mu_s \sin\theta}\ .$$

(c) We minimize the above expression for F by working through the $\frac{dF}{d\theta} = 0$ condition:

$$\frac{dF}{d\theta} = \frac{\mu_s W (\sin\theta - \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0$$

which leads to the result $\theta = \tan^{-1}\mu_s = 23°$.

(d) Plugging $\theta = 23°$ into the above result for F, with $\mu_s = 0.42$ and $W = 180$ N, yields $F = 70$ N.

76. (a) We note that $N = mg$ in this situation, so $f_{s,\max} = \mu_s mg = (0.52)(11)(9.8) = 56\,\text{N}$. Consequently, the horizontal force $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than 56 N.

(b) Analyzing vertical forces when $\vec{F}$ is at nonzero θ yields

$$F\sin\theta + N = mg \implies f_{s,\max} = \mu_s (mg - F\sin\theta)\ .$$

Now, the horizontal component of $\vec{F}$ needed to initiate motion must be (at minimum) slightly more than this, so

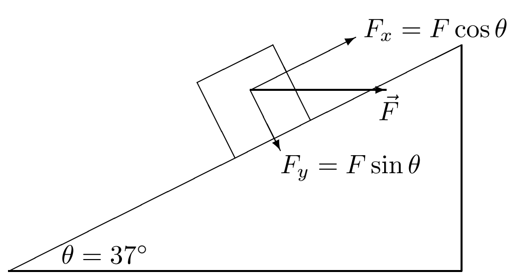
$$F\cos\theta = \mu_s (mg - F\sin\theta) \implies F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

which yields $F = 59$ N when $\theta = 60°$.

(c) We now set $\theta = -60°$ and obtain

$$F = \frac{(0.52)(11)(9.8)}{\cos(-60°) + (0.52)\sin(-60°)} = 1.1 \times 10^3\,\text{N}\ .$$

77. The coordinate system we wish to use is shown in Fig. 5-18 in the textbook, so we resolve this horizontal force into appropriate components.



(a) Applying Newton's second law to the x (directed uphill) and y (directed away from the incline surface) axes, we obtain

$$\begin{aligned} F\cos\theta - f_k - mg\sin\theta &= ma \\ N - F\sin\theta - mg\cos\theta &= 0\ . \end{aligned}$$

Using $f_k = \mu_k N$, these equations lead to

$$a = \frac{F}{m}(\cos\theta - \mu_k \sin\theta) - g(\sin\theta + \mu_k \cos\theta)$$

which yields $a = -2.1\ \text{m/s}^2$ for $\mu_k = 0.30$, $F = 50$ N and $m = 5.0$ kg.

(b) With $v_0 = +4.0$ m/s and $v = 0$, Eq. 2-16 gives

$$\Delta x = -\frac{4.0^2}{2(-2.1)} = 3.9\ \text{m}\ .$$

(c) We expect $\mu_s \geq \mu_k$; otherwise, an object started into motion would immediately start decelerating (before it gained any speed)! In the minimal expectation case, where $\mu_s = 0.30$, the maximum possible (downhill) static friction is, using Eq. 6-1,

$$f_{s,\max} = \mu_s N = \mu_s (F \sin\theta + mg\cos\theta)$$

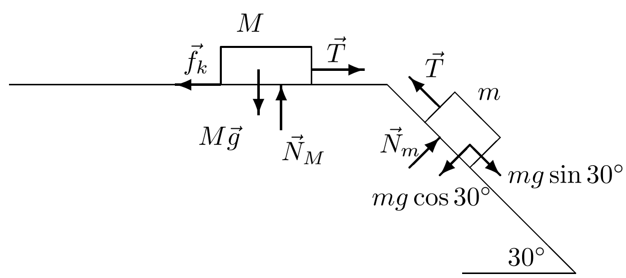
which turns out to be 21 N. But in order to have no acceleration along the x axis, we must have

$$f_s = F\cos\theta - mg\sin\theta = 10 \text{ N}$$

(the fact that this is positive reinforces our suspicion that $\vec{f_s}$ points downhill). Since the f_s needed to remain at rest is less than $f_{s,\max}$ then it stays at that location.

78. Since the problem is allowing for student creativity and research here, we only present a problem and solution for part (a).

(a) We show below two blocks M and m, the first on a horizontal surface with $\mu_k = 0.25$ and the second on a frictionless incline. They are connected by a rope (not shown) in which the tension is T. The goal is to find T given $M = 2.0$ kg and $m = 3.0$ kg. We assume f_s is not relevant to this computation.



Solution: We apply Newton's second law to each block's x axis, which for M is positive rightward and for m is positive downhill:

$$\begin{aligned} T - f_k &= Ma \\ mg\sin 30° - T &= ma \end{aligned}$$

Adding the equations, we obtain the acceleration.

$$a = \frac{mg\sin 30° - f_k}{m + M}$$

For $f_k = \mu_k N_M = \mu_k Mg$, we obtain $a = 1.96$ m/s^2. Returning this value to either of the above equations, we find $T = 8.8$ N.

79. (First problem in **Cluster 1**)
Since the block remains stationary, then $\sum \vec{F} = 0$, and we have (along the horizontal x axis) $f_s = 25$ N, where $\vec{f_s}$ points left.

80. (Second problem in **Cluster 1**)
To keep the block stationary, we require $\sum \vec{F} = 0$ (equilibrium of forces), which leads (along the horizontal x axis) to $f_s = 50$ N. Now, we take $f_s = f_{s,\max} = \mu_s N$ and find that N must equal $50/0.4 = 125$ N. Equilibrium of forces along the y axis implies $N - mg - F = 0$, so that (with $mg = 98$ N) we must have $F = 27$ N.

81. (Third problem in **Cluster 1**)
A useful diagram (where some of these forces are analyzed) is Fig. 6-5 in the textbook. Using that figure for this problem, W is the weight (equal to $mg = 98$ N), and $\theta = 25°$.

   (a) The maximum static friction is given by Eq. 6-1:

$$f_{s,\,\max} = \mu_s N = (0.60)W\cos\theta = 53 \text{ N} \quad .$$

   (b) $W\sin\theta = 41$ N.

   (c) If there is *no* motion, then $\sum\vec{F} = 0$ along the incline, so $f_s - W\sin\theta - F = 0$ (if uphill is positive). And if the system verges on motion, then $f_s = f_{s,\,\max}$. Therefore, in that case we find $F = 53 - 41 = 12$ N.

   (d) With the block sliding, with no applied force F, then Newton's second law yields $f_k - W\sin\theta = ma$ (if uphill is positive) where $f_k = \mu_k N = (0.20)W\cos\theta = 18$ N. We thus obtain $a = -2.4$ m/s^2. Therefore, the magnitude of $\vec{a}$ is 2.4 m/s^2 and the direction is downhill.

82. (Fourth problem in **Cluster 1**)
A useful diagram (where some of these forces are analyzed) is Fig. 6-5 in the textbook; however, since the block is about to move uphill, one must imagine $\vec{f_s}$ turned around (so that it points downhill). Using that figure for this problem, W is the weight (equal to $mg = 98$ N), and $\theta = 25°$.

   (a) If there is *no* motion, then $\sum\vec{F} = 0$ along the incline, so $F - f_s - W\sin\theta = 0$ (if uphill is positive). And if the system verges on motion, then $f_s = f_{s,\,\max} = \mu_s W\cos\theta = 53$ N. Therefore, in that case we find $F = 95$ N.

   (b) With the block sliding, and the applied force F still equal to the value found in part (a), then Newton's second law yields $F - f_k - W\sin\theta = ma$ (if uphill is positive) where $f_k = \mu_k N = (0.20)W\cos\theta = 18$ N. We thus obtain $a = 3.6$ m/s^2. Therefore, the magnitude of $\vec{a}$ is 3.6 m/s^2 and the direction is uphill.

   (c) With the block sliding uphill, but with no applied force F, then Newton's second law yields $-f_k - W\sin\theta = ma$ (if uphill is positive) where $f_k = 18$ N. We thus obtain $a = -5.9$ m/s^2. Therefore, the magnitude of $\vec{a}$ is 5.9 m/s^2 and the direction is downhill. It is decelerating and will ultimately come to a stop and remain at there at equilibrium.

83. (Fifth problem in **Cluster 1**)
A useful diagram (where these forces are analyzed) is Fig. 6-5 in the textbook. In that figure, W is the weight (equal to $mg = 98$ N).

   (a) Since there is no motion, then $\sum\vec{F} = 0$ along the incline, so $f_s - W\sin\theta = 0$ (if uphill is positive, which is the direction assumed for $\vec{f_s}$). We therefore obtain $f_s = 25$ N. Our result is positive, so it indeed points uphill as we had assumed. One can check that this value of f_s does not exceed the maximum possible value $f_{s,\,\max}$ (see next part).

   (b) As in part (a), we have $f_s - W\sin\theta = 0$, but since the system is on the verge of motion we also have $f_s = f_{s,\,\max} = \mu_s W\cos\theta$. Therefore,

$$\mu_s W\cos\theta - W\sin\theta = 0 \quad\Longrightarrow\quad \mu_s = \tan\theta$$

   which leads to $\theta_s = \tan^{-1}\mu_s = 31°$ (this is often called "the angle of repose").

   (c) If the block slides with no acceleration then we have $f_k - W\sin\theta = 0$ from Newton's second law applied along the incline surface. With $f_k = \mu_k W\cos\theta$ we are led to $\theta_k = \tan^{-1}\mu_k$ as the condition for this constant velocity sliding downhill. Since $\mu_k < \mu_s$ then we see that $\theta_k < \theta_s$ from part (b).

   (d) We find $\theta_k = \tan^{-1}\mu_k = 11°$.

Chapter 7

1. The kinetic energy is given by $K = \frac{1}{2}mv^2$, where m is the mass and v is the speed of the electron. The speed is therefore

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(6.7 \times 10^{-19}\,\text{J})}{9.11 \times 10^{-31}\,\text{kg}}} = 1.2 \times 10^6 \text{ m/s} .$$

2. (a) The change in kinetic energy for the meteorite would be

$$\begin{aligned} \Delta K &= K_f - K_i = -K_i = -\frac{1}{2}m_i v_i^2 \\ &= -\frac{1}{2}\left(4 \times 10^6\,\text{kg}\right)\left(15 \times 10^3\,\text{m/s}\right)^2 \\ &= -5 \times 10^{14} \text{ J} \end{aligned}$$

where the negative sign indicates that kinetic energy is lost.

(b) The energy loss in units of megatons of TNT would be

$$-\Delta K = \left(5 \times 10^{14}\,\text{J}\right)\left(\frac{1\,\text{megaton TNT}}{4.2 \times 10^{15}\,\text{J}}\right) = 0.1 \text{ megaton TNT} .$$

(c) The number of bombs N that the meteorite impact would correspond to is found by noting that megaton = 1000 kilotons and setting up the ratio:

$$N = \frac{0.1 \times 1000\,\text{kiloton TNT}}{13\,\text{kiloton TNT}} = 8 .$$

3. We convert to SI units (where necessary) and use $K = \frac{1}{2}mv^2$.

(a) $K = \frac{1}{2}(110)(8.1)^2 = 3.6 \times 10^3$ J.

(b) Since 1000 g = kg, we find

$$K = \frac{1}{2}\left(4.2 \times 10^{-3}\,\text{kg}\right)(950\,\text{m/s})^2 = 1.9 \times 10^3 \text{ J} .$$

(c) We note that the conversion from knots to m/s can be obtained from the information in Appendix D (knot = 1.688 ft/s where ft = 0.3048 m), which is also where the ton → kilogram conversion can be found. Therefore,

$$K = \frac{1}{2}\left((91400\,\text{tons})\,\frac{907.2\,\text{kg}}{\text{ton}}\right)\left((32\,\text{knots})\,\frac{0.515\,\text{m/s}}{\text{knot}}\right)^2 = 1.1 \times 10^{10} \text{ J} .$$

4. We denote the mass of the father as m and his initial speed v_i. The initial kinetic energy of the father is

$$K_i = \frac{1}{2} K_{\text{son}}$$

and his final kinetic energy (when his speed is $v_f = v_i + 1.0$ m/s) is

$$K_f = K_{\text{son}} \ .$$

We use these relations along with Eq. 7-1 in our solution.

(a) We see from the above that $K_i = \frac{1}{2} K_f$ which (with SI units understood) leads to

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left(\frac{1}{2} m (v_i + 1.0)^2 \right) \ .$$

The mass cancels and we find a second-degree equation for v_i :

$$\frac{1}{2} v_i^2 - v_i - \frac{1}{2} = 0 \ .$$

The positive root (from the quadratic formula) yields $v_i = 2.4$ m/s.

(b) From the first relation above ($K_i = \frac{1}{2} K_{\text{son}}$), we have

$$\frac{1}{2} m v_i^2 = \frac{1}{2} \left(\frac{1}{2} \left(\frac{m}{2} \right) v_{\text{son}}^2 \right)$$

and (after canceling m and one factor of 1/2) are led to $v_{\text{son}} = 2 v_i = 4.8$ m/s.

5. (a) From Table 2-1, we have $v^2 = v_0^2 + 2a\Delta x$. Thus,

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{(2.4 \times 10^7)^2 + 2\,(3.6 \times 10^{15})\,(0.035)} = 2.9 \times 10^7 \text{ m/s} \ .$$

(b) The initial kinetic energy is

$$K_i = \frac{1}{2} m v_0^2 = \frac{1}{2} \left(1.67 \times 10^{-27} \text{ kg}\right) \left(2.4 \times 10^7 \text{ m/s}\right)^2 = 4.8 \times 10^{-13} \text{ J} \ .$$

The final kinetic energy is

$$K_f = \frac{1}{2} m v^2 = \frac{1}{2} \left(1.67 \times 10^{-27} \text{ kg}\right) \left(2.9 \times 10^7 \text{ m/s}\right)^2 = 6.9 \times 10^{-13} \text{ J} \ .$$

The change in kinetic energy is $\Delta K = 6.9 \times 10^{-13} - 4.8 \times 10^{-13} = 2.1 \times 10^{-13}$ J.

6. Using Eq. 7-8 (and Eq. 3-23), we find the work done by the water on the ice block:

$$\begin{aligned} W &= \vec{F} \cdot \vec{d} \\ &= \left(210\,\hat{\imath} - 150\,\hat{\jmath}\right) \cdot \left(15\,\hat{\imath} - 12\,\hat{\jmath}\right) \\ &= (210)(15) + (-150)(-12) \\ &= 5.0 \times 10^3 \text{ J} \ . \end{aligned}$$

7. (a) The force of the worker on the crate is constant, so the work it does is given by $W_F = \vec{F} \cdot \vec{d} = Fd\cos\phi$, where $\vec{F}$ is the force, $\vec{d}$ is the displacement of the crate, and ϕ is the angle between the force and the displacement. Here $F = 210\,\text{N}$, $d = 3.0\,\text{m}$, and $\phi = 20°$. Thus $W_F = (210\,\text{N})(3.0\,\text{m})\cos 20° = 590\,\text{J}$.

(b) The force of gravity is downward, perpendicular to the displacement of the crate. The angle between this force and the displacement is 90° and $\cos 90° = 0$, so the work done by the force of gravity is zero.

(c) The normal force of the floor on the crate is also perpendicular to the displacement, so the work done by this force is also zero.

(d) These are the only forces acting on the crate, so the total work done on it is 590 J.

8. Since this involves constant-acceleration motion, we can apply the equations of Table 2-1, such as $x = v_0 t + \frac{1}{2}at^2$ (where $x_0 = 0$). We choose to analyze the third and fifth points, obtaining

$$\begin{aligned} 0.2\,\mathrm{m} &= v_0(1.0\,\mathrm{s}) + \frac{1}{2}a\,(1.0\,\mathrm{s})^2 \\ 0.8\,\mathrm{m} &= v_0(2.0\,\mathrm{s}) + \frac{1}{2}a\,(2.0\,\mathrm{s})^2 \end{aligned}$$

Simultaneous solution of the equations leads to $v_0 = 0$ and $a = 0.40\,\mathrm{m/s^2}$. We now have two ways to finish the problem. One is to compute force from $F = ma$ and then obtain the work from Eq. 7-7. The other is to find ΔK as a way of computing W (in accordance with Eq. 7-10). In this latter approach, we find the velocity at $t = 2.0$ s from $v = v_0 + at$ (so $v = 0.80$ m/s). Thus,

$$W = \Delta K = \frac{1}{2}(1.0\,\mathrm{kg})(0.80\,\mathrm{m/s})^2 = 0.32\ \mathrm{J}\ .$$

9. We choose $+x$ as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\mathrm{kg})(-2.0\,\mathrm{m/s^2})$ so that $F = |\vec{F}| = 170$ N.

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \quad \Longrightarrow \quad \Delta x = -\frac{(37\,\mathrm{m/s})^2}{2(-2.0\,\mathrm{m/s^2})}$$

which gives $\Delta x = 3.4 \times 10^2$ m. Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8 \times 10^4$ J.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\mathrm{kg})(-4.0\,\mathrm{m/s^2})$ so that $F = |\vec{F}| = 340$ N.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\mathrm{m/s})^2}{2(-4.0\,\mathrm{m/s^2})} = 1.7 \times 10^2\ \mathrm{m}\ .$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8 \times 10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

10. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is $v_i = 3.0$ m/s and the speed at $t = 4$ s is $v_f = 19$ m/s. The change in kinetic energy for the object of mass $m = 3.0$ kg is therefore

$$\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = 528\ \mathrm{J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3 \times 10^2$ J.

11. (a) The forces are constant, so the work done by any one of them is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{d}$ is the displacement. Force $\vec{F}_1$ is in the direction of the displacement, so

$$W_1 = F_1 d \cos\phi_1 = (5.00\,\text{N})(3.00\,\text{m})\cos 0^\circ = 15.0 \text{ J} .$$

Force $\vec{F}_2$ makes an angle of 120° with the displacement, so

$$W_2 = F_2 d \cos\phi_2 = (9.00\,\text{N})(3.00\,\text{m})\cos 120^\circ = -13.5 \text{ J} .$$

Force $\vec{F}_3$ is perpendicular to the displacement, so $W_3 = F_3 d \cos\phi_3 = 0$ since $\cos 90^\circ = 0$. The net work done by the three forces is

$$W = W_1 + W_2 + W_3 = 15.0\,\text{J} - 13.5\,\text{J} + 0 = +1.5 \text{ J} .$$

(b) If no other forces do work on the box, its kinetic energy increases by 1.5 J during the displacement.

12. By the work-kinetic energy theorem,

$$\begin{aligned} W &= \Delta K \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(2.0\,\text{kg})\left((6.0\,\text{m/s})^2 - (4.0\,\text{m/s})^2\right) \\ &= 20 \text{ J} . \end{aligned}$$

We note that the *directions* of $\vec{v}_f$ and $\vec{v}_i$ play no role in the calculation.

13. The forces are all constant, so the total work done by them is given by $W = F_{\text{net}}\,\Delta x$, where F_{net} is the magnitude of the net force and Δx is the magnitude of the displacement. We add the three vectors, finding the x and y components of the net force:

$$\begin{aligned} F_{\text{net}\,x} &= -F_1 - F_2 \sin 50^\circ + F_3 \cos 35^\circ \\ &= -3.00\,\text{N} - (4.00\,\text{N})\sin 35^\circ + (10.0\,\text{N})\cos 35^\circ \\ &= 2.127 \text{ N} \\ F_{\text{net}\,y} &= -F_2 \cos 50^\circ + F_3 \sin 35^\circ \\ &= -(4.00\,\text{N})\cos 50^\circ + (10.0\,\text{N})\sin 35^\circ \\ &= 3.165 \text{ N} . \end{aligned}$$

The magnitude of the net force is

$$F_{\text{net}} = \sqrt{F_{\text{net}\,x}^2 + F_{\text{net}\,y}^2} = \sqrt{2.127^2 + 3.165^2} = 3.813 \text{ N} .$$

The work done by the net force is

$$W = F_{\text{net}}\, d = (3.813\,\text{N})(4.00\,\text{m}) = 15.3 \text{ J}$$

where we have used the fact that $\vec{d} \parallel \vec{F}_{\text{net}}$ (which follows from the fact that the canister started from rest and moved horizontally under the action of horizontal forces – the resultant effect of which is expressed by $\vec{F}_{\text{net}}$).

14. In both cases, there is no acceleration, so the lifting force is equal to the weight of the object.

(a) Eq. 7-8 leads to $W = \vec{F} \cdot \vec{d} = (360\,\text{kN})(0.10\,\text{m}) = 36$ kJ.

(b) In this case, we find $W = (4000\,\text{N})(0.050\,\text{m}) = 200$ J.

15. There is no acceleration, so the lifting force is equal to the weight of the object. We note that the person's pull $\vec{F}$ is equal (in magnitude) to the tension in the cord.

    (a) As indicated in the *hint*, tension contributes twice to the lifting of the canister: $2T = mg$. Since, $|\vec{F}| = T$, we find $|\vec{F}| = 98$ N.

    (b) To rise 0.020 m, two segments of the cord (see Fig. 7-28) must shorten by that amount. Thus, the amount of string pulled down at the left end (this is the magnitude of $\vec{d}$, the downward displacement of the hand) is $d = 0.040$ m.

    (c) Since (at the left end) both $\vec{F}$ and $\vec{d}$ are downward, then Eq. 7-7 leads to $W = \vec{F} \cdot \vec{d} = (98)(0.040) =$ 3.9 J.

    (d) Since the force of gravity $\vec{F}_g$ (with magnitude mg) is opposite to the displacement $\vec{d}_c = 0.020$ m (up) of the canister, Eq. 7-7 leads to $W = \vec{F}_g \cdot \vec{d}_c = -(196)(0.020) = -3.9$ J. This is consistent with Eq. 7-15 since there is no change in kinetic energy.

16. (a) The component of the force of gravity exerted on the ice block (of mass m) along the incline is $mg \sin\theta$, where $\theta = \sin^{-1}(0.91/1.5)$ gives the angle of inclination for the inclined plane. Since the ice block slides down with uniform velocity, the worker must exert a force $\vec{F}$ "uphill" with a magnitude equal to $mg \sin\theta$. Consequently,

    $$F = mg\sin\theta = (45\,\text{kg})\left(9.8\,\text{m/s}^2\right)\left(\frac{0.91\,\text{m}}{1.5\,\text{m}}\right) = 2.7 \times 10^2\ \text{N}\ .$$

    (b) Since the "downhill" displacement is opposite to $\vec{F}$, the work done by the worker is

    $$W_1 = -\left(2.7 \times 10^2\,\text{N}\right)(1.5\,\text{m}) = -4.0 \times 10^2\ \text{J}\ .$$

    (c) Since the displacement has a vertically downward component of magnitude 0.91 m (in the same direction as the force of gravity), we find the work done by gravity to be

    $$W_2 = (45\,\text{kg})\left(9.8\,\text{m/s}^2\right)(0.91\,\text{m}) = 4.0\ \times 10^2\ \text{J}\ .$$

    (d) Since $\vec{N}$ is perpendicular to the direction of motion of the block, and $\cos 90° = 0$, work done by the normal force is $W_3 = 0$ by Eq. 7-7.

    (e) The resultant force $\vec{F}_{\text{net}}$ is zero since there is no acceleration. Thus, it's work is zero, as can be checked by adding the above results $W_1 + W_2 + W_3 = 0$.

17. (a) We use $\vec{F}$ to denote the upward force exerted by the cable on the astronaut. The force of the cable is upward and the force of gravity is mg downward. Furthermore, the acceleration of the astronaut is $g/10$ upward. According to Newton's second law, $F - mg = mg/10$, so $F = 11mg/10$. Since the force $\vec{F}$ and the displacement $\vec{d}$ are in the same direction, the work done by $\vec{F}$ is

    $$W_F = Fd = \frac{11mgd}{10} = \frac{11(72\,\text{kg})\left(9.8\,\text{m/s}^2\right)(15\,\text{m})}{10} = 1.164 \times 10^4\ \text{J}$$

    which (with respect to significant figures) should be quoted as 1.2×10^4 J.

    (b) The force of gravity has magnitude mg and is opposite in direction to the displacement. Thus, using Eq. 7-7, the work done by gravity is

    $$W_g = -mgd = -(72\,\text{kg})\left(9.8\,\text{m/s}^2\right)(15\,\text{m}) = -1.058 \times 10^4\ \text{J}$$

    which should be quoted as -1.1×10^4 J.

(c) The total work done is $W = 1.164 \times 10^4\,\mathrm{J} - 1.058 \times 10^4\,\mathrm{J} = 1.06 \times 10^3\,\mathrm{J}$. Since the astronaut started from rest, the work-kinetic energy theorem tells us that this (which we round to 1.1×10^3 J) is her final kinetic energy.

(d) Since $K = \frac{1}{2}mv^2$, her final speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06 \times 10^3\,\mathrm{J})}{72\,\mathrm{kg}}} = 5.4\ \mathrm{m/s}\ .$$

18. We use d to denote the magnitude of the spelunker's displacement during each stage. The mass of the spelunker is $m = 80.0$ kg. The work done by the lifting force is denoted W_i where $i = 1, 2, 3$ for the three stages. We apply the work-energy theorem, Eq. 17-15.

(a) For stage 1, $W_1 - mgd = \Delta K_1 = \frac{1}{2}mv_1^2$, where $v_1 = 5.00\,\mathrm{m/s}$. This gives

$$W_1 = mgd + \frac{1}{2}mv_1^2 = (80.0)(9.8)(10.0) + \frac{1}{2}(80.0)(5.00)^2 = 8.84 \times 10^3\ \mathrm{J}\ .$$

(b) For stage 2, $W_2 - mgd = \Delta K_2 = 0$, which leads to

$$W_2 = mgd = (80.0\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(10.0\,\mathrm{m}) = 7.84 \times 10^3\ \mathrm{J}\ .$$

(c) For stage 3, $W_3 - mgd = \Delta K_3 = -\frac{1}{2}mv_1^2$. We obtain

$$W_3 = mgd - \frac{1}{2}mv_1^2 = (80.0)(9.8)(10.0) - \frac{1}{2}(80.0)(5.00)^2 = 6.84 \times 10^3\ \mathrm{J}\ .$$

19. (a) We use F to denote the magnitude of the force of the cord on the block. This force is upward, opposite to the force of gravity (which has magnitude Mg). The acceleration is $\vec{a} = g/4$ downward. Taking the downward direction to be positive, then Newton's second law yields

$$\vec{F}_{\mathrm{net}} = m\vec{a} \quad \Longrightarrow \quad Mg - F = M\left(\frac{g}{4}\right)$$

so $F = 3Mg/4$. The displacement is downward, so the work done by the cord's force is $W_F = -Fd = -3Mgd/4$, using Eq. 7-7.

(b) The force of gravity is in the same direction as the displacement, so it does work $W_g = Mgd$.

(c) The total work done on the block is $-3Mgd/4 + Mgd = Mgd/4$. Since the block starts from rest, we use Eq. 7-15 to conclude that this $(Mgd/4)$ is the block's kinetic energy K at the moment it has descended the distance d.

(d) Since $K = \frac{1}{2}Mv^2$, the speed is

$$v = \sqrt{\frac{2K}{M}} = \sqrt{\frac{2\,(Mgd/4)}{M}} = \sqrt{\frac{gd}{2}}$$

at the moment the block has descended the distance d.

20. The spring constant is $k = 100$ N/m and the maximum elongation is $x_i = 5.00$ m. Using Eq. 7-25 with $x_f = 0$, the work is found to be

$$W = \frac{1}{2}kx_i^2 = \frac{1}{2}(100)(5.00)^2 = 1.25 \times 10^3\ \mathrm{J}\ .$$

21. (a) The spring constant is $k = 1500$ N/m and the elongation is $x = 0.0076$ m. Our $+x$ direction is rightward. Using Eq. 7-26, the work is found to be

$$W = -\frac{1}{2}kx^2 = -\frac{1}{2}(1500)(0.0076)^2 = -0.043\ \mathrm{J}\ .$$

(b) We use Eq. 7-25 with $x_i = x = 0.0076$ m and $x_f = 2x = 0.0152$ m to find the additional work:

$$\begin{aligned} W &= \frac{1}{2}k\left(x_i^2 - x_f^2\right) \\ &= \frac{1}{2}k\left(x^2 - 4x^2\right) \\ &= -\frac{3}{2}kx^2 \\ &= -\frac{3}{2}(1500)(0.0076)^2 = -0.13 \text{ J} . \end{aligned}$$

We note that this is greater (in magnitude) than the work done in the first interval (even though the displacements have the same magnitude), due to the fact that the force is larger throughout the second interval.

22. (a) The compression of the spring is $d = 0.12$ m. The work done by the force of gravity (acting on the block) is, by Eq. 7-12,

$$W_1 = mgd = (0.25\,\text{kg})\left(9.8\,\text{m/s}^2\right)(0.12\,\text{m}) = 0.29 \text{ J} .$$

(b) The work done by the spring is, by Eq. 7-26,

$$W_2 = -\frac{1}{2}kd^2 = -\frac{1}{2}(250\,\text{N/m})(0.12\,\text{m})^2 = -1.8 \text{ J} .$$

(c) The speed v_i of the block just before it hits the spring is found from the work-kinetic energy theorem (Eq. 7-15).

$$\Delta K = 0 - \frac{1}{2}mv_i^2 = W_1 + W_2$$

which yields

$$v_i = \sqrt{\frac{(-2)(W_1 + W_2)}{m}} = \sqrt{\frac{(-2)(0.29 - 1.8)}{0.25}} = 3.5 \text{ m/s} .$$

(d) If we instead had $v_i' = 7$ m/s, we reverse the above steps and solve for d'. Recalling the theorem used in part (c), we have

$$\begin{aligned} 0 - \frac{1}{2}mv_i'^{\,2} &= W_1' + W_2' \\ &= mgd' - \frac{1}{2}kd'^{\,2} \end{aligned}$$

which (choosing the positive root) leads to

$$d' = \frac{mg + \sqrt{m^2g^2 + mkv_i'^{\,2}}}{k}$$

which yields $d' = 0.23$ m. In order to obtain this, we have used more digits in our intermediate results than are shown above (so $v_i = \sqrt{12.048} = 3.471$ m/s and $v_i' = 6.942$ m/s).

23. (a) As the body moves along the x axis from $x_i = 3.0\,\text{m}$ to $x_f = 4.0\,\text{m}$ the work done by the force is

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x\,dx \\ &= \int_{x_i}^{x_f} -6x\,dx = -3(x_f^2 - x_i^2) \\ &= -3\left(4.0^2 - 3.0^2\right) = -21 \text{ J} . \end{aligned}$$

According to the work-kinetic energy theorem, this gives the change in the kinetic energy:

$$W = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

where v_i is the initial velocity (at x_i) and v_f is the final velocity (at x_f). The theorem yields

$$v_f = \sqrt{\frac{2W}{m} + v_i^2} = \sqrt{\frac{2(-21)}{2.0} + 8.0^2} = 6.6 \text{ m/s} .$$

(b) The velocity of the particle is $v_f = 5.0\,\text{m/s}$ when it is at $x = x_f$. The work-kinetic energy theorem is used to solve for x_f. The net work done on the particle is $W = -3(x_f^2 - x_i^2)$, so the theorem leads to

$$-3\left(x_f^2 - x_i^2\right) = \frac{1}{2}m\left(v_f^2 - v_i^2\right) .$$

Thus,

$$\begin{aligned} x_f &= \sqrt{-\frac{m}{6}\left(v_f^2 - v_i^2\right) + x_i^2} \\ &= \sqrt{-\frac{2.0\,\text{kg}}{6\,\text{N/m}}\left((5.0\,\text{m/s})^2 - (8.0\,\text{m/s})^2\right) + (3.0\,\text{m})^2} \\ &= 4.7 \text{ m} . \end{aligned}$$

24. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. Finding that area (in terms of rectangular [length×width] and triangular [$\frac{1}{2}$base×height] areas) we obtain

$$\begin{aligned} W &= W_{0<x<2} + W_{2<x<4} + W_{4<x<6} + W_{6<x<8} \\ &= 20 + 10 + 0 - 5 = 25 \text{ J} . \end{aligned}$$

25. According to the graph the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph. Numerically,

$$\alpha = \frac{20\,\text{m/s}^2}{8.0\,\text{m}} = 2.5 \text{ s}^{-2} .$$

The force on the brick is in the positive x direction and, according to Newton's second law, its magnitude is given by $F = a/m = (\alpha/m)x$. If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F\,dx = \frac{\alpha}{m}\int_0^{x_f} x\,dx = \frac{\alpha}{2m}x_f^2 = \frac{2.5}{2(10)}(8.0)^2 = 800 \text{ J} .$$

26. From Eq. 7-32, we see that the"area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length×width] and triangular [$\frac{1}{2}$base×height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be $x = 0$, where $v_0 = 4.0$ m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16$ J, we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4 \text{ J}$$

so that K_3 (the kinetic energy when $x = 3.0$ m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x\Delta x = (-4)(x_f - 3.0)$ and apply the work-kinetic energy theorem:

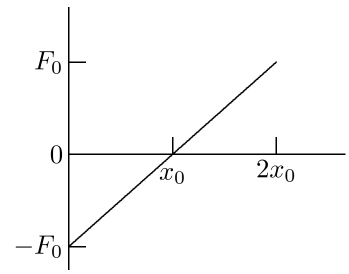
$$\begin{aligned} K_{x_f} - K_3 &= W_{3<x<x_f} \\ K_{x_f} - 12 &= (-4)(x_f - 3.0) \end{aligned}$$

so that the requirement $K_{x_f} = 8$ J leads to $x_f = 4.0$ m.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until $x = 1.0$ m. At that location, the kinetic energy is

$$\begin{aligned} K_1 &= K_0 + W_{0<x<1} \\ &= 16 + 2 \quad = \quad 18 \text{ J} . \end{aligned}$$

27. (a) The graph shows F as a function of x assuming x_0 is positive. The work is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 2x_0$. Since the area of a triangle is $\frac{1}{2}$(base)(altitude), the work done from $x = 0$ to $x = x_0$ is $-\frac{1}{2}(x_0)(F_0)$ and the work done from $x = x_0$ to $x = 2x_0$ is $\frac{1}{2}(2x_0 - x_0)(F_0) = \frac{1}{2}(x_0)(F_0)$. The total work is the sum, which is zero.



(b) The integral for the work is

$$W = \int_0^{2x_0} F_0 \left(\frac{x}{x_0} - 1 \right) dx = F_0 \left(\frac{x^2}{2x_0} - x \right) \Bigg|_0^{2x_0} = 0 .$$

28. (a) Using the work-kinetic energy theorem

$$K_f = K_i + \int_0^2 \left(2.5 - x^2\right) dx = 0 + (2.5)(2) - \frac{1}{3}(2)^3$$

we obtain $K_f = 2.3$ J.

(b) For a variable end-point, we have K_f as a function of x, which could be differentiated to find the extremum value, but we recognize that this is equivalent to solving $F = 0$ for x:

$$F = 0 \implies 2.5 - x^2 = 0$$

Thus, K is extremized at $x = \sqrt{2.5}$ and we compute

$$K_f = K_i + \int_0^{\sqrt{2.5}} \left(2.5 - x^2\right) dx = 0 + (2.5)(\sqrt{2.5}) - \frac{1}{3}(\sqrt{2.5})^3 .$$

Therefore, $K = 2.6$ J at $x = \sqrt{2.5} = 1.6$ m. Recalling our answer for part (a), it is clear that this extreme value is a maximum.

29. One approach is to assume a "path" from $\vec{r}_i$ to $\vec{r}_f$ and do the line-integral accordingly. Another approach is to simply use Eq. 7-36, which we demonstrate:

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x\, dx + \int_{y_i}^{y_f} F_y\, dy \\ &= \int_2^{-4} (2x)\, dx + \int_3^{-3} (3)\, dy \end{aligned}$$

with SI units understood. Thus, we obtain $W = 12 - 18 = -6$ J.

30. Recognizing that the force in the cable must equal the total weight (since there is no acceleration), we employ Eq. 7-47:

$$P = Fv\cos\theta = mg\left(\frac{\Delta x}{\Delta t}\right)$$

where we have used the fact that $\theta = 0°$ (both the force of the cable and the elevator's motion are upward). Thus,

$$P = \left(3.0 \times 10^3\,\mathrm{kg}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(\frac{210\,\mathrm{m}}{23\,\mathrm{s}}\right) = 2.7 \times 10^5\ \mathrm{W}\ .$$

31. The power associated with force $\vec{F}$ is given by $P = \vec{F}\cdot\vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F}\cdot\vec{v} = Fv\cos\phi = (122\,\mathrm{N})(5.0\,\mathrm{m/s})\cos 37° = 490\ \mathrm{W}\ .$$

32. (a) Using Eq.7-48 and Eq. 3-23, we obtain

$$P = \vec{F}\cdot\vec{v} = (4.0\,\mathrm{N})(-2.0\,\mathrm{m/s}) + (9.0\,\mathrm{N})(4.0\,\mathrm{m/s}) = 28\ \mathrm{W}\ .$$

(b) We again use Eq.7-48 and Eq. 3-23, but with a one-component velocity: $\vec{v} = v\hat{\mathrm{j}}$.

$$\begin{aligned} P &= \vec{F}\cdot\vec{v} \\ -12\,\mathrm{W} &= (-2.0\,\mathrm{N})v \end{aligned}$$

which yields $v = 6$ m/s.

33. (a) The power is given by $P = Fv$ and the work done by $\vec{F}$ from time t_1 to time t_2 is given by

$$W = \int_{t_1}^{t_2} P\,\mathrm{d}t = \int_{t_1}^{t_2} Fv\,\mathrm{d}t\,.$$

Since $\vec{F}$ is the net force, the magnitude of the acceleration is $a = F/m$, and, since the initial velocity is $v_0 = 0$, the velocity as a function of time is given by $v = v_0 + at = (F/m)t$. Thus

$$W = \int_{t_1}^{t_2} (F^2/m)t\,\mathrm{d}t = \frac{1}{2}(F^2/m)(t_2^2 - t_1^2)\,.$$

For $t_1 = 0$ and $t_2 = 1.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\,\mathrm{N})^2}{15\,\mathrm{kg}}\right)(1.0\,\mathrm{s})^2 = 0.83\,\mathrm{J}\ .$$

(b) For $t_1 = 1.0\,\mathrm{s}$ and $t_2 = 2.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\,\mathrm{N})^2}{15\,\mathrm{kg}}\right)\left((2.0\,\mathrm{s})^2 - (1.0\,\mathrm{s})^2\right) = 2.5\ \mathrm{J}\ .$$

(c) For $t_1 = 2.0\,\mathrm{s}$ and $t_2 = 3.0\,\mathrm{s}$,

$$W = \frac{1}{2}\left(\frac{(5.0\,\mathrm{N})^2}{15\,\mathrm{kg}}\right)\left((3.0\,\mathrm{s})^2 - (2.0\,\mathrm{s})^2\right) = 4.2\ \mathrm{J}\ .$$

(d) Substituting $v = (F/m)t$ into $P = Fv$ we obtain $P = F^2t/m$ for the power at any time t. At the end of the third second

$$P = \frac{(5.0\,\mathrm{N})^2(3.0\,\mathrm{s})}{15\,\mathrm{kg}} = 5.0\ \mathrm{W}\ .$$

34. (a) Since constant speed implies $\Delta K = 0$, we require $W_a = -W_g$, by Eq. 7-15. Since W_g is the same in both cases (same weight and same path), then $W_a = 900$ J just as it was in the first case.

(b) Since the speed of 1.0 m/s is constant, then 8.0 meters is traveled in 8.0 seconds. Using Eq. 7-42, and noting that average power is *the* power when the work is being done at a steady rate, we have

$$P = \frac{W}{\Delta t} = \frac{900 \text{ J}}{8.0 \text{ s}}$$

which results in $P = 113$ W.

(c) Since the speed of 2.0 m/s is constant, 8.0 meters is traveled in 4.0 seconds. Using Eq. 7-42, with *average power* replaced by *power*, we have

$$P = \frac{W}{\Delta t} = \frac{900 \text{ J}}{4.0 \text{ s}}$$

from which we obtain $P = 225$ W.

35. The total work is the sum of the work done by gravity on the elevator, the work done by gravity on the counterweight, and the work done by the motor on the system: $W_T = W_e + W_c + W_s$. Since the elevator moves at constant velocity, its kinetic energy does not change and according to the work-kinetic energy theorem the total work done is zero. This means $W_e + W_c + W_s = 0$. The elevator moves upward through 54 m, so the work done by gravity on it is

$$W_e = -m_e g d = -(1200\,\text{kg})(9.8\,\text{m/s}^2)(54\,\text{m}) = -6.35 \times 10^5\,\text{J}\,.$$

The counterweight moves downward the same distance, so the work done by gravity on it is

$$W_c = m_c g d = (950\,\text{kg})\left(9.8\,\text{m/s}^2\right)(54\,\text{m}) = 5.03 \times 10^5 \text{ J}\,.$$

Since $W_T = 0$, the work done by the motor on the system is

$$W_s = -W_e - W_c = 6.35 \times 10^5\,\text{J} - 5.03 \times 10^5\,\text{J} = 1.32 \times 10^5 \text{ J}\,.$$

This work is done in a time interval of $\Delta t = 3.0\,\text{min} = 180\,\text{s}$, so the power supplied by the motor to lift the elevator is

$$P = \frac{W_s}{\Delta t} = \frac{1.32 \times 10^5\,\text{J}}{180\,\text{s}} = 7.4 \times 10^2 \text{ W}\,.$$

36. (a) Since the force exerted by the spring on the mass is zero when the mass passes through the equilibrium position of the spring, the rate at which the spring is doing work on the mass at this instant is also zero.

(b) The rate is given by $P = \vec{F}\cdot\vec{v} = -Fv$, where the minus sign corresponds to the fact that $\vec{F}$ and $\vec{v}$ are antiparallel to each other. The magnitude of the force is given by $F = kx = (500\,\text{N/m})(0.10\,\text{m}) = 50\,\text{N}$, while v is obtained from conservation of energy for the spring-mass system:

$$E = K + U = 10\,\text{J} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.30\,\text{kg})v^2 + \frac{1}{2}(500\,\text{N/m})(0.10\,\text{m})^2$$

which gives $v = 7.1\,\text{m/s}$. Thus

$$P = -Fv = -(50\,\text{N})(7.1\,\text{m/s}) = -3.5\,\times 10^2\,\text{W}\,.$$

37. We write the force as $F = \alpha v$, where v is the speed and α is a constant of proportionality. The power required is $P = Fv = \alpha v^2$. Let P_1 be the power required for speed v_1 and P_2 be the power required for speed v_2. Dividing $P_2 = \alpha v_2^2$ by $P_1 = \alpha v_1^2$, we find

$$P_2 = \left(\frac{v_2}{v_1}\right)^2 P_1\,.$$

Since $P_1 = 7.5\,\text{kW}$ and $v_2 = 3v_1$,

$$P_2 = (3)^2(7.5\,\text{kW}) = 68 \text{ kW}\,.$$

38. (a) The force $\vec{F}$ of the incline is a combination of normal and friction force which is serving to "cancel" the tendency of the box to fall downward (due to its 19.6 N weight). Thus, $\vec{F} = mg$ upward. In this part of the problem, the angle ϕ between the belt and $\vec{F}$ is 80°. From Eq. 7-47, we have

$$P = Fv\cos\phi = (19.6)(0.50)\cos 80^\circ$$

which leads to $P = 1.7$ W.

(b) Now the angle between the belt and $\vec{F}$ is 90°, so that $P = 0$.

(c) In this part, the angle between the belt and $\vec{F}$ is 100°, so that $P = (19.6)(0.50)\cos 100^\circ = -1.7$ W.

39. (a) In 10 min the cart moves

$$\left(6.0\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right)(10\,\text{min}) = 5280\text{ ft}$$

so that Eq. 7-7 yields

$$W = Fd\cos\phi = (40\,\text{lb})(5280\,\text{ft})\cos 30^\circ = 1.8\times 10^5\text{ ft}\cdot\text{lb}\ .$$

(b) The average power is given by Eq. 7-42, and the conversion to horsepower (hp) can be found on the inside back cover. We note that 10 min is equivalent to 600 s.

$$P_{\text{avg}} = \frac{1.8\times 10^5\text{ ft}\cdot\text{lb}}{600\,\text{s}} = 305\text{ ft}\cdot\text{lb/s}$$

which (upon dividing by 550) converts to $P_{\text{avg}} = 0.55$ hp.

40. The acceleration is constant, so we may use the equations in Table 2-1. We choose the direction of motion as $+x$ and note that the displacement is the same as the distance traveled, in this problem. We designate the force (assumed singular) along the x direction acting on the $m = 2.0$ kg object as F.

(a) With $v_0 = 0$, Eq. 2-11 leads to $a = v/t$. And Eq. 2-17 gives $\Delta x = \frac{1}{2}vt$ Newton's second law yields the force $F = ma$. Eq. 7-8, then, gives the work:

$$W = F\Delta x = m\left(\frac{v}{t}\right)\left(\frac{1}{2}vt\right) = \frac{1}{2}mv^2$$

as we expect from the work-kinetic energy theorem. With $v = 10$ m/s, this yields $W = 100$ J.

(b) Instantaneous power is defined in Eq. 7-48. With $t = 3.0$ s, we find

$$P = Fv = m\left(\frac{v}{t}\right)v = 67\text{ W}\ .$$

(c) The velocity at $t' = 1.5$ s is $v' = at' = 5.0$ m/s. Thus,

$$P' = Fv' = 33\text{ W}\ .$$

41. The total weight is $(100)(660) = 6.6\times 10^4$ N, and the words "raises ... at constant speed" imply zero acceleration, so the lift-force is equal to the total weight. Thus $P = Fv = (6.6\times 10^4)(150/60) = 1.65\times 10^5$ W.

42. Using Eq. 7-32, we find

$$W = \int_{0.25}^{1.25} e^{-4x^2}\,dx = 0.21\text{ J}$$

where the result has been obtained numerically. Many modern calculators have that capability, as well as most math software packages that a great many students have access to.

43. (a) and (b) Hooke's law and the work done by a spring is discussed in the chapter. We apply Work-kinetic energy theorem, in the form of $\Delta K = W_a + W_s$, to the points in Figure 7-48 at $x = 1.0$ m and $x = 2.0$ m, respectively. The "applied" work W_a is that due to the constant force $\vec{P}$.

$$\begin{aligned} 4 &= P(1.0) - \frac{1}{2}k(1.0)^2 \\ 0 &= P(2.0) - \frac{1}{2}k(2.0)^2 \end{aligned}$$

Simultaneous solution leads to $P = 8.0$ N and $k = 8.0$ N/m.

44. Using Eq. 7-8, we find

$$W = \vec{F} \cdot \vec{d} = \left(F\cos\theta\,\hat{\imath} + F\sin\theta\,\hat{\jmath}\right) \cdot \left(x\hat{\imath} + y\hat{\jmath}\right) = Fx\cos\theta + Fy\sin\theta$$

where $x = 2.0$ m, $y = -4.0$ m, $F = 10$ N, and $\theta = 150°$. Thus, we obtain $W = -37$ J. Note that the given mass value (2.0 kg) is not used in the computation.

45. (a) Estimating the initial speed from the slope of the graph near the origin is somewhat difficult, and it may be simpler to determine it from the constant-acceleration equations from chapter 2: $v = v_0 + at$ and $x = v_0 + \frac{1}{2}at^2$, where $x_0 = 0$ has been used. Applying these to the last point on the graph (where the slope is apparently zero) or applying just the x equation to any two points on the graph, leads to a pair of simultaneous equations from which $a = -2$ m/s^2 and $v_0 = 10$ m/s can be found. Then,

$$K_0 = \frac{1}{2}mv_0^2 = 2.5 \times 10^3 \text{ J} = 2.5 \text{ kJ} .$$

(b) The speed at $t = 3.0$ s is obtained by

$$v = v_0 + at = 10 + (-2)(3) = 4 \text{ m/s}$$

or by estimating the slope from the graph (not recommended). Then the work-kinetic energy theorem yields

$$W = \Delta K = \frac{1}{2}(50 \text{ kg})(4 \text{ m/s})^2 - 2.5 \times 10^3 \text{ J} = -2.1 \text{ kJ} .$$

46. (a) Using Eq. 7-8 and SI units, we find

$$W = \vec{F} \cdot \vec{d} = \left(2\,\hat{\imath} - 4\,\hat{\jmath}\right) \cdot \left(8\hat{\imath} + c\hat{\jmath}\right) = 16 - 4c$$

which, if equal zero, implies $c = 16/4 = 4$ m.

(b) If $W > 0$ then $16 > 4c$, which implies $c < 4$ m.

(c) If $W < 0$ then $16 < 4c$, which implies $c > 4$ m.

47. With speed $v = 11200$ m/s, we find

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.9 \times 10^5\right)(11200)^2 = 1.8 \times 10^{13} \text{ J} .$$

48. (a) Hooke's law and the work done by a spring is discussed in the chapter. Taking absolute values, and writing that law in terms of differences ΔF and Δx, we analyze the first two pictures as follows:

$$\begin{aligned} |\Delta F| &= k\,|\Delta x| \\ 240\,\text{N} - 110\,\text{N} &= k(60\,\text{mm} - 40\,\text{mm}) \end{aligned}$$

which yields $k = 6.5$ N/mm. Designating the relaxed position (as read by that scale) as x_o we look again at the first picture:

$$110\,\text{N} = k\,(40\,\text{mm} - x_o)$$

which (upon using the above result for k) yields $x_o = 23$ mm.

(b) Using the results from part (a) to analyze that last picture, we find

$$W = k\,(30\,\text{mm} - x_\text{o}\,) = 45 \text{ N} .$$

49. (a) To hold the crate at equilibrium in the final situation, $\vec{F}$ must have the same magnitude as the horizontal component of the rope's tension $T\sin\theta$, where θ is the angle between the rope (in the final position) and vertical:

$$\theta = \sin^{-1}\left(\frac{4.00}{12.0}\right) = 19.5^\circ .$$

But the vertical component of the tension supports against the weight: $T\cos\theta = mg$. Thus, the tension is $T = (230)(9.8)/\cos 19.5^\circ = 2391\,\text{N}$ and $F = (2391)\sin 19.5^\circ = 797\,\text{N}$. An alternative approach based on drawing a vector triangle (of forces) in the final situation provides a quick solution.

(b) Since there is no change in kinetic energy, the net work on it is zero.

(c) The work done by gravity is $W_g = \vec{F}_g \cdot \vec{d} = -mgh$, where $h = L(1-\cos\theta)$ is the vertical component of the displacement. With $L = 12.0$ m, we obtain $W_g = -1547$ J which should be rounded to three figures: -1.55 kJ.

(d) The tension vector is everywhere perpendicular to the direction of motion, so its work is zero (since $\cos 90^\circ = 0$).

(e) The implication of the previous three parts is that the work due to $\vec{F}$ is $-W_g$ (so the net work turns out to be zero). Thus, $W_F = -W_g = 1.55$ kJ.

(f) Since $\vec{F}$ does not have constant magnitude, we cannot expect Eq. 7-8 to apply.

50. (a) In the work-kinetic energy theorem, we include both the work due to an applied force W_a and work done by gravity W_g in order to find the latter quantity.

$$\Delta K = W_a + W_g \quad\Longrightarrow\quad 30 = (100)(1.8)\cos 180^\circ + W_g$$

leading to $W_g = 210$ J.

(b) The value of W_g obtained in part (a) still applies since the weight and the path of the child remain the same, so $\Delta K = W_g = 210$ J.

51. Using Eq. 7-7, we have $W = Fd\cos\phi = 1504$ J. Then, by the work-kinetic energy theorem, we find the kinetic energy $K_f = K_i + W = 0 + 1504$ J. The answer is therefore 1.5 kJ.

52. (a) Before the cord is cut, each spring (which might be described as being "in series" in this case) is stretched by the force $F = 100$ N. Thus, each spring is stretched by $x = 100/500 = 0.20\,\text{m}$ in the initial configuration. Since the relaxed length of each spring is 0.50 m, then the full length of each spring in the initial configuration is $0.20 + 0.50 = 0.70$ m. Therefore (including that 0.10 m length of string) the distance from the box to the ceiling is $2(0.70) + 0.10 = 1.50$ m, before the string is cut. In the moments after the short string is cut, there is some "transient motion" that is difficult to analyze, but after it has settled down again (in its new equilibrium position) the springs (which now might be described as being "in parallel") are sharing half the weight, so the force stretching each one is $F/2 = 50$ N. This means the elongation of each is $x/2 = 0.10$ m. The total distance (recalling that the longer cords are each of length 0.85 m) of the box to the ceiling is now $0.85 + 0.10 + 0.50 = 1.45$ m. Thus, the box is closer to the ceiling now than it was before. It has moved up.

(b) The distance moved up by the box is $d = 1.50 - 1.45 = 0.05$ m.

(c) To avoid worrying about friction-related (dissipative) processes which are involved in making the "transient motion" ultimately disappear, we consider that the person who cut the cord (and has predicted the new equilibrium position) very carefully and gradually moves it up to that new position, in which case the work being done on the system is due to the person. In this variation of the problem, it is easy to see that the work done by the person *against gravity* is $-W_g = mgd = 5.0$ J (though this is not the full work done by the person, since Eq. 7-25 hasn't been used). Returning to the problem in its original form, we can say that the work done on the block in raising it the distance d is 5.0 J, regardless of the agent doing the work (and in its original form, that agent is the pair of springs, and this represents part of the full work they do).

53. (a) We set up the ratio

$$\frac{50\,\text{km}}{1\,\text{km}} = \left(\frac{E}{1\,\text{megaton}}\right)^{1/3}$$

and find $E = 50^3 \approx 1 \times 10^5$ megatons of TNT.

(b) We note that 15 kilotons is equivalent to 0.015 megatons. Dividing the result from part (a) by 0.013 yields about ten million bombs.

54. (a) With SI units (and three significant figures) understood, the object's displacement is

$$\vec{d} = \vec{d_f} - \vec{d_i} = -8\,\hat{\text{i}} + 6\,\hat{\text{j}} + 2\,\hat{\text{k}}\ .$$

Thus, Eq. 7-8 gives

$$W = \vec{F} \cdot \vec{d} = (3)(-8) + (7)(6) + (7)(2) = 32.0 \text{ J}\ .$$

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32}{4} = 8.00 \text{ W}\ .$$

(c) The distance from the coordinate origin to the initial position is $d_i = \sqrt{3^2 + (-2)^2 + 5^2} = 6.16$ m, and the magnitude of the distance from the coordinate origin to the final position is $d_f = \sqrt{(-5)^2 + 4^2 + 7^2} = 9.49$ m. Their scalar (dot) product is

$$\vec{d_i} \cdot \vec{d_f} = (3)(-5) + (-2)(4) + (5)(7) = 12.0 \text{ m}^2\ .$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d_i} \cdot \vec{d_f}}{d_i\, d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right)$$

which yields $\phi = 78°$.

Chapter 8

1. The potential energy stored by the spring is given by $U = \frac{1}{2}kx^2$, where k is the spring constant and x is the displacement of the end of the spring from its position when the spring is in equilibrium. Thus

$$k = \frac{2U}{x^2} = \frac{2(25\,\text{J})}{(0.075\,\text{m})^2} = 8.9 \times 10^3 \text{ N/m} .$$

2. (a) Noting that the vertical displacement is $10.0 - 1.5 = 8.5$ m downward (same direction as $\vec{F}_g$), Eq. 7-12 yields

$$W_g = mgd\cos\phi = (2.00)(9.8)(8.5)\cos 0^\circ = 167 \text{ J} .$$

   (b) One approach (which is fairly trivial) is to use Eq. 8-1, but we feel it is instructive to instead calculate this as ΔU where $U = mgy$ (with upwards understood to be the $+y$ direction).

$$\Delta U = mgy_f - mgy_i = (2.00)(9.8)(1.5) - (2.00)(9.8)(10.0) = -167 \text{ J} .$$

   (c) In part (b) we used the fact that $U_i = mgy_i = 196$ J.

   (d) In part (b), we also used the fact $U_f = mgy_f = 29$ J.

   (e) The computation of W_g does not use the new information (that $U = 100$ J at the ground), so we again obtain $W_g = 167$ J.

   (f) As a result of Eq. 8-1, we must again find $\Delta U = -W_g = -167$ J.

   (g) With this new information (that $U_0 = 100$ J where $y = 0$) we have $U_i = mgy_i + U_0 = 296$ J.

   (h) With this new information (that $U_0 = 100$ J where $y = 0$) we have $U_f = mgy_f + U_0 = 129$ J. We can check part (f) by subtracting the new U_i from this result.

3. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the flake, so this reduces to $W = mgh$, where h is the height from which the flake falls. This is equal to the radius r of the bowl. Thus

$$W = mgr = (2.00 \times 10^{-3}\,\text{kg})(9.8\,\text{m/s}^2)(22.0 \times 10^{-2}\,\text{m}) = 4.31 \times 10^{-3} \text{ J} .$$

   (b) The force of gravity is conservative, so the change in gravitational potential energy of the flake-Earth system is the negative of the work done: $\Delta U = -W = -4.31 \times 10^{-3}$ J.

   (c) The potential energy when the flake is at the top is greater than when it is at the bottom by $|\Delta U|$. If $U = 0$ at the bottom, then $U = +4.31 \times 10^{-3}$ J at the top.

   (d) If $U = 0$ at the top, then $U = -4.31 \times 10^{-3}$ J at the bottom.

   (e) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

4. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and A is horizontal, so $\phi = 90°$ and $W_g = 0$ (since $\cos 90° = 0$).

(b) The displacement between the initial point and B has a vertical component of $h/2$ downward (same direction as $\vec{F}_g$), so we obtain $W_g = \vec{F}_g \cdot \vec{d} = mgh/2$.

(c) The displacement between the initial point and C has a vertical component of h downward (same direction as $\vec{F}_g$), so we obtain $W_g = \vec{F}_g \cdot \vec{d} = mgh$.

(d) With the reference position at C, we obtain $U_B = mgh/2$.

(e) Similarly, we find $U_A = mgh$.

(f) All the answers are proportional to the mass of the object. If the mass is doubled, all answers are doubled.

5. (a) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length L of the rod, so the work done by the force of gravity is $W = mgL$.

(b) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to L, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is $W = -mgL$.

(c) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement.

(d) The force of gravity is conservative. The change in the gravitational potential energy of the ball-Earth system is the negative of the work done by gravity: $\Delta U = -mgL$ as the ball goes to the lowest point

(e) Continuing this line of reasoning, we find $\Delta U = mgL$ as it goes to the highest point.

(f) Continuing this line of reasoning, we have $\Delta U = 0$ as it goes to the point at the same height.

(g) The change in the gravitational potential energy depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the potential energy is the *same* since the initial and final positions are the same.

6. We use Eq. 7-12 for W_g and Eq. 8-9 for U.

(a) The displacement between the initial point and Q has a vertical component of $h - R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain $W_g = \vec{F}_g \cdot \vec{d} = 4mgR$.

(b) The displacement between the initial point and the top of the loop has a vertical component of $h - 2R$ downward (same direction as $\vec{F}_g$), so (with $h = 5R$) we obtain $W_g = \vec{F}_g \cdot \vec{d} = 3mgR$.

(c) With $y = h = 5R$, we find $U = 5mgR$ at P.

(d) With $y = R$, we find $U = mgR$ at Q.

(e) With $y = 2R$, we find $U = 2mgR$ at the top of the loop.

(f) The new information ($v_i \neq 0$) is not involved in any of the preceding computations; the above results are unchanged.

7. (a) The force of gravity is constant, so the work it does is given by $W = \vec{F} \cdot \vec{d}$, where $\vec{F}$ is the force and $\vec{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the snowball. The expression for the work reduces to $W = mgh$, where h is the height through which the snowball drops. Thus

$$W = mgh = (1.50\,\text{kg})(9.8\,\text{m/s}^2)(12.5\,\text{m}) = 184\ \text{J}\ .$$

(b) The force of gravity is conservative, so the change in the potential energy of the snowball-Earth system is the negative of the work it does: $\Delta U = -W = -184\,\mathrm{J}$.

(c) The potential energy when it reaches the ground is less than the potential energy when it is fired by $|\Delta U|$, so $U = -184\,\mathrm{J}$ when the snowball hits the ground.

8. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the swing) $h = L - L\cos\theta$ (for angle θ measured from vertical as shown in Fig. 8-29). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U).

(a) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$W_g = \vec{F}_g \cdot \vec{d} = mgh = mgL\,(1 - \cos\theta)\ .$$

(b) From Eq. 8-1, we have $\Delta U = -W_g = -mgL(1 - \cos\theta)$.

(c) With $y = h$, Eq. 8-9 yields $U = mgL(1 - \cos\theta)$.

(d) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos\theta$ decreases so that $1 - \cos\theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.

9. (a) If K_i is the kinetic energy of the flake at the edge of the bowl, K_f is its kinetic energy at the bottom, U_i is the gravitational potential energy of the flake-Earth system with the flake at the top, and U_f is the gravitational potential energy with it at the bottom, then $K_f + U_f = K_i + U_i$. Taking the potential energy to be zero at the bottom of the bowl, then the potential energy at the top is $U_i = mgr$ where $r = 0.220$ m is the radius of the bowl and m is the mass of the flake. $K_i = 0$ since the flake starts from rest. Since the problem asks for the speed at the bottom, we write $\frac{1}{2}mv^2$ for K_f. Energy conservation leads to

$$mgr = \frac{1}{2}mv^2 \implies v = \sqrt{2gr} = \sqrt{2(9.8)(0.220)} = 2.08 \text{ m/s}\ .$$

(b) We note that the expression for the speed ($v = \sqrt{2gr}$) does not contain the mass of the flake. The speed would be the same, 2.08 m/s, regardless of the mass of the flake.

(c) The final kinetic energy is given by $K_f = K_i + U_i - U_f$. Since K_i is greater than before, K_f is greater. This means the final speed of the flake is greater.

10. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to exercise 2 (to which this problem refers), we found $U_i = mgy_i = 196$ J and $U_f = mgy_f = 29$ J (assuming the reference position is at the ground). Since $K_i = 0$ in this case, we have

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 + 196 &= K_f + 29 \end{aligned}$$

which gives $K_f = 167$ J and thus leads to

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(167)}{2.00}} = 12.9 \text{ m/s}\ .$$

(b) If we proceed algebraically through the calculation in part (a), we find $K_f = -\Delta U = mgh$ where $h = y_i - y_f$ and is positive-valued. Thus,

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gh}$$

as we might also have derived from the equations of Table 2-1 (particularly Eq. 2-16). The fact that the answer is independent of mass means that the answer to part (b) is identical to that of part (a).

(c) If $K_i \neq 0$, then we find $K_f = mgh + K_i$ (where K_i is necessarily positive-valued). This represents a larger value for K_f than in the previous parts, and thus leads to a larger value for v.

11. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to exercise 5 (to which this problem refers), we found $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ K_{\text{top}} - K_0 + mgL &= 0 \end{aligned}$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL}\ .$$

(b) We also found in the solution to exercise 5 that the potential energy change is $\Delta U = -mgL$ in going from the initial point to the lowest point (the bottom). Thus,

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ K_{\text{bottom}} - K_0 - mgL &= 0 \end{aligned}$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{\frac{2K_{\text{bottom}}}{m}} = \sqrt{4gL}$$

which simplifies to $2\sqrt{gL}$.

(c) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U = 0$, which implies $\Delta K = 0$. Consequently, the speed is the same as what it was initially ($\sqrt{2gL}$).

(d) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.

12. We use Eq. 8-17, representing the conservation of mechanical energy (which neglects friction and other dissipative effects).

(a) In the solution to exercise 4, we found $U_A = mgh$ (with the reference position at C). Referring again to Fig. 8-26, we see that this is the same as U_0 which implies that $K_A = K_0$ and thus that $v_A = v_0$.

(b) In the solution to exercise 4, we also found $U_B = mgh/2$. In this case, we have

$$\begin{aligned} K_0 + U_0 &= K_B + U_B \\ \frac{1}{2}mv_0^2 + mgh &= \frac{1}{2}mv_B^2 + mg\left(\frac{h}{2}\right) \end{aligned}$$

which leads to $v_B = \sqrt{v_0^2 + gh}$.

(c) Similarly, $v_C = \sqrt{v_0^2 + 2gh}$.

(d) To find the "final" height, we set $K_f = 0$. In this case, we have

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ \frac{1}{2}mv_0^2 + mgh &= 0 + mgh_f \end{aligned}$$

which leads to $h_f = h + v_0^2/2g$.

(e) It is evident that the above results do not depend on mass. Thus, a different mass for the coaster must lead to the same results.

13. We neglect any work done by friction. We work with SI units, so the speed is converted: $v = 130(1000/3600) = 36.1$ m/s.

(a) We use Eq. 8-17: $K_f + U_f = K_i + U_i$ with $U_i = 0$, $U_f = mgh$ and $K_f = 0$. Since $K_i = \frac{1}{2}mv^2$, where v is the initial speed of the truck, we obtain

$$\frac{1}{2}mv^2 = mgh \implies h = \frac{v^2}{2g} = \frac{36.1^2}{2(9.8)} = 66.5 \text{ m} .$$

If L is the length of the ramp, then $L \sin 15° = 66.5$ m so that $L = 66.5/\sin 15° = 257$ m. Therefore, the ramp must be about 260 m long if friction is negligible.

(b) The answers do not depend on the mass of the truck. They remain the same if the mass is reduced.

(c) If the speed is decreased, h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).

14. We use Eq. 8-18, representing the conservation of mechanical energy. The reference position for computing U is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) In the solution to problem 8 (to which this problem refers), we found $U = mgL(1 - \cos\theta)$ at the position shown in Fig. 8-29 (which we consider to be the initial position). Thus, we have

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 + mgL(1 - \cos\theta) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to

$$v = \sqrt{\frac{2mgL(1 - \cos\theta)}{m}} = \sqrt{2gL(1 - \cos\theta)} .$$

Plugging in $L = 2.00$ m and $\theta = 30.0°$ we find $v = 2.29$ m/s.

(b) It is evident that the result for v does not depend on mass. Thus, a different mass for the ball must not change the result.

15. We use Eq. 8-18, representing the conservation of mechanical energy. We choose the reference position for computing U to be at the ground below the cliff; it is also regarded as the "final" position in our calculations.

(a) Using Eq. 8-9, the initial potential energy is $U_i = mgh$ where $h = 12.5$ m and $m = 1.50$ kg. Thus, we have

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ \frac{1}{2}mv_i^2 + mgh &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to the speed of the snowball at the instant before striking the ground:

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_i^2 + mgh\right)} = \sqrt{v_i^2 + 2gh}$$

where $v_i = 14.0$ m/s is the magnitude of its initial velocity (not just one component of it). Thus we find $v = 21.0$ m/s.

(b) As noted above, v_i is the magnitude of its initial velocity and not just one component of it; therefore, there is no dependence on launch angle. The answer is again 21.0 m/s.

(c) It is evident that the result for v in part (a) does not depend on mass. Thus, changing the mass of the snowball does not change the result for v.

16. We convert to SI units and choose upward as the $+y$ direction. Also, the relaxed position of the top end of the spring is the origin, so the initial compression of the spring (defining an equilibrium situation between the spring force and the force of gravity) is $y_0 = -0.100$ m and the additional compression brings it to the position $y_1 = -0.400$ m.

(a) When the stone is in the equilibrium ($a = 0$) position, Newton's second law becomes

$$\begin{aligned} \vec{F}_{\text{net}} &= ma \\ F_{\text{spring}} - mg &= 0 \\ -k(-0.100) - (8.00)(9.8) &= 0 \end{aligned}$$

where Hooke's law (Eq. 7-21) has been used. This leads to a spring constant equal to $k = 784$ N/m.

(b) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upwards, turning some of its elastic potential energy (stored in the spring) into kinetic energy. The amount of elastic potential energy at the moment of release is, using Eq. 8-11,

$$U = \frac{1}{2}ky_1^2 = \frac{1}{2}(784)(-0.400)^2 = 62.7 \text{ J} .$$

(c) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the gravitational potential energy, then

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + \frac{1}{2}ky_1^2 &= 0 + mgh \end{aligned}$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the gravitational potential energy) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

(d) We find $h = ky_1^2/2mg = 0.800$ m, or 80.0 cm.

17. We take the reference point for gravitational potential energy at the position of the marble when the spring is compressed.

(a) The gravitational potential energy when the marble is at the top of its motion is $U_g = mgh$, where $h = 20$ m is the height of the highest point. Thus,

$$U_g = \left(5.0 \times 10^{-3}\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right)(20\,\text{m}) = 0.98 \text{ J} .$$

(b) Since the kinetic energy is zero at the release point and at the highest point, then conservation of mechanical energy implies $\Delta U_g + \Delta U_s = 0$, where ΔU_s is the change in the spring's elastic potential energy. Therefore, $\Delta U_s = -\Delta U_g = -0.98$ J.

(c) We take the spring potential energy to be zero when the spring is relaxed. Then, our result in the previous part implies that its initial potential energy is $U_s = 0.98\,\text{J}$. This must be $\frac{1}{2}kx^2$, where k is the spring constant and x is the initial compression. Consequently,

$$k = \frac{2U_s}{x^2} = \frac{2(0.98\,\text{J})}{(0.080\,\text{m})^2} = 3.1 \times 10^2\ \text{N/m} = 3.1\ \text{N/cm}\ .$$

18. We use Eq. 8-18, representing the conservation of mechanical energy (which neglects friction and other dissipative effects). The reference position for computing U (and height h) is the lowest point of the swing; it is also regarded as the "final" position in our calculations.

(a) Careful examination of the figure leads to the trigonometric relation $h = L - L\cos\theta$ when the angle is measured from vertical as shown. Thus, the gravitational potential energy is $U = mgL(1-\cos\theta_0)$ at the position shown in Fig. 8-32 (the initial position). Thus, we have

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

which leads to

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0)\right)} = \sqrt{v_0^2 + 2gL(1-\cos\theta_0)}\ .$$

(b) We look for the initial speed required to barely reach the horizontal position – described by $v_h = 0$ and $\theta = 90°$ (or $\theta = -90°$, if one prefers, but since $\cos(-\phi) = \cos\phi$, the sign of the angle is not a concern).

$$\begin{aligned} K_0 + U_0 &= K_h + U_h \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= 0 + mgL \end{aligned}$$

which leads to $v_0 = \sqrt{2gL\cos\theta_0}$.

(c) For the cord to remain straight, then the centripetal force (at the top) must be (at least) equal to gravitational force:

$$\frac{mv_t^2}{r} = mg \implies mv_t^2 = mgL$$

where we recognize that $r = L$. We plug this into the expression for the kinetic energy (at the top, where $\theta = 180°$).

$$\begin{aligned} K_0 + U_0 &= K_t + U_t \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}mv_t^2 + mg\,(1-\cos 180°) \\ \frac{1}{2}mv_0^2 + mgL(1-\cos\theta_0) &= \frac{1}{2}(mgL) + mg(2L) \end{aligned}$$

which leads to $v_0 = \sqrt{gL(3 + 2\cos\theta_0)}$.

(d) The more initial potential energy there is, the less initial kinetic energy there needs to be, in order to reach the positions described in parts (b) and (c). Increasing θ_0 amounts to increasing U_0, so we see that a greater value of θ_0 leads to smaller results for v_0 in parts (b) and (c).

19. The reference point for the gravitational potential energy U_g (and height h) is at the block when the spring is maximally compressed. When the block is moving to its highest point, it is first accelerated by the spring; later, it separates from the spring and finally reaches a point where its speed v_f is (momentarily) zero. The x axis is along the incline, pointing uphill (so x_0 for the initial compression is negative-valued); its origin is at the relaxed position of the spring. We use SI units, so $k = 1960$ N/m and $x_0 = -0.200$ m.

(a) The elastic potential energy is $\frac{1}{2}kx_0^2 = 39.2$ J.

(b) Since initially $U_g = 0$, the change in U_g is the same as its final value mgh where $m = 2.00$ kg. That this must equal the result in part (a) is made clear in the steps shown in the next part. Thus, $\Delta U_g = U_g = 39.2$ J.

(c) The principle of mechanical energy conservation leads to

$$\begin{aligned} K_0 + U_0 &= K_f + U_f \\ 0 + \frac{1}{2}kx_0^2 &= 0 + mgh \end{aligned}$$

which yields $h = 2.00$ m. The problem asks for the distance *along the incline*, so we have $d = h/\sin 30° = 4.00$ m.

20. (a) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$\begin{aligned} K_P + U_P &= K_Q + U_Q \\ 0 + mgh &= \frac{1}{2}mv^2 + mgR \end{aligned}$$

Using the fact (mentioned in problem 6) that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is $mv^2/R = 8mg$ and points left (in the same direction as $\vec{a}$).

(b) The downward component of the net force on the block at Q is the force of gravity mg downward.

(c) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$\frac{mv_t^2}{R} = mg \implies mv_t^2 = mgR$$

This requires a different value of h than was used above.

$$\begin{aligned} K_P + U_P &= K_t + U_t \\ 0 + mgh &= \frac{1}{2}mv_t^2 + mgh_t \\ mgh &= \frac{1}{2}(mgR) + mg(2R) \end{aligned}$$

Consequently, $h = 2.5R$.

(d) The normal force N, for speeds v_t greater than $\sqrt{gR}$ (which are the only possibilities for non-zero N – see the solution in the previous part), obeys

$$N = \frac{mv_t^2}{R} - mg$$

from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_P + U_P = K_t + U_t \implies gh = \frac{1}{2}v_t^2 + 2gR$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ – see solution in previous part), becomes

$$N = \frac{2mg}{R}h - 5mg \ .$$

Thus, the graph for $h \geq 2.5R$ consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). For $h \leq 2.5R$, the normal force is zero. In the interest of saving space, we do not show the graph here.

21. We refer to its starting point as A, the point where it first comes into contact with the spring as B, and the point where the spring is compressed $|x| = 0.055\,\text{m}$ as C. Point C is our reference point for computing gravitational potential energy. Elastic potential energy (of the spring) is zero when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = \frac{F}{x} = \frac{270\,\text{N}}{0.02\,\text{m}} = 1.35 \times 10^4 \text{ N/m} .$$

(a) The distance between points A and B is ℓ, and we note that the total sliding distance $\ell + |x|$ is related to the initial height h of the block (measured relative to C) by

$$\frac{h}{\ell + |x|} = \sin\theta$$

where the incline angle θ is $30°$. Mechanical energy conservation leads to

$$\begin{aligned} K_A + U_A &= K_C + U_C \\ 0 + mgh &= 0 + \frac{1}{2}kx^2 \end{aligned}$$

which yields

$$h = \frac{kx^2}{2mg} = \frac{(1.35 \times 10^4\,\text{N/m})(0.055\,\text{m})^2}{2(12\,\text{kg})\left(9.8\,\text{m/s}^2\right)} = 0.174 \text{ m} .$$

Therefore,

$$\ell + |x| = \frac{h}{\sin 30°} = \frac{0.174\,\text{m}}{\sin 30°} = 0.35 \text{ m} .$$

(b) From this result, we find $\ell = 0.35 - 0.055 = 0.29$ m, which means $\Delta y = -\ell\sin\theta = -0.15$ m in sliding from point A to point B. Thus, Eq. 8-18 gives

$$\begin{aligned} \Delta K + \Delta U &= 0 \\ \frac{1}{2}mv_B^2 + mg\Delta h &= 0 \end{aligned}$$

which yields

$$v_B = \sqrt{-2g\Delta h} = \sqrt{-(9.8)(-0.15)} = 1.7 \text{ m/s} .$$

22. Since time does not directly enter into the energy formulations, we return to Chapter 4 (or Table 2-1 in Chapter 2) to find the change of height during this $t = 6$ s flight.

$$\Delta y = v_{0_y}t - \frac{1}{2}gt^2$$

This leads to $\Delta y = -32$ m. Therefore $\Delta U = mg\Delta y = -318 \approx -320$ J.

23. (a) As the string reaches its lowest point, its original potential energy $U = mgL$ (measured relative to the lowest point) is converted into kinetic energy. Thus,

$$mgL = \frac{1}{2}mv^2 \implies v = \sqrt{2gL} .$$

With $L = 1.20$ m we obtain $v = 4.85$ m/s.

(b) In this case, the total mechanical energy is shared between kinetic $\frac{1}{2}mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L - d = 0.450$ m. Energy conservation leads to

$$mgL = \frac{1}{2}mv_b^2 + mgy_b$$

which yields

$$v_b = \sqrt{2gL - 2g(2r)} = 2.42 \text{ m/s} .$$

24. From Chapter 4, we know the height h of the skier's jump can be found from $v_y^2 = 0 = v_{0\,y}^2 - 2gh$ where $v_{0\,y} = v_0 \sin 28^\circ$ is the upward component of the skier's "launch velocity." To find v_0 we use energy conservation.

    (a) The skier starts at rest $y = 20$ m above the point of "launch" so energy conservation leads to

    $$mgy = \frac{1}{2}mv^2 \implies v = \sqrt{2gy} = 20 \text{ m/s}$$

    which becomes the initial speed v_0 for the launch. Hence, the above equation relating h to v_0 yields

    $$h = \frac{(v_0 \sin 28^\circ)^2}{2g} = 4.4 \text{ m} .$$

    (b) We see that all reference to mass cancels from the above computations, so a new value for the mass will yield the same result as before.

25. We denote m as the mass of the block, $h = 0.40$ m as the height from which it dropped (measured from the relaxed position of the spring), and x the compression of the spring (measured downward so that it yields a positive value). Our reference point for the gravitational potential energy is the initial position of the block. The block drops a total distance $h + x$, and the final gravitational potential energy is $-mg(h + x)$. The spring potential energy is $\frac{1}{2}kx^2$ in the final situation, and the kinetic energy is zero both at the beginning and end. Since energy is conserved

    $$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 &= -mg(h + x) + \frac{1}{2}kx^2 \end{aligned}$$

    which is a second degree equation in x. Using the quadratic formula, its solution is

    $$x = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} .$$

    Now $mg = 19.6\,\text{N}$, $h = 0.40\,\text{m}$, and $k = 1960\,\text{N/m}$, and we choose the positive root so that $x > 0$.

    $$x = \frac{19.6 + \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10 \text{ m} .$$

26. To find out whether or not the vine breaks, it is sufficient to examine it at the moment Tarzan swings through the lowest point, which is when the vine – if it didn't break – would have the greatest tension. Choosing upward positive, Newton'second law leads to

    $$T - mg = m\frac{v^2}{r}$$

    where $r = 18$ m and $m = W/g = 688/9.8 = 70.2$ kg. We find the v^2 from energy conservation (where the reference position for the potential energy is at the lowest point).

    $$mgh = \frac{1}{2}mv^2 \implies v^2 = 2gh$$

    where $h = 3.2$ m. Combining these results, we have

    $$T = mg + m\frac{2gh}{r} = mg\left(1 + \frac{2h}{r}\right)$$

    which yields 933 N. Thus, the vine does not break. And rounding to an appropriate number of significant figures, we see the maximum tension is roughly 930 N.

27. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal distance to the point where the marble lands. Then $x = v_0 t$ and $h = \frac{1}{2}gt^2$ (since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0\sqrt{2h/g}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote $v_{0\,1}$ be the initial speed of the first shot and $x_1 = 1.93$ m be the horizontal distance to its landing point; similarly, $v_{0\,2}$ is the initial speed of the second shot and $x_2 = 2.20$ m is the horizontal distance to its landing spot. Then

$$\frac{v_{0\,2}}{v_{0\,1}} = \frac{x_2}{x_1} \quad\Longrightarrow\quad v_{0\,2} = \frac{x_2}{x_1}\,v_{0\,1}\ .$$

When the spring is compressed an amount ℓ, the elastic potential energy is $\frac{1}{2}k\ell^2$. When the marble leaves the spring its kinetic energy is $\frac{1}{2}mv_0^2$. Mechanical energy is conserved: $\frac{1}{2}mv_0^2 = \frac{1}{2}k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{02} = (\ell_2/\ell_1)v_{01}$. Relating this to the previous result, we obtain

$$\ell_2 = \frac{x_2}{x_1}\,\ell_1 = \left(\frac{2.20\,\text{m}}{1.93\,\text{m}}\right)(1.10\,\text{cm}) = 1.25\ \text{cm}\ .$$

28. We place the reference position for evaluating gravitational potential energy at the relaxed position of the spring. We use x for the spring's compression, measured positively downwards (so $x > 0$ means it is compressed).

    (a) With $x = 0.190$ m, Eq. 7-26 gives $W_s = -\frac{1}{2}kx^2 = -7.22$ J for the work done by the spring force. Using Newton's third law, we conclude the work done on the spring is 7.22 J.

    (b) As noted above, $W_s = -7.22$ J.

    (c) Energy conservation leads to

    $$\begin{aligned} K_i + U_i &= K_f + U_f \\ mgh_0 &= -mgx + \frac{1}{2}kx^2 \end{aligned}$$

    which (with $m = 0.700$ kg) yields $h_0 = 0.862$ m.

    (d) With a new value for the height $h_0' = 2h_0 = 1.72$ m, we solve for a new value of x using the quadratic formula (taking its positive root so that $x > 0$).

    $$mgh_0' = -mgx + \frac{1}{2}kx^2 \quad\Longrightarrow\quad x = \frac{mg + \sqrt{(mg)^2 + 2mgkh_0'}}{k}$$

    which yields $x = 0.261$ m.

29. We use conservation of mechanical energy: the mechanical energy must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the kinetic energy, at the top. There the tension force T of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written $T + mg = mv^2/(L - d)$, where v is the speed and m is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m\frac{v^2}{L-d} \quad\Longrightarrow\quad v = \sqrt{g(L-d)}\ .$$

We take the gravitational potential energy of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial potential energy is mgL. The initial kinetic energy is zero since the ball starts from rest. The final potential energy, at the top of the swing, is $2mg(L-d)$ and the final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}mg(L-d)$ using the above result for v. Conservation of energy yields

$$mgL = 2mg(L-d) + \frac{1}{2}mg(L-d) \implies d = 3L/5 \ .$$

If d is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If d is less, the ball cannot go around. Thus the value we found for d is a lower limit.

30. The connection between angle θ (measured from vertical – see Fig. 8-29) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy) is given by $h = L(1-\cos\theta)$ where L is the length of the pendulum.

(a) We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mgL\,(1-\cos\theta_1) &= \frac{1}{2}mv_2^2 + mgL\,(1-\cos\theta_2) \end{aligned}$$

This leads to

$$v_2 = \sqrt{2gL\,(\cos\theta_2 - \cos\theta_1)} = 1.4 \text{ m/s}$$

since $L = 1.4$ m, $\theta_1 = 30°$, and $\theta_2 = 20°$.

(b) The maximum speed v_3 is at the lowest point. Our formula for h gives $h_3 = 0$ when $\theta_3 = 0°$, as expected.

$$\begin{aligned} K_1 + U_1 &= K_3 + U_3 \\ 0 + mgL\,(1-\cos\theta_1) &= \frac{1}{2}mv_3^2 + 0 \end{aligned}$$

This yields $v_3 = 1.9$ m/s.

(c) We look for an angle θ_4 such that the speed there is $v_4 = v_3/3$. To be as accurate as possible, we proceed algebraically (substituting $v_3^2 = 2gL\,(1-\cos\theta_1)$ at the appropriate place) and plug numbers in at the end. Energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_4 + U_4 \\ 0 + mgL\,(1-\cos\theta_1) &= \frac{1}{2}mv_4^2 + mgL\,(1-\cos\theta_4) \\ mgL\,(1-\cos\theta_1) &= \frac{1}{2}m\,\frac{v_3^2}{9} + mgL\,(1-\cos\theta_4) \\ -gL\cos\theta_1 &= \frac{1}{2}\frac{2gL\,(1-\cos\theta_1)}{9} - gL\cos\theta_4 \end{aligned}$$

where in the last step we have subtracted out mgL and then divided by m. Thus, we obtain

$$\theta_4 = \cos^{-1}\left(\frac{1}{9} + \frac{8}{9}\cos\theta_1\right) = 28.2°$$

where we have quoted the answer to three significant figures since the problem gives θ_1 to three figures.

31. The connection between angle θ (measured from vertical – see Fig. 8-29) and height h (measured from the lowest point, which is our choice of reference position in computing the gravitational potential energy mgh) is given by $h = L(1-\cos\theta)$ where L is the length of the pendulum.

(a) Using this formula (or simply using intuition) we see the initial height is $h_1 = 2L$, and of course $h_2 = 0$. We use energy conservation in the form of Eq. 8-17.

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mg\,(2L) &= \frac{1}{2}mv^2 + 0 \end{aligned}$$

This leads to $v = 2\sqrt{gL}$.

(b) The ball is in circular motion with the center of the circle above it, so $\vec{a} = v^2/r$ upward, where $r = L$. Newton's second law leads to

$$T - mg = m\,\frac{v^2}{r} \implies T = m\left(g + \frac{4gL}{L}\right) = 5mg\ .$$

(c) The pendulum is now started (with zero speed) at $\theta_i = 90°$ (that is, $h_i = L$), and we look for an angle θ such that $T = mg$. When the ball is moving through a point at angle θ, then Newton's second law applied to the axis along the rod yields

$$T - mg\cos\theta = m\,\frac{v^2}{r}$$

which (since $r = L$) implies $v^2 = gL(1 - \cos\theta)$ at the position we are looking for. Energy conservation leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 + mgL &= \frac{1}{2}mv^2 + mgL\,(1 - \cos\theta) \\ gL &= \frac{1}{2}\,(gL(1 - \cos\theta)) + gL\,(1 - \cos\theta) \end{aligned}$$

where we have divided by mass in the last step. Simplifying, we obtain

$$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5°\ .$$

32. All heights h are measured from the lower end of the incline (which is our reference position for computing gravitational potential energy mgh). Our x axis is along the incline, with $+x$ being uphill (so spring compression corresponds to $x > 0$) and its origin being at the relaxed end of the spring. The 1.00 m distance indicated in Fig. 8-40 will be referred to as ℓ, and the 37.0° angle will be referred to as θ. Thus, the height that corresponds to the canister's initial position (with spring compressed amount $x = 0.200$ m) is given by $h_1 = (\ell + x)\sin\theta$.

(a) Energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ 0 + mg(\ell + x)\sin\theta + \frac{1}{2}kx^2 &= \frac{1}{2}mv_2^2 + mg\ell\sin\theta \end{aligned}$$

which yields $v_2 = \sqrt{2gx\sin\theta + kx^2/m} = 2.40$ m/s using the data $m = 2.00$ kg and $k = 170$ N/m.

(b) In this case, energy conservation leads to

$$\begin{aligned} K_1 + U_1 &= K_3 + U_3 \\ 0 + mg(\ell + x)\sin\theta + \frac{1}{2}kx^2 &= \frac{1}{2}mv_3^2 + 0 \end{aligned}$$

which yields $v_3 = \sqrt{2g(\ell + x)\sin\theta + kx^2/m} = 4.19$ m/s.

33. The work required is the change in the gravitational potential energy as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)\,dy$ and the change in potential energy of a segment when it is a distance $|y|$ below the table top is $dU = (m/L)g|y|\,dy = -(m/L)gy\,dy$ since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total potential energy change is

$$\Delta U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}(L/4)^2 = mgL/32 \ .$$

The work required to pull the chain onto the table is therefore $W = \Delta U = mgL/32$.

34. If the description of the scenario seems confusing, reference to Figure 8-31 in the textbook is helpful. We note that the block being unattached means that for $y > 0.25$ m, the elastic potential energy vanishes. With $k = 400$ N/m, $m = 40.0/9.8 = 4.08$ kg and length in meters, the energy equation is

$$E = \begin{cases} \frac{1}{2}k\left(\frac{1}{4}\right)^2 & y = 0 \\ K + mgy + \frac{1}{2}k\left(\frac{1}{4} - y\right)^2 & 0 \le y \le \frac{1}{4} \\ K + mgy & \frac{1}{4} \le y \end{cases}$$

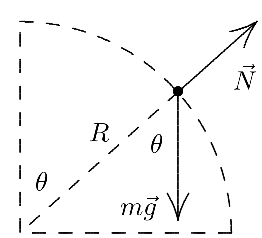
In this way, the kinetic energy K for each region is related to E – which by conservation of energy is always equal to the value 12.5 J that it had at $y = 0$. We arrange our results in a table (with energies in Joules) where it is clear that the sum of each column (of energies) is 12.5 J:

part	(a)	(b)	(c)	(d)	(e)	(f)	(g)
position y	0	0.05	0.10	0.15	0.20	0.25	0.30
U_g	0	2.0	4.0	6.0	8.0	10.0	12.0
U_e	12.5	8.0	4.5	2.0	0.5	0	0
K	0	2.5	4.0	4.5	4.0	2.5	0.5

Finally (for part (h)), where $y \ge 0.25$ m, we have $K = E - mgy$, so that $K = 0$ occurs when $y = (12.5\text{ J})/(40\text{ N}) = 0.313$ m.

35. The free-body diagram for the boy is shown below. $\vec{N}$ is the normal force of the ice on him and m is his mass. The net inward force is $mg\cos\theta - N$ and, according to Newton's second law, this must be equal to mv^2/R, where v is the speed of the boy. At the point where the boy leaves the ice $N = 0$, so $g\cos\theta = v^2/R$. We wish to find his speed. If the gravitational potential energy is taken to be zero when he is at the top of the ice mound, then his potential energy at the time shown is $U = -mgR(1 - \cos\theta)$.

He starts from rest and his kinetic energy at the time shown is $\frac{1}{2}mv^2$. Thus conservation of energy gives $0 = \frac{1}{2}mv^2 - mgR(1-\cos\theta)$, or $v^2 = 2gR(1-\cos\theta)$. We substitute this expression into the equation developed from the second law to obtain $g\cos\theta = 2g(1-\cos\theta)$. This gives $\cos\theta = 2/3$. The height of the boy above the bottom of the mound is $R\cos\theta = 2R/3$.



36. We use Eq. 8-20 and various observations made in §8-5.

(a) The force at $x = 2.0$ m is

$$F = -\frac{dU}{dx} \approx -\frac{(-17.5) - (-2.8)}{4.0 - 1.0} = 4.9 \text{ N}$$

in the $+x$ direction (but there is some uncertainty in reading the graph which makes the last digit not very significant).

(b) The total mechanical energy at $x = 2.0$ m is

$$E = \frac{1}{2}mv^2 + U \approx \frac{1}{2}(2.0)(-1.5)^2 - 7.7 = -5.5$$

in SI units (Joules). Again, there is some uncertainty in reading the graph which makes the last digit not very significant. At that level (-5.5 J) on the graph, we find two points where the potential energy curve has that value – at $x \approx 1.5$ m and $x \approx 13.5$ m. Therefore, the particle remains in the region $1.5 < x < 13.5$ m.

(c) At $x = 7.0$ m, we read $U \approx -17.5$ J. Thus, if its total energy (calculated in the previous part) is $E \approx -5.5$ J, then we find

$$\frac{1}{2}mv^2 = E - U \approx 12\,\mathrm{J} \implies v = \sqrt{\frac{2}{m}(E-U)} \approx 3.5 \text{ m/s}$$

where there is certainly room for disagreement on that last digit for the reasons cited above.

37. We use Eq. 8-20 and various observations made in §8-5.

(a) The force at the equilibrium position $r = r_{\rm eq}$ is

$$\begin{aligned} F = -\left.\frac{dU}{dr}\right|_{r=r_{\rm eq}} &= 0 \\ -\frac{12A}{r_{\rm eq}^{13}} + \frac{6B}{r_{\rm eq}^{7}} &= 0 \end{aligned}$$

which leads to the result

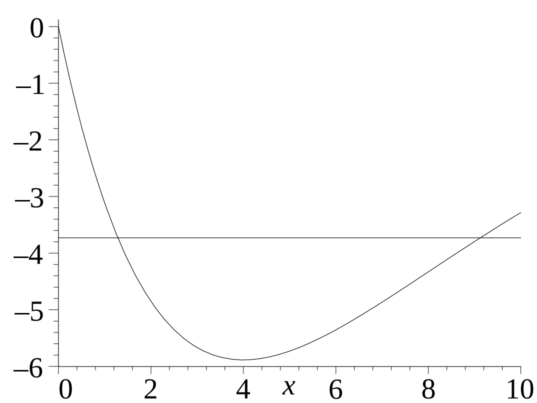
$$r_{\rm eq} = \left(\frac{2A}{B}\right)^{\frac{1}{6}} = 1.12\left(\frac{A}{B}\right)^{\frac{1}{6}} .$$

(b) This defines a minimum in the potential energy curve (as can be verified either by a graph or by taking another derivative and verifying that it is concave upward at this point), which means that for values of r slightly smaller than $r_{\rm eq}$ the slope of the curve is negative (so the force is positive, repulsive).

(c) And for values of r slightly larger than $r_{\rm eq}$ the slope of the curve must be positive (so the force is negative, attractive).

38. (a) The energy at $x = 5.0$ m is $E = K + U = 2.0 - 5.7 = -3.7$ J.

(b) A plot of the potential energy curve (SI units understood) and the energy E (the horizontal line) is shown for $0 \le x \le 10$ m.



(c) The problem asks for a graphical determination of the turning points, which are the points on the curve corresponding to the total energy computed in part (a). The result for the smallest turning point (determined, to be honest, by more careful means) is $x = 1.29$ m

(d) And the result for the largest turning point is $x = 9.12$ m.

(e) Since $K = E - U$, then maximizing K involves finding the minimum of U. A graphical determination suggests that this occurs at $x = 4.0$ m, which plugs into the expression $E - U = -3.7 - \left(-4xe^{-x/4}\right)$ to give $K = 2.16$ J. Alternatively, one can measure from the graph from the minimum of the U curve up to the level representing the total energy E and thereby obtain an estimate of K at that point.

(f) As mentioned in the previous part, the minimum of the U curve occurs at $x = 4.0$ m.

(g) The force (understood to be in Newtons) follows from the potential energy, using Eq. 8-20 (and Appendix E if students are unfamiliar with such derivatives).

$$F = \frac{dU}{dx} = (4 - x)\,e^{-x/4}$$

(h) This revisits the considerations of parts (d) and (e) (since we are returning to the minimum of $U(x)$) – but now with the advantage of having the analytic result of part (g). We see that the location which produces $F = 0$ is exactly $x = 4$ m.

39. (a) Using Eq. 7-8, we have

$$W_{\text{applied}} = (8.0\,\text{N})(0.70\,\text{m}) = 5.6\ \text{J} .$$

(b) Using Eq. 8-29, the thermal energy generated is

$$\Delta E_{\text{th}} = f_k d = (5.0\,\text{N})(0.70\,\text{m}) = 3.5\ \text{J} .$$

40. Since the speed is constant $\Delta K = 0$ and Eq. 8-31 (an application of the energy conservation concept) implies

$$W_{\text{applied}} = \Delta E_{\text{th}} = \Delta E_{\text{th (cube)}} + \Delta E_{\text{th (floor)}} .$$

Thus, if $W_{\text{applied}} = (15)(3.0) = 45$ J, and we are told that $\Delta E_{\text{th (cube)}} = 20$ J, then we conclude that $\Delta E_{\text{th (floor)}} = 25$ J.

41. (a) The work done on the block by the force in the rope is, using Eq. 7-7,

$$W = Fd\cos\theta = (7.68\,\text{N})(4.06\,\text{m})\cos 15.0^\circ = 30.1\ \text{J} .$$

(b) Using f for the magnitude of the kinetic friction force, Eq. 8-29 reveals that the increase in thermal energy is

$$\Delta E_{\text{th}} = fd = (7.42\,\text{N})(4.06\,\text{m}) = 30.1\ \text{J} .$$

(c) We can use Newton's second law of motion to obtain the frictional and normal forces, then use $\mu_k = f/N$ to obtain the coefficient of friction. Place the x axis along the path of the block and the y axis normal to the floor. The x component of Newton's second law is $F\cos\theta - f = 0$ and the y component is $N + F\sin\theta - mg = 0$, where m is the mass of the block, F is the force exerted by the rope, and θ is the angle between that force and the horizontal. The first equation gives $f = F\cos\theta = (7.68)\cos 15.0^\circ = 7.42\,\text{N}$ and the second gives $N = mg - F\sin\theta = (3.57)(9.8) - (7.68)\sin 15.0^\circ = 33.0\,\text{N}$. Thus

$$\mu_k = \frac{f}{N} = \frac{7.42\,\text{N}}{33.0\,\text{N}} = 0.22 .$$

42. Since the velocity is constant, $\vec{a} = 0$ and the horizontal component of the worker's push $F\cos\theta$ (where $\theta = 32°$) must equal the friction force magnitude $f_k = \mu_k N$. Also, the vertical forces must cancel, implying

$$N = mg + F\sin\theta \quad \Longrightarrow \quad F\cos\theta = \mu_k\,(mg + F\sin\theta)$$

which is solved to find $F = 71$ N.

(a) The work done on the block by the worker is, using Eq. 7-7,

$$W = Fd\cos\theta = (71\,\text{N})(9.2\,\text{m})\cos 32° = 5.6 \times 10^2 \text{ J} .$$

(b) Since $f_k = \mu_k\,(mg + F\sin\theta)$, we find

$$\Delta E_{\text{th}} = f_k d = (60\,\text{N})(9.2\,\text{m}) = 5.6 \times 10^2 \text{ J} .$$

43. (a) We take the initial gravitational potential energy to be $U_i = 0$. Then the final gravitational potential energy is $U_f = -mgL$, where L is the length of the tree. The change is

$$U_f - U_i = -mgL = -(25\,\text{kg})\left(9.8\,\text{m/s}^2\right)(12\,\text{m}) = -2.9 \times 10^3 \text{ J} .$$

(b) The kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(25\,\text{kg})(5.6\,\text{m/s})^2 = 3.9 \times 10^2 \text{ J} .$$

(c) The changes in the mechanical and thermal energies must sum to zero. The change in thermal energy is $\Delta E_{\text{th}} = fL$, where f is the magnitude of the average frictional force; therefore,

$$f = -\frac{\Delta K + \Delta U}{L} = -\frac{3.92 \times 10^2\,\text{J} - 2.94 \times 10^3\,\text{J}}{12\,\text{m}} = 210 \text{ N} .$$

44. We use SI units so $m = 0.030$ kg and $d = 0.12$ m.

(a) Since there is no change in height (and we assume no changes in elastic potential energy), then $\Delta U = 0$ and we have

$$\Delta E_{\text{mech}} = \Delta K = -\frac{1}{2}mv_0^2 = -3.8 \times 10^3 \text{ J}$$

where $v_0 = 500$ m/s and the final speed is zero.

(b) By Eq. 8-31 (with $W = 0$) we have $\Delta E_{\text{th}} = 3.8 \times 10^3$ J, which implies

$$f = \frac{\Delta E_{\text{th}}}{d} = 3.1 \times 10^4 \text{ N}$$

using Eq. 8-29 with f_k replaced by f (effectively generalizing that equation to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

45. Equation 8-31 provides $\Delta E_{\text{th}} = -\Delta E_{\text{mec}}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}
\Delta E_{\text{th}} &= \frac{1}{2}m\left(v_i^2 - v_f^2\right) + mg\,(y_i - y_f) \\
&= \frac{1}{2}(60)(24^2 - 22^2) + (60)(9.8)(14) \\
&= 1.1 \times 10^4 \text{ J} .
\end{aligned}$$

That the angle of 25° is nowhere used in this calculation is indicative of the fact that energy is a scalar quantity.

46. We use SI units so $m = 0.075$ kg. Equation 8-30 provides $\Delta E_{\rm th} = -\Delta E_{\rm mec}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\rm th} &= \frac{1}{2}m\left(v_i^2 - v_f^2\right) + mg\left(y_i - y_f\right) \\ &= \frac{1}{2}(0.075)(12^2 - 10.5^2) + (0.075)(9.8)(1.1 - 2.1) \\ &= 0.53 \text{ J} .\end{aligned}$$

47. We work this using the English units (with $g = 32$ ft/s), but for consistency we convert the weight to pounds

$$mg = (9.0\,\text{oz})\left(\frac{1\,\text{lb}}{16\,\text{oz}}\right) = 0.56 \text{ lb}$$

which implies $m = 0.018$ lb·s^2/ft (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_i = (81.8\,\text{mi/h})\left(\frac{5280\,\text{ft/mi}}{3600\,\text{s/h}}\right) = 120 \text{ ft/s}$$

or a more "direct" conversion from Appendix D can be used. Equation 8-30 provides $\Delta E_{\rm th} = -\Delta E_{\rm mec}$ for the energy "lost" in the sense of this problem. Thus,

$$\begin{aligned}\Delta E_{\rm th} &= \frac{1}{2}m\left(v_i^2 - v_f^2\right) + mg\left(y_i - y_f\right) \\ &= \frac{1}{2}(0.018)(120^2 - 110^2) + 0 \\ &= 20 \text{ ft} \cdot \text{lb} .\end{aligned}$$

48. (a) During one second, the decrease is potential energy is

$$-\Delta U = mg(-\Delta y) = \left(5.5 \times 10^6\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right)(50\,\text{m}) = 2.7 \times 10^9 \text{ J}$$

where $+y$ is upward and $\Delta y = y_f - y_i$.

(b) The information relating mass to volume is not needed in the computation. By Eq. 8-36 (and the SI relation W = J/s), the result follows: $(2.7 \times 10^9\text{ J})/(1\,\text{s}) = 2.7 \times 10^9$ W.

(c) One year is equivalent to $24 \times 365.25 = 8766$ h which we write as 8.77 kh. Thus, the energy supply rate multiplied by the cost and by the time is

$$\left(2.7 \times 10^9\,\text{W}\right)(8.77\,\text{kh})\left(\frac{1\,\text{cent}}{1\,\text{kWh}}\right) = 2.4 \times 10^{10}\,\text{cents}$$

which equals $\$2.4 \times 10^8$.

49. (a) The initial potential energy is

$$U_i = mgy_i = (520\,\text{kg})\left(9.8\,\text{m/s}^2\right)(300\,\text{m}) = 1.53 \times 10^6 \text{ J}$$

where $+y$ is upward and $y = 0$ at the bottom (so that $U_f = 0$).

(b) Since $f_k = \mu_k N = \mu_k mg\cos\theta$ we have

$$\Delta E_{\rm th} = f_k d = \mu_k mgd\cos\theta$$

from Eq. 8-29. Now, the hillside surface (of length $d = 500$ m) is treated as an hypotenuse of a 3-4-5 triangle, so $\cos\theta = x/d$ where $x = 400$ m. Therefore,

$$\Delta E_{\rm th} = \mu_k mgd\frac{x}{d} = \mu_k mgx = (0.25)(520)(9.8)(400) = 5.1 \times 10^5 \text{ J} .$$

(c) Using Eq. 8-31 (with $W = 0$) we find

$$\begin{aligned} K_f &= K_i + U_i - U_f - \Delta E_{\text{th}} \\ &= 0 + 1.53\times10^6 - 0 - 5.1\times10^5 \\ &= 0 + 1.02\times10^6 \text{ J} . \end{aligned}$$

(d) From $K_f = \frac{1}{2}mv^2$ we obtain $v = 62.6$ m/s.

50. Energy conservation, as expressed by Eq. 8-31 (with $W = 0$) leads to

$$\begin{aligned} \Delta E_{\text{th}} &= K_i - K_f + U_i - U_f \\ f_k d &= 0 - 0 + \frac{1}{2}kx^2 - 0 \\ \mu_k mgd &= \frac{1}{2}(200\,\text{N/m})(0.15\,\text{m})^2 \\ \mu_k(2.0\,\text{kg})\left(9.8\,\text{m/s}^2\right)(0.75\,\text{m}) &= 2.25 \text{ J} \end{aligned}$$

which yields $\mu_k = 0.15$ as the coefficient of kinetic friction.

51. (a) The vertical forces acting on the block are the normal force, upward, and the force of gravity, downward. Since the vertical component of the block's acceleration is zero, Newton's second law requires $N = mg$, where m is the mass of the block. Thus $f = \mu_k N = \mu_k mg$. The increase in thermal energy is given by $\Delta E_{\text{th}} = fd = \mu_k mgd$, where d is the distance the block moves before coming to rest. Using Eq. 8-29, we have

$$\Delta E_{\text{th}} = (0.25)(3.5\,\text{kg})\left(9.8\,\text{m/s}^2\right)(7.8\,\text{m}) = 67 \text{ J} .$$

(b) The block has its maximum kinetic energy K_{max} just as it leaves the spring and enters the region where friction acts. Therefore, the maximum kinetic energy equals the thermal energy generated in bringing the block back to rest, 67 J.

(c) The energy that appears as kinetic energy is originally in the form of potential energy in the compressed spring. Thus $K_{\text{max}} = U_i = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression. Thus,

$$x = \sqrt{\frac{2K_{\text{max}}}{k}} = \sqrt{\frac{2(67\,\text{J})}{640\,\text{N/m}}} = 0.46 \text{ m} .$$

52. We use Eq. 8-29

$$\Delta E_{\text{th}} = f_k d = (10\,\text{N})(5.0\,\text{m}) = 50 \text{ J}$$

and Eq. 7-8

$$W = Fd = (2.0\,\text{N})(5.0\,\text{m}) = 10 \text{ J}$$

and Eq. 8-31

$$\begin{aligned} W &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ 10 &= 35 + \Delta U + 50 \end{aligned}$$

which yields $\Delta U = -75$ J. By Eq. 8-1, then, the work done by gravity is $W = -\Delta U = 75$ J.

53. (a) To stretch the spring an external force, equal in magnitude to the force of the spring but opposite to its direction, is applied. Since a spring stretched in the positive x direction exerts a force in the negative x direction, the applied force must be $F = 52.8x + 38.4x^2$, in the $+x$ direction. The work it does is

$$W = \int_{0.50}^{1.00} (52.8x + 38.4x^2)\,dx = \left[\frac{52.8}{2}x^2 + \frac{38.4}{3}x^3\right]_{0.50}^{1.00} = 31.0 \text{ J} .$$

(b) The spring does 31.0 J of work and this must be the increase in the kinetic energy of the particle. Its speed is then

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(31.0\,\mathrm{J})}{2.17\,\mathrm{kg}}}=5.35\ \mathrm{m/s}\ .$$

(c) The force is conservative since the work it does as the particle goes from any point x_1 to any other point x_2 depends only on x_1 and x_2, not on details of the motion between x_1 and x_2.

54. We look for the distance along the incline d which is related to the height ascended by $\Delta h = d\sin\theta$. By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $N = mg\cos\theta$ which means $f_k = \mu_k mg\cos\theta$. Thus, Eq. 8-31 (with $W = 0$) leads to

$$\begin{aligned} 0 &= K_f - K_i + \Delta U + \Delta E_{\mathrm{th}} \\ &= 0 - K_i + mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which leads to

$$d=\frac{K_i}{mg(\sin\theta+\mu_k\cos\theta)}=\frac{128}{(4.0)(9.8)(\sin 30^\circ+0.30\cos 30^\circ)}=4.3\ \mathrm{m}\ .$$

55. (a) We take the gravitational potential energy of the skier-Earth system to be zero when the skier is at the bottom of the peaks. The initial potential energy is $U_i = mgh_i$, where m is the mass of the skier, and h_i is the height of the higher peak. The final potential energy is $U_f = mgh_f$, where h_f is the height of the lower peak. The skier initially has a kinetic energy of $K_i = 0$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the skier at the top of the lower peak. The normal force of the slope on the skier does no work and friction is negligible, so mechanical energy is conserved.

$$\begin{aligned} U_i + K_i &= U_f + K_f \\ mgh_i &= mgh_f + \frac{1}{2}mv^2 \end{aligned}$$

Thus,

$$v=\sqrt{2g\,(h_i-h_f)}=\sqrt{2(9.8)(850-750)}=44\ \mathrm{m/s}\ .$$

(b) We recall from analyzing objects sliding down inclined planes that the normal force of the slope on the skier is given by $N = mg\cos\theta$, where θ is the angle of the slope from the horizontal, 30° for each of the slopes shown. The magnitude of the force of friction is given by $f = \mu_k N = \mu_k mg\cos\theta$. The thermal energy generated by the force of friction is $fd = \mu_k mgd\cos\theta$, where d is the total distance along the path. Since the skier gets to the top of the lower peak with no kinetic energy, the increase in thermal energy is equal to the decrease in potential energy. That is, $\mu_k mgd\cos\theta = mg(h_i - h_f)$. Consequently,

$$\mu_k=\frac{(h_i-h_f)}{d\cos\theta}=\frac{(850-750)}{(3.2\times10^3)\cos 30^\circ}=0.036\ .$$

56. (a) By a force analysis in the style of Chapter 6, we find the normal force $N = mg\cos\theta$ (where $mg = 267$ N) which means $f_k = \mu_k mg\cos\theta$. Thus, Eq. 8-29 yields

$$\Delta E_{\mathrm{th}}=f_k d=\mu_k mgd\cos\theta=(0.10)(267)(6.1)\cos 20^\circ=1.5\times10^2\ \mathrm{J}\ .$$

(b) The potential energy change is $\Delta U = mg(-d\sin\theta) = (267)(-6.1\sin 20^\circ) = -5.6\times10^2$ J. The initial kinetic energy is

$$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}\left(\frac{267\,\mathrm{N}}{9.8\,\mathrm{m/s^2}}\right)(0.457\,\mathrm{m/s})^2=2.8\ \mathrm{J}\ .$$

Therefore, using Eq. 8-31 (with $W = 0$), the final kinetic energy is

$$K_f = K_i - \Delta U - \Delta E_{\text{th}} = 2.8 - \left(-5.6 \times 10^2\right) - 1.5 \times 10^2 = 4.1 \times 10^2 \text{ J} .$$

Consequently, the final speed is $v_f = \sqrt{2K_f/m} = 5.5$ m/s

57. (a) With $x = 0.075$ m and $k = 320$ N/m, Eq. 7-26 yields $W_s = -\frac{1}{2}kx^2 = -0.90$ J. For later reference, this is equal to the negative of ΔU.

(b) Analyzing forces, we find $N = mg$ which means $f_k = \mu_k mg$. With $d = x$, Eq. 8-29 yields

$$\Delta E_{\text{th}} = f_k d = \mu_k mgx = (0.25)(2.5)(9.8)(0.075) = 0.46 \text{ J} .$$

(c) Eq. 8-31 (with $W = 0$) indicates that the initial kinetic energy is

$$K_i = \Delta U + \Delta E_{\text{th}} = 0.90 + 0.46 = 1.36 \text{ J}$$

which leads to $v_i = \sqrt{2K_i/m} = 1.0$ m/s.

58. This can be worked entirely by the methods of Chapters 2-6, but we will use energy methods in as many steps as possible.

(a) By a force analysis in the style of Chapter 6, we find the normal force has magnitude $N = mg\cos\theta$ (where $\theta = 39°$) which means $f_k = \mu_k mg\cos\theta$ where $\mu_k = 0.28$. Thus, Eq. 8-29 yields $\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta$. Also, elementary trigonometry leads us to conclude that $\Delta U = -mgd\sin\theta$ where $d = 3.7$ m. Since $K_i = 0$, Eq. 8-31 (with $W = 0$) indicates that the final kinetic energy is

$$K_f = -\Delta U - \Delta E_{\text{th}} = mgd\left(\sin\theta - \mu_k\cos\theta\right)$$

which leads to the speed at the bottom of the ramp

$$v = \sqrt{\frac{2K_f}{m}} = \sqrt{2gd\left(\sin\theta - \mu_k\cos\theta\right)} = 5.5 \text{ m/s} .$$

(b) This speed begins its horizontal motion, where $f_k = \mu_k mg$ and $\Delta U = 0$. It slides a distance d' before it stops. According to Eq. 8-31 (with $W = 0$),

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= 0 - \frac{1}{2}mv^2 + 0 + \mu_k mgd' \\ &= -\frac{1}{2}\left(2gd\left(\sin\theta - \mu_k\cos\theta\right)\right) + \mu_k gd' \end{aligned}$$

where we have divided by mass and substituted from part (a) in the last step. Therefore,

$$d' = \frac{d\left(\sin\theta - \mu_k\cos\theta\right)}{\mu_k} = 5.4 \text{ m} .$$

(c) We see from the algebraic form of the results, above, that the answers do not depend on mass. A 90 kg crate should have the same speed at the bottom and sliding distance across the floor, to the extent that the friction relations in Ch. 6 are accurate. Interestingly, since g does not appear in the relation for d', the sliding distance would seem to be the same if the experiment were performed on Mars!

59. Since the valley is frictionless, the only reason for the speed being less when it reaches the higher level is the gain in potential energy $\Delta U = mgh$ where $h = 1.1$ m. Sliding along the rough surface of the higher level, the block finally stops since its remaining kinetic energy has turned to thermal energy

$$\Delta E_{\text{th}} = f_k d = \mu mgd$$

where $\mu = 0.60$. Thus, Eq. 8-31 (with $W = 0$) provides us with an equation to solve for the distance d:

$$K_i = \Delta U + \Delta E_{\text{th}} = mg\,(h + \mu d)$$

where $K_i = \frac{1}{2}mv_i^2$ and $v_i = 6.0$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2\mu g} - \frac{h}{\mu} = 1.2 \text{ m} .$$

60. This can be worked entirely by the methods of Chapters 2-6, but we will use energy methods in as many steps as possible.

(a) By a force analysis of the style done in Ch. 6, we find the normal force has magnitude $N = mg\cos\theta$ (where $\theta = 40°$) which means $f_k = \mu_k mg\cos\theta$ where $\mu_k = 0.15$. Thus, Eq. 8-29 yields $\Delta E_{\text{th}} = f_k d = \mu_k mgd\cos\theta$. Also, elementary trigonometry leads us to conclude that $\Delta U = mgd\sin\theta$. Eq. 8-31 (with $W = 0$ and $K_f = 0$) provides an equation for determining d:

$$\begin{aligned} K_i &= \Delta U + \Delta E_{\text{th}} \\ \frac{1}{2}mv_i^2 &= mgd\,(\sin\theta + \mu_k\cos\theta) \end{aligned}$$

where $v_i = 1.4$ m/s. Dividing by mass and rearranging, we obtain

$$d = \frac{v_i^2}{2g\,(\sin\theta + \mu_k\cos\theta)} = 0.13 \text{ m} .$$

(b) Now that we know where on the incline it stops ($d' = 0.13 + 0.55 = 0.68$ m from the bottom), we can use Eq. 8-31 again (with $W = 0$ and now with $K_i = 0$,) to describe the final kinetic energy (at the bottom):

$$\begin{aligned} K_f &= -\Delta U - \Delta E_{\text{th}} \\ \frac{1}{2}mv^2 &= mgd'\,(\sin\theta - \mu_k\cos\theta) \end{aligned}$$

which – after dividing by the mass and rearranging – yields

$$v = \sqrt{2gd'\,(\sin\theta - \mu_k\cos\theta)} = 2.7 \text{ m/s} .$$

(c) In part (a) it is clear that d increases if μ_k decreases – both mathematically (since it is a positive term in the denominator) and intuitively (less friction – less energy "lost"). In part (b), there are two terms in the expression for v which imply that it should increase if μ_k were smaller: the increased value of $d' = d_0 + d$ and that last factor $\sin\theta - \mu_k\cos\theta$ which indicates that less is being subtracted from $\sin\theta$ when μ_k is less (so the factor itself increases in value).

61. (a) The maximum height reached is h. The thermal energy generated by air resistance as the stone rises to this height is $\Delta E_{\text{th}} = fh$ by Eq. 8-29. We use energy conservation in the form of Eq. 8-31 (with $W = 0$):

$$K_f + U_f + \Delta E_{\text{th}} = K_i + U_i$$

and we take the potential energy to be zero at the throwing point (ground level). The initial kinetic energy is $K_i = \frac{1}{2}mv_0^2$, the initial potential energy is $U_i = 0$, the final kinetic energy is $K_f = 0$, and the final potential energy is $U_f = wh$. Thus $wh + fh = \frac{1}{2}mv_0^2$, and we solve for the height:

$$h = \frac{mv_0^2}{2(w + f)} = \frac{wv_0^2}{2g(w + f)} = \frac{v_0^2}{2g(1 + f/w)} .$$

(b) We notice that the force of the air is downward on the trip up and upward on the trip down, since it is opposite to the direction of motion. Over the entire trip the increase in thermal energy is $\Delta E_{\text{th}} = 2fh$. The final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the speed of the stone just before it hits the ground. The final potential energy is $U_f = 0$. Thus, using Eq. 8-31 (with $W = 0$), we find

$$\frac{1}{2}mv^2 + 2fh = \frac{1}{2}mv_0^2 .$$

We substitute the expression found for h to obtain

$$-\frac{2fv_0^2}{2g(1+f/w)} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

which leads to

$$v^2 = v_0^2 - \frac{2fv_0^2}{mg(1+f/w)} = v_0^2 - \frac{2fv_0^2}{w(1+f/w)} = v_0^2\left(1 - \frac{2f}{w+f}\right) = v_0^2\,\frac{w-f}{w+f}$$

where w was substituted for mg and some algebraic manipulations were carried out. Therefore,

$$v = v_0\sqrt{\frac{w-f}{w+f}} .$$

62. (a) The assumption is that the slope of the bottom of the slide is horizontal, like the ground. A useful analogy is that of the pendulum of length $R = 12$ m that is pulled leftward to an angle θ (corresponding to being at the top of the slide at height $h = 4.0$ m) and released so that the pendulum swings to the lowest point (zero height) gaining speed $v = 6.2$ m/s. Exactly as we would analyze the trigonometric relations in the pendulum problem, we find

$$h = R\,(1 - \cos\theta) \quad\Longrightarrow\quad \theta = \cos^{-1}\left(1 - \frac{h}{R}\right) = 48^\circ$$

or 0.84 radians. The slide, representing a circular arc of length $s = R\theta$, is therefore $(12)(0.84) = 10$ m long.

(b) To find the magnitude f of the frictional force, we use Eq. 8-31 (with $W = 0$):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + fs \end{aligned}$$

so that (with $m = 25$ kg) we obtain $f = 49$ N.

(c) The assumption is no longer that the slope of the bottom of the slide is horizontal, but rather that the slope of the top of the slide is vertical (and 12 m to the left of the center of curvature). Returning to the pendulum analogy, this corresponds to releasing the pendulum from horizontal (at $\theta_1 = 90^\circ$ measured from vertical) and taking a snapshot of its motion a few moments later when it is at angle θ_2 with speed $v = 6.2$ m/s. The difference in height between these two positions is (just as we would figure for the pendulum of length R)

$$\Delta h = R\,(1 - \cos\theta_2) - R\,(1 - \cos\theta_1) = -R\cos\theta_2$$

where we have used the fact that $\cos\theta_1 = 0$. Thus, with $\Delta h = -4.0$ m, we obtain $\theta_2 = 70.5^\circ$ which means the arc subtends an angle of $|\Delta\theta| = 19.5^\circ$ or 0.34 radians. Multiplying this by the radius gives a slide length of $s' = 4.1$ m.

(d) We again find the magnitude f' of the frictional force by using Eq. 8-31 (with $W = 0$):

$$\begin{aligned} 0 &= \Delta K + \Delta U + \Delta E_{\text{th}} \\ &= \frac{1}{2}mv^2 - mgh + f's' \end{aligned}$$

so that we obtain $f' = 1.2 \times 10^2$ N.

63. The initial and final kinetic energies are zero, and we set up energy conservation in the form of Eq. 8-31 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the thermal energy generated is $\Delta E_{\text{th}} = f_k d$ where $d \leq L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total thermal energy is $\Delta E_{\text{th}} = \mu_k mg(L + d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \leq d \leq L$). Generalizing to the n^{th} pass through, we see that $\Delta E_{\text{th}} = \mu_k mg((n-1)L + d)$. In this way, Eq. 8-39 leads to

$$mgh = \mu_k mg\left((n-1)L + d\right)$$

which simplifies (when h = L/2 is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n \ .$$

The first two terms give $1 + 1/2\mu_k = 3.5$, so that the requirement $0 \leq d/L \leq 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = \frac{1}{2}$ and that this occurs on its third pass through the flat region.

64. We observe that the last line of the problem indicates that static friction is not to be considered a factor in this problem. The friction force of magnitude $f = 4400$ N mentioned in the problem is kinetic friction and (as mentioned) is constant (and directed upward), and the thermal energy change associated with it is $\Delta E_{\text{th}} = fd$ (Eq. 8-29) where $d = 3.7$ m in part (a) (but will be replaced by x, the spring compression, in part (b)).

   (a) With $W = 0$ and the reference level for computing $U = mgy$ set at the top of the (relaxed) spring, Eq. 8-31 leads to

   $$U_i = K + \Delta E_{\text{th}} \implies v = \sqrt{2d\left(g - \frac{f}{m}\right)}$$

   which yields $v = 7.4$ m/s for $m = 1800$ kg.

   (b) We again utilize Eq. 8-31 (with $W = 0$), now relating its kinetic energy at the moment it makes contact with the spring to the system energy at the bottom-most point. Using the same reference level for computing $U = mgy$ as we did in part (a), we end up with gravitational potential energy equal to $mg(-x)$ at that bottom-most point, where the spring (with spring constant $k = 1.5 \times 10^5$ N/m) is fully compressed.

   $$K = mg(-x) + \frac{1}{2}kx^2 + fx$$

   where $K = \frac{1}{2}mv^2 = 4.9 \times 10^4$ J using the speed found in part (a). Using the abbreviation $\xi = mg - f = 1.3 \times 10^4$ N, the quadratic formula yields

   $$x = \frac{\xi \pm \sqrt{\xi^2 + 2kK}}{k} = 0.90 \text{ m}$$

   where we have taken the positive root.

   (c) We relate the energy at the bottom-most point to that of the highest point of rebound (a distance d' above the relaxed position of the spring). We assume $d' > x$. We now use the bottom-most point as the reference level for computing gravitational potential energy.

   $$\frac{1}{2}kx^2 = mgd' + fd' \implies d' = \frac{kx^2}{2(mg + d)} = 2.8 \text{ m} \ .$$

(d) The non-conservative force (§8-1) is friction, and the energy term associated with it is the one that keeps track of the total distance traveled (whereas the potential energy terms, coming as they do from conservative forces, depend on positions – but not on the paths that led to them). We assume the elevator comes to final rest at the equilibrium position of the spring, with the spring compressed an amount $d_{\rm eq}$ given by

$$mg = kd_{\rm eq} \implies d_{\rm eq} = \frac{mg}{k} = 0.12 \text{ m} .$$

In this part, we use that final-rest point as the reference level for computing gravitational potential energy, so the original $U = mgy$ becomes $mg(d_{\rm eq} + d)$. In that final position, then, the gravitational energy is zero and the spring energy is $\frac{1}{2}kd_{\rm eq}^2$. Thus, Eq. 8-31 becomes

$$\begin{aligned} mg\,(d_{\rm eq} + d) &= \frac{1}{2}kd_{\rm eq}^2 + fd_{\rm total} \\ (1800)(9.8)(0.12 + 3.7) &= \frac{1}{2}\left(1.5 \times 10^5\right)(0.12)^2 + (4400)d_{\rm total} \end{aligned}$$

which yields $d_{\rm total} = 15$ m.

65. (a) Since the speed of the crate of mass m increases from 0 to 1.20 m/s relative to the factory ground, the kinetic energy supplied to it is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(300\,\text{kg})(1.20\,\text{m/s})^2 = 216 \text{ J}.$$

(b) The magnitude of the kinetic frictional force is

$$f = \mu N = \mu mg = (0.400)(300\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 1.18 \times 10^3 \text{ N} .$$

(c) and (d) The energy supplied by the motor is the work W it does on the system, and must be greater than the kinetic energy gained by the crate computed in part (b). This is due to the fact that part of the energy supplied by the motor is being used to compensate for the energy dissipated $\Delta E_{\rm th}$ while it was slipping. Let the distance the crate moved relative to the conveyor belt before it stops slipping be d, then from Eq. 2-16 ($v^2 = 2ad = 2(f/m)d$) we find

$$\Delta E_{\rm th} = fd = \frac{1}{2}mv^2 = K .$$

Thus, the total energy that must be supplied by the motor is

$$W = K + \Delta E_{\rm th} = 2K = (2)(216\,\text{J}) = 432 \text{ J} .$$

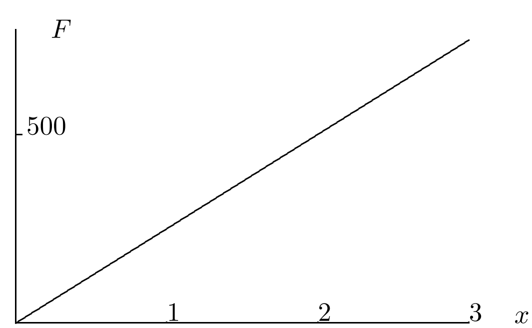
66. (a) The compression is "spring-like" so the maximum force relates to the distance x by Hooke's law:

$$F_{\rm x} = kx \implies x = \frac{750}{2.5 \times 10^5} = 0.0030 \text{ m} .$$

(b) The work is what produces the "spring-like" potential energy associated with the compression. Thus, using Eq. 8-11,

$$W = \frac{1}{2}kx^2 = \frac{1}{2}\left(2.5 \times 10^5\right)(0.0030)^2 = 1.1 \text{ J} .$$

(c) By Newton's third law, the force F exerted by the tooth is equal and opposite to the "spring-like" force exerted by the licorice, so the graph of F is a straight line of slope k. We plot F (in Newtons) versus x (in millimeters); both are taken as positive.



(d) As mentioned in part (b), the spring potential energy expression is relevant. Now, whether or not we can ignore dissipative processes is a deeper question. In other words, it seems unlikely that – if the tooth at any moment were to reverse its motion – that the licorice could "spring back" to its original shape. Still, to the extent that $U = \frac{1}{2}kx^2$ applies, the graph is a parabola (not shown here) which has its vertex at the origin and is either concave upward or concave downward depending on how one wishes to define the sign of F (the connection being $F = -dU/dx$).

(e) As a crude estimate, the area under the curve is roughly half the area of the entire plotting-area (8000 N by 12 mm). This leads to an approximate work of $\frac{1}{2}(8000)(0.012) \approx 50$ J. Estimates in the range $40 \le W \le 50$ J are acceptable.

(f) Certainly dissipative effects dominate this process, and we cannot assign it a meaningful potential energy.

67. (a) The drawings in the Figure (especially pictures (b) and (c)) show this geometric relationship very clearly. But we can work out the details, if need be. If ℓ is the length we are to compute (that of the still moving upper section) and ℓ' is the length of the lower (motionless) section, then clearly $\ell + \ell' = L$. Also, (as is especially easy to see in picture (c)) $x + \ell$ must equal ℓ'. These two equations, then, lead to the conclusion $\ell = \frac{1}{2}(L - x)$.

(b) The mass of the still moving upper section is

$$m = \rho\ell = \frac{\rho}{2}(L - x) \ .$$

(c) The assumptions stated in the problem lead to

$$\frac{1}{2}(\rho L + m_f)\, v_0^2 = \frac{1}{2}\left(\frac{\rho}{2}(L - x) + m_f\right) v^2$$

which yields the speed of the still moving upper section:

$$v = v_0\sqrt{\frac{\rho L + m_f}{\rho(L - x)/2 + m_f}} \ .$$

(d) As x approaches L, we obtain

$$v_f = v_0\sqrt{\frac{\rho L + m_f}{m_f}} = (6.0)\sqrt{\frac{(1.3)(20) + 0.8}{0.8}}$$

which yields $v_f = 35$ m/s.

68. (a) The effect of a (sliding) friction is described in terms of energy dissipated as shown in Eq. 8-29. We have

$$\Delta E = K + \frac{1}{2}k(0.08)^2 - \frac{1}{2}k(0.10)^2 = -f_k(0.02)$$

where distances are in meters and energies are in Joules. With $k = 4000$ N/m and $f_k = 80$ N, we obtain $K = 5.6$ J.

(b) In this case, we have $d = 0.10$ m. Thus,

$$\Delta E = K + 0 - \frac{1}{2}k(0.10)^2 = -f_k(0.10)$$

which leads to $K = 12$ J.

(c) We can approach this two ways. One way is to examine the dependence of energy on the variable d:

$$\Delta E = K + \frac{1}{2}k\,(d_0 - d)^2 - \frac{1}{2}kd_0^2 = -f_k d$$

where $d_0 = 0.10$ m, and solving for K as a function of d:

$$K = -\frac{1}{2}kd^2 + (kd_0)\,d - f_k d \ .$$

In this first approach, we could work through the $\frac{dK}{dd} = 0$ condition (or with the special capabilities of a graphing calculator) to obtain the answer $K_{\max} = \frac{1}{2k}(kd_0 - f_k)^2$. In the second (and perhaps easier) approach, we note that K is maximum where v is maximum – which is where $a = 0 \implies$ equilibrium of forces. Thus, the second approach simply solves for the equilibrium position

$$|F_{\text{spring}}| = f_k \quad \implies \quad kx = 80 \ \ .$$

Thus, with $k = 4000$ N/m we obtain $x = 0.02$ m. But $x = d_0 - d$ so this corresponds to $d = 0.08$ m. Then the methods of part (a) lead to the answer $K_{\max} = 12.8 \approx 13$ J.

69. Let the amount of stretch of the spring be x. For the object to be in equilibrium

$$kx - mg = 0 \quad \implies \quad x = mg/k \ .$$

Thus the gain in elastic potential energy for the spring is

$$\Delta U_e = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{m^2g^2}{2k}$$

while the loss in the gravitational potential energy of the system is

$$-\Delta U_g = mgx = mg\left(\frac{mg}{k}\right) = \frac{m^2g^2}{k}$$

which we see (by comparing with the previous expression) is equal to $2\Delta U_e$. The reason why $|\Delta U_g| \neq \Delta U_e$ is that, since the object is slowly lowered, an upward external force (e.g., due to the hand) must have been exerted on the object during the lowering process, preventing it from accelerating downward. This force does *negative* work on the object, reducing the total mechanical energy of the system.

70. (a) The rate of change of the gravitational potential energy is

$$\frac{dU}{dt} = mg\frac{dy}{dt} = -mg|v| = -(68)(9.8)(59) = -3.9 \times 10^4 \text{ J/s} \ .$$

Thus, the gravitational energy is being reduced at the rate of 3.9×10^4 W.

(b) Since the velocity is constant, the rate of change of the kinetic energy is zero. Thus the rate at which the mechanical energy is being dissipated is the same as that of the gravitational potential energy (3.9×10^4 W).

71. The power generation (assumed constant, so average power is the same as instantaneous power) is

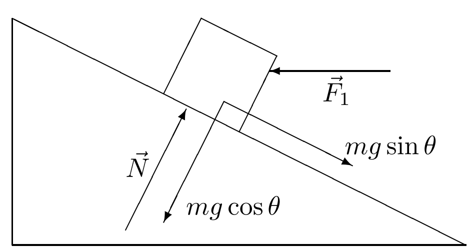
$$P = \frac{mgh}{t} = \frac{(3/4)\left(1200\,\text{m}^3\right)\left(10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(100\,\text{m})}{1.0\,\text{s}}$$

which yields $P = 8.8 \times 10^8$ W.

72. The free-body diagram for the trunk is shown.

The x and y applications of Newton's second law provide two equations:

$$\begin{aligned} F_1\cos\theta - f_k - mg\sin\theta &= ma \\ N - F_1\sin\theta - mg\cos\theta &= 0 \ . \end{aligned}$$



(a) The trunk is moving up the incline at constant velocity, so $a = 0$. Using $f_k = \mu_k N$, we solve for the push-force F_1 and obtain

$$F_1 = \frac{mg(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta} \ .$$

The work done by the push-force $\vec{F}_1$ as the trunk is pushed through a distance ℓ up the inclined plane is therefore

$$\begin{aligned} W_1 &= F_1 \ell \cos\theta = \frac{(mg\ell\cos\theta)(\sin\theta + \mu_k\cos\theta)}{\cos\theta - \mu_k\sin\theta} \\ &= \frac{(50\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(6.0\,\mathrm{m})(\cos 30^\circ)\,(\sin 30^\circ + (0.20)\cos 30^\circ)}{\cos 30^\circ - (0.20)\sin 30^\circ} \\ &= 2.2 \times 10^3 \ \mathrm{J} \ . \end{aligned}$$

(b) The increase in the gravitational potential energy of the trunk is

$$\Delta U = mg\ell\sin\theta = (50\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)(6.0\,\mathrm{m})\sin 30^\circ = 1.5 \times 10^3 \ \mathrm{J} \ .$$

Since the speed (and, therefore, the kinetic energy) of the trunk is unchanged, Eq. 8-31 leads to

$$W_1 = \Delta U + \Delta E_{\mathrm{th}} \ .$$

Thus, using more precise numbers than are shown above, the increase in thermal energy (generated by the kinetic friction) is $2.24 \times 10^3 - 1.47 \times 10^3 = 7.7 \times 10^2$ J. An alternate way to this result is to use $\Delta E_{\mathrm{th}} = f_k \ell$ (Eq. 8-29).

73. The style of reasoning used here is presented in §8-5.

(a) The horizontal line representing E_1 intersects the potential energy curve at a value of $r \approx 0.07$ nm and seems not to intersect the curve at larger r (though this is somewhat unclear since $U(r)$ is graphed only up to $r = 0.4$ nm). Thus, if m were propelled towards M from large r with energy E_1 it would "turn around" at 0.07 nm and head back in the direction from which it came.

(b) The line representing E_2 has two intersections points $r_1 \approx 0.16$ nm and $r_2 \approx 0.28$ nm with the $U(r)$ plot. Thus, if m starts in the region $r_1 < r < r_2$ with energy E_2 it will bounce back and forth between these two points, presumably forever.

(c) At $r = 0.3$ nm, the potential energy is roughly $U = -1.1 \times 10^{-19}$ J.

(d) With $M >> m$, the kinetic energy is essentially just that of m. Since $E = 1 \times 10^{-19}$ J, its kinetic energy is $K = E - U \approx 2.1 \times 10^{-19}$ J.

(e) Since force is related to the slope of the curve, we must (crudely) estimate $|F| \approx 1 \times 10^{-9}$ N at this point. The sign of the slope is positive, so by Eq. 8-20, the force is negative-valued. This is interpreted to mean that the atoms are attracted to each other.

(f) Recalling our remarks in the previous part, we see that the sign of F is positive (meaning it's repulsive) for $r < 0.2$ nm.

(g) And the sign of F is negative (attractive) for $r > 0.2$ nm.

(h) At $r = 0.2$ nm, the slope (hence, F) vanishes.

74. We take her original elevation to be the $y = 0$ reference level and observe that the top of the hill must consequently have $y_A = R(1 - \cos 20^\circ) = 1.2$ m, where R is the radius of the hill. The mass of the skier is $600/9.8 = 61$ kg.

(a) Applying energy conservation, Eq. 8-17, we have

$$K_B + U_B = K_A + U_A \quad \Longrightarrow \quad K_B + 0 = K_A + m\ .gy_A$$

Using $K_B = \frac{1}{2}(61\,\text{kg})(8.0\,\text{m/s})^2$, we obtain $K_A = 1.2 \times 10^3$ J. Thus, we find the speed at the hilltop is $v = \sqrt{2K/m} = 6.4$ m/s. (Note: one might wish to check that the skier stays in contact with the hill – which is indeed the case, here. For instance, at A we find $v^2/r \approx 2$ m/s^2 which is considerably less than g.)

(b) With $K_A = 0$, we have

$$K_B + U_B = K_A + U_A \implies K_B + 0 = 0 + mgy_A$$

which yields $K_B = 724$ J, and the corresponding speed is $v = \sqrt{2K/m} = 4.9$ m/s.

(c) Expressed in terms of mass, we have

$$\begin{aligned} K_B + U_B &= K_A + U_A \implies \\ \frac{1}{2}mv_B^2 + mgy_B &= \frac{1}{2}mv_A^2 + mgy_A \ . \end{aligned}$$

Thus, the mass m cancels, and we observe that solving for speed does not depend on the value of mass (or weight).

75. The spring is relaxed at $y = 0$, so the elastic potential energy (Eq. 8-11) is $U_{\text{el}} = \frac{1}{2}ky^2$. The total energy is conserved, and is zero (determined by evaluating it at its initial position). We note that U is the same as ΔU in these manipulations. Thus, we have

$$0 = K + U_g + U_e \implies K = -U_g - U_e$$

where $U_g = mgy = (20\,\text{N})y$ with y in meters (so that the energies are in Joules). We arrange the results in a table:

position y	-0.05	-0.10	-0.15	-0.20
U_g	-1.0	-2.0	-3.0	-4.0
U_e	0.25	1.0	2.25	4.0
K	0.75	1.0	0.75	0

76. From Eq. 8-6, we find (with SI units understood)

$$U(\xi) = -\int_0^{\xi} \left(-3x - 5x^2\right) dx = \frac{3}{2}\xi^2 + \frac{5}{3}\xi^3 \ .$$

(a) Using the above formula, we obtain $U(2) \approx 19$ J.

(b) When its speed is $v = 4$ m/s, its mechanical energy is $\frac{1}{2}mv^2 + U(5)$. This must equal the energy at the origin:

$$\frac{1}{2}mv^2 + U(5) = \frac{1}{2}mv_{\text{o}}^2 + U(0)$$

so that the speed at the origin is

$$v_{\text{o}} = \sqrt{v^2 + \frac{2}{m}\left(U(5) - U(0)\right)} \ .$$

Thus, with $U(5) = 246$ J, $U(0) = 0$ and $m = 20$ kg, we obtain $v_{\text{o}} = 6.4$ m/s.

(c) Our original formula for U is changed to $U(x) = -8 + \frac{3}{2}x^2 + \frac{5}{3}x^3$ in this case. Therefore, $U(2) = 11$ J. But we still have $v_{\text{o}} = 6.4$ m/s since that calculation only depended on the difference of potential energy values (specifically, $U(5) - U(0)$).

77. (a) At the top of its flight, the vertical component of the velocity vanishes, and the horizontal component (neglecting air friction) is the same as it was when it was thrown. Thus,

$$K_{\text{top}} = \frac{1}{2}mv_x^2 = \frac{1}{2}(0.050\,\text{kg})\left((8.0\,\text{m/s})\cos 30°\right)^2$$

which yields $K_{\text{top}} = 1.2$ J.

(b) We choose the point 3.0 m below the window as the reference level for computing the potential energy. Thus, equating the mechanical energy when it was thrown to when it is at this reference level, we have (with SI units understood)

$$\begin{aligned} mgy_0 + K_0 &= K \\ m(9.8)(3.0) + \frac{1}{2}m(8.0)^2 &= \frac{1}{2}mv^2 \end{aligned}$$

which yields (after canceling m and simplifying) $v = 11$ m/s.

(c) As mentioned, m cancels – and is therefore not relevant to that computation.

(d) The v in the kinetic energy formula is the magnitude of the velocity vector; it does not depend on the direction.

78. From the slope of the graph, we find the spring constant

$$k = \frac{\Delta F}{\Delta x} = 0.10 \text{ N/cm} = 10 \text{ N/m} .$$

(a) Equating the potential energy of the compressed spring to the kinetic energy of the cork at the moment of release, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \implies v = x\sqrt{\frac{k}{m}}$$

which yields $v = 2.8$ m/s for $m = 0.0038$ kg and $x = 0.055$ m.

(b) The new scenario involves some potential energy at the moment of release. With $d = 0.015$ m, energy conservation becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}kd^2 \implies v = \sqrt{\frac{k}{m}\left(x^2 - d^2\right)}$$

which yields $v = 2.7$ m/s.

79. We assume his initial kinetic energy (when he jumps) is negligible. Then, his initial gravitational potential energy measured relative to where he momentarily stops is what becomes the elastic potential energy of the stretched net (neglecting air friction). Thus,

$$U_{\text{net}} = U_{\text{grav}} = mgh$$

where $h = 11.0 + 1.5 = 12.5$ m. Consequently, with $m = 70$ kg, we obtain $U_{\text{net}} = 8.6 \times 10^3$ J.

80. The work done by $\vec{F}$ is the negative of its potential energy change (see Eq. 8-6), so $U_B = U_A - 25 = 15$ J.

81. (a) During the final $d = 12$ m of motion, we use

$$K_1 + U_1 = K_2 + U_2 + f_k d \implies \frac{1}{2}mv^2 + 0 = 0 + 0 + f_k d$$

where $v = 4.2$ m/s. This gives $f_k = 0.31$ N. Therefore, the thermal energy change is $f_k d = 3.7$ J.

(b) Using $f_k = 0.31$ N we obtain $f_k d_{\text{total}} = 4.3$ J for the thermal energy generated by friction; here, $d_{\text{total}} = 14$ m.

(c) During the initial $d' = 2$ m of motion, we have

$$K_0 + U_0 + W_{\text{app}} = K_1 + U_1 + f_k d' \implies 0 + 0 + W_{\text{app}} = \frac{1}{2}mv^2 + 0 + f_k d'$$

which essentially combines Eq. 8-31 and Eq. 8-29. This leads to the result $W_{\text{app}} = 4.3$ J, and – reasonably enough – is the same as our answer in part (b).

82. (a) This part is essentially a free-fall problem, which can be easily done with Chapter 2 methods. Instead, choosing energy methods, we take $y = 0$ to be the ground level.

$$K_i + U_i = K + U \implies 0 + mgy_i = \frac{1}{2}mv^2 + 0$$

Therefore $v = \sqrt{2gy_i} = 9.2$ m/s, where $y_i = 4.3$ m.

(b) Eq. 8-29 provides $\Delta E_{\text{th}} = f_k d$ for thermal energy generated by the kinetic friction force. We apply Eq. 8-31:

$$K_i + U_i = K + U \implies 0 + mgy_i = \frac{1}{2}mv^2 + 0 + f_k d$$

With $d = y_i$, $m = 70$ kg and $f_k = 500$ N, this yields $v = 4.8$ m/s.

83. We want to convert (at least in theory) the water that falls through $h = 500$ m into electrical energy. The problem indicates that in one year, a volume of water equal to $A\Delta z$ lands in the form of rain on the country, where $A = 8 \times 10^{12}$ m^2 and $\Delta z = 0.75$ m. Multiplying this volume by the density $\rho = 1000\,\text{kg/m}^3$ leads to

$$m_{\text{total}} = \rho A \Delta z = (1000)\left(8 \times 10^{12}\right)(0.75) = 6 \times 10^{15}\,\text{kg}$$

for the mass of rainwater. One-third of this "falls" to the ocean, so it is $m = 2 \times 10^{15}$ kg that we want to use in computing the gravitational potential energy mgh (which will turn into electrical energy during the year). Since a year is equivalent to 3.2×10^7 s, we obtain

$$P_{\text{avg}} = \frac{\left(2 \times 10^{15}\right)(9.8)(500)}{3.2 \times 10^7} = 3.1 \times 10^{11}\,\text{W} .$$

84. With the potential energy reference level set at the point of throwing, we have (with SI units understood)

$$\Delta E = mgh - \frac{1}{2}mv_0^2 = m\left((9.8)(8.1) - \frac{1}{2}(14)^2\right)$$

which yields $\Delta E = -12$ J for $m = 0.63$ kg. This "loss" of mechanical energy is presumably due to air friction.

85. We note that in one second, the block slides $d = 1.34$ m up the incline, which means its height increase is $h = d\sin\theta$ where

$$\theta = \tan^{-1}\left(\frac{30}{40}\right) = 37^\circ .$$

We also note that the force of kinetic friction in this inclined plane problem is $f_k = \mu_k mg\cos\theta$ where $\mu_k = 0.40$ and $m = 1400$ kg. Thus, using Eq. 8-31 and Eq. 8-29, we find

$$W = mgh + f_k d = mgd\,(\sin\theta + \mu_k\cos\theta)$$

or $W = 1.69 \times 10^4$ J for this one-second interval. Thus, the power associated with this is

$$P = \frac{1.69 \times 10^4\,\text{J}}{1\,\text{s}} = 1.69 \times 10^4\,\text{W} .$$

86. We take the original height of the box to be the $y = 0$ reference level and observe that, in general, the height of the box (when the box has moved a distance d downhill) is $y = -d\sin 40^\circ$.

(a) Using the conservation of energy, we have

$$K_i + U_i = K + U \implies 0 + 0 = \frac{1}{2}mv^2 + mgy + \frac{1}{2}kd^2 .$$

Therefore, with $d = 0.10$ m, we obtain $v = 0.81$ m/s.

(b) We look for a value of $d \neq 0$ such that $K = 0$.

$$K_i + U_i = K + U \implies 0 + 0 = 0 + mgy + \frac{1}{2}kd^2$$

Thus, we obtain $mgd\sin 40° = \frac{1}{2}kd^2$ and find $d = 0.21$ m.

(c) The uphill force is caused by the spring (Hooke's law) and has magnitude $kd = 25.2$ N. The downhill force is the component of gravity $mg\sin 40° = 12.6\,\text{N}$. Thus, the net force on the box is $25.2 - 12.6 = 12.6$ N uphill, and the acceleration is uphill with magnitude $12.6/2 = 6.3\ \text{m/s}^2$.

87. Equating the mechanical energy at his initial position (as he emerges from the canon, where we set the reference level for computing potential energy) to his energy as he lands, we obtain

$$\begin{aligned} K_i &= K_f + U_f \\ \frac{1}{2}(60\,\text{kg})(16\,\text{m/s})^2 &= K_f + (60\,\text{kg})\left(9.8\,\text{m/s}^2\right)(3.9\,\text{m}) \end{aligned}$$

which leads to $K_f = 5.4 \times 10^3$ J.

88. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(20)^2 = 300\,\text{J}$.

(b) At the point of maximum height, the vertical component of velocity vanishes but the horizontal component remains what it was when it was "shot" (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(1.5)\left(20\cos 34°\right)^2 = 206\ \text{J}\ .$$

Thus, $\Delta U = K_i - K = 300 - 206 = 94$ J.

(c) Since $\Delta U = mg\Delta y$, we obtain

$$\Delta y = \frac{94\,\text{J}}{(1.5\,\text{kg})\,(9.8\,\text{m/s}^2)} = 6.4\ \text{m}\ .$$

89. We note that if the larger mass ($M = 2$ kg) falls $d = 0.25$ m, then the smaller mass ($m = 1$ kg) must increase its height by $h = d\sin 30°$. Thus, by mechanical energy conservation, the kinetic energy of the system is

$$K_{\text{total}} = Mgd - mgh = 3.7\ \text{J}\ .$$

90. (a) At the point of maximum height, where $y = 140$ m, the vertical component of velocity vanishes but the horizontal component remains what it was when it was launched (if we neglect air friction). Its kinetic energy at that moment is

$$K = \frac{1}{2}(0.55\,\text{kg})v_x^2\ .$$

Also, its potential energy (with the reference level chosen at the level of the cliff edge) at that moment is $U = mgy = 755$ J. Thus, by mechanical energy conservation,

$$K = K_i - U = 1550 - 755 \implies v_x = \sqrt{\frac{2(1550 - 755)}{0.55}}$$

which yields $v_x = 54$ m/s.

(b) As mentioned $v_x = v_{i\,x}$ so that the initial kinetic energy

$$K_i = \frac{1}{2}m\left(v_{i\,x}^2 + v_{i\,y}^2\right)$$

can be used to find $v_{i\,y}$. We obtain $v_{i\,y} = 52$ m/s.

(c) Applying Eq. 2-16 to the vertical direction (with $+y$ upward), we have

$$\begin{aligned} v_y^2 &= v_{i\,y}^2 - 2g\Delta y \\ 65^2 &= 52^2 - 2(9.8)\Delta y \end{aligned}$$

which yields $\Delta y = -76$ m. The minus sign tells us it is below its launch point.

91. (a) The initial kinetic energy is $K_i = \frac{1}{2}(1.5)(3)^2 = 6.75\,\text{J}$.

(b) The work of gravity is the negative of its change in potential energy. At the highest point, all of K_i has converted into U (if we neglect air friction) so we conclude the work of gravity is $-6.75\,\text{J}$.

(c) And we conclude that $\Delta U = 6.75\,\text{J}$.

(d) The potential energy there is $U_f = U_i + \Delta U = 6.75\,\text{J}$.

(e) If $U_f = 0$, then $U_i = U_f - \Delta U = -6.75\,\text{J}$.

(f) Since $mg\Delta y = \Delta U$, we obtain $\Delta y = 0.46$ m.

92. (a) With energy in Joules and length in meters, we have

$$\Delta U = U(x) - U(0) = -\int_0^x (6x' - 12)\,dx' \ .$$

Therefore, with $U(0) = 27$ J, we obtain $U(x)$ (written simply as U) by integrating and rearranging:

$$U = 27 + 12x - 3x^2 \ .$$

(b) We can maximize the above function by working through the $\frac{dU}{dx} = 0$ condition, or we can treat this as a force equilibrium situation – which is the approach we show.

$$F = 0 \quad \Longrightarrow \quad 6x_{eq} - 12 = 0$$

Thus, $x_{eq} = 2.0$ m, and the above expression for the potential energy becomes $U = 39$ J.

(c) Using the quadratic formula or using the polynomial solver on an appropriate calculator, we find the values of x for which $U = 0$ to be 5.6 m and -1.6 m.

93. Since the aim of this problem is to invite student creativity (and possibly some research), we "invent a problem" (and give its solution) somewhat along the lines of part (b) (in fact, the student might consider running our example "in reverse"). Consider a block of mass M that falls from rest a distance H to a vertical spring of spring constant k. The spring compresses by x_c in order to halt the block, but on the rebound (due to the fact that the block is stuck on the end of the spring) the spring stretches (relative to its original relaxed length) an amount x_s before the block is momentarily at rest again. Take both values of x to be positive. Find x_c and x_s and their difference.

Solution: The height to which the spring reaches when it is relaxed is our $y = 0$ reference level. We relate the initial situation (when the block is dropped) to the situation of maximum compression using energy conservation.

$$K_0 + U_0 = K_c + U_c \quad \Longrightarrow \quad 0 + MgH = 0 + Mg(-x_c) + \frac{1}{2}kx_c^2$$

The positive root stemming from a quadratic formula solution for x_c yields

$$x_c = \frac{Mg}{k}\left(1 + \sqrt{1 + \frac{2kH}{Mg}}\right) \ .$$

Next, we relate the initial situation to the final situation (of maximal stretch) using energy conservation.

$$K_0 + U_0 = K_s + U_s \quad \Longrightarrow \quad 0 + MgH = 0 + Mgx_s + \frac{1}{2}kx_s^2$$

The positive root stemming from a quadratic formula solution for x_s yields

$$x_s = \frac{Mg}{k}\left(-1+\sqrt{1+\frac{2kH}{Mg}}\right) .$$

Finally, we note that $x_c > x_s$ with the difference being $x_c - x_s = 2Mg/k$.

94. (First problem in **Cluster 1**)
We take the bottom of the incline to be the $y = 0$ reference level. The incline angle is $\theta = 30°$. The distance along the incline d (measured from the bottom) is related to height y by the relation $y = d\sin\theta$.

(a) Using the conservation of energy, we have

$$K_0 + U_0 = K_{\text{top}} + U_{\text{top}} \implies \frac{1}{2}mv_0^2 + 0 = 0 + mgy$$

with $v_0 = 5.0$ m/s. This yields $y = 1.3$ m, from which we obtain $d = 2.6$ m.

(b) An analysis of forces in the manner of Chapter 6 reveals that the magnitude of the friction force is $f_k = \mu_k mg\cos\theta$. Now, we write Eq. 8-31 as

$$\begin{aligned} K_0 + U_0 &= K_{\text{top}} + U_{\text{top}} + f_k d \\ \frac{1}{2}mv_0^2 + 0 &= 0 + mgy + f_k d \\ \frac{1}{2}mv_0^2 &= mgd\sin\theta + \mu_k mgd\cos\theta \end{aligned}$$

which – upon cancelling the mass and rearranging – provides the result for d:

$$d = \frac{v_0^2}{2g\left(\mu_k\cos\theta + \sin\theta\right)} = 1.5 \text{ m} .$$

(c) The thermal energy generated by friction is $f_k d = \mu_k mgd\cos\theta = 26$ J.

(d) The slide back down, from the height $y = 1.5\sin 30°$ is also described by Eq. 8-31. With ΔE_{th} again equal to 26 J, we have

$$K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}} + f_k d \implies 0 + mgy = \frac{1}{2}mv_{\text{bot}}^2 + 0 + 26$$

from which we find $v_{\text{bot}} = 2.1$ m/s.

95. (Second problem in **Cluster 1**)
Converting to SI units, $v_0 = 8.3$ m/s and $v = 11.1$ m/s. The incline angle is $\theta = 5.0°$. The height difference between the car's highest and lowest points is $(50 \text{ m})\sin\theta = 4.4$ m. We take the lowest point (the car's final reported location) to correspond to the $y = 0$ reference level.

(a) Using Eq. 8-31 and Eq. 8-29, we find

$$f_k d = -\Delta K - \Delta U \implies f_k d = \frac{1}{2}m\left(v_0^2 - v^2\right) + mgy_0 .$$

Therefore, the mechanical energy reduction (due to friction) is $f_k d = 2.4 \times 10^4$ J.

(b) With $d = 50$ m, we solve for f_k and obtain 471 N, which can be rounded to 470 N.

96. (Third problem in **Cluster 1**)

(a) When there is no change in potential energy, Eq. 8-24 leads to

$$W_{\text{app}} = \Delta K = \frac{1}{2}m\left(v^2 - v_0^2\right) .$$

Therefore, $\Delta E = 6.0 \times 10^3$ J.

(b) From the above manipulation, we see $W_{\text{app}} = 6.0 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{6.0 \times 10^3}{10} = 600 \text{ W} .$$

(c) and (d) The constant applied force is $ma = 30$ N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 300 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 900 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

We note that the average of these two values agrees with the result in part (b).

97. (Fourth problem in **Cluster 1**)
The distance traveled up the incline can be figured with Chapter 2 techniques: $v^2 = v_0^2 + 2a\Delta x \longrightarrow \Delta x = 200\,\text{m}$. This corresponds to an increase in height equal to $y = 200 \sin\theta = 17$ m, where $\theta = 5.0°$. We take its initial height to be $y = 0$.

(a) Eq. 8-24 leads to

$$W_{\text{app}} = \Delta E = \frac{1}{2} m \left(v^2 - v_0^2\right) + mgy .$$

Therefore, $\Delta E = 8.6 \times 10^3$ J.

(b) From the above manipulation, we see $W_{\text{app}} = 8.6 \times 10^3$ J. Also, from Chapter 2, we know that $\Delta t = \Delta v/a = 10$ s. Thus, using Eq. 7-42,

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{8.6 \times 10^3}{10} = 860 \text{ W}$$

where the answer has been rounded off (from the 856 value that is provided by the calculator).

(c) and (d) Taking into account the component of gravity along the incline surface, the applied force is $ma + mg \sin\theta = 43$ N and clearly in the direction of motion, so Eq. 7-48 provides the results for instantaneous power

$$P = \vec{F} \cdot \vec{v} = \begin{cases} 430 \text{ W} & \text{for } v = 10 \text{ m/s} \\ 1300 \text{ W} & \text{for } v = 30 \text{ m/s} \end{cases}$$

where these answers have been rounded off (from 428 and 1284, respectively). We note that the average of these two values agrees with the result in part (b).

Chapter 9

1. (a) We locate the coordinate origin at the center of Earth. Then the distance r_{com} of the center of mass of the Earth-Moon system is given by

$$r_{\text{com}} = \frac{m_M r_M}{m_M + m_E}$$

where m_M is the mass of the Moon, m_E is the mass of Earth, and r_M is their separation. These values are given in Appendix C. The numerical result is

$$r_{\text{com}} = \frac{(7.36 \times 10^{22}\,\text{kg})(3.82 \times 10^{8}\,\text{m})}{7.36 \times 10^{22}\,\text{kg} + 5.98 \times 10^{24}\,\text{kg}} = 4.64 \times 10^{6}\ \text{m}\ .$$

(b) The radius of Earth is $R_E = 6.37 \times 10^6\,\text{m}$, so $r_{\text{com}} = 0.73 R_E$.

2. We locate the coordinate origin at the center of the carbon atom, and we consider both atoms to be "point particles." We will use the non-SI units for mass found in Appendix F; since they will cancel they will not prevent the answer from being in SI units.

$$r_{\text{com}} = \frac{(15.9994\,\text{grams/mole})(1.131 \times 10^{-10}\,\text{m})}{12.01115\,\text{grams/mole} + 15.9994\,\text{grams/mole}} = 6.46 \times 10^{-11}\ \text{m}\ .$$

3. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0\,\text{kg}$ particle; $x_2 = 1.0\,\text{m}$ and $y_2 = 2.0\,\text{m}$ are the coordinates of the $m_2 = 8.0\,\text{kg}$ particle; and, $x_3 = 2.0\,\text{m}$ and $y_3 = 1.0\,\text{m}$ are the coordinates of the $m_3 = 4.0\,\text{kg}$ particle.

(a) The x coordinate of the center of mass is

$$\begin{aligned} x_{\text{com}} &= \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_2} \\ &= \frac{0 + (8.0\,\text{kg})(1.0\,\text{m}) + (4.0\,\text{kg})(2.0\,\text{m})}{3.0\,\text{kg} + 8.0\,\text{kg} + 4.0\,\text{kg}} \\ &= 1.1\,\text{m}\ . \end{aligned}$$

(b) The y coordinate of the center of mass is

$$\begin{aligned} y_{\text{com}} &= \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} \\ &= \frac{0 + (8.0\,\text{kg})(2.0\,\text{m}) + (4.0\,\text{kg})(1.0\,\text{m})}{3.0\,\text{kg} + 8.0\,\text{kg} + 4.0\,\text{kg}} \\ &= 1.3\ \text{m}\ . \end{aligned}$$

(c) As the mass of the topmost particle is increased, the center of mass shifts toward that particle. As we approach the limit as the topmost particle is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of that particle.

4. We will refer to the arrangement as a "table." We locate the coordinate origin at the center of the tabletop and note that the center of mass of each "leg" is a distance $L/2$ below the top. With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x, y) = (+L/2, -L/2)$ and the center of mass of the left leg is at $(x, y) = (-L/2, -L/2)$. Thus, the x coordinate of the (whole table) center of mass is
$$x_{\text{com}} = \frac{M\,(+L/2) + M\,(-L/2)}{M + M + 3M} = 0$$
as expected. And the y coordinate of the (whole table) center of mass is
$$y_{\text{com}} = \frac{M\,(-L/2) + M\,(-L/2)}{M + M + 3M} = -\frac{L}{5}$$
so that the whole table center of mass is a small distance $(0.2L)$ directly below the middle of the tabletop.

5. First, we imagine that the small square piece (of mass m) that was cut from the large plate is returned to it so that the large plate is again a complete $6\,\text{m} \times 6\,\text{m}$ square plate (which has its center of mass at the origin). Then we "add" a square piece of "negative mass" $(-m)$ at the appropriate location to obtain what is shown in Fig. 9-24. If the mass of the whole plate is M, then the mass of the small square piece cut from it is obtained from a simple ratio of areas:
$$m = \left(\frac{2.0\,\text{m}}{6.0\,\text{m}}\right)^2 M \quad \Longrightarrow \quad M = 9m\ .$$
   (a) The x coordinate of the small square piece is $x = 2.0\,\text{m}$ (the middle of that square "gap" in the figure). Thus the x coordinate of the center of mass of the remaining piece is
$$x_{\text{com}} = \frac{(-m)x}{M + (-m)} = \frac{-m(2.0\,\text{m})}{9m - m} = -0.25\ \text{m}\ .$$
   (b) Since the y coordinate of the small square piece is zero, we have $y_{\text{com}} = 0$.

6. We locate the coordinate origin at the lower left corner of the iron side of the composite slab. We orient the x axis along the length of the slab (the 22.0-cm side); the y axis along the width of the slab (the 13.0-cm side); and, the z axis along the height of the slab (the 2.80-cm side). The coordinates for the opposite corner on the aluminum side are then $x = 22.0\,\text{cm}$, $y = 13.0\,\text{cm}$, and $z = 2.80\,\text{cm}$. By symmetry $y_{\text{com}} = 13.0\,\text{cm}/2 = 6.50\,\text{cm}$ and $z_{\text{com}} = 2.80\,\text{cm}/2 = 1.40\,\text{cm}$. We use Eq. 9-5 to find x_{com}:
$$\begin{aligned} x_{\text{com}} &= \frac{m_i x_{\text{com},i} + m_a x_{\text{com},a}}{m_i + m_a} = \frac{\rho_i V_i x_{\text{com},i} + \rho_a V_a x_{\text{cm},a}}{\rho_i V_i + \rho_a V_a} \\ &= \frac{(11.0\,\text{cm}/2)\left(7.85\,\text{g/cm}^3\right) + 3(11.0\,\text{cm}/2)\left(2.70\,\text{g/cm}^3\right)}{7.85\,\text{g/cm}^3 + 2.70\,\text{g/cm}^3} = 8.30\,\text{cm}\ . \end{aligned}$$
Therefore, the center of mass is at $11.0\,\text{cm} - 8.3\,\text{cm} = 2.7\,\text{cm}$ from the midpoint of the slab.

7. By symmetry the center of mass is located on the axis of symmetry of the molecule. We denote the distance between the nitrogen atom and the center of mass of NH_3 as x. Then $m_{\text{N}}x = 3m_{\text{H}}(d - x)$, where d is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:
$$d = \sqrt{(10.14 \times 10^{-11}\,\text{m})^2 - (9.4 \times 10^{-11}\text{m})^2} = 3.803 \times 10^{-11}\,\text{m}\ .$$
Thus,
$$x = \frac{3m_{\text{H}}d}{m_{\text{N}} + 3m_{\text{H}}} = \frac{3(1.00797)(3.803 \times 10^{-11}\text{m})}{14.0067 + 3(1.00797)} = 6.8 \times 10^{-12}\,\text{m}$$
where Appendix F has been used to find the masses.

8. The centers of mass (with centimeters understood) for each of the five sides are as follows:

$$\begin{array}{rl} (x_1, y_1, z_1) = (0, 20, 20) & \text{for the side in the } yz \text{ plane} \\ (x_2, y_2, z_2) = (20, 0, 20) & \text{for the side in the } xz \text{ plane} \\ (x_3, y_3, z_3) = (20, 20, 0) & \text{for the side in the } xy \text{ plane} \\ (x_4, y_4, z_4) = (40, 20, 20) & \text{for the remaining side parallel to side 1} \\ (x_5, y_5, z_5) = (20, 40, 20) & \text{for the remaining side parallel to side 2} \end{array}$$

Recognizing that all sides have the same mass m, we plug these into Eq. 9-5 to obtain the results (the first two being expected based on the symmetry of the problem).

(a)

$$x_{\text{com}} = \frac{mx_1 + mx_2 + mx_3 + mx_4 + mx_5}{5m} = \frac{0 + 20 + 20 + 40 + 20}{5} = 20\,\text{cm}$$

(b)

$$y_{\text{com}} = \frac{my_1 + my_2 + my_3 + my_4 + my_5}{5m} = \frac{20 + 0 + 20 + 20 + 40}{5} = 20\,\text{cm}$$

(c)

$$z_{\text{com}} = \frac{mz_1 + mz_2 + mz_3 + mz_4 + mz_5}{5m} = \frac{20 + 20 + 0 + 20 + 20}{5} = 16\,\text{cm}$$

9. (a) Since the can is uniform, its center of mass is at its geometrical center, a distance $H/2$ above its base. The center of mass of the soda alone is at its geometrical center, a distance $x/2$ above the base of the can. When the can is full this is $H/2$. Thus the center of mass of the can and the soda it contains is a distance

$$h = \frac{M(H/2) + m(H/2)}{M + m} = \frac{H}{2}$$

above the base, on the cylinder axis.

(b) We now consider the can alone. The center of mass is $H/2$ above the base, on the cylinder axis.

(c) As x decreases the center of mass of the soda in the can at first drops, then rises to $H/2$ again.

(d) When the top surface of the soda is a distance x above the base of the can, the mass of the soda in the can is $m_p = m(x/H)$, where m is the mass when the can is full ($x = H$). The center of mass of the soda alone is a distance $x/2$ above the base of the can. Hence

$$h = \frac{M(H/2) + m_p(x/2)}{M + m_p} = \frac{M(H/2) + m(x/H)(x/2)}{M + (mx/H)} = \frac{MH^2 + mx^2}{2(MH + mx)} \;.$$

We find the lowest position of the center of mass of the can and soda by setting the derivative of h with respect to x equal to 0 and solving for x. The derivative is

$$\frac{dh}{dx} = \frac{2mx}{2(MH + mx)} - \frac{(MH^2 + mx^2)m}{2(MH + mx)^2} = \frac{m^2x^2 + 2MmHx - MmH^2}{2(MH + mx)^2} \;.$$

The solution to $m^2x^2 + 2MmHx - MmH^2 = 0$ is

$$x = \frac{MH}{m}\left(-1 + \sqrt{1 + \frac{m}{M}}\right) \;.$$

The positive root is used since x must be positive. Next, we substitute the expression found for x into $h = (MH^2 + mx^2)/2(MH + mx)$. After some algebraic manipulation we obtain

$$h = \frac{HM}{m}\left(\sqrt{1 + \frac{m}{M}} - 1\right) \;.$$

10. Since the center of mass of the two-skater system does not move, both skaters will end up at the center of mass of the system. Let the center of mass be a distance x from the 40-kg skater, then

$$(65\,\text{kg})(10\,\text{m} - x) = (40\,\text{kg})x \quad \Longrightarrow \quad x = 6.2\,\text{m} \ .$$

Thus the 40-kg skater will move by 6.2 m.

11. Let m_c be the mass of the Chrysler and v_c be its velocity. Let m_f be the mass of the Ford and v_f be its velocity. Then the velocity of the center of mass is

$$v_{\text{com}} = \frac{m_c v_c + m_f v_f}{m_c + m_f} = \frac{(2400\,\text{kg})(80\,\text{km/h}) + (1600\,\text{kg})(60\,\text{km/h})}{2400\,\text{kg} + 1600\,\text{kg}} = 72\,\text{km/h}\,.$$

We note that the two velocities are in the same direction, so the two terms in the numerator have the same sign.

12. (a) Since the center of mass of the man-balloon system does not move, the balloon will move downward with a certain speed u relative to the ground as the man climbs up the ladder. The speed of the man relative to the ground is $v_g = v - u$. Thus, the speed of the center of mass of the system is

$$v_{\text{com}} = \frac{m v_g - Mu}{M + m} = \frac{m(v-u) - Mu}{M + m} = 0 \ .$$

This yields $u = mv/(M + m)$.

(b) Now that there is no relative motion within the system, the speed of both the balloon and the man is equal to v_{com}, which is zero. So the balloon will again be stationary.

13. We use the constant-acceleration equations of Table 2-1 (with $+y$ downward and the origin at the release point), Eq. 9-5 for y_{com} and Eq. 9-17 for $\vec{v}_{\text{com}}$.

(a) The location of the first stone (of mass m_1) at $t = 300\times10^{-3}\,\text{s}$ is $y_1 = (1/2)gt^2 = (1/2)(9.8)\left(300 \times 10^{-3}\right)^2 = 0.44\,\text{m}$, and the location of the second stone (of mass $m_2 = 2m_1$) at $t = 300 \times 10^{-3}\,\text{s}$ is $y_2 = (1/2)gt^2 = (1/2)(9.8)(300 \times 10^{-3} - 100 \times 10^{-3})^2 = 0.20\,\text{m}$. Thus, the center of mass is at

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1(0.44\,\text{m}) + 2m_1(0.20\,\text{m})}{m_1 + 2m_2} = 0.28 \text{ m} \ .$$

(b) The speed of the first stone at time t is $v_1 = gt$, while that of the second stone is $v_2 = g(t - 100 \times 10^{-3}\,\text{s})$. Thus, the center-of-mass speed at $t = 300 \times 10^{-3}\,\text{s}$ is

$$\begin{aligned} v_{\text{com}} &= \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \\ &= \frac{m_1(9.8)\left(300 \times 10^{-3}\right) + 2m_1(9.8)\left(300 \times 10^{-3} - 100 \times 10^{-3}\right)}{m_1 + 2m_1} \\ &= 2.3 \text{ m/s} \ . \end{aligned}$$

14. We use the constant-acceleration equations of Table 2-1 (with the origin at the traffic light), Eq. 9-5 for x_{com} and Eq. 9-17 for $\vec{v}_{\text{com}}$. At $t = 3.0\,\text{s}$, the location of the automobile (of mass m_1) is $x_1 = \frac{1}{2}at^2 = \frac{1}{2}(4.0\,\text{m/s}^2)(3.0\,\text{s})^2 = 18\,\text{m}$, while that of the truck (of mass m_2) is $x_2 = vt = (8.0\,\text{m/s})(3.0\,\text{s}) = 24\,\text{m}$. The speed of the automobile then is $v_1 = at = \left(4.0\,\text{m/s}^2\right)(3.0\,\text{s}) = 12\,\text{m/s}$, while the speed of the truck remains $v_2 = 8.0\,\text{m/s}$.

(a) The location of their center of mass is

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(1000\,\text{kg})(18\,\text{m}) + (2000\,\text{kg})(24\,\text{m})}{1000\,\text{kg} + 2000\,\text{kg}} = 22 \text{ m} \ .$$

(b) The speed of the center of mass is

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1000\,\text{kg})(12\,\text{m/s}) + (2000\,\text{kg})(8.0\,\text{m/s})}{1000\,\text{kg} + 2000\,\text{kg}} = 9.3\,\text{m/s}\,.$$

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0\,y} - gt$ and this is zero at time $t = v_{0\,y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{0x}t = v_0 t\cos\theta_0 = \frac{v_0^2}{g}\sin\theta_0\cos\theta_0 = \frac{(20\,\text{m/s})^2}{9.8\,\text{m/s}^2}\sin 60^\circ\cos 60^\circ = 17.7\text{ m}$$

and

$$y = v_{0\,y}t - \frac{1}{2}gt^2 = \frac{1}{2}\frac{v_0^2}{g}\sin^2\theta_0 = \frac{1}{2}\frac{(20\,\text{m/s})^2}{9.8\,\text{m/s}^2}\sin^2 60^\circ = 15.3\text{ m}\ .$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2v_0\cos\theta_0 = 2(20\,\text{m/s})\cos 60^\circ = 20\text{ m/s}\ .$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7\,\text{m}$, $y_0 = 15.3\,\text{m}$. Its y coordinate is given by $y = y_0 - \frac{1}{2}gt^2$, and when it lands this is zero. The time of landing is $t = \sqrt{2y_0/g}$ and the x coordinate of the landing point is

$$x = x_0 + V_0 t = x_0 + V_0\sqrt{\frac{2y_0}{g}} = 17.7\,\text{m} + (20\,\text{m/s})\sqrt{\frac{2(15.3\,\text{m})}{9.8\,\text{m/s}^2}} = 53\text{ m}\ .$$

16. The implication in the problem regarding $\vec{v}_0$ is that the olive and the nut start at rest. Although we could proceed by analyzing the forces on each object, we prefer to approach this using Eq. 9-14. The total force on the nut-olive system is $\vec{F}_{\mathbf{o}} + \vec{F}_{\mathbf{n}} = -\,\hat{\imath} + \hat{\jmath}$ with the unit newton understood. Thus, Eq. 9-14 becomes

$$-\,\hat{\imath} + \hat{\jmath} = M\vec{a}_{\text{com}}$$

where $M = 2.0$ kg. Thus, $\vec{a}_{\text{com}} = -\frac{1}{2}\,\hat{\imath} + \frac{1}{2}\,\hat{\jmath}$ in SI units. Each component is constant, so we apply the equations discussed in Chapters 2 and 4.

$$\Delta\vec{r}_{\text{com}} = \frac{1}{2}\,\vec{a}_{\text{com}}\,t^2 = -4.0\,\hat{\imath} + 4.0\,\hat{\jmath}$$

(in meters) when $t = 4.0$ s. It is perhaps instructive to work through this problem the *long way* (separate analysis for the olive and the nut and then application of Eq. 9-5) since it helps to point out the computational advantage of Eq. 9-14.

17. (a) We place the origin of a coordinate system at the center of the pulley, with the x axis horizontal and to the right and with the y axis downward. The center of mass is halfway between the containers, at $x = 0$ and $y = \ell$, where ℓ is the vertical distance from the pulley center to either of the containers. Since the diameter of the pulley is 50 mm, the center of mass is 25 mm from each container.

(b) Suppose 20 g is transferred from the container on the left to the container on the right. The container on the left has mass $m_1 = 480\,\text{g}$ and is at $x_1 = -25\,\text{mm}$. The container on the right has mass $m_2 = 520\,\text{g}$ and is at $x_2 = +25\,\text{mm}$. The x coordinate of the center of mass is then

$$x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(480\,\text{g})(-25\,\text{mm}) + (520\,\text{g})(25\,\text{mm})}{480\,\text{g} + 520\,\text{g}} = 1.0\,\text{mm}\,.$$

The y coordinate is still ℓ. The center of mass is 26 mm from the lighter container, along the line that joins the bodies.

(c) When they are released the heavier container moves downward and the lighter container moves upward, so the center of mass, which must remain closer to the heavier container, moves downward.

(d) Because the containers are connected by the string, which runs over the pulley, their accelerations have the same magnitude but are in opposite directions. If a is the acceleration of m_2, then $-a$ is the acceleration of m_1. The acceleration of the center of mass is

$$a_{\text{com}} = \frac{m_1(-a) + m_2a}{m_1 + m_2} = a\,\frac{m_2 - m_1}{m_1 + m_2}\ .$$

We must resort to Newton's second law to find the acceleration of each container. The force of gravity m_1g, down, and the tension force of the string T, up, act on the lighter container. The second law for it is $m_1g - T = -m_1a$. The negative sign appears because a is the acceleration of the heavier container. The same forces act on the heavier container and for it the second law is $m_2g - T = m_2a$. The first equation gives $T = m_1g + m_1a$. This is substituted into the second equation to obtain $m_2g - m_1g - m_1a = m_2a$, so $a = (m_2 - m_1)g/(m_1 + m_2)$. Thus

$$a_{\text{com}} = \frac{g(m_2 - m_1)^2}{(m_1 + m_2)^2} = \frac{(9.8\,\text{m/s}^2)(520\,\text{g} - 480\,\text{g})^2}{(480\,\text{g} + 520\,\text{g})^2} = 1.6 \times 10^{-2}\,\text{m/s}^2\,.$$

The acceleration is downward.

18. We denote the mass of Ricardo as M_R and that of Carmelita as M_C. Let the center of mass of the two-person system (assumed to be closer to Ricardo) be a distance x from the middle of the canoe of length L and mass m. Then $M_R(L/2 - x) = mx + M_C(L/2 + x)$. Now, after they switch positions, the center of the canoe has moved a distance $2x$ from its initial position. Therefore, $x = 40\,\text{cm}/2 = 0.20\,\text{m}$, which we substitute into the above equation to solve for M_C:

$$M_C = \frac{M_R(L/2 - x) - mx}{L/2 + x} = \frac{(80)\left(\frac{3.0}{2} - 0.20\right) - (30)(0.20)}{(3.0/2) + 0.20} = 58\ \text{kg}\ .$$

19. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16,

$$M\Delta x_{\text{com}} = 0 = m_b\Delta x_b + m_d\Delta x_d$$

which implies

$$|\Delta x_b| = \frac{m_d}{m_b}\,|\Delta x_d|\ .$$

Now we express the geometrical condition that *relative to the boat* the dog has moved a distance $d = 2.4$ m:

$$|\Delta x_b| + |\Delta x_d| = d$$

which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}\,|\Delta x_d| + |\Delta x_d| = d$$

which leads to

$$|\Delta x_d| = \frac{d}{1 + \frac{m_d}{m_b}} = \frac{2.4}{1 + \frac{4.5}{18}} = 1.92\ \text{m}\ .$$

The dog is therefore 1.9 m closer to the shore than initially (where it was 6.1 m from it). Thus, it is now 4.2 m from the shore.

20. We apply Eq. 9-22 ($p = mv$) and Eq. 7-1 ($K = \frac{1}{2}mv^2$).

    (a) The speed of the VW Beetle of mass m is

$$v = \frac{p}{m} = \frac{(2650\,\text{kg})(16\,\text{km/h})}{816\,\text{kg}} = 52\,\text{km/h}\ .$$

    (b) In this case, the speed of the VW Beetle must be

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(2650\,\text{kg})(16\,\text{km/h})^2/2}{816\,\text{kg}}} = 29\,\text{km/h}\ .$$

21. Using Eq. 9-22, the necessary speed v is

$$v = \frac{p}{m} = \frac{(1600\,\text{kg})(1.2\,\text{km/h})}{80\,\text{kg}} = 24\,\text{km/h}\ .$$

22. The magnitude of the ball's momentum change is

$$\Delta p = |mv_i - mv_f| = (0.70\,\text{kg})\,|5.0\,\text{m/s} - (-2.0\,\text{m/s})| = 4.9\ \text{kg}\cdot\text{m/s}\ .$$

23. (a) The change in kinetic energy is

$$\begin{aligned}\Delta K &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}(2100\,\text{kg})\left((51\,\text{km/h})^2 - (41\,\text{km/h})^2\right) \\ &= 9.66\times10^4\,\text{kg}\cdot(\text{km/h})^2\left((10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})\right)^2 \\ &= 7.5\times10^4\ \text{J}\ .\end{aligned}$$

    (b) The magnitude of the change in velocity is

$$\begin{aligned}|\Delta\vec{v}| &= \sqrt{(-v_i)^2 + (v_f)^2} \\ &= \sqrt{(-41\,\text{km/h})^2 + (51\,\text{km/h})^2} \\ &= 65.4\ \text{km/h}\end{aligned}$$

    so the magnitude of the change in momentum is

$$|\Delta\vec{p}| = m\,|\Delta\vec{v}| = (2100\,\text{kg})(65.4\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right) = 3.8\times10^4\,\text{kg}\cdot\text{m/s}\ .$$

    (c) The vector $\Delta\vec{p}$ points at an angle θ south of east, where

$$\theta = \tan^{-1}\left(\frac{v_i}{v_f}\right) = \tan^{-1}\left(\frac{41\,\text{km/h}}{51\,\text{km/h}}\right) = 39^\circ\ .$$

24. (a) Since the force of impact on the ball is in the y direction, p_x is conserved: $p_{x\,i} = mv_i\sin 30^\circ = p_{x\,f} = mv_i\sin\theta$. Thus $\theta = 30^\circ$.

    (b) The momentum change is

$$\begin{aligned}\Delta\vec{p} &= mv_i\cos\theta\,(-\hat{\jmath}\,) - mv_i\cos\theta\,(+\hat{\jmath}\,) \\ &= -2(0.165\,\text{kg})(2.00\,\text{m/s})(\cos 30^\circ)\,\hat{\jmath} \\ &= -0.572\,\hat{\jmath}\quad \text{kg}\cdot\text{m/s}\ .\end{aligned}$$

25. The velocity of the object is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left((3500 - 160t)\,\hat{\imath} + 2700\,\hat{\jmath} + 300\,\hat{\mathrm{k}}\right) = -160\,\hat{\imath}\ \text{m/s}\ .$$

(a) The linear momentum is

$$\vec{p} = m\vec{v} = (250)(-160\,\hat{\imath}) = -4.0\times 10^4\,\hat{\imath}\quad \text{kg·m/s}\ .$$

(b) The object is moving west (our $-\,\hat{\imath}$ direction).

(c) Since the value of $\vec{p}$ does not change with time, the net force exerted on the object is zero, by Eq. 9-23.

26. We use coordinates with $+x$ horizontally toward the pitcher and $+y$ upward. Angles are measured counterclockwise from the $+x$ axis. Mass, velocity and momentum units are SI. Thus, the initial momentum can be written $\vec{p}_0 = (4.5\ \angle\ 215^\circ)$ in magnitude-angle notation.

(a) In magnitude-angle notation, the momentum change is $(6.0\ \angle -90^\circ) - (4.5\ \angle\ 215^\circ) = (5.0\ \angle -43^\circ)$ (efficiently done with a vector capable calculator in polar mode). The magnitude of the momentum change is therefore 5.0 kg·m/s.

(b) The momentum change is $(6.0\ \angle\ 0^\circ) - (4.5\ \angle\ 215^\circ) = (10\ \angle\ 15^\circ)$. Thus, the magnitude of the momentum change is 10 kg·m/s.

27. No external forces with horizontal components act on the man-stone system and the vertical forces sum to zero, so the total momentum of the system is conserved. Since the man and the stone are initially at rest, the total momentum is zero both before and after the stone is kicked. Let m_s be the mass of the stone and v_s be its velocity after it is kicked; let m_m be the mass of the man and v_m be his velocity after he kicks the stone. Then $m_s v_s + m_m v_m = 0 \;\rightarrow\; v_m = -m_s v_s/m_m$. We take the axis to be positive in the direction of motion of the stone. Then

$$v_m = -\frac{(0.068\,\text{kg})(4.0\,\text{m/s})}{91\,\text{kg}} = -3.0\times 10^{-3}\ \text{m/s}\ .$$

The negative sign indicates that the man moves in the direction opposite to the direction of motion of the stone.

28. The fact that they are connected by a spring is not used in the solution. We use Eq. 9-17 for $\vec{v}_{\text{com}}$:

$$\begin{aligned} M\vec{v}_{\text{com}} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ 0 &= (1.0)(1.7) + (3.0)\vec{v}_2 \end{aligned}$$

which yields $|\vec{v}_2| = 0.57$ m/s. The direction of $\vec{v}_2$ is opposite that of $\vec{v}_1$ (that is, they are both headed towards the center of mass, but from opposite directions).

29. No external forces with horizontal components act on the cart-man system and the vertical forces sum to zero, so the total momentum of the system is conserved. Let m_c be the mass of the cart, v be its initial velocity, and v_c be its final velocity (after the man jumps off). Let m_m be the mass of the man. His initial velocity is the same as that of the cart and his final velocity is zero. Conservation of momentum yields $(m_m + m_c)v = m_c v_c$. Consequently, the final speed of the cart is

$$v_c = \frac{v(m_m + m_c)}{m_c} = \frac{(2.3\,\text{m/s})(75\,\text{kg} + 39\,\text{kg})}{39\,\text{kg}} = 6.7\,\text{m/s}\ .$$

The cart speeds up by $6.7 - 2.3 = 4.4$ m/s. In order to slow himself, the man gets the cart to push backward on him by pushing forward on it, so the cart speeds up.

30. We apply Eq. 9-17, with $M = \sum m = 1.3$ kg,

$$\begin{aligned} M\vec{v}_{\rm com} &= m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C \\ (1.3)\,(-0.40\,\hat{\imath}) &= (0.50)\vec{v}_A + (0.60)\,(0.20\,\hat{\imath}) + (0.20)\,(0.30\,\hat{\imath}) \end{aligned}$$

which leads to $\vec{v}_A = -1.4\,\hat{\imath}$ in SI units (m/s).

31. Our notation is as follows: the mass of the motor is M; the mass of the module is m; the initial speed of the system is v_0; the relative speed between the motor and the module is v_r; and, the speed of the module relative to the Earth is v after the separation. Conservation of linear momentum requires $(M+m)v_0 = mv + M(v - v_r)$. Therefore,

$$v = v_0 + \frac{Mv_r}{M+m} = 4300\,\text{km/h} + \frac{(4m)(82\,\text{km/h})}{4m+m} = 4.4\times 10^3\,\text{km/h}\ .$$

32. Denoting the new speed of the car as v, then the new speed of the man relative to the ground is $v - v_{\rm rel}$. Conservation of momentum requires

$$\left(\frac{W}{g} + \frac{w}{g}\right)v_0 = \left(\frac{W}{g}\right)v + \left(\frac{w}{g}\right)(v - v_{\rm rel})\ .$$

Consequently, the change of velocity is

$$\Delta\vec{v} = v - v_0 = \frac{w\,v_{\rm rel}}{W+w}\ .$$

33. We assume no external forces act on the system composed of the two parts of the last stage. Hence, the total momentum of the system is conserved. Let m_c be the mass of the rocket case and m_p be the mass of the payload. At first they are traveling together with velocity v. After the clamp is released m_c has velocity v_c and m_p has velocity v_p. Conservation of momentum yields $(m_c + m_p)v = m_cv_c + m_pv_p$.

(a) After the clamp is released the payload, having the lesser mass, will be traveling at the greater speed. We write $v_p = v_c + v_{\rm rel}$, where $v_{\rm rel}$ is the relative velocity. When this expression is substituted into the conservation of momentum condition, the result is

$$(m_c + m_p)\,v = m_cv_c + m_pv_c + m_pv_{\rm rel}\ .$$

Therefore,

$$\begin{aligned} v_c &= \frac{(m_c+m_p)\,v - m_pv_{\rm rel}}{m_c+m_p} \\ &= \frac{(290.0\,\text{kg} + 150.0\,\text{kg})(7600\,\text{m/s}) - (150.0\,\text{kg})(910.0\,\text{m/s})}{290.0\,\text{kg} + 150.0\,\text{kg}} \\ &= 7290\ \text{m/s}\ . \end{aligned}$$

(b) The final speed of the payload is $v_p = v_c + v_{\rm rel} = 7290\,\text{m/s} + 910.0\,\text{m/s} = 8200\,\text{m/s}$.

(c) The total kinetic energy before the clamp is released is

$$K_i = \frac{1}{2}\,(m_c+m_p)v^2 = \frac{1}{2}(290.0\,\text{kg} + 150.0\,\text{kg})(7600\,\text{m/s})^2 = 1.271\times 10^{10}\ \text{J}\ .$$

(d) The total kinetic energy after the clamp is released is

$$\begin{aligned} K_f &= \frac{1}{2}\,m_cv_c^2 + \frac{1}{2}\,m_pv_p^2 \\ &= \frac{1}{2}(290.0\,\text{kg})(7290\,\text{m/s})^2 + \frac{1}{2}(150.0\,\text{kg})(8200\,\text{m/s})^2 \\ &= 1.275\times 10^{10}\ \text{J}\ . \end{aligned}$$

The total kinetic energy increased slightly. Energy originally stored in the spring is converted to kinetic energy of the rocket parts.

34. Our $+x$ direction is east and $+y$ direction is north. The linear momenta for the two $m = 2.0\,\mathrm{kg}$ parts are then

$$\vec{p}_1 = m\vec{v}_1 = mv_1\,\hat{\jmath}$$

where $v_1 = 3.0\,\mathrm{m/s}$, and

$$\vec{p}_2 = m\vec{v}_2 = m(v_{2x}\,\hat{\imath} + v_{2y}\,\hat{\jmath}) = mv_2(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})$$

where $v_2 = 5.0\,\mathrm{m/s}$ and $\theta = 30°$. The combined linear momentum of both parts is then

$$\begin{aligned}\vec{P} &= \vec{p}_1 + \vec{p}_2 \\ &= mv_1\,\hat{\jmath} + mv_2\left(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}\right) = (mv_2\cos\theta)\,\hat{\imath} + (mv_1 + mv_2\sin\theta)\,\hat{\jmath} \\ &= (2.0\,\mathrm{kg})(5.0\,\mathrm{m/s})(\cos 30°)\hat{\imath} + (2.0\,\mathrm{kg})\,(3.0\,\mathrm{m/s} + (5.0\,\mathrm{m/s})(\sin 30°))\,\hat{\jmath} \\ &= \left(8.66\,\hat{\imath} + 11\,\hat{\jmath}\right)\ \mathrm{kg{\cdot}m/s}\ .\end{aligned}$$

From conservation of linear momentum we know that this is also the linear momentum of the whole kit before it splits. Thus the speed of the 4.0-kg kit is

$$v = \frac{P}{M} = \frac{\sqrt{P_x^2 + P_y^2}}{M} = \frac{\sqrt{(8.66\,\mathrm{kg{\cdot}m/s})^2 + (11\,\mathrm{kg{\cdot}m/s})^2}}{4.0\,\mathrm{kg}} = 3.5\ \mathrm{m/s}\ .$$

35. We establish a coordinate system with the origin at the position of initial nucleus of mass $m_{m\,i}$ (which was stationary), with the electron momentum $\vec{p}_e$ in the $-x$ direction and the neutrino momentum $\vec{p}_\nu$ in the $-y$ direction. We will use unit-vector notation, although the problem does not specifically request it.

(a) We find the momentum $\vec{p}_{n\,r}$ of the residual nucleus from momentum conservation.

$$\begin{aligned}\vec{p}_{n\,i} &= \vec{p}_e + \vec{p}_\nu + \vec{p}_{n\,r} \\ 0 &= -1.2\times10^{-22}\,\hat{\imath} - 6.4\times10^{-23}\,\hat{\jmath} + \vec{p}_{n\,r}\end{aligned}$$

Thus, $\vec{p}_{n\,r} = 1.2\times10^{-22}\,\hat{\imath} + 6.4\times10^{-23}\,\hat{\jmath}$ in SI units (kg·m/s). Its magnitude is

$$|\vec{p}_{n\,r}| = \sqrt{(1.2\times10^{-22})^2 + (6.4\times10^{-23})^2} = 1.4\times10^{-22}\ \mathrm{kg{\cdot}m/s}\ .$$

(b) The angle measured from the $+x$ axis to $\vec{p}_{n\,r}$ is

$$\theta = \tan^{-1}\left(\frac{6.4\times10^{-23}}{1.2\times10^{-22}}\right) = 28°\ .$$

Therefore, the angle between $\vec{p}_e$ (which is in the $-x$ direction) and $\vec{p}_{n\,r}$ is $180° - 28° \approx 150°$.

(c) Measuring clockwise (but not using the "traditional" minus sign with that sense) we find the angle between $\vec{p}_{n\,r}$ and $\vec{p}_\nu$ (which points in the $-y$ direction) is $90° + 28° \approx 120°$.

(d) Combining the two equations $p = mv$ and $K = \frac{1}{2}mv^2$, we obtain (with $p = p_{n\,r}$ and $m = m_{n\,r}$)

$$K = \frac{p^2}{2m} = \frac{\left(1.4\times10^{-22}\right)^2}{2\left(5.8\times10^{-26}\right)} = 1.6\times10^{-19}\ \mathrm{J}\ .$$

36. This problem involves both mechanical energy conservation

$$U_i = K_1 + K_2$$

where $U_i = 60$ J, and momentum conservation

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $m_2 = 2m_1$. From the second equation, we find $|\vec{v}_1| = 2\,|\vec{v}_2|$ which in turn implies (since $v_1 = |\vec{v}_1|$ and likewise for v_2)

$$K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}\left(\frac{1}{2}m_2\right)(2v_2)^2 = 2\left(\frac{1}{2}m_2v_2^2\right) = 2K_2\ .$$

(a) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \implies K_2 = \frac{1}{3}U_i = 20\text{ J}\ .$$

(b) And we obtain $K_1 = 2(20) = 40$ J.

37. Our notation is as follows: the mass of the original body is $M = 20.0$ kg; its initial velocity is $\vec{v}_0 = 200\hat{\imath}$ in SI units (m/s); the mass of one fragment is $m_1 = 10.0$ kg; ; its velocity is $\vec{v}_1 = 100\hat{\jmath}$ in SI units; the mass of the second fragment is $m_2 = 4.0$ kg; ; its velocity is $\vec{v}_2 = -500\hat{\imath}$ in SI units; and, the mass of the third fragment is $m_3 = 6.00$ kg.

(a) Conservation of linear momentum requires

$$M\vec{v}_0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$$

which (using the above information) leads to

$$\vec{v}_3 = 1000\,\hat{\imath} - 167\,\hat{\jmath}$$

in SI units. The magnitude of $\vec{v}_3$ is $v_3 = \sqrt{1000^2 + (-167)^2} = 1.01 \times 10^3$ m/s. It points at $\tan^{-1}(-167/1000) = -9.48°$ (that is, at 9.5° measured clockwise from the $+x$ axis).

(b) We are asked to calculate ΔK or

$$\left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2\right) - \frac{1}{2}Mv_0^2 = 3.23 \times 10^6\text{ J}\ .$$

38. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of the original body is m; its initial velocity is $\vec{v}_0 = v\,\hat{\imath}$; the mass of the less massive piece is m_1; ; its velocity is $\vec{v}_1 = 0$; and, the mass of the more massive piece is m_2. We note that the conditions $m_2 = 3m_1$ (specified in the problem) and $m_1 + m_2 = m$ generally assumed in classical physics (before Einstein) lead us to conclude

$$m_1 = \frac{1}{4}\,m \quad \text{and} \quad m_2 = \frac{3}{4}\,m\ .$$

Conservation of linear momentum requires

$$\begin{aligned} m\vec{v}_0 &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ mv\,\hat{\imath} &= 0 + \frac{3}{4}\,m\,\vec{v}_2 \end{aligned}$$

which leads to

$$\vec{v}_2 = \frac{4}{3}\,v\ \hat{\imath}\ .$$

The increase in the system's kinetic energy is therefore

$$\begin{aligned} \Delta K &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}mv_0^2 \\ &= 0 + \frac{1}{2}\left(\frac{3}{4}\,m\right)\left(\frac{4}{3}\,v\right)^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{6}\,m\,v^2\ . \end{aligned}$$

39. Our notation (and, implicitly, our choice of coordinate system) is as follows: the mass of one piece is $m_1 = m$; ; its velocity is $\vec{v}_1 = -30\,\hat{\imath}$ in SI units (m/s); the mass of the second piece is $m_2 = m$; ; its velocity is $\vec{v}_2 = -30\,\hat{\jmath}$ in SI units; and, the mass of the third piece is $m_3 = 3m$. Conservation of linear momentum requires

$$\begin{aligned} m\vec{v}_0 &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \\ 0 &= m\,(-30\,\hat{\imath}) + m\left(-30\,\hat{\jmath}\right) + 3m\vec{v}_3 \end{aligned}$$

which leads to

$$\vec{v}_3 = 10\,\hat{\imath} + 10\,\hat{\jmath}$$

in SI units. Its magnitude is $v_3 = 10\sqrt{2} \approx 14$ m/s and its angle is 45° counterclockwise from $+x$ (in this system where we have m_1 flying off in the $-x$ direction and m_2 flying off in the $-y$ direction).

40. One approach is to choose a *moving* coordinate system which travels the center of mass of the body, and another is to do a little extra algebra analyzing it in the original coordinate system (in which the speed of the $m = 8.0$ kg mass is $v_0 = 2$ m/s, as given). Our solution is in terms of the latter approach since we are assuming that this is the approach most students would take. Conservation of linear momentum (along the direction of motion) requires

$$\begin{aligned} mv_0 &= m_1v_1 + m_2v_2 \\ (8.0)(2.0) &= (4.0)v_1 + (4.0)v_2 \end{aligned}$$

which leads to

$$v_2 = 4 - v_1$$

in SI units (m/s). We require

$$\begin{aligned} \Delta K &= \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \frac{1}{2}mv_0^2 \\ 16 &= \left(\frac{1}{2}(4.0)v_1^2 + \frac{1}{2}(4.0)v_2^2\right) - \frac{1}{2}(8.0)(2.0)^2 \end{aligned}$$

which simplifies to

$$v_2^2 = 16 - v_1^2$$

in SI units. If we substitute for v_2 from above, we find

$$(4 - v_1)^2 = 16 - v_1^2$$

which simplifies to

$$2v_1^2 - 8v_1 = 0$$

and yields either $v_1 = 0$ or $v_1 = 4$ m/s. If $v_1 = 0$ then $v_2 = 4 - v_1 = 4$ m/s, and if $v_1 = 4$ then $v_2 = 0$. Stated more simply, one of the chunks has zero speed and the other has a velocity of 4.0 m/s (along the original direction of motion).

41. We use Eq. 9-43. Then

$$\begin{aligned} v_f &= v_i + v_{\text{rel}} \ln \frac{M_i}{M_f} \\ &= 105\,\text{m/s} + (253\,\text{m/s}) \ln \frac{6090\,\text{kg}}{6010\,\text{kg}} \\ &= 108\ \text{m/s}\ . \end{aligned}$$

42. (a) We use Eq. 9-42. The thrust is

$$\begin{aligned} R\,v_{\rm rel} &= Ma \\ &= \left(4.0\times 10^4\ {\rm kg}\right)\left(2.0\,{\rm m/s^2}\right) \\ &= 8.0\times 10^4\ {\rm N}\ . \end{aligned}$$

(b) Since $v_{\rm rel} = 3000$ m/s, we see from part (a) that $R \approx 27$ kg/s.

43. (a) The thrust of the rocket is given by $T = Rv_{\rm rel}$, where R is the rate of fuel consumption and $v_{\rm rel}$ is the speed of the exhaust gas relative to the rocket. For this problem $R = 480\,{\rm kg/s}$ and $v_{\rm rel} = 3.27\times 10^3\,{\rm m/s}$, so

$$T = (480\,{\rm kg/s})(3.27\times 10^3\,{\rm m/s}) = 1.57\times 10^6\ {\rm N}\ .$$

(b) The mass of fuel ejected is given by $M_{\rm fuel} = R\Delta t$, where Δt is the time interval of the burn. Thus, $M_{\rm fuel} = (480\,{\rm kg/s})(250\,{\rm s}) = 1.20\times 10^5\,{\rm kg}$. The mass of the rocket after the burn is $M_f = M_i - M_{\rm fuel} = 2.55\times 10^5\,{\rm kg} - 1.20\times 10^5\,{\rm kg} = 1.35\times 10^5\,{\rm kg}$.

(c) Since the initial speed is zero, the final speed is given by

$$v_f = v_{\rm rel}\,\ln\frac{M_i}{M_f} = \left(3.27\times 10^3\right)\ln\left(\frac{2.55\times 10^5}{1.35\times 10^5}\right) = 2.08\times 10^3\ {\rm m/s}\ .$$

44. We use Eq. 9-43 and simplify with $v_i = 0$, $v_f = v$, and $v_{\rm rel} = u$.

$$v_f - v_i = v_{\rm rel}\,\ln\frac{M_i}{M_f} \quad\Longrightarrow\quad \frac{M_i}{M_f} = e^{v/u}$$

(a) If $v = u$, we obtain $\frac{M_i}{M_f} = e^1 \approx 2.7\,.$

(b) If $v = 2u$, we obtain $\frac{M_i}{M_f} = e^2 \approx 7.4\,.$

45. We use Eq. 9-43 and simplify with $v_f - v_i = \Delta v$, and $v_{\rm rel} = u$.

$$v_f - v_i = v_{\rm rel}\,\ln\frac{M_i}{M_f} \quad\Longrightarrow\quad \frac{M_f}{M_i} = e^{-\Delta v/u}$$

If $\Delta v = 2.2$ m/s and $u = 1000$ m/s, we obtain $\frac{M_i - M_f}{M_i} = 1 - e^{-0.0022} \approx 0.0022$.

46. We convert mass rate to SI units: $R = 540/60 = 9.00$ kg/s. In the absence of the asked-for additional force, the car would decelerate with a magnitude given by Eq. 9-42:

$$R\,v_{\rm rel} = M\,|a|$$

so that if $a = 0$ is desired then the additional force must have a magnitude equal to $R\,v_{\rm rel}$ (so as to cancel that effect).

$$F = R\,v_{\rm rel} = (9.00)(3.20) = 28.8\ {\rm N}\ .$$

47. (a) We consider what must happen to the coal that lands on the faster barge during one minute ($\Delta t = 60\,{\rm s}$). In that time, a total of $m = 1000\,{\rm kg}$ of coal must experience a change of velocity

$$\Delta v = 20\,{\rm km/h} - 10\,{\rm km/h} = 10\,{\rm km/h} = 2.8\ {\rm m/s}$$

where rightwards is considered the positive direction. The rate of change in momentum for the coal is therefore

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t} = \frac{(1000)(2.8)}{60} = 46\ {\rm N}$$

which, by Eq. 9-23, must equal the force exerted by the (faster) barge on the coal. The processes (the shoveling, the barge motions) are constant, so there is no ambiguity in equating $\frac{\Delta p}{\Delta t}$ with $\frac{dp}{dt}$.

(b) The problem states that the frictional forces acting on the barges does not depend on mass, so the loss of mass from the slower barge does not affect its motion (so no extra force is required as a result of the shoveling).

48. (a) The thrust is $Rv_{\rm rel}$ where $v_{\rm rel} = 1200$ m/s. For this to equal the weight Mg where $M = 6100$ kg, we must have $R = (6100)(9.8)/1200 \approx 50$ kg/s.

(b) Using Eq. 9-42 with the additional effect due to gravity, we have

$$Rv_{\rm rel} - Mg = Ma$$

so that requiring $a = 21$ m/s^2 leads to $R = (6100)(9.8 + 21)/1200 = 1.6 \times 10^2$ kg/s.

49. (a) We assume his mass is between $m_1 = 50$ kg and $m_2 = 70$ kg (corresponding to a weight between 110 lb and 154 lb). His increase in gravitational potential energy is therefore in the range

$$\begin{aligned} m_1gh \leq & \quad \Delta U \quad \leq m_2gh \\ 2 \times 10^5 \leq & \quad \Delta U \quad \leq 3 \times 10^5 \end{aligned}$$

in SI units (J), where $h = 443$ m.

(b) The problem only asks for the amount of internal energy which converts into gravitational potential energy, so this result is the same as in part (a). But if we were to consider his *total* internal energy "output" (much of which converts to heat) we can expect that external climb is quite different from taking the stairs.

50. (a) The (internal) energy the climber must convert to gravitational potential energy is

$$\Delta U = mgh = (90)(9.8)(8850) = 7.8 \times 10^6 \text{ J} .$$

(b) The number of candy bars this corresponds to is

$$N = \frac{7.8 \times 10^6 \text{ J}}{1.25 \times 10^6 \text{ J/bar}} \approx 6 \text{ bars} .$$

51. (a) The acceleration of the sprinter is (using Eq. 2-15)

$$a = \frac{2\Delta x}{t^2} = \frac{(2)(7.0\,\text{m})}{(1.6\,\text{s})^2} = 5.47 \text{ m/s}^2 .$$

Consequently, the speed at $t = 1.6$ s is

$$v = at = \left(5.47\,\text{m/s}^2\right)(1.6\,\text{s}) = 8.8 \text{ m/s} .$$

Alternatively, Eq. 2-17 could be used.

(b) The kinetic energy of the sprinter (of weight w and mass $m = w/g$) is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 = \frac{(670)(8.8)^2}{2(9.8)} = 2.6 \times 10^3 \text{ J} .$$

(c) The average power is

$$P_{\rm avg} = \frac{\Delta K}{\Delta t} = \frac{2.6 \times 10^3\,\text{J}}{1.6\,\text{s}} = 1.6 \times 10^3 \text{ W} .$$

52. We use $P = Fv$ (Eq. 7-48) to compute the force:

$$F = \frac{P}{v} = \frac{92 \times 10^6 \text{ W}}{(32.5\,\text{knot})\left(1.852\,\frac{\text{km/h}}{\text{knot}}\right)\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)} = 5.5 \times 10^6 \text{ N} .$$

53. To swim at constant velocity the swimmer must push back against the water with a force of 110 N. Relative to him the water is going at 0.22 m/s toward his rear, in the same direction as his force. Using Eq. 7-48, his power output is obtained:

$$P = \vec{F} \cdot \vec{v} = Fv = (110\,\text{N})(0.22\,\text{m/s}) = 24\ \text{W}\ .$$

54. The initial kinetic energy of the automobile of mass m moving at speed v_i is $K_i = \frac{1}{2}mv_i^2$, where $m = 16400/9.8 = 1673$ kg. Using Eq. 8-29 and Eq. 8-31, this relates to the effect of friction force f in stopping the auto over a distance d by

$$K_i = fd$$

where the road is assumed level (so $\Delta U = 0$). Thus,

$$d = \frac{K_i}{f} = \frac{mv_i^2}{2f} = \frac{(1673\,\text{kg})\left((113\,\text{km/h})\left(\frac{1000\,\text{m/km}}{3600\,\text{s/h}}\right)\right)^2}{2(8230\,\text{N})} = 100\ \text{m}\ .$$

55. (a) By combining Newton's second law $F - mg = ma$ (where F is the force exerted up on her by the floor) and Eq. 2-16 $v^2 = 2ad_1$ (where $d_1 = 0.90 - 0.40 = 0.50\,\text{m}$ is the distance her center of mass moves while her feet are on the floor) it is straightforward to derive the equation

$$K_{\text{launch}} = (F - mg)d_1$$

where $K_{\text{launch}} = \frac{1}{2}mv^2$ is her kinetic energy as her feet leave the floor. We mention this method of deriving that equation (which also follows from the work-kinetic energy theorem Eq. 7-10, or – suitably interpreted – from energy conservation as expressed by Eq. 8-31) since the energy approaches might seem paradoxical (one might sink into the quagmire of questions such as "how can the floor possibly provide energy to the person?"); the Newton's law approach leads to no such quandaries. Next, her feet leave the floor and this kinetic energy is converted to gravitational potential energy. Then mechanical energy conservation leads straightforwardly to

$$K_{\text{launch}} = mgd_2$$

where $d_2 = 1.20 - 0.90 = 0.30\,\text{m}$ is the distance her center of mass rises from the time her feet leave the floor to the time she reaches the top of her leap. Now we combine these two equations and solve $(F - mg)d_1 = mgd_2$ for the force:

$$F = \frac{mg(d_1 + d_2)}{d_1} = \frac{(55\,\text{kg})(9.8\,\text{m/s}^2)(0.50\,\text{m} + 0.30\,\text{m})}{0.50\,\text{m}} = 860\ \text{N}\ .$$

(b) She has her maximum speed at the time her feet leave the floor (this is her "launch" speed). Consequently, the equation derived above becomes

$$\frac{1}{2}mv^2 = (F - mg)d_1$$

from which we obtain

$$\begin{aligned} v &= \sqrt{\frac{2(F - mg)d_1}{m}} \\ &= \sqrt{\frac{2\left(860\,\text{N} - (55\,\text{kg})\left(9.8\,\text{m/s}^2\right)\right)(0.50\,\text{m})}{55\,\text{kg}}} \\ &= 2.4\ \text{m/s}\ . \end{aligned}$$

56. (a) The kinetic energy K of the automobile of mass m at $t = 30\,\mathrm{s}$ is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1500\,\mathrm{kg})\left((72\,\mathrm{km/h})\left(\frac{1000\,\mathrm{m/km}}{3600\,\mathrm{s/h}}\right)\right)^2 = 3.0\times10^5\ \mathrm{J}\ .$$

(b) The average power required is

$$P_{\mathrm{avg}} = \frac{\Delta K}{\Delta t} = \frac{3.0\times10^5\,\mathrm{J}}{30\,\mathrm{s}} = 1.0\times10^4\ \mathrm{W}\ .$$

(c) We use Eq. 7-48 ($P = Fv$) for the instantaneous power delivered at t. Since the acceleration a is constant, the power is $P = Fv = mav = ma(at) = ma^2t$, using Eq. 2-11. By contrast, from part (b), the average power is $P_{\mathrm{avg}} = \frac{mv^2}{2t}$ which becomes $\frac{1}{2}ma^2t$ when $v = at$ is again utilized. Thus, the instantaneous power at the end of the interval is twice the average power during it:

$$P = 2P_{\mathrm{avg}} = (2)\left(1.0\times10^4\,\mathrm{W}\right) = 2.0\times10^4\ \mathrm{W}\ .$$

57. (a) With $P = 1.5\,\mathrm{MW} = 1.5\times10^6\,\mathrm{W}$ (assumed constant) and $t = 6.0\,\mathrm{min} = 360\,\mathrm{s}$, the work-kinetic energy theorem (along with Eq. 7-48) becomes

$$W = Pt = \Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right)\ .$$

The mass of the locomotive is then

$$m = \frac{2Pt}{v_f^2 - v_i^2} = \frac{(2)(1.5\times10^6\,\mathrm{W})(360\,\mathrm{s})}{(25\,\mathrm{m/s})^2 - (10\,\mathrm{m/s})^2} = 2.1\times10^6\,\mathrm{kg}\,.$$

(b) With t arbitrary, we use $Pt = \frac{1}{2}m\left(v^2 - v_i^2\right)$ to solve for the speed $v = v(t)$ as a function of time and obtain

$$v(t) = \sqrt{v_i^2 + \frac{2Pt}{m}} = \sqrt{(10)^2 + \frac{(2)(1.5\times10^6)t}{2.1\times10^6}} = \sqrt{100+1.5t}$$

in SI units (v in m/s and t in s).

(c) Using Eq. 7-48, the force $F(t)$ as a function of time is

$$F(t) = \frac{P}{v(t)} = \frac{1.5\times10^6}{\sqrt{100+1.5t}}$$

in SI units (F in N and t in s).

(d) The distance d the train moved is given by

$$d = \int_0^t v(t')\,dt' = \int_0^{360}\left(100 + \frac{3}{2}t\right)^{\frac{1}{2}}\,dt = \frac{4}{9}\left(100+\frac{3}{2}t\right)^{\frac{3}{2}}\Bigg|_0^{360}$$

which yields 6.7×10^3 m.

58. We work this in SI units and convert to horsepower in the last step. Thus,

$$v = (80\,\mathrm{km/h})\left(\frac{1000\,\mathrm{m/km}}{3600\,\mathrm{s/h}}\right) = 22.2\ \mathrm{m/s}\ .$$

The force F_P needed to propel the car (of weight w and mass $m = w/g$) is found from Newton's second law:

$$F_{\mathrm{net}} = F_P - F = ma = \frac{wa}{g}$$

where $F = 300 + 1.8v^2$ in SI units. Therefore, the power required is

$$\begin{aligned} P &= \vec{F}_P \cdot \vec{v} \\ &= \left(F + \frac{wa}{g}\right) v \\ &= \left(300 + 1.8(22.2)^2 + \frac{(12000)(0.92)}{9.8}\right)(22.2) \\ &= 5.14 \times 10^4 \text{ W} \\ &= \left(5.14 \times 10^4 \text{ W}\right)\left(\frac{1\,\text{hp}}{746\,\text{W}}\right) = 69 \text{ hp} . \end{aligned}$$

59. The third-to-last statement in the problem about the peeling-off rate of the top layer and the thickening rate of the bottom layer is best interpreted, we feel, in the rest frame of the layer. Thus, imagining that we are in a reference frame moving up at v_t, then it is clear from the uniform nature of the described peeling-off of the top and thickening of the bottom that *in this moving reference frame* the center of mass of the layer must move downward with a speed $2v_f$ (if the rates were denoted R and were different then this would be $R_{\text{bottom}} + R_{\text{top}}$). Returning to the original reference frame, where we see the trapped bubbles rising at v_t, we find (with $+y$ upward) the center of mass velocity is

$$v_{\text{com}} = v_t - 2v_f = -1.5 \text{ cm/s} .$$

60. (a) Since the initial momentum is zero, then the final momenta must add (in the vector sense) to 0. Therefore, with SI units understood, we have

$$\begin{aligned} \vec{p}_3 &= -\vec{p}_1 - \vec{p}_2 \\ &= -m_1\vec{v}_1 - m_2\vec{v}_2 \\ &= -\left(16.7 \times 10^{-27}\right)\left(6.00 \times 10^6\,\hat{\imath}\right) - \left(8.35 \times 10^{-27}\right)\left(-8.00 \times 10^6\,\hat{\jmath}\right) \\ &= -1.00 \times 10^{-19}\,\hat{\imath} + 0.67 \times 10^{-19}\,\hat{\jmath} \text{ kg}\cdot\text{m/s} . \end{aligned}$$

(b) Dividing by $m_3 = 11.7 \times 10^{-27}$ kg and using Pythagorean's theorem we find the speed of the third particle to be $v_3 = 1.03 \times 10^7$ m/s. The total amount of kinetic energy is

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 = 1.19 \times 10^{-12} \text{ J} .$$

61. By conservation of momentum, the final speed v of the sled satisfies

$$(2900\,\text{kg})(250\,\text{m/s}) = (2900\,\text{kg} + 920\,\text{kg})v$$

which gives $v = 190$ m/s.

62. We denote the mass of the car as M and that of the sumo wrestler as m. Let the initial velocity of the sumo wrestler be $v_0 > 0$ and the final velocity of the car be v. We apply the momentum conservation law.

(a) From $mv_0 = (M + m)v$ we get

$$v = \frac{mv_0}{M + m} = \frac{(242\,\text{kg})(5.3\,\text{m/s})}{2140\,\text{kg} + 242\,\text{kg}} = 0.54 \text{ m/s} .$$

(b) Since $v_{\text{rel}} = v_0$, we have

$$mv_0 = Mv + m\left(v + v_{\text{rel}}\right) = mv_0 + (M + m)v$$

and obtain $v = 0$ for the final speed of the flatcar.

(c) Now $mv_0 = Mv + m(v - v_{\rm rel})$, which leads to

$$v = \frac{m(v_0 + v_{\rm rel})}{m + M} = \frac{(242\,{\rm kg})(5.3\,{\rm m/s} + 5.3\,{\rm m/s})}{242\,{\rm kg} + 2140\,{\rm kg}} = 1.1 \text{ m/s} .$$

63. (a) We use coordinates with $+x$ eastward and $+y$ northward, and employ magnitude-angle notation which is well suited for computations with vector-capable calculators. Positive angles are measured counterclockwise from the $+x$ axis (negative angles are clockwise). Length is in meters and time is in seconds. The mass of each piece is designated m. Thus, the conservation of momentum becomes

$$\begin{aligned}\vec{p}_0 &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 \\ \vec{p}_0 &= m(7.0 \angle\, 90^\circ) + m(4.0 \angle\, 210^\circ) + m(4.0 \angle - 30^\circ) \\ \vec{p}_0 &= m(3.0 \angle\, 90^\circ)\end{aligned}$$

which implies that the velocity of the package had magnitude $|\vec{p}|/(3m) = 1.0$ m/s and was directed north.

(b) The center of mass proceeds at $1.0\,{\rm m/s}$ unaffected by the explosion. Its displacement during the 3.0 s interval is $(1.0\,{\rm m/s})(3.0\,{\rm s}) = 3.0$ m. The displacement is directed north, in accordance with its velocity.

64. The width ℓ of the pyramid measured at variable height z is seen to decrease from L at the base (where $z = 0$) to zero at the top (where $z = H$). This is a linear decrease, so we must have

$$\ell = L\left(1 - \frac{z}{H}\right) .$$

If we imagine the pyramid layered into a large number N of horizontal (square) slabs (each of thickness Δz) then the volume of each slab is $V' = \ell^2 \Delta z$ and the mass of each slab is $m' = \rho V' = \rho \ell^2 \Delta z$. If we make the continuum approximation ($N \to \infty$ while $\Delta z \to dz$) and substitute from above for ℓ, the mass element becomes

$$dm = \rho L^2 \left(1 - \frac{z}{H}\right)^2 dz .$$

We note, for later use, that the total mass M is given by $\rho L^2 H/3$ using the volume relation mentioned in the problem, but this can also be derived by integrating the above expression for dm.

(a) Using Eq. 9-9 we find

$$z_{\rm com} = \frac{1}{M}\int z\, dm = \frac{3}{\rho L^2 H}\int_0^H z\rho L^2 \left(1 - \frac{z}{H}\right)^2 dz$$

where ρ and L^2 are constants (and, in fact, cancel) so we obtain

$$z_{\rm com} = \frac{3}{H}\int_0^H \left(z - \frac{2z^2}{H} + \frac{z^3}{H}\right) dz = \frac{H}{4} = 36.8 \text{ m} .$$

(b) Although we could do the integral $\int dU = \int gz\, dm$ to find the work done against gravity, it is easier to use the conclusion drawn in the book that this should be equivalent to lifting a point mass M to height $z_{\rm com}$.

$$W = \Delta U = Mgz_{\rm com} = \left(\frac{\rho L^2 H}{3}\right) g\, \frac{H}{4} = 1.7 \times 10^{12} \text{ J} .$$

65. Although it is expected that the boat will have a slight downward recoil (of brief duration) from the upward component of the father's leap, the problem's intent is to concentrate only on the horizontal components, since – if the effects of friction are small – the boat can continue moving horizontally for a significant time. Mass, velocity and momentum units are SI. We use coordinates with $+x$ eastward and

$+y$ northward. Angles are positive if measured counterclockwise from the $+x$ axis. Using magnitude-angle notation, momentum conservation is expressed as

$$\begin{aligned}\vec{p}_0 &= \vec{p}_\mathrm{c} + \vec{p}_\mathrm{f} + \vec{p}_\mathrm{b} \\ (0 \angle 0^\circ) &= (80 \angle 0^\circ) + (90 \angle -90^\circ) + \vec{p}_\mathrm{b}\end{aligned}$$

where it must be stressed that the relevant component of the father's momentum is $\vec{p}_\mathrm{f} = (75)(1.5)\cos 37^\circ$ *south* (represented as $(90 \angle -90^\circ)$ in the expression above). Thus, we obtain $\vec{p}_\mathrm{b} = (120 \angle 132^\circ)$, which implies that the boat's (horizontal) velocity is $|\vec{p}|/m = 120/100 = 1.2$ m/s at an angle of 132° counterclockwise from east; this can also be expressed as 48° north of west.

66. (a) Ignoring air friction amounts to assuming that the ball has the same speed v when it returns to its original height.

$$K_i = K_f = \frac{1}{2}mv^2 = \frac{1}{2}(0.050\,\mathrm{kg})(16\,\mathrm{m/s})^2 = 6.4 \text{ J} .$$

(b) The momentum at the moment it is thrown (taking $+y$ upward) is

$$|\vec{p}_i| = |\vec{p}_f| = mv = (0.050\,\mathrm{kg})(16\,\mathrm{m/s}) = 0.80 \text{ kg}\cdot\text{m/s} .$$

The vector $\vec{p}_i$ is $\theta = 30^\circ$ above the horizontal, while $\vec{p}_f$ is 30° below the horizontal (since the vertical component is now downward). We note for later reference that the magnitude of the change in momentum is

$$|\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i| = 2mv\sin\theta = 0.80 \text{ kg}\cdot\text{m/s}$$

and $\Delta\vec{p}$ points vertically downward.

(c) The time of flight for the ball is $t = 2v_i \sin\theta/g$, thus

$$mgt = mg\left(\frac{2v\sin\theta}{g}\right) = 2mv\sin\theta = 2p_i\sin\theta = 0.80 \text{ kg}\cdot\text{m/s}$$

which (recalling our result in part (b)) illustrates the relation $|\Delta p| = Ft$ where $F = mg$.

67. Choosing downward as the $+y$ direction and placing the coordinate origin at the top of the building, we apply the equations from Table 2-1 to this two-block system:

$$\begin{aligned} y_1 &= \frac{1}{2}gt^2 &\quad& \text{for} \quad 0 \le t \le 5 \\ y_2 &= \frac{1}{2}g(t-1)^2 && \text{for} \quad 1 \le t \le 6 \\ v_1 &= gt && \text{for} \quad 0 \le t \le 5 \\ v_2 &= g(t-1) && \text{for} \quad 1 \le t \le 6 \end{aligned}$$

with SI units understood.

(a) With $m_1 = 2.00$ kg and $m_2 = 3.00$ kg, Eq. 9-5 provides

$$y_\mathrm{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{1}{2}gt^2 - \frac{3}{5}gt + \frac{3}{10}g$$

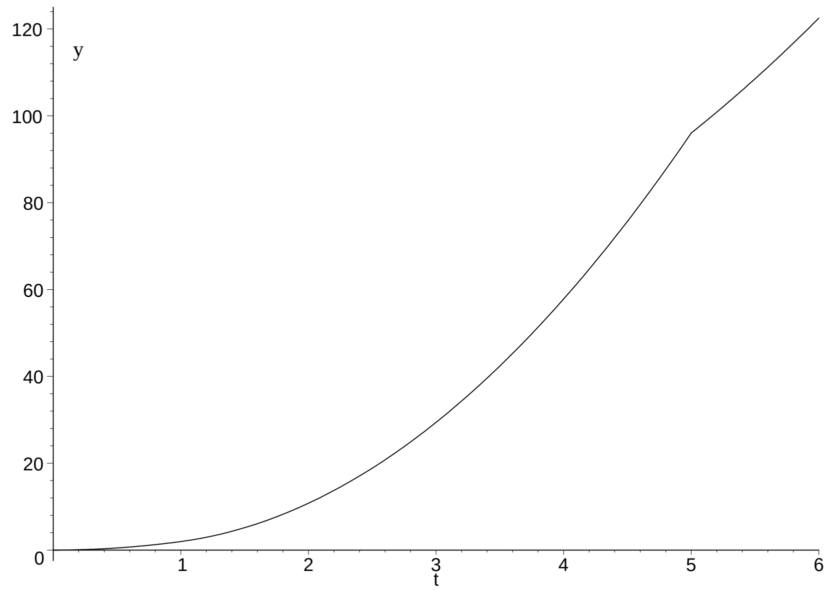
while they are both in free fall ($1 \le t \le 5$). But during the interval when m_2 is "waiting" at the top of the building, we have

$$y_\mathrm{com} = \frac{m_1 y_1 + m_2(0)}{m_1 + m_2} = \frac{1}{5}gt^2 \quad \text{for } 0 \le t \le 1$$

and during the interval where m_1 is sitting on the ground (at $y = \frac{1}{2}(9.8)(5)^2$) we have

$$y_{\text{com}} = \frac{m_1\left(\frac{25g}{2}\right) + m_2 y_2}{m_1 + m_2} = \frac{3}{10}gt^2 - \frac{3}{5}gt + \frac{53}{10}g$$

for $5 \le t \le 6$. This behavior is plotted below, with y_{com} in meters and t in seconds.



(b) We turn now to Eq. 9-17 which gives

$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = gt - \frac{3}{5}g$$

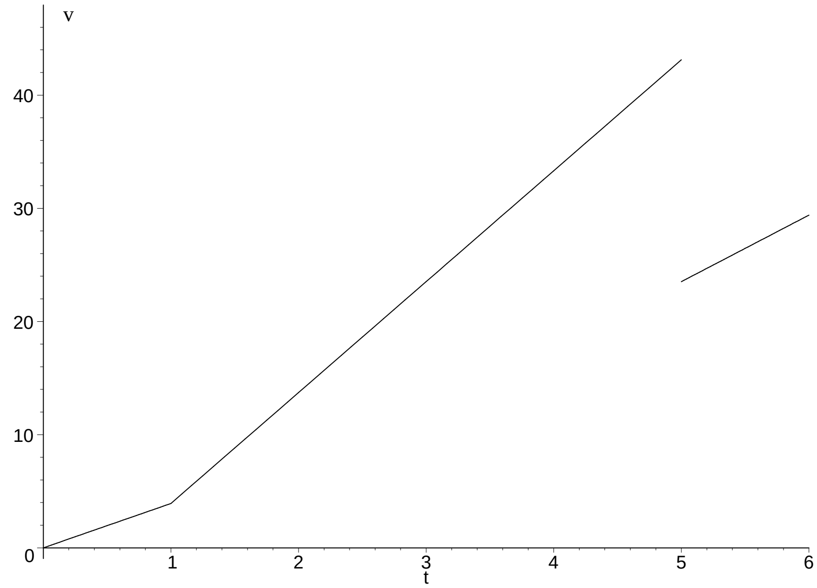
while they are both in free fall ($1 \le t \le 5$). We note that we could have easily gotten this by taking the derivative of the corresponding y_{com} expression in part (a). During the interval when m_2 sits on the top of the building, we have

$$v_{\text{com}} = \frac{m_1 v_1 + m_2(0)}{m_1 + m_2} = \frac{2}{5}gt \quad \text{for } 0 \le t \le 1$$

and during the interval where m_1 sits on the ground we have

$$v_{\text{com}} = \frac{m_1(0) + m_2 v_2}{m_1 + m_2} = \frac{3}{5}gt - \frac{3}{5}g$$

for $5 \le t \le 6$. This behavior is plotted below, with v_{com} in m/s and t in s. The sudden drop at $t = 5$ s is understandable, since m_1 stops, but it should be noted that we are ignoring the dynamics of how the ground decelerates that block – the effects of which might be to (slightly) smooth out that transition.



68. The velocity of the first particle (of mass $m_1 = 3.0$ kg) is $\vec{v}_1 = -6.0\hat{\jmath}$ m/s while that of the second one (of mass $m_2 = 4.0$ kg) is $\vec{v}_2 = 7.0\,\hat{\imath}$ m/s. The center-of-mass velocity is then

$$\vec{v}_{\rm com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{(3.0)(-6.0\,\hat{\jmath}\,) + (4.0)(7.0\,\hat{\imath}\,)}{3.0+4.0} = -2.6\,\hat{\imath} + 4.0\,\hat{\jmath}$$

in SI units. The corresponding speed is

$$v_{\rm com} = \sqrt{v_x^2 + v_y^2} = \sqrt{(-2.6)^2 + (4.0)^2} = 4.8 \text{ m/s} .$$

69. We use Eq. 9-17, or – equivalently – we differentiate Eq. 9-5.

$$\begin{aligned} v_{{\rm com}\,x} &= \frac{1}{M}\left((1500\,{\rm kg})(0 \text{ m/s}) + (4000\,{\rm kg})v_{\rm truck}\right) \\ v_{{\rm com}\,y} &= \frac{1}{M}\left((1500\,{\rm kg})v_{\rm car} + (4000\,{\rm kg})(0\,{\rm m/s})\right) \end{aligned}$$

where $M = 5500$ kg. From $v_{{\rm com}\,x} = (11)\cos 55° = 6.3$ m/s and $v_{{\rm com}\,y} = (11)\sin 55° = 9.0$ m/s, we get the following results for $v_{\rm truck}$ and $v_{\rm car}$ from the above formulas.

(a) $v_{\rm car} = 33$ m/s.

(b) $v_{\rm truck} = 8.7$ m/s.

70. (a) We use Eq. 7-48:

$$P = Fv \implies F = \frac{16.0\,{\rm kW}}{15.0\,{\rm m/s}} = 1.07 \text{ kN} .$$

(b) We add to our previous result the downhill pull of gravity $mg\sin\theta$ where $\theta = \tan^{-1}(8/100)$.

$$F' = 1.07 \times 10^3 + (1710)(9.8)\sin 4.57° = 2.40 \times 10^3$$

in SI units (N). Therefore,

$$P' = F'v = (2.40\,{\rm kN}))(15.0\,{\rm m/s}) = 36 \text{ kW} .$$

(c) For the engine to be off but the (downhill) velocity to remain constant, the downhill component of gravity must equal the magnitude of the retarding forces:

$$mg\sin\theta = F \ .$$

Using F from part (a), we find $\theta = 3.65°$ which corresponds to $\tan\theta = 0.0638 \approx 6.4\%$.

71. Using Eq. 2-15 for both object j (the jelly jar) and object p (the peanut butter), with $y = 0$ designating the base of the building in both cases, we have

$$\begin{aligned} y_j &= 40t - \frac{1}{2}gt^2 \\ y_p - 50 &= 0 - \frac{1}{2}gt^2 \end{aligned}$$

with SI units understood. Thus, using Eq. 9-5, the center of mass of this system is at

$$y_{\text{com}} = \frac{1}{3.0\ \text{kg}}\left((1.0\ \text{kg})y_j + (2.0\ \text{kg})y_p\right) = \frac{100}{3} + \frac{40}{3}t - \frac{1}{2}gt^2 \ .$$

(a) With $t = 3.0$ s, the above equation gives $y_{\text{com}} = 29$ m.

(b) We maximize y_{com} by working through the condition

$$\frac{dy_{\text{com}}}{dt} = 0 = \frac{40}{3} - gt \ .$$

Thus, we find $t = 1.4$ s, which produces $y_{\text{com}} = 42$ m as its highest value.

72. (a) We denote the mass of the car (and cannon) as M (excluding that of the cannonballs) and the mass of all the cannonballs as m. For concreteness, we assume that before firing all the cannonballs are at the front (left side of Fig. 9-52) of the car, which we choose to be the origin of the x axis; we choose $+x$ rightward. The coordinate of the center of mass of the car-cannonball system is

$$x_{\text{com}} = \frac{(0)m + \left(\frac{L}{2}\right)M}{M+m} = \frac{LM}{2(M+m)} \ .$$

After the firing, we assume all the cannonballs are at the other end of the car; the train will have moved (in the negative x direction) by a distance d, at which time

$$x_{\text{com}} = \frac{\left(\frac{L}{2} - d\right)M + (L-d)m}{M+m} \ .$$

Equating the two expressions, we obtain $d = \frac{mL}{M+m} < L$. If $m \gg M$, the distance d can be very close to (but can never exceed) L. Thus $d_{\text{max}} = L$.

(b) After each impact, there is no relative motion in the system; thus, the final speed of the car is equal to that of the center of mass of the system, which is zero.

73. Let the velocity of the shell (of mass m_s) relative to the ground be $\vec{v}_s$, the recoiling velocity of the cannon (of mass m_c) be $\vec{v}_c$ (pointed in our $-x$ direction), and the velocity of the shell relative to the muzzle be $\vec{v}_s'$, where $\vec{v}_s\,' + \vec{v}_c = \vec{v}_s$. In component form, this becomes

$$\begin{aligned} v_s'\cos 39.0° - v_c &= v_{sx} \\ v_s'\sin 39.0° &= v_{sy} \end{aligned}$$

where $v_c = |\vec{v}_c|$. Conservation of linear momentum in the horizontal direction provides us with the additional relation $m_s v_{sx} = m_c v_c$. We solve these equations for the components of $\vec{v}_s$:

$$\begin{aligned} v_{sx} &= \frac{m_c v_s'\cos 39.0°}{m_s + m_c} = \frac{(1400\,\text{kg})(556\,\text{m/s})\cos 39.0°}{1400\,\text{kg} + 70.0\,\text{kg}} = 412\,\text{m/s} \\ v_{sy} &= v_s'\sin 39.0° = (556\,\text{m/s})(\sin 39.0°) = 350\,\text{m/s} \ . \end{aligned}$$

(a) The speed of the shell relative to the Earth is then

$$v_s = \sqrt{v_{sx}^2 + v_{sy}^2} = \sqrt{412^2 + 350^2} = 540 \text{ m/s} .$$

(b) The angle (relative to a stationary observer) at which the shell is fired is given by

$$\theta = \tan^{-1}\left(\frac{v_{sy}}{v_{sx}}\right) = \tan^{-1}\left(\frac{350}{412}\right) = 40.4^\circ .$$

74. The value 0.368 comes from rounding off e^{-1}. We will use e^{-1} in our solution. The speed of the rocket v as a function of the instantaneous rocket mass M' is given by $v = v_{\text{rel}} \ln(M/M')$ (Eq. 9-43 with $v_i = 0$). Thus, when $M' = e^{-1} M$, the speed of the fuel as measured by observers in the initial reference frame (defined when the rocket was at rest with $M' = M$) is

$$v_{\text{fuel}} = v - v_{\text{rel}} = v_{\text{rel}}\left(\ln\frac{M}{M'} - 1\right) = v(\ln\left(\frac{1}{e^{-1}}\right) - 1) = 0 .$$

75. We use momentum conservation choosing $+x$ forward and recognizing that the initial momentum is zero. We analyze this from the point of view of an observer at rest on the ice.

(a) If $v_{1\text{ and }2}$ is the speed of the stones, then the speeds are related by $v_{1\text{ and }2} + v_{\text{boat}} = v_{\text{rel}}$. Thus, with $m_1 = 2m_2$ and $M = 12m_2$, we obtain

$$\begin{aligned} 0 &= (m_1 + m_2)(-v_{1\text{ and }2}) + Mv_{\text{boat}} \\ &= (2m_2 + m_2)(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2 v_{\text{boat}} \\ &= -3m_2 v_{\text{rel}} + 15m_2 v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{1}{5} v_{\text{rel}} = 0.2000 v_{\text{rel}}$.

(b) Using $v_1 + v'_{\text{boat}} = v_{\text{rel}}$, we find – as a result of the first throw – the boat's speed:

$$\begin{aligned} 0 &= m_1(-v_1) + (M + m_2)\, v'_{\text{boat}} \\ &= 2m_2(-v_{\text{rel}} + v'_{\text{boat}}) + (12m_2 + m_2)\, v'_{\text{boat}} \\ &= -2m_2 v_{\text{rel}} + 15m_2 v'_{\text{boat}} \end{aligned}$$

which yields $v'_{\text{boat}} = \frac{2}{15} v_{\text{rel}} \approx 0.133 v_{\text{rel}}$. Then, using $v_2 + v_{\text{boat}} = v_{\text{rel}}$, we consider the second throw:

$$\begin{aligned} (M + m_2)\, v'_{\text{boat}} &= m_2(-v_2) + Mv_{\text{boat}} \\ (12m_2 + m_2)\left(\frac{2}{15} v_{\text{rel}}\right) &= m_2(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2 v_{\text{boat}} \\ \frac{26}{15} m_2 v_{\text{rel}} &= -m_2 v_{\text{rel}} + 13m_2 v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{41}{195} v_{\text{rel}} \approx 0.2103 v_{\text{rel}}$.

(c) Finally, using $v_2 + v'_{\text{boat}} = v_{\text{rel}}$, we find – as a result of the first throw – the boat's speed:

$$\begin{aligned} 0 &= m_2(-v_2) + (M + m_1)\, v'_{\text{boat}} \\ &= m_2(-v_{\text{rel}} + v'_{\text{boat}}) + (12m_2 + 2m_2)\, v'_{\text{boat}} \\ &= -m_2 v_{\text{rel}} + 15m_2 v'_{\text{boat}} \end{aligned}$$

which yields $v'_{\text{boat}} = \frac{1}{15} v_{\text{rel}} \approx 0.0673 v_{\text{rel}}$. Then, using $v_1 + v_{\text{boat}} = v_{\text{rel}}$, we consider the second throw:

$$\begin{aligned} (M + m_1)\, v'_{\text{boat}} &= m_1(-v_1) + Mv_{\text{boat}} \\ (12m_2 + 2m_2)\left(\frac{1}{15} v_{\text{rel}}\right) &= 2m_2(-v_{\text{rel}} + v_{\text{boat}}) + 12m_2 v_{\text{boat}} \\ \frac{14}{15} m_2 v_{\text{rel}} &= -2m_2 v_{\text{rel}} + 14m_2 v_{\text{boat}} \end{aligned}$$

which yields $v_{\text{boat}} = \frac{22}{105}\, v_{\text{rel}} \approx 0.2095 v_{\text{rel}}$.

76. (a) It is clear from the problem that $\vec{v}_{\text{air,plane}} = -180\,\hat{\imath}$ m/s where $+\hat{\imath}$ is the plane's direction of motion (relative to the ground).

(b) Let ΔM_a be the mass of air taken in and ejected and let ΔM_f be the mass of fuel ejected in time Δt. From the viewpoint of ground-based observers, the initial velocity of the air is zero and its final velocity is $v - u$, where u is the exhaust speed (labeled v_{rel} in the textbook) and v is the velocity of the plane. The initial velocity of the fuel is v and its final velocity is $v - u$. The velocity of the plane changes from v to $v + \Delta v$ over this time interval. The change in the total momentum of the plane-fuel-air system is $\Delta P = M_p \Delta v + \Delta M_f(u) + \Delta M_a(u - v)$ so the net external force is

$$\frac{\Delta P}{\Delta t} = M_p \frac{\Delta v}{\Delta t} - u \frac{\Delta M_f}{\Delta t} + (v - u) \frac{\Delta M_a}{\Delta t} \ .$$

We examine some of these terms individually. The $v\,\Delta M_a/\Delta t$ term gives the magnitude of the force on the plane due to air intake (most easily seen from the point of view of observers on the plane) and is equal to $(180)(70) = 1.3 \times 10^4$ N.

(c) We interpret the question as asking for the force due to ejection of both the air and the combustion products due to consuming the fuel. This corresponds then to the $u\,\Delta M_a/\Delta t$ and $u\,\Delta M_f/\Delta t$ terms above, and is equal to $(490)(70 + 2.9) = 3.6 \times 10^4$ N.

(d) We require $\Delta P/\Delta t = 0$ since this (the air, plane and fuel) forms an isolated system (Eq. 9-29). Therefore, our equation above leads to

$$M_p \frac{\Delta v}{\Delta t} = u \frac{\Delta M_f}{\Delta t} + (u - v) \frac{\Delta M_a}{\Delta t}$$

with all the terms on the right hand side constituting the net thrust (compare Eq. 9-42). These are the values (with appropriate signs) found in parts (b) and (c), so we obtain $3.6 \times 10^4 - 1.3 \times 10^4 = 2.3 \times 10^4$ N.

(e) Using Eq. 7-48, we multiply the net thrust by the plane speed and obtain $\left(2.3 \times 10^4\right)(180) = 4.2 \times 10^6$ W.

77. Using Eq. 9-5, we have

$$\begin{aligned} x_{\text{com}} &= 0 = \frac{1}{M}\left((4.0 \text{ kg})(0 \text{ m}) + (3.0 \text{ kg})(3.0 \text{ m}) + (2.0 \text{ kg})x\right) \\ y_{\text{com}} &= 0 = \frac{1}{M}\left((4.0 \text{ kg})(2.0 \text{ m}) + (3.0 \text{ kg})(1.0 \text{ m}) + (2.0 \text{ kg})y\right) \end{aligned}$$

where $M = 9.0$ kg.

(a) Evaluating the above, we find $x = -4.5$ m.

(b) And we find $y = -5.5$ m.

78. (a) We use Eq. 9-5 to compute the center of mass coordinates.

$$\begin{aligned} x_{\text{com}} &= \frac{(4\,\text{kg})(0) + (3\,\text{kg})(7\,\text{m}) + (5\,\text{kg})(3\,\text{m})}{4\,\text{kg} + 3\,\text{kg} + 5\,\text{kg}} = 3.00 \text{ m} \\ y_{\text{com}} &= \frac{(4\,\text{kg})(0) + (3\,\text{kg})(3\,\text{m}) + (5\,\text{kg})(2\,\text{m})}{4\,\text{kg} + 3\,\text{kg} + 5\,\text{kg}} = 1.58 \text{ m} \end{aligned}$$

(b) Using Eq. 9-17 and SI units, we obtain

$$\begin{aligned} \vec{v} &= \frac{(4\,\text{kg})\left(1.5\,\hat{\imath} - 2.5\,\hat{\jmath}\right) + (3\,\text{kg})(0) + (5\,\text{kg})\left(2.0\,\hat{\imath} - 1.0\,\hat{\jmath}\right)}{4\,\text{kg} + 3\,\text{kg} + 5\,\text{kg}} \\ &= 1.33\,\hat{\imath} - 1.25\,\hat{\jmath} \ \text{ m/s} \ . \end{aligned}$$

(c) Multiplying the previous result by the total mass yields $\vec{P} = 16.0\,\hat{\imath} - 15.0\,\hat{\jmath}$ in SI units (kg·m/s). This can also be gotten by adding up the individual momenta.

79. Although we do not show graphs here, we do jot down down an idea for each part.

(a) Find the center of mass of a rod in which the density is not uniform. If the rod extends along the x axis from the origin to $x = 5$ m, then with mass-per-unit-length (as a function of x) equal to e^{-x} in SI units, use Eq. 9-9 to find x_{com}. *A sketch of the solution is*

$$x_{\text{com}} = \frac{1}{M}\int_0^5 xe^{-x}dx \approx 37 \text{ m}$$

where M was figured from $\int_0^5 e^{-x}dx \approx 1$ kg.

(b) A firecracker is dropped from a height of 20 m. Halfway down it explodes into two identical pieces. As a result of the explosion, one of the pieces is (momentarily) at rest. What is the speed of the other piece immediately after the explosion? *A sketch of the solution is*

$$v_{\text{firecrack}} = \sqrt{2g(10\,\text{m})} = 14 \text{ m/s}$$

and we use momentum conservation:

$$mv_{\text{firecrack}} = \frac{m}{2}v_{\text{piece}} \quad\Longrightarrow\quad v_{\text{piece}} = 28 \text{ m/s} .$$

(c) An 80 kg person is climbing a ladder at a steady rate of 25 cm/s. If we assume his total power output P is three times his rate of gaining gravitational potential energy, then compute P. *The solution is*

$$P = 3\frac{\Delta U}{\Delta t} = 3\frac{mg\Delta y}{\Delta t} \approx 590 \text{ J}$$

where $\Delta t = 1$ s and $\Delta y = 0.25$ m.

(d) Unlike the ideal physics of point particles moving through a vacuum, a runner cannot continue at a constant velocity effortlessly. If a runner's total power output is 650 W while running at 5.8 m/s, then what is the force retarding him (which includes several friction-related effects)? *The solution is*

$$P = Fv = \;\Longrightarrow\; F = \frac{P}{v} \approx 110 \text{ N} .$$

80. (First problem in **Cluster**)

(a) The length of each of the tall sides is $\ell = \sqrt{H^2 + (B/2)^2}$, so that the total length of the wire is $L = 2\sqrt{H^2 + (B/2)^2} + B$. If A is the cross-section area and ρ is the density, then the total mass of the wire is $M = \rho AL$ and the mass of each of the tall sides is

$$m_\ell = \rho A\sqrt{H^2 + (B/2)^2} = M\frac{\sqrt{H^2 + (B/2)^2}}{2\sqrt{H^2 + (B/2)^2} + B} .$$

It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. Note that the base does not contribute to this computation:

$$y_{\text{com}} = \frac{1}{M}\left(m_\ell\frac{H}{2} + m_\ell\frac{H}{2}\right)$$

which can be 'simplified' to the following form.

$$y_{\text{com}} = \frac{H}{2 + \frac{B}{\sqrt{H^2 + (B/2)^2}}}$$

(b) The element of mass on the left-hand tall side is related to $d\ell = \sqrt{dx^2 + dy^2}$ and ultimately to the individual coordinate elements (since $dy = (2H/B)dx$):

$$dm_\ell = \rho A d\ell = \begin{cases} \rho A\sqrt{1+(2H/B)^2}\,dx \\ \rho A\sqrt{(B/2H)^2+1}\,dy \end{cases}$$

where $\rho A = m_\ell/\sqrt{H^2+(B/2)^2}$ (see part (a)). Therefore, using Eq. 9-9, we have

$$\begin{aligned} x_{\text{com}} &= \frac{1}{m_\ell}\int_0^{B/2} x\,\frac{m_\ell}{\sqrt{H^2+(B/2)^2}}\sqrt{1+(2H/B)^2}\,dx \\ &= \frac{\sqrt{1+(2H/B)^2}}{\sqrt{H^2+(B/2)^2}}\int_0^{B/2} x\,dx \\ &= \frac{2}{B}\,\frac{\sqrt{(B/2)^2+H^2}}{\sqrt{H^2+(B/2)^2}}\left(\frac{B^2}{8}-0\right) \\ &= \frac{B}{4} \quad \text{and} \\ y_{\text{com}} &= \frac{1}{m_\ell}\int_0^{H} y\,\frac{m_\ell}{\sqrt{H^2+(B/2)^2}}\sqrt{(B/2H)^2+1}\,dy \\ &= \frac{\sqrt{(B/2H)^2+1}}{\sqrt{H^2+(B/2)^2}}\int_0^{H} y\,dy \\ &= \frac{1}{H}\,\frac{\sqrt{(B/2)^2+H^2}}{\sqrt{H^2+(B/2)^2}}\left(\frac{H^2}{2}-0\right) \\ &= \frac{H}{2}\,. \end{aligned}$$

81. (Second problem in **Cluster**)
It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. If the thickness is Δz and the density is ρ, then the relation between the mass element dm and a height element dy is

$$dm = \rho\Delta z \ell_y\,dy = \frac{M}{A_\triangle}\ell_y\,dy$$

where the area of the triangle is $A_\triangle = \frac{1}{2}BH$ and the length of each horizontal "strip" at height y is $\ell_y = B(1 - y/H)$. Therefore, using Eq. 9-9, we have

$$\begin{aligned} y_{\text{com}} &= \frac{1}{M}\int_0^{H} y\,\frac{M}{A_\triangle}\,B\left(1-\frac{y}{H}\right)\,dy \\ &= \frac{B}{\frac{1}{2}BH}\int_0^{H} y\left(1-\frac{y}{H}\right)\,dy \\ &= \frac{2}{H}\left(\frac{H^2}{2}-\frac{H^3}{3H}\right) \\ &= \frac{H}{3}\,. \end{aligned}$$

82. (Third problem in **Cluster**)
It is clear by symmetry that $x_{\text{com}} = B/2$ for the system, but the value of y_{com} is not obvious. If the cross-section area of the wire is A and the density is ρ, then in one quadrant the relation between the mass element dm and height element dy is

$$dm = \rho A\frac{R}{\sqrt{R^2-y^2}}\,dy = \frac{M}{\ell_\cap}\,\frac{R}{\sqrt{R^2-y^2}}\,dy$$

where the length of the semicircle is $\ell_\cap = \pi R$. To include the contributions from both quadrants shown, we multiply by 2, and Eq. 9-9 becomes

$$\begin{aligned} y_{\text{com}} &= \frac{2}{M}\int_0^R y\,\frac{M}{\ell_\cap}\,\frac{R}{\sqrt{R^2-y^2}}\,dy \\ &= \frac{2}{\pi}\int_0^R \frac{y}{\sqrt{R^2-y^2}}\,dy \\ &= \frac{2}{\pi}\left[-\sqrt{R^2-y^2}\right]_0^R \\ &= \frac{2R}{\pi}\ . \end{aligned}$$

83. (Fourth problem in **Cluster**)
It is clear by symmetry that $x_{\text{com}} = B/2$ for the system. The value of y_{com} is found as follows. If the thickness is Δz and the density is ρ, then the relation between the mass element dm and a height element dy is

$$dm = \rho\Delta z \ell_y\,dy = \frac{M}{A}\ell_y\,dy$$

where the area of the semicircle is $A = \frac{1}{2}\pi R^2$ and the length of each horizontal "strip" at height y is $\ell_y = 2\sqrt{R^2-y^2}$. Therefore, using Eq. 9-9, we find

$$\begin{aligned} y_{\text{com}} &= \frac{1}{M}\int_0^R y\,\frac{M}{A}\,2\sqrt{R^2-y^2}\,dy \\ &= \frac{2}{\frac{1}{2}\pi R^2}\int_0^R y\,\sqrt{R^2-y^2}\,dy \\ &= \frac{4}{\pi R^2}\left[-\frac{1}{3}\left(R^2-y^2\right)^{3/2}\right]_0^R \\ &= \frac{4R}{3\pi}\ . \end{aligned}$$

Chapter 10

1. If $F_{\rm avg}$ is the magnitude of the average force, then the magnitude of the impulse is $J = F_{\rm avg}\Delta t$, where Δt is the time interval over which the force is exerted (see Eq. 10-8). This equals the magnitude of the change in the momentum of the ball. Since the ball is initially at rest, J is equal to the magnitude of the final momentum mv. When $F_{\rm avg}\Delta t = mv$ is solved for the speed, the result is

$$v = \frac{F_{\rm avg}\Delta t}{m} = \frac{(50\,{\rm N})\left(10\times10^{-3}\,{\rm s}\right)}{0.20\,{\rm kg}} = 2.5 \text{ m/s} .$$

2. The magnitude of the average force is

$$\left|\vec{F}_{\rm avg}\right| = \frac{|\Delta\vec{p}|}{m} = \frac{m|\Delta\vec{v}|}{\Delta t} = \frac{(2300\,{\rm kg})(15\,{\rm m/s})}{0.56\,{\rm s}} = 6.2\times10^4 \text{ N} .$$

3. We take the final direction of motion to be the $+\hat{\imath}$ direction (when it is headed back to the pitcher) so that $\vec{v}_f = +60\,\hat{\imath}$ and $\vec{v}_i = -40\,\hat{\imath}$ in SI units. Therefore, $\Delta\vec{v} = 60 - (-40) = 100\,\hat{\imath}$ m/s. The magnitude of the average force is

$$\left|\vec{F}_{\rm avg}\right| = \frac{|\Delta\vec{p}|}{\Delta t} = \frac{m|\Delta\vec{v}|}{\Delta t} = \frac{(0.150\,{\rm kg})(100\,{\rm m/s})}{5.0\times10^{-3}\,{\rm s}} = 3.0\times10^3 \text{ N} .$$

4. We estimate his mass in the neighborhood of 70 kg and compute the upward force F of the water from Newton's second law.

$$F - mg = ma$$

where we have chosen $+y$ upward, so that $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the water). His speed when he arrives at the surface of the water is found either from Eq. 2-16 or from energy conservation:

$$v = \sqrt{2gh}$$

where $h = 12$ m, and since the deceleration a reduces the speed to zero over a distance $d = 0.30$ m we also obtain $v = \sqrt{2ad}$. We use these observations in the following.

   (a) Equating our two expressions for v leads to $a = gh/d$. Our force equation, then, leads to

$$F = mg + m\left(g\,\frac{h}{d}\right) = mg\left(1 + \frac{h}{d}\right)$$

   which yields $F \approx 2.8\times10^4$ kg. Since we are not at all certain of his mass, we express this as a guessed-at range (in kN) $25 < F < 30$.

   (b) Since $F \gg mg$, the impulse $\vec{J}$ due to the net force (while he is in contact with the water) is overwhelmingly caused by the upward force of the water: $\int F\,dt = \vec{J}$ to a good approximation. Thus, by Eq. 10-2,

$$\int F\,dt = \vec{p}_f - \vec{p}_i = 0 - m\left(-\sqrt{2gh}\right)$$

(the minus sign with the initial velocity is due to the fact that downward is the negative direction) which yields $(70)\sqrt{2(9.8)(12)} = 1.1 \times 10^3$ kg·m/s. Expressing this as a range (in kN·s) we estimate $1.0 < \int F\,dt < 1.2$.

5. We take the initial direction of motion to be positive and use F_{avg} to denote the magnitude of the average force, Δt as the duration of the force, m as the mass of the ball, v_i as the initial velocity of the ball, and v_f as the final velocity of the ball. The force is in the negative direction and the impulse-momentum theorem (Eq. 10-4 with Eq. 10-8) yields $-F_{\text{avg}}\Delta t = mv_f - mv_i$. Thus,

$$v_f = \frac{mv_i - F_{\text{avg}}\Delta t}{m} = \frac{(0.40\,\text{kg})(14\,\text{m/s}) - (1200\,\text{N})(27 \times 10^{-3}\,\text{s})}{0.40\,\text{kg}} = -67\,\text{m/s} .$$

The final speed of the ball is 67 m/s. The negative sign indicates that the velocity is opposite to the initial direction of travel.

6. We choose $+y$ upward, which implies $a > 0$ (the acceleration is upward since it represents a deceleration of his downward motion through the snow).

   (a) The maximum deceleration a_{max} of the paratrooper (of mass m and initial speed $v = 56$ m/s) is found from Newton's second law

   $$F_{\text{snow}} - mg = ma_{\text{max}}$$

   where we require $F_{\text{snow}} = 1.2 \times 10^5$ N. Using Eq. 2-15 $v^2 = 2a_{\text{max}}d$, we find the minimum depth of snow for the man to survive:

   $$d = \frac{v^2}{2a_{\text{max}}} = \frac{mv^2}{2\,(F_{\text{snow}} - mg)} \approx \frac{(85\,\text{kg})(56\,\text{m/s})^2}{2(1.2 \times 10^5\,\text{N})} = 1.1\,\text{m} .$$

   (b) His short trip through the snow involves a change in momentum

   $$\vec{p}_f - \vec{p}_i = 0 - (85\,\text{kg})(-56\,\text{m/s})$$

   (the negative value of the initial velocity is due to the fact that downward is the negative direction) which yields 4.8×10^3 kg·m/s. By the impulse-momentum theorem, this equals the impulse due to the net force $F_{\text{snow}} - mg$, but since $F_{\text{snow}} \gg mg$ we can approximate this as the impulse on him just from the snow.

7. We choose $+y$ upward, which means $\vec{v}_i = -25$ m/s and $\vec{v}_f = +10$ m/s. During the collision, we make the reasonable approximation that the net force on the ball is equal to F_{avg} – the average force exerted by the floor up on the ball.

   (a) Using the impulse momentum theorem (Eq. 10-4) we find

   $$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10) - (1.2)(-25) = 42 \text{ kg·m/s} .$$

   (b) From Eq. 10-8, we obtain

   $$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{42}{0.020} = 2.1 \times 10^3 \text{ N} .$$

8. We choose the positive direction in the direction of rebound so that $\vec{v}_f > 0$ and $\vec{v}_i < 0$. Since they have the same speed v, we write this as $\vec{v}_f = v$ and $\vec{v}_i = -v$. Therefore, the change in momentum for each bullet of mass m is $\Delta\vec{p} = m\Delta v = 2mv$. Consequently, the total change in momentum for the 100 bullets (each minute) $\Delta\vec{P} = 100\Delta\vec{p} = 200mv$. The average force is then

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{P}}{\Delta t} = \frac{(200)(3 \times 10^{-3}\,\text{kg})(500\,\text{m/s})}{(1\,\text{min})(60\,\text{s/min})} \approx 5\,\text{N} .$$

9. (a) The initial momentum of the car is $\vec{p}_i = m\vec{v}_i = (1400\,\text{kg})(5.3\,\text{m/s})\,\hat{\jmath} = (7400\,\text{kg}\cdot\text{m/s})\,\hat{\jmath}$ and the final momentum is $\vec{p}_f = (7400\,\text{kg}\cdot\text{m/s})\,\hat{\imath}$. The impulse on it equals the change in its momentum:

$$\vec{J} = \vec{p}_f - \vec{p}_i = (7400\,\text{N}\cdot\text{s})(\hat{\imath} - \hat{\jmath}) \ .$$

(b) The initial momentum of the car is $\vec{p}_i = (7400\,\text{kg}\cdot\text{m/s})\,\hat{\imath}$ and the final momentum is $\vec{p}_f = 0$. The impulse acting on it is

$$\vec{J} = \vec{p}_f - \vec{p}_i = -7400\,\hat{\imath}\ \ \text{N}\cdot\text{s} \ .$$

(c) The average force on the car is

$$\vec{F}_{\text{avg}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{J}}{\Delta t} = \frac{(7400\,\text{kg}\cdot\text{m/s})(\hat{\imath} - \hat{\jmath})}{4.6\,\text{s}} = (1600\,\text{N})(\hat{\imath} - \hat{\jmath})$$

and its magnitude is $F_{\text{avg}} = (1600\,\text{N})\sqrt{2} = 2300\,\text{N}$.

(d) The average force is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(-7400\,\text{kg}\cdot\text{m/s})\,\hat{\imath}}{350\times 10^{-3}\,\text{s}} = (-2.1\times 10^4\,\text{N})\,\hat{\imath}$$

and its magnitude is $F_{\text{avg}} = 2.1\times 10^4\,\text{N}$.

(e) The average force is given above in unit vector notation. Its x and y components have equal magnitudes. The x component is positive and the y component is negative, so the force is 45° below the positive x axis.

10. We use coordinates with $+x$ rightward and $+y$ upward, with the usual conventions for measuring the angles (so that the initial angle becomes $180+35=215°$). Using SI units and magnitude-angle notation (efficient to work with when using a vector capable calculator), the change in momentum is

$$\vec{p}_f - \vec{p}_i = (3.0\ \angle\ 90°) - (3.6\ \angle\ 215°) = (5.9\ \angle\ 60°) \ .$$

This equals the impulse delivered to the ball (by the bat). Then, Eq. 10-8 leads to

$$F_{\text{avg}}\Delta t = 5.9 \quad\Longrightarrow\quad F_{\text{avg}} = \frac{5.9}{2.0\times 10^{-3}} \ \approx\ 2.9\times 10^3\ \text{N} \ .$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision.

11. We take the magnitude of the force to be $F = At$, where A is a constant of proportionality. The condition that $F = 50\,\text{N}$ when $t = 4.0\,\text{s}$ leads to $A = (50\,\text{N})/(4.0\,\text{s}) = 12.5\,\text{N/s}$. The magnitude of the impulse exerted on the object is

$$J = \int_0^{4.0} F\,dt = \int_0^{4.0} At\,dt = \frac{1}{2}At^2\bigg|_0^{4.0} = \frac{1}{2}(12.5)(4.0)^2 = 100\ \text{N}\cdot\text{s} \ .$$

This equals the magnitude of the change in the momentum of the object (by the impulse-momentum theorem), and since the ball started from rest, we have $J = mv_f$. Therefore, $v_f = J/m = (100\,\text{N}\cdot\text{s})/(10\,\text{kg}) = 10\,\text{m/s}$.

12. (a) The mass of each spherical hailstone of radius $r = 0.5$ cm and density $\rho = 0.92$ g/cm^3 is

$$m = \rho\left(\frac{4\pi r^3}{3}\right) = 0.48\ \text{g} = 4.8\times 10^{-4}\ \text{kg} \ .$$

(b) If the final speed is zero, then Eq. 10-4 and Eq. 10-8 (with $+y$ upward) lead to

$$\vec{F}_{\rm avg}\Delta t = -m\vec{v}_i = -\left(4.8\times10^{-4}\right)(-25) = 0.012$$

in SI units (N·s). This gives the impulse imparted to a single hailstone by the roof (and is equal to the magnitude of the force on the roof by the hailstone, by Newton's third law). An imagined "cube" of falling air, $\ell = 1$ m on each side (falling with the hail at $v = 25$ m/s), takes a time of

$$\Delta t = \frac{\ell}{v} = \frac{1\,\rm m}{25\,\rm m/s} = 0.04\ \rm s$$

to fully "collapse" onto a square meter of roof top (delivering its load of 120 hailstones). We can cover an area of $10\,\rm m\times 20\,\rm m$ with 200 of these "collapsing cubes" of air. Therefore, in this time, the total impulse is of magnitude

$$\vec{F}_{\rm avg,total}\Delta t = 200(120)(0.012\,\rm N\cdot s) \approx 290\,\rm N\cdot s$$

which leads to $\vec{F}_{\rm avg,total} = 290/0.04 = 7.2\times10^3$ N.

13. (a) If m is the mass of a pellet and v is its velocity as it hits the wall, then its momentum is $p = mv = (2.0\times10^{-3}\,\rm kg)(500\,\rm m/s) = 1.0\,\rm kg\cdot m/s$, toward the wall.

(b) The kinetic energy of a pellet is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(2.0\times10^{-3}\,\rm kg\right)(500\,\rm m/s)^2 = 2.5\times10^2\ \rm J\ .$$

(c) The force on the wall is given by the rate at which momentum is transferred from the pellets to the wall. Since the pellets do not rebound, each pellet that hits transfers $p = 1.0\,\rm kg\cdot m/s$. If ΔN pellets hit in time Δt, then the average rate at which momentum is transferred is

$$F_{\rm avg} = \frac{p\,\Delta N}{\Delta t} = (1.0\,\rm kg\cdot m/s)\left(10\,\rm s^{-1}\right) = 10\ \rm N\ .$$

The force on the wall is in the direction of the initial velocity of the pellets.

(d) If Δt is the time interval for a pellet to be brought to rest by the wall, then the average force exerted on the wall by a pellet is

$$F_{\rm avg} = \frac{p}{\Delta t} = \frac{1.0\,\rm kg\cdot m/s}{0.6\times10^{-3}\,\rm s} = 1.7\times10^3\,\rm N\ .$$

The force is in the direction of the initial velocity of the pellet.

(e) In part (d) the force is averaged over the time a pellet is in contact with the wall, while in part (c) it is averaged over the time for many pellets to hit the wall. During the majority of this time, no pellet is in contact with the wall, so the average force in part (c) is much less than the average force in part (d).

14. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued). We evaluate the integral $J = \int F\,dt$ by adding the appropriate areas (of a triangle, a rectangle, and another triangle) shown in the graph (but with the t converted to seconds). With $m = 0.058\,\rm kg$ and $v = 34\,\rm m/s$, we apply the impulse-momentum theorem:

$$\begin{aligned}
\int F_{\rm wall}\,dt &= m\vec{v}_f - m\vec{v}_i\\
\int_0^{0.002} F\,dt + \int_{0.002}^{0.004} F\,dt + \int_{0.004}^{0.006} F\,dt &= m(+v) - m(-v)\\
\frac{1}{2}F_{\rm max}\,(0.002\,\rm s) + F_{\rm max}\,(0.002\,\rm s) + \frac{1}{2}F_{\rm max}\,(0.002\,\rm s) &= 2mv\\
F_{\rm max}\,(0.004\,\rm s) &= 2\,(0.058\,\rm kg)\,(34\,\rm m/s)
\end{aligned}$$

which yields $F_{\rm max} = 9.9\times10^2$ N.

15. We first consider the 1200 kg part. The impulse has magnitude J and is (by our choice of coordinates) in the positive direction. Let m_1 be the mass of the part and v_1 be its velocity after the bolts are exploded. We assume both parts are at rest before the explosion. Then $J = m_1 v_1$, so

$$v_1 = \frac{J}{m_1} = \frac{300\,\mathrm{N\cdot s}}{1200\,\mathrm{kg}} = 0.25\ \mathrm{m/s}\ .$$

The impulse on the 1800 kg part has the same magnitude but is in the opposite direction, so $-J = m_2 v_2$, where m_2 is the mass and v_2 is the velocity of the part. Therefore,

$$v_2 = -\frac{J}{m_2} = -\frac{300\,\mathrm{N\cdot s}}{1800\,\mathrm{kg}} = -0.167\ \mathrm{m/s}\ .$$

Consequently, the relative speed of the parts after the explosion is $0.25\,\mathrm{m/s} - (-0.167\,\mathrm{m/s}) = 0.417\,\mathrm{m/s}$.

16. We choose our positive direction in the direction of the rebound (so the ball's initial velocity is negative-valued: $\vec{v}_i = -5.2$ m/s).

(a) The speed of the ball right after the collision is

$$\begin{aligned} v_f &= \sqrt{\frac{2K_f}{m}} \\ &= \sqrt{\frac{2\left(\frac{1}{2}K_i\right)}{m}} \\ &= \sqrt{\frac{\frac{1}{2}mv_i^2}{m}} \\ &= \frac{v_i}{\sqrt{2}} \approx 3.7\ \mathrm{m/s}\ . \end{aligned}$$

(b) With $m = 0.15$ kg, the impulse-momentum theorem (Eq. 10-4) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (0.15)(3.7) - (0.15)(-5.2) = 1.3$$

in SI units (N·s).

(c) Eq. 10-8 leads to $F_{\mathrm{avg}} = J/\Delta t = 1.3/0.0076 = 1.8 \times 10^2$ N.

17. We choose $+y$ in the direction of the rebound (directly away from the wall) and $+x$ towards the right in the figure (parallel to the wall; see Fig. 10-30). Using unit-vector notation, the the ball's initial and final velocities are

$$\begin{aligned} \vec{v}_i &= v\cos\theta\,\hat{\imath} - v\sin\theta\,\hat{\jmath} = 5.2\,\hat{\imath} - 3.0\,\hat{\jmath} \\ \vec{v}_f &= v\cos\theta\,\hat{\imath} + v\sin\theta\,\hat{\jmath} = 5.2\,\hat{\imath} + 3.0\,\hat{\jmath} \end{aligned}$$

respectively (with SI units understood).

(a) With $m = 0.30$ kg, the impulse-momentum theorem (Eq. 10.4) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2(0.30)(3.0\,\hat{\jmath})$$

so that the magnitude of the impulse delivered on the ball by the wall is 1.8 N·s and its direction is directly away from the wall (which, in terms of Fig. 10-30, is "up").

(b) Using Eq. 10-8, the force on the ball by the wall is $\vec{J}/\Delta t = 1.8\hat{\jmath}/0.010 = 180\,\hat{\jmath}$ N. By Newton's third law, the force on the wall by the ball is $-180\,\hat{\jmath}$ N (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 10-30).

18. (a) Regardless of the direction of the thrust, the change in linear momentum of the space probe is given by the impulse-momentum theorem (also using Eq. 10-8):

$$\Delta p = (3000\,\text{N})(65.0\,\text{s}) = 1.95 \times 10^5\,\text{kg}\cdot\text{m/s} \ .$$

(b) The change in speed for the probe of mass m is

$$\Delta v = \frac{\Delta p}{m} = \frac{1.95 \times 10^5\,\text{kg}\cdot\text{m/s}}{2500\,\text{kg}} = 78.0\,\text{m/s} \ .$$

Let the initial and final speeds of the probe be v_i and v_f, respectively. Then, the change in its kinetic energy is $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$. If the thrust is backward then $v_f = v_i - \Delta v$, and

$$\begin{aligned}
\Delta K &= \frac{1}{2}m\left((v_i - \Delta v)^2 - v_i^2\right) \\
&= \frac{1}{2}(2500\,\text{kg})\left((300\,\text{m/s} - 78.0\,\text{m/s})^2 - (300\,\text{m/s})^2\right) \\
&= -5.09 \times 10^7\ \text{J}
\end{aligned}$$

If the thrust is forward then $v_f = v_i + \Delta v$, and

$$\begin{aligned}
\Delta K &= \frac{1}{2}m\left((v_i + \Delta v)^2 - v_i^2\right) \\
&= \frac{1}{2}(2500\,\text{kg})\left((300\,\text{m/s} + 78.0\,\text{m/s})^2 - (300\,\text{m/s})^2\right) \\
&= 6.61 \times 10^7\ \text{J} \ .
\end{aligned}$$

If the thrust is sideways then $v_f = \sqrt{(\Delta v)^2 + v_i^2}$, and

$$\Delta K = \frac{1}{2}m\left((\Delta v)^2 + v_i^2 - v_i^2\right) = \frac{1}{2}(2500\,\text{kg})(78.0\,\text{m/s})^2 = 7.61 \times 10^6\,\text{J} \ .$$

19. (a) We take the force to be in the positive direction, at least for earlier times. Then the impulse is

$$\begin{aligned}
J &= \int_0^{3.0\times10^{-3}} F\,dt \\
&= \int_0^{3.0\times10^{-3}} \left(6.0 \times 10^6\right)t - \left(2.0 \times 10^9\right)t^2\ dt \\
&= \left[\frac{1}{2}(6.0 \times 10^6)t^2 - \frac{1}{3}(2.0 \times 10^9)t^3\right]_0^{3.0\times10^{-3}} \\
&= 9.0\ \text{N}\cdot\text{s} \ .
\end{aligned}$$

(b) Since $J = F_{\text{avg}}\,\Delta t$, we find

$$F_{\text{avg}} = \frac{J}{\Delta t} = \frac{9.0\,\text{N}\cdot\text{s}}{3.0 \times 10^{-3}\,\text{s}} = 3.0 \times 10^3\ \text{N} \ .$$

(c) To find the time at which the maximum force occurs, we set the derivative of F with respect to time equal to zero – and solve for t. The result is $t = 1.5 \times 10^{-3}\,\text{s}$. At that time the force is

$$F_{\text{max}} = (6.0 \times 10^6)(1.5 \times 10^{-3}) - (2.0 \times 10^9)(1.5 \times 10^{-3})^2 = 4.5 \times 10^3\,\text{N} \ .$$

(d) Since it starts from rest, the ball acquires momentum equal to the impulse from the kick. Let m be the mass of the ball and v be its speed as it leaves the foot. Then,

$$v = \frac{p}{m} = \frac{J}{m} = \frac{9.0\,\text{N}\cdot\text{s}}{0.45\,\text{kg}} = 20\ \text{m/s} \ .$$

20. (a) We choose $+x$ along the initial direction of motion and apply momentum conservation:

$$\begin{aligned} m_{\text{bullet}}\vec{v}_i &= m_{\text{bullet}}\vec{v}_1 + m_{\text{block}}\vec{v}_2 \\ (5.2\,\text{g})(672\,\text{m/s}) &= (5.2\,\text{g})(428\,\text{m/s}) + (700\,\text{g})\vec{v}_2 \end{aligned}$$

which yields $v_2 = 1.81$ m/s.

(b) It is a consequence of momentum conservation that the velocity of the center of mass is unchanged by the collision. We choose to evaluate it before the collision:

$$\vec{v}_{\text{com}} = \frac{m_{\text{bullet}}\vec{v}_i}{m_{\text{bullet}} + m_{\text{block}}} = \frac{(5.2\,\text{g})(672\,\text{m/s})}{5.2\,\text{g} + 700\,\text{g}}$$

which gives the result $\vec{v}_{\text{com}} = 4.96$ m/s.

21. We examine the horizontal components of the momenta of the package and sled. Let m_s be the mass of the sled and v_s be its initial velocity. Let m_p be the mass of the package and let v be the final velocity of the sled and package together. The horizontal component of the total momentum is conserved, so $m_s v_s = (m_s + m_p)v$ and

$$v = \frac{v_s m_s}{m_s + m_p} = \frac{(9.0\,\text{m/s})(6.0\,\text{kg})}{6.0\,\text{kg} + 12\,\text{kg}} = 3.0 \text{ m/s} .$$

22. We refer to the discussion in the textbook (see Sample Problem 10-2, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{2.010}{0.010}\sqrt{2(9.8)(0.12)} = 3.1 \times 10^2 \text{ m/s} .$$

23. Let m_m be the mass of the meteor and m_e be the mass of Earth. Let v_m be the velocity of the meteor just before the collision and let v be the velocity of Earth (with the meteor) just after the collision. The momentum of the Earth-meteor system is conserved during the collision. Thus, in the reference frame of Earth before the collision, $m_m v_m = (m_m + m_e)v$, so

$$v = \frac{v_m m_m}{m_m + m_e} = \frac{(7200\,\text{m/s})(5 \times 10^{10}\,\text{kg})}{5.98 \times 10^{24}\,\text{kg} + 5 \times 10^{10}\,\text{kg}} = 6 \times 10^{-11}\,\text{m/s} .$$

We convert this as follows:

$$\left(6 \times 10^{-11}\,\frac{\text{m}}{\text{s}}\right)\left(\frac{1000\,\text{mm}}{\text{m}}\right)\left(\frac{3.2 \times 10^7\,\text{s}}{\text{y}}\right) = 2 \text{ mm/y} .$$

24. (a) To relate the sliding distance to the speed V of the bullet-plus-block at the instant it has finished embedding itself in the block, we can either use Eq. 2-16 and $\vec{F}_{\text{net}} = m\vec{a}$, or energy conservation as expressed by Eq. 8-31 (with $W = 0$ and $f_k = \mu_k(m+M)g$ using Eq. 6-2). We choose the latter approach:

$$\begin{aligned} K_{\text{bullet plus block}} &= \Delta E_{\text{th}} \\ \frac{1}{2}(m+M)V^2 &= \mu_k(m+M)gd \end{aligned}$$

which yields $V = \sqrt{2\mu_k\, gd} = 2.7$ m/s.

(b) For the collision itself, we use momentum conservation (with the direction of motion being positive).

$$\begin{aligned} m_{\text{bullet}}v_i &= (m+M)V \\ (0.0045\,\text{kg})v_i &= (2.4045\,\text{kg})(2.7\text{m/s}) \end{aligned}$$

which gives the result $v_i = 1.4 \times 10^3$ m/s.

25. (a) The magnitude of the deceleration of each of the cars is $a = f/m = \mu_k mg/m = \mu_k g$. If a car stops in distance d, then its speed v just after impact is obtained from Eq. 2-16:

$$v^2 = v_0^2 + 2ad \implies v = \sqrt{2ad} = \sqrt{2\mu_k gd}$$

since $v_0 = 0$ (this could alternatively have been derived using Eq. 8-31). Thus,

$$v_A = \sqrt{2(0.13)(9.8)(8.2)} = 4.6 \text{ m/s} \quad , \quad \text{and}$$

(b) $v_B = \sqrt{2(0.13)(9.8)(6.1)} = 3.9$ m/s.

(c) Let the speed of car B be v just before the impact. Conservation of linear momentum gives $m_B v = m_A v_A + m_B v_B$, or

$$v = \frac{(m_A v_A + m_B v_B)}{m_B} = \frac{(1100)(4.6) + (1400)(3.9)}{1400} = 7.5 \text{ m/s} .$$

The conservation of linear momentum during the impact depends on the fact that the only significant force (during impact of duration Δt) is the force of contact between the bodies. In this case, that implies that the force of friction exerted by the road on the cars is neglected during the brief Δt. This neglect would introduce some error in the analysis. Related to this is the assumption we are making that the transfer of momentum occurs at one location – that the cars do not slide appreciably during Δt – which is certainly an approximation (though probably a good one). Another source of error is the application of the friction relation Eq. 6-2 for the sliding portion of the problem (after the impact); friction is a complex force that Eq. 6-2 only partially describes.

26. We note that the "(a)" and "(b)" in Fig. 10-32 do not correspond to parts (a) and (b) (in fact, it's somewhat the reverse). Our $+x$ direction is to the right (so all velocities are positive-valued).

(a) We apply momentum conservation to relate the situation just before the bullet strikes the second block to the situation where the bullet is embedded within the block.

$$(0.0035\,\text{kg})v = (1.8035\,\text{kg})(1.4\,\text{m/s}) \implies v = 721 \text{ m/s} .$$

(b) We apply momentum conservation to relate the situation just before the bullet strikes the first block to the instant it has passed through it (having speed v found in part (a)).

$$(0.0035\,\text{kg})v_0 = (1.2\,\text{kg})(0.63\,\text{m/s}) + (0.0035\,\text{kg})(721\,\text{m/s})$$

which yields $v_0 = 937$ m/s.

27. (a) We want to calculate the force that the scale exerts on the marbles. This is the sum of the force that holds the marbles already on the scale against the downward force of gravity and the force that brings the falling marbles to rest. At the end of time t, the number of marbles on the scale is Rt. At this moment, the gravitational force on them is $Rtmg$ and the upward force of the scale that holds them is $F_1 = Rtmg$. Just before striking the scale, a marble that fell from height h has speed $v = \sqrt{2gh}$ and momentum $p = m\sqrt{2gh}$. To stop the falling marbles, the scale must exert an upward force $F_2 = Rp = Rm\sqrt{2gh}$. The total force of the scale on the marbles is

$$F = F_1 + F_2 = Rtmg + Rm\sqrt{2gh} = Rm\left(gt + \sqrt{2gh}\right) .$$

(b) For the given data (using SI units, so $m = 0.0045$ kg), we find

$$F = (100)(0.0045)\left((9.8)(10.0) + \sqrt{2(9.8)(7.60)}\right)$$

which yields $F = 49.6$ N. Assuming the scale is calibrated to read in terms of an equivalent mass, its reading is $F/g = 49.6/9.8 = 5.06$ kg.

28. We choose $+x$ in the direction of (initial) motion of the blocks, which have masses $m_1 = 5$ kg and $m_2 = 10$ kg. Where units are not shown in the following, SI units are to be understood.

(a) Momentum conservation leads to

$$\begin{aligned} m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \\ (5)(3) + (10)(2) &= 5\vec{v}_{1f} + (10)(2.5) \end{aligned}$$

which yields $\vec{v}_{1f} = 2$. Thus, the speed of the 5 kg block immediately after the collision is 2.0 m/s.

(b) We find the reduction in total kinetic energy:

$$K_i - K_f = \frac{1}{2}(5)(3)^2 + \frac{1}{2}(10)(2)^2 - \frac{1}{2}(5)(2)^2 - \frac{1}{2}(10)(2.5)^2$$

which gives the result 1.25 J. Rounding to two figures and recalling that $\Delta K = K_f - K_i$ then our answer is $\Delta K = -1.3$ J.

(c) In this new scenario where $\vec{v}_{2f} = 4.0$ m/s, momentum conservation leads to $\vec{v}_{1f} = -1.0$ m/s and we obtain $\Delta K = +40$ J.

(d) The creation of additional kinetic energy is possible if, say, some gunpowder were on the surface where the impact occurred (initially stored chemical energy would then be contributing to the result).

29. Let m_F be the mass of the freight car and v_F be its initial velocity. Let m_C be the mass of the caboose and v be the common final velocity of the two when they are coupled. Conservation of the total momentum of the two-car system leads to $m_F v_F = (m_F + m_C)v$, so $v = v_F m_F/(m_F + m_C)$. The initial kinetic energy of the system is

$$K_i = \frac{1}{2}m_F v_F^2$$

and the final kinetic energy is

$$K_f = \frac{1}{2}(m_F + m_C)v^2 = \frac{1}{2}(m_F + m_C)\frac{m_F^2 v_F^2}{(m_F + m_C)^2} = \frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} .$$

Since 27% of the original kinetic energy is lost, we have $K_f = 0.73K_i$. Thus,

$$\frac{1}{2}\frac{m_F^2 v_F^2}{(m_F + m_C)} = (0.73)\left(\frac{1}{2}m_F v_F^2\right) .$$

Simplifying, we obtain $m_F/(m_F + m_C) = 0.73$, which we use in solving for the mass of the caboose:

$$m_C = \frac{0.27}{0.73}m_F = 0.37\,m_F = (0.37)\left(3.18 \times 10^4\,\text{kg}\right) = 1.18 \times 10^4\ \text{kg} .$$

30. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\,\text{kg})(1000\,\text{m/s}) = (5.0\,\text{kg})\vec{v} + (0.01\,\text{kg})(400\,\text{m/s})$$

which yields $\vec{v} = 1.2$ m/s. The second part involves either the free-fall equations from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch. 8. Choosing the latter approach, we have

$$\frac{1}{2}(5.0\,\text{kg})(1.2\,\text{m/s})^2 = (5.0\,\text{kg})\left(9.8\,\text{m/s}^2\right)h$$

which gives the result $h = 0.073$ m.

31. (a) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m+M)v$. Therefore, $v = mv_i/(m+M)$.

    (b) The initial kinetic energy is $K_i = \frac{1}{2}mv_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(m+M)v^2 = \frac{1}{2}m^2v_i^2/(m+M)$. The problem indicates $\Delta E_{\text{th}} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\frac{m^2v_i^2}{(m+M)} = \frac{1}{2}mv_i^2\left(1 - \frac{m}{m+M}\right) = \frac{1}{2}mv_i^2\frac{M}{m+M} \ .$$

    Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is $U_s/K_i = M/(m+M)$.

32. For a picture of this one-dimensional example of an "explosion" involving two objects ($m_1 = 4.0$ kg and $m_2 = 6.0$ kg), see Fig. 9-40 (but reverse the velocity arrows). Since the system was initially at rest, momentum conservation leads to

$$0 = m_2\vec{v}_2 + m_1\vec{v}_1 \quad \Longrightarrow \quad |\vec{v}_1| = \frac{m_2}{m_1}\,|\vec{v}_2|$$

which yields 6.0 m/s for the speed of the physics book. Mechanical energy conservation tells us that the initial potential energy is

$$U_i = K_{f\,\text{total}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

which gives the result $U_i = 120$ J.

33. As hinted in the problem statement, the velocity v of the system as a whole – when the spring reaches the maximum compression x_{m} – satisfies $m_1v_{1i} + m_2v_{2i} = (m_1+m_2)v$. The change in kinetic energy of the system is therefore

$$\begin{aligned}\Delta K &= \frac{1}{2}(m_1+m_2)v^2 - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2 \\ &= \frac{(m_1v_{1i}+m_2v_{2i})^2}{2\,(m_1+m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2\end{aligned}$$

which yields $\Delta K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $|\Delta K| = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_{\text{rel}}^2$ where $v_{\text{rel}} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_{\text{m}}^2 = -\Delta K \quad \Longrightarrow \quad x_{\text{m}} = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35)}{1120}}$$

which gives the result $x_{\text{m}} = 0.25$ m.

34. We think of this as having two parts: the first is the collision itself – where the blocks "join" so quickly that the 1.0-kg block has not had time to move through any distance yet – and then the subsequent motion of the 3.0 kg system as it compresses the spring to the maximum amount x_{m}. The first part involves momentum conservation (with $+x$ rightward):

$$(2.0\,\text{kg})(4.0\,\text{m/s}) = (3.0\,\text{kg})\vec{v}$$

which yields $\vec{v} = 2.7$ m/s. The second part involves mechanical energy conservation:

$$\frac{1}{2}(3.0\,\text{kg})(2.7\,\text{m/s})^2 = \frac{1}{2}(200\,\text{N/m})x_{\text{m}}^2$$

which gives the result $x_{\text{m}} = 0.33$ m.

35. (a) Let m_1 be the mass of the block on the left, v_{1i} be its initial velocity, and v_{1f} be its final velocity. Let m_2 be the mass of the block on the right, v_{2i} be its initial velocity, and v_{2f} be its final velocity. The momentum of the two-block system is conserved, so $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ and

$$v_{1f} = \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1} = \frac{(1.6)(5.5) + (2.4)(2.5) - (2.4)(4.9)}{1.6}$$

which yields $v_{1f} = 1.9$ m/s. The block continues going to the right after the collision.

(b) To see if the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}(1.6)(5.5)^2 + \frac{1}{2}(2.4)(2.5)^2 = 31.7 \text{ J} .$$

The total kinetic energy after is

$$K_f = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}(1.6)(1.9)^2 + \frac{1}{2}(2.4)(4.9)^2 = 31.7 \text{ J} .$$

Since $K_i = K_f$ the collision is found to be elastic.

(c) Now $v_{2i} = -2.5\,\text{m/s}$ and

$$v_{1f} = \frac{m_1v_{1i} + m_2v_{2i} - m_2v_{2f}}{m_1} = \frac{(1.6)(5.5) + (2.4)(-2.5) - (2.4)(4.9)}{1.6}$$

which yields $v_{1f} = -5.6$ m/s. Thus, the velocity is opposite to the direction shown in Fig. 10-37.

36. We use m_1 for the mass of the electron and $m_2 = 1840m_1$ for the mass of the hydrogen atom. Using Eq. 10-31,

$$v_{2f} = \frac{2m_1}{m_1 + 1840m_1}\,v_{1i} = \frac{2}{1841}\,v_{1i}$$

we compute the final kinetic energy of the hydrogen atom:

$$K_{2f} = \frac{1}{2}(1840m_1)\left(\frac{2\,v_{1i}}{1841}\right)^2 = \frac{(1840)(4)}{1841^2}\left(\frac{1}{2}(1840m_1)v_{1i}^2\right)$$

so we find the fraction to be $(1840)(4)/1841^2 \approx 2.2 \times 10^{-3}$, or 0.22%.

37. (a) Let m_1 be the mass of the cart that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the cart that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 10-30,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} .$$

Using SI units (so $m_1 = 0.34$ kg), we obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1i} + v_{1f}}m_1 = \left(\frac{1.2 - 0.66}{1.2 + 0.66}\right)(0.34) = 0.099 \text{ kg} .$$

(b) The velocity of the second cart is given by Eq. 10-31:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} = \left(\frac{2(0.34)}{0.34 + 0.099}\right)(1.2) = 1.9 \text{ m/s} .$$

(c) The speed of the center of mass is

$$v_{\rm com} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(0.34)(1.2) + 0}{0.34 + 0.099} = 0.93 \text{ m/s} .$$

Values for the initial velocities were used but the same result is obtained if values for the final velocities are used.

38. No mechanical energy is "lost" in this encounter, so we analyze it with the elastic collision equations, particularly Eq. 10-38. Thus,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \approx \; -v_{1i} + 2v_{2i}$$

where we have made the (certainly reasonable) approximation that $m_2 \gg m_1$ and simplified accordingly. Thus, $v_{1\,f} = -12 + 2(-13) = -38$, resulting in a final speed (relative to the Sun) of 38 km/s.

39. We use $m_1 = 4$ u for the mass of the alpha particle and $m_2 = 197$ u for the mass of the gold nucleus in Eq. 10-31:

$$v_{2f} = \frac{2(4)}{4 + 197} v_{1i} = \frac{8}{201} v_{1i}$$

we compute the final kinetic energy of the gold nucleus (which must be the same as the kinetic energy lost by the alpha particle – since this is an elastic collision)

$$K_{2f} = \frac{1}{2} m_2 v_{2f}^2 = \frac{1}{2} (197\,\text{u}) \left(\frac{8\, v_{1i}}{201} \right)^2 .$$

We divide this by the initial alpha particle energy $K_i = \frac{1}{2}(4\,\text{u})v_{1i}^2$ to obtain

$$\frac{K_{2f}}{K_i} = \frac{(197)(8)^2}{(4)(201)^2} \approx 0.078$$

so we find the percentage is 7.8%.

40. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with $h = 0.700$ m) leads to

$$m_1 g h = \frac{1}{2} m_1 v^2 \quad \Longrightarrow \quad v = \sqrt{2gh} = 3.7 \text{ m/s} .$$

(a) We now treat the elastic collision (with SI units) using Eq. 10-30:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 - 2.5}{0.5 + 2.5} (3.7) = -2.47$$

which means the final speed of the ball is 2.47 m/s.

(b) Finally, we use Eq. 10-31 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5)}{0.5 + 2.5} (3.7) = 1.23 \text{ m/s} .$$

41. (a) Let m_1 be the mass of the body that is originally moving, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the body that is originally at rest and v_{2f} be its velocity after the collision. Then, according to Eq. 10–30,

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} .$$

We solve for m_2 to obtain

$$m_2 = \frac{v_{1i} - v_{1f}}{v_{1f} + v_{1i}} m_1 .$$

We combine this with $v_{1f} = v_{1i}/4$ to obtain $m_2 = 3m_1/5 = 3(2.0)/5 = 1.2\,\text{kg}$.

(b) The speed of the center of mass is

$$v_{\rm com} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(2.0)(4.0)}{2.0 + 1.2} = 2.5 \text{ m/s} .$$

42. We refer to the discussion in the textbook (Sample Problem 10-4, which uses the same notation that we use here) for some important details in the reasoning. We choose rightward in Fig. 10-15 as our $+x$ direction. We use the notation $\vec{v}$ when we refer to velocities and v when we refer to speeds (which are necessarily positive). Since the algebra is fairly involved, we find it convenient to introduce the notation $\Delta m = m_2 - m_1$ (which, we note for later reference, is a positive-valued quantity).

(a) Since $\vec{v}_{1i} = +\sqrt{2gh_1}$ where $h_1 = 9.0$ cm, we have

$$\vec{v}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\, v_{1i} = -\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}$$

which is to say that the *speed* of sphere 1 immediately after the collision is $v_{1f} = (\Delta m/(m_1 + m_2))\sqrt{2gh_1}$ and that $\vec{v}_{1f}$ points in the $-x$ direction. This leads (by energy conservation $m_1 g h_{1f} = \frac{1}{2} m_1 v_{1f}^2$) to

$$h_{1f} = \frac{v_{1f}^2}{2g} = \left(\frac{\Delta m}{m_1 + m_2}\right)^2 h_1 .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{1f} \approx 0.6$ cm.

(b) Eq. 10-31 gives

$$v_{2f} = \frac{2m_1}{m_1 + m_2}\, v_{1i} = \frac{2m_1}{m_1 + m_2}\sqrt{2gh_1}$$

which leads (by energy conservation $m_2 g h_{2f} = \frac{1}{2} m_2 v_{2f}^2$) to

$$h_{2f} = \frac{v_{2f}^2}{2g} = \left(\frac{2m_1}{m_1 + m_2}\right)^2 h_1 .$$

With $m_1 = 50$ g and $m_2 = 85$ g, this becomes $h_{2f} \approx 4.9$ cm.

(c) Fortunately, they hit again at the lowest point (as long as their amplitude of swing was "small" – this is further discussed in Chapter 16). At the risk of using cumbersome notation, we refer to the *next* set of heights as h_{1ff} and h_{2ff}. At the lowest point (before this second collision) sphere 1 has velocity $+\sqrt{2gh_{1f}}$ (rightward in Fig. 10-15) and sphere 2 has velocity $-\sqrt{2gh_{1f}}$ (that is, it points in the $-x$ direction). Thus, the velocity of sphere 1 immediately after the second collision is, using Eq. 10-38,

$$\begin{aligned}
\vec{v}_{1ff} &= \frac{m_1 - m_2}{m_1 + m_2}\sqrt{2gh_{1f}} + \frac{2m_2}{m_1 + m_2}\left(-\sqrt{2gh_{2f}}\right) \\
&= \frac{-\Delta m}{m_1 + m_2}\left(\frac{\Delta m}{m_1 + m_2}\sqrt{2gh_1}\right) - \frac{2m_2}{m_1 + m_2}\left(\frac{2m_1}{m_1 + m_2}\sqrt{2gh_1}\right) \\
&= -\frac{(\Delta m)^2 + 4m_1 m_2}{(m_1 + m_2)^2}\sqrt{2gh_1} .
\end{aligned}$$

This can be greatly simplified (by expanding $(\Delta m)^2$ and $(m_1 + m_2)^2$) to arrive at the conclusion that the speed of sphere 1 immediately after the second collision is simply $v_{1ff} = \sqrt{2gh_1}$ and that $\vec{v}_{1ff}$ points in the $-x$ direction. Energy conservation ($m_1 g h_{1ff} = \frac{1}{2} m_1 v_{1ff}^2$) leads to

$$h_{1ff} = \frac{v_{1ff}^2}{2g} = h_1 = 9.0 \text{ cm} .$$

(d) One can reason (energy-wise) that $h_{1\,ff} = 0$ simply based on what we found in part (c). Still, it might be useful to see how this shakes out of the algebra. Eq. 10-39 gives the velocity of sphere 2 immediately after the second collision:

$$\begin{aligned} v_{2\,ff} &= \frac{2m_1}{m_1+m_2}\sqrt{2gh_{1\,f}} + \frac{m_2-m_1}{m_1+m_2}\left(-\sqrt{2gh_{2\,f}}\right) \\ &= \frac{2m_1}{m_1+m_2}\left(\frac{\Delta m}{m_1+m_2}\sqrt{2gh_1}\right) + \frac{\Delta m}{m_1+m_2}\left(\frac{-2m_1}{m_1+m_2}\sqrt{2gh_1}\right) \end{aligned}$$

which vanishes since $(2m_1)(\Delta m) - (\Delta m)(2m_1) = 0$. Thus, the second sphere (after the second collision) stays at the lowest point, which basically recreates the conditions at the start of the problem (so all subsequent swings-and-impacts, neglecting friction, can be easily predicted – as they are just replays of the first two collisions).

43. (a) Let m_1 be the mass of one sphere, v_{1i} be its velocity before the collision, and v_{1f} be its velocity after the collision. Let m_2 be the mass of the other sphere, v_{2i} be its velocity before the collision, and v_{2f} be its velocity after the collision. Then, according to Eq. 10–38,

$$v_{1f} = \frac{m_1-m_2}{m_1+m_2}v_{1i} + \frac{2m_2}{m_1+m_2}v_{2i}\,.$$

Suppose sphere 1 is originally traveling in the positive direction and is at rest after the collision. Sphere 2 is originally traveling in the negative direction. Replace v_{1i} with v, v_{2i} with $-v$, and v_{1f} with zero to obtain $0 = m_1 - 3m_2$. Thus $m_2 = m_1/3 = (300\,\text{g})/3 = 100\,\text{g}$.

(b) We use the velocities before the collision to compute the velocity of the center of mass:

$$v_{\text{com}} = \frac{m_1v_{1i} + m_2v_{2i}}{m_1+m_2} = \frac{(300\,\text{g})(2.0\,\text{m/s}) + (100\,\text{g})(-2.0\,\text{m/s})}{300\,\text{g} + 100\,\text{g}}$$

which yields $v_{\text{com}} = 1.0$ m/s.

44. The velocities of m_1 and m_2 just after the collision with each other are given by Eq. 10-38 and Eq. 10-39 (setting $v_{1i} = 0$).

$$\begin{aligned} v_{1f} &= \frac{2m_2}{m_1+m_2}\,v_{2i} \\ v_{2f} &= \frac{m_2-m_1}{m_1+m_2}\,v_{2i} \end{aligned}$$

After bouncing off the wall, the velocity of m_2 becomes $-v_{2f}$ (see *a massive target* in §10-5). In these terms, the problem requires

$$\begin{aligned} v_{1f} &= -v_{2f} \\ \frac{2m_2}{m_1+m_2}\,v_{2i} &= -\frac{m_2-m_1}{m_1+m_2}\,v_{2i} \end{aligned}$$

which simplifies to

$$2m_2 = -\left(m_2 - m_1\right) \implies m_2 = \frac{m_1}{3}\,.$$

45. (a) We use conservation of mechanical energy to find the speed of either ball after it has fallen a distance h. The initial kinetic energy is zero, the initial gravitational potential energy is Mgh, the final kinetic energy is $\frac{1}{2}Mv^2$, and the final potential energy is zero. Thus $Mgh = \frac{1}{2}Mv^2$ and $v = \sqrt{2gh}$. The collision of the ball of M with the floor is an elastic collision of a light object with a stationary massive object. The velocity of the light object reverses direction without change in magnitude. After the collision, the ball is traveling upward with a speed of $\sqrt{2gh}$. The ball of

mass m is traveling downward with the same speed. We use Eq. 10-38 to find an expression for the velocity of the ball of mass M after the collision:

$$\begin{aligned} v_{Mf} &= \frac{M-m}{M+m}\, v_{Mi} + \frac{2m}{M+m}\, v_{mi} \\ &= \frac{M-m}{M+m}\sqrt{2gh} - \frac{2m}{M+m}\sqrt{2gh} \\ &= \frac{M-3m}{M+m}\sqrt{2gh}\ . \end{aligned}$$

For this to be zero, $M = 3m$.

(b) We use the same equation to find the velocity of the ball of mass m after the collision:

$$v_{mf} = -\frac{m-M}{M+m}\sqrt{2gh} + \frac{2M}{M+m}\sqrt{2gh} = \frac{3M-m}{M+m}\sqrt{2gh}$$

which becomes (upon substituting $M = 3m$) $v_{mf} = 2\sqrt{2gh}$. We next use conservation of mechanical energy to find the height h' to which the ball rises. The initial kinetic energy is $\frac{1}{2}mv_{mf}^2$, the initial potential energy is zero, the final kinetic energy is zero, and the final potential energy is mgh'. Thus

$$\frac{1}{2}mv_{mf}^2 = mgh' \implies h' = \frac{v_{mf}^2}{2g} = 4h$$

where $2\sqrt{2gh}$ is substituted for v_{mf}.

46. (a) Conservation of linear momentum implies $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$. Since $m_A = m_B = m = 2.0\,\text{kg}$, the masses divide out and we obtain (in m/s)

$$\begin{aligned} \vec{v}\,'_B &= \vec{v}_A + \vec{v}_B - \vec{v}\,'_A \\ &= (15\hat{\imath} + 30\hat{\jmath}) + (-10\hat{\imath} + 5\hat{\jmath}) - (-5\hat{\imath} + 20\hat{\jmath}) \\ &= 10\hat{\imath} + 15\hat{\jmath}\ . \end{aligned}$$

(b) The final and initial kinetic energies are

$$\begin{aligned} K_f &= \frac{1}{2}mv_A'^2 + \frac{1}{2}mv_B'^2 = \frac{1}{2}(2.0)\left((-5)^2 + 20^2 + 10^2 + 15^2\right) = 8.0 \times 10^2\ \text{J} \\ K_i &= \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2}(2.0)\left(15^2 + 30^2 + (-10)^2 + 5^2\right) = 1.3 \times 10^3\ \text{J}\ . \end{aligned}$$

The change kinetic energy is then $\Delta K = -5.0 \times 10^2$ J (that is, 500 J of the initial kinetic energy is lost).

47. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +64°$ for the alpha (α) particle (after collision) and $\phi = -51°$ for the oxygen nucleus (o) (which is going into the fourth quadrant, in our scenario). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_\alpha v_\alpha &= m_\alpha v'_\alpha \cos\theta + m_o v'_o \cos\phi \\ 0 &= m_\alpha v'_\alpha \sin\theta + m_o v'_o \sin\phi \end{aligned}$$

We are given $v'_o = 1.2 \times 10^5$ m/s, which leaves us two unknowns and two equations, which is sufficient for solving.

(a) We solve for the final alpha particle speed using the y-momentum equation:

$$v'_\alpha = -\frac{m_\alpha v'_\alpha \sin\theta}{m_o \sin\phi} = -\frac{(16)\left(1.2 \times 10^5\right)\sin(-51°)}{(4)\sin(64°)}$$

which yields $v'_\alpha = 4.15 \times 10^5$ m/s.

(b) Plugging our result from part (a) into the x-momentum equation produces the initial alpha particle speed:

$$\begin{aligned} m_\alpha v_\alpha &= \frac{m_\alpha v'_\alpha \cos\theta + m_o v'_o \cos\phi}{m_\alpha} \\ &= \frac{(4)\left(4.15 \times 10^5\right)\cos(64^\circ) + (16)\left(1.2 \times 10^5\right)\cos(-51^\circ)}{4} \\ &= 4.84 \times 10^5 \text{ m/s} . \end{aligned}$$

48. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +60^\circ$ for the proton (1) which is assumed to scatter into the first quadrant and $\phi = -30^\circ$ for the target proton (2) which scatters into the fourth quadrant (recall that the problem has told us that this is perpendicular to θ). We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_1 v_1 &= m_1 v'_1 \cos\theta + m_2 v'_2 \cos\phi \\ 0 &= m_1 v'_1 \sin\theta + m_2 v'_2 \sin\phi \end{aligned}$$

We are given $v_1 = 500$ m/s, which provides us with two unknowns and two equations, which is sufficient for solving. Since $m_1 = m_2$ we can cancel the mass out of the equations entirely.

(a) Combining the above equations and solving for v'_2 we obtain

$$v'_2 = \frac{v_1 \sin(\theta)}{\sin(\theta - \phi)} = \frac{500 \sin(60^\circ)}{\sin(90^\circ)} = 433$$

in SI units (m/s). We used the identity $\sin(\theta)\cos(\phi) - \cos(\theta)\sin(\phi) = \sin(\theta - \phi)$ in simplifying our final expression.

(b) In a similar manner, we find

$$v'_1 = \frac{v_1 \sin(\phi)}{\sin(\phi - \theta)} = \frac{500 \sin(-30^\circ)}{\sin(-90^\circ)} = 250 \text{ m/s} .$$

49. (a) We use Fig. 10-16 of the text (which treats both angles are positive-valued, even though one of them is in the fourth quadrant; this is why there is an explicit minus sign in Eq. 10-43 as opposed to it being implicitly in the angle). We take the cue ball to be body 1 and the other ball to be body 2. Conservation of the x component of the total momentum of the two-ball system leads to $mv_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$ and conservation of the y component leads to $0 = -mv_{1f}\sin\theta_1 + mv_{2f}\sin\theta_2$. The masses are the same and cancel from the equations. We solve the second equation for $\sin\theta_2$:

$$\sin\theta_2 = \frac{v_{1f}}{v_{2f}} \sin\theta_1 = \left(\frac{3.50 \text{ m/s}}{2.00 \text{ m/s}}\right) \sin 22.0^\circ = 0.656 .$$

Consequently, the angle between the second ball and the initial direction of the first is $\theta_2 = 41.0^\circ$.

(b) We solve the first momentum conservation equation for the initial speed of the cue ball.

$$\begin{aligned} v_{1i} &= v_{1f}\cos\theta_1 + v_{2f}\cos\theta_2 \\ &= (3.50 \text{ m/s})\cos 22.0^\circ + (2.00 \text{ m/s})\cos 41.0^\circ \\ &= 4.75 \text{ m/s} . \end{aligned}$$

(c) With SI units understood, the initial kinetic energy is

$$K_i = \frac{1}{2} m v_i^2 = \frac{1}{2} m (4.75)^2 = 11.3m$$

and the final kinetic energy is

$$K_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 = \frac{1}{2}m\left((3.50)^2 + (2.00)^2\right) = 8.1m \ .$$

Kinetic energy is not conserved.

50. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = -90°$ for the particle B which is assumed to scatter "downward" and $\phi > 0$ for particle A which presumably goes into the first quadrant. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} m_B v_B &= m_B v'_B \cos\theta + m_A v'_A \cos\phi \\ 0 &= m_B v'_B \sin\theta + m_A v'_A \sin\phi \end{aligned}$$

(a) Setting $v_B = v$ and $v'_B = v/2$, the y-momentum equation yields

$$m_A v'_A \sin\phi = m_B \, \frac{v}{2}$$

and the x-momentum equation yields

$$m_A v'_A \cos\phi = m_B \, v \ .$$

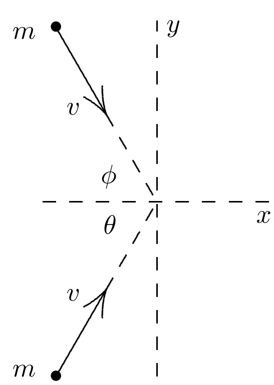
Dividing these two equations, we find $\tan\phi = \frac{1}{2}$ which yields $\phi = 27°$. If we choose to measure this from the final direction of motion for B, then this becomes $90° + 27° = 117°$.

(b) We can *formally* solve for v'_A (using the y-momentum equation and the fact that $\sin\phi = 1/\sqrt{5}$)

$$v'_A = \frac{\sqrt{5}}{2} \frac{m_B}{m_A} \, v$$

but lacking numerical values for v and the mass ratio, we cannot fully determine the final speed of A. Note: substituting $\cos\phi = 2/\sqrt{5}$, into the x-momentum equation leads to exactly this same relation (that is, no new information is obtained which might help us determine an answer).

51. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the x axis, as shown in the diagram below. Both have the same mass m and the same initial speed v.



We suppose that after the collision the combined object moves in the positive x direction with speed V. Since the y component of the total momentum of the two-object system is conserved, $mv\sin\theta - mv\sin\phi = 0$. This means $\phi = \theta$. Since the x component is conserved, $2mv\cos\theta = 2mV$. We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is $120°$.

52. We orient our $+x$ axis along the initial direction of motion, and specify angles in the "standard" way – so $\theta = +60°$ for one ball (1) which is assumed to go into the first quadrant with speed $v'_1 = 1.1$ m/s, and $\phi < 0$ for the other ball (2) which presumably goes into the fourth quadrant. The mass of each ball is m, and the initial speed of one of the balls is $v_0 = 2.2$ m/s. We apply the conservation of linear momentum to the x and y axes respectively.

$$\begin{aligned} mv_0 &= mv'_1 \cos\theta + mv'_2 \cos\phi \\ 0 &= mv'_1 \sin\theta + mv'_2 \sin\phi \end{aligned}$$

The mass m cancels out of these equations, and we are left with two unknowns and two equations, which is sufficient to solve.

(a) With SI units understood, the y-momentum equation can be rewritten as

$$v_2' \sin\phi = -v_1' \sin 60° = -0.95$$

and the x-momentum equation yields

$$v_2' \cos\phi = v_0 - v_1' \cos 60° = 1.65$$

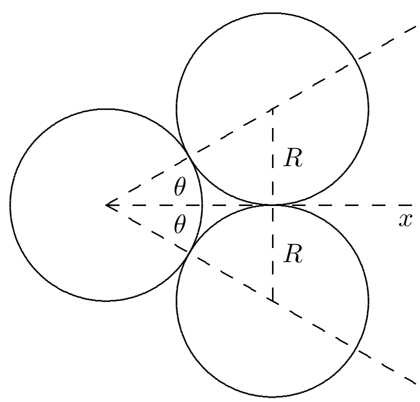
Dividing these two equations, we find $\tan\phi = -0.577$ which yields $\phi = -30°$. If we choose to measure this as a positive-valued angle (as the textbook does in §10-6), then this becomes 30°. We plug $\phi = -30°$ into either equation and find $v_2' \approx 1.9$ m/s.

(b) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_0^2 \quad \text{and} \quad \frac{2K_f}{m} = {v_1'}^2 + {v_2'}^2$$

and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed "exactly" as $v_2' = \frac{1}{2}v_0\sqrt{3}$ (and of course $v_1' = \frac{1}{2}v_0$ "exactly" – which makes it clear that these two kinetic energy expressions are indeed equal).

53. The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two. It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dotted lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m.



Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta$$

and since the total kinetic energy is conserved,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mV^2 + 2\left(\frac{1}{2}mv^2\right) .$$

We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum equation gives $V = v_0 - 2v\cos\theta$, so $V^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta$ and the energy equation becomes $v_0^2 = v_0^2 - 4v_0 v\cos\theta + 4v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2v_0\cos\theta}{1 + 2\cos^2\theta} = \frac{2(10\,\text{m/s})\cos 30°}{1 + 2\cos^2 30°} = 6.93 \text{ m/s} .$$

(a) The discussion and computation above determines the final velocity of ball 2 (as labeled in Fig. 10-41) to be 6.9 m/s at 30° counterclockwise from the $+x$ axis.

(b) Similarly, the final velocity of ball 3 is 6.9 m/s at 30° clockwise from the $+x$ axis.

(c) Now we use the momentum equation to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10\,\text{m/s} - 2(6.93\,\text{m/s})\cos 30^\circ = -2.0\,\text{m/s}\ .$$

The minus sign indicates that it bounces back in the $-x$ direction.

54. The problem involves the completely inelastic collision of the two children followed by their completely inelastic collision with the (already moving) man. Speeds are given but no angles, so we are free to orient our $-x$ axis along the direction of motion of the man before his collision with the children (so his angle is 180°). The magnitude of the man's momentum before that collision is $(75\,\text{kg})(2.0\,\text{m/s}) = 150$ kg·m/s. Thus, with SI units understood, the second collision is described by momentum conservation:

$$\vec{p} + (150\ \angle\ 180^\circ) = 0$$

which yields the momentum of the stuck-together children $\vec{p} = (150\ \angle\ 0^\circ)$ in magnitude-angle notation. We now describe the first collision (of the two children) using momentum conservation:

$$\vec{p}_1 + \vec{p}_2 = (150\ \angle\ 0^\circ) \quad \text{or} \quad 150\,\hat{\imath}$$

where the unit-vector notation has also been used, in case the magnitude-angle notation is less familiar. Now, since $m_1 = m_2 = 30$ kg and $|\vec{p}_1| = |\vec{p}_2| = 120$ kg·m/s, we see that the y components of the children's initial velocities must be equal and opposite. Therefore, if child 1 has an initial velocity angle θ then child 2 has an initial velocity angle $-\theta$. The previous equation becomes

$$120\cos(\theta) + 120\cos(-\theta) = 150$$

which has the solution $\theta = 51^\circ$. The angle between the children (initially) is therefore $2\theta \approx 103^\circ$.

55. Let $m_n = 1.0$ u be the mass of the neutron and $m_d = 2.0$ u be the mass of the deuteron. In our manipulations we treat these masses as "exact", so, for instance, we write $m_n/m_d = \frac{1}{2}$ in our simplifying steps. We assume the neutron enters with a velocity $\vec{v}_\text{o}$ pointing in the $+x$ direction and leaves along the positive y axis with speed v_n. The deuteron goes into the fourth quadrant with velocity components $v_{dx} > 0$ and $v_{dy} < 0$. Conservation of the x component of momentum leads to

$$m_n v_\text{o} = m_d v_{dx} \quad \Longrightarrow \quad v_{dx} = \frac{1}{2} v_\text{o}$$

and conservation of the y component leads to

$$0 = m_n v_n + m_d v_{fy} \quad \Longrightarrow \quad v_{dy} = -\frac{1}{2} v_\text{n}\ .$$

Also, the collision is elastic, so kinetic energy "conservation" leads to

$$\frac{1}{2} m_n v_\text{o}^2 = \frac{1}{2} m_n v_n^2 + \frac{1}{2} m_d v_d^2$$

which we simplify by multiplying through with $2/m_n$ and using $v_d^2 = v_{dx}^2 + v_{dy}^2$

$$v_\text{o}^2 = v_n^2 + \frac{m_d}{m_n}\left(v_{dx}^2 + v_{dy}^2\right)\ .$$

Now we substitute in the relations found from the momentum conditions:

$$v_\text{o}^2 = v_n^2 + 2\left(\frac{v_\text{o}^2}{4} + \frac{v_n^2}{4}\right) \quad \Longrightarrow \quad v_n = v_\text{o}\sqrt{\frac{1}{3}}\ .$$

Finally, we set up a ratio expressing the (relative) loss of kinetic energy (by the neutron).

$$\frac{K_\text{o} - K_n}{K_\text{o}} = 1 - \frac{v_n^2}{v_\text{o}^2} = 1 - \frac{1}{3} = \frac{2}{3}\ .$$

56. (a) Choosing upward as the positive direction, the momentum change of the foot is

$$\Delta\vec{p} = 0 - m_{\text{foot}}\,\vec{v}_i = -(0.003\,\text{kg})(-1.5\text{m/s})$$

which yields an impulse of 4.50×10^{-3} N·s.

(b) Using Eq. 10-8 and now treating *downward* as the positive direction, we have

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = m_{\text{lizard}}\,g\,\Delta t = (0.090)(9.8)(0.6)$$

which yields $\vec{J} = 0.529$ N·s.

(c) Considering the large difference between the answers for part (a) and part (b), we see that the slap cannot account for the support; we infer, then, that the push does the job.

57. From mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward) we obtain

$$v = \sqrt{2gh} = \sqrt{2(9.8)(1.5)} = 5.4 \text{ m/s}$$

for the speed just as the body makes contact with the ground.

(a) During the compression of the body, the center of mass must decelerate over a distance $d = 0.30$ m. Choosing $+y$ downward, the deceleration a is found using Eq. 2-16

$$0 = v^2 + 2ad \quad\Longrightarrow\quad a = -\frac{v^2}{2d} = -\frac{5.4^2}{2(0.30)}$$

which yields $a = -49$ m/s^2. Thus, the magnitude of the net (vertical) force is $m|a| = 49m$ in SI units, which (since $49 = 5(9.8)$) can be expressed as $5mg$.

(b) During the deceleration process, the forces on the dinosaur are (in the vertical direction) $\vec{N}$ and $m\vec{g}$. If we choose $+y$ upward, and use the final result from part (a), we therefore have $N - mg = 5mg$, or $N = 6mg$. In the horizontal direction, there is also a deceleration (from $v_o = 19$ m/s to zero), in this case due to kinetic friction $f_k = \mu_k N = \mu_k(6mg)$. Thus, the net force exerted by the ground on the dinosaur is

$$F_{\text{ground}} = \sqrt{f_k^2 + N^2} \approx 7mg\ .$$

(c) We can applying Newton's second law in the horizontal direction (with the sliding distance denoted as Δx) and then use Eq. 2-16, or we can apply the general notions of energy conservation. The latter approach is shown:

$$\frac{1}{2}mv_o^2 = \mu_k(6mg)\Delta x \quad\Longrightarrow\quad \Delta x = \frac{19^2}{2(6)(0.6)(9.8)} \approx 5 \text{ m}\ .$$

58. (a) As explained in the problem, the height of the n^{th} domino is $h_n = 1.5^{n-1}$ in centimeters. Therefore, $h_{32} = 1.5^{31} = 2.9 \times 10^5$ cm $= 2.9$ km (!).

(b) When the center of the domino is directly over the corner, the height of the center-point is

$$h_c = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = \frac{d}{2}\sqrt{101}$$

where $h = 10d$ has been used in that last step. While the domino is in its usual resting position, the height of that point is only $h_o = h/2$ which can be written as $5d$. Since the answer is requested to be in terms of U_1 then

$$U_1 = mg(5d) \quad\Longrightarrow\quad d = \frac{U_1}{5mg}\ .$$

Therefore, the energy needed to push over the domino is

$$\Delta U = mgh_c - U_1 = mg\left(\frac{d}{2}\sqrt{101}\right) - U_1 = \frac{U_1}{10}\sqrt{101} - U_1$$

which yields approximately $0.005U_1$; the problem refers to this as $\Delta E_{1,\text{in}}$.

(c) The "loss" of potential energy equal to

$$mgh_c - mg\left(\frac{h}{2}\sin\theta\right)$$

becomes the kinetic energy (denoted $\Delta E_{1,\text{out}}$ in the problem). Therefore, we obtain

$$\Delta E_{1,\text{out}} = mg\left(\frac{d}{2}\sqrt{101}\right) - mg\left(\frac{10d}{2}\sin\theta\right)$$

which (using $\theta = 45°$) simplifies to $1.49mgd$. Since $d = U_1/5mg$ this becomes roughly $\Delta E_{1,\text{out}} = 0.30U_1$.

(d) We see from part (b) that $\Delta E_{n,\text{in}}$ is directly proportional to $m_n d_n$ and consequently (since the density is assumed the same for all of them and the volume of a domino is hdw where w is the width) is proportional to $w_n h_n d_n^2$. The width also scales like the other quantities, so $\Delta E_{n,\text{in}}$ is proportional to $1.5^{4(n-1)}$. Therefore, $\Delta E_{2,\text{in}} = 1.5^4 \Delta E_{1,\text{in}}$ which implies $\Delta E_{2,\text{in}} = 0.025U_1$.

(e) Therefore,

$$\frac{\Delta E_{1,\text{out}}}{\Delta E_{2,\text{in}}} = \frac{0.30U_1}{0.025U_1} = 12 .$$

59. (a) We choose $+x$ to be away from the armor (pointing back towards the gun). The velocity is there negative-valued and the acceleration is positive-valued. Using Eq. 2-11,

$$0 = \vec{v}_0 + \vec{a}t \implies \vec{a} = -\frac{\vec{v}_0}{t} = -\frac{-300}{40\times 10^{-6}} = 7.5\times 10^6 \text{ m/s}^2 .$$

(b) Since the final momentum is zero, the momentum change is

$$\Delta\vec{p} = 0 - m\vec{v}_0 = -(0.0102\,\text{kg})(-300\,\text{m/s}) = 3.1 \text{ kg}\cdot\text{m/s} .$$

(c) We compute $K_f - K_i = 0 - \frac{1}{2}mv_0^2$ and obtain $-\frac{1}{2}(0.0102)(300)^2 \approx -460$ J.

(d) If we assume uniform deceleration, Eq. 2-17 gives

$$\Delta x = \frac{1}{2}(\vec{v}_0 + 0)\,t = \frac{1}{2}(-300)\left(40\times 10^{-6}\right)$$

so that the distance is $|\Delta x| = 0.0060$ m.

(e) By the impulse-momentum theorem, the impulse of the armor on the bullet is $\vec{J} = \Delta\vec{p} = 3.1$ N·s. By Newton's third law, the impulse of the bullet on the armor must have that same magnitude.

(f) Using Eq. 10-8, we find the magnitude of the (average) force exerted by the bullet on the armor:

$$F_{\text{avg}} = \frac{J}{t} = \frac{3.1}{40\times 10^{-6}} = 7.7\times 10^4 \text{ N} .$$

(g) From Newton's second law, we find $a_p = F_{\text{avg}}/M$ (where $M = 65$ kg) to be 1.2×10^3 m/s^2.

(h) Momentum conservation leads to $V = mv_0/M = 0.047$ m/s. (This result can be gotten a number of ways, given the information available at this point in the problem.)

(i) Shortening the distance means decreasing the stopping time (Eq. 2-17 shows this clearly) which (recalling our calculation in part (a)) means the magnitude of the bullet's deceleration increases. It does not change the answer to part (b) (for the change in momentum), nor does it affect part (c) (the change in kinetic energy). Since $\vec{J}$ is determined by $\Delta\vec{p}$, part (e) is unchanged. But with t smaller, $J/t = F_{\text{avg}}$ is larger, as is a_p. Finally, v_p is the same as before since momentum conservation describes the input/output of the collision and not the inner dynamics of it.

60. From mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward) we obtain

$$v = \sqrt{2gh} = \sqrt{2(9.8)(6.0)} = 10.8 \text{ m/s}$$

for the speed just as the $m = 3000$-kg block makes contact with the pile. At the moment of "joining", they are a system of mass $M = 3500$ kg and speed V. With downward positive, momentum conservation leads to

$$mv = MV \implies V = \frac{(3000)(10.8)}{3500} = 9.3 \text{ m/s} .$$

Now this block-pile "object" must be rapidly decelerated over the small distance $d = 0.030$ m. Using Eq. 2-16 and choosing $+y$ downward, we have

$$0 = V^2 + 2ad \implies a = -\frac{9.3^2}{2(0.030)} = -1440$$

in SI units (m/s^2). Thus, the net force during the decelerating process has magnitude $M\,|a| = 5.0\times10^6$ N.

61. Using Eq. 10-31 with $m_1 = 3.0$ kg, $v_{1i} = 8.0$ m/s and $v_{2f} = 6.0$ m/s, then

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \implies m_2 = m_1 \left(\frac{2v_{1i}}{v_{2f}} - 1 \right)$$

leads to $m_2 = M = 5.0$ kg.

62. In the momentum relationships, we could as easily work with weights as with masses, but because part (b) of this problem asks for kinetic energy – we will find the masses at the outset: $m_1 = 280 \times 10^3/9.8 = 2.86 \times 10^4$ kg and $m_2 = 210 \times 10^3/9.8 = 2.14 \times 10^4$ kg. Both cars are moving in the $+x$ direction: $v_{1i} = 1.52$ m/s and $v_{2i} = 0.914$ m/s.

   (a) If the collision is completely elastic, momentum conservation leads to a final speed of

$$V = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = 1.26 \text{ m/s} .$$

   (b) We compute the total initial kinetic energy and subtract from it the final kinetic energy.

$$K_i - K_f = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 - \frac{1}{2}(m_1 + m_2) V^2 = 2.25 \times 10^3 \text{ J} .$$

   (c) and (d) Using Eq. 10-38 and Eq. 10-39, we find

$$\begin{aligned} v_{2\,f} &= \frac{2m_1}{m_1 + m_2} v_{1\,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2\,i} = 1.61 \text{ m/s} \quad \text{and} \\ v_{1\,f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1\,i} + \frac{2m_2}{m_1 + m_2} v_{2\,i} = 1.00 \text{ m/s} . \end{aligned}$$

63. We choose coordinates with $+x$ East and $+y$ North, with the standard conventions for measuring the angles. With SI units understood, we write the initial magnitude of the man's momentum as $(60)(6.0) = 360$ and the final momentum of the two of them together as $(98)(3.0) = 294$. Using magnitude-angle notation (quickly implemented using a vector capable calculator in polar mode), momentum conservation becomes

$$\begin{aligned} \vec{p}_{\text{man}} + \vec{p}_{\text{child}} &= \vec{p}_{\text{together}} \\ (360 \angle 90^\circ) + \vec{p} &= (294 \angle 35^\circ) \end{aligned}$$

Therefore, the momentum of the 38 kg child before the collision is $\vec{p} = (308 \angle -38^\circ)$. Thus, the child's velocity has magnitude equal to $308/38 = 8.1$ m/s and direction of 38° south of east.

64. (a) We choose a coordinate system with $+x$ downriver and $+y$ in the initial direction of motion of the second barge. The velocities in component forms are $\vec{v}_{1i} = (6.2\,\text{m/s})\hat{\imath}$ and $\vec{v}_{2i} = (4.3\,\text{m/s})\hat{\jmath}$ before collision. After the collision, barge 2 has velocity

$$\vec{v}_{2f} = (5.1\,\text{m/s})\left((\sin 18^\circ)\hat{\imath} + (\cos 18^\circ)\hat{\jmath}\right) .$$

Writing $\vec{v}_{1f} = v_{1f}\left((\cos\theta)\hat{\imath} + (\sin\theta)\hat{\jmath}\right)$, with θ we express the component form of the conservation of momentum:

$$\begin{aligned} m_1 v_{1i} &= m_1 v_{1f}\cos\theta + m_2 v_{2f}\sin 18^\circ \\ m_2 v_{2i} &= m_1 v_{1f}\sin\theta + m_2 v_{2f}\cos 18^\circ . \end{aligned}$$

Substituting $v_{1i} = 6.2\,\text{m/s}$, $v_{2i} = 4.3\,\text{m/s}$, and $v_{2f} = 5.1\,\text{m/s}$, we find: $v_{1f} = 3.4\,\text{m/s}$, $\theta = 17^\circ$ (from the point of view of someone on that barge, this deflection is toward the left).

(b) The loss of kinetic energy is

$$K_i - K_f = \left(\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2\right) - \left(\frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2\right)$$

which yields 9.5×10^5 J.

65. Let the mass of each ball be m. Conservation of (kinetic) energy in elastic collisions requires that $K_i = K_f$ which leads to

$$\frac{1}{2}mV^2 = \frac{1}{2}(16m)v^2$$

which yields $v = V/4$.

66. The speed of each particle of mass m upon impact with the scale is found from mechanical energy conservation (or simply using Eq. 2-16 with $\vec{a} = g$ downward): $v = \sqrt{2gh}$, where $h = 3.5\,\text{m}$. With $+y$ upward, the change in momentum for the particle is therefore

$$\Delta\vec{p} = m\Delta\vec{v} = 2mv = 2m\sqrt{2gh} .$$

During a time interval Δt, the number of collisions is $N = R\Delta t$ where $R = 42\,\text{s}^{-1}$. Thus, using the impulse-momentum theorem and Eq. 10-8, the average force is

$$\begin{aligned} \vec{F}_{\text{avg}} &= \frac{N\Delta\vec{p}}{\Delta t} \\ &= 2mR\sqrt{2gh} \\ &= 2(0.110)(42)\sqrt{2(9.8)(3.5)} \\ &= 77\ \text{N} \end{aligned}$$

which corresponds to a mass reading of $77/9.8 = 7.8\,\text{kg}$.

67. The momentum before the collision (with $+x$ rightward) is

$$(6.0\ \text{kg})(8.0\ \text{m/s}) + (4.0\ \text{kg})(2.0\ \text{m/s}) = 56\ \text{kg}\cdot\text{m/s} .$$

(a) The total momentum at this instant is $(6.0\,\text{kg})(6.4\,\text{m/s}) + (4.0\,\text{kg})\vec{v}$. Since this must equal the initial total momentum (56, using SI units), then we find $\vec{v} = 4.4$ m/s.

(b) The initial kinetic energy was

$$\frac{1}{2}(6.0\ \text{kg})(8.0\ \text{m/s})^2 + \frac{1}{2}(4.0\ \text{kg})(2.0\ \text{m/s})^2 = 200\ \text{J} .$$

The kinetic energy at the instant described in part (a) is

$$\frac{1}{2}(6.0\text{ kg})(6.4\text{ m/s})^2 + \frac{1}{2}(4.0\text{ kg})(4.4\text{ m/s})^2 = 162\text{ J} .$$

The "missing" 38 J is not dissipated since there is no friction; it is the energy stored in the spring at this instant when it is compressed. Thus, $U_e = 38$ J.

68. This is a completely inelastic collision, followed by projectile motion. In the collision, we use momentum conservation.

$$\begin{aligned} \vec{p}_{\text{shoes}} &= \vec{p}_{\text{together}} \\ (3.2\,\text{kg})(3.0\,\text{m/s}) &= (5.2\,\text{kg})\vec{v} \end{aligned}$$

Therefore, $\vec{v} = 1.8$ m/s toward the right as the combined system is projected from the edge of the table. Next, we can use the projectile motion material from Ch. 4 or the energy techniques of Ch. 8; we choose the latter.

$$\begin{aligned} K_{\text{edge}} + U_{\text{edge}} &= K_{\text{floor}} + U_{\text{floor}} \\ \frac{1}{2}(5.2\,\text{kg})(1.8\,\text{m/s})^2 + (5.2\,\text{kg})(9.8\text{ m/s}^2)(0.40\,\text{m}) &= K_{\text{floor}} + 0 \end{aligned}$$

Therefore, the kinetic energy of the system right before hitting the floor is $K_{\text{floor}} = 29$ J.

69. We use the impulse-momentum theorem $\vec{J} = \Delta\vec{p}$ where $\vec{J} = \int \vec{F}\,dt$. Integrating the given expression for force from the moment it starts from rest up to a variable upper limit t, we have $\vec{J} = \left(16t - \frac{1}{3}t^3\right)\hat{\imath}$ with SI units understood.

(a) Since $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ with $m = 1.6$, we obtain $\vec{v} = 24\,\hat{\imath}$ in meters-per-second, for $t = 3.0$ s.

(b) Setting $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ equal to zero leads to $t = 6.9$ s as the positive root.

(c) We can work through the $\frac{d\vec{v}}{dt} = 0$ condition using our $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ relation, or more simply observe, from the outset, that this is equivalent to finding when the acceleration, hence the force, is zero. We obtain $t = 4.0$ s as the positive root, which we plug into the $\left(16t - \frac{1}{3}t^3\right)\hat{\imath} = m\vec{v}$ relation and find $\vec{v}_{\max} = 27\,\hat{\imath}$ m/s.

70. (a) We use coordinates with $+x$ rightward and $+y$ upward, with the usual conventions for measuring the angles (so that the final angle is written $90° - 40° = 50°$). With SI units understood, the magnitude of the diver's momentum before contact is $(60.0)(3.00) = 180$ and after contact is $(60.0)(5.00) = 300$. Using magnitude-angle notation (quickly implemented using a vector capable calculator in polar mode), the change in momentum is

$$(300 \angle 50°) - (180 \angle -90°) = (453 \angle 65°) .$$

This equals the *total* impulse delivered to the diver (by the board and by gravity). If F_{net} denotes the magnitude of the average *net* force exerted on the diver, then we have

$$F_{\text{net}}\Delta t = 453 \implies F_{\text{net}} = \frac{453}{1.2} = 377\text{ N} .$$

(b) Since $\vec{F}_{\text{net}} = (377 \angle 65°)$ and the weight of the diver is $(588 \angle -90)$, we obtain

$$(377 \angle 65°) - (588 \angle -90°) = (943 \angle 80°) .$$

Therefore, the magnitude of the average force exerted by the board on the diver is 943 N.

71. The magnitude of the impulse exerted by the gunner on the gun per minute is $J = F_{\text{avg}}\Delta t$, where $F_{\text{avg}} = 180\,\text{N}$ and $\Delta t = 60\,\text{s}$. The impulse exerted on the gun by each bullet of mass m and speed v is $J' = mv$. The maximum number of bullets N that he could fire per minute satisfies $J = NJ'$. Thus

$$N = \frac{J}{J'} = \frac{F_{\text{avg}}\Delta t}{mv} = \frac{(180)(60)}{(50\times10^{-3})(1000)} = 216 \ .$$

72. (a) The magnitude of the force is

$$F = \frac{\Delta p}{\Delta t} = \frac{9.0\times10^{3}\,\text{kg}\cdot\text{m/s}}{12\,\text{s}} = 750\,\text{N} \ .$$

(b) Assuming this is one-dimensional motion (so that any acceleration implies a change in the magnitude of the velocity), we find the speed increase to be

$$\Delta v = \frac{\Delta p}{m} = \frac{9.0\times10^{3}\,\text{kg}\cdot\text{m/s}}{1500\,\text{kg}} = 6.0\,\text{m/s} \ .$$

73. (a) The momentum conservation equation (for this completely inelastic collision) $m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{V}$ can be written in terms of weights by multiplying through by g:

$$w_A\vec{v}_A + w_B\vec{v}_B = (w_A + w_B)\vec{V} \ .$$

Our $\hat{\text{i}}$ direction is West and $\hat{\text{j}}$ is South, so we have (with weights in kN and speeds in km/h)

$$\begin{aligned}\vec{V} &= \frac{(12.0)\,(64.4\,\hat{\text{i}}) + (16.0)\left(96.6\,\hat{\text{j}}\right)}{12.0 + 16.0} \\ &= 27.6\,\hat{\text{i}} + 55.2\,\hat{\text{j}}\end{aligned}$$

which implies that the final speed is 61.7 km/h.

(b) And the angle for the final velocity is $\tan^{-1}(55.2/27.6) = 63.4^\circ$ South of West.

74. We choose $\hat{\text{i}}$ East and $\hat{\text{j}}$ North, and use SI units (kg for mass and m/s for speed). The initially moving tin cookie has mass $m_1 = 2.0$ and velocity $\vec{v}_{\text{o}} = 8.0\,\hat{\text{i}}$, and the initially stationary cookie tin has mass $m_2 = 4.0$.

(a) Momentum conservation leads to

$$\begin{aligned}m_1\vec{v}_{\text{o}} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ 16\,\hat{\text{i}} &= 8\cos(37^\circ)\,\hat{\text{i}} + 8\sin(37^\circ)\,\hat{\text{j}} + (4.0)\vec{v}_2\end{aligned}$$

which leads to

$$\vec{v}_2 = 2.4\,\hat{\text{i}} - 1.2\,\hat{\text{j}} \implies \vec{v}_2 = (2.7\ \angle\ 27^\circ)$$

where magnitude-angle notation is used. Thus, the speed of the cookie tin is 2.7 m/s.

(b) And its angle is $\tan^{-1}(-1.2/2.4) = -27^\circ$ which can be expressed as 27° south of east.

75. We choose $\hat{\text{i}}$ East and $\hat{\text{j}}$ North, and use SI units. The ball initially moving eastward has mass $m_1 = 5.0$ kg and initial velocity $\vec{v}_{1i} = 4.0\,\hat{\text{i}}$ m/s, and the ball initially moving westward has mass $m_2 = 4.0$ kg and velocity $\vec{v}_{2i} = -3.0\,\hat{\text{i}}$ m/s. The final velocity of m_1 is $\vec{v}_{1f} = -1.2\,\hat{\text{j}}$.

(a) Momentum conservation leads to

$$\begin{aligned}m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ 20\,\hat{\text{i}} - 12\,\hat{\text{i}} &= -6\,\hat{\text{j}} + 4\vec{v}_2\end{aligned}$$

which leads to

$$\vec{v}_2 = 2.0\,\hat{\text{i}} + 1.5\,\hat{\text{j}} \implies \vec{v}_2 = (2.5\ \angle\ 37^\circ)$$

where magnitude-angle notation is used. Thus, the speed of the 4.0 kg ball just after the collision is 2.5 m/s.

(b) We compute the decrease in total kinetic energy:

$$K_i - K_f = \frac{1}{2}(5)(4)^2 + \frac{1}{2}(4)(3)^2 - \frac{1}{2}(5)(1.2)^2 - \frac{1}{2}(4)(2.5)^2$$

which gives the result 42 J.

76. Using mechanical energy conservation, we find the speed v of a pendulum at the bottom of its swing is related to the height h it was released from (or that it swings up to) by $v^2 = 2gh$. Thus, the conservation of momentum at the instant they collide can be expressed as

$$m_1\sqrt{2gd} = (m_1 + m_2)\sqrt{2gh_f}\ .$$

Therefore, the "final" height of the system (which it swings to shortly after the collision) is

$$h_f = \left(\frac{m_1}{m_1 + m_2}\right)^2 d\ .$$

77. If we neglect the time required for the spring to decelerate the leftward moving glider m_2 and re-accelerate it (rightward), then we are effectively assuming that glider bounces elastically off the wall (with the spring playing no dynamic role). Thus, we assume the time t required for m_2 to travel distance $d + x$ (to the wall and then rightward to position x, assuming the origin is at the wall) is simply $t = (d + x)/v$ where $v = |v_{2f}|$ is its speed resulting from the first elastic collision. This velocity is found from Eq. 10-31:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}\,v_{1i} = \frac{2(590)}{940}(-75)$$

which yields -94 cm/s. Thus, with $d = 53$ cm, we have the relation $t = (53 + x)/94$ with x in cm and t in s. During that time, glider m_1 has a displacement $\Delta x = x - d$ due its velocity v_{1f} where

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}\,v_{1i} = \frac{240}{940}(-75)$$

which yields $v_{1f} = -19$ cm/s. This provides another relation between t and x: $t = (x - d)/v_{1f} = (53 - x)/19$. Equating these to relations, we obtain

$$\frac{53 + x}{94} = \frac{53 - x}{19} \quad \Longrightarrow \quad x = 35 \text{ cm}\ .$$

78. Eq. 10-31, for situations where $m_1 \gg m_2$, reduces simply to $v_{2f} \approx 2v_{1i}$. Thus, the speed of the fly after the collision is $2(2.1) = 4.2$ m/s.

79. (a) We find the velocity $\vec{v}_{1f}$ of the 1200 kg car after the collision (taking the direction of motion as positive) using momentum conservation (with mass in kg and speed in km/h).

$$\begin{aligned} m_1\vec{v}_{1i} + m_2\vec{v}_{2i} &= m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \\ (1200)(70) + (900)(60) &= (1200)\vec{v}_{1f} + (900)(70) \end{aligned}$$

This gives the result $\vec{v}_{1f} = 62.5$ km/h.

(b) We compute the reduction of total kinetic energy in the collision:

$$Q = K_i - K_f = \frac{1}{2}(1200)(70)^2 + \frac{1}{2}(900)(60)^2 - \frac{1}{2}(1200)(62.5)^2 - \frac{1}{2}(900)(70)^2$$

which gives the result 11250 in mixed units (kg·km^2/h^2). We set up the requested ratio (where $v_o = 5$ km/h):

$$\frac{Q}{\frac{1}{2}m_1 v_o^2} = \frac{11250}{15000} = \frac{3}{4}\ .$$

80. We refer to the discussion in the textbook (see Sample Problem 10-2, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units).

    (a) The bullet's initial kinetic energy is

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{m+M}{m}\sqrt{2gh}\right)^2 = \frac{m+M}{m}U_f$$

    where $U_f = (m+M)gh$ is the system's final potential energy (equal to its total mechanical energy since its speed is zero at height h). Thus,

$$\frac{U_f}{\frac{1}{2}mv^2} = \frac{m}{m+M} = \frac{0.008}{7.008} = 0.00114 \ .$$

    (b) The fraction $m/(m+M)$ shown in part (a) has no v-dependence. The answer remains the same.

    (c) As we found in part (a), the fraction is $m/(m+M)$. The numerical value of h given in the problem statement has not been used in this solution.

81. (a) Since $\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$ (Eq. 9-23), we read from value of F_x (see graph) that the rate of change of momentum is 4.0 kg·m/s^2 at $t = 3.0$ s.

    (b) The impulse, which causes the change in momentum, is equivalent to the area under the curve in this graph (see Eq. 10-3). We break the area into that of a triangle $\frac{1}{2}(2.0\,\text{s})(4.0\,\text{N})$ plus that of a rectangle $(1.0\,\text{s})(4.0\,\text{N})$, which yields a total of 8.0 N·s. Since the car started from rest, its momentum at $t = 3.0$ s must therefore be 8.0 kg·m/s.

82. We use $J = \int F dt = m\Delta v = mv_f$. The integral $\int F\,dt$ is estimated from the area under the curve in Fig. 10-61 as approximately 4 N·s. (If one doesn't want to "count squares" one can assume the curve to be a parabola, in which case $F = \xi(t-3.25)(t-0.35)$ (with t in milliseconds) will fit it once the parameter ξ is adjusted so that $F = 2200$ N when t is midway between 0.35 ms and 3.25 ms. Then the integral can be done explicitly.) Thus, the final speed of the ball is

$$v_f = \frac{J}{m} = \frac{4\,\text{N}\cdot\text{s}}{0.5\,\text{kg}} = 8 \text{ m/s} \ .$$

83. (a) The impulse on the ball is

$$\vec{J} = \Delta\vec{p} = m\vec{v} - 0 = (46\times10^{-3}\,\text{kg})(50\,\text{m/s})\hat{\imath} = (2.3\,\text{N}\cdot\text{s})\hat{\imath}$$

    where we choose $\hat{\imath}$ to be in the direction of the velocity $\vec{v}$ of the ball as it leaves the club (at 30° above horizontal – so it is like the x axis of an inclined plane problem).

    (b) The impulse on the club is, by Newton's third law, $\vec{J}\,' = -\vec{J} = -(2.3\,\text{N}\cdot\text{s})\hat{\imath}$. We note that it is directed opposite to the direction of motion.

    (c) Using Eq. 10-8, the average force on the ball is

$$\vec{F}_{\text{avg}} = \frac{\vec{J}}{\Delta t} = \frac{(2.3)\hat{\imath}}{1.7\times10^{-3}} = 1400\,\hat{\imath} \text{ N} \ .$$

    (d) The work done on the ball is

$$W = \Delta K = \frac{1}{2}mv^2 = \frac{1}{2}(46\times10^{-3})(50)^2 = 58 \text{ J} \ .$$

84. We first note that when the the velocity of a projectile is simply reversed as a result of collision, its change in momentum (in magnitude) is $2mv$ (where v is its speed). If this collision takes time Δt, then the average force involved is (using Eq. 10-8) $F_{\text{avg}} = 2mv/\Delta t$. To relate this observation to the present situation, we replace m with Δm (representing just that amount of the water stream which is in contact with the blade during Δt, and since the impinging flow rate dm/dt is constant (and no water is lost or "splattered away" in the process) then we conclude $dm/dt = \Delta m/\Delta t$. Therefore,

$$F_{\text{avg}} = 2v\,\frac{dm}{dt} \ .$$

85. One could reason as in §9-7 (with the thrust concept) or proceed with Eq. 10-8. Choosing the latter approach, we note that (with the final momentum being zero) the average force is (in magnitude)

$$F_{\text{avg}} = v\,\frac{\Delta m}{\Delta t}$$

where Δm is the portion of the water that is decelerated (by the wall) from speed $v = 500$ cm/s to zero during time Δt. If the impinging mass flow rate dm/dt is constant, then we conclude $dm/dt = \Delta m/\Delta t$. Thus, $F_{\text{avg}} = v\,dm/dt$. We are given the volume flow rate $dV/dt = 300\ \text{cm}^3/\text{s}$, and we use the concept of density to relate mass and volume: $m = \rho V$ where $\rho = 1.0\ \text{g/cm}^3$ for water (most students have seen density in previous courses). Thus,

$$F_{\text{avg}} = v\,\frac{dm}{dt} = \rho\,v\,\frac{dV}{dt} = (1.0)(500)(300)$$

which yields $F_{\text{avg}} = 1.5 \times 10^5\ \text{g}\cdot\text{cm/s}^2$ which we convert to SI, giving the result $F_{\text{avg}} = 1.5$ N.

86. Although we do not present problems and solutions here, we share a few thoughts on the matter.

(a) This might be more like part (b) of problem 80, in which energy is "liberated" in the collision, but this depends on what particular sort of pinball collision one has in mind.

(b) This is a good example of an inelastic (but not completely so) collision and might be similar to part (a) of problem 80.

(c) This might be similar to problem 85, finding the average force on the car in the hailstorm. Instead of having the hail be halted completely by the collision (as is done with the water in problem 85) there should be some small rebound speed.

(d) An interesting comparison can be made here between the impact of fist with face with glove, and without the glove. The increase in contact time with the glove certainly decreases the force of impact.

(e) If baseball is chosen as one's example, it might be of interest to refer to the article by Howard Brody in the August 1990 issue of the *American Journal of Physics*, where he considers that the bat may be viewed as a relatively free body in the batting process.

87. (First problem in **Cluster 1**)
Instead of using V for final speed in completely inelastic collisions (as is used in Eq. 10-18), we use $v_{1f} = v_{2f}$, since that facilitates comparison of the results of parts (a) and (b). When we make comparisons, we assume $v_{1i} > 0$.

(a) When they stick together, we have

$$v_{1f} = v_{2f} = \frac{m_1}{m_1 + m_2}\,v_{1i} \ .$$

(b) Eq. 10-30 and Eq. 10-31 provide the elastic collision results:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$
$$v_{1f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

from which it is evident that $v_{1f\,\text{elastic}} < v_{1f\,\text{inelastic}}$ and $v_{2f\,\text{elastic}} > v_{2f\,\text{inelastic}}$.

88. (Second problem in **Cluster 1**)
We note that the problem has implicitly chosen the initial direction of motion (of m_1) as the positive direction. The questions to find "greatest" and "least" values are understood in terms of that axis choice (*greatest* = largest positive value, and *least* = the negative value of greatest magnitude or the smallest non-negative value). In addition to the assumptions mentioned in the problem, we also assume that m_1 cannot pass through m_2 (like a bullet might be able to). We are only able to use momentum conservation, since no assumptions about the total kinetic energy can be made.

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

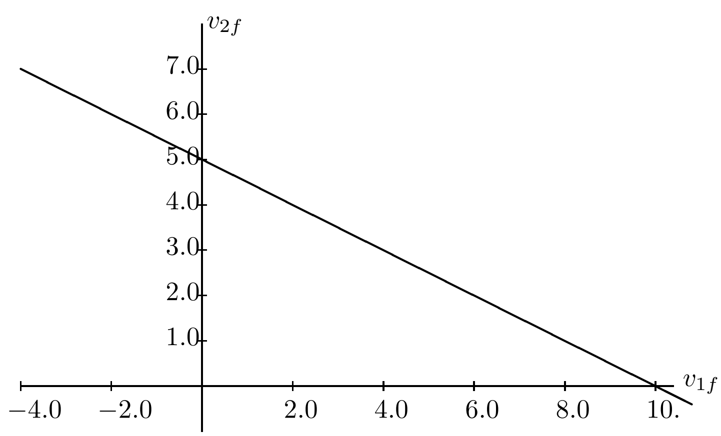
This (since $m_2 = 2.00 m_1$) simplifies to

$$v_{1i} = v_{1f} + 2.00 v_{2f} \ .$$

(a) Using $v_{1i} = 10.0$ m/s, we have

$$v_{2f} = (5.00\,\text{m/s}) - 0.500 v_{1f} \ .$$

(b) Ignoring physics considerations, our function is a line of infinite extent with negative slope.



(c) The greatest possible value of v_{1f} occurs in the completely inelastic case (reasons mentioned in the next several parts) where (see solution to part (a) of previous problem) its value would be $(10.0)(1/3) \approx 3.33$ m/s.

(d) Clearly, this is also the value of v_{2f} in this case.

(e) They stick together (completely inelastic collision).

(f) As mentioned above, we assume m_1 does not pass through m_2 and the problem states that there's no energy production so that $K_{1f} \le K_{1i}$ which implies $v_{1f} \le v_{1i}$.

(g) The plot is shown below, in part (ℓ).

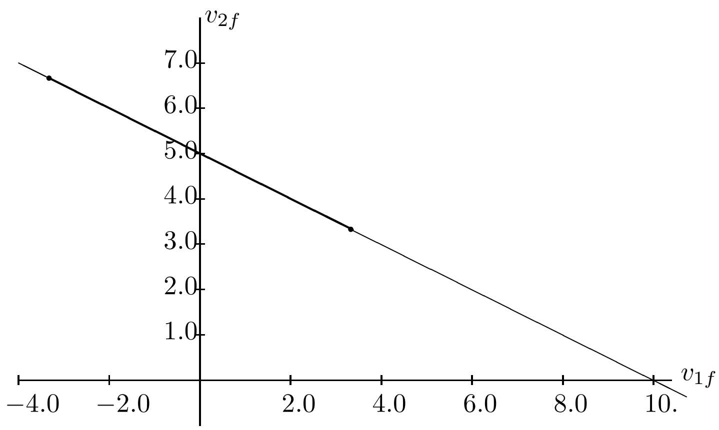
(h) With energy production not a possibility, then the "hardest rebound" m_1 can suffer is in an elastic collision, in which its final velocity (see part (b) of the previous problem) is $(10.0)(1 - 2)/3 \approx -3.33$ m/s.

(i) Eq. 10-31 gives the velocity of m_2 as $(10.0)(2/3) \approx 6.67$ m/s (see also part (b) of previous problem).

(j) As mentioned, this is an elastic collision (no "loss" of kinetic energy).

(k) The problem states that there's no energy production so that $K_{1i} - K_{1f} = K_{2f}$ and any greater value of $|v_{2f}|$ would violate this condition.

(l) The above graph is redrawn here, with the dark part representing the physically allowed region; the small circles bounding the dark segment correspond to the values calculated in the previous parts of this problem.



89. (Third problem in **Cluster 1**)
We note that the problem has implicitly chosen the initial direction of motion (of m_1) as the positive direction. The questions to find "greatest" and "least" values are understood in terms of that axis choice (*greatest* = largest positive value, and *least* = the negative value of greatest magnitude or the smallest non-negative value). In addition to the assumptions mentioned in the problem, we also assume that m_1 cannot pass through m_2 (like a bullet might be able to). We are only able to use momentum conservation, since no assumptions about the total kinetic energy can be made.

$$m_1 \, v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

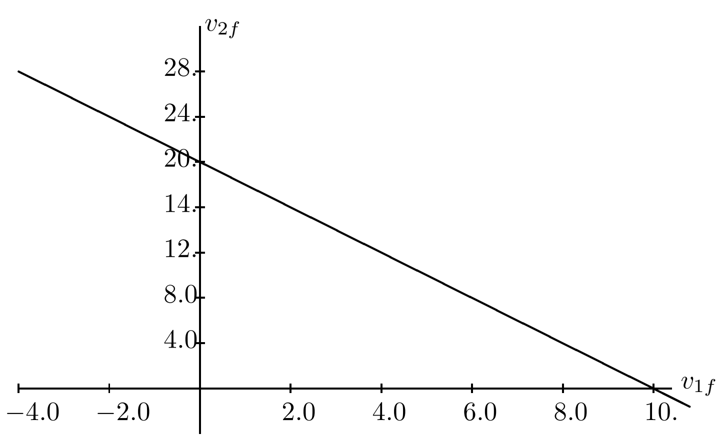
This (since $m_2 = 0.500 m_1$) simplifies to

$$v_{1i} = v_{1f} + 0.500 v_{2f} \ .$$

(a) Using $v_{1i} = 10.0$ m/s, we have

$$v_{2f} = (20.0\,\mathrm{m/s}) - 2.00 v_{1f} \ .$$

(b) Ignoring physics considerations, our function is a line of infinite extent with negative slope.



(c) The greatest possible value of v_{1f} occurs in the completely inelastic case (reasons mentioned in the next several parts) where (see solution to part (a) of previous problem) its value would be $(10.0)(2/3) \approx 6.67$ m/s.

(d) Clearly, this is also the value of v_{2f} in this case.

(e) They stick together (completely inelastic collision).

(f) As mentioned above, we assume m_1 does not pass through m_2 and the problem states that there's no energy production so that $K_{1f} \le K_{1i}$ which implies $v_{1f} \le v_{1i}$.

(g) The plot is shown below, in part (ℓ).

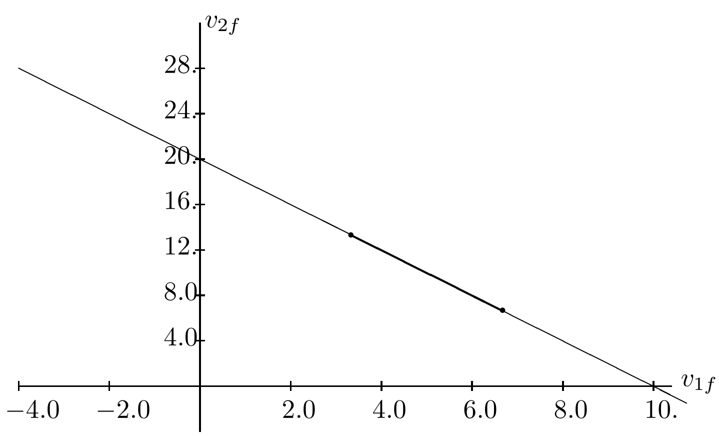
(h) With energy production not a possibility, then the "hardest rebound" m_1 can suffer is in an elastic collision, in which its final velocity (see part (b) of the previous problem) is $(10.0)(2-1)/3 \approx 3.33$ m/s.

(i) Eq. 10-31 gives the velocity of m_2 as $(10.0)(4/3) \approx 13.3$ m/s (see also part (b) of previous problem).

(j) As mentioned, this is an elastic collision (no "loss" of kinetic energy).

(k) The problem states that there's no energy production so that $K_{1i} - K_{1f} = K_{2f}$ and any greater value of $|v_{2f}|$ would violate this condition.

(l) The above graph is redrawn here, with the dark part representing the physically allowed region; the small circles bounding the dark segment correspond to the values calculated in the previous parts of this problem.



90. (First problem in **Cluster 2**)
The setup for this cluster refers to Fig. 10-16 in the chapter that assumes both angles are positive (at least, this is what is assumed in writing down Eq. 10-43) regardless of whether they are measured clockwise or counterclockwise. In this solution, we adopt that same convention.

(a) We first examine conservation of the y components of momentum:

$$\begin{aligned} 0 &= -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2 \\ 0 &= -m_1(5.00 \text{ m/s}) \sin 30^\circ + (2m_1)\, v_{2f} \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1 v_{1i} &= m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1(5.00 \text{ m/s}) \cos 30^\circ + (2m_1)\, v_{2f} \cos\theta_2 \end{aligned}$$

From the y equation, we obtain $1.25 = v_{2f} \sin\theta_2$ with SI units understood; similarly, the x equation yields $2.83 = v_{2f} \cos\theta_2$. Squaring these two relations and adding them leads to

$$1.25^2 + 2.83^2 = v_{2f}^2 \left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and consequently to $v_{2f} = \sqrt{1.25^2 + 2.83^2} = 3.10$ m/s. Plugging back in to either the x or y equation yields the angle $\theta_2 = 23.8°$.

(b) We compute decrease in total kinetic energy:

$$K_i - K_f = 27.9\, m_1$$

so that the collision is seen to be inelastic. We find that

$$\frac{27.9\, m_1}{\frac{1}{2}\, m_1\, 10^2} = 0.558\ ,$$

or roughly 56%, of the initial energy has been "lost."

91. (Second problem in **Cluster 2**)
As explained in the previous solution, we take both angles θ_1 and θ_2 to be positive-valued.

(a) We first examine conservation of the y components of momentum.

$$\begin{aligned} 0 &= -m_1\, v_{1f}\, \sin\theta_1 + m_2\, v_{2f}\, \sin\theta_2 \\ 0 &= -m_1 v_{1f}\, \sin 30° + 2m_1\, v_{2f}\, \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1\, v_{1i} &= m_1\, v_{1f}\, \cos\theta_1 + m_2\, v_{2f}\, \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1 v_{1f}\, \cos 30° + 2m_1\, v_{2f}\, \cos\theta_2 \end{aligned}$$

From the y equation, we obtain $v_{1f} = 4\, v_{2f}\, \sin\theta_2$; similarly, the x equation yields $20 - v_{1f}\sqrt{3} = 4v_{2f}\, \cos\theta_2$ with SI units understood (also, $\cos 30° = \sqrt{3}/2$ has been used). Squaring these two relations and adding them leads to

$$v_{1f}^2\, (1+3) - 40v_{1f}\sqrt{3} + 400 = 16\, v_{2f}^2\, \left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and thus to $v_{2f}^2 = v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25$. We plug this into the condition of total kinetic energy "conservation."

$$\begin{aligned} K_i &= K_f \\ \frac{1}{2}\, m_1\, v_{1i}^2 &= \frac{1}{2}\, m_1\, v_{1f}^2 + \frac{1}{2}\, m_2\, v_{2f}^2 \\ \frac{1}{2} m_1 \left(10\ \frac{\text{m}}{\text{s}}\right)^2 &= \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2}\, (2m_1) \left(\frac{v_{1f}^2}{4} - \frac{5\sqrt{3}}{2} v_{1f} + 25\right) \end{aligned}$$

This leads to an equation of second degree (in the variable v_{1f}):

$$\frac{3}{4}\, v_{1f}^2 - \frac{5\sqrt{3}}{2}\, v_{1f} - 25 = 0$$

which has a positive root $v_{1f} = \frac{5}{3}\sqrt{3}\left(1+\sqrt{5}\right) \approx 9.34$ m/s.

(b) We plug our result for v_{1f} into the relation $v_{2f} = \sqrt{v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25}$ derived above and obtain $v_{2f} = \frac{5}{6}\sqrt{6}\left(\sqrt{5} - 1\right) \approx 2.52$ m/s.

(c) Plugging these values of v_{1f} and v_{2f} into, say, the $v_{1f} = 4\, v_{2f}\, \sin\theta_2$ relation, we find $\theta_2 = 67.8°$.

92. (Second problem in **Cluster 2**)
As explained in the first solution in this cluster, we take both angles θ_1 and θ_2 to be positive-valued.

(a) We first examine conservation of the y components of momentum.

$$\begin{aligned} 0 &= -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2 \\ 0 &= -m_1 v_{1f} \sin 30^\circ + 2m_1 v_{2f} \sin\theta_2 \end{aligned}$$

Next, we examine conservation of the x components of momentum.

$$\begin{aligned} m_1 v_{1i} &= m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2 \\ m_1(10.0 \text{ m/s}) &= m_1 v_{1f} \cos 30^\circ + 2m_1 v_{2f} \cos\theta_2 \end{aligned}$$

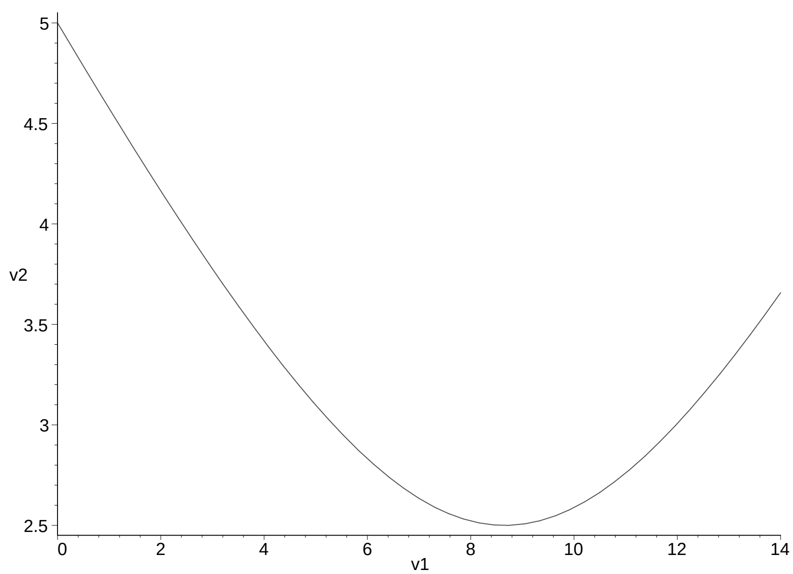
From the y equation, we obtain $v_{1f} = 4\,v_{2f} \sin\theta_2$; similarly, the x equation yields $20 - v_{1f}\sqrt{3} = 4v_{2f} \cos\theta_2$ with SI units understood (and the fact that $\cos 30^\circ = \sqrt{3}/2$ has been used). Squaring these two relations and adding them leads to

$$v_{1f}^2\,(1+3) - 40v_{1f}\sqrt{3} + 400 = 16\,v_{2f}^2\left(\sin^2\theta_2 + \cos^2\theta_2\right)$$

and thus to

$$v_{2f}^2 = v_{1f}^2/4 - 5v_{1f}\sqrt{3}/2 + 25\ .$$

(b) The plot (v_{2f} versus v_{1f}) is shown below. The units for both axes are meters/second.



(c) Simply from the total kinetic energy requirement that $K_i \geq K_f$ we see immediately that $v_{1f} \leq v_{1i} = 10.0$ m/s (where the upper bound represents the trivial case where it passes m_2 by completely with $K_i = K_f$), and with the more stringent requirement that it does strike m_2 and scatters at $\theta_1 = 30^\circ$ we again find that it is bounded by the $K_i = K_f$ case. The elastic collision scenario was worked in the previous problem with the result $v_{1f} = 9.34$ m/s.

(d) And we also found the result $v_{2f} = 2.52$ m/s.

(e) As mentioned, this is an elastic collision.

(f) A higher speed for v_{1f} would require energy conversion into kinetic form (say, from an explosion) since $K_i < K_f$ would be the result.

(g) To save space, a separate graph for this part is not shown.

(h) Returning to the x and y equations derived in part (a), we divide them to obtain

$$\frac{v_{1f}}{20 - v_{1f}\sqrt{3}} = \frac{4v_{2f}\sin\theta_2}{4v_{2f}\,\cos\theta_2} = \tan\theta_2$$

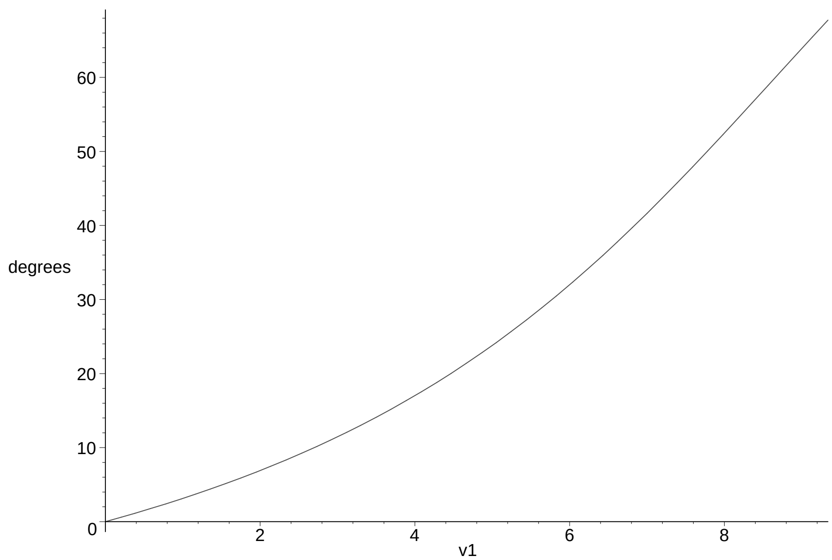
which leads to

$$\theta_2 = \tan^{-1}\left(\frac{v_{1f}}{20 - v_{1f}\sqrt{3}}\right) .$$

(i) See part (k).

(j) The value for the elastic case was computed in the previous problem; we find $\theta_2 = 67.8°$ when $v_{1f} = 9.34$ m/s.

(k) This corresponds to the upper righthand point of the curve shown below.



(l) , (m), (n), and (o)
Now, unlike the notation used in the one-dimensional collisions, this v_{1f} cannot be negative (it is the *magnitude* of the velocity). This suggests that its smallest value is zero, but the requirement that it scatter at $\theta_1 = 30°$ might seem to conflict with this. However, if one considers the (smooth) limit

of $v_{1f} \to 0$, we find there is nothing inconsistent with $\theta_1 = 30°$ in setting $v_{1f} = 0$. It is certainly inelastic (but not completely so! A completely inelastic collision *would* be inconsistent with the $\theta_1 = 30°$ condition!); we find from $v_{2f} = 5.00$ m/s (see the graph for part (b)) that $K_i < K_f$ in this case. Clearly, $\theta_2 = 0°$ in this circumstance (see, e.g., the graph for part(i)).

Chapter 11

1. (a) Eq. 11-6 leads to
$$\omega = \frac{d}{dt}\left(at + bt^3 - ct^4\right) = a + 3bt^2 - 4ct^3 \ .$$
   (b) And Eq. 11-8 gives
$$\alpha = \frac{d}{dt}\left(a + 3bt^2 - 4ct^3\right) = 6bt - 12ct^2 \ .$$

2. (a) The second hand of the smoothly running watch turns through 2π radians during 60 s. Thus,
$$\omega = \frac{2\pi}{60} = 0.105 \text{ rad/s} \ .$$
   (b) The minute hand of the smoothly running watch turns through 2π radians during 3600 s. Thus,
$$\omega = \frac{2\pi}{3600} = 1.75 \times 10^{-3} \text{ rad/s} \ .$$
   (c) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus,
$$\omega = \frac{2\pi}{43200} = 1.45 \times 10^{-4} \text{ rad/s} \ .$$

3. (a) The time for one revolution is the circumference of the orbit divided by the speed v of the Sun: $T = 2\pi R/v$, where R is the radius of the orbit. We convert the radius:
$$R = \left(2.3 \times 10^4 \,\text{ly}\right)\left(9.46 \times 10^{12} \,\text{km/ly}\right) = 2.18 \times 10^{17} \text{ km}$$
   where the ly $\leftrightarrow$ km conversion can be found in Appendix D or figured "from basics" (knowing the speed of light). Therefore, we obtain
$$T = \frac{2\pi\left(2.18 \times 10^{17} \,\text{km}\right)}{250 \,\text{km/s}} = 5.5 \times 10^{15} \text{ s} \ .$$
   (b) The number of revolutions N is the total time t divided by the time T for one revolution; that is, $N = t/T$. We convert the total time from years to seconds and obtain
$$N = \frac{\left(4.5 \times 10^9 \,\text{y}\right)\left(3.16 \times 10^7 \,\text{s/y}\right)}{5.5 \times 10^{15} \text{ s}} = 26 \ .$$

4. If we make the units explicit, the function is
$$\theta = (4.0 \,\text{rad/s})t - \left(3.0 \,\text{rad/s}^2\right)t^2 + \left(1.0 \,\text{rad/s}^3\right)t^3$$
but generally we will proceed as shown in the problem – letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 11-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2 \ .$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 11-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad/s}^2 \ .$$

(d) And Eq. 11-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t \ .$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

5. If we make the units explicit, the function is

$$\theta = 2\,\text{rad} + \left(4\,\text{rad/s}^2\right)t^2 + \left(2\,\text{rad/s}^3\right)t^3$$

but in some places we will proceed as indicated in the problem – by letting these units be understood.

(a) We evaluate the function θ at $t = 0$ to obtain $\theta_0 = 2$ rad.

(b) The angular velocity as a function of time is given by Eq. 11-6:

$$\omega = \frac{d\theta}{dt} = \left(8\,\text{rad/s}^2\right)t + \left(6\,\text{rad/s}^3\right)t^2$$

which we evaluate at $t = 0$ to obtain $\omega_0 = 0$.

(c) For $t = 4$ s, the function found in the previous part is $\omega_4 = (8)(4) + (6)(4)^2 = 128$ rad/s. If we round this to two figures, we obtain $\omega_4 \approx 130$ rad/s.

(d) The angular acceleration as a function of time is given by Eq. 11-8:

$$\alpha = \frac{d\omega}{dt} = 8\,\text{rad/s}^2 + \left(12\,\text{rad/s}^3\right)t$$

which yields $\alpha_2 = 8 + (12)(2) = 32\,\text{rad/s}^2$ at $t = 2$ s.

(e) The angular acceleration, given by the function obtained in the previous part, depends on time; it is not constant.

6. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8 \text{ rev}}{2.5\,\text{rev/s}} = 0.050 \text{ s} \ .$$

The minimum speed of the arrow is then

$$v_{\text{min}} = \frac{20\,\text{cm}}{0.050\,\text{s}} = 400\,\text{cm/s} = 4.0 \text{ m/s} \ .$$

(b) No – there is no dependence on radial position in the above computation.

7. Applying Eq. 2-15 to the vertical axis (with $+y$ downward) we obtain the free-fall time:

$$\Delta y = v_{0y}t + \frac{1}{2}gt^2 \implies t = \sqrt{\frac{2(10)}{9.8}} = 1.4 \text{ s} .$$

Thus, by Eq. 11-5, the magnitude of the average angular velocity is

$$\omega_{\text{avg}} = \frac{(2.5)(2\pi)}{1.4} = 11 \text{ rad/s} .$$

8. (a) We assume the sense of rotation is positive. Applying Eq. 11-12, we obtain

$$\omega = \omega_0 + \alpha t \implies \alpha = \frac{3000 - 1200}{12/60} = 9000 \text{ rev/min}^2 .$$

(b) And Eq. 11-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\, t = \frac{1}{2}(1200 + 3000)\left(\frac{12}{60}\right)$$

which yields $\theta = 420$ rev.

9. We assume the sense of initial rotation is positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued.

(a) The angular acceleration is constant, so we can apply Eq. 11-12 ($\omega = \omega_0 + \alpha t$). To obtain the requested units, we have $t = 30/60 = 0.50\,\text{min}$. Thus,

$$\alpha = -\frac{33.33\,\text{rev/min}}{0.50\,\text{min}} = -66.7 \text{ rev/min}^2 .$$

(b) We use Eq. 11-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (33.33)(0.50) + \frac{1}{2}(-66.7)(0.50)^2 = 8.3 \text{ rev} .$$

10. We assume the sense of initial rotation is positive. Then, with $\omega_0 = +120$ rad/s and $\omega = 0$ (since it stops at time t), our angular acceleration ("deceleration") will be negative-valued: $\alpha = -4.0$ rad/s^2.

(a) We apply Eq. 11-12 to obtain t.

$$\omega = \omega_0 + \alpha t \implies t = \frac{0 - 120}{-4.0} = 30 \text{ s} .$$

(b) And Eq. 11-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\, t = \frac{1}{2}(120 + 0)(30)$$

which yields $\theta = 1800$ rad. Alternatively, Eq. 11-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular equation in Table 11-1 (except Eq. 11-12) can be used to find θ.

11. We apply Eq. 11-12 twice, assuming the sense of rotation is positive. We have $\omega > 0$ and $\alpha < 0$. Since the angular velocity at $t = 1$ min is $\omega_1 = (0.90)(250) = 225$ rev/min, we have

$$\omega_1 = \omega_0 + \alpha t \implies \alpha = \frac{225 - 250}{1} = -25 \text{ rev/min}^2 .$$

Next, between $t = 1$ min and $t = 2$ min we have the interval $\Delta t = 1$ min. Consequently, the angular velocity at $t = 2$ min is

$$\omega_2 = \omega_1 + \alpha\Delta t = 225 + (-25)(1) = 200 \text{ rev/min} .$$

12. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

   (a) The angular acceleration satisfies Eq. 11-13:

$$25\,\text{rad} = \frac{1}{2}\alpha(5.0\,\text{s})^2 \implies \alpha = 2.0\ \text{rad/s}^2\ .$$

   (b) The average angular velocity is given by Eq. 11-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25\,\text{rad}}{5.0\,\text{s}} = 5.0\ \text{rad/s}\ .$$

   (c) Using Eq. 11-12, the instantaneous angular velocity at $t = 5.0$ s is

$$\omega = \left(2.0\,\text{rad/s}^2\right)(5.0\,\text{s}) = 10\ \text{rad/s}\ .$$

   (d) According to Eq. 11-13, the angular displacement at $t = 10$ s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0)(10)^2 = 100\ \text{rad}\ .$$

   Thus, the displacement between $t = 5$ s and $t = 10$ s is $\Delta\theta = 100 - 25 = 75$ rad.

13. We take $t = 0$ at the start of the interval and take the sense of rotation as positive. Then at the end of the $t = 4.0\,\text{s}$ interval, the angular displacement is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. We solve for the angular velocity at the start of the interval:

$$\omega_0 = \frac{\theta - \frac{1}{2}\alpha t^2}{t} = \frac{120\,\text{rad} - \frac{1}{2}\left(3.0\,\text{rad/s}^2\right)(4.0\,\text{s})^2}{4.0\,\text{s}} = 24\ \text{rad/s}\ .$$

   We now use $\omega = \omega_0 + \alpha t$ (Eq. 11-12) to find the time when the wheel is at rest:

$$t = -\frac{\omega_0}{\alpha} = -\frac{24\,\text{rad/s}}{3.0\,\text{rad/s}^2} = -8.0\ \text{s}\ .$$

   That is, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

14. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = 2.00\ \text{rad/s}^2$. Between t_1 and t_2 it turns through $\Delta\theta = 90.0$ rad, where $t_2 - t_1 = \Delta t = 3.00$ s.

   (a) We use Eq. 11-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha\,(\Delta t)^2 \implies \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2}$$

   which we plug into Eq. 11-12, set up to describe the motion during $0 \le t \le t_1$:

$$\begin{aligned} \omega_1 &= \omega_0 + \alpha t_1 \\ \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} &= \alpha t_1 \\ \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} &= (2.00)t_1 \end{aligned}$$

   yielding $t_1 = 13.5$ s.

   (b) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0\ \text{rad/s}\ .$$

15. The problem has (implicitly) specified the positive sense of rotation. The angular acceleration of magnitude 0.25 rad/s^2 in the negative direction is assumed to be constant over a large time interval, including negative values (for t).

    (a) We specify $\theta_{\rm max}$ with the condition $\omega = 0$ (this is when the wheel reverses from positive rotation to rotation in the negative direction). We obtain $\theta_{\rm max}$ using Eq. 11-14:

    $$\theta_{\rm max} = -\frac{\omega_{\rm o}^2}{2\alpha} = -\frac{4.7^2}{2(-0.25)} = 44 \text{ rad} .$$

    (b) We find values for t_1 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = 22$ rad (or 22.09 rad if we wish to keep track of accurate values in all intermediate steps and only round off on the final answers). Using Eq. 11-13 and the quadratic formula, we have

    $$\theta_1 = \omega_{\rm o} t_1 + \frac{1}{2}\alpha t_1^2 \implies t_1 = \frac{-\omega_{\rm o} \pm \sqrt{\omega_{\rm o}^2 + 2\theta_1\alpha}}{\alpha}$$

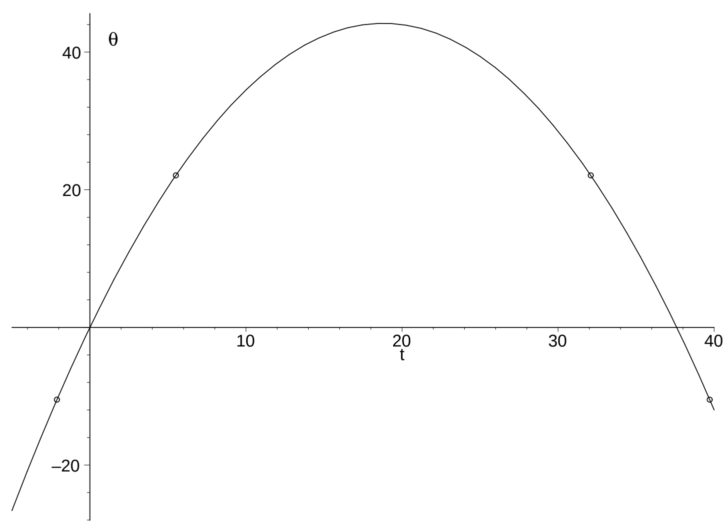
    which yields the two roots 5.5 s and 32 s.

    (c) We find values for t_2 when the angular displacement (relative to its orientation at $t = 0$) is $\theta_2 = -10.5$ rad. Using Eq. 11-13 and the quadratic formula, we have

    $$\theta_2 = \omega_{\rm o} t_2 + \frac{1}{2}\alpha t_2^2 \implies t_2 = \frac{-\omega_{\rm o} \pm \sqrt{\omega_{\rm o}^2 + 2\theta_2\alpha}}{\alpha}$$

    which yields the two roots -2.1 s and 40 s.

    (d) With radians and seconds understood, the graph of θ versus t is shown below (with the points found in the previous parts indicated as small circles).



16. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha > 0$, which makes our choice for positive sense of rotation. At t_1 its angular velocity is $\omega_1 = +10$ rev/s, and at t_2 its angular velocity is $\omega_2 = +15$ rev/s. Between t_1 and t_2 it turns through $\Delta\theta = 60$ rev, where $t_2 - t_1 = \Delta t$.

(a) We find α using Eq. 11-14:

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta \implies \alpha = \frac{15^2 - 10^2}{2(60)}$$

which yields $\alpha = 1.04$ rev/s^2 which we round off to 1.0 rev/s^2.

(b) We find Δt using Eq. 11-15:

$$\Delta\theta = \frac{1}{2}(\omega_1 + \omega_2)\,\Delta t \implies \Delta t = \frac{2(60)}{10+15} = 4.8 \text{ s} .$$

(c) We obtain t_1 using Eq. 11-12:

$$\omega_1 = \omega_0 + \alpha t_1 \implies t_1 = \frac{10}{1.04} = 9.6 \text{ s} .$$

(d) Any equation in Table 11-1 involving θ can be used to find θ_1 (the angular displacement during $0 \le t \le t_1$); we select Eq. 11-14.

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1 \implies \theta_1 = \frac{10^2}{2(1.04)} = 48 \text{ rev} .$$

17. The wheel has angular velocity $\omega_0 = +1.5$ rad/s $= +0.239$ rev/s^2 at $t = 0$, and has constant value of angular acceleration $\alpha < 0$, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at $t = 0$) is $\theta_1 = +20$ rev, and at t_2 its angular displacement is $\theta_2 = +40$ rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 11-15:

$$\theta_2 = \frac{1}{2}(\omega_0 + \omega_2)\,t_2 \implies t_2 = \frac{2(40)}{0.239}$$

which yields $t_2 = 335$ s which we round off to $t_2 \approx 340$ s.

(b) Any equation in Table 11-1 involving α can be used to find the angular acceleration; we select Eq. 11-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2}\alpha t_2^2 \implies \alpha = -\frac{2(40)}{335^2}$$

which yields $\alpha = -7.12 \times 10^{-4}$ rev/s^2 which we convert to $\alpha = -4.5 \times 10^{-3}$ rad/s^2.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$ (Eq. 11-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1\alpha}}{\alpha} = \frac{-0.239 \pm \sqrt{0.239^2 + 2(20)\left(-7.12 \times 10^{-4}\right)}}{-7.12 \times 10^{-4}}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is $t_1 = 98$ s.

18. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = +4.0$ rad/s^2, which makes our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at $t = 0$) is θ_1, and at t_2 its angular velocity is θ_2, where $\theta_2 - \theta_1 = \Delta\theta = 80$ rad. Also, $t_2 - t_1 = \Delta t = 4.0$ s.

(a) We find the angular velocity at t_1 using Eq. 11-13 (set up to describe the interval $t_1 \le t \le t_2$).

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha\,(\Delta t)^2 \implies \omega_1 = \frac{80 - \frac{1}{2}(4.0)(4.0)^2}{4.0}$$

which yields $\omega_1 = 12$ rad/s.

(b) We obtain t_1 using Eq. 11-12:

$$\omega_1 = \omega_0 + \alpha t_1 \implies t_1 = \frac{12}{4.0} = 3.0 \text{ s} .$$

19. The magnitude of the acceleration is given by $a = \omega^2 r$ (Eq. 11-23) where r is the distance from the center of rotation and ω is the angular velocity. We convert the given angular velocity to rad/s:

$$\omega = \frac{(33.33\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 3.49 \text{ rad/s} .$$

Therefore,

$$a = \left(3.49\,\text{rad/s}^2\right)^2 (0.15\,\text{m}) = 1.8 \text{ m/s}^2 .$$

The acceleration vector is toward the center of the record.

20. (a) We obtain

$$\omega = \frac{(33.33\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 3.49 \text{ rad/s} .$$

(b) Using Eq. 11-18, we have

$$v = r\omega = (15)(3.49) = 52 \text{ cm/s} .$$

(c) Similarly, when $r = 7.4$ cm we find $v = r\omega = 26$ cm/s. The goal of this exercise to observe what is and is not the same at different locations on a body in rotational motion (ω is the same, v is not), as well as to emphasize the importance of radians when working with equations such as Eq. 11-18.

21. With $v = 50(1000/3600) = 13.9$ m/s, Eq. 11-18 leads to

$$\omega = \frac{v}{r} = \frac{13.9}{110} = 0.13 \text{ rad/s} .$$

22. (a) We obtain

$$\omega = \frac{(200\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 20.9 \text{ rad/s} .$$

(b) With $r = 1.20/2 = 0.60$ m, Eq. 11-18 leads to

$$v = r\omega = (0.60)(20.9) = 12.6 \text{ m/s} .$$

(c) With $t = 1$ min, $\omega = 1000$ rev/min and $\omega_\text{o} = 200$ rev/min, Eq. 11-12 gives

$$\alpha = \frac{\omega - \omega_\text{o}}{t} = 800 \text{ rev/min}^2 .$$

(d) With the same values used in part (c), Eq. 11-15 becomes

$$\theta = \frac{1}{2}(\omega_\text{o} + \omega)\,t = \frac{1}{2}(200 + 1000)(1) = 600 \text{ rev} .$$

23. (a) Using Eq. 11-6, the angular velocity at $t = 5.0$ s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \left.\frac{d}{dt}\left(0.30t^2\right)\right|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad/s} .$$

(b) Eq. 11-18 gives the linear speed at $t = 5.0$ s:

$$v = \omega r = (3.0\,\text{rad/s})(10\,\text{m}) = 30 \text{ m/s} .$$

(c) The angular acceleration is, from Eq. 11-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad/s}^2 \ .$$

Then, the tangential acceleration at $t = 5.0\,\text{s}$ is, using Eq. 11-22,

$$a_t = r\alpha = (10\,\text{m})\left(0.60\,\text{rad/s}^2\right) = 6.0 \text{ m/s}^2 \ .$$

(d) The radial (centripetal) acceleration is given by Eq. 11-23:

$$a_r = \omega^2 r = (3.0\,\text{rad/s})^2(10\,\text{m}) = 90 \text{ m/s}^2 \ .$$

24. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 11-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90 \times 10^4\,\text{km/h}\right)\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right)}{3.22 \times 10^3\,\text{km}} = 2.50 \times 10^{-3}\,\text{rad/s} \ .$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 11-23:

$$a_r = \omega^2 r = \left(2.50 \times 10^{-3}\,\text{rad/s}\right)^2\left(3.22 \times 10^6\,\text{m}\right) = 20.2 \text{ m/s}^2 \ .$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \quad \text{and} \quad a_t = r\alpha = 0 \ .$$

25. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}\,\text{rad}$. That time is

$$t = \frac{2\ell}{c} = \frac{2(500\,\text{m})}{2.998 \times 10^8\,\text{m/s}} = 3.34 \times 10^{-6} \text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26 \times 10^{-2}\,\text{rad}}{3.34 \times 10^{-6}\,\text{s}} = 3.8 \times 10^3 \text{ rad/s} \ .$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = \left(3.8 \times 10^3\,\text{rad/s}\right)(0.05\,\text{m}) = 190 \text{ m/s} \ .$$

26. (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 150\,\text{rev/min}}{(2.2\,\text{h})(60\,\text{min/1 h})} = -1.14 \text{ rev/min}^2 \ .$$

(b) Using Eq. 11-13 with $t = (2.2)(60) = 132$ min, the number of revolutions is

$$\begin{aligned} \theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ &= (150\,\text{rev/min})(132\,\text{min}) + \frac{1}{2}\left(-1.14\,\text{rev/min}^2\right)(132\,\text{min})^2 \\ &= 9.9 \times 10^3 \text{ rev} \ . \end{aligned}$$

(c) With $r = 500$ mm, the tangential acceleration is

$$a_t = \alpha\, r = \left(-1.14\,\text{rev/min}^2\right)\left(\frac{2\pi\,\text{rad}}{1\text{ rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)^2 (500\,\text{mm})$$

which yields $a_t = -0.99$ mm/s^2.

(d) With $r = 0.50$ m, the radial (or centripetal) acceleration is given by Eq. 11-23:

$$a_r = \omega^2 r = \left((75\,\text{rev/min})\left(\frac{2\pi\,\text{rad/rev}}{1\,\text{min}/60\,\text{s}}\right)\right)^2 (0.50\,\text{m})$$

which yields $a_r = 31$ in SI units – and is seen to be much bigger than a_t. Consequently, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} \approx a_r = 31\text{ m/s}^2 .$$

27. (a) Earth makes one rotation per day and $1\,d$ is $(24\,\text{h})(3600\,\text{s/h}) = 8.64 \times 10^4\,\text{s}$, so the angular speed of Earth is

$$\omega = \frac{2\pi\,\text{rad}}{8.64 \times 10^4\,\text{s}} = 7.27 \times 10^{-5}\text{ rad/s} .$$

(b) We use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R\cos 40°$, where R is the radius of Earth (6.37×10^6 m). Therefore, its speed is

$$v = \omega\,(R\cos 40°) = \left(7.27 \times 10^{-5}\,\text{rad/s}\right)\left(6.37 \times 10^6\,\text{m}\right)\cos 40° = 355\text{ m/s} .$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.27×10^{-5} rad/s).

(d) The latitude is 0° and the speed is

$$v = \omega R = \left(7.27 \times 10^{-5}\,\text{rad/s}\right)\left(6.37 \times 10^6\,\text{m}\right) = 463\text{ m/s} .$$

28. (a) The tangential acceleration, using Eq. 11-22, is

$$a_t = \alpha r = \left(14.2\,\text{rad/s}^2\right)(2.83\,\text{cm}) = 40.2\text{ cm/s}^2 .$$

(b) In rad/s, the angular velocity is $\omega = (2760)(2\pi/60) = 289$, so

$$a_r = \omega^2 r = (289\,\text{rad/s})^2(0.0283\,\text{m}) = 2.36 \times 10^3\text{ m/s}^2 .$$

(c) The angular displacement is, using Eq. 11-14,

$$\theta = \frac{\omega^2}{2\alpha} = \frac{289^2}{2(14.2)} = 2.94 \times 10^3\text{ rad} .$$

Then, using Eq. 11-1, the distance traveled is

$$s = r\theta = (0.0283\text{ m})\left(2.94 \times 10^3\,\text{rad}\right) = 83.2\text{ m} .$$

29. Since the belt does not slip, a point on the rim of wheel C has the same tangential acceleration as a point on the rim of wheel A. This means that $\alpha_A r_A = \alpha_C r_C$, where α_A is the angular acceleration of wheel A and α_C is the angular acceleration of wheel C. Thus,

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_A = \left(\frac{10\,\text{cm}}{25\,\text{cm}}\right)(1.6\,\text{rad/s}^2) = 0.64\text{ rad/s}^2 .$$

Since the angular speed of wheel C is given by $\omega_C = \alpha_C t$, the time for it to reach an angular speed of $\omega = 100\,\text{rev/min} = 10.5\,\text{rad/s}$ starting from rest is

$$t = \frac{\omega_C}{\alpha_C} = \frac{10.5\,\text{rad/s}}{0.64\,\text{rad/s}^2} = 16\text{ s} .$$

30. The function $\theta = \xi e^{\beta t}$ where $\xi = 0.40$ rad and $\beta = 2\,\text{s}^{-1}$ is describing the angular coordinate of a line (which is marked in such a way that all points on it have the same value of angle at a given time) on the object. Taking derivatives with respect to time leads to $\frac{d\theta}{dt} = \xi\,\beta\,e^{\beta t}$ and $\frac{d^2\theta}{dt^2} = \xi\,\beta^2\,e^{\beta t}$.

   (a) Using Eq. 11-22, we have
   $$a_t = \alpha r = \frac{d^2\theta}{dt^2}\,r = \; 6.4\ \text{cm/s}^2 \ .$$

   (b) Using Eq. 11-23, we have
   $$a_r = \omega^2 r = \left(\frac{d\theta}{dt}\right)^2 r = \; 2.6\ \text{cm/s}^2 \ .$$

31. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by
   $$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt} \ .$$
   For the pulsar described in the problem, we have
   $$\frac{dT}{dt} = \frac{1.26\times10^{-5}\,\text{s/y}}{3.16\times10^{7}\,\text{s/y}} = 4.00\times10^{-13} \ .$$
   Therefore,
   $$\alpha = -\left(\frac{2\pi}{(0.033\,\text{s})^2}\right)(4.00\times10^{-13}) = -2.3\times10^{-9}\,\text{rad/s}^2 \ .$$
   The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

   (b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:
   $$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3\times10^{-9}\,\text{rad/s}^2)(0.033\,\text{s})} = 8.3\times10^{10}\,\text{s} \ .$$
   This is about 2600 years.

   (c) The pulsar was born $1992 - 1054 = 938$ years ago. This is equivalent to $(938\,\text{y})(3.16\times10^7\,\text{s/y}) = 2.96\times10^{10}\,\text{s}$. Its angular velocity at that time was
   $$\omega = \omega_0 + \alpha t = \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033\,\text{s}} + (-2.3\times10^{-9}\,\text{rad/s}^2)(-2.96\times10^{10}\,\text{s}) = 258\,\text{rad/s} \ .$$
   Its period was
   $$T = \frac{2\pi}{\omega} = \frac{2\pi}{258\,\text{rad/s}} = 2.4\times10^{-2}\,\text{s} \ .$$

32. (a) The angular speed in rad/s is
   $$\omega = \left(33\frac{1}{3}\,\text{rev/min}\right)\left(\frac{2\pi\,\text{rad/rev}}{60\,\text{s/min}}\right) = 3.49\ \text{rad/s} \ .$$
   Consequently, the radial (centripetal) acceleration is (using Eq. 11-23)
   $$a = \omega^2 r = (3.49\,\text{rad/s})^2\left(6.0\times10^{-2}\,\text{m}\right) = 0.73\,\text{m/s}^2 \ .$$

   (b) Using Ch. 6 methods, we have $ma = f_s \le f_{s,\,\text{max}} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:
   $$\mu_{s,\,\text{min}} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075 \ .$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2} \ .$$

If the object is not to slip at any time, we require

$$f_{s,\max} = \mu_s mg = ma_{\max} = mr\sqrt{\omega_{\max}^4 + \alpha^2} \ .$$

Thus, since $\alpha = \omega/t$ (from Eq. 11-12), we find

$$\begin{aligned}\mu_{s,\min} &= \frac{r\sqrt{\omega_{\max}^4 + \alpha^2}}{g} \\ &= \frac{r\sqrt{\omega_{\max}^4 + (\omega_{\max}/t)^2}}{g} \\ &= \frac{(0.060)\sqrt{3.49^4 + (3.49/0.25)^2}}{9.8} \\ &= 0.11 \ .\end{aligned}$$

33. The kinetic energy (in J) is given by $K = \frac{1}{2}I\omega^2$, where I is the rotational inertia (in kg·m^2) and ω is the angular velocity (in rad/s). We have

$$\omega = \frac{(602\,\text{rev/min})(2\pi\,\text{rad/rev})}{60\,\text{s/min}} = 63.0\,\text{rad/s} \ .$$

Consequently, the rotational inertia is

$$I = \frac{2K}{\omega^2} = \frac{2(24400\,\text{J})}{(63.0\,\text{rad/s})^2} = 12.3\ \text{kg}\cdot\text{m}^2 \ .$$

34. The translational kinetic energy of the molecule is

$$K_t = \frac{1}{2}mv^2 = \frac{1}{2}\left(5.30\times10^{-26}\right)(500)^2 = 6.63\times10^{-21}\ \text{J} \ .$$

With $I = 1.94\times10^{-46}$ kg·m^2, we employ Eq. 11-27:

$$\begin{aligned}K_r &= \frac{2}{3}K_t \\ \frac{1}{2}I\omega^2 &= \frac{2}{3}\left(6.63\times10^{-21}\right)\end{aligned}$$

which leads to $\omega = 6.75\times10^{12}$ rad/s.

35. Since the rotational inertia of a cylinder is $I = \frac{1}{2}MR^2$ (Table 11-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{4}MR^2\omega^2 \ .$$

For the first cylinder, we have $K = \frac{1}{4}(1.25)(0.25)^2(235)^2 = 1.1\times10^3$ J. For the second cylinder, we obtain $K = \frac{1}{4}(1.25)(0.75)^2(235)^2 = 9.7\times10^3$ J.

36. (a) Using Table 11-2(c), the rotational inertia is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(1210\,\text{kg})\left(\frac{1.21\,\text{m}}{2}\right)^2 = 221\ \text{kg}\cdot\text{m}^2 \ .$$

(b) The rotational kinetic energy is, by Eq. 11-27,

$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(2.21\times10^2\,\mathrm{kg\cdot m^2}\right)\left((1.52\,\mathrm{rev/s})(2\pi\,\mathrm{rad/rev})\right)^2 \\ &= 1.10\times10^4\ \mathrm{J}\ . \end{aligned}$$

37. The particles are treated "point-like" in the sense that Eq. 11-26 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 11-2(e) and the parallel-axis theorem (Eq. 11-29).

(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$\begin{aligned} I &= I_1+I_2+I_3+I_4 \\ &= \left(\frac{1}{12}Md^2+M\left(\frac{1}{2}d\right)^2\right)+md^2+\left(\frac{1}{12}Md^2+M\left(\frac{3}{2}d\right)^2\right)+m(2d)^2 \\ &= \frac{8}{3}Md^2+5md^2\ . \end{aligned}$$

(b) Using Eq. 11-27, we have

$$K=\frac{1}{2}I\omega^2=\left(\frac{4}{3}Md^2+\frac{5}{2}md^2\right)\omega^2\ .$$

38. (a) The rotational inertia of the three blades (each of mass m and length L) is

$$I=3\left(\frac{1}{3}mL^2\right)=mL^2=(240\,\mathrm{kg})(5.2\,\mathrm{m})^2=6.49\times10^3\ \mathrm{kg\cdot m^2}\ .$$

(b) The rotational kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(6.49\times10^3\,\mathrm{kg\cdot m^2}\right)\left((350\,\mathrm{rev/min})\left(\frac{2\pi\,\mathrm{rad/rev}}{60\,\mathrm{s/min}}\right)\right)^2 \\ &= 4.36\times10^6\ \mathrm{J}=\ 4.36\ \mathrm{MJ}\ . \end{aligned}$$

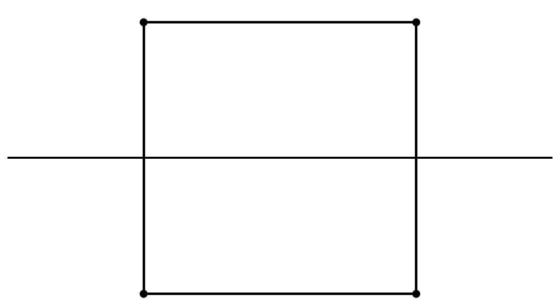
39. We use the parallel axis theorem: $I=I_{\mathrm{com}}+Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 11-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies $h=0.50\,\mathrm{m}-0.20\,\mathrm{m}=0.30\,\mathrm{m}$. We find

$$I_{\mathrm{com}}=\frac{1}{12}ML^2=\frac{1}{12}(0.56\,\mathrm{kg})(1.0\,\mathrm{m})^2=4.67\times10^{-2}\ \mathrm{kg\cdot m^2}\ .$$

Consequently, the parallel axis theorem yields

$$I=4.67\times10^{-2}\,\mathrm{kg\cdot m^2}+(0.56\,\mathrm{kg})(0.30\,\mathrm{m})^2=9.7\times10^{-2}\ \mathrm{kg\cdot m^2}\ .$$

40. (a) We show the figure with its axis of rotation (the thin horizontal line).



We note that each mass is $r = 1.0$ m from the axis. Therefore, using Eq. 11-26, we obtain

$$I = \sum m_i r_i^2 = 4(0.50 \text{ kg})(1.0 \text{ m})^2 = 2 \text{ kg}\cdot\text{m}^2 \ .$$

(b) In this case, the two masses nearest the axis are $r = 1.0$ m away from it, but the two furthest from the axis are $r = \sqrt{1.0^2 + 2.0^2}$ m from it. Here, then, Eq. 11-26 leads to

$$I = \sum m_i r_i^2 = 2(0.50 \text{ kg})(1.0 \text{ m}^2) + 2(0.50 \text{ kg})(5.0 \text{ m}^2) = 6.0 \text{ kg}\cdot\text{m}^2 \ .$$

(c) Now, two masses are on the axis (with $r = 0$) and the other two are a distance $r = \sqrt{1.0^2 + 1.0^2}$ m away. Now we obtain $I = 2.0 \text{ kg} \cdot \text{m}^2$.

41. We use the parallel-axis theorem. According to Table 11-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}(a^2 + b^2) \ .$$

A parallel axis through the corner is a distance $h = \sqrt{(a/2)^2 + (b/2)^2}$ from the center. Therefore,

$$I = I_{\text{com}} + Mh^2 = \frac{M}{12}\left(a^2 + b^2\right) + \frac{M}{4}\left(a^2 + b^2\right) = \frac{M}{3}\left(a^2 + b^2\right) \ .$$

42. (a) We apply Eq. 11-26:

$$I_x = \sum_{i=1}^{4} m_i y_i^2 = 50(2.0)^2 + (25)(4.0)^2 + 25(-3.0)^2 + 30(4.0)^2 = 1.3 \times 10^3 \text{ g}\cdot\text{cm}^2 \ .$$

(b) For rotation about the y axis we obtain

$$I_y = \sum_{i=1}^{4} m_i x_i^2 = 50(2.0)^2 + (25)(0)^2 + 25(3.0)^2 + 30(2.0)^2 = 5.5 \times 10^2 \text{ g}\cdot\text{cm}^2 \ .$$

(c) And about the z axis, we find (using the fact that the distance from the z axis is $\sqrt{x^2 + y^2}$)

$$I_z = \sum_{i=1}^{4} m_i (x_i^2 + y_i^2) = I_x + I_y = 1.3 \times 10^3 + 5.5 \times 10^2 = 1.9 \times 10^2 \text{ g}\cdot\text{cm}^2 \ .$$

(d) Clearly, the answer to part (c) is $A + B$.

43. (a) According to Table 11-2, the rotational inertia formulas for the cylinder (radius R) and the hoop (radius r) are given by

$$I_C = \frac{1}{2}MR^2 \quad \text{and} \quad I_H = Mr^2 \ .$$

Since the two bodies have the same mass, then they will have the same rotational inertia if $R^2/2 = R_H^2$, or $R_H = R/\sqrt{2}$.

(b) We require the rotational inertia to be written as $I = Mk^2$, where M is the mass of the given body and k is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I/M}$.

44. (a) Using Table 11-2(c) and Eq. 11-27, the rotational kinetic energy is

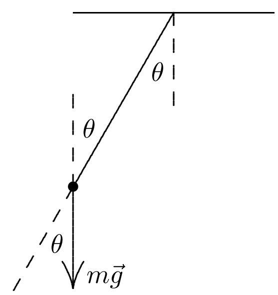
$$\begin{aligned} K &= \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 \\ &= \frac{1}{4}(500\,\text{kg})(200\pi\,\text{rad/s})^2(1.0\,\text{m})^2 \\ &= 4.9 \times 10^7 \text{ J} \ . \end{aligned}$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9 \times 10^7 \text{ J}}{8.0 \times 10^3 \text{ W}} = 6.2 \times 10^3 \text{ s}$$

which we rewrite as $t \approx 100$ min.

45. Two forces act on the ball, the force of the rod and the force of gravity. No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is $mg \sin\theta$. If ℓ is the length of the rod, then the torque associated with this force has magnitude $\tau = mg\ell \sin\theta = (0.75)(9.8)(1.25)\sin 30° = 4.6 \text{ N}\cdot\text{m}$. For the position shown, the torque is counterclockwise.



46. We compute the torques using $\tau = rF\sin\phi$.

$$\begin{aligned}
\tau_a &= (0.152\text{ m})(111\text{ N})\sin 30° = 8.4 \text{ N}\cdot\text{m} \\
\tau_b &= (0.152\text{ m})(111\text{ N})\sin 90° = 17 \text{ N}\cdot\text{m} \\
\tau_c &= (0.152\text{ m})(111\text{ N})\sin 180° = 0
\end{aligned}$$

47. (a) We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1F_1\sin\theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2F_2\sin\theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2 \ .$$

(b) Substituting the given values, we obtain

$$\tau = (1.30\text{ m})(4.20\text{ N})\sin 75° - (2.15\text{ m})(4.90\text{ N})\sin 60° = -3.85 \text{ N}\cdot\text{m} \ .$$

48. The net torque is

$$\begin{aligned}
\tau &= \tau_A + \tau_B + \tau_C \\
&= F_Ar_A\sin\phi_A - F_Br_B\sin\phi_B + F_Cr_C\sin\phi_C \\
&= (10)(8.0)\sin 135° - (16)(4.0)\sin 90° + (19)(3.0)\sin 160° \\
&= 12 \text{ N}\cdot\text{m} \ .
\end{aligned}$$

49. (a) We use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, and t is the time. This gives

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad/s}}{220 \times 10^{-3}\text{ s}} = 28.2 \text{ rad/s}^2 \ .$$

(b) If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is

$$\tau = I\alpha = \left(12.0\,\text{kg}\cdot\text{m}^2\right)\left(28.2\,\text{rad/s}^2\right) = 3.38\times 10^2\,\text{N}\cdot\text{m} .$$

50. The rotational inertia is found from Eq. 11-37.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28\ \text{kg}\cdot\text{m}^2$$

51. (a) We use $\tau = I\alpha$, where τ is the net torque acting on the shell, I is the rotational inertia of the shell, and α is its angular acceleration. Therefore,

$$I = \frac{\tau}{\alpha} = \frac{960\,\text{N}\cdot\text{m}}{6.20\,\text{rad/s}^2} = 155\,\text{kg}\cdot\text{m}^2 .$$

(b) The rotational inertia of the shell is given by $I = (2/3)MR^2$ (see Table 11-2 of the text). This implies

$$M = \frac{3I}{2R^2} = \frac{3(155\,\text{kg}\cdot\text{m}^2)}{2(1.90\,\text{m})^2} = 64.4\,\text{kg} .$$

52. According to the sign conventions used in the book, the magnitude of the net torque exerted on the cylinder of mass m and radius R_2 is

$$\begin{aligned}\tau_{\text{net}} &= F_1R_2 - F_2R_2 - F_3R_1\\ &= (6.0\,\text{N})(0.12\,\text{m}) - (4.0\,\text{N})(0.12\,\text{m}) - (2.0\,\text{N})(0.05\,\text{m})\\ &= 71\ \text{N}\cdot\text{m} .\end{aligned}$$

The resulting angular acceleration of the cylinder (with $I = \frac{1}{2}MR^2$ according to Table 11-2(c)) is

$$\begin{aligned}\alpha &= \frac{\tau_{\text{net}}}{I}\\ &= \frac{71\ \text{N}\cdot\text{m}}{\frac{1}{2}(2.0\,\text{kg})(0.12\,\text{m})^2}\\ &= 9.7\ \text{rad/s}^2\end{aligned}$$

and is counterclockwise (which is the positive sense of rotation).

53. We use $\tau = Fr = I\alpha$, where α satisfies $\theta = \frac{1}{2}\alpha t^2$ (Eq. 11-13). Here $\theta = 90° = \frac{\pi}{2}$ rad and $t = 30\,\text{s}$. The force needed is consequently

$$F = \frac{I\alpha}{r} = \frac{I\left(2\theta/t^2\right)}{r} = \frac{(8.7\times 10^4)\left(2(\pi/2)/30^2\right)}{2.4} = 1.3\times 10^2\,\text{N} .$$

54. With rightward positive for the block and clockwise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to

$$P - T = ma \qquad\qquad \text{where } m = 2.0\ \text{kg} .$$

Applying Newton's second law (for rotation) to the wheel leads to

$$-TR = I\alpha \qquad\qquad \text{where } I = 0.050\ \text{kg}\cdot\text{m}^2 .$$

Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \implies T = a\,\frac{I}{R^2} .$$

Adding this to the above equation (for the block) leads to

$$P = \left(m + \frac{I}{R^2}\right) a \ .$$

Thus, $a = 0.92 \text{ m/s}^2$ and therefore $\alpha = -4.6 \text{ rad/s}^2$, where the negative sign should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

55. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750\,\text{m})}{(5.00\,\text{s})^2} = 6.00 \times 10^{-2} \text{ m/s}^2 \ .$$

The lighter block has an acceleration of $6.00 \times 10^{-2}\,\text{m/s}^2$ upward.

(b) Newton's second law for the heavier block is $m_h g - T_h = m_h a$, where m_h is its mass and T_h is the tension force on the block. Thus,

$$T_h = m_h(g - a) = (0.500\,\text{kg})\left(9.8\,\text{m/s}^2 - 6.00 \times 10^{-2}\,\text{m/s}^2\right) = 4.87 \text{ N} \ .$$

(c) Newton's second law for the lighter block is $m_l g - T_l = -m_l a$, where T_l is the tension force on the block. Thus,

$$T_l = m_l(g + a) = (0.460\,\text{kg})\left(9.8\,\text{m/s}^2 + 6.00 \times 10^{-2}\,\text{m/s}^2\right) = 4.54\,\text{N} \ .$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00 \times 10^{-2}\,\text{m/s}^2}{5.00 \times 10^{-2}\,\text{m}} = 1.20 \text{ rad/s}^2 \ .$$

(e) The net torque acting on the pulley is $\tau = (T_h - T_l)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$\begin{aligned} I &= \frac{(T_h - T_l)R}{\alpha} \\ &= \frac{(4.87\,\text{N} - 4.54\,\text{N})(5.00 \times 10^{-2}\,\text{m})}{1.20\,\text{rad/s}^2} \\ &= 1.38 \times 10^{-2} \text{ kg}\cdot\text{m}^2 \ . \end{aligned}$$

56. Since the force acts tangentially at $r = 0.10$ m, the angular acceleration (presumed positive) is

$$\alpha = \frac{\tau}{I} = \frac{Fr}{I} = \frac{\left(0.5t + 0.3t^2\right)(0.10)}{1.0 \times 10^{-3}} = 50t + 30t^2$$

in SI units (rad/s^2).

(a) At $t = 3$ s, the above expression becomes $\alpha = 420$ rad/s^2.

(b) We integrate the above expression, noting that $\omega_o = 0$, to obtain the angular speed at $t = 3$ s:

$$\omega = \int_0^3 \alpha\, dt = \left(25t^2 + 10t^3\right)\Big|_0^3 = 5.0 \times 10^2\,\text{rad/s} \ .$$

57. With counterclockwise positive, the angular acceleration α for both masses satisfies $\tau = mgL_1 - mgL_2 = I\alpha = (mL_1^2 + mL_2^2)\alpha$, by combining Eq. 11-37 with Eq. 11-32 and Eq. 11-26. Therefore, using SI units,

$$\alpha = \frac{g\,(L_1 - L_2)}{L_1^2 + L_2^2} = \frac{(9.8)(0.20 - 0.80)}{0.80^2 + 0.20^2} = -8.65 \text{ rad/s}^2$$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 11-22 and obtain the respective answers for parts (a) and (b):

$$\begin{aligned} |\vec{a}_1| = |\alpha| L_1 &= \left(8.65\,\text{rad/s}^2\right)(0.80\,\text{m}) = 6.9 \text{ m/s}^2 \\ |\vec{a}_2| = |\alpha| L_2 & \\ &= \left(8.65\,\text{rad/s}^2\right)(0.20\,\text{m}) \\ &= 1.7 \text{ m/s}^2 . \end{aligned}$$

58. (a) The speed of v of the mass m after it has descended $d = 50\,\text{cm}$ is given by $v^2 = 2ad$ (Eq. 2-16) where a is calculated as in Sample Problem 11-7 except that here we choose $+y$ downward (so $a > 0$). Thus, using $g = 980$ cm/s^2, we have

$$v = \sqrt{2ad} = \sqrt{\frac{2(2mg)d}{M + 2m}} = \sqrt{\frac{4(50)(980)(50)}{400 + 2(50)}} = 1.4 \times 10^2\,\text{cm/s} .$$

(b) The answer is still $1.4 \times 10^2\,\text{cm/s} = 1.4\,\text{m/s}$, since it is independent of R.

59. With $\omega = (1800)(2\pi/60) = 188.5$ rad/s, we apply Eq. 11-47:

$$P = \tau\omega \implies \tau = \frac{74600\,\text{W}}{188.5\,\text{rad/s}}$$

which yields $\tau = 396$ N·m.

60. The initial angular speed is $\omega = (280)(2\pi/60) = 29.3$ rad/s. We use Eq. 11-44 for the work and Eq. 7-42 for the average power.

(a) Since the rotational inertia is (Table 11-2(a)) $I = (32)(1.2)^2 = 46.1$ kg·m^2, the work done is

$$W = \Delta K = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}(46.1)(29.3)^2$$

which yields $|W| = 19.8 \times 10^3$ J.

(b) The average power (in absolute value) is therefore

$$|P| = \frac{|W|}{\Delta t} = \frac{19.8 \times 10^3}{15} = 1.32 \times 10^3\,\text{W} .$$

61. (a) We apply Eq. 11-27:

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 = \frac{1}{6}mL^2\omega^2 .$$

(b) Simple conservation of mechanical energy leads to $K = mgh$. Consequently, the center of mass rises by

$$h = \frac{K}{mg} = \frac{mL^2\omega^2}{6mg} = \frac{L^2\omega^2}{6g} .$$

62. (a) The angular speed ω associated with Earth's spin is $\omega = 2\pi/T$, where $T = 86400\,\text{s}$ (one day). Thus

$$\omega = \frac{2\pi}{86400\,\text{s}} = 7.27 \times 10^{-5}\,\text{rad/s}$$

and the angular acceleration α required to accelerate the Earth from rest to ω in one day is $\alpha = \omega/T$. The torque needed is then

$$\tau = I\alpha = \frac{I\omega}{T} = \frac{(9.71 \times 10^{27})\,(7.27 \times 10^{-5})}{86400} = 8.17 \times 10^{28}\ \text{N}\cdot\text{m}$$

where we used

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left(5.98 \times 10^{24}\right)\left(6.37 \times 10^{6}\right)^2$$

for Earth's rotational inertia.

(b) Using the values from part (a), the kinetic energy of the Earth associated with its rotation about its own axis is $K = \frac{1}{2}I\omega^2 = 2.57 \times 10^{29}\,\text{J}$. This is how much energy would need to be supplied to bring it (starting from rest) to the current angular speed.

(c) The associated power is

$$P = \frac{K}{T} = \frac{2.57 \times 10^{29}\,\text{J}}{86400\,\text{s}} = 2.97 \times 10^{24}\ \text{W}\ .$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $\frac{1}{2}mg\ell$, where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \quad\Longrightarrow\quad \omega = \sqrt{\frac{mg\ell}{I}}\ .$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 11-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}\ .$$

Using Table 11-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8\,\text{m/s}^2\right)(1.00\,\text{m})} = 5.42\ \text{m/s}\ .$$

64. (a) We use the parallel-axis theorem to find the rotational inertia:

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{1}{2}MR^2 + Mh^2 \\ &= \frac{1}{2}(20\,\text{kg})(0.10\,\text{m})^2 + (20\,\text{kg})(0.50\,\text{m})^2 \\ &= 0.15\ \text{kg}\cdot\text{m}^2\ . \end{aligned}$$

(b) Conservation of energy requires that $Mgh = \frac{1}{2}I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20)(9.8)(0.050)}{0.15}} = 11\ \text{rad/s}\ .$$

65. We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by $L/2$, where L is the length of any one of the rods. The gravitational potential energy decreases by $MgL/2$, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written $\frac{1}{2}I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus

$$0 = -MgL/2 + \frac{1}{2}I\omega^2 \implies \omega = \sqrt{MgL/I} \ .$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is $I = (ML^2/3) + (ML^2/9) = 4ML^2/9$. Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} \ .$$

66. From Table 11-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega_{\text{sphere}}^2 + \frac{1}{2}I\omega_{\text{pulley}}^2 + \frac{1}{2}mv^2 \ .$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

67. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2}I\omega^2 \implies \omega^2 = (MgH/I)(1 - \cos\theta) \ .$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 11-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1 - \cos\theta)} \ .$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so $a_r = 3g(1 - \cos\theta)$.

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 11-1 since the acceleration is not uniform. Hence, we differentiate $\omega^2 = (3g/H)(1 - \cos\theta)$ with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \implies \alpha = (3g/2H)\sin\theta \ .$$

Consequently, $a_t = H\alpha = \frac{3g}{2}\sin\theta$.

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

68. (a) The longitudinal separation between Helsinki and the explosion site is $\Delta\theta = 102° - 25° = 77°$. The spin of the earth is constant at

$$\omega = \frac{1\,\text{rev}}{1\,\text{day}} = \frac{360°}{24\,\text{h}}$$

so that an angular displacement of $\Delta\theta$ corresponds to a time interval of

$$\Delta t = (77°)\left(\frac{24\,\text{h}}{360°}\right) = 5.1\text{ h} .$$

(b) Now $\Delta\theta = 102° - (-20°) = 122°$ so the required time shift would be

$$\Delta t = (122°)\left(\frac{24\,\text{h}}{360°}\right) = 8.1\text{ h} .$$

69. Analyzing the forces tending to drag the $M = 5124$ kg stone down the oak beam, we find

$$F = Mg\,(\sin\theta + \mu_s\cos\theta)$$

where $\mu_s = 0.22$ (static friction is assumed to be at its maximum value) and the incline angle θ for the oak beam is $\sin^{-1}(3.9/10) = 23°$ (but the incline angle for the spruce log is the complement of that). We note that the component of the weight of the workers (N of them) which is perpendicular to the spruce log is $Nmg\cos(90° - \theta) = Nmg\sin\theta$, where $m = 85$ kg. The corresponding torque is therefore $Nmg\ell\sin\theta$ where $\ell = 4.5 - 0.7 = 3.8$ m (see figure). This must (at least) equal the magnitude of torque due to F, so with $r = 0.7$ m, we have

$$Mgr\,(\sin\theta + \mu_s\cos\theta) = Ngm\ell\sin\theta .$$

This expression yields $N \approx 17$ for the number of workers.

70. (a) We apply Eq. 11-18, using the subscript J for the Jeep.

$$\omega = \frac{v_J}{r_J} = \frac{114\,\text{km/h}}{0.100\,\text{km}}$$

which yields 1140 rad/h or (dividing by 3600) 0.32 rad/s for the value of the angular speed ω.

(b) Since the cheetah has the same angular speed, we again apply Eq. 11-18, using the subscript c for the cheetah.

$$v_c = r_c\omega = (92\,\text{m})(1140\,\text{rad/h})$$

which yields 1.05×10^5 m/h or 105 km/h for the cheetah's speed.

71. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or – simply – in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass dm located a distance r from the rotational axis is (Newton's second law) $dF = (dm)\omega^2 r$, where dm can be written as $(M/L)dr$ and the angular speed is $\omega = (320)(2\pi/60) = 33.5$ rad/s. Thus for the entire blade of mass M and length L the total force is given by

$$\begin{aligned} F &= \int dF = \int \omega^2\, r\, dm \\ &= \frac{M}{L}\int_0^L \omega^2\, r\, dr \end{aligned}$$

$$
\begin{aligned}
&= \left.\frac{M\omega^2 r^2}{2L}\right|_0^L = \frac{M\omega^2 L}{2} \\
&= \frac{(110\,\text{kg})(33.5\,\text{rad/s})^2(7.80\,\text{m})}{2} \\
&= 4.8\times 10^5 \text{ N} .
\end{aligned}
$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 11-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 11-37, the torque (assumed constant) is

$$
\begin{aligned}
\tau &= I\alpha \\
&= \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right) \\
&= \frac{1}{3}(110\,\text{kg})(7.8\,\text{m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right) \\
&= 1.1\times 10^4 \text{ N}\cdot\text{m} .
\end{aligned}
$$

(c) Using Eq. 11-44, the work done is

$$
\begin{aligned}
W &= \Delta K = \frac{1}{2}I\omega^2 - 0 \\
&= \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \\
&= \frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2 \\
&= 1.3\times 10^6 \text{ J} .
\end{aligned}
$$

72. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration α. If ω_0 is the initial angular velocity and t is the time to come to rest, then

$$0 = \omega_0 + \alpha t \quad \Longrightarrow \quad \alpha = -\frac{\omega_0}{t}$$

which yields $-39/32 = -1.2$ rev/s or (multiplying by 2π) $-7.66\,\text{rad/s}^2$ for the value of α.

(b) We use $\tau = I\alpha$, where τ is the torque and I is the rotational inertia. The contribution of the rod to I is $M\ell^2/12$ (Table 11-2(e)), where M is its mass and ℓ is its length. The contribution of each ball is $m(\ell/2)^2$, where m is the mass of a ball. The total rotational inertia is

$$I = \frac{M\ell^2}{12} + 2\frac{m\ell^2}{4} = \frac{(6.40\,\text{kg})(1.20\,\text{m})^2}{12} + \frac{(1.06\,\text{kg})(1.20\,\text{m})^2}{2}$$

which yields $I = 1.53$ kg$\cdot$m^2. The torque, therefore, is

$$\tau = \left(1.53\,\text{kg}\cdot\text{m}^2\right)\left(-7.66\,\text{rad/s}^2\right) = -11.7 \text{ N}\cdot\text{m} .$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i = \frac{1}{2}I\omega_0^2 = \frac{1}{2}\left(1.53\,\text{kg}\cdot\text{m}^2\right)\left((2\pi)(39)\,\text{rad/s}\right)^2 = 4.59\times 10^4 \text{ J} .$$

(d) We apply Eq. 11-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \left((2\pi)(39)\,\text{rad/s}\right)(32.0\,\text{s}) + \frac{1}{2}\left(-7.66\,\text{rad/s}^2\right)(32.0\,\text{s})^2$$

which yields 3920 rad or (dividing by 2π) 624 rev for the value of angular displacement θ.

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is 4.59×10^4 J no matter how τ varies with time, as long as the system comes to rest.

73. We assume the given rate of 1.2×10^{-3} m/y is the linear speed of the top; it is also possible to interpret it as just the horizontal component of the linear speed but the difference between these interpretations is arguably negligible. Thus, Eq. 11-18 leads to

$$\omega = \frac{1.2 \times 10^{-3}\,\text{m/y}}{55\,\text{m}} = 2.18 \times 10^{-5}\ \text{rad/y}$$

which we convert (since there are about 3.16×10^7 s in a year) to $\omega = 6.9 \times 10^{-13}$ rad/s.

74. The rotational inertia of the passengers is (to a good approximation) given by Eq. 11-26: $I = \sum mR^2 = NmR^2$ where N is the number of people and m is the (estimated) mass per person. We apply Eq. 11-44:

$$W = \frac{1}{2}I\omega^2 = \frac{1}{2}NmR^2\omega^2 \ .$$

where $R = 38$ m and $N = 36 \times 60 = 2160$ persons. The rotation rate is constant so that $\omega = \theta/t$ which leads to $\omega = 2\pi/120 = 0.052$ rad/s. The mass (in kg) of the average person is probably in the range $50 \leq m \leq 100$, so the work should be in the range

$$\begin{aligned} \frac{1}{2}(2160)(50)(38)^2(0.052)^2 \ &\leq \ W \ \leq \ \frac{1}{2}(2160)(100)(38)^2(0.052)^2 \\ 2 \times 10^5\ \text{J} \ &\leq \ W \ \leq \ 4 \times 10^5\ \text{J} \ . \end{aligned}$$

75. (a) The axis of rotation is at the bottom right edge of the rod along the ground, a horizontal distance of $d_3 + d_2 + d_1/2$ from the middle of the table assembly (mass $m = 90$ kg). The linebacker's center of mass at that critical moment was a horizontal distance of $d_4 + d_5$ from the axis of rotation. For the clockwise torque caused by the linebacker (mass M) to overcome the counterclockwise torque of the table assembly, we require (using Eq. 11-33)

$$Mg\,(d_4 + d_5) > mg\left(d_3 + d_2 + \frac{d_1}{2}\right) \ .$$

With the values given in the problem, we do indeed find the inequality is satisfied.

(b) Replacing our inequality with an equality and solving for M, we obtain

$$M = m\,\frac{d_3 + d_2 + \frac{1}{2}d_1}{d_4 + d_5} = 114\ \text{kg} \ .$$

76. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2 and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 11-37) to M, respectively, we arrive at the following three equations.

$$\begin{aligned} T_1 - m_1 g &= m_1 a_1 \\ m_2 g - T_2 &= m_2 a_2 \\ T_2 R - T_1 R &= I\alpha \end{aligned}$$

(a) The rotational inertia of the disk is $I = \frac{1}{2}MR^2$ (Table 11-2(c)), so we divide the third equation (above) by R, add them all, and use the earlier equality among accelerations – to obtain:

$$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

which yields $a = \frac{4}{25}\,g = 1.6$ m/s^2.

(b) Plugging back in to the first equation, we find $T_1 = \frac{29}{24} m_1 g = 4.6$ N (where it is important in this step to have the mass in SI units: $m_1 = 0.40$ kg).

(c) Similarly, with $m_2 = 0.60$ kg, we find $T_2 = \frac{5}{6} m_2 g = 4.9$ N.

77. We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$$K_{\text{box}} = \frac{1}{2} m_{\text{box}} v^2 \implies v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = 1.41 \text{ m/s}$$

implies that the angular speed is $\omega = 1.41/0.20 = 0.71$ rad/s. Thus, the kinetic energy of rotation is $\frac{1}{2} I\omega^2 = 10.0$ J.

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$\begin{aligned} K_0 + U_0 &= K + U \\ 0 + 0 &= (6.0 + 10.0) + m_{\text{box}} g(-h) \ . \end{aligned}$$

Therefore, $h = 16.0/58.8 = 0.27$ m.

78. The distances from P to the particles are as follows:

$$\begin{aligned} r_1 &= a \quad \text{for } \ m_1 = 2M \quad \text{(lower left)} \\ r_2 &= \sqrt{b^2 - a^2} \quad \text{for } \ m_2 = M \quad \text{(top)} \\ r_3 &= a \quad \text{for } \ m_1 = 2M \quad \text{(lower right)} \end{aligned}$$

The rotational inertia of the system about P is

$$I = \sum_{i=1}^{3} m_i r_i^2 = \left(3a^2 + b^2\right) M$$

which yields $I = 0.208$ kg·m^2 for $M = 0.40$ kg, $a = 0.30$ m and $b = 0.50$ m. Applying Eq. 11-44, we find

$$W = \frac{1}{2} I\omega^2 = \frac{1}{2}(0.208)(5.0)^2 = 2.6 \text{ J} \ .$$

79. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 11-37) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2).

$$\begin{aligned} m_1 g - T_1 &= m_1 a_1 \\ T_2 - f_2 &= m_2 a_2 \\ T_1 R - T_2 R &= I\alpha \end{aligned}$$

(a) From Eq. 11-13 (with $\omega_0 = 0$) we find

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \implies \alpha = \frac{2\theta}{t^2} \ .$$

(b) From the fact that $a = R\alpha$ (noted above), we obtain $a = 2R\theta/t^2$.

(c) From the first of the above equations, we find

$$T_1 = m_1 (g - a_1) = M \left(g - \frac{2R\theta}{t^2} \right) .$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = M \left(g - \frac{2R\theta}{t^2} \right) - \frac{2I\theta}{Rt^2}$$

80. (a) With $r = 0.780$ m, the rotational inertia is

$$I = Mr^2 = (1.30\,\text{kg})(0.780\,\text{m})^2 = 0.791 \text{ kg}\cdot\text{m}^2 .$$

(b) The torque that must be applied to counteract the effect of the drag is

$$\tau = rf = (0.780\,\text{m})(2.30 \times 10^{-2}\,\text{N}) = 1.79 \times 10^{-2} \text{ N}\cdot\text{m} .$$

81. (a) The rotational inertia relative to the specified axis is

$$I = \sum m_i r_i^2 = (2M)L^2 + (2M)L^2 + M(2L)^2$$

which is found to be $I = 4.6$ kg·m^2. Then, with $\omega = 1.2$ rad/s, we obtain the kinetic energy from Eq. 11-27:

$$K = \frac{1}{2} I\omega^2 = 3.3 \text{ J} .$$

(b) In this case the axis of rotation would appear as a standard y axis with origin at P. Each of the $2M$ balls are a distance of $r = L\cos 30°$ from that axis. Thus, the rotational inertia in this case is

$$I = \sum m_i r_i^2 = (2M)r^2 + (2M)r^2 + M(2L)^2$$

which is found to be $I = 4.0$ kg·m^2. Again, from Eq. 11-27 we obtain the kinetic energy

$$K = \frac{1}{2} I\omega^2 = 2.9 \text{ J} .$$

82. We make use of Table 11-2(e) as well as the parallel-axis theorem, Eq. 11-27, where needed. We use ℓ (as a subscript) to refer to the long rod and s to refer to the short rod.

(a) The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12} m_s L_s^2 + \frac{1}{3} m_\ell L_\ell^2 = 0.019 \text{ kg}\cdot\text{m}^2 .$$

(b) We note that the center of the short rod is a distance of $h = 0.25$ m from the axis. The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12} m_s L_s^2 + m_s h^2 + \frac{1}{12} m_\ell L_\ell^2$$

which again yields $I = 0.019$ kg·m^2.

83. This may be derived from Eq. 11-28 or (suitably interpreted) from Eq. 11-26. Since every element of the hoop has the same radius $r = R$, the integration (or summation, if preferred) is trivial: $I = \int r^2\, dm = R^2 \int dm = MR^2$.

84. (a) Using Eq. 11-15 with $\omega = 0$, we have

$$\theta = \frac{\omega_0 + \omega}{2} t = 2.8 \text{ rad} .$$

(b) One ingredient in this calculation is $\alpha = (0 - 3.5\,\text{rad/s})/(1.6\,\text{s}) = -2.2\,\text{rad/s}^2$, so that the tangential acceleration is $r\alpha = 0.33\ \text{m/s}^2$. Another ingredient is $\omega = \omega_0 + \alpha t = 1.3$ rad/s for $t = 1.0$ s, so that the radial (centripetal) acceleration is $\omega^2 r = 0.26\,\text{m/s}^2$. Thus, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{0.33^2 + 0.26^2} = 0.42\ \text{m/s}^2 .$$

85. (a) Using $T = 1$ yr $= 3.16 \times 10^7$ s for the time to make one full revolution (or 2π rad), we obtain

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.16 \times 10^7} = 2.0 \times 10^{-7}\,\text{rad/s} .$$

(b) The radius r of Earth's orbit can be found in Appendix C or the inside front cover. Eq. 11-18 gives

$$v = \alpha r = (2.0 \times 10^{-7}\,\text{rad/s})(1.49 \times 10^{11}\,\text{m}) = 3.0 \times 10^4\,\text{m/s} .$$

(c) The (radial, or centripetal) acceleration is

$$a = \omega^2 r = \left(2.0 \times 10^{-7}\,\text{rad/s}\right)^2 (1.49 \times 10^{11}\,\text{m}) = 5.9 \times 10^{-3}\,\text{m/s}^2 .$$

The direction of $\vec{a}$ is toward the sun.

86. Using Eq. 11-12, we have

$$\omega = \omega_0 + \alpha t \implies \alpha = \frac{2.6 - 8.0}{3.0}$$

which yields $\alpha = -1.8$ rad/s^2. Using this value in Eq. 11-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \implies \theta = \frac{0^2 - 8.0^2}{2\,(-1.8)} = 18 \text{ rad} .$$

87. The motion consists of two stages. The first, the interval $0 \le t \le 20$ s, consists of constant angular acceleration given by

$$\alpha = \frac{5.0\,\text{rad/s}}{2.0\,\text{s}} = 2.5\ \text{rad/s}^2 .$$

The second stage, $20 < t \le 40$ s, consists of constant angular velocity $\omega = \Delta\theta/\Delta t$. Analyzing the first stage, we find

$$\begin{aligned} \theta_1 &= \left.\frac{1}{2}\alpha t^2\right|_{t=20} = 500 \text{ rad} \\ \omega &= \left.\alpha t\right|_{t=20} = 50 \text{ rad/s} . \end{aligned}$$

Analyzing the second stage, we obtain

$$\theta_2 = \theta_1 + \omega\Delta t = 500 + (50)(20) = 1500 \text{ rad} .$$

88. (a) Eq. 11-12 leads to $\alpha = -\omega_o/t = -25.0/20.0 = -1.25\,\text{rad/s}^2$.

(b) Eq. 11-15 leads to $\theta = \frac{1}{2}\omega_o t = \frac{1}{2}(25.0)(20.0) = 250\,\text{rad}$.

(c) Dividing the previous result by 2π we obtain $\theta = 39.8$ rev.

89. (a) We integrate the angular acceleration (as a function of τ) with respect to τ to find the angular velocity as a function of $t > 0$.

$$\omega = \omega_0 + \int_0^t \left(4a\tau^3 - 3b\tau^2\right) d\tau = \omega_0 + at^4 - bt^3 \ .$$

(b) We integrate the angular velocity (as a function of τ) with respect to τ to find the angular position as a function of $t > 0$.

$$\theta = \theta_0 + \int_0^t \left(4a\tau^3 - 3b\tau^2\right) d\tau = \theta_0 + \omega_0 t + \frac{a}{5}t^5 - \frac{b}{4}t^4 \ .$$

90. (a) The particle at A has $r = 0$ with respect to the axis of rotation. The particle at B is $r = L = 0.50$ m from the axis; similarly for the particle directly above A in the figure. The particle diagonally opposite A is a distance $r = \sqrt{2}\,L = 0.71$ m from the axis. Therefore,

$$I = \sum m_i r_i^2 = 2mL^2 + m\left(\sqrt{2}\,L\right)^2 = 0.20 \text{ kg}\cdot\text{m}^2 \ .$$

(b) One imagines rotating the figure (about point A) clockwise by 90° and noting that the center of mass has fallen a distance equal to L as a result. If we let our reference position for gravitational potential be the height of the center of mass at the instant AB swings through vertical orientation, then

$$\begin{aligned} K_0 + U_0 &= K + U \\ 0 + (4m)gh_0 &= K + 0 \ . \end{aligned}$$

Since $h_0 = L = 0.50$ m, we find $K = 3.9$ J. Then, using Eq. 11-27, we obtain

$$K = \frac{1}{2} I_A\, \omega^2 \implies \omega = 6.3 \ \frac{\text{rad}}{\text{s}} \ .$$

91. The center of mass is initially at height $h = \frac{L}{2}\sin 40°$ when the system is released (where $L = 2.0$ m). The corresponding potential energy Mgh (where $M = 1.5$ kg) becomes rotational kinetic energy $\frac{1}{2}I\omega^2$ as it passes the horizontal position (where I is the rotational inertia about the pin). Using Table 11-2(e) and the parallel axis theorem, we find $I = \frac{1}{12}ML^2 + M(L/2)^2 = \frac{1}{3}ML^2$. Therefore,

$$Mg\frac{L}{2}\sin 40° = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \implies \omega = \sqrt{\frac{3g\sin 40°}{L}}$$

which yields $\omega = 3.1$ rad/s.

92. We choose $\pm$ directions such that the initial angular velocity is $\omega_0 = -317$ rad/s and the values for α, τ and F are positive.

(a) Combining Eq. 11-12 with Eq. 11-37 and Table 11-2(f) (and using the fact that $\omega = 0$) we arrive at the expression

$$\tau = \left(\frac{2}{5}MR^2\right)\left(-\frac{\omega_0}{t}\right) = -\frac{2}{5}\,\frac{MR^2\omega_0}{t} \ .$$

With $t = 15.5$ s, $R = 0.226$ m and $M = 1.65$ kg, we obtain $\tau = 0.689$ N·m.

(b) From Eq. 11-32, we find $F = \tau/R = 3.05$ N.

(c) Using again the expression found in part (a), but this time with $R = 0.854$ m, we get $\tau = 9.84$ N·m.

(d) Now, $F = \tau/R = 11.5$ N.

93. We choose positive coordinate directions so that each is accelerating positively, which will allow us to set $a_{\text{box}} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the $m = 2.0$ kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 11-37) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).

$$\begin{aligned} mg\sin\theta - T &= ma \\ TR &= I\alpha \end{aligned}$$

Since the problem gives $a = 2.0$ m/s^2, the first equation gives the tension $T = m(g\sin\theta - a) = 2.7$ N. Plugging this and $R = 0.20$ m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia $I = TR^2/a = 0.054\,\text{kg}\cdot\text{m}^2$.

94. Eq. 11-32 leads to $\tau = mgr = (70)(9.8)(0.20)$ in SI units, which yields $\tau = 1.4 \times 10^2$ N.

95. The disk centered on A has $I = \frac{1}{2}MR^2$ (Table 11-2(c)) about that point, but the rotational inertia of the other disk is found using the parallel-axis theorem $I = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$ about point A. Adding these two results, we obtain

$$\frac{1}{2}MR^2 + \frac{9}{2}MR^2 = 5MR^2 = 5(4.0)(0.40)^2$$

which yields $3.2\,\text{kg}\cdot\text{m}^2$.

96. (a) One particle is on the axis, so $r = 0$ for it. For each of the others, the distance from the axis is $r = (0.60\text{ m})\sin 60° = 0.52$ m. Therefore, the rotational inertia is $I = \sum m_i r_i^2 = 0.27\text{ kg}\cdot\text{m}^2$.

(b) The two particles that are nearest the axis are each a distance of $r = 0.30$ m from it. The particle "opposite" from that side is a distance $r = (0.60\text{ m})\sin 60° = 0.52$ m from the axis. Thus, the rotational inertia is $I = \sum m_i r_i^2 = 0.22\text{ kg}\cdot\text{m}^2$.

(c) The distance from the axis for each of the particles is $r = \frac{1}{2}(0.60\,\text{m})\sin 60°$. Now, $I = 3(0.50\,\text{kg})(0.26\,\text{m})^2 = 0.10\,\text{kg}\cdot\text{m}^2$.

97. The parallel axis theorem gives $I = I_{\text{com}} + Mh^2$ for the rotational inertia about any axis (parallel to the axis used to compute I_{com}). Let us assume that an axis has already been chosen through the center of mass of the body such that I_{com} is as small as it possibly can be. Since $Mh^2 >$ for all nonzero values of h, then $I > I_{\text{com}}$ from the parallel axis theorem as long as $h \neq 0$. Thus, with $h = 0$ we get $I = I_{\text{com}}$ and therefore the smallest possible value of rotational inertia.

98. (a) The linear speed at $t = 15.0$ s is

$$v = a_t t = \left(0.500\,\text{m/s}^2\right)(15.0\,\text{s}) = 7.50\text{ m/s} .$$

The radial (centripetal) acceleration at that moment is

$$a_r = \frac{v^2}{r} = \frac{(7.50\,\text{m/s})^2}{30.0\,\text{m}} = 1.875\text{ m/s}^2 .$$

Thus, the net acceleration has magnitude:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(0.500\,\text{m/s}^2\right)^2 + \left(1.875\,\text{m/s}^2\right)^2} = 1.94\text{ m/s}^2 .$$

(b) We note that $\vec{a}_t \parallel \vec{v}$. Therefore, the angle between $\vec{v}$ and $\vec{a}$ is

$$\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.875}{0.5}\right) = 75.1°$$

so that the vector is pointing more toward the center of the track than in the direction of motion.

99. First, we convert the angular velocity: $\omega = (2000)(2\pi/60) = 209$ rad/s. Also, we convert the plane's speed to SI units: $(480)(1000/3600) = 133$ m/s. We use Eq. 11-18 in part (a) and (implicitly) Eq. 4-39 in part (b).

(a) The speed of the tip as seen by the pilot is

$$v_t = \omega r = (209\,\text{rad/s})(1.5\,\text{m}) = 314 \text{ m/s}$$

which (since the radius is given to only two significant figures) we write as $v = 3.1 \times 10^2$ m/s.

(b) The plane's velocity $\vec{v}_p$ and the velocity of the tip $\vec{v}_t$ (found in the plane's frame of reference), in any of the tip's positions, must be perpendicular to each other. Thus, the speed as seen by an observer on the ground is

$$v = \sqrt{v_p^2 + v_t^2} = \sqrt{(133\,\text{m/s})^2 + (314\,\text{m/s})^2} \;=\; 3.4 \times 10^2 \text{ m/s} .$$

100. Using Eq. 11-7 and Eq. 11-18, the average angular acceleration is

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} = \frac{25 - 12}{(0.75/2)(6.2)} = 5.6 \text{ rad/s}^2 .$$

101. (a) Eq. 11-15 gives

$$90\,\text{rev} = \frac{1}{2}\left(\omega_0 + 10\,\text{rev/s}\right)(15\,\text{s})$$

which leads to $\omega_0 = 2.0\,\text{rev/s}$.

(b) From Eq. 11-12, the angular acceleration is

$$\alpha = \frac{10\,\text{rev/s} - 2.0\,\text{rev/s}}{15\,\text{s}} = 0.53 \text{ rev/s}^2 .$$

Using the equation again (with the same value for α) we seek a *negative* value of t (meaning an earlier time than that when $\omega_0 = 2.0$ rev/s) such that $\omega = 0$. Thus,

$$t = -\frac{\omega_0}{\alpha} = -\frac{2.0\,\text{rev/s}}{0.53\,\text{rev/s}^2} = -3.8 \text{ s}$$

which means that the wheel was at rest 3.8 s before the 15 s interval began.

102. (a) Using Eq. 11-1, the angular displacement is

$$\theta = \frac{5.6\,\text{m}}{8.0 \times 10^{-2}\,\text{m}} = 1.4 \times 10^2 \text{ rad} .$$

(b) We use $\theta = \frac{1}{2}\alpha t^2$ (Eq. 11-13) to obtain t:

$$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2(1.4 \times 10^2\,\text{rad})}{1.5\,\text{rad/s}^2}} = 14 \text{ s} .$$

103. The problem asks us to assume v_{com} and ω are constant. For consistency of units, we write

$$v_{\text{com}} = (85\,\text{mi/h})\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right) = 7480 \text{ ft/min} .$$

Thus, with $\Delta x = 60$ ft, the time of flight is $t = \Delta x/v_{\text{com}} = 60/7480 = 0.00802$ min. During that time, the angular displacement of a point on the ball's surface is

$$\theta = \omega t = (1800\,\text{rev/min})(0.00802\,\text{min}) \approx 14 \text{ rev} .$$

Chapter 12

1. The initial speed of the car is $v = (80.0)(1000/3600) = 22.2$ m/s. The tire radius is $R = 0.750/2 = 0.375$ m.

   (a) The initial speed of the car is the initial speed of the center of mass of the tire, so Eq. 12-2 leads to

   $$\omega_0 = \frac{v_{\text{com}\,0}}{R} = \frac{22.2}{0.375} = 59.3 \text{ rad/s} .$$

   (b) With $\theta = (30.0)(2\pi) = 188$ rad and $\omega = 0$, Eq. 11-14 leads to

   $$\omega^2 = \omega_0^2 + 2\alpha\theta \implies |\alpha| = \frac{59.3^2}{2(188)} = 9.31 \text{ rad/s}^2 .$$

   (c) Eq. 12-1 gives $R\theta = 70.7$ m for the distance traveled.

2. We define the direction of motion of the car as the $+x$ direction. The velocity of the car is a constant $\vec{v} = +(80)(1000/3600) = +22$ m/s, and the radius of the wheel is $r = 0.66/2 = 0.33$ m.

   (a) In the car's reference frame (where the lady perceives herself to be at rest) the road is moving towards the rear at $\vec{v}_{\text{road}} = -v = -22$ m/s, and the motion of the tire is purely rotational. In this frame, the center of the tire is "fixed" so $v_{\text{center}} = 0$.

   (b) This frame of reference is not accelerating, so "fixed" points within it have zero acceleration; thus, $a_{\text{center}} = 0$.

   (c) Since the tire's motion is only rotational (not translational) in this frame, Eq. 11-18 gives $\vec{v}_{\text{top}} = +v = +22$ m/s.

   (d) Not only is the motion purely rotational in this frame, but we also have $\omega =$ constant, which means the only acceleration for points on the rim is radial (centripetal). Therefore, the magnitude of the acceleration is

   $$a_{\text{top}} = \frac{v^2}{r} = \frac{22^2}{0.33} = 1.5 \times 10^3 \text{ m/s}^2 .$$

   (e) The bottom-most point of the tire is (momentarily) in firm contact with the road (not skidding) and has the same velocity as the road: $\vec{v}_{\text{bottom}} = -22$ m/s. This also follows from Eq. 11-18.

   (f) The magnitude of the acceleration is the same as in part (d): $a_{\text{bottom}} = 1.5 \times 10^3$ m/s^2.

   (g) Now we examine the situation in the road's frame of reference (where the road is "fixed" and it is the car that appears to be moving). The center of the tire undergoes purely translational motion while points at the rim undergo a combination of translational and rotational motions. The velocity of the center of the tire is $\vec{v} = +v = +22$ m/s.

   (h) The translational motion of the center is constant; it does not accelerate.

(i) In part (c), we found $\vec{v}_{\text{top,car}} = +v$ and we use Eq. 4-39:

$$\begin{aligned}\vec{v}_{\text{top,ground}} &= \vec{v}_{\text{top,car}} + \vec{v}_{\text{car,ground}} \\ &= v + v\end{aligned}$$

which yields $2v = +44$ m/s. This is consistent with Fig. 12-3(c).

(j) Since we are transforming between constant-velocity frames of reference, the accelerations are unaffected. The answer is as it was in part (d): $1.5 \times 10^3\,\text{m/s}^2$.

(k) We can proceed as in part (i) or simply recall that the bottom-most point is in firm contact with the (zero-velocity) road. Either way – the answer is zero.

(l) As explained in part (j), $a = 1.5 \times 10^3\,\text{m/s}^2$.

3. By Eq. 11-44, the work required to stop the hoop is the negative of the initial kinetic energy of the hoop. The initial kinetic energy is $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$ (Eq. 12-5), where $I = mR^2$ is its rotational inertia about the center of mass, $m = 140$ kg, and $v = 0.150$ m/s is the speed of its center of mass. Eq. 12-2 relates the angular speed to the speed of the center of mass: $\omega = v/R$. Thus,

$$K = \frac{1}{2}mR^2\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2 = mv^2 = (140)(0.150)^2$$

which implies that the work required is -3.15 J.

4. The rotational kinetic energy is $K = \frac{1}{2}I\omega^2$, where $I = mR^2$ is its rotational inertia about the center of mass (Table 11-2(a)), $m = 140$ kg, and $\omega = v_{\text{com}}/R$ (Eq. 12-2). The asked-for ratio is

$$\frac{K_{\text{transl}}}{K_{\text{rot}}} = \frac{\frac{1}{2}mv_{\text{com}}^2}{\frac{1}{2}\left(mR^2\right)\left(v_{\text{com}}/R\right)^2} = 1\ .$$

5. Let M be the mass of the car (presumably including the mass of the wheels) and v be its speed. Let I be the rotational inertia of one wheel and ω be the angular speed of each wheel. The kinetic energy of rotation is

$$K_{\text{rot}} = 4\left(\frac{1}{2}I\omega^2\right)$$

where the factor 4 appears because there are four wheels. The total kinetic energy is given by $K = \frac{1}{2}Mv^2 + 4\left(\frac{1}{2}I\omega^2\right)$. The fraction of the total energy that is due to rotation is

$$\text{fraction} = \frac{K_{\text{rot}}}{K} = \frac{4I\omega^2}{Mv^2 + 4I\omega^2}\ .$$

For a uniform disk (relative to its center of mass) $I = \frac{1}{2}mR^2$ (Table 11-2(c)). Since the wheels roll without sliding $\omega = v/R$ (Eq. 12-2). Thus the numerator of our fraction is

$$4I\omega^2 = 4\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = 2mv^2$$

and the fraction itself becomes

$$\text{fraction} = \frac{2mv^2}{Mv^2 + 2mv^2} = \frac{2m}{M + 2m} = \frac{2(10)}{1000} = \frac{1}{50}\ .$$

The wheel radius cancels from the equations and is not needed in the computation.

6. Interpreting h as the height increase for the center of mass of the body, then (using Eq. 12-5) mechanical energy conservation leads to

$$\begin{aligned} K_i &= U_f \\ \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 &= mgh \\ \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 &= mg\left(\frac{3v^2}{4g}\right) \end{aligned}$$

from which v cancels and we obtain $I = \frac{1}{2}mR^2$ (solid cylinder – Table 11-2(c)).

7. Rather than reproduce the analysis in §12-3, we simply use the results from that section.

(a) We substitute $I = \frac{2}{5}MR^2$ (Table 11-2(f)) and $a = -0.10g$ into Eq. 12-10:

$$-0.10g = -\frac{g\sin\theta}{1 + \left(\frac{2}{5}MR^2\right)/MR^2} = -\frac{g\sin\theta}{7/5}$$

which yields $\theta = \sin^{-1}(0.14) = 8.0°$.

(b) The acceleration would be more. We can look at this in terms of forces or in terms of energy. In terms of forces, the uphill static friction would then be absent so the downhill acceleration would be due only to the downhill gravitational pull. In terms of energy, the rotational term in Eq. 12-5 would be absent so that the potential energy it started with would simply become $\frac{1}{2}mv^2$ (without it being "shared" with another term) resulting in a greater speed (and, because of Eq. 2-16, greater acceleration).

8. We choose $+x$ rightward (so $\vec{F} = 10\,\hat{\imath}$ in Newtons) and apply Eq. 9-14 and Eq. 11-37.

(a) Newton's second law in the x direction leads to

$$F - f_s = ma \quad\Longrightarrow\quad f_s = 10\,\text{N} - (10\,\text{kg})(0.60\,\text{m/s}^2)$$

which yields $f_s = 4.0$ N. As assumed in setting up the equation, $\vec{f_s}$ points leftward.

(b) With $R = 0.30$ m, we find the magnitude of the angular acceleration to be $|\alpha| = |a_{\text{com}}|/R = 2.0\,\text{rad/s}^2$, from Eq. 12-6. The only force not directed towards (or away from) the center of mass is $\vec{f_s}$, and the torque it produces is clockwise:

$$\begin{aligned} |\tau| &= I\,|\alpha| \\ (0.30\,\text{m})(4.0\,\text{N}) &= I\left(2.0\,\text{rad/s}^2\right) \end{aligned}$$

which yields the wheel's rotational inertia about its center of mass: $I = 0.60\ \text{kg}\cdot\text{m}^2$.

9. To find where the ball lands, we need to know its speed as it leaves the track (using conservation of energy). Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = MgH$. Its final kinetic energy (as it leaves the track) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 12-5) and its final potential energy is Mgh. Here we use v to denote the speed of its center of mass and ω is its angular speed – at the moment it leaves the track. Since (up to that moment) the ball rolls without sliding we can set $\omega = v/R$. Using $I = \frac{2}{5}MR^2$ (Table 11-2(f)), conservation of energy leads to

$$\begin{aligned} MgH &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + Mgh \\ &= \frac{1}{2}Mv^2 + \frac{2}{10}Mv^2 + Mgh \\ &= \frac{7}{10}Mv^2 + Mgh\ . \end{aligned}$$

The mass M cancels from the equation, and we obtain

$$v = \sqrt{\frac{10}{7} g(H-h)} = \sqrt{\frac{10}{7}\left(9.8\,\mathrm{m/s^2}\right)(6.0\,\mathrm{m} - 2.0\,\mathrm{m})} = 7.48\,\mathrm{m/s}\ .$$

Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ rightward and $+y$ downward. Then (since the initial velocity is purely horizontal) the projectile motion equations become

$$x = vt \quad \text{and} \quad y = -\frac{1}{2} g t^2\ .$$

Solving for x at the time when $y = h$, the second equation gives $t = \sqrt{2h/g}$. Then, substituting this into the first equation, we find

$$x = v\sqrt{\frac{2h}{g}} = (7.48)\sqrt{\frac{2(2.0)}{9.8}} = 4.8\ \mathrm{m}\ .$$

10. (a) When the small sphere is released at the edge of the large "bowl" (the hemisphere of radius R), its center of mass is at the same height at that edge, but when it is at the bottom of the "bowl" its center of mass is a distance r above the the bottom surface of the hemisphere. Since the small sphere descends by $R - r$, its loss in gravitational potential energy is $mg(R - r)$, which, by conservation of mechanical energy, is equal to its kinetic energy at the bottom of the track.

(b) Using Eq. 12-5 for K, the asked-for fraction becomes

$$\frac{K_{\text{rot}}}{K} = \frac{\frac{1}{2} I\omega^2}{\frac{1}{2} I\omega^2 + \frac{1}{2} M v_{\text{com}}^2} = \frac{1}{1 + \left(\frac{M}{I}\right)\left(\frac{v_{\text{com}}}{\omega}\right)^2}\ .$$

Substituting $v_{\text{com}} = R\omega$ (Eq. 12-2) and $I = \frac{2}{5} M R^2$ (Table 11-2(f)), we obtain

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1 + \left(\frac{5}{2R^2}\right) R^2} = \frac{2}{7}\ .$$

(c) The small sphere is executing circular motion so that when it reaches the bottom, it experiences a radial acceleration upward (in the direction of the normal force which the "bowl" exerts on it). From Newton's second law along the vertical axis, the normal force N satisfies $N - mg = m a_{\text{com}}$ where $a_{\text{com}} = v_{\text{com}}^2/(R - r)$. Therefore,

$$N = mg + \frac{m v_{\text{com}}^2}{R - r} = \frac{mg(R - r) + m v_{\text{com}}^2}{R - r}\ .$$

But from part (a), $mg(R - r) = K$, and from Eq. 12-5, $\frac{1}{2} m v_{\text{com}}^2 = K - K_{\text{rot}}$. Thus,

$$N = \frac{K + 2\left(K - K_{\text{rot}}\right)}{R - r} = 3\frac{K}{R - r} - 2\frac{K_{\text{rot}}}{R - r}\ .$$

We now plug in $R - r = K/mg$ and use the result of part (b):

$$N = 3mg - 2mg\left(\frac{2}{7}\right) = \frac{17}{7} mg\ .$$

11. (a) We find its angular speed as it leaves the roof using conservation of energy. Its initial kinetic energy is $K_i = 0$ and its initial potential energy is $U_i = Mgh$ where $h = 6.0 \sin 30^\circ = 3.0$ m (we are using the edge of the roof as our reference level for computing U). Its final kinetic energy (as it leaves

the roof) is $K_f = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ (Eq. 12-5). Here we use v to denote the speed of its center of mass and ω is its angular speed – at the moment it leaves the roof. Since (up to that moment) the ball rolls without sliding we can set $v = R\omega = v$ where $R = 0.10$ m. Using $I = \frac{1}{2}MR^2$ (Table 11-2(c)), conservation of energy leads to

$$\begin{aligned} Mgh &= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \\ &= \frac{1}{2}MR^2\omega^2 + \frac{1}{4}MR^2\omega^2 \\ &= \frac{3}{4}MR^2\omega^2 . \end{aligned}$$

The mass M cancels from the equation, and we obtain

$$\omega = \frac{1}{R}\sqrt{\frac{4}{3}gh} = \frac{1}{0.10\,\text{m}}\sqrt{\frac{4}{3}\left(9.8\,\text{m/s}^2\right)(3.0\,\text{m})} = 63\,\text{rad/s} .$$

(b) Now this becomes a projectile motion of the type examined in Chapter 4. We put the origin at the position of the center of mass when the ball leaves the track (the "initial" position for this part of the problem) and take $+x$ leftward and $+y$ downward. The result of part (a) implies $v_0 = R\omega = 6.3$ m/s, and we see from the figure that (with these positive direction choices) its components are

$$\begin{aligned} v_{0x} &= v_0 \cos 30^\circ = 5.4 \text{ m/s} \quad \text{and} \\ v_{0y} &= v_0 \sin 30^\circ = 3.1 \text{ m/s} . \end{aligned}$$

The projectile motion equations become

$$x = v_{0x}t \quad \text{and} \quad y = v_{0y}t + \frac{1}{2}gt^2 .$$

We first find the time when $y = 5.0$ m from the second equation (using the quadratic formula, choosing the positive root):

$$t = \frac{-v_{0y} + \sqrt{v_{0y}^2 + 2gy}}{g} = 0.74 \text{ s} .$$

Then we substitute this into the x equation and obtain

$$x = (5.4\,\text{m/s})(0.74\,\text{s}) = 4.0 \text{ m} .$$

12. Using the floor as the reference position for computing potential energy, mechanical energy conservation leads to

$$\begin{aligned} U_{\text{release}} &= K_{\text{top}} + U_{\text{top}} \\ mgh &= \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}I\omega^2 + mg(2R) . \end{aligned}$$

Substituting $I = \frac{2}{5}mr^2$ (Table 11-2(f)) and $\omega = v_{\text{com}}/r$ (Eq. 12-2), we obtain

$$\begin{aligned} mgh &= \frac{1}{2}mv_{\text{com}}^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_{\text{com}}}{r}\right)^2 + 2mgR \\ gh &= \frac{7}{10}v_{\text{com}}^2 + 2gR \end{aligned}$$

where we have canceled out mass m in that last step.

(a) To be on the verge of losing contact with the loop (at the top) means the normal force is vanishingly small. In this case, Newton's second law along the vertical direction ($+y$ downward) leads to

$$mg = ma_r \implies g = \frac{v_{\rm com}^2}{R-r}$$

where we have used Eq. 11-23 for the radial (centripetal) acceleration (of the center of mass, which at this moment is a distance $R-r$ from the center of the loop). Plugging the result $v_{\rm com}^2 = g(R-r)$ into the previous expression stemming from energy considerations gives

$$gh = \frac{7}{10}(g)(R-r) + 2gR$$

which leads to

$$h = 2.7R - 0.7r \quad \approx \quad 2.7R\ .$$

(b) The energy considerations shown above (now with $h = 6R$) can be applied to point Q (which, however, is only at a height of R) yielding the condition

$$g(6R) = \frac{7}{10}v_{\rm com}^2 + gR$$

which gives us $v_{\rm com}^2 = 50gR/7$. Recalling previous remarks about the radial acceleration, Newton's second law applied to the horizontal axis at Q ($+x$ leftward) leads to

$$\begin{aligned} N &= m\,\frac{v_{\rm com}^2}{R-r} \\ &= m\,\frac{50gR}{7(R-r)} \end{aligned}$$

which (for $R \gg r$) gives $N \approx 50mg/7$.

13. From $I = \frac{2}{3}MR^2$ (Table 11-2(g)) we find

$$M = \frac{3I}{2R^2} = \frac{3(0.040)}{2(0.15)^2} = 2.7\ \text{kg}\ .$$

It also follows from the rotational inertia expression that $\frac{1}{2}I\omega^2 = \frac{1}{3}MR^2\omega^2$. Furthermore, it rolls without slipping, $v_{\rm com} = R\omega$, and we find

$$\frac{K_{\rm rot}}{K_{\rm com} + K_{\rm rot}} = \frac{\frac{1}{3}MR^2\omega^2}{\frac{1}{2}mR^2\omega^2 + \frac{1}{3}MR^2\omega^2}\ .$$

(a) Simplifying the above ratio, we find $K_{\rm rot}/K = 0.4$. Thus, 40% of the kinetic energy is rotational, or $K_{\rm rot} = (0.4)(20) = 8.0$ J.

(b) From $K_{\rm rot} = \frac{1}{3}MR^2\omega^2 = 8.0$ J (and using the above result for M) we find

$$\omega = \frac{1}{0.15\,\text{m}}\sqrt{\frac{3(8.0\,\text{J})}{2.7\,\text{kg}}} = 20\ \text{rad/s}$$

which leads to $v_{\rm com} = (0.15)(20) = 3.0$ m/s.

(c) We note that the inclined distance of 1.0 m corresponds to a height $h = 1.0\sin 30° = 0.50$ m. Mechanical energy conservation leads to

$$\begin{aligned} K_i &= K_f + U_f \\ 20\,\text{J} &= K_f + Mgh \end{aligned}$$

which yields (using the values of M and h found above) $K_f = 6.9$ J.

(d) We found in part (a) that 40% of this must be rotational, so

$$\frac{1}{3}MR^2\omega_f^2 = (0.40)K_f \implies \omega_f = \frac{1}{0.15}\sqrt{\frac{3(0.40)(6.9)}{2.7}}$$

which yields $\omega_f = 12$ rad/s and leads to

$$v_{\text{com}f} = R\omega_f = (0.15)(12) = 1.8 \text{ m/s} .$$

14. (a) We choose clockwise as the negative rotational sense and rightwards as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 12-2 becomes

$$v_{\text{com}} = -R\omega = (-0.11\,\text{m})\omega .$$

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in Fig. 12-34.

(b) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{\text{com}} = -\mu g = -(0.21)\left(9.8\,\text{m/s}^2\right) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 11-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 11-37)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)(9.8)}{2(0.11)} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up").

(d) The center-of-mass of the sliding ball decelerates from $v_{\text{com},0}$ to v_{com} during time t according to Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} - \mu gt .$$

During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 11-12:

$$|\omega| = |\alpha|\,t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{\text{com},0}}{7\mu g} = \frac{2(8.5)}{7(0.21)(9.8)} = 1.2 \text{ s} .$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{com},0}t - \frac{1}{2}(\mu g)t^2 = (8.5)(1.2) - \frac{1}{2}(0.21)(9.8)(1.2)^2 = 8.6 \text{ m} .$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{\text{com}} = v_{\text{com},0} - \mu gt = 8.5 - (0.21)(9.8)(1.2) = 6.1 \text{ m/s} .$$

15. (a) The derivation of the acceleration is found in §12-4; Eq. 12-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = 950\,\text{g}\cdot\text{cm}^2$, $M = 120\,\text{g}$, $R_0 = 0.32\,\text{cm}$ and $g = 980\,\text{cm/s}^2$ and obtain

$$|a_{\text{com}}| = \frac{980}{1 + (950)/(120)(0.32)^2} = 12.5\ \text{cm/s}^2\ .$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{\text{com}} = \frac{1}{2}a_{\text{com}}t^2$. Thus, we set $y_{\text{com}} = -120\,\text{cm}$, and find

$$t = \sqrt{\frac{2y_{\text{com}}}{a_{\text{com}}}} = \sqrt{\frac{2(-120\,\text{cm})}{-12.5\,\text{cm/s}^2}} = 4.38\ \text{s}\ .$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11: $v_{\text{com}} = a_{\text{com}}t = (-12.5\,\text{cm/s}^2)(4.38\,\text{s}) = -54.8\,\text{cm/s}$, so its linear speed then is approximately 55 cm/s.

(d) The translational kinetic energy is $\frac{1}{2}mv_{\text{com}}^2 = \frac{1}{2}(0.120\,\text{kg})(0.548\,\text{m/s})^2 = 1.8\times10^{-2}\,\text{J}$.

(e) The angular velocity is given by $\omega = -v_{\text{com}}/R_0$ and the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}I_{\text{com}}\frac{v_{\text{com}}^2}{R_0^2} = \frac{1}{2}\frac{(9.50\times10^{-5}\,\text{kg}\cdot\text{m}^2)(0.548\,\text{m/s})^2}{(3.2\times10^{-3}\,\text{m})^2}$$

which yields $K_{\text{rot}} = 1.4$ J.

(f) The angular speed is $\omega = |v_{\text{com}}|/R_0 = (0.548\,\text{m/s})/(3.2\times10^{-3}\,\text{m}) = 1.7\times10^2\,\text{rad/s} = 27\,\text{rev/s}$.

16. (a) The acceleration is given by Eq. 12-13:

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where upward is the positive translational direction. Taking the coordinate origin at the initial position, Eq. 2-15 leads to

$$y_{\text{com}} = v_{\text{com},0}\,t + \frac{1}{2}a_{\text{com}}t^2 = v_{\text{com},0}\,t - \frac{\frac{1}{2}gt^2}{1 + I_{\text{com}}/MR_0^2}$$

where $y_{\text{com}} = -1.2\,\text{m}$ and $v_{\text{com},0} = -1.3$ m/s. Substituting $I_{\text{com}} = 0.000095\,\text{kg}\cdot\text{m}^2$, $M = 0.12\,\text{kg}$, $R_0 = 0.0032\,\text{m}$ and $g = 9.8\,\text{m/s}^2$, we use the quadratic formula and find

$$\begin{aligned} t &= \frac{\left(1 + \frac{I_{\text{com}}}{MR_0^2}\right)\left(v_{\text{com},0} \mp \sqrt{v_{\text{com},0}^2 - \frac{2gy_{\text{com}}}{1+I_{\text{com}}/MR_0^2}}\right)}{g} \\ &= \frac{\left(1 + \frac{0.000095}{(0.12)(0.0032)^2}\right)\left(-1.3 \mp \sqrt{1.3^2 - \frac{2(9.8)(-1.2)}{1+0.000095/(0.12)(0.0032)^2}}\right)}{9.8} \\ &= -21.7 \quad \text{or} \quad 0.885 \end{aligned}$$

where we choose $t = 0.89$ s as the answer.

(b) We note that the initial potential energy is $U_i = Mgh$ and $h = 1.2$ m (using the bottom as the reference level for computing U). The initial kinetic energy is as shown in Eq. 12-5, where the initial angular and linear speeds are related by Eq. 12-2. Energy conservation leads to

$$\begin{aligned} K_f &= K_i + U_i \\ &= \frac{1}{2}mv_{\text{com},0}^2 + \frac{1}{2}I\left(\frac{v_{\text{com},0}}{R_0}\right)^2 + Mgh \\ &= \frac{1}{2}(0.12)(1.3)^2 + \frac{1}{2}\left(9.5\times 10^{-5}\right)\left(\frac{1.3}{0.0032}\right)^2 + (0.12)(9.8)(1.2) \\ &= 9.4 \text{ J} . \end{aligned}$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{\text{com}} = v_{\text{com},0} + a_{\text{com}}t = v_{\text{com},0} - \frac{gt}{1 + I_{\text{com}}/MR_0^2} .$$

Thus, we obtain

$$v_{\text{com}} = -1.3 - \frac{(9.8)(0.885)}{1 + \frac{0.000095}{(0.12)(0.0032)^2}} = -1.41 \text{ m/s}$$

so its linear speed at that moment is approximately 1.4 m/s.

(d) The translational kinetic energy is $\frac{1}{2}mv_{\text{com}}^2 = \frac{1}{2}(0.12)(1.41)^2 = 0.12\,\text{J}$.

(e) The angular velocity at that moment is given by

$$\omega = -\frac{v_{\text{com}}}{R_0} = -\frac{-1.41}{0.0032} = 441$$

or approximately 440 rad/s.

(f) And the rotational kinetic energy is

$$\frac{1}{2}I_{\text{com}}\omega^2 = \frac{1}{2}\left(9.50\times 10^{-5}\,\text{kg}\cdot\text{m}^2\right)(441\,\text{rad/s})^2 = 9.2 \text{ J} .$$

17. One method is to show that $\vec{r}\cdot\left(\vec{r}\times\vec{F}\right) = \vec{F}\cdot\left(\vec{r}\times\vec{F}\right) = 0$, but we choose here a more pedestrian approach: without loss of generality we take $\vec{r}$ and $\vec{F}$ to be in the xy plane – and will show that $\vec{\tau}$ has no x and y components (that it is parallel to the $\hat{\text{k}}$ direction). We proceed as follows: in the general expression $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, we will set $z = 0$ to constrain $\vec{r}$ to the xy plane, and similarly for $\vec{F}$. Using Eq. 3-30, we find $\vec{r}\times\vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\text{i}} + (zF_x - xF_z)\,\hat{\text{j}} + (xF_y - yF_x)\,\hat{\text{k}}$$

and once we set $z = 0$ and $F_z = 0$ we obtain

$$\vec{\tau} = \vec{r}\times\vec{F} = (xF_y - yF_x)\,\hat{\text{k}}$$

which demonstrates that $\vec{\tau}$ has no component in the xy plane.

18. If we write $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\hat{\text{k}}$, then (using Eq. 3-30) we find $\vec{r}\times\vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\text{i}} + (zF_x - xF_z)\,\hat{\text{j}} + (xF_y - yF_x)\,\hat{\text{k}} .$$

(a) In the above expression, we set (with SI units understood) $x = -2$, $y = 0$, $z = 4$, $F_x = 6$, $F_y = 0$ and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r}\times\vec{F} = 24\,\hat{\text{j}}$ N·m.

(b) The values are just as in part (a) with the exception that now $F_x = -6$. We find $\vec{\tau} = \vec{r}\times\vec{F} = -24\,\hat{\text{j}}$ N·m.

(c) In the above expression, we set $x = -2$, $y = 0$, $z = 4$, $F_x = 0$, $F_y = 0$ and $F_z = 6$. We get $\vec{\tau} = \vec{r} \times \vec{F} = 12\,\hat{\jmath}$ N·m.

(d) The values are just as in part (c) with the exception that now $F_z = -6$. We find $\vec{\tau} = \vec{r} \times \vec{F} = -12\,\hat{\jmath}$ N·m.

19. If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k} .$$

(a) In the above expression, we set (with SI units understood) $x = 0$, $y = -4$, $z = 3$, $F_x = 2$, $F_y = 0$ and $F_z = 0$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = \left(6\,\hat{\jmath} + 8\,\hat{k}\right)$ N·m. This has magnitude $\sqrt{6^2 + 8^2} = 10$ N·m and is seen to be parallel to the yz plane. Its angle (measured counterclockwise from the $+y$ direction) is $\tan^{-1}(8/6) = 53°$.

(b) In the above expression, we set $x = 0$, $y = -4$, $z = 3$, $F_x = 0$, $F_y = 2$ and $F_z = 4$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = -22\,\hat{\imath}$ N·m. This has magnitude 22 N·m and points in the $-x$ direction.

20. We use the notation $\vec{r}\,'$ to indicate the vector pointing from the axis of rotation directly to the position of the particle. If we write $\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\,\hat{k}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$(y'F_z - z'F_y)\,\hat{\imath} + (z'F_x - x'F_z)\,\hat{\jmath} + (x'F_y - y'F_x)\,\hat{k} .$$

(a) Here, $\vec{r}\,' = \vec{r}$. Dropping the primes in the above expression, we set (with SI units understood) $x = 0$, $y = 0.5$, $z = -2.0$, $F_x = 2$, $F_y = 0$ and $F_z = -3$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F} = \left(-1.5\,\hat{\imath} - 4\,\hat{\jmath} - \hat{k}\right)$ N·m.

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2\,\hat{\imath} - 3\,\hat{k}$. Therefore, in the above expression, we set $x' = -2.0$, $y' = 0.5$, $z' = 1.0$, $F_x = 2$, $F_y = 0$ and $F_z = -3$. Thus, we obtain $\vec{\tau} = \vec{r}\,' \times \vec{F} = \left(-1.5\,\hat{\imath} - 4\,\hat{\jmath} - \hat{k}\right)$ N·m.

21. If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k} .$$

(a) Plugging in, we find

$$\vec{\tau} = \left((3.0\,\mathrm{m})(6.0\,\mathrm{N}) - (4.0\,\mathrm{m})(-8.0\,\mathrm{N})\right)\hat{k} = 50\,\hat{k}\ \mathrm{N{\cdot}m} .$$

(b) We use Eq. 3-27, $|\vec{r} \times \vec{F}| = rF\sin\phi$, where ϕ is the angle between $\vec{r}$ and $\vec{F}$. Now $r = \sqrt{x^2 + y^2} = 5.0\,\mathrm{m}$ and $F = \sqrt{F_x^2 + F_y^2} = 10\,\mathrm{N}$. Thus $rF = (5.0\,\mathrm{m})(10\,\mathrm{N}) = 50\,\mathrm{N \cdot m}$, the same as the magnitude of the vector product calculated in part (a). This implies $\sin\phi = 1$ and $\phi = 90°$.

22. If we write $\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath} + z'\,\hat{k}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{F}$ is equal to

$$(y'F_z - z'F_y)\,\hat{\imath} + (z'F_x - x'F_z)\,\hat{\jmath} + (x'F_y - y'F_x)\,\hat{k} .$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 4\,\hat{k}$, and $\vec{F} = \vec{F}_1$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3$, $y = -2$, $z = 4$, $F_x = 3$, $F_y = -4$ and $F_z = 5$. Then we obtain $\vec{\tau} = \vec{r} \times \vec{F}_1 = \left(6.0\,\hat{\imath} - 3.0\,\hat{\jmath} - 6.0\,\hat{k}\right)$ N·m.

(b) This is like part (a) but with $\vec{F} = \vec{F}_2$. We plug in $F_x = -3$, $F_y = -4$ and $F_z = -5$ and obtain $\vec{\tau} = \vec{r} \times \vec{F}_2 = \left(26\,\hat{\imath} + 3.0\,\hat{\jmath} - 18\,\hat{k}\right)$ N·m.

(c) We can proceed in either of two ways. We can add (vectorially) the answers from parts (a) and (b), or we can first add the two force vectors and then compute $\vec{\tau} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$ (these total force components are computed in the next part). The result is $\left(32\,\hat{\imath} - 24\,\hat{k}\right)$ N·m.

(d) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 3\,\hat{\mathrm{i}} + 2\,\hat{\mathrm{j}} + 4\,\hat{\mathrm{k}}$. Therefore, in the above expression, we set $x' = 0$, $y' = -4$, $z' = 0$, $F_x = 3 - 3 = 0$, $F_y = -4 - 4 = -8$ and $F_z = 5 - 5 = 0$. We get $\vec{\tau} = \vec{r}\,' \times \left(\vec{F}_1 + \vec{F}_2\right) = 0$.

23. We could proceed formally by setting up an xyz coordinate system and using Eq. 3-30 for the vector cross product, or we can approach this less formally in the style of Sample Problem 12-4 (which is our choice). For the 3.1 kg particle, Eq. 12-21 yields

$$\ell_1 = r_{\perp 1}\, m v_1 = (2.8)(3.1)(3.6) = 31.2 \text{ kg}\cdot\text{m}^2/\text{s} .$$

Using the right-hand rule for vector products, we find this $(\vec{r}_1 \times \vec{p}_1)$ is out of the page, perpendicular to the plane of Fig. 12-35. And for the 6.5 kg particle, we find

$$\ell_2 = r_{\perp 2}\, m v_2 = (1.5)(6.5)(2.2) = 21.4 \text{ kg}\cdot\text{m}^2/\text{s} .$$

And we use the right-hand rule again, finding that this $(\vec{r}_2 \times \vec{p}_2)$ is into the page. Consequently, the two angular momentum vectors are in opposite directions, so their vector sum is the *difference* of their magnitudes:

$$L = \ell_1 - \ell_2 = 9.8 \text{ kg}\cdot\text{m}^2/\text{s} .$$

24. We note that the component of $\vec{v}$ perpendicular to $\vec{r}$ has magnitude $v \sin\phi$ where $\phi = 30°$. A similar observation applies to $\vec{F}$.

(a) Eq. 12-20 leads to

$$\ell = r m v_\perp = (3.0)(2.0)(4.0)\sin 30° = 12 \text{ kg}\cdot\text{m}^2/\text{s} .$$

Using the right-hand rule for vector products, we find $\vec{r} \times \vec{p}$ points out of the page, perpendicular to the plane of Fig. 12-36.

(b) Eq. 11-31 (which is the same as Eq. 12-15) leads to

$$\tau = rF\sin\phi = (3.0)(2.0)\sin 30° = 3.0 \text{ N}\cdot\text{m} .$$

Using the right-hand rule for vector products, we find $\vec{r} \times \vec{F}$ is also out of the page.

25. (a) We use $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the object, $\vec{v}$ is its velocity vector, and m is its mass. Only the x and z components of the position and velocity vectors are nonzero, so Eq. 3-30 leads to $\vec{r} \times \vec{v} = (-xv_z + zv_x)\,\hat{\mathrm{j}}$. Therefore,

$$\begin{aligned} \vec{\ell} &= m\,(-xv_z + zv_x)\,\hat{\mathrm{j}} \\ &= (0.25\,\text{kg})\,(-(2.0\,\text{m})(5.0\,\text{m/s}) + (-2.0\,\text{m})(-5.0\,\text{m/s}))\,\hat{\mathrm{j}} \\ &= 0\,. \end{aligned}$$

(b) If we write $\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\mathrm{i}} + (zF_x - xF_z)\,\hat{\mathrm{j}} + (xF_y - yF_x)\,\hat{\mathrm{k}} .$$

With $x = 2.0$, $z = -2.0$, $F_y = 4.0$ and all other components zero (and SI units understood) the expression above yields $\vec{\tau} = \vec{r} \times \vec{F} = \left(8.0\,\hat{\mathrm{i}} + 8.0\,\hat{\mathrm{k}}\right)$ N·m.

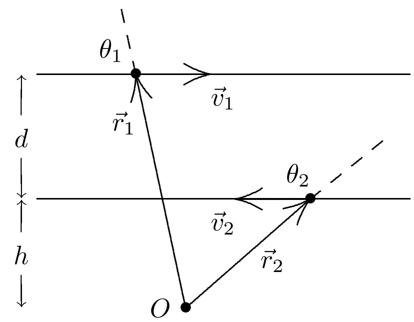
26. If we write $\vec{r}\,' = x'\,\hat{\mathrm{i}} + y'\,\hat{\mathrm{j}} + z'\,\hat{\mathrm{k}}$, then (using Eq. 3-30) we find $\vec{r}\,' \times \vec{v}$ is equal to

$$(y'v_z - z'v_y)\,\hat{\mathrm{i}} + (z'v_x - x'v_z)\,\hat{\mathrm{j}} + (x'v_y - y'v_x)\,\hat{\mathrm{k}} .$$

(a) Here, $\vec{r}\,' = \vec{r}$ where $\vec{r} = 3\,\hat{\mathrm{i}} - 4\,\hat{\mathrm{j}}$. Thus, dropping the primes in the above expression, we set (with SI units understood) $x = 3$, $y = -4$, $z = 0$, $v_x = 30$, $v_y = 60$ and $v_z = 0$. Then (with $m = 2.0$ kg) we obtain $\vec{\ell} = m\,(\vec{r} \times \vec{v}) = 600\,\hat{\mathrm{k}}$ kg·m²/s.

(b) Now $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = -2\,\hat{\imath} - 2\,\hat{\jmath}$. Therefore, in the above expression, we set $x' = 5$, $y' = -2$, $z' = 0$, $v_x = 30$, $v_y = 60$ and $v_z = 0$. We get $\vec{\ell} = m\,(\vec{r}\,' \times \vec{v}) = 720\,\hat{k}$ kg·m²/s.

27. (a) The diagram below shows the particles and their lines of motion. The origin is marked O and may be anywhere. The angular momentum of particle 1 has magnitude $\ell_1 = mvr_1 \sin\theta_1 = mv(d+h)$ and it is into the page. The angular momentum of particle 2 has magnitude $\ell_2 = mvr_2 \sin\theta_2 = mvh$ and it is out of the page. The net angular momentum has magnitude $L = mv(d+h) - mvh = mvd$ and is into the page. This result is independent of the location of the origin.



(b) As indicated above, the expression does not change.

(c) Suppose particle 2 is traveling to the right. Then $L = mv(d+h) + mvh = mv(d+2h)$. This result depends on h, the distance from the origin to one of the lines of motion. If the origin is midway between the lines of motion, then $h = -d/2$ and $L = 0$.

28. (a) With $\vec{p} = m\vec{v} = -16\,\hat{\jmath}$ kg · m/s, we take the vector cross product (using either Eq. 3-30 or, more simply, Eq. 12-20 and the right-hand rule):

$$\vec{\ell} = \vec{r} \times \vec{p} = -32\,\hat{k}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(b) Now the axis passes through the point $\vec{R} = 4.0\,\hat{\jmath}$ m, parallel with the z axis. With $\vec{r}\,' = \vec{r} - \vec{R} = 2.0\,\hat{\imath}$ m, we again take the cross product and arrive at the same result as before:

$$\vec{\ell}\,' = \vec{r}\,' \times \vec{p} = -32\,\hat{k}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(c) Torque is defined in Eq. 12-14: $\vec{\tau} = \vec{r} \times \vec{F} = 12\,\hat{k}$ N·m.

(d) Using the notation from part (b),

$$\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = 0\quad .$$

29. If we write (for the general case) $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{v}$ is equal to

$$(yv_z - zv_y)\,\hat{\imath} + (zv_x - xv_z)\,\hat{\jmath} + (xv_y - yv_x)\,\hat{k}\ .$$

(a) The angular momentum is given by the vector product $\vec{\ell} = m\vec{r} \times \vec{v}$, where $\vec{r}$ is the position vector of the particle, $\vec{v}$ is its velocity, and $m = 3.0$ kg is its mass. Substituting (with SI units understood) $x = 3$, $y = 8$, $z = 0$, $v_x = 5$, $v_y = -6$ and $v_z = 0$ into the above expression, we obtain

$$\vec{\ell} = (3.0)\,((3)(-6) - (8.0)(5.0))\,\hat{k} = -1.7 \times 10^2\,\hat{k}\ \text{kg}\cdot\text{m}^2/\text{s}\ .$$

(b) The torque is given by Eq. 12-14, $\vec{\tau} = \vec{r} \times \vec{F}$. We write $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and $\vec{F} = F_x\,\hat{\imath}$ and obtain

$$\vec{\tau} = \left(x\,\hat{\imath} + y\,\hat{\jmath}\right) \times (F_x\,\hat{\imath}) = -yF_x\,\hat{k}$$

since $\hat{\imath} \times \hat{\imath} = 0$ and $\hat{\jmath} \times \hat{\imath} = -\hat{k}$. Thus, we find $\vec{\tau} = -(8.0\,\text{m})(-7.0\,\text{N})\,\hat{k} = 56\,\hat{k}$ N·m.

(c) According to Newton's second law $\vec{\tau} = d\vec{\ell}/dt$, so the rate of change of the angular momentum is $56\,\text{kg}\cdot\text{m}^2/\text{s}^2$, in the positive z direction.

30. The rate of change of the angular momentum is

$$\frac{d\vec{\ell}}{dt} = \vec{\tau}_1 + \vec{\tau}_2 = 2.0\hat{\imath} - 4.0\hat{\jmath}\ \mathrm{N{\cdot}m}\ .$$

Consequently, the vector $d\vec{\ell}/dt$ has a magnitude $\sqrt{2.0^2 + (-4.0)^2} = 4.5\,\mathrm{N\cdot m}$ and is at an angle θ (in the xy plane, or a plane parallel to it) measured from the positive x axis, where $\theta = \tan^{-1}\left(\frac{-4.0}{2.0}\right) = -63°$, the negative sign indicating that the angle is measured clockwise as viewed "from above" (by a person on the $+z$ axis).

31. We use a right-handed coordinate system with $+\hat{\mathrm{k}}$ directed out of the xy plane so as to be consistent with counterclockwise rotation (and the right-hand rule). Thus, all the angular momenta being considered are along the $-\hat{\mathrm{k}}$ direction; for example, in part (b) $\vec{\ell} = -4.0t^2\,\hat{\mathrm{k}}$ in SI units. We use Eq. 12-23.

(a) The angular momentum is constant so its derivative is zero. There is no torque in this instance.

(b) Taking the derivative with respect to time, we obtain the torque:

$$\vec{\tau} = \frac{d\vec{\ell}}{dt} = (-4.0\,\hat{\mathrm{k}})\,\frac{dt^2}{dt} = -8.0t\ \hat{\mathrm{k}}$$

in SI units (N·m). This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$.

(c) With $\vec{\ell} = -4.0\sqrt{t}\,\hat{\mathrm{k}}$ in SI units, the torque is

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{d\sqrt{t}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

which yields $\vec{\tau} = -2.0/\sqrt{t}\,\hat{\mathrm{k}}$ in SI units. This vector points in the $-\hat{\mathrm{k}}$ direction (causing the clockwise motion to speed up) for all $t > 0$ (and it is undefined for $t < 0$).

(d) Finally, we have

$$\vec{\tau} = \left(-4.0\hat{\mathrm{k}}\right)\frac{dt^{-2}}{dt} = \left(-4.0\hat{\mathrm{k}}\right)\left(\frac{-2}{t^3}\right)$$

which yields $\vec{\tau} = 8.0/t^3\,\hat{\mathrm{k}}$ in SI units. This vector points in the $+\hat{\mathrm{k}}$ direction (causing the initially clockwise motion to slow down) for all $t > 0$.

32. Both $\vec{r}$ and $\vec{v}$ lie in the xy plane. The position vector $\vec{r}$ has an x component that is a function of time (being the integral of the x component of velocity, which is itself time-dependent) and a y component that is constant ($y = -2.0$ m). In the cross product $\vec{r}\times\vec{v}$, all that matters is the y component of $\vec{r}$ since $v_x \neq 0$ but $v_y = 0$:

$$\vec{r}\times\vec{v} = -yv_x\,\hat{\mathrm{k}}\ .$$

(a) The angular momentum is $\vec{\ell} = m\,(\vec{r}\times\vec{v})$ where the mass is $m = 2.0$ kg in this case. With SI units understood and using the above cross-product expression, we have

$$\vec{\ell} = (2.0)\left(-(-2.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = -24t^2\,\hat{\mathrm{k}}$$

in kg·m²/s. This implies the particle is moving clockwise (as observed by someone on the $+z$ axis) for $t > 0$.

(b) The torque is caused by the (net) force $\vec{F} = m\vec{a}$ where

$$\vec{a} = \frac{d\vec{v}}{dt} = -12t\,\hat{\imath}\ \mathrm{m/s^2}\ .$$

The remark above that only the y component of $\vec{r}$ still applies, since $a_y = 0$. We use $\vec{\tau} = \vec{r} \times \vec{F} = m(\vec{r} \times \vec{a})$ and obtain

$$\vec{\tau} = (2.0)\,(-(-2.0)(-12t))\,\hat{\mathrm{k}} = -48t\,\hat{\mathrm{k}}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is clockwise, causing the particle motion (which was clockwise to begin with) to increase.

(c) We replace $\vec{r}$ with $\vec{r}\,'$ (measured relative to the new reference point) and note (again) that only its y component matters in these calculations. Thus, with $y' = -2.0 - (-3.0) = 1.0$ m, we find

$$\vec{\ell}\,' = (2.0)\left(-(1.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}} = 12t^2\,\hat{\mathrm{k}}$$

in kg·m^2/s. The fact that this is positive implies that the particle is moving counterclockwise relative to the new reference point.

(d) Using $\vec{\tau}\,' = \vec{r}\,' \times \vec{F} = m(\vec{r}\,' \times \vec{a})$, we obtain

$$\vec{\tau} = (2.0)\,(-(1.0)\,(-12t))\,\hat{\mathrm{k}} = 24t\,\hat{\mathrm{k}}$$

in N·m. The torque on the particle (as observed by someone on the $+z$ axis) is counterclockwise, relative to the new reference point.

33. (a) Since $\tau = dL/dt$, the average torque acting during any interval Δt is given by $\tau_{\text{avg}} = (L_f - L_i)/\Delta t$, where L_i is the initial angular momentum and L_f is the final angular momentum. Thus

$$\tau_{\text{avg}} = \frac{0.800\,\mathrm{kg{\cdot}m^2/s} - 3.00\,\mathrm{kg{\cdot}m^2/s}}{1.50\,\mathrm{s}}$$

which yields $\tau_{\text{avg}} = -1.467 \approx= -1.47\,\mathrm{N{\cdot}m}$. In this case the negative sign indicates that the direction of the torque is opposite the direction of the initial angular momentum, implicitly taken to be positive.

(b) The angle turned is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. If the angular acceleration α is uniform, then so is the torque and $\alpha = \tau/I$. Furthermore, $\omega_0 = L_i/I$, and we obtain

$$\begin{aligned}\theta &= \frac{L_i t + \frac{1}{2}\tau t^2}{I} \\ &= \frac{(3.00\,\mathrm{kg{\cdot}m^2/s})(1.50\,\mathrm{s}) + \frac{1}{2}(-1.467\,\mathrm{N{\cdot}m})(1.50\,\mathrm{s})^2}{0.140\,\mathrm{kg{\cdot}m^2}} \\ &= 20.4 \text{ rad}\ .\end{aligned}$$

(c) The work done on the wheel is

$$W = \tau\theta = (-1.47\,\mathrm{N{\cdot}m})(20.4\,\mathrm{rad}) = -29.9 \text{ J}$$

where more precise values are used in the calculation than what is shown here. An equally good method for finding W is Eq. 11-44, which, if desired, can be rewritten as $W = (L_f^2 - L_i^2)/2I$.

(d) The average power is the work done by the flywheel (the negative of the work done on the flywheel) divided by the time interval:

$$P_{\text{avg}} = -\frac{W}{\Delta t} = -\frac{-29.8\,\mathrm{J}}{1.50\,\mathrm{s}} = 19.9 \text{ W}\ .$$

34. (a) Eq. 11-27 gives $\alpha = \tau/I$ and Eq. 11-12 leads to $\omega = \alpha t = \tau t/I$. Therefore, the angular momentum at $t = 0.033$ s is

$$I\omega = \tau t = (16\,\mathrm{N\cdot m})(0.033\,\mathrm{s}) = 0.53 \text{ kg·m}^2/\text{s}$$

where this is essentially a derivation of the angular version of the impulse-momentum theorem.

(b) We find

$$\omega = \frac{\tau t}{I} = \frac{(16)(0.033)}{1.2 \times 10^{-3}} = 440 \text{ rad}$$

which we convert as follows: $\omega = (440)(60/2\pi) \approx 4200$ rev/min.

35. (a) A particle contributes mr^2 to the rotational inertia. Here r is the distance from the origin O to the particle. The total rotational inertia is

$$I = m(3d)^2 + m(2d)^2 + m(d)^2 = 14md^2 \ .$$

(b) The angular momentum of the middle particle is given by $L_m = I_m\omega$, where $I_m = 4md^2$ is its rotational inertia. Thus $L_m = 4md^2\omega$.

(c) The total angular momentum is $I\omega = 14md^2\omega$.

36. We integrate Eq. 12-29 (for a single torque) over the time interval (where the angular speed at the beginning is ω_i and at the end is ω_f)

$$\int \tau\, dt = \int \frac{dL}{dt}\, dt = L_f - L_i = I\,(\omega_f - \omega_i)$$

and if we use the calculus-based notion of the average of a function f

$$f_{\text{avg}} = \frac{1}{\Delta t} \int f\, dt$$

then (using Eq. 12-16) we obtain

$$\int \tau\, dt = \tau_{\text{avg}}\Delta t = F_{\text{avg}} R \Delta t \ .$$

Inserting this into the top line proves the relationship shown in the problem.

37. Suppose cylinder 1 exerts a uniform force of magnitude F on cylinder 2, tangent to the cylinder's surface at the point of contact. The torque applied to cylinder 2 is $\tau_2 = R_2F$ and the angular acceleration of that cylinder is $\alpha_2 = \tau_2/I_2 = R_2F/I_2$. As a function of time its angular velocity is

$$\omega_2 = \alpha_2 t = \frac{R_2Ft}{I_2} \ .$$

The forces of the cylinders on each other obey Newton's third law, so the magnitude of the force of cylinder 2 on cylinder 1 is also F. The torque exerted by cylinder 2 on cylinder 1 is $\tau_1 = R_1F$ and the angular acceleration of cylinder 1 is $\alpha_1 = \tau_1/I_1 = R_1F/I_1$. This torque slows the cylinder. As a function of time, its angular velocity is $\omega_1 = \omega_0 - R_1Ft/I_1$. The force ceases and the cylinders continue rotating with constant angular speeds when the speeds of points on their rims are the same ($R_1\omega_1 = R_2\omega_2$). Thus,

$$R_1\omega_0 - \frac{R_1^2Ft}{I_1} = \frac{R_2^2Ft}{I_2} \ .$$

When this equation is solved for the product of force and time, the result is

$$Ft = \frac{R_1I_1I_2}{I_1R_2^2 + I_2R_1^2}\,\omega_0 \ .$$

Substituting this expression for Ft in the ω_2 equation above, we obtain

$$\omega_2 = \frac{R_1R_2I_1}{I_1R_2^2 + I_2R_1^2}\,\omega_0 \ .$$

38. (a) For the hoop, we use Table 11-2(h) and the parallel-axis theorem to obtain

$$I_1 = I_{\text{com}} + mh^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 \ .$$

Of the thin bars (in the form of a square), the member along the rotation axis has (approximately) no rotational inertia about that axis (since it is thin), and the member farthest from it is very much like it (by being parallel to it) except that it is displaced by a distance h; it has rotational inertia given by the parallel axis theorem:

$$I_2 = I_{\text{com}} + mh^2 = 0 + mR^2 = mR^2 \ .$$

Now the two members of the square perpendicular to the axis have the same rotational inertia (that is, $I_3 = I_4$). We find I_3 using Table 11-2(e) and the parallel-axis theorem:

$$I_3 = I_{\text{com}} + mh^2 = \frac{1}{12}mR^2 + m\left(\frac{R}{2}\right)^2 = \frac{1}{3}mR^2 \ .$$

Therefore, the total rotational inertia is

$$I_1 + I_2 + I_3 + I_4 = \frac{19}{6}mR^2 = 1.6 \text{ kg}\cdot\text{m}^2 \ .$$

(b) The angular speed is constant:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.5} = 2.5 \text{ rad/s} \ .$$

Thus, $L = I_{\text{total}}\omega = 4.0 \text{ kg}\cdot\text{m}^2/\text{s}$.

39. (a) No external torques act on the system consisting of the man, bricks, and platform, so the total angular momentum of the system is conserved. Let I_i be the initial rotational inertia of the system and let I_f be the final rotational inertia. Then $I_i\omega_i = I_f\omega_f$ and

$$\begin{aligned} \omega_f &= \left(\frac{I_i}{I_f}\right)\omega_i \\ &= \left(\frac{6.0\,\text{kg}\cdot\text{m}^2}{2.0\,\text{kg}\cdot\text{m}^2}\right)(1.2\,\text{rev/s}) \\ &= 3.6 \text{ rev/s} \ . \end{aligned}$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_i\omega_i^2$, the final kinetic energy is $K_f = \frac{1}{2}I_f\omega_f^2$, and their ratio is

$$\frac{K_f}{K_i} = \frac{I_f\omega_f^2}{I_i\omega_i^2} = \frac{(2.0\,\text{kg}\cdot\text{m}^2)(3.6\,\text{rev/s})^2}{(6.0\,\text{kg}\cdot\text{m}^2)(1.2\,\text{rev/s})^2} = 3.0 \ .$$

(c) The man did work in decreasing the rotational inertia by pulling the bricks closer to his body. This energy came from the man's store of internal energy.

40. We use conservation of angular momentum: $I_m\omega_m = I_p\omega_p$. The respective angles θ_m and θ_p by which the motor and probe rotate are therefore related by

$$\int I_m\omega_m dt = I_m\theta_m = \int I_p\omega_p dt = I_p\theta_p$$

which gives

$$\theta_m = \frac{I_p\theta_p}{I_m} = \frac{\left(12\,\text{kg}\cdot\text{m}^2\right)(30^\circ)}{2.0\times10^{-3}\,\text{kg}\cdot\text{m}^2} = 180000^\circ \ .$$

The number of revolutions for the rotor is then $1.8\times10^5/360 = 500\,\text{rev}$.

41. (a) No external torques act on the system consisting of the two wheels, so its total angular momentum is conserved. Let I_1 be the rotational inertia of the wheel that is originally spinning (at ω_i) and I_2 be the rotational inertia of the wheel that is initially at rest. Then $I_1\omega_i = (I_1 + I_2)\omega_f$ and

$$\omega_f = \frac{I_1}{I_1 + I_2}\,\omega_i$$

where ω_f is the common final angular velocity of the wheels. Substituting $I_2 = 2I_1$ and $\omega_i = 800\,\text{rev/min}$, we obtain $\omega_f = 267\,\text{rev/min}$.

(b) The initial kinetic energy is $K_i = \frac{1}{2}I_1\omega_i^2$ and the final kinetic energy is $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$. We rewrite this as

$$K_f = \frac{1}{2}(I_1 + 2I_1)\left(\frac{I_1\omega_i}{I_1 + 2I_1}\right)^2 = \frac{1}{6}I\omega_i^2\ .$$

Therefore, the fraction lost, $(K_i - K_f)/K_i$, is

$$1 - \frac{K_f}{K_i} = 1 - \frac{\frac{1}{6}I\omega_i^2}{\frac{1}{2}I\omega_i^2} = \frac{2}{3}\ .$$

42. (a) We apply conservation of angular momentum: $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$. The angular speed after coupling is therefore

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{\left(3.3\,\text{kg}\cdot\text{m}^2\right)(450\,\text{rev/min}) + \left(6.6\,\text{kg}\cdot\text{m}^2\right)(900\,\text{rev/min})}{3.3\,\text{kg}\cdot\text{m}^2 + 6.6\,\text{kg}\cdot\text{m}^2} = 750\ \text{rev/min}\ .$$

(b) In this case, we obtain

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{(3.3)(450) + (6.6)(-900)}{3.3 + 6.6} = -450\,\text{rev/min}$$

where the minus sign indicates that $\vec{\omega}$ is in the direction of the second disk's initial angular velocity.

43. (a) In terms of the radius of gyration k, the rotational inertia of the merry-go-round is $I = Mk^2$. We obtain $I = (180\,\text{kg})(0.910\,\text{m})^2 = 149\,\text{kg}\cdot\text{m}^2$.

(b) An object moving along a straight line has angular momentum about any point that is not on the line. The magnitude of the angular momentum of the child about the center of the merry-go-round is given by Eq. 12-21, mvR, where R is the radius of the merry-go-round. Therefore,

$$\left|\vec{L}_{\text{child}}\right| = (44.0\,\text{kg})(3.00\,\text{m/s})(1.20\,\text{m}) = 158\,\text{kg}\cdot\text{m}^2/\text{s}\ .$$

(c) No external torques act on the system consisting of the child and the merry-go-round, so the total angular momentum of the system is conserved. The initial angular momentum is given by mvR; the final angular momentum is given by $\left(I + mR^2\right)\omega$, where ω is the final common angular velocity of the merry-go-round and child. Thus $mvR = (I + mR^2)\omega$ and

$$\omega = \frac{mvR}{I + mR^2} = \frac{158\,\text{kg}\cdot\text{m}^2/\text{s}}{149\,\text{kg}\cdot\text{m}^2 + (44.0\,\text{kg})(1.20\,\text{m})^2} = 0.744\ \text{rad/s}\ .$$

44. Angular momentum conservation $I_i\omega_i = I_f\omega_f$ leads to

$$\frac{\omega_f}{\omega_i} = \frac{I_i}{I_f}\omega_i = 3$$

which implies

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}I_f\omega_f^2}{\frac{1}{2}I_i\omega_i^2} = \frac{I_f}{I_i}\left(\frac{\omega_f}{\omega_i}\right)^2 = 3\ .$$

45. No external torques act on the system consisting of the train and wheel, so the total angular momentum of the system (which is initially zero) remains zero. Let $I = MR^2$ be the rotational inertia of the wheel. Its final angular momentum is $= I\omega\hat{\mathrm{k}} = -MR^2|\omega|\hat{\mathrm{k}}$, where $\hat{\mathrm{k}}$ is *up* in Fig. 12-40 and that last step (with the minus sign) is done in recognition that the wheel's clockwise rotation implies a negative value for ω. The linear speed of a point on the track is ωR and the speed of the train (going counterclockwise in Fig. 12-40 with speed v' relative to an outside observer) is therefore $v' = v - |\omega|R$ where v is its speed relative to the tracks. Consequently, the angular momentum of the train is $m(v-|\omega|R)R\,\hat{\mathrm{k}}$. Conservation of angular momentum yields

$$0 = -MR^2|\omega|\,\hat{\mathrm{k}} + m\,(v - |\omega|R)\,R\,\hat{\mathrm{k}} \ .$$

When this equation is solved for the angular speed, the result is

$$|\omega| = \frac{mvR}{(M+m)R^2} = \frac{mv}{(M+m)R} \ .$$

46. We assume that from the moment of grabbing the stick onward, they maintain rigid postures so that the system can be analyzed as a symmetrical rigid body with center of mass midway between the skaters.

(a) The total linear momentum is zero (the skaters have the same mass and equal-and-opposite velocities). Thus, their center of mass (the middle of the 3.0 m long stick) remains fixed and they execute circular motion (of radius $r = 1.5$ m) about it. Using Eq. 11-18, their angular velocity (counterclockwise as seen in Fig. 12-41) is

$$\omega = \frac{v}{r} = \frac{1.4}{1.5} = 0.93 \text{ rad/s} \ .$$

(b) Their rotational inertia is that of two particles in circular motion at $r = 1.5$ m, so Eq. 11-26 yields

$$I = \sum mr^2 = 2(50)(1.5)^2 = 225 \text{ kg}\cdot\text{m}^2 \ .$$

Therefore, Eq. 11-27 leads to

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(225)(0.93)^2 = 98 \text{ J} \ .$$

(c) Angular momentum is conserved in this process. If we label the angular velocity found in part (a) ω_i and the rotational inertia of part (b) as I_i, we have

$$I_i\omega_i = (225)(0.93) = I_f\omega_f \ .$$

The final rotational inertia is $\sum mr_f^2$ where $r_f = 0.5$ m so $I_f = 25$ kg·m^2. Using this value, the above expression gives $\omega_f = 8.4$ rad/s.

(d) We find

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(25)(8.4)^2 = 8.8 \times 10^2 \text{ J} \ .$$

(e) We account for the large increase in kinetic energy (part (d) minus part (b)) by noting that the skaters do a great deal of work (converting their internal energy into mechanical energy) as they pull themselves closer – "fighting" what appears to them to be large "centrifugal forces" trying to keep them apart.

47. So that we don't get confused about $\pm$ signs, we write the angular *speed* of the lazy Susan as $|\omega|$ and reserve the ω symbol for the angular velocity (which, using a common convention, is negative-valued when the rotation is clockwise). When the roach "stops" we recognize that it comes to rest relative to the lazy Susan (not relative to the ground).

(a) Angular momentum conservation leads to

$$mvR + I\omega_0 = \left(mR^2 + I\right)\omega_f$$

which we can write (recalling our discussion about angular speed versus angular velocity) as

$$mvR - I\,|\omega_0| = -\left(mR^2 + I\right)|\omega_f| \ .$$

We solve for the final angular speed of the system:

$$|\omega_f| = \frac{mvR - I\,|\omega_0|}{mR^2 + I} \ .$$

(b) No, $K_f \neq K_i$ and – if desired – we can solve for the difference:

$$K_i - K_f = \frac{m\,I}{2}\,\frac{v^2 + \omega_0^2 R^2 + 2Rv\,|\omega_0|}{mR^2 + I}$$

which is clearly positive. Thus, some of the initial kinetic energy is "lost" – that is, transferred to another form. And the culprit is the roach, who must find it difficult to stop (and "internalize" that energy).

48. The initial angular momentum of the system is zero. The final angular momentum of the girl-plus-merry-go-round is $\left(I + MR^2\right)\omega$ which we will take to be positive. The final angular momentum we associate with the thrown rock is negative: $-mRv$, where v is the speed (positive, by definition) of the rock relative to the ground.

(a) Angular momentum conservation leads to

$$0 = \left(I + MR^2\right)\omega - mRv \implies \omega = \frac{mRv}{I + MR^2} \ .$$

(b) The girl's linear speed is given by Eq. 11-18:

$$R\omega = \frac{mvR^2}{I + MR^2} \ .$$

49. For simplicity, we assume the record is turning freely, without any work being done by its motor (and without any friction at the bearings or at the stylus trying to slow it down). Before the collision, the angular momentum of the system (presumed positive) is $I_i\omega_i$ where $I_i = 5.0 \times 10^{-4}$ kg·m^2 and $\omega_i = 4.7$ rad/s. The rotational inertia afterwards is $I_f = I_i + mR^2$ where $m = 0.020$ kg and $R = 0.10$ m. The mass of the record (0.10 kg), although given in the problem, is not used in the solution. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \implies \omega_f = \frac{I_i\omega_i}{I_i + mR^2} = 3.4 \text{ rad/s} \ .$$

50. The axis of rotation is in the middle of the rod, $r = 0.25$ m from either end. By Eq. 12-19, the initial angular momentum of the system (which is just that of the bullet, before impact) is $rmv\sin\phi$ where $m = 0.003$ kg and $\phi = 60°$. Relative to the axis, this is counterclockwise and thus (by the common convention) positive. After the collision, the moment of inertia of the system is $I = I_{\text{rod}} + mr^2$ where $I_{\text{rod}} = ML^2/12$ by Table 11-2(e), with $M = 4.0$ kg and $L = 0.5$ m. Angular momentum conservation leads to

$$rmv\sin\phi = \left(\frac{1}{12}ML^2 + mr^2\right)\omega \ .$$

Thus, with $\omega = 10$ rad/s, we obtain

$$v = \frac{\left(\frac{1}{12}(4.0)(0.5)^2 + (0.003)(0.25)^2\right)(10)}{(0.25)(0.003)\sin 60°} = 1.3 \times 10^3 \text{ m/s} \ .$$

51. (a) If we consider a short time interval from just before the wad hits to just after it hits and sticks, we may use the principle of conservation of angular momentum. The initial angular momentum is the angular momentum of the falling putty wad. The wad initially moves along a line that is $d/2$ distant from the axis of rotation, where $d = 0.500\,\text{m}$ is the length of the rod. The angular momentum of the wad is $mvd/2$ where $m = 0.0500\,\text{kg}$ and $v = 3.00$ m/s are the mass and initial speed of the wad. After the wad sticks, the rod has angular velocity ω and angular momentum $I\omega$, where I is the rotational inertia of the system consisting of the rod with the two balls and the wad at its end. Conservation of angular momentum yields $mvd/2 = I\omega$ where $I = (2M + m)(d/2)^2$ and $M = 2.00$ kg is the mass of each of the balls. We solve $mvd/2 = (2M + m)(d/2)^2\omega$ for the angular speed:

$$\omega = \frac{2mv}{(2M+m)d} = \frac{2(0.0500)(3.00)}{(2(2.00)+0.0500)(0.500)} = 0.148 \text{ rad/s} .$$

(b) The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, the final kinetic energy is $K_f = \frac{1}{2}I\omega^2$, and their ratio is $K_f/K_i = I\omega^2/mv^2$. When $I = (2M + m)d^2/4$ and $\omega = 2mv/(2M + m)d$ are substituted, this becomes

$$\frac{K_f}{K_i} = \frac{m}{2M+m} = \frac{0.0500}{2(2.00)+0.0500} = 0.0123 .$$

(c) As the rod rotates, the sum of its kinetic and potential energies is conserved. If one of the balls is lowered a distance h, the other is raised the same distance and the sum of the potential energies of the balls does not change. We need consider only the potential energy of the putty wad. It moves through a 90° arc to reach the lowest point on its path, gaining kinetic energy and losing gravitational potential energy as it goes. It then swings up through an angle θ, losing kinetic energy and gaining potential energy, until it momentarily comes to rest. Take the lowest point on the path to be the zero of potential energy. It starts a distance $d/2$ above this point, so its initial potential energy is $U_i = mgd/2$. If it swings up to the angular position θ, as measured from its lowest point, then its final height is $(d/2)(1 - \cos\theta)$ above the lowest point and its final potential energy is $U_f = mg(d/2)(1 - \cos\theta)$. The initial kinetic energy is the sum of that of the balls and wad: $K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M + m)(d/2)^2\omega^2$. At its final position, we have $K_f = 0$. Conservation of energy provides the relation:

$$mg\,\frac{d}{2} + \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2\omega^2 = mg\,\frac{d}{2}\,(1-\cos\theta) .$$

When this equation is solved for $\cos\theta$, the result is

$$\begin{aligned}\cos\theta &= -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2 \\ &= -\frac{1}{2}\left(\frac{2(2.00\,\text{kg})+0.0500\,\text{kg}}{(0.0500\,\text{kg})(9.8\,\text{m/s}^2)}\right)\left(\frac{0.500\,\text{m}}{2}\right)(0.148\,\text{rad/s})^2 \\ &= -0.0226 .\end{aligned}$$

Consequently, the result for θ is 91.3°. The total angle through which it has swung is $90° + 91.3° = 181°$.

52. We denote the cockroach with subscript 1 and the disk with subscript 2.

(a) Initially the angular momentum of the system consisting of the cockroach and the disk is

$$L_i = m_1 v_{1i} r_{1i} + I_2\omega_{2i} = m_1\omega_0 R^2 + \frac{1}{2}m_2\omega_0 R^2 .$$

After the cockroach has completed its walk, its position (relative to the axis) is $r_{1f} = R/2$ so the final angular momentum of the system is

$$L_f = m_1\omega_f\left(\frac{R}{2}\right)^2 + \frac{1}{2}m_2\omega_f R^2 .$$

Then from $L_f = L_i$ we obtain

$$\omega_f\left(\frac{1}{4}m_1R^2 + \frac{1}{2}m_2R\right) = \omega_0\left(m_1R^2 + \frac{1}{2}m_2R^2\right) \ .$$

Thus,

$$\begin{aligned}\omega_f - \omega_0 &= \omega_0\left(\frac{m_1R^2 + m_2R^2/2}{m_1R^2/4 + m_2R^2/2}\right) - \omega_0 \\ &= \omega_0\left(\frac{m + 10m/2}{m/4 + 10m/2} - 1\right) \\ &= \omega_0(1.14 - 1)\end{aligned}$$

which yields $\Delta\omega = 0.14\omega_0$. For later use, we note that $\omega_f/\omega_i = 1.14$.

(b) We substitute $I = L/\omega$ into $K = \frac{1}{2}I\omega^2$ and obtain $K = \frac{1}{2}L\omega$. Since we have $L_i = L_f$, the the kinetic energy ratio becomes

$$\frac{K}{K_0} = \frac{\frac{1}{2}L_f\omega_f}{\frac{1}{2}L_i\omega_i} = \frac{\omega_f}{\omega_i} = 1.14 \ .$$

(c) The cockroach does positive work while walking toward the center of the disk, increasing the total kinetic energy of the system.

53. If the polar cap melts, the resulting body of water will effectively increase the equatorial radius of the Earth from R_e to $R'_e = R_e + \Delta R$, thereby increasing the moment of inertia of the Earth and slowing its rotation (by conservation of angular momentum), causing the duration T of a day to increase by ΔT. We note that (in rad/s) $\omega = 2\pi/T$ so

$$\frac{\omega'}{\omega} = \frac{2\pi/T'}{2\pi/T} = \frac{T}{T'}$$

from which it follows that

$$\frac{\Delta\omega}{\omega} = \frac{\omega'}{\omega} - 1 = \frac{T}{T'} - 1 = -\frac{\Delta T}{T'} \ .$$

We can approximate that last denominator as T so that we end up with the simple relationship $|\Delta\omega|/\omega = \Delta T/T$. Now, conservation of angular momentum gives us

$$\Delta L = 0 = \Delta(I\omega) \approx I(\Delta\omega) + \omega(\Delta I)$$

so that $|\Delta\omega|/\omega = \Delta I/I$. Thus, using our expectation that rotational inertia is proportional to the equatorial radius squared (supported by Table 11-2(f) for a perfect uniform sphere, but then this isn't a perfect uniform sphere) we have

$$\begin{aligned}\frac{\Delta T}{T} &= \frac{\Delta I}{I} \\ &= \frac{\Delta(R_e^2)}{R_e^2} \approx \frac{2\Delta R_e}{R_e} \\ &= \frac{2(30\,\text{m})}{6.37 \times 10^6\,\text{m}}\end{aligned}$$

so with $T = 86400\,\text{s}$ we find (approximately) that $\Delta T = 0.8$ s. The radius of the earth can be found in Appendix C or on the inside front cover of the textbook.

54. The initial rotational inertia of the system is $I_i = I_{\text{disk}} + I_{\text{student}}$ where $I_{\text{disk}} = 300\ \text{kg}\cdot\text{m}^2$ (which, incidentally, does agree with Table 11-2(c)) and $I_{\text{student}} = mR^2$ where $m = 60$ kg and $R = 2.0$ m. The rotational inertia when the student reaches $r = 0.5$ m is $I_f = I_{\text{disk}} + mr^2$. Angular momentum conservation leads to

$$I_i\omega_i = I_f\omega_f \implies \omega_f = \omega_i\,\frac{I_{\text{disk}} + mR^2}{I_{\text{disk}} + mr^2}$$

which yields, for $\omega_i = 1.5$ rad/s, a final angular velocity of $\omega_f = 2.6$ rad/s.

55. Their angular velocities, when they are stuck to each other, are equal, regardless of whether they share the same central axis. The initial rotational inertia of the system is

$$I_0 = I_{\text{big disk}} + I_{\text{small disk}} \qquad \text{where} \quad I_{\text{big disk}} = \frac{1}{2}MR^2$$

using Table 11-2(c). Similarly, since the small disk is initially concentric with the big one, $I_{\text{small disk}} = \frac{1}{2}mr^2$. After it slides, the rotational inertia of the small disk is found from the parallel axis theorem (using $h = R - r$). Thus, the new rotational inertia of the system is

$$I = \frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m(R-r)^2 \; .$$

(a) Angular momentum conservation, $I_0\omega_0 = I\omega$, leads to the new angular velocity:

$$\omega = \omega_0 \, \frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2}{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + m(R-r)^2} \; .$$

Substituting $M = 10m$ and $R = 3r$, this becomes $\omega = \omega_0(91/99)$. Thus, with $\omega_0 = 20$ rad/s, we find $\omega = 18$ rad/s.

(b) From the previous part, we know that

$$\frac{I_0}{I} = \frac{91}{99} \quad \text{and} \quad \frac{\omega}{\omega_0} = \frac{91}{99} \; .$$

Plugging these into the ratio of kinetic energies, we have

$$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I_0\omega_0^2} = \frac{I}{I_0}\left(\frac{\omega}{\omega_0}\right)^2 = \frac{99}{91}\left(\frac{91}{99}\right)^2$$

which yields $K/K_0 = 0.92$.

56. This is a completely inelastic collision which we analyze using angular momentum conservation. Let m and v_0 be the mass and initial speed of the ball and R the radius of the merry-go-round. The initial angular momentum is

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 \quad \Longrightarrow \quad \ell_0 = R\,(mv_0)\sin 53^\circ$$

where 53° is the angle between the radius vector pointing to the child and the direction of $\vec{v}_0$. Thus, $\ell_0 = 19\,\text{kg}\cdot\text{m}^2/\text{s}$. Now, with SI units understood,

$$\begin{aligned} \ell_0 &= L_f \\ 19 &= I\omega \\ &= \left(150 + (30)R^2 + (1.0)R^2\right)\omega \end{aligned}$$

so that $\omega = 0.070$ rad/s.

57. (a) With $r = 0.60$ m, we obtain $I = 0.060 + (0.501)r^2 = 0.24\ \text{kg}\cdot\text{m}^2$.

(b) Invoking angular momentum conservation, with SI units understood,

$$\begin{aligned} \ell_0 &= L_f \\ mv_0r &= = I\omega \\ (0.001)v_0(0.60) &= (0.24)(4.5) \end{aligned}$$

which leads to $v_0 = 1.8 \times 10^3$ m/s.

58. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities in this problem are positive. With $r = 0.60$ m and $I_0 = 0.12\ \text{kg}\cdot\text{m}^2$, the rotational inertia of the putty-rod system (after the collision) is $I = I_0 + (0.20)r^2 = 0.19\ \text{kg}\cdot\text{m}^2$. Invoking angular momentum conservation, with SI units understood, we have

$$\begin{aligned} L_0 &= L_f \\ I_0\omega_0 &= I\omega \\ (0.12)(2.4) &= (0.19)\omega \end{aligned}$$

which yields $\omega = 1.5$ rad/s.

59. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocities (and angles) in this problem are positive. Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \implies v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega$$

where I_{rod} is found using Table 11-2(e) and the parallel axis theorem:

$$I_{\text{rod}} = \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2 = \frac{1}{3}Md^2 .$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{\frac{1}{3}Md^2 + md^2}$$

which means the system has kinetic energy $\frac{1}{2}(I_{\text{rod}} + md^2)\omega^2$ which will turn into potential energy in the final position, where the block has reached a height H (relative to the lowest point) and the center of mass of the stick has increased its height by $H/2$. From trigonometric considerations, we note that $H = d(1 - \cos\theta)$, so we have

$$\begin{aligned} \frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 &= mgH + Mg\frac{H}{2} \\ \frac{1}{2}\frac{m^2d^2(2gh)}{\frac{1}{3}Md^2 + md^2} &= \left(m + \frac{M}{2}\right)gd\,(1 - \cos\theta) \end{aligned}$$

from which we obtain

$$\theta = \cos^{-1}\left(1 - \frac{m^2h}{\left(m + \frac{1}{2}M\right)\left(m + \frac{1}{3}M\right)}\right) .$$

60. (a) Since the motorcycle is going leftward across our field of view, then when its wheels are rolling they must be going counterclockwise (which we take as the positive sense of rotation, which is the usual convention).

(b) Just before the rear wheel spins up to ω_{wf} it has the angular velocity necessary for rolling $\omega_{wR} = v/R$ where $v = 32$ m/s and $R = 0.30$ m. Since $\omega_{wf} > \omega_{wR}$ the system would seem to have suddenly acquired an increase in (positive) angular momentum − without the action of external torques! Since this is not possible, then the other constituents of the system (the man and the motorcycle body, which the problem just refers to as "the motorcycle") must have acquired some (negative) angular momentum. Thus, the motorcycle rotated clockwise.

(c) Assuming the system's (translational) projectile motion is symmetrical (as in Fig. 4-34 in the textbook) then (with $+y$ upward) it starts with $v_{0y} = v\sin 15°$ and returns with $v_y = -v\sin 15°$. Substituting these into Eq. 2-11 (with $a = -g$) leads to

$$-v\sin 15° = v\sin 15° - gt \implies t = \frac{2v\sin 15°}{g} = 1.7 \text{ s} .$$

(d) As noted in our solution of part (b), $\omega_{wR} = v/R$ which yields the value $\omega_{wR} = 32/0.30 = 106.7$ rad/s. In keeping with the significant figures rules, we round this to 1.1×10^2 rad/s.

(e) We have $L_w = I_w\omega_{wR} = (0.40)(106.7) = 43$ kg·m^2/s.

(f) Recalling our discussion in part (b), we apply angular momentum conservation:

$$I_w\omega_{wR} = I_w\omega_{wf} + I_c\omega_c \implies \omega_c = -\frac{I_w\left(\omega_{wf} - \omega_{wR}\right)}{I_c}$$

which yields $\omega_c = -1.067$ rad/s or $|\omega_c| \approx 1.1$ rad/s.

(g) The problem states that the spin up occurs immediately – the moment this becomes a projectile motion problem (for the center of mass). We assume the motorcycle turns at the (constant) rate $|\omega_c|$ for the duration of the motion. Using the more precise values from our previous results, we are led to

$$\theta = \omega_c t = -1.80 \text{ rad}$$

which we convert (multiplying by $180/\pi$) to $-103°$. Rounding off, we find $|\theta| \approx 100°$.

61. (a) The derivation of the acceleration is found in §12-4; Eq. 12-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = \frac{1}{2}MR^2$ where the radius is $R = 0.32$ m and $M = 116$ kg is the *total* mass (thus including the fact that there are two disks) and obtain

$$a = -\frac{g}{1 + \frac{1}{2}MR^2/MR_0^2} = \frac{g}{1 + \frac{1}{2}\left(\frac{R}{R_0}\right)^2}$$

which yields $a = -g/51$ upon plugging in $R_0 = R/10 = 0.032$ m. Thus, the magnitude of the center of mass acceleration is 0.19 m/s^2 and the direction of that vector is down.

(b) As observed in §12-4, our result in part (a) applies to both the descending and the rising yoyo motions.

(c) The external forces on the center of mass consist of the cord tension (upward) and the pull of gravity (downward). Newton's second law leads to

$$T - Mg = ma \implies T = M\left(g - \frac{g}{51}\right)$$

which yields $T = 1.1 \times 10^3$ N.

(d) Our result in part (c) indicates that the tension is well below the ultimate limit for the cord.

(e) As we saw in our acceleration computation, all that mattered was the ratio R/R_0 (and, of course, g). So if it's a scaled-up version, then such ratios are unchanged and we obtain the same result.

(f) Since the tension also depends on mass, then the larger yoyo will involve a larger cord tension.

62. We denote the wheel with subscript 1 and the whole system with subscript 2. We take clockwise as the negative sense for rotation (as is the usual convention). Conservation of angular momentum gives $L = I_1\omega_1 = I_2\omega_2$, where $I_1 = m_1 R_1^2$. Thus

$$\omega_2 = \omega_1 \frac{I_1}{I_2} = (-57.7\,\text{rad/s})\frac{\left(37\,\text{N}/9.8\,\text{m/s}^2\right)(0.35\,\text{m})^2}{2.1\,\text{kg}\cdot\text{m}^2}$$

which yields $\omega_2 = -12.7\,\text{rad/s}$. The system therefore rotates clockwise (as seen from above) at the rate of 12.7 rad/s.

63. We use $L = I\omega$ and $K = \frac{1}{2}I\omega^2$ and observe that the speed of points on the rim (corresponding to the speed of points on the belt) of wheels A and B must be the same (so $\omega_A R_A = \omega_B r_B$).

(a) If $L_A = L_B$ (call it L) then the ratio of rotational inertias is

$$\frac{I_A}{I_B} = \frac{L/\omega_A}{L/\omega_B} = \frac{\omega_A}{\omega_B} = \frac{R_A}{R_B} = \frac{1}{3}\ .$$

(b) If we have $K_A = K_B$ (call it K) then the ratio of rotational inertias becomes

$$\frac{I_A}{I_B} = \frac{2K/\omega_A^2}{2K/\omega_B^2} = \left(\frac{\omega_B}{\omega_A}\right)^2 = \left(\frac{R_A}{R_B}\right)^2 = \frac{1}{9}\ .$$

64. Since we will be taking the vector cross product in the course of our calculations, below, we note first that when the two vectors in a cross product $\vec{A} \times \vec{B}$ are in the xy plane, we have $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$, and Eq. 3-30 leads to

$$\vec{A} \times \vec{B} = (A_x B_y - A_y B_x)\,\hat{k}\ .$$

(a) We set up a coordinate system with its origin at the firing point, the positive x axis in the horizontal direction of motion of the projectile and the positive y axis vertically upward. The projectile moves in the xy plane, and if $+x$ is to our right then the "rotation" sense will be clockwise. Thus, we expect our answer to be negative. The position vector for the projectile (as a function of time) is given by

$$\vec{r} = (v_{0x}t)\hat{\imath} + \left(v_{0y}t - \frac{1}{2}gt^2\right)\hat{\jmath} = (v_0\cos\theta_0 t)\hat{\imath} + (v_0\sin\theta_0 - gt)\hat{\jmath}$$

and the velocity vector is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (v_0\cos\theta_0)\,\hat{\imath} + (v_0\sin\theta_0 - gt)\hat{\jmath}\ .$$

Thus (using the above observation about the cross product of vectors in the xy plane) the angular momentum of the projectile as a function of time is

$$\vec{\ell} = m\vec{r} \times \vec{v} = -\frac{1}{2}mv_0\cos\theta_0 gt^2\hat{k}\,.$$

(b) We take the derivative of our result in part (a): $\frac{d\vec{\ell}}{dt} = -v_0 mgt\cos\theta_0\hat{k}$.

(c) Again using the above observation about the cross product of vectors in the xy plane, we find

$$\vec{r} \times \vec{F} = \left((v_0\cos\theta_0 t)\hat{\imath} + r_y\hat{\jmath}\right) \times (-mg\hat{\jmath}) = -v_0 mgt\cos\theta_0\,\hat{k}$$

which is the same as the result in part (b).

(d) They are the same because $d\vec{\ell}/dt = \tau = \vec{r} \times \vec{F}$.

65. The problem asks that we put the origin of coordinates at point O but compute all the angular momenta and torques relative to point A. This requires some care in defining $\vec{r}$ (which occurs in the angular momentum and torque formulas). If $\vec{r}_O$ locates the point (where the block is) in the prescribed coordinates, and $\vec{r}_{OA} = -1.2\,\hat{\jmath}$ points from O to A, then $\vec{r} = \vec{r}_O - \vec{r}_{OA}$ gives the position of the block relative to point A. SI units are used throughout this problem.

(a) Here, the momentum is $\vec{p}_0 = m\vec{v}_0 = 1.5\,\hat{\imath}$ and $\vec{r}_0 = 1.2\,\hat{\jmath}$, so that

$$\vec{\ell}_0 = \vec{r}_0 \times \vec{p}_0 = -1.8\,\hat{\mathrm{k}}\ \mathrm{kg{\cdot}m^2/s}\ .$$

(b) The horizontal component of momentum doesn't change in projectile motion (without friction), and its vertical component depends on how far its fallen. From either the free-fall equations of Ch. 2 or the energy techniques of Ch. 8, we find the vertical momentum component after falling a distance h to be $-m\sqrt{2gh}$. Thus, with $m = 0.50$ and $h = 1.2$, the momentum just before the block hits the floor is $\vec{p} = 1.5\,\hat{\imath} - 2.4\,\hat{\jmath}$. Now, $\vec{r} = R\,\hat{\imath}$ where R is figured from the projectile motion equations of Ch. 4 to be $R = v_0\sqrt{\frac{2h}{g}} = 1.5$ m. Consequently,

$$\vec{\ell} = \vec{r} \times \vec{p} = -3.6\,\hat{\mathrm{k}}\ \mathrm{kg{\cdot}m^2/s}\ .$$

(c) and (d) The only force on the object is its weight $m\vec{g} = -4.9\,\hat{\jmath}$. Thus,

$$\begin{aligned} \vec{\tau}_0 &= \vec{r}_0 \times \vec{F} = 0 \\ \vec{\tau} &= \vec{r} \times \vec{F} = -7.3\,\hat{\mathrm{k}}\ \mathrm{N{\cdot}m}\ . \end{aligned}$$

66. Since we will be taking the vector cross product in the course of our calculations, below, we note first that when the two vectors in a cross product $\vec{A} \times \vec{B}$ are in the xy plane, we have $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$, and Eq. 3-30 leads to

$$\vec{A} \times \vec{B} = (A_xB_y - A_yB_x)\,\hat{\mathrm{k}}\ .$$

Now, we choose coordinates centered on point O, with $+x$ rightwards and $+y$ upwards. In unit-vector notation, the initial position of the particle, then, is $\vec{r}_0 = s\,\hat{\imath}$ and its later position (halfway to the ground) is $\vec{r} = s\,\hat{\imath} - \frac{1}{2}h\,\hat{\jmath}$. Using either the free-fall equations of Ch. 2 or the energy techniques of Ch. 8, we find the speed at its later position to be $v = \sqrt{2g|\Delta y|} = \sqrt{gh}$. Its momentum there is $\vec{p} = -M\sqrt{gh}\,\hat{\jmath}$. We find the angular momentum using Eq. 12-18 and our observation, above, about the cross product of two vectors in the xy plane.

$$\vec{\ell} = \vec{r} \times \vec{p} = -sM\sqrt{gh}\ \hat{\mathrm{k}}$$

Therefore, its magnitude is $|\vec{\ell}| = sM\sqrt{gh}$.

67. We may approximate the planets and their motions as particles in circular orbits, and use Eq. 12-26

$$L = \sum_{i=1}^{9} \ell_i = \sum_{i=1}^{9} m_i r_i^2 \omega_i$$

to compute the total angular momentum. Since we assume the angular speed of each one is constant, we have (in rad/s) $\omega_i = 2\pi/T_i$ where T_i is the time for that planet to go around the Sun (this and related information is found in Appendix C but there, the T_i are expressed in years and we'll need to convert with 3.156×10^7 s/y, and the M_i are expressed as multiples of M_{earth} which we'll convert by multiplying by 5.98×10^{24} kg).

(a) Using SI units, we find (with $i = 1$ designating Mercury)

$$\begin{aligned} L &= \sum_{i=1}^{9} m_i r_i^2 \left(\frac{2\pi}{T_i}\right) \\ &= 2\pi \frac{3.34 \times 10^{23}}{7.61 \times 10^6} \left(57.9 \times 10^9\right)^2 + 2\pi \frac{4.87 \times 10^{24}}{19.4 \times 10^7} \left(108 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{5.98 \times 10^{24}}{3.156 \times 10^7} \left(150 \times 10^9\right)^2 + 2\pi \frac{6.40 \times 10^{23}}{5.93 \times 10^7} \left(228 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{1.9 \times 10^{27}}{3.76 \times 10^8} \left(778 \times 10^9\right)^2 + 2\pi \frac{5.69 \times 10^{26}}{9.31 \times 10^8} \left(1430 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{8.67 \times 10^{25}}{2.65 \times 10^9} \left(2870 \times 10^9\right)^2 + 2\pi \frac{1.03 \times 10^{26}}{5.21 \times 10^9} \left(4500 \times 10^9\right)^2 + \\ &\quad 2\pi \frac{1.2 \times 10^{22}}{7.83 \times 10^9} \left(5900 \times 10^9\right)^2 \\ &= 3.14 \times 10^{43} \text{ kg}\cdot\text{m}^2/\text{s} . \end{aligned}$$

(b) The fractional contribution of Jupiter is

$$\frac{\ell_5}{L} = \frac{2\pi \frac{1.9 \times 10^{27}}{3.76 \times 10^8} \left(778 \times 10^9\right)^2}{3.14 \times 10^{43}} = 0.61 .$$

68. If we write $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then (using Eq. 3-30) we find $\vec{r} \times \vec{F}$ is equal to

$$(yF_z - zF_y)\,\hat{\imath} + (zF_x - xF_z)\,\hat{\jmath} + (xF_y - yF_x)\,\hat{k} .$$

With (using SI units) $x = 0$, $y = -4.0$, $z = 5.0$, $F_x = 0$, $F_y = -2.0$ and $F_z = 3.0$ (these latter terms being the individual forces that contribute to the net force), the expression above yields

$$\vec{\tau} = \vec{r} \times \vec{F} = -2.0\,\hat{\imath} \text{ N}\cdot\text{m} .$$

69. We make the unconventional choice of *clockwise* sense as positive, so that the angular acceleration are positive (as is the linear acceleration of the center of mass, since we take rightwards as positive).

(a) We approach this in the manner of Eq. 12-3 (*pure rotation* about point P) but use torques instead of energy:

$$\tau = I_P\,\alpha \quad \text{where } I_P = \frac{1}{2}MR^2 + MR^2$$

where the parallel-axis theorem and Table 11-2(c) has been used. The torque (relative to point P) is due to the $F = 12$ N force and is $\tau = F(2R)$. In this way, we find

$$\alpha = \frac{(12)(0.20)}{0.05 + 0.10} = 16 \text{ rad/s}^2 .$$

Hence, $a_{\text{com}} = R\alpha = 1.6 \text{ m/s}^2$.

(b) As shown above, $\alpha = 16 \text{ rad/s}^2$.

(c) Applying Newton's second law in its linear form yields

$$(12 \text{ N}) - f = Ma_{\text{com}} \quad .$$

Therefore, $f = -4.0$ N. Contradicting what we assumed in setting up our force equation, the friction force is found to point *rightward* (with magnitude 4.0 N).

70. The speed of the center of mass of the car is $v = (40)(1000/3600) = 11$ m/s. The angular speed of the wheels is given by Eq. 12-2: $\omega = v/R$ where the wheel radius R is not given (but will be seen to cancel in these calculations).

    (a) For one wheel of mass $M = 32$ kg, Eq. 11-27 gives (using Table 11-2(c))

    $$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

    which yields $K_{\text{rot}} = 9.9 \times 10^2$ J. The time given in the problem (10 s) is not used in the solution.

    (b) Adding the above to the wheel's translational kinetic energy, $\frac{1}{2}Mv^2$, leads to

    $$K_{\text{wheel}} = \frac{1}{2}Mv^2 + \frac{1}{4}Mv^2 = \frac{3}{4}(32)(11)^2 = 3.0 \times 10^3 \text{ J} .$$

    (c) With $M_{\text{car}} = 1700$ kg and the fact that there are four wheels, we have

    $$\frac{1}{2}M_{\text{car}}v^2 + 4\left(\frac{3}{4}Mv^2\right) = 1.2 \times 10^5 \text{ J} .$$

71. Information relevant to this calculation can be found in Appendix C. We apply angular momentum conservation using Table 11-2(f):

    $$I_i\omega_i = I_f\omega_f \quad\Longrightarrow\quad \frac{\omega_i}{\omega_f} = \frac{I_f}{I_i} = \frac{\frac{2}{5}MR_f^2}{\frac{2}{5}MR_i^2}$$

    and we note that $\omega = 2\pi/T$ in rad/min if T is the period in minutes. Plugging this into to our expression above (and simplifying) yields

    $$\frac{T_f}{T_i} = \left(\frac{R_f}{R_i}\right)^2 .$$

    Substituting $T_i = 25(24)(60) = 36000$ min, $R_f = 6.37 \times 10^6$ m and $R_i = 6.96 \times 10^8$ m into this relation, we obtain $T_f = 3.0$ min.

72. (a) We use Table 11-2(e) and the parallel-axis theorem to obtain the rod's rotational inertia about an axis through one end:

    $$I = I_{\text{com}} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$$

    where $L = 6.00$ m and $M = 10.0/9.8 = 1.02$ kg. Thus, $I = 12.2$ kg$\cdot$m^2.

    (b) Using $\omega = (240)(2\pi/60) = 25.1$ rad/s, Eq. 12-31 gives the magnitude of the angular momentum as $I\omega = (12.2)(25.1) = 308$ kg$\cdot$m^2/s. Since it is rotating clockwise as viewed from above, then the right-hand rule indicates that its direction is down.

73. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r}\,' = x'\,\hat{\imath} + y'\,\hat{\jmath}$, then (using Eq. 3-30) we find

    $$\vec{r}\,' \times \vec{v} = (x'v_y - y'v_x)\,\hat{\text{k}} .$$

    (a) Here, $\vec{r}\,'$ points in either the $+\hat{\imath}$ or the $-\hat{\imath}$ direction (since the particle moves along the x axis). It has no y' or z' components, and neither does $\vec{v}$, so it is clear from the above expression (or, more simply, from the fact that $\hat{\imath} \times \hat{\imath} = 0$) that $\vec{\ell} = m(\vec{r}\,' \times \vec{v}) = 0$ in this case.

    (b) The net force is in the $-\hat{\imath}$ direction (as one finds from differentiating the velocity expression, yielding the acceleration), so, similar to what we found in part (a), we obtain $\tau = \vec{r}\,' \times \vec{F} = 0$.

(c) Now, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\,\hat{\imath} + 5.0\,\hat{\jmath}$ (with SI units understood) and points from $(2.0, 5.0, 0)$ to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_o \times \vec{v}\right) = -(3.0)\left((2.0)(0) - (5.0)\left(-2.0t^3\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = -30t^3\,\hat{\mathrm{k}}$ in SI units (kg·m^2/s).

(d) The acceleration vector is given by $\vec{a} = \frac{d\vec{v}}{dt} = -6.0t^2\,\hat{\imath}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_o \times \vec{a}\right) = -(3.0)\left((2.0)(0) - (5.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = -90t^2\,\hat{\mathrm{k}}$ in SI units (N·m).

(e) In this situation, $\vec{r}\,' = \vec{r} - \vec{r}_o$ where $\vec{r}_o = 2.0\,\hat{\imath} - 5.0\,\hat{\jmath}$ (with SI units understood) and points from $(2.0, -5.0, 0)$ to the instantaneous position of the car (indicated by $\vec{r}$ which points in either the $+x$ or $-x$ directions, or nowhere (if the car is passing through the origin)). Since $\vec{r} \times \vec{v} = 0$ we have (plugging into our general expression above)

$$\vec{\ell} = m\left(\vec{r}\,' \times \vec{v}\right) = -m\left(\vec{r}_o \times \vec{v}\right) = -(3.0)\left((2.0)(0) - (-5.0)\left(-2.0t^3\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\ell} = 30t^3\,\hat{\mathrm{k}}$ in SI units (kg·m^2/s).

(f) Again, the acceleration vector is given by $\vec{a} = -6.0t^2\,\hat{\imath}$ in SI units, and the net force on the car is $m\vec{a}$. In a similar argument to that given in the previous part, we have

$$\vec{\tau} = m\left(\vec{r}\,' \times \vec{a}\right) = -m\left(\vec{r}_o \times \vec{a}\right) = -(3.0)\left((2.0)(0) - (-5.0)\left(-6.0t^2\right)\right)\hat{\mathrm{k}}$$

which yields $\vec{\tau} = 90t^2\,\hat{\mathrm{k}}$ in SI units (N·m).

74. This problem involves the vector cross product of vectors lying in the xy plane. For such vectors, if we write $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath}$, then (using Eq. 3-30) we find

$$\vec{r} \times \vec{p} = \left(\Delta x p_y - \Delta y p_x\right)\hat{\mathrm{k}}\ .$$

The momentum components are $p_x = p\cos\theta$ and $p_y = p\sin\theta$ where $p = 2.4$ (SI units understood) and $\theta = 115°$. The mass (0.80 kg) given in the problem is not used in the solution. Thus, with $x = 2.0$, $y = 3.0$ and the momentum components described above, we obtain

$$\vec{\ell} = \vec{r} \times \vec{p} = 7.4\,\hat{\mathrm{k}}\ \mathrm{kg{\cdot}m^2/s}\ .$$

75. Information relevant to this calculation can be found in Appendix C or on the inside front cover of the textbook. The angular speed is constant so

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.3 \times 10^{-5}\ \mathrm{rad/s}\ .$$

Thus, with $m = 84$ kg and $R = 6.37 \times 10^6$ m, we find $\ell = mR^2\omega = 2.5 \times 10^{11}$ kg·m^2/s.

76. With $r_\perp = 1300$ m, Eq. 12-21 gives

$$\ell = r_\perp m v = (1300)(1200)(80) = 1.2 \times 10^8\ \mathrm{kg{\cdot}m^2/s}\ .$$

77. The result follows immediately from Eq. 3-30. If what is desired to show here is basically a derivation of Eq. 3-30, then (with the slight change to position and force notation) that is shown in some detail in our solution to problem 32 of Chapter 3.

78. (a) Using Eq. 2-16 for the translational (center-of-mass) motion, we find

$$v^2 = v_0^2 + 2a\Delta x \implies a = -\frac{v_0^2}{2\Delta x}$$

which yields $a = -4.11$ for $v_0 = 43$ and $\Delta x = 225$ (SI units understood). The magnitude of the linear acceleration of the center of mass is therefore 4.11 m/s^2.

(b) With $R = 0.250$ m, Eq. 12-6 gives $|\alpha| = |a|/R = 16.4$ rad/s^2. If the wheel is going rightward, it is rotating in a clockwise sense. Since it is slowing down, this angular acceleration is counterclockwise (opposite to ω) so (with the usual convention that counterclockwise is positive) there is no need for the absolute value signs for α.

(c) Eq. 12-8 applies with Rf_s representing the magnitude of the frictional torque. Thus, $Rf_s = I\alpha = (0.155)(16.4) = 2.55\,\text{N}\cdot\text{m}$.

79. We note that its mass is $M = 36/9.8 = 3.67$ kg and its rotational inertia is $I_{\text{com}} = \frac{2}{5}MR^2$ (Table 11-2(f)).

(a) Using Eq. 12-2, Eq. 12-5 becomes

$$\begin{aligned} K &= \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 \\ &= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{\text{com}}}{R}\right)^2 + \frac{1}{2}Mv_{\text{com}}^2 \\ &= \frac{7}{10}Mv_{\text{com}}^2 \end{aligned}$$

which yields $K = 61.7$ J for $v_{\text{com}} = 4.9$ m/s.

(b) This kinetic energy turns into potential energy Mgh at some height $h = d\sin\theta$ where the sphere comes to rest. Therefore, we find the distance traveled up the $\theta = 30°$ incline from energy conservation:

$$\frac{7}{10}Mv_{\text{com}}^2 = Mgd\sin\theta \implies d = \frac{7v_{\text{com}}^2}{10g\sin\theta} = 3.43 \text{ m} .$$

(c) As shown in the previous part, M cancels in the calculation for d. Since the answer is independent of mass, then, it is also independent of the sphere's weight.

80. Although we will not be "working" this problem, we do – briefly – share a few thoughts about it.

(a) A figure in the textbook that may be referred to is Fig. 8-16. The idea, crudely stated, is to show that although all bodies will return to the same height they're released from (in the absence of dissipative effects), the one with the least rotational inertia (say, a sphere) will get there the fastest because its speed is greatest at every point inbetween.

(b) Several people might be pulling on ropes attached to a merry-go-round to set it into motion. The ropes should be at different angles (measured relative to tangent lines at the appropriate points). The idea is to calculate the net torque using Eq. 12-15 and then to find the angular acceleration (using Eq. 11-37) of the merry-go-round.

(c) This might require particular care in the wording, especially regarding a clown "falling off." If he falls off in what might described as the "natural way" (simply letting go and pursuing a straight-line trajectory tangent to the merry-go-round) then there is no change in the angular momentum. It's easier to see that there'd be a change in angular momentum in the case of a clown (initially at rest) stepping onto the moving merry-go-round.

(d) This is an important astrophysical application of the angular momentum concept (angular momentum is conserved in gravitational-dominated situations such as binary star systems). When the masses of the stars are similar and the mass transfer is relatively steady, they are often known as Algol binaries, and realistic numerical values can be found in many astronomy textbooks (and, probably, on the Web).

81. (First problem in **Cluster 1**)

(a) Applying Newton's second law in its linear form yields

$$(200\text{ N}) - f = M_{\text{cart}}\, a \quad .$$

Therefore, $f = 200 - (50.0)(3.00) = 50$ N.

(b) The torque associated with the friction is $\tau_f = fR = (50)(0.200) = 10$ N·m. (We make the unconventional choice of the clockwise sense as positive, so that the frictional torque and this angular acceleration are positive.)

(c) Applying the rotational form of Newton's second law (relative to the axle) yields

$$\tau_f = I\alpha \qquad\qquad \text{where } \alpha = \frac{a}{R} = 15.0\text{ rad/s}^2 \ .$$

Therefore, $I = 0.667$ kg·m^2.

82. (Second problem in **Cluster 1**)

(a) If we interpret this "one-wheel cart" which has a wheel that is a "long cylinder" as simply the cylinder itself, then an appropriate picture for this problem is Fig. 12-30 in the textbook. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocity in this problem is positive; we choose *downhill* positive for the x axis (which is parallel to the incline surface) so that $a_{\text{com}} = R\alpha$ holds. We can combine the rotational (about the center of mass) and linear forms of Newton's second law, or we can more simply adopt the view of pure rotation (see, for example, Eq. 12-3) and examine torques about the bottom-most point P:

$$MgR\sin\theta = I_P\,\alpha = I_P\,\frac{a_{\text{com}}}{R}$$

We have assumed that the center of mass of the cart-wheel system is at the center of the wheel (the axle), although this is not stated in the problem. Now, $\theta = 30.0°$, $R = 0.200$ m, $M = 50.0$ kg, and $I_P = 0.667\text{ kg}\cdot\text{m}^2 + MR^2 = 2.67\text{ kg}\cdot\text{m}^2$ (using the parallel-axis theorem and the result of the previous problem). Thus, we find $a_{\text{com}} = 3.68\,\text{m/s}^2$.

(b) If we apply the linear form of Newton's law, we have

$$\begin{aligned}\sum F_x &= Mg\sin\theta - f_{s,\,\max} = Ma_{\text{com}}\\ \sum F_y &= N - Mg\sin\theta = 0\end{aligned}$$

Solving for $f_{s,\,\max}$ and N and dividing, we obtain

$$\mu_s = \frac{f_{s,\,\max}}{N} = 0.14 \quad .$$

83. (Third problem in **Cluster 1**)
An appropriate picture for this problem is Fig. 12-7 in the textbook. We make the unconventional choice of *clockwise* sense as positive, so that the angular velocity in this problem is positive; we choose *downhill* positive for the x axis (which is parallel to the incline surface) so that $a_{\text{com}} = R\alpha$ holds. For simplicity, we refer to a_{com} as a. We examine the rotational (about the center of mass) and linear forms of Newton's second law:

$$\begin{aligned}\sum \tau_z &= f_sR = I\alpha = I\frac{a}{R}\\ \sum F_x &= Mg\sin\theta - f_s = Ma\\ \sum F_y &= N - Mg\cos\theta = 0\end{aligned}$$

Combining the first two of these equations, we obtain

$$f_s = \frac{Mg\sin\theta}{1+\frac{MR^2}{I}} \quad .$$

We now let $f_s = f_{s,\max} = \mu_s N$ and combine this with the third equation above:

$$\mu_s Mg\cos\theta = \frac{Mg\sin\theta}{1+\frac{MR^2}{I}} \implies \theta = \tan^{-1}\left(\mu_s + \frac{MR^2\mu_s}{I}\right) .$$

84. (Fourth problem in **Cluster 1**)

(a) We take the tangential acceleration of the bottom-most point on the (positively) accelerating disk to equal $R\alpha + a_{\text{com}}$. This in turn must equal the (forward) acceleration of the truck $a_{\text{truck}} = a > 0$. Since the disk is rolling toward the back of the truck, $a_{\text{com}} < a$ which implies that α is positive. If the forward direction is *rightward*, then this makes it consistent to choose counterclockwise as the positive rotational sense, which is the usual convention. Thus, $\sum \tau = I\alpha$ becomes

$$f_s R = I\alpha \qquad\qquad \text{where } I = \frac{1}{2}MR^2$$

and $\sum F_x = Ma_{\text{com}}$ becomes

$$f_s = M\,(a - R\alpha) \quad .$$

Combining these two equations, we find $R\alpha = \frac{2}{3}\,a$. From the previous discussion, we see acceleration of the disk relative to the truck bed is $a_{\text{com}} - a = -R\alpha$, so this has a magnitude of $\frac{2}{3}$ and is directed leftward.

(b) Returning to $R\alpha + a_{\text{com}} = a$ with our result that $R\alpha = \frac{2}{3}\,a$, we find $a_{\text{com}} = \frac{1}{3}\,a$. This is positive, hence rightward.

85. (First problem in **Cluster 2**)
The last line of the problem indicates our choice of positive directions: up for m_2, down for m_1 and counterclockwise for the two-pulley device. This allows us to write $R_2\alpha = a_2$ and $R_1\alpha = a_1$ with all terms positive. We apply Newton's second law to the elements of this system:

$$\begin{aligned} T_2 - m_2 g &= m_2 a_2 = m_2 R_2 \alpha \\ m_1 g - T_1 &= m_1 a_1 = m_1 R_1 \alpha \\ T_1 R_1 - T_2 R_2 &= I\alpha \end{aligned}$$

Multiplying the first equation by R_2, the second by R_1 and adding the equations leads to

$$\alpha = \frac{m_1 g R_1 - m_2 g R_2}{I + m_1 R_1^2 + m_2 R_2^2} \quad .$$

(a) Therefore, again using $R_1\alpha = a_1$, we obtain

$$a_1 = \frac{m_1 g R_1^2 - m_2 g R_1 R_2}{I + m_1 R_1^2 + m_2 R_2^2} \quad .$$

(b) Once more, we use $R_2\alpha = a_2$ and find

$$a_2 = \frac{m_1 g R_1 R_2 - m_2 g R_2^2}{I + m_1 R_1^2 + m_2 R_2^2} \quad .$$

86. (Second problem in **Cluster 2**)
This system is extensively discussed in §12-4. Rather than repeat those steps here, we refer to their conclusion, Eq. 12-13.

   (a) The magnitude of the result in Eq. 12-13 is

$$|a| = g\,\frac{1}{1+\frac{I}{MR^2}} \quad .$$

   (b) The relation $a = a_{\text{com}} = -R\alpha$ used in §12-3 must now be modified to read $a_f - a_{\text{com}} = R\alpha$ where a_f is the acceleration of the finger. With this in mind, the linear and angular versions of Newton's second law become

$$\begin{aligned} T - Mg &= Ma_{\text{com}} \\ TR &= I\alpha \qquad \text{where } \alpha = \frac{a_f - a_{\text{com}}}{R} \end{aligned}$$

   If we require $a_{\text{com}} = 0$ then these equations yield

$$a_f = g\,\frac{MR^2}{I} \quad .$$

87. (Third problem in **Cluster 2**)
Our analysis of spool 2 is exactly as in the solution of part (b) of the previous problem, but with a_f replaced with $-a_s$. The negative sign is due to the wording of the problem (which refers to a "downward acceleration a_s"):

$$\begin{aligned} T - Mg &= Ma_1 \\ TR_1 &= I_1\alpha_1 = I_1\left(\frac{-a_s - a_1}{R_1}\right) \end{aligned}$$

In our analysis of spool 1, we pay close attention to signs: positive (downward) a_s corresponds to clockwise (conventionally taken to be negative) rotation of spool 1; hence, $R_2\alpha_2 = -a_s$. For spool 1, we therefore have

$$\sum \tau_z = -TR_2 = I_2\alpha_1 = I_2\left(\frac{-a_s}{R_2}\right) \quad .$$

   (a) Simultaneous solution (certainly non-trivial) of these three equations yields

$$a_1 = -\frac{g}{1+\dfrac{1}{\dfrac{MR_1^2}{I_1}+\dfrac{MR_2^2}{I_2}}} \quad .$$

   The problem asks for the magnitude of this (which eliminates the negative sign).

   (b) This amounts to eliminating the $\frac{MR_2^2}{I_2}$ term in the expression for a_1.

Chapter 13

1. From $\vec{\tau} = \vec{r} \times \vec{F}$, we note that persons 1 through 4 exert torques pointing out of the page (relative to the fulcrum), and persons 5 through 8 exert torques pointing into the page.

   (a) Among persons 1 through 4, the largest magnitude of torque is $(330\,\text{N})(3\,\text{m}) = 990\,\text{N}\cdot\text{m}$, due to the weight of person 2.

   (b) Among persons 5 through 8, the largest magnitude of torque is $(330\,\text{N})(3\,\text{m}) = 990\,\text{N}\cdot\text{m}$, due to the weight of person 7.

2. (a) If it were not leaning (the ideal case), its center of mass would be directly above the center of its base – that is, 3.5 m from the edge. Thus, to move the center of mass from that ideal location to a point directly over the bottom edge requires moving the center of the tower 3.5 m horizontally. Measured at the top, this would correspond to a displacement of twice as much: 7.0 m. Now, the top of the tower is already displaced (according to the problem) by 4.5 m, so what is needed to put it on the verge of toppling is an additional shift of $7.0 - 4.5 = 2.5$ m.

   (b) The angle measured from vertical is $\tan^{-1}(7.0/55) = 7.3°$.

3. (a) The forces are balanced when they sum to zero: $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. This means

$$\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -(10\,\text{N})\,\hat{\imath} + (4\,\text{N})\,\hat{\jmath} - (17\,\text{N})\,\hat{\imath} - (2\,\text{N})\,\hat{\jmath} = (-27\,\text{N})\,\hat{\imath} + (2\,\text{N})\,\hat{\jmath}\ .$$

   (b) If θ is the angle the vector makes with the x axis then

$$\tan\theta = \frac{F_{3y}}{F_{3x}} = \frac{2\,\text{N}}{-27\,\text{N}} = -0.741\ .$$

   The angle is either $-4.2°$ or $176°$. The second solution yields a negative x component and a positive y component and is therefore the correct solution.

4. The situation is somewhat similar to that depicted for problem 10 (see the figure that accompanies that problem). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T\sin\theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are colinear). Setting $T = F$ therefore yields $\theta = 30°$. Since $\alpha = 180° - 2\theta$ is the angle between the two segments, then we find $\alpha = 120°$.

5. The object exerts a downward force of magnitude $F = 3160$ N at the midpoint of the rope, causing a "kink" similar to that shown for problem 10 (see the figure that accompanies that problem). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T\sin\theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are colinear). In this problem, we have

$$\theta = \tan^{-1}\left(\frac{0.35\,\text{m}}{1.72\,\text{m}}\right) = 11.5°\ .$$

   Therefore, $T = F/2\sin\theta = 7.92 \times 10^3$ N.

6. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = 5.0 - 1.5 = 3.5\,\text{m}$. We denote the tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

   (a) Taking torques about the end of the plank farthest from the window washer, we find

   $$F_1 = \frac{m_w g \ell_2 + m_s g \ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})\left(9.8\,\text{m/s}^2\right)(3.5\,\text{m}) + (60\,\text{kg})\left(9.8\,\text{m/s}^2\right)(2.5\,\text{m})}{5.0\,\text{m}} = 8.4 \times 10^2\ \text{N}\ .$$

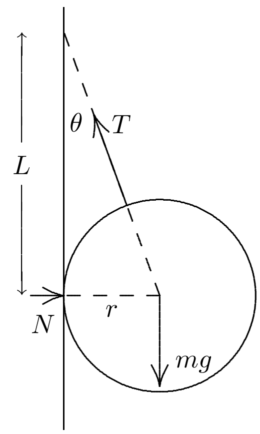
   (b) Equilibrium of forces leads to

   $$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 1.4 \times 10^3\ \text{N}$$

   which (using our result from part (a)) yields $F_2 = 5.3 \times 10^2\,\text{N}$.

7.

   Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{N}$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then, the vertical component of Newton's second law is $T\cos\theta - mg = 0$. The horizontal component is $N - T\sin\theta = 0$.



   (a) We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2 + r^2}$ to obtain $T = mg\sqrt{L^2 + r^2}/L$.

   (b) We solve the second equation for the normal force: $N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2 + r^2}$, we obtain

   $$N = \frac{Tr}{\sqrt{L^2 + r^2}} = \frac{mg\sqrt{L^2 + r^2}}{L}\,\frac{r}{\sqrt{L^2 + r^2}} = \frac{mgr}{L}\ .$$

8. Our notation is as follows: $M = 1360\,\text{kg}$ is the mass of the automobile; $L = 3.05\,\text{m}$ is the horizontal distance between the axles; $\ell = 3.05 - 1.78 = 1.27\,\text{m}$ is the horizontal distance from the rear axle to the center of mass; F_1 is the force exerted on each front wheel; and, F_2 is the force exerted on each back wheel.

   (a) Taking torques about the rear axle, we find

   $$F_1 = \frac{Mg\ell}{2L} = \frac{(1360\text{kg})\left(9.8\text{m/s}^2\right)(1.27\,\text{m})}{2(3.05\,\text{m})} = 2.77 \times 10^3\ \text{N}\ .$$

   (b) Equilibrium of forces leads to $2F_1 + 2F_2 = Mg$, from which we obtain $F_2 = 3.89 \times 10^3\,\text{N}$.

9. We take the force of the left pedestal to be F_1 at $x = x_1$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = x_2$. W is the weight of the diver, located at $x = x_3$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

   $$\begin{aligned} F_1 + F_2 - W &= 0 \\ F_1(x_2 - x_1) + W(x_3 - x_2) &= 0 \end{aligned}$$

(a) The second equation gives

$$F_1 = -\frac{x_3 - x_2}{x_2 - x_1} W = -\left(\frac{3.0\,\text{m}}{1.5\,\text{m}}\right)(580\,\text{N}) = -1160\,\text{N} .$$

The result is negative, indicating that this force is downward.

(b) The first equation gives

$$F_2 = W - F_1 = 580\,\text{N} + 1160\,\text{N} = 1740\,\text{N} .$$

The result is positive, indicating that this force is upward.

(c) and (d) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched. The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

10. The angle of each half of the rope, measured from the dashed line, is

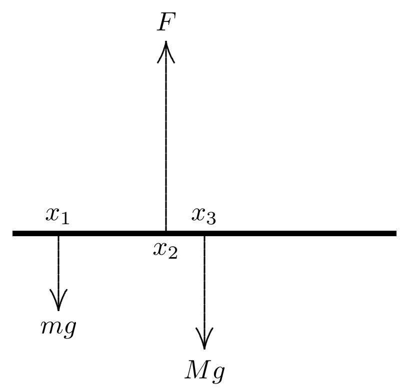
$$\theta = \tan^{-1}\left(\frac{0.3\,\text{m}}{9\,\text{m}}\right) = 1.9^\circ .$$

Analyzing forces at the "kink" (where $\vec{F}$ is exerted) we find

$$T = \frac{F}{2\sin\theta} = \frac{550\,\text{N}}{2\sin 1.9^\circ} = 8.3 \times 10^3 \text{ N} .$$

11.

The x axis is along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown on the diagram to the right. The nickels are at $x = x_1 = 0.120\,\text{m}$, and m is their total mass. The knife edge is at $x = x_2 = 0.455\,\text{m}$ and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3 = 0.500\,\text{m}$. Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish: $Mg(x_3 - x_2) - mg(x_2 - x_1) = 0$. Thus,



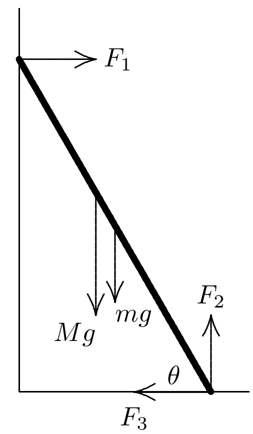
$$M = \frac{x_2 - x_1}{x_3 - x_2} m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}}\right)(10.0\,\text{g}) = 74 \text{ g} .$$

12. The forces exerted horizontally by the obstruction and vertically (upward) by the floor are applied at the bottom front corner C of the crate, as it verges on tipping. The center of the crate, which is where we locate the gravity force of magnitude $mg = 500\,\text{N}$, is a horizontal distance $\ell = 0.375\,\text{m}$ from C. The applied force of magnitude $F = 350\,\text{N}$ is a vertical distance h from C. Taking torques about C, we obtain

$$h = \frac{mg\ell}{F} = \frac{(500\,\text{N})(0.375\,\text{m})}{350\,\text{N}} = 0.536 \text{ m} .$$

13. The forces on the ladder are shown in the diagram below.

F_1 is the force of the window, horizontal because the window is frictionless. F_2 and F_3 are components of the force of the ground on the ladder. M is the mass of the window cleaner and m is the mass of the ladder. The force of gravity on the man acts at a point 3.0 m up the ladder and the force of gravity on the ladder acts at the center of the ladder. Let θ be the angle between the ladder and the ground. We use $\cos\theta = d/L$ or $\sin\theta = \sqrt{L^2 - d^2}/L$ to find $\theta = 60°$. Here L is the length of the ladder (5.0 m) and d is the distance from the wall to the foot of the ladder (2.5 m).



(a) Since the ladder is in equilibrium the sum of the torques about its foot (or any other point) vanishes. Let ℓ be the distance from the foot of the ladder to the position of the window cleaner. Then, $Mg\ell\cos\theta + mg(L/2)\cos\theta - F_1 L\sin\theta = 0$, and

$$\begin{aligned} F_1 &= \frac{(M\ell + mL/2)g\cos\theta}{L\sin\theta} \\ &= \frac{((75\,\mathrm{kg})(3.0\,\mathrm{m}) + (10\,\mathrm{kg})(2.5\,\mathrm{m}))\,(9.8\,\mathrm{m/s^2})\cos 60°}{(5.0\,\mathrm{m})\sin 60°} = 2.8\times10^2\ \mathrm{N}\ . \end{aligned}$$

This force is outward, away from the wall. The force of the ladder on the window has the same magnitude but is in the opposite direction: it is approximately 280 N, inward.

(b) The sum of the horizontal forces and the sum of the vertical forces also vanish:

$$\begin{aligned} F_1 - F_3 &= 0 \\ F_2 - Mg - mg &= 0 \end{aligned}$$

The first of these equations gives $F_3 = F_1 = 2.8\times10^2$ N and the second gives

$$F_2 = (M+m)g = (75\,\mathrm{kg} + 10\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 8.3\times10^2\ \mathrm{N}$$

The magnitude of the force of the ground on the ladder is given by the square root of the sum of the squares of its components:

$$F = \sqrt{F_2^2 + F_3^2} = \sqrt{(2.8\times10^2\,\mathrm{N})^2 + (8.3\times10^2\,\mathrm{N})^2} = 8.8\times10^2\ \mathrm{N}\ .$$

The angle ϕ between the force and the horizontal is given by $\tan\phi = F_3/F_2 = 830/280 = 2.94$, so $\phi = 71°$. The force points to the left and upward, 71° above the horizontal. We note that this force is not directed along the ladder.

14. The (vertical) forces at points A, B and P are F_A, F_B and F_P, respectively. We note that $F_P = W$ and is upward. Equilibrium of forces and torques (about point B) lead to

$$\begin{aligned} F_A + F_B + W &= 0 \\ bW - aF_A &= 0 \end{aligned}$$

(a) From the second equation, we find $F_A = bW/a = (15/5)W = 3W$.

(b) Using this result in the first equation above, we obtain $F_B = W - F_A = -4W$, pointing downward (as indicated by the minus sign).

15. (a) The forces acting on bucket are the force of gravity, down, and the tension force of cable A, up. Since the bucket is in equilibrium and its weight is $W_B = m_B g = (817\,kg)(9.8\,\mathrm{m/s^2}) = 8.01\times10^3$ N, the tension force of cable A is $T_A = 8.01\times10^3$ N.

(b) We use the coordinates axes defined in the diagram. Cable A makes an angle of 66° with the negative y axis, cable B makes an angle of 27° with the positive y axis, and cable C is along the x axis. The y components of the forces must sum to zero since the knot is in equilibrium. This means $T_B \cos 27° - T_A \cos 66° = 0$ and

$$T_B = \frac{\cos 66°}{\cos 27°} T_A = \left(\frac{\cos 66°}{\cos 27°}\right)(8.01 \times 10^3\,\text{N}) = 3.65 \times 10^3\ \text{N}\ .$$

(c) The x components must also sum to zero. This means $T_C + T_B \sin 27° - T_A \sin 66° = 0$ and

$$T_C = T_A \sin 66° - T_B \sin 27° = (8.01 \times 10^3\,\text{N}) \sin 66° - (3.65 \times 10^3\,\text{N}) \sin 27° = 5.66 \times 10^3\ \text{N}\ .$$

16. (a) Analyzing vertical forces where string 1 and string 2 meet, we find

$$T_1 = \frac{40\,\text{N}}{\cos 35°} = 49\ \text{N}\ .$$

(b) Looking at the horizontal forces at that point leads to

$$T_2 = T_1 \sin 35° = (49\,\text{N}) \sin 35° = 28\ \text{N}\ .$$

(c) We denote the components of T_3 as T_x (rightward) and T_y (upward). Analyzing horizontal forces where string 2 and string 3 meet, we find $T_x = T_2 = 28\,\text{N}$. From the vertical forces there, we conclude $T_y = 50\,\text{N}$. Therefore,

$$T_3 = \sqrt{T_x^2 + T_y^2} = 57\ \text{N}\ .$$

(d) The angle of string 3 (measured from vertical) is

$$\theta = \tan^{-1}\left(\frac{T_x}{T_y}\right) = \tan^{-1}\left(\frac{28}{50}\right) = 29°\ .$$

17. The cable that goes around the lowest pulley is cable 1 and has tension $T_1 = F$. That pulley is supported by the cable 2 (so $T_2 = 2T_1 = 2F$) and goes around the middle pulley. The middle pulley is supported by cable 3 (so $T_3 = 2T_2 = 4F$) and goes around the top pulley. The top pulley is supported by the upper cable with tension T, so $T = 2T_3 = 8F$. Three cables are supporting the block (which has mass $m = 6.40\,\text{kg}$):

$$T_1 + T_2 + T_3 = mg \implies F = \frac{mg}{7} = 8.96\ \text{N}\ .$$

Therefore, $T = 8(8.96) = 71.7\,\text{N}$.

18. (a) All forces are vertical and all distances are measured along an axis inclined at 30°. Thus, any trigonometric factor cancels out and the application of torques about the contact point (referred to in the problem) leads to

$$F_{\text{tripcep}} = \frac{(15\,\text{kg})\left(9.8\,\text{m/s}^2\right)(35\,\text{cm}) - (2.0\,\text{kg})\left(9.8\,\text{m/s}^2\right)(15\,\text{cm})}{2.5\,\text{cm}} = 1.9 \times 10^3\ \text{N}\ .$$

(b) Equilibrium of forces (with upwards positive) leads to

$$F_{\text{tripcep}} + F_{\text{humer}} + (15\,\text{kg})\left(9.8\,\text{m/s}^2\right) - (2.0\,\text{kg})\left(9.8\,\text{m/s}^2\right) = 0$$

and thus to $F_{\text{humer}} = -1.9 \times 10^3\,\text{N}$, with the minus sign implying that it points downward.

19. (a) Analyzing the horizontal forces (which add to zero) we find $F_h = F_3 = 5.0$ N.

(b) Equilibrium of vertical forces leads to $F_v = F_1 + F_2 = 30$ N.

(c) Computing torques about point O, we obtain

$$F_v d = F_2 b + F_3 a \implies d = \frac{(10)(3.0) + (5.0)(2.0)}{30} = 1.3 \text{ m} .$$

20. (a) The sign is attached in two places: at $x_1 = 1.00\,\text{m}$ (measured rightward from the hinge) and at $x_2 = 3.00\,\text{m}$. We assume the downward force due to the sign's weight is equal at these two attachment points: each being *half* the sign's weight of mg. The angle where the cable comes into contact (also at x_2) is $\theta = \tan^{-1}(4/3)$ and the force exerted there is the tension T. Computing torques about the hinge, we find

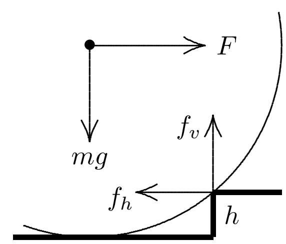
$$T = \frac{\frac{1}{2}mgx_1 + \frac{1}{2}mgx_2}{x_2 \sin\theta} = \frac{\frac{1}{2}(50.0)(9.8)(1.00) + \frac{1}{2}(50.0)(9.8)(3.00)}{(3.00)(0.800)} = 408 \text{ N} .$$

(b) Equilibrium of horizontal forces requires the (rightward) horizontal hinge force be $F_x = T\cos\theta = 245$ N.

(c) And equilibrium of vertical forces requires the (upward) vertical hinge force be $F_y = mg - T\sin\theta = 163$ N.

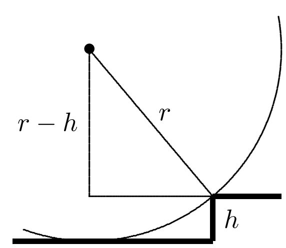
21.

We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.



We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step. The distance from the line of mg to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus $F(r-h) - mg\sqrt{2rh - h^2} = 0$. The solution for F is

$$F = \frac{\sqrt{2rh - h^2}}{r - h}\, mg .$$



22. (a) The problem asks for the person's pull (his force exerted on the rock) but since we are examining forces and torques *on the person*, we solve for the reaction force N_1 (exerted leftward on the hands by the rock). At that point, there is also an upward force of static friction on his hands f_1 which we will take to be at its maximum value $\mu_1 N_1$. We note that equilibrium of horizontal forces requires $N_1 = N_2$ (the force exerted leftward on his feet); on this feet there is also an upward static friction force of magnitude $\mu_2 N_2$. Equilibrium of vertical forces gives

$$f_1 + f_2 - mg = 0 \implies N_1 = \frac{mg}{\mu_1 + \mu_2} = 3.4 \times 10^2 \text{ N} .$$

(b) Computing torques about the point where his feet come in contact with the rock, we find

$$mg(d + w) - f_1 w - N_1 h = 0 \implies h = \frac{mg(d+w) - \mu_1 N_1 w}{N_1} = 0.88 \text{ m} .$$

(c) Both intuitively and mathematically (since both coefficients are in the denominator) we see from part (a) that N_1 would increase in such a case. As for part (b), it helps to plug part (a) into part (b) and simplify:

$$h = (d + w)\mu_2 + d\mu_1$$

from which it becomes apparent that h should decrease if the coefficients decrease.

23. The beam is in equilibrium: the sum of the forces and the sum of the torques acting on it each vanish. As we see in the figure, the beam makes an angle of 60° with the vertical and the wire makes an angle of 30° with the vertical.

(a) We calculate the torques around the hinge. Their sum is $TL\sin 30° - W(L/2)\sin 60° = 0$. Here W is the force of gravity acting at the center of the beam, and T is the tension force of the wire. We solve for the tension:

$$T = \frac{W\sin 60°}{2\sin 30°} = \frac{(222\,\text{N})\sin 60°}{2\sin 30°} = 192.3\ \text{N}\ .$$

(b) Let F_h be the horizontal component of the force exerted by the hinge and take it to be positive if the force is outward from the wall. Then, the vanishing of the horizontal component of the net force on the beam yields $F_h - T\sin 30° = 0$ or

$$F_h = T\sin 30° = (192.3\,\text{N})\sin 30° = 96.1\ \text{N}\ .$$

(c) Let F_v be the vertical component of the force exerted by the hinge and take it to be positive if it is upward. Then, the vanishing of the vertical component of the net force on the beam yields $F_v + T\cos 30° - W = 0$ or

$$F_v = W - T\cos 30° = 222\,\text{N} - (192.3\,\text{N})\cos 30° = 65.5\ \text{N}\ .$$

24. (a) The top brick's center of mass cannot be further (to the right) with respect to the brick below it (brick 2) than $L/2$; otherwise, its center of gravity is past any point of support and it will fall. So $a_1 = L/2$ in the maximum case.

(b) With brick 1 (the top brick) in the maximum situation, then the combined center of mass of brick 1 and brick 2 is halfway between the middle of brick 2 and its right edge. That point (the combined com) must be supported, so in the maximum case, it is just above the right edge of brick 3. Thus, $a_2 = L/4$.

(c) Now the total center of mass of bricks 1, 2 and 3 is one-third of the way between the middle of brick 3 and its right edge, as shown by this calculation:

$$x_{\text{com}} = \frac{2m(0) + m(-L/2)}{3m} = -\frac{L}{6}$$

where the origin is at the right edge of brick 3. This point is above the right edge of brick 4 in the maximum case, so $a_3 = L/6$.

(d) A similar calculation

$$x'_{\text{com}} = \frac{3m(0) + m(-L/2)}{4m} = -\frac{L}{8}$$

shows that $a_4 = L/8$.

(e) We find $h = \sum_{i=1}^{4} a_i = 25L/24$.

25. (a) We note that the angle θ between the cable and the strut is $45° - 30° = 15°$. The angle ϕ between the strut and any vertical force (like the weights in the problem) is $90° - 45° = 45°$. Denoting $M = 225$ kg and $m = 45.0$ kg, and ℓ as the length of the boom, we compute torques about the hinge and find

$$T = \frac{Mg\ell\sin\phi + mg\left(\frac{\ell}{2}\right)\sin\phi}{\ell\sin\theta}\ .$$

The unknown length ℓ cancels out and we obtain $T = 6.63 \times 10^3$ N.

(b) Since the cable is at 30° from horizontal, then horizontal equilibrium of forces requires that the horizontal hinge force be

$$F_x = T\cos 30° = 5.74 \times 10^3 \text{ N} .$$

(c) And vertical equilibrium of forces gives the vertical hinge force component:

$$F_y = Mg + mg + T\sin 30° = 5.96 \times 10^3 \text{ N} .$$

26. (a) The problem states that each hinge supports half the door's weight, so each vertical hinge force component is $F_y = mg/2 = 1.3 \times 10^2$ N.

(b) Computing torques about the top hinge, we find the horizontal hinge force component (at the bottom hinge) is

$$F_h = \frac{(27\,\text{kg})\,(9.8\,\text{m/s}^2)\,\left(\frac{0.91\,\text{m}}{2}\right)}{2.1\,\text{m} - 2(0.30\,\text{m})} = 80 \text{ N} .$$

Equilibrium of horizontal forces demands that the horizontal component of the top hinge force has the same magnitude (though opposite direction).

27. The bar is in equilibrium, so the forces and the torques acting on it each sum to zero. Let T_l be the tension force of the left-hand cord, T_r be the tension force of the right-hand cord, and m be the mass of the bar. The equations for equilibrium are:

$$\begin{array}{rc} \text{vertical force components} & T_l\cos\theta + T_r\cos\phi - mg = 0 \\ \text{horizontal force components} & -T_l\sin\theta + T_r\sin\phi = 0 \\ \text{torques} & mgx - T_rL\cos\phi = 0 \ . \end{array}$$

The origin was chosen to be at the left end of the bar for purposes of calculating the torque.

The unknown quantities are T_l, T_r, and x. We want to eliminate T_l and T_r, then solve for x. The second equation yields $T_l = T_r\sin\phi/\sin\theta$ and when this is substituted into the first and solved for T_r the result is $T_r = mg\sin\theta/(\sin\phi\cos\theta + \cos\phi\sin\theta)$. This expression is substituted into the third equation and the result is solved for x:

$$x = L\,\frac{\sin\theta\cos\phi}{\sin\phi\cos\theta + \cos\phi\sin\theta} = L\,\frac{\sin\theta\cos\phi}{\sin(\theta+\phi)} \ .$$

The last form was obtained using the trigonometric identity $\sin(A+B) = \sin A\cos B + \cos A\sin B$. For the special case of this problem $\theta + \phi = 90°$ and $\sin(\theta+\phi) = 1$. Thus,

$$x = L\,\sin\theta\,\cos\phi = (6.10\,\text{m})\sin 36.9°\cos 53.1° = 2.20 \text{ m} .$$

28. (a) Computing torques about the hinge, we find the tension in the wire:

$$TL\sin\theta - Wx = 0 \implies T = \frac{Wx}{L\sin\theta} \ .$$

(b) The horizontal component of the tension is $T\cos\theta$, so equilibrium of horizontal forces requires that the horizontal component of the hinge force is

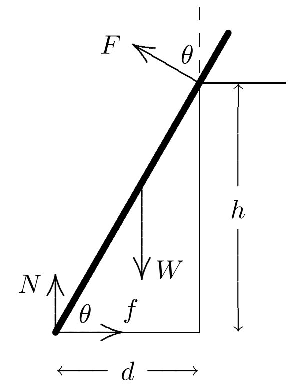
$$F_x = \left(\frac{Wx}{L\sin\theta}\right)\cos\theta = \frac{Wx}{L\tan\theta} \ .$$

(c) The vertical component of the tension is $T\sin\theta$, so equilibrium of vertical forces requires that the vertical component of the hinge force is

$$F_y = W - \left(\frac{Wx}{L\sin\theta}\right)\sin\theta = W\left(1 - \frac{x}{L}\right) \ .$$

29.

The diagram on the right shows the forces acting on the plank. Since the roller is frictionless the force it exerts is normal to the plank and makes the angle θ with the vertical. Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$. The equations of equilibrium are:



horizontal force components	$F\sin\theta - f = 0$
vertical force components	$F\cos\theta - W + N = 0$
torques	$Nd - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0$.

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70^\circ$ the plank just begins to slip and $f = \mu_s N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute N and f for $\theta = 70^\circ$, then use $\mu_s = f/N$ to compute the coefficient of friction.

The second equation gives $F = (W - N)/\cos\theta$ and this is substituted into the first to obtain $f = (W - N)\sin\theta/\cos\theta = (W - N)\tan\theta$. This is substituted into the third equation and the result is solved for N:

$$N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta} W \ .$$

Now replace d with $h/\tan\theta$ and multiply both numerator and denominator by $\tan\theta$. The result is

$$N = \frac{h(1 + \tan^2\theta) - (L/2)\sin\theta}{h(1 + \tan^2\theta)} W \ .$$

We use the trigonometric identity $1 + \tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right) \ .$$

Now we use this expression for N in $f = (W - N)\tan\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta \ .$$

We substitute these expressions for f and N into $\mu_s = f/N$ and obtain

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta} \ .$$

Evaluating this expression for $\theta = 70^\circ$, we obtain

$$\mu_s = \frac{(6.1\,\mathrm{m})\sin^2 70^\circ\cos 70^\circ}{2(3.05\,\mathrm{m}) - (6.1\,\mathrm{m})\sin 70^\circ\cos^2 70^\circ} = 0.34 \ .$$

30. (a) Computing torques about point A, we find

$$T_{\max} L \sin\theta = W x_{\max} + W_b \left(\frac{L}{2}\right) .$$

We solve for the maximum distance:

$$x_{\max} = \frac{T_{\max}\sin\theta - \frac{W_b}{2}}{W} L = \frac{500\sin 30^\circ - \frac{200}{2}}{300}(3.0) = 1.5 \text{ m} .$$

(b) Equilibrium of horizontal forces gives

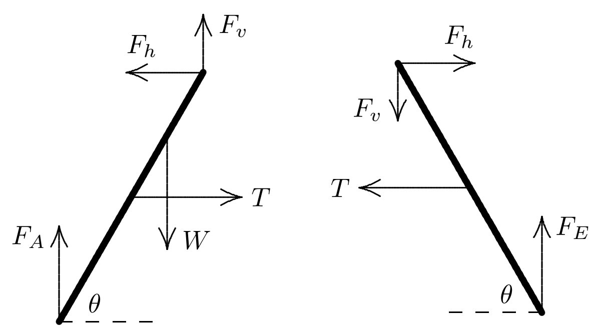
$$F_x = T_{\max}\cos\theta = 433 \text{ N} .$$

(c) And equilibrium of vertical forces gives

$$F_y = W + W_b - T_{\max}\sin\theta = 250 \text{ N} .$$

31.

The diagrams to the right show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction.



Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero: $F_v + F_A - W = 0$. The horizontal components must sum to zero: $T - F_h = 0$. The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then $F_A L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0$. Here we recognize that the man is one-fourth the length of the ladder side from the top and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L\cos\theta - T(L/2)\sin\theta = 0$.

There are 5 different equations:

$$\begin{gathered} F_v + F_A - W = 0\,, \\ T - F_h = 0 \\ F_A L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0 \\ F_E - F_v = 0 \\ F_E L\cos\theta - T(L/2)\sin\theta = 0\ . \end{gathered}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$\begin{gathered} T - F_h = 0 \\ WL\cos\theta - F_E L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0 \\ F_E L\cos\theta - T(L/2)\sin\theta = 0\ . \end{gathered}$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{3W}{4\tan\theta} .$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of $\sqrt{(1.22\,\text{m})^2 - (0.381\,\text{m})^2} = 1.16\,\text{m}$. This means $\tan\theta = (1.16\,\text{m})/(0.381\,\text{m}) = 3.04$. Thus,

$$T = \frac{3(854\,\text{N})}{4(3.04)} = 211\text{ N} .$$

(b) We now solve for F_A. Since $F_v = F_E$ and $F_E = T\sin\theta/2\cos\theta$, $F_v = 3W/8$. We substitute this into $F_v + F_A - W = 0$ and solve for F_A. We find

$$F_A = W - F_v = W - 3W/8 = 5W/8 = 5(884\,\text{N})/8 = 534\text{ N} .$$

(c) We have already obtained an expression for F_E: $F_E = 3W/8$. Evaluating it, we get $F_E = 320$ N.

32. The phrase "loosely bolted" means that there is no torque exerted by the bolt at that point (where A connects with B). The force exerted on A at the hinge has x and y components F_x and F_y. The force exerted on A at the bolt has components G_x and G_y and those exerted on B are simply $-G_x$ and $-G_y$ by Newton's third law. The force exerted on B at its hinge has components H_x and H_y. If a horizontal force is positive, it points rightward, and if a vertical force is positive it points upward.

(a) We consider the combined $A \cup B$ system, which has a combined weight of Mg where $M = 122$ kg and the line of action of that downward force of gravity is $x = 1.20$ m from the wall. The vertical distance between the hinges is $y = 1.80$ m. We compute torques about the bottom hinge and find

$$F_x = -\frac{Mgx}{y} = -797\text{ N} .$$

If we examine the forces on A alone and compute torques about the bolt, we instead find

$$F_y = \frac{m_A g x}{\ell} = 265\text{ N}$$

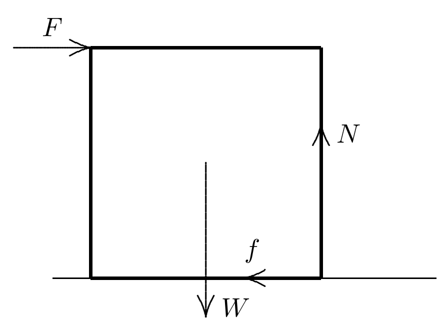
where $m_A = 54.0$ kg and $\ell = 2.40$ m (the length of beam A).

(b) Equilibrium of horizontal and vertical forces on beam A readily yields $G_x = -F_x = 797\,\text{N}$ and $G_y = m_A g - F_y = 265\,\text{N}$.

(c) Considering again the combined $A \cup B$ system, equilibrium of horizontal and vertical forces readily yields $H_x = -F_x = 797\,\text{N}$ and $H_y = Mg - F_y = 931\,\text{N}$.

(d) As mentioned above, Newton's third law (and the results from part (b)) immediately provide $-G_x = -797\,\text{N}$ and $-G_y = -265\,\text{N}$ for the force components acting on B at the bolt.

33. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled N on the diagram to the right. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $N - W = 0$, and the sum of the torques vanishes: $FL - WL/2 = 0$. Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.



(a) From the torque equation, we find

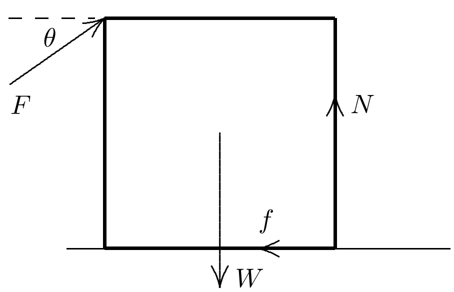
$$F = \frac{W}{2} = \frac{890\,\text{N}}{2} = 445\ \text{N}\ .$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/N$. According to the equations of equilibrium $N = W = 890\,\text{N}$ and $f = F = 445\,\text{N}$, so

$$\mu_s = \frac{445\,\text{N}}{890\,\text{N}} = 0.50\ .$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes $FL\cos\theta + FL\sin\theta - WL/2 = 0$, with the solution

$$F = \frac{W}{2(\cos\theta + \sin\theta)}\ .$$



We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

$$F = \frac{W}{4\cos 45°} = \frac{890\,\text{N}}{4\cos 45°} = 315\,\text{N}\ .$$

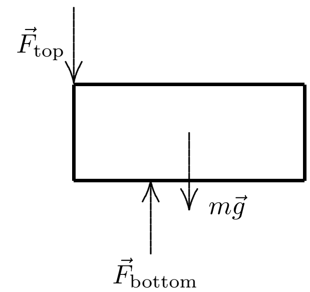
34. We locate the origin of the x axis at the edge of the table and choose rightwards positive. The criterion (in part (a)) is that the center of mass of the block above another must be no further than the edge of the one below; the criterion in part (b) is more subtle and is discussed below. Since the edge of the table corresponds to $x = 0$ then the total center of mass of the blocks must be zero.

(a) We treat this as three items: one on the upper left (composed of two bricks, one directly on top of the other) of mass $2m$ whose center is above the left edge of the bottom brick; a single brick at the upper right of mass m which necessarily has its center over the right edge of the bottom brick (so $a_1 = L/2$ trivially); and, the bottom brick of mass m. The total center of mass is

$$\frac{(2m)(a_2 - L) + ma_2 + m(a_2 - L/2)}{4m} = 0$$

which leads to $a_2 = 5L/8$. Consequently, $h = a_2 + a_1 = 9L/8$.

(b) We have four bricks (each of mass m) where the center of mass of the top and the center of mass of the bottom one have the same value $x_{cm} = b_2 - L/2$. The middle layer consists of two bricks, and we note that it is possible for each of their centers of mass to be beyond the respective edges of the bottom one! This is due to the fact that the top brick is exerting downward forces (each equal to half its weight) on the middle blocks – and in the extreme case, this may be thought of as a pair of concentrated forces exerted at the innermost edges of the middle bricks. Also, in the extreme case, the support force (upward) exerted on a middle block (by the bottom one) may be thought of as a concentrated force located at the edge of the bottom block (which is the point about which we compute torques, in the following). If (as indicated in our sketch, where $\vec{F}_{\rm top}$ has magnitude $mg/2$) we consider equilibrium of torques on the rightmost brick, we obtain

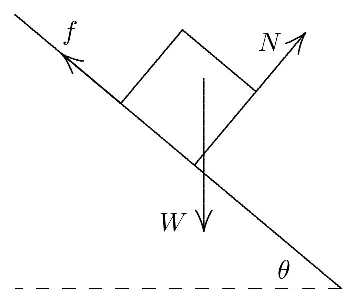


$$mg\left(b_1 - \frac{1}{2}L\right) = \frac{mg}{2}(L - b_1)$$

which leads to $b_1 = 2L/3$. Once we conclude from symmetry that $b_2 = L/2$ then we also arrive at $h = b_2 + b_1 = 7L/6$.

35.

The force diagram shown on the right depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, N is the normal force of the plane on the crate, and f is the force of friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(a) The x and y components of Newton's second law are

$$W\sin\theta - f = 0 \quad \text{and} \quad N - W\cos\theta = 0$$

respectively. The y equation gives $N = W\cos\theta$. Since the crate is about to slide $f = \mu_s N = \mu_s W\cos\theta$, where μ_s is the coefficient of static friction. We substitute into the x equation and find

$$W\sin\theta - \mu_s W\cos\theta = 0 \implies \tan\theta = \mu_s \ .$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.60 = 31.0°$.

In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude fh, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $N\ell/2$, where ℓ is the length of a edge. Since the total torque vanishes, $fh = N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W\sin\theta$ and $N = W\cos\theta$. Substituting these expressions into the torque equation, we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7° \ .$$

As θ is increased from zero the crate slides before it tips. It starts to slide when $\theta = 31.0°$.

(b) The analysis is the same. The crate begins to slide when $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.70 = 35.0°$ and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased. Tipping begins at $\theta = 33.7°$.

36. (a) The Young's modulus is given by

$$\begin{aligned} E &= \frac{\text{stress}}{\text{strain}} = \text{slope of the stress-strain curve} \\ &= \frac{150 \times 10^6\,\text{N/m}^2}{0.002} = 7.5 \times 10^{10}\ \text{N/m}^2\ . \end{aligned}$$

(b) Since the linear range of the curve extends to about $2.9 \times 10^8\,\text{N/m}^2$, this is approximately the yield strength for the material.

37. (a) The shear stress is given by F/A, where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross-sectional area of the rod. In this case F is the weight of the object hung on the end: $F = mg$, where m is the mass of the object. If r is the radius of the rod then $A = \pi r^2$. Thus, the shear stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.024\,\text{m})^2} = 6.5 \times 10^6\ \text{N/m}^2\ .$$

(b) The shear modulus G is given by

$$G = \frac{F/A}{\Delta x/L}$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F/A)L}{G} = \frac{(6.5 \times 10^6\,\text{N/m}^2)(0.053\,\text{m})}{3.0 \times 10^{10}\,\text{N/m}^2} = 1.1 \times 10^{-5}\ \text{m}\ .$$

38. (a) Since the brick is now horizontal and the cylinders were initially the same length ℓ, then both have been compressed an equal amount $\Delta\ell$. Thus,

$$\frac{\Delta\ell}{\ell} = \frac{F_A}{A_A E_A} \qquad \text{and} \qquad \frac{\Delta\ell}{\ell} = \frac{F_B}{A_B E_B}$$

which leads to

$$\frac{F_A}{F_B} = \frac{A_A E_A}{A_B E_B} = \frac{(2A_B)\,(2E_B)}{A_B E_B} = 4\ .$$

When we combine this ratio with the equation $F_A + F_B = W$, we find $F_A = \frac{4}{5}W$.

(b) This also leads to the result $F_B = W/5$.

(c) Computing torques about the center of mass, we find $F_A d_A = F_B d_B$ which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{1}{4}\ .$$

39. (a) Let F_A and F_B be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A/AE$, where A is the cross-sectional area of the wire and E is Young's modulus for steel ($200 \times 10^9\,\text{N/m}^2$). Similarly, $\Delta L_B = F_B L_B/AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell\ .$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}\ .$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B} \ .$$

The cross-sectional area of a wire is $A = \pi r^2 = \pi(1.20 \times 10^{-3}\,\text{m})^2 = 4.52 \times 10^{-6}\,\text{m}^2$. Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$\begin{aligned} F_A &= \frac{(103\,\text{kg})(9.8\,\text{m/s}^2)(2.50\,\text{m}) + (4.52 \times 10^{-6}\,\text{m}^2)(200 \times 10^9\,\text{N/m}^2)(2.0 \times 10^{-3}\,\text{m})}{2.50\,\text{m} + 2.50\,\text{m}} \\ &= 866 \text{ N} \ . \end{aligned}$$

(b) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_A = (103\,\text{kg})(9.8\,\text{m/s}^2) - 866\,\text{N} = 143 \text{ N} \ .$$

(c) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque equation becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165 \ .$$

40. The flat roof (as seen from the air) has area $A = 150 \times 5.8 = 870\,\text{m}^2$. The volume of material directly above the tunnel (which is at depth $d = 60\,\text{m}$) is therefore $V = A \times d = 870 \times 60 = 52200\,\text{m}^3$. Since the density is $\rho = 2.8\,\text{g/cm}^3 = 2800\,\text{kg/m}^3$, we find the mass of material supported by the steel columns to be $m = \rho V = 1.46 \times 10^8 \text{ m}^3$.

(a) The weight of the material supported by the columns is $mg = 1.4 \times 10^9$ N.

(b) The number of columns needed is

$$n = \frac{1.43 \times 10^9\,\text{N}}{\frac{1}{2}\,(400 \times 10^6\,\text{N/m}^2)\,(960 \times 10^{-4}\,\text{m}^2)} = 75 \ .$$

41. When the log is on the verge of moving (just before its left edge begins to lift) we take the system to be in equilibrium with the static friction at its maximum value $f_{s,\max} = \mu_s N$. Thus, our force and torque equations yield

$$\begin{aligned} F\cos\theta &= f_{s,\max} && \text{horizontal forces} \\ F\sin\theta + N &= Mg && \text{vertical forces} \\ FL\sin\theta &= Mg\left(\tfrac{L}{2}\right) && \text{torques about rightmost edge} \end{aligned}$$

where L is the length of the log (and cancels out of that last equation).

(a) Solving the three equations simultaneously yields

$$\theta = \tan^{-1}\left(\frac{1}{\mu_s}\right) = 51^\circ$$

when $\mu_s = 0.8$.

(b) And the tension is found to be

$$T = \frac{Mg}{2}\sqrt{1 + \mu^2} = 0.64Mg \ .$$

42. (a) The volume occupied by the sand within $r \le \frac{1}{2}r_m$ is that of a cylinder of height h' plus a cone atop that of height h. To find h, we consider

$$\tan\theta = \frac{h}{\frac{1}{2}r_m} \quad \Longrightarrow \quad h = \frac{1.82\,\mathrm{m}}{2}\tan 33^\circ = 0.59\ \mathrm{m}\ .$$

Therefore, since $h' = H - h$, the volume V contained within that radius is

$$\pi\left(\frac{r_m}{2}\right)^2 (H-h) + \frac{\pi}{3}\left(\frac{r_m}{2}\right)^2 h = \pi\left(\frac{r_m}{2}\right)^2\left(H - \frac{2}{3}h\right)$$

which yields $V = 6.78\ \mathrm{m}^3$.

(b) Since weight W is mg, and mass m is ρV, we have

$$W = \rho V g = \left(1800\,\mathrm{kg/m^3}\right)\left(6.78\,\mathrm{m^3}\right)\left(9.8\,\mathrm{m/s^2}\right) = 1.20\times 10^5\ \mathrm{N}\ .$$

(c) Since the slope is $(\sigma_m - \sigma_\mathrm{o})/r_m$ and the y-intercept is σ_o we have

$$\sigma = \left(\frac{\sigma_m - \sigma_\mathrm{o}}{r_m}\right) r + \sigma_\mathrm{o} \qquad \text{for } r \le r_m$$

or (with numerical values, SI units assumed) $\sigma \approx 13r + 40000$.

(d) The length of the circle is $2\pi r$ and it's "thickness" is dr, so the infinitesimal area of the ring is $dA = 2\pi r\,dr$.

(e) The force results from the product of stress and area (if both are well-defined). Thus, with SI units understood,

$$dF = \sigma\,dA = \left(\left(\frac{\sigma_m - \sigma_\mathrm{o}}{r_m}\right) r + \sigma_\mathrm{o}\right)(2\pi r\,dr) \approx 83r^2 dr + 2.5\times 10^5 r dr\ .$$

(f) We integrate our expression (using the precise numerical values) for dF and find

$$F = \int_0^{r_m/2} \left(82.855r^2 + 251327r\right) dr = \frac{82.855}{3}\left(\frac{r_m}{2}\right)^3 + \frac{251327}{2}\left(\frac{r_m}{2}\right)^2$$

which yields $F = 104083 \approx 1.04\times 10^5$ N for $r_m = 1.82$ m.

(g) The fractional reduction is

$$\frac{F-W}{W} = \frac{F}{W} - 1 = \frac{104083}{1.20\times 10^5} - 1 = -0.13\ .$$

43. (a) If L $(= 1500\,\mathrm{cm})$ is the unstretched length of the rope and $\Delta L = 2.8\,\mathrm{cm}$ is the amount it stretches then the strain is $\Delta L/L = (2.8\,\mathrm{cm})/(1500\,\mathrm{cm}) = 1.9\times 10^{-3}$.

(b) The stress is given by F/A where F is the stretching force applied to one end of the rope and A is the cross-sectional area of the rope. Here F is the force of gravity on the rock climber. If m is the mass of the rock climber then $F = mg$. If r is the radius of the rope then $A = \pi r^2$. Thus the stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(95\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{\pi(4.8\times 10^{-3}\,\mathrm{m})^2} = 1.3\times 10^7\ \mathrm{N/m^2}\ .$$

(c) Young's modulus is the stress divided by the strain: $E = (1.3\times 10^7\,\mathrm{N/m^2})/(1.9\times 10^{-3}) = 6.9\times 10^9\,\mathrm{N/m^2}$.

44. To support a load of $W = mg = (670)(9.8) = 6566$ N, the steel cable must stretch an amount proportional to its "free" length:

$$\Delta L = \left(\frac{W}{AY}\right) L \qquad \text{where } A = \pi r^2$$

and $r = 0.0125$ m.

(a) If $L = 12$ m, then

$$\Delta L = \left(\frac{6566}{\pi(0.0125)^2\,(2.0\times 10^{11})}\right)(12) = 8.0\times 10^{-4} \text{ m} .$$

(b) Similarly, when $L = 350$ m, we find $\Delta L = 0.023$ m.

45. The force F exerted on the beam is $F = 7900$ N, as computed in the Sample Problem. Let $F/A = S_u/6$, then

$$A = \frac{6F}{S_u} = \frac{6(7900)}{50\times 10^6} = 9.5\times 10^{-4} \text{ m}^2 .$$

Thus the thickness is $\sqrt{A} = \sqrt{9.5\times 10^{-4}} = 0.031$ m.

46. We denote the tension in the upper left string (bc) as T' and the tension in the lower right string (ab) as T. The supported weight is $Mg = 19.6$ N. The force equilibrium conditions lead to

$$\begin{aligned} T'\cos 60^\circ &= T\cos 20^\circ && \text{horizontal forces} \\ T'\sin 60^\circ &= W + T\sin 20^\circ && \text{vertical forces} . \end{aligned}$$

(a) We solve the above simultaneous equations and find

$$T = \frac{W}{\tan 60^\circ\ \cos 20^\circ - \sin 20^\circ} = 15 \text{ N} .$$

(b) Also, we obtain $T' = T\cos 20^\circ/\cos 60^\circ = 29$ N.

47. We choose an axis through the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $\frac{1}{5}\,8.0 = 1.6$ m; in other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is $r_\perp = \frac{4}{5}\,8.0 = 6.4$ m. The moment arm for the weight is half the horizontal distance from the wall to the base of the ladder; this works out to be $\frac{1}{2}\sqrt{10^2 - 8^2} = 3.0$ m. Similarly, the moment arms for the x and y components of the force at the ground ($\vec{F}_g$) are 8.0 m and 6.0 m, respectively. Thus, with lengths in meters, we have

$$\sum \tau_z = F(6.4) + W(3.0) + F_{gx}(8.0) - F_{gy}(6.0) = 0 \quad .$$

In addition, from balancing the vertical forces we find that $W = F_{gy}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum \tau_z = F(6.4) + W(3.0) + F_{gx}(8.0) - W(6.0) = 0 \quad .$$

(a) With $F = 50$ N and $W = 200$ N, the above equation yields $F_{gx} = 35$ N. Thus, in unit vector notation (with the unit Newton understood) we obtain

$$\vec{F}_g = 35\,\hat{\imath} + 200\,\hat{\jmath} .$$

(b) With $F = 150$ N and $W = 200$ N, the above equation yields $F_{gx} = -45$ N. Therefore, in unit vector notation (with the unit Newton understood) we obtain

$$\vec{F}_g = -45\,\hat{\imath} + 200\,\hat{\jmath} .$$

(c) Note that the phrase "start to move towards the wall" implies that the friction force is pointed away from the wall (in the $-\hat{\imath}$ direction). Now, if $f = -F_{gx}$ and $N = F_{gy} = 200$ N are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s N$) with $\mu_s = 0.38$, then we find $F_{gx} = -76$ N. Returning this to the above equation, we obtain

$$F = \frac{(200\,\mathrm{N})(3.0\,\mathrm{m}) + (76\,\mathrm{N})(8.0\,\mathrm{m})}{6.4\,\mathrm{m}} = 1.9 \times 10^2 \text{ N} .$$

48. (a) Computing the torques about the hinge, we have

$$TL \sin 40° = W\,\frac{L}{2} \sin 50°$$

where the length of the beam is $L = 12$ m and the tension is $T = 400$ N. Therefore, the weight is $W = 671$ N.

(b) Equilibrium of horizontal and vertical forces yields, respectively,

$$\begin{aligned} F_{\text{hinge}\,x} &= T = 400 \text{ N} \\ F_{\text{hinge}\,y} &= W \approx 670 \text{ N} \end{aligned}$$

where the hinge force components are rightward (for x) and upward (for y).

49. We denote the mass of the slab as m, its density as ρ, and volume as V. The angle of inclination is $\theta = 26°$.

(a) The component of the weight of the slab along the incline is

$$\begin{aligned} F_1 &= mg \sin\theta = \rho V g \sin\theta \\ &= (3.2 \times 10^3\,\mathrm{kg/m^3})(43\,\mathrm{m})(2.5\,\mathrm{m})(12\,\mathrm{m})(9.8\,\mathrm{m/s^2}) \sin 26° = 1.77 \times 10^7 \text{ N} . \end{aligned}$$

(b) The static force of friction is

$$\begin{aligned} f_s &= \mu_s N = \mu_s mg \cos\theta = \mu_s \rho V g \cos\theta \\ &= (0.39)(3.2 \times 10^3\,\mathrm{kg/m^3})(43\,\mathrm{m})(2.5\,\mathrm{m})(12\,\mathrm{m})(9.8\,\mathrm{m/s^2}) \cos 26° = 1.42 \times 10^7 \text{ N} . \end{aligned}$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77 \times 10^7\,\mathrm{N} - 1.42 \times 10^7\,\mathrm{N} = 3.5 \times 10^6 \text{ N} .$$

If the minimum number of bolts needed is n, then $F_2/nA \le 3.6 \times 10^8\,\mathrm{N/m^2}$, or

$$n \ge \frac{3.5 \times 10^6\,\mathrm{N}}{(3.6 \times 10^8\mathrm{N/m^2})(6.4 \times 10^{-4}\,\mathrm{m^2})} = 15.2 .$$

Thus 16 bolts are needed.

50. (a) Choosing an axis through the hinge, perpendicular to the plane of the figure and taking torques that would cause counterclockwise rotation as positive, we require the net torque to vanish:

$$FL \sin 90° - Th \sin 65° = 0$$

where the length of the beam is $L = 3.2$ m and the height at which the cable attaches is $h = 2.0$ m. Note that the weight of the beam does not enter this equation since its line of action is directed towards the hinge. With $F = 50$ N, the above equation yields $T = 88$ N.

(b) To find the components of $\vec{F}_p$ we balance the forces:

$$\sum F_x = 0 \quad \Longrightarrow \quad F_{px} = T\cos 25° - F$$
$$\sum F_y = 0 \quad \Longrightarrow \quad F_{py} = T\sin 25° + W$$

where W is the weight of the beam (60 N). Thus, we find that the hinge force components are $F_{px} = 30$ N rightward and $F_{py} = 97$ N upward.

51. (a) For computing torques, we choose the axis to be at support 2 and consider torques which encourage counterclockwise rotation to be positive. Let $m =$ mass of gymnast and $M =$ mass of beam. Thus, equilibrium of torques leads to

$$Mg(1.96 \text{ m}) - mg(0.54 \text{ m}) - F_1(3.92 \text{ m}) = 0 \ .$$

Therefore, the upward force at support 1 is $F_1 = 1163\,\text{N}$ (quoting more figures than are significant – but with an eye toward using this result in the remaining calculation).

(b) Balancing forces in the vertical direction, we have

$$F_1 + F_2 - Mg - mg = 0$$

so that the upward force at support 2 is $F_2 = 1.74 \times 10^3$ N.

52. The cube has side length l and volume $V = l^3$. We use $p = B\Delta V/V$ for the pressure p. We note that

$$\frac{\Delta V}{V} = \frac{\Delta l^3}{l^3} = \frac{(l+\Delta l)^3 - l^3}{l^3} \approx \frac{3l^2\Delta l}{l^3} = 3\frac{\Delta l}{l} \ .$$

Thus, the pressure required is

$$p = \frac{3B\Delta l}{l} = \frac{3\left(1.4 \times 10^{11}\,\text{N/m}^2\right)(85.5\,\text{cm} - 85.0\,\text{cm})}{85.5\,\text{cm}} = 2.4 \times 10^9 \ \text{N/m}^2 \ .$$

53. When it is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{N}$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point $\mathcal{O}$ where P is being applied. Thus, the line of action of $\vec{N}$ passes through point $\mathcal{O}$ and exerts no torque about $\mathcal{O}$ (of course, a similar observation applied to the pull P). Since $N = mg$ in this problem, we have $f_{s\max} = \mu mg$ applied a distance h away from $\mathcal{O}$. And the line of action of force of gravity (of magnitude mg), which is best pictured as a concentrated force at the center of the cube, is a distance $L/2$ away from $\mathcal{O}$. Therefore, equilibrium of torques about $\mathcal{O}$ produces

$$\mu mgh = mg\left(\frac{L}{2}\right) \quad \Longrightarrow \quad \mu = \frac{L}{2h}$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ.

(a) For it to slide but not tip, a value of μ *less* than that derived above is needed, since then – static friction will be exceeded for a smaller value of P, before the pull is strong enough to cause it to tip. Thus, $\mu < L/2h$ is required.

(b) And for it to tip but not slide, we need μ *greater* than that derived above is needed, since now – static friction will not be exceeded even for the value of P which makes the cube rotate about its front lower corner. That is, we need to have $\mu > L/2h$ in this case.

54. Adopting the usual convention that torques that would produce counterclockwise rotation are positive, we have (with axis at the hinge)

$$\sum \tau_z = 0 \implies TL\sin 60^\circ - Mg\left(\frac{L}{2}\right) = 0$$

where $L = 5.0$ m and $M = 53$ kg. Thus, $T = 300$ N. Now (with F_p for the force of the hinge)

$$\sum F_x = 0 \implies F_{px} = -T\cos\theta = -150 \text{ N}$$
$$\sum F_y = 0 \implies F_{py} = Mg - T\sin\theta = 260 \text{ N}$$

where $\theta = 60^\circ$. Therefore (in newtons),

$$\vec{F}_p = -150\,\hat{\imath} + 260\,\hat{\jmath} \ .$$

55. Let the forces that compress stoppers A and B be F_A and F_B, respectively. Then equilibrium of torques about the axle requires $FR = r_AF_A + r_BF_B$. If the stoppers are compressed by amounts $|\Delta y_A|$ and $|\Delta y_B|$ respectively, when the rod rotates a (presumably small) angle θ (in radians), then

$$|\Delta y_A| = r_A\theta \qquad \text{and} \qquad |\Delta y_B| = r_B\theta \ .$$

Furthermore, if their "spring constants" k are identical, then $k = |F/\Delta y|$ leads to the condition $F_A/r_A = F_B/r_B$ which provides us with enough information to solve.

(a) Simultaneous solution of the two conditions leads to

$$F_A = \frac{Rr_A}{r_A^2 + r_B^2}\,F \ .$$

(b) It also yields

$$F_B = \frac{Rr_B}{r_A^2 + r_B^2}\,F \ .$$

56. Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_\perp = (40\,\text{N})\frac{2.6\,\text{cm}}{12\,\text{cm}} = 8.7 \text{ N} \ .$$

57. Analyzing forces at the knot (particularly helpful is a graphical view of the vector right-triangle with horizontal "side" equal to the static friction force f_s and vertical "side" equal to the weight W_5 of the 5.0-kg mass), we find $f_s = W_5\tan\theta$ where $\theta = 30^\circ$. For f_s to be at its maximum value, then it must equal μ_sW_10 where the weight of the 10 kg object is $W_10 = (10\,\text{kg})(9.8\,\text{m/s}^2)$. Therefore,

$$\mu_sW_10 = W_5\tan\theta \implies \mu_s = \frac{5}{10}\tan 30^\circ = 0.29 \ .$$

58. (a) Setting up equilibrium of torques leads to a simple "level principle" ratio:

$$F_{\text{catch}} = (11\,\text{kg})(9.8\,\text{m/s}^2)\frac{(91/2 - 10)\,\text{cm}}{91\,\text{cm}} = 42 \text{ N} \ .$$

(b) Then, equilibrium of vertical forces provides

$$F_{\text{hinge}} = (11\,\text{kg})(9.8\,\text{m/s}^2) - F_{\text{catch}} = 66 \text{ N} \ .$$

59. One arm of the balance has length ℓ_1 and the other has length ℓ_2. The two cases described in the problem are expressed (in terms of torque equilibrium) as

$$m_1\ell_1 = m\ell_2 \qquad \text{and} \qquad m\ell_1 = m_2\ell_2 \ .$$

We divide equations and solve for the unknown mass: $m = \sqrt{m_1 m_2}$.

60. Since all surfaces are frictionless, the contact force $\vec{F}$ exerted by the lower sphere on the upper one is along that 45° line, and the forces exerted by walls and floors are "normal" (perpendicular to the wall and floor surfaces, respectively). Equilibrium of forces on the top sphere lead to the two conditions

$$N_{\text{wall}} = F\cos 45^\circ \qquad \text{and} \qquad F\sin 45^\circ = mg \ .$$

And (using Newton's third law) equilibrium of forces on the bottom sphere lead to the two conditions

$$N'_{\text{wall}} = F\cos 45^\circ \qquad \text{and} \qquad N'_{\text{floor}} = F\sin 45^\circ + mg \ .$$

(a) Solving the above equations, we find $N'_{\text{floor}} = 2mg$.

(b) Also, we obtain $N'_{\text{wall}} = N_{\text{wall}} = mg$.

(c) And we get $F = mg/\sin 45^\circ = mg\sqrt{2}$.

61. (a) Setting up equilibrium of torques leads to

$$F_{\text{far end}}L = (73\,\text{kg})(9.8\,\text{m/s}^2)\frac{L}{4} + (2700\,\text{N})\frac{L}{2}$$

which yields $F_{\text{far end}} = 1.5 \times 10^3$ N.

(b) Then, equilibrium of vertical forces provides

$$F_{\text{near end}} = (73)(9.8) + 2700 - F_{\text{far end}} = 1.9 \times 10^3 \text{ N} \ .$$

62. Since GA exerts a leftward force T at the corner A, then (by equilibrium of horizontal forces at that point) the force F_{diag} in CA must be pulling with magnitude

$$F_{\text{diag}} = \frac{T}{\sin 45^\circ} = T\sqrt{2} \ .$$

This analysis applies equally well to the force in DB. And these diagonal bars are pulling on the bottom horizontal bar exactly as they do to the top bar, so the bottom bar is the "mirror image" of the top one (it is also under tension T). Since the figure is symmetrical (except for the presence of the turnbuckle) under 90° rotations, we conclude that the side bars also are under tension T (a conclusion that also follows from considering the vertical components of the pull exerted at the corners by the diagonal bars).

63. Where the crosspiece comes into contact with the beam, there is an upward force of $2F$ (where F is the upward force exerted by each man). By equilibrium of vertical forces, $W = 3F$ where W is the weight of the beam. If the beam is uniform, its center of gravity is a distance $L/2$ from the man in front, so that computing torques about the front end leads to

$$W\frac{L}{2} = 2F\,x = 2\left(\frac{W}{3}\right)x$$

which yields $x = 3L/4$ for the distance from the crosspiece to the front end. It is therefore a distance $L/4$ from the rear end (the "free" end).

Chapter 14

1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})(5.2\,\mathrm{kg})(2.4\,\mathrm{kg})}{2.3 \times 10^{-12}\,\mathrm{N}}} = 19\ \mathrm{m}\ .$$

2. (a) The gravitational force exerted on the baby (denoted with subscript b) by the obstetrician (denoted with subscript o) is given by

$$F_{bo} = \frac{Gm_om_b}{r_{bo}^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})(70\,\mathrm{kg})(3\,\mathrm{kg})}{(1\,\mathrm{m})^2} = 1 \times 10^{-8}\ \mathrm{N}\ .$$

   (b) The maximum (minimum) forces exerted by Jupiter on the baby occur when it is separated from the Earth by the shortest (longest) distance $r_{\min}$ ($r_{\max}$), respectively. Thus

$$F_{bJ}^{\max} = \frac{Gm_Jm_b}{r_{\min}^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})(2 \times 10^{27}\,\mathrm{kg})(3\,\mathrm{kg})}{(6 \times 10^{11}\,\mathrm{m})^2} = 1 \times 10^{-6}\ \mathrm{N}\ .$$

   (c) And we obtain

$$F_{bJ}^{\min} = \frac{Gm_Jm_b}{r_{\max}^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})(2 \times 10^{27}\,\mathrm{kg})(3\mathrm{kg})}{(9 \times 10^{11}\,\mathrm{m})^2} = 5 \times 10^{-7}\ \mathrm{N}\ .$$

   (d) No. The gravitational force exerted by Jupiter on the baby is greater than that by the obstetrician by a factor of up to $1 \times 10^{-6}\,\mathrm{N}/1 \times 10^{-8}\,\mathrm{N} = 100$.

3. We use $F = Gm_sm_m/r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15\,\mathrm{m} + 3\,\mathrm{m} = 18\,\mathrm{m}$. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor. Thus,

$$F = \frac{(6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})(20\,\mathrm{kg})(7.0\,\mathrm{kg})}{(18\,\mathrm{m})^2} = 2.9 \times 10^{-11}\ \mathrm{N}\ .$$

4. We use subscripts s, e, and m for the Sun, Earth and Moon, respectively.

$$\frac{F_{sm}}{F_{em}} = \frac{\frac{Gm_sm_m}{r_{sm}^2}}{\frac{Gm_em_m}{r_{em}^2}} = \frac{m_s}{m_e}\left(\frac{r_{em}}{r_{sm}}\right)^2$$

   Plugging in the numerical values (say, from Appendix C) we find

$$\frac{1.99 \times 10^{30}}{5.98 \times 10^{24}}\left(\frac{3.82 \times 10^{8}}{1.50 \times 10^{11}}\right)^2 = 2.16\ .$$

5. The gravitational force between the two parts is

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(mM - m^2\right)$$

which we differentiate with respect to m and set equal to zero:

$$\frac{dF}{dm} = 0 = \frac{G}{r^2}(M-2m) \implies M = 2m$$

which leads to the result $m/M = 1/2$.

6. Let the distance from Earth to the spaceship be r. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{(R_{em}-r)^2} = F_E = \frac{GM_e m}{r^2},$$

where m is the mass of the spaceship. Solving for r, we obtain

$$\begin{aligned} r &= \frac{R_{em}}{\sqrt{M_m/M_e}+1} \\ &= \frac{3.82 \times 10^8\,\text{m}}{\sqrt{(7.36\times 10^{22}\,\text{kg})/(5.98\times 10^{24}\,\text{kg})}+1} = 3.44 \times 10^8\,\text{m}\ . \end{aligned}$$

7. At the point where the forces balance $GM_e m/r_1^2 = GM_s m/r_2^2$, where M_e is the mass of Earth, M_s is the mass of the Sun, m is the mass of the space probe, r_1 is the distance from the center of Earth to the probe, and r_2 is the distance from the center of the Sun to the probe. We substitute $r_2 = d - r_1$, where d is the distance from the center of Earth to the center of the Sun, to find

$$\frac{M_e}{r_1^2} = \frac{M_s}{(d-r_1)^2}\ .$$

Taking the positive square root of both sides, we solve for r_1. A little algebra yields

$$r_1 = \frac{d\sqrt{M_e}}{\sqrt{M_s}+\sqrt{M_e}} = \frac{(150\times 10^9\,\text{m})\sqrt{5.98\times 10^{24}\,\text{kg}}}{\sqrt{1.99\times 10^{30}\,\text{kg}}+\sqrt{5.98\times 10^{24}\,\text{kg}}} = 2.6 \times 10^8\ \text{m}\ .$$

Values for M_e, M_s, and d can be found in Appendix C.

8. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^{-8} N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^{-8} N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{\text{net}} = \left(1.0\times 10^{-8}, 1.9\times 10^{-8}\right) \implies \left(2.1\times 10^{-8}\ \angle\ 61^\circ\right)\ .$$

The magnitude of the force is 2.1×10^{-8} N.

9. The gravitational forces on m_5 from the two 500-kg masses cancel each other. Contributions to the net force on m_5 come from the remaining two masses:

$$F_{\text{net}} = \frac{(6.67\times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(250\,\text{kg})(300\,\text{kg}-100\,\text{kg})}{(\sqrt{2}\times 10^{-2}\,\text{m})^2} = 0.017\ \text{N}\ .$$

The force is directed along the diagonal between the 300 kg and 100 kg masses, towards the 300-kg mass.

10. (a) The distance between any of the spheres at the corners and the sphere at the center is $r = \ell/2\cos 30° = \ell/\sqrt{3}$ where ℓ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\left(\frac{Gm_4m}{r^2}\right)\sin 30° = 3\frac{Gm_4m}{\ell^2} \ .$$

This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4m}{\ell^2} = \frac{Gm_4M}{(\ell/\sqrt{3})^2}$$

which readily yields $m = M$.

(b) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

11. We use m_1 for the 20 kg of the sphere at $(x_1, y_1) = (0.5, 1.0)$ (SI units understood), m_2 for the 40 kg of the sphere at $(x_2, y_2) = (-1.0, -1.0)$, and m_3 for the 60 kg of the sphere at $(x_3, y_3) = (0, -0.5)$. The mass of the 20 kg object at the origin is simply denoted m. We note that $r_1 = \sqrt{1.25}$, $r_2 = \sqrt{2}$, and $r_3 = 0.5$ (again, with SI units understood). The force $\vec{F}_n$ that the n^{th} sphere exerts on m has magnitude Gm_nm/r_n^2 and is directed from the origin towards m_n , so that it is conveniently written as

$$\vec{F}_n = \frac{Gm_nm}{r_n^2}\left(\frac{x_n}{r_n}\,\hat{\imath} + \frac{y_n}{r_n}\,\hat{\jmath}\right) = \frac{Gm_nm}{r_n^3}\left(x_n\hat{\imath} + y_n\hat{\jmath}\right) \ .$$

Consequently, the vector addition to obtain the net force on m becomes

$$\begin{aligned}\vec{F}_{\text{net}} &= \sum_{n=1}^{3}\vec{F}_n \\ &= Gm\left(\left(\sum_{n=1}^{3}\frac{m_nx_n}{r_n^3}\right)\hat{\imath} + \left(\sum_{n=1}^{3}\frac{m_ny_n}{r_n^3}\right)\hat{\jmath}\right) \\ &= -9.3\times10^{-9}\,\hat{\imath} - 3.2\times10^{-7}\,\hat{\jmath}\end{aligned}$$

in SI units. Therefore, we find the net force magnitude is $|\vec{F}_{\text{net}}| = 3.2\times10^{-7}$ N.

12. We note that r_A (the distance from the origin to sphere A, which is the same as the separation between A and B) is 0.5, $r_C = 0.8$, and $r_D = 0.4$ (with SI units understood). The force $\vec{F}_k$ that the k^{th} sphere exerts on m_B has magnitude Gm_km_B/r_k^2 and is directed from the origin towards m_k so that it is conveniently written as

$$\vec{F}_k = \frac{Gm_km_B}{r_k^2}\left(\frac{x_k}{r_k}\,\hat{\imath} + \frac{y_k}{r_k}\,\hat{\jmath}\right) = \frac{Gm_km_B}{r_k^3}\left(x_k\hat{\imath} + y_k\hat{\jmath}\right) \ .$$

Consequently, the vector addition (where k equals A, B and D) to obtain the net force on m_B becomes

$$\begin{aligned}\vec{F}_{\text{net}} &= \sum_k\vec{F}_k \\ &= Gm_B\left(\left(\sum_k\frac{m_kx_k}{r_k^3}\right)\hat{\imath} + \left(\sum_k\frac{m_ky_k}{r_k^3}\right)\hat{\jmath}\right) \\ &= 3.7\times10^{-5}\,\hat{\jmath}\ \text{N} \ .\end{aligned}$$

13. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GMm/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere.

The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere. That is $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = \left(\frac{r^3}{R^3}\right) M = \left(\frac{R^3}{8R^3}\right) M = \frac{M}{8} \ .$$

The center of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is

$$F_2 = \frac{G(M/8)m}{(d - R/2)^2} \ .$$

The force of the hollowed sphere on m is

$$F = F_1 - F_2 = GMm\left(\frac{1}{d^2} - \frac{1}{8(d - R/2)^2}\right) = \frac{GMm}{d^2}\left(1 - \frac{1}{8(1 - R/2d)^2}\right) \ .$$

14. We follow the method shown in Sample Problem 14-3. Thus,

$$a_g = \frac{GM_E}{r^2} \implies da_g = -2\frac{GM_E}{r^3}dr$$

which implies that the change in weight is

$$W_{\text{top}} - W_{\text{bottom}} \approx m\,(da_g) \ .$$

But since $W_{\text{bottom}} = GmM_E/R^2$ (where R is Earth's mean radius), we have

$$m da_g = -2\frac{GmM_E}{R^3}dr = -2W_{\text{bottom}}\frac{dr}{R} = -2(530\,\text{N})\frac{410\,\text{m}}{6.37 \times 10^6\,\text{m}}$$

which yields -0.068 N for the weight change (the minus sign indicating that it is a decrease in W). We are not including any effects due to the Earth's rotation (as treated in Eq. 14-12).

15. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R + h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R + h)^2$. We solve for h and obtain $h = \sqrt{GM/a_g} - R$. According to Appendix C, $R = 6.37 \times 10^6$ m and $M = 5.98 \times 10^{24}$ kg, so

$$h = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(5.98 \times 10^{24}\,\text{kg})}{4.9\,\text{m/s}^2}} - 6.37 \times 10^6\,\text{m} = 2.6 \times 10^6\ \text{m} \ .$$

16. (a) The gravitational acceleration at the surface of the Moon is $g_{\text{moon}} = 1.67$ m/s^2 (see Appendix C). The ratio of weights (for a given mass) is the ratio of g-values, so $W_{\text{moon}} = (100\,\text{N})(1.67/9.8) = 17\,\text{N}$.

    (b) For the force on that object caused by Earth's gravity to equal 17 N, then the free-fall acceleration at its location must be $a_g = 1.67$ m/s^2. Thus,

$$a_g = \frac{GM_E}{r^2} \implies r = \sqrt{\frac{GM_E}{a_g}} = 1.5 \times 10^7\,\text{m}$$

    so the object would need to be a distance of $r/R_E = 2.4$ "radii" from Earth's center.

17. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space.

(a) The magnitude of the gravitational force exerted by the planet on an object of mass m at its surface is given by $F = GmM/R^2$, where M is the mass of the planet and R is its radius. According to Newton's second law this must equal mv^2/R, where v is the speed of the object. Thus,

$$\frac{GM}{R^2} = \frac{v^2}{R} .$$

Replacing M with $(4\pi/3)\rho R^3$ (where ρ is the density of the planet) and v with $2\pi R/T$ (where T is the period of revolution), we find

$$\frac{4\pi}{3} G\rho R = \frac{4\pi^2 R}{T^2} .$$

We solve for T and obtain

$$T = \sqrt{\frac{3\pi}{G\rho}} .$$

(b) The density is $3.0 \times 10^3\ \mathrm{kg/m^3}$. We evaluate the equation for T:

$$T = \sqrt{\frac{3\pi}{(6.67 \times 10^{-11}\ \mathrm{m^3/s^2 \cdot kg})(3.0 \times 10^3\ \mathrm{kg/m^3})}} = 6.86 \times 10^3\ \mathrm{s} = 1.9\ \mathrm{h} .$$

18. (a) The gravitational acceleration is

$$a_g = \frac{GM}{R^2} = 7.6\ \mathrm{m/s^2} .$$

(b) Note that the total mass is $5M$. Thus,

$$a_g = \frac{G(5M)}{(3R)^2} = 4.2\ \mathrm{m/s^2} .$$

19. (a) The forces acting on an object being weighed are the downward force of gravity and the upward force of the spring balance. Let F_g be the magnitude of the force of Earth's gravity and let W be the magnitude of the force exerted by the spring balance. The reading on the balance gives the value of W. The object is traveling around a circle of radius R and so has a centripetal acceleration. Newton's second law becomes $F_g - W = mV^2/R$, where V is the speed of the object as measured in an inertial frame and m is the mass of the object. Now $V = R\omega \pm v$, where ω is the angular velocity of Earth as it rotates and v is the speed of the ship relative to Earth. We note that the first term gives the speed of a point fixed to the rotating Earth. The plus sign is used if the ship is traveling in the same direction as the portion of Earth under it (west to east) and the negative sign is used if the ship is traveling in the opposite direction (east to west).

Newton's second law is now $F_g - W = m(R\omega \pm v)^2/R$. When we expand the parentheses we may neglect the term v^2 since v is much smaller than $R\omega$. Thus, $F_g - W = m(R^2\omega^2 \pm 2R\omega v)/R$ and $W = F_g - mR\omega^2 \mp 2m\omega v$. When $v = 0$ the scale reading is $W_0 = F_g - mR\omega^2$, so $W = W_0 \mp 2m\omega v$. We replace m with W_0/g to obtain $W = W_0(1 \mp 2\omega v/g)$.

(b) The upper sign $(-)$ is used if the ship is sailing eastward and the lower sign $(+)$ is used if the ship is sailing westward.

20. (a) Plugging $R_h = 2GM_h/c^2$ into the indicated expression, we find

$$a_g = \frac{GM_h}{(1.001R_h)^2} = \frac{GM_h}{(1.001)^2\left(2GM_h/c^2\right)^2} = \frac{c^4}{(2.002)^2 G}\,\frac{1}{M_h}$$

which yields $a_g = \left(3.02 \times 10^{43}\ \mathrm{kg \cdot m/s^2}\right)/M_h$.

(b) Since M_h is in the denominator of the above result, a_g decreases as M_h increases.

(c) With $M_h = \left(1.55 \times 10^{12}\right)\left(1.99 \times 10^{30}\ \mathrm{kg}\right)$, we obtain $a_g = 9.8\ \mathrm{m/s^2}$.

(d) This part refers specifically to the very large black hole treated in the previous part. With that mass for M in Eq. 14-15, and $r = 2.002GM/c^2$, we obtain

$$da_g = -2\frac{GM}{(2.002GM/c^2)^3}\,dr = -\frac{2c^6}{(2.002)^3(GM)^2}\,dr$$

where $dr \rightarrow 1.70$ m as in the Sample Problem. This yields (in absolute value) an acceleration difference of 7.3×10^{-15} m/s^2.

(e) The miniscule result of the previous part implies that, in this case, any effects due to the differences of gravitational forces on the body are negligible.

21. From Eq. 14-13, we see the extreme case is when "g" becomes zero, and plugging in Eq. 14-14 leads to

$$0 = \frac{GM}{R^2} - R\omega^2 \implies M = \frac{R^3\omega^2}{G} \ .$$

Thus, with $R = 20000$ m and $\omega = 2\pi$ rad/s, we find $M = 4.7 \times 10^{24}$ kg.

22. (a) What contributes to the GmM/r^2 force on m is the (spherically distributed) mass M contained within r (where r is measured from the center of M). At point A we see that $M_1 + M_2$ is at a smaller radius than $r = a$ and thus contributes to the force:

$$|F_{\text{on } m}| = \frac{G\,(M_1 + M_2)\,m}{a^2} \quad .$$

(b) In the case $r = b$, only M_1 is contained within that radius, so the force on m becomes GM_1m/b^2.

(c) If the particle is at C, then no other mass is at smaller radius and the gravitational force on it is zero.

23. Using the fact that the volume of a sphere is $4\pi R^3/3$, we find the density of the sphere:

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{1.0 \times 10^4\,\text{kg}}{\frac{4}{3}\pi(1.0\,\text{m})^3} = 2.4 \times 10^3 \ \text{kg/m}^3 \ .$$

When the particle of mass m (upon which the sphere, or parts of it, are exerting a gravitational force) is at radius r (measured from the center of the sphere), then whatever mass M is at a radius less than r must contribute to the magnitude of that force (GMm/r^2).

(a) At $r = 1.5$ m, all of M_{total} is at a smaller radius and thus all contributes to the force:

$$|F_{\text{on } m}| = \frac{GmM_{\text{total}}}{r^2} = m\left(3.0 \times 10^{-7}\,\text{N/kg}\right) \ .$$

(b) At $r = 0.50$ m, the portion of the sphere at radius smaller than that is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.3 \times 10^3 \ \text{kg} \ .$$

Thus, the force on m has magnitude $GMm/r^2 = m\left(3.3 \times 10^{-7}\,\text{N/kg}\right)$.

(c) Pursuing the calculation of part (b) algebraically, we find

$$|F_{\text{on } m}| = \frac{Gm\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = mr\left(6.7 \times 10^{-7}\,\frac{\text{N}}{\text{kg}\cdot\text{m}}\right) \ .$$

24. Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = \frac{M_{\text{total}}}{\frac{4}{3}\pi R^3} = \frac{3M_{\text{total}}}{4\pi R^3} \ .$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M which is at radius less than r is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r \le R$ then we can write this result as

$$\frac{G\left(\frac{3M_{\text{total}}}{4\pi R^3}\right)\left(\frac{4\pi r^3}{3}\right)}{r^2} = \frac{GM_{\text{total}}\, r}{R^3}$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2} \ ,$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}\, r}{R^3} \quad \Longrightarrow \quad r = \frac{R}{3} \ .$$

Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \quad \Longrightarrow \quad r = R\sqrt{3} \ .$$

25. (a) The magnitude of the force on a particle with mass m at the surface of Earth is given by $F = GMm/R^2$, where M is the total mass of Earth and R is Earth's radius. The acceleration due to gravity is

$$a_g = \frac{F}{m} = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(5.98 \times 10^{24}\,\text{kg})}{(6.37 \times 10^6\,\text{m})^2} = 9.83\ \text{m/s}^2 \ .$$

(b) Now $a_g = GM/R^2$, where M is the total mass contained in the core and mantle together and R is the outer radius of the mantle (6.345×10^6 m, according to Fig. 14–36). The total mass is $M = 1.93 \times 10^{24}\,\text{kg} + 4.01 \times 10^{24}\,\text{kg} = 5.94 \times 10^{24}\,\text{kg}$. The first term is the mass of the core and the second is the mass of the mantle. Thus,

$$a_g = \frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(5.94 \times 10^{24}\,\text{kg})}{(6.345 \times 10^6\,\text{m})^2} = 9.84\ \text{m/s}^2 \ .$$

(c) A point 25 km below the surface is at the mantle-crust interface and is on the surface of a sphere with a radius of $R = 6.345 \times 10^6$ m. Since the mass is now assumed to be uniformly distributed the mass within this sphere can be found by multiplying the mass per unit volume by the volume of the sphere: $M = (R^3/R_e^3)M_e$, where M_e is the total mass of Earth and R_e is the radius of Earth. Thus,

$$M = \left(\frac{6.345 \times 10^6\,\text{m}}{6.37 \times 10^6\,\text{m}}\right)^3 (5.98 \times 10^{24}\,\text{kg}) = 5.91 \times 10^{24}\,\text{kg} \ .$$

The acceleration due to gravity is

$$a_g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(5.91 \times 10^{24}\,\text{kg})}{(6.345 \times 10^6\,\text{m})^2} = 9.79\ \text{m/s}^2 \ .$$

26. (a) The gravitational potential energy is

$$U = -\frac{GMm}{r} = -\frac{\left(6.67 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2\right)(5.2\,\text{kg})(2.4\,\text{kg})}{19\,\text{m}} = -4.4 \times 10^{-11}\ \text{J}\ .$$

(b) Since the change in potential energy is

$$\Delta U = -\frac{GMm}{3r} - \left(-\frac{GMm}{r}\right) = -\frac{2}{3}(-4.4 \times 10^{-11}\,\text{J}) = 2.9 \times 10^{-11}\ \text{J}\ ,$$

the work done by the gravitational force is $W = -\Delta U = -2.9 \times 10^{-11}$ J.

(c) The work done by you is $W' = \Delta U = 2.9 \times 10^{-11}$ J.

27. (a) We note that r_C (the distance from the origin to sphere C, which is the same as the separation between C and B) is 0.8, $r_D = 0.4$, and the separation between spheres C and D is $r_{CD} = 1.2$ (with SI units understood). The total potential energy is therefore

$$-\frac{GM_BM_C}{r_C^2} - \frac{GM_BM_D}{r_D^2} - \frac{GM_CM_D}{r_{CD}^2} = -1.3 \times 10^{-4}\ \text{J}$$

using the mass-values given in problem 12.

(b) Since any gravitational potential energy term (of the sort considered in this chapter) is necessarily negative ($-GmM/r^2$ where all variables are positive) then having another mass to include in the computation can only lower the result (that is, make the result more negative).

(c) The observation in the previous part implies that the work I do in removing sphere A (to obtain the case considered in part (a)) must lead to an increase in the system energy; thus, I do positive work.

(d) To put sphere A back in, I do negative work, since I am causing the system energy to become more negative.

28. The gravitational potential energy is

$$U = -\frac{Gm(M-m)}{r} = -\frac{G}{r}\left(Mm - m^2\right)$$

which we differentiate with respect to m and set equal to zero (in order to minimize). Thus, we find $M - 2m = 0$ which leads to the ratio $m/M = 1/2$ to obtain the least potential energy. (Note that a second derivative of U with respect to m would lead to a positive result regardless of the value of m – which means its graph is everywhere concave upward and thus its extremum is indeed a minimum).

29. (a) The density of a uniform sphere is given by $\rho = 3M/4\pi R^3$, where M is its mass and R is its radius. The ratio of the density of Mars to the density of Earth is

$$\frac{\rho_M}{\rho_E} = \frac{M_M}{M_E}\frac{R_E^3}{R_M^3} = 0.11\left(\frac{0.65 \times 10^4\,\text{km}}{3.45 \times 10^3\,\text{km}}\right)^3 = 0.74\ .$$

(b) The value of a_g at the surface of a planet is given by $a_g = GM/R^2$, so the value for Mars is

$$a_{gM} = \frac{M_M}{M_E}\frac{R_E^2}{R_M^2}a_{gE} = 0.11\left(\frac{0.65 \times 10^4\,\text{km}}{3.45 \times 10^3\,\text{km}}\right)^2 (9.8\,\text{m/s}^2) = 3.8\ \text{m/s}^2\ .$$

(c) If v is the escape speed, then, for a particle of mass m

$$\tfrac{1}{2}mv^2 = G\frac{mM}{R}$$

and

$$v = \sqrt{\frac{2GM}{R}} \ .$$

For Mars

$$v = \sqrt{\frac{2(6.67 \times 10^{-11}\,\mathrm{m^3/s^2{\cdot}kg})(0.11)(5.98 \times 10^{24}\,\mathrm{kg})}{3.45 \times 10^6\,\mathrm{m}}} = 5.0 \times 10^3 \ \mathrm{m/s} \ .$$

30. The amount of (kinetic) energy needed to escape is the same as the (absolute value of the) gravitational potential energy at its original position. Thus, an object of mass m on a planet of mass M and radius R needs $K = GmM/R$ in order to (barely) escape.

(a) Setting up the ratio, we find

$$\frac{K_m}{K_E} = \frac{M_m}{M_E}\frac{R_E}{R_m} = 0.045$$

using the values found in Appendix C.

(b) Similarly, for the Jupiter escape energy (divided by that for Earth) we obtain

$$\frac{K_J}{K_E} = \frac{M_J}{M_E}\frac{R_E}{R_J} = 28 \ .$$

31. (a) The work done by you in moving the sphere of mass m_2 equals the change in the potential energy of the three-sphere system. The initial potential energy is

$$U_i = -\frac{Gm_1m_2}{d} - \frac{Gm_1m_3}{L} - \frac{Gm_2m_3}{L-d}$$

and the final potential energy is

$$U_f = -\frac{Gm_1m_2}{L-d} - \frac{Gm_1m_3}{L} - \frac{Gm_2m_3}{d} \ .$$

The work done is

$$\begin{aligned} W &= U_f - U_i = Gm_2\left(m_1\left(\frac{1}{d} - \frac{1}{L-d}\right) + m_3\left(\frac{1}{L-d} - \frac{1}{d}\right)\right) \\ &= (6.67 \times 10^{-11}\,\mathrm{m^3/s^2{\cdot}kg})(0.10\,\mathrm{kg})\left[(0.80\,\mathrm{kg})\left(\frac{1}{0.040\,\mathrm{m}} - \frac{1}{0.080\,\mathrm{m}}\right)\right. \\ &\qquad \left. +(0.20\,\mathrm{kg})\left(\frac{1}{0.080\,\mathrm{m}} - \frac{1}{0.040\,\mathrm{m}}\right)\right] \\ &= +5.0 \times 10^{-11}\ \mathrm{J} \ . \end{aligned}$$

(b) The work done by the force of gravity is $-(U_f - U_i) = -5.0 \times 10^{-11}\,\mathrm{J}$.

32. Energy conservation for this situation may be expressed as follows:

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ K_1 - \frac{GmM}{r_1} &= K_2 - \frac{GmM}{r_2} \end{aligned}$$

where $M = 5.0 \times 10^{23}\,\mathrm{kg}$, $r_1 = R = 3.0 \times 10^6\,\mathrm{m}$ and $m = 10\,\mathrm{kg}$.

(a) If $K_1 = 5.0 \times 10^7\,\mathrm{J}$ and $r_2 = 4.0 \times 10^6\,\mathrm{m}$, then the above equation leads to

$$K_2 = K_1 + GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 2.2 \times 10^7\ \mathrm{J} \ .$$

(b) In this case, we require $K_2 = 0$ and $r_2 = 8.0 \times 10^6$ m, and solve for K_1:

$$K_1 = K_2 + GmM\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 6.9 \times 10^7 \text{ J} .$$

33. (a) We use the principle of conservation of energy. Initially the rocket is at Earth's surface and the potential energy is $U_i = -GMm/R_e = -mgR_e$, where M is the mass of Earth, m is the mass of the rocket, and R_e is the radius of Earth. The relationship $g = GM/R_e^2$ was used. The initial kinetic energy is $\frac{1}{2}mv^2 = 2mgR_e$, where the substitution $v = 2\sqrt{gR_e}$ was made. If the rocket can escape then conservation of energy must lead to a positive kinetic energy no matter how far from Earth it gets. We take the final potential energy to be zero and let K_f be the final kinetic energy. Then, $U_i + K_i = U_f + K_f$ leads to $K_f = U_i + K_i = -mgR_e + 2mgR_e = mgR_e$. The result is positive and the rocket has enough kinetic energy to escape the gravitational pull of Earth.

(b) We write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Then, $\frac{1}{2}mv_f^2 = mgR_e$ and $v_f = \sqrt{2gR_e}$.

34. Energy conservation for this situation may be expressed as follows:

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} &= \frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \end{aligned}$$

where $M = 7.0 \times 10^{24}$ kg, $r_2 = R = 1.6 \times 10^6$ m and $r_1 = \infty$ (which means that $U_1 = 0$). We are told to assume the meteor starts at rest, so $v_1 = 0$. Thus, $K_1 + U_1 = 0$ and the above equation is rewritten as

$$\frac{1}{2}mv_2^2 = \frac{GmM}{r_2} \implies v_2 = \sqrt{\frac{2GM}{R}} = 2.4 \times 10^4 \text{ m/s} .$$

35. (a) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = -GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $\frac{1}{2}mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $-GMm/R + \frac{1}{2}mv^2 = 0$. We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy equation becomes $-a_gR + \frac{1}{2}v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m})} = 1.7 \times 10^3 \text{ m/s} .$$

(b) Initially the particle is at the surface; the potential energy is $U_i = -GMm/R$ and the kinetic energy is $K_i = \frac{1}{2}mv^2$. Suppose the particle is a distance h above the surface when it momentarily comes to rest. The final potential energy is $U_f = -GMm/(R+h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} .$$

We replace GM with a_gR^2 and cancel m in the energy equation to obtain

$$-a_gR + \frac{1}{2}v^2 = -\frac{a_gR^2}{(R+h)} .$$

The solution for h is

$$\begin{aligned} h &= \frac{2a_gR^2}{2a_gR - v^2} - R \\ &= \frac{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m})^2}{2(3.0\,\text{m/s}^2)(500 \times 10^3\,\text{m}) - (1000\,\text{m/s})^2} - (500 \times 10^3\,\text{m}) \\ &= 2.5 \times 10^5 \text{ m} . \end{aligned}$$

(c) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = -GMm/(R+h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = -GMm/R$. Write $\frac{1}{2}mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h} = -\frac{GMm}{R} + \frac{1}{2}mv^2 \ .$$

We substitute $a_g R^2$ for GM and cancel m, obtaining

$$-\frac{a_g R^2}{R+h} = -a_g R + \frac{1}{2}v^2 \ .$$

The solution for v is

$$\begin{aligned} v &= \sqrt{2a_g R - \frac{2a_g R^2}{R+h}} \\ &= \sqrt{2(3.0\,\mathrm{m/s^2})(500\times 10^3\,\mathrm{m}) - \frac{2(3.0\,\mathrm{m/s^2})(500\times 10^3\,\mathrm{m})^2}{500\times 10^3\,\mathrm{m} + 1000\times 10^3\,\mathrm{m}}} \\ &= 1.4\times 10^3\ \mathrm{m/s}\ . \end{aligned}$$

36. (a) We note that *height* $= R - R_{\mathrm{Earth}}$ where $R_{\mathrm{Earth}} = 6.37\times 10^6$ m. With $M = 5.98\times 10^{24}$ kg, $R_0 = 6.57\times 10^6$ m and $R = 7.37\times 10^6$ m, we have

$$K_i + U_i = K + U \quad\Longrightarrow\quad \frac{1}{2}m\left(3.7\times 10^3\right)^2 - \frac{GmM}{R_0} = K - \frac{GmM}{R}$$

Solving, we find $K = 3.8\times 10^7$ J.

(b) Again, we use energy conservation.

$$K_i + U_i = K_f + U_f \quad\Longrightarrow\quad \frac{1}{2}m\left(3.7\times 10^3\right)^2 - \frac{GmM}{R_0} = 0 - \frac{GmM}{R_f}$$

Therefore, we find $R_f = 7.40\times 10^6$ m. This corresponds to a distance of $1034.9 \approx 1.03\times 10^3$ km above the earth's surface.

37. (a) The momentum of the two-star system is conserved, and since the stars have the same mass, their speeds and kinetic energies are the same. We use the principle of conservation of energy. The initial potential energy is $U_i = -GM^2/r_i$, where M is the mass of either star and r_i is their initial center-to-center separation. The initial kinetic energy is zero since the stars are at rest. The final potential energy is $U_f = -2GM^2/r_i$ since the final separation is $r_i/2$. We write Mv^2 for the final kinetic energy of the system. This is the sum of two terms, each of which is $\frac{1}{2}Mv^2$. Conservation of energy yields

$$-\frac{GM^2}{r_i} = -\frac{2GM^2}{r_i} + Mv^2 \ .$$

The solution for v is

$$v = \sqrt{\frac{GM}{r_i}} = \sqrt{\frac{(6.67\times 10^{-11}\,\mathrm{m^3/s^2\cdot kg})(10^{30}\,\mathrm{kg})}{10^{10}\,\mathrm{m}}} = 8.2\times 10^4\,\mathrm{m/s}\ .$$

(b) Now the final separation of the centers is $r_f = 2R = 2\times 10^5$ m, where R is the radius of either of the stars. The final potential energy is given by $U_f = -GM^2/r_f$ and the energy equation becomes

$-GM^2/r_i = -GM^2/r_f + Mv^2$. The solution for v is

$$\begin{aligned} v &= \sqrt{GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)} \\ &= \sqrt{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(10^{30}\,\mathrm{kg})\left(\frac{1}{2 \times 10^5\,\mathrm{m}} - \frac{1}{10^{10}\,\mathrm{m}}\right)} \\ &= 1.8 \times 10^7\ \mathrm{m/s}\ . \end{aligned}$$

38. (a) The initial gravitational potential energy is

$$U_i = -\frac{GM_AM_B}{r_i} = -\frac{\left(6.67 \times 10^{-11}\right)(20)(10)}{0.80} = -1.67 \times 10^{-8}\ \mathrm{J}\ .$$

(b) We use conservation of energy (with $K_i = 0$):

$$\begin{aligned} U_i &= K + U \\ -1.67 \times 10^{-8} &= K - \frac{\left(6.67 \times 10^{-11}\right)(20)(10)}{0.60} \end{aligned}$$

which yields $K = 5.6 \times 10^{-9}$ J. Note that the value of r is the difference between 0.80 m and 0.20 m.

39. Energy conservation for this situation may be expressed as follows:

$$\begin{aligned} K_1 + U_1 &= K_2 + U_2 \\ \frac{1}{2}mv_1^2 - \frac{GmM}{r_1} &= \frac{1}{2}mv_2^2 - \frac{GmM}{r_2} \end{aligned}$$

where $M = 5.98 \times 10^{24}$ kg, $r_1 = R = 6.37 \times 10^6$ m and $v_1 = 10000$ m/s. Setting $v_2 = 0$ to find the maximum of its trajectory, we solve the above equation (noting that m cancels in the process) and obtain $r_2 = 3.2 \times 10^7$ m. This implies that its *altitude* is $r_2 - R = 2.5 \times 10^7$ m.

40. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{a_M}{a_E}\right)^3 = \left(\frac{T_M}{T_E}\right)^2 \implies 1.52^3 = \left(\frac{T_M}{1\,\mathrm{y}}\right)^2$$

where we have substituted the mean-distance (from Sun) ratio for the semimajor axis ratio. This yields $T_M = 1.87$ y. The value in Appendix C (1.88 y) is quite close, and the small apparent discrepancy is not significant, since a more precise value for the semimajor axis ratio is $a_M/a_E = 1.523$ which does lead to $T_M = 1.88$ y using Kepler's law. A question can be raised regarding the use of a ratio of mean distances for the ratio of semimajor axes, but this requires a more lengthy discussion of what is meant by a "mean distance" than is appropriate here.

41. The period T and orbit radius r are related by the law of periods: $T^2 = (4\pi^2/GM)r^3$, where M is the mass of Mars. The period is 7 h 39 min, which is 2.754×10^4 s. We solve for M:

$$\begin{aligned} M &= \frac{4\pi^2 r^3}{GT^2} \\ &= \frac{4\pi^2(9.4 \times 10^6\,\mathrm{m})^3}{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(2.754 \times 10^4\,\mathrm{s})^2} = 6.5 \times 10^{23}\ \mathrm{kg}\ . \end{aligned}$$

42. With $T = 27.3(86400) = 2.36 \times 10^6$ s, Kepler's law of periods becomes

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 \implies M_E = \frac{4\pi^2\left(3.82 \times 10^8\right)^3}{\left(6.67 \times 10^{-11}\right)\left(2.36 \times 10^6\right)^2}$$

which yields $M_E = 5.93 \times 10^{24}$ kg for the mass of Earth.

43. Let N be the number of stars in the galaxy, M be the mass of the Sun, and r be the radius of the galaxy. The total mass in the galaxy is NM and the magnitude of the gravitational force acting on the Sun is $F = GNM^2/r^2$. The force points toward the galactic center. The magnitude of the Sun's acceleration is $a = v^2/R$, where v is its speed. If T is the period of the Sun's motion around the galactic center then $v = 2\pi R/T$ and $a = 4\pi^2 R/T^2$. Newton's second law yields $GNM^2/R^2 = 4\pi^2 MR/T^2$. The solution for N is

$$N = \frac{4\pi^2 R^3}{GT^2 M} \ .$$

The period is 2.5×10^8 y, which is 7.88×10^{15} s, so

$$N = \frac{4\pi^2 (2.2 \times 10^{20}\,\mathrm{m})^3}{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(7.88 \times 10^{15}\,\mathrm{s})^2 (2.0 \times 10^{30}\,\mathrm{kg})} = 5.1 \times 10^{10} \ .$$

44. Kepler's law of periods, expressed as a ratio, is

$$\left(\frac{r_s}{r_m}\right)^3 = \left(\frac{T_s}{T_m}\right)^2 \implies \left(\frac{1}{2}\right)^3 = \left(\frac{T_s}{1\,\text{lunar month}}\right)^2$$

which yields $T_s = 0.35$ lunar month for the period of the satellite.

45. (a) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m the orbit radius is $r = 6.37 \times 10^6\,\mathrm{m} + 160 \times 10^3\,\mathrm{m} = 6.53 \times 10^6\,\mathrm{m}$. The solution for v is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2 \cdot kg})(5.98 \times 10^{24}\,\mathrm{kg})}{6.53 \times 10^6\,\mathrm{m}}} = 7.82 \times 10^3 \ \mathrm{m/s} \ .$$

(b) Since the circumference of the circular orbit is $2\pi r$, the period is

$$T = \frac{2\pi r}{v} = \frac{2\pi (6.53 \times 10^6\,\mathrm{m})}{7.82 \times 10^3\,\mathrm{m/s}} = 5.25 \times 10^3 \ \mathrm{s} \ .$$

This is equivalent to 87.4 min.

46. (a) The distance from the center of an ellipse to a focus is ae where a is the semimajor axis and e is the eccentricity. Thus, the separation of the foci (in the case of Earth's orbit) is

$$2ae = 2\left(1.50 \times 10^{11}\,\mathrm{m}\right)(0.0167) = 5.01 \times 10^9 \ \mathrm{m} \ .$$

(b) To express this in terms of solar radii (see Appendix C), we set up a ratio:

$$\frac{5.01 \times 10^9\,\mathrm{m}}{6.96 \times 10^8\,\mathrm{m}} = 7.2 \ .$$

47. (a) The greatest distance between the satellite and Earth's center (the apogee distance) is $R_a = 6.37 \times 10^6\,\mathrm{m} + 360 \times 10^3\,\mathrm{m} = 6.73 \times 10^6\,\mathrm{m}$. The least distance (perigee distance) is $R_p = 6.37 \times 10^6\,\mathrm{m} + 180 \times 10^3\,\mathrm{m} = 6.55 \times 10^6\,\mathrm{m}$. Here 6.37×10^6 m is the radius of Earth. From Fig. 14-13, we see that the semimajor axis is

$$a = \frac{R_a + R_p}{2} = \frac{6.73 \times 10^6\,\mathrm{m} + 6.55 \times 10^6\,\mathrm{m}}{2} = 6.64 \times 10^6 \ \mathrm{m} \ .$$

(b) The apogee and perigee distances are related to the eccentricity e by $R_a = a(1+e)$ and $R_p = a(1-e)$. Add to obtain $R_a + R_p = 2a$ and $a = (R_a + R_p)/2$. Subtract to obtain $R_a - R_p = 2ae$. Thus,

$$e = \frac{R_a - R_p}{2a} = \frac{R_a - R_p}{R_a + R_p} = \frac{6.73 \times 10^6\,\mathrm{m} - 6.55 \times 10^6\,\mathrm{m}}{6.73 \times 10^6\,\mathrm{m} + 6.55 \times 10^6\,\mathrm{m}} = 0.0136\ .$$

48. To "hover" above Earth ($M_E = 5.98 \times 10^{24}$ kg) means that it has a period of 24 hours (86400 s). By Kepler's law of periods,

$$86400^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3 \implies r = 4.225 \times 10^7\ \mathrm{m}\ .$$

Its altitude is therefore $r - R_E$ (where $R_E = 6.37 \times 10^6$ m) which yields 3.59×10^7 m.

49. (a) The period of the comet is 1420 years (and one month), which we convert to $T = 4.48 \times 10^{10}$ s. Since the mass of the Sun is 1.99×10^{30} kg, then Kepler's law of periods gives

$$\left(4.48 \times 10^{10}\right)^2 = \left(\frac{4\pi^2}{\left(6.67 \times 10^{-11}\right)\left(1.99 \times 10^{30}\right)}\right) a^3 \implies a = 1.89 \times 10^{13}\ \mathrm{m}\ .$$

(b) Since the distance from the focus (of an ellipse) to its center is ea and the distance from center to the aphelion is a, then the comet is at a distance of

$$ea + a = (0.11 + 1)\left(1.89 \times 10^{13}\,\mathrm{m}\right) = 2.1 \times 10^{13}\ \mathrm{m}$$

when it is farthest from the Sun. To express this in terms of Pluto's orbital radius (found in Appendix C), we set up a ratio:

$$\left(\frac{2.1 \times 10^{13}}{5.9 \times 10^{12}}\right) R_P = 3.6 R_P\ .$$

50. (a) The period is $T = 27(3600) = 97200$ s, and we are asked to assume that the orbit is circular (of radius $r = 100000$ m). Kepler's law of periods provides us with an approximation to the asteroid's mass:

$$(97200)^2 = \left(\frac{4\pi^2}{GM}\right)(100000)^3 \implies M = 6.3 \times 10^{16}\ \mathrm{kg}\ .$$

(b) Dividing the mass M by the given volume yields an average density equal to $6.3 \times 10^{16}/1.41 \times 10^{13} = 4.4 \times 10^3\,\mathrm{kg/m^3}$, which is about 20% less dense than Earth (the average density of Earth is given in a Table in Chapter 15).

51. (a) If we take the logarithm of Kepler's law of periods, we obtain

$$2\log(T) = \log\left(4\pi^2/GM\right) + 3\log(a) \implies \log(a) = \frac{2}{3}\log(T) - \frac{1}{3}\log\left(4\pi^2/GM\right)$$

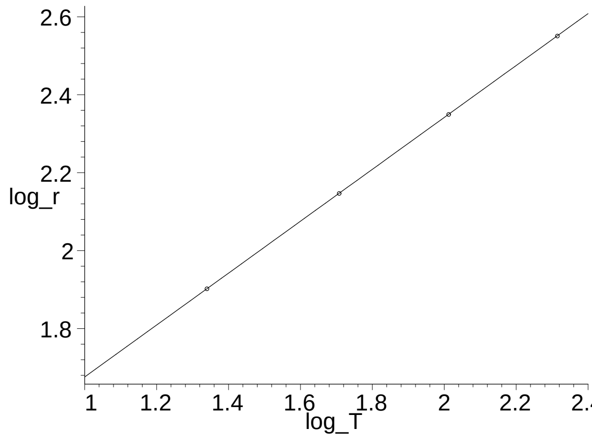
where we are ignoring an important subtlety about units (the arguments of logarithms cannot have units, since they are transcendental functions). Although the problem can be continued in this way, we prefer to set it up without units, which requires taking a ratio. If we divide Kepler's law (applied to the Jupiter-moon system, where M is mass of Jupiter) by the law applied to Earth orbiting the Sun (of mass M_o), we obtain

$$(T/T_E)^2 = \left(\frac{M_\mathrm{o}}{M}\right)\left(\frac{a}{r_E}\right)^3$$

where $T_E = 365.25$ days is Earth's orbital period and $r_E = 1.50 \times 10^{11}$ m is its mean distance from the Sun. In this case, it is perfectly legitimate to take logarithms and obtain

$$\log\left(\frac{r_E}{a}\right) = \frac{2}{3}\log\left(\frac{T_E}{T}\right) + \frac{1}{3}\log\left(\frac{M_\mathrm{o}}{M}\right)$$

(written to make each term positive) which is the way we plot the data ($\log(r_E/a)$ on the vertical axis and $\log(T_E/T)$ on the horizontal axis).



(b) When we perform a least-squares fit to the data, we obtain $\log(r_E/a) = 0.666\log(T_E/T) + 1.01$, which confirms the expectation of slope $= 2/3$ based on the above equation.

(c) And the 1.01 intercept corresponds to the term $\frac{1}{3}\log\left(\frac{M_o}{M}\right)$ which implies

$$\frac{M_o}{M} = 10^{3.03} \implies M = \frac{M_o}{1.07 \times 10^3} .$$

Plugging in $M_o = 1.99 \times 10^{30}$ kg (see Appendix C), we obtain $M = 1.86 \times 10^{27}$ kg for Jupiter's mass. This is reasonably consistent with the value 1.90×10^{27} kg found in Appendix C.

52. From Kepler's law of periods (where $T = 2.4(3600) = 8640$ s), we find the planet's mass M:

$$(8640\,\text{s})^2 = \left(\frac{4\pi^2}{GM}\right)\left(8.0 \times 10^6\,\text{m}\right)^3 \implies M = 4.06 \times 10^{24}\text{ kg} .$$

But we also know $a_g = GM/R^2 = 8.0\,\text{m/s}^2$ so that we are able to solve for the planet's radius:

$$R = \sqrt{\frac{GM}{a_g}} = 5.8 \times 10^6\text{ m} .$$

53. We follow the approach shown in Sample Problem 14-7. In our system, we have $m_1 = m_2 = M$ (the mass of our Sun, 1.99×10^{30} kg). From Eq. 14-37, we see that $r = 2r_1$ in this system (so r_1 is one-half the Earth-to-Sun distance r). And Eq. 14-39 gives $v = \pi r/T$ for the speed. Plugging these observations into Eq. 14-35 leads to

$$\frac{Gm_1m_2}{r^2} = m_1\frac{(\pi r/T)^2}{r/2} \implies T = \sqrt{\frac{2\pi^2 r^3}{GM}} .$$

With $r = 1.5 \times 10^{11}$ m, we obtain $T = 2.2 \times 10^7$ s. We can express this in terms of Earth-years, by setting up a ratio:

$$T = \left(\frac{T}{1\,\text{y}}\right)(1\,\text{y}) = \left(\frac{2.2 \times 10^7\,\text{s}}{3.156 \times 10^7\,\text{s}}\right)(1\,\text{y}) = 0.71\text{ y} .$$

54. The magnitude of the net gravitational force on one of the smaller stars (of mass m) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2}\left(M + \frac{m}{4}\right) .$$

This supplies the centripetal force needed for the motion of the star:

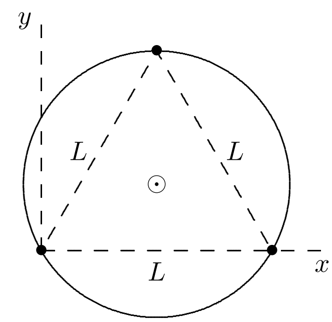
$$\frac{Gm}{r^2}\left(M+\frac{m}{4}\right)=m\frac{v^2}{r} \qquad \text{where } v=\frac{2\pi r}{T}\ .$$

Plugging in for speed v, we arrive at an equation for period T:

$$T=\frac{2\pi r^{3/2}}{\sqrt{G(M+m/4)}}\ .$$

55. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos 30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos 30° = Mv^2/R$.

The stars rotate about their center of mass (marked by $\odot$ on the diagram to the right) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $(\sqrt{3}/2)L$, so the stars are located at $x=0$, $y=0$; $x=L$, $y=0$; and $x=L/2$, $y=\sqrt{3}L/2$. The x coordinate of the center of mass is $x_c=(L+L/2)/3=L/2$ and the y coordinate is $y_c=(\sqrt{3}L/2)/3=L/2\sqrt{3}$. The distance from a star to the center of mass is $R=\sqrt{x_c^2+y_c^2}=\sqrt{(L^2/4)+(L^2/12)}=L/\sqrt{3}$.



Once the substitution for R is made Newton's second law becomes $(2GM^2/L^2)\cos 30° = \sqrt{3}Mv^2/L$. This can be simplified somewhat by recognizing that $\cos 30° = \sqrt{3}/2$, and we divide the equation by M. Then, $GM/L^2 = v^2/L$ and $v=\sqrt{GM/L}$.

56. (a) From Eq. 14-44, we see that the energy of each satellite is $-GM_Em/2r$. The total energy of the two satellites is twice that result; $-GM_Em/r$.

(b) We note that the speed of the wreckage will be zero (immediately after the collision), so it has no kinetic energy at that moment. Replacing m with $2m$ in the potential energy expression, we therefore find the total energy of the wreckage at that instant is $-2GM_Em/r$.

(c) An object with zero speed at that distance from Earth will simply fall towards the Earth, its trajectory being toward the center of the planet.

57. (a) We use the law of periods: $T^2=(4\pi^2/GM)r^3$, where M is the mass of the Sun (1.99×10^{30} kg) and r is the radius of the orbit. The radius of the orbit is twice the radius of Earth's orbit: $r=2r_e=2(150\times 10^9\,\text{m})=300\times 10^9\,\text{m}$. Thus,

$$\begin{aligned} T &= \sqrt{\frac{4\pi^2 r^3}{GM}} \\ &= \sqrt{\frac{4\pi^2(300\times 10^9\,\text{m})^3}{(6.67\times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(1.99\times 10^{30}\,\text{kg})}} = 8.96\times 10^7\,\text{s}\ . \end{aligned}$$

Dividing by (365 d/y)(24 h/d)(60 min/h)(60 s/min), we obtain $T=2.8$ y.

(b) The kinetic energy of any asteroid or planet in a circular orbit of radius r is given by $K=GMm/2r$, where m is the mass of the asteroid or planet. We note that it is proportional to m and inversely proportional to r. The ratio of the kinetic energy of the asteroid to the kinetic energy of Earth is $K/K_e=(m/m_e)(r_e/r)$. We substitute $m=2.0\times 10^{-4}m_e$ and $r=2r_e$ to obtain $K/K_e=1.0\times 10^{-4}$.

58. Although altitudes are given, it is the orbital radii which enter the equations. Thus, $r_A = 6370 + 6370 = 12740$ km, and $r_B = 19110 + 6370 = 25480$ km

(a) The ratio of potential energies is

$$\frac{U_B}{U_A} = \frac{-\frac{GmM}{r_B}}{-\frac{GmM}{r_A}} = \frac{r_A}{r_B} = \frac{1}{2} .$$

(b) Using Eq. 14-42, the ratio of kinetic energies is

$$\frac{K_B}{K_A} = \frac{\frac{GmM}{2r_B}}{\frac{GmM}{2r_A}} = \frac{r_A}{r_B} = \frac{1}{2} .$$

(c) From Eq. 14-44, it is clear that the satellite with the largest value of r has the smallest value of $|E|$ (since r is in the denominator). And since the values of E are negative, then the smallest value of $|E|$ corresponds to the largest energy E. Thus, satellite B has the largest energy, by an amount

$$\Delta E = E_B - E_A = -\frac{GmM}{2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) .$$

Being careful to convert the r values to meters, we obtain $\Delta E = 1.1 \times 10^8$ J. The mass M of Earth is found in Appendix C.

59. The total energy is given by $E = -GMm/2a$, where M is the mass of the central attracting body (the Sun, for example), m is the mass of the object (a planet, for example), and a is the semimajor axis of the orbit. If the object is a distance r from the central body the potential energy is $U = -GMm/r$. We write $\frac{1}{2}mv^2$ for the kinetic energy. Then, $E = K + U$ becomes $-GMm/2a = \frac{1}{2}mv^2 - GMm/r$. We solve for v^2. The result is

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right) .$$

60. (a) For $r = R_p$,

$$\begin{aligned} v_p^2 &= GM_s\left(\frac{2}{R_p} - \frac{1}{a}\right) \\ &= \left(6.67 \times 10^{-11}\,\mathrm{m^3/s^2{\cdot}kg}\right)\left(1.99 \times 10^{30}\,\mathrm{kg}\right)\left(\frac{2}{8.9 \times 10^{10}\,\mathrm{m}} - \frac{1}{2.7 \times 10^{12}\,\mathrm{m}}\right) \\ v_p &= 5.4 \times 10^4 \ \mathrm{m/s} . \end{aligned}$$

(b) For $r = R_a$,

$$\begin{aligned} v_a^2 &= GM_s\left(\frac{2}{R_a} - \frac{1}{a}\right) \\ &= \left(6.67 \times 10^{-11}\,\mathrm{m^3/s^2{\cdot}kg}\right)\left(1.99 \times 10^{30}\,\mathrm{kg}\right)\left(\frac{2}{5.3 \times 10^{12}\,\mathrm{m}} - \frac{1}{2.7 \times 10^{12}\,\mathrm{m}}\right) \\ v_a &= 9.6 \times 10^2 \ \mathrm{m/s} . \end{aligned}$$

(c) We appeal to angular momentum conservation:

$$L = mvr = mv_aR_a = mv_pR_p = \text{constant} \qquad \Longrightarrow \quad \frac{v_a}{v_p} = \frac{R_p}{R_a} .$$

61. The energy required to raise a satellite of mass m to an altitude h (at rest) is given by

$$E_1 = \Delta U = GM_E m \left(\frac{1}{R_E} - \frac{1}{R_E + h} \right) ,$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2} m v_{\text{orb}}^2 = \frac{GM_E m}{2(R_E + h)} .$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m \left[\frac{1}{R_E} - \frac{3}{2(R_E + h)} \right] .$$

(a) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370\,\text{km}} - \frac{3}{2(6370\,\text{km} + 1500\,\text{km})} < 0$$

the answer is no ($E_1 < E_2$).

(b) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370\,\text{km}} - \frac{3}{2(6370\,\text{km} + 3185\,\text{km})} = 0$$

we have $E_1 = E_2$.

(c) Since

$$\frac{1}{R_E} - \frac{3}{2(R_E + h)} = \frac{1}{6370\,\text{km}} - \frac{3}{2(6370\,\text{km} + 4500\,\text{km})} > 0$$

the answer is yes ($E_1 > E_2$).

62. (a) The pellets will have the same speed v but opposite direction of motion, so the *relative speed* between the pellets and satellite is $2v$. Replacing v with $2v$ in Eq. 14-42 is equivalent to multiplying it by a factor of 4. Thus,

$$K_{\text{rel}} = 4 \left(\frac{GM_E m}{2r} \right) = \frac{2 \left(6.67 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2\right) \left(5.98 \times 10^{24}\,\text{kg}\right) (0.0040\,\text{kg})}{(6370 + 500) \times 10^3\,\text{m}} = 4.6 \times 10^5\ \text{J} .$$

(b) We set up the ratio of kinetic energies:

$$\frac{K_{\text{rel}}}{K_{\text{bullet}}} = \frac{4.6 \times 10^5\,\text{J}}{\frac{1}{2}(0.0040\text{kg})(950\,\text{m/s})^2} = 2.6 \times 10^2 .$$

63. (a) The force acting on the satellite has magnitude GMm/r^2, where M is the mass of Earth, m is the mass of the satellite, and r is the radius of the orbit. The force points toward the center of the orbit. Since the acceleration of the satellite is v^2/r, where v is its speed, Newton's second law yields $GMm/r^2 = mv^2/r$ and the speed is given by $v = \sqrt{GM/r}$. The radius of the orbit is the sum of Earth's radius and the altitude of the satellite: $r = 6.37 \times 10^6 + 640 \times 10^3 = 7.01 \times 10^6$ m. Thus,

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{m}^3/\text{s}^2\cdot\text{kg})(5.98 \times 10^{24}\,\text{kg})}{7.01 \times 10^6\,\text{m}}} = 7.54 \times 10^3\ \text{m/s} .$$

(b) The period is $T = 2\pi r/v = 2\pi(7.01 \times 10^6\,\text{m})/(7.54 \times 10^3\,\text{m/s}) = 5.84 \times 10^3\,\text{s}$. This is 97 min.

(c) If E_0 is the initial energy then the energy after n orbits is $E = E_0 - nC$, where $C = 1.4\times10^5$ J/orbit. For a circular orbit the energy and orbit radius are related by $E = -GMm/2r$, so the radius after n orbits is given by $r = -GMm/2E$.

The initial energy is

$$E_0 = -\frac{(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg})(5.98\times10^{24}\,\mathrm{kg})(220\,\mathrm{kg})}{2(7.01\times10^6\,\mathrm{m})} = -6.26\times10^9\ \mathrm{J}\ ,$$

the energy after 1500 orbits is

$$E = E_0 - nC = -6.26\times10^9\,\mathrm{J} - (1500\,\mathrm{orbit})(1.4\times10^5\,\mathrm{J/orbit}) = -6.47\times10^9\ \mathrm{J}\ ,$$

and the orbit radius after 1500 orbits is

$$r = -\frac{(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg})(5.98\times10^{24}\,\mathrm{kg})(220\,\mathrm{kg})}{2\,(-6.47\times10^9\,\mathrm{J})} = 6.78\times10^6\ \mathrm{m}\ .$$

The altitude is $h = r - R = 6.78\times10^6\,\mathrm{m} - 6.37\times10^6\,\mathrm{m} = 4.1\times10^5\,\mathrm{m}$. Here R is the radius of Earth. This torque is internal to the satellite-Earth system, so the angular momentum of that system is conserved.

(d) The speed is

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67\times10^{-11}\,\mathrm{m^3/s^2\cdot kg})(5.98\times10^{24}\,\mathrm{kg})}{6.78\times10^6\,\mathrm{m}}} = 7.67\times10^3\,\mathrm{m/s}\ .$$

(e) The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.78\times10^6\,\mathrm{m})}{7.67\times10^3\,\mathrm{m/s}} = 5.6\times10^3\ \mathrm{s}\ .$$

This is equivalent to 93 min.

(f) Let F be the magnitude of the average force and s be the distance traveled by the satellite. Then, the work done by the force is $W = -Fs$. This is the change in energy: $-Fs = \Delta E$. Thus, $F = -\Delta E/s$. We evaluate this expression for the first orbit. For a complete orbit $s = 2\pi r = 2\pi(7.01\times10^6\,\mathrm{m}) = 4.40\times10^7\,\mathrm{m}$, and $\Delta E = -1.4\times10^5$ J. Thus,

$$F = -\frac{\Delta E}{s} = \frac{1.4\times10^5\,\mathrm{J}}{4.40\times10^7\,\mathrm{m}} = 3.2\times10^{-3}\ \mathrm{N}\ .$$

(g) The resistive force exerts a torque on the satellite, so its angular momentum is not conserved.

(h) The satellite-Earth system is essentially isolated, so its momentum is very nearly conserved.

64. We define the "effective gravity" in his environment as $g = 220/60 = 3.67\,\mathrm{m/s^2}$. Thus, using equations from Chapter 2 (and selecting downwards as the positive direction), we find the the "fall-time" to be

$$\Delta y = v_0 t + \frac{1}{2}at^2 \implies t = \sqrt{\frac{2(2.1)}{3.67}} = 1.1\ \mathrm{s}\ .$$

65. We estimate the planet to have radius $r = 10$ m. To estimate the mass m of the planet, we require its density equal that of Earth (and use the fact that the volume of a sphere is $4\pi r^3/3$).

$$\frac{m}{4\pi r^3/3} = \frac{M_E}{4\pi R_E^3/3} \implies m = M_E\left(\frac{r}{R_E}\right)^3$$

which yields (with $M_E \approx 6\times10^{24}$ kg and $R_E \approx 6.4\times10^6$ m) $m = 2.3\times10^7$ kg.

(a) With the above assumptions, the acceleration due to gravity is

$$a_g = \frac{Gm}{r^2} = \frac{\left(6.7 \times 10^{-11}\right)\left(2.3 \times 10^7\right)}{10^2} = 1.5 \times 10^{-5} \text{ m/s}^2 .$$

(b) Eq. 14-27 gives the escape speed:

$$v = \sqrt{\frac{2Gm}{r}} \approx 0.02 \text{ m/s} .$$

66. From Eq. 14-41, we obtain $v = \sqrt{GM/r}$ for the speed of an object in circular orbit (of radius r) around a planet of mass M. In this case, $M = 5.98 \times 10^{24}$ kg and $r = 700 + 6370 = 7070\,\text{km} = 7.07 \times 10^6\,\text{m}$. The speed is found to be $v = 7.51 \times 10^3$ m/s. After multiplying by 3600 s/h and dividing by 1000 m/km this becomes $v = 2.7 \times 10^4$ km/h.

(a) For a head-on collision, the relative speed of the two objects must be $2v = 5.4 \times 10^4$ km/h.

(b) A perpendicular collision is possible if one satellite is, say, orbiting above the equator and the other is following a longitudinal line. In this case, the relative speed is given by the Pythagorean theorem: $\sqrt{v^2 + v^2} = 3.8 \times 10^4$ km/h.

67. (a) It is possible to use $v^2 = v_0^2 + 2a\Delta y$ as we did for free-fall problems in Chapter 2 because the acceleration can be considered approximately constant over this interval. However, our approach will not assume constant acceleration; we use energy conservation:

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{r} \implies v = \sqrt{\frac{2GM(r_0 - r)}{r_0\, r}}$$

which yields $v = 1.4 \times 10^6$ m/s.

(b) We estimate the height of the apple to be $h = 7\,\text{cm} = 0.07\,\text{m}$. We may find the answer by evaluating Eq. 14-10 at the surface (radius r in part (a)) and at radius $r + h$, being careful not to round off, and then taking the difference of the two values, or we may take the differential of that equation – setting dr equal to h. We illustrate the latter procedure:

$$|da_g| = \left| -2\frac{GM}{r^3}\, dr \right| \approx 2\frac{GM}{r^3}\, h = 3 \times 10^6 \text{ m/s}^2 .$$

68. (a) We partition the full range into arcs of 3° each: 360°/3° = 120. Thus, the maximum number of geosynchronous satellites is 120.

(b) Kepler's law of periods, applied to a satellite around Earth, gives

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3$$

where $T = 24\,\text{h} = 86400\,\text{s}$ for the geosynchronous case. Thus, we obtain $r = 4.23 \times 10^7$ m.

(c) Arclength s is related to angle of arc θ (in radians) by $s = r\theta$. Thus, with $theta = 3(\pi/180) = 0.052$ rad, we find $s = 2.2 \times 10^6$ m.

(d) Points on the surface (which, of course, is not in orbit) are moving toward the east with a period of 24 h. If the satellite is found to be east of its expected position (above some point on the surface for which it used to stay directly overhead), then its period must now be *smaller* than 24 h.

(e) From Kepler's law of periods, it is evident that smaller T requires smaller r. The storm moved the satellite towards Earth.

69. (a) Their initial potential energy is $-Gm^2/R_i$ and they started from rest, so energy conservation leads to

$$-\frac{Gm^2}{R_i} = K_{\text{total}} - \frac{Gm^2}{0.5R_i} \implies K_{\text{total}} = \frac{Gm^2}{R_i} .$$

(b) The have equal mass, and this is being viewed in the center-of-mass frame, so their speeds are identical and their kinetic energies are the same. Thus,

$$K = \frac{1}{2}K_{\text{total}} = \frac{Gm^2}{2R_i} .$$

(c) With $K = \frac{1}{2}mv^2$, we solve the above equation and find $v = \sqrt{Gm/R_i}$.

(d) Their relative speed is $2v = 2\sqrt{Gm/R_i}$. This is the (instantaneous) rate at which the gap between them is closing.

(e) The premise of this part is that we assume we are not moving (that is, that body A acquires no kinetic energy in the process). Thus, $K_{\text{total}} = K_B$ and the logic of part (a) leads to $K_B = Gm^2/R_i$.

(f) And $\frac{1}{2}mv_B^2 = K_B$ yields $v_B = \sqrt{2Gm/R_i}$.

(g) The answer to part (f) is incorrect, due to having ignored the accelerated motion of "our" frame (that of body A). Our computations were therefore carried out in a noninertial frame of reference, for which the energy equations of Chapter 8 are not directly applicable.

70. (a) The equation preceding Eq. 14-40 is adapted as follows:

$$\frac{m_2^3}{(m_1+m_2)^2} = \frac{v^3T}{2\pi G}$$

where $m_1 = 0.9M_{\text{Sun}}$ is the estimated mass of the star. With $v = 70\,\text{m/s}$ and $T = 1500$ days (or $1500 \times 86400 = 1.3 \times 10^8$ s), we find

$$\frac{m_2^3}{(0.9M_{\text{Sun}}+m_2)^2} = 1.06 \times 10^{23} \text{ kg} .$$

Since $M_{\text{Sun}} \approx 2 \times 10^{30}$ kg, we find $m_2 \approx 7 \times 10^{27}$ kg. This solution may be reached in several ways (see discussion in the Sample Problem). Dividing by the mass of Jupiter (see Appendix C), we obtain $m \approx 3.7m_J$.

(b) Since $v = 2\pi r_1/T$ is the speed of the star, we find

$$r_1 = \frac{vT}{2\pi} = \frac{(70\,\text{m/s})\left(1.3 \times 10^8\,\text{s}\right)}{2\pi} = 1.4 \times 10^9 \text{ m}$$

for the star's orbital radius. If r is the distance between the star and the planet, then $r_2 = r - r_1$ is the orbital radius of the planet. And r can be figured from Eq. 14-37, which leads to

$$r_2 = r_1\left(\frac{m_1+m_2}{m_2} - 1\right) = r_1\,\frac{m_1}{m_2} = 3.7 \times 10^{11} \text{ m} .$$

Dividing this by 1.5×10^{11} m (Earth's orbital radius, r_E) gives $r_2 = 2.5r_E$.

71. (a) From Ch. 2, we have $v^2 = v_0^2 + 2a\Delta x$, where a may be interpreted as an average acceleration in cases where the acceleration is not uniform. With $v_0 = 0$, $v = 11000$ m/s and $\Delta x = 220$ m, we find $a = 2.75 \times 10^5\,\text{m/s}^2$. Therefore,

$$a = \left(\frac{2.75 \times 10^5\,\text{m/s}^2}{9.8\,\text{m/s}^2}\right) g = 2.8 \times 10^4 g$$

which is certainly enough to kill the passengers.

(b) Again using $v^2 = v_0^2 + 2a\Delta x$, we find

$$a = \frac{7000^2}{2(3500)} = 7000\,\mathrm{m/s^2} = 714g\ .$$

(c) Energy conservation gives the craft's speed v (in the absence of friction and other dissipative effects) at altitude $h = 700\,\mathrm{km}$ after being launched from $R = 6.37\times10^6\,\mathrm{m}$ (the surface of Earth) with speed $v_0 = 7000$ m/s. That altitude corresponds to a distance from Earth's center of $r = R + h = 7.07\times10^6\,\mathrm{m}$.

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}\ .$$

With $M = 5.98\times10^{24}\,\mathrm{kg}$ (the mass of Earth) we find $v = 6.05\times10^3\,\mathrm{m/s}$. But to orbit at that radius requires (by Eq. 14-41) $v' = \sqrt{GM/r} = 7.51\times10^3\,\mathrm{m/s}$. The difference between these is $v' - v = 1.46\times10^3$ m/s, which presumably is accounted for by the action of the rocket engine.

72. We apply the work-energy theorem to the object in question. It starts from a point at the surface of the Earth with zero initial speed and arrives at the center of the Earth with final speed v_f. The corresponding increase in its kinetic energy, $\frac{1}{2}mv_f^2$, is equal to the work done on it by Earth's gravity: $\int F\,dr = \int(-Kr)dr$ (using the notation of that Sample Problem referred to in the problem statement). Thus,

$$\frac{1}{2}mv_f^2 = \int_R^0 F\,dr = \int_R^0 (-Kr)\,dr = \frac{1}{2}KR^2$$

where R is the radius of Earth. Solving for the final speed, we obtain $v_f = R\sqrt{K/m}$. We note that the acceleration of gravity $a_g = g = 9.8\,\mathrm{m/s^2}$ on the surface of Earth is given by $a_g = GM/R^2 = G(4\pi R^3/3)\rho/R^2$, where ρ is Earth's average density. This permits us to write $K/m = 4\pi G\rho/3 = g/R$. Consequently,

$$\begin{aligned} v_f &= R\sqrt{\frac{K}{m}} = R\sqrt{\frac{g}{R}} = \sqrt{gR} \\ &= \sqrt{(9.8\,\mathrm{m/s^2})\,(6.37\times10^6\,\mathrm{m})} = 7.9\times10^3\ \mathrm{m/s}\ . \end{aligned}$$

73. Equating Eq. 14-18 with Eq. 14-10, we find

$$a_{gs} - a_g = \frac{4\pi G\rho R}{3} - \frac{4\pi G\rho r}{3} = \frac{4\pi G\rho(R-r)}{3}$$

which yields $a_{gs} - a_g = 4\pi G\rho D/3$. Since $4\pi G\rho/3 = a_{gs}/R$ this is equivalent to

$$a_{gs} - a_g = a_{gs}\frac{D}{R} \implies a_g = a_{gs}\left(1 - \frac{D}{R}\right)\ .$$

74. Let v and V be the speeds of particles m and M, respectively. These are measured in the frame of reference described in the problem (where the particles are seen as initially at rest). Now, momentum conservation demands

$$mv = MV \implies v + V = v\left(1 + \frac{m}{M}\right)$$

where $v+V$ is their relative speed (the instantaneous rate at which the gap between them is shrinking). Energy conservation applied to the two-particle system leads to

$$\begin{aligned} K_i + U_i &= K + U \\ 0 - \frac{GmM}{r} &= \frac{1}{2}mv^2 + \frac{1}{2}MV^2 - \frac{GmM}{d} \\ -\frac{GmM}{r} &= \frac{1}{2}mv^2\left(1 + \frac{m}{M}\right) - \frac{GmM}{d}\ . \end{aligned}$$

If we take the initial separation r to be large enough that GmM/r is approximately zero, then this yields a solution for the speed of particle m:

$$v = \sqrt{\frac{2GM}{d\left(1+\frac{m}{M}\right)}} \ .$$

Therefore, the relative speed is

$$v + V = \sqrt{\frac{2GM}{d\left(1+\frac{m}{M}\right)}}\left(1+\frac{m}{M}\right) = \sqrt{\frac{2G(M+m)}{d}} \ .$$

75. The initial distance from each fixed sphere to the ball is $r_0 = \infty$, which implies the initial gravitational potential energy is zero. The distance from each fixed sphere to the ball when it is at $x = 0.30$ m is $r = 0.50$ m, by the Pythagorean theorem.

   (a) With $M = 20$ kg and $m = 10$ kg, energy conservation leads to

   $$K_i + U_i = K + U \implies 0 + 0 = K - 2\frac{GmM}{r}$$

   which yields $K = 2GmM/r = 5.3 \times 10^{-8}$ J.

   (b) Since the y-component of each force will cancel, the force will be $-2F_x = -2\left(GmM/r^2\right)\cos\theta$, where $\theta = \tan^{-1} 4/3 = 53°$. Thus, the result (in Newtons – and using unit-vector notation) is $\vec{F}_{\text{net}} = -6.4 \times 10^{-8}\,\hat{\imath}$.

76. Energy conservation leads to

$$K_i + U_i = K + U \implies \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 - \frac{GmM}{R} = 0 - \frac{GmM}{R_{\text{max}}}$$

Consequently, we find $R_{\text{max}} = 2R$.

77. Consider that the leftmost rod is made of point-like particles (*mass elements*) of infinitesimal mass $dm = (M/L)dx$. The force on each of these, adapting the result of Sample Problem 14-9, is

$$\frac{G(dm)M}{x(L+x)} = \frac{G(M/L)(dx)M}{x(L+x)}$$

where x is the distance from the leftmost edge of the rightmost rod to a particular mass element of the leftmost rod. We take $+x$ to be leftward in this calculation. The magnitude of the net gravitational force exerted by the rightmost rod on the leftmost rod is therefore

$$\left|\vec{F}_{\text{net}}\right| = \frac{GM^2}{L}\int_d^{d+L}\frac{dx}{x(L+x)}$$

and is the same (by Newton's third law) as that exerted by the leftmost rod on the rightmost one. The integral can be evaluated (though the problem does not require us to do this), and the result is

$$\left|\vec{F}_{\text{net}}\right| = \frac{GM^2}{L^2}\ln\left(\frac{(d+L)^2}{d(d+2L)}\right) \ .$$

78. See Appendix C. We note that, since $v = 2\pi r/T$, the centripetal acceleration may be written as $a = 4\pi^2 r/T^2$. To express the result in terms of g, we divide by $9.8\,\text{m/s}^2$.

   (a) The acceleration associated with Earth's spin ($T = 24\,\text{h} = 86400\,\text{s}$) is

   $$a = g\,\frac{4\pi^2\left(6.37\times10^6\,\text{m}\right)}{(86400\,\text{s})^2\,(9.8\,\text{m/s}^2)} = 0.0034g \ .$$

(b) The acceleration associated with Earth's motion around the Sun ($T = 1\,\text{y} = 3.156 \times 10^7\,\text{s}$) is

$$a = g\,\frac{4\pi^2\left(1.5 \times 10^{11}\,\text{m}\right)}{\left(3.156 \times 10^7\,\text{s}\right)^2\left(9.8\,\text{m/s}^2\right)} = 0.00061g\ .$$

(c) The acceleration associated with the Solar System's motion around the galactic center ($T = 2.5 \times 10^8\,\text{y} = 7.9 \times 10^{15}\,\text{s}$) is

$$a = g\,\frac{4\pi^2\left(2.2 \times 10^{20}\,\text{m}\right)}{\left(7.9 \times 10^{15}\,\text{s}\right)^2\left(9.8\,\text{m/s}^2\right)} = 1.4 \times 10^{-11}g\ .$$

79. (a) We convert distances to meters, and use $v = \sqrt{GM/r}$ for speed when the probe is in circular orbit (this equation is readily obtained from Eq. 14-41). Our notations for the speeds are: v_o for the original speed of the probe when it is in a circular Venus-like orbit (of radius r_o); v_p for the speed when the rockets have fired and it is at the perihelion ($r_p = r_o$) of its subsequent elliptical orbit; and, v_f for its final speed once it is in a circular Earth-like orbit (of radius r_f which coincides with the aphelion distance r_a of the aforementioned ellipse). We find

$$v_o = \sqrt{\frac{GM}{r_o}} = \sqrt{\frac{\left(6.67 \times 10^{-11}\right)\left(1.99 \times 10^{30}\right)}{1.08 \times 10^{11}}} = 3.51 \times 10^4\ \text{m/s}\ .$$

With $m = 6000$ kg, the original energy is given by Eq. 14-44:

$$E_o = -\frac{GMm}{2r_o} = -3.69 \times 10^{12}\ \text{J}\ .$$

Once the rockets have fired, the probe starts on an elliptical path with semimajor axis

$$a = \frac{r_p + r_a}{2} = \frac{r_o + r_f}{2} = 1.29 \times 10^{11}\ \text{m}$$

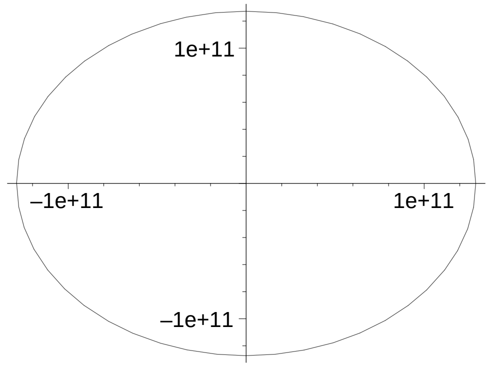
where $r_f = 1.5 \times 10^{11}$ m. By Eq. 14-46, its energy is now

$$E_{\text{ellipse}} = -\frac{GMm}{2a} = -3.09 \times 10^{12}\ \text{J}\ .$$

The energy "boost" required when the probe is at r_o is therefore $E_{\text{ellipse}} - E_o = 6.0 \times 10^{11}$ J. The speed of the probe at the moment it has received this boost is figured from the kinetic energy ($v_p = \sqrt{2K/m}$) where $K = E_{\text{ellipse}} - U$. Thus,

$$v_p = \sqrt{\frac{2}{m}\left(-\frac{GMm}{2a} + \frac{GMm}{r_p}\right)} = 3.78 \times 10^4\ \text{m/s}$$

which means the speed increase is $v_p - v_o = 2.75 \times 10^3$ m/s. The orbit (if it were allowed to complete one full revolution) is plotted below. The Sun is not shown; it is not exactly at the center but rather 2.1×10^{10} m to the right of origin (if we are assuming the perihelion is the rightmost point shown and the aphelion is the leftmost point shown).



(b) When the probe reaches $r_f = r_a$ it still has energy E_{ellipse} but now has speed

$$v_a = \frac{r_p v_p}{r_a} = \frac{\left(1.08 \times 10^{11}\right)\left(3.78 \times 10^4\right)}{1.5 \times 10^{11}} = 2.722 \times 10^4 \text{ m/s}$$

as a result of angular momentum conservation (see discussion of Kepler's law of areas), though this could also be figured similarly to the way we found v_p in the previous part. To be in circular motion at that radius, the speed must be

$$v_f = \sqrt{\frac{GM}{r_f}} = 2.975 \times 10^4 \text{ m/s} .$$

Thus, the speed increase needed at this stage must be $v_f - v_a = 2.53 \times 10^3$ m/s. Thus, using Eq. 14-44 again, the necessary energy increase here is

$$-\frac{GMm}{2\,r_f} - E_{\text{ellipse}} = 4.3 \times 10^{11} \text{ J} .$$

80. (a) Taking the differential of $F = GmM/r^2$ and approximating dF and dr as ΔW and $-h$, respectively, we arrive at

$$\Delta W = \frac{2GMmh}{r^3} = \frac{2G\left(4\pi\rho r^3/3\right)mh}{r^3}$$

where in the last step we have used the definition of average density ($\rho = M/V$ where $V_{\text{sphere}} = 4\pi r^3/3$). The above expression is easily simplified to yield the desired expression.

(b) We divide the previous result by $W = mg$ and obtain

$$\frac{\Delta W}{W} = \frac{8\pi G\rho h}{3g} .$$

We replace the lefthand side with 1×10^{-6} and set $\rho = 5500\,\text{kg/m}^3$, and obtain $h = 3.2$ m.

81. He knew that some force F must point toward the center of the orbit in order to hold the Moon in orbit around Earth, and that the approximation of a circular orbit with constant speed means the acceleration must be

$$a = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r^2}{T^2\,r} .$$

Plugging in $T^2 = Cr^3$ (where C is some constant) this leads to

$$F = ma = m\,\frac{4\pi^2 r^2}{Cr^4} = \frac{4\pi^2 m}{C\,r^2}$$

which indicates a force inversely proportional to the square of r.

82. (a) Kepler's law of periods is

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \ .$$

Thus, with $M = 6.0 \times 10^{30}$ kg and $T = 300(86400) = 2.6 \times 10^7$ s, we obtain $r = 1.9 \times 10^{11}$ m.

(b) That its orbit is circular suggests that its speed is constant, so

$$v = \frac{2\pi r}{T} = 4.6 \times 10^4 \text{ m/s} \ .$$

83. (a) Using Kepler's law of periods, we obtain

$$T = \sqrt{\left(\frac{4\pi^2}{GM}\right) r^3} = 2.15 \times 10^4 \text{ s} \ .$$

(b) The speed is constant (before she fires the thrusters), so $v_o = 2\pi r/T = 1.23 \times 10^4$ m/s.

(c) A two percent reduction in the previous value gives $v = 0.98 v_o = 1.20 \times 10^4$ m/s.

(d) The kinetic energy is $K = \frac{1}{2}mv^2 = 2.17 \times 10^{11}$ J.

(e) The potential energy is $U = -GmM/r = -4.53 \times 10^{11}$ J.

(f) Adding these two results gives $E = K + U = -2.35 \times 10^{11}$ J.

(g) Using Eq. 14-46, we find the semimajor axis to be

$$a = \frac{-GMm}{2E} = 4.04 \times 10^7 \text{ m} \ .$$

(h) Using Kepler's law of periods for elliptical orbits (using a instead of r) we find the new period is

$$T' = \sqrt{\left(\frac{4\pi^2}{GM}\right) a^3} = 2.03 \times 10^4 \text{ s} \ .$$

This is smaller than our result for part (a) by $T - T' = 1.22 \times 10^3$ s.

84. (a) With $M = 2.0 \times 10^{30}$ kg and $r = 10000$ m, we find

$$a_g = \frac{GM}{r^2} = 1.3 \times 10^{12} \text{ m/s}^2 \ .$$

(b) Although a close answer may be gotten by using the constant acceleration equations of Chapter 2, we show the more general approach (using energy conservation):

$$K_o + U_o = K + U$$

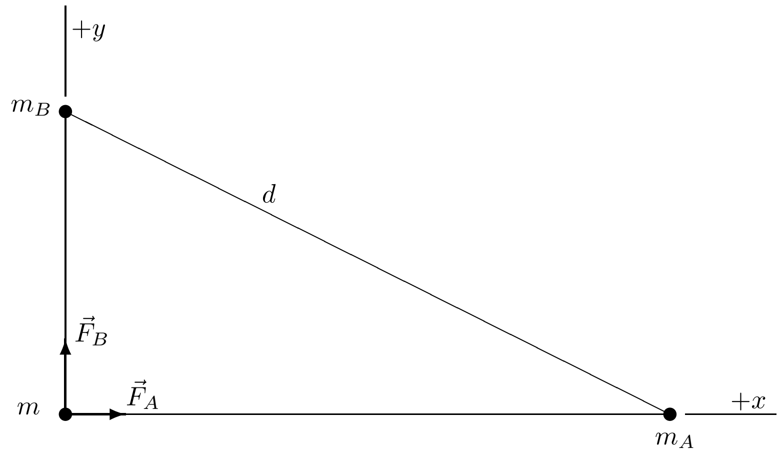
where $K_o = 0$, $K = \frac{1}{2}mv^2$ and U given by Eq. 14-20. Thus, with $r_o = 10001$ m, we find

$$v = \sqrt{2GM\left(\frac{1}{r} - \frac{1}{r_o}\right)} = 1.6 \times 10^6 \text{ m/s} \ .$$

85. It is clear from the given data that the $m = 2.0$ kg sphere cannot be along the line between m_A and m_B (that is, it is "off-axis"). The magnitudes of the individual forces (acting on m, exerted by m_A and m_B respectively) are

$$F_A = \frac{Gm_A m}{r_A^2} = 2.7 \times 10^{-6} \text{ N} \quad \text{and} \quad F_B = \frac{Gm_B m}{r_B^2} = 3.6 \times 10^{-6} \text{ N}$$

where $r_A = 0.20$ m and $r_B = 0.15$ m. Letting d stand for the distance between m_A and m_B then we note that $d^2 = r_A^2 + r_B^2$ (that is, the line between m_A and m_B forms the hypotenuse of a right triangle with m at the right-angle corner, as illustrated in the figure below).



Choosing x and y axes as shown above, then (in Newtons) $\vec{F}_A = 2.7 \times 10^{-6}\,\hat{\imath}$ and $\vec{F}_B = 3.6 \times 10^{-6}\,\hat{\jmath}$, which makes the vector addition very straightforward: we find

$$F_{\text{net}} = \sqrt{F_A^2 + F_B^2} = 4.4 \times 10^{-6}\ \text{N}$$

and (as measured counterclockwise from the x axis) $\theta = 53°$. It is not difficult to check that the direction of $\vec{F}_{\text{net}}$ (given by θ) is along a line that is perpendicular to the segment d.

86. (a) We use Eq. 14-27:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\,(6.67 \times 10^{-11})\,(1.99 \times 10^{30})}{1.50 \times 10^{11}}} = 4.21 \times 10^4\ \text{m/s}\ .$$

(b) Earth's orbital speed is gotten by solving Eq. 14-41:

$$v_{\text{orb}} = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.67 \times 10^{-11})\,(1.99 \times 10^{30})}{1.50 \times 10^{11}}} = 2.97 \times 10^4\ \text{m/s}\ .$$

The difference is therefore $v_{\text{esc}} - v_{\text{orb}} = 1.23 \times 10^4$ m/s.

(c) To obtain the speed (relative to Earth) mentioned above, the object must be launched with initial speed

$$v_0 = \sqrt{(1.23 \times 10^4)^2 + 2\frac{GM_E}{R_E}} = 1.66 \times 10^4\ \text{m/s}\ .$$

However, this is not precisely the same as the speed it would need to be launched at if it is desired that the object be just able to escape the solar system. The computation needed for that is shown below.

Including the Sun's gravitational influence as well as that of Earth (and accounting for the fact that Earth is moving around the Sun) the object at moment of launch has energy

$$K + U_E + U_S = \frac{1}{2}m\,(v_{\text{launch}} + v_{\text{orb}})^2 - \frac{GmM_E}{R_E} - \frac{GmM_S}{R}$$

which must equate to zero for the object to (barely) escape the solar system. Consequently,

$$v_{\text{launch}} = \sqrt{2G\left(\frac{M_E}{R_E} + \frac{M_S}{R}\right)} - v_{\text{orb}} = \sqrt{2\,(6.67 \times 10^{-11})\left(\frac{5.98 \times 10^{24}}{6.37 \times 10^6} + \frac{1.99 \times 10^{30}}{1.50 \times 10^{11}}\right)} - 2.97 \times 10^4$$

which yields $v_{\text{launch}} = 1.38 \times 10^4$ m/s.

87. (a) Converting T to seconds (by multiplying by 3.156×10^7) we do a linear fit of T^2 versus a^3 by the method of least squares. We obtain (with SI units understood)

$$T^2 = -7.4 \times 10^{15} + 2.982 \times 10^{-19}\, a^3 \ .$$

The coefficient of a^3 should be $4\pi^2/GM$ so that this result gives the mass of the Sun as

$$M = \frac{4\pi^2}{(6.67 \times 10^{-11}\,\mathrm{m^3/kg{\cdot}s^2})\,(2.982 \times 10^{-19}\,\mathrm{s^2/m^3})} = 1.98 \times 10^{30}\ \mathrm{kg}\ .$$

(b) Since $\log T^2 = 2\log T$ and $\log a^3 = 3\log a$ then the coefficient of $\log a$ in this next fit should be close to $3/2$, and indeed we find

$$\log T = -9.264 + 1.50007 \log a \ .$$

In order to compute the mass, we recall the property $\log AB = \log A + \log B$, which when applied to Eq. 14-33 leads us to identify

$$-9.264 = \frac{1}{2}\log\left(\frac{4\pi^2}{GM}\right) \quad\Longrightarrow\quad M = 1.996 \times 10^{30} \approx 2.00 \times 10^{30}\ \mathrm{kg}\ .$$

88. (a) We write the centripetal acceleration (which is the same for each, since they have identical mass) as $r\omega^2$ where ω is the unknown angular speed. Thus,

$$\frac{G(M)(M)}{(2r)^2} = \frac{GM^2}{4r^2} = Mr\omega^2$$

which gives $\omega = \frac{1}{2}\sqrt{MG/r^3} = 2.2 \times 10^{-7}$ rad/s.

(b) To barely escape means to have total energy equal to zero (see discussion prior to Eq. 14-27). If m is the mass of the meteoroid, then

$$\frac{1}{2}mv^2 - \frac{GmM}{r} - \frac{GmM}{r} = 0 \quad\Longrightarrow\quad v = \sqrt{\frac{4GM}{r}} = 8.9 \times 10^4\ \mathrm{m/s}\ .$$

89. (a) Circular motion requires that the force in Newton's second law provide the necessary centripetal acceleration:

$$\frac{GmM}{r^2} = m\frac{v^2}{r}$$

which is identical to Eq. 14-39 in the textbook. Since the left-hand side of this equation is the force given as 80 N, then we can solve for the combination mv^2 by multiplying both sides by $r = 2.0 \times 10^7$ m. Thus, $mv^2 = \left(2.0 \times 10^7\right)(80) = 1.6 \times 10^9$ J. Therefore,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.6 \times 10^9\right) = 8.0 \times 10^8\ \mathrm{J}\ .$$

(b) Since the gravitational force is inversely proportional to the square of the radius, then

$$\frac{F'}{F} = \left(\frac{r}{r'}\right)^2 \ .$$

Thus, $F' = (80)(2/3)^2 = 36$ N.

90. (a) Because it is moving in a circular orbit, F/m must equal the centripetal acceleration:

$$\frac{80\ \mathrm{N}}{50\ \mathrm{kg}} = \frac{v^2}{r}$$

But $v = 2\pi r/T$, where $T = 21600$ s, so we are led to

$$1.6\ \mathrm{m/s^2} = \frac{4\pi^2}{T^2}\,r$$

which yields $r = 1.9 \times 10^7$ m.

(b) From the above calculation, we infer $v^2 = (1.6 \text{ m/s}^2)r$ which leads to $v^2 = 3.0 \times 10^7 \text{ m}^2/\text{s}^2$. Thus, $K = \frac{1}{2}mv^2 = 7.6 \times 10^8$ J.

(c) As discussed in §14-4, F/m also tells us the gravitational acceleration:

$$a_g = 1.6 \text{ m/s}^2 = \frac{GM}{r^2}$$

We therefore find $M = 8.6 \times 10^{24}$ kg.

91. (a) The total energy is conserved, so there is no difference between its values at aphelion and perihelion.

(b) Since the change is small, we use differentials:

$$dU = \left(\frac{GM_E M_S}{r^2}\right) dr \approx \left(\frac{(6.67 \times 10^{-11})(1.99 \times 10^{30})(5.98 \times 10^{24})}{(1.5 \times 10^{11})^2}\right)(5 \times 10^9)$$

which yields $\Delta U \approx 1.8 \times 10^{32}$ J. A more direct subtraction of the values of the potential energies leads to the same result.

(c) and (d) From the previous two parts, we see that the variation in the kinetic energy ΔK must also equal 1.8×10^{32} J. So, with $\Delta K \approx dK = mv\,dv$, where $v \approx 2\pi R/T$, we have

$$1.8 \times 10^{32} \approx (5.98 \times 10^{24})\left(\frac{2\pi(1.5 \times 10^{11})}{3.156 \times 10^7}\right)\Delta v$$

which yields a difference of $\Delta v \approx 1$ km/s in Earth's speed (relative to the Sun) between aphelion and perihelion.

92. (a) From Kepler's law of periods, we see that T is proportional to $r^{3/2}$.

(b) Eq. 14-42 shows that K is inversely proportional to r.

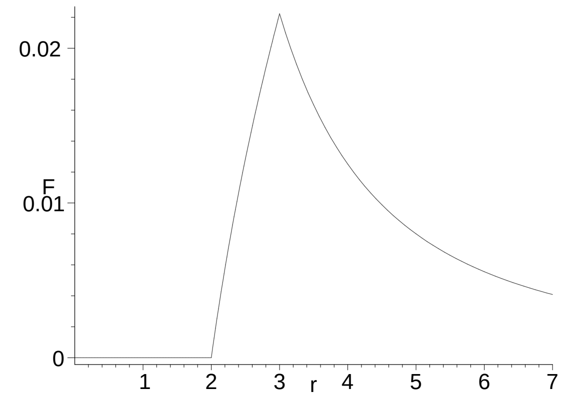
(c) and (d) From the previous part, knowing that K is proportional to v^2, we find that v is proportional to $1/\sqrt{r}$. Thus, by Eq. 14-30, the angular momentum (which depends on the product rv) is proportional to $r/\sqrt{r} = \sqrt{r}$.

93. The orbital speed is readily found from Eq. 14-41 to be $v_{\text{orb}} = \sqrt{GM/r}$. Comparing this with the expression for the escape velocity, Eq. 14-27, we immediately obtain the desired result.

94. (a) When testing for a gravitational force at $r < b$, none is registered. But at points within the shell $b \le r \le a$, the force will increase according to how much mass M' of the shell is at smaller radius. Specifically, for $b \le r \le a$, we find

$$F = \frac{GmM'}{r^2} = \frac{GmM\left(\frac{r^3-b^3}{a^3-b^3}\right)}{r^2}\ .$$

Once $r = a$ is reached, the force takes the familiar form GmM/r^2 and continues to have this form for $r > a$. We have chosen $m = 1$ kg, $M = 3 \times 10^9$ kg, $b = 2$ m and $a = 3$ m in order to produce the following graph of F versus r (in SI units).



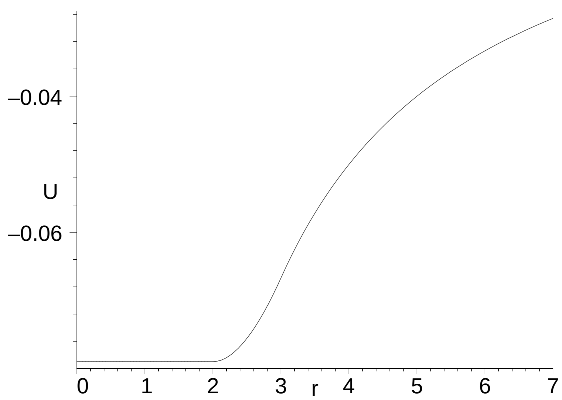
(b) Starting with the large r formula for force, we integrate and obtain the expected $U = -GmM/r$ (for $r \geq a$). Integrating the force formula indicated above for $b \leq r \leq a$ produces

$$U = \frac{GmM\left(r^3 + 2b^3\right)}{2r\left(a^3 - b^3\right)} + C$$

where C is an integration constant that we determine to be

$$C = -\frac{3GmMa^2}{2a\left(a^3 - b^3\right)}$$

so that this U and the large r formula for U agree at $r = a$. Finally, the $r < a$ formula for U is a constant (since the corresponding force vanishes), and we determine its value by evaluating the previous U at $r = b$. The resulting graph is shown below.



Chapter 15

1. The pressure increase is the applied force divided by the area: $\Delta p = F/A = F/\pi r^2$, where r is the radius of the piston. Thus $\Delta p = (42\,\text{N})/\pi(0.011\,\text{m})^2 = 1.1 \times 10^5\,\text{Pa}$. This is equivalent to 1.1 atm.

2. We note that the container is cylindrical, the important aspect of this being that it has a uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquids. Using the fact that $1\,\text{L} = 1000\,\text{cm}^3$, we find the weight of the first liquid to be

$$\begin{aligned} W_1 &= m_1 g = \rho_1 V_1 g \\ &= (2.6\,\text{g/cm}^3)(0.50\,\text{L})(1000\,\text{cm}^3/\,\text{L})(980\,\text{cm/s}^2) = 1.27 \times 10^6\,\text{g}\cdot\text{cm/s}^2 = 12.7\text{ N} . \end{aligned}$$

In the last step, we have converted grams to kilograms and centimeters to meters. Similarly, for the second and the third liquids, we have

$$W_2 = m_2 g = \rho_2 V_2 g = (1.0\,\text{g/cm}^3)(0.25\,\text{L})(1000\,\text{cm}^3/\,\text{L})(980\,\text{cm/s}^2) = 2.5\,\text{N}$$

and

$$W_3 = m_3 g = \rho_3 V_3 g = (0.80\,\text{g/cm}^3)(0.40\,\text{L})(1000\,\text{cm}^3/\text{L})(980\,\text{cm/s}^2) = 3.1\text{ N} .$$

The total force on the bottom of the container is therefore $F = W_1 + W_2 + W_3 = 18\,\text{N}$.

3. The air inside pushes outward with a force given by $p_i A$, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by $p_o A$, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$. Since $1\,\text{atm} = 1.013 \times 10^5\,\text{Pa}$,

$$F = (1.0\,\text{atm} - 0.96\,\text{atm})(1.013 \times 10^5\,\text{Pa/atm})(3.4\,\text{m})(2.1\,\text{m}) = 2.9 \times 10^4\text{ N} .$$

4. Knowing the standard air pressure value in several units allows us to set up a variety of conversion factors:

   (a) $P = \left(28\,\text{lb/in.}^2\right)\left(\frac{1.01\times10^5\,\text{Pa}}{14.7\,\text{lb/in}^2}\right) = 190\text{ kPa}$.

   (b) $(120\,\text{mmHg})\left(\frac{1.01\times10^5\,\text{Pa}}{760\,\text{mmHg}}\right) = 15.9\text{ kPa}$,

   $(80\,\text{mmHg})\left(\frac{1.01\times10^5\,\text{Pa}}{760\,\text{mmHg}}\right) = 10.6\text{ kPa}$.

5. Let the volume of the expanded air sacs be V_a and that of the fish with its air sacs collapsed be V. Then

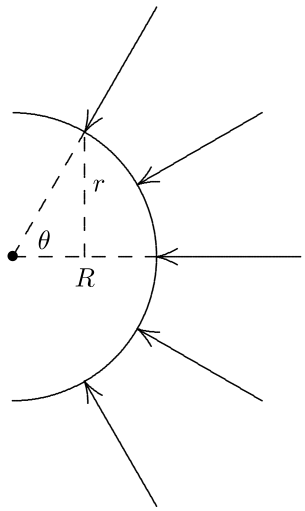
$$\rho_{\text{fish}} = \frac{m_{\text{fish}}}{V} = 1.08\,\text{g/cm}^3 \quad \text{and} \quad \rho_w = \frac{m_{\text{fish}}}{V + V_a} = 1.00\,\text{g/cm}^3 .$$

where ρ_w is the density of the water. This implies $\rho_{\text{fish}} V = \rho_w (V + V_a)$ or $(V + V_a)/V = 1.08/1.00$, which gives $V_a/(V + V_a) = 7.4\%$.

6. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\,\text{N/m}^2 = 1\,\text{Pa}$, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5\,\text{Pa} - \frac{480\,\text{N}}{77 \times 10^{-4}\,\text{m}^2} = 3.8 \times 10^4\ \text{Pa}\ .$$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.



We consider a force vector at angle θ. Its leftward component is $\Delta p \cos\theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R\sin\theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\,d\theta$ and its area is $dA = 2\pi R^2 \sin\theta\,d\theta$. Thus the net horizontal component of the force of the air is given by

$$\begin{aligned} F_h &= 2\pi R^2\,\Delta p \int_0^{\pi/2} \sin\theta\cos\theta\,d\theta \\ &= \pi R^2\,\Delta p \sin^2\theta\,\Big|_0^{\pi/2} = \pi R^2\,\Delta p\ . \end{aligned}$$

(b) We use $1\,\text{atm} = 1.01 \times 10^5\,\text{Pa}$ to show that $\Delta p = 0.90\,\text{atm} = 9.09 \times 10^4\,\text{Pa}$. The sphere radius is $R = 0.30\,\text{m}$, so $F_h = \pi(0.30\,\text{m})^2(9.09 \times 10^4\,\text{Pa}) = 2.6 \times 10^4\,\text{N}$.

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

8. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho g h = (1.06 \times 10^3\,\text{kg/m}^3)\left(9.8\,\text{m/s}^2\right)(1.83\,\text{m}) = 1.90 \times 10^4\ \text{Pa}\ .$$

9. The pump must work against the hydrostatic pressure exerted by the column of sewage (of density ρ and height $\ell = 8.2\,\text{m} - 2.1\,\text{m} = 6.1\,\text{m}$). The (minimum) pressure difference that must be maintained by the pump is $\Delta p = \rho g \ell = (900\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(6.1\,\text{m}) = 5.4 \times 10^4\,\text{Pa}$.

10. From the Figure, we see that the minimum pressure for diamond to form at 1000°C is $p_{\min} = 4.0\,\text{GPa}$. This pressure occurs at a minimum depth of $h_{\min}$ given by $p_{\min} = \rho g h_{\min}$. Thus,

$$h_{\min} = \frac{p_{\min}}{\rho g} = \frac{4.0 \times 10^9\,\text{Pa}}{\left(3.1 \times 10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)} = 1.3 \times 10^5\ \text{m}\ .$$

11. (a) We note that the pool has uniform cross-section (as viewed from above); this allows us to relate the pressure at the bottom simply to the total weight of the liquid. Thus,

$$F_{\text{bottom}} = mg = \rho g V = \left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(540\,\text{m}^3\right) = 5.3 \times 10^6\ \text{N}\ .$$

(b) The average pressure due to the water (that is, averaged over depth h) is

$$p_{\text{avg}} = \rho g \left(\frac{h}{2}\right)$$

where $h = 2.5$ m. Thus, the force on a short side (of area $A = 9.0 \times 2.5$ in SI units) is

$$F_{\text{short side}} = \rho g \left(\frac{h}{2}\right) A = 2.8 \times 10^5 \text{ N} .$$

(c) The area of a long side is $A' = 24 \times 2.5$ in SI units. Therefore, the force exerted by the water pressure on a long side is

$$F_{\text{long side}} = \rho g \left(\frac{h}{2}\right) A' = 7.4 \times 10^5 \text{ N} .$$

(d) If the pool is above ground, then it is clear that the air pressure outside the walls "cancels" any contribution of air pressure to the water pressure exerted by the liquid in the pool. If the pool is, as is often the case, surrounded by soil, then the situation may be more subtle, but our expectation is under normal circumstances the push from the soil certainly compensates for any atmospheric contribution to the water pressure (due to a "liberal interpretation" of Pascal's principle).

12. (a) The total weight is

$$W = \rho g h A = \left(1.00 \times 10^3 \,\text{kg/m}^3\right)\left(9.8 \,\text{m/s}^2\right)(200 \,\text{m})\left(3000 \,\text{m}^2\right) = 6.06 \times 10^9 \text{ N} .$$

(b) The water pressure is

$$p = \rho g h = \left(1.03 \times 10^3 \,\text{kg/m}^3\right)\left(9.8 \,\text{m/s}^2\right)(200 \,\text{m})\left(\frac{1 \,\text{atm}}{1.01 \times 10^5 \,\text{Pa}}\right) = 20 \text{ atm}$$

which is too much for anybody to endure without special equipment.

13. The pressure p at the depth d of the hatch cover is $p_0 + \rho g d$, where ρ is the density of ocean water and p_0 is atmospheric pressure. The downward force of the water on the hatch cover is $(p_0 + \rho g d)A$, where A is the area of the cover. If the air in the submarine is at atmospheric pressure then it exerts an upward force of $p_0 A$. The minimum force that must be applied by the crew to open the cover has magnitude $F = (p_0 + \rho g d)A - p_0 A = \rho g d A = (1025 \,\text{kg/m}^3)(9.8 \,\text{m/s}^2)(100 \,\text{m})(1.2 \,\text{m})(0.60 \,\text{m}) = 7.2 \times 10^5$ N.

14. Since the pressure (caused by liquid) at the bottom of the barrel is doubled due to the presence of the narrow tube, so is the hydrostatic force. The ratio is therefore equal to 2.0. The difference between the hydrostatic force and the weight is accounted for by the additional upward force exerted by water on the top of the barrel due to the increased pressure introduced by the water in the tube.

15. When the levels are the same the height of the liquid is $h = (h_1 + h_2)/2$, where h_1 and h_2 are the original heights. Suppose h_1 is greater than h_2. The final situation can then be achieved by taking liquid with volume $A(h_1 - h)$ and mass $\rho A(h_1 - h)$, in the first vessel, and lowering it a distance $h - h_2$. The work done by the force of gravity is $W = \rho A(h_1 - h)g(h - h_2)$. We substitute $h = (h_1 + h_2)/2$ to obtain $W = \frac{1}{4}\rho g A(h_1 - h_2)^2$.

16. Letting $p_a = p_b$, we find $\rho_c g(6.0 \,\text{km} + 32 \,\text{km} + D) + \rho_m(y - D) = \rho_c g(32 \,\text{km}) + \rho_m(y)$ and obtain

$$D = \frac{(6.0 \,\text{km})\rho_c}{\rho_m - \rho_c} = \frac{(6.0 \,\text{km})\left(2.9 \,\text{g/cm}^3\right)}{3.3 \,\text{g/cm}^3 - 2.9 \,\text{g/cm}^3} = 44 \text{ km} .$$

17. We assume that the pressure is the same at all points that are the distance $d = 20\,\text{km}$ below the surface. For points on the left side of Fig. 15-31, this pressure is given by $p = p_0 + \rho_o g d_o + \rho_c g d_c + \rho_m g d_m$, where p_0 is atmospheric pressure, ρ_o and d_o are the density and depth of the ocean, ρ_c and d_c are the density and thickness of the crust, and ρ_m and d_m are the density and thickness of the mantle (to a depth of 20 km). For points on the right side of the figure p is given by $p = p_0 + \rho_c g d$. We equate the two expressions for p and note that g cancels to obtain $\rho_c d = \rho_o d_o + \rho_c d_c + \rho_m d_m$. We substitute $d_m = d - d_o - d_c$ to obtain

$$\rho_c d = \rho_o d_o + \rho_c d_c + \rho_m d - \rho_m d_o - \rho_m d_c \ .$$

We solve for d_o:

$$\begin{aligned} d_o &= \frac{\rho_c d_c - \rho_c d + \rho_m d - \rho_m d_c}{\rho_m - \rho_o} = \frac{(\rho_m - \rho_c)(d - d_c)}{\rho_m - \rho_o} \\ &= \frac{\left(3.3\,\text{g/cm}^3 - 2.8\,\text{g/cm}^3\right)(20\,\text{km} - 12\,\text{km})}{3.3\,\text{g/cm}^3 - 1.0\,\text{g/cm}^3} = 1.7\ \text{km} \ . \end{aligned}$$

18. (a) The force on face A of area A_A is

$$\begin{aligned} F_A &= p_A A_A = \rho_w g h_A A_A = 2\rho_w g d^3 \\ &= 2\left(1.0\times10^3\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(5.0\,\text{m})^3 = 2.5\times10^6\ \text{N} \ . \end{aligned}$$

(b) The force on face B is

$$\begin{aligned} F_B &= p_{\text{avg}B} A_B = \rho_w g\left(\frac{5d}{2}\right)d^2 = \frac{5}{2}\rho_w g d^3 \\ &= \frac{5}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(5.0\,\text{m})^3 = 3.1\times10^6\ \text{N} \ . \end{aligned}$$

Note that these figures are due to the water pressure only. If you add the contribution from the atmospheric pressure, then you need to add $F' = (1.0\times10^5\,\text{Pa})(5.0\,\text{m})^2 = 2.5\times10^6\,\text{N}$ to each of the figures above. The results would then be $5.0\times10^6\,\text{N}$ and $5.6\times10^6\,\text{N}$, respectively.

19. (a) At depth y the gauge pressure of the water is $p = \rho g y$, where ρ is the density of the water. We consider a horizontal strip of width W at depth y, with (vertical) thickness dy, across the dam. Its area is $dA = W\,dy$ and the force it exerts on the dam is $dF = p\,dA = \rho g y W\,dy$. The total force of the water on the dam is

$$F = \int_0^D \rho g y W\,dy = \frac{1}{2}\rho g W D^2 \ .$$

(b) Again we consider the strip of water at depth y. Its moment arm for the torque it exerts about O is $D - y$ so the torque it exerts is $d\tau = dF(D-y) = \rho g y W(D-y)dy$ and the total torque of the water is

$$\tau = \int_0^D \rho g y W(D-y)\,dy = \rho g W\left(\frac{1}{2}D^3 - \frac{1}{3}D^3\right) = \frac{1}{6}\rho g W D^3 \ .$$

(c) We write $\tau = rF$, where r is the effective moment arm. Then,

$$r = \frac{\tau}{F} = \frac{\frac{1}{6}\rho g W D^3}{\frac{1}{2}\rho g W D^2} = \frac{D}{3} \ .$$

20. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\,\text{kg/m}^3\right)(9.8\,\text{m/s}^2)(4.0\times10^{-2}\,\text{m})}{1.01\times10^5\,\text{Pa/atm}} = -3.9\times10^{-3}\ \text{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

21. (a) We use the expression for the variation of pressure with height in an incompressible fluid: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. For this calculation, we take the density to be uniformly $1.3\,\mathrm{kg/m^3}$. Then,

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5\,\mathrm{Pa}}{\left(1.3\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)} = 7.9 \times 10^3\,\mathrm{m} = 7.9\ \mathrm{km}\ .$$

(b) Let h be the height of the atmosphere. Now, since the density varies with altitude, we integrate

$$p_2 = p_1 - \int_0^h \rho g\, dy\ .$$

Assuming $\rho = \rho_0(1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8\,\mathrm{m/s^2}$ for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 g h\ .$$

Since $p_2 = 0$, this implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01 \times 10^5\,\mathrm{Pa})}{(1.3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})} = 16 \times 10^3\,\mathrm{m} = 16\ \mathrm{km}\ .$$

22. (a) According to Pascal's principle $F/A = f/a \ \rightarrow\ F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80\,\mathrm{cm})^2}{(53.0\,\mathrm{cm})^2}(20.0 \times 10^3\,\mathrm{N}) = 103\ \mathrm{N}\ .$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

23. We assume the fluid in the press is incompressible. Then, the work done by the output force is the same as the work done by the input force. If the large piston moves a distance D and the small piston moves a distance d, then $fd = FD$ and

$$D = \frac{fd}{F} = \frac{(103\,\mathrm{N})(0.85\,\mathrm{m})}{20.0 \times 10^3\,\mathrm{N}} = 4.4 \times 10^{-3}\,\mathrm{m} = 4.4\ \mathrm{mm}\ .$$

24. (a) Archimedes' principle makes it clear that a body, in order to float, displaces an amount of the liquid which corresponds to the weight of the body. The problem (indirectly) tells us that the weight of the boat is $W = 35.6$ kN. In salt water of density $\rho' = 1100\,\mathrm{kg/m^3}$, it must displace an amount of liquid having weight equal to 35.6 kN.

(b) The displaced volume of salt water is equal to

$$V' = \frac{W}{\rho' g} = \frac{35600}{(1100)(9.8)} = 3.30\ \mathrm{m^3}\ .$$

In freshwater, it displaces a volume of $V = W/\rho g = 3.63\,\mathrm{m^3}$, where $\rho = 1000\,\mathrm{kg/m^3}$. The difference is $V - V' = 0.33\,\mathrm{m^3}$.

25. (a) The anchor is completely submerged in water of density ρ_w. Its effective weight is $W_{\text{eff}} = W - \rho_w g V$, where W is its actual weight (mg). Thus,

$$V = \frac{W - W_{\text{eff}}}{\rho_w g} = \frac{200\,\mathrm{N}}{\left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)} = 2.04 \times 10^{-2}\ \mathrm{m^3}\ .$$

(b) The mass of the anchor is $m = \rho V$, where ρ is the density of iron (found in Table 15-1). Its weight in air is

$$W = mg = \rho V g = \left(7870\,\mathrm{kg/m^3}\right)\left(2.04 \times 10^{-2}\,\mathrm{m^3}\right)\left(9.8\,\mathrm{m/s^2}\right) = 1.6 \times 10^3\ \mathrm{N}\ .$$

26. (a) The pressure (including the contribution from the atmosphere) at a depth of $h_{\mathrm{top}} = L/2$ (corresponding to the top of the block) is

$$p_{\mathrm{top}} = p_{\mathrm{atm}} + \rho g h_{\mathrm{top}} = 1.01 \times 10^5 + (1030)(9.8)(0.300) = 1.04 \times 10^5\ \mathrm{Pa}$$

where the unit Pa (Pascal) is equivalent to $\mathrm{N/m^2}$. The force on the top surface (of area $A = L^2 = 0.36\,\mathrm{m^2}$) is $F_{\mathrm{top}} = p_{\mathrm{top}} A = 3.75 \times 10^4\,\mathrm{N}$.

(b) The pressure at a depth of $h_{\mathrm{bot}} = 3L/2$ (that of the bottom of the block) is

$$p_{\mathrm{bot}} = p_{\mathrm{atm}} + \rho g h_{\mathrm{bot}} = 1.01 \times 10^5 + (1030)(9.8)(0.900) = 1.10 \times 10^5\ \mathrm{Pa}$$

where we recall that the unit Pa (Pascal) is equivalent to $\mathrm{N/m^2}$. The force on the bottom surface is $F_{\mathrm{bot}} = p_{\mathrm{bot}} A = 3.96 \times 10^4\,\mathrm{N}$.

(c) Taking the difference $F_{\mathrm{bot}} - F_{\mathrm{top}}$ cancels the contribution from the atmosphere (including any numerical uncertainties associated with that value) and leads to

$$F_{\mathrm{bot}} - F_{\mathrm{top}} = \rho g \left(h_{\mathrm{bot}} - h_{\mathrm{top}}\right) A = \rho g L^3 = 2180\ \mathrm{N}$$

which is to be expected on the basis of Archimedes' principle. Two other forces act on the block: an upward tension T and a downward pull of gravity mg. To remain stationary, the tension must be

$$T = mg - (F_{\mathrm{bot}} - F_{\mathrm{top}}) = (450)(9.8) - 2180 = 2230\ \mathrm{N}\ .$$

(d) This has already been noted in the previous part: $F_b = 2180\,\mathrm{N}$, and $T + F_b = mg$.

27. (a) Let V be the volume of the block. Then, the submerged volume is $V_s = 2V/3$. Since the block is floating, the weight of the displaced water is equal to the weight of the block, so $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block. We substitute $V_s = 2V/3$ to obtain $\rho_b = 2\rho_w/3 = 2(1000\,\mathrm{kg/m^3})/3 \approx 670\,\mathrm{kg/m^3}$.

(b) If ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s = \rho_b V$. We substitute $V_s = 0.90V$ to obtain $\rho_o = \rho_b/0.90 = 740\,\mathrm{kg/m^3}$.

28. The weight of the additional cargo ΔW the blimp could carry is equal to the difference between the weight of the helium and that of the hydrogen gas inside the blimp:

$$\begin{aligned}\Delta W &= W_{\mathrm{He}} - W_{\mathrm{H_2}} = (\rho_{\mathrm{He}} - \rho_{\mathrm{H_2}}) g V \\ &= \left(0.16\,\mathrm{kg/m^3} - 0.081\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(5000\,\mathrm{m^3}\right) \\ &= 3.9 \times 10^3\ \mathrm{N}\end{aligned}$$

which corresponds to about 400 kg of mass. The reason why helium is used is because it is safer (non-flammable).

29. (a) The downward force of gravity mg is balanced by the upward buoyant force of the liquid: $mg = \rho g V_s$. Here m is the mass of the sphere, ρ is the density of the liquid, and V_s is the submerged volume. Thus $m = \rho V_s$. The submerged volume is half the total volume of the sphere, so $V_s = \frac{1}{2}(4\pi/3) r_o^3$, where r_o is the outer radius. Therefore,

$$m = \frac{2\pi}{3} \rho r_o^3 = \left(\frac{2\pi}{3}\right)\left(800\,\mathrm{kg/m^3}\right)(0.090\,\mathrm{m})^3 = 1.22\ \mathrm{kg}\ .$$

(b) The density ρ_m of the material, assumed to be uniform, is given by $\rho_m = m/V$, where m is the mass of the sphere and V is its volume. If r_i is the inner radius, the volume is

$$V = \frac{4\pi}{3}\left(r_o^3 - r_i^3\right) = \frac{4\pi}{3}\left((0.090\,\mathrm{m})^3 - (0.080\,\mathrm{m})^3\right) = 9.09 \times 10^{-4}\ \mathrm{m}^3 \ .$$

The density is

$$\rho_m = \frac{1.22\,\mathrm{kg}}{9.09 \times 10^{-4}\,\mathrm{m}^3} = 1.3 \times 10^3\ \mathrm{kg/m}^3 \ .$$

30. Equilibrium of forces (on the floating body) is expressed as

$$F_b = m_{\mathrm{body}}g \implies \rho_{\mathrm{liquid}}gV_{\mathrm{submerged}} = \rho_{\mathrm{body}}gV_{\mathrm{total}}$$

which leads to

$$\frac{V_{\mathrm{submerged}}}{V_{\mathrm{total}}} = \frac{\rho_{\mathrm{body}}}{\rho_{\mathrm{liquid}}} \ .$$

We are told (indirectly) that two-thirds of the body is below the surface, so the fraction above is 2/3. Thus, with $\rho_{\mathrm{body}} = 0.98\,\mathrm{g/cm}^3$, we find $\rho_{\mathrm{liquid}} \approx 1.5\,\mathrm{g/cm}^3$ – certainly much more dense than normal seawater (the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region).

31. For our estimate of $V_{\mathrm{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\mathrm{submerged}} \approx \frac{4}{3}\pi r_{\mathrm{o}}^3 \qquad \text{where} \quad r_{\mathrm{o}} = 60\ \mathrm{cm} \ .$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\mathrm{iron}}g \implies \rho_{\mathrm{water}}gV_{\mathrm{submerged}} = \rho_{\mathrm{iron}}g\left(\frac{4}{3}\pi r_{\mathrm{o}}^3 - \frac{4}{3}\pi r_{\mathrm{i}}^3\right)$$

where r_{i} is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\mathrm{submerged}}$ as well as the densities of water ($1.0\,\mathrm{g/cm}^3$) and iron ($7.87\,\mathrm{g/cm}^3$), we obtain the inner diameter:

$$2r_{\mathrm{i}} = 2r_{\mathrm{o}}\left(1 - \frac{1}{7.87}\right)^{1/3} = 57.3\ \mathrm{cm} \ .$$

32. (a) Since the lead is not displacing any water (of density ρ_w), the lead's volume is not contributing to the buoyant force F_b. If the immersed volume of wood is V_i, then

$$F_b = \rho_w V_i g = 0.90\rho_w V_{\mathrm{wood}} g = 0.90\rho_w g\left(\frac{m_{\mathrm{wood}}}{\rho_{\mathrm{wood}}}\right) ,$$

which, when floating, equals the weights of the wood and lead:

$$F_b = 0.90\rho_w g\left(\frac{m_{\mathrm{wood}}}{\rho_{\mathrm{wood}}}\right) = (m_{\mathrm{wood}} + m_{\mathrm{lead}})g \ .$$

Thus,

$$\begin{aligned} m_{\mathrm{lead}} &= 0.90\rho_w\left(\frac{m_{\mathrm{wood}}}{\rho_{\mathrm{wood}}}\right) - m_{\mathrm{wood}} \\ &= \frac{(0.90)(1000\,\mathrm{kg/m}^3)(3.67\,\mathrm{kg})}{600\,\mathrm{kg/m}^3} - 3.67\,\mathrm{kg} = 1.84\,\mathrm{kg} \ \approx 1.8\ \mathrm{kg} \ . \end{aligned}$$

(b) In this case, the volume $V_{\text{lead}} = m_{\text{lead}}/\rho_{\text{lead}}$ also contributes to F_b. Consequently,

$$F_b = 0.90\rho_w g\left(\frac{m_{\text{wood}}}{\rho_{\text{wood}}}\right) + \left(\frac{\rho_w}{\rho_{\text{lead}}}\right) m_{\text{lead}} g = (m_{\text{wood}} + m_{\text{lead}})g\,,$$

which leads to

$$\begin{aligned} m_{\text{lead}} &= \frac{0.90(\rho_w/\rho_{\text{wood}})m_{\text{wood}} - m_{\text{wood}}}{1 - \rho_w/\rho_{\text{lead}}} \\ &= \frac{1.84\,\text{kg}}{1 - \left(1.00\times10^3\,\text{kg/m}^3/1.13\times10^4\,\text{kg/m}^3\right)} = 2.0\text{ kg}\,. \end{aligned}$$

33. The volume V_{cav} of the cavities is the difference between the volume V_{cast} of the casting as a whole and the volume V_{iron} contained: $V_{\text{cav}} = V_{\text{cast}} - V_{\text{iron}}$. The volume of the iron is given by $V_{\text{iron}} = W/g\rho_{\text{iron}}$, where W is the weight of the casting and ρ_{iron} is the density of iron. The effective weight in water (of density ρ_w) is $W_{\text{eff}} = W - g\rho_w V_{\text{cast}}$. Thus, $V_{\text{cast}} = (W - W_{\text{eff}})/g\rho_w$ and

$$\begin{aligned} V_{\text{cav}} &= \frac{W - W_{\text{eff}}}{g\rho_w} - \frac{W}{g\rho_{\text{iron}}} \\ &= \frac{6000\,\text{N} - 4000\,\text{N}}{(9.8\,\text{m/s}^2)(1000\,\text{kg/m}^3)} - \frac{6000\,\text{N}}{(9.8\,\text{m/s}^2)(7.87\times10^3\,\text{kg/m}^3)} \\ &= 0.126\text{ m}^3\,. \end{aligned}$$

34. Let F_o be the buoyant force of air exerted on the object (of mass m and volume V), and F_{brass} be the buoyant force on the brass weights (of total mass m_{brass} and volume V_{brass}). Then we have

$$F_o = \rho_{\text{air}} V g = \rho_{\text{air}}\left(\frac{mg}{\rho}\right)$$

and

$$F_{\text{brass}} = \rho_{\text{air}} V_{\text{brass}} g = \rho_{\text{air}}\left(\frac{m_{\text{brass}}}{\rho_{\text{brass}}}\right)\,.$$

For the two arms of the balance to be in mechanical equilibrium, we require $mg - F_o = m_{\text{brass}}g - F_{\text{brass}}$, or

$$mg - mg\left(\frac{\rho_{\text{air}}}{\rho}\right) = m_{\text{brass}}g - m_{\text{brass}}g\left(\frac{\rho_{\text{air}}}{\rho_{\text{brass}}}\right),$$

which leads to

$$m_{\text{brass}} = \left(\frac{1 - \rho_{\text{air}}/\rho}{1 - \rho_{\text{air}}/\rho_{\text{brass}}}\right)m\,.$$

Therefore, the percent error in the measurement of m is

$$\begin{aligned} \frac{\Delta m}{m} &= \frac{m - m_{\text{brass}}}{m} = 1 - \frac{1 - \rho_{\text{air}}/\rho}{1 - \rho_{\text{air}}/\rho_{\text{brass}}} = \frac{\rho_{\text{air}}(1/\rho - 1/\rho_{\text{brass}})}{1 - \rho_{\text{air}}/\rho_{\text{brass}}} \\ &= \frac{0.0012(1/\rho - 1/8.0)}{1 - 0.0012/8.0} \approx 0.0012\left(\frac{1}{\rho} - \frac{1}{8.0}\right)\,, \end{aligned}$$

where ρ is in g/cm^3. Stating this as a *percent* error, our result is 0.12% multiplied by $\left(\frac{1}{\rho} - \frac{1}{8.0}\right)$.

35. (a) We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface. Let M be the mass of the automobile, ρ_i be the density of ice, and ρ_w be the density of water. Suppose the ice slab has area A and thickness h. Since the volume of

ice is Ah, the downward force of gravity on the automobile and ice is $(M + \rho_i Ah)g$. The buoyant force of the water is $\rho_w Ahg$, so the condition of equilibrium is $(M + \rho_i Ah)g - \rho_w Ahg = 0$ and

$$A = \frac{M}{(\rho_w - \rho_i)h} = \frac{1100\,\text{kg}}{(998\,\text{kg/m}^3 - 917\,\text{kg/m}^3)(0.30\,\text{m})} = 45\ \text{m}^2\ .$$

These density values are found in Table 15-1 of the text.

(b) It does matter where the car is placed since the ice tilts if the automobile is not at the center of its surface.

36. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log: $V = \pi(0.15\,\text{m})^2(1.80\,\text{m}) = 0.13\,\text{m}^3$. The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$ where we require $V_{\text{submerged}} \le NV$. The density of water is $1000\,\text{kg/m}^3$. To obtain the minimum value of N we set $V_{\text{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \quad\Longrightarrow\quad N = \frac{3(356\,\text{N})}{gV\,(\rho_{\text{water}} - \rho_{\text{wood}})}$$

which yields $N = 4.28 \;\rightarrow\; 5$ logs.

37. (a) We assume the center of mass is closer to the right end of the rod, so the distance from the left end to the center of mass is $\ell = 0.60$ m. Four forces act on the rod: the upward force of the left rope T_L, the upward force of the right rope T_R, the downward force of gravity mg, and the upward buoyant force F_b. The force of gravity (effectively) acts at the center of mass, and the buoyant force acts at the geometric center of the rod (which has length $L = 0.80$ m). Computing torques about the left end of the rod, we find

$$T_R L + F_b\left(\frac{L}{2}\right) - mg\ell = 0 \quad\Longrightarrow\quad T_R = \frac{mg\ell - F_b L/2}{L}\ .$$

Now, the buoyant force is equal to the weight of the displaced water (where the volume of displacement is $V = AL$). Thus,

$$F_b = \rho_w gAL = \left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(6.0\times 10^{-4}\,\text{m}^2\right)(0.80\,\text{m}) = 4.7\ \text{N}\ .$$

Consequently, the tension in the right rope is

$$T_R = \frac{(1.6\,\text{kg})\left(9.8\,\text{m/s}^2\right)(0.60\,\text{m}) - (4.7\,\text{N})(0.40\,\text{m})}{0.80\,\text{m}} = 9.4\ \text{N}\ .$$

(b) Newton's second law (for the case of zero acceleration) leads to

$$T_L + T_R + F_B - mg = 0 \quad\Longrightarrow\quad T_L = mg - F_B - T_R = (1.6\,\text{kg})\left(9.8\,\text{m/s}^2\right) - 4.69\,\text{N} - 9.4\,\text{N} = 1.6\ \text{N}\ .$$

38. (a) If the volume of the car below water is V_1 then $F_b = \rho_w V_1 g = W_{\text{car}}$, which leads to

$$V_1 = \frac{W_{\text{car}}}{\rho_w g} = \frac{(1800\,\text{kg})\left(9.8\,\text{m/s}^2\right)}{\left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)} = 1.80\ \text{m}^3\ .$$

(b) We denote the total volume of the car as V and that of the water in it as V_2. Then

$$F_b = \rho_w V g = W_{\text{car}} + \rho_w V_2 g$$

which gives

$$\begin{aligned} V_2 &= V - \frac{W_{\text{car}}}{\rho_w g} \\ &= \left(0.750\,\text{m}^3 + 5.00\,\text{m}^3 + 0.800\,\text{m}^3\right) - \frac{1800\,\text{kg}}{1000\,\text{kg/m}^3} \\ &= 4.75\ \text{m}^3\ . \end{aligned}$$

39. We use the equation of continuity. Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. $A_1 = \pi R^2$ is the cross-sectional area of the hose. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 (N A_2) \implies v_2 = \frac{A_1}{N A_2} v_1 = \frac{R^2}{N r^2} v_1$$

where R is the radius of the hose and r is the radius of a hole. Noting that $R/r = D/d$ (the ratio of diameters) we find

$$v_2 = \frac{D^2}{N d^2} v_1 = \frac{(1.9\,\text{cm})^2}{24(0.13\,\text{cm})^2}(0.91\,\text{m/s}) = 8.1\ \text{m/s}\ .$$

40. We use the equation of continuity and denote the depth of the river as h. Then,

$$(8.2\,\text{m})(3.4\,\text{m})(2.3\,\text{m/s}) + (6.8\,\text{m})(3.2\text{m})(2.6\,\text{m/s}) = h(10.5\,\text{m})(2.9\,\text{m/s})$$

which leads to $h = 4.0\,\text{m}$.

41. Suppose that a mass Δm of water is pumped in time Δt. The pump increases the potential energy of the water by $\Delta m g h$, where h is the vertical distance through which it is lifted, and increases its kinetic energy by $\frac{1}{2}\Delta m v^2$, where v is its final speed. The work it does is $\Delta W = \Delta m g h + \frac{1}{2}\Delta m v^2$ and its power is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta m}{\Delta t}\left(gh + \frac{1}{2}v^2\right)\ .$$

Now the rate of mass flow is $\Delta m/\Delta t = \rho_w A v$, where ρ_w is the density of water and A is the area of the hose. The area of the hose is $A = \pi r^2 = \pi(0.010\,\text{m})^2 = 3.14\times10^{-4}\,\text{m}^2$ and $\rho_w A v = (1000\,\text{kg/m}^3)(3.14\times10^{-4}\,\text{m}^2)(5.0\,\text{m/s}) = 1.57\,\text{kg/s}$. Thus,

$$\begin{aligned} P &= \rho A v\left(gh + \frac{1}{2}v^2\right) \\ &= (1.57\,\text{kg/s})\left((9.8\,\text{m/s}^2)(3.0\,\text{m}) + \frac{(5.0\,\text{m/s})^2}{2}\right) = 66\ \text{W}\ . \end{aligned}$$

42. (a) The equation of continuity provides $26 + 19 + 11 = 56$ L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{\frac{56}{\pi(1.9)^2/4}}{\frac{26}{\pi(1.3)^2/4}} \approx 1\ .$$

43. (a) We use the equation of continuity: $A_1 v_1 = A_2 v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. Thus $v_2 = (A_1/A_2)v_1 = \left((4.0\,\text{cm}^2)/(8.0\,\text{cm}^2)\right)(5.0\,\text{m/s}) = 2.5\,\text{m/s}$.

(b) We use the Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude. Thus

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2) \\ &= 1.5\times 10^5\,\text{Pa} + \frac{1}{2}(1000\,\text{kg/m}^3)\left((5.0\,\text{m/s})^2 - (2.5\,\text{m/s})^2\right) + (1000\,\text{kg/m}^3)(9.8\,\text{m/s}^2)(10\,\text{m}) \\ &= 2.6\times 10^5\ \text{Pa}\ . \end{aligned}$$

44. (a) We use $Av =$ const. The speed of water is

$$v = \frac{(25.0\,\text{cm})^2 - (5.00\,\text{cm})^2}{(25.0\,\text{cm})^2}(2.50\,\text{m/s}) = 2.40\ \text{m/s}\ .$$

(b) Since $p + \frac{1}{2}\rho v^2 =$ const., the pressure difference is

$$\Delta p = \frac{1}{2}\rho\Delta v^2 = \frac{1}{2}(1000\,\text{kg/m}^3)[(2.50\,\text{m/s})^2 - (2.40\,\text{m/s})^2] = 245\ \text{Pa}\ .$$

45. (a) The equation of continuity leads to

$$v_2 A_2 = v_1 A_1 \quad\Longrightarrow\quad v_2 = v_1\left(\frac{r_1^2}{r_2^2}\right)$$

which gives $v_2 = 3.9\,\text{m/s}$.

(b) With $h = 7.6\,\text{m}$ and $p_1 = 1.7\times 10^5\,\text{Pa}$, Bernoulli's equation reduces to

$$p_2 = p_1 - \rho g h + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 8.8\times 10^4\ \text{Pa}\ .$$

46. We use Bernoulli's equation:

$$p_2 - p_i = \rho g h + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

where $\rho = 1000\,\text{kg/m}^3$, $h = 180\,\text{m}$, $v_1 = 0.40\,\text{m/s}$ and $v_2 = 9.5\,\text{m/s}$. Therefore, we find $\Delta p = 1.7\times 10^6$ Pa, or 1.7 MPa. The SI unit for pressure is the Pascal (Pa) and is equivalent to N/m^2.

47. (a) We use the Bernoulli equation: $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$, where h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. ρ is the density of water. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then becomes $\rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2$ and

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2(9.8\,\text{m/s}^2)(0.30\,\text{m})} = 2.42\ \text{m/s}\ .$$

The flow rate is $A_2 v_2 = (6.5\times 10^{-4}\,\text{m}^2)(2.42\,\text{m/s}) = 1.6\times 10^{-3}\ \text{m}^3/\text{s}$.

(b) We use the equation of continuity: $A_2 v_2 = A_3 v_3$, where $A_3 = \frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole. Thus $v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\,\text{m/s}$. The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84\,\text{m/s})^2 - (2.42\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 0.90\ \text{m}\ .$$

48. The lift force follows from the pressure difference (large pressure on the bottom surface than on the top) and the fact that the pressure difference is related to force through the relation $\Delta p = F/A$ where we are asked to use L for F. From Bernoulli's equation, we have

$$p_u - p_t = \frac{1}{2}\rho v_t^2 - \frac{1}{2}\rho v_u^2 + \rho g \Delta z$$

where Δz is the thickness of the wing. The last term makes a negligible contribution (we will return to this point in a moment) and can be ignored. We then have

$$\Delta p = \frac{1}{2}\rho\left(v_t^2 - v_u^2\right) \implies L = \frac{1}{2}\rho A\left(v_t^2 - v_u^2\right)$$

as desired. The contribution of the "potential" term would have been $\rho g A \Delta z$ which we can estimate as follows: let $\rho \approx 1$ kg/m^3, $A \approx 100$ m^2, and $\Delta z \approx 1$ m. Then $\rho g A \Delta z \approx 1000$ N which perhaps corresponds to the weight of a couple of adults, and is at least an order of magnitude less than the weight of an airplane with wings (the size of which are as estimated above) and equipment and crew.

49. We use the Bernoulli equation: $p_\ell + \frac{1}{2}\rho v_\ell^2 = p_u + \frac{1}{2}\rho v_u^2$, where p_ℓ is the pressure at the lower surface, p_u is the pressure at the upper surface, v_ℓ is the air speed at the lower surface, v_u is the air speed at the upper surface, and ρ is the density of air. The two tubes of flow are essentially at the same altitude. We want to solve for v_u such that $p_\ell - p_u = 900$ Pa. That is,

$$v_u = \sqrt{\frac{2(p_\ell - p_u)}{\rho} + v_\ell^2} = \sqrt{\frac{2(900\,\text{Pa})}{1.30\,\text{kg/m}^3} + (110\,\text{m/s})^2} = 116 \text{ m/s} .$$

50. (a) The speed v of the fluid flowing out of the hole satisfies $\frac{1}{2}\rho v^2 = \rho g h$ or $v = \sqrt{2gh}$. Thus, $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$, which leads to

$$\rho_1\sqrt{2gh}A_1 = \rho_2\sqrt{2gh}A_2 \implies \frac{\rho_1}{\rho_2} = \frac{A_2}{A_1} = 2 .$$

(b) The ratio of volume flow is

$$\frac{R_1}{R_2} = \frac{v_1 A_1}{v_2 A_2} = \frac{A_1}{A_2} = \frac{1}{2} .$$

(c) Letting $R_1/R_2 = 1$, we obtain $v_1/v_2 = A_2/A_1 = 2 = \sqrt{h_1/h_2}$. Thus $h_2 = h_1/4$.

51. (a) The volume of water (during 10 minutes) is

$$V = (v_1 t)A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4 \text{ m}^3 .$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4 \text{ m/s} .$$

(c) Since $p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$ and $h_1 = h_2$, $p_1 = p_0$ (= atmospheric pressure),

$$\begin{aligned} p_2 &= p_0 + \frac{1}{2}\rho(v_1^2 - v_2^2) \\ &= 1.01 \times 10^5\,\text{Pa} + \frac{1}{2}(1.0 \times 10^3\,\text{kg/m}^3)[(15\,\text{m/s})^2 - (5.4\,\text{m/s})^2] \\ &= 1.99 \times 10^5\,\text{Pa} = 1.97 \text{ atm} . \end{aligned}$$

Thus the gauge pressure is $1.97\,\text{atm} - 1.00\,\text{atm} = 0.97\,\text{atm} = 9.8 \times 10^4\,\text{Pa}$.

52. (a) We denote a point at the top surface of the liquid A and a point at the opening B. Point A is a vertical distance $h = 0.50$ m above B. Bernoulli's equation yields $p_A = p_B + \frac{1}{2}\rho v_B^2 - \rho g h$. Noting that $p_A = p_B$ we obtain

$$\begin{aligned} v_B &= \sqrt{2gh + \frac{2}{\rho}(p_A - p_B)} \\ &= \sqrt{2(9.8\,\text{m/s}^2)(0.50\,\text{m})} = 3.1 \text{ m/s} . \end{aligned}$$

(b)

$$\begin{aligned} v_B &= \sqrt{2gh + \frac{2}{\rho}(p_A - p_B)} \\ &= \sqrt{2(9.8\,\text{m/s}^2)(0.50\,\text{m}) + \frac{2(1.40\,\text{atm} - 1.00\,\text{atm})}{1.0 \times 10^3\,\text{kg/m}^3}} = 9.5 \text{ m/s} . \end{aligned}$$

53. (a) The friction force is

$$\begin{aligned} f &= A\Delta p = \rho_w g h A \\ &= \left(1.0 \times 10^3\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(6.0\text{m})\left(\frac{\pi}{4}\right)(0.040\,\text{m})^2 = 74 \text{ N} . \end{aligned}$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gh}$. Thus, the volume of water flowing out of the pipe in $t = 3.0\,\text{h}$ is

$$\begin{aligned} V &= Avt = \frac{\pi d v t}{4} \\ &= \frac{\pi^2}{4}(0.040\,\text{m})^2\sqrt{2\,(9.8\,\text{m/s}^2)\,(6.0\,\text{m})}(3.0\,\text{h})(3600\,\text{s/h}) \\ &= 1.5 \times 10^2 \text{ m}^3 . \end{aligned}$$

54. (a) Since Sample Problem 15-9 deals with a similar situation, we use the final equation (labeled "Answer") from it:

$$v = \sqrt{2gh} \implies v = v_o \text{ for the projectile motion.}$$

The stream of water emerges horizontally ($\theta_o = 0°$ in the notation of Chapter 4), and setting $y - y_o = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)} .$$

Using this in Eq. 4-21, where $x_o = 0$ by choice of coordinate origin, we find

$$x = v_o t = \sqrt{2gh}\sqrt{\frac{2}{g}(H-h)} = 2\sqrt{h(H-h)} .$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \implies h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H .$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$.

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \implies h = \frac{H}{2}$$

as the depth from which an emerging stream of water will travel the maximum horizontal distance.

55. (a) The continuity equation yields $Av = aV$, and Bernoulli's equation yields $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho V^2$, where $\Delta p = p_1 - p_2$. The first equation gives $V = (A/a)v$. We use this to substitute for V in the second equation, and obtain $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho(A/a)^2 v^2$. We solve for v. The result is

$$v = \sqrt{\frac{2\,\Delta p}{\rho\left(\frac{A^2}{a^2} - 1\right)}} = \sqrt{\frac{2a^2\,\Delta p}{\rho(A^2 - a^2)}} .$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32 \times 10^{-4}\,\mathrm{m}^2)^2(55 \times 10^3\,\mathrm{Pa} - 41 \times 10^3\,\mathrm{Pa})}{(1000\,\mathrm{kg/m}^3)\left((64 \times 10^{-4}\,\mathrm{m}^2)^2 - (32 \times 10^{-4}\,\mathrm{m}^2)^2\right)}} = 3.06\ \mathrm{m/s} .$$

Consequently, the flow rate is

$$Av = \left(64 \times 10^{-4}\,\mathrm{m}^2\right)(3.06\,\mathrm{m/s}) = 2.0 \times 10^{-2}\ \mathrm{m}^3/\mathrm{s} .$$

56. We use the result of part (a) in the previous problem.

(a) In this case, we have $\Delta p = p_1 = 2.0\,\mathrm{atm}$. Consequently,

$$v = \sqrt{\frac{2\Delta p}{\rho\left((A/a)^2 - 1\right)}} = \sqrt{\frac{4(1.01 \times 10^5\,\mathrm{Pa})}{(1000\,\mathrm{kg/m}^3)\left((5a/a)^2 - 1\right)}} = 4.1\ \mathrm{m/s} .$$

(b) And the equation of continuity yields $V = (A/a)v = (5a/a)v = 5v = 21\,\mathrm{m/s}$.

(c) The flow rate is given by

$$Av = \frac{\pi}{4}\left(5.0 \times 10^{-4}\,\mathrm{m}^2\right)(4.1\,\mathrm{m/s}) = 8.0 \times 10^{-3}\ \mathrm{m}^3/\mathrm{s} .$$

57. (a) Bernoulli's equation gives $p_A = p_B + \frac{1}{2}\rho_{\mathrm{air}}v^2$. But $\Delta p = p_A - p_B = \rho g h$ in order to balance the pressure in the two arms of the U-tube. Thus $\rho g h = \frac{1}{2}\rho_{\mathrm{air}}v^2$, or

$$v = \sqrt{\frac{2\rho g h}{\rho_{\mathrm{air}}}} .$$

(b) The plane's speed relative to the air is

$$v = \sqrt{\frac{2\rho g h}{\rho_{\mathrm{air}}}} = \sqrt{\frac{2\left(810\,\mathrm{kg/m}^3\right)(9.8\,\mathrm{m/s}^2)(0.260\,\mathrm{m})}{1.03\,\mathrm{kg/m}^3}} = 63.3\ \mathrm{m/s} .$$

58. We use the formula for v obtained in the previous problem:

$$v = \sqrt{\frac{2\Delta p}{\rho_{\text{air}}}} = \sqrt{\frac{2(180\,\text{Pa})}{0.031\,\text{kg/m}^3}} = 1.1 \times 10^2 \text{ m/s} .$$

59. (a) To avoid confusing weight with work, we write out the word instead of using the symbol W. Thus,

$$\text{weight} = mg = \left(1.85 \times 10^4\,\text{kg}\right)\left(9.8\,\text{m/s}^2\right) \approx 1.8 \times 10^5 \text{ N} .$$

(b) The buoyant force is $F_b = \rho_w g V_w$ where $\rho_w = 1000\,\text{kg/m}^3$ is the density of water and V_w is the volume of water displaced by the dinosaur. If we use f for the fraction of the dinosaur's total volume V which is submerged, then $V_w = fV$. We can further relate V to the dinosaur's mass using the assumption that the density of the dinosaur is 90% that of water: $V = m/(0.9\rho_w)$. Therefore, the apparent weight of the dinosaur is

$$\text{weight}_{\text{app}} = \text{weight} - \rho_w g \left(f \frac{m}{0.9\rho_w}\right) = \text{weight} - gf\frac{m}{0.9} .$$

If $f = 0.50$, this yields 81 kN for the apparent weight.

(c) If $f = 0.80$, our formula yields 20 kN for the apparent weight.

(d) If $f = 0.90$, we find the apparent weight is zero (it floats).

(e) Eq. 15-8 indicates that the water pressure at that depth is greater than standard air pressure (the assumed pressure at the surface) by $\rho_w gh = (1000)(9.8)(8) = 7.8 \times 10^4$ Pa. If we assume the pressure of air in the dinosaur's lungs is approximately standard air pressure, then this value represents the pressure difference which the lung muscles would have to work against.

(f) Assuming the maximum pressure difference the muscles can work with is 8 kPa, then our previous result (78 kPa) spells doom to the wading Diplodocus hypothesis.

60. The volume rate of flow is $R = vA$ where $A = \pi r^2$ and $r = d/2$. Solving for speed, we obtain

$$v = \frac{R}{A} = \frac{R}{\pi(d/2)^2} = \frac{4R}{\pi d^2} .$$

(a) With $R = 7.0 \times 10^{-3}\,\text{m}^3/\text{s}$ and $d = 14 \times 10^{-3}$ m, our formula yields $v = 45$ m/s, which is about 13% of the speed of sound (which we establish by setting up a ratio: v/v_s where $v_s = 343$ m/s).

(b) With the contracted trachea ($d = 5.2 \times 10^{-3}$ m) we obtain $v = 330$ m/s, or 96% of the speed of sound.

61. To be as general as possible, we denote the ratio of body density to water density as f (so that $f = \rho/\rho_w = 0.95$ in this problem). Floating involves an equilibrium of vertical forces acting on the body (Earth's gravity pulls down and the buoyant force pushes up). Thus,

$$F_b = F_g \implies \rho_w g V_w = \rho g V$$

where V is the total volume of the body and V_w is the portion of it which is submerged.

(a) We rearrange the above equation to yield

$$\frac{V_w}{V} = \frac{\rho}{\rho_w} = f$$

which means that 95% of the body is submerged and therefore 5% is above the water surface.

(b) We replace ρ_w with $1.6\rho_w$ in the above equilibrium of forces relationship, and find

$$\frac{V_w}{V} = \frac{\rho}{1.6\rho_w} = \frac{f}{1.6}$$

which means that 59% of the body is submerged and thus 41% is above the quicksand surface.

(c) The answer to part (b) suggests that a person in that situation is able to breathe.

(d) The thixotropic property is warning that slow motions are best. Reasonable steps are: lay back on the surface, slowly pull your legs free, and then roll to the shore.

62. (a) The volume rate of flow is related to speed by $R = vA$. Thus,

$$v_1 = \frac{R_1}{\pi r_{\text{stream}}^2} = \frac{7.9\,\text{cm}^3/\text{s}}{\pi(0.13\,\text{cm})^2} = 148.8\,\text{cm/s} = 1.5 \text{ m/s} .$$

(b) The depth d of spreading water becomes smaller as r (the distance from the impact point) increases due to the equation of continuity (and the assumption that the water speed remains equal to v_1 in this region). The water that has reached radius r (with perimeter $2\pi r$) is crossing an area of $2\pi rd$. Thus, the equation of continuity gives

$$R_1 = v_1 2\pi rd \implies d = \frac{R}{2\pi r v_1} .$$

(c) As noted above, d is a decreasing function of r.

(d) At $r = r_J$ we apply the formula from part (b):

$$d_J = \frac{R_1}{2\pi r_J v_1} = \frac{7.9\,\text{cm}^3/\text{s}}{2\pi(2.0\,\text{cm})(148.8\,\text{cm/s})} = 0.0042 \text{ cm} .$$

(e) We are told "the depth just after the jump is 2.0 mm" which means $d_2 = 0.20$ cm, and we are asked to find v_2. We use the equation of continuity:

$$R_1 = R_2 \implies 2\pi r_J\, v_1 d_J = 2\pi r'_J\, v_2 d_2$$

where r'_J is some very small amount greater than r_J (and for calculation purposes is taken to be the same numerical value, 2.0 cm). This yields

$$v_2 = v_1\left(\frac{d_1}{d_2}\right) = (148.8\,\text{cm/s})\left(\frac{0.0042\,\text{cm}}{0.20\,\text{cm}}\right) = 3.1 \text{ cm/s} .$$

(f) The kinetic energy per unit volume at $r = r_J$ with $v = v_1$ is

$$\frac{1}{2}\rho_w v_1^2 = \frac{1}{2}\left(1000\,\text{kg/m}^3\right)(1.488\,\text{m/s})^2 = 1.1\times10^3 \text{ J/m}^3 .$$

(g) The kinetic energy per unit volume at $r = r'_J$ with $v = v_2$ is

$$\frac{1}{2}\rho_w v_2^2 = \frac{1}{2}\left(1000\,\text{kg/m}^3\right)(0.031\,\text{m/s})^2 = 0.49 \text{ J/m}^3 .$$

(h) The hydrostatic pressure change is due to the change in depth:

$$\Delta p = \rho_w g\,(d_2 - d_1) = \left(1000\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)(0.0020\,\text{m} - 0.000042\,\text{m}) = 19 \text{ Pa} .$$

(i) Certainly, $\frac{1}{2}\rho_w v_1^2 + \rho_w g d_1 + p_1$ is greater than $\frac{1}{2}\rho_w v_2^2 + \rho_w g d_2 + p_2$ which is not unusual with "shock-like" fluids structures such as this hydraulic jump. Not only does Bernoulli's equation not apply but the very concept of a streamline becomes difficult to define in this circumstance.

63. (a) We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the chocolate cylinder. On a per unit mass basis (utilizing the equation for density $\rho = m/V$) we have

$$\frac{W}{m} = p\left(\frac{V}{m}\right) = \frac{p}{\rho} \ .$$

(b) If $p = 5.5 \times 10^6$ Pa and $\rho = 1200\,\text{kg/m}^3$, we obtain $W/m = p/\rho = 4.6 \times 10^3$ J/kg.

64. (a) When the model is suspended (in air) the reading is F_g (its true weight, neglecting any buoyant effects caused by the air). When the model is submerged in water, the reading is lessened because of the buoyant force: $F_g - F_b$. We denote the difference in readings as Δm. Thus,

$$(F_g) - (F_g - F_b) = \Delta m g$$

which leads to $F_b = \Delta m g$. Since $F_b = \rho_w g V_m$ (the weight of water displaced by the model) we obtain

$$V_m = \frac{\Delta m}{\rho_w} = \frac{0.63776\,\text{kg}}{1000\,\text{kg/m}^3} = 6.3776 \times 10^{-4}\ \text{m}^3 \ .$$

(b) The $\frac{1}{20}$ scaling factor is discussed in the problem (and for purposes of significant figures is treated as exact). The actual volume of the dinosaur is

$$V_{\text{dino}} = 20^3\, V_m = 5.1021\ \text{m}^3 \ .$$

(c) Using $\rho \approx \rho_w = 1000\,\text{kg/m}^3$, we find

$$\rho = \frac{m_{\text{dino}}}{V_{\text{dino}}} \implies m_{\text{dino}} = \left(1000\,\text{kg/m}^3\right)\left(5.1021\,\text{m}^3\right)$$

which yields 5.1×10^3 kg for the *T. Rex* mass.

(d) We estimate the mass range for college students as $50 \le m \le 115$ kg. Dividing these values into the previous result leads to ratios r in the range of roughly $100 \ge r \ge 45$.

65. We apply Bernoulli's equation to the central streamline:

$$p_1 + \frac{1}{2}\rho_{\text{air}} v_1^2 = p_{\text{o}} + \frac{1}{2}\rho_{\text{air}} v_{\text{o}}^2 \implies p_1 - p_{\text{o}} = \frac{1}{2}\rho_{\text{air}}\left(v_{\text{o}}^2 - v_1^2\right)$$

where $v_{\text{o}} = 65\,\text{m/s}$, $v_1 = 2\,\text{m/s}$ and the density of air is $\rho_{\text{air}} = 1.2\,\text{kg/m}^3$ (see Table 15-1). Thus, we obtain $p_1 - p_{\text{o}} \approx 2500$ Pa.

66. The pressure (relative to standard air pressure) is given by Eq. 15-8:

$$\rho g h = \left(1024\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(6.0 \times 10^3\,\text{m}\right) = 6.02 \times 10^7\ \text{Pa} \ .$$

67. Recalling that $1\,\text{atm} = 1.01 \times 10^5\,\text{atm}$, Eq. 15-8 leads to

$$\rho g h = \left(1024\,\text{kg/m}^3\right)\left(9.8\,\text{m/s}^2\right)\left(10.9 \times 10^3\,\text{m}\right)\left(\frac{1\,\text{atm}}{1.01 \times 10^5\,\text{atm}}\right) \approx 1080\ \text{atm} \ .$$

68. (a) We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

which leads to

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2} \quad \approx \quad \sqrt{2g\,(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{\text{air}}$ and $v_D/v_C \approx 0$.

(b) We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C \ .$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{\text{air}}$, Bernoulli's equation becomes

$$p_B = p_C + \rho g(h_C - h_B) = p_{\text{air}} - \rho g(h_1 + h_2 + d) \ .$$

(c) Since $p_B \geq 0$, we must let $p_{\text{air}} - \rho g(h_1 + d + h_2) \geq 0$, which yields

$$h_1 \leq h_{1,\max} = \frac{p_{\text{air}}}{\rho} - d - h_2 \leq \frac{p_{\text{air}}}{\rho} \ = \ 10.3 \text{ m} \ .$$

69. An object of mass $m = \rho V$ floating in a liquid of density ρ_{liquid} is able to float if the downward pull of gravity mg is equal to the upward buoyant force $F_b = \rho_{\text{liquid}} g V_{\text{sub}}$ where V_{sub} is the portion of the object which is submerged. This readily leads to the relation:

$$\frac{\rho}{\rho_{\text{liquid}}} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged of a floating object. When the liquid is water, as described in this problem, this relation leads to

$$\frac{\rho}{\rho_w} = 1$$

since the object "floats fully submerged" in water (thus, the object has the same density as water). We assume the block maintains an "upright" orientation in each case (which is not necessarily realistic).

(a) For liquid A,

$$\frac{\rho}{\rho_A} = \frac{1}{2}$$

so that, in view of the fact that $\rho = \rho_w$, we obtain $\rho_A/\rho_w = 2$.

(b) For liquid B, noting that two-thirds *above* means one-third *below*,

$$\frac{\rho}{\rho_B} = \frac{1}{3}$$

so that $\rho_B/\rho_w = 2$.

(c) For liquid C, noting that one-fourth *above* means three-fourths *below*,

$$\frac{\rho}{\rho_C} = \frac{3}{4}$$

so that $\rho_C/\rho_w = 4/3$.

70. In this case, Bernoulli's equation, reduces to Eq. 15-10. Thus,

$$p_g = \rho g(-h) = -(1800)(9.8)(1.5) = -2.6 \times 10^4 \text{ Pa} .$$

71. The downward force on the balloon is mg and the upward force is $F_b = \rho_{\text{out}} V g$. Newton's second law (with $m = \rho_{\text{in}} V$) leads to

$$\rho_{\text{out}} V g - \rho_{\text{in}} V g = \rho_{\text{in}} V a \implies \left(\frac{\rho_{\text{out}}}{\rho_{\text{in}}} - 1\right) g = a .$$

The problem specifies $\rho_{\text{out}}/\rho_{\text{in}} = 1.39$ (the outside air is cooler and thus more dense than the hot air inside the balloon). Thus, the upward acceleration is $(1.39 - 1)(9.8) = 3.8\,\text{m/s}^2$.

72. We rewrite the formula for work W (when the force is constant in a direction parallel to the displacement d) in terms of pressure:

$$W = Fd = \left(\frac{F}{A}\right)(Ad) = pV$$

where V is the volume of the water being forced through, and p is to be interpreted as the pressure difference between the two ends of the pipe. Thus,

$$W = \left(1.01 \times 10^5\,\text{Pa}\right)\left(1.4\,\text{m}^3\right) = 1.5 \times 10^5 \text{ J} .$$

73. (a) Using Eq. 15-10, we have $p_g = \rho g h = 1.21 \times 10^7$ Pa.

(b) By definition, $p = p_g + p_{\text{atm}} = 1.22 \times 10^7$ Pa.

(c) We interpret the question as asking for the total force *compressing* the sphere's surface, and we multiply the pressure by total area:

$$p\left(4\pi r^2\right) = 3.82 \times 10^5 \text{ N} .$$

(d) The (upward) buoyant force exerted on the sphere by the seawater is

$$F_b = \rho_w g V \qquad\qquad \text{where } V = \frac{4}{3}\pi r^3 .$$

Therefore, $F_b = 5.26$ N.

(e) Newton's second law applied to the sphere (of mass $m = 7.0$ kg) yields

$$F_b - mg = ma$$

which results in $a = -9.04$, which means the acceleration vector has a magnitude of 9.04 m/s^2 and is directed downward.

74. Neglecting the buoyant force caused by air, then the 30 N value is interpreted as the true weight W of the object. The buoyant force of the water on the object is therefore $30 - 20 = 10$ N, which means

$$F_b = \rho_w V g \implies V = \frac{10\text{N}}{(1000\,\text{kg/m}^3)\,(9.8\,\text{m/s}^2)} = 1.02 \times 10^{-3} \text{ m}^3$$

is the volume of the object. When the object is in the second liquid, the buoyant force is $30 - 24 = 6$ N, which implies

$$\rho_2 = \frac{6\,\text{N}}{(9.8\,\text{m/s}^2)\,(1.02 \times 10^{-3}\,\text{m}^3)} = 600 \text{ kg/m}^3 .$$

75. The beaker is indicated by the subscript b. The volume of the glass of which the beaker walls and base are made is $V_b = m_b/\rho_b$. We consider the case where the beaker is slightly more than half full (which, for calculation purposes, will be simply set equal to half-volume) and thus remains on the bottom of the sink – as the water around it reaches its rim. At this point, the force of buoyancy exerted on it is given by $F = (V_b + V)\rho_w g$, where V is the interior volume of the beaker. Thus $F = (V_b + V)\rho_w g = \rho_w g(V/2) + m_b$, which we solve for ρ_b:

$$\rho_b = \frac{2m_b\rho_w}{2m_b - \rho_w V} = \frac{2(390\,\mathrm{g})(1.00\,\mathrm{g/cm^3})}{2(390\,\mathrm{g}) - \left(1.00\,\mathrm{g/cm^3}\right)(500\,\mathrm{cm^3})} = 2.79\ \mathrm{g/cm^3}\ .$$

76. If the mercury level in one arm of the tube is lowered by an amount x, it will rise by x in the other arm. Thus, the net difference in mercury level between the two arms is $2x$, causing a pressure difference of $\Delta p = 2\rho_{\mathrm{Hg}}gx$, which should be compensated for by the water pressure $p_w = \rho_w gh$, where $h = 11.2\,\mathrm{cm}$. In these units, $\rho_w = 1\,\mathrm{g/cm^3}$ and $\rho_{\mathrm{Hg}} = 13.6\,\mathrm{g/cm^3}$ (see Table 15-1). We obtain

$$x = \frac{\rho_w gh}{2\rho_{\mathrm{Hg}}g} = \frac{\left(1.00\,\mathrm{g/cm^3}\right)(11.2\,\mathrm{cm})}{2\,(13.6\,\mathrm{g/cm^3})} = 0.412\ \mathrm{cm}\ .$$

77. (a) Since the pressure (due to the water) increases linearly with depth, we use its average (multiplied by the dam area) to compute the force exerts on the face of the dam, its average being simply half the pressure value near the bottom (at depth $d = 48$ m). The maximum static friction will be μN where the normal force N (exerted upward by the portion of the bedrock directly underneath the concrete) is equal to the weight mg of the dam. Since $m = \rho_c V$ with ρ_c being the density of the concrete and V being the volume (thickness times width times height: ℓwh), we write $N = \rho_c \ell whg$. Thus, the safety factor is

$$\frac{\mu\rho_c\ell whg}{\frac{1}{2}\rho_w g dA_{\mathrm{face}}} = \frac{2\mu\rho_c\ell wh}{\rho_w d(wd)} = \frac{2\mu\rho_c\ell h}{\rho_w d^2}$$

which (since $\rho_w = 1\,\mathrm{g/cm^3}$) yields $2(.47)(3.2)(24)(71)/48^2 = 2.2$.

(b) To compute the torque due to the water pressure, we will need to integrate Eq. 15-7 (multiplied by $(d-y)$ and the dam width w) as shown below. The countertorque due to the weight of the concrete is the weight multiplied by half the thickness ℓ, since we take the center of mass of the dam is at its geometric center and the axis of rotation at A. Thus, the safety factor relative to rotation is

$$\frac{mg\frac{\ell}{2}}{\int_{\mathrm{o}}^{d}\rho_w gy(d-y)w\,dy} = \frac{\rho_c\ell whg\frac{\ell}{2}}{\frac{1}{6}\rho_w gwd^3} = \frac{3\rho_c\ell^2 h}{\rho_w d^3}$$

which yields $3(3.2)(24)^2(71)/(48)^3 = 3.55$.

78. We use $p = p_{\mathrm{air}} = \rho gh$ to obtain

$$h = \frac{p_{\mathrm{air}}}{\rho g} = \frac{1.01\times 10^5\ \mathrm{Pa}}{\left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)} = 10.3\ \mathrm{m}\ .$$

79. We consider the can with nearly its total volume submerged, and just the rim above water. For calculation purposes, we take its submerged volume to be $V = 1200\,\mathrm{cm^3}$. To float, the total downward force of gravity (acting on the tin mass m_t and the lead mass m_ℓ) must be equal to the buoyant force upward:

$$(m_t + m_\ell)\,g = \rho_w Vg \implies m_\ell = \left(1\,\mathrm{g/cm^3}\right)\left(1200\,\mathrm{cm^3}\right) - 130\,\mathrm{g}$$

which yields 1070 g for the (maximum) mass of the lead (for which the can still floats). The given density of lead is not used in the solution.

80. The force f that is required to tether the airship of volume V and weight W is given by

$$\begin{aligned} f &= F_b - W = \rho_{\rm air} gV - \rho_{\rm gas} gV \\ &= \left(1.21\,{\rm kg/m^3} - 0.80\,{\rm kg/m^3}\right)\left(9.8\,{\rm m/s^2}\right)\left(1.0\times 10^6\,{\rm m^3}\right) \\ &= 4.0\times 10^6\ {\rm N}\ . \end{aligned}$$

81. The weight of the air inside the balloon of volume V is $W = \rho_{\rm gas} V g$, and the buoyant force exerted on it is given by $F_b = \rho_{\rm air} V g$. Thus, we have $F_b = W + mg$, where m is the mass of the payload. we have $\rho_{\rm air} V g = \rho_{\rm gas} V g + mg$, which gives

$$V = \frac{m}{\rho_{\rm air} - \rho_{\rm gas}} = \frac{40\,{\rm kg} + 15\,{\rm kg}}{0.035\,{\rm kg/m^3} - 0.0051\,{\rm kg/m^3}} = 1.8\times 10^3\ {\rm m^3}\ .$$

82. (a) We consider a thin slab of water with bottom area A and infinitesimal thickness dh. We apply Newton's second law to the slab:

$$\begin{aligned} dF_{\rm net} &= (p+dp)A - pA \\ &= dp\cdot A - dm\cdot g \\ &= Adp - \rho g A dh \\ &= dm\cdot a = \rho a A dh \end{aligned}$$

which gives

$$\frac{dp}{dh} = \rho(g+a)\ .$$

Integrating over the range $(0,h)$, we get

$$p = \int_0^h \rho(g+a)dh = \rho h(g+a)\ .$$

(b) We reverse the direction of the acceleration, from that in part (a). This amounts to changing a to $-a$. Thus,

$$p = \rho(g-a)\ .$$

(c) In a free fall, we use the above equation with $a = g$, which gives $p = 0$. The internal pressure p in the water totally disappears, because there is no force of interaction among different portions of the water in the bucket to make their acceleration different from g.

83. The absolute pressure is

$$\begin{aligned} p &= p_0 + \rho g h \\ &= 1.01\times 10^5\,{\rm N/m^2} + (1.03\times 10^3\,{\rm kg/m^3})(9.8\,{\rm m/s^2})(150\,{\rm m}) = 1.62\times 10^6\ {\rm Pa}\ . \end{aligned}$$

84. The area facing down (and up) is $A = (0.050\,{\rm m})(0.040\,{\rm m}) = 0.0020\,{\rm m^2}$. The submerged volume is $V = Ad$ where $d = 0.015\,{\rm m}$. In order to float, the downward pull of gravity mg must equal the upward buoyant force exerted by the seawater of density ρ:

$$mg = \rho V g \quad\Longrightarrow\quad m = \rho V = (1025)(0.0020)(0.015) = 0.031\,{\rm kg}\ .$$

85. Using Eq. 15-8, the maximum depth is

$$h_{\rm max} = \frac{\Delta p}{\rho g} = \frac{(0.050)\left(1.01\times 10^5\,{\rm Pa}\right)}{(1000\,{\rm kg/m^3})\,(9.8\,{\rm m/s^2})} = 0.52\ {\rm m}\ .$$

86. Note that "surface area" refers to the *total* surface area of all six faces, so that the area of each (square) face is $24/6 = 4$ m^2. From Archimedes' principle and the requirement that the cube (of total volume V and density ρ) floats, we find

$$\rho V g = \rho_w V_{\text{sub}} g \implies \frac{\rho}{\rho_w} = \frac{V_{\text{sub}}}{V}$$

for the fraction of volume submerged. The assumption that the cube floats upright, as described in this problem, simplifies this relation to

$$\frac{\rho}{\rho_w} = \frac{h_{\text{sub}}}{h}$$

where h is the length of one side, and $\rho_w = 4\rho$ is given. With $h = \sqrt{4} = 2$ m, we find $h_{\text{sub}} = h/4 = 0.50$ m.

87. We equate the buoyant force F_b to the combined weight of the cork and sinker:

$$\rho_w V_w g = \rho_c V_c g + \rho_s V_s g$$

With $V_w = \frac{1}{2} V_c$ and $\rho_w = 1.00$ g/cm^3, we obtain

$$V_c = \frac{2\rho_s V_s}{\rho_w - 2\rho_c} = \frac{2(11.4)(0.4)}{1 - 2(0.2)} = 15.2 \text{ cm}^3 .$$

Using the formula for the volume of a sphere (Appendix E), we have

$$r = \left(\frac{3V_c}{4\pi}\right)^{1/3} = 1.54 \text{ cm} .$$

88. The equation of continuity is

$$A_i v_i = A_f v_f$$

where $A = \pi r^2$. Therefore,

$$v_f = v_i \left(\frac{r_i}{r_f}\right)^2 = (0.09)\left(\frac{0.2}{0.6}\right)^2 .$$

Consequently, $v_f = 1.00 \times 10^{-2}$ m/s.

89. (a) This is similar to the situation treated in Sample Problem 15-8, and we refer to some of its steps (and notation). Combining Eq. 15-35 and Eq. 15-36 in a manner very similar to that shown in the textbook, we find

$$R = A_1 A_2 \sqrt{\frac{2\Delta p}{\rho \left(A_1^2 - A_2^2\right)}} .$$

for the flow rate expressed in terms of the pressure difference and the cross-sectional areas. Note that this reduces to Eq. 15-38 for the case $A_2 = A_1/2$ treated in the Sample Problem. Note that $\Delta p = p_1 - p_2 = -7.2 \times 10^3$ Pa and $A_1^2 - A_2^2 = -8.66 \times 10^{-3}$ m^4, so that the square root is well defined. Therefore, we obtain $R = 0.0776$ m^3/s.

(b) The mass rate of flow is $\rho R = 68.9$ kg/s.

90. (a) The equation of continuity is $A_1 v_1 = A_2 v_2$ where $A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2 = \pi \left(r_1/2\right)^2$. Consequently, we find $v_2 = 4v_1$.

(b) $\Delta\left(\frac{1}{2}\rho v^2\right)$ is equal to

$$\frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left(16v_1^2 - v_1^2\right) = \frac{15}{2}\rho v_1^2 .$$

Chapter 16

1. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$ must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

   (b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

   (c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

2. (a) The problem describes the time taken to execute one cycle of the motion. The period is $T = 0.75$ s.

   (b) Frequency is simply the reciprocal of the period: $f = 1/T \approx 1.3$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second.

   (c) Since 2π radians are equivalent to a cycle, the angular frequency ω (in radians-per-second) is related to frequency f by $\omega = 2\pi f$ so that $\omega \approx 8.4$ rad/s.

3. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

   (b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500\,\text{s}) = 2.00\,\text{Hz}$.

   (c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00\,\text{Hz}) = 12.57\,\text{rad/s}$.

   (d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k: $k = m\omega^2 = (0.500\,\text{kg})(12.57\,\text{rad/s})^2 = 79.0\,\text{N/m}$.

   (e) Let x_m be the amplitude. The maximum speed is $v_m = \omega x_m = (12.57\,\text{rad/s})(0.350\,\text{m}) = 4.40\,\text{m/s}$.

   (f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0\,\text{N/m})(0.350\,\text{m}) = 27.6\,\text{N}$.

4. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = (2\pi(6.60\,\text{Hz}))^2\,(0.0220\,\text{m}) = 37.8\ \text{m/s}^2\ .$$

5. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude. The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2}{1.0\times10^{-6}\,\text{m}}} = 500\ \text{Hz}\ .$$

   For frequencies greater than 500 Hz, the acceleration exceeds g for some part of the motion.

6. (a) Hooke's law readily yields $k = (15\,\text{kg})(9.8\,\text{m/s}^2)/(0.12\,\text{m}) = 1225\,\text{N/m}$. Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

   (b) We are told $f = 2.00\,\text{Hz} = 2.00\,\text{cycles/sec}$. Since a cycle is equivalent to 2π radians, we have $\omega = 2\pi(2.00) = 4\pi$ rad/s (understood to be valid to three significant figures). Using Eq. 16-12, we find

$$\omega = \sqrt{\frac{k}{m}} \implies m = \frac{1225\,\text{N/m}}{(4\pi\,\text{rad/s})^2} = 7.76 \text{ kg} .$$

   Consequently, the weight of the package is $mg = 76$ N.

7. (a) The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus $\omega = 2\pi/(1.00 \times 10^{-5}\,\text{s}) = 6.28 \times 10^5\,\text{rad/s}$.

   (b) The maximum speed v_m and maximum displacement x_m are related by $v_m = \omega x_m$, so

$$x_m = \frac{v_m}{\omega} = \frac{1.00 \times 10^3\,\text{m/s}}{6.28 \times 10^5\,\text{rad/s}} = 1.59 \times 10^{-3}\,\text{m} .$$

8. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{\text{max}} = ma_m$. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{\text{max}} = m\omega^2 x_m = (0.12\,\text{kg})(10\pi\,\text{rad/s})^2(0.085\,\text{m}) = 10 \text{ N} .$$

   (b) Using Eq. 16-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \implies k = (0.12\,\text{kg})(10\pi\,\text{rad/s})^2 = 1.2 \times 10^2 \text{ N/m} .$$

9. (a) The amplitude is half the range of the displacement, or $x_m = 1.0\,\text{mm}$.

   (b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi(120\,\text{Hz})\left(1.0 \times 10^{-3}\,\text{m}\right) = 0.75 \text{ m/s} .$$

   (c) The maximum acceleration is

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = (2\pi(120\,\text{Hz}))^2\left(1.0 \times 10^{-3}\,\text{m}\right) = 570 \text{ m/s}^2 .$$

10. (a) The problem gives the frequency $f = 440$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second. The angular frequency ω is similar to frequency except that ω is in radians-per-second. Recalling that 2π radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2800$ rad/s.

    (b) In the discussion immediately after Eq. 16-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With $x_m = 0.00075$ m and the above value for ω, this expression yields $v_m = 2.1$ m/s.

    (c) In the discussion immediately after Eq. 16-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value $\omega = 2765$ rad/s is used) yields $a_m = 5.7$ km/s.

11. (a) Since the problem gives the frequency $f = 3.00$ Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass m_{car} so that Eq. 16-12 leads to

$$\omega = \sqrt{\frac{k}{\frac{1}{4}m_{\text{car}}}} \implies k = \left(\frac{1}{4}(1450\,\text{kg})\right)(6\pi\,\text{rad/s})^2 = 1.29 \times 10^5 \text{ N/m} .$$

(b) If the new mass being supported by the four springs is $m_{\text{total}} = 1450 + 5(73) = 1815$ kg, then Eq. 16-12 leads to

$$\omega_{\text{new}} = \sqrt{\frac{k}{\frac{1}{4}\,m_{\text{total}}}} \implies f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{1.29 \times 10^5}{1815/4}} = 2.68 \text{ Hz} .$$

12. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0 \cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = 3.0 \text{ m} .$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left(3\pi(2.0) + \frac{\pi}{3}\right) = -49 \text{ m/s} .$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = -2.7 \times 10^2 \text{ m/s}^2 .$$

(d) In the second paragraph after Eq. 16-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \frac{\pi}{3} \approx 20$ rad.

(e) Comparing with Eq. 16-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

13. We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is $180/(60\,\text{s}) = 3.0\,\text{Hz}$ and the amplitude is half the stroke, or $x_m = 0.38\,\text{m}$. Thus, $v_m = 2\pi(3.0\,\text{Hz})(0.38\,\text{m}) = 7.2\,\text{m/s}$.

14. (a) For a total mass of $m + M$, Eq. 16-12 becomes

$$\omega = \sqrt{\frac{k}{m+M}} \implies M = \frac{k}{\omega^2} - m .$$

Eq. 16-5 ($\omega = 2\pi/T$) is used to put this into its final form:

$$M = \frac{k}{(2\pi/T)^2} - m = \left(\frac{k}{4\pi^2}\right)T^2 - m .$$

(b) With $T = 0.90149$ s, $k = 605.6$ N/m and $M = 0$ in the above expression, we obtain $m = 12.47$ kg.

(c) With the same k and m, we plug $T = 2.08832$ s into the expression and obtain $M = 54.43$ kg.

15. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 16-5. Thus, $x_m = \frac{1}{2}d$ and $\omega = 0.503$ rad/h. The phase constant ϕ in Eq. 16-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x = \frac{1}{2}x_m = \frac{1}{4}d$.

$$\begin{aligned} x &= x_m \cos(\omega t + \phi) \\ \frac{1}{4}d &= \left(\frac{1}{2}d\right)\cos(0.503t + 0) \\ \frac{1}{2} &= \cos(0.503t) \end{aligned}$$

which has $t = 2.08$ h as the smallest positive root. The calculator is in radians mode during this calculation.

16. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{\max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = \sqrt{k/(m+M)}$ from Eq. 16-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \implies \frac{k}{m+M} x_m = \mu_s\, g$$

which leads to $x_m = 0.22$ m.

17. The maximum force that can be exerted by the surface must be less than $\mu_s N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by $F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m$, where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $N = mg$ into $F < \mu_s N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\,\mathrm{m/s^2})}{(2\pi \times 2.0\,\mathrm{Hz})^2} = 0.031\,\mathrm{m}\ .$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

18. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8\,\mathrm{m/s^2}$.

(a) Using Eq. 16-5 and $T = 1.0$ s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m = g \implies x_m = \frac{gT^2}{4\pi^2} = 0.25 \text{ m}\ .$$

(b) Since $\omega = 2\pi f$, and $x_m = 0.050$ m is given, we find

$$\left(2\pi f)^2\right) x_m = g \implies f = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = 2.2 \text{ Hz}\ .$$

19. (a) Eq. 16-8 leads to

$$a = -\omega^2 x \implies \omega = \sqrt{\frac{-a}{x}} = \sqrt{\frac{123}{0.100}}$$

which yields $\omega = 35.07$ rad/s. Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Eq. 16-12 provides a relation between ω (found in the previous part) and the mass:

$$\omega = \sqrt{\frac{k}{m}} \implies m = \frac{400}{35.07^2} = 0.325 \text{ kg}\ .$$

(c) By energy conservation, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t described in the problem.

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \implies x_m = \frac{m}{k}v^2 + x^2\ .$$

Consequently, $x_m = \sqrt{(0.325/400)(13.6)^2 + 0.1^2} = 0.400$ m.

20. Eq. 16-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{2.00}} = 7.07 \text{ rad/s} .$$

Energy methods (discussed in §16-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 16-3 and Eq. 16-6.

(a) Dividing Eq. 16-6 by Eq. 16-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase $(\omega t + \phi)$ is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415}{(7.07)(0.129)}\right) .$$

With the calculator in radians mode, this gives the phase equal to -1.31 rad. Plugging this back into Eq. 16-3 leads to

$$0.129\,\text{m} = x_m \cos(-1.31) \implies 0.500\,\text{m} = x_m .$$

(b) Since $\omega t + \phi = -1.31$ rad at $t = 1.00$ s. We can use the above value of ω to solve for the phase constant ϕ. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of ϕ, we find $x_\text{o} = x_m \cos\phi = -0.251$ m.

(c) And we obtain $v_\text{o} = -x_m \omega \sin\phi = 3.06$ m/s.

21. (a) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If ℓ is the elongation of the spring at equilibrium, then $k\ell = mg$, where k is the spring constant and m is the mass of the object. Thus $k/m = g/\ell$ and $f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}$. Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,\text{cm} = 0.050\,\text{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}} = 2.23\,\text{Hz} .$$

(b) Use conservation of energy. We take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. We take the y axis to be positive in the downward direction and let $y = 0.080\,\text{m}$. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes $0 = \frac{1}{2}ky^2 - mgy + \frac{1}{2}mv^2$. We solve for the speed.

$$\begin{aligned} v &= \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} \\ &= \sqrt{2(9.8\,\text{m/s}^2)(0.080\,\text{m}) - \left(\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}\right)(0.080\,\text{m})^2} = 0.56 \text{ m/s} \end{aligned}$$

(c) Let m be the original mass and Δm be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m + \Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. We solve $\sqrt{k/(m + \Delta m)} = \frac{1}{2}\sqrt{k/m}$ for m. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives $m = \Delta m/3 = (300\,\text{g})/3 = 100\,\text{g}$.

(d) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m + \Delta m)g$. Thus $y = (m + \Delta m)g/k$. We will need to find the value of the spring constant k. Use $k = m\omega^2 = m(2\pi f)^2$. Then

$$y = \frac{(m + \Delta m)g}{m(2\pi f)^2} = \frac{(0.10\,\mathrm{kg} + 0.30\,\mathrm{kg})\left(9.8\,\mathrm{m/s^2}\right)}{(0.10\,\mathrm{kg})(2\pi \times 2.24\,\mathrm{Hz})^2} = 0.20\,\mathrm{m}\ .$$

This is measured from the initial position.

22. They pass each other at time t, at $x_1 = x_2 = \frac{1}{2}x_m$ where

$$x_1 = x_m \cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m \cos(\omega t + \phi_2)\ .$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m\omega \sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m\omega \sin(\omega t + \phi_2)\ .$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the ωt term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

23. (a) Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50\,\mathrm{s}$,

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}}\right) = -0.250A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50\,\mathrm{s}}{1.5\,\mathrm{s}} + \frac{\pi}{6}\right) = -0.433A\ .$$

Their separation at that time is $x_1 - x_2 = -0.250A + 0.433A = 0.183A$.

(b) The velocities of the particles are given by

$$v_1 = \frac{\mathrm{d}x_1}{\mathrm{d}t} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{\mathrm{d}x_2}{\mathrm{d}t} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)\ .$$

We evaluate these expressions for $t = 0.50\,\mathrm{s}$ and find they are both negative-valued, indicating that the particles are moving in the same direction.

24. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\,\frac{d^2x}{dt^2} = -2kx \ .$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{2k}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \ .$$

25. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1x + k_2x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\,\frac{d^2x}{dt^2} = -k_1x - k_2x \ .$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \ .$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} \quad \text{and} \quad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} \ ,$$

respectively. Comparing these expressions, it is clear that $f = \sqrt{f_1^2 + f_2^2}$.

26. (a) The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = (2\pi(1000\,\text{Hz}))^2\,(0.00040\,\text{m}) = 1.6 \times 10^4 \ \text{m/s}^2 \ .$$

(b) Similarly, in the discussion after Eq. 16-6, we find $v_m = \omega x_m$ so that

$$v_m = (2\pi(1000\,\text{Hz}))\,(0.00040\,\text{m}) = 2.5 \ \text{m/s} \ .$$

(c) From Eq. 16-8, we have (in absolute value)

$$|a| = (2\pi(1000\,\text{Hz}))^2\,(0.00020\,\text{m}) = 7.9 \times 10^3 \ \text{m/s}^2 \ .$$

(d) This can be approached with the energy methods of §16-4, but here we will use trigonometric relations along with Eq. 16-3 and Eq. 16-6. Thus, allowing for both roots stemming from the square root,

$$\begin{aligned} \sin(\omega t + \phi) &= \pm\sqrt{1 - \cos^2(\omega t + \phi)} \\ -\frac{v}{\omega x_m} &= \pm\sqrt{1 - \frac{x^2}{x_m^2}} \ . \end{aligned}$$

Taking absolute values and simplifying, we obtain

$$|v| = 2\pi f\sqrt{x_m^2 - x^2} = 2\pi(1000)\sqrt{0.00040^2 - 0.00020^2} = 2.2 \text{ m/s} .$$

27. We wish to find the effective spring constant for the combination of springs shown in Fig. 16–31. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\,\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\,\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on the block and its magnitude is $F = k\,\Delta x_\ell$. Thus $k_{\text{eff}} = k\,\Delta x_\ell/2\Delta x_r = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} .$$

28. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \implies x = \frac{14.0\sin 40.0^\circ}{120} = 0.075 \text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore $0.450 + 0.75 = 0.525$ m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 16-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0/9.8}{120}} = 0.686 \text{ s} .$$

29. (a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by $\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is $\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n+1)$, where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n+1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n+1)/n$ into $k\,\Delta x = k_1\,\Delta x_1$ and solve for k_1. The result is $k_1 = k(n+1)/n$.

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\,\Delta x = k_2\,\Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n+1)$, then solve for k_2. The result is $k_2 = k(n+1)$.

(c) To find the frequency when spring 1 is attached to mass m, we replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n+1)/n$ to obtain

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}f$$

where the substitution $f = (1/2\pi)\sqrt{k/m}$ was made.

(d) To find the frequency when spring 2 is attached to the mass, we replace k with $k(n+1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}f$$

where the same substitution was made.

30. The magnitude of the downhill component of the gravitational force acting on each ore car is

$$w_x = (10000\,\text{kg})\left(9.8\,\text{m/s}^2\right)\sin\theta$$

where $\theta = 30°$ (and it is important to have the calculator in degrees mode during this problem). We are told that a downhill pull of $3w_x$ causes the cable to stretch $x = 0.15\,\text{m}$. Since the cable is expected to obey Hooke's law, its spring constant is

$$k = \frac{3w_x}{x} = 9.8\times10^5\ \text{N/m}\ .$$

(a) Noting that the oscillating mass is that of *two* of the cars, we apply Eq. 16-12 (divided by 2π).

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8\times10^5\,\text{N/m}}{20000\,\text{kg}}} = 1.1\ \text{Hz}\ .$$

(b) The difference between the equilibrium positions of the end of the cable when supporting two as opposed to three cars is

$$\Delta x = \frac{3w_x - 2w_x}{k} = 0.050\ \text{m}\ .$$

31. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E = \frac{1}{2}kx_m^2 = \frac{1}{2}(1.3\times10^2\,\text{N/m})(0.024\,\text{m})^2 = 3.7\times10^{-2}\ \text{J}\ .$$

32. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where $E = 1.00$ J and $x_m = 0.100$ m. Thus,

$$k = \frac{2E}{x_m^2} = 200\ \text{N/m}\ .$$

(b) The energy as the block passes through the equilibrium position (with speed $v_m = 1.20$ m/s)is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \implies m = \frac{2E}{v_m^2} = 1.39\ \text{kg}\ .$$

(c) Eq. 16-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.91\ \text{Hz}\ .$$

33. (a) Eq. 16-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000\,\text{N/m}}{5.00\,\text{kg}}} = 2.25\ \text{Hz}\ .$$

(b) With $x_o = 0.500$ m, we have $U_o = \frac{1}{2}kx_o^2 = 125$ J.

(c) With $v_o = 10.0$ m/s, the initial kinetic energy is $K_o = \frac{1}{2}mv_o^2 = 250$ J.

(d) Since the total energy $E = K_o + U_o = 375$ J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \implies x_m = \sqrt{\frac{2E}{k}} = 0.866 \text{ m} .$$

34. (a) We require

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \implies k = m\left(\frac{v_m}{x_m}\right)^2$$

where $m = 0.130$ kg, $v_m = 11200$ m/s and $x_m = 1.50$ m. This yields $k = 7.25 \times 10^6$ N/m.

(b) The force required to produce an elongation x_m if the spring constant is k is $kx_m = 1.087 \times 10^7$ N. Dividing this among N persons, each one exerting a force of 220 N, requires $N = 1.087 \times 10^7/220 \approx 49400$.

35. (a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where $y = 0.096$ m is the elongation of the spring at equilibrium, k is the spring constant, and $m = 1.3$ kg is the mass of the block. Thus $k = mg/y = (1.3)(9.8)/0.096 = 133\,\text{N/m}$.

(b) The period is given by $T = 1/f = 2\pi/\omega = 2\pi\sqrt{m/k} = 2\pi\sqrt{1.3/133} = 0.62\,\text{s}$.

(c) The frequency is $f = 1/T = 1/0.62\,\text{s} = 1.6\,\text{Hz}$.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. At the initial position, the block is not moving but it has potential energy

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3)(9.8)(0.146) + \frac{1}{2}(133)(0.146)^2 = -0.44 \text{ J} .$$

When the block is at the equilibrium point, the elongation of the spring is $y = 9.6\,\text{cm}$ and the potential energy is

$$U_f = -mgy + \frac{1}{2}ky^2 = -(1.3)(9.8)(0.096) + \frac{1}{2}(133)(0.096)^2 = -0.61 \text{ J} .$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\,\text{J} + 0.61\,\text{J})}{1.3\,\text{kg}}} = 0.51 \text{ m/s} .$$

36. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant k).

(a) Momentum conservation readily yields $v' = mv/(m + M)$.

(b) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\begin{aligned} \frac{1}{2}(m + M)\,(v')^2 &= \frac{1}{2}kx_m^2 \\ \frac{1}{2}(m + M)\frac{m^2v^2}{(m + M)^2} &= \frac{1}{2}kx_m^2 \end{aligned}$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} \ .$$

37. The total energy is given by $E = \frac{1}{2}kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

   (a) The fraction of the energy that is kinetic is

   $$\frac{K}{E} = \frac{E-U}{E} = 1 - \frac{U}{E} = 1 - \frac{1}{4} = \frac{3}{4}$$

   where the result from part (b) has been used.

   (b) When $x = \frac{1}{2}x_m$ the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{8}kx_m^2$. The ratio is

   $$\frac{U}{E} = \frac{\frac{1}{8}kx_m^2}{\frac{1}{2}kx_m^2} = \frac{1}{4} \ .$$

   (c) Since $E = \frac{1}{2}kx_m^2$ and $U = \frac{1}{2}kx^2$, $U/E = x^2/x_m^2$. We solve $x^2/x_m^2 = 1/2$ for x. We should get $x = x_m/\sqrt{2}$.

38. The textbook notes (in the discussion immediately after Eq. 16-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency and $x_m = 0.0020$ m is the amplitude. Thus, $a_m = 8000\,\text{m/s}^2$ leads to $\omega = 2000$ rad/s.

   (a) Using Newton's second law with $m = 0.010$ kg, we have

   $$F = ma = m\left(-a_m \cos\left(\omega t + \phi\right)\right) = -(80\,\text{N})\cos\left(2000t - \frac{\pi}{3}\right)$$

   where t is understood to be in seconds.

   (b) Eq. 16-5 gives $T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

   (c) The relation $v_m = \omega x_m$ can be used to solve for v_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter. By Eq. 16-12, the spring constant is $k = \omega^2 m = 40000$ N/m. Then, energy conservation leads to

   $$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \implies v_m = x_m\sqrt{\frac{k}{m}} = 4.0 \text{ m/s} \ .$$

   (d) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080$ J.

39. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet-block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m+M)v$, so

   $$v = \frac{mv_0}{m+M} = \frac{(0.050\,\text{kg})(150\,\text{m/s})}{0.050\,\text{kg} + 4.0\,\text{kg}} = 1.85\,\text{m/s} \ .$$

   When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance $\ell = (M+m)g/k$, somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M+m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k(y_m - \ell)^2 + (M+m)gy_m \ .$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$\begin{aligned} y_m &= \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)} \\ &= \sqrt{\frac{(0.050\,\mathrm{kg} + 4.0\,\mathrm{kg})(1.85\,\mathrm{m/s})^2}{500\,\mathrm{N/m}} - \frac{(0.050\,\mathrm{kg})(9.8\,\mathrm{m/s^2})^2}{(500\,\mathrm{N/m})^2}[2(4.0\,\mathrm{kg}) + 0.050\,\mathrm{kg}]} \\ &= 0.166 \text{ m} \ . \end{aligned}$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050\,\mathrm{kg})(150\,\mathrm{m/s})^2 = 563\,\mathrm{J}$. The kinetic energy of the bullet-block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050\,\mathrm{kg} + 4.0\,\mathrm{kg})(1.85\,\mathrm{m/s})^2 = 6.94 \text{ J} \ .$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is $E/E_0 = (6.94\,\mathrm{J})/(563\,\mathrm{J}) = 0.0123$ or 1.23%.

40. (a) The rotational inertia is $I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00\,\mathrm{kg})(0.700\,\mathrm{m})^2 = 0.735\,\mathrm{kg\cdot m^2}$.

(b) Using Eq. 16-22 (in absolute value), we find

$$\kappa = \frac{\tau}{\theta} = \frac{0.0600\,\mathrm{N\cdot m}}{2.5\,\mathrm{rad}} = 0.024 \text{ N}\cdot\text{m} \ .$$

(c) Using Eq. 16-5, Eq. 16-23 leads to

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.024\,\mathrm{N\cdot m}}{0.735\,\mathrm{kg\cdot m^2}}} 0.181 \text{ rad/s} \ .$$

41. (a) We take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m \sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi\,\mathrm{rad})}{0.500\,\mathrm{s}} = 39.5 \text{ rad/s} \ .$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1 - \cos^2(2\pi t/T)} = \sqrt{1 - (1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500\,\mathrm{s}}\right)(\pi\,\mathrm{rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2 \text{ rad/s} \ .$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta \ .$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500\,\text{s}}\right)^2 \left(\frac{\pi}{4}\right) = -124 \text{ rad/s}^2 \ .$$

42. (a) Eq. 16-28 gives

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{17\,\text{m}}{9.8\,\text{m/s}^2}} = 8.3 \text{ s} \ .$$

(b) Plugging $I = mL^2$ into Eq. 16-25, we see that the mass m cancels out. Thus, the characteristics (such as the period) of the periodic motion do not depend on the mass.

43. The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$, where L is its length. Thus,

$$L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\,\text{s})^2(9.8\,\text{m/s}^2)}{4\pi^2} = 0.99 \text{ m} \ .$$

44. From Eq. 16-28, we find the length of the pendulum when the period is $T = 8.85$ s:

$$L = \frac{gT^2}{4\pi^2} \ .$$

The new length is $L' = L - d$ where $d = 0.350\,\text{m}$. The new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L}{g} - \frac{d}{g}} = 2\pi\sqrt{\frac{T^2}{4\pi^2} - \frac{d}{g}}$$

which yields $T' = 8.77$ s.

45. We use Eq. 16-29 and the parallel-axis theorem $I = I_{\text{cm}} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{\text{cm}} = mL^2/12$ where $L = 1.0$ m. Thus, for $T = 2.5$ s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}} \ .$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g\,(T/2\pi)^2 \pm \sqrt{d^2\,(T/2\pi)^4 - L^2/3}}{2} \ .$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d = 0.056$ m.

46. We use Eq. 16-29 and the parallel-axis theorem $I = I_{\text{cm}} + mh^2$ where $h = d$. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{\text{cm}} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}} \ .$$

47. (a) The period of the pendulum is given by $T = 2\pi\sqrt{I/mgd}$, where I is its rotational inertia, m is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ and, according to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus

$$T = 2\pi\sqrt{\frac{m(L^2/12 + d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2 + 12d^2}{12gd}} .$$

(b) $(L^2 + 12d^2)/12gd$, considered as a function of d, has a minimum at $d = L/\sqrt{12}$, so the period increases as d decreases if $d < L/\sqrt{12}$ and decreases as d decreases if $d > L/\sqrt{12}$.

(c) L occurs only in the numerator of the expression for the period, so T increases as L increases.

(d) The period does not depend on the mass of the pendulum, so T does not change when m increases.

48. (a) We use Eq. 16-29 and the parallel-axis theorem $I = I_{\text{cm}} + mh^2$ where $h = R = 0.125$ m. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{\text{cm}} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.869 \text{ s} .$$

(b) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{3R \pm \sqrt{(3R)^2 - 8R^2}}{4} = R \quad \text{or} \quad \frac{R}{2} .$$

Thus, our result is $r = 0.125/2 = 0.0625$ m.

49. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}MR^2$, where M is its mass and R is its radius. The disk of this problem rotates about a point that is displaced from its center by $R + L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}MR^2 + M(L+R)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}MR^2 + M(L+R)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\,\text{kg})(0.100\,\text{m})^2 + (0.500\,\text{kg})(0.500\,\text{m} + 0.100\,\text{m})^2 + \frac{1}{3}(0.270\,\text{kg})(0.500\,\text{m})^2 \\ &= 0.205 \text{ kg}\cdot\text{m}^2 . \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + R = 0.500\,\text{m} + 0.100\,\text{m} = 0.600 \text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\,\text{m})/2 = 0.250\,\text{m}$ away, on the same line. The distance from the pivot point to the center of mass of the disk-rod system is

$$d = \frac{M\ell_d + m\ell_r}{M + m} = \frac{(0.500\,\text{kg})(0.600\,\text{m}) + (0.270\,\text{kg})(0.250\,\text{m})}{0.500\,\text{kg} + 0.270\,\text{kg}} = 0.477 \text{ m} .$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\,\mathrm{kg\cdot m^2}}{(0.500\,\mathrm{kg}+0.270\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.447\,\mathrm{m})}} = 1.50\ \mathrm{s}\ .$$

50. (a) Referring to Sample Problem 16-5, we see that the distance between P and C is $h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. The parallel axis theorem (see Eq. 16-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = \left(\frac{1}{12} + \frac{1}{36}\right) mL^2 = \frac{1}{9}mL^2\ .$$

And Eq. 16-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L^2/9}{gL/6}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which yields $T = 1.64$ s for $L = 1.00$ m.

(b) Comparing with Eq. 16-32, we note that this T is identical to that computed in Sample Problem 16-5. As far as the characteristics of the periodic motion are concerned, the center of oscillation provides a pivot which is equivalent to that chosen in the Sample Problem (pivot at the edge of the stick).

51. We require

$$T = 2\pi\sqrt{\frac{L_o}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

similar to the approach taken in part (b) of Sample Problem 16-5, but treating in our case a more general possibility for I. Canceling 2π, squaring both sides, and canceling g leads directly to the result; $L_o = I/mh$.

52. (a) This is similar to the situation treated in Sample Problem 16-5, except that O is no longer at the end of the stick. Referring to the center of mass as C (assumed to be the geometric center of the stick), we see that the distance between O and C is $h = x$. The parallel axis theorem (see Eq. 16-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right)\ .$$

And Eq. 16-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{(L^2 + 12x^2)}{12gx}}\ .$$

(b) Minimizing T by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of T (it should be clear that $12gT^2/2\pi$ is a minimum whenever T is a minimum).

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields $x = L/\sqrt{12}$ as the value of x which should produce the smallest possible value of T. Stated as a ratio, this means $x/L = 0.289$.

(c) With $L = 1.00$ m and $x = 0.289$ m, we obtain $T = 1.53$ s from the expression derived in part (a).

53. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is $T = 2\pi/\omega = 2\pi\sqrt{I/C}$, where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved $(L/2)(1-\cos\theta)$ to the right. The length of the spring is now $\sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}$. If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\,\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 11–2. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}} \,.$$

54. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2} \,.$$

Then, since frequency is the reciprocal of the period, Eq. 16-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{\text{eff}}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}} \,.$$

55. (a) The frequency for small amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives $f = (1/2\pi)\sqrt{(9.80\,\text{m/s}^2)/(2.0\,\text{m})} = 0.35\,\text{Hz}$.

(b) The forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}\,'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}\,'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}\,'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g} - \vec{a}_e$. Since $\vec{g}$ and $\vec{a}_e$ are along the same line and in opposite directions we can find the frequency for small amplitude oscillations by replacing g with $g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8\,\text{m/s}^2 + 2.0\,\text{m/s}^2}{2.0\,\text{m}}} = 0.39\text{ Hz} \,.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

56. For simple harmonic motion, Eq. 16-24 must reduce to

$$\tau = -L\,(F_g \sin\theta) \;\longrightarrow\; -L\,(F_g\theta)$$

where θ is in radians. We take the percent difference (in absolute value)

$$\left|\frac{(-LF_g \sin\theta) - (-LF_g\theta)}{-LF_g \sin\theta}\right| = \left|1 - \frac{\theta}{\sin\theta}\right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for θ (since this is not possible "in closed form"), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin\theta \approx \theta - \theta^3/6$ (valid for small θ) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left|1 - \frac{\theta}{\theta - \theta^3/6}\right| \approx 0.010 \quad \Longrightarrow \quad \frac{1}{1-\theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24\,\text{rad} = 14°$. A more accurate value (found numerically) for the θ value which results in a 1.0% deviation is 13.986°.

57. Careful consideration of how the angle θ relates to height h (measured from the lowest position) gives $h = R(1 - \cos\theta)$. The energy at the amplitude point is equal to the energy as it swings through the lowest position:

$$\begin{aligned} mgh &= \frac{1}{2}mv^2 \\ gR\,(1-\cos\theta_m) &= \frac{1}{2}v^2 \end{aligned}$$

where the mass has been canceled in the last step. The tension (acting upward on the bob when it swings through the lowest position) is related to the bob's weight mg and the centripetal acceleration using Newton's second law:

$$T - mg = m\,\frac{v^2}{R}\ .$$

From the above, we substitute for v^2:

$$T - mg = m\,\frac{2gR\,(1-\cos\theta_m)}{R} = 2mg\,(1-\cos\theta_m)\ .$$

(a) This provides an "exact" answer for the tension, but the problem directs us to examine the small angle behavior: $\cos\theta \approx 1-\theta^2/2$ (where θ is in radians). Solving for T and using this approximation, we find

$$T \approx mg + 2mg\left(\frac{\theta_m^2}{2}\right) = mg\left(1+\theta_m^2\right)\ .$$

(b) At other values of θ (other than the lowest position, where $\theta = 0$), Newton's second law yields

$$T' - mg\cos\theta = m\,\frac{v^2}{R} \quad \text{or} \quad T' - mg\left(1-\frac{\theta^2}{2}\right) \approx m\,\frac{v^2}{R}\ .$$

Making the same substitutions as before, we obtain

$$T' \approx mg\left(1+\theta_m^2-\theta^2\right)$$

which is clearly smaller than the result of part (a).

58. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

and θ is in radians. We note that $r\omega = v$ (where $v = \frac{dx}{dt}$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{m\,R^2}{r^2}\right)v^2 + \frac{1}{2}kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in §16-4 *if* we identify the mass m in that section with the term mR^2/r^2 appearing in this problem. Making this identification, Eq. 16-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}} \ .$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back towards its equilibrium position).

59. Referring to the numbers in Sample Problem 16-7, we have $m = 0.25$ kg, $b = 0.070$ kg/s and $T = 0.34$ s. Thus, when $t = 20T$, the damping factor becomes

$$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)} = 0.39 \ .$$

60. Since the energy is proportional to the amplitude squared (see Eq. 16-21), we find the fractional change (assumed small) is

$$\frac{E' - E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m dx_m}{x_m^2} = 2\frac{dx_m}{x_m} \ .$$

Thus, if we approximate the fractional change in x_m as dx_m/x_m, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if x_m decreases by 3%, then E must decrease by 6%.

61. (a) We want to solve $e^{-bt/2m} = 1/3$ for t. We take the natural logarithm of both sides to obtain $-bt/2m = \ln(1/3)$. Therefore, $t = -(2m/b)\ln(1/3) = (2m/b)\ln 3$. Thus,

$$t = \frac{2(1.50\,\text{kg})}{0.230\,\text{kg/s}}\ln 3 = 14.3 \text{ s} \ .$$

(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\,\text{N/m}}{1.50\,\text{kg}} - \frac{(0.230\,\text{kg/s})^2}{4(1.50\,\text{kg})^2}} = 2.31 \text{ rad/s} \ .$$

The period is $T = 2\pi/\omega' = (2\pi)/(2.31\,\text{rad/s}) = 2.72\,\text{s}$ and the number of oscillations is $t/T = (14.3\,\text{s})/(2.72\,\text{s}) = 5.27$.

62. (a) From Hooke's law, we have

$$k = \frac{(500\,\text{kg})\left(9.8\,\text{m/s}^2\right)}{10\,\text{cm}} = 490 \text{ N/cm} \ .$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad \text{where} \quad T = \frac{2\pi}{\omega'} \ .$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000\,\text{N/m}}{500\,\text{kg}}} \approx 9.9 \text{ rad/s} \ ,$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T} \ln 2 \approx \frac{2(500)}{0.63}(0.69) = 1.1 \times 10^3 \text{ kg/s} .$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 16-41 in its full form and solve for b. The result would be (quoting more figures than are significant)

$$b = \frac{2 \ln 2 \sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086 \text{ kg/s}$$

which is in good agreement with the value gotten "the easy way" above.

63. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

    (b) In the discussion immediately after Eq. 16-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

64. With $M = 1000$ kg and $m = 82$ kg, we adapt Eq. 16-12 to this situation by writing

$$\omega = \sqrt{\frac{k}{M + 4m}} \quad \text{where} \quad \omega = \frac{2\pi}{T} .$$

If $d = 4.0$ m is the distance traveled (at constant car speed v) between impulses, then we may write $T = v/d$, in which case the above equation may be solved for the spring constant:

$$\frac{2\pi v}{d} = \sqrt{\frac{k}{M + 4m}} \implies k = (M + 4m)\left(\frac{2\pi v}{d}\right)^2 .$$

Before the people got out, the equilibrium compression is $x_i = (M + 4m)g/k$, and afterward it is $x_f = Mg/k$. Therefore, with $v = 16000/3600 = 4.44$ m/s, we find the rise of the car body on its suspension is

$$x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M + 4m}\left(\frac{d}{2\pi v}\right)^2 = 0.050 \text{ m} .$$

65. The rotational inertia for an axis through A is $I_{\text{cm}} + mh_A^2$ and that for an axis through B is $I_{\text{cm}} + mh_B^2$. Using Eq. 16-29, we require

$$2\pi\sqrt{\frac{I_{\text{cm}} + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{I_{\text{cm}} + mh_B^2}{mgh_B}}$$

which (after canceling 2π and squaring both sides) becomes

$$\frac{I_{\text{cm}} + mh_A^2}{mgh_A} = \frac{I_{\text{cm}} + mh_B^2}{mgh_B} .$$

Cross-multiplying and rearranging, we obtain

$$I_{\text{cm}}(h_B - h_A) = m\left(h_A h_B^2 - h_B h_A^2\right) = mh_A h_B (h_B - h_A)$$

which simplifies to $I_{\text{cm}} = mh_A h_B$. We plug this back into the first period formula above and obtain

$$T = 2\pi\sqrt{\frac{mh_A h_B + mh_A^2}{mgh_A}} = 2\pi\sqrt{\frac{h_B + h_A}{g}} .$$

From the figure, we see that $h_B + h_A = L$, and (after squaring both sides) we can solve the above equation for the gravitational acceleration:

$$g = \left(\frac{2\pi}{T}\right)^2 L = \frac{4\pi^2 L}{T^2} .$$

66. (a) The net horizontal force is F since the batter is assumed to exert no horizontal force on the bat. Thus, the horizontal acceleration (which applies as long as F acts on the bat) is $a = F/m$.

(b) The only torque on the system is that due to F, which is exerted at P, at a distance $L_o - \frac{1}{2}L$ from C. Since $L_o = 2L/3$ (see Sample Problem 16-5), then the distance from C to P is $\frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. Since the net torque is equal to the rotational inertia ($I = \frac{1}{12}mL^2$ about the center of mass) multiplied by the angular acceleration, we obtain

$$\alpha = \frac{\tau}{I} = \frac{F\left(\frac{1}{6}L\right)}{\frac{1}{12}mL^2} = \frac{2F}{mL} \ .$$

(c) The distance from C to O is $r = L/2$, so the contribution to the acceleration at O stemming from the angular acceleration (in the counterclockwise direction of Fig. 16-11) is $\alpha r = \frac{1}{2}\alpha L$ (leftward in that figure). Also, the contribution to the acceleration at O due to the result of part (a) is F/m (rightward in that figure). Thus, if we choose rightward as positive, then the net acceleration of O is

$$a_O = \frac{F}{m} - \frac{1}{2}\alpha L = \frac{F}{m} - \frac{1}{2}\left(\frac{2F}{mL}\right)L = 0 \ .$$

(d) Point O stays relatively stationary in the batting process, and that might be possible due to a force exerted by the batter or due to a finely tuned cancellation such as we have shown here. We assumed that the batter exerted no force, and our first expectation is that the impulse delivered by the impact would make all points on the bat go into motion, but for this particular choice of impact point, we have seen that the point being held by the batter is naturally stationary and exerts no force on the batter's hands which would otherwise have to "fight" to keep a good hold of it.

67. Since $\omega = 2\pi f$ where $f = 2.2$ Hz, we find that the angular frequency is $\omega = 13.8$ rad/s. Thus, with $x = 0.010$ m, the acceleration amplitude is $a_m = x_m\omega^2 = 1.91\,\text{m/s}^2$. We set up a ratio:

$$a_m = \left(\frac{a_m}{g}\right)g = \left(\frac{1.91}{9.8}\right)g = 0.19g \ .$$

68. We adjust the phase constant ϕ in Eq. 16-3 so that $x = -x_m$ when $t = 0$.

$$-x_m = x_m\cos\phi \implies \phi = \pi \text{ rad} \ .$$

We also note that $\omega = 2\pi/T = 5\pi$ rad/s.

(a) With this information, Eq. 16-3 becomes

$$x = 0.10\cos\left(5\pi t + \pi\right)$$

where t is in seconds and x is in meters.

(b) By taking the derivative of the previous expression (or by plugging into Eq. 16-6) we have

$$v = -0.50\pi\sin\left(5\pi t + \pi\right)$$

with SI units again understood. Both of these expression can be simplified using standard trig identities.

69. (a) We are told

$$e^{-bt/2m} = \frac{3}{4} \quad \text{where} \quad t = 4T$$

where $T = 2\pi/\omega' \approx 2\pi\sqrt{m/k}$ (neglecting the second term in Eq. 16.41). Thus,

$$T \approx 2\pi\sqrt{(2.00\,\text{kg})/(10.0\,\text{N/m})} = 2.81\,\text{s}$$

and we find

$$\frac{b(4T)}{2m} = \ln\left(\frac{4}{3}\right) = 0.288 \implies b = \frac{2(2.00)(0.288)}{4(2.81)} = 0.102 \text{ kg/s} \ .$$

(b) Initially, the energy is $E_o = \frac{1}{2}kx_{m\,o}^2 = \frac{1}{2}(10.0)(0.250)^2 = 0.313$ J. At $t = 4T$, $E = \frac{1}{2}k(\frac{3}{4}x_{m\,o})^2 = 0.176$ J. Therefore, $E_o - E = 0.137$ J.

70. (a) Sample Problem 16-7 gives $b = 0.070$ kg/s, $m = 0.25$ kg and $k = 85$ N/m, and notes that $b \ll \sqrt{km}$ which implies $\omega' \approx \omega = \sqrt{k/m}$ (and, as will be important below, $b/m \ll \omega$). Thus, from Eq. 16-40, we find

$$v = \frac{dx}{dt} = x_m \left(\frac{-b}{2m}\right) e^{-bt/2m} \cos(\omega t + \phi) - \omega x_m e^{-bt/2m} \sin(\omega t + \phi)$$

where the first term is considered negligible ($b/2m \ll \omega$) and we write

$$v \approx -\omega x_m e^{-bt/2m} \sin(\omega t + \phi) \quad \Longrightarrow \quad v_m = \omega x_m e^{-bt/2m} \; .$$

Thus, the ratio of maximum values of forces is

$$\frac{F_{m\,\text{damp}}}{F_{m\,\text{spring}}} = \frac{bv_m}{kx_m e^{-bt/2m}} \approx \frac{b\omega}{k} = \frac{b}{\sqrt{km}} = 0.015 \; .$$

(b) The ratio of force amplitudes found in part (a) displays no time dependence, implying there is no (or, since approximations were made, approximately no) change in the ratio as the system undergoes further oscillations.

71. We take derivatives and let $dg \approx \Delta g$ and $dT \approx \Delta T$. The derivative of Eq. 16-28 is

$$\frac{dT}{dg} = 2\pi \left(\frac{1}{2}\right) \frac{-L/g^2}{\sqrt{L/g}}$$

which (after dividing the left side by T and the right side by $2\pi\sqrt{L/g}$) can be written

$$\frac{\Delta T}{T} = -\frac{1}{2}\,\frac{\Delta g}{g}$$

where both sides have also been multiplied by $dg \to \Delta g$. To make the units consistent, we write

$$\frac{\Delta T}{T} = \frac{2.5\,\text{min}}{1\,\text{day}} = \frac{2.5\,\text{min}}{1440\,\text{min}} = 0.00174 \; .$$

Therefore, with $g = 9.81$ m/s^2, we obtain

$$0.00174 = -\frac{1}{2}\,\frac{\Delta g}{9.81\,\text{m/s}^2} \quad \Longrightarrow \quad \Delta g = -0.034 \text{ m/s}^2$$

which yields $g' = g + \Delta g = 9.78\,\text{m/s}^2$.

72. The speed of the submarine going eastward is

$$v_{\text{east}} = v_{\text{equator}} + v_{\text{sub}}$$

where $v_{\text{sub}} = 16000/3600 = 4.44$ m/s. The term v_{equator} is the speed that any point at the equator (at radius $R = 6.37 \times 10^6$ m) would have in order to keep up with the spinning earth. With $T = 1\,\text{day} = 86400\,\text{s}$, we note that $v_{\text{equator}} = R\omega = R2\pi/T = 463$ m/s and is much larger than v_{sub}. Similarly, when it travels westward, its speed is

$$v_{\text{west}} = v_{\text{equator}} - v_{\text{sub}} \; .$$

The effective gravity g_e (or apparent gravity) combines the gravitational pull of the earth g (which cancels when we take the difference) and the effect of the centripetal acceleration v^2/R. Considering the two motions of the submarine, the difference is therefore

$$\begin{aligned} \Delta g_e = g'_e - g_e &= \frac{v_{\text{east}}^2}{R} - \frac{v_{\text{west}}^2}{R} \\ &= \frac{1}{R}\left((v_{\text{equator}} + v_{\text{sub}})^2 - (v_{\text{equator}} - v_{\text{sub}})^2\right) \\ &= \frac{4v_{\text{equator}}v_{\text{sub}}}{R} \quad = \quad \frac{8\pi v_{\text{sub}}}{T} \end{aligned}$$

where in the last step we have used $v_{\rm equator} = R2\pi/T$. Consequently, we find

$$\frac{\Delta g_e}{g} = \frac{8\pi v_{\rm sub}}{gT} = \frac{8\pi(4.44)}{(9.8)(86400)} = 1.3 \times 10^{-4} \ .$$

The problem asks for $\Delta g/g$ for *either* travel direction, and since our computation examines eastward travel as opposed to westward travel, then we infer that either-way travel versus no-travel should be half of our result. Thus, the answer to the problem is $\frac{1}{2}(1.3 \times 10^{-4}) = 6.6 \times 10^{-5}$.

73. (a) The graph makes it clear that the period is $T = 0.20$ s.

(b) Eq. 16-13 states

$$T = 2\pi\sqrt{\frac{m}{k}} \ .$$

Thus, using the result from part (a) with $k = 200$ N/m, we obtain $m = 0.203 \approx 0.20$ kg.

(c) The graph indicates that the speed is (momentarily) zero at $t = 0$, which implies that the block is at $x_0 = \pm x_m$. From the graph we also note that the slope of the velocity curve (hence, the acceleration) is positive at $t = 0$, which implies (from $ma = -kx$) that the value of x is negative. Therefore, with $x_m = 0.20$ m, we obtain $x_0 = -0.20$ m.

(d) We note from the graph that $v = 0$ at $t = 0.10$ s, which implied $a = \pm a_m = \pm\omega^2 x_m$. Since acceleration is the instantaneous slope of the velocity graph, then (looking again at the graph) we choose the negative sign. Recalling $\omega^2 = k/m$ we obtain $a = -197 \approx -200$ m/s^2.

(e) The graph shows $v_m = 6.28$ m/s, so

$$K_m = \frac{1}{2}mv_m^2 = 4.0 \text{ J} \ .$$

74. (a) The Hooke's law force (of magnitude $(100)(0.30) = 30$ N) is directed upward and the weight (20 N) is downward. Thus, the net force is 10 N upward.

(b) The equilibrium position is where the upward Hooke's law force balances the weight, which corresponds to the spring being stretched (from unstretched length) by 20 N/100 N/m = 0.20 m. Thus, relative to the equilibrium position, the block (at the instant described in part (a)) is at what one might call *the bottom turning point* (since $v = 0$) at $x = -x_m$ where the amplitude is $x_m = 0.30 - 0.20 = 0.10$ m.

(c) Using Eq. 16-13 with $m = W/g \approx 2.0$ kg, we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.90 \text{ s} \ .$$

(d) The maximum kinetic energy is equal to the maximum potential energy $\frac{1}{2}kx_m^2$. Thus,

$$K_m = U_m = \frac{1}{2}(100 \text{ N/m})(0.10 \text{ m})^2 = 0.50 \text{ J} \ .$$

75. (a) Comparing with Eq. 16-3, we see $\omega = 10$ rad/s in this problem. Thus, $f = \omega/2\pi = 1.6$ Hz.

(b) Since $v_m = \omega x_m$ and $x_m = 10$ cm (see Eq. 16-3), then $v_m = (10\,\text{rad/s})(10\,\text{cm}) = 100$ cm/s or 1.0 m/s.

(c) Since $a_m = \omega^2 x_m$ then $v_m = (10\,\text{rad/s})^2(10\,\text{cm}) = 1000$ cm/s^2 or 10 m/s^2.

(d) The acceleration extremes occur at the displacement extremes: $x = \pm x_m$ or $x = \pm 10$ cm.

(e) Using Eq. 16-12, we find

$$\omega = \sqrt{\frac{k}{m}} \implies k = (0.10\,\text{kg})(10\,\text{rad/s})^2 = 10 \text{ N/m} \ .$$

Thus, Hooke's law gives $F = -kx = -10x$ in SI units.

76. (a) We take the x axis along the tunnel, with $x = 0$ at the middle. At any instant during the train's motion, it is a distance r from the center of Earth, and we can think of this as a vector $\vec{r}$ pointing from the train to the Earth's center. We neglect any effects associated with the spinning of Earth (which has mass M and radius R). Based on the theory of Ch. 14, we know that the magnitude of gravitational force on the train of mass m_o at any instant is

$$|F_g| = \frac{Gm_oM\left(r^3/R^3\right)}{r^2} = \frac{Gm_oMr}{R^3} .$$

It is only the horizontal component of this force which leads to acceleration/deceleration of the train, so a $\cos\theta$ factor (with θ giving the angle of $\vec{r}$ measured from the x axis) must be included, and we can relate $\cos\theta = x/r$ and obtain

$$m_oa = F_x = -\frac{Gm_oMr}{R^3}\frac{x}{r}$$

where the minus sign is necessary because the force pulls towards the $x = 0$ position, so when the train is, say, at a large negative value of x the force is in the positive x direction (towards the origin of the x axis). The above expression simplifies to exactly the form (Eq. 16-8) required for simple harmonic motion:

$$a = -\omega^2 x \quad \text{where} \quad \omega = \sqrt{\frac{GM}{R^3}} .$$

Since a full cycle of the motion would return the train to its starting point, then a half cycle is required to travel from the departure city to the destination city. Therefore, $t_{\text{travel}} = \frac{1}{2}T$.

(b) Since $T = 2\pi/\omega$, we obtain

$$t_{\text{travel}} = \pi\sqrt{\frac{R^3}{GM}} = \pi\sqrt{\frac{\left(6.37\times10^6\right)^3}{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}}$$

which yields 2530 s or 42 min.

77. Using $\Delta m = 2.0$ kg, $T_1 = 2.0$ s and $T_2 = 3.0$ s, we write

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}} .$$

Dividing one relation by the other, we obtain

$$\frac{T_2}{T_1} = \sqrt{\frac{m+\Delta m}{m}}$$

which (after squaring both sides) simplifies to

$$m = \frac{\Delta m}{\left(\frac{T_2}{T_1}\right)^2 - 1} = 1.6 \text{ kg} .$$

78. (a) Hooke's law readily yields $(0.300\,\text{kg})(9.8\,\text{m/s}^2)/(0.0200\,\text{m}) = 147$ N/m.

(b) With $m = 2.00$ kg, the period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.733 \text{ s} .$$

79. Since $T = 0.500$ s, we note that $\omega = 2\pi/T = 4\pi$ rad/s. We work with SI units, so $m = 0.0500$ kg and $v_m = 0.150$ m/s.

(a) Since $\omega = \sqrt{k/m}$, the spring constant is

$$k = \omega^2 m = (4\pi)^2(0.0500) = 7.90 \text{ N/m} .$$

(b) We use the relation $v_m = x_m\omega$ and obtain

$$x_m = \frac{v_m}{\omega} = \frac{0.150}{4\pi} = 0.0119 \text{ m} .$$

(c) The frequency is $f = \omega/2\pi = 2.00$ Hz (which is equivalent to $f = 1/T$).

80. (a) Hooke's law provides the spring constant: $k = (20\,\text{N})/(0.20\,\text{m}) = 100$ N/m.

(b) The attached mass is $m = (5.0\,\text{N})/(9.8\,\text{m/s}^2) = 0.51$ kg. Consequently, Eq. 16-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.51}{100}} = 0.45 \text{ s} .$$

81. Since a mole of silver atoms has a mass of 0.108 kg, then the mass of one atom is

$$m = \frac{0.108\,\text{kg}}{6.02 \times 10^{23}} = 1.8 \times 10^{-25} \text{ kg} .$$

Using Eq. 16-12 and the fact that $f = \omega/2\pi$, we have

$$1 \times 10^{13}\,\text{Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \implies k = \left(2\pi \times 10^{13}\right)^2 \left(1.8 \times 10^{-25}\right) \approx 7.1 \times 10^2 \text{ N/m} .$$

82. (a) Hooke's law provides the spring constant: $k = (4.00\,\text{kg})(9.8\,\text{m/s}^2)/(0.160\,\text{m}) = 245$ N/m.

(b) The attached mass is $m = 0.500$ kg. Consequently, Eq. 16-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.500}{245}} = 0.284 \text{ s} .$$

83. (a) By Eq. 16-13, the mass of the block is

$$m_b = \frac{kT_0^2}{4\pi^2} = 2.43 \text{ kg} .$$

Therefore, with $m_p = 0.50$ kg, the new period is

$$T = 2\pi\sqrt{\frac{m_p + m_b}{k}} = 0.44 \text{ s} .$$

(b) The speed before the collision (since it is at its maximum, passing through equilibrium) is $v_0 = x_m\omega_0$ where $\omega_0 = 2\pi/T_0$; thus, $v_0 = 3.14$ m/s. Using momentum conservation (along the horizontal direction) we find the speed after the collision.

$$V = v_0\,\frac{m_b}{m_p + m_b} = 2.61 \text{ m/s} .$$

The equilibrium position has not changed, so (for the new system of greater mass) this represents the maximum speed value for the subsequent harmonic motion: $V = x'_m\omega$ where $\omega = 2\pi/T = 14.3$ rad/s. Therefore, $x'_m = 0.18$ m.

84. The period is the time for one oscillation: $T = 180/72 = 2.5$ s. Thus, by Eq. 16-28, we have

$$T = 2\pi\sqrt{\frac{L}{g}} \implies g = L\left(\frac{2\pi}{T}\right)^2 = 9.47 \text{ m/s}^2 .$$

85. Using Eq. 16-12, we find $\omega = \sqrt{k/m} = 10$ rad/s. We also use $v_m = x_m\omega$ and $a_m = x_m\omega^2$.

(a) The amplitude (meaning "displacement amplitude") is $x_m = v_m/\omega = 3/10 = 0.30$ m.

(b) The acceleration-amplitude is $a_m = (0.30)(10)^2 = 30\,\mathrm{m/s^2}$.

(c) One interpretation of this question is "what is the most negative value of the acceleration?" in which case the answer is $-a_m = -30\,\mathrm{m/s^2}$. Another interpretation is "what is the smallest value of the absolute-value of the acceleration?" in which case the answer is zero.

(d) Since the period is $T = 2\pi/\omega = 0.628$ s. Therefore, seven cycles of the motion requires $t = 7T = 4.4$ s.

86. We find that the spring constant is $k = mg/h$. Thus, Eq. 16-13 becomes

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(mg/h)}}2\pi\sqrt{\frac{h}{g}}$$

which we recognize as the period formula for a simple pendulum of length h (see Eq. 16-28).

87. Using Eq. 16-28, we obtain

$$L = g\left(\frac{T}{2\pi}\right)^2 = (9.75)\left(\frac{1.50}{2\pi}\right)^2 = 0.556 \text{ m} .$$

88. Using Eq. 16-29 and the parallel-axis formula for rotational inertia, we have

$$I = 2\pi\sqrt{\frac{I_{\mathrm{cm}} + mh^2}{mgh}} = 2\pi\sqrt{\frac{L^2}{12gh} + \frac{h}{g}}$$

where we have used the fact (from Ch. 11) that $I_{\mathrm{cm}} = mL^2/12$ for a uniform rod. We wish to minimize by taking the derivative and setting equal to zero, but we observe that this is done more easily if we consider I^2 (the square of the above expression) instead of I. Thus,

$$\frac{dI^2}{dh} = 0 = 4\pi^2\left(-\frac{L^2}{12gh^2} + \frac{1}{g}\right)$$

which leads to

$$0 = -\frac{L^2}{12h^2} + 1 \implies h = \frac{L}{\sqrt{12}} \approx 0.29L .$$

89. To use Eq. 16-29 we need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass so the center of mass of the system is $h = \frac{1}{2}(0.50) = 0.25$ m below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where $L = 1.00$ m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 16-29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, $T = 1.83$ s.

90. The period formula, Eq. 16-29, requires knowing the distance h from the axis of rotation and the center of mass of the system. We also need the rotational inertia I about the axis of rotation. From Figure 16-53, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M(L+R)^2 \quad \text{where } M = 1.0 \text{ kg} .$$

Thus, Eq. 16-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M(L+R)^2}{Mg(L+R)}}$$

which leads to $L = 0.8315$ m.

91. (a) From Eq. 16-12, $T = 2\pi\sqrt{m/k} = 0.45$ s.

(b) For a vertical spring, the distance between the unstretched length and the equilibrium length (with a mass m attached) is mg/k, where in this problem $mg = 10$ N and $k = 200$ N/m (so that the distance is 0.05 m). During simple harmonic motion, the convention is to establish $x = 0$ at the equilibrium length (the middle level for the oscillation) and to write the total energy without any gravity term; i.e.,

$$E = K + U \qquad \text{where } U = \frac{1}{2}kx^2 \; .$$

Thus, as the block passes through the unstretched position, the energy is $E = 2.0 + \frac{1}{2}k(0.05)^2 = 2.25$ J. At its topmost and bottommost points of oscillation, the energy (using this convention) is all elastic potential: $\frac{1}{2}kx_m^2$. Therefore, by energy conservation,

$$2.25 = \frac{1}{2}kx_m^2 \implies x_m = \pm 0.15 \text{ m} .$$

This gives the amplitude of oscillation as 0.15 m, but how far are these points from the *unstretched* position? We add (or subtract) the 0.05 m value found above and obtain 0.10 m for the top-most position and 0.20 m for the bottom-most position.

(c) As noted in part (b), $x_m = \pm 0.15$ m.

(d) The maximum kinetic energy equals the maximum potential energy (found in part (b)) and is equal to 2.25 J.

92. (a) Eq. 16-21 leads to

$$E = \frac{1}{2}kx_m^2 \implies x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(4.0)}{200}} = 0.020 \text{ m} .$$

(b) Since $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.80/200} \approx 0.4$ s, then the block completes $10/0.4 = 25$ cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \implies 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 .$$

Therefore, when $x = 0.15$ m, we find $v = 2.1$ m/s.

93. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = \frac{1}{3}ML^2$. Therefore, Eq. 16-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} \implies L = \frac{3gT^2}{8\pi^2}$$

so that $L = 0.84$ m.

(b) By energy conservation

$$\begin{aligned} E_{\text{bottom of swing}} &= E_{\text{end of swing}} \\ K_m &= U_m \end{aligned}$$

where $U = Mg\ell\,(1-\cos\theta)$ with ℓ being the distance from the axis of rotation to the center of mass. If we use the small angle approximation ($\cos\theta \approx 1 - \frac{1}{2}\theta^2$ with θ in radians (Appendix E)), we obtain

$$U_m = (0.5)(9.8)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where $\theta_m = 0.17$ rad. Thus, $K_m = U_m = 0.031$ J. If we calculate $(1-\cos\theta)$ straightforwardly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

94. From Eq. 16-23 (in absolute value) we find the torsion constant:

$$\kappa = \left|\frac{\tau}{\theta}\right| = \frac{0.20}{0.85} = 0.235$$

in SI units. With $I = 2mR^2/5$ (the rotational inertia for a solid sphere – from Chapter 11), Eq. 16-23 leads to

$$T = 2\pi\sqrt{\frac{\frac{2}{5}\,m\,R^2}{\kappa}} = 2\pi\sqrt{\frac{\frac{2}{5}(95)(0.15)^2}{0.235}} = 12\ \text{s}\ .$$

95. The time for one cycle is $T = (50\,\text{s})/20 = 2.5$ s. Thus, from Eq. 16-23, we find

$$I = \kappa\left(\frac{T}{2\pi}\right)^2 = (0.50)\left(\frac{2.5}{2\pi}\right)^2 = 0.079\ \text{kg}\cdot\text{m}^2\ .$$

96. The distance from the relaxed position of the bottom end of the spring to its equilibrium position when the body is attached is given by Hooke's law: $\Delta x = F/k = (0.20\,\text{kg})(9.8\,\text{m/s}^2)/(19\,\text{N/m}) = 0.103$ m.

(a) The body, once released, will not only fall through the Δx distance but continue through the equilibrium position to a "turning point" equally far on the other side. Thus, the total descent of the body is $2\Delta x = 0.21$ m.

(b) Since $f = \omega/2\pi$, Eq. 16-12 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.6\ \text{Hz}\ .$$

(c) The maximum distance from the equilibrium position is the amplitude: $x_m = \Delta x = 0.10$ m.

97. The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = \frac{1}{3}ML^2$. Therefore, Eq. 16-29 leads to

$$T_0 = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}}\ .$$

If we replace L with $L/2$ (for the case where half has been cut off) then the new period is $T = 2\pi\sqrt{L/3g}$. Since frequency is the reciprocal of the period, then $T_0/T = f/f_0$ which leads to

$$\frac{f}{f_0} = \frac{2\pi\sqrt{2L/3g}}{2\pi\sqrt{L/3g}} \implies f = f_0\sqrt{2}\ .$$

98. We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given $v = 5.2$ m/s at $x = 0$ is the maximum value v_m (which equals ωx_m where $\omega = \sqrt{k/m} = 20$ rad/s).

(a) Since $\omega = 2\pi f$, we find $f = 3.2$ Hz.

(b) We have $v_m = 5.2 = (20)x_m$, which leads to $x_m = 0.26$ m.

(c) With meters, seconds and radians understood,

$$\begin{aligned} x &= 0.26\cos(20t + \phi) \\ v &= -5.2\sin(20t + \phi) \end{aligned}$$

The requirement that $x = 0$ at $t = 0$ implies (from the first equation above) that either $\phi = +\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when $t = 0$; that choice is $\phi = -\pi/2$. Therefore,

$$x = 0.26\cos\left(20t - \frac{\pi}{2}\right) = 0.26\sin(20t) \ .$$

99. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\begin{aligned} \frac{1}{2}kx_m^2 &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 \\ &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2 \\ &= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{4}Mv_{\text{cm}}^2 = \frac{3}{4}Mv_{\text{cm}}^2 \ . \end{aligned}$$

which leads to $Mv_{\text{cm}}^2 = 2kx_m^2/3 = 0.125$ J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\text{cm}}^2 = kx_m^2/3 = 0.0625$ J.

(b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\text{cm}}^2 = kx_m^2/6 = 0.03125$ J.

(c) In this part, we use v_{cm} to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\begin{aligned} \frac{dE}{dt} &= 0 \\ \frac{d}{dt}\left(\frac{3}{4}Mv_{\text{cm}}^2\right)\frac{d}{dt}\left(\frac{1}{2}kx^2\right) &= 0 \\ \frac{3}{2}Mv_{\text{cm}}a_{\text{cm}} + kxv_{\text{cm}} &= 0 \end{aligned}$$

which leads to

$$a_{\text{cm}} = -\left(\frac{2k}{3M}\right)x \ .$$

Comparing with Eq. 16-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.

100. Eq. 16-28 gives $T = 2\pi\sqrt{L/g}$. Replacing L by $L/2$ gives the new period $T' = 2\pi\sqrt{L/2g}$. The ratio is

$$\frac{T'}{T} = \frac{2\pi\sqrt{L/2g}}{2\pi\sqrt{L/g}} = \frac{1}{\sqrt{2}} \ .$$

Therefore, we conclude that $T' = T/\sqrt{2}$.

101. (a) We require $U = \frac{1}{2}E$ at some value of x. Using Eq. 16-21, this becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right) \implies x = \frac{x_m}{\sqrt{2}} \ .$$

We compare the given expression x as a function of t with Eq. 16-3 and find $x_m = 5.0$ m. Thus, the value of x we seek is $x = 5.0/\sqrt{2} \approx 3.5$ m.

(b) We solve the given expression (with $x = 5.0/\sqrt{2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54 \text{ s} .$$

Since we are asked for the interval $t_{eq} - t$ where t_{eq} specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t_{eq} = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29 \text{ s} .$$

Consequently, the time interval is $t_{eq} - t = 0.75$ s.

102. (a) From the graph, it is clear that $x_m = 0.30$ m.

(b) With $F = -kx$, we see k is the (negative) slope of the graph – which is $75/0.30 = 250$ N/m. Plugging this into Eq. 16-13 yields

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.28 \text{ s} \quad .$$

(c) As discussed in §16-2, the maximum acceleration is

$$a_m = \omega^2 x_m = \frac{k}{m}\, x_m = 150 \text{ m/s}^2 \;.$$

Alternatively, we could arrive at this result using $a_m = \left(\frac{2\pi}{T}\right)^2 x_m$.

(d) Also in §16-2 is $v_m = \omega x_m$ so that the maximum kinetic energy is

$$K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\omega^2 x_m^2 = \frac{1}{2}kx_m^2$$

which yields $11.3 \approx 11$ J. We note that the above manipulation reproduces the notion of energy conservation for this system (maximum kinetic energy being equal to the maximum potential energy).

103. Since the particle has zero speed (momentarily) at $x \neq 0$, then it must be at its turning point; thus, $x_o = x_m = 0.37$ cm. It is straightforward to infer from this that the phase constant ϕ in Eq. 16-2 is zero. Also, $f = 0.25$ Hz is given, so we have $\omega = 2\pi f = \pi/2$ rad/s. The variable t is understood to take values in seconds.

(a) The period is $T = 1/f = 4.0$ s.

(b) As noted above, $\omega = \frac{\pi}{2}$ rad/s.

(c) The amplitude, as observed above, is 0.37 cm.

(d) Eq. 16-3 becomes $x = (0.37)\cos(\pi t/2)$ in centimeters.

(e) The derivative of x is $v = -(0.37)(\pi/2)\sin(\pi t/2) \approx (-0.58)\sin(\pi t/2)$ in centimeters-per-second.

(f) From the previous part, we conclude $v_m = 0.58$ cm/s.

(g) The acceleration-amplitude is $a_m = \omega^2 x_m = 0.91$ cm/s^2.

(h) Making sure our calculator is in radians mode, we find $x = (0.37)\cos(\pi(3.0)/2) = 0$. It is important to avoid rounding off the value of π in order to get precisely zero, here.

(i) With our calculator still in radians mode, we obtain $v = -(0.58)\sin(\pi(3.0)/2) = 0.58$ cm/s.

104. (a) Since no torque is being applied to the system, the angular momentum is constant.

(b) The maximum ω occurs when the maximum speed v occurs (as it passes through vertical: $\theta = 0$). The angular momentum of the "particle" may be written as $mvr = mr^2\omega$ so that conservation of momentum (applied to the $\theta = 0$ position) leads to

$$mr^2\omega_{\rm max} = mr_0^2\omega_{0,\rm max} \implies \omega_{\rm max} = \left(\frac{r_0}{r}\right)^2 \omega_{0,\rm max}$$

which becomes (with $r_0 = 0.80$ m and $\omega_{0,\rm max} = 1.30$ rad/s) $\omega_{\rm max} = 0.832/r^2$ in SI units.

(c) The maximum kinetic energy occurs at this same position: $K_{\rm max} = \frac{1}{2}mv_{\rm max}^2$ which we write as

$$K_{\rm max} = \frac{1}{2}mr^2\omega_{\rm max}^2 = \frac{1}{2}mr^2\left(\left(\frac{r_0}{r}\right)^2\omega_{0,\rm max}\right)^2 = \frac{mr_0^4\omega_{0,\rm max}^2}{2\,r^2} .$$

(d) We note from the previous result that $K_{\rm max}$ depends *inversely* on r^2, so it decreases as r increases.

(e) Measuring height h from the low point of the swing, consideration of the geometry leads to the relation $h = r(1-\cos\theta)$. The maximum height is therefore related to the maximum angle (measured from vertical) by

$$h_{\rm max} = r\,(1 - \cos\theta_{\rm max}\,)$$

which means the maximum potential energy (which must equal the same numerical value as the maximum kinetic energy if we assume mechanical energy conservation) is

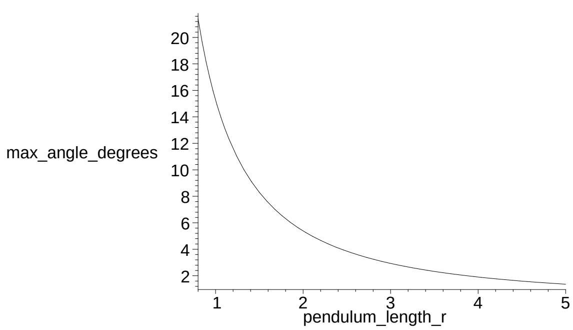
$$U_{\rm max} = K_{\rm max} = mgh_{\rm max} = mgr\,(1 - \cos\theta_{\rm max}\,) .$$

(f) Combining the results of part (c) and part (e), we obtain

$$\frac{mr_0^4\omega_{0,\rm max}^2}{2\,r^2} = mgr\,(1 - \cos\theta_{\rm max}\,) \implies \theta_{\rm max} = \cos^{-1}\left(1 - \frac{r_0^4\omega_{0,\rm max}^2}{2gr^3}\right)$$

which evaluates to be $\theta_{\rm max} = \cos^{-1}(1 - 0.0353/r^3\,)$ in SI units.

(g) As can be seen in the graph below, the angle of the pendulum "turning point" decreases as the pendulum lengthens (note that r is in meters).



(h) The original value of $\theta_{\rm max}$ is $\cos^{-1}(1 - 0.0353/r_0^3)$ where $r_0 = 0.80$ m. This gives 21.4° as the initial "turning point" angle. The question, then, asks us to solve for r in the case that $\theta_{\rm max} = \frac{1}{2}(21.4°) = 10.7°$. We know to look for half the initial value (as opposed to one twice as big) because the previous part shows $\theta_{\rm max}$ decreases with r. This value of the turning point angle occurs for

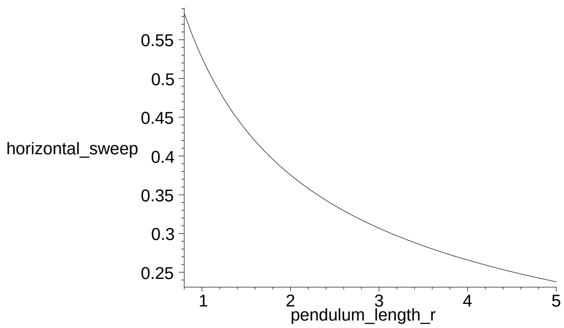
$$r = \left(\frac{0.0353}{1 - \cos 10.7°}\right)^{1/3} = 1.27 \text{ m} .$$

(i) The angle $\theta_{\rm max}$ is measured from vertical, so the horizontal sweep involves $\sin\theta_{\rm max}$. From one turning point to the opposite one covers a horizontal distance of $\Delta x = 2r\sin\theta_{\rm max}$.

(j) Plugging in from part (f), we find

$$\Delta x = 2r \sin\left(\cos^{-1}\left(1 - \frac{r_0^4 \omega_{0,\max}^2}{2gr^3}\right)\right) = \frac{r_0^2 \omega_{0,\max}}{gr^2}\sqrt{4gr^3 - r_0^4 \omega_{0,\max}^2}$$

where that last equality is (depending on one's viewpoint) a simplification and should not viewed as a necessary step. With $r_0 = 0.80$ m and $\omega_{0,\max} = 1.30$ rad/s, we plot this expression (with r and Δx (the horizontal sweep) in meters) and see that it is a decreasing function.

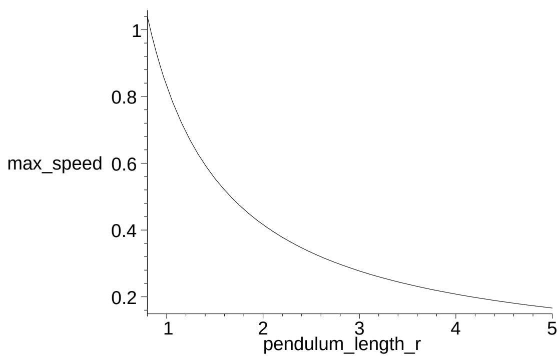


(k) When $r = r_0$ we find $\Delta x_0 = 0.584$ m. Any later value must be smaller (according to the above graph), so we seek a value of r that gives half of Δx_0 (that is, $\Delta x = 0.292$ m). If we numerically solve the expression in the previous part for r in the range $0.8\,\text{m} \le r \le 5\,\text{m}$, we obtain $r = 3.31$ m.

(l) Returning to part (b) with $v_{\max} = r\omega_{\max}$ we obtain

$$v_{\max} = \left(\frac{r_0^2}{r}\right)\omega_{0,\max} = \frac{0.832\,\text{m}^2/\text{s}}{r} \; .$$

(m) This result is again a decreasing function of r. We graph $v_{\max}$ versus r (with SI units understood) below.



(n) When $r = r_0$ we find $v_{\max} = 1.04$ m/s. Any later value must be smaller (according to the above graph), so we seek a value of r that gives $v_{\max} = 0.520$ m. This can be solved for algebraically:

$$r = \frac{0.832}{0.520} = 1.60 \text{ m} \; .$$

(o) If we do not examine changes in perspective (the fact that the blade is getting closer to the observer), then Poe's description must be considered misleading. We have found that the angular swing, the horizontal sweep and the maximum speed should decrease as r increases, which is contrary to the description given in Poe's story.

Chapter 17

1. (a) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\ \text{m}^{-1}\ .$$

   (b) The speed of the wave is

$$v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.8\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\ \text{m/s}\ .$$

2. (a) For visible light

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^{8}\,\text{m/s}}{700\times10^{-9}\,\text{m}} = 4.3\times10^{14}\ \text{Hz}$$

   and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^{8}\,\text{m/s}}{400\times10^{-9}\,\text{m}} = 7.5\times10^{14}\ \text{Hz}\ .$$

   (b) For radio waves

$$\lambda_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^{8}\,\text{m/s}}{300\times10^{6}\,\text{Hz}} = 1.0\,\text{m}$$

   and

$$\lambda_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^{8}\,\text{m/s}}{1.5\times10^{6}\,\text{Hz}} = 2.0\times10^{2}\ \text{m}\ .$$

   (c) For X rays

$$f_{\text{min}} = \frac{c}{\lambda_{\text{max}}} = \frac{3.0\times10^{8}\,\text{m/s}}{5.0\times10^{-9}\,\text{m}} = 6.0\times10^{16}\,\text{Hz}$$

   and

$$f_{\text{max}} = \frac{c}{\lambda_{\text{min}}} = \frac{3.0\times10^{8}\,\text{m/s}}{1.0\times10^{-11}\,\text{m}} = 3.0\times10^{19}\ \text{Hz}\ .$$

3. (a) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is $T = 4(0.170\,\text{s}) = 0.680\,\text{s}$.

   (b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\ \text{Hz}\ .$$

   (c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\ \text{m/s}\ .$$

4. Since the wave is traveling in the $-x$ direction, the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 17-2).

$$\begin{aligned} y(x,t) &= y_{\rm m}\sin(kx+\omega t) = y_{\rm m}\sin\left[2\pi f\left(\frac{x}{v}+t\right)\right] \\ &= (0.010\,{\rm m})\sin\left[2\pi(550\,{\rm Hz})\left(\frac{x}{330\,{\rm m/s}}+t\right)\right] \\ &= 0.010\,{\rm m}\sin[\pi(3.33x+1100t)] \end{aligned}$$

where x is in meters and t is in seconds.

5. We substitute $\omega = kv$ into $y = y_m \sin(kx - \omega t)$ to obtain

$$y = y_m \sin(kx - kvt) = y_m \sin k(x - vt) \ .$$

We put $k = 2\pi/\lambda$ and $\omega = 2\pi f$ into $y = y_m \sin(kx - \omega t)$ and obtain

$$y = y_m \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right) = y_m \sin 2\pi\left(\frac{x}{\lambda} - ft\right) \ .$$

When we substitute $k = \omega/v$ into $y = y_m \sin(kx - \omega t)$, we find

$$y = y_m \sin\left(\frac{\omega x}{v} - \omega t\right) = y_m \sin\omega\left(\frac{x}{v} - t\right) \ .$$

Finally, we substitute $k = 2\pi/\lambda$ and $\omega = 2\pi/T$ into $y = y_m \sin(kx - \omega t)$ to get

$$y = y_m \sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right) = y_m \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \ .$$

6. (a) The amplitude is $y_m = 6.0\,{\rm cm}$.
   (b) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 100\,{\rm cm}$.
   (c) Solving $2\pi f = \omega = 4.0\pi$, we obtain$f = 2.0\,{\rm Hz}$.
   (d) The wavespeed is $v = \lambda f = (100\,{\rm cm})(2.0\,{\rm Hz}) = 200\,{\rm cm/s}$.
   (e) The wave propagates in the negative x direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 17-2).
   (f) The maximum transverse speed (found from the time derivative of y) is

$$u_{\rm max} = 2\pi f y_m = \left(4.0\pi\,{\rm s}^{-1}\right)(6.0\,{\rm cm}) = 75\ {\rm cm/s}\ .$$

   (g) $y(3.5\,{\rm cm}, 0.26\,{\rm s}) = (6.0\,{\rm cm})\sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0\,{\rm cm}$.

7. (a) We write the expression for the displacement in the form $y(x,t) = y_m \sin(kx - \omega t)$. A negative sign is used before the ωt term in the argument of the sine function because the wave is traveling in the positive x direction. The angular wave number k is $k = 2\pi/\lambda = 2\pi/(0.10\,{\rm m}) = 62.8\,{\rm m}^{-1}$ and the angular frequency is $\omega = 2\pi f = 2\pi(400\,{\rm Hz}) = 2510\,{\rm rad/s}$. Here λ is the wavelength and f is the frequency. The amplitude is $y_m = 2.0\,{\rm cm}$. Thus

$$y(x,t) = (2.0\,{\rm cm})\sin\left(\left(62.8\,{\rm m}^{-1}\right)x - \left(2510\,{\rm s}^{-1}\right)t\right) \ .$$

   (b) The (transverse) speed of a point on the cord is given by taking the derivative of y:

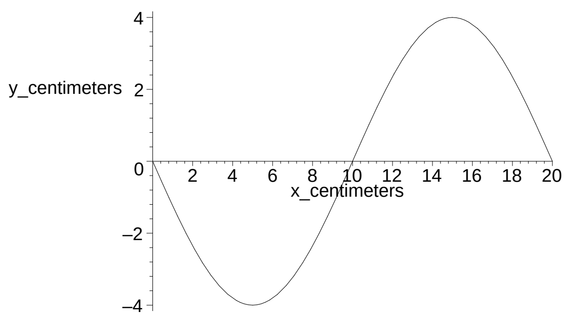
$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

   which leads to a maximum speed of $u_m = \omega y_m = (2510\,{\rm rad/s})(0.020\,{\rm m}) = 50\,{\rm m/s}$.

(c) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{m}^{-1}} = 40\text{ m/s} .$$

8. (a) The figure in the book makes it clear that the period is $T = 10$ s and the amplitude is $y_m = 4.0$ cm. The phase constant ϕ is more subtly determined by that figure: what is shown is $4\sin\omega t$, yet what follows from Eq. 17-2 (without the phase constant) should be $4\sin(-\omega t)$ at $x = 0$. Thus, we need the phase constant $\phi = \pi$ since $4\sin(-\omega t + \pi) = 4\sin(\omega t)$). Therefore, we use Eq. 17-2 (modified by the inclusion of ϕ) with $k = 2\pi/\lambda = \pi/10$ (in inverse centimeters) and $\omega = 2\pi/T = \pi/5$ (in inverse seconds). In the graph below we plot the equation for $t = 0$ over the range $0 \le x \le 20$ cm, making sure our calculator is in radians mode.



(b) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(c) Using the observations made in part (a), Eq. 17-2 becomes

$$y = 4.0\sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0\sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

where y and x are in centimeters and t is in seconds.

(d) Taking the derivative of y with respect to t, we find

$$u = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which (evaluated at $(x, t) = (0, 5.0)$, making sure our calculator is in radians mode) yields $u = -2.5$ cm/s.

9. Using $v = f\lambda$, we find the length of one cycle of the wave is $\lambda = 350/500 = 0.700\,\text{m} = 700$ mm. From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}\,\text{s} = 2.00$ ms.

(a) A cycle is equivalent to 2π radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6 = 700/6 = 117$ mm.

(b) The interval 1.00 ms is half of T and thus corresponds to half of one cycle, or half of 2π rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

10. The volume of a cylinder of height $\ell$is $V = \pi r^2 \ell = \pi d^2 \ell/4$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho\pi d^2\ell/4}{\ell} = \frac{\pi\rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho d_1^2/4}{\pi\rho d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2 .$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2 \ .$$

11. The wave speed v is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope: $\mu = m/L = (0.0600\,\text{kg})/(2.00\,\text{m}) = 0.0300\,\text{kg/m}$. Thus

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129 \text{ m/s} \ .$$

12. From $v = \sqrt{\tau/\mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2} \ .$$

13. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency $(2\pi/T)$, and k is the angular wave number $(2\pi/\lambda)$. The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\,\text{m}^{-1}$ and $\omega = 30\,\text{rad/s}$. Thus $v = (30\,\text{rad/s})/(2.0\,\text{m}^{-1}) = 15\,\text{m/s}$.

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6 \times 10^{-4}\,\text{kg/m}\right)(15\,\text{m/s})^2 = 0.036 \text{ N} \ .$$

14. (a) Comparing with Eq. 17-2, we see that $k = 20/\text{m}$ and $\omega = 600/\text{s}$. Therefore, the speed of the wave is (see Eq. 17-12) $v = \omega/k = 30$ m/s.

(b) From Eq. 17-25, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017\,\text{kg/m} = 17 \text{ g/m} \ .$$

15. We write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction. The frequency is $f = 100\,\text{Hz}$, so the angular frequency is $\omega = 2\pi f = 2\pi(100\,\text{Hz}) = 628\,\text{rad/s}$. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1} \ .$$

The amplitude is $y_m = 0.12\,\text{mm}$. Thus

$$y = (0.12\,\text{mm})\sin\left[(141\,\text{m}^{-1})x + (628\,\text{s}^{-1})t\right] \ .$$

16. Let the cross-sectional area of the wire be A and the density of steel be ρ. The tensile stress is given by τ/A where τ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$\begin{aligned} v_{\text{max}} &= \sqrt{\frac{\tau_{\text{max}}}{\mu}} = \sqrt{\frac{\tau_{\text{max}}/A}{\rho}} \\ &= \sqrt{\frac{7.0 \times 10^8\,\text{N/m}^2}{7800\,\text{kg/m}^3}} = 3.0 \times 10^2 \text{ m/s} \end{aligned}$$

which is indeed independent of the diameter of the wire.

17. (a) We take the form of the displacement to be $y(x,t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is $u(x,t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$ and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega/k} = k y_m = \frac{2\pi y_m}{\lambda} \ .$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds , linear densities of the cords, and the tensions in the cords.

18. (a) The general expression for $y(x,t)$ for the wave is $y(x,t) = y_m \sin(kx - \omega t)$, which, at $x = 10\,\text{cm}$, becomes $y(x = 10\,\text{cm}, t) = y_m \sin[k(10\,\text{cm} - \omega t)]$. Comparing this with the expression given, we find $\omega = 4.0\,\text{rad/s}$, or $f = \omega/2\pi = 0.64\,\text{Hz}$.

(b) Since $k(10\,\text{cm}) = 1.0$, the wave number is $k = 0.10/\text{cm}$. Consequently, the wavelength is $\lambda = 2\pi/k = 63\,\text{cm}$.

(c) Substituting the values of k and ω into the general expression for $y(x,t)$, with centimeters and seconds understood, we obtain

$$y(x,t) = 5.0 \sin(0.10x - 4.0t) \ .$$

(d) Since $v = \omega/k = \sqrt{\tau/\mu}$, the tension is

$$\tau = \frac{\omega^2\mu}{k^2} = \frac{(4.0\,\text{g/cm})\left(4.0\,\text{s}^{-1}\right)^2}{(0.10\,\text{cm}^{-1})^2} = 6400\,\text{g}\cdot\text{cm/s}^2 = 0.064\ \text{N} \ .$$

19. (a) We read the amplitude from the graph. It is about $5.0\,\text{cm}$.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15\,\text{cm}$ and again with the same slope at about $x = 55\,\text{cm}$, so $\lambda = 55\,\text{cm} - 15\,\text{cm} = 40\,\text{cm} = 0.40\,\text{m}$.

(c) The wave speed is $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6\,\text{N}}{25 \times 10^{-3}\,\text{kg/m}}} = 12\,\text{m/s} \ .$$

(d) The frequency is $f = v/\lambda = (12\,\text{m/s})/(0.40\,\text{m}) = 30\,\text{Hz}$ and the period is $T = 1/f = 1/(30\,\text{Hz}) = 0.033\,\text{s}$.

(e) The maximum string speed is $u_m = \omega y_m = 2\pi f y_m = 2\pi(30\,\text{Hz})(5.0\,\text{cm}) = 940\,\text{cm/s} = 9.4\,\text{m/s}$.

(f) The string displacement is assumed to have the form $y(x,t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative x direction. The amplitude is $y_m = 5.0 \times 10^{-2}\,\text{m}$, the angular frequency is $\omega = 2\pi f = 2\pi(30\,\text{Hz}) = 190\,\text{rad/s}$, and the angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40\,\text{m}) = 16\,\text{m}^{-1}$. According to the graph, the displacement at $x = 0$ and $t = 0$ is $4.0 \times 10^{-2}\,\text{m}$. The formula for the displacement gives $y(0,0) = y_m \sin\phi$. We wish to select ϕ so that $5.0 \times 10^{-2} \sin\phi = 4.0 \times 10^{-2}$. The solution is either $0.93\,\text{rad}$ or $2.21\,\text{rad}$. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93\,\text{rad}$. The expression for the displacement is

$$y(x,t) = (5.0 \times 10^{-2}\,\text{m}) \sin\left[(16\,\text{m}^{-1})x + (190\,\text{s}^{-1})t + 0.93\right] \ .$$

20. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})\,(9.8\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\ \text{m/s} \ .$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\mathrm{g})\,(9.8\,\mathrm{m/s^2})}{2(5.00\,\mathrm{g/m})}} = 22.1\ \mathrm{m/s}\ .$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for M_1 and obtain

$$M_1 = \frac{M}{1+\mu_2/\mu_1} = \frac{500\,\mathrm{g}}{1+5.00/3.00} = 187.5\,\mathrm{g} \approx 188\ \mathrm{g}\ .$$

(d) And we solve for the second mass: $M_2 = M - M_1 = 500\,\mathrm{g} - 187.5\,\mathrm{g} \approx 313\,\mathrm{g}$.

21. The pulses have the same speed v. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time t is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where L is the length of the wire, at time $t = 30\,\mathrm{ms}$. If this time is denoted by t_0 then the coordinate of this wave at time t is $x_2 = L - v(t-t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t-t_0)$. We solve for the time they meet: $t = (L+vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L+vt_0)/2$. Now, we calculate the wave speed:

$$v = \sqrt{\frac{\tau L}{m}} = \sqrt{\frac{(250\,\mathrm{N})(10.0\,\mathrm{m})}{0.100\,\mathrm{kg}}} = 158\,\mathrm{m/s}\ .$$

Here τ is the tension in the wire and L/m is the linear mass density of the wire. The coordinate of the meeting point is

$$x = \frac{10.0\,\mathrm{m} + (158\,\mathrm{m/s})(30\times10^{-3}\,\mathrm{s})}{2} = 7.37\,\mathrm{m}\ .$$

This is the distance from the left end of the wire. The distance from the right end is $L - x = 10\,\mathrm{m} - 7.37\,\mathrm{m} = 2.63\,\mathrm{m}$.

22. (a) The wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{k\Delta\ell}{m/(\ell+\Delta\ell)}} = \sqrt{\frac{k\Delta\ell(\ell+\Delta\ell)}{m}}\ .$$

(b) The time required is

$$t = \frac{2\pi(\ell+\Delta\ell)}{v} = \frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}} = 2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}\ .$$

Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t \simeq 2\pi\sqrt{m/k}$=const.

23. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by $\mu g y$, so the tension is $\tau = \mu g y$. The wave speed is $v = \sqrt{\mu g y/\mu} = \sqrt{gy}$.

(b) The time $\mathrm{d}t$ for the wave to move past a length $\mathrm{d}y$, a distance y from the bottom end, is $\mathrm{d}t = \mathrm{d}y/v = \mathrm{d}y/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{\mathrm{d}y}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\,\bigg|_0^L = 2\sqrt{\frac{L}{g}}\ .$$

24. Using Eq. 17-32 for the average power and Eq. 17-25 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$\begin{aligned} f &= \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} \\ &= \frac{1}{2\pi(7.7\times10^{-3}\,\text{m})}\sqrt{\frac{2(85\,\text{W})}{\sqrt{(36\,\text{N})(0.260\,\text{kg}/2.7\,\text{m})}}} = 198\ \text{Hz}\ . \end{aligned}$$

25. (a) The displacement of the string is assumed to have the form $y(x,t) = y_m \sin(kx-\omega t)$. The velocity of a point on the string is $u(x,t) = \partial y/\partial t = -\omega y_m \cos(kx-\omega t)$ and its maximum value is $u_m = \omega y_m$. For this wave the frequency is $f = 120\,\text{Hz}$ and the angular frequency is $\omega = 2\pi f = 2\pi(120\,\text{Hz}) = 754\,\text{rad/s}$. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00\times10^{-3}\,\text{m}$. The maximum speed is $u_m = (754\,\text{rad/s})(5.00\times10^{-3}\,\text{m}) = 3.77\,\text{m/s}$.

(b) Consider the string at coordinate x and at time t and suppose it makes the angle θ with the x axis. The tension is along the string and makes the same angle with the x axis. Its transverse component is $\tau_{\text{trans}} = \tau\sin\theta$. Now θ is given by $\tan\theta = \partial y/\partial x = ky_m\cos(kx-\omega t)$ and its maximum value is given by $\tan\theta_m = ky_m$. We must calculate the angular wave number k. It is given by $k = \omega/v$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the rope and μ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}\ .$$

Thus

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times10^{-3}\,\text{m}) = 0.138$$

and $\theta = 7.83°$. The maximum value of the transverse component of the tension in the string is $\tau_{\text{trans}} = (90.0\,\text{N})\sin 7.83° = 12.3\,\text{N}$. We note that $\sin\theta$ is nearly the same as $\tan\theta$ because θ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at x. The transverse component of the tension pulling on it due to the string to the left is $-\tau\partial y/\partial x = -\tau k y_m\cos(kx-\omega t)$ and it reaches its maximum value when $\cos(kx-\omega t) = -1$. The wave speed is $u = \partial y/\partial t = -\omega y_m\cos(kx-\omega t)$ and it also reaches its maximum value when $\cos(kx-\omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx-\omega t) = -1$ the value of $\sin(kx-\omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement Δy, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}} u\ .$$

P has its maximum value when the transverse component τ_{trans} of the tension and the string speed u have their maximum values. Hence the maximum power is $(12.3\,\text{N})(3.77\,\text{m/s}) = 46.4\,\text{W}$.

(e) As shown above $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx-\omega t) = 0$ and $\sin(kx-\omega t) = \pm1$ at that time. The string displacement is $y = \pm y_m = \pm0.50\,\text{cm}$.

26. (a) Let the phase difference be ϕ. Then from Eq. 17-39, $2y_m \cos(\phi/2) = 1.50 y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8^\circ \ .$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to $1.45\,\text{rad}/2\pi = 0.23$ wavelength.

27. The displacement of the string is given by $y = y_m \sin(kx-\omega t)+y_m \sin(kx-\omega t+\phi) = 2y_m \cos(\frac{1}{2}\phi)\sin(kx-\omega t + \frac{1}{2}\phi)$, where $\phi = \pi/2$. The amplitude is $A = 2y_m\cos(\frac{1}{2}\phi) = 2y_m \cos(\pi/4) = 1.41y_m$.

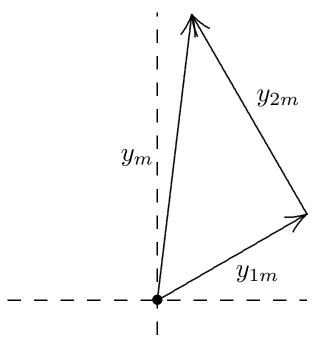
28. We compare the resultant wave given with the standard expression (Eq. 17-39) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda$, $2y_m\cos(\frac{1}{2}\phi) = 3.0$ mm, and $\frac{1}{2}\phi = 0.820$ rad.

(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

(b) The phase difference is $\phi = 1.64$ rad.

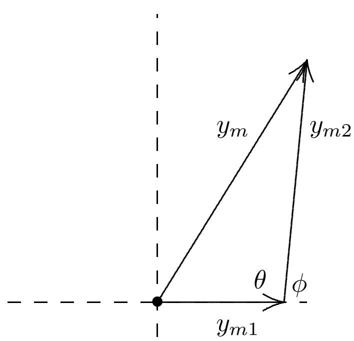
(c) And the amplitude is $y_m = 2.2$ mm.

29. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives $y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = 25\,\text{cm}^2$. Thus $y_m = 5.0$ cm.



30. The phasor diagram is shown below. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi \ .$$



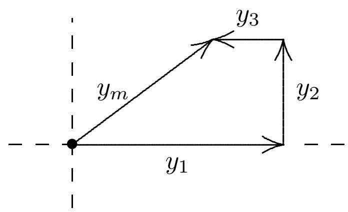
We solve for $\cos\phi$:

$$\begin{aligned}\cos\phi &= \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}y_{m2}} \\ &= \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} \\ &= 0.10 \ .\end{aligned}$$

The phase constant is therefore $\phi = 84°$.

31. (a) The phasor diagram is shown to the right: y_1, y_2, and y_3 represent the original waves and y_m represents the resultant wave. The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1 .$$



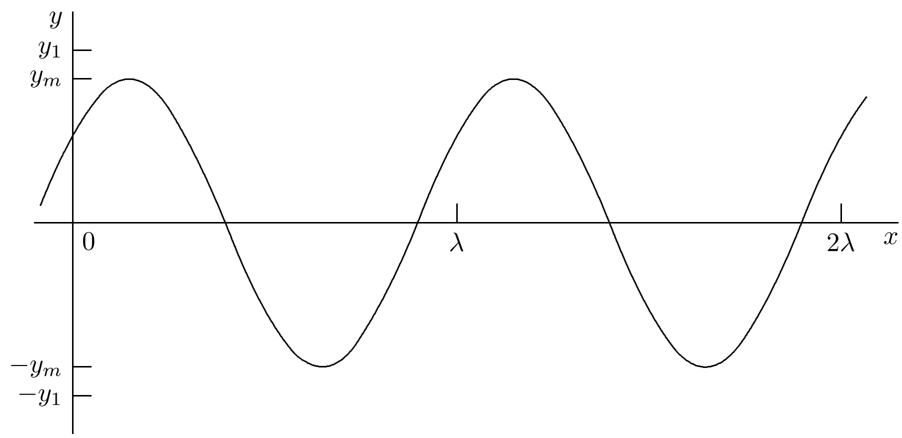
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\frac{y_{mv}}{y_{mh}} = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\frac{3}{4} = 0.644\,\text{rad} = 37° .$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644\,\text{rad}) .$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



32. Use Eq. 17-53 (for the resonant frequencies) and Eq. 17-25 ($v = \sqrt{\tau/\mu}$) to find f_n:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$.

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3' = \frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}} = 2f_3 .$$

(b) And we get the new wavelength

$$\lambda_3' = \frac{v'}{f_3'} = \frac{2L}{3} = \lambda_3 \ .$$

33. (a) Eq. 17-25 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\mathrm{N}}{7.2\times 10^{-3}\,\mathrm{kg/m}}} = 1.4\times 10^2 \ \mathrm{m/s} \ .$$

(b) From the Figure, we find the wavelength of the standing wave to be $\lambda = (2/3)(90\,\mathrm{cm}) = 60\,\mathrm{cm}$.

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.4\times 10^2\,\mathrm{m/s}}{0.60\,\mathrm{m}} = 2.4\times 10^2 \ \mathrm{Hz} \ .$$

34. The string is flat each time the particles passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude $T = 2(0.50\,\mathrm{s}) = 1.0\,\mathrm{s}$. Thus, $f = 1/T = 1.0$ Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\mathrm{cm/s}}{1.0\,\mathrm{Hz}} = 10 \ \mathrm{cm} \ .$$

35. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\,\mathrm{N})(8.40\,\mathrm{m})}{0.120\,\mathrm{kg}}} = 82.0 \ \mathrm{m/s} \ .$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40\,\mathrm{m}) = 16.8\,\mathrm{m}$.

(c) The frequency is $f = v/\lambda = (82.0\,\mathrm{m/s})/(16.8\,\mathrm{m}) = 4.88\,\mathrm{Hz}$.

36. (a) The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\mathrm{N}}{2.00\times 10^{-3}\,\mathrm{kg}/1.25\mathrm{m}}} = 66.1 \ \mathrm{m/s} \ .$$

(b) The wavelength of the wave with the lowest resonant frequency f_1 is $\lambda_1 = 2L$, where $L = 125\,\mathrm{cm}$. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1\,\mathrm{m/s}}{2(1.25\,\mathrm{m})} = 26.4 \ \mathrm{Hz} \ .$$

37. Possible wavelengths are given by $\lambda = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\,\mathrm{N}}{(10.0\,\mathrm{m})(0.100\,\mathrm{kg})}} = n(7.91\,\mathrm{Hz}) \ .$$

For $n = 1$, $f = 7.91\,\mathrm{Hz}$; for $n = 2$, $f = 15.8\,\mathrm{Hz}$; and for $n = 3$, $f = 23.7\,\mathrm{Hz}$.

38. The n^{th} resonant frequency of string A is

$$f_{n,A} = \frac{v_A}{2l_A}n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} ,$$

while for string B it is

$$f_{n,B} = \frac{v_B}{2l_B}n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4}f_{n,A} .$$

Thus, we see $f_{1,A} = f_{4,B}$ and $f_{2,A} = f_{8,B}$.

39. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n+1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n+1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n} .$$

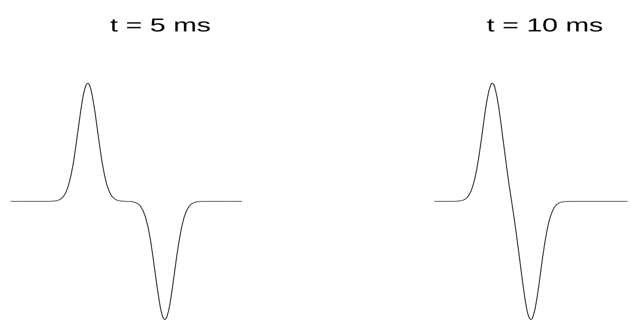
The solution for n is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315\,\text{Hz}}{420\,\text{Hz} - 315\,\text{Hz}} = 3 .$$

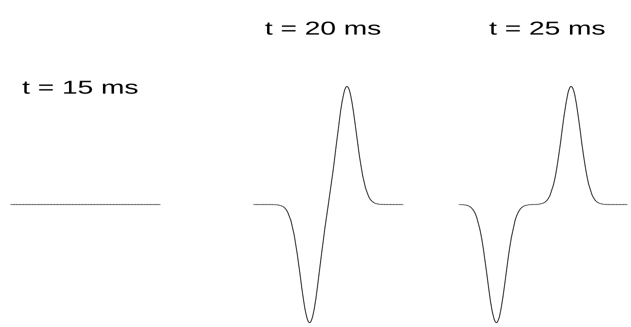
The lowest possible resonant frequency is $f = v/2L = f_1/n = (315\,\text{Hz})/3 = 105\,\text{Hz}$.

(b) The longest possible wavelength is $\lambda = 2L$. If f is the lowest possible frequency then $v = \lambda f = 2Lf = 2(0.75\,\text{m})(105\,\text{Hz}) = 158\,\text{m/s}$.

40. (a) We note that each pulse travels 1 cm during each $\Delta t = 5$ ms interval. Thus, in these first two pictures, their peaks are closer to each other by 2 cm, successively.



And the next pictures show the (momentary) complete cancellation of the visible pattern at $t = 15$ ms, and the pulses moving away from each other after that.



(b) The particles of the string are moving rapidly as they pass (transversely) through their equilibrium positions; the energy at $t = 15$ ms is purely kinetic.

41. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\,\mathrm{cm}^{-1}$. The wave speed is $v = \omega/k = (40\pi\,\mathrm{rad/s})/(\pi/3\,\mathrm{cm}^{-1}) = 120\,\mathrm{cm/s}$.

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\,\mathrm{cm}^{-1}) = 3.0\,\mathrm{cm}$. Here $2\pi/k$ was substituted for λ.

(d) The string speed is given by $u(x,t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t)$. For the given coordinate and time,

$$u = -(40\pi\,\mathrm{rad/s})(0.50\,\mathrm{cm})\sin\left[\left(\frac{\pi}{3}\,\mathrm{cm}^{-1}\right)(1.5\,\mathrm{cm})\right]\sin\left[\left(40\pi\,\mathrm{s}^{-1}\right)\left(\frac{9}{8}\,\mathrm{s}\right)\right] = 0\ .$$

42. Repeating the steps of Eq. 17-34 $\longrightarrow$ Eq. 17-40, but applying

$$\cos\alpha + \cos\beta = 2\,\cos\left(\frac{\alpha+\beta}{2}\right)\,\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 17-37, we obtain

$$y' = [0.10\cos\pi x]\cos 4\pi t$$

with SI units understood.

(a) For non-negative x, the smallest value to produce $\cos\pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x]\,(-4\pi\sin 4\pi t)$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ which leads to the answers $t = 0$, $t = 0.25$ s, and $t = 0.50$ s.

43. (a) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\mathrm{m/s})}{2(3.0\,\mathrm{m})} = 50\ \mathrm{Hz}\ .$$

(b) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be $y_1 = y_m\sin(kx - \omega t)$ and $y_2 = y_m\sin(kx + \omega t)$. The amplitude y_m is half the maximum displacement of the standing wave, or $5.0\times10^{-3}\,\mathrm{m}$. The angular frequency is the same as that of the standing wave, or $\omega = 2\pi f = 2\pi(50\,\mathrm{Hz}) = 314\,\mathrm{rad/s}$. The angular wave number is $k = 2\pi/\lambda = 2\pi/(2.0\,\mathrm{m}) = 3.14\,\mathrm{m}^{-1}$. Thus,

$$y_1 = (5.0\times10^{-3}\,\mathrm{m})\sin\left[\left(3.14\,\mathrm{m}^{-1}\right)x - \left(314\,\mathrm{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\mathrm{m}\right)\sin\left[\left(3.14\,\mathrm{m}^{-1}\right)x + \left(314\,\mathrm{s}^{-1}\right)t\right]\ .$$

44. To oscillate in four loops means $n = 4$ in Eq. 17-52 (treating both ends of the string as effectively "fixed'). Thus, $\lambda = 2(0.90\,\mathrm{m})/4 = 0.45\,\mathrm{m}$. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is $\mu = m/L = (0.044\,\mathrm{kg})/(0.90\,\mathrm{m}) = 0.049$ kg/m. Thus, using Eq. 17-25, we obtain the tension: $\tau = v^2\mu = (27)^2(0.049) = 36$ N.

45. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and $L = 2v/f = 2(400\,\text{m/s})/(600\,\text{Hz}) = 1.3\,\text{m}$.

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx)\cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency. The angular wave number is $k = 2\pi/\lambda = 2\pi f/v = 2\pi(600\,\text{Hz})/(400\,\text{m/s}) = 9.4\,\text{m}^{-1}$ and the angular frequency is $\omega = 2\pi f = 2\pi(600\,\text{Hz}) = 3800\,\text{rad/s}$. y_m is 2.0 mm. The displacement is given by

$$y(x,t) = (2.0\,\text{mm})\sin[(9.4\,\text{m}^{-1})x]\cos\left[(3800\,\text{s}^{-1})t\right] \ .$$

46. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 17-21(b), where

$$\lambda = L \qquad \text{and} \qquad f = \frac{v}{L}$$

(see Eq. 17-52 and Eq. 17-53).

(a) Comparing the given function with Eq. 17-47, we obtain $k = \pi/2$ and $\omega = 12\pi$ (SI units understood). Since $k = 2\pi/\lambda$ then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \implies \lambda = 4\text{ m} \implies L = 4\text{ m} \ .$$

(b) Since $\omega = 2\pi f$ then

$$2\pi f = 12\pi \implies f = 6\text{ Hz} \implies v = f\lambda = 24\text{ m/s} \ .$$

(c) Using Eq. 17-25, we have

$$\begin{aligned} v &= \sqrt{\frac{\tau}{\mu}} \\ 24 &= \sqrt{\frac{200}{m/L}} \end{aligned}$$

with leads to $m = 1.4$ kg.

(d) Now, "third-harmonic ... pattern" draws our attention to Figure 17-22(c), where

$$f = \frac{3v}{2L} = \frac{3(24)}{2(4)} = 9\text{ Hz}$$

so that $T = 1/f = 0.11$ s.

47. (a) The angular frequency is $\omega = 8.0\pi/2 = 4.0\pi\,\text{rad/s}$, so the frequency is $f = \omega/2\pi = (4.0\pi\,\text{rad/s})/2\pi = 2.0\,\text{Hz}$.

(b) The angular wave number is $k = 2.0\pi/2 = 1.0\pi\,\text{m}^{-1}$, so the wavelength is $\lambda = 2\pi/k = 2\pi/(1.0\pi\,\text{m}^{-1}) = 2.0\,\text{m}$.

(c) The wave speed is

$$v = \lambda f = (2.0\,\text{m})(2.0\,\text{Hz}) = 4.0\,\text{m/s} \ .$$

(d) We need to add two cosine functions. First convert them to sine functions using $\cos\alpha = \sin(\alpha+\pi/2)$, then apply Eq. 42. The steps are as follows:

$$\begin{aligned} \cos\alpha + \cos\beta &= \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \\ &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \end{aligned}$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m \cos(kx + \omega t) + y_m \cos(kx - \omega t) = 2y_m \cos(kx)\cos(\omega t) \ .$$

Nodes occur where $\cos(kx) = 0$ or $kx = n\pi + \pi/2$, where n is an integer (including zero). Since $k = 1.0\pi\,\mathrm{m}^{-1}$, this means $x = (n + \frac{1}{2})(1.0\,\mathrm{m})$. Nodes occur at $x = 0.50\,\mathrm{m}$, $1.5\,\mathrm{m}$, $2.5\,\mathrm{m}$, etc.

(e) The displacement is a maximum where $\cos(kx) = \pm 1$. This means $kx = n\pi$, where n is an integer. Thus, $x = n(1.0\,\mathrm{m})$. Maxima occur at $x = 0$, $1.0\,\mathrm{m}$, $2.0\,\mathrm{m}$, $3.0\,\mathrm{m}$, etc.

48. (a) The nodes are located from vanishing of the spatial factor $\sin 5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Longrightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \ldots$$

so that the values of x lying in the allowed range are $x = 0$, $x = 0.20$ m, and $x = 0.40$ m.

(b) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(c) Comparing the given function with Eq. 17-45 through Eq. 17-47, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(d) And we see the amplitude is $y_m = 0.020$ m.

(e) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -\,(0.040)\,(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all x) at times such $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Longrightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \ldots$$

so that the values of t lying in the allowed range are $t = 0$, $t = 0.025$ s, and $t = 0.050$ s.

49. We consider an infinitesimal segment of a string oscillating in a standing wave pattern. Its length is $\mathrm{d}x$ and its mass is $\mathrm{d}m = \mu\,\mathrm{d}x$, where μ is its linear mass density. If it is moving with speed u its kinetic energy is $\mathrm{d}K = \frac{1}{2}u^2\,\mathrm{d}m = \frac{1}{2}\mu u^2\,\mathrm{d}x$. If the segment is located at x its displacement at time t is $y = 2y_m \sin(kx)\cos(\omega t)$ and its velocity is $u = \partial y/\partial t = -2\omega y_m \sin(kx)\sin(\omega t)$, so its kinetic energy is

$$\mathrm{d}K = \left(\frac{1}{2}\right)\left(4\mu\omega^2 y_m^2\right)\sin^2(kx)\sin^2(\omega t) = 2\mu\omega^2 y_m^2 \sin^2(kx)\sin^2(\omega t) \ .$$

Here y_m is the amplitude of each of the traveling waves that combine to form the standing wave. The infinitesimal segment has maximum kinetic energy when $\sin^2(\omega t) = 1$ and the maximum kinetic energy is given by the differential amount

$$dK_m = 2\mu\omega^2 y_m^2 \sin^2(kx) \ .$$

Note that every portion of the string has its maximum kinetic energy at the same time although the values of these maxima are different for different parts of the string. If the string is oscillating with n loops, the length of string in any one loop is L/n and the kinetic energy of the loop is given by the integral

$$K_m = 2\mu\omega^2 y_m^2 \int_0^{L/n} \sin^2(kx)\,dx \ .$$

We use the trigonometric identity $\sin^2(kx) = \frac{1}{2}[1 + 2\cos(2kx)]$ to obtain

$$K_m = \mu\omega^2 y_m^2 \int_0^{L/n} [1 + 2\cos(2kx)]\,\mathrm{d}x = \mu\omega^2 y_m^2 \left[\frac{L}{n} + \frac{1}{k}\sin\frac{2kL}{n}\right] .$$

For a standing wave of n loops the wavelength is $\lambda = 2L/n$ and the angular wave number is $k = 2\pi/\lambda = n\pi/L$, so $2kL/n = 2\pi$ and $\sin(2kL/n) = 0$, no matter what the value of n. Thus,

$$K_m = \frac{\mu\omega^2 y_m^2 L}{n} .$$

To obtain the expression given in the problem statement, we first make the substitutions $\omega = 2\pi f$ and $L/n = \lambda/2$, where f is the frequency and λ is the wavelength. This produces $K_m = 2\pi^2\mu y_m^2 f^2\lambda$. We now substitute the wave speed v for $f\lambda$ and obtain $K_m = 2\pi^2\mu y_m^2 f v$.

50. From the $x = 0$ plot (and the requirement of an antinode at $x = 0$), we infer a standing wave function of the form

$$y = -(0.04)\cos(kx)\sin(\omega t) \quad \text{where} \quad \omega = \frac{2\pi}{T} = \pi \text{ rad/s}$$

with length in meters and time in seconds. The parameter k is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive x direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y = -0.04\,\cos(kx)\sin(\omega t) = 0.04 \text{ m} \quad \text{at } x = 0.20 \text{ m} .$$

(b) The above equation yields zero at $x = 0.30$ m.

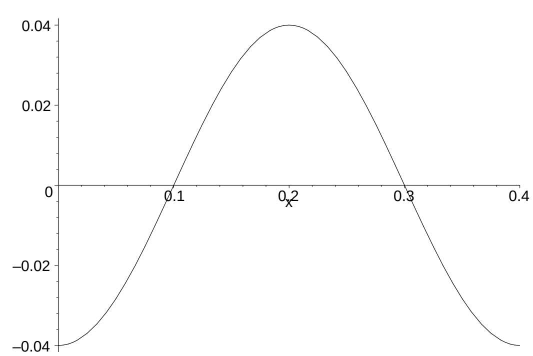
(c) We take the derivative with respect to time and obtain

$$u = \frac{dy}{dt} = -0.04\,\omega\,\cos(kx)\cos(\omega t) = 0 \quad \text{at } t = 0.50 \text{ s}$$

where $x = 0.20$ m.

(d) The above equation yields $u = -0.126$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown.



51. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then, $L_1 = n_1\lambda_1/2 = n_1 v_1/2f$, where λ_1 is the wavelength and v_1 is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where f is the frequency. Thus $f = n_1 v_1/2L_1$. A similar expression holds for the steel section: $f = n_2 v_2/2L_2$. Since the frequency is the same for the two sections, $n_1 v_1/L_1 = n_2 v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where μ_1 is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where ρ_1 is the mass density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus $\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$ and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where A has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\mathrm{m})\sqrt{7.80\times 10^3\,\mathrm{kg/m^3}}}{(0.600\,\mathrm{m})\sqrt{2.60\times 10^3\,\mathrm{kg/m^3}}} = 2.5\ .$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is $f = n_1 v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}$. The tension is provided by the hanging block and is $\tau = mg$, where m is the mass of the block. Thus

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\mathrm{m})}\sqrt{\frac{(10.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(2.60\times 10^3\,\mathrm{kg/m^3})(1.00\times 10^{-6}\,\mathrm{m^2})}} = 324\,\mathrm{Hz}\ .$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

52. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = (2000\,\mathrm{m/s})\left(40\times 10^{-6}\,\mathrm{s}\right) = 8.0\times 10^{-2}\ \mathrm{m}\ .$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve $a = 300/40\times 10^{-6}$) we find the stopping distance d:

$$v^2 = v_o^2 + 2ad \implies d = \frac{(300)^2\left(40\times 10^{-6}\right)}{2(300)}$$

which gives $d = 6.0\times 10^{-3}\,\mathrm{m}$. This and the radius r form the legs of a right triangle (where r is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \implies r = d\tan 60° = 1.0\times 10^{-2}\ \mathrm{m}\ .$$

53. We refer to the points where the rope is attached as A and B, respectively. When A and B are not displaced horizontal, the rope is in its initial state (neither stretched (under tension) nor slack). If they are displaced away from each other, the rope is clearly stretched. When A and B are displaced in the same direction, by amounts (in absolute value) $|\xi_A|$ and $|\xi_B|$, then if $|\xi_A| < |\xi_B|$ then the rope is stretched, and if $|\xi_A| > |\xi_B|$ the rope is slack. We must be careful about the case where one is displaced but the other is not, as will be seen below.

(a) The standing wave solution for the shorter cable, appropriate for the initial condition $\xi = 0$ at $t = 0$, and the boundary conditions $\xi = 0$ at $x = 0$ and $x = L$ (the x axis runs vertically here), is $\xi_A = \xi_m \sin(k_A x)\sin(\omega_A t)$. The angular frequency is $\omega_A = 2\pi/T_A$, and the wave number is $k_A = 2\pi/\lambda_A$ where $\lambda_A = 2L$ (it begins oscillating in its fundamental mode) where the point of attachment is $x = L/2$. The displacement of what we are calling point A at time $t = \eta T_A$ (where η is a pure number) is

$$\xi_A = \xi_m \sin\left(\frac{2\pi}{2L}\frac{L}{2}\right) \sin\left(\frac{2\pi}{T_A}\eta T_A\right) = \xi_m \sin(2\pi\eta) \ .$$

The fundamental mode for the longer cable has wavelength $\lambda_B = 2\lambda_A = 2(2L) = 4L$, which implies (by $v = f\lambda$ and the fact that both cables support the same wave speed v) that $f_B = \frac{1}{2} f_A$ or $\omega_B = \frac{1}{2}\omega_A$. Thus, the displacement for point B is

$$\xi_B = \xi_m \sin\left(\frac{2\pi}{4L}\frac{L}{2}\right) \sin\left(\frac{1}{2}\left(\frac{2\pi}{T_A}\right)\eta T_A\right) = \frac{\xi_m}{\sqrt{2}} \sin(\pi\eta) \ .$$

Running through the possibilities ($\eta = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}$, and 2) we find the rope is under tension in the following cases. The first case is one we must be very careful in our reasoning, since A is not displaced but B is displaced in the positive direction; we interpret that as the direction *away from* A (rightwards in the figure) – thus making the rope stretch.

$$\begin{array}{lll} \eta = \dfrac{1}{2} & \xi_A = 0 & \xi_B = \dfrac{\xi_m}{\sqrt{2}} > 0 \\ \eta = \dfrac{3}{4} & \xi_A = -\xi_m < 0 & \xi_B = \dfrac{\xi_m}{2} > 0 \\ \eta = \dfrac{7}{4} & \xi_A = -\xi_m < 0 & \xi_B = -\dfrac{\xi_m}{2} < 0 \end{array}$$

where in the last case they are both displaced leftward but A more so than B so that the rope is indeed stretched.

(b) The values of η (where we have defined $\eta = t/T_A$) which reproduce the initial state are

$$\begin{array}{llll} \eta = 1 & \xi_A = 0 & \xi_B = 0 & \text{and} \\ \eta = 2 & \xi_A = 0 & \xi_B = 0 & . \end{array}$$

(c) The values of η for which the rope is slack are given below. In the first case, both displacements are to the right, but point A is farther to the right than B. In the second case, they are displaced towards each other.

$$\begin{array}{lll} \eta = \dfrac{1}{4} & \xi_A = x_m > 0 & \xi_B = \dfrac{\xi_m}{\sqrt{2}} > 0 \\ \eta = \dfrac{5}{4} & \xi_A = \xi_m > 0 & \xi_B = -\dfrac{\xi_m}{2} < 0 \\ \eta = \dfrac{3}{2} & \xi_A = 0 & \xi_B = -\dfrac{\xi_m}{\sqrt{2}} < 0 \end{array}$$

where in the third case B is displaced leftward toward the undisplaced point A.

(d) The first design works effectively to damp fundamental modes of vibration in the two cables (especially in the shorter one which would have an antinode at that point), whereas the second one only damps the fundamental mode in the longer cable.

54. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as u_m) is 5.0 cm/s. Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \implies y_m = 3.2 \text{ cm} \ .$$

(c) As already noted, $f = 0.25$ Hz.

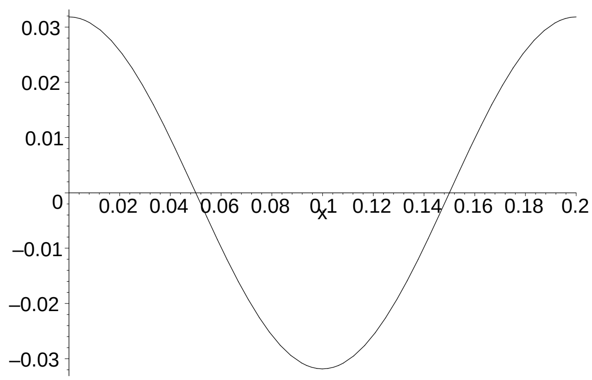
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the t and x terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050 \sin \frac{\pi}{2} t$. Therefore, the function $u(x, t)$ is

$$u = 0.050 \sin\left(\frac{\pi}{2} t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y = -\frac{2(0.050)}{\pi} \cos\left(\frac{\pi}{2} t - 10\pi x\right) + C$$

where C is an integration constant (which we will assume to be zero). The sketch of this function at $t = 2.0$ s for $0 \le x \le 0.20$ m is shown.



55. Using Eq. 17-37, we have

$$y' = \left[0.60 \cos\frac{\pi}{6}\right] \sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 17-42 for comparison).

(a) The amplitude is seen to be

$$0.60 \cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52 \text{ m} \quad .$$

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 17-11)

$$v = \frac{\omega}{k} = 40 \text{ m/s} \quad .$$

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

56. We orient one phasor along the x axis with length 4.0 mm and angle 0 and the other at $0.8\pi \text{ rad} = 144°$ (in the second quadrant) with length 7.0 mm. Adding the components, we obtain

$$\begin{aligned} 4.0 + 7.0\cos(144°) &= -1.66 \text{ mm} && \text{along } x \text{ axis} \\ 7.0\sin(144°) &= 4.11 \text{ mm} && \text{along } y \text{ axis} . \end{aligned}$$

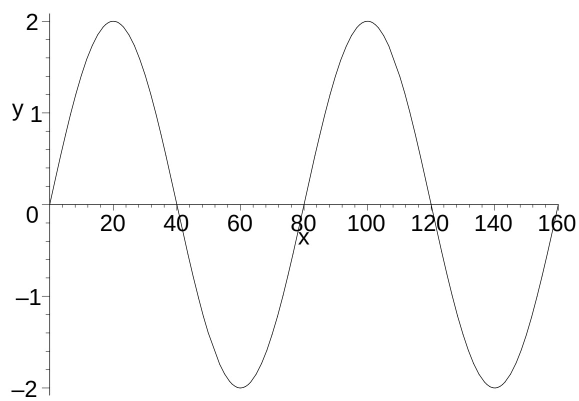
(a) The amplitude of the resultant wave is consequently

$$\sqrt{(-1.66)^2 + 4.11^2} = 4.4 \text{ mm} .$$

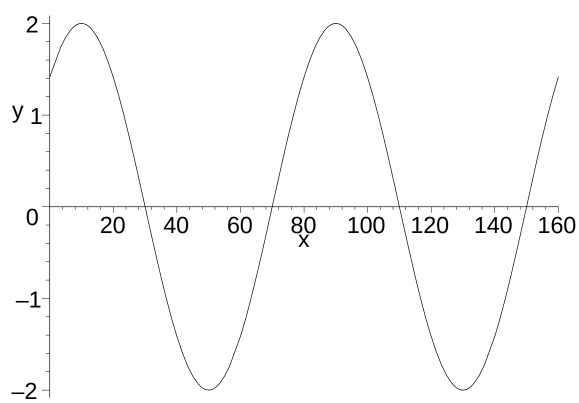
(b) And the phase constant (an angle, measured counterclockwise from the $+x$ axis) is

$$180^\circ + \tan^{-1}\left(\frac{4.11}{-1.66}\right) = 112^\circ .$$

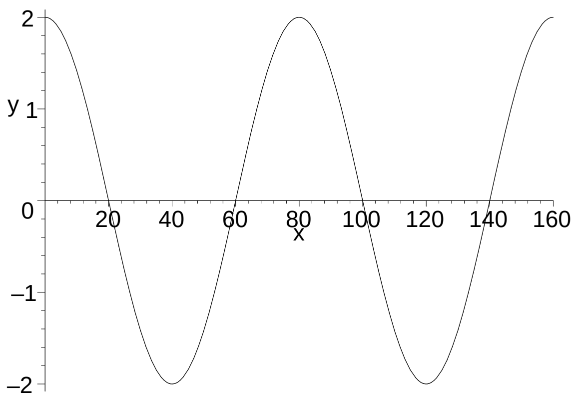
57. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find



(b) The previous graph is at $t = 0$, and this next one is at $t = 0.050$ s.



And the final one, shown below, is at $t = 0.010$ s.



(c) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the $-x$ direction.

58. We use Eq. 17-2, Eq. 17-5, Eq. 17-9, Eq. 17-12, and take the derivative to obtain the transverse speed u.

(a) The amplitude is $y_m = 2.0$ mm.

(b) Since $\omega = 600$ rad/s, the frequency is found to be $f = 600/2\pi \approx 95$ Hz.

(c) Since $k = 20$ rad/m, the velocity of the wave is $v = \omega/k = 600/20 = 30$ m/s in the $+x$ direction.

(d) The wavelength is $\lambda = 2\pi/k \approx 0.31$ m, or 31 cm.

(e) We obtain

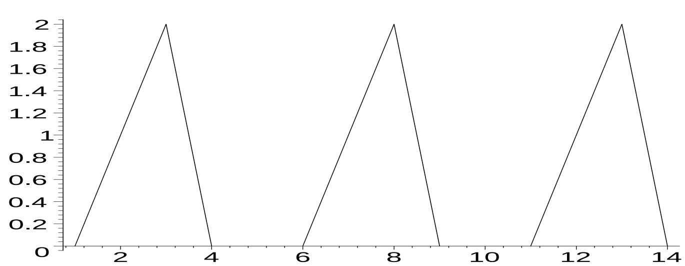
$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \quad \Longrightarrow \quad u_m = \omega y_m$$

so that the maximum transverse speed is $u_m = (600)(2.0) = 1200$ mm/s, or 1.2 m/s.

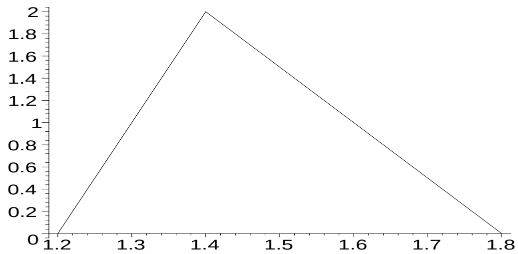
59. (a) Recalling the discussion in §17-5, we see that the speed of the wave given by a function with argument $x - 5t$ (where x is in centimeters and t is in seconds) must be 5 cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 17-47 (at $t = 0$), the middle one is at $t = 1.0$ s, and the rightmost one is at $t = 2.0$ s. It is clear that the wave is traveling to the right (the $+x$ direction).

(c) The third picture in the sequence below shows the pulse at 2 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.



(d) The leading edge of the pulse reaches $x = 10$ cm at $t = (10 - 4)/5 = 1.2$ s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h = 2$ cm at $t = (10 - 3)/5 = 1.4$ s. Finally, the the trailing edge of the pulse departs from $x = 10$ cm at $t = (10 - 1)/5 = 1.8$ s. Thus, we find for $h(t)$ at $x = 10$ cm (with the horizontal axis, t, in seconds):



60. We use $P = \frac{1}{2}\mu v\omega^2 y_m^2 \propto vf^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then

$$P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1 \ .$$

(b) If the frequency is halved, then

$$P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1 \ .$$

61. We use $v = \sqrt{\tau/\mu} \propto \sqrt{\tau}$ to obtain

$$\tau_2 = \tau_1\left(\frac{v_2}{v_1}\right)^2 = (120\,\text{N})\left(\frac{180\,\text{m/s}}{170\,\text{m/s}}\right)^2 = 135 \text{ N} \ .$$

62. (a) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{120\,\text{N}}{8.70\times 10^{-3}\,\text{kg}/1.50\,\text{m}}} = 144 \text{ m/s} \ .$$

(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\,\text{m}) = 3.00\,\text{m}$. For the two-loop standing wave $\lambda_2 = L = 1.50\,\text{m}$.

(c) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\,\text{m/s})/(3.00\,\text{m}) = 48.0\,\text{Hz}$ and that for the two-loop wave is $f_2 = v/\lambda_2 = (144\,\text{m/s})/(1.50\,\text{m}) = 96.0\,\text{Hz}$.

63. (a) At $x = 2.3\,\text{m}$ and $t = 0.16\,\text{s}$ the displacement is

$$y(x,t) = 0.15\sin[(0.79)(2.3) - 13(0.16)]\,\text{m} = -0.039\,\text{m} \, .$$

(b) The wave we are looking for must be traveling in $-x$ direction with the same speed and frequency. Thus, its general form is $y'(x,t) = y_m\sin(0.79x + 13t + \phi)$, where y_m is its amplitude and ϕ is its initial phase. In particular, if $y_m = 0.15\,\text{m}$, then there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) In the special case when $y_m = 0.15\,\text{m}$ and $\phi = 0$, the displacement of the standing wave at $x = 2.3\,\text{m}$ and $t = 0.16\,\text{s}$ is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14 \text{ m} \ .$$

64. (a) Let the displacements of the wave at (y,t) be $z(y,t)$. Then $z(y,t) = z_m\sin(ky - \omega t)$, where $z_m = 3.0\,\text{mm}$, $k = 60\,\text{cm}^{-1}$, and $\omega = 2\pi/T = 2\pi/0.20\,\text{s} = 10\pi\,\text{s}^{-1}$. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right] \ .$$

(b) The maximum transverse speed is

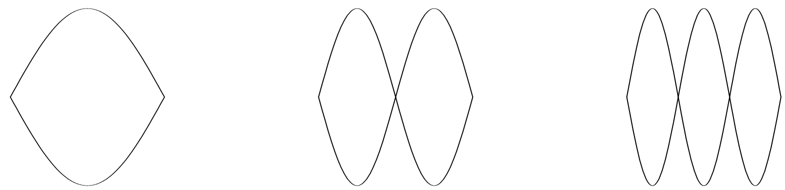
$$u_m = \omega z_m = (2\pi/0.20\,\mathrm{s})(3.0\,\mathrm{mm}) = 94\ \mathrm{mm/s}\ .$$

65. (a) Using Eq. 17-52 with $L = 120$ cm, we find

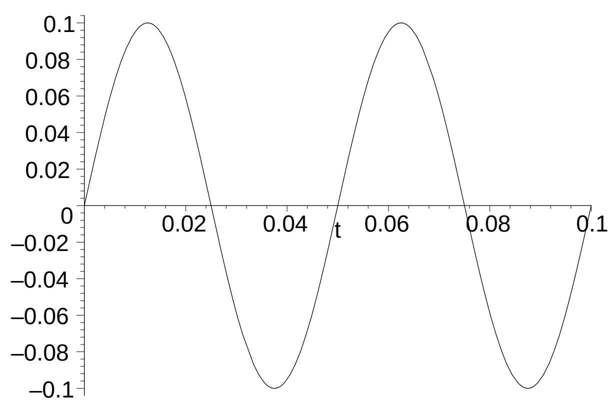
$$\lambda_1 = \frac{2L}{1} = 240 \qquad \lambda_2 = \frac{2L}{2} = 120 \qquad \lambda_3 = \frac{2L}{3} = 80$$

with all values understood to be in centimeters.

(b) The three standing waves are shown below.



66. It is certainly possible to simplify (in the trigonometric sense) the expressions at $x = 3$ m (since $k = 1/2$ in inverse-meters), but there is no particular need to do so, if the goal is to plot the time-dependence of the wave superposition at this value of x. Still, it is worth mentioning the end result of such simplification if it provides some insight into the nature of the graph (shown below): $y_1 + y_2 = (0.10\,\mathrm{m})\sin(40\pi t)$ with t in seconds.



67. By Eq. 17-53, the higher frequencies are integer multiples of the lowest (the fundamental). Therefore, $f_2 = 2(440) = 880$ Hz and $f_3 = 3(440) = 1320$ Hz are the second and third harmonics, respectively.

68. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\mathrm{m/s}}{3.2\,\mathrm{m}} = 75\ \mathrm{Hz}\ .$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\mathrm{Hz}} = 0.0133\,\mathrm{s} \approx 13\ \mathrm{ms}\ .$$

69. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi\cos\left(\frac{\pi x}{8} - 4\pi t\right)\ .$$

Thus, when $x = 6$ and $t = \frac{1}{4}$, we obtain

$$u = -60\pi \cos \frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2 \sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x = 6$ and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin \frac{-\pi}{4}$ which yields $a = 16.7$ m/s^2.

(d) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is 23.7 m/s^2.

70. (a) Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16}{0.04} = 400 \text{ rad/s} .$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = 80/64 = 1.26$ m.

(c) Now, $k = 2\pi/\lambda = 5$ rad/m, so the function describing the wave becomes

$$y = 0.04 \sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.04$ at $x = t = 0$. Therefore, $\sin\phi = 1$, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.04 \sin\left(5x - 400t + \frac{\pi}{2}\right) .$$

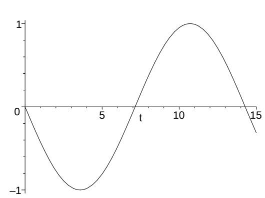
71. We orient one phasor along the x axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

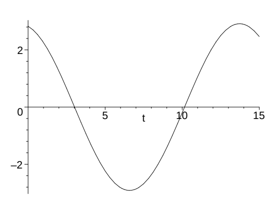
$$\begin{aligned} 3.0 + 5.0\cos(70°) &= 4.71 \text{ mm} \qquad && \text{along } x \text{ axis} \\ 5.0\sin(70°) &= 4.70 \text{ mm} \qquad && \text{along } y \text{ axis} . \end{aligned}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{4.71^2 + 4.70^2} = 6.7$ mm.

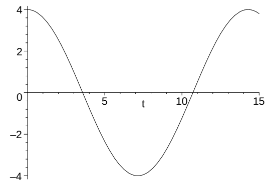
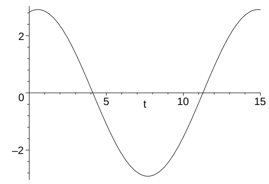
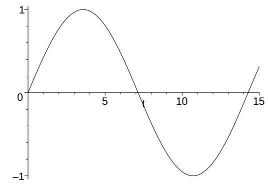
(b) And the angle (phase constant) is $\tan^{-1}(4.70/4.71) = 45°$.

72. (a) The wave number for each wave is $k = 25.1$/m, which means $\lambda = 2\pi/k = 250$ mm. The angular frequency is $\omega = 440$/s; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis (y) is in millimeters.





The following three graphs show the oscillation at $x = \lambda/4 \approx 63$ mm (graph on the left), at $x = 3\lambda/8 \approx 94$ mm (middle graph), and at $x = \lambda/2 \approx 125$ mm.

(b) If we think of wave y_1 as being made of two smaller waves going in the same direction, a wave y_{1a} of amplitude 1.50 mm (the same as y_2) and a wave y_{1b} of amplitude 1.00 mm. It is made clear in §17-11 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves y_{1a} and y_2 form a standing wave, which leaves y_{1b} as the remaining traveling wave. Since the argument of y_{1b} involves the subtraction $kx - \omega t$, then y_{1b} travels in the $+x$ direction.

(c) If y_2 (which travels in the $-x$ direction, which for simplicity will be called “leftward”) had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{\max} = 4.0$ mm and occurs at $x = \lambda/4$, and the smallest amplitude of oscillation is $y_{\min} = 1.0$ mm and occurs at $x = 0$ (and at $x = \lambda/2$).

(e) The largest and smallest amplitudes can be related to the amplitudes of y_1 and y_2 in a simple way: $y_{\max} = y_{1m} + y_{2m}$ and $y_{\min} = y_{1m} - y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

Chapter 18

1. The rule: if you divide the time (in seconds) by 3, then you get (approximately) the straight-line distance d. We note that the speed of sound we are to use is given at the beginning of the problem section in the textbook, and that the speed of light is very much larger than the speed of sound. The proof of our rule is as follows:
$$t = t_{\text{sound}} - t_{\text{light}} \approx t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{d}{343\,\text{m/s}} = \frac{d}{0.343\,\text{km/s}}\ .$$
Cross-multiplying yields (approximately) $(0.3\,\text{km/s})t = d$ which (since $1/3 \approx 0.3$) demonstrates why the rule works fairly well.

2. We denote the speed of light $c = 3.0 \times 10^8$ m/s. The time t_1 it takes for you to hear the music is $t_1 = D_1/v_s = (300\,\text{m})/(343\,\text{m/s}) = 0.87\,\text{s}$. The time t_2 it takes for a listener 5000 km away to hear the music is $t_2 = D_2/c = 5000\,\text{km}/(3 \times 10^5\,\text{km/s}) = 0.02\,\text{s}$. So the listener 5000 km away actually hears the music first! The time difference is $\Delta t = t_1 - t_2 = 0.87\,\text{s} - 0.02\,\text{s} = 0.85\,\text{s}$.

3. (a) The time for the sound to travel from the kicker to a spectator is given by d/v, where d is the distance and v is the speed of sound. The time for light to travel the same distance is given by d/c, where c is the speed of light. The delay between seeing and hearing the kick is $\Delta t = (d/v) - (d/c)$. The speed of light is so much greater than the speed of sound that the delay can be approximated by $\Delta t = d/v$. This means $d = v\,\Delta t$. The distance from the kicker to the first spectator is $d_1 = v\,\Delta t_1 = (343\,\text{m/s})(0.23\,\text{s}) = 79\,\text{m}$. The distance from the kicker to the second spectator is $d_2 = v\,\Delta t_2 = (343\,\text{m/s})(0.12\,\text{s}) = 41\,\text{m}$.

   (b) Lines from the kicker to each spectator and from one spectator to the other form a right triangle with the line joining the spectators as the hypotenuse, so the distance between the spectators is $D = \sqrt{d_1^2 + d_2^2} = \sqrt{(79\,\text{m})^2 + (41\,\text{m})^2} = 89\,\text{m}$.

4. The time it takes for a soldier in the rear end of the column to switch from the left to the right foot to stride forward is $t = 1\,\text{min}/120 = 1/120\,\text{min} = 0.5\,\text{s}$. This is also the time for the sound of the music to reach from the musicians (who are in the front) to the rear end of the column. Thus the length of the column is
$$l = vt = (343\,\text{m/s})(0.5\,\text{s}) = 1.7 \times 10^2\ \text{m}\ .$$

5. If d is the distance from the location of the earthquake to the seismograph and v_s is the speed of the S waves then the time for these waves to reach the seismograph is $t_s = d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is $\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p$, so
$$d = \frac{v_s v_p\,\Delta t}{(v_p - v_s)} = \frac{(4.5\,\text{km/s})(8.0\,\text{km/s})(3.0\,\text{min})(60\,\text{s/min})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1900\ \text{km}\ .$$
We note that values for the speeds were substituted as given, in km/s, but that the value for the time delay was converted from minutes to seconds.

6. (a) The time it takes for sound to travel in air is $t_a = L/v$, while it takes $t_m = L/V$ for the sound to travel in the metal. Thus

$$t = t_a - t_m = \frac{L}{v} - \frac{L}{V} = \frac{L(V - v)}{Vv} .$$

(b) Using the values indicated (see Table 18-1), we obtain

$$L = \frac{t}{1/v - 1/V} = \frac{1.00\,\text{s}}{1/(343\,\text{m/s}) - 1/(5941\,\text{m/s})} = 364\,\text{m} .$$

7. Let t_f be the time for the stone to fall to the water and t_s be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If d is the depth of the well, then the kinematics of free fall gives $d = \frac{1}{2}gt_f^2$, or $t_f = \sqrt{2d/g}$. The sound travels at a constant speed v_s, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for d. Rewrite it as $\sqrt{2d/g} = t - d/v_s$ and square both sides to obtain $2d/g = t^2 - 2(t/v_s)d + (1/v_s^2)d^2$. Now multiply by gv_s^2 and rearrange to get $gd^2 - 2v_s(gt + v_s)d + gv_s^2 t^2 = 0$. This is a quadratic equation for d. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2 v_s^2 t^2}}{2g} .$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7\,\text{m}$ is obtained.

8. At $f = 20\,\text{Hz}$,

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{20\,\text{Hz}} = 17\ \text{m} ,$$

and at $f = 20\,\text{kHz}$,

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{20 \times 10^3\,\text{Hz}} = 1.7 \times 10^{-2}\ \text{m} .$$

9. (a) Using $\lambda = v/f$, where v is the speed of sound in air and f is the frequency, we find

$$\lambda = \frac{343\,\text{m/s}}{4.5 \times 10^6\,\text{Hz}} = 7.62 \times 10^{-5}\ \text{m} .$$

(b) Now, $\lambda = v/f$, where v is the speed of sound in tissue. The frequency is the same for air and tissue. Thus $\lambda = (1500\,\text{m/s})/(4.5 \times 10^6\,\text{Hz}) = 3.33 \times 10^{-4}\,\text{m}$.

10. (a) Since $\lambda = 24\,\text{cm}$, the wave speed is $v = \lambda f = (0.24\text{m})(25\,\text{Hz}) = 6.0\,\text{m/s}$.

(b) With x in centimeters and t in seconds, the equation for the wave is

$$y = A\sin[2\pi(x/\lambda + ft)] = (0.30\,\text{cm})\sin\left(\frac{\pi}{12}x + 50\pi t\right) .$$

11. (a) The amplitude of a sinusoidal wave is the numerical coefficient of the sine (or cosine) function: $p_m = 1.50$ Pa.

(b) From the theory presented in Ch. 17, we identify $k = 0.9\pi$ and $\omega = 315\pi$ (in SI units), which leads to $f = \omega/2\pi = 158$ Hz.

(c) We also obtain $\lambda = 2\pi/k = 2.22$ m.

(d) The speed of the wave is $v = \omega/k = 350$ m/s.

12. It is useful to study Sample Problem 18-3 before working this problem. We label the two point sources 1 and 2 and assume they are on the x axis (a distance $D = 2\lambda$ apart). When we refer to the circle of large radius, we are assuming that a line drawn from source 1 to a point on the circle and a line drawn to it from source 2 are approximately parallel (and thus both at angle θ measured from the y axis). In terms of the theory developed in §18-4, we find that the phase difference at P (on the large circle of radius R) for the two waves emitted from 1 and 2 is

$$\Delta\phi \approx \frac{2\pi\Delta x}{\lambda} = \frac{2\pi D\sin\theta}{\lambda} = 4\pi\sin\theta\ .$$

   (a) For maximum signal, we set $\Delta\phi = 2m\pi$ ($m = 0, \pm 1, \pm 2, \ldots$) to obtain $\sin\theta = m/2$. Thus we get a total of 8 possible values of θ between 0 and 2π, given by $\theta = 0,\ \sin^{-1}(1/2) = 30°,\ \sin^{-1}(1) = 90°$ and (using symmetry properties of the sine function) $150°, 180°, 210°, 270°$, and $330°$.

   (b) Since there must be a mininum in between two successive maxima, the total number of minima is also eight.

13. Let L_1 be the distance from the closer speaker to the listener. The distance from the other speaker to the listener is $L_2 = \sqrt{L_1^2 + d^2}$, where d is the distance between the speakers. The phase difference at the listener is $\phi = 2\pi(L_2 - L_1)/\lambda$, where λ is the wavelength.

   (a) For a minimum in intensity at the listener, $\phi = (2n+1)\pi$, where n is an integer. Thus $\lambda = 2(L_2 - L_1)/(2n+1)$. The frequency is

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{2\left(\sqrt{L_1^2+d^2} - L_1\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz})\ .$$

   Now $20,000/343 = 58.3$, so $2n+1$ must range from 0 to 57 for the frequency to be in the audible range. This means n ranges from 1 to 28 and $f = 1029,\ 1715,\ \ldots,\ 19550\,\text{Hz}$.

   (b) For a maximum in intensity at the listener, $\phi = 2n\pi$, where n is any positive integer. Thus $\lambda = (1/n)\left(\sqrt{L_1^2+d^2} - L_1\right)$ and

$$f = \frac{v}{\lambda} = \frac{nv}{\sqrt{L_1^2+d^2} - L_1} = \frac{n(343\,\text{m/s})}{\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}} = n(686\,\text{Hz})\ .$$

   Since $20,000/686 = 29.2$, n must be in the range from 1 to 29 for the frequency to be audible and $f = 686,\ 1372,\ \ldots,\ 19890\,\text{Hz}$.

14. Let the separation between the point and the two sources (labeled 1 and 2) be x_1 and x_2, respectively. Then the phase difference is

$$\begin{aligned}\Delta\phi &= \phi_1 - \phi_2 = 2\pi\left(\frac{x_1}{\lambda} + ft\right) - 2\pi\left(\frac{x_2}{\lambda} + ft\right) = \frac{2\pi(x_1 - x_2)}{\lambda} \\ &= \frac{2\pi(4.40\,\text{m} - 4.00\,\text{m})}{(330\,\text{m/s})/540\,\text{Hz}} = 4.12\ \text{rad}\ .\end{aligned}$$

15. (a) Building on the theory developed in $\S 18-4$, we set $\Delta L/\lambda = \frac{1}{2}$(odd numbers) in order to have destructive interference. Since $v = f\lambda$, we can write this in terms of frequency:

$$f = \frac{(\text{odd number})v}{2\Delta L} = \begin{cases} 143\,\text{Hz} & \text{for } n = 1 \\ 429\,\text{Hz} & \text{for } n = 3 \\ 715\,\text{Hz} & \text{for } n = 5 \end{cases}$$

   where we have used $v = 343$ m/s (note the remarks made in the textbook at the beginning of the exercises and problems section) and $\Delta L = 19.5 - 18.3 = 1.2$ m.

(b) Now we set $\Delta L/\lambda = \frac{1}{2}$(even numbers) – which can be written more simply as "(all integers)" – in order to establish constructive interference. Thus,

$$f = \frac{(\text{integer})v}{\Delta L} = \begin{cases} 286\,\text{Hz} & \text{for } n = 1 \\ 572\,\text{Hz} & \text{for } n = 2 \\ 858\,\text{Hz} & \text{for } n = 3 \end{cases} .$$

16. At the location of the detector, the phase difference between the wave which traveled straight down the tube and the other one which took the semi-circular detour is

$$\Delta\phi = k\Delta d = \frac{2\pi}{\lambda}(\pi r - 2r) .$$

For $r = r_{\min}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus

$$r_{\min} = \frac{\lambda}{2(\pi - 2)} = \frac{40.0\,\text{cm}}{2(\pi - 2)} = 17.5 \text{ cm} .$$

17. The intensity is the rate of energy flow per unit area perpendicular to the flow. The rate at which energy flows across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If P is the power output and I is the intensity a distance r from the source, then $P = IA = 4\pi r^2 I$, where A $(= 4\pi r^2)$ is the surface area of a sphere of radius r. Thus $P = 4\pi(2.50\,\text{m})^2(1.91 \times 10^{-4}\,\text{W/m}^2) = 1.50 \times 10^{-2}\,\text{W}$.

18. (a) Since intensity is power divided by area, and for an isotropic source the area may be written $A = 4\pi r^2$ (the area of a sphere), then we have

$$I = \frac{P}{A} = \frac{1.0\,\text{W}}{4\pi(1.0\,\text{m})^2} = 0.080 \text{ W/m}^2 .$$

(b) This calculation may be done exactly as shown in part (a) (but with $r = 2.5$ m instead of $r = 1.0$ m), or it may be done by setting up a ratio. We illustrate the latter approach. Thus,

$$\frac{I'}{I} = \frac{P/4\pi(r')^2}{P/4\pi r^2} = \left(\frac{r}{r'}\right)^2$$

leads to $I' = (0.080\,\text{W/m}^2)(1/2.5)^2 = 0.013\,\text{W/m}^2$.

19. The intensity is given by $I = \frac{1}{2}\rho v \omega^2 s_m^2$, where ρ is the density of air, v is the speed of sound in air, ω is the angular frequency, and s_m is the displacement amplitude for the sound wave. Replace ω with $2\pi f$ and solve for s_m:

$$s_m = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \sqrt{\frac{1.00 \times 10^{-6}\,\text{W/m}^2}{2\pi^2(1.21\,\text{kg/m}^3)(343\,\text{m/s})(300\,\text{Hz})^2}} = 3.68 \times 10^{-8} \text{ m} .$$

20. Sample Problem 18-5 shows that a decibel difference $\Delta\beta$ is directly related to an intensity ratio (which we write as $\mathcal{R} = I'/I$). Thus,

$$\Delta\beta = 10\log(\mathcal{R}) \implies \mathcal{R} = 10^{\Delta\beta/10} = 10^{0.1} = 1.26 .$$

21. (a) Let I_1 be the original intensity and I_2 be the final intensity. The original sound level is $\beta_1 = (10\,\text{dB})\log(I_1/I_0)$ and the final sound level is $\beta_2 = (10\,\text{dB})\log(I_2/I_0)$, where I_0 is the reference intensity. Since $\beta_2 = \beta_1 + 30\,\text{dB}$, $(10\,\text{dB})\log(I_2/I_0) = (10\,\text{dB})\log(I_1/I_0) + 30\,\text{dB}$, or $(10\,\text{dB})\log(I_2/I_0) - (10\,\text{dB})\log(I_1/I_0) = 30\,\text{dB}$. Divide by 10 dB and use $\log(I_2/I_0) - \log(I_1/I_0) = \log(I_2/I_1)$ to obtain $\log(I_2/I_1) = 3$. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2/I_1$. The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of 1000.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} = 32$.

22. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at $r = 3.00$ m,

$$I = \frac{1.00 \times 10^{-6}\,\text{W}}{4\pi(3.00\,\text{m})^2} = 8.84 \times 10^{-9}\ \text{W/m}^2 .$$

(b) The sound level there is

$$\beta = 10\log\left(\frac{8.84 \times 10^{-9}\,\text{W/m}^2}{1.00 \times 10^{-12}\,\text{W/m}^2}\right) = 39.5\,\text{dB} .$$

23. (a) The intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where ρ is the density of the medium, v is the speed of sound, ω is the angular frequency, and s_m is the displacement amplitude. The displacement and pressure amplitudes are related by $\Delta p_m = \rho v\omega s_m$, so $s_m = \Delta p_m/\rho v\omega$ and $I = (\Delta p_m)^2/2\rho v$. For waves of the same frequency the ratio of the intensity for propagation in water to the intensity for propagation in air is

$$\frac{I_w}{I_a} = \left(\frac{\Delta p_{mw}}{\Delta p_{ma}}\right)^2 \frac{\rho_a v_a}{\rho_w v_w} ,$$

where the subscript a denotes air and the subscript w denotes water. Since $I_a = I_w$,

$$\frac{\Delta p_{mw}}{\Delta p_{ma}} = \sqrt{\frac{\rho_w v_w}{\rho_a v_a}} = \sqrt{\frac{(0.998 \times 10^3\,\text{kg/m}^3)(1482\,\text{m/s})}{(1.21\,\text{kg/m}^3)(343\,\text{m/s})}} = 59.7 .$$

The speeds of sound are given in Table 18–1 and the densities are given in Table 15–1.

(b) Now, $\Delta p_{mw} = \Delta p_{ma}$, so

$$\frac{I_w}{I_a} = \frac{\rho_a v_a}{\rho_w v_w} = \frac{(1.21\,\text{kg/m}^3)(343\,\text{m/s})}{(0.998 \times 10^3\,\text{kg/m}^3)(1482\,\text{m/s})} = 2.81 \times 10^{-4} .$$

24. Since the power of the sound emitted from a section of the source with unit length is related to I by $P = IA = 2\pi r I(r)$, then we have $I(r) = P/(2\pi r) \propto r^{-1}$. And since $s_m \propto \sqrt{I}$ (by Eq. 18-27), then the fact that $I \propto r^{-1}$ in this situation leads to $s_m \propto r^{-1/2}$.

25. (a) We take the wave to be a plane wave and consider a region formed by the surface of a rectangular solid, with two plane faces of area A perpendicular to the direction of travel and separated by a distance d, along the direction of travel. The energy contained in this region is $U = uAd$. If the wave speed is v then all the energy passes through one end of the region in time $t = d/v$. The energy passing through per unit time is $U/t = uAdv/d = uvA$. The intensity is the energy passing through per unit time, per unit area, or $I = U/tA = uv$.

(b) The power output P of the source equals the rate at which energy crosses the surface of any sphere centered at the source. It is related to the intensity I a distance r away by $P = AI = 4\pi r^2 I$, where A $(= 4\pi r^2)$ is the surface area of a sphere of radius r. Substitute $I = uv$ to obtain $P = 4\pi r^2 uv$, then solve for u:

$$u = \frac{P}{4\pi r^2 v} = \frac{50{,}000\,\text{W}}{4\pi(480 \times 10^3\,\text{m})^2(3.00 \times 10^8\,\text{m/s})} = 5.76 \times 10^{-17}\ \text{J/m}^3 .$$

26. We use $\Delta\beta_{12} = \beta_1 - \beta_2 = (10\,\text{dB})\log(I_1/I_2)$.

(a) Since $\Delta\beta_{12} = (10\,\text{dB})\log(I_1/I_2) = 37\,\text{dB}$, we get $I_1/I_2 = 10^{37\,\text{dB}/10\,\text{dB}} = 10^{3.7} = 5.0 \times 10^3$.

(b) Since $\Delta p_m \propto s_m \propto \sqrt{I}$, we have $\Delta p_{m\,1}/\Delta p_{m\,2} = \sqrt{I_1/I_2} = \sqrt{5.0 \times 10^3} = 71$.

(c) The displacement amplitude ratio is $s_{m\,1}/s_{m\,2} = \sqrt{I_1/I_2} = 71$.

27. (a) Let P be the power output of the source. This is the rate at which energy crosses the surface of any sphere centered at the source and is therefore equal to the product of the intensity I at the sphere surface and the area of the sphere. For a sphere of radius r, $P = 4\pi r^2 I$ and $I = P/4\pi r^2$. The intensity is proportional to the square of the displacement amplitude s_m. If we write $I = Cs_m^2$, where C is a constant of proportionality, then $Cs_m^2 = P/4\pi r^2$. Thus $s_m = \sqrt{P/4\pi r^2 C} = \left(\sqrt{P/4\pi C}\right)(1/r)$. The displacement amplitude is proportional to the reciprocal of the distance from the source. We take the wave to be sinusoidal. It travels radially outward from the source, with points on a sphere of radius r in phase. If ω is the angular frequency and k is the angular wave number then the time dependence is $\sin(kr - \omega t)$. Letting $b = \sqrt{P/4\pi C}$, the displacement wave is then given by

$$s(r,t) = \sqrt{\frac{P}{4\pi C}}\,\frac{1}{r}\sin(kr - \omega t) = \frac{b}{r}\sin(kr - \omega t)\ .$$

(b) Since s and r both have dimensions of length and the trigonometric function is dimensionless, the dimensions of b must be length squared.

28. (a) The intensity is

$$I = \frac{P}{4\pi r^2} = \frac{30.0\,\mathrm{W}}{(4\pi)(200\,\mathrm{m})^2} = 5.97\times 10^{-5}\ \mathrm{W/m^2}\ .$$

(b) Let $A\,(= 0.750\,\mathrm{cm^2})$ be the cross-sectional area of the microphone. Then the power intercepted by the microphone is

$$P' = IA = 0 = \left(6.0\times 10^{-5}\,\mathrm{W/m^2}\right)\left(0.750\,\mathrm{cm^2}\right)\left(10^{-4}\,\mathrm{m^2/cm^2}\right) = 4.48\times 10^{-9}\ \mathrm{W}\ .$$

29. (a) When the right side of the instrument is pulled out a distance d the path length for sound waves increases by $2d$. Since the interference pattern changes from a minimum to the next maximum, this distance must be half a wavelength of the sound. So $2d = \lambda/2$, where λ is the wavelength. Thus $\lambda = 4d$ and, if v is the speed of sound, the frequency is $f = v/\lambda = v/4d = (343\,\mathrm{m/s})/4(0.0165\,\mathrm{m}) = 5.2\times 10^3\,\mathrm{Hz}$.

(b) The displacement amplitude is proportional to the square root of the intensity (see Eq. 18–27). Write $\sqrt{I} = Cs_m$, where I is the intensity, s_m is the displacement amplitude, and C is a constant of proportionality. At the minimum, interference is destructive and the displacement amplitude is the difference in the amplitudes of the individual waves: $s_m = s_{SAD} - s_{SBD}$, where the subscripts indicate the paths of the waves. At the maximum, the waves interfere constructively and the displacement amplitude is the sum of the amplitudes of the individual waves: $s_m = s_{SAD} + s_{SBD}$. Solve $\sqrt{100} = C(s_{SAD} - s_{SBD})$ and $\sqrt{900} = C(s_{SAD} + s_{SBD})$ for s_{SAD} and s_{SBD}. Add the equations to obtain $s_{SAD} = (\sqrt{100} + \sqrt{900})/2C = 20/C$, then subtract them to obtain $s_{SBD} = (\sqrt{900} - \sqrt{100})/2C = 10/C$. The ratio of the amplitudes is $s_{SAD}/s_{SBD} = 2$.

(c) Any energy losses, such as might be caused by frictional forces of the walls on the air in the tubes, result in a decrease in the displacement amplitude. Those losses are greater on path B since it is longer than path A.

30. (a) From Eq. 17-53, we have

$$f = \frac{nv}{2L} = \frac{(1)(250\,\mathrm{m/s})}{2(0.150\,\mathrm{m})} = 833\ \mathrm{Hz}\ .$$

(b) The frequency of the wave on the string is the same as the frequency of the sound wave it produces during its vibration. Consequently, the wavelength in air is

$$\lambda = \frac{v_{\mathrm{sound}}}{f} = \frac{348\,\mathrm{m/s}}{833\,\mathrm{Hz}} = 0.418\ \mathrm{m}\ .$$

31. At the beginning of the exercises and problems section in the textbook, we are told to assume $v_{\text{sound}} =$ 343 m/s unless told otherwise. The second harmonic of pipe A is found from Eq. 18-39 with $n = 2$ and $L = L_A$, and the third harmonic of pipe B is found from Eq. 18-41 with $n = 3$ and $L = L_B$. Since these frequencies are equal, we have

$$\frac{2v_{\text{sound}}}{2L_A} = \frac{3v_{\text{sound}}}{4L_B} \implies L_B = \frac{3}{4}L_A .$$

   (a) Since the fundamental frequency for pipe A is 300 Hz, we immediately know that the second harmonic has $f = 2(300) = 600$ Hz. Using this, Eq. 18-39 gives $L_A = (2)(343)/2(600) = 0.572$ m.

   (b) The length of pipe B is $L_B = \frac{3}{4}L_A = 0.429$ m.

32. The frequency is $f = 686$ Hz. At the beginning of the exercises and problems section in the textbook, we are told to assume $v_{\text{sound}} = 343$ m/s unless told otherwise. If L is the length of the air-column (so that the water height is $h = 1.00\,\text{m} - L$) then Eq. 18-41 leads to

$$L = \frac{nv}{4f} \implies h = 1.00 - L = \begin{cases} 0.875\,\text{m} & \text{for } n = 1 \\ 0.625\,\text{m} & \text{for } n = 3 \\ 0.375\,\text{m} & \text{for } n = 5 \\ 0.125\,\text{m} & \text{for } n = 7 \end{cases} .$$

33. (a) When the string (fixed at both ends) is vibrating at its lowest resonant frequency, exactly one-half of a wavelength fits between the ends. Thus, $\lambda = 2L$. We obtain $v = f\lambda = 2Lf = 2(0.220\,\text{m})(920\,\text{Hz}) = 405\,\text{m/s}$.

   (b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. If M is the mass of the (uniform) string, then $\mu = M/L$. Thus $\tau = \mu v^2 = (M/L)v^2 = \left[(800 \times 10^{-6}\,\text{kg})/(0.220\,\text{m})\right](405\,\text{m/s})^2 = 596\,\text{N}$.

   (c) The wavelength is $\lambda = 2L = 2(0.220\,\text{m}) = 0.440\,\text{m}$.

   (d) The frequency of the sound wave in air is the same as the frequency of oscillation of the string. The wavelength is different because the wave speed is different. If v_a is the speed of sound in air the wavelength in air is $\lambda_a = v_a/f = (343\,\text{m/s})/(920\,\text{Hz}) = 0.373\,\text{m}$.

34. (a) The fundamental frequency of a string can be increased (for instance, going from A up to C) by shortening the length of the vibrating portion of the string. When the note C is played, the vibrating length is (using Eq. 17-53)

$$\frac{f'}{f} = \frac{nv/2L'}{nv/2L} \implies L = (30\,\text{cm})\left(\frac{440\,\text{Hz}}{528\,\text{Hz}}\right) = 25\text{ cm} .$$

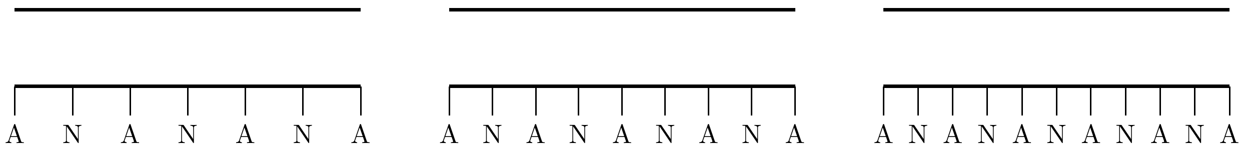
   Thus, one should place his finger a distance of $30\,\text{cm} - 25\,\text{cm} = 5\,\text{cm}$ from one end of the string.

   (b) Since $v = f\lambda$, the ratio of wavelengths is the reciprocal of the frequency ratio, so that $\lambda_{\text{A}}/\lambda_{\text{C}} = 528\,\text{Hz}/440\,\text{Hz} = 1.2$.

   (c) This has the same answer as part (b), due to the fact that the frequencies are the same on the string and the air (transmitting a signal from one medium to another does not generally change its frequency. Both wavelengths are larger (*much* larger) in the air than on the string, but their ratio (due to $v = f\lambda$) remains the same.

35. (a) Since the pipe is open at both ends there are displacement antinodes at both ends and an integer number of half-wavelengths fit into the length of the pipe. If L is the pipe length and λ is the wavelength then $\lambda = 2L/n$, where n is an integer. If v is the speed of sound then the resonant frequencies are given by $f = v/\lambda = nv/2L$. Now $L = 0.457\,\text{m}$, so $f = n(344\,\text{m/s})/2(0.457\,\text{m}) = 376.4n\,\text{Hz}$. To find the resonant frequencies that lie between 1000 Hz and 2000 Hz, first set $f = 1000\,\text{Hz}$ and solve for n, then set $f = 2000\,\text{Hz}$ and again solve for n. You should get 2.66 and 5.32. This means $n = 3$, 4, and 5 are the appropriate values of n. For $n = 3$, $f = 3(376.4\,\text{Hz}) = 1129\,\text{Hz}$; for $n = 4$, $f = 4(376.4\,\text{Hz}) = 1526\,\text{Hz}$; and for $n = 5$, $f = 5(376.4\,\text{Hz}) = 1882\,\text{Hz}$.

(b) For any integer value of n the displacement has n nodes and $n+1$ antinodes, counting the ends. The nodes (N) and antinodes (A) are marked on the diagrams below for the three resonances found in part (a).



$n = 3$ $n = 4$ $n = 5$

36. (a) Using Eq. 17-53 with $n = 1$ (for the fundamental mode of vibration), we obtain

$$\frac{f'}{f} = \frac{(1)v/2L'}{(1)v/2L} = \frac{L}{L'}$$

so that $f' = rf$ (where r is a pure number) implies $L' = L/r$. Thus, the amount if must be shortened is $l = \Delta L = L - L' = L(1 - 1/r)$.

(b) With $L = 80$ cm and $r = 1.2$, this yields $l = 13$ cm.

(c) Since $v = f\lambda$, the ratio of wavelengths is the reciprocal of the ratio of frequencies: $\lambda'/\lambda = f/f' = 1/1.2 = 5/6$. This ratio applies to the wavelength ratio for the vibrating string and also for the wavelength ratio for the emitted sound waves (due to the fact that the frequency of a signal is generally not altered when transmitted from one medium to another).

37. The top of the water is a displacement node and the top of the well is a displacement antinode. At the lowest resonant frequency exactly one-fourth of a wavelength fits into the depth of the well. If d is the depth and λ is the wavelength then $\lambda = 4d$. The frequency is $f = v/\lambda = v/4d$, where v is the speed of sound. The speed of sound is given by $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of air in the well. Thus $f = (1/4d)\sqrt{B/\rho}$ and

$$d = \frac{1}{4f}\sqrt{\frac{B}{\rho}} = \left[\frac{1}{4(7.00\,\text{Hz})}\right]\sqrt{\frac{1.33\times10^5\,\text{Pa}}{1.10\,\text{kg/m}^3}} = 12.4 \text{ m} .$$

38. (a) Using Eq. 18-39 with $n = 1$ (for the fundamental mode of vibration) and 343 m/s for the speed of sound, we obtain

$$f = \frac{(1)v_{\text{sound}}}{4L_{\text{tube}}} = \frac{343\,\text{m/s}}{4(1.20\,\text{m})} = 71.5 \text{ Hz} .$$

(b) For the wire (using Eq. 17-53) we have

$$f' = \frac{nv_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{\tau}{\mu}}$$

where $\mu = m_{\text{wire}}/L_{\text{wire}}$. Recognizing that $f = f'$ (both the wire and the air in the tube vibrate at the same frequency), we solve this for the tension τ:

$$\tau = (2L_{\text{wire}}f)^2\left(\frac{m_{\text{wire}}}{L_{\text{wire}}}\right) = 4f^2 m_{\text{wire}}L_{\text{wire}} = 4(71.5\,\text{Hz})^2\left(9.60\times10^{-3}\,\text{kg}\right)(0.33\,\text{m}) = 64.8 \text{ N} .$$

39. (a) We expect the center of the star to be a displacement node. The star has spherical symmetry and the waves are spherical. If matter at the center moved it would move equally in all directions and this is not possible.

(b) We assume the oscillation is at the lowest resonance frequency. Then, exactly one-fourth of a wavelength fits the star radius. If λ is the wavelength and R is the star radius then $\lambda = 4R$. The frequency is $f = v/\lambda = v/4R$, where v is the speed of sound in the star. The period is $T = 1/f = 4R/v$.

(c) The speed of sound is $v = \sqrt{B/\rho}$, where B is the bulk modulus and ρ is the density of stellar material. The radius is $R = 9.0 \times 10^{-3} R_s$, where R_s is the radius of the Sun (6.96×10^8 m). Thus

$$T = 4R\sqrt{\frac{\rho}{B}} = 4(9.0 \times 10^{-3})(6.96 \times 10^8\,\text{m})\sqrt{\frac{1.0 \times 10^{10}\,\text{kg/m}^3}{1.33 \times 10^{22}\,\text{Pa}}} = 22\ \text{s}\ .$$

40. We observe that "third lowest ... frequency" corresponds to harmonic number $n = 3$ for a pipe open at both ends. Also, "second lowest ... frequency" corresponds to harmonic number $n = 3$ for a pipe closed at one end.

(a) Since $\lambda = 2L/n$ for pipe A, where $L = 1.2$ m, then $\lambda = 0.80$ m for this mode. The change from node to antinode requires a distance of $\lambda/4$ so that every increment of 0.20 m along the x axis involves a switch between node and antinode. Since the opening is a displacement antinode, then the locations for displacement nodes are at $x = 0.20$ m, $x = 0.60$ m, and $x = 1.0$ m.

(b) The waves in both pipes have the same wavespeed (sound in air) and frequency, so the standing waves in both pipes have the same wavelength (0.80 m). Therefore, using Eq. 18-38 for pipe B, we find $L = 3\lambda/4 = 0.60$ m.

(c) Using $v = 343$ m/s, we find $f_3 = v/\lambda = 429$ Hz. Now, we find the fundamental resonant frequency by dividing by the harmonic number, $f_1 = f_3/3 = 143$ Hz.

41. The string is fixed at both ends so the resonant wavelengths are given by $\lambda = 2L/n$, where L is the length of the string and n is an integer. The resonant frequencies are given by $f = v/\lambda = nv/2L$, where v is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Thus $f = (n/2L)\sqrt{\tau/\mu}$. Suppose the lower frequency is associated with $n = n_1$ and the higher frequency is associated with $n = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus $f_1 = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_2 = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_1 + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}\ .$$

This means $f_2 - f_1 = (1/2L)\sqrt{\tau/\mu}$ and

$$\begin{aligned} \tau &= 4L^2\mu(f_2 - f_1)^2 \\ &= 4(0.300\,\text{m})^2(0.650 \times 10^{-3}\,\text{kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 \\ &= 45.3\ \text{N}\ . \end{aligned}$$

42. Let the period be T. Then the beat frequency is $\frac{1}{T} - 440\,\text{Hz} = 4.00\,\text{beats/s}$. Therefore, $T = 2.25 \times 10^{-3}$ s. The string that is "too tightly stretched" has the higher tension and thus the higher (fundamental) frequency.

43. Since the beat frequency equals the difference between the frequencies of the two tuning forks, the frequency of the first fork is either 381 Hz or 387 Hz. When mass is added to this fork its frequency decreases (recall, for example, that the frequency of a mass-spring oscillator is proportional to $1/\sqrt{m}$). Since the beat frequency also decreases the frequency of the first fork must be greater than the frequency of the second. It must be 387 Hz.

44. (a) The number of different ways of picking up a pair of tuning forks out of a set of five is $5!/(2!3!) = 10$. For each of the pairs selected, there will be one beat frequency. If these frequencies are all different from each other, we get the maximum possible number of 10.

(b) First, we note that the minimum number occurs when the frequencies of these forks, labeled 1 through 5, increase in equal increments: $f_n = f_1 + n\Delta f$, where $n = 2, 3, 4, 5$. Now, there are only 4 different beat frequencies: $f_{\text{beat}} = n\Delta f$, where $n = 1, 2, 3, 4$.

45. Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is $f = v/\lambda = (1/2L)\sqrt{\tau/\mu}$, where v $(= \sqrt{\tau/\mu})$ is the wave speed for the wire, τ is the tension in the wire, and μ is the linear mass density of the wire. Suppose the tension in one wire is τ and the oscillation frequency of that wire is f_1. The tension in the other wire is $\tau + \Delta\tau$ and its frequency is f_2. You want to calculate $\Delta\tau/\tau$ for $f_1 = 600\,\text{Hz}$ and $f_2 = 606\,\text{Hz}$. Now, $f_1 = (1/2L)\sqrt{\tau/\mu}$ and $f_2 = (1/2L)\sqrt{(\tau + \Delta\tau)/\mu}$, so

$$f_2/f_1 = \sqrt{(\tau + \Delta\tau)/\tau} = \sqrt{1 + (\Delta\tau/\tau)} \ .$$

This leads to

$$\Delta\tau/\tau = (f_2/f_1)^2 - 1 = [(606\,\text{Hz})/(600\,\text{Hz})]^2 - 1 = 0.020 \ .$$

46. The Doppler effect formula, Eq. 18-47, and its accompanying rule for choosing $\pm$ signs, are discussed in §18-8. Using that notation, we have $v = 343$ m/s, $v_D = v_S = 160000/3600 = 44.4$ m/s, and $f = 500$ Hz. Thus,

$$f' = (500)\left(\frac{343 - 44.4}{343 - 44.4}\right) = 500\,\text{Hz} \quad \Longrightarrow \quad \Delta f = 0 \ .$$

47. The detector (the second plane) is moving toward the source (the first plane). This tends to increase the frequency, so we use the plus sign in the numerator of Eq. 18-47. The source is moving away from the detector. This tends to decrease the frequency, so we use the plus sign in the denominator of Eq. 18-47. Thus

$$f' = f\,\frac{v + v_D}{v + v_S} = (16000\,\text{Hz})\left(\frac{343\,\text{m/s} + 250\,\text{m/s}}{343\,\text{m/s} + 200\,\text{m/s}}\right) = 17500\ \text{Hz} \ .$$

48. The Doppler effect formula, Eq. 18-47, and its accompanying rule for choosing $\pm$ signs, are discussed in §18-8. Using that notation, we have $v = 343$ m/s, $v_D = 2.44$ m/s, $f' = 1590$ Hz and $f = 1600$ Hz. Thus,

$$f' = f\left(\frac{v + v_D}{v + v_S}\right) \quad \Longrightarrow \quad v_S = \frac{f}{f'}\,(v + v_D) - v = 4.61\ \text{m/s} \ .$$

49. We use $v_S = r\omega$ (with $r = 0.600$ m and $\omega = 15.0$ rad/s) for the linear speed during circular motion, and Eq. 18-47 for the Doppler effect (where $f = 540$ Hz, and $v = 343$ m/s for the speed of sound).

$$f' = f\left(\frac{v + 0}{v \pm v_S}\right) = \begin{cases} 526\,\text{Hz} & \text{for + choice} \\ 555\,\text{Hz} & \text{for − choice} \end{cases}$$

50. We are combining two effects: the reception of a moving object (the truck of speed $u = 45.0$ m/s) of waves emitted by a stationary object (the motion detector), and the subsequent emission of those waves by the moving object (the truck) which are picked up by the stationary detector. This could be figured in two steps, but is more compactly computed in one step as shown here:

$$f_{\text{final}} = f_{\text{initial}}\left(\frac{v + u}{v - u}\right) = (0.150\,\text{MHz})\left(\frac{343\,\text{m/s} + 45\,\text{m/s}}{343\,\text{m/s} - 45\,\text{m/s}}\right) = 0.195\ \text{MHz} \ .$$

51. We denote the speed of the French submarine by u_1 and that of the U.S. sub by u_2.

(a) The frequency as detected by the U.S. sub is

$$f_1' = f_1\left(\frac{v + u_2}{v - u_1}\right) = (1000\,\text{Hz})\left(\frac{5470 + 70}{5470 - 50}\right) = 1.02 \times 10^3\ \text{Hz} \ .$$

(b) If the French sub were stationary, the frequency of the reflected wave would be $f_r = f_1(v+u_2)/(v-u_2)$. Since the French sub is moving towards the reflected signal with speed u_1, then

$$\begin{aligned} f_r' &= f_r\left(\frac{v+u_1}{v}\right) = f_1\frac{(v+u_1)(v+u_2)}{v(v-u_2)} \\ &= \frac{(1000\,\text{Hz})(5470+50)(5470+70)}{(5470)(5470-70)} \\ &= 1.04\times 10^3 \text{ Hz} . \end{aligned}$$

52. We use Eq. 18-47 with $f = 1200$ Hz and $v = 329$ m/s.

(a) In this case, $v_D = 65.8$ m/s and $v_S = 29.9$ m/s, and we choose signs so that f' is larger than f:

$$f' = f\left(\frac{329+65.8}{329-29.9}\right) = 1584 \text{ Hz} .$$

(b) The wavelength is $\lambda = v/f' = 0.208$ m.

(c) The wave (of frequency f') "emitted" by the moving reflector (now treated as a "source," so $v_S = 65.8$ m/s) is returned to the detector (now treated as a detector, so $v_D = 29.9$ m/s) and registered as a new frequency f'':

$$f'' = f'\left(\frac{329+29.9}{329-65.8}\right) = 2160 \text{ Hz} .$$

(d) This has wavelength $v/f'' = 0.152$ m.

53. In this case, the intruder is moving *away* from the source with a speed u satisfying $u/v \ll 1$. The Doppler shift (with $u = -0.950\,\text{m/s}$) leads to

$$f_{\text{beat}} = |f_r - f_s| \approx \frac{2|u|}{v}f_s = \frac{2(0.95\,\text{m/s})(28.0\,\text{kHz})}{343\,\text{m/s}}) = 155 \text{ Hz} .$$

54. As a result of the Doppler effect, the frequency of the reflected sound as heard by the bat is

$$f_r = f'\left(\frac{v+u_{\text{bat}}}{v-u_{\text{bat}}}\right) = (39000\,\text{Hz})\left(\frac{v+v/40}{v-v/40}\right) = 41000 \text{ Hz} .$$

55. (a) The expression for the Doppler shifted frequency is

$$f' = f\,\frac{v \pm v_D}{v \mp v_S} ,$$

where f is the unshifted frequency, v is the speed of sound, v_D is the speed of the detector (the uncle), and v_S is the speed of the source (the locomotive). All speeds are relative to the air. The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is $v_S = 10\,\text{m/s}$. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\,\frac{v}{v+v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s}+10.00\,\text{m/s}}\right) = 485.8 \text{ Hz} .$$

(b) The girl is now the detector. Relative to the air she is moving with speed $v_D = 10.00\,\text{m/s}$ toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at $v_S = 10.00\,\text{m/s}$ away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus $(v+v_D) = (v+v_S)$ and $f' = f = 500.0\,\text{Hz}$.

(c) Relative to the air the locomotive is moving at $v_S = 20.00\,\text{m/s}$ away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at $v_D = 10.00\,\text{m/s}$ toward the locomotive. Use the plus sign in the numerator. Thus

$$f' = f\,\frac{v + v_D}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s} + 10.00\,\text{m/s}}{343\,\text{m/s} + 20.00\,\text{m/s}}\right) = 486.2\,\text{Hz}\ .$$

(d) Relative to the air the locomotive is moving at $v_S = 20.00\,\text{m/s}$ away from the girl and the girl is moving at $v_D = 20.00\,\text{m/s}$ toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0\,\text{Hz}$.

56. The Doppler shift formula, Eq. 18-47, is valid only when both u_S and u_D are measured with respect to a stationary medium (i.e., no wind). To modify this formula in the presence of a wind, we switch to a new reference frame in which there is no wind.

(a) When the wind is blowing from the source to the observer with a speed w, we have $u'_S = u'_D = w$ in the new reference frame that moves together with the wind. Since the observer is now approaching the source while the source is backing off from the observer, we have, in the new reference frame,

$$f' = f\left(\frac{v + u'_D}{v + u'_S}\right) = f\left(\frac{v + w}{v + w}\right) = 2000 \text{ Hz}\ .$$

In other words, there is no Doppler shift.

(b) In this case, all we need to do is to reverse the signs in front of both u'_D and u'_S. The result is that there is still no Doppler shift:

$$f' = f\left(\frac{v - u'_D}{v - u'_S}\right) = f\left(\frac{v - w}{v - w}\right) = 2000 \text{ Hz}\ .$$

In general, there will always be no Doppler shift as long as there is no relative motion between the observer and the source, regardless of whether a wind is present or not.

57. We use Eq. 18-47 with $f = 500$ Hz and $v = 343$ m/s. We choose signs to produce $f' > f$.

(a) The frequency heard in still air is

$$f' = 500\left(\frac{343 + 30.5}{343 - 30.5}\right) = 598 \text{ Hz}\ .$$

(b) In a frame of reference where the air seems still, the velocity of the detector is $30.5 - 30.5 = 0$, and that of the source is $2(30.5)$. Therefore,

$$f' = 500\left(\frac{343 + 0}{343 - 2(30.5)}\right) = 608 \text{ Hz}\ .$$

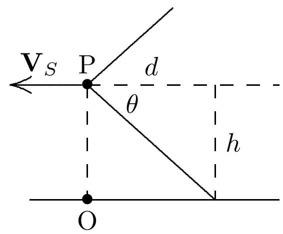
(c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is $30.5 - 30.5 = 0$, and that of the detector is $2(30.5)$. Consequently,

$$f' = 500\left(\frac{343 + 2(30.5)}{343 - 0)}\right) = 589 \text{ Hz}\ .$$

58. The angle is $\sin^{-1}(v/v_s) = sin^{-1}(343/685) = 30°$.

59. (a) The half angle θ of the Mach cone is given by $\sin\theta = v/v_S$, where v is the speed of sound and v_S is the speed of the plane. Since $v_S = 1.5v$, $\sin\theta = v/1.5v = 1/1.5$. This means $\theta = 42°$.

(b) Let h be the altitude of the plane and suppose the Mach cone intersects Earth's surface a distance d behind the plane. The situation is shown on the diagram below, with P indicating the plane and O indicating the observer. The cone angle is related to h and d by $\tan\theta = h/d$, so $d = h/\tan\theta$. The shock wave reaches O in the time the plane takes to fly the distance d: $t = d/v = h/v\tan\theta = (5000\,\text{m})/1.5(331\,\text{m/s})\tan 42^\circ = 11\,\text{s}$.



60. The altitude H and the horizontal distance x for the legs of a right triangle, so we have

$$H = x\tan\theta = v_p t\tan\theta = 1.25vt\sin\theta$$

where v is the speed of sound, v_p is the speed of the plane and

$$\theta = \sin^{-1}\left(\frac{v}{v_p}\right) = \sin^{-1}\left(\frac{v}{1.25v}\right) = 53.1^\circ .$$

Thus the altitude is

$$H = x\tan\theta = (1.25)(330\,\text{m/s})(60\,\text{s})(\tan 53.1^\circ) = 3.30\times 10^4 \text{ m} .$$

61. We use $\beta = 10\log(I/I_\text{o})$ with $I_\text{o} = 1\times 10^{-12}\,\text{W/m}^2$ and $I = P/4\pi r^2$ (an assumption we are asked to make in the problem). We estimate $r \approx 0.3$ m (distance from knuckle to ear) and find

$$P \approx 4\pi(0.3\,\text{m})^2\left(1\times 10^{-12}\,\text{W/m}^2\right)10^{6.2} = 2\times 10^{-6} \text{ W} .$$

62. (a) Using Eq. 18-39 with $v = 343$ m/s and $n = 1$, we find $f = nv/2L = 86$ Hz for the fundamental frequency in a nasal passage of length $L = 2.0$ m (subject to various assumptions about the nature of the passage as a "bent tube open at both ends").

(b) The sound would be perceptible as *sound* (as opposed to just a general vibration) of very low frequency.

(c) Smaller L implies larger f by the formula cited above. Thus, the female's sound is of higher pitch (frequency).

63. (a) Since $\omega = 2\pi f$, Eq. 18-15 leads to

$$\Delta p_m = v\rho(2\pi f)s_m \implies s_m = \frac{1.13\times 10^{-3}\,\text{Pa}}{2\pi(1665\,\text{Hz})(343\,\text{m/s})(1.21\,\text{kg/m}^3)}$$

which yields $s_m = 0.26$ nm. The nano prefix represents 10^{-9}. We use the speed of sound and air density values given at the beginning of the exercises and problems section in the textbook.

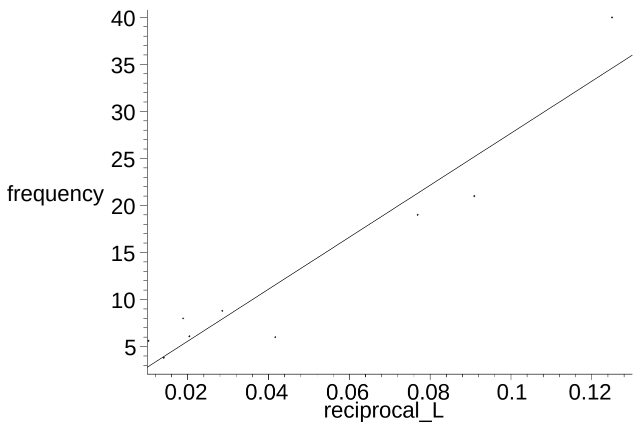
(b) We can plug into Eq. 18-27 or into its equivalent form, rewritten in terms of the pressure amplitude:

$$I = \frac{1}{2}\frac{(\Delta p_m)^2}{\rho v} = \frac{1}{2}\frac{\left(1.13\times 10^{-3}\,\text{Pa}\right)^2}{(1.21\,\text{kg/m}^3)\,(343\,\text{m/s})} = 1.5 \text{ nW/m}^2 .$$

64. We use $\beta = 10\log(I/I_\text{o})$ with $I_\text{o} = 1\times 10^{-12}\,\text{W/m}^2$ and Eq. 18-27 with $\omega = 2\pi f = 2\pi(260\,\text{Hz})$, $v = 343$ m/s and $\rho = 1.21\,\text{kg/m}^3$.

$$I = I_\text{o}\left(10^{8.5}\right) = \frac{1}{2}\rho v(2\pi f)^2 s_m^2 \implies s_m = 7.6\times 10^{-7} \text{ m} .$$

65. The points and the least-squares fit is shown in the graph below. The graph has frequency in Hertz along the vertical axis and $1/L$ in inverse meters along the horizontal axis. The function found by the least squares fit procedure is $f = 276(1/L) + 0.037$. Assuming this fits either the model of an open organ pipe (mathematically similar to a string fixed at both ends) or that of a pipe closed at one end, as discussed in the textbook, then $f = v/2L$ in the former case or $f = v/4L$ in the latter. Thus, if the least-squares slope of 276 fits the first model, then a value of $v = 2(276) = 553$ m/s is implied. In the second model (the pipe with only one end open) we find $v = 4(276) = 1106$ m/s which is more "in the ballpark" of the 1400 m/s value cited in the problem. This suggests that the acoustic resonance involved in this situation is more closely related to the $n = 1$ case of Figure 18-15(b) than to Figure 18-14.



66. The round-trip time is $t = 2L/v$ where we estimate from the chart that the time between clicks is 3 ms. Thus, with $v = 1372$ m/s, we find $L = \frac{1}{2}vt = 2.1$ m.

67. (a) In regions where the speed is constant, it is equal to distance divided by time. Thus, we conclude that the time difference is

$$\Delta t = \left(\frac{L-d}{V} + \frac{d}{V-\Delta V}\right) - \frac{L}{V}$$

where the first term is the travel time through bone and rock and the last term is the expected travel time purely through rock. Solving for d and simplifying, we obtain

$$d = \Delta t\,\frac{V(V-\Delta V)}{\Delta V} \approx \Delta t\,\frac{V^2}{\Delta V}\ .$$

(b) If we estimate $d \approx 10$ cm (as the lower limit of a range that goes up to a diameter of 20 cm), then the above expression (with the numerical values given in the problem) leads to $\Delta t = 0.8\,\mu$s (as the lower limit of a range that goes up to a time difference of $1.6\,\mu$s).

68. (a) Using $m = 7.3 \times 10^7$ kg, the initial gravitational potential energy is $U = mgy = 3.9 \times 10^{11}$ J, where $h = 550$ m. Assuming this converts primarily into kinetic energy during the fall, then $K = 3.9 \times 10^{11}$ J just before impact with the ground. Using instead the mass estimate $m = 1.7 \times 10^8$ kg, we arrive at $K = 9.2 \times 10^{11}$ J.

(b) The process of converting this kinetic energy into other forms of energy (during the impact with the ground) is assumed to take $\Delta t = 0.50$ s (and in the average sense, we take the "power" P to be wave-energy/Δt). With 20% of the energy going into creating a seismic wave, the intensity of the body wave is estimated to be

$$I = \frac{P}{A_{\text{hemisphere}}} = \frac{(0.20)K/\Delta t}{\frac{1}{2}\,(4\pi r^2)} = 0.63\ \text{W/m}^2$$

using $r = 200 \times 10^3$ m and the smaller value for K from part (a). Using instead the larger estimate for K, we obtain $I = 1.5$ W/m^2.

(c) The surface area of a cylinder of "height" d is $2\pi r d$, so the intensity of the surface wave is

$$I = \frac{P}{A_{\text{cylinder}}} = \frac{(0.20)K/\Delta t}{(2\pi r d)} = 25 \times 10^3 \text{ W/m}^2$$

using $d = 5.0$ m, $r = 200 \times 10^3$ m and the smaller value for K from part (a). Using instead the larger estimate for K, we obtain $I = 58$ kW/m^2.

(d) Although several factors are involved in determining which seismic waves are most likely to be detected, we observe that on the basis of the above findings we should expect the more intense waves (the surface waves) to be more readily detected.

69. (a) The period is the reciprocal of the frequency: $T = 1/f = 1/(90\,\text{Hz}) = 1.1 \times 10^{-2}$ s.

(b) Using $v = 343$ m/s, we find $\lambda = v/f = 3.8$ m.

70. (a) The blood is moving towards the right (towards the detector), because the Doppler shift in frequency is an *increase*: $\Delta f > 0$.

(b) The reception of the ultrasound by the blood and the subsequent remitting of the signal by the blood back toward the detector is a two step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + v_x}{v - v_x}\right) \qquad \text{where} \quad v_x = v_{\text{blood}} \cos\theta \ .$$

If we write the ratio of frequencies as $R = (f + \Delta f)/f$, then the solution of the above equation for the speed of the blood is

$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta} = 0.90 \text{ m/s}$$

where $v = 1540$ m/s, $\theta = 20°$, and $R = 1 + 5495/5 \times 10^6$.

(c) We interpret the question as asking how Δf (still taken to be positive, since the detector is in the "forward" direction) changes as the detection angle θ changes. Since larger θ means smaller horizontal component of velocity v_x then we expect Δf to decrease towards zero as θ is increased towards 90°.

71. (a) When the speed is constant, we have $v = d/t$ where $v = 343$ m/s is assumed. Therefore, with $t = \frac{1}{2}(15\,\text{s})$ (the time for sound to travel to the far wall) we obtain $d = (343)(15/2)$ which yields a distance of 2.6 km!

(b) Just as the $\frac{1}{2}$ factor in part (a) was $1/(n+1)$ for $n = 1$ reflection, so also can we write

$$d = (343\,\text{m/s})\left(\frac{15\,\text{s}}{n+1}\right) \implies n = \frac{(343)(15)}{d} - 1$$

for multiple reflections (with d in meters). For $d = 25.7$ m, we find $n = 199$.

72. Any phase changes associated with the reflections themselves are rendered inconsequential by the fact that there are an even number of reflections. The additional path length traveled by wave A consists of the vertical legs in the zig-zag path: $2L$. To be (minimally) out of phase means, therefore, that $2L = \lambda/2$ (corresponding to a half-cycle, or 180°, phase difference). Thus, $L = \lambda/4$.

73. The reception of the ultrasound by the structure (moving with speed u) and the subsequent remitting of the signal by the structure back toward the detector is a two step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + u}{v - u}\right) \implies v = \left(\frac{2 + \xi}{\xi}\right) u$$

with $\xi = \Delta f/f$ and where we have assumed that the structure is moving toward the detector. If $u = 1.00 \times 10^{-3}$ m/s and $\xi = 1.30 \times 10^{-6}$, we get $v = 1.54 \times 10^3$ m/s.

74. (a) The wavelength of the sound wave is

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{1000\,\text{Hz}} = 0.343 \text{ m} .$$

(b) From $\Delta p_m = v^2 \rho k s_m = 2\pi v \rho f s_m$ we find

$$s_m = \frac{\Delta p_m}{2\pi v \rho f} = \frac{10.0\,\text{Pa}}{(2\pi)(343\,\text{m/s})(1.21\,\text{kg/m}^3)(1000\,\text{Hz})} = 3.83 \times 10^{-6} \text{ m} .$$

(c) The velocity of the particle is the derivative of the sinusoidal wave function with respect to time. Its maximum value is

$$v_{\text{m}} = 2\pi f s_m = (3.60 \times 10^{-6}\,\text{m})(2\pi)(1000\,\text{Hz}) = 2.41 \times 10^{-2} \text{ m/s}.$$

(d) From Eq. 18-38, we obtain

$$L = \frac{\lambda}{2} = \frac{0.343\,\text{m}}{2} = 0.172 \text{ m} .$$

75. (a) With the detector stationary, we seek a value of source speed v_S such that the frequency ratio (heard/emitted) is $r = (20\,\text{kHz})/(30\,\text{kHz}) = 2/3$. From the Doppler effect formula, we find

$$f' = f\left(\frac{v+0}{v+v_S}\right) \implies v_S = \left(\frac{1-r}{r}\right) v .$$

If $v = 343$ m/s, we get $v_S = 171.5$ m/s which converts to 617 km/h.

(b) If $r = 20/22$, we find $v_S = 34.3\,\text{m/s} = 123$ km/h.

76. Let the frequencies of sound heard by the person from the left and right forks be f_l and f_r, respectively.

(a) If the speeds of both forks are u, then $f_{l,r} = fv/(v \pm u)$ and

$$\begin{aligned} f_{\text{beat}} &= |f_r - f_l| = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right) = \frac{2fuv}{v^2-u^2} \\ &= \frac{2(440\,\text{Hz})(30.0\,\text{m/s})(343\,\text{m/s})}{(343\,\text{m/s})^2 - (30.0\,\text{m/s})^2} \\ &= 77.6 \text{ Hz} . \end{aligned}$$

(b) If the speed of the listener is u, then $f_{l,r} = f(v \pm u)/v$ and

$$f_{\text{beat}} = |f_l - f_r| = 2f\left(\frac{u}{v}\right) = 2(440\,\text{Hz})\left(\frac{30.0\,\text{m/s}}{343\,\text{m/s}}\right) = 77.0 \text{ Hz} .$$

77. (a) Since the source is moving toward the wall, the frequency of the sound as received at the wall is

$$f' = f\left(\frac{v}{v-v_S}\right) = (440\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} - 20.0\,\text{m/s}}\right) = 467 \text{ Hz} .$$

(b) Since the person is moving with a speed u toward the reflected sound with frequency f', the frequency registered at the source is

$$f_r = f'\left(\frac{v+u}{v}\right) = (467\,\text{Hz})\left(\frac{343\,\text{m/s} + 20.0\,\text{m/s}}{343\,\text{m/s}}\right) = 494 \text{ Hz} .$$

78. (a) If the destroyer drifts with the current, then it will detect a signal with frequency f' given by

$$\begin{aligned} f' &= f\left(\frac{v}{v-u_1}\right) \\ &= \frac{(1000\,\mathrm{Hz})(5470\,\mathrm{km/h})}{5470\,\mathrm{km/h}-(75.0\,\mathrm{km/h}-30.0\,\mathrm{km/h})} \\ &= 1008.29\ \mathrm{Hz}\ . \end{aligned}$$

Thus, $\Delta f = f' - f = 8.29\,\mathrm{Hz}$.

(b) If the destroyer is stationary with respect to the ocean floor, then it is moving at $u_2 = 30.0\,\mathrm{km/h}$ relative to the current. The detected frequency then becomes

$$\begin{aligned} f'' &= f\left(\frac{v+u_2}{v-u_1}\right) = \frac{(1000\,\mathrm{Hz})(5470\,\mathrm{km/h}+30.0\,\mathrm{km/h})}{5470\,\mathrm{km/h}-(75.0\,\mathrm{km/h}-30.0\,\mathrm{km/h})} \\ &= 1013.9\ \mathrm{Hz}\ . \end{aligned}$$

Thus, $\Delta f = f'' - f = 13.9\,\mathrm{Hz}$.

79. (a) With $r = 10$ m in Eq. 18-28, we have

$$I = \frac{P}{4\pi r^2} \implies P = 10\,\mathrm{W}\ .$$

(b) Using that value of P in Eq. 18-28 with a new value for r, we obtain

$$I = \frac{P}{4\pi(5.0)^2} = 0.032\ \frac{\mathrm{W}}{\mathrm{m}^2}.$$

Alternatively, a ratio $I'/I = (r/r')^2$ could have been used.

(c) Using Eq. 18-29 with $I = 0.0080\ \mathrm{W/m^2}$, we have

$$\beta = 10\log\frac{I}{I_0} = 99\ \mathrm{dB}$$

where $I_0 = 1\times10^{-12}\ \mathrm{W/m^2}$.

80. (a) We proceed by dividing the (velocity) equation involving the new (fundamental) frequency f' by the equation when the frequency f is 440 Hz to obtain

$$\frac{f'\lambda}{f\lambda} = \sqrt{\frac{\frac{\tau'}{\mu}}{\frac{\tau}{\mu}}} \implies \frac{f'}{f} = \sqrt{\frac{\tau'}{\tau}}$$

where we are making an assumption that the mass-per-unit-length of the string does not change significantly. Thus, with $\tau' = 1.2\tau$, we have $f'/440 = \sqrt{1.2}$. Therefore, $f' = 482$ Hz.

(b) In this case, neither tension nor mass-per-unit-length change, so the wavespeed v is unchanged. Hence,

$$f'\lambda' = f\lambda \implies f'\,(2L') = f\,(2L)$$

where Eq. 18-38 with $n = 1$ has been used. Since $L' = \frac{2}{3}\,L$, we obtain $f' = \frac{3}{2}(440) = 660$ Hz.

81. We find the difference in the two applications of the Doppler formula:

$$f_2 - f_1 = 37 = f\left(\frac{340+25}{340-15} - \frac{340}{340-15}\right) = f\left(\frac{25}{340-15}\right)$$

which leads to $f = 481 \approx 480$ Hz.

82. (a) If point P is infinitely far away, then the small distance d between the two sources is of no consequence (they seem effectively to be the same distance away from P). Thus, there is no perceived phase difference.

(b) Since the sources oscillate in phase, then the situation described in part (a) produces constructive interference.

(c) For finite values of x, the difference in source positions becomes significant. The path lengths for waves to travel from S_1 and S_2 become is now different. We interpret the question as asking for the behavior of the absolute value of the phase difference $|\Delta\phi|$, in which case any change from zero (the answer for part (a)) is certainly an increase.

(d) The path length difference for waves traveling from S_1 and S_2 is

$$\Delta\ell = \sqrt{d^2 + x^2} - x \qquad \text{for} \quad x > 0 \; .$$

The phase difference in "cycles" (in absolute value) is therefore

$$|\Delta\phi| = \frac{\Delta\ell}{\lambda} = \frac{\sqrt{d^2 + x^2} - x}{\lambda} \; .$$

Thus, in terms of λ, the phase difference is identical to the path length difference: $|\Delta\phi| = \Delta\ell > 0$. Consider $\Delta\ell = \lambda/2$. Then $\sqrt{d^2 + x^2} = x + \lambda/2$. Squaring both sides, rearranging, and solving, we find

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4} \; .$$

In general, if $\Delta\ell = \xi\lambda$ for some multiplier $\xi > 0$, we find

$$x = \frac{d^2}{2\xi\lambda} - \frac{1}{2}\xi\lambda \; .$$

Using $d = 16$ m and $\lambda = 2.0$ m, we insert $\xi = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}$ into this expression and find the respective values (in meters) $x = 128, 63, 41, 30, 23$. Since whole cycle phase differences are equivalent (as far as the wave superposition goes) to zero phase difference, then the $\xi = 1, 2$ cases give constructive interference. A shift of a half-cycle brings "troughs" of one wave in superposition with "crests" of the other, thereby canceling the waves; therefore, the $\xi = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ cases produce destructive interference.

83. We use $v = \sqrt{B/\rho}$ to find the bulk modulus B:

$$B = v^2\rho = \left(5.4 \times 10^3 \text{ m/s}\right)^2 \left(2.7 \times 10^3 \text{ kg/m}^3\right) = 7.9 \times 10^{10} \text{ Pa} \; .$$

84. Let ℓ be the length of the rod. Then the time of travel for sound in air (speed v_s) will be $t_s = \ell/v_s$. And the time of travel for compressional waves in the rod (speed v_r) will be $t_r = \ell/v_r$. In these terms, the problem tells us that

$$t_s - t_r = 0.12 \text{ s} = \ell\left(\frac{1}{v_s} - \frac{1}{v_r}\right) \; .$$

Thus, with $v_s = 343$ m/s and $v_r = 15v_s = 5145$ m/s, we find $\ell = 44$ m.

85. (a) The frequency at which $\lambda = D$ is

$$f_1 = \frac{v}{D} = \frac{343 \text{ m/s}}{15.0 \times 10^{-2} \text{ m}} = 2.29 \times 10^3 \text{ Hz} \; ,$$

the frequency at which $\lambda = 10D$ is $f_2 = 2.29 \times 10^2$ Hz, and the frequency at which $\lambda = 0.1D$ is $f_3 = 2.29 \times 10^4$ Hz.

(b) Now, $D' = 30.0\,\text{cm}$. The frequency at which $\lambda = D'$ is $f_1' = v/D' = (343\,\text{m/s})/(30.0\times10^{-2}\,\text{m}) = 1.14\times10^3\,\text{Hz}$, the frequency at which $\lambda = 10D'$ is $f_2' = 1.14\times10^2\,\text{Hz}$, and the frequency at which $\lambda = 0.1D'$ is $f_3' = 1.14\times10^4\,\text{Hz}$.

86. When $\phi = 0$ it is clear that the superposition wave has amplitude $2\Delta p_m$. For the other cases, it is useful to write

$$\Delta p_1 + \Delta p_2 = \Delta p_m\left(\sin(\omega t) + \sin(\omega t - \phi)\right) = \left(2\Delta p_m \cos\frac{\phi}{2}\right)\sin\left(\omega t - \frac{\phi}{2}\right) .$$

The factor in front of the sine function gives the amplitude for all cases considered: $\phi = \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4}$ gives $\Delta p_m\sqrt{2}, \Delta p_m\sqrt{3}, \Delta p_m\sqrt{2+\sqrt{2}}$, respectively.

87. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

(a) With $I_1 = 9.60\times10^{-4}$ W/m^2, $r_1 = 6.10$ m, and $r_2 = 30.0$ m, we find $I_2 = 0.960(6.10/30.0)^2 = 3.97\times10^{-5}$ W/m^2.

(b) Using Eq. 18-27 with $I_1 = 9.60\times10^{-4}$ W/m^2, $\omega = 2\pi(2000\,\text{Hz})$, $v = 343$ m/s and $\rho = 1.21\,\text{kg/m}^3$, we obtain

$$s_m = \sqrt{\frac{2I}{\rho v\omega^2}} = 1.71\times10^{-7}\ \text{m} .$$

(c) Eq. 18-15 gives the pressure amplitude:

$$\Delta p_m = \rho v\omega s_m = 0.893\ \text{Pa} .$$

88. The source being a "point source" means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

From the discussion in §18-5, we know that the intensity ratio between "barely audible" and the "painful threshold" is $10^{-12} = I_2/I_1$. Thus, with $r_2 = 10000$ m, we find $r_1 = r_2\sqrt{10^{-12}} = 0.01$ m.

89. The density of oxygen gas is

$$\rho = \frac{0.0320\,\text{kg}}{0.0224\,\text{m}^3} = 1.43\,\text{kg/m}^3 .$$

From $v = \sqrt{B/\rho}$ we find

$$B = v^2\rho = (317\,\text{m/s})^2\left(1.43\,\text{kg/m}^3\right) = 1.44\times10^5\ \text{Pa} .$$

90. The wavelength is

$$\lambda = \frac{v}{f} = \frac{240\,\text{m/s}}{4.2\times10^9\,\text{Hz}} = 5.7\times10^{-8}\,\text{m} = 57\ \text{nm} .$$

91. Since they oscillate out of phase, then their waves will cancel (producing a node) at a point exactly midway between them (the midpoint of the system, where we choose $x = 0$). We note that Figure 18-14, and the $n = 3$ case of Figure 18-15(a) have this property (of a node at the midpoint). The distance Δx between nodes is $\lambda/2$, where $\lambda = v/f$ and $f = 300$ Hz and $v = 343$ m/s. Thus, $\Delta x == v/2f = 0.572$ m. Therefore, nodes are found at the following positions:

$$\begin{aligned} x &= \pm\Delta x = \pm0.57\ \text{m} \\ x &= \pm2\Delta x = \pm1.14\ \text{m} \\ x &= \pm3\Delta x = \pm1.72\ \text{m} \end{aligned}$$

92. (a) Consider a string of pulses returning to the stage. A pulse which came back just before the previous one has traveled an extra distance of $2w$, taking an extra amount of time $\Delta t = 2w/v$. The frequency of the pulse is therefore

$$f = \frac{1}{\Delta t} = \frac{v}{2w} = \frac{343\,\text{m/s}}{2(0.75\,\text{m})} = 2.3 \times 10^2 \text{ Hz} .$$

(b) Since $f \propto 1/w$, the frequency would be higher if w were smaller.

93. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$. Since intensity is proportional to the square of the amplitude (see Eq. 18-27), this further implies

$$\frac{I_2}{I_1} = \left(\frac{s_{m2}}{s_{m1}}\right)^2 = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2$$

or $s_{m2}/s_{m1} = r_1/r_2$.

(a) With $I = P/4\pi r^2 = (10\,\text{W})/4\pi(3.0\,\text{m})^2 = 0.088\ \text{W/m}^2$.

(b) Using the notation A instead of s_m for the amplitude, we find

$$\frac{A_4}{A_3} = \frac{3.0\,\text{m}}{4.0\,\text{m}} \implies A_4 = \frac{3}{4}A_3 .$$

94. We use $I \propto r^{-2}$ appropriate for an isotropic source. We have

$$\frac{I_{r=d}}{I_{r=D-d}} = \frac{(D-d)^2}{D^2} = \frac{1}{2} ,$$

where $d = 50.0\,\text{m}$. We solve for D: $D = \sqrt{2}d/(\sqrt{2}-1) = \sqrt{2}(50.0\,\text{m})/(\sqrt{2}-1) = 171\,\text{m}$.

95. Let the original tension be τ_1 and the new tension be τ_2. Then

$$\frac{\lambda_{s2}}{\lambda_{s1}} = \frac{v_s/f_2}{v_s/f_1} = \frac{f_1}{f_2} = \frac{v_2/\lambda_1}{v_2/\lambda_2} = \frac{v_1}{v_2} = \sqrt{\frac{\tau_1}{\tau_2}} = \frac{1}{2} .$$

Thus, $\tau_2/\tau_1 = 4$. That is, the tension must be increased by a factor of 4.

96. Since they are approaching each other, the sound produced (of emitted frequency f) by the flatcar-trumpet received by an observer on the ground will be of higher pitch f'. In these terms, we are told $f' - f = 4.0$ Hz, and consequently that $f'/f = 444/440 = 1.0091$. With v_S designating the speed of the flatcar and $v = 343$ m/s being the speed of sound, the Doppler equation leads to

$$\frac{f'}{f} = \frac{v+0}{v-v_S} \implies v_S = (343)\frac{1.0091-1}{1.0091} = 3.1 \text{ m/s} .$$

97. The siren is between you and the cliff, moving away from you and towards the cliff. Both "detectors" (you and the cliff) are stationary, so $v_D = 0$ in Eq. 18-47 (and see the discussion in the textbook immediately after that equation regarding the selection of $\pm$ signs). The source is the siren with $v_S = 10$ m/s. The problem asks us to use $v = 330$ m/s for the speed of sound.

(a) With $f = 1000$ Hz, the frequency f_y you hear becomes

$$f_y = f\left(\frac{v+0}{v+v_S}\right) = 970.6 \approx 9.7 \times 10^2 \text{ Hz} .$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = f\left(\frac{v+0}{v-v_S}\right) = 1031.3 \approx 1.03 \times 10^2 \text{ Hz} .$$

(c) The beat frequency is $f_c - f_y = 61$ beats/s (which, due to specific features of the human ear, is too large to be perceptible).

98. (a) We observe that "third lowest ... frequency" corresponds to harmonic number $n = 5$ for such a system. Using Eq. 18-41, we have

$$f = \frac{nv}{4L} \implies 750 = \frac{5v}{4(0.60)}$$

so that $v = 360$ m/s.

(b) As noted, $n = 5$; therefore, $f_1 = 750/5 = 150$ Hz.

99. (a) The problem asks for the source frequency f. We use Eq. 18-47 with great care (regarding its $\pm$ sign conventions).

$$f' = f\left(\frac{340-16}{340-40}\right)$$

Therefore, with $f' = 950$ Hz, we obtain $f = 880$ Hz.

(b) We now have

$$f' = f\left(\frac{340+16}{340+40}\right)$$

so that with $f = 880$ Hz, we find $f' = 824$ Hz.

100. (a) Since the speed of sound is lower in air than in water, the speed of sound in the air-water mixture is lower than in pure water (see Table 18-1). Frequency is proportional to the speed of sound (see Eq. 18-39 and Eq. 18-41), so the decrease in speed is "heard" due to the accompanying decrease in frequency.

(b) This follows from Eq. 18-3 and Eq. 18-2 (with Δ's replaced by derivatives). Thus,

$$\frac{1}{v^2} = \frac{\rho}{B} = \frac{\rho}{V\left|\frac{dp}{dV}\right|} = \frac{\rho}{V}\left|\frac{dV}{dp}\right| .$$

(c) Returning to the Δ notation, and letting the absolute values be "understood," we write $\Delta V = \Delta V_w + \Delta V_a$ as indicated in the problem. Subject to the approximations mentioned in the problem, our equation becomes

$$\frac{1}{v^2} = \frac{\rho_w}{V_w}\left(\frac{\Delta V_w}{\Delta p} + \frac{\Delta V_a}{\Delta p}\right) = \frac{\rho_w}{V_w}\frac{\Delta V_w}{\Delta p} + \frac{\rho_w}{\rho_a}\frac{V_a}{V_w}\left(\frac{\rho_a}{V_a}\frac{\Delta V_a}{\Delta p}\right) .$$

In a pure water system or a pure air system, we would have

$$\frac{1}{v_w^2} = \frac{\rho_w}{V_w}\frac{\Delta V_w}{\Delta p} \qquad \text{or} \qquad \frac{1}{v_a^2} = \frac{\rho_a}{V_a}\frac{\Delta V_a}{\Delta p} .$$

Substituting these into the above equation, and using the notation $r = V_a/V_w$, we arrive at

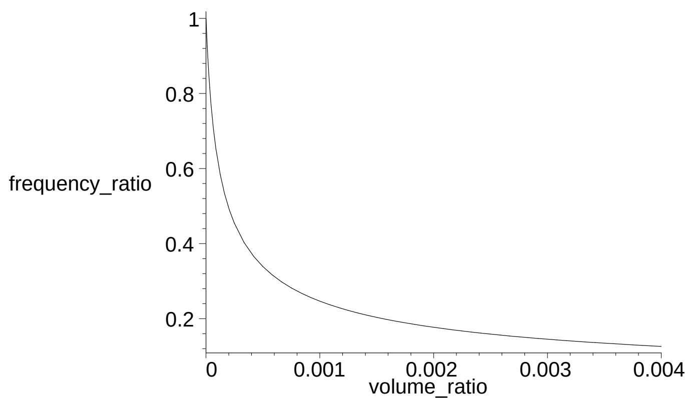
$$\frac{1}{v^2} = \frac{1}{v_w^2} + \frac{\rho_w}{\rho_a}\frac{r}{v_a^2} \implies v = \frac{1}{\sqrt{1/v_w^2 + r(\rho_w/\rho_a)/v_a^2}} .$$

(d) Dividing our result in the previous part by v_w and using the fact that the wave speed is proportional to the frequency, we find

$$\frac{v}{v_w} = \frac{f_{\text{shift}}}{f} = \frac{1}{v_w\sqrt{1/v_w^2 + r(\rho_w/\rho_a)/v_a^2}} = \frac{1}{\sqrt{1 + r(\rho_w/\rho_a)(v_w/v_a)^2}}$$

which becomes the expression shown in the problem when we plug in $\rho_w = 1000\,\text{kg/m}^3$, $\rho_a = 1.21\,\text{kg/m}^3$, $v_w = 1482$ m/s and $v_a = 343$ m/s, and round to three significant figures.

(e) The graph of f_{shift}/f versus r is shown below.



(f) From the graph (or more accurately by solving the equation itself) we find $r = 5.2\times10^{-4}$ corresponds to $f_{\text{shift}}/f = 1/3$.

Chapter 19

1. We take p_3 to be 80 kPa for both thermometers. According to Fig. 19-6, the nitrogen thermometer gives 373.35 K for the boiling point of water. Use Eq. 19-5 to compute the pressure:

$$p_{\rm N} = \frac{T}{273.16\,{\rm K}} p_3 = \left(\frac{373.35\,{\rm K}}{273.16\,{\rm K}}\right)(80\,{\rm kPa}) = 109.343 \text{ kPa} .$$

The hydrogen thermometer gives 373.16 K for the boiling point of water and

$$p_{\rm H} = \left(\frac{373.16\,{\rm K}}{273.16\,{\rm K}}\right)(80\,{\rm kPa}) = 109.287 \text{ kPa} .$$

The pressure in the nitrogen thermometer is higher than the pressure in the hydrogen thermometer by 0.056 kPa.

2. From Eq. 19-6, we see that the limiting value of the pressure ratio is the same as the absolute temperature ratio: $(373.15\,{\rm K})/(273.16\,{\rm K}) = 1.366$.

3. Let T_L be the temperature and p_L be the pressure in the left-hand thermometer. Similarly, let T_R be the temperature and p_R be the pressure in the right-hand thermometer. According to the problem statement, the pressure is the same in the two thermometers when they are both at the triple point of water. We take this pressure to be p_3. Writing Eq. 19-5 for each thermometer,

$$T_L = (273.16\,{\rm K})\left(\frac{p_L}{p_3}\right) \qquad \text{and} \quad T_R = (273.16\,{\rm K})\left(\frac{p_R}{p_3}\right) ,$$

we subtract the second equation from the first to obtain

$$T_L - T_R = (273.16\,{\rm K})\left(\frac{p_L - p_R}{p_3}\right) .$$

First, we take $T_L = 373.125$ K (the boiling point of water) and $T_R = 273.16$ K (the triple point of water). Then, $p_L - p_R = 120$ torr. We solve

$$373.125\,{\rm K} - 273.16\,{\rm K} = (273.16\,{\rm K})\left(\frac{120\,{\rm torr}}{p_3}\right)$$

for p_3. The result is $p_3 = 328$ torr. Now, we let $T_L = 273.16$ K (the triple point of water) and T_R be the unknown temperature. The pressure difference is $p_L - p_R = 90.0$ torr. Solving

$$273.16\,{\rm K} - T_R = (273.16\,{\rm K})\left(\frac{90.0\,{\rm torr}}{328\,{\rm torr}}\right)$$

for the unknown temperature, we obtain $T_R = 348$ K.

4. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. If we require $y = 2x$, then we have

$$2x = \frac{9}{5}x + 32 \implies x = (5)(32) = 160°\text{C}$$

which yields $y = 2x = 320°$F.

(b) In this case, we require $y = \frac{1}{2}x$ and find

$$\frac{1}{2}x = \frac{9}{5}x + 32 \implies x = -\frac{(10)(32)}{13} \approx -24.6°\text{C}$$

which yields $y = x/2 = -12.3°$F.

5. (a) Fahrenheit and Celsius temperatures are related by $T_F = (9/5)T_C + 32°$. T_F is numerically equal to T_C if $T_F = (9/5)T_F + 32°$. The solution to this equation is $T_F = -(5/4)(32°) = -40°$F.

(b) Fahrenheit and Kelvin temperatures are related by $T_F = (9/5)T_C + 32° = (9/5)(T - 273.15) + 32°$. The Fahrenheit temperature T_F is numerically equal to the Kelvin temperature T if $T_F = (9/5)(T_F - 273.15) + 32°$. The solution to this equation is

$$T_F = \frac{5}{4}\left(\frac{9}{5} \times 273.15 - 32°\right) = 575°\text{F} .$$

(c) Since $T_C = T - 273.15$ the Kelvin and Celsius temperatures can never have the same numerical value.

6. (a) Let the reading on the Celsius scale be x and the reading on the Fahrenheit scale be y. Then $y = \frac{9}{5}x + 32$. For $x = -71$, this gives $y = -96$.

(b) The relationship between y and x may be inverted to yield $x = \frac{5}{9}(y - 32)$. Thus, for $y = 134$ we find $x \approx 56.7$ on the Celsius scale.

7. (a) Changes in temperature take place by means of radiation, conduction, and convection. The constant A can be reduced by placing the object in isolation, by surrounding it with a vacuum jacket, for example. This reduces conduction and convection. Absorption of radiation can be reduced by polishing the surface to a mirror finish. We note that A depends on the condition of the surface and on the ability of the environment to conduct or convect energy to or from the object. A has the dimensions of reciprocal time.

(b) We rearrange the equation to obtain

$$\frac{1}{\Delta T}\frac{d\Delta T}{dt} = -A .$$

Now, we integrate with respect to time and recognize that

$$\int \frac{1}{\Delta T}\frac{d\,\Delta T}{dt}\,dt = \int \frac{1}{\Delta T}\,d(\Delta T) .$$

Thus,

$$\int_{\Delta T_0}^{\Delta T} \frac{1}{\Delta T}\,d(\Delta T) = -\int_0^t A\,dt .$$

The integral on the right side yields $-At$ and the integral on the left yields $\ln \Delta T|_{\Delta T_0}^{\Delta T} = \ln(\Delta T) - \ln(\Delta T_0) = \ln(\Delta T/\Delta T_0)$, so

$$\ln\frac{\Delta T}{\Delta T_0} = -At .$$

We use each side as the exponent of e, the base of the natural logarithms, to obtain

$$\frac{\Delta T}{\Delta T_0} = e^{-At}$$

or

$$\Delta T = \Delta T_0 \, e^{-At} \ .$$

8. From $\Delta T = \Delta T_0 e^{-At}$, we have $\Delta T/\Delta T_0 = e^{-A_1 t_1}$ (before insulation) and $\Delta T/\Delta T_0 = e^{-A_2 t_2}$ (after insulation). Thus the ratio is given by $A_2/A_1 = t_1/t_2 = 1/2$.

9. We assume scale X is a linear scale in the sense that if its reading is x then it is related to a reading y on the Kelvin scale by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$\begin{aligned} 373.15 &= m(-53.5) + b \\ 273.15 &= m(-170) + b \end{aligned}$$

which yield the solutions $m = 100/(170 - 53.5) = 0.858$ and $b = 419$. With these values, we find x for $y = 340$:

$$x = \frac{y - b}{m} = \frac{340 - 419}{0.858} = -92.1°X \ .$$

10. The change in length for the aluminum pole is

$$\Delta\ell = \ell_0 \alpha_{\text{Al}} \Delta T = (33\,\text{m})(23 \times 10^{-6}/\text{C}°)(15\text{C}°) = 0.011 \text{ m} \ .$$

11. When the temperature changes from T to $T + \Delta T$ the diameter of the mirror changes from D to $D + \Delta D$, where $\Delta D = \alpha D \, \Delta T$. Here α is the coefficient of linear expansion for Pyrex glass ($3.2 \times 10^{-6}/\text{C}°$, according to Table 19–2). The range of values for the diameters can be found by setting ΔT equal to the temperature range. Thus $\Delta D = (3.2 \times 10^{-6}\,/\text{C}°)(200\,\text{in.})(60\,\text{C}°) = 3.84 \times 10^{-2}\,\text{in.}$ Since $1\,\text{in.} = 2.50\,\text{cm} = 2.50 \times 10^4\,\mu\text{m}$, this is $960\,\mu\text{m}$.

12. (a) The coefficient of linear expansion α for the alloy is

$$\alpha = \Delta L/L\Delta T = \frac{10.015\,\text{cm} - 10.000\,\text{cm}}{(10.01\,\text{cm})(100°\text{C} - 20.000°\text{C})} = 1.88 \times 10^{-5}/\text{C}° \ .$$

Thus, from 100°C to 0°C we have

$$\Delta L = L\alpha\Delta T = (10.015\,\text{cm})\left(1.88 \times 10^{-5}/\text{C}°\right)(0°\text{C} - 100°\text{C}) = -1.88 \times 10^{-2} \text{ cm} \ .$$

The length at 0°C is therefore $L' = L + \Delta L = 10.015\,\text{cm} - 0.0188\,\text{cm} = 9.996\,\text{cm}$.

(b) Let the temperature be T_x. Then from 20°C to T_x we have

$$\Delta L = 10.009\,\text{cm} - 10.000\,\text{cm} = \alpha L \Delta T = (1.88 \times 10^{-5}/\text{C}°)(10.000\,\text{cm})\Delta T \ ,$$

giving $\Delta T = 48\,\text{C}°$. Thus, $T_x = 20°\text{C} + 48\,\text{C}° = 68°\text{C}$.

13. The new diameter is

$$\begin{aligned} D &= D_0(1 + \alpha_{\text{Al}} \Delta T) \\ &= (2.725\,\text{cm})[1 + (23 \times 10^{-6}/\text{C}°)(100.0°\text{C} - 0.000°\text{C})] = 2.731 \text{ cm} \ . \end{aligned}$$

14. The volume at 30°C is given by

$$\begin{aligned} V' &= V(1 + \beta\Delta T) = V(1 + 3\alpha\Delta T) \\ &= (50\,\text{cm}^3)[1 + 3(29 \times 10^{-6}/\text{C}°)(30°\text{C} - 60°\text{C})] = 49.87 \text{ cm}^3 \ . \end{aligned}$$

where we have used $\beta = 3\alpha$.

15. Since a volume is the product of three lengths, the change in volume due to a temperature change ΔT is given by $\Delta V = 3\alpha V\,\Delta T$, where V is the original volume and α is the coefficient of linear expansion. See Eq. 19–11. Since $V = (4\pi/3)R^3$, where R is the original radius of the sphere, then

$$\Delta V = 3\alpha\left(\frac{4\pi}{3}R^3\right)\Delta T = (23\times10^{-6}\,/\mathrm{C}^\circ)(4\pi)(10\,\mathrm{cm})^3(100\,\mathrm{C}^\circ) = 29\,\mathrm{cm}^3\ .$$

The value for the coefficient of linear expansion is found in Table 19-2.

16. The change in area for the plate is

$$\Delta A = (a+\Delta a)(b+\Delta b) - ab \approx a\Delta b + b\Delta a = 2ab\alpha\Delta T = 2\alpha A\Delta T\ .$$

17. If V_c is the original volume of the cup, α_a is the coefficient of linear expansion of aluminum, and ΔT is the temperature increase, then the change in the volume of the cup is $\Delta V_c = 3\alpha_a V_c\,\Delta T$. See Eq. 19–11. If β is the coefficient of volume expansion for glycerin then the change in the volume of glycerin is $\Delta V_g = \beta V_c\,\Delta T$. Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\begin{aligned}\Delta V_g - \Delta V_c &= (\beta - 3\alpha_a)\,V_c\,\Delta T\\ &= \left[\left(5.1\times10^{-4}\,/\mathrm{C}^\circ\right) - 3\left(23\times10^{-6}\,/\mathrm{C}^\circ\right)\right]\left(100\,\mathrm{cm}^3\right)\left(6\,\mathrm{C}^\circ\right)\\ &= 0.26\ \mathrm{cm}^3\ .\end{aligned}$$

18. The change in length for the section of the steel ruler between its 20.05 cm mark and 20.11 cm mark is

$$\Delta L_s = L_s\alpha_s\Delta T = (20.11\,\mathrm{cm})(11\times10^{-6}/\mathrm{C}^\circ)(270^\circ\mathrm{C} - 20^\circ\mathrm{C}) = 0.055\ \mathrm{cm}\ .$$

Thus, the actual change in length for the rod is $\Delta L = (20.11\,\mathrm{cm} - 20.05\,\mathrm{cm}) + 0.055\,\mathrm{cm} = 0.115\,\mathrm{cm}$. The coefficient of thermal expansion for the material of which the rod is made of is then

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{0.115\,\mathrm{cm}}{270^\circ\mathrm{C} - 20^\circ\mathrm{C}} = 23\times10^{-6}/\mathrm{C}^\circ\ .$$

19. After the change in temperature the diameter of the steel rod is $D_s = D_{s0} + \alpha_s D_{s0}\,\Delta T$ and the diameter of the brass ring is $D_b = D_{b0} + \alpha_b D_{b0}\,\Delta T$, where D_{s0} and D_{b0} are the original diameters, α_s and α_b are the coefficients of linear expansion, and ΔT is the change in temperature. The rod just fits through the ring if $D_s = D_b$. This means $D_{s0} + \alpha_s D_{s0}\,\Delta T = D_{b0} + \alpha_b D_{b0}\,\Delta T$. Therefore,

$$\begin{aligned}\Delta T &= \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}}\\ &= \frac{3.000\,\mathrm{cm} - 2.992\,\mathrm{cm}}{(19\times10^{-6}\,/\mathrm{C}^\circ)(2.992\,\mathrm{cm}) - (11\times10^{-6}\,/\mathrm{C}^\circ)(3.000\,\mathrm{cm})} = 335\,\mathrm{C}^\circ\ .\end{aligned}$$

The temperature is $T = 25^\circ\mathrm{C} + 335\,\mathrm{C}^\circ = 360^\circ\mathrm{C}$.

20. (a) We use $\rho = m/V$ and $\Delta\rho = \Delta(m/V) = m\Delta(1/V) \simeq -m\Delta V/V^2 = -\rho(\Delta V/V) = -3\rho(\Delta L/L)$. The percent change in density is

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta L}{L} = -3(0.23\%) = -0.69\%\ .$$

(b) Since $\alpha = \Delta L/(L\Delta T) = 0.23\times10^{-2}/(100^\circ\mathrm{C} - 0.0^\circ\mathrm{C}) = 23\times10^{-6}/\mathrm{C}^\circ$, the metal is aluminum (using Table 19-2).

21. The change in volume of the liquid is given by $\Delta V = \beta V\,\Delta T$. If A is the cross-sectional area of the tube and h is the height of the liquid, then $V = Ah$ is the original volume and $\Delta V = A\,\Delta h$ is the change in volume. Since the tube does not change the cross-sectional area of the liquid remains the same. Therefore, $A\,\Delta h = \beta Ah\,\Delta T$ or $\Delta h = \beta h\,\Delta T$.

22. (a) Since $A = \pi D^2/4$, we have the differential $dA = 2(\pi D/4)dD$. Dividing the latter relation by the former, we obtain $dA/A = 2\,dD/D$. In terms of Δ's, this reads

$$\frac{\Delta A}{A} = 2\frac{\Delta D}{D} \qquad \text{for} \quad \frac{\Delta D}{D} \ll 1 \ .$$

We can think of the factor of 2 as being due to the fact that area is a two-dimensional quantity. Therefore, the area increases by $2(0.18\%) = 0.36\%$.

(b) Assuming that all dimension are allowed to freely expand, then the thickness increases by 0.18%.

(c) The volume (a three-dimensional quantity) increases by $3(0.18\%) = 0.54\%$.

(d) The mass does not change.

(e) The coefficient of linear expansion is

$$\alpha = \frac{\Delta D}{D\Delta T} = \frac{0.18 \times 10^{-2}}{100°\mathrm{C}} = 18 \times 10^{-6}/\mathrm{C}° \ .$$

23. We note that if the pendulum shortens, its frequency of oscillation will increase, thereby causing it to record more units of time ("ticks") than have actually passed during an interval. Thus, as the pendulum contracts (this problem involves cooling the brass wire), the pendulum will "run fast." Since the "direction" of the error has now been discussed, the remaining calculations are understood to be in absolute value. The differential of the equation for the pendulum period in Chapter 16 is

$$dT = \frac{1}{2}(2\pi)\frac{dL}{\sqrt{gL}}$$

which we divide by the period equation $T = 2\pi\sqrt{L/g}$ (and replace differentials with $|\Delta|$'s) to obtain

$$\frac{|\Delta T|}{T} = \frac{1}{2}\frac{|\Delta L|}{L} = \frac{1}{2}\alpha|\Delta T|$$

where we use Eq. 19-9 (in absolute value) in the last step. Thus, the (unitless) fractional change in period is

$$\frac{|\Delta T|}{T} = \frac{1}{2}\left(19 \times 10^{-6}/\mathrm{C}°\right)\left(20\,\mathrm{C}°\right) = 1.9 \times 10^{-4}$$

using Table 19-2. We can express this in "mixed units" fashion by recalling that there are 3600 s in an hour. Thus, $(3600\,\mathrm{s/h})(1.9 \times 10^{-4}) = 0.68$ s/h.

24. We divide Eq. 19-9 by the time increment Δt and equate it to the (constant) speed $v = 100 \times 10^{-9}$ m/s.

$$v = \alpha L_0 \frac{\Delta T}{\Delta t}$$

where $L_0 = 0.0200$ m and $\alpha = 23 \times 10^{-6}/\mathrm{C}°$. Thus, we obtain

$$\frac{\Delta T}{\Delta t} = 0.217\,\frac{\mathrm{C}°}{\mathrm{s}} = 0.217\,\frac{\mathrm{K}}{\mathrm{s}} \ .$$

25. Consider half the bar. Its original length is $\ell_0 = L_0/2$ and its length after the temperature increase is $\ell = \ell_0 + \alpha\ell_0\,\Delta T$. The old position of the half-bar, its new position, and the distance x that one end is displaced form a right triangle, with a hypotenuse of length ℓ, one side of length ℓ_0, and the other side of length x. The Pythagorean theorem yields $x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1 + \alpha\,\Delta T)^2 - \ell_0^2$. Since the change in length is small we may approximate $(1 + \alpha\,\Delta T)^2$ by $1 + 2\alpha\,\Delta T$, where the small term $(\alpha\,\Delta T)^2$ was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2\alpha\,\Delta T - \ell_0^2 = 2\ell_0^2\alpha\,\Delta T$$

and

$$x = \ell_0\sqrt{2\alpha\,\Delta T} = \frac{3.77\,\mathrm{m}}{2}\sqrt{2(25 \times 10^{-6}\,/\mathrm{C}°)(32\,\mathrm{C}°)} = 7.5 \times 10^{-2}\ \mathrm{m} \ .$$

26. We use $Q = cm\Delta T$. The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The mass m of the water that must be consumed is

$$m = \frac{Q}{c\Delta T} = \frac{3500 \times 10^3\,\mathrm{cal}}{(1\,\mathrm{g/cal}\cdot \mathrm{C}^\circ)(37.0^\circ\mathrm{C} - 0.0^\circ\mathrm{C})} = 94.6 \times 10^4 \text{ g},$$

which is equivalent to $9.46 \times 10^4\,\mathrm{g}/(1000\,\mathrm{g/liter}) = 94.6$ liters of water. This is certainly too much to drink in a single day!

27. (a) The specific heat is given by $c = Q/m(T_f - T_i)$, where Q is the heat added, m is the mass of the sample, T_i is the initial temperature, and T_f is the final temperature. Thus, recalling that a change in Celsius degrees is equal to the corresponding change on the Kelvin scale,

$$c = \frac{314\,\mathrm{J}}{(30.0 \times 10^{-3}\,\mathrm{kg})(45.0^\circ\mathrm{C} - 25.0^\circ\mathrm{C})} = 523 \text{ J/kg}\cdot\text{K}.$$

(b) The molar specific heat is given by

$$c_m = \frac{Q}{N(T_f - T_i)} = \frac{314\,\mathrm{J}}{(0.600\,\mathrm{mol})(45.0^\circ\mathrm{C} - 25.0^\circ\mathrm{C})} = 26.2 \text{ J/mol}\cdot\text{K}.$$

(c) If N is the number of moles of the substance and M is the mass per mole, then $m = NM$, so

$$N = \frac{m}{M} = \frac{30.0 \times 10^{-3}\,\mathrm{kg}}{50 \times 10^{-3}\,\mathrm{kg/mol}} = 0.600 \text{ mol}.$$

28. The amount of water m which is frozen is

$$m = \frac{Q}{L_F} = \frac{50.2\,\mathrm{kJ}}{333\,\mathrm{kJ/kg}} = 0.151\,\mathrm{kg} = 151\,\mathrm{g}.$$

Therefore the amount of water which remains unfrozen is $260\,\mathrm{g} - 151\,\mathrm{g} = 109\,\mathrm{g}$.

29. The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0°C (= 288 K) to 1235 K. This requires heat

$$Q = cm(T_f - T_i) = (236\,\mathrm{J/kg}\cdot\mathrm{K})(0.130\,\mathrm{kg})(1235^\circ\mathrm{C} - 288^\circ\mathrm{C}) = 2.91 \times 10^4 \text{ J}.$$

Now the silver at its melting point must be melted. If L_F is the heat of fusion for silver this requires

$$Q = mL_F = (0.130\,\mathrm{kg})(105 \times 10^3\,\mathrm{J/kg}) = 1.36 \times 10^4 \text{ J}.$$

The total heat required is $2.91 \times 10^4\,\mathrm{J} + 1.36 \times 10^4\,\mathrm{J} = 4.27 \times 10^4\,\mathrm{J}$.

30. Recalling that a Watt is a Joule-per-second, the heat Q which is added to the room in 1 h is

$$Q = 4(100\,\mathrm{W})(0.90)(1.00\,\mathrm{h})\left(\frac{3600\,\mathrm{s}}{1.00\,\mathrm{h}}\right) = 1.30 \times 10^6 \text{ J}.$$

31. The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The athlete's rate of dissipating energy is

$$P = 4000\,\mathrm{Cal/day} = \frac{(4000 \times 10^3\,\mathrm{cal})(4.18\,\mathrm{J/cal})}{(1\,\mathrm{day})(86400\,\mathrm{s/day})} = 194 \text{ W},$$

which is 1.9 times as much as the power of a 100 W light bulb.

32. The work the man has to do to climb to the top of Mt. Everest is given by $W = mgy = (73)(9.8)(8840) = 6.3 \times 10^6$ J. Thus, the amount of butter needed is

$$m = \frac{(6.3 \times 10^6\,\mathrm{J})\left(\frac{1.00\,\mathrm{cal}}{4.186\,\mathrm{J}}\right)}{6000\,\mathrm{cal/g}} \approx 250 \text{ g} .$$

33. (a) The heat generated is the power output of the drill multiplied by the time: $Q = Pt$. We use $1\,\mathrm{hp} = 2545\,\mathrm{Btu/h}$ to convert the given value of the power to Btu/h and $1\,\mathrm{min} = (1/60)\,\mathrm{h}$ to convert the given value of the time to hours. Then,

$$Q = \frac{(0.400\,\mathrm{hp})(2545\,\mathrm{Btu/h})(2.00\,\mathrm{min})}{60\,\mathrm{min/h}} = 33.9 \text{ Btu} .$$

(b) We use $0.75Q = cm\,\Delta T$ to compute the rise in temperature. Here c is the specific heat of copper and m is the mass of the copper block. Table 19-3 gives $c = 386\,\mathrm{J/kg{\cdot}K}$. We use $1\,\mathrm{J} = 9.481 \times 10^{-4}\,\mathrm{Btu}$ and $1\,\mathrm{kg} = 6.852 \times 10^{-2}\,\mathrm{slug}$ (see Appendix D) to show that

$$c = \frac{(386\,\mathrm{J/kg{\cdot}K})(9.481 \times 10^{-4}\,\mathrm{Btu/J})}{6.852 \times 10^{-2}\,\mathrm{slug/kg}} = 5.341 \text{ Btu/slug}{\cdot}\text{K} .$$

The mass of the block is its weight W divided by the gravitational acceleration (which is 32 ft/s^2 in customary units, which uses “slugs” for mass):

$$m = \frac{W}{g} = \frac{1.60\,\mathrm{lb}}{32\,\mathrm{ft/s}^2} = 0.0500 \text{ slug} .$$

Thus,

$$\Delta T = \frac{0.750Q}{cm} = \frac{(0.750)(33.9\,\mathrm{Btu})}{(5.341\,\mathrm{Btu/slug{\cdot}K})(0.0500\,\mathrm{slug})} = 95.3\,\mathrm{K} = 95.3\,\mathrm{C}^\circ .$$

This is equivalent to $(9/5)(95.3) = 172\,\mathrm{F}^\circ$.

34. (a) The water (of mass m) releases energy in two steps, first by lowering its temperature from 20°C to 0°C, and then by freezing into ice. Thus the total energy transferred from the water to the surroundings is

$$Q = c_w m\Delta T + L_F m = (4190\,\mathrm{J/kg{\cdot}K})(125\,\mathrm{kg})(20°\mathrm{C}) + (333\,\mathrm{kJ/kg})(125\,\mathrm{kg}) = 5.2 \times 10^7 \text{ J} .$$

(b) Before all the water freezes, the lowest temperature possible is 0°C, below which the water must have already turned into ice.

35. The mass $m = 0.100$ kg of water, with specific heat $c = 4190\,\mathrm{J/kg{\cdot}K}$, is raised from an initial temperature $T_i = 23°\mathrm{C}$ to its boiling point $T_f = 100°\mathrm{C}$. The heat input is given by $Q = cm(T_f - T_i)$. This must be the power output of the heater P multiplied by the time t; $Q = Pt$. Thus,

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190\,\mathrm{J/kg{\cdot}K})(0.100\,\mathrm{kg})(100°\mathrm{C} - 23°\mathrm{C})}{200\,\mathrm{J/s}} = 160 \text{ s} .$$

36. (a) Using Eq. 19-17, the heat transferred to the water is

$$\begin{aligned} Q_w &= c_w m_w \Delta T + L_V m_s \\ &= (1\,\mathrm{cal/g{\cdot}C^\circ})(220\,\mathrm{g})(100°\mathrm{C} - 20.0°\mathrm{C}) + (539\,\mathrm{cal/g})(5.00\,\mathrm{g}) \\ &= 20.3\,\mathrm{kcal} . \end{aligned}$$

(b) The heat transferred to the bowl is

$$Q_b = c_b m_b \Delta T = (0.0923\,\mathrm{cal/g{\cdot}C^\circ})(150\,\mathrm{g})(100°\mathrm{C} - 20.0°\mathrm{C}) = 1.11 \text{ kcal} .$$

(c) If the original temperature of the cylinder be T_i, then $Q_w + Q_b = c_c m_c (T_i - T_f)$, which leads to

$$T_i = \frac{Q_w + Q_b}{c_c m_c} + T_f = \frac{20.3\,\text{kcal} + 1.11\,\text{kcal}}{(0.0923\,\text{cal/g}\cdot\text{C}^\circ)(300\,\text{g})} + 100^\circ\text{C} = 873^\circ\text{C} \ .$$

37. Mass m of water must be raised from an initial temperature $T_i = 59^\circ\text{F} = 15^\circ\text{C}$ to a final temperature $T_f = 100^\circ\text{C}$. If c is the specific heat of water then the energy required is $Q = cm(T_f - T_i)$. Each shake supplies energy mgh, where h is the distance moved during the downward stroke of the shake. If N is the total number of shakes then $Nmgh = Q$. If t is the time taken to raise the water to its boiling point then $(N/t)mgh = Q/t$. We note that N/t is the rate R of shaking (30 shakes/min). This leads to $Rmgh = Q/t$. The distance h is 1.0 ft = 0.3048 m. Consequently,

$$\begin{aligned} t &= \frac{Q}{Rmgh} = \frac{cm(T_f - T_i)}{Rmgh} = \frac{c(T_f - T_i)}{Rgh} \\ &= \frac{(4190\,\text{J/kg}\cdot\text{K})(100^\circ\text{C} - 15^\circ\text{C})}{(30\,\text{shakes/min})(9.8\,\text{m/s}^2)(0.3048\,\text{m})} \\ &= 3.97 \times 10^3\,\text{min} = 2.8\ \text{days} \ . \end{aligned}$$

38. We note from Eq. 19-12 that 1 Btu = 252 cal. The heat relates to the power, and to the temperature change, through $Q = Pt = cm\Delta T$. Therefore, the time t required is

$$\begin{aligned} t &= \frac{cm\Delta T}{P} = \frac{(1000\,\text{cal/kg}\cdot\text{C}^\circ)(40\,\text{gal})(1000\,\text{kg/264\,gal})(100^\circ\text{F} - 70^\circ\text{F})(5\text{C}^\circ/9\text{F}^\circ)}{(2.0 \times 10^5\,\text{Btu/h})(252.0\,\text{cal/Btu})(1\,\text{h}/60\,\text{min})} \\ &= 3.0\ \text{min} \ . \end{aligned}$$

The metric version proceeds similarly:

$$\begin{aligned} t &= \frac{c\rho V\Delta T}{P} = \frac{(4190\,\text{J/kg}\cdot\text{C}^\circ)(1000\,\text{kg/m}^3)(150\,\text{L})(1\,\text{m}^3/1000\,\text{L})(38^\circ\text{C} - 21^\circ\text{C})}{(59000\,\text{J/s})(60\,\text{s}/1\,\text{min})} \\ &= 3.0\ \text{min} \ . \end{aligned}$$

39. To accomplish the phase change at 78°C, $Q = L_V\, m = (879)(0.510) = 448.29$ kJ must be removed. To cool the liquid to −114°C, $Q = cm|\Delta T| = (2.43)(0.510)(192) = 237.95$ kJ, must be removed. Finally, to accomplish the phase change at −114°C, $Q = L_F\, m = (109)(0.510) = 55.59$ kJ must be removed. The grand total of heat removed is therefore $448.29 + 237.95 + 55.59 = 742$ kJ.

40. The deceleration a of the car is given by $v_f^2 - v_i^2 = -v_i^2 = 2ad$, or

$$a = -\frac{[(90\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})]^2}{2(80\,\text{m})} = -3.9\ \text{m/s}^2 \ .$$

The time t it takes for the car to stop is then

$$t = \frac{v_f - v_i}{a} = -\frac{(90\,\text{km/h})(10^3\,\text{m/km})(1\,\text{h}/3600\,\text{s})}{-3.9\,\text{m/s}^2} = 6.4\ \text{s} \ .$$

The average rate at which thermal energy is produced is then

$$P = \frac{\frac{1}{2}mv_i^2}{t} = \frac{(1500\,\text{kg})[(90\,\text{km/h})(1000\,\text{m/km})(1\,\text{h}/3600\,\text{s})]^2}{2(6.4\,\text{s})} = 7.3 \times 10^4\ \text{W} \ .$$

41. The heat needed is found by integrating the heat capacity:

$$\begin{aligned} Q &= \int_{T_i}^{T_f} cm\,dT = m\int_{T_i}^{T_f} c\,dT \\ &= (2.09)\int_{5.0^\circ\mathrm{C}}^{15.0^\circ\mathrm{C}} (0.20 + 0.14T + 0.023T^2)\,dT \\ &= (2.0)(0.20T + 0.070T^2 + 0.00767T^3)\Big|_{5.0}^{15.0} \text{ (cal)} \\ &= 82 \text{ cal} . \end{aligned}$$

42. The power consumed by the system is

$$\begin{aligned} P &= \left(\frac{1}{20\%}\right)\frac{cm\Delta T}{t} \\ &= \left(\frac{1}{20\%}\right)\frac{(4.18\,\mathrm{J/g\cdot{}^\circ C})(200\times 10^3\,\mathrm{cm^3})(1\,\mathrm{g/cm^3})(40^\circ\mathrm{C} - 20^\circ\mathrm{C})}{(1.0\,\mathrm{h})(3600\,\mathrm{s/h})} \\ &= 2.3\times 10^4 \text{ W} . \end{aligned}$$

The area needed is then

$$A = \frac{2.3\times 10^4\,\mathrm{W}}{700\,\mathrm{W/m^2}} = 33 \text{ m}^2 .$$

43. Let the mass of the steam be m_s and that of the ice be m_i. Then $L_F m_c + c_w m_c(T_f - 0.0^\circ\mathrm{C}) = L_s m_s + c_w m_s(100^\circ\mathrm{C} - T_f)$, where $T_f = 50^\circ\mathrm{C}$ is the final temperature. We solve for m_s:

$$\begin{aligned} m_s &= \frac{L_F m_c + c_w m_c(T_f - 0.0^\circ\mathrm{C})}{L_s + c_w(100^\circ\mathrm{C} - T_f)} \\ &= \frac{(79.7\,\mathrm{cal/g})(150\,\mathrm{g}) + (1\,\mathrm{cal/g\cdot{}^\circ C})(150\,\mathrm{g})(50^\circ\mathrm{C} - 0.0^\circ\mathrm{C})}{539\,\mathrm{cal/g} + (1\,\mathrm{cal/g\cdot C^\circ})(100^\circ\mathrm{C} - 50^\circ\mathrm{C})} \\ &= 33 \text{ g} . \end{aligned}$$

44. We compute with Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 19-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. If the equilibrium temperature is T_f then the energy absorbed as heat by the ice is $Q_I = L_F m_I + c_w m_I(T_f - 0^\circ\mathrm{C})$, while the energy transferred as heat from the water is $Q_w = c_w m_w(T_f - T_i)$. The system is insulated, so $Q_w + Q_I = 0$, and we solve for T_f:

$$T_f = \frac{c_w m_w T_i - L_F m_I}{(m_I + m_c)\,c_w} .$$

(a) Now $T_i = 90^\circ\mathrm{C}$ so

$$T_f = \frac{(4190\,\mathrm{J/kg\cdot C^\circ})(0.500\,\mathrm{kg})(90^\circ\mathrm{C}) - \left(333\times 10^3\,\mathrm{J/kg}\right)(0.500\,\mathrm{kg})}{(0.500\,\mathrm{kg} + 0.500\,\mathrm{kg})(4190\,\mathrm{J/kg\cdot C^\circ})} = 5.3^\circ\mathrm{C} .$$

(b) If we were to use the formula above with $T_i = 70^\circ\mathrm{C}$, we would get $T_f < 0$, which is impossible. In fact, not all the ice has melted in this case (and the equilibrium temperature is 0°C) The amount of ice that melts is given by

$$m_I' = \frac{c_w m_w\,(T_i - 0^\circ\mathrm{C})}{L_F} = \frac{(4190\,\mathrm{J/kg\cdot C^\circ})\,(0.500\,\mathrm{kg})\,(70\,\mathrm{C^\circ})}{333\times 10^3\,\mathrm{J/kg}} = 0.440 \text{ kg} .$$

Therefore, there amount of (solid) ice remaining is $\Delta m_I = m_I - m_I' = 500\,\mathrm{g} - 440\,\mathrm{g} = 60\,\mathrm{g}$, and (as mentioned) we have $T_f = 0^\circ\mathrm{C}$ (because the system is an ice-water mixture in thermal equilibrium).

45. (a) We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 19-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

- None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.
- The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.
- All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, we suppose that no ice melts. The temperature of the water decreases from $T_{Wi} = 25°\text{C}$ to some final temperature T_f and the temperature of the ice increases from $T_{Ii} = -15°\text{C}$ to T_f. If m_W is the mass of the water and c_W is its specific heat then the water rejects heat

$$|Q| = c_W m_W \left(T_{Wi} - T_f\right) \ .$$

If m_I is the mass of the ice and c_I is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{Ii}) \ .$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_W m_W \left(T_{Wi} - T_f\right) = c_I m_I \left(T_f - T_{Ii}\right) \ .$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{c_W m_W + c_I m_I} \\ &= \frac{(4190\,\text{J/kg·K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg·K})(0.100\,\text{kg})(-15°\text{C})}{(4190\,\text{J/kg·K})(0.200\,\text{kg}) + (2220\,\text{J/kg·K})(0.100\,\text{kg})} \\ &= 16.6°\text{C} \ . \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at $T_f = 0°\text{C}$, with mass m ($< m_I$) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_W m_W T_{Wi} \ ,$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{Ii}) + m L_F \ ,$$

where L_F is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to 0°C and the second term is the energy required to melt mass m of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{Ii} + m L_F \ .$$

This equation can be solved for the mass m of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii}}{L_F} \\ &= \frac{(4190\,\text{J/kg·K})(0.200\,\text{kg})(25°\text{C}) + (2220\,\text{J/kg·K})(0.100\,\text{kg})(-15°\text{C})}{333 \times 10^3\,\text{J/kg}} \\ &= 5.3 \times 10^{-2}\,\text{kg} = 53\ \text{g} \ . \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to 0°C. This is then the solution: the ice and water reach thermal equilibrium at a temperature of 0°C with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{Ii}) + c_W m_I (T_f - 0) + m_I L_F \ .$$

The first term is the energy required to raise the temperature of the ice to 0°C, the second term is the energy required to raise the temperature of the melted ice from 0°C to T_f, and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{Ii}) + c_W m_I T_f + m_I L_F \ .$$

The solution for T_f is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{Ii} - m_I L_F}{c_W (m_W + m_I)} \ .$$

Inserting the given values, we obtain $T_f = 2.5$°C.

46. We denote the ice with subscript I and the coffee with c, respectively. Let the final temperature be T_f. The heat absorbed by the ice is $Q_I = \lambda_F m_I + m_I c_w \left(T_f - 0°\text{C}\right)$, and the heat given away by the coffee is $|Q_c| = m_w c_w \left(T_I - T_f\right)$. Setting $Q_I = |Q_c|$, we solve for T_f:

$$\begin{aligned} T_f &= \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c) c_w} \\ &= \frac{(130\,\text{g})\,(4190\,\text{J/kg}\cdot\text{C}°)\,(80.0°\text{C}) - \left(333 \times 10^3\,\text{J/g}\right)(12.0\,\text{g})}{(12.0\,\text{g} + 130\,\text{g})\,(4190\,\text{J/kg}\cdot\text{C}°)} \\ &= 66.5°\text{C} \ . \end{aligned}$$

Note that we work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 19-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by $|\Delta T| = 80.0°\text{C} - 66.5°\text{C} = 13.5\text{C}°$.

47. If the ring diameter at 0.000°C is D_{r0} then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0}(1 + \alpha_c T_f) \,,$$

where T_f is the final temperature and α_c is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at T_i (= 100.0°C) is D_{s0} then its diameter at the final temperature is

$$D_s = D_{s0}[1 + \alpha_a (T_f - T_i)] \,,$$

where α_a is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1 + \alpha_c T_f) = D_{s0}[1 + \alpha_a (T_f - T_i)] \ .$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\ &= \frac{2.54000\,\text{cm} - 2.54508\,\text{cm} + (2.54508\,\text{cm})\left(23 \times 10^{-6}/\text{C}°\right)(100°\text{C})}{(2.54508\,\text{cm})\,(23 \times 10^{-6}/\text{C}°) - (2.54000\,\text{cm})\,(17 \times 10^{-6}/\text{C}°)} \\ &= 50.38°\text{C} \ . \end{aligned}$$

The expansion coefficients are from Table 19-2 of the text. Since the initial temperature of the ring is 0°C, the heat it absorbs is

$$Q = c_c m_r T_f \ ,$$

where c_c is the specific heat of copper and m_r is the mass of the ring. The heat rejected up by the sphere is

$$|Q| = c_a m_s (T_i - T_f)$$

where c_a is the specific heat of aluminum and m_s is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s (T_i - T_f) \ ,$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a (T_i - T_f)} = \frac{(386\,\text{J/kg}\cdot\text{K})(0.0200\,\text{kg})(50.38°\text{C})}{(900\,\text{J/kg}\cdot\text{K})(100°\text{C} - 50.38°\text{C})} = 8.71 \times 10^{-3}\ \text{kg} \ .$$

48. (a) Since work is done *on* the system (perhaps to compress it) we write $W = -200\,\text{J}$.

    (b) Since heat leaves the system, we have $Q = -70.0\,\text{cal} = -293\,\text{J}$.

    (c) The change in internal energy is $\Delta E_{\text{int}} = Q - W = -293\,\text{J} - (-200\,\text{J}) = -93\,\text{J}$.

49. One part of path A represents a constant pressure process. The volume changes from $1.0\,\text{m}^3$ to $4.0\,\text{m}^3$ while the pressure remains at 40 Pa. The work done is

$$W_A = p\,\Delta V = (40\,\text{Pa})\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 120\ \text{J} \ .$$

The other part of the path represents a constant volume process. No work is done during this process. The total work done over the entire path is 120 J. To find the work done over path B we need to know the pressure as a function of volume. Then, we can evaluate the integral $W = \int p\,dV$. According to the graph, the pressure is a linear function of the volume, so we may write $p = a + bV$, where a and b are constants. In order for the pressure to be 40 Pa when the volume is $1.0\,\text{m}^3$ and 10 Pa when the volume is $4.00\,\text{m}^3$ the values of the constants must be $a = 50\,\text{Pa}$ and $b = -10\,\text{Pa/m}^3$. Thus $p = 50\,\text{Pa} - (10\,\text{Pa/m}^3)V$ and

$$\begin{aligned} W_B &= \int_1^4 p\,dV = \int_1^4 (50 - 10V)\,dV = \left(50V - 5V^2\right)\Big|_1^4 \\ &= 200\,\text{J} - 50\,\text{J} - 80\,\text{J} + 5\,\text{J} = 75\ \text{J} \ . \end{aligned}$$

One part of path C represents a constant pressure process in which the volume changes from $1.0\,\text{m}^3$ to $4.0\,\text{m}^3$ while p remains at 10 Pa. The work done is

$$W_C = p\,\Delta V = (10\,\text{Pa})(4.0\,\text{m}^3 - 1.0\,\text{m}^3) = 30\ \text{J} \ .$$

The other part of the process is at constant volume and no work is done. The total work is 30 J. We note that the work is different for different paths.

50. (a)
    - During process $A \to B$, the system is expanding, doing work on its environment, so $W > 0$, and since $\Delta E_{\text{int}} > 0$ is given then $Q = W + \Delta E_{\text{int}}$ must also be positive.
    - During process $B \to C$, the system is neither expanding nor contracting, so $W = 0$; therefore, the sign of ΔE_{int} must be the same (by the first law of thermodynamics) as that of Q (which is given as positive).
    - During process $C \to A$, the system is contracting (the environment is doing work on the system), which implies $W < 0$. Also, $\Delta E_{\text{int}} < 0$ because $\sum \Delta E_{\text{int}} = 0$ (for the whole cycle) and the other values of ΔE_{int} (for the other processes) were positive. Therefore, $Q = W + \Delta E_{\text{int}}$ must also be negative.

(b) The area of a triangle is $\frac{1}{2}$(base)(height). Applying this to the figure, we find $|W_{\rm net}| = \frac{1}{2}(2.0\,{\rm m}^3)(20\,{\rm Pa}) =$ 20 J. Since process $C \to A$ involves larger negative work (it occurs at higher average pressure) than the positive work done during process $A \to B$, then the net work done during the cycle must be negative. The answer is therefore $W_{\rm net} = -20$ J.

51. Over a cycle, the internal energy is the same at the beginning and end, so the heat Q absorbed equals the work done: $Q = W$. Over the portion of the cycle from A to B the pressure p is a linear function of the volume V and we may write

$$p = \frac{10}{3}\,{\rm Pa} + \left(\frac{20}{3}\,{\rm Pa/m^3}\right) V\,,$$

where the coefficients were chosen so that $p = 10\,{\rm Pa}$ when $V = 1.0\,{\rm m}^3$ and $p = 30\,{\rm Pa}$ when $V = 4.0\,{\rm m}^3$. The work done by the gas during this portion of the cycle is

$$\begin{aligned} W_{AB} &= \int_1^4 p\,dV = \int_1^4 (\frac{10}{3} + \frac{20}{3}V)\,dV = \left(\frac{10}{3}V + \frac{10}{3}V^2\right)\Big|_1^4 \\ &= \frac{40}{3} + \frac{160}{3} - \frac{10}{3} - \frac{10}{3} = 60\ {\rm J}\,. \end{aligned}$$

The BC portion of the cycle is at constant pressure and the work done by the gas is $W_{BC} = p\,\Delta V =$ $(30\,{\rm Pa})(1.0\,{\rm m}^3 - 4.0\,{\rm m}^3) = -90\,{\rm J}$. The CA portion of the cycle is at constant volume, so no work is done. The total work done by the gas is $W = W_{AB} + W_{BC} + W_{CA} = 60\,{\rm J} - 90\,{\rm J} + 0 = -30\,{\rm J}$ and the total heat absorbed is $Q = W = -30\,{\rm J}$. This means the gas loses 30 J of energy in the form of heat.

52. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero and the heat absorbed by the gas is equal to the work done by the gas: $Q = W$. In terms of the contributions of the individual parts of the cycle $Q_{AB} + Q_{BC} + Q_{CA} = W$ and $Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0\,{\rm J} - 20.0\,{\rm J} - 0 = -5.0\,{\rm J}$. This means 5.0 J of energy leaves the gas in the form of heat.

53. (a) The change in internal energy $\Delta E_{\rm int}$ is the same for path iaf and path ibf. According to the first law of thermodynamics, $\Delta E_{\rm int} = Q - W$, where Q is the heat absorbed and W is the work done by the system. Along iaf $\Delta E_{\rm int} = Q - W = 50\,{\rm cal} - 20\,{\rm cal} = 30\,{\rm cal}$. Along ibf $W = Q - \Delta E_{\rm int} =$ $36\,{\rm cal} - 30\,{\rm cal} = 6\,{\rm cal}$.

(b) Since the curved path is traversed from f to i the change in internal energy is $-30\,{\rm cal}$ and $Q =$ $\Delta E_{\rm int} + W = -30\,{\rm cal} - 13\,{\rm cal} = -43\,{\rm cal}$.

(c) Let $\Delta E_{\rm int} = E_{{\rm int},\,f} - E_{{\rm int},\,i}$. Then, $E_{{\rm int},\,f} = \Delta E_{\rm int} + E_{{\rm int},\,i} = 30\,{\rm cal} + 10\,{\rm cal} = 40\,{\rm cal}$.

(d) The work W_{bf} for the path bf is zero, so $Q_{bf} = E_{{\rm int},\,f} - E_{{\rm int},\,b} = 40\,{\rm cal} - 22\,{\rm cal} = 18\,{\rm cal}$. For the path ibf $Q = 36\,{\rm cal}$ so $Q_{ib} = Q - Q_{bf} = 36\,{\rm cal} - 18\,{\rm cal} = 18\,{\rm cal}$.

54. We use $P_{\rm cond} = kA(T_H - T_C)/L$. The temperature T_H at a depth of 35.0 km is

$$T_H = \frac{P_{\rm cond}L}{kA} + T_C = \frac{(54.0\times10^{-3}\,{\rm W/m^2})(35.0\times10^3\,{\rm m})}{2.50\,{\rm W/m\cdot K}} + 10.0^\circ{\rm C} = 766^\circ{\rm C}\,.$$

55. We refer to the polyurethane foam with subscript p and silver with subscript s. We use Eq 19-32 to find $L = kR$.

(a) From Table 19-6 we find $k_p = 0.024\,{\rm W/m\cdot K}$ so

$$\begin{aligned} L_p &= k_pR_p \\ &= (0.024\,{\rm W/m\cdot K})(30\,{\rm ft^2\cdot F^\circ\cdot h/Btu})(1\,{\rm m}/3.281\,{\rm ft})^2(5{\rm C^\circ}/9{\rm F^\circ})(3600\,{\rm s/h})(1\,{\rm Btu}/1055\,{\rm J}) \\ &= 0.13\ {\rm m}\,. \end{aligned}$$

(b) For silver $k_s = 428\,\mathrm{W/m\cdot K}$, so

$$L_s = k_s R_s = \left(\frac{k_s R_s}{k_p R_p}\right) L_p = \left[\frac{428(30)}{0.024(30)}\right](0.13\,\mathrm{m}) = 2.3 \times 10^3 \text{ m} .$$

56. (a) The rate of heat flow is

$$P_{\mathrm{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(0.040\,\mathrm{W/m\cdot K})(1.8\,\mathrm{m}^2)(33°\mathrm{C} - 1.0°\mathrm{C})}{1.0 \times 10^{-2}\,\mathrm{m}} = 2.3 \times 10^2 \text{ J/s} .$$

(b) The new rate of heat flow is

$$P'_{\mathrm{cond}} = \frac{k' P_{\mathrm{cond}}}{k} = \frac{(0.60\,\mathrm{W/m\cdot K})(230\,\mathrm{J/s})}{0.040\,\mathrm{W/m\cdot K}} = 3.5 \times 10^3 \text{ J/s} ,$$

which is about 15 times as fast as the original heat flow.

57. The rate of heat flow is given by

$$P_{\mathrm{cond}} = kA\,\frac{T_H - T_C}{L} ,$$

where k is the thermal conductivity of copper ($401\,\mathrm{W/m\cdot K}$), A is the cross-sectional area (in a plane perpendicular to the flow), L is the distance along the direction of flow between the points where the temperature is T_H and T_C. Thus,

$$P_{\mathrm{cond}} = \frac{(401\,\mathrm{W/m\cdot K})(90.0 \times 10^{-4}\,\mathrm{m}^2)(125°\mathrm{C} - 10.0°\mathrm{C})}{0.250\,\mathrm{m}} = 1.66 \times 10^3 \text{ J/s} .$$

The thermal conductivity is found in Table 19-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

58. (a) We estimate the surface area of the average human body to be about $2\,\mathrm{m}^2$ and the skin temperature to be about 300 K (somewhat less than the internal temperature of 310 K). Then from Eq. 19-37

$$P_r = \sigma\varepsilon A T^4 \approx \left(5.67 \times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\right)(0.9)\left(2.0\,\mathrm{m}^2\right)(300\,\mathrm{K})^4 = 8 \times 10^2 \text{ W} .$$

(b) The energy lost is given by

$$\Delta E = P_r \Delta t = (8 \times 10^2\,\mathrm{W})(30\,\mathrm{s}) = 2 \times 10^4 \text{ J} .$$

59. (a) Recalling that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale, we find that the rate of heat conduction is

$$P_{\mathrm{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(401\,\mathrm{W/m\cdot K})(4.8 \times 10^{-4}\,\mathrm{m}^2)(100\,\mathrm{C°})}{1.2\,\mathrm{m}} = 16 \text{ J/s} .$$

(b) Using Table 19-4, the rate at which ice melts is

$$\left|\frac{dm}{dt}\right| = \frac{P_{\mathrm{cond}}}{L_F} = \frac{16\,\mathrm{J/s}}{333\,\mathrm{J/g}} = 0.048 \text{ g/s} .$$

60. With arrangement (a), the rate of the heat flow is

$$\begin{aligned} P_{\mathrm{cond}\,a} &= P_{\mathrm{cond}\,1} + P_{\mathrm{cond}\,2} = \frac{Ak_1}{2L}(T_H - T_C) + \frac{Ak_2}{2L}(T_H - T_C) \\ &= \frac{A}{2L} k_a (T_H - T_C) \end{aligned}$$

where $k_a = 4K_1 + k_2$. With arrangement (b), we use Eq. 19-36 to find the rate of heat flow:

$$P_{\text{cond}\,b} = \frac{2A\,(T_H - T_C)}{(L/k_1) + (L/k_2)} = \frac{A}{2L} k_b\,(T_H - T_C)$$

where $k_b = fk_1k_2/(k_1 + k_2)$. Since $k_1 \neq k_2$, we see that $(k_1 - k_2)^2 = (k_1 + k_2)^2 - 4k_1k_2 > 0$, or

$$\frac{k_b}{k_a} = \frac{4k_1 + k_2}{(k_1 + k_2)^2} < 0 \ .$$

Therefore, $P_{\text{cond}\,b} < P_{\text{cond}\,a}$. That is, arrangement (b) would give the lower heat flow.

61. We use $P_{\text{cond}} = kA\Delta T/L \propto A/L$. Comparing cases (a) and (b) in Figure 19-40, we have

$$P_{\text{cond}\,b} = \left(\frac{A_bL_a}{A_aL_b}\right) P_{\text{cond}\,a} = 4P_{\text{cond}\,a} \quad .$$

Consequently, it would take $2.0\,\text{min}/4 = 0.5\,\text{min}$ for the same amount of heat to be conducted through the rods welded as shown in Fig. 19-42(b).

62. We use Eqs. 19-38 through 19-40. Note that the surface area of the sphere is given by $A = 4\pi r^2$, where $r = 0.500\,\text{m}$ is the radius.

(a) The temperature of the sphere is $T = 273.15 + 27.00 = 300.15\,\text{K}$. Thus

$$\begin{aligned} P_r &= \sigma\varepsilon AT^4 \\ &= \left(5.67 \times 10^{-8}\,\text{W/m}^2{\cdot}\text{K}^4\right)(0.850)(4\pi)(0.500\,\text{m})^2(300.15\,\text{K})^4 \\ &= 1.23 \times 10^3 \ \text{W} \ . \end{aligned}$$

(b) Now, $T_{\text{env}} = 273.15 + 77.00 = 350.15\,\text{K}$ so

$$\begin{aligned} P_a &= \sigma\varepsilon AT_{\text{env}}^4 \\ &= \left(5.67 \times 10^{-8}\,\text{W/m}^2{\cdot}\text{K}^4\right)(0.850)(4\pi)(0.500\,\text{m})^2(350.15\,\text{K})^4 \\ &= 2.28 \times 10^3 \ \text{W} \ . \end{aligned}$$

(c) From Eq. 19-40, we have

$$P_n = P_a - P_r = 2.28 \times 10^3\,\text{W} - 1.23 \times 10^3\,\text{W} = 1.05 \times 10^3 \ \text{W} \ .$$

63. (a) We use

$$P_{\text{cond}} = kA\,\frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 19-6 as $1.0\,\text{W/m}{\cdot}\text{K}$. We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72^\circ\text{F} - (-20^\circ\text{F}) = 92\,\text{F}^\circ$$

is equivalent to $\frac{5}{9}(92) = 51.1\,\text{C}^\circ$. This, in turn, is equal to 51.1 K since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{\text{cond}}}{A} = k\,\frac{T_H - T_C}{L} = (1.0\,\text{W/m}{\cdot}\text{K})\left(\frac{51.1\,\text{C}^\circ}{3.0 \times 10^{-3}\,\text{m}}\right) = 1.7 \times 10^4 \ \text{W/m}^2 \ .$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate P is the same in each layer and is given by

$$P_{\text{cond}} = \frac{A\,(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If L_g is the thickness of a glass layer, L_a is the thickness of the air layer, k_g is the thermal conductivity of glass, and k_a is the thermal conductivity of air, then the denominator is

$$\sum \frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g} \; .$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\begin{aligned} \frac{P_{\text{cond}}}{A} &= \frac{(T_H - T_C)k_a k_g}{2L_g k_a + L_a k_g} \\ &= \frac{(51.1\,\mathrm{C}^\circ)\,(0.026\,\mathrm{W/m{\cdot}K})(1.0\,\mathrm{W/m{\cdot}K})}{2(3.0\times10^{-3}\,\mathrm{m})(0.026\,\mathrm{W/m{\cdot}K}) + (0.075\,\mathrm{m})(1.0\,\mathrm{W/m{\cdot}K})} \\ &= 18\ \mathrm{W/m^2} \; . \end{aligned}$$

64. We divide both sides of Eq. 19-32 by area A, which gives us the (uniform) rate of heat conduction per unit area:

$$\frac{P_{\text{cond}}}{A} = k_1\,\frac{T_H - T_1}{L_1} = k_4\,\frac{T - T_C}{L_4}$$

where $T_H = 30°\mathrm{C}$, $T_1 = 25°\mathrm{C}$ and $T_C = -10°\mathrm{C}$. We solve for the unknown T.

$$T = T_C + \frac{k_1\,L_4}{k_4\,L_1}\,(T_H - T_1) = -4.2°\mathrm{C} \; .$$

65. Let h be the thickness of the slab and A be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA\,(T_H - T_C)}{h}$$

where k is the thermal conductivity of ice, T_H is the temperature of the water (0°C), and T_C is the temperature of the air above the ice (−10°C). The heat leaving the water freezes it, the heat required to freeze mass m of water being $Q = L_F m$, where L_F is the heat of fusion for water. Differentiate with respect to time and recognize that $dQ/dt = P_{\text{cond}}$ to obtain

$$P_{\text{cond}} = L_F\,\frac{dm}{dt} \; .$$

Now, the mass of the ice is given by $m = \rho A h$, where ρ is the density of ice and h is the thickness of the ice slab, so $dm/dt = \rho A(dh/dt)$ and

$$P_{\text{cond}} = L_F \rho A\,\frac{dh}{dt} \; .$$

We equate the two expressions for P_{cond} and solve for $dh/\mathrm{d}t$:

$$\frac{dh}{dt} = \frac{k(T_H - T_C)}{L_F \rho h} \; .$$

Since $1\,\mathrm{cal} = 4.186\,\mathrm{J}$ and $1\,\mathrm{cm} = 1\times10^{-2}\,\mathrm{m}$, the thermal conductivity of ice has the SI value $k = (0.0040\,\mathrm{cal/s{\cdot}cm{\cdot}K})(4.186\,\mathrm{J/cal})/(1{\times}10^{-2}\,\mathrm{m/cm}) = 1.674\,\mathrm{W/m{\cdot}K}$. The density of ice is $\rho = 0.92\,\mathrm{g/cm^3} = 0.92\times10^3\,\mathrm{kg/m^3}$. Thus,

$$\frac{dh}{dt} = \frac{(1.674\,\mathrm{W/m{\cdot}K})(0°\mathrm{C} + 10°\mathrm{C})}{(333\times10^3\,\mathrm{J/kg})(0.92\times10^3\,\mathrm{kg/m^3})(0.050\,\mathrm{m})} = 1.1\times10^{-6}\,\mathrm{m/s} = 0.40\ \mathrm{cm/h} \; .$$

66. We assume (although this should be viewed as a "controversial" assumption) that the top surface of the ice is at $T_C = -5.0°\text{C}$. Less controversial are the assumptions that the bottom of the body of water is at $T_H = 4.0°\text{C}$ and the interface between the ice and the water is at $T_X = 0.0°\text{C}$. The primary mechanism for the heat transfer through the total distance $L = 1.4$ m is assumed to be conduction, and we use Eq. 19-34:

$$\begin{aligned}\frac{k_{\text{water}}A\,(T_H - T_X)}{L - L_{\text{ice}}} &= \frac{k_{\text{ice}}A\,(T_X - T_C)}{L_{\text{ice}}} \\ \frac{(0.12)A\,(4.0° - 0.0°)}{1.4 - L_{\text{ice}}} &= \frac{(0.40)A\,(0.0° + 5.0°)}{L_{\text{ice}}}\ .\end{aligned}$$

We cancel the area A and solve for thickness of the ice layer: $L_{\text{ice}} = 1.1$ m.

67. For a cylinder of height h, the surface area is $A_c = 2\pi rh$, and the area of a sphere is $A_o = 4\pi R^2$. The net radiative heat transfer is given by Eq. 19-40.

   (a) We estimate the surface area A of the body as that of a cylinder of height 1.8 m and radius $r = 0.15$ m plus that of a sphere of radius $R = 0.10$ m. Thus, we have $A \approx A_c + A_o = 1.8\ \text{m}^2$. The emissivity $\varepsilon = 0.80$ is given in the problem, and the Stefan-Boltzmann constant is found in §19-11: $\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. The "environment" temperature is $T_{\text{env}} = 303$ K, and the skin temperature is $T = \frac{5}{9}(102 - 32) + 273 = 312$ K. Therefore,

   $$P_{\text{net}} = \sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right) = -86\ \text{W}\ .$$

   The corresponding sign convention is discussed in the textbook immediately after Eq. 19-40. We conclude that heat is being lost by the body at a rate of roughly 90 W.

   (b) Half the body surface area is roughly $A = 1.8/2 = 0.9\ \text{m}^2$. Now, with $T_{\text{env}} = 248$ K, we find

   $$|P_{\text{net}}| = \left|\sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right)\right| \approx 230\ \text{W}\ .$$

   (c) Finally, with $T_{\text{env}} = 193$ K (and still with $A = 0.9\ \text{m}^2$) we obtain $|P_{\text{net}}| = 330$ W.

68. (a) The top surface area is that of a circle $A_o = \pi r^2$. Since the problem directs us to denote this as "a" then the radius is

   $$r = \sqrt{\frac{a}{\pi}}\ .$$

   The side surface of a cylinder of height h is $A_c = 2\pi rh$. Therefore, the total radiating surface area is

   $$A = A_o + A_c = a + 2\pi\left(\sqrt{\frac{a}{\pi}}\right)h = a + 2h\sqrt{\pi a}\ .$$

   Consequently, Eq. 19-38 leads to

   $$P_i = \sigma\varepsilon AT^4 = \sigma\varepsilon T^4\left(a + 2h\sqrt{\pi a}\right)\ .$$

   (b) Packing together N rigid cylinders as close as possible into a large cylinder-like arrangement can involve some subtle mathematics, which we will avoid by simply assuming that (perhaps due to the fact that these "cylinders" are certainly not rigid!) they somehow become a large-radius (R) cylinder of height h. With the top surface area being Na, the large radius is

   $$R = \sqrt{\frac{Na}{\pi}}\ .$$

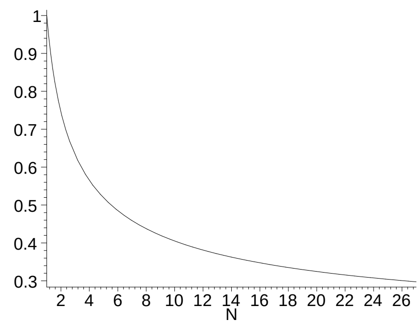
   The side surface of the large-radius cylinder is $A_c = 2\pi Rh$. Therefore, the total radiating surface area is

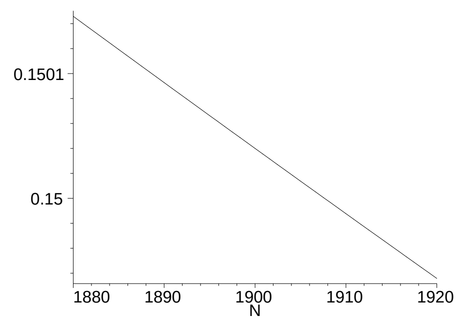
   $$A = A_o + A_c = Na + 2\pi\left(\sqrt{\frac{Na}{\pi}}\right)h = Na + 2h\sqrt{N\pi a}\ .$$

Consequently, Eq. 19-38 leads to

$$P_h = \sigma\varepsilon A T^4 = \sigma\varepsilon T^4\left(Na + 2h\sqrt{N\pi a}\right) \ .$$

(c) The graphs below shows P_h/NP_i (vertical axis) versus the number of penguins N (horizontal axis).





(d) This can be estimated from the graph, in which case we $N \approx 5$, or algebraically solved for (in which case $N = 5.53$ which should be rounded to 5 or 6).

(e) From the graph, we estimate $N \approx 10$. If we algebraically solve for it, we get $N = 10.4$ which should be rounded to 10 or 11.

(f) From the graph, we estimate $N \approx 26$. If we algebraically solve for it, we get $N = 26.2$ which should be rounded to 26.

(g) A graph over the appropriate range is not shown above (but would be straightforward to generate). If we algebraically solve for it, we get $N = 154.8$ which should be rounded to 150 or 160.

(h) From the second graph above, we estimate N is slightly more than 1900. If we algebraically solve for it, we get $N = 1907.65$ which should be rounded to 1900.

(i) The $N \to \infty$ limit of the ratio

$$\frac{Na + 2h\sqrt{\pi N a}}{N(a + 2h\sqrt{\pi a})}\frac{a + 2h\sqrt{\pi a/N}}{a + 2h\sqrt{\pi a}} \to \frac{a}{a + 2h\sqrt{\pi a}}$$

is 0.13. We note that this value depends on the ratio of $h/\sqrt{a}$.

69. We denote $T_H = 100°\text{C}$, $T_C = 0°\text{C}$, the temperature of the copper-aluminum junction by T_1 and that of the aluminum-brass junction by T_2. Then,

$$P_{\text{cond}} = \frac{k_c A}{L}(T_H - T_1) = \frac{k_a A}{L}(T_1 - T_2) = \frac{k_b A}{L}(T_2 - T_c) \ .$$

We solve for T_1 and T_2 to obtain

$$\begin{aligned} T_1 &= T_H + \frac{T_C - T_H}{1 + k_c(k_a + k_b)/k_a k_b} \\ &= 100°\text{C} + \frac{0.00°\text{C} - 100°\text{C}}{1 + 401(235 + 109)/[(235)(109)]} = 84.3°\text{C} \end{aligned}$$

and

$$\begin{aligned} T_2 &= T_c + \frac{T_H - T_C}{1 + k_b(k_c + k_a)/k_c k_a} \\ &= 0.00°\text{C} + \frac{100°\text{C} - 0.00°\text{C}}{1 + 109(235 + 401)/[(235)(401)]} \\ &= 57.6°\text{C} \ . \end{aligned}$$

70. The heat conducted is

$$\begin{aligned} Q &= P_{\text{cond}}t = \frac{kAt\Delta T}{L} \\ &= \frac{(67\,\text{W/m}\cdot\text{K})(\pi/4)(1.7\,\text{m})^2(5.0\,\text{min})(60\,\text{s/min})(2.3\,\text{C}^\circ)}{5.2\times10^{-3}\,\text{m}} \\ &= 2.0\times10^7\ \text{J}\ . \end{aligned}$$

71. The problem asks for 0.5% of E, where $E = Pt$ with $t = 120$ s and P given by Eq. 19-38. Therefore, with $A = 4\pi r^2 = 5.0\times10^{-3}$ m^2, we obtain

$$(0.005)Pt = (0.005)\sigma\varepsilon AT^4 t = 8.6\ \text{J}\ .$$

72. We denote the total mass M and the melted mass m. The problem tells us that Work/$M = p/\rho$, and that all the work is assumed to contribute to the phase change $Q = Lm$ where $L = 150\times10^3$ J/kg. Thus,

$$\frac{p}{\rho}M = Lm \quad\Longrightarrow\quad m = \frac{5.5\times10^6}{1200}\,\frac{M}{150\times10^3}$$

which yields $m = 0.0306M$. Dividing this by 0.30M (the mass of the fats, which we are told is equal to 30% of the total mass), leads to a percentage $0.0306/0.30 = 10\%$.

73. The net work may be computed as a sum of works (for the individual processes involved) or as the "area" (with appropriate $\pm$ sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in pV diagrams:

$$\text{for straight line}\quad \text{Work} = \frac{p_i + p_f}{2}\,\Delta V$$

which is easily verified using the definition Eq. 19-25. The cycle represented by the "triangle" BC consists of three processes:

- "tilted" straight line from $(1.0\,\text{m}^3, 40\,\text{Pa})$ to $(4.0\,\text{m}^3, 10\,\text{Pa})$, with

$$\text{Work} = \frac{40\,\text{Pa} + 10\,\text{Pa}}{2}\left(4.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 75\ \text{J}$$

- horizontal line from $(4.0\,\text{m}^3, 10\,\text{Pa})$ to $(1.0\,\text{m}^3, 10\,\text{Pa})$, with

$$\text{Work} = (10\,\text{Pa})\left(1.0\,\text{m}^3 - 4.0\,\text{m}^3\right) = -30\ \text{J}$$

- vertical line from $(1.0\,\text{m}^3, 10\,\text{Pa})$ to $(1.0\,\text{m}^3, 40\,\text{Pa})$, with

$$\text{Work} = \frac{10\,\text{Pa} + 40\,\text{Pa}}{2}\left(1.0\,\text{m}^3 - 1.0\,\text{m}^3\right) = 0$$

Thus, the total work during the BC cycle is $75 - 30 = 45$ J. During the BA cycle, the "tilted" part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with Work $= (40\,\text{Pa})(-3.0\,\text{m}^3) = -120$ J. Therefore, the total work during the BA cycle is $75 - 120 = -45$ J.

74. The work (the "area under the curve") for process 1 is $4p_iV_i$, so that $U_b - U_a = Q_1 - W_1 = 6p_iV_i$ by the First Law of Thermodynamics.

   (a) Path 2 involves more work than path 1 (note the triangle in the figure of area $\frac{1}{2}(4V_i)(p_i/2) = p_iV_i$). With $W_2 = 4p_iV_i + p_iV_i = 5p_iV_i$, we obtain

$$Q_2 = W_2 + U_b - U_a = 5p_iV_i + 6p_iV_i = 11p_iV_i\ \ .$$

(b) Path 3 starts at a and ends at b so that $\Delta U = U_b - U_a = 6p_iV_i$.

75. We use $Q = -\lambda_F m_{ice} = W + \Delta E_{\text{int}}$. In this case $\Delta E_{\text{int}} = 0$. Since $\Delta T = 0$ for the idea gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333\,\text{J/g})(100\,\text{g}) = 33.3\text{ kJ} .$$

76. Consider the object of mass m_1 falling through a distance h. The loss of its mechanical energy is $\Delta E = m_1 gh$. This amount of energy is then used to heat up the temperature of water of mass m_2: $\Delta E = m_1 gh = Q = m_2 c\Delta T$. Thus, the maximum possible rise in water temperature is

$$\begin{aligned}\Delta T &= \frac{m_1 gh}{m_2 c} \\ &= \frac{(6.00\,\text{kg})\left(9.8\,\text{m/s}^2\right)(50.0\,\text{m})}{(0.600\,\text{kg})\left(4190\,\text{J/kg}\cdot\text{C}^\circ\right)} \\ &= 1.17\text{ C}^\circ .\end{aligned}$$

77. The change in length of the rod is

$$\Delta L = L\alpha\Delta T = (20\,\text{cm})(11\times 10^{-6}/\text{C}^\circ)(50^\circ\text{C} - 30^\circ\text{C}) = 4.4\times 10^{-3}\text{ cm} .$$

78. The diameter of the brass disk in the dry ice is

$$\begin{aligned}D' &= D(1+\alpha\Delta T) \\ &= (80.00\,\text{mm})\left[1+\left(19\times 10^{-6}/\text{C}^\circ\right)(-57.00^\circ\text{C} - 43.00^\circ\text{C})\right] \\ &= 79.85\text{ mm} .\end{aligned}$$

79. The increase in the surface area of the brass cube (which has six faces), which had side length is L at 20°, is

$$\begin{aligned}\Delta A &= 6(L+\Delta L)^2 - 6L^2 \approx 12L\Delta L = 12\alpha_{\text{b}}L^2\Delta T \\ &= 12\left(19\times 10^{-6}/\text{C}^\circ\right)(30\,\text{cm})^2(75^\circ\text{C} - 20^\circ\text{C}) \\ &= 11\text{ cm}^2 .\end{aligned}$$

80. No, the doctor is probably using the Kelvin scale, in which case your temperature is $310 - 273 = 37^\circ$C. This is equivalent to $\frac{9}{5}(37) + 32 = 98.6^\circ$F.

81. We use $T_C = T_K - 273 = (5/9)[T_F - 32]$. The results are:

(a) $T = 10000^\circ$F;

(b) $T = 37.0^\circ$C;

(c) $T = -57^\circ$C;

(d) $T = -297^\circ$F;

(e) $28^\circ\text{C} = 82^\circ$F (for example).

82. The heat needed is

$$\begin{aligned}Q &= (10\%)mL_F \\ &= \left(\frac{1}{10}\right)(200,000\text{ metric tons})(1000\,\text{kg/metric ton})(333\,\text{kJ/kg}) \\ &= 6.7\times 10^{12}\text{ J} .\end{aligned}$$

83. For isotropic materials, the coefficient of linear expansion α is related to that for volume expansion by $\alpha = \frac{1}{3}\beta$ (Eq. 19-11). The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$$\begin{aligned}\Delta R_E &= R_E \alpha \Delta T \\ &= \left(6.4 \times 10^3\,\text{km}\right)\left(\frac{1}{3}\right)\left(3.0 \times 10^{-5}/\,\text{K}\right)\left(3000\,\text{K} - 300\,\text{K}\right) \\ &= 1.7 \times 10^3\ \text{km}\ .\end{aligned}$$

84. If the window is L_1 high and L_2 wide at the lower temperature and $L_1 + \Delta L_1$ high and $L_2 + \Delta L_2$ wide at the higher temperature then its area changes from $A_1 = L_1 L_2$ to

$$A_2 = (L_1 + \Delta L_1)(L_2 + \Delta L_2) \approx L_1 L_2 + L_1\,\Delta L_2 + L_2\,\Delta L_1$$

where the term $\Delta L_1\,\Delta L_2$ has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A = A_2 - A_1 = L_1\,\Delta L_2 + L_2\,\Delta L_1\ .$$

If ΔT is the change in temperature then $\Delta L_1 = \alpha L_1\,\Delta T$ and $\Delta L_2 = \alpha L_2\,\Delta T$, where α is the coefficient of linear expansion. Thus

$$\begin{aligned}\Delta A &= \alpha(L_1 L_2 + L_1 L_2)\,\Delta T = 2\alpha L_1 L_2\,\Delta T \\ &= 2\left(9 \times 10^{-6}/\text{C}^\circ\right)(30\,\text{cm})(20\,\text{cm})(30^\circ\text{C}) \\ &= 0.32\ \text{cm}^2\ .\end{aligned}$$

85. (a) Recalling that a Watt is a Joule-per-second, and that a change in Celsius temperature is equivalent (numerically) to a change in Kelvin temperature, we convert the value of k to SI units, using Eq. 19-12.

$$\left(2.9 \times 10^{-3}\,\frac{\text{cal}}{\text{cm}\cdot\text{C}^\circ\cdot\text{s}}\right)\left(\frac{4.186\,\text{J}}{1\ \text{cal}}\right)\left(\frac{100\,\text{cm}}{1\ \text{m}}\right) = 1.2\ \frac{\text{W}}{\text{m}\cdot\text{K}}\ .$$

(b) Now, a change in Celsius is equivalent to five-ninths of a Fahrenheit change, so

$$\left(2.9 \times 10^{-3}\,\frac{\text{cal}}{\text{cm}\cdot\text{C}^\circ\cdot\text{s}}\right)\left(\frac{0.003969\,\text{Btu}}{1\ \text{cal}}\right)\left(\frac{5\,\text{C}^\circ}{9\,\text{F}^\circ}\right)\left(\frac{3600\,\text{s}}{1\ \text{h}}\right)\left(\frac{30.48\,\text{cm}}{1\ \text{ft}}\right) = 0.70\ \frac{\text{Btu}}{\text{ft}\cdot\text{F}^\circ\cdot\text{h}}\ .$$

(c) Using Eq. 19-33, we obtain

$$R = \frac{L}{k} = \frac{0.0064\,\text{m}}{1.2\,\text{W/m}\cdot\text{K}} = 0.0053\ \text{m}^2\cdot\text{K/W}\ .$$

86. Its initial volume is $5^3 = 125\ \text{cm}^3$, and using Table 19-2, Eq. 19-10 and Eq. 19-11, we find

$$\Delta V = \left(125\,\text{m}^3\right)\left(3 \times 23 \times 10^{-6}/\text{C}^\circ\right)\left(50\,\text{C}^\circ\right) = 0.43\ \text{cm}^3\ .$$

87. The cube has six faces, each of which has an area of $\left(6.0 \times 10^{-6}\,\text{m}\right)^2$. Using Kelvin temperatures and Eq. 19-40, we obtain

$$\begin{aligned}P_{\text{net}} &= \sigma\varepsilon A\left(T_{\text{env}}^4 - T^4\right) \\ &= \left(5.67 \times 10^{-8}\,\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(0.75)\left(2.16 \times 10^{-10}\,\text{m}^2\right)\left((123.15\,\text{K})^4 - (173.15\,\text{K})^4\right) \\ &= -6.1 \times 10^{-9}\ \text{W}\ .\end{aligned}$$

88. We take absolute values of Eq. 19-9 and Eq. 13-25:

$$|\Delta L| = L\alpha|\Delta T| \qquad \text{and} \qquad \left|\frac{F}{A}\right| = E\left|\frac{\Delta L}{L}\right| \ .$$

The ultimate strength for steel is $(F/A)_{\text{rupture}} = S_u = 400 \times 10^6\,\text{N/m}^2$ from Table 13-1. Combining the above equations (eliminating the ratio $\Delta L/L$), we find the rod will rupture if the temperature change exceeds

$$|\Delta T| = \frac{S_u}{E\alpha} = \frac{400 \times 10^6\,\text{N/m}^2}{\left(200 \times 10^9\,\text{N/m}^2\right)(11 \times 10^{-6}/\text{C}^\circ)} = 182^\circ\text{C} \ .$$

Since we are dealing with a temperature decrease, then, the temperature at which the rod will rupture is $T = 25.0^\circ\text{C} - 182^\circ\text{C} = -157^\circ\text{C}$.

89. (a) At -40°F the tuning fork is shorter and takes less time to execute a "tick." The record of the clock assumes every "tick" corresponds to some standard unit of time – the net effect being that its time-record is "fast" or "ahead" of the correct time. We write the (absolute value of) relative error as

$$\left|\frac{t_{\text{fork}} - t_{\text{correct}}}{t_{\text{correct}}}\right| = \frac{T_{\text{fork}}}{T_{\text{correct}}} - 1 \ .$$

Using Eq. 16-28, this becomes

$$\left|\frac{\Delta t}{t_{\text{correct}}}\right| = \sqrt{\frac{L_{-40}}{L_{25}}} - 1 \ ,$$

where we have used the fact that the tuning fork would be accurate if the temperature were 25°F. Now, Eq. 19-9 tells us that $L_{-40} = L_{25}(1 + \alpha\Delta T)$, where $\Delta T = -65\,\text{F}^\circ$. Also, $\alpha = 5 \times 10^{-7}/\text{C}^\circ$ according to Table 19-2, which we convert to $\alpha = 2.8 \times 10^{-7}/\text{F}^\circ$ for the needed computations. Now, the above equation becomes

$$\left|\frac{\Delta t}{t_{\text{correct}}}\right| = \sqrt{1 + \alpha\Delta T} - 1 \ .$$

We can expand this with the binomial theorem (Appendix E) or compute it the "brute force" way; in any case we find $|\Delta t/t_{\text{correct}}| = 9 \times 10^{-6}$. Since the clock, as mentioned above, is "fast" we say the relative *gain* in time is 9×10^{-6}. *Note:* a more elegant approach to this problem in terms of differentials is as follows (with k some constant of proportionality).

$$\begin{aligned} t_{\text{fork}} &= k\sqrt{L} \qquad\qquad k = \text{constant} \\ dt_{\text{fork}} &= \frac{1}{2}kL^{-1/2}\,dL \\ dt_{\text{fork}} &= \frac{\alpha}{2}t_{\text{fork}}\,dT \\ \frac{dt_{\text{fork}}}{t_{\text{fork}}} &= \frac{\alpha}{2}\,dT \end{aligned}$$

At this point $dT \to \Delta T$ and the previous results are obtained.

(b) This proceeds very similarly to part (a), but with the tuning fork longer – and thus ticking more slowly, and with $\Delta T = 95\,\text{F}^\circ$. The result is a relative *loss* in time of magnitude 13×10^{-6}.

90. We require $\sum Q = 0$ (which amounts to assuming the system is isolated). There are both temperature changes (with $Q = cm\Delta T$) and phase changes ($Q = L_F\,m$). Masses are in kilograms and heat in Joules, with temperatures measured on the Celsius scale. We refer to the ice (which melts and becomes (liquid) water) as H_2O to avoid confusion; note that it involves *three* terms. The ice has mass m

and the tea has a 1.0 kg mass (the density of tea is taken to be the same as the density of water $\rho_w = 1000\ \text{kg/m}^3 = 1.0\ \text{kg/L}$).

$$\begin{aligned} Q_{H_2O} + Q_{\text{tea}} &= 0 \\ (2220)m\,(10^\circ) + (333000)m + (4190)m\,(10^\circ) + (4190)(1.0)\,(10^\circ - 90^\circ) &= 0 \\ 397100m - 335200 &= 0 \end{aligned}$$

Therefore, $m = 0.84$ kg which amounts to *forty-two* 20 g ice cubes.

91. We have $W = \int p\,dV$ (Eq. 19-24). Therefore,

$$W = a \int V^2\,dV = \frac{a}{3}\left(V_f^3 - V_i^3\right) = 23\ \text{J}\ .$$

92. (a) The length change of bar 1 is ΔL_1 and that of bar 2 is ΔL_2. The total length change is given by

$$\begin{aligned} \alpha L \Delta T &= \Delta L \\ &= \Delta L_1 + \Delta L_2 \\ &= \alpha_1 L_1 \Delta T + \alpha_2 L_2 \Delta T \end{aligned}$$

which leads to the desired expression after dividing through by ΔT and solving for α.

(b) Substituting $L_2 = L - L_1$ into the expression, we have

$$\alpha = \frac{\alpha_1 L_1 + \alpha_2\,(L - L_1)}{L} \quad \Longrightarrow \quad L_1 = L\frac{\alpha - \alpha_2}{\alpha_1 - \alpha_2}\ .$$

Therefore, if $\alpha_1 = 19 \times 10^{-6}/\text{C}^\circ$ (brass, from Table 19-2), $\alpha_2 = 11 \times 10^{-6}/\text{C}^\circ$ (steel, also from Table 19-2), $L = 52.4$ cm and $\alpha = 13 \times 10^{-6}/\text{C}^\circ$, we obtain $L_1 = 13.1$ cm for the length of brass and $L_2 = L - L_1 = 39.3$ cm for the steel.

93. (a) The surface are of the cylinder is given by $A_1 = 2\pi r_1^2 + 2\pi r_1 h_1 = 2\pi(2.5 \times 10^{-2}\,\text{m})^2 + 2\pi(2.5 \times 10^{-2}\,\text{m})(5.0 \times 10^{-2}\,\text{m}) = 1.18 \times 10^{-2}\,\text{m}^2$, its temperature is $T_1 = 273 + 30 = 303\,\text{K}$, and the temperature of the environment is $T_{\text{env}} = 273 + 50 = 323\,\text{K}$. From Eq. 19-39 we have

$$\begin{aligned} P_1 &= \sigma\varepsilon A_1(T_{\text{env}}^4 - T^4) \\ &= ()\,(0.85)(1.18 \times 10^{-2}\,\text{m}^2)\left((323\,\text{K})^4 - (303\,\text{K})^4\right) \\ &= 1.39\,\text{W}\ . \end{aligned}$$

(b) Let the new height of the cylinder be h_2. Since the volume V of the cylinder is fixed, we must have $V = \pi r_1^2 h_1 = \pi r_2^2 h_2$. We solve for h_2:

$$\begin{aligned} h_2 = \left(\frac{r_1}{r_2}\right)^2 h_1 & \\ &= \left(\frac{2.5\,\text{cm}}{0.50\,\text{cm}}\right)^2 (5.0\,\text{cm}) \\ &= 125\,\text{cm} = 1.25\ \text{m}\ . \end{aligned}$$

The corresponding new surface area A_2 of the cylinder is

$$A_2 = 2\pi r_2^2 + 2\pi r_2 h_2 = 2\pi(0.50 \times 10^{-2}\,\text{m})^2 + 2\pi(0.50 \times 10^{-2}\,\text{m})(1.25\,\text{m}) = 3.94 \times 10^{-2}\ \text{m}^2\ .$$

Consequently,

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{3.94 \times 10^{-2}\,\text{m}^2}{1.18 \times 10^{-2}\,\text{m}^2} = 3.3\ .$$

94. We denote the density of the liquid as ρ, the rate of liquid flowing in the calorimeter as μ, the specific heat of the liquid as c, the rate of heat flow as P, and the temperature change as ΔT. Consider a time duration dt, during this time interval, the amount of liquid being heated is $dm = \mu\rho dt$. The energy required for the heating is $dQ = Pdt = c(dm)\Delta T = c\mu\Delta T dt$. Thus,

$$c = \frac{P}{\rho\mu\Delta T} = \frac{250\,\text{W}}{(8.0\times10^{-6}\,\text{m}^3/\text{s})(0.85\times10^3\,\text{kg/m}^3)(15°\text{C})} = 2.5\times10^3\,\text{J/kg}\cdot\text{C}° \ .$$

95. This follows from Eq. 19-35 by dividing numerator and denominator by the product k_1k_2 as shown below:

$$T_X = \frac{\frac{1}{k_1k_2}(k_1L_2T_C + k_2L_1T_H)}{\frac{1}{k_1k_2}(k_1L_2 + k_2L_1)} = \frac{\frac{L_2}{k_2}T_C + \frac{L_1}{k_1}T_H}{\frac{L_2}{k_2} + \frac{L_1}{k_1}} = \frac{R_2T_C + R_1T_H}{R_2 + R_1}$$

where the definition $Eq.\ 19 - 33$ has also been used.

96. We note that there is no work done in process $c \to b$, since there is no change of volume. We also note that the *magnitude* of work done in process $b \to c$ is given, but not its sign (which we identify as negative as a result of the discussion in §19-8). The total (or *net*) heat transfer is $Q_{\text{net}} = (-40) + (-130) + (+400) =$ 230 J. By the First Law of Thermodynamics (or, equivalently, conservation of energy), we have

$$\begin{aligned} Q_{\text{net}} &= W_{\text{net}} \\ 230\,\text{J} &= W_{a\,\to\,c} + W_{c\,\to\,b} + W_{b\,\to\,a} \\ &= W_{a\,\to\,c} + 0 + (-80\,\text{J}) \end{aligned}$$

Therefore, $W_{a\,\to\,c} = 310$ J.

97. (a) and (b) Regarding part (a), it is important to recognize that the problem is asking for the total work done during the two-step "path": $a \to b$ followed by $b \to c$. During the latter part of this "path" there is no volume change and consequently no work done. Thus, the answer to part (b) is also the answer to part (a). Since ΔU for process $c \to a$ is -160 J, then $U_c - U_a = 160$ J. Therefore, using the First Law of Thermodynamics, we have

$$\begin{aligned} 160 &= U_c - U_b + U_b - U_a \\ &= Q_{b\,\to\,c} - W_{b\,\to\,c} + Q_{a\,\to\,b} - W_{a\,\to\,b} \\ &= 40 - 0 + 200 - W_{a\,\to\,b} \end{aligned}$$

Therefore, $W_{a\,\to\,b} = 80$ J.

98. Let the initial water temperature be T_{wi} and the initial thermometer temperature be T_{ti}. Then, the heat absorbed by the thermometer is equal (in magnitude) to the heat lost by the water:

$$c_tm_t\,(T_f - T_{ti}) = c_wm_w\,(T_{wi} - T_f) \ .$$

We solve for the initial temperature of the water:

$$\begin{aligned} T_{wi} &= \frac{c_tm_t(T_f - T_{ti})}{c_wm_w} + T_f \\ &= \frac{(0.0550\,\text{kg})(0.837\,\text{kJ/kg}\cdot\text{K})(44.4 - 15.0)\,\text{K}}{(4.18\,\text{kJ/kg}\cdot\text{C}°)(0.300\,\text{kg})} + 44.4°\text{C} \\ &= 45.5°\text{C} \ . \end{aligned}$$

99. (a) A change of five Celsius degrees is equivalent to a change of nine Fahrenheit degrees. Using Table 19-2,

$$\alpha = \left(23\times10^{-6}/\text{C}°\right)\left(\frac{5\,\text{C}°}{9\,\text{F}°}\right) = 13\times10^{-6}/\text{F}° \ .$$

(b) For $\Delta T = 55\,\mathrm{F}^\circ$ and $L = 6.0$ m, we find $\Delta L = L\alpha\Delta T = 0.0042$ m.

100. The initial volume V_0 of the liquid is h_0A_0 where A_0 is the initial cross-section area and $h_0 = 0.64$ m. Its final volume is $V = hA$ where $h - h_0$ is what we wish to compute. Now, the area expands according to how the glass expands, which is we analyze as follows.

$$\begin{aligned} A &= \pi r^2 \\ dA &= 2\pi r\, dr \\ dA &= 2\pi r\,(r\alpha\, dT) \\ dA &= 2\alpha A\, dT \end{aligned}$$

Therefore, the height is

$$h = \frac{V}{A} = \frac{V_0\,(1 + \beta_{\rm liquid}\Delta T)}{A_0\,(1 + 2\alpha_{\rm glass}\Delta T)}\ .$$

Thus, with $V_0/A_0 = h_0$ we obtain

$$\begin{aligned} h - h_0 &= h_0\left(\frac{1+\beta_{\rm liquid}\Delta T}{1+2\alpha_{\rm glass}\Delta T} - 1\right) \\ &= (0.64)\left(\frac{1+\left(4\times10^{-5}\right)(10^\circ)}{1+2\,(1\times10^{-5})\,(10^\circ)}\right) \\ &= 1.3\times10^{-4}\ \mathrm{m}\ . \end{aligned}$$

101. The heat required to warm up to the melting point is $Q = cm\Delta T = (2220)(15.0)(20.0) = 666$ kJ, which is less than the total 7000 kJ added to the sample. Therefore, 6334 kJ remain for melting the block and warming the sample (now in the form of liquid water) further. Melting the block requires

$$Q = L_F m = (333\,\mathrm{kJ/kg})(15.0\,\mathrm{kg}) = 4995\ \mathrm{kJ}$$

which leaves $6334 - 4995 = 1339$ kJ. The final temperature of the (liquid) water, which has $c = 4190\,\mathrm{J/kg{\cdot}C^\circ}$, is found from

$$Q = cm\,(T_f - 0^\circ\mathrm{C}) \quad\Longrightarrow\quad T_f = \frac{1339\times10^3}{(4190)(15.0)} = 21.3^\circ\mathrm{C}\ .$$

102. Using Eq. 19-40 with $T = 323$ K and $T_{\rm env} = 293$ K, we find

$$P_{\rm net} = \sigma\varepsilon A\left(T_{\rm env}^4 - T^4\right) = -3.8\times10^{-7}\ \mathrm{W}$$

where we have used the fact that the surface area of the cube is $A = 6A_{\rm face} = 6\left(2.0\times10^{-5}\,\mathrm{m}\right)^2 = 2.4\times10^{-9}\ \mathrm{m}^2$.

103. Let $m_w = 14\,\mathrm{kg}$, $m_c = 3.6\,\mathrm{kg}$, $m_m = 1.8\,\mathrm{kg}$, $T_{i1} = 180^\circ\mathrm{C}$, $T_{i2} = 16.0^\circ\mathrm{C}$, and $T_f = 18.0^\circ\mathrm{C}$. The specific heat c_m of the metal then satisfies

$$(m_wc_w + m_cc_m)\,(T_f - T_{i2}) + m_mc_m\,(T_f - T_{i1}) = 0$$

which we solve for c_m:

$$\begin{aligned} c_m &= \frac{m_wc_w(T_{i2}-T_f)}{m_c(T_f-T_{i2}) + m_m(T_f-T_{i1})} \\ &= \frac{(14\,\mathrm{kg})(4.18\,\mathrm{kJ/kg{\cdot}K})(16.0^\circ\mathrm{C} - 18.0^\circ\mathrm{C})}{(3.6\,\mathrm{kg})(18.0^\circ\mathrm{C} - 16.0^\circ\mathrm{C}) + (1.8\,\mathrm{kg})(18.0^\circ\mathrm{C} - 180^\circ\mathrm{C})} \\ &= 0.41\ \mathrm{kJ/kg{\cdot}C^\circ}\ . \end{aligned}$$

104. The energy (which was originally in the form $K = \frac{1}{2}mv^2$) dissipated as a result of friction melts a portion of mass m. Therefore,

$$\begin{aligned}\frac{1}{2}(50.0\,\text{kg})(5.38\,\text{m/s})^2 &= mL_F \\ 723\,\text{J} &= m(333\,\text{kJ/kg}\end{aligned}$$

which, for consistency of the energy units, is best written $723\,\text{J} = m(333\,\text{J/g})$. This yields $m = 2.17$ g.

105. We demand $\sum Q = 0$ as an expression of the fact that the system is isolated. Only temperature changes (with $Q = cm\Delta T$) are involved (no phase changes). Let masses be in kilograms, heat in Joules and temperature on the Celsius scale.

$$\begin{aligned}Q_{\text{copper}} + Q_{\text{water}} &= 0 \\ (386)(3.00)\,(T_f - 70.0°) + (4190)(4.00)\,(T_f - 10.0°) &= 0\end{aligned}$$

Therefore, we find

$$T_f = \frac{(386)(3.00)\,(70.0°) + (4190)(4.00)\,(10.0°)}{(386)(3.00) + (4190)(4.00)} = 13.9°\text{C} \;.$$

106. We use $Q = cm\Delta T$ and $m = \rho V$. The volume of water needed is

$$V = \frac{m}{\rho} = \frac{Q}{\rho C\Delta T} = \frac{(1.00 \times 10^6\,\text{kcal/day})(5\,\text{days})}{(1.00 \times 10^3\,\text{kg/m}^3)(1.00\,\text{kcal/kg})(50.0°\text{C} - 22.0°\text{C})} = 35.7\ \text{m}^3 \;.$$

107. (a) Let the number of weight lift repetitions be N. Then $Nmgh = Q$, or (using Eq. 19-12 and the discussion preceding it)

$$N = \frac{Q}{mgh} = \frac{(3500\,\text{Cal})(4186\,\text{J/Cal})}{(80.0\,\text{kg})\,(9.8\,\text{m/s}^2)\,(1.00\,\text{m})} \approx\ 18700 \;.$$

(b) The time required is

$$t = (18700)(2.00\,\text{s})\left(\frac{1.00\,\text{h}}{3600\,\text{s}}\right) = 10.4\ \text{h} \;.$$

108. We assume scales X and Y are linearly related in the sense that reading is x is related to reading y by a linear relationship $y = mx + b$. We determine the constants m and b by solving the simultaneous equations:

$$\begin{aligned}-70.00 &= m(-125.0) + b \\ -30.00 &= m(375.0) + b\end{aligned}$$

which yield the solutions $m = 40.00/500.0 = 8.000 \times 10^{-2}$ and $b = -60.00$. With these values, we find x for $y = 50.00$:

$$x = \frac{y-b}{m} = \frac{50.00 + 60.00}{0.08000} = 1375°X \;.$$

109. (a) The 8.0 cm thick layer of air in front of the glass conducts heat at a rate of

$$P_{\text{cond}} = kA\frac{T_H - T_C}{L} = (0.026)(0.36)\frac{15}{0.08} = 1.8\ \text{W}$$

which must be the same as the heat conduction current through the glass if a steady-state heat transfer situation is assumed.

(b) For the glass pane,

$$\begin{aligned} P_{\text{cond}} &= kA\frac{T_H - T_C}{L} \\ 1.8 &= (1.0)(0.36)\frac{T_H - T_C}{0.005} \end{aligned}$$

which yields $T_H - T_C = 0.024\,\text{C}^\circ$.

110. One method is to simply compute the change in length in each edge ($x_0 = 0.200$ m and $y_0 = 0.300$ m) from Eq. 19-9 ($\Delta x = 3.6 \times 10^{-5}$ m and $\Delta y = 5.4 \times 10^{-5}$ m) and then compute the area change:

$$A - A_0 = (x_0 + \Delta x)\,(y_0 + \Delta y) - x_0 y_0 = 2.16 \times 10^{-5}\ \text{m}^2\ .$$

Another (though related) method uses $\Delta A = 2\alpha A_0 \Delta T$ (valid for $\Delta A/A \ll 1$) which can be derived by taking the differential of $A = xy$ and replacing d's with Δ's.

Chapter 20

1. Each atom has a mass of $m = M/N_{\rm A}$, where M is the molar mass and $N_{\rm A}$ is the Avogadro constant. The molar mass of arsenic is 74.9 g/mol or 74.9×10^{-3} kg/mol. 7.50×10^{24} arsenic atoms have a total mass of $(7.50 \times 10^{24})(74.9 \times 10^{-3}\,{\rm kg/mol})/(6.02 \times 10^{23}\,{\rm mol^{-1}}) = 0.933\,{\rm kg}$.

2. (a) Eq. 20-3 yields $n = M_{\rm sam}/M = 2.5/197 = 0.0127$.

   (b) The number of atoms is found from Eq. 20-2: $N = nN_{\rm A} = (0.0127)(6.02 \times 10^{23}) = 7.64 \times 10^{21}$.

3. The surface area of a sphere is $4\pi R^2$, and we find the radius of Earth in Appendix C ($R_E = 6.37\times 10^6\,{\rm m} = 6.37 \times 10^8\,{\rm cm}$). Therefore, the number of square "patches" (with one centimeter side length) needed to cover Earth is

$$A = 4\pi \left(6.37 \times 10^8\right)^2 = 5.1 \times 10^{18} \ .$$

   The number of molecules that we want to distribute as evenly as possible among all those patches is (using Eqs. 20-2, 20-3, with $M = 18$ g/mol)

$$N = N_{\rm A}\,\frac{M_{\rm sam}}{M} = \left(6.02 \times 10^{23}\right)\frac{1.00\,{\rm g}}{18\,{\rm g/mol}} = 3.3 \times 10^{22} \ .$$

   Therefore, we have $N/A = 6.56 \times 10^3$ molecules in each patch. Note: students are encouraged to figure $M = 18$ g/mol (for water) based on what they have learned in their chemistry courses, but it should be mentioned that this can also be gleaned from Table 20-1.

4. The number of molecules in $M_{\rm sam} = 1\,\mu{\rm g} = 10^{-6}$ g of ink is (using Eqs. 20-2, 20-3, with $M = 18$ g/mol)

$$N = N_{\rm A}\,\frac{M_{\rm sam}}{M} = \left(6.02 \times 10^{23}/{\rm mol}\right)\left(\frac{1 \times 10^{-6}\,{\rm g}}{18\,{\rm g/mol}}\right) \approx 3 \times 10^{16} \ .$$

   The number of creatures in our galaxy, with the assumption made in the problem, is about $N' = 5 \times 10^9 \times 10^{11} = 5 \times 10^{20}$. So the statement is wrong by a factor of about 20,000.

5. (a) We solve the ideal gas law $pV = nRT$ for n:

$$n = \frac{pV}{RT} = \frac{(100\,{\rm Pa})(1.0 \times 10^{-6}\,{\rm m^3})}{(8.31\,{\rm J/mol\cdot K})(220\,{\rm K})} = 5.47 \times 10^{-8}\ {\rm mol} \ .$$

   (b) Using Eq. 20-2, the number of molecules N is

$$N = nN_{\rm A} = \left(5.47 \times 10^{-6}\,{\rm mol}\right)\left(6.02 \times 10^{23}\,{\rm mol^{-1}}\right) = 3.29 \times 10^{16}\ {\rm molecules} \ .$$

6. With $V = 1.0 \times 10^{-6}$ m^3, $p = 1.01 \times 10^{-13}$ Pa, and $T = 293$ K, the ideal gas law gives

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^{-13}\right)\left(1.0 \times 10^{-6}\right)}{(8.31)(293)} = 4.1 \times 10^{-23}\ {\rm mole} \ .$$

   Consequently, Eq. 20-2 yields $N = nN_{\rm A} = 25$ molecules. We can express this as a ratio (with V now written as $1\,{\rm cm^3}$) $N/V = 25$ molecules/cm^3.

7. (a) In solving $pV = nRT$ for n, we first convert the temperature to the Kelvin scale: $T = 40.0+273.15 = 313.15\,\text{K}$. And we convert the volume to SI units: $1000\,\text{cm}^3 = 1000 \times 10^{-6}\,\text{m}^3$. Now, according to the ideal gas law,

$$n = \frac{pV}{RT} = \frac{(1.01 \times 10^5\,\text{Pa})(1000 \times 10^{-6}\,\text{m}^3)}{(8.31\,\text{J/mol}\cdot\text{K})(313.15\,\text{K})} = 3.88 \times 10^{-2}\ \text{mol}\ .$$

(b) The ideal gas law $pV = nRT$ leads to

$$T = \frac{pV}{nR} = \frac{(1.06 \times 10^5\,\text{Pa})(1500 \times 10^{-6}\,\text{m}^3)}{(3.88 \times 10^{-2}\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})} = 493\,\text{K}\ .$$

We note that the final temperature may be expressed in degrees Celsius as 220°C.

8. Since (standard) air pressure is 101 kPa, then the initial (absolute) pressure of the air is $p_i = 266$ kPa. Setting up the gas law in ratio form (where $n_i = n_f$ and thus cancels out – see Sample Problem 20-1), we have

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad\Longrightarrow\quad p_f = (266\,\text{kPa})\left(\frac{1.64 \times 10^{-2}\,\text{m}^3}{1.67 \times 10^{-2}\,\text{m}^3}\right)\left(\frac{300\,\text{K}}{273\,\text{K}}\right)$$

which yields $p_f = 287$ kPa. Expressed as a gauge pressure, we subtract 101 kPa and obtain 186 kPa.

9. (a) With $T = 283$ K, we obtain

$$n = \frac{pV}{RT} = \frac{\left(100 \times 10^3\,\text{Pa}\right)\left(2.50\,\text{m}^3\right)}{\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(283\,\text{K})} = 106\ \text{mol}\ .$$

(b) We can use the answer to part (a) with the new values of pressure and temperature, and solve the ideal gas law for the new volume, or we could set up the gas law in ratio form as in Sample Problem 20-1 (where $n_i = n_f$ and thus cancels out):

$$\frac{p_f V_f}{p_i V_i} = \frac{T_f}{T_i} \quad\Longrightarrow\quad V_f = \left(2.50\,\text{m}^3\right)\left(\frac{100\,\text{kPa}}{300\,\text{kPa}}\right)\left(\frac{303\,\text{K}}{283\,\text{K}}\right)$$

which yields a final volume of $V_f = 0.892$ m^3.

10. We write $T = 273$ K and use Eq. 20-14:

$$W = (1.00\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K})\,\ln\!\left(\frac{16.8}{22.4}\right)$$

which yields $W = -653$ J. Recalling the sign conventions for work stated in Chapter 19, this means an external agent does 653 J of work *on* the ideal gas during this process.

11. Since the pressure is constant the work is given by $W = p(V_2 - V_1)$. The initial volume is $V_1 = (AT_1 - BT_1^2)/p$, where T_1 is the initial temperature. The final volume is $V_2 = (AT_2 - BT_2^2)/p$. Thus $W = A(T_2 - T_1) - B(T_2^2 - T_1^2)$.

12. The pressure p_1 due to the first gas is $p_1 = n_1RT/V$, and the pressure p_2 due to the second gas is $p_2 = n_2RT/V$. So the total pressure on the container wall is

$$p = p_1 + p_2 = \frac{n_1RT}{V} + \frac{n_2RT}{V} = (n_1 + n_2)\frac{RT}{V}\ .$$

The fraction of P due to the second gas is then

$$\frac{p_2}{p} = \frac{n_2RT/V}{(n_1+n_2)(RT/V)} = \frac{n_2}{n_1+n_2} = \frac{0.5}{2+0.5} = \frac{1}{5}\ .$$

13. Suppose the gas expands from volume V_i to volume V_f during the isothermal portion of the process. The work it does is

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i}\,,$$

where the ideal gas law $pV = nRT$ was used to replace p with nRT/V. Now $V_i = nRT/p_i$ and $V_f = nRT/p_f$, so $V_f/V_i = p_i/p_f$. Also replace nRT with p_iV_i to obtain

$$W = p_iV_i \ln \frac{p_i}{p_f}\,.$$

Since the initial gauge pressure is 1.03×10^5 Pa, $p_i = 1.03 \times 10^5\,\text{Pa} + 1.013 \times 10^5\,\text{Pa} = 2.04 \times 10^5$ Pa. The final pressure is atmospheric pressure: $p_f = 1.013 \times 10^5$ Pa. Thus

$$W = (2.04 \times 10^5\,\text{Pa})(0.14\,\text{m}^3) \ln \frac{2.04 \times 10^5\,\text{Pa}}{1.013 \times 10^5\,\text{Pa}} = 2.00 \times 10^4 \text{ J}\,.$$

During the constant pressure portion of the process the work done by the gas is $W = p_f(V_i - V_f)$. The gas starts in a state with pressure p_f, so this is the pressure throughout this portion of the process. We also note that the volume decreases from V_f to V_i. Now $V_f = p_iV_i/p_f$, so

$$\begin{aligned} W &= p_f\left(V_i - \frac{p_iV_i}{p_f}\right) = (p_f - p_i)V_i \\ &= (1.013 \times 10^5\,\text{Pa} - 2.04 \times 10^5\,\text{Pa})(0.14\,\text{m}^3) = -1.44 \times 10^4 \text{ J}\,. \end{aligned}$$

The total work done by the gas over the entire process is $W = 2.00 \times 10^4\,\text{J} - 1.44 \times 10^4\,\text{J} = 5.6 \times 10^3\,\text{J}$.

14. (a) At point a, we know enough information to compute n:

$$n = \frac{pV}{RT} = \frac{(2500\,\text{Pa})\,(1.0\,\text{m}^3)}{(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}})\,(200\,\text{K})} = 1.5 \text{ mol}\,.$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law as in Sample Problem 20-1 in terms of ratios (note: $n_a = n_b$ and cancels out):

$$\frac{p_bV_b}{p_aV_a} = \frac{T_b}{T_a} \implies T_b = (200\,\text{K})\left(\frac{7.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at b of $T_b = 1800$ K.

(c) As in the previous part, we choose to approach this using the gas law in ratio form (see Sample Problem 20-1):

$$\frac{p_cV_c}{p_aV_a} = \frac{T_c}{T_a} \implies T_c = (200\,\text{K})\left(\frac{2.5\,\text{kPa}}{2.5\,\text{kPa}}\right)\left(\frac{3.0\,\text{m}^3}{1.0\,\text{m}^3}\right)$$

which yields an absolute temperature at c of $T_c = 600$ K.

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the pV diagram shown in Fig. 20-19). This, in turn, is related to $\pm$ "area" inside that triangle (with area $= \frac{1}{2}$(base)(height)), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2}\,(2.0\,\text{m}^3)\,(5.0 \times 10^3\,\text{Pa}) = 5000 \text{ J}\,.$$

15. We assume that the pressure of the air in the bubble is essentially the same as the pressure in the surrounding water. If d is the depth of the lake and ρ is the density of water, then the pressure at the bottom of the lake is $p_1 = p_0 + \rho g d$, where p_0 is atmospheric pressure. Since $p_1 V_1 = nRT_1$, the number of moles of gas in the bubble is $n = p_1 V_1/RT_1 = (p_0 + \rho g d)V_1/RT_1$, where V_1 is the volume of the bubble at the bottom of the lake and T_1 is the temperature there. At the surface of the lake the pressure is p_0 and the volume of the bubble is $V_2 = nRT_2/p_0$. We substitute for n to obtain

$$\begin{aligned} V_2 &= \frac{T_2}{T_1}\frac{p_0 + \rho g d}{p_0} V_1 \\ &= \left(\frac{293\,\mathrm{K}}{277\,\mathrm{K}}\right)\left(\frac{1.013\times10^5\,\mathrm{Pa} + (0.998\times10^3\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})(40\,\mathrm{m})}{1.013\times10^5\,\mathrm{Pa}}\right)(20\,\mathrm{cm^3}) \\ &= 100\ \mathrm{cm^3}\ . \end{aligned}$$

16. Consider the open end of the pipe. The balance of the pressures inside and outside the pipe requires that $p + \rho_w(L/2)g = p_0 + \rho_w h g$, where p_0 is the atmospheric pressure, and p is the pressure of the air inside the pipe, which satisfies $p(L/2) = p_0 L$, or $p = 2p_0$. We solve for h:

$$h = \frac{p - p_0}{\rho_w g} + \frac{L}{2} = \frac{1.01\times10^5\,\mathrm{Pa}}{(1.00\times10^3\,\mathrm{kg/m^3})\left(9.8\,\mathrm{m/s^2}\right)} + \frac{25.0\,\mathrm{m}}{2} = 22.8\ \mathrm{m}\ .$$

17. When the valve is closed the number of moles of the gas in container A is $n_A = p_A V_A/RT_A$ and that in container B is $n_B = 4p_B V_A/RT_B$. The total number of moles in both containers is then

$$n = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right) = \text{const.}$$

After the valve is opened the pressure in container A is $p'_A = Rn'_A T_A/V_A$ and that in container B is $p'_B = Rn'_B T_B/4V_A$. Equating p'_A and p'_B, we obtain $Rn'_A T_A/V_A = Rn'_B T_B/4V_A$, or $n'_B = (4T_A/T_B)n'_A$. Thus,

$$n = n'_A + n'_B = n'_A\left(1 + \frac{4T_A}{T_B}\right) = n_A + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right)\ .$$

We solve the above equation for n'_A:

$$n'_A = \frac{V}{R}\frac{(p_A/T_A + 4p_B/T_B)}{(1 + 4T_A/T_B)}\ .$$

Substituting this expression for n'_A into $p'V_A = n'_A RT_A$, we obtain the final pressure:

$$p' = \frac{n'_A RT_A}{V_A} = \frac{p_A + 4p_B T_A/T_B}{1 + 4T_A/T_B} = 2.0\times10^5\ \mathrm{Pa}\ .$$

18. Appendix F gives $M = 4.00\times10^{-3}$ kg/mol (Table 20-1 gives this to fewer significant figures). Using Eq. 20-22, we obtain

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.31\,\frac{\mathrm{J}}{\mathrm{mol\cdot K}}\right)(1000\,\mathrm{K})}{4.00\times10^{-3}\,\mathrm{kg/mol}}} = 2.50\times10^3\ \mathrm{m/s}\ .$$

19. According to kinetic theory, the rms speed is

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}}$$

where T is the temperature and M is the molar mass. See Eq. 20-34. According to Table 20-1, the molar mass of molecular hydrogen is $2.02\,\mathrm{g/mol} = 2.02 \times 10^{-3}\,\mathrm{kg/mol}$, so

$$v_{\mathrm{rms}} = \sqrt{\frac{3\,(8.31\,\mathrm{J/mol{\cdot}K})\,(2.7\,\mathrm{K})}{2.02 \times 10^{-3}\,\mathrm{kg/mol}}} = 180\ \mathrm{m/s}\ .$$

20. The molar mass of argon is 39.95 g/mol. Eq. 20-22 gives

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.31\,\frac{\mathrm{J}}{\mathrm{mol{\cdot}K}}\right)(313\,\mathrm{K})}{39.95 \times 10^{-3}\,\mathrm{kg/mol}}} = 442\ \mathrm{m/s}\ .$$

21. First we rewrite Eq. 20-22 using Eq. 20-4 and Eq. 20-7:

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\,(kN_{\mathrm{A}})\,T}{(m\,N_{\mathrm{A}})}} = \sqrt{\frac{3kT}{M}}\ .$$

The mass of the electron is given in the problem, and $k = 1.38 \times 10^{-23}$ J/K is given in the textbook. With $T = 2.00 \times 10^6$ K, the above expression gives $v_{\mathrm{rms}} = 9.53 \times 10^6$ m/s. The pressure value given in the problem is not used in the solution.

22. Table 20-1 gives $M = 28.0$ g/mol for Nitrogen. This value can be used in Eq. 20-22 with T in Kelvins to obtain the results. A variation on this approach is to set up ratios, using the fact that Table 20-1 also gives the rms speed for nitrogen gas at 300 K (the value is 517 m/s). Here we illustrate the latter approach, using v for v_{rms}:

$$\frac{v_2}{v_1} = \frac{\sqrt{\frac{3RT_2}{M}}}{\sqrt{\frac{3RT_1}{M}}} = \sqrt{\frac{T_2}{T_1}}\ .$$

(a) With $T_2 = 20.0 + 273.15 \approx 293$ K, we obtain

$$v_2 = (517\,\mathrm{m/s})\sqrt{\frac{293\,\mathrm{K}}{300\,\mathrm{K}}} = 511\ \mathrm{m/s}\ .$$

(b) In this case, we set $v_3 = \frac{1}{2}v_2$ and solve $v_3/v_2 = \sqrt{T_3/T_2}$ for T_3:

$$T_3 = T_2\left(\frac{v_3}{v_2}\right)^2 = (293\,\mathrm{K})\left(\frac{1}{2}\right)^2 = 73\ \mathrm{K}$$

which we write as $73 - 273 = -200°$C.

(c) Now we have $v_4 = 2v_2$ and obtain

$$T_4 = T_2\left(\frac{v_4}{v_2}\right)^2 = (293\,\mathrm{K})(4) = 1.17 \times 10^3\,\mathrm{K}$$

which is equivalent to 899°.

23. In the reflection process, only the normal component of the momentum changes, so for one molecule the change in momentum is $2mv\cos\theta$, where m is the mass of the molecule, v is its speed, and θ is the angle between its velocity and the normal to the wall. If N molecules collide with the wall, then the change in their total momentum is $2Nmv\cos\theta$, and if the total time taken for the collisions is Δt, then

the average rate of change of the total momentum is $2(N/\Delta t)mv\cos\theta$. This is the average force exerted by the N molecules on the wall, and the pressure is the average force per unit area:

$$\begin{aligned} p &= \frac{2}{A}\left(\frac{N}{\Delta t}\right) mv\cos\theta \\ &= \left(\frac{2}{2.0\times 10^{-4}\,\mathrm{m}^2}\right)(1.0\times 10^{23}\,\mathrm{s}^{-1})(3.3\times 10^{-27}\,\mathrm{kg})(1.0\times 10^{3}\,\mathrm{m/s})\cos 55^\circ \\ &= 1.9\times 10^{3}\ \mathrm{Pa}\ . \end{aligned}$$

We note that the value given for the mass was converted to kg and the value given for the area was converted to m^2.

24. We can express the ideal gas law in terms of density using $n = M_{\mathrm{sam}}/M$:

$$pV = \frac{M_{\mathrm{sam}}RT}{M} \implies \rho = \frac{pM}{RT}\ .$$

We can also use this to write the rms speed formula in terms of density:

$$v_{\mathrm{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(pM/\rho)}{M}} = \sqrt{\frac{3p}{\rho}}\ .$$

(a) We convert to SI units: $\rho = 1.24\times 10^{-2}$ kg/m^3 and $p = 1.01\times 10^{3}$ Pa. The rms speed is $\sqrt{3(1010)/0.0124} = 494$ m/s.

(b) We find M from $\rho = pM/RT$ with $T = 273$ K.

$$M = \frac{\rho RT}{p} = \frac{\left(0.0124\,\mathrm{kg/m^3}\right)\left(8.31\,\frac{\mathrm{J}}{\mathrm{mol{\cdot}K}}\right)(273\,\mathrm{K})}{1.01\times 10^{3}\,\mathrm{Pa}}$$

This yields $M = 0.028$ kg/mol, which converts to 28 g/mol.

25. The average translational kinetic energy is given by $K_{\mathrm{avg}} = \frac{3}{2}kT$, where k is the Boltzmann constant (1.38×10^{-23} J/K) and T is the temperature on the Kelvin scale. Thus

$$K_{\mathrm{avg}} = \frac{3}{2}(1.38\times 10^{-23}\,\mathrm{J/K})(1600\,\mathrm{K}) = 3.31\times 10^{-20}\ \mathrm{J}\ .$$

26. (a) Eq. 20-24 gives

$$K_{\mathrm{avg}} = \frac{3}{2}\left(1.38\times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}}\right)(273\,\mathrm{K}) = 5.65\times 10^{-21}\ \mathrm{J}\ .$$

(b) Similarly, for $T = 373$ K, the average translational kinetic energy is $K_{\mathrm{avg}} = 7.72\times 10^{-21}$ J.

(c) The unit mole may be thought of as a (large) collection: 6.02×10^{23} molecules of ideal gas, in this case. Each molecule has energy specified in part (a), so the large collection has a total kinetic energy equal to

$$K_{\mathrm{mole}} = N_{\mathrm{A}}K_{\mathrm{avg}} = \left(6.02\times 10^{23}\right)\left(5.65\times 10^{-21}\,\mathrm{J}\right) = 3.40\times 10^{3}\ \mathrm{J}\ .$$

(d) Similarly, the result from part (b) leads to

$$K_{\mathrm{mole}} = \left(6.02\times 10^{23}\right)\left(7.72\times 10^{-21}\,\mathrm{J}\right) = 4.65\times 10^{3}\ \mathrm{J}\ .$$

27. (a) We use $\epsilon = L_V/N$, where L_V is the heat of vaporization and N is the number of molecules per gram. The molar mass of atomic hydrogen is 1 g/mol and the molar mass of atomic oxygen is 16 g/mol so the molar mass of H_2O is $1+1+16 = 18$ g/mol. There are $N_{\mathrm{A}} = 6.02\times 10^{23}$ molecules in a mole so the number of molecules in a gram of water is $(6.02\times 10^{23}\,\mathrm{mol}^{-1})/(18\,\mathrm{g/mol}) = 3.34\times 10^{22}$ molecules/g. Thus $\epsilon = (539\,\mathrm{cal/g})/(3.34\times 10^{22}/\mathrm{g}) = 1.61\times 10^{-20}$ cal. This is $(1.61\times 10^{-20}\,\mathrm{cal})(4.186\,\mathrm{J/cal}) = 6.76\times 10^{-20}$ J.

(b) The average translational kinetic energy is

$$K_{\text{avg}} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\,\text{J/K})\,[(32.0 + 273.15)\,\text{K}] = 6.32 \times 10^{-21}\,\text{J} .$$

The ratio ϵ/K_{avg} is $(6.76 \times 10^{-20}\,\text{J})/(6.32 \times 10^{-21}\,\text{J}) = 10.7$.

28. We express the ideal gas law in terms of density using $\rho = M_{\text{sam}}/V$ and $n = M_{\text{sam}}/M$:

$$pV = \frac{M_{\text{sam}}RT}{M} \implies p = \frac{\rho RT}{M} .$$

29. They are not equivalent. Avogadro's law does not tell how the pressure, volume, and temperature are related, so you cannot use it, for example, to calculate the change in volume when the pressure increases at constant temperature. The ideal gas law, however, implies Avogadro's law. It yields $N = nN_A = (pV/RT)N_A = pV/kT$, where $k = R/N_A$ was used. If the two gases have the same volume, the same pressure, and the same temperature, then pV/kT is the same for them. This implies that N is also the same.

30. We solve Eq. 20-25 for d:

$$d = \sqrt{\frac{1}{\lambda\pi\sqrt{2}\,(N/V)}} = \sqrt{\frac{1}{(0.80 \times 10^5\,\text{cm})\,\pi\sqrt{2}\,(2.7 \times 10^{19}/\text{cm}^3)}}$$

which yields $d = 3.2 \times 10^{-8}$ cm, or 0.32 nm.

31. (a) According to Eq. 20–25, the mean free path for molecules in a gas is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} ,$$

where d is the diameter of a molecule and N is the number of molecules in volume V. Substitute $d = 2.0 \times 10^{-10}\,\text{m}$ and $N/V = 1 \times 10^6\,\text{molecules/m}^3$ to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(2.0 \times 10^{-10}\,\text{m})^2(1 \times 10^6\,\text{m}^{-3})} = 6 \times 10^{12}\ \text{m} .$$

(b) At this altitude most of the gas particles are in orbit around Earth and do not suffer randomizing collisions. The mean free path has little physical significance.

32. Using $v = f\lambda$ with $v = 331$ m/s (see Table 18-1) with Eq. 20-2 and Eq. 20-25 leads to

$$f = \frac{v}{\left(\frac{1}{\sqrt{2}\,\pi\,d^2(N/V)}\right)} = (331\,\text{m/s})\pi\sqrt{2}\left(3.0 \times 10^{-10}\,\text{m}\right)^2\left(\frac{nN_A}{V}\right) = \left(8.0 \times 10^7\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{n}{V}\right)$$

Using the ideal gas law, we substitute $n/V = p/RT$ into the above expression and find

$$f = \left(8.0 \times 10^7\,\frac{\text{m}^3}{\text{s}\cdot\text{mol}}\right)\left(\frac{1.01 \times 10^5\,\text{Pa}}{\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273.15\,\text{K})}\right) = 3.5 \times 10^9\ \text{Hz} .$$

If we instead use $v = 343$ m/s (the "default value" for speed of sound in air, used repeatedly in Ch. 18), then the answer is 3.7×10^9 Hz.

33. We substitute $d = 1.0 \times 10^{-2}\,\text{m}$ and $N/V = 15/(1.0 \times 10^{-3}\,\text{m}^3) = 15 \times 10^3\,\text{beans/m}^3$ into Eq. 20–25

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V}$$

to obtain

$$\lambda = \frac{1}{\sqrt{2}\pi(1.0 \times 10^{-2}\,\text{m})^2(15 \times 10^3/\text{m}^3)} = 0.15\ \text{m} .$$

The conversion $1.00\,\text{L} = 1.00 \times 10^{-3}\,\text{m}^3$ is used.

34. (a) We set up a ratio using Eq. 20-25:

$$\frac{\lambda_{\text{Ar}}}{\lambda_{\text{N}_2}} = \frac{1/\left(\pi\sqrt{2}d_{\text{Ar}}^2(N/V)\right)}{1/\left(\pi\sqrt{2}d_{\text{N}_2}^2(N/V)\right)} = \left(\frac{d_{\text{N}_2}}{d_{\text{Ar}}}\right)^2 .$$

Therefore, we obtain

$$\frac{d_{\text{Ar}}}{d_{\text{N}_2}} = \sqrt{\frac{\lambda_{\text{N}_2}}{\lambda_{\text{Ar}}}} = \sqrt{\frac{27.5}{9.9}} = 1.7 .$$

(b) Using Eq. 20-2 and the ideal gas law, we substitute $N/V = N_{\text{A}}n/V = N_{\text{A}}p/RT$ into Eq. 20-25 and find

$$\lambda = \frac{RT}{\pi\sqrt{2}d^2pN_{\text{A}}} .$$

Comparing (for the same species of molecule) at two different pressures and temperatures, this leads to

$$\frac{\lambda_2}{\lambda_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{p_1}{p_2}\right) .$$

With $\lambda_1 = 9.9\times 10^{-6}$ cm, $T_1 = 293$ K (the same as T_2 in this part), $p_1 = 750$ torr and $p_2 = 150$ torr, we find $\lambda_2 = 5.0 \times 10^{-5}$ cm.

(c) The ratio set up in part (b), using the same values for quantities with subscript 1, leads to $\lambda_2 = 7.9 \times 10^{-6}$ cm for $T_2 = 233$ K and $p_2 = 750$ torr.

35. (a) We use the ideal gas law $pV = nRT = NkT$, where p is the pressure, V is the volume, T is the temperature, n is the number of moles, and N is the number of molecules. The substitutions $N = nN_{\text{A}}$ and $k = R/N_{\text{A}}$ were made. Since 1 cm of mercury = 1333 Pa, the pressure is $p = (10^{-7})(1333) = 1.333 \times 10^{-4}$ Pa. Thus,

$$\begin{aligned}\frac{N}{V} &= \frac{p}{kT} = \frac{1.333\times 10^{-4}\,\text{Pa}}{(1.38\times 10^{-23}\,\text{J/K})(295\,\text{K})} \\ &= 3.27\times 10^{16}\,\text{molecules/m}^3 = 3.27\times 10^{10}\ \text{molecules/cm}^3 .\end{aligned}$$

(b) The molecular diameter is $d = 2.00 \times 10^{-10}$ m, so, according to Eq. 20–25, the mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 N/V} = \frac{1}{\sqrt{2}\pi(2.00\times 10^{-10}\,\text{m})^2(3.27\times 10^{16}\,\text{m}^{-3})} = 172\ \text{m} .$$

36. (a) The average speed is

$$v_{\text{avg}} = \frac{\sum n_i v_i}{\sum n_i} = \frac{2(1.0)+4(2.0)+6(3.0)+8(4.0)+2(5.0)}{2+4+6+8+2} = 3.2\ \text{cm/s} .$$

(b) From $v_{\text{rms}} = \sqrt{\sum n_i {v_i}^2/\sum n_i}$ we get

$$v_{\text{rms}} = \sqrt{\frac{2(1.0)^2+4(2.0)^2+6(3.0)^2+8(4.0)^2+2(5.0)^2}{2+4+6+8+2}} = 3.4\ \text{cm/s} .$$

(c) There are eight particles at $v = 4.0$ cm/s, more than the number of particles at any other single speed. So 4.0 cm/s is the most probable speed.

37. (a) The average speed is

$$\overline{v} = \frac{\sum v}{N} ,$$

where the sum is over the speeds of the particles and N is the number of particles. Thus

$$\overline{v} = \frac{(2.0+3.0+4.0+5.0+6.0+7.0+8.0+9.0+10.0+11.0)\,\text{km/s}}{10} = 6.5\ \text{km/s} .$$

(b) The rms speed is given by

$$v_{\text{rms}} = \sqrt{\frac{\sum v^2}{N}}\ .$$

Now

$$\begin{aligned}\sum v^2 &= (2.0)^2 + (3.0)^2 + (4.0)^2 + (5.0)^2 + (6.0)^2 \\ &\quad +(7.0)^2 + (8.0)^2 + (9.0)^2 + (10.0)^2 + (11.0)^2 = 505\ \text{km}^2/\text{s}^2\end{aligned}$$

so

$$v_{\text{rms}} = \sqrt{\frac{505\,\text{km}^2/\text{s}^2}{10}} = 7.1\ \text{km/s}\ .$$

38. (a) The average and rms speeds are as follows:

$$\begin{aligned}v_{\text{avg}} &= \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{10}[4(200\,\text{m/s}) + 2(500\,\text{m/s}) + 4(600\,\text{m/s})] = 420\,\text{m/s}\,, \\ v_{\text{rms}} &= \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} = \sqrt{\frac{1}{10}[4(200\,\text{m/s})^2 + 2(500\,\text{m/s})^2 + 4(600\,\text{m/s})^2]} = 458\ \text{m/s}\ .\end{aligned}$$

From these results, we see that $v_{\text{rms}} > v_{\text{avg}}$.

(b) One may check the validity of the inequality $v_{\text{rms}} \geq v_{\text{avg}}$ for any speed distribution. For example, we consider a set of ten particles divided into two groups of five particles each, with the first group of particles moving at speed v_1 and the second group at v_2 where both v_1 and v_2 are positive-valued (by the definition of speed). In this case, $v_{\text{avg}} = (v_1 + v_2)/2$ and

$$v_{\text{rms}} = \sqrt{\frac{v_1^2 + v_2^2}{2}}\ .$$

To show this must be greater than (or equal to) v_{avg} we examine the difference in the squares of the quantities:

$$\begin{aligned}v_{\text{rms}}^2 - v_{\text{avg}}^2 &= \frac{v_1^2 + v_2^2}{2} - \frac{1}{4}\left(v_1^2 + v_2^2 + 2v_1v_2\right) \\ &= \frac{v_1^2 + v_2^2 - 2v_1v_2}{4} \\ &= \frac{1}{4}\left(v_1 - v_2\right)^2 \quad \geq\ 0\end{aligned}$$

which demonstrates that $v_{\text{rms}} \geq v_{\text{avg}}$ in this situation.

(c) As one can infer from our manipulation in the previous part, we will obtain $v_{\text{rms}} = v_{\text{avg}}$ if all speeds are the same (if $v_1 = v_2$ in the previous part).

39. (a) The rms speed of molecules in a gas is given by $v_{\text{rms}} = \sqrt{3RT/M}$, where T is the temperature and M is the molar mass of the gas. See Eq. 20–34. The speed required for escape from Earth's gravitational pull is $v = \sqrt{2gr_e}$, where g is the acceleration due to gravity at Earth's surface and r_e $(= 6.37 \times 10^6\,\text{m})$ is the radius of Earth. To derive this expression, take the zero of gravitational potential energy to be at infinity. Then, the gravitational potential energy of a particle with mass m at Earth's surface is $U = -GMm/r_e^2 = -mgr_e$, where $g = GM/r_e^2$ was used. If v is the speed of the particle, then its total energy is $E = -mgr_e + \frac{1}{2}mv^2$. If the particle is just able to travel far away, its kinetic energy must tend toward zero as its distance from Earth becomes large without bound. This means $E = 0$ and $v = \sqrt{2gr_e}$. We equate the expressions for the speeds to

obtain $\sqrt{3RT/M} = \sqrt{2gr_e}$. The solution for T is $T = 2gr_eM/3R$. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8\,\text{m/s}^2)(6.37 \times 10^6\,\text{m})(2.02 \times 10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 1.0 \times 10^4\ \text{K}\ .$$

(b) The molar mass of oxygen is 32.0×10^{-3} kg/mol, so for that gas

$$T = \frac{2(9.8\,\text{m/s}^2)(6.37 \times 10^6\,\text{m})(32.0 \times 10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 1.6 \times 10^5\ \text{K}\ .$$

(c) Now, $T = 2g_mr_mM/3R$, where r_m $(= 1.74 \times 10^6\,\text{m})$ is the radius of the Moon and g_m $(= 0.16g)$ is the acceleration due to gravity at the Moon's surface. For hydrogen

$$T = \frac{2(0.16)(9.8\,\text{m/s}^2)(1.74 \times 10^6\,\text{m})(2.02 \times 10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 4.4 \times 10^2\,\text{K}\ .$$

For oxygen

$$T = \frac{2(0.16)(9.8\,\text{m/s}^2)(1.74 \times 10^6\,\text{m})(32.0 \times 10^{-3}\,\text{kg/mol})}{3(8.31\,\text{J/mol}\cdot\text{K})} = 7.0 \times 10^3\ \text{K}\ .$$

(d) The temperature high in Earth's atmosphere is great enough for a significant number of hydrogen atoms in the tail of the Maxwellian distribution to escape. As a result the atmosphere is depleted of hydrogen. On the other hand, very few oxygen atoms escape.

40. We divide Eq. 20-35 by Eq. 20-22:

$$\frac{v_P}{v_{\text{rms}}} = \frac{\sqrt{2RT_2/M}}{\sqrt{3RT_1/M}} = \sqrt{\frac{2T_2}{3T_1}}$$

which leads to

$$\frac{T_2}{T_1} = \frac{3}{2}\left(\frac{v_P}{v_{\text{rms}}}\right)^2 = \frac{3}{2} \qquad \text{if } v_P = v_{\text{rms}}\ .$$

41. (a) The root-mean-square speed is given by $v_{\text{rms}} = \sqrt{3RT/M}$. See Eq. 20–34. The molar mass of hydrogen is 2.02×10^{-3} kg/mol, so

$$v_{\text{rms}} = \sqrt{\frac{3(8.31\,\text{J/mol}\cdot\text{K})(4000\,\text{K})}{2.02 \times 10^{-3}\,\text{kg/mol}}} = 7.0 \times 10^3\,\text{m/s}\ .$$

(b) When the surfaces of the spheres that represent an H_2 molecule and an Ar atom are touching, the distance between their centers is the sum of their radii: $d = r_1 + r_2 = 0.5 \times 10^{-8}\,\text{cm} + 1.5 \times 10^{-8}\,\text{cm} = 2.0 \times 10^{-8}\,\text{cm}$.

(c) The argon atoms are essentially at rest so in time t the hydrogen atom collides with all the argon atoms in a cylinder of radius d and length vt, where v is its speed. That is, the number of collisions is $\pi d^2vtN/V$, where N/V is the concentration of argon atoms. The number of collisions per unit time is

$$\frac{\pi d^2vN}{V} = \pi(2.0 \times 10^{-10}\,\text{m})^2(7.0 \times 10^3\,\text{m/s})(4.0 \times 10^{25}\,\text{m}^{-3}) = 3.5 \times 10^{10}\,\text{collisions/s}\ .$$

42. We divide Eq. 20-31 by Eq. 20-22:

$$\frac{v_{\rm avg2}}{v_{\rm rms1}} = \frac{\sqrt{8RT/\pi M_2}}{\sqrt{3RT/M_1}} = \sqrt{\frac{8M_1}{3\pi M_2}}$$

which leads to

$$\frac{m_1}{m_2} = \frac{M_1}{M_2} = \frac{3\pi}{8}\left(\frac{v_{\rm avg2}}{v_{\rm rms1}}\right)^2 = \frac{3\pi}{2} \qquad \text{if } v_{\rm avg2} = 2v_{\rm rms1} \ .$$

43. (a) The distribution function gives the fraction of particles with speeds between v and $v + dv$, so its integral over all speeds is unity: $\int P(v)\,dv = 1$. Evaluate the integral by calculating the area under the curve in Fig. 20–22. The area of the triangular portion is half the product of the base and altitude, or $\frac{1}{2}av_0$. The area of the rectangular portion is the product of the sides, or av_0. Thus $\int P(v)\,dv = \frac{1}{2}av_0 + av_0 = \frac{3}{2}av_0$, so $\frac{3}{2}av_0 = 1$ and $a = 2/3v_0$.

(b) The number of particles with speeds between $1.5v_0$ and $2v_0$ is given by $N\int_{1.5v_0}^{2v_0} P(v)\,dv$. The integral is easy to evaluate since $P(v) = a$ throughout the range of integration. Thus the number of particles with speeds in the given range is $Na(2.0v_0 - 1.5v_0) = 0.5Nav_0 = N/3$, where $2/3v_0$ was substituted for a.

(c) The average speed is given by

$$v_{\rm avg} = \int vP(v)\,dv \ .$$

For the triangular portion of the distribution $P(v) = av/v_0$, and the contribution of this portion is

$$\frac{a}{v_0}\int_0^{v_0} v^2\,dv = \frac{a}{3v_0}\,v_0^3 = \frac{av_0^2}{3} = \frac{2}{9}v_0 \, ,$$

where $2/3v_0$ was substituted for a. $P(v) = a$ in the rectangular portion, and the contribution of this portion is

$$a\int_{v_0}^{2v_0} v\,dv = \frac{a}{2}\,(4v_0^2 - v_0^2) = \frac{3a}{2}v_0^2 = v_0 \ .$$

Therefore,

$$v_{\rm avg} = \frac{2}{9}v_0 + v_0 = 1.22v_0 \ .$$

(d) The mean-square speed is given by

$$v_{\rm rms}^2 = \int v^2 P(v)\,{\rm d}v \ .$$

The contribution of the triangular section is

$$\frac{a}{v_0}\int_0^{v_0} v^3\,dv = \frac{a}{4v_0}\,v_0^4 = \frac{1}{6}\,v_0^2 \ .$$

The contribution of the rectangular portion is

$$a\int_{v_0}^{2v_0} v^2\,dv = \frac{a}{3}\,(8v_0^3 - v_0^3) = \frac{7a}{3}v_0^3 = \frac{14}{9}\,v_0^2 \ .$$

Thus,

$$v_{\rm rms} = \sqrt{\frac{1}{6}v_0^2 + \frac{14}{9}v_0^2} = 1.31v_0 \ .$$

44. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K}) = 3.4 \times 10^3\ \text{J}\ .$$

45. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Since the process is isothermal $\Delta E_{\text{int}} = 0$ (the internal energy of an ideal gas depends only on the temperature) and $Q = W$. The work done by the gas as its volume expands from V_i to V_f at temperature T is

$$W = \int_{V_i}^{V_f} p\,dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i}$$

where the ideal gas law $pV = nRT$ was used to substitute for p. For 1 mole $Q = W = RT \ln(V_f/V_i)$.

46. (a) According to the first law of thermodynamics $Q = \Delta E_{\text{int}} + W$. When the pressure is a constant $W = p\,\Delta V$. So

$$\begin{aligned}\Delta E_{\text{int}} &= Q - p\Delta V \\ &= 20.9\,\text{J} - \left(1.01 \times 10^5\,\text{Pa}\right)\left(100\,\text{cm}^3 - 50\,\text{cm}^3\right)\left(\frac{1 \times 10^{-6}\,\text{m}^3}{1\,\text{cm}^3}\right) \\ &= 15.9\ \text{J}\ .\end{aligned}$$

(b) The molar specific heat at constant pressure is

$$\begin{aligned}C_p &= \frac{Q}{n\Delta T} \\ &= \frac{Q}{n\left(\frac{p\Delta V}{nR}\right)} = \frac{R}{p}\frac{Q}{\Delta V} \\ &= \frac{\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(20.9\,\text{J})}{(1.01 \times 10^5\,\text{Pa})(50 \times 10^{-6}\,\text{m}^3)} = 34.4\ \text{J/mol}\cdot\text{K}\ .\end{aligned}$$

(c) Using Eq. 20-49, $C_V = C_p - R = 26.1\,\text{J/mol}\cdot\text{K}$.

47. When the temperature changes by ΔT the internal energy of the first gas changes by $n_1 C_1\,\Delta T$, the internal energy of the second gas changes by $n_2 C_2\,\Delta T$, and the internal energy of the third gas changes by $n_3 C_3\,\Delta T$. The change in the internal energy of the composite gas is $\Delta E_{\text{int}} = (n_1C_1 + n_2C_2 + n_3C_3)\,\Delta T$. This must be $(n_1 + n_2 + n_3)C\,\Delta T$, where C is the molar specific heat of the mixture. Thus

$$C = \frac{n_1C_1 + n_2C_2 + n_3C_3}{n_1 + n_2 + n_3}\ .$$

48. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 20-11, that for any process that is depicted as a *straight line* on the pV diagram – the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Further, Eq. 20-44 with Eq. 20-51 gives

$$E_{\text{int}} = n\left(\frac{f}{2}\right)RT = \left(\frac{f}{2}\right)pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in Joules, pressure should be in Pascals (N/m^2) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is $f = 5$.

(a) The internal energy change is

$$\begin{aligned} E_{\text{int}\,c} - E_{\text{int}\,a} &= \frac{5}{2}\left(p_c V_c - p_a V_a\right) \\ &= \frac{5}{2}\left((2000\,\text{Pa})\left(4.0\,\text{m}^3\right) - (5000\,\text{Pa})\left(2.0\,\text{m}^3\right)\right) \\ &= -5000\ \text{J}\ . \end{aligned}$$

(b) The work done during the process represented by the diagonal path is

$$W_{\text{diag}} = \left(\frac{p_a + p_c}{2}\right)(V_c - V_a) = (3500\,\text{Pa})\left(2.0\,\text{m}^3\right)$$

which yields $W_{\text{diag}} = 7000$ J. Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = -5000 + 7000 = 2000\ \text{J}\ .$$

(c) The fact that ΔE_{int} only depends on the initial and final states, and not on the details of the "path" between them, means we can write

$$\Delta E_{\text{int}} = E_{\text{int}\,c} - E_{\text{int}\,a} = -5000\ \text{J}$$

for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = (5000\,\text{Pa})\left(2.0\,\text{m}^3\right) + 0 = 10000\ \text{J}\ .$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = -5000 + 10000 = 5000\ \text{J}\ .$$

49. Argon is a monatomic gas, so $f = 3$ in Eq. 20-51, which provides

$$C_V = \left(\frac{3}{2}\right)R = \left(\frac{3}{2}\right)\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left(\frac{1\,\text{cal}}{4.186\,\text{J}}\right) = 2.98\,\frac{\text{cal}}{\text{mol}\cdot\text{C}^\circ}$$

where we have converted Joules to calories (Eq. 19-12), and taken advantage of the fact that a Celsius degree is equivalent to a unit change on the Kelvin scale. Since (for a given substance) M is effectively a conversion factor between grams and moles, we see that c_V (see units specified in the problem statement) is related to C_V by

$$C_V = c_V M \qquad \text{where} \quad M = mN_\text{A}$$

where m is the mass of a single atom (see Eq. 20-4).

(a) From the above discussion, we obtain

$$m = \frac{M}{N_\text{A}} = \frac{C_V/c_V}{N_\text{A}} = \frac{2.98/0.075}{6.02\times 10^{23}} = 6.6\times 10^{-23}\ \text{g}\ .$$

(b) The molar mass is found to be $M = C_V/c_V = 2.98/0.075 = 39.7$ g/mol which should be rounded to 40 since the given value of c_V is specified to only two significant figures.

50. Referring to Table 20-3, Eq. 20-45 and Eq. 20-46, we have

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V\Delta T = \frac{5}{2}nR\Delta T \\ \text{and}\qquad Q &= nC_p\Delta T = \frac{7}{2}nR\Delta T\ . \end{aligned}$$

Dividing the equations, we obtain

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{5}{7} \ .$$

Thus, the given value $Q = 70$ J leads to

$$\Delta E_{\text{int}} = 50 \text{ J} \ .$$

51. The fact that they rotate but do not oscillate means that the value of f given in Table 20-3 is relevant. Thus, Eq. 20-46 leads to

$$Q = nC_p\Delta T = n\left(\frac{7}{2}R\right)(T_f - T_i) = nRT_i\left(\frac{7}{2}\right)\left(\frac{T_f}{T_i} - 1\right)$$

where $T_i = 273$ K and $n = 1$ mol. The ratio of absolute temperatures is found from the gas law in ratio form (see Sample Problem 20-1). With $p_f = p_i$ we have

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2 \ .$$

Therefore, the energy added as heat is

$$Q = (1\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K})\left(\frac{7}{2}\right)(2-1) \approx 8 \times 10^3 \text{ J} \ .$$

52. (a) Using $M = 32.0$ g/mol from Table 20-1 and Eq. 20-3, we obtain

$$n = \frac{M_{\text{sam}}}{M} = \frac{12.0\,\text{g}}{32.0\,\text{g/mol}} = 0.375 \text{ mol} \ .$$

(b) This is a constant pressure process with a diatomic gas, so we use Eq. 20-46 and Table 20-3. We note that a change of Kelvin temperature is numerically the same as a change of Celsius degrees.

$$\begin{aligned} Q &= nC_p\Delta T = n\left(\frac{7}{2}R\right)\Delta T \\ &= (0.375\,\text{mol})\left(\frac{7}{2}\right)\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(100\,\text{K}) \\ &= 1.09 \times 10^3 \text{ J} \ . \end{aligned}$$

(c) We could compute a value of ΔE_{int} from Eq. 20-45 and divide by the result from part (b), or perform this manipulation algebraically to show the generality of this answer (that is, many factors will be seen to cancel). We illustrate the latter approach:

$$\frac{\Delta E_{\text{int}}}{Q} = \frac{n\left(\frac{5}{2}R\right)\Delta T}{n\left(\frac{7}{2}R\right)\Delta T} = \frac{5}{7} \approx 0.714 \ .$$

53. (a) Since the process is at constant pressure energy transferred as heat to the gas is given by $Q = nC_p\,\Delta T$, where n is the number of moles in the gas, C_p is the molar specific heat at constant pressure, and ΔT is the increase in temperature. For a diatomic ideal gas $C_p = \frac{7}{2}R$. Thus

$$Q = \frac{7}{2}nR\,\Delta T = \frac{7}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 6.98 \times 10^3 \text{ J} \ .$$

(b) The change in the internal energy is given by $\Delta E_{\text{int}} = nC_V\,\Delta T$, where C_V is the specific heat at constant volume. For a diatomic ideal gas $C_V = \frac{5}{2}R$, so

$$\Delta E_{\text{int}} = \frac{5}{2}nR\,\Delta T = \frac{5}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 4.99 \times 10^3 \text{ J} \ .$$

(c) According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$, so

$$W = Q - \Delta E_{\text{int}} = 6.98 \times 10^3\,\text{J} - 4.99 \times 10^3\,\text{J} = 1.99 \times 10^3\ \text{J}\ .$$

(d) The change in the total translational kinetic energy is

$$\Delta K = \frac{3}{2} nR\,\Delta T = \frac{3}{2}(4.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(60.0\,\text{K}) = 2.99 \times 10^3\ \text{J}\ .$$

54. (a) We use Eq. 20-54 with $V_f/V_i = \frac{1}{2}$ for the gas (assumed to obey the ideal gas law).

$$p_i V_i^\gamma = p_f V_f^\gamma \implies \frac{p_f}{p_i} = \left(\frac{V_i}{V_f}\right)^\gamma = 2^{1.3}$$

which yields $p_f = (2.46)(1.0\,\text{atm}) = 2.5$ atm. Similarly, Eq. 20-56 leads to

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (273\,\text{K})(1.23) = 336\ \text{K}\ .$$

(b) We use the gas law in ratio form (see Sample Problem 20-1) and note that when $p_1 = p_2$ then the ratio of volumes is equal to the ratio of (absolute) temperatures. Consequently, with the subscript 1 referring to the situation (of small volume, high pressure, and high temperature) the system is in at the end of part (a), we obtain

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{273\,\text{K}}{336\,\text{K}} = 0.81\ .$$

The volume V_1 is half the original volume of one liter, so

$$V_2 = 0.81(0.50\,\text{L}) = 0.41\ \text{L}\ .$$

55. (a) Let p_i, V_i, and T_i represent the pressure, volume, and temperature of the initial state of the gas. Let p_f, V_f, and T_f represent the pressure, volume, and temperature of the final state. Since the process is adiabatic $p_i V_i^\gamma = p_f V_f^\gamma$, so

$$p_f = \left(\frac{V_i}{V_f}\right)^\gamma p_i = \left(\frac{4.3\,\text{L}}{0.76\,\text{L}}\right)^{1.4} (1.2\,\text{atm}) = 13.6\ \text{atm}\ .$$

We note that since V_i and V_f have the same units, their units cancel and p_f has the same units as p_i.

(b) The gas obeys the ideal gas law $pV = nRT$, so $p_i V_i / p_f V_f = T_i / T_f$ and

$$T_f = \frac{p_f V_f}{p_i V_i} T_i = \left[\frac{(13.6\,\text{atm})(0.76\,\text{L})}{(1.2\,\text{atm})(4.3\,\text{L})}\right](310\,\text{K}) = 620\ \text{K}\ .$$

56. The fact that they rotate but do not oscillate means that the value of f given in Table 20-3 is relevant. In §20-11, it is noted that $\gamma = C_p/C_V$ so that we find $\gamma = 7/5$ in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(300\,\text{K})}{1.01 \times 10^5\,\text{N/m}^2}$$

which yields $V = 0.049\ \text{m}^3$. Consequently,

$$pV^\gamma = \left(1.01 \times 10^5\,\text{N/m}^2\right)\left(0.049\,\text{m}^3\right)^{1.4} = 1.5 \times 10^3\ \text{N}\cdot\text{m}^{2.2}\ .$$

57. We use the first law of thermodynamics: $\Delta E_{\text{int}} = Q - W$. The change in internal energy is $\Delta E_{\text{int}} = nC_V(T_2 - T_1)$, where C_V is the molar heat capacity for a constant volume process. Since the process is adiabatic $Q = 0$. Thus, $W = -\Delta E_{\text{int}} = nC_V(T_1 - T_2)$.

58. (a) Differentiating Eq. 20-53, we obtain

$$\frac{dp}{dV} = (\text{constant})\frac{-\gamma}{V^{\gamma+1}} \implies B = --V\frac{dp}{dV} = (\text{constant})\frac{\gamma}{V^{\gamma}}$$

which produces the desired result upon using Eq. 20-53 again ($p = (\text{constant})/V^{\gamma}$).

(b) Due to the fact that $v = \sqrt{B/\rho}$ (from Chapter 18) and $p = nRT/V = (M_{\text{sam}}/M)RT/V$ (from this chapter) with $\rho = M_{\text{sam}}/V$ (the definition of density), the speed of sound in an ideal gas becomes

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma(M_{\text{sam}}/M)RT/V}{M_{\text{sam}}/V}} = \sqrt{\frac{\gamma RT}{M}} .$$

59. With $p = 1.01 \times 10^5$ Pa and $\rho = 1.29$ kg/m^3, we use the result of part (b) of the previous problem to obtain

$$\gamma = \frac{\rho v^2}{p} = \frac{(1.29\,\text{kg/m}^3)\,(331\,\text{m/s})^2}{1.01 \times 10^5\,\text{Pa}} = 1.40 .$$

60. (a) In the free expansion from state 0 to state 1 we have $Q = W = 0$, so $\Delta E_{\text{int}} = 0$, which means that the temperature of the ideal gas has to remain unchanged. Thus the final pressure is

$$p_1 = \frac{p_0 V_0}{V_1} = \frac{p_0 V_0}{3V_0} = \frac{1}{3}p_0 .$$

(b) For the adiabatic process from state 1 to 2 we have $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$, i.e.,

$$\frac{1}{3}p_0(3V_0)^{\gamma} = (3.00)^{\frac{1}{3}} p_0 V_0^{\gamma}$$

which gives $\gamma = 4/3$. The gas is therefore polyatomic.

(c) From $T = pV/nR$ we get

$$\frac{\bar{K}_2}{\bar{K}_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1} = (3.00)^{\frac{1}{3}} = 1.44 .$$

61. In the following $C_V = \frac{3}{2}R$ is the molar specific heat at constant volume, $C_p = \frac{5}{2}R$ is the molar specific heat at constant pressure, ΔT is the temperature change, and n is the number of moles.

(a) The process $1 \to 2$ takes place at constant volume. The heat added is

$$\begin{aligned} Q &= nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K} - 300\,\text{K}) = 3.74 \times 10^3\ \text{J} . \end{aligned}$$

Since the process takes place at constant volume the work W done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74 \times 10^3\,\text{J} .$$

The process $2 \to 3$ is adiabatic. The heat added is zero. The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K} - 600\,\text{K}) = -1.81 \times 10^3\ \text{J} . \end{aligned}$$

According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81 \times 10^3 \text{ J} .$$

The process $3 \to 1$ takes place at constant pressure. The heat added is

$$\begin{aligned} Q &= nC_p\,\Delta T = \frac{5}{2}nR\,\Delta T \\ &= \frac{5}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -3.22 \times 10^3 \text{ J} . \end{aligned}$$

The change in the internal energy is

$$\begin{aligned} \Delta E_{\text{int}} &= nC_V\,\Delta T = \frac{3}{2}nR\,\Delta T \\ &= \frac{3}{2}(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K} - 455\,\text{K}) = -1.93 \times 10^3 \text{ J} . \end{aligned}$$

According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22 \times 10^3\,\text{J} + 1.93 \times 10^3\,\text{J} = -1.29 \times 10^3 \text{ J} .$$

For the entire process the heat added is

$$Q = 3.74 \times 10^3\,\text{J} + 0 - 3.22 \times 10^3\,\text{J} = 520 \text{ J} ,$$

the change in the internal energy is

$$\Delta E_{\text{int}} = 3.74 \times 10^3\,\text{J} - 1.81 \times 10^3\,\text{J} - 1.93 \times 10^3\,\text{J} = 0 ,$$

and the work done by the gas is

$$W = 0 + 1.81 \times 10^3\,\text{J} - 1.29 \times 10^3\,\text{J} = 520 \text{ J} .$$

(b) We first find the initial volume. Use the ideal gas law $p_1V_1 = nRT_1$ to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(300\,\text{K})}{(1.013 \times 10^5\,\text{Pa})} = 2.46 \times 10^{-2}\,\text{m}^3 .$$

Since $1 \to 2$ is a constant volume process $V_2 = V_1 = 2.46 \times 10^{-2}\,\text{m}^3$. The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(600\,\text{K})}{2.46 \times 10^{-2}\,\text{m}^3} = 2.02 \times 10^5 \text{ Pa} .$$

This is equivalent to 1.99 atm. Since $3 \to 1$ is a constant pressure process, the pressure for state 3 is the same as the pressure for state 1: $p_3 = p_1 = 1.013 \times 10^5\,\text{Pa}$ (1.00 atm). The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00\,\text{mol})(8.31\,\text{J/mol}\cdot\text{K})(455\,\text{K})}{1.013 \times 10^5\,\text{Pa}} = 3.73 \times 10^{-2} \text{ m}^3 .$$

62. We note that $\Delta K = n\left(\frac{3}{2}R\right)\Delta T$ according to the discussion in §20-5 and §20-9. Also, $\Delta E_{\text{int}} = nC_V\Delta T$ can be used for each of these processes (since we are told this is an ideal gas). Finally, we note that Eq. 20-49 leads to $C_p = C_V + R \approx 8.0\,\text{cal/mol}\cdot\text{K}$ after we convert Joules to calories in the ideal gas constant value (Eq. 20-6): $R \approx 2.0\,\text{cal/mol}\cdot\text{K}$. The first law of thermodynamics $Q = \Delta E_{\text{int}} + W$ applies to each process.

- Constant volume process with $\Delta T = 50$ K and $n = 3.0$ mol.
  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450$ cal
  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal
  $W = 0$ for constant volume processes since the application of force (from the pressure) is not associated with an displacements (see §7-2 and §7-3).
  The first law gives $Q = 900 + 0 = 900$ cal.

- Constant pressure process with $\Delta T = 50$ K and $n = 3.0$ mol.
  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450$ cal.
  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal
  $W = p\Delta V$ for constant pressure processes, so (using the ideal gas law) $W = nR\Delta T = (3.0)(2.0)(50) = 300$ cal.
  The first law gives $Q = 900 + 300 = 1200$ cal.
- Adiabiatic process with $\Delta T = 50$ K and $n = 3.0$ mol.
  $\Delta K = (3.0)\left(\frac{3}{2}(2.0)\right)(50) = 450$ cal.
  $\Delta E_{\text{int}} = (3.0)(6.00)(50) = 900$ cal
  $Q = 0$ by definition of "adiabatic."
  The first law leads to $W = Q - E_{\text{int}} = 0 - 900 = -900$ cal.

63. (a) We use $p_i V_i^\gamma = p_f V_f^\gamma$ to compute γ:

$$\gamma = \frac{\log(p_i/p_f)}{\log(V_f/V_i)} = \frac{\log(1.0\,\text{atm}/1.0\times 10^5\,\text{atm})}{\log(1.0\times 10^3\,\text{L}/1.0\times 10^6\,\text{L})} = \frac{5}{3}\ .$$

Therefore the gas is monatomic.

(b) Using the gas law in ratio form (see Sample Problem 20-1), the final temperature is

$$T_f = T_i \frac{p_f V_f}{p_i V_i} = (273\,\text{K})\frac{(1.0\times 10^5\,\text{atm})(1.0\times 10^3\,\text{L})}{(1.0\,\text{atm})(1.0\times 10^6\,\text{L})} = 2.7\times 10^4\ \text{K}\ .$$

(c) The number of moles of gas present is

$$n = \frac{p_i V_i}{RT_i} = \frac{(1.01\times 10^5\,\text{Pa})\left(1.0\times 10^3\,\text{cm}^3\right)}{\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K})} = 4.5\times 10^4\ \text{mol}\ .$$

(d) The total translational energy per mole before the compression is

$$K_i = \frac{3}{2}RT_i = \frac{3}{2}\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K}) = 3.4\times 10^3\ \text{J}\ .$$

After the compression,

$$K_f = \frac{3}{2}RT_f = \frac{3}{2}\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left(2.7\times 10^4\,\text{K}\right) = 3.4\times 10^5\ \text{J}\ .$$

(e) Since $v_{\text{rms}}^2 \propto T$, we have

$$\frac{v_{\text{rms},i}^2}{v_{\text{rms},f}^2} = \frac{T_i}{T_f} = \frac{273\,\text{K}}{2.7\times 10^4\,\text{K}} = 0.01\ .$$

64. (a) For the isothermal process the final temperature of the gas is $T_f = T_i = 300\,\text{K}$. The final pressure is

$$p_f = \frac{p_i V_i}{V_f} = \frac{(32\,\text{atm})(1.0\,\text{L})}{4.0\,\text{L}} = 8.0\ \text{atm}\ ,$$

and the work done is

$$\begin{aligned} W &= nRT_i \ln\left(\frac{V_f}{V_i}\right) = p_i V_i \ln\left(\frac{V_f}{V_i}\right) \\ &= (32\,\text{atm})(1.01\times 10^5\,\text{Pa/atm})(1.0\times 10^{-3}\,\text{m}^3)\ln\left(\frac{4.0\,\text{L}}{1.0\,\text{L}}\right) \\ &= 4.4\times 10^3\ \text{J}\ . \end{aligned}$$

(b) For the adiabatic process $p_i V_i^\gamma = p_f V_f^\gamma$. Thus,

$$\begin{aligned}
p_f &= p_f \left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm}) \left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{5/3} = 3.2\,\text{atm}\ , \\
T_f &= \frac{p_f V_f T_i}{p_i V_i} = \frac{(3.2\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 120\,\text{K}\ , \quad \text{and} \\
W &= Q - \Delta E_{\text{int}} = -\Delta E_{\text{int}} = -\frac{3}{2}nR\Delta T = -\frac{3}{2}(p_f V_f - p_i V_i) \\
&= -\frac{3}{2}[(3.2\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})](1.01 \times 10^5\,\text{Pa/atm})(10^{-3}\,\text{m}^3/\text{L}) \\
&= 2.9 \times 10^3\ \text{J}\ .
\end{aligned}$$

(c) Now, $\gamma = 1.4$ so

$$\begin{aligned}
p_f &= p_i \left(\frac{V_i}{V_f}\right)^\gamma = (32\,\text{atm}) \left(\frac{1.0\,\text{L}}{4.0\,\text{L}}\right)^{1.4} = 4.6\ \text{atm}\ , \\
T_f &= \frac{p_f V_f T_i}{p_i V_i} = \frac{(4.6\,\text{atm})(4.0\,\text{L})(300\,\text{K})}{(32\,\text{atm})(1.0\,\text{L})} = 170\ \text{K}\ , \quad \text{and} \\
W &= Q - \Delta E_{\text{int}} = -\frac{5}{2}nR\Delta T = -\frac{5}{2}(p_f V_f - p_i V_i) \\
&= -\frac{5}{2}[(4.6\,\text{atm})(4.0\,\text{L}) - (32\,\text{atm})(1.0\,\text{L})](1.01 \times 10^5\,\text{Pa/atm})(10^{-3}\,\text{m}^3/\text{L}) \\
&= 3.4 \times 10^3\ \text{J}\ .
\end{aligned}$$

65. We label the various states of the ideal gas as follows: it starts expanding adiabatically from state 1 until it reaches state 2, with $V_2 = 4\,\text{m}^3$; then continues onto state 3 isothermally, with $V_3 = 10\,\text{m}^3$; and eventually getting compressed adiabatically to reach state 4, the final state. For the adiabatic process $1 \to 2$ $\quad p_1 V_1^\gamma = p_2 V_2^\gamma$, for the isothermal process $2 \to 3$ $\quad p_2 V_2 = p_3 V_3$, and finally for the adiabatic process $3 \to 4$ $\quad p_3 V_3^\gamma = p_4 V_4^\gamma$. These equations yield

$$p_4 = p_3 \left(\frac{V_3}{V_4}\right)^\gamma = p_2 \left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma = p_1 \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_2}{V_3}\right)\left(\frac{V_3}{V_4}\right)^\gamma\ .$$

We substitute this expression for p_4 into the equation $p_1 V_1 = p_4 V_4$ (since $T_1 = T_4$) to obtain $V_1 V_3 = V_2 V_4$. Solving for V_4 we obtain

$$V_4 = \frac{V_1 V_3}{V_2} = \frac{(2\,\text{m}^3)(10\,\text{m}^3)}{4\,\text{m}^3} = 5\ \text{m}^3\ .$$

66. We use the result of exercise 58 to set up the ratio

$$\frac{v_1}{v_2} = \frac{\sqrt{\gamma_1 RT/M_1}}{\sqrt{\gamma_2 RT/M_2}} = \sqrt{\frac{M_2}{M_1}} \quad \text{if } \gamma_1 = \gamma_2\ .$$

That final condition (equality of the γ's) is reasonable if we are comparing diatomic gas to diatomic gas, or monatomic gas to monatomic gas. That is, all diatomic gases have $\gamma = 1.4$ (or very nearly so), and all monatomic gases have $\gamma \approx 1.7$.

67. (a) We use the result of exercise 58 to express γ in terms of the speed of sound $v = f\lambda$.

$$\gamma = \frac{Mv^2}{RT} = \frac{M\lambda^2 f^2}{RT}\ .$$

The distance between nodes is half of a wavelength $\lambda = 2 \times 0.0677\,\text{m}$, and the molar mass in SI units is $M = 0.127$ kg/mol. Consequently,

$$\gamma = \frac{(0.127)(2 \times 0.0677)^2(1400)^2}{(8.31)(400)} = 1.37\ .$$

(b) Using Table 20-3, we find $\gamma = 5/3 \approx 1.7$ for monatomic gases, $\gamma = 7/5 = 1.4$ for diatomic gases, and $\gamma = 4/3 \approx 1.3$ for polyatomic gases. Our result in part (a) suggests that iodine is a diatomic gas.

68. We assume this to be an ideal gas, so that $C_p = C_V + R = 6.0R$. Therefore, $\gamma = C_p/C_V = 1.2$, and the result of exercise 58 divided by Eq. 20-2 becomes

$$\frac{v_s}{v_{\rm rms}} = \frac{\sqrt{\gamma RT/M}}{\sqrt{3RT/M}} = \sqrt{\frac{\gamma}{3}} = \sqrt{0.40} = 0.63 \; .$$

69. The initial data concerning the balloon is indicated by the subscript 1. As in Sample Problem 1, we use the gas law in ratio form:

$$\frac{p_1 V_1}{p_2 V_2} = \frac{T_1}{T_2} \implies V_2 = \left(2.2\,{\rm m}^3\right)\left(\frac{760\,{\rm torr}}{380\,{\rm torr}}\right)\left(\frac{225\,{\rm K}}{293\,{\rm K}}\right) = 3.4\ {\rm m}^3 \; .$$

70. (a) We use $pV = nRT$. The volume of the tank is

$$\begin{aligned} V &= \frac{nRT}{p} = \frac{\left(\frac{300\,{\rm g}}{17\,{\rm g/mol}}\right)\left(8.31\,\frac{\rm J}{\rm mol\cdot K}\right)(350\,{\rm K})}{1.35\times 10^6\,{\rm Pa}} \\ &= 3.8\times 10^{-2}\,{\rm m}^3 = 38\ {\rm L}\; . \end{aligned}$$

(b) The number of moles of the remaining gas is

$$n' = \frac{p'V}{RT'} = \frac{(8.7\times 10^5\,{\rm Pa})(3.8\times 10^{-2}\,{\rm m}^3)}{\left(8.31\,\frac{\rm J}{\rm mol\cdot K}\right)(293\,{\rm K})} = 13.5\ {\rm mol}\; .$$

The mass of the gas that leaked out is then $\Delta m = 300\,{\rm g} - (13.5\,{\rm mol})(17\,{\rm g/mol}) = 71\,{\rm g}$.

71. (a) Since an ideal gas is involved, then $\Delta E_{\rm int} = 0$ implies $T_1 = T_0$ (see Eq. 20-62). Consequently, the ideal gas law leads to

$$p_1 = p_0\left(\frac{V_0}{V_1}\right) = \frac{p_0}{5}$$

for the pressure at the end of the sudden expansion. Now, the (slower) adiabatic process is described by Eq. 20-54:

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = p_1\left(5^{\gamma}\right)$$

as a result of the fact that $V_2 = V_0$. Therefore,

$$p_2 = \left(\frac{p_0}{5}\right)\left(5^{\gamma}\right) = \left(5^{\gamma-1}\right)p_0$$

which is compared with the problem requirement that $p_2 = 5^{0.4}\,p_0$. Thus, we find that $\gamma = 1.4 = \frac{7}{5}$. Since $\gamma = C_p/C_V$, we see from Table 20-3 that this is a diatomic gas with rotation of the molecules.

(b) The direct connection between $E_{\rm int}$ and $K_{\rm avg}$ is explained at the beginning of §20-8. Since $\Delta E_{\rm int} = 0$ in the free expansion, then $K_1 = K_0$.

(c) In the (slower) adiabatic process, Eq. 20-56 indicates

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 5^{0.4}\,T_0 \qquad \implies \qquad \frac{(E_{\rm int})_2}{(E_{\rm int})_0} = \frac{T_2}{T_0} = 5^{0.4} \approx 1.9 \; .$$

Therefore, $K_2 = 1.9K_0$.

72. A molecule with speed v_{avg} will (typically) suffer a collision after a time $t = \lambda/v_{\text{avg}}$ by definition of the mean free path λ. Thus, we think of $1/t$ as the collision frequency f and use Eq. 20-25.

$$f = \frac{v_{\text{avg}}}{\lambda} = \frac{v_{\text{avg}}}{\frac{1}{\pi\sqrt{2}d^2 N/V}} = \pi\sqrt{2}d^2 v_{\text{avg}}\left(\frac{N}{V}\right) .$$

73. From Table 20-3, $C_V = \frac{3}{2}R = 12.5\,\frac{\text{J}}{\text{mol·K}}$ for a monatomic gas such as helium. To obtain the desired result c_V we need to effectively "convert" mol $\rightarrow$ kg, which can be done using the molar mass M expressed in kilograms per mole. Although we could look up M for helium in Table 20-1 or Appendix F, the problem gives us m so that we can use Eq. 20-4 to find M. That is,

$$M = mN_{\text{A}} = \left(6.66 \times 10^{-27}\,\text{kg}\right)\left(6.02 \times 10^{23}/\text{mol}\right) = 4.01 \times 10^{-3}\,\frac{\text{kg}}{\text{mol}} .$$

Therefore, $c_V = C_V/M = 3.11 \times 10^3$ J/kg·K.

74. (a) When $n = 1$, $V = V_m = RT/p$, where V_m is the molar volume of the gas. So

$$V_m = \frac{RT}{p} = \frac{(8.31\,\text{J/mol}\cdot\text{K})(273.15\,\text{K})}{1.01 \times 10^5\,\text{Pa}} = 22.5\ \text{L} .$$

(b) We use $v_{\text{rms}} = \sqrt{3RT/M}$. The ratio is given by

$$\frac{v_{\text{rms,He}}}{v_{\text{rms,Ne}}} = \sqrt{\frac{M_{\text{Ne}}}{M_{\text{He}}}} = \sqrt{\frac{20\,\text{g}}{4.0\,\text{g}}} = 2.25 .$$

(c) We use $\lambda_{\text{He}} = (\sqrt{2}\pi d^2 N/V)^{-1}$, where the number of particles per unit volume is given by $N/V = N_A n/V = N_A p/RT = p/kT$. So

$$\begin{aligned}\lambda_{\text{He}} &= \frac{1}{\sqrt{2}\pi d^2 (p/kT)} = \frac{kT}{\sqrt{2}\pi d^2 p} \\ &= \frac{(1.38 \times 10^{-23}\,\text{J/K})(273.15\,\text{K})}{1.414\pi(1 \times 10^{-10}\,\text{m})^2(1.01 \times 10^5\,\text{Pa})} = 0.84\,\mu\text{m} .\end{aligned}$$

(d) $\lambda_{\text{Ne}} = \lambda_{\text{He}} = 0.84\,\mu$m.

75. Since ΔE_{int} does not depend on the type of process,

$$(\Delta E_{\text{int}})_{\text{path 2}} = (\Delta E_{\text{int}})_{\text{path 1}} .$$

Also, since (for an ideal gas) it only depends on the temperature variable (so $\Delta E_{\text{int}} = 0$ for isotherms), then

$$(\Delta E_{\text{int}})_{\text{path 1}} = \sum (\Delta E_{\text{int}})_{\text{adiabat}} .$$

Finally, since $Q = 0$ for adiabatic processes, then (for path 1)

$$\begin{aligned}(\Delta E_{\text{int}})_{\text{adiabatic expansion}} &= -W = -40\ \text{J} \qquad \text{and} \\ (\Delta E_{\text{int}})_{\text{adiabatic compression}} &= -W = -(-25) = 25\ \text{J} .\end{aligned}$$

Therefore,

$$(\Delta E_{\text{int}})_{\text{path 2}} = -40\,\text{J} + 25\,\text{J} = -15\ \text{J} .$$

76. For convenience, the "int" subscript for the internal energy will be omitted in this solution. Recalling Eq. 19-28, we note that

$$\begin{aligned} \sum_{\text{cycle}} E &= 0 \\ \Delta E_{A\to B} + \Delta E_{B\to C} + \Delta E_{C\to D} + \Delta E_{D\to E} + \Delta E_{E\to A} &= 0 \ . \end{aligned}$$

Since a gas is involved (assumed to be ideal), then the internal energy does not change when the temperature does not change, so

$$\Delta E_{A\to B} = \Delta E_{D\to E} = 0 \ \ .$$

Now, with $\Delta E_{E\to A} = 8.0$ J given in the problem statement, we have

$$\Delta E_{B\to C} + \Delta E_{C\to D} + 8.0 = 0 \ \ .$$

In an adiabatic process, $\Delta E = -W$, which leads to

$$-5.0 + \Delta E_{C\to D} + 8.0 = 0 \ \ ,$$

and we obtain $\Delta E_{C\to D} = -3.0$ J.

77. We solve

$$\sqrt{\frac{3RT}{M_{\text{helium}}}} = \sqrt{\frac{3R(293\,\text{K})}{M_{\text{hydrogen}}}}$$

for T. With the molar masses found in Table 20-1, we obtain

$$T = (293\,\text{K})\left(\frac{4.0}{2.02}\right) = 580 \text{ K}$$

which is equivalent to 307°C.

78. It is straightforward to show, from Eq. 20-11, that for any process that is depicted as a straight line on the pV diagram, the work is

$$W_{\text{straight}} = \left(\frac{p_i + p_f}{2}\right)\Delta V$$

which includes, as special cases, $W = p\Delta V$ for constant-pressure processes and $W = 0$ for constant-volume processes. Also, from the ideal gas law in ratio form (see Sample Problem 1), we find the final temperature:

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = 4T_1 \ .$$

(a) With $\Delta V = V_2 - V_1 = 2V_1 - V_1 = V_1$ and $p_1 + p_2 = p_1 + 2p_1 = 3p_1$, we obtain

$$W_{\text{straight}} = \frac{3}{2}(p_1V_1) = \frac{3}{2}nRT_1$$

where the ideal gas law is used in that final step.

(b) With $\Delta T = T_2 - T_1 = 4T_1 - T_1 = 3T_1$ and $C_V = \frac{3}{2}R$, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)(3T_1) = \frac{9}{2}nRT_1 \ .$$

(c) The energy added as heat is $Q = \Delta E_{\text{int}} + W_{\text{straight}} = 6nRT_1$.

(d) The molar specific heat for this process may be defined by

$$C_{\text{straight}} = \frac{Q}{n\Delta T} = \frac{6nRT_1}{n\,(3T_1)} = 2R \ .$$

79. (a) The ideal gas law leads to

$$V = \frac{nRT}{p} = \frac{(1.00\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273\,\text{K})}{1.01\times 10^5\,\text{Pa}}$$

which yields $V = 0.0225\,\text{m}^3 = 22.5$ L. If we use the standard pressure value given in Appendix D, $1\,\text{atm} = 1.013\times 10^5\,\text{Pa}$, then our answer rounds more properly to 22.4 L.

(b) From Eq. 20-2,we have $N = 6.02\times 10^{23}$ molecules in the volume found in part (a) (which may be expressed as $V = 2.24\times 10^4\,\text{cm}^3$), so that

$$\frac{N}{V} = \frac{6.02\times 10^{23}}{2.24\times 10^4\,\text{cm}^3} = 2.69\times 10^{19}\ \text{molecules/cm}^3\ .$$

80. The gas law in ratio form (see Sample Problem 20-1) leads to

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (5.67\,\text{Pa})\left(\frac{4.00\,\text{m}^3}{7.00\,\text{m}^3}\right)\left(\frac{313\,\text{K}}{217\,\text{K}}\right) = 4.67\ \text{Pa}\ .$$

81. It is recommended to look over §20-7 before doing this problem.

(a) We normalize the distribution function as follows:

$$\int_0^{v_\text{o}} P(v)\,dv = 1 \quad\Longrightarrow\quad C = \frac{3}{v_\text{o}^3}\ .$$

(b) The average speed is

$$\int_0^{v_\text{o}} v\,P(v)\,dv = \int_0^{v_\text{o}} v\left(\frac{3v^2}{v_\text{o}^3}\right)dv = \frac{3}{4}\,v_\text{o}\ .$$

(c) The rms speed is the square root of

$$\int_0^{v_\text{o}} v^2\,P(v)\,dv = \int_0^{v_\text{o}} v^2\left(\frac{3v^2}{v_\text{o}^3}\right)dv = \frac{3}{5}\,v_\text{o}^2\ .$$

Therefore, $v_\text{rms} = \sqrt{3/5}\,v_\text{o} \approx 0.775v_\text{o}$.

82. (a) From Table 20-3, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. Thus, Eq. 20-46 yields

$$Q = nC_p\Delta T = (3.00)\left(\frac{7}{2}(8.31)\right)(40.0) = 3490\ \text{J}\ .$$

(b) Eq. 20-45 leads to

$$\Delta E_\text{int} = nC_V\Delta T = (3.00)\left(\frac{5}{2}(8.31)\right)(40.0) = 2493 \approx 2490\ \text{J}\ .$$

(c) From either $W = Q - \Delta E_\text{int}$ or $W = p\Delta T = nR\Delta T$, we find $W = 997$ J.

(d) Eq. 20-24 is written in more convenient form (for this problem) in Eq. 20-38. Thus, we obtain

$$\Delta K_\text{trans} = \Delta\left(NK_\text{avg}\right) = n\left(\frac{3}{2}\,R\right)\Delta T \approx 1500\ \text{J}\ .$$

83. The average kinetic energy is related to the absolute temperature by

$$\begin{aligned} K_\text{avg} &= \frac{3}{2}kT \\ 4.0\times 10^{-19}\ \text{J} &= \frac{3}{2}\left(1.38\times 10^{-23}\ \text{J/K}\right)T \end{aligned}$$

which yields $T = 19.3\times 10^3$ K.

84. Using the ideal gas law, one mole occupies a volume equal to

$$V = \frac{nRT}{p} = \frac{(1)(8.31)(50)}{1 \times 10^{-8}} = 4.2 \times 10^{10}\ \text{m}^3 \ .$$

Therefore, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = \frac{(1)\left(6.02 \times 10^{23}\right)}{4.2 \times 10^{10}} = 1.4 \times 10^{13}\ \frac{\text{molecules}}{\text{m}^3} \ .$$

Using $d = 20.0 \times 10^{-9}$ m, Eq. 20-25 yields

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 \left(\frac{N}{V}\right)} = 39\ \text{m} \ .$$

85. The mass of hot air is $M_\text{hot} = nM$ by Eq. 20-3, where the number of moles contained within the envelope is

$$n = \frac{pV}{RT} = \frac{\left(1.01 \times 10^5\right)\left(2.18 \times 10^3\right)}{(8.31)T} = \frac{2.65 \times 10^7}{T}$$

with SI units understood. The magnitude of the gravitational force acting on the balloon is

$$F_g = (M_\text{envelope} + M_\text{basket} + M_\text{hot})\,g = \left(249 + M\frac{2.65 \times 10^7}{T}\right)(9.8)$$

with SI units, again, understood (which implies $M = 0.028$). The problem requires that the buoyant force (equal to the weight of the displaced air of density $\rho = 1.21\,\text{kg/m}^3$) is equal to 2700 N plus the magnitude of the gravitational force. Therefore,

$$\rho V g = 2700 + \left(249 + (0.028)\frac{2.65 \times 10^7}{T}\right)(9.8) \qquad \text{where } V = 2.18 \times 10^3 \ .$$

Solving this for the temperature, we obtain

$$T = \frac{(0.028)\left(2.65 \times 10^7\right)}{\frac{(1.21)(2.18\times 10^3)(9.8) - 2700}{9.8} - 249} = 351\ \text{K}$$

which is equivalent to 78°.

86. (a) The temperature is 10°C $\rightarrow T = 283$ K. Then, with $n = 3.5$ mol and $V_f/V_0 = 3/4$, we use Eq. 20-14:

$$W = nRT \ln\left(\frac{V_f}{V_0}\right) = -2369\ \text{J} \approx -2.4\ \text{kJ} \ .$$

(b) The internal energy change ΔE_int vanishes (for an ideal gas) when $\Delta T = 0$ so that the First Law of Thermodynamics leads to $Q = W = -2.4$ kJ. The negative value implies that the heat transfer is from the sample to its environment.

87. (a) Since $n/V = p/RT$, the number of molecules per unit volume is

$$\frac{N}{V} = \frac{nN_\text{A}}{V} = N_\text{A}\left(\frac{p}{RT}\right)\left(6.02 \times 10^{23}\right)\frac{1.01 \times 10^5\,\text{Pa}}{\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(293\,\text{K})} = 2.5 \times 10^{25}\ \frac{\text{molecules}}{\text{m}^3} \ .$$

(b) Three-fourths of the 2.5×10^{25} value found in part (a) are nitrogen molecules with $M = 28.0$ g/mol (using Table 20-1), and one-fourth of that value are oxygen molecules with $M = 32.0$ g/mol. Consequently, we generalize the $M_\text{sam} = NM/N_\text{A}$ expression for these two species of molecules and write

$$\frac{3}{4}\left(2.5 \times 10^{25}\right)\frac{28.0}{6.02 \times 10^{23}} + \frac{1}{4}\left(2.5 \times 10^{25}\right)\frac{32.0}{6.02 \times 10^{23}} = 1.2 \times 10^3\ \text{g} \ .$$

88. • Using a ruler, we find the diameter of the period D to be roughly 0.5 mm. Therefore, its area is $A = \pi D^2/4 \approx 2\times 10^{-7}\,\mathrm{m}^2$. Meanwhile, we estimate the diameter d of an air molecule to be roughly 2×10^{-10} m (this is "in the ballpark" of the value used in Sample Problem 20-4). So the area an air molecule covers is $a = \pi d^2/4 \approx 3\times 10^{-20}\,\mathrm{m}^2$. Thus

$$\frac{A}{a} \approx \frac{2\times 10^{-7}}{3\times 10^{-20}} \approx 10^{13} \ .$$

This tells us that rouhgly 10^{13} air molecules are needed to cover the period.

• Assume that every second there are N air molecules which collide with the period. If each one of them bounces back elastically after the collision then the change in linear momentum per molecule per collision is $2mv_x$, where m is the molecular mass and v_x is the component of the molecular velocity in the direction perpendicular to the surface of the paper containing the period. We take v_x to mean the *average* velocity x-component. Thus, the pressure exerted by the air molecules on the period is

$$p = \frac{2mNv_x}{A\Delta t} \qquad \text{where} \quad \Delta t = 1\,\mathrm{s}$$

and $v_x \approx v_{\mathrm{rms}}/\sqrt{3}$ (see the discussion immediately preceding Eq. 20-20). Also we have $m = M/N_A$, where M is the average molar mass of the air molecules. We solve for N:

$$\begin{aligned} N &= \frac{\sqrt{3}pAN_\mathrm{A}\Delta t}{2Mv_{\mathrm{rms}}} = \frac{pAN_\mathrm{A}\Delta t}{2\sqrt{MRT}} \\ &= \frac{(1.01\times 10^5\,\mathrm{Pa})(2\times 10^{-7}\,\mathrm{m}^2)(6.02\times 10^{23}/\mathrm{mol})(1\,\mathrm{s})}{2\sqrt{(0.028\,\mathrm{kg/mol})\left(8.31\,\frac{\mathrm{J}}{\mathrm{mol\cdot K}}\right)(300\,\mathrm{K})}} \approx 7\times 10^{20} \ . \end{aligned}$$

89. (a) The work done in a constant-pressure process is $W = p\Delta V$. Therefore,

$$W = \left(25\,\mathrm{N/m}^2\right)\left(1.8\,\mathrm{m}^3 - 3.0\,\mathrm{m}^3\right) = -30 \text{ J} \ .$$

The sign conventions discussed in the textbook for Q indicate that we should write -75 J for the energy which leaves the system in the form of heat. Therefore, the first law of thermodynamics leads to

$$\Delta E_{\mathrm{int}} = Q - W = (-75\,\mathrm{J}) - (-30\,\mathrm{J}) = -45 \text{ J} \ .$$

(b) Since the pressure is constant (and the number of moles is presumed constant), the ideal gas law in ratio form (see Sample Problem 20-1) leads to

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (300\,\mathrm{K})\left(\frac{1.8\,\mathrm{m}^3}{3.0\,\mathrm{m}^3}\right) = 180 \text{ K} \ .$$

It should be noted that this is consistent with the gas being monatomic (that is, if one assumes $C_V = \frac{3}{2}R$ and uses Eq. 20-45, one arrives at this same value for the final temperature).

90. In a constant-pressure process, the work done is $W = p\Delta V$. Using the ideal gas law (assuming the number of moles is constant) this becomes $W = nR\Delta T$. Therefore,

$$W = (3.00 \text{ mol})\left(8.31 \, \frac{\mathrm{J}}{\mathrm{mol\cdot K}}\right)(-75K) \approx -1870 \text{ J} \ .$$

Now, the First Law of Thermodynamics (Eq.19-24) yields

$$\Delta E_{\mathrm{int}} = Q - W = (-4670) - (-1870) = -2800 \text{ J} \ .$$

91. Since no heat is transferred in an adiabatic process, then

$$Q_{\text{total}} = Q_{\text{isotherm}} = W_{\text{isotherm}} = nRT \ln\left(\frac{3}{12}\right)$$

where the First Law of Thermodynamics (with $\Delta E_{\text{int}} = 0$ during the isothermal process) and Eq. 20-14 have been used. With $n = 2.0$ mol and $T = 300$ K, we obtain $Q = -6912\,\text{J} \approx -6.9$ kJ.

92. (a) We recall the sign convention for heat exchange developed in Chapter 19: the value of Q is positive when the system absorbs heat and negative when it releases heat. Thus, in part (a) we have $Q = -300$ kJ, which is used in $Q = c_{\text{liquid}}\, m\, \Delta T$ to produce $\Delta T = -18.75$ C° so that the resulting temperature is $T_{\text{a}} = T_0 + \Delta T = 61.25°$C ("block P").

(b) With $Q = -400$ kJ in $Q = c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = -25$ C°, yielding $T_{\text{b}} = T_{\text{a}} + \Delta T = 36.25°$C ("block S").

(c) With $Q = -820$ kJ in $Q = c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = -51.25$ C°, yielding $T_{\text{c}} = T_{\text{b}} + \Delta T = -15.00°$C ("block X").

(d) We adapt the change-of-state equation to the sign convention adopted for Q (so that the equation is generally $Q = \pm\, L\, m$). With $Q = -820$ kJ in $Q = -\, L_F\, m'$, we find $m' = 1.67$ kg, so that not quite half the material has solidified (still in "block X" at $-15.00°$C).

(e) As a result of part (d), there is $m'' = m - m' = 4.00 - 1.67 = 2.33$ kg of liquid material which remains to solidify before the system may continue lowering temperature (as a solid). With $Q = -670.0$ kJ in $Q = -\, L_F\, m'' + c_{\text{solid}}\, m\, \Delta T$, we find $\Delta T = -40$ C°, yielding $T_{\text{e}} = T_{\text{d}} + \Delta T = -55.00°$C ("block BB").

(f) Now the system is absorbing heat: with $Q = 1240.0$ kJ and $\Delta T' = 40$ C° in $Q = c_{\text{solid}}\, m\, \Delta T' + L_F\, m + c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = 20$ C°, yielding $T_{\text{f}} = T_{\text{e}} + \Delta T' + \Delta T = 5.00°$C ("block V").

(g) With $Q = 1280$ kJ in $Q = c_{\text{liquid}}\, m\, \Delta T$, we find $\Delta T = 80$ C°, yielding $T_{\text{g}} = T_{\text{f}} + \Delta T = 85.00°$C ("block N").

(h) With $Q = 820.0$ kJ and $\Delta T = 20$ C° in $Q = c_{\text{liquid}}\, m\, \Delta T + L_V\, m'$, we find $m' = 1.00$ kg, so one-fourth of the material has vaporized at $T_{\text{h}} = T_{\text{g}} + \Delta T = 105.0°$C ("block L").

(i) With $Q = 1000$ kJ in $Q = L_V\, m''$, we find $m'' = 2.00$ kg, so three-fourths of the material has now vaporized at $T_{\text{i}} = T_{\text{h}} = 105.0°$C ("block L").

(j) We are careful to interpret the given "Molar mass = 3.000" as that of the element (the single atoms), so that for a diatomic gaseous configuration we must use 6.000 g/mol when converting between mass m and moles n. Note that the temperature has not reached the point where rotational modes are excited, so $C_V = (3/2)R$. In the equation that follows, m''' is the mass (at the end of the preceding step) remaining to vaporize (1.000 kg) and $n = m/(6.000) = 666.7$ mol. With $Q = 583.1$ kJ in $Q = L_V\, m''' + nC_V \Delta T$, we find $\Delta T = 10$ C°, yielding $T_{\text{j}} = T_{\text{i}} + \Delta T = 115.0°$C ("block K").

(k) With $Q = 166.2$ kJ in $Q = nC_V \Delta T$, we find $\Delta T = 20$ C°, yielding $T_{\text{k}} = T_{\text{j}} + \Delta T = 135.0°$C ("block I").

(l) Note that the temperature is now in the range where rotational modes are excited, so $C_V = (5/2)R$. With $Q = 277.0$ kJ in $Q = nC_V \Delta T$, we find $\Delta T = 20$ C°, yielding $T_{\text{l}} = T_{\text{k}} + \Delta T = 155.0°$C ("block G").

(m) With the temperature in the range where rotational modes are excited *and* expanding at constant pressure, we have $C_p = (7/2)R$. With $Q = 581.7$ kJ in $Q = nC_p \Delta T$, we find $\Delta T = 30$ C°, yielding $T_{\text{m}} = T_{\text{l}} + \Delta T = 185.0°$ C ("block D").

(n) Finally, we are in the temperature range where vibrational modes are excited (and expanding at constant pressure), so that we have $C_p = (9/2)R$. With $Q = 249.3$ kJ in $Q = nC_p \Delta T$, we find $\Delta T = 10$ C°, yielding $T_{\text{n}} = T_{\text{m}} + \Delta T = 195.0°$C ("block C").

Chapter 21

1. An isothermal process is one in which $T_i = T_f$ which implies $\ln(T_f/T_i) = 0$. Therefore, with $V_f/V_i = 2$, Eq. 21-4 leads to

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (2.50\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln(2) = 14.4\ \text{J/K}\ .$$

2. From Eq. 21-2, we obtain

$$Q = T\Delta S = (405\,\text{K})(46.0\,\text{J/K}) = 1.86\times 10^4\ \text{J}\ .$$

3. (a) Since the gas is ideal, its pressure p is given in terms of the number of moles n, the volume V, and the temperature T by $p = nRT/V$. The work done by the gas during the isothermal expansion is

$$W = \int_{V_1}^{V_2} p\,dV = nRT\int_{V_1}^{V_2}\frac{dV}{V} = nRT\ln\frac{V_2}{V_1}\ .$$

We substitute $V_2 = 2V_1$ to obtain

$$W = nRT\ln 2 = (4.00\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(400\,\text{K})\ln 2 = 9.22\times 10^3\ \text{J}\ .$$

(b) Since the expansion is isothermal, the change in entropy is given by $\Delta S = \int(1/T)\,dQ = Q/T$, where Q is the heat absorbed. According to the first law of thermodynamics, $\Delta E_{\text{int}} = Q - W$. Now the internal energy of an ideal gas depends only on the temperature and not on the pressure and volume. Since the expansion is isothermal, $\Delta E_{\text{int}} = 0$ and $Q = W$. Thus,

$$\Delta S = \frac{W}{T} = \frac{9.22\times 10^3\,\text{J}}{400\,\text{K}} = 23.1\ \text{J/K}\ .$$

(c) $\Delta S = 0$ for all reversible adiabatic processes.

4. An isothermal process is one in which $T_i = T_f$ which implies $\ln(T_f/T_i) = 0$. Therefore, Eq. 21-4 leads to

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) \implies n = \frac{22.0}{(8.31)\ln(3.4/1.3)} = 2.75\ \text{mol}\ .$$

5. We use the following relation derived in Sample Problem 21-2:

$$\Delta S = mc\,\ln\!\left(\frac{T_f}{T_i}\right)\ .$$

(a) The energy absorbed as heat is given by Eq. 19-14. Using Table 19-3, we find

$$Q = cm\Delta T = \left(386\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(2.00\,\text{kg})(75\,\text{K}) = 5.79\times 10^4\ \text{J}$$

where we have used the fact that a change in Kelvin temperature is equivalent to a chance in Celsius degrees.

(b) With $T_f = 373.15$ K and $T_i = 298.15$ K, we obtain

$$\Delta S = (2.00\,\text{kg})\left(386\,\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\ln\left(\frac{373.15}{298.15}\right) = 173\ \text{J/K}\ .$$

6. (a) Isothermal means that the temperature remains constant during the process. ON a graph with temperature plotted along the vertical axis, this means that the points representing that process must lie on a horizontal line (all corresponding to a single value of T). Therefore, process AE is isothermal. This conclusion does not depend on the nature of the material (that is, AE is isothermal irrespective of this substance being a monatomic ideal gas).

(b) Isobaric means that the pressure stays constant during the process. Knowing that this is an ideal gas, and assuming (as usual) that n stays constant during the process, then the gas law in ratio form (see Sample Problem 20-1) leads to

$$\frac{T_f}{T_i} = \frac{V_f}{V_i} = 2 \qquad \text{(see Figure 21-21)}\ .$$

Consequently, we see that process AC is isobaric for this ideal gas. That it should be linear is implied by the simple proportionality between T and V shown above.

(c) For a monatomic gas, $\gamma = 5/3$ (see the discussion in Chapter 20). Therefore,

$$T_f = T_i\left(\frac{V_i}{V_f}\right)^{\gamma-1} = T_0\left(\frac{1}{2}\right)^{2/3} = 0.63T_0$$

which implied process AF is adiabatic.

(d) Since $\ln(x)$ is positive for all $x > 1$, then Eq. 21-4 makes it clear that all processes (with the possible exception of AF) have $\Delta S > 0$. We assume process AF to be reversibly adiabatic, in which case Eq. 21-1 gives $\Delta S = 0$ (since $Q = 0$ for the process, or any small portion of the process); in fact, if AF represented (in some sense) an irreversible process which generated entropy, then we would still end up with the overall conclusion that none of the processes shown are accompanied by an entropy decrease.

7. (a) This may be considered a reversible process (as well as isothermal), so we use $\Delta S = Q/T$ where $Q = Lm$ with $L = 333$ J/g from Table 19-4. Consequently,

$$\Delta S = \frac{(333\,\text{J/g})(12.0\,\text{g})}{273\,\text{K}} = 14.6\ \text{J/K}\ .$$

(b) The situation is similar to that described in part (a), except with $L = 2256$ J/g, $m = 5.00$ g, and $T = 373$ K. We therefore find $\Delta S = 30.2$ J/K.

8. (a) It is possible to motivate, starting from Eq. 21-3, the notion that heat may be found from the integral (or "area under the curve") of a curve in a TS diagram, such as this one. Either from calculus, or from geometry (area of a trapezoid), it is straightforward to find the result for a "straight-line" path in the TS diagram:

$$Q_{\text{straight}} = \left(\frac{T_i + T_f}{2}\right)\Delta S$$

which could, in fact, be *directly* motivated from Eq. 21-3 (but it is important to bear in mind that this is rigorously true only for a process which forms a straight line in a graph that plots T versus S). This leads to $(300\,\text{K})(15\,\text{J/K}) = 4500$ J for the energy absorbed as heat by the gas.

(b) Using Table 20-3 and Eq. 20-45, we find

$$\Delta E_{\text{int}} = n\left(\frac{3}{2}R\right)\Delta T = (2.0\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(200\,\text{K} - 400\,\text{K}) = -5.0\times10^{3}\ \text{J}\ .$$

(c) By the first law of thermodynamics,

$$W = Q - \Delta E_{\text{int}} = 4.5\,\text{kJ} - (-5.0\,\text{kJ}) = 9.5\ \text{kJ}\ .$$

9. (a) The energy that leaves the aluminum as heat has magnitude $Q = m_a c_a (T_{ai} - T_f)$, where m_a is the mass of the aluminum, c_a is the specific heat of aluminum, T_{ai} is the initial temperature of the aluminum, and T_f is the final temperature of the aluminum-water system. The energy that enters the water as heat has magnitude $Q = m_w c_w (T_f - T_{wi})$, where m_w is the mass of the water, c_w is the specific heat of water, and T_{wi} is the initial temperature of the water. The two energies are the same in magnitude since no energy is lost. Thus,

$$m_a c_a (T_{ai} - T_f) = m_w c_w (T_f - T_{wi}) \implies T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w}\ .$$

The specific heat of aluminum is 900 J/kg·K and the specific heat of water is 4190 J/kg·K. Thus,

$$\begin{aligned} T_f &= \frac{(0.200\,\text{kg})(900\,\text{J/kg}\cdot\text{K})(100°\text{C}) + (0.0500\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})(20°\text{C})}{(0.200\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) + (0.0500\,\text{kg})(4190\,\text{J/kg}\cdot\text{K})} \\ &= 57.0°\text{C} \qquad \text{or} \quad 330\ \text{K}\ . \end{aligned}$$

(b) Now temperatures must be given in Kelvins: $T_{ai} = 393\,\text{K}$, $T_{wi} = 293\,\text{K}$, and $T_f = 330\,\text{K}$. For the aluminum, $dQ = m_a c_a\, dT$ and the change in entropy is

$$\begin{aligned} \Delta S_a &= \int \frac{dQ}{T} = m_a c_a \int_{T_{ai}}^{T_f} \frac{dT}{T} = m_a c_a \ln \frac{T_f}{T_{ai}} \\ &= (0.200\,\text{kg})(900\,\text{J/kg}\cdot\text{K}) \ln\left(\frac{330\,\text{K}}{373\,\text{K}}\right) = -22.1\ \text{J/K}\ . \end{aligned}$$

(c) The entropy change for the water is

$$\begin{aligned} \Delta S_w &= \int \frac{dQ}{T} = m_w c_w \int_{T_{wi}}^{T_f} \frac{dT}{T} = m_w c_w \ln \frac{T_f}{T_{wi}} \\ &= (0.0500\,\text{kg})(4190\,\text{J/kg}\cdot\text{K}) \ln\left(\frac{330\,\text{K}}{293\,\text{K}}\right) = +24.9\ \text{J/K}\ . \end{aligned}$$

(d) The change in the total entropy of the aluminum-water system is $\Delta S = \Delta S_a + \Delta S_w = -22.1\,\text{J/K} + 24.9\,\text{J/K} = +2.8\,\text{J/K}$.

10. This problem is similar to Sample Problem 21-2. The only difference is that we need to find the mass m of each of the blocks. Since the two blocks are identical the final temperature T_f is the average of the initial temperatures:

$$T_f = \frac{1}{2}(T_i + T_f) = \frac{1}{2}(305.5\,\text{K} + 294.5\,\text{K}) = 300.0\,\text{K}\ .$$

Thus from $Q = mc\Delta T$ we find the mass m:

$$m = \frac{Q}{c\Delta T} = \frac{215\,\text{J}}{(386\,\text{J/kg}\cdot\text{K})(300.0\,\text{K} - 294.5\,\text{K})} = 0.101\ \text{kg}\ .$$

(a)

$$\Delta S_L = mc \ln\left(\frac{T_f}{T_{iL}}\right) = (0.101\,\text{kg})(386\,\text{J/kg}\cdot\text{K}) \ln\left(\frac{300.0\,\text{K}}{305.5\,\text{K}}\right) = -0.710\ \text{J/K}\ .$$

(b) Since the temperature of the reservoir is virtually the same as that of the block, which gives up the same amount of heat as the reservoir absorbs, the change in entropy $\Delta S'_L$ of the reservoir connected to the left block is the opposite of that of the left block: $\Delta S'_L = -\Delta S_L = +0.710\,\text{J/K}$.

(c) The entropy change for block R is

$$\Delta S_R = mc \ln\left(\frac{T_f}{T_{iR}}\right) = (0.101\,\text{kg})(386\,\text{J/kg}\cdot\text{K}) \ln\left(\frac{300.0\,\text{K}}{294.5\,\text{K}}\right) = +0.723 \text{ J/K} .$$

(d) Similar to the case in part (b) above, the change in entropy $\Delta S'_R$ of the reservoir connected to the right block is given by $\Delta S'_R = -\Delta S_R = -0.723\,\text{J/K}$.

(e) The change in entropy for the two-block system is $\Delta S_L + \Delta S_R = -0.710\,\text{J/K} + 0.723\,\text{J/K} = +0.013\,\text{J/K}$.

(f) The entropy change for the entire system is given by $\Delta S = \Delta S_L + \Delta S'_L + \Delta S_R + \Delta S'_R = \Delta S_L - \Delta S_L + \Delta S_R - \Delta S_R = 0$, which is expected of a reversible process.

11. From problem #10 we know that, if the process in Fig. 21-5 should happen in reverse, then the change in entropy for the left block, which now absorbs energy, is $\Delta S_L = +0.710\,\text{J/K}$; while for the right block $\Delta S_R = -0.723\,\text{J/K}$. The net change in entropy of the two-block system would then be

$$\Delta S = \Delta S_L + \Delta S_R = +0.710\,\text{J/K} - 0.723\,\text{J/K} = -0.013\,\text{J/K} \;\; < \; 0 .$$

This is a clear violation of the second law.

12. The connection between molar heat capacity and the degrees of freedom of a diatomic gas is given by setting $f = 5$ in Eq. 20-51. Thus, $C_V = \frac{5}{2}R$, $C_p = \frac{7}{2}R$, and $\gamma = \frac{7}{5}$. In addition to various equations from Chapter 20, we also make use of Eq. 21-4 of this chapter. We note that we are asked to use the ideal gas constant as R and not plug in its numerical value. We also recall that isothermal means constant-temperature, so $T_2 = T_1$ for the $1 \to 2$ process. The statement (at the end of the problem) regarding "per mole" may be taken to mean that n may be set identically equal to 1 wherever it appears.

(a) The gas law in ratio form (see Sample Problem 20-1) as well as the adiabatic relations Eq. 20-54 and Eq. 20-56 are used to obtain

$$\begin{aligned}
p_2 &= p_1\left(\frac{V_1}{V_2}\right) = \frac{p_1}{3} , \\
p_3 &= p_1\left(\frac{V_1}{V_3}\right)^{\gamma} = \frac{p_1}{3^{1.4}} , \\
T_3 &= T_1\left(\frac{V_1}{V_3}\right)^{\gamma-1} = \frac{T_1}{3^{0.4}} .
\end{aligned}$$

(b) The energy and entropy contributions from all the processes are

- process $1 \to 2$
  The internal energy change is $\Delta E_{\text{int}} = 0$ since this is an ideal gas process without a temperature change (see Eq. 20-45).
  The work is given by Eq. 20-14: $W = nRT_1 \ln (V_2/V_1) = RT_1 \ln 3$ which is approximately $1.10RT_1$.
  The energy absorbed as heat is given by the first law of thermodynamics: $Q = \Delta E_{\text{int}} + W \approx 1.10RT_1$.
  The entropy change is $\Delta S = Q/T_1 = 1.10R$.
- process $2 \to 3$
  The work is zero since there is no volume change.
  The internal energy change is

$$\Delta E_{\text{int}} = nC_V\,(T_3 - T_2) = (1)\left(\frac{5}{2}R\right)\left(\frac{T_1}{3^{0.4}} - T_1\right) \approx -0.889RT_1 .$$

This $(-0.889RT_1)$ is also the value for Q (by either the first law of thermodynamics or by the definition of C_V).
For the entropy change, we obtain

$$\begin{aligned}\Delta S &= nR\ln\left(\frac{V_3}{V_1}\right) + nC_V\ln\left(\frac{T_3}{T_1}\right) \\ &= (1)R\ln(1) + (1)\left(\frac{5}{2}R\right)\ln\left(\frac{T_1/3^{0.4}}{T_1}\right) \\ &= 0 + \frac{5}{2}R\ln\left(3^{-0.4}\right) \approx -1.10R\ .\end{aligned}$$

- process $3 \to 1$
By definition, $Q = 0$ in an adiabatic process, which also implies an absence of entropy change (taking this to be a reversible process). The internal change must be the negative of the value obtained for it in the previous process (since all the internal energy changes must add up to zero, for an entire cycle, and its change is zero for process $1 \to 2$), so $\Delta E_{\text{int}} = +0.889RT_1$. By the first law of thermodynamics, then, $W = Q - \Delta E_{\text{int}} = 0.889RT_1$.

13. (a) We refer to the copper block as block 1 and the lead block as block 2. The equilibrium temperature T_f satisfies $m_1c_1(T_f - T_{i,1}) + m_2c_2(T_f - T_{i,2}) = 0$, which we solve for T_f:

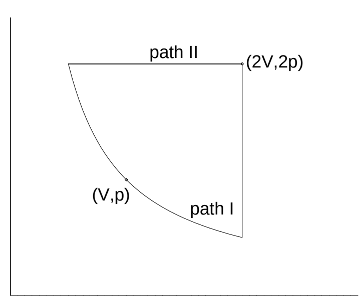
$$\begin{aligned}T_f &= \frac{m_1c_1T_{i,1} + m_2c_2T_{i,2}}{m_1c_1 + m_2c_2} \\ &= \frac{(50\,\text{g})(386\,\text{J/kg}\cdot\text{K})(400\,\text{K}) + (100\,\text{g})(128\,\text{J/kg}\cdot\text{K})(200\,\text{K})}{(50\,\text{g})(386\,\text{J/kg}\cdot\text{K}) + (100\,\text{g})(128\,\text{J/kg}\cdot\text{K})} \\ &= 320\ \text{K}\ .\end{aligned}$$

(b) Since the two-block system in thermally insulated from the environment, the change in internal energy of the system is zero.

(c) The change in entropy is

$$\begin{aligned}\Delta S &= \Delta S_1 + \Delta S_2 = m_1c_1\ln\left(\frac{T_f}{T_{i,1}}\right) + m_2c_2\ln\left(\frac{T_f}{T_{i,2}}\right) \\ &= (50\,\text{g})(386\,\text{J/kg}\cdot\text{K})\ln\left(\frac{320\,\text{K}}{400\,\text{K}}\right) + (100\,\text{g})(128\,\text{J/kg}\cdot\text{K})\ln\left(\frac{320\,\text{K}}{200\,\text{K}}\right) \\ &= +1.72\ \text{J/K}\ .\end{aligned}$$

14. (a) The pV diagram depicting the two "paths" is shown:



(b) "Path I" consists of an isothermal (constant T) process in which the volume doubles, followed by a constant-volume process. We consider the Q for each of these steps. We note that the connection between molar heat capacity and the degrees of freedom of a monatomic gas is given by setting $f = 3$ in Eq. 20-51. Thus, $C_V = \frac{3}{2}R$, $C_p = \frac{5}{2}R$, and $\gamma = \frac{5}{3}$.

- Isothermal: Since this is an ideal gas, Eq. 20-45 holds, which implies $\Delta E_{\rm int} = 0$ for this process. Eq. 20-14 also applies, so that by the first law of thermodynamics, $Q = 0 + W = nRT \ln V_f/V_i = pV \ln 2$. The ideal gas law is used in the last step.
- Constant-volume: The gas law in ratio form (see Sample Problem 20-1) implies that the pressure decreased by a factor of 2 during the isothermal portion, so that it needs to increase by a factor of 4 in this portion of "path I." That same ratio form now applied to this constant-volume process, yielding $4 = T_f/T_i$ which is used in the following:

$$\begin{aligned} Q &= nC_V\Delta T = n\left(\frac{3}{2}R\right)(T_f - T_i) \\ &= \frac{3}{2}nRT_i\left(\frac{T_f}{T_i} - 1\right) \\ &= \frac{3}{2}pV\,(4-1) \quad = \frac{9}{2}pV \ . \end{aligned}$$

"Path II" consists of an isothermal (constant T) process in which the volume halves, followed by a isobaric (constant p) process. We again consider the Q for each of these steps.

- Isothermal: Here the gas law applied to the isothermal portion leads to a volume half as big as the original. Since $\ln\left(\frac{1}{2}\right) = -\ln 2$, the reasoning used above leads to $Q = -pV \ln 2$.
- Isobaric: To obtain a final volume twice as big as the original, then this portion of the "path" needs to increase the volume by a factor of 4. Now, the gas law applied to this isobaric portion leads to a temperature ratio $T_f/T_i = 4$. Thus,

$$\begin{aligned} Q &= nC_p\Delta T = n\left(\frac{5}{2}R\right)(T_f - T_i) \\ &= \frac{5}{2}nRT_i\left(\frac{T_f}{T_i} - 1\right) \\ &= \frac{5}{2}pV\,(4-1) = \frac{15}{2}pV \ . \end{aligned}$$

(c) Much of the reasoning has been given in part (b). Here and in the next part, we will be brief.

- Path I – Isothermal expansion: Eq. 20-14 gives $W = nRT \ln V_f/V_i = pV \ln 2$.
- Path I – constant-volume part: $W = 0$.
- Path II – Isothermal compression: Eq. 20-14 gives $W = nRT \ln V_f/V_i = pV \ln 1/2 = -pV \ln 2$.
- Path II – isobaric: The initial value of the volume, for this part of the process, is $V_i = \frac{1}{2}V$, and the final volume is $V_f = 2V$. The pressure maintained during this process is $p' = 2p$. The work is given by Eq. 20-16:

$$W = p'\Delta V = p'\,(V_f - V_i) = (2p)\left(2V - \frac{1}{2}V\right) = 3pV \ .$$

(d) The change in internal energy between the very beginning and end of Path I is the same as that for Path II. We can calculate it directly from Eq. 20-45 (in which case the computation is very similar to one done in part (b)) or (indirectly) from the first law of thermodynamics. We illustrate the indirect approach, using information relevant to Path I:

$$\Delta E_{\rm int} = Q_{\rm total1} - W_{\rm total1} = \left(pV\ln 2 + \frac{9}{2}pV\right) - (pV\ln 2 + 0) = \frac{9}{2}pV \ .$$

(e) The change in entropy energy between the very beginning (when the temperature is T_i) and the end of Path I (when the temperature is $T_f = 4T_i$ – as noted in part (b)) is the same as the entropy change for Path II. We compute it using Eq. 21-4:

$$\Delta S = nR\,\ln\!\left(\frac{2V}{V}\right) + nC_V\,\ln\!\left(\frac{4T}{T}\right) = nR\ln 2 + n\left(\frac{3}{2}R\right)\ln 2^2 = nR\ln 2 + 3nR\ln 2 = 4nR\ln 2 \ .$$

15. The ice warms to 0° C, then melts, and the resulting water warms to the temperature of the lake water, which is 15° C. As the ice warms, the energy it receives as heat when the temperature changes by dT is $dQ = mc_I\,dT$, where m is the mass of the ice and c_I is the specific heat of ice. If T_i ($= 263\,\text{K}$) is the initial temperature and T_f ($= 273\,\text{K}$) is the final temperature, then the change in its entropy is

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} = mc_I \int_{T_i}^{T_f} \frac{dT}{T} = mc_I \ln\frac{T_f}{T_i} \\ &= (0.010\,\text{kg})(2220\,\text{J/kg·K}) \ln\left(\frac{273\,\text{K}}{263\,\text{K}}\right) = 0.828\ \text{J/K}\ .\end{aligned}$$

Melting is an isothermal process. The energy leaving the ice as heat is mL_F, where L_F is the heat of fusion for ice. Thus, $\Delta S = Q/T = mL_F/T = (0.010\,\text{kg})(333 \times 10^3\,\text{J/kg})/(273\,\text{K}) = 12.20\,\text{J/K}$. For the warming of the water from the melted ice, the change in entropy is

$$\Delta S = mc_w \ln\frac{T_f}{T_i}\ ,$$

where c_w is the specific heat of water ($4190\,\text{J/kg}\cdot\text{K}$). Thus,

$$\Delta S = (0.010\,\text{kg})(4190\,\text{J/kg·K}) \ln\left(\frac{288\,\text{K}}{273\,\text{K}}\right) = 2.24\ \text{J/K}\ .$$

The total change in entropy for the ice and the water it becomes is

$$\Delta S = 0.828\,\text{J/K} + 12.20\,\text{J/K} + 2.24\,\text{J/K} = 15.27\ \text{J/K}\ .$$

Since the temperature of the lake does not change significantly when the ice melts, the change in its entropy is $\Delta S = Q/T$, where Q is the energy it receives as heat (the negative of the energy it supplies the ice) and T is its temperature. When the ice warms to 0° C,

$$Q = -mc_I(T_f - T_i) = -(0.010\,\text{kg})(2220\,\text{J/kg·K})(10\,\text{K}) = -222\ \text{J}\ .$$

When the ice melts,

$$Q = -mL_F = -(0.010\,\text{kg})(333 \times 10^3\,\text{J/kg}) = -3.33 \times 10^3\ \text{J}\ .$$

When the water from the ice warms,

$$Q = -mc_w(T_f - T_i) = -(0.010\,\text{kg})(4190\,\text{J/kg·K})(15\,\text{K}) = -629\ \text{J}\ .$$

The total energy leaving the lake water is $Q = -222\,\text{J} - 3.33 \times 10^3\,\text{J} - 6.29 \times 10^2\,\text{J} = -4.18 \times 10^3\,\text{J}$. The change in entropy is

$$\Delta S = \frac{-4.18 \times 10^3\,\text{J}}{288\,\text{K}} = -14.51\ \text{J/K}\ .$$

The change in the entropy of the ice-lake system is $\Delta S = (15.27 - 14.51)\,\text{J/K} = 0.76\,\text{J/K}$.

16. In coming to equilibrium, the heat lost by the $100\,\text{cm}^3$ of liquid water (of mass $m_w = 100$ g and specific heat capacity $c_w = 4190\,\text{J/kg·K}$) is absorbed by the ice (of mass m_i which melts and reaches $T_f > 0°\text{C}$). We begin by finding the equilibrium temperature:

$$\begin{aligned}\sum Q &= 0 \\ Q_{\text{warm water cools}} + Q_{\text{ice warms to }0°} + Q_{\text{ice melts}} + Q_{\text{melted ice warms}} &= 0 \\ c_w m_w\,(T_f - 20°) + c_i m_i\,(0° - (-10°)) + L_F\,m_i + c_w m_i\,(T_f - 0°) &= 0\end{aligned}$$

which yields, after using $L_F = 333000$ J/kg and values cited in the problem, $T_f = 12.24°$ which is equivalent to $T_f = 285.39$ K. Sample Problem 20-2 shows that

$$\Delta S_{\text{temp change}} = mc\,\ln\left(\frac{T_2}{T_1}\right)$$

for processes where $\Delta T = T_2 - T_1$, and Eq. 21-2 gives

$$\Delta S_{\text{melt}} = \frac{L_F\, m}{T_{\text{o}}}$$

for the phase change experienced by the ice (with $T_{\text{o}} = 273.15$ K). The total entropy change is (with T in Kelvins)

$$\begin{aligned}\Delta S_{\text{system}} &= m_w c_w \ln\left(\frac{285.39}{293.15}\right) + m_i c_i \ln\left(\frac{273.15}{263.15}\right) + m_i c_w \ln\left(\frac{285.39}{273.15}\right) + \frac{L_F\, m_i}{273.15} \\ &= -11.24 + 0.66 + 1.47 + 9.75 \;=\; 0.64 \text{ J/K} .\end{aligned}$$

17. (a) The final mass of ice is $(1773\,\text{g} + 227\,\text{g})/2 = 1000\,\text{g}$. This means 773 g of water froze. Energy in the form of heat left the system in the amount mL_F, where m is the mass of the water that froze and L_F is the heat of fusion of water. The process is isothermal, so the change in entropy is $\Delta S = Q/T = -mL_F/T = -(0.773\,\text{kg})(333 \times 10^3\,\text{J/kg})/(273\,\text{K}) = -943\,\text{J/K}$.

(b) Now, 773 g of ice is melted. The change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{mL_F}{T} = +943 \text{ J/K} .$$

(c) Yes, they are consistent with the second law of thermodynamics. Over the entire cycle, the change in entropy of the water-ice system is zero even though part of the cycle is irreversible. However, the system is not closed. To consider a closed system, we must include whatever exchanges energy with the ice and water. Suppose it is a constant-temperature heat reservoir during the freezing portion of the cycle and a Bunsen burner during the melting portion. During freezing the entropy of the reservoir increases by 943 J/K. As far as the reservoir-water-ice system is concerned, the process is adiabatic and reversible, so its total entropy does not change. The melting process is irreversible, so the total entropy of the burner-water-ice system increases. The entropy of the burner either increases or else decreases by less than 943 J/K.

18. (a) In an adiabatic process $Q = 0$. This can be done by placing the gas in a thermally insulated container whose volume can be adjusted (say, by means of a movable piston). If the volume is *slowly* increased from V_i to V_x, then the process is reversible. To realize the reversible, constant-volume process from x to f, we would place the gas in a rigid container, which has a fixed volume V_f and is in thermal contact with a heat reservoir. If we *gradually* increase the temperature of the reservoir from T_x to T_f, the gas will undergo the desired reversible process from x to f.

(b) For the two states i and x we have $p_iV_i/T_i = p_xV_x/T_x$ and $p_iV_i^\gamma = p_xV_x^\gamma$. We eliminate p_i and p_x from these equations to obtain

$$\frac{T_x}{T_i} = \left(\frac{V_i}{V_x}\right)^{\gamma-1} .$$

For monatomic ideal gases $\gamma = 5/3$ (see §20-8 and §20-11), so $\gamma - 1 = 2/3$. Also $V_x = V_f$. Substituting these into the equation above, we obtain $T_x = T_i(V_i/V_f)^{2/3}$.

(c) For an ideal gas undergoing an isothermal process, Eq. 20-45 implies $\Delta E_{\text{int}} = 0$. And Eq. 20-14 gives $W = nRT \ln(V_f/V_i)$ for such a process. Therefore, the first law of thermodynamics leads to

$$Q_{\text{path I}} = \Delta E_{\text{int}\,\text{I}} + W_{\text{I}} = nRT_i \ln\left(\frac{V_f}{V_i}\right) .$$

And for path II, we have

$$Q_{\text{path II}} = Q_{\text{adiabat}} + Q_{\text{const vol}} = 0 + nC_V\Delta T .$$

But $C_V = \frac{3}{2}R$ (see Eq. 20-43), so we obtain

$$Q_{\text{path II}} = \frac{3}{2}nR\,(T_f - T_x) \ .$$

We see that $Q_{\text{path I}} \neq Q_{\text{path II}}$.

(d) Since the first part of path II is reversibly adiabatic, then the entropy changes only during the second, constant-volume, part of the path:

$$\Delta S = \int_{T_x}^{T_f} \frac{nC_V\,dT}{T} = nC_V\,\ln\left(\frac{T_f}{T_x}\right) = \frac{3}{2}nR\ln\left(\frac{T_f}{T_x}\right) \ .$$

Entropy is a function of "where you are" on the pV diagram, not "how you got there." Since the beginning and ending point of path I are the same as those of path II, then ΔS is the same for both.

(e) Using the result in part (b) with $V_i/V_f = \frac{1}{2}$ and $T_i = 500$ K, we find

$$T_x = (500\,\mathrm{K})\left(\frac{1}{2}\right)^{2/3} = 315\ \mathrm{K}\ .$$

For path I, Eq. 21-2 gives $Q_I = (\Delta S)\,T$ where $T = T_i = T_f$ and ΔS is the expression calculated in the part (d). Thus,

$$Q_I = \frac{3}{2}nRT_f\,\ln\left(\frac{T_f}{T_x}\right)$$

which can be alternatively derived from Eq. 20-14 and the first law of thermodynamics. With $n = 1$ mol, $T_f = T_i = 500$ K, we find

$$Q_I = \frac{3}{2}(1)(8.31)(500)\ln\left(\frac{500}{315}\right) = 2880\ \mathrm{J}\ .$$

For path II, $Q = Q_{\text{constant volume}} = nC_V\Delta T$ and we obtain

$$Q_{II} = (1)\left(\frac{3}{2}(8.31)\right)(500 - 315) = 2306\ \mathrm{J}\ .$$

The issue of significant figures is problematic since the given "$n = 1$" could be interpreted various ways (exact value, or just one figure?). From part (d),

$$\Delta S = \frac{3}{2}(1)(8.31)\ln\left(\frac{500}{315}\right) = 5.76\ \mathrm{J/K}\ .$$

19. (a) Work is done only for the ab portion of the process. This portion is at constant pressure, so the work done by the gas is

$$W = \int_{V_0}^{4V_0} p_0\,dV = p_0(4V_0 - V_0) = 3p_0V_0\ .$$

(b) We use the first law: $\Delta E_{\text{int}} = Q - W$. Since the process is at constant volume, the work done by the gas is zero and $E_{\text{int}} = Q$. The energy Q absorbed by the gas as heat is $Q = nC_V\,\Delta T$, where C_V is the molar specific heat at constant volume and ΔT is the change in temperature. Since the gas is a monatomic ideal gas, $C_V = \frac{3}{2}R$. Use the ideal gas law to find that the initial temperature is $T_b = p_bV_b/nR = 4p_0V_0/nR$ and that the final temperature is $T_c = p_cV_c/nR = (2p_0)(4V_0)/nR = 8p_0V_0/nR$. Thus,

$$Q = \frac{3}{2}nR\left(\frac{8p_0V_0}{nR} - \frac{4p_0V_0}{nR}\right) = 6p_0V_0\,.$$

The change in the internal energy is $\Delta E_{\text{int}} = 6p_0V_0$. Since $n = 1\,\text{mol}$, this can also be written $Q = 6RT_0$. Since the process is at constant volume, use $dQ = nC_V\,dT$ to obtain

$$\Delta S = \int \frac{dQ}{T} = nC_V \int_{T_b}^{T_c} \frac{dT}{T} = nC_V \ln \frac{T_c}{T_b} \ .$$

Substituting $C_V = \frac{3}{2}R$ and using the ideal gas law, we write

$$\frac{T_c}{T_b} = \frac{p_cV_c}{p_bV_b} = \frac{(2p_0)(4V_0)}{p_0(4V_0)} = 2 \ .$$

Thus, $\Delta S = \frac{3}{2}nR\ln 2$. Since $n = 1$, this is $\Delta S = \frac{3}{2}R\ln 2$.

(c) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $\Delta S = 0$.

20. (a) The final pressure is

$$p_f = (5.00\,\text{kPa})e^{(V_i - V_f)/a} = (5.00\,\text{kPa})e^{(1.00\,\text{m}^3 - 2.00\,\text{m}^3)/1.00\,\text{m}^3} = 1.84\ \text{kPa} \ .$$

(b) We use the ratio form of the gas law (see Sample Problem 20-1) to find the final temperature of the gas:

$$T_f = T_i\left(\frac{p_fV_f}{p_iV_i}\right) = (600\,\text{K})\frac{(1.84\,\text{kPa})(2.00\,\text{m}^3)}{(5.00\,\text{kPa})(1.00\,\text{m}^3)} = 441\ \text{K} \ .$$

For later purposes, we note that this result can be written "exactly" as $T_f = T_i\left(2e^{-1}\right)$. In our solution, we are avoiding using the "one mole" datum since it is not clear how precise it is.

(c) The work done by the gas is

$$\begin{aligned} W &= \int_i^f p\,dV = \int_{V_i}^{V_f} (5.00\,\text{kPa})e^{(V_i - V)/a}\,dV \\ &= (5.00\,\text{kPa})e^{V_i/a} \cdot \Big[-ae^{-V/a}\Big]_{V_i}^{V_f} \\ &= (5.00\,\text{kPa})e^{1.00}(1.00\,\text{m}^3)\left(e^{-1.00} - e^{-2.00}\right) \\ &= 3.16\ \text{kJ} \ . \end{aligned}$$

(d) Consideration of a two-stage process as suggested in the hint, brings us simply to Eq. 21-4. Consequently, with $C_V = \frac{3}{2}R$ (see Eq. 20-43), we find

$$\begin{aligned} \Delta S &= nR\ln\left(\frac{V_f}{V_i}\right) + n\left(\frac{3}{2}R\right)\ln\left(\frac{T_f}{T_i}\right) \\ &= nR\left(\ln 2 + \frac{3}{2}\ln\left(2e^{-1}\right)\right) \\ &= \frac{p_iV_i}{T_i}\left(\ln 2 + \frac{3}{2}\ln 2 + \frac{3}{2}\ln e^{-1}\right) \\ &= \frac{(5000\,\text{Pa})(1.00\,\text{m}^3)}{600\,\text{K}}\left(\frac{5}{2}\ln 2 - \frac{3}{2}\right) \\ &= 1.94\ \text{J/K} \ . \end{aligned}$$

21. The answers to this exercise do not depend on the engine being of the Carnot design. Any heat engine that in-takes energy as heat (from, say, consuming fuel) equal to $|Q_{\text{H}}| = 52$ kJ and exhausts (or discards) energy as heat equal to $|Q_{\text{L}}| = 36$ kJ will have these values of efficiency ε and net work W.

(a) Eq. 21-10 gives

$$\varepsilon = 1 - \left|\frac{Q_L}{Q_H}\right| = 0.31 = 31\% \ .$$

(b) Eq. 21-6 gives

$$W = |Q_H| - |Q_L| = 16 \text{ J} \ .$$

22. With $T_L = 290$ k, we find

$$\varepsilon = 1 - \frac{T_L}{T_H} \implies T_H = \frac{T_L}{1-\varepsilon} = \frac{290\,\text{K}}{1 - 0.40}$$

which yields the (initial) temperature of the high-temperature reservoir: $T_H = 483$ K. If we replace $\varepsilon = 0.40$ in the above calculation with $\varepsilon = 0.50$, we obtain a (final) high temperature equal to $T'_H = 580$ K. The difference is

$$T'_H - T_H = 580\,\text{K} - 483\,\text{K} = 97 \text{ K} \ .$$

23. (a) The efficiency is

$$\varepsilon = \frac{T_H - T_L}{T_H} = \frac{(235 - 115)\,\text{K}}{(235 + 273)\,\text{K}} = 0.236 = 23.6\% \ .$$

We note that a temperature difference has the same value on the Kelvin and Celsius scales. Since the temperatures in the equation must be in Kelvins, the temperature in the denominator is converted to the Kelvin scale.

(b) Since the efficiency is given by $\varepsilon = |W|/|Q_H|$, the work done is given by

$$|W| = \varepsilon|Q_H| = 0.236(6.30 \times 10^4 \text{ J}) = 1.49 \times 10^4 \text{ J} \ .$$

24. Eq. 21-11 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{373\,\text{K}}{7 \times 10^8\,\text{K}} = 0.9999995$$

quoting more figures than are significant. As a percentage, this is $\varepsilon = 99.99995\%$.

25. For an Carnot engine, the efficiency is related to the reservoir temperatures by Eq. 21-11. Therefore,

$$T_H = \frac{T_H - T_L}{\varepsilon} = \frac{75\,\text{K}}{0.22} = 341 \text{ K}$$

which is equivalent to 68°C. The temperature of the cold reservoir is $T_L = T_H - 75 = 341\,\text{K} - 75\,\text{K} = 266\,\text{K}$.

26. (a) Eq. 21-11 leads to

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{333\,\text{K}}{373\,\text{K}} = 0.107 \ .$$

We recall that a Watt is Joule-per-second. Thus, the (net) work done by the cycle per unit time is the given value 500 J/s. Therefore, by Eq. 21-9, we obtain the heat input per unit time:

$$\varepsilon = \frac{W}{|Q_H|} \implies \frac{0.500\,\text{kJ/s}}{0.107} = 4.66 \text{ kJ/s} \ .$$

(b) Considering Eq. 21-6 on a per unit time basis, we find $4.66 - 0.500 = 4.16$ kJ/s for the rate of heat exhaust.

27. (a) Energy is added as heat during the portion of the process from a to b. This portion occurs at constant volume (V_b), so $Q_{\text{in}} = nC_V\,\Delta T$. The gas is a monatomic ideal gas, so $C_V = \frac{3}{2}R$ and the ideal gas law gives $\Delta T = (1/nR)(p_bV_b - p_aV_a) = (1/nR)(p_b - p_a)V_b$. Thus, $Q_{\text{in}} = \frac{3}{2}(p_b - p_a)V_b$. V_b

and p_b are given. We need to find p_a. Now p_a is the same as p_c and points c and b are connected by an adiabatic process. Thus, $p_c V_c^\gamma = p_b V_b^\gamma$ and

$$p_a = p_c = \left(\frac{V_b}{V_c}\right)^\gamma p_b = \left(\frac{1}{8.00}\right)^{5/3} (1.013 \times 10^6 \text{ Pa}) = 3.167 \times 10^4 \text{ Pa} .$$

The energy added as heat is

$$Q_{\text{in}} = \frac{3}{2}(1.013 \times 10^6 \text{ Pa} - 3.167 \times 10^4 \text{ Pa})(1.00 \times 10^{-3} \text{ m}^3) = 1.47 \times 10^3 \text{ J} .$$

(b) Energy leaves the gas as heat during the portion of the process from c to a. This is a constant pressure process, so

$$\begin{aligned} Q_{\text{out}} &= nC_p\,\Delta T = \frac{5}{2}(p_a V_a - p_c V_c) = \frac{5}{2} p_a (V_a - V_c) \\ &= \frac{5}{2}(3.167 \times 10^4 \text{ Pa})(-7.00)(1.00 \times 10^{-3} \text{ m}^3) = -5.54 \times 10^2 \text{ J} . \end{aligned}$$

The substitutions $V_a - V_c = V_a - 8.00V_a = -7.00V_a$ and $C_p = \frac{5}{2}R$ were made.

(c) For a complete cycle, the change in the internal energy is zero and $W = Q = 1.47 \times 10^3 \text{ J} - 5.54 \times 10^2 \text{ J} = 9.18 \times 10^2 \text{ J}$.

(d) The efficiency is $\varepsilon = W/Q_{\text{in}} = (9.18 \times 10^2 \text{ J})/(1.47 \times 10^3 \text{ J}) = 0.624$.

28. During the adiabatic processes (the vertical lines in Fig. 21-9) there is no heat transfer, so we only consider the isothermal processes (the horizontal lines). We can interpret Eq. 21-2, $Q = T\Delta S$, as represent the "area" (with appropriate $\pm$ sign) under the horizontal lines. Since $a \to b$ is in the positive direction while $c \to d$ is in the negative direction, then there is a partial cancellation in the "areas" under the two lines, and the net contribution is the rectangular area between them. This can be seen explicitly as follows:

$$Q_{\text{net}} = T_{\text{H}}\,(S_b - S_a) + T_{\text{L}}\,(S_d - S_c) = (T_{\text{H}} - T_{\text{L}})\,(S_{\text{max}} - S_{\text{min}})$$

where we have used the fact that $S_b = S_c = S_{\text{max}}$ and $S_a = S_d = S_{\text{min}}$.

29. (a) The net work done is the rectangular "area" enclosed in the pV diagram:

$$W = (V - V_0)\,(p - p_0) = (2V_0 - V_0)\,(2p_0 - p_0) = V_0 p_0 .$$

Inserting the values stated in the problem, we obtain $W = 2.27$ kJ.

(b) We compute the energy added as heat during the "heat-intake" portions of the cycle using Eq. 20-39, Eq. 20-43, and Eq. 20-46:

$$\begin{aligned} Q_{abc} &= nC_V\,(T_b - T_a) + nC_p\,(T_c - T_b) \\ &= n\left(\frac{3}{2}R\right) T_a \left(\frac{T_b}{T_a} - 1\right) + n\left(\frac{5}{2}R\right) T_a \left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right) \\ &= nRT_a \left(\frac{3}{2}\left(\frac{T_b}{T_a} - 1\right) + \frac{5}{2}\left(\frac{T_c}{T_a} - \frac{T_b}{T_a}\right)\right) \\ &= p_0 V_0 \left(\frac{3}{2}(2-1) + \frac{5}{2}(4-2)\right) \quad = \quad \frac{13}{2} p_0 V_0 \end{aligned}$$

where, to obtain the last line, the gas law in ratio form has been used (see Sample Problem 20-1). Therefore, since $W = p_0 V_0$, we have $Q_{abc} = 13W/2 = 14.8$ kJ.

(c) The efficiency is given by Eq. 21-9:

$$\varepsilon = \frac{W}{|Q_\mathrm{H}|} = \frac{2}{13} = 0.154 = 15.4\% \ .$$

(d) A Carnot engine operating between T_c and T_a has efficiency equal to

$$\varepsilon = 1 - \frac{T_a}{T_c} = 1 - \frac{1}{4} = 0.750 = 75.0\%$$

where the gas law in ratio form has been used. This is greater than our result in part (c), as expected from the second law of thermodynamics.

30. All terms are assumed to be positive. The total work done by the two-stage system is $W_1 + W_2$. The heat-intake (from, say, consuming fuel) of the system is Q_1 so we have (by Eq. 21-9 and Eq. 21-6)

$$\varepsilon = \frac{W_1 + W_2}{Q_1} = \frac{(Q_1 - Q_2) + (Q_2 - Q_3)}{Q_1} = 1 - \frac{Q_3}{Q_1} \ .$$

Now, Eq. 21-8 leads to

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}$$

where we assume Q_2 is absorbed by the second stage at temperature T_2. This implies the efficiency can be written

$$\varepsilon = 1 - \frac{T_3}{T_1} = \frac{T_1 - T_3}{T_1} \ .$$

31. (a) If T_H is the temperature of the high-temperature reservoir and T_L is the temperature of the low-temperature reservoir, then the maximum efficiency of the engine is

$$\varepsilon = \frac{T_\mathrm{H} - T_\mathrm{L}}{T_\mathrm{H}} = \frac{(800 + 40)\,\mathrm{K}}{(800 + 273)\ \mathrm{K}} = 0.78 \ .$$

(b) The efficiency is defined by $\varepsilon = |W|/|Q_\mathrm{H}|$, where W is the work done by the engine and Q_H is the heat input. W is positive. Over a complete cycle, $Q_\mathrm{H} = W + |Q_\mathrm{L}|$, where Q_L is the heat output, so $\varepsilon = W/(W + |Q_\mathrm{L}|)$ and $|Q_\mathrm{L}| = W[(1/\varepsilon) - 1]$. Now $\varepsilon = (T_\mathrm{H} - T_\mathrm{L})/T_\mathrm{H}$, where T_H is the temperature of the high-temperature heat reservoir and T_L is the temperature of the low-temperature reservoir. Thus,

$$\frac{1}{\varepsilon} - 1 = \frac{T_\mathrm{L}}{T_\mathrm{H} - T_\mathrm{L}} \qquad \text{and} \qquad |Q_\mathrm{L}| = \frac{W T_\mathrm{L}}{T_\mathrm{H} - T_\mathrm{L}} \ .$$

The heat output is used to melt ice at temperature $T_i = -40°\mathrm{C}$. The ice must be brought to $0°\mathrm{C}$, then melted, so $|Q_\mathrm{L}| = mc(T_f - T_i) + mL_F$, where m is the mass of ice melted, T_f is the melting temperature ($0°$ C), c is the specific heat of ice, and L_F is the heat of fusion of ice. Thus, $WT_\mathrm{L}/(T_\mathrm{H} - T_\mathrm{L}) = mc(T_f - T_i) + mL_F$. We differentiate with respect to time and replace dW/dt with P, the power output of the engine, and obtain $PT_\mathrm{L}/(T_\mathrm{H} - T_\mathrm{L}) = (dm/dt)[c(T_f - T_i) + L_F]$. Thus,

$$\frac{dm}{dt} = \left(\frac{PT_\mathrm{L}}{T_\mathrm{H} - T_\mathrm{L}}\right)\left(\frac{1}{c(T_f - T_i) + L_F}\right) \ .$$

Now, $P = 100 \times 10^6\,\mathrm{W}$, $T_\mathrm{L} = 0 + 273 = 273\,\mathrm{K}$, $T_\mathrm{H} = 800 + 273 = 1073\,\mathrm{K}$, $T_i = -40 + 273 = 233\,\mathrm{K}$, $T_f = 0 + 273 = 273\,\mathrm{K}$, $c = 2220\,\mathrm{J/kg{\cdot}K}$, and $L_F = 333 \times 10^3\,\mathrm{J/kg}$, so

$$\begin{aligned} \frac{dm}{dt} &= \left[\frac{(100 \times 10^6\,\mathrm{J/s})(273\,\mathrm{K})}{1073\,\mathrm{K} - 273\,\mathrm{K}}\right]\left[\frac{1}{(2220\,\mathrm{J/kg\cdot K})(273\,\mathrm{K} - 233\,\mathrm{K}) + 333 \times 10^3\,\mathrm{J/kg}}\right] \\ &= 82\ \mathrm{kg/s} \ . \end{aligned}$$

We note that the engine is now operated between $0°\mathrm{C}$ and $800°\mathrm{C}$.

32. (a) Using Eq. 20-54 for process $D \to A$ gives

$$\begin{aligned} p_D V_D^\gamma &= p_A V_A^\gamma \\ \frac{p_0}{32}(8V_0)^\gamma &= p_0 V_0^\gamma \end{aligned}$$

which leads to

$$8^\gamma = 32 \implies \gamma = \frac{5}{3}$$

which (see §20-9 and §20-11) implies the gas is monatomic.

(b) The input heat is that absorbed during process $A \to B$:

$$Q_\mathrm{H} = nC_p\Delta T = n\left(\frac{5}{2}R\right)T_A\left(\frac{T_B}{T_A}-1\right) = nRT_A\left(\frac{5}{2}\right)(2-1) = p_0V_0\left(\frac{5}{2}\right)$$

and the exhaust heat is that liberated during process $C \to D$:

$$Q_\mathrm{L} = nC_p\Delta T = n\left(\frac{5}{2}R\right)T_D\left(1-\frac{T_\mathrm{L}}{T_D}\right) = nRT_D\left(\frac{5}{2}\right)(1-2) = -\frac{1}{4}p_0V_0\left(\frac{5}{2}\right)$$

where in the last step we have used the fact that $T_D = \frac{1}{4}T_A$ (from the gas law in ratio form – see Sample Problem 20-1). Therefore, Eq. 21-10 leads to

$$\varepsilon = 1 - \left|\frac{Q_\mathrm{L}}{Q_\mathrm{H}}\right| = 1 - \frac{1}{4} = 0.75 = 75\% \ .$$

33. (a) The pressure at 2 is $p_2 = 3p_1$, as given in the problem statement. The volume is $V_2 = V_1 = nRT_1/p_1$. The temperature is

$$T_2 = \frac{p_2V_2}{nR} = \frac{3p_1V_1}{nR} = 3T_1 \ .$$

The process $4 \to 1$ is adiabatic, so $p_4V_4^\gamma = p_1V_1^\gamma$ and

$$p_4 = \left(\frac{V_1}{V_4}\right)^\gamma p_1 = \frac{p_1}{4^\gamma} \ ,$$

since $V_4 = 4V_1$. The temperature at 4 is

$$T_4 = \frac{p_4V_4}{nR} = \left(\frac{p_1}{4^\gamma}\right)\left(\frac{4nRT_1}{p_1}\right)\left(\frac{1}{nR}\right) = \frac{T_1}{4^{\gamma-1}} \ .$$

The process $2 \to 3$ is adiabatic, so $p_2V_2^\gamma = p_3V_3^\gamma$ and $p_3 = (V_2/V_3)^\gamma p_2$. Substitute $V_3 = 4V_1$, $V_2 = V_1$, and $p_2 = 3p_1$ to obtain

$$p_3 = \frac{3p_1}{4^\gamma} \ .$$

The temperature is

$$T_3 = \frac{p_3V_3}{nR} = \left(\frac{1}{nR}\right)\left(\frac{3p_1}{4^\gamma}\right)\left(\frac{4nRT_1}{p_1}\right) = \frac{3T_1}{4^{\gamma-1}} \ ,$$

where $V_3 = V_4 = 4V_1 = 4nRT/p_1$ is used.

(b) The efficiency of the cycle is $\varepsilon = W/Q_{12}$, where W is the total work done by the gas during the cycle and Q_{12} is the energy added as heat during the $1 \to 2$ portion of the cycle, the only portion in which energy is added as heat. The work done during the portion of the cycle from 2 to 3 is $W_{23} = \int p\,\mathrm{d}V$. Substitute $p = p_2V_2^\gamma/V^\gamma$ to obtain

$$W_{23} = p_2V_2^\gamma \int_{V_2}^{V_3} V^{-\gamma}\,dV = \left(\frac{p_2V_2^\gamma}{\gamma-1}\right)\left(V_2^{1-\gamma} - V_3^{1-\gamma}\right) \ .$$

Substitute $V_2 = V_1$, $V_3 = 4V_1$, and $p_3 = 3p_1$ to obtain

$$W_{23} = \left(\frac{3p_1V_1}{1-\gamma}\right)\left(1 - \frac{1}{4^{\gamma-1}}\right) = \left(\frac{3nRT_1}{\gamma-1}\right)\left(1 - \frac{1}{4^{\gamma-1}}\right) .$$

Similarly, the work done during the portion of the cycle from 4 to 1 is

$$W_{41} = \left(\frac{p_1V_1^\gamma}{\gamma-1}\right)\left(V_4^{1-\gamma} - V_1^{1-\gamma}\right) = -\left(\frac{p_1V_1}{\gamma-1}\right)\left(1 - \frac{1}{4^{\gamma-1}}\right) = -\left(\frac{nRT_1}{\gamma-1}\right)\left(1 - \frac{1}{4^{\gamma-1}}\right) .$$

No work is done during the $1 \to 2$ and $3 \to 4$ portions, so the total work done by the gas during the cycle is

$$W = W_{23} + W_{41} = \left(\frac{2nRT_1}{\gamma-1}\right)\left(1 - \frac{1}{4^{\gamma-1}}\right) .$$

The energy added as heat is $Q_{12} = nC_V(T_2 - T_1) = nC_V(3T_1 - T_1) = 2nC_VT_1$, where C_V is the molar specific heat at constant volume. Now $\gamma = C_p/C_V = (C_V + R)/C_V = 1 + (R/C_V)$, so $C_V = R/(\gamma - 1)$. Here C_p is the molar specific heat at constant pressure, which for an ideal gas is $C_p = C_V + R$. Thus, $Q_{12} = 2nRT_1/(\gamma - 1)$. The efficiency is

$$\varepsilon = \frac{2nRT_1}{\gamma-1}\left(1 - \frac{1}{4^{\gamma-1}}\right)\frac{\gamma-1}{2nRT_1} = 1 - \frac{1}{4^{\gamma-1}} .$$

34. (a) We use Eq. 21-12,

$$K = \frac{|Q_\mathrm{L}|}{|W|} = \frac{600}{200} = 3 .$$

(b) Energy conservation for a refrigeration cycle requires $|Q_\mathrm{L}| + |W| = |Q_\mathrm{H}|$, so that the result is 800 J.

35. A Carnot refrigerator working between a hot reservoir at temperature T_H and a cold reservoir at temperature T_L has a coefficient of performance K that is given by $K = T_\mathrm{L}/(T_\mathrm{H} - T_\mathrm{L})$. For the refrigerator of this problem, $T_\mathrm{H} = 96°\,\mathrm{F} = 309\,\mathrm{K}$ and $T_\mathrm{L} = 70°\,\mathrm{F} = 294\,\mathrm{K}$, so $K = (294\,\mathrm{K})/(309\,\mathrm{K} - 294\,\mathrm{K}) = 19.6$. The coefficient of performance is the energy Q_L drawn from the cold reservoir as heat divided by the work done: $K = |Q_\mathrm{L}|/|W|$. Thus, $|Q_\mathrm{L}| = K|W| = (19.6)(1.0\,\mathrm{J}) = 20\,\mathrm{J}$.

36. Eq. 21-8 still holds (particularly due to its use of absolute values), and energy conservation implies $|W| + Q_\mathrm{L} = Q_\mathrm{H}$. Therefore, with $T_\mathrm{L} = 268.15$ K and $T_\mathrm{H} = 290.15$ K, we find

$$|Q_\mathrm{H}| = |Q_\mathrm{L}|\left(\frac{T_\mathrm{H}}{T_\mathrm{L}}\right) = (|Q_\mathrm{H}| - |W|)\left(\frac{290.15}{268.15}\right)$$

which (with $|W| = 1.0$ J) leads to

$$|Q_\mathrm{H}| = |W|\left(\frac{1}{1 - \frac{268.15}{290.15}}\right) = 13 \text{ J} .$$

37. The coefficient of performance for a refrigerator is given by $K = |Q_\mathrm{L}|/|W|$, where Q_L is the energy absorbed from the cold reservoir as heat and W is the work done during the refrigeration cycle, a negative value. The first law of thermodynamics yields $Q_\mathrm{H} + Q_\mathrm{L} - W = 0$ for an integer number of cycles. Here Q_H is the energy ejected to the hot reservoir as heat. Thus, $Q_\mathrm{L} = W - Q_\mathrm{H}$. Q_H is negative and greater in magnitude than W, so $|Q_\mathrm{L}| = |Q_\mathrm{H}| - |W|$. Thus,

$$K = \frac{|Q_\mathrm{H}| - |W|}{|W|} .$$

The solution for $|W|$ is $|W| = |Q_\mathrm{H}|/(K + 1)$. In one hour,

$$|W| = \frac{7.54\,\mathrm{MJ}}{3.8 + 1} = 1.57 \text{ MJ} .$$

The rate at which work is done is $(1.57 \times 10^6\,\mathrm{J})/(3600\,\mathrm{s}) = 440\,\mathrm{W}$.

38. (a) Using Eq. 21-12 and Eq. 21-14, we obtain

$$|W| = \frac{|Q_{\rm L}|}{K_C} = (1.0\,{\rm J})\left(\frac{300\,{\rm K} - 280\,{\rm K}}{280\,{\rm K}}\right) = 0.071\ {\rm J}\ .$$

(b) A similar calculation (being sure to use absolute temperature) leads to 0.50 J in this case.

(c) with $T_{\rm L} = 100$ K, we obtain $|W| = 2.0$ J.

(d) Finally, with the low temperature reservoir at 50 K, an amount of work equal to $|W| = 5.0$ J is required.

39. We are told $K = 0.27K_C$ where

$$K_C = \frac{T_{\rm L}}{T_{\rm H} - T_{\rm L}} = \frac{294\,{\rm K}}{307\,{\rm K} - 294\,{\rm K}} = 23$$

where the Fahrenheit temperatures have been converted to Kelvins. Expressed on a per unit time basis, Eq. 21-12 leads to

$$\frac{|W|}{t} = \frac{\left(\frac{|Q_{\rm L}|}{t}\right)}{K} = \frac{4000\,{\rm Btu/h}}{(0.27)(23)} = 643\ {\rm Btu/h}\ .$$

Appendix D indicates $1\,{\rm But/h} = 0.0003929\,{\rm hp}$, so our result may be expressed as $|W|/t = 0.25$ hp.

40. The work done by the motor in $t = 10.0\,{\rm min}$ is $|W| = Pt = (200\,{\rm W})(10.0\,{\rm min})(60\,{\rm s/min}) = 1.20 \times 10^5$ J. The heat extracted is then

$$|Q_{\rm L}| = K|W| = \frac{T_{\rm L}|W|}{T_{\rm H} - T_{\rm L}} = \frac{(270\,{\rm K})\left(1.20 \times 10^5\,{\rm J}\right)}{300\,{\rm K} - 270\,{\rm K}} = 1.08 \times 10^6\ {\rm J}\ .$$

41. The efficiency of the engine is defined by $\varepsilon = W/Q_1$ and is shown in the text to be $\varepsilon = (T_1 - T_2)/T_1$, so $W/Q_1 = (T_1 - T_2)/T_1$. The coefficient of performance of the refrigerator is defined by $K = Q_4/W$ and is shown in the text to be $K = T_4/(T_3 - T_4)$, so $Q_4/W = T_4/(T_3 - T_4)$. Now $Q_4 = Q_3 - W$, so $(Q_3 - W)/W = T_4/(T_3 - T_4)$. The work done by the engine is used to drive the refrigerator, so W is the same for the two. Solve the engine equation for W and substitute the resulting expression into the refrigerator equation. The engine equation yields $W = (T_1 - T_2)Q_1/T_1$ and the substitution yields

$$\frac{T_4}{T_3 - T_4} = \frac{Q_3}{W} - 1 = \frac{Q_3 T_1}{Q_1(T_1 - T_2)} - 1\,.$$

Solve for Q_3/Q_1:

$$\frac{Q_3}{Q_1} = \left(\frac{T_4}{T_3 - T_4} + 1\right)\left(\frac{T_1 - T_2}{T_1}\right) = \left(\frac{T_3}{T_3 - T_4}\right)\left(\frac{T_1 - T_2}{T_1}\right) = \frac{1 - (T_2/T_1)}{1 - (T_4/T_3)}\ .$$

42. We need nine labels:

Label I for 8 molecules on side 1	and	0 on the side 2
Label II for 7 molecules on side 1	and	1 on the side 2
Label III for 6 molecules on side 1	and	2 on the side 2
Label IV for 5 molecules on side 1	and	3 on the side 2
Label V for 4 molecules on side 1	and	4 on the side 2
Label VI for 3 molecules on side 1	and	5 on the side 2
Label VII for 2 molecules on side 1	and	6 on the side 2
Label VIII for 1 molecules on side 1	and	7 on the side 2
Label IX for 0 molecules on side 1	and	8 on the side 2

The multiplicity W is computing using Eq. 21-18. For example, the multiplicity for label IV is

$$W = \frac{8!}{(5!)(3!)} = \frac{40320}{(120)(6)} = 56$$

and the corresponding entropy is (using Eq. 21-19)

$$S = k \ln W = \left(1.38 \times 10^{-23}\ \text{J/K}\right) \ln(56) = 5.6 \times 10^{-23}\ \text{J/K} .$$

In this way, we generate the following table:

Label	W	S
I	1	0
II	8	2.9×10^{-23} J/K
III	28	4.6×10^{-23} J/K
IV	56	5.6×10^{-23} J/K
V	70	5.9×10^{-23} J/K
VI	56	5.6×10^{-23} J/K
VII	28	4.6×10^{-23} J/K
VIII	8	2.9×10^{-23} J/K
IX	1	0

43. There are 2 possible choices for each molecules: it can either be in side 1 or in side 2 of the box. Since there are a total of N independent molecules, the total number of available states of the N-particle system is

$$\mathcal{N}_{\text{total}} = 2 \times 2 \times 2 \times \cdots \times 2 = 2^N .$$

For instance, in the solution of problem #42, above, there are a total of $2^8 = 256$ states, as one can readily verify. It is possible to check this with the textbook example, too, but it is important to realize that there are three additional configurations beyond what are shown in Table 21-1: one with $n_1 = 0$ and $n_2 = 6$, another with $n_1 = 1$ and $n_2 = 5$, and so on. When all these are included, there are a total of $2^6 = 64$ microstates.

44. (a) We denote the configuration with n heads out of N trials as $(n; N)$. We use Eq. 21-18:

$$W(25; 50) = \frac{50!}{(25!)(50-25)!} = 1.26 \times 10^{14} .$$

(b) We use the result of problem #43: $\mathcal{N}_{\text{total}} = 2^{50} = 1.13 \times 10^{15}$.

(c) The percentage of time in question is equal to the probability for the system to be in the central configuration:

$$p(25; 50) = \frac{W(25; 50)}{2^{50}} = \frac{1.26 \times 10^{14}}{1.13 \times 10^{15}} = 11.1\% .$$

(d) We use $W(N/2, N) = N!/[(N/2)!]^2$, $\mathcal{N}_{\text{total}} = 2^N$ and $p(N/2; N) = W(N/2, N)/\mathcal{N}_{\text{total}}$. The results are as follows: For $N = 100$, $W(N/2, N) = 1.01 \times 10^{29}$, $\mathcal{N}_{\text{total}} = 1.27 \times 10^{30}$, and $p(N/2; N) = 8.0\%$.

(e) Similarly, for $N = 250$, we obtain $W(N/2, N) = 9.25 \times 10^{58}$, $\mathcal{N}_{\text{total}} = 1.61 \times 10^{60}$, and $p(N/2; N) = 5.7\%$.

(f) As N increases the number of available microscopic states increase as 2^N, so there are more states to be occupied, leaving the probability less for the system to remain in its central configuration.

45. (a) Suppose there are n_L molecules in the left third of the box, n_C molecules in the center third, and n_R molecules in the right third. There are $N!$ arrangements of the N molecules, but $n_L!$ are simply rearrangements of the n_L molecules in the right third, $n_C!$ are rearrangements of the n_C molecules

in the center third, and $n_R!$ are rearrangements of the n_R molecules in the right third. These rearrangements do not produce a new configuration. Thus, the multiplicity is

$$W = \frac{N!}{n_L!\, n_C!\, n_R!} \ .$$

(b) If half the molecules are in the right half of the box and the other half are in the left half of the box, then the multiplicity is

$$W_B = \frac{N!}{(N/2)!\,(N/2)!} \ .$$

If one-third of the molecules are in each third of the box, then the multiplicity is

$$W_A = \frac{N!}{(N/3)!\,(N/3)!\,(N/3)!} \ .$$

The ratio is

$$\frac{W_A}{W_B} = \frac{(N/2)!\,(N/2)!}{(N/3)!\,(N/3)!\,(N/3)!} \ .$$

(c) For $N = 100$,

$$\frac{W_A}{W_B} = \frac{50!\,50!}{33!\,33!\,34!} = 4.16 \times 10^{16} \ .$$

46. The first law requires that $Q_\mathrm{H} = W + Q_\mathrm{L}$, while the second law requires that

$$\varepsilon = \frac{W}{Q_\mathrm{H}} \le 1 - \frac{T_\mathrm{L}}{T_\mathrm{H}} \ .$$

Thus, we see that the first law is violated in engine A; both laws are violated in B; the second law is violated in C; and, neither of the laws is violated in D.

47. (a) We use $\varepsilon = |W/Q_\mathrm{H}|$. The heat absorbed is

$$|Q_\mathrm{H}| = \frac{|W|}{\varepsilon} = \frac{8.2\,\mathrm{kJ}}{0.25} = 33 \text{ kJ} \ .$$

(b) The heat exhausted is then

$$|Q_\mathrm{L}| = |Q_\mathrm{H}| - |W| = 33\,\mathrm{kJ} - 8.2\,\mathrm{kJ} = 25 \text{ kJ} \ .$$

(c) Now we have

$$\begin{aligned} |Q_\mathrm{H}| &= \frac{|W|}{\varepsilon} = \frac{8.2\,\mathrm{kJ}}{0.31} = 26 \text{ kJ} \\ \text{and} \quad |Q_\mathrm{C}| &= |Q_\mathrm{H}| - |W| = 26\,\mathrm{kJ} - 8.2\,\mathrm{kJ} = 18 \text{ kJ} \ . \end{aligned}$$

48. We find the "percent error" in the use of Stirling's approximation by computing

$$\frac{(N(\ln N) - N) - \ln(N!)}{\ln(N!)} = \frac{(N(\ln N) - N)}{\ln(N!)} - 1$$

which would be multiplied by 100% to be expressed as a percentage.

(a) For $N = 50$, the percent error is

$$\frac{50\ln(50) - 50}{\ln(50!)} - 1 = \frac{145.6}{\ln(3.04 \times 10^{64})} - 1 = \frac{145.6}{148.5} - 1$$

which yields -1.9%, meaning Stirling's approximation produces a value that is 1.9% lower than the correct one.

(b) For $N = 100$, this procedure gives the result -0.89%.

(c) And for $N = 250$, we obtain -0.32%.

(d) The trend is such that Stirling's approximation becomes a better estimate of $\ln(N!)$ for larger values of N.

49. (a) From problem #43, we have $\mathcal{N}_{\text{total}} = 2^{100} = 1.27 \times 10^{30}$ microstates.

(b) Using Eq. 21-18, we find

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(50!)(50!)}\right)}{1.27 \times 10^{30}} = \frac{126114180681955241668515621 57}{1584563250285286751870879006 72} \\ &= 0.079589 \approx 8.0\% \ .\end{aligned}$$

(c) Similarly, for $n_1 = 48$ and $n_2 = 52$ we obtain

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(48!)(52!)}\right)}{1.27 \times 10^{30}} = \frac{2330163971876246923739542027 5}{3169126500570573503741758013 44} \\ &= 0.073527 \approx 7.4\% \ .\end{aligned}$$

(d) With $n_1 = 52$ and $n_2 = 48$, we obtain the same result as in part (c).

(e) For $n_1 = 40$ and $n_2 = 60$ we obtain

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(40!)(60!)}\right)}{1.27 \times 10^{30}} = \frac{171827926822535143765842121 5}{1584563250285286751870879006 72} \\ &= 0.010844 \approx 1.1\% \ .\end{aligned}$$

(f) Finally, for $n_1 = 30$ and $n_2 = 70$ we find

$$\begin{aligned}\frac{W}{\mathcal{N}_{\text{total}}} &= \frac{\left(\frac{100!}{(30!)(70!)}\right)}{1.27 \times 10^{30}} = \frac{1835771238850684051497735}{79228162514264337593543950336} \\ &= 0.00002317 \approx 0.0023\% \ .\end{aligned}$$

50. (a) We use Eq. 21-14. For configuration A

$$W_A = \frac{N!}{(N/2)!(N/2)!} = \frac{50!}{(25!)(25!)} = 1.26 \times 10^{14} \ .$$

(b) For configuration B

$$W_B = \frac{N!}{(0.6N)!(0.4N)!} = \frac{50!}{[0.6(50)]![0.4(50)]!} = 4.71 \times 10^{13} \ .$$

(c) Since all microstates are equally probable,

$$f = \frac{W_B}{W_A} = \frac{1265}{3393} \approx 0.37 \ .$$

(d) We use these formulas for $N = 100$: $W_A = 1.01 \times 10^{29}$, $W_B = 1.37 \times 10^{28}$, and $f \approx 0.14$.

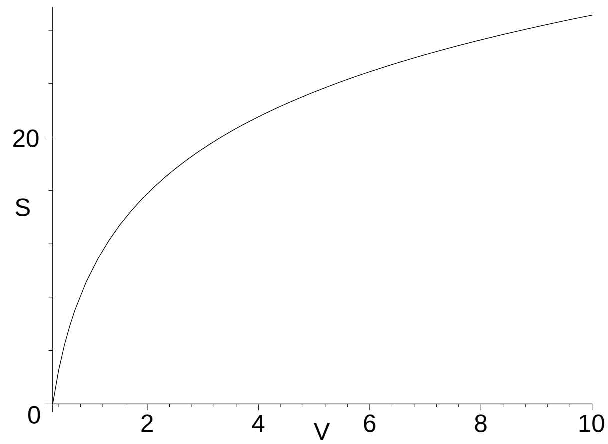
(e) For $N = 200$ we have $W_A = 9.05 \times 10^{58}$, $W_B = 1.64 \times 10^{57}$, and $f = 0.018$.

(f) We see from the calculation above that f decreases as N increases, as expected.

51. Since isothermal means constant temperature, then this would be a flat horizontal line on a T versus S graph (with T being the vertical axis). Since this concerns an ideal gas (also see Figure 21-3) then $\Delta E_{\rm int} = 0$ (by Eq. 20-45), so this isothermal process would be a vertical line on an S versus $E_{\rm int}$ graph (with $E_{\rm int}$ being the horizontal axis). When $T = T_i$ Eq. 21-4 reduces to

$$S - S_i = nR\,\ln\left(\frac{V}{V_i}\right)$$

which is shown in the graph below for $n = 1$ mol, $V_1 = 1\,\mathrm{m}^3$, and $S_i = 10$ J/K (arbitrarily picked).



52. The change in entropy for the ideal gas is found from Eq. 21-2, Eq. 20-14, and the first law of thermodynamics (using the fact that $\Delta E_{\rm int} = 0$ for an ideal gas isothermal process).

$$\Delta S = \frac{Q}{T} = \frac{W}{T} = \frac{nRT}{T}\ln\left(\frac{V_f}{V_i}\right) = nR\ln 2\ ,$$

which is independent of the temperature T of the reservoir. Thus the change in entropy of the reservoir, $\Delta S' = -\Delta S = -nR\ln 2$, is also independent of T. Here we noticed that the net change in entropy for the entire system (the ideal gas plus the reservoir) is $\Delta S_{\rm total} = \Delta S + \Delta S' = 0$ for a reversible process so $\Delta S' = -\Delta S$.

53. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = 40.9^\circ\mathrm{C}\ ,$$

which is equivalent to 314 K.

From Eq. 21-1, we have

$$\Delta S_{\rm copper} = \int_{353}^{314} \frac{c\,m\,dT}{T} = (386)(0.6)\ln\left(\frac{314}{353}\right) = -27.2\ \mathrm{J/K}\ .$$

(b) Also,

$$\Delta S_{\rm water} = \int_{283}^{314} \frac{c\,m\,dT}{T} = (4190)(0.07)\ln\left(\frac{314}{283}\right) = 30.4\ \mathrm{J/K}\ .$$

(c) The net result for the system is $30.3 - 27.2 = 3.2$ J/K. (Note: these calculations are fairly sensitive to round-off errors. To arrive at this final answers, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

54. (a) Starting from $\sum Q = 0$ (for calorimetry problems) we can derive (when no phase changes are involved)

$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2} = -44.2°\text{C} \ ,$$

which is equivalent to 229 K.

(b) From Eq. 21-1, we have

$$\Delta S_{\text{tungsten}} = \int_{303}^{229} \frac{c\, m\, dT}{T} = (134)(0.045)\ln\left(\frac{229}{303}\right) = -1.69 \text{ J/K} \ .$$

(c) Also,

$$\Delta S_{\text{silver}} = \int_{153}^{229} \frac{c\, m\, dT}{T} = (236)(0.025)\ln\left(\frac{229}{153}\right) = 2.37 \text{ J/K} \ .$$

(d) The net result for the system is $2.37 - 1.69 = 0.68$ J/K. (Note: these calculations are fairly sensitive to round-off errors. To arrive at this final answers, the value 273.15 was used to convert to Kelvins, and all intermediate steps were retained to full calculator accuracy.)

55. The Carnot efficiency (Eq. 21-11) depends linearly on T_{L} so that we can take a derivative

$$\varepsilon = 1 - \frac{T_{\text{L}}}{T_{\text{H}}} \quad\Longrightarrow\quad \frac{d\varepsilon}{dT_{\text{L}}} = -\frac{1}{T_{\text{H}}}$$

and quickly get to the result. With $d\varepsilon \to \Delta\varepsilon = 0.100$ and $T_{\text{H}} = 400$ K, we find $dT_{\text{L}} \to \Delta T_{\text{L}} = -40$ K.

56. (a) Processes 1 and 2 both require the input of heat, which is denoted Q_{H}. Noting that rotational degrees of freedom are not involved, then, from the discussion in Chapter 20, $C_V = \frac{3}{2}R$, $C_p = \frac{5}{2}R$, and $\gamma = \frac{5}{3}$. We further note that since the working substance is an ideal gas, process 2 (being isothermal) implies $Q_2 = W_2$. Finally, we note that the volume ratio in process 2 is simply 8/3. Therefore,

$$Q_{\text{H}} = Q_1 + Q_2 = nC_V\,(T' - T) + nRT' \ln\frac{8}{3}$$

which yields (for $T = 300$ K and $T' = 800$ K) the result $Q_{\text{H}} = 25.5 \times 10^3$ J.

(b) The net work is the net heat $(Q_1 + Q_2 + Q_3)$. We find Q_3 from $nC_p\,(T - T') = -20.8 \times 10^3$ J. Thus, $W = 4.73 \times 10^3$ J.

(c) Using Eq. 21-9, we find that the efficiency is

$$\varepsilon = \frac{|W|}{|Q_{\text{H}}|} = \frac{4.73 \times 10^3}{25.5 \times 10^3} = 0.185 \ .$$

57. The change in entropy in transferring a certain amount of heat Q from a heat reservoir at T_1 to another one at T_2 is $\Delta S = \Delta S_1 + \Delta S_2 = Q(1/T_2 - 1/T_1)$.

(a) $\Delta S = (260\,\text{J})(1/100\,\text{K} - 1/400\,\text{K}) = 1.95\,\text{J}$.

(b) $\Delta S = (260\,\text{J})(1/200\,\text{K} - 1/400\,\text{K}) = 0.650\,\text{J}$.

(c) $\Delta S = (260\,\text{J})(1/300\,\text{K} - 1/400\,\text{K}) = 0.217\,\text{J}$.

(d) $\Delta S = (260\,\text{J})(1/360\,\text{K} - 1/400\,\text{K}) = 0.072\,\text{J}$.

(e) We see that as the temperature difference between the two reservoirs decreases, so does the change in entropy.

58. Since the volume of the monatomic ideal gas is kept constant it does not do any work in the heating process. Therefore the heat Q it absorbs is equal to the change in its inertial energy: $dQ = dE_{\rm int} = \frac{3}{2}nR\,dT$. Thus

$$\begin{aligned}\Delta S &= \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{(3nR/2)dT}{T} = \frac{3}{2}nR\ln\left(\frac{T_f}{T_i}\right) \\ &= \frac{3}{2}(1.0\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{400\,\text{K}}{300\,\text{K}}\right) = 3.59\ \text{J/K}\ .\end{aligned}$$

59. Now

$$dQ = nC_p dT = n(C_V + R)dT = \left(\frac{3}{2}nR + nR\right)dT = \frac{5}{2}nR\,dT\ ,$$

so we need to replace the factor 3/2 in the last problem by 5/2. The rest is the same. Thus the answer now is

$$\Delta S = \frac{5}{2}nR\ln\left(\frac{T_f}{T_i}\right) = \frac{5}{2}(1.00\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{400\,\text{K}}{300\,\text{K}}\right) = 5.98\ \text{J/K}\ .$$

60. (a) The ideal gas is diatomic, so $f = 5$ (see Table 20-3). Since this is an isobaric (constant pressure) process, with no change in the number of moles, then the ideal gas in ratio form (see Sample Problem 20-1) leads to

$$\frac{V_f}{V_i} = \frac{T_f}{T_i} = \frac{8}{3}\ .$$

With $C_V = \frac{f}{2}R$, Eq. 21-4 gives

$$\Delta S_{\rm gas} = nR\ln\left(\frac{8}{3}\right) + n\left(\frac{5}{2}R\right)\ln\left(\frac{8}{3}\right)$$

where n is the number of moles (25 mol), not to be confused with the number of reservoirs (also denoted "n" in the later parts of this problem). Consequently, we obtain

$$\Delta S_{\rm gas} = \frac{7}{2}(25\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\ln\left(\frac{8}{3}\right) = 713\ \text{J/K}\ .$$

Since $Q = nC_p\Delta T$ for this process, the entropy change of the reservoir (which transfers energy Q to the gas, so it (the heat) is negative-valued in this context) is (using Eq. 21-2)

$$\Delta S_{\rm res} = \frac{-Q}{T} = -\frac{n\left(\frac{7}{2}R\right)(800\,\text{K} - 300\,\text{K})}{800\,\text{K}} = -454\ \text{J/K}\ .$$

Therefore, $\Delta S_{\rm system} = \Delta S_{\rm gas} + \Delta S_{\rm res} = 259$ J/K.

(b) The change in entropy of the gas is the same regardless of the number of intermediate reservoirs, so long as the beginning state and final state of the gas is unchanged. The difference (relative to part (a)) is that the sum of these *two* reservoirs' entropy changes is not equivalent to that of the one reservoir in the previous part:

$$\begin{aligned}\Delta S_{\rm res1} + \Delta S_{\rm res2} &= \frac{-Q_1}{T_1} + \frac{-Q_2}{T_2} \\ &= -\frac{(25\,\text{mol})\left(\frac{7}{2}R\right)(550\,\text{K} - 300\,\text{K})}{550\,\text{K}} - \frac{(25\,\text{mol})\left(\frac{7}{2}R\right)(800\,\text{K} - 550\,\text{K})}{800\,\text{K}} \\ &= -(25\,\text{mol})\left(\frac{7}{2}R\right)(250\,\text{K})\left(\frac{1}{550\,\text{K}} + \frac{1}{800\,\text{K}}\right)\end{aligned}$$

which yields -558 J/K for the total loss of entropy from the reservoirs. The entire system change in entropy is therefore $713 - 558 = 155$ J/K.

(c) Towards the end of the calculation in part (b), a pattern emerges in the computation of the total entropy loss from the original high-temperature reservoir plus the n intermediate reservoirs:

$$\Delta S_{\text{res total}} = -(25\,\text{mol})\left(\frac{7}{2}R\right)\left(\frac{500\,\text{K}}{n+1}\right)\left(\sum_{\text{reservoirs}}^{n+1}\frac{1}{T}\right)$$

where the temperature of a particular reservoir (the j^{th} reservoir, where $1 \le j \le n+1$) is $T = 300 + \left(\frac{500}{n+1}\right)$ (in Kelvins). For $n = 10$, this leads to $\Delta S_{\text{res total}} = -680$ J/K and therefore $713 - 680 = 33$ J/K for the entire system (including the gas) entropy change.

(d) For $n = 50$, this leads to $\Delta S_{\text{res total}} = -705.82$ J/K and therefore $713.19 - 705.82 = 7.37$ J/K for the entire system (including the gas) entropy change.

(e) For $n = 100$, this leads to $\Delta S_{\text{res total}} = -709.45$ J/K and therefore $713.19 - 709.45 = 3.74$ J/K for the entire system (including the gas) entropy change.

61. (a) It is a reversible set of processes returning the system to its initial state; clearly, $\Delta S_{\text{net}} = 0$.

(b) Process 1 is adiabatic and reversible (as opposed to, say, a free expansion) so that Eq. 21-1 applies with $dQ = 0$ and yields $\Delta S_1 = 0$.

(c) Since the working substance is an ideal gas, then an isothermal process implies $Q = W$, which further implies (regarding Eq. 21-1) $dQ = p\,dV$. Therefore,

$$\int \frac{dQ}{T} = \int \frac{p\,dV}{\left(\frac{pV}{nR}\right)} = nR\int\frac{dV}{V}$$

which leads to $\Delta S_3 = nR\ln\frac{1}{2} = -23.0$ J/K.

(d) By part (a), $\Delta S_1 + \Delta S_2 + \Delta S_3 = 0$. Then, part (b) implies $\Delta S_2 = -\Delta S_3$. Therefore, $\Delta S_2 = 23.0$ J/K.

62. A metric ton is 1000 kg, so that the heat generated by burning 380 metric tons during one hour is

$$(380000\,\text{kg})(28\,\text{MJ/kg}) = 10.6 \times 10^6\ \text{MJ}\ .$$

The work done in one hour is

$$W = (750\,\text{MJ/s})(3600\,\text{s}) = 2.7 \times 10^6\ \text{MJ}$$

where we use the fact that a Watt is a Joule-per-second. By Eq. 21-9, the efficiency is

$$\varepsilon = \frac{2.7 \times 10^6\,\text{MJ}}{10.6 \times 10^6\,\text{MJ}} = 0.253 = 25\%\ .$$

63. We adapt the discussion of §21-7 to 3 and 5 particles (as opposed to the 6 particle situation treated in that section).

(a) The least multiplicity configuration is when all the particles are in the same half of the box. In this case, using Eq. 21-18, we have

$$W = \frac{3!}{3!\,0!} = 1\ .$$

(b) Similarly for box B, $W = 5!/(5!\,0!) = 1$ in the "least" case.

(c) The most likely configuration in the 3 particle case is to have 2 on one side and 1 on the other. Thus,

$$W = \frac{3!}{2!\,1!} = 3\ .$$

(d) The most likely configuration in the 5 particle case is to have 3 on one side and 2 on the other. Thus,

$$W = \frac{5!}{3!\,2!} = 10 \ .$$

(e) We use Eq. 21-19 with our result in part (c) to obtain

$$S = k \ln W = \left(1.38 \times 10^{-23}\right) \ln 3 = 1.5 \times 10^{-23} \text{ J/K} \ .$$

(f) Similarly for the 5 particle case (using the result from part (d)), we find $S = k \ln 10 = 3.2 \times 10^{-23}$ J/K.

64. (a) The most obvious input-heat step is the constant-volume process. Since the gas is monatomic, we know from Chapter 20 that $C_V = \frac{3}{2}R$. Therefore,

$$\begin{aligned} Q_V &= nC_V \Delta T \\ &= (1\,\text{mol})\left(\frac{3}{2}\right)\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600\,\text{K} - 300\,\text{K}) \\ &= 3740 \text{ J} \ . \end{aligned}$$

Since the heat transfer during the isothermal step is positive, we may consider it also to be an input-heat step. The isothermal Q is equal to the isothermal work (calculated in the next part) because $\Delta E_{\text{int}} = 0$ for an ideal gas isothermal process (see Eq. 20-45). Borrowing from the part (b) computation, we have

$$Q_{\text{isotherm}} = nRT_{\text{H}} \ln 2 = (1\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(600\,\text{K}) \ln 2 = 3456 \text{ J} \ .$$

Therefore, $Q_{\text{H}} = Q_V + Q_{\text{isotherm}} = 7.2 \times 10^3$ J.

(b) We consider the sum of works done during the processes (noting that no work is done during the constant-volume step). Using Eq. 20-14 and Eq. 20-16, we have

$$W = nRT_{\text{H}} \ln\left(\frac{V_{\text{max}}}{V_{\text{min}}}\right) + p_{\text{min}}\left(V_{\text{min}} - V_{\text{max}}\right)$$

where (by the gas law in ratio form, as illustrated in Sample Problem 20-1) the volume ratio is

$$\frac{V_{\text{max}}}{V_{\text{min}}} = \frac{T_{\text{H}}}{T_{\text{L}}} = \frac{600\,\text{K}}{300\,\text{K}} = 2 \ .$$

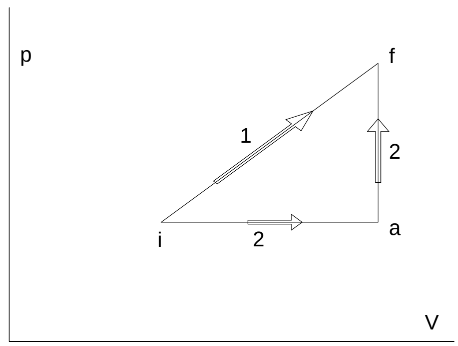
Thus, the net work is

$$\begin{aligned} W &= nRT_{\text{H}} \ln 2 + p_{\text{min}} V_{\text{min}} \left(1 - \frac{V_{\text{max}}}{V_{\text{min}}}\right) \\ &= nRT_{\text{H}} \ln 2 + nRT_{\text{L}}(1-2) \\ &= nR\left(T_{\text{H}} \ln 2 - T_{\text{L}}\right) \\ &= (1\,\text{mol})\left(8.31\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)((600\,\text{K})\ln 2 - (300\,\text{K})) \\ &= 9.6 \times 10^2 \text{ J} \ . \end{aligned}$$

(c) Eq. 21-9 gives

$$\varepsilon = \frac{W}{Q_{\text{H}}} = 0.134 \quad \approx \ 13\% \ .$$

65. First we show that $\int dQ$ is path-dependent. To do this all we need is to show that $\int dQ$ is different for at least two separate paths, say path 1 and 2, as depicted in the figure below. We write $\int dQ = \int pdV + \int nC_V dT$. The second term on the right, $\int nC_V dT$, yields $nC_V \Delta T$ upon integration and is obviously path-independent. The first term, $\int pdV$, however, is different for the two paths. In fact $\int_i^f pdV$ along path 1 is greater than that along path 2, by the area of the shaded triangle enclosed by the two paths. Therefore, $\int dQ$ is indeed path-dependent.



Now we consider $\int TdQ = \int pTdV + \int nC_V TdT$. Once again the second term on the right, $\int nC_V TdT$, yields $\frac{1}{2}nC_V \Delta T^2$ upon integration and is path-independent. The first term, $\int pTdV$, however, yields a higher value along path 1 than path 2. To see that, note that

$$\int_2 pT\,dV = \int_{i\to a} pT\,dV + \int_{a\to f} pT\,dV = \int_{i\to a} pT\,dV \ .$$

Now, if we compare the two integrals, $\int_1 pTdV$ and $\int_{i\to a} pT\,dV$, we realize that the average values of both T and p along path 1 are greater than their respective corresponding values along the $i \to a$ segment of path 2. Hence, the integrand $f(p,T) = pT$ is always greater along path 1. Thus, the two integrals over V, which have the same upper and lower limits, are not equal to each other:

$$\int_1 pT\,dV > \int_{i\to a} pT\,dV = \int_2 pT\,dV \ .$$

We see then that $\int TdQ$ is greater along path 1 than path 2 and is therefore path-dependent. Similarly, one can show that for $\int dQ/T^2 = \int pdV/T^2 + \int nC_V dT/T^2$, the second term on the right is path-independent, while for the first term

$$\int pdV/T^2 = nR\int \frac{dV}{TV} \ ,$$

we have

$$nR\int_2 \frac{dV}{TV} = nR\int_{i\to a} \frac{dV}{TV} > nR\int_1 \frac{dV}{TV} \ ,$$

since the average value of $1/T$ is greater along along the $i \to a$ segment of path 2 than on path 1. Consequently, $\int dQ/T^2$ is less along path 1 than path 2 and is therefore path-dependent.

66. We consider a three-step reversible process as follows: the supercooled water drop (of mass m) starts at state 1 ($T_1 = 268\,\mathrm{K}$), moves on to state 2 (still in liquid form but at $T_2 = 273\,\mathrm{K}$), freezes to state 3 ($T_3 = T_2$), and then cools down to state 4 (in solid form, with $T_4 = T_1$). The change in entropy for each of the stages is given as follows: $\Delta S_{12} = mc_w \ln(T_2/T_1)$, $\Delta S_{23} = -mL_F/T_2$, and $\Delta S_{34} = mc_I \ln(T_4/T_3) = mc_I \ln(T_1/T_2) = -mc_I \ln(T_2/T_1)$. Thus the net entropy change for the water

drop is

$$\begin{aligned}\Delta S &= \Delta S_{12} + \Delta S_{23} + \Delta S_{34} = m(c_w - c_I)\ln\left(\frac{T_2}{T_1}\right) - \frac{mL_F}{T_2} \\ &= (1.00\,\text{g})(4.19\,\text{J/g·K} - 2.22\,\text{J/g·K})\ln\left(\frac{273\,\text{K}}{268\,\text{K}}\right) - \frac{(1.00\,\text{g})(333\,\text{J/g})}{273\,\text{K}} \\ &= -1.18\,\text{J/K} .\end{aligned}$$

67. Eq. 21-8 gives

$$\left|\frac{Q_{\rm H}}{Q_{\rm L}}\right| = \frac{T_{\rm H}}{T_{\rm L}} = \frac{300\,\text{K}}{4.0\,\text{K}} = 75 .$$

68. (a) Eq. 21-13 provides

$$K_C = \frac{|Q_{\rm L}|}{|Q_{\rm H}| - |Q_{\rm L}|} \implies |Q_{\rm H}| = |Q_{\rm L}|\left(\frac{1+K_C}{K_C}\right)$$

which yields $|Q_{\rm H}| = 49$ kJ when $K_C = 5.7$ and $|Q_{\rm L}| = 42$ kJ.

(b) From §21-5 we obtain

$$|W| = |Q_{\rm H}| - |Q_{\rm L}| = 49.4\,\text{kJ} - 42.0\,\text{kJ} = 7.4\ \text{kJ}$$

if we take the initial 42 kJ datum to be accurate to three figures. The given temperatures are not used in the calculation; in fact, it is possible that the given room temperature value is not meant to be the high temperature for the (reversed) Carnot cycle – since it does not lead to the given K_C using Eq. 21-14.

69. (a) Combining Eq. 21-9 with Eq. 21-11, we obtain

$$|W| = |Q_{\rm H}|\left(1 - \frac{T_{\rm L}}{T_{\rm H}}\right) = (500\,\text{J})\left(1 - \frac{260\,\text{K}}{320\,\text{K}}\right) = 94\ \text{J} .$$

(b) Combining Eq. 21-12 with Eq. 21-14, we find

$$|W| = \frac{|Q_{\rm L}|}{\left(\frac{T_{\rm L}}{T_{\rm H}-T_{\rm L}}\right)} = \frac{1000\,\text{J}}{\left(\frac{260\,\text{K}}{320\,\text{K}-260\,\text{K}}\right)} = 231\ \text{J} .$$

Chapter 22

1. Eq. 22-1 gives Coulomb's Law, $F = k\,\frac{|q_1||q_2|}{r^2}$, which we solve for the distance:

$$\begin{aligned} r &= \sqrt{\frac{k|q_1||q_2|}{F}} \\ &= \sqrt{\frac{(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})\,(26.0\times 10^{-6}\,\mathrm{C})\,(47.0\times 10^{-6}\,\mathrm{C})}{5.70\,\mathrm{N}}} = 1.39\ \mathrm{m}\ . \end{aligned}$$

2. The magnitude of the mutual force of attraction at $r = 0.120$ m is

$$F = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times 10^9\right)\frac{\left(3.00\times 10^{-6}\right)\left(1.50\times 10^{-6}\right)}{0.120^2} = 2.81\ \mathrm{N}\ .$$

3. (a) With a understood to mean the magnitude of acceleration, Newton's second and third laws lead to

$$m_2a_2 = m_1a_1 \implies m_2 = \frac{\left(6.3\times 10^{-7}\,\mathrm{kg}\right)\left(7.0\,\mathrm{m/s^2}\right)}{9.0\,\mathrm{m/s^2}} = 4.9\ \times 10^{-7}\ \mathrm{kg}\ .$$

(b) The magnitude of the (only) force on particle 1 is

$$F = m_1a_1 = k\frac{|q_1||q_2|}{r^2} = \left(8.99\times 10^9\right)\frac{|q|^2}{0.0032^2}\ .$$

Inserting the values for $m_1$and a_1 (see part (a)) we obtain $|q| = 7.1\times 10^{-11}$ C.

4. The fact that the spheres are identical allows us to conclude that when two spheres are in contact, they share equal charge. Therefore, when a charged sphere (q) touches an uncharged one, they will (fairly quickly) each attain half that charge ($q/2$). We start with spheres 1 and 2 each having charge q and experiencing a mutual repulsive force $F = kq^2/r^2$. When the neutral sphere 3 touches sphere 1, sphere 1's charge decreases to $q/2$. Then sphere 3 (now carrying charge $q/2$) is brought into contact with sphere 2, a total amount of $q/2 + q$ becomes shared equally between them. Therefore, the charge of sphere 3 is $3q/4$ in the final situation. The repulsive force between spheres 1 and 2 is finally

$$F' = k\frac{\left(\frac{q}{2}\right)\left(\frac{3q}{4}\right)}{r^2} = \frac{3}{8}\,k\frac{q^2}{r^2} = \frac{3}{8}F\ .$$

5. We put the origin of a coordinate system at the lower left corner of the square and take $+x$ rightward and $+y$ upward. The force exerted by the charge $+q$ on the charge $+2q$ is

$$\vec{F}_1 = k\frac{q(2q)}{a^2}\,(-\hat{\mathrm{j}})\ .$$

The force exerted by the charge $-q$ on the $+2q$ charge is directed along the diagonal of the square and has magnitude

$$F_2 = k\frac{q(2q)}{(a\sqrt{2})^2}$$

which becomes, upon finding its components (and using the fact that $\cos 45° = 1/\sqrt{2}$),

$$\vec{F}_2 = k\frac{q(2q)}{2\sqrt{2}\,a^2}\ \hat{\imath} + k\frac{q(2q)}{2\sqrt{2}\,a^2}\ \hat{\jmath} \ .$$

Finally, the force exerted by the charge $-2q$ on $+2q$ is

$$\vec{F}_3 = k\frac{(2q)(2q)}{a^2}\ \hat{\imath} \ .$$

(a) Therefore, the horizontal component of the resultant force on $+2q$ is

$$\begin{aligned} F_x &= F_{1x} + F_{2x} + F_{3x} = k\frac{q^2}{a^2}\left(\frac{1}{\sqrt{2}} + 4\right) \\ &= \left(8.99 \times 10^9\right)\frac{\left(1.0 \times 10^{-7}\right)^2}{0.050^2}\left(\frac{1}{\sqrt{2}} + 4\right) = 0.17\ \text{N} \ . \end{aligned}$$

(b) The vertical component of the net force is

$$F_y = F_{1y} + F_{2y} + F_{3y} = k\frac{q^2}{a^2}\left(-2 + \frac{1}{\sqrt{2}}\right) = -0.046\ \text{N} \ .$$

6. (a) The individual force magnitudes (acting on Q) are, by Eq. 22-1,

$$k\,\frac{|q_1|\,Q}{\left(-a - \frac{a}{2}\right)^2} = k\,\frac{|q_2|\,Q}{\left(a - \frac{a}{2}\right)^2}$$

which leads to $|q_1| = 9\,|q_2|$. Since Q is located between q_1 and q_2, we conclude q_1 and q_2 are like-sign. Consequently, $q_1 = 9q_2$.

(b) Now we have

$$k\,\frac{|q_1|\,Q}{\left(-a - \frac{3a}{2}\right)^2} = k\,\frac{|q_2|\,Q}{\left(a - \frac{3a}{2}\right)^2}$$

which yields $|q_1| = 25\,|q_2|$. Now, Q is not located between q_1 and q_2, one of them must push and the other must pull. Thus, they are unlike-sign, so $q_1 = -25q_2$.

7. We assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges. We choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Then, the force on q_2 is

$$F_a = -\frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r^2} = -k\,\frac{q_1 q_2}{r^2}$$

where $r = 0.500$ m. The negative sign indicates that the spheres attract each other. After the wire is connected, the spheres, being identical, acquire the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\varepsilon_0}\,\frac{\left(\frac{q_1+q_2}{2}\right)\left(\frac{q_1+q_2}{2}\right)}{r^2} = k\,\frac{(q_1 + q_2)^2}{4r^2} \ .$$

We solve the two force equations simultaneously for q_1 and q_2. The first gives the product

$$q_1 q_2 = -\frac{r^2 F_a}{k} = -\frac{(0.500\,\text{m})^2(0.108\,\text{N})}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -3.00 \times 10^{-12}\ \text{C}^2 \ ,$$

and the second gives the sum

$$q_1 + q_2 = 2r\sqrt{\frac{F_b}{k}} = 2(0.500\,\mathrm{m})\sqrt{\frac{0.0360\,\mathrm{N}}{8.99\times 10^9\,\mathrm{N\cdot m^2/C^2}}} = 2.00\times 10^{-6}\ \mathrm{C}$$

where we have taken the positive root (which amounts to assuming $q_1 + q_2 \geq 0$). Thus, the product result provides the relation

$$q_2 = \frac{-(3.00\times 10^{-12}\,\mathrm{C^2})}{q_1}$$

which we substitute into the sum result, producing

$$q_1 - \frac{3.00\times 10^{-12}\,\mathrm{C^2}}{q_1} = 2.00\times 10^{-6}\ \mathrm{C}\ .$$

Multiplying by q_1 and rearranging, we obtain a quadratic equation

$$q_1^2 - (2.00\times 10^{-6}\,\mathrm{C})q_1 - 3.00\times 10^{-12}\,\mathrm{C^2} = 0\ .$$

The solutions are

$$q_1 = \frac{2.00\times 10^{-6}\,\mathrm{C} \pm \sqrt{(-2.00\times 10^{-6}\,\mathrm{C})^2 - 4(-3.00\times 10^{-12}\,\mathrm{C^2})}}{2}\ .$$

If the positive sign is used, $q_1 = 3.00\times 10^{-6}\,\mathrm{C}$, and if the negative sign is used, $q_1 = -1.00\times 10^{-6}\,\mathrm{C}$. Using $q_2 = (-3.00\times 10^{-12})/q_1$ with $q_1 = 3.00\times 10^{-6}\,\mathrm{C}$, we get $q_2 = -1.00\times 10^{-6}\,\mathrm{C}$. If we instead work with the $q_1 = -1.00\times 10^{-6}\,\mathrm{C}$ root, then we find $q_2 = 3.00\times 10^{-6}\,\mathrm{C}$. Since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00\times 10^{-6}\,\mathrm{C}$ and the other had charge $+3.00\times 10^{-6}\,\mathrm{C}$. What if we had not made the assumption, above, that $q_1 + q_2 \geq 0$? If the signs of the charges were reversed (so $q_1 + q_2 < 0$), then the forces remain the same, so a charge of $+1.00\times 10^{-6}\,\mathrm{C}$ on one sphere and a charge of $-3.00\times 10^{-6}\,\mathrm{C}$ on the other also satisfies the conditions of the problem.

8. With rightwards positive, the net force on q_3 is

$$k\frac{q_1 q_3}{(2d)^2} + k\frac{q_2 q_3}{d^2}\ .$$

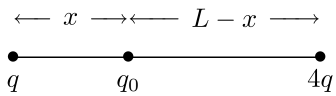
We note that each term exhibits the proper sign (positive for rightward, negative for leftward) for all possible signs of the charges. For example, the first term (the force exerted on q_3 by q_1) is negative if they are unlike charges, indicating that q_3 is being pulled toward q_1, and it is positive if they are like charges (so q_3 would be repelled from q_1). Setting the net force equal to zero and canceling k, q_3 and d^2 leads to

$$\frac{q_1}{4} + q_2 = 0 \implies q_1 = -4q_2\ .$$

9. (a) If the system of three charges is to be in equilibrium, the force on each charge must be zero. Let the third charge be q_0. It must lie between the other two or else the forces acting on it due to the other charges would be in the same direction and q_0 could not be in equilibrium. Suppose q_0 is a distance x from q, as shown on the diagram below. The force acting on q_0 is then given by

$$F_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{qq_0}{x^2} - \frac{4qq_0}{(L-x)^2}\right)$$

where the positive direction is rightward. We require $F_0 = 0$ and solve for x. Canceling common factors yields $1/x^2 = 4/(L-x)^2$ and taking the square root yields $1/x = 2/(L-x)$. The solution is $x = L/3$.



The force on q is

$$F_q = \frac{-1}{4\pi\varepsilon_0}\left(\frac{qq_0}{x^2} + \frac{4q^2}{L^2}\right) .$$

The signs are chosen so that a negative force value would cause q to move leftward. We require $F_q = 0$ and solve for q_0:

$$q_0 = -\frac{4qx^2}{L^2} = -\frac{4}{9}\,q$$

where $x = L/3$ is used. We now examine the force on $4q$:

$$\begin{aligned} F_{4q} &= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4qq_0}{(L-x)^2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} + \frac{4(-4/9)q^2}{(4/9)L^2}\right) \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{4q^2}{L^2} - \frac{4q^2}{L^2}\right) \end{aligned}$$

which we see is zero. Thus, with $q_0 = -(4/9)q$ and $x = L/3$, all three charges are in equilibrium.

(b) If q_0 moves toward q the force of attraction exerted by q is greater in magnitude than the force of attraction exerted by $4q$. This causes q_0 to continue to move toward q and away from its initial position. The equilibrium is unstable.

10. There is no equilibrium position for q_3 *between* the two fixed charges, because it is being pulled by one and pushed by the other (since q_1 and q_2 have different signs); in this region this means the two force arrows on q_3 are in the same direction and cannot cancel. It should also be clear that off-axis (with the axis defined as that which passes through the two fixed charges) there are no equilibrium positions. On the semi-infinite region of the axis which is nearest q_2 and furthest from q_1 an equilibrium position for q_3 cannot be found because $|q_1| < |q_2|$ and the magnitude of force exerted by q_2 is everywhere (in that region) stronger than that exerted by q_1 on q_3. Thus, we must look in the semi-infinite region of the axis which is nearest q_1 and furthest from q_2, where the net force on q_3 has magnitude

$$\left| k\frac{|q_1q_3|}{x^2} - k\frac{|q_2q_3|}{(d+x)^2} \right|$$

with $d = 10$ cm and x assumed positive. We set this equal to zero, as required by the problem, and cancel k and q_3. Thus, we obtain

$$\frac{|q_1|}{x^2} - \frac{|q_2|}{(d+x)^2} = 0 \implies \left(\frac{d+x}{x}\right)^2 = \left|\frac{q_2}{q_1}\right| = 3$$

which yields (after taking the square root)

$$\frac{d+x}{x} = \sqrt{3} \implies x = \frac{d}{\sqrt{3}-1} \approx 14 \text{ cm}$$

for the distance between q_3 and q_1, so $x + d$ (the distance between q_2 and q_3) is approximately 24 cm.

11. (a) The magnitudes of the gravitational and electrical forces must be the same:

$$\frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{r^2} = G\,\frac{mM}{r^2}$$

where q is the charge on either body, r is the center-to-center separation of Earth and Moon, G is the universal gravitational constant, M is the mass of Earth, and m is the mass of the Moon. We solve for q:

$$q = \sqrt{4\pi\varepsilon_0 GmM} .$$

According to Appendix C of the text, $M = 5.98 \times 10^{24}$ kg, and $m = 7.36 \times 10^{22}$ kg, so (using $4\pi\varepsilon_0 = 1/k$) the charge is

$$q = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(7.36 \times 10^{22}\,\mathrm{kg})(5.98 \times 10^{24}\,\mathrm{kg})}{8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}}} = 5.7 \times 10^{13}\ \mathrm{C} .$$

We note that the distance r cancels because both the electric and gravitational forces are proportional to $1/r^2$.

(b) The charge on a hydrogen ion is $e = 1.60 \times 10^{-19}$ C, so there must be

$$\frac{q}{e} = \frac{5.7 \times 10^{13}\,\mathrm{C}}{1.6 \times 10^{-19}\,\mathrm{C}} = 3.6 \times 10^{32}\,\text{ions} .$$

Each ion has a mass of 1.67×10^{-27} kg, so the total mass needed is

$$(3.6 \times 10^{32})(1.67 \times 10^{-27}\,\mathrm{kg}) = 6.0 \times 10^5\ \mathrm{kg} .$$

12. (a) The distance between q_1 and q_2 is

$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-0.020 - 0.035)^2 + (0.015 - 0.005)^2} = 0.0559\ \mathrm{m} .$$

The magnitude of the force exerted by q_1 on q_2 is

$$F_{21} = k\frac{|q_1 q_2|}{r_{12}^2} = \frac{(8.99 \times 10^9)\left(3.0 \times 10^{-6}\right)\left(4.0 \times 10^{-6}\right)}{0.0559^2} = 34.5\ \mathrm{N} .$$

The vector $\vec{F}_{21}$ is directed towards q_1 and makes an angle θ with the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \tan^{-1}\left(\frac{1.5 - 0.5}{-2.0 - 3.5}\right) = -10.3^\circ .$$

(b) Let the third charge be located at (x_3, y_3), a distance r from q_2. We note that q_1, q_2 and q_3 must be colinear; otherwise, an equilibrium position for any one of them would be impossible to find. Furthermore, we cannot place q_3 on the same side of q_2 where we also find q_1 , since in that region both forces (exerted on q_2 by q_3 and q_1) would be in the same direction (since q_2 is attracted to both of them). Thus, in terms of the angle found in part (a), we have $x_3 = x_2 - r\cos\theta$ and $y_3 = y_2 - r\sin\theta$ (which means $y_3 > y_2$ since θ is negative). The magnitude of force exerted on q_2 by q_3 is $F_{23} = k|q_2 q_3|/r^2$, which must equal that of the force exerted on it by q_1 (found in part (a)). Therefore,

$$k\frac{|q_2 q_3|}{r^2} = k\frac{|q_1 q_2|}{r_{12}^2} \implies r = r_{12}\sqrt{\frac{q_3}{q_1}} = 0.0645\ \mathrm{cm} .$$

Consequently, $x_3 = x_2 - r\cos\theta = -2.0\,\mathrm{cm} - (6.45\,\mathrm{cm})\cos(-10.3^\circ) = -8.4\,\mathrm{cm}$ and $y_3 = y_2 - r\sin\theta = 1.5\,\mathrm{cm} - (6.45\,\mathrm{cm})\sin(-10.3^\circ) = 2.7\,\mathrm{cm}$.

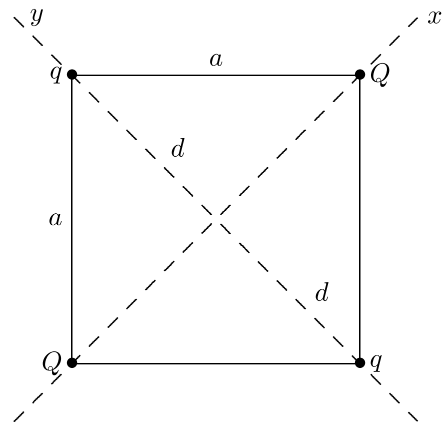
13. The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q(Q - q)}{r^2}$$

where r is the distance between the charges. We want the value of q that maximizes the function $f(q) = q(Q - q)$. Setting the derivative df/dq equal to zero leads to $Q - 2q = 0$, or $q = Q/2$.

14. (a) We choose the coordinate axes as shown in the diagram below. For ease of presentation (of the computations below) we assume $Q > 0$ and $q < 0$ (although the final result does not depend on this particular choice). The repulsive force between the diagonally opposite Q's is along our (tilted)

x axis. The attractive force between each pair of Q and q is along the sides (of length a). In our drawing, the distance between the center to the corner is d, where $d = a/\sqrt{2}$, and the diagonal itself is therefore of length $2d = a\sqrt{2}$.



Since the angle between each attractive force and the x axis is 45° (note: $\cos 45° = 1/\sqrt{2}$), then the net force on Q is

$$\begin{aligned} F_x &= \frac{1}{4\pi\varepsilon_0}\left(\frac{(Q)(Q)}{(2d)^2} - 2\frac{(|q|)\,(Q)}{a^2}\cos 45°\right) \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^2}{2a^2} - 2\frac{|q|\cdot Q}{a^2}\frac{1}{\sqrt{2}}\right) \end{aligned}$$

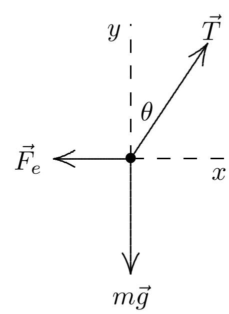
which (upon requiring $F_x = 0$) leads to $|q| = Q/2\sqrt{2}$ or $q = -\frac{Q}{2\sqrt{2}}$.

(b) The net force on q, examined along the y axis is

$$\begin{aligned} F_y &= \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{(2d)^2} - 2\frac{(|q|)\,(Q)}{a^2}\sin 45°\right) \\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{2a^2} - 2\frac{|q|\cdot Q}{a^2}\frac{1}{\sqrt{2}}\right) \end{aligned}$$

which (if we demand $F_y = 0$) leads to $q = -2Q\sqrt{2}$ which is inconsistent with the result of part (a). Thus, we are unable to construct an equilibrium configuration with this geometry, where the only forces acting are given by Eq. 22-1.

15. (a) A force diagram for one of the balls is shown below. The force of gravity $m\vec{g}$ acts downward, the electrical force $\vec{F}_e$ of the other ball acts to the left, and the tension in the thread acts along the thread, at the angle θ to the vertical. The ball is in equilibrium, so its acceleration is zero. The y component of Newton's second law yields $T\cos\theta - mg = 0$ and the x component yields $T\sin\theta - F_e = 0$. We solve the first equation for T and obtain $T = mg/\cos\theta$. We substitute the result into the second to obtain $mg\tan\theta - F_e = 0$.



Examination of the geometry of Figure 22-19 leads to

$$\tan\theta = \frac{x/2}{\sqrt{L^2 - (x/2)^2}} .$$

If L is much larger than x (which is the case if θ is very small), we may neglect $x/2$ in the denominator and write $\tan\theta \approx x/2L$. This is equivalent to approximating $\tan\theta$ by $\sin\theta$. The magnitude of the electrical force of one ball on the other is

$$F_e = \frac{q^2}{4\pi\varepsilon_0 x^2}$$

by Eq. 22-4. When these two expressions are used in the equation $mg\tan\theta = F_e$, we obtain

$$\frac{mgx}{2L} \approx \frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} \implies x \approx \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3} .$$

(b) We solve $x^3 = 2kq^2L/mg)$ for the charge (using Eq. 22-5):

$$q = \sqrt{\frac{mgx^3}{2kL}} = \sqrt{\frac{(0.010\,\mathrm{kg})(9.8\,\mathrm{m/s^2})(0.050\,\mathrm{m})^3}{2(8.99\times10^9\,\mathrm{N{\cdot}m^2/C^2})(1.20\,\mathrm{m})}} = \pm 2.4\times10^{-8}\ \mathrm{C} .$$

16. If one of them is discharged, there would no electrostatic repulsion between the two balls and they would both come to the position $\theta = 0$, making contact with each other. A redistribution of the remaining charge would then occur, with each of the balls getting $q/2$. Then they would again be separated due to electrostatic repulsion, which results in the new equilibrium separation

$$x' = \left[\frac{(q/2)^2 L}{2\pi\varepsilon_0 mg}\right]^{1/3} = \left(\frac{1}{4}\right)^{1/3} x = \left(\frac{1}{4}\right)^{1/3} (5.0\,\mathrm{cm}) = 3.1\ \mathrm{cm} .$$

17. (a) Since the rod is in equilibrium, the net force acting on it is zero, and the net torque about any point is also zero. We write an expression for the net torque about the bearing, equate it to zero, and solve for x. The charge Q on the left exerts an upward force of magnitude $(1/4\pi\varepsilon_0)(qQ/h^2)$, at a distance $L/2$ from the bearing. We take the torque to be negative. The attached weight exerts a downward force of magnitude W, at a distance $x - L/2$ from the bearing. This torque is also negative. The charge Q on the right exerts an upward force of magnitude $(1/4\pi\varepsilon_0)(2qQ/h^2)$, at a distance $L/2$ from the bearing. This torque is positive. The equation for rotational equilibrium is

$$\frac{-1}{4\pi\varepsilon_0}\frac{qQ}{h^2}\frac{L}{2} - W\left(x - \frac{L}{2}\right) + \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2}\frac{L}{2} = 0 .$$

The solution for x is

$$x = \frac{L}{2}\left(1 + \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2W}\right) .$$

(b) If N is the magnitude of the upward force exerted by the bearing, then Newton's second law (with zero acceleration) gives

$$W - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{h^2} - \frac{1}{4\pi\varepsilon_0}\frac{2qQ}{h^2} - N = 0 .$$

We solve for h so that $N = 0$. The result is

$$h = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{3qQ}{W}} .$$

18. The magnitude of the force is

$$F = k\frac{e^2}{r^2} = \left(8.99 \times 10^9 \, \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) \frac{(1.60 \times 10^{-19}\,\text{C})^2}{(2.82 \times 10^{-10}\,\text{m})^2} = 2.89 \times 10^{-9} \text{ N} .$$

19. The mass of an electron is $m = 9.11 \times 10^{-31}$ kg, so the number of electrons in a collection with total mass $M = 75.0$ kg is

$$N = \frac{M}{m} = \frac{75.0\,\text{kg}}{9.11 \times 10^{-31}\,\text{kg}} = 8.23 \times 10^{31} \text{ electrons} .$$

The total charge of the collection is

$$q = -Ne = -(8.23 \times 10^{31})(1.60 \times 10^{-19}\,\text{C}) = -1.32 \times 10^{13} \text{ C} .$$

20. There are two protons (each with charge $q = +e$) in each molecule, so

$$Q = N_A q = (6.02 \times 10^{23})(2)(1.60 \times 10^{-19}\,\text{C}) = 1.9 \times 10^5\,\text{C} = 0.19 \text{ MC} .$$

21. (a) The magnitude of the force between the (positive) ions is given by

$$F = \frac{(q)(q)}{4\pi\varepsilon_0 r^2} = k\,\frac{q^2}{r^2}$$

where q is the charge on either of them and r is the distance between them. We solve for the charge:

$$q = r\sqrt{\frac{F}{k}} = (5.0 \times 10^{-10}\,\text{m})\sqrt{\frac{3.7 \times 10^{-9}\,\text{N}}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = 3.2 \times 10^{-19} \text{ C} .$$

(b) Let N be the number of electrons missing from each ion. Then, $Ne = q$, or

$$N = \frac{q}{e} = \frac{3.2 \times 10^{-19}\,\text{C}}{1.6 \times 10^{-19}\,\text{C}} = 2 .$$

22. (a) Eq. 22-1 gives

$$F = \frac{\left(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.00 \times 10^{-16}\,\text{C}\right)^2}{\left(1.00 \times 10^{-2}\,\text{m}\right)^2} = 8.99 \times 10^{-19} \text{ N} .$$

(b) If n is the number of excess electrons (of charge $-e$ each) on each drop then

$$n = -\frac{q}{e} = -\frac{-1.00 \times 10^{-16}\,\text{C}}{1.60 \times 10^{-19}\,\text{C}} = 625 .$$

23. Eq. 22-11 (in absolute value) gives

$$n = \frac{|q|}{e} = \frac{1.0 \times 10^{-7}\,\text{C}}{1.6 \times 10^{-19}\,\text{C}} = 6.3 \times 10^{11} .$$

24. With $F = m_e g$, Eq. 22-1 leads to

$$r^2 = \frac{ke^2}{m_e g} = \frac{\left(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\,\text{C}\right)^2}{(9.11 \times 10^{-31}\,\text{kg})\,(9.8\,\text{m/s}^2)}$$

which leads to $r = 5.1$ m. The second electron should be below the first one, so that the repulsive force (acting on the first) is in the direction opposite to the pull of Earth's gravity.

25. The unit Ampere is discussed in §22-4. The proton flux is given as 1500 protons per square meter per second, where each proton provides a charge of $q = +e$. The current through the spherical area $4\pi R^2 = 4\pi(6.37 \times 10^6\,\text{m})^2 = 5.1 \times 10^{14}\,\text{m}^2$ would be

$$i = \left(5.1 \times 10^{14}\,\text{m}^2\right)\left(1500\,\frac{\text{protons}}{\text{s}\cdot\text{m}^2}\right)\left(1.6 \times 10^{-19}\,\text{C/proton}\right) = 0.122 \text{ A} .$$

26. The volume of $250\,\text{cm}^3$ corresponds to a mass of $250\,\text{g}$ since the density of water is $1.0\,\text{g/cm}^3$. This mass corresponds to $250/18 = 14$ moles since the molar mass of water is 18. There are ten protons (each with charge $q = +e$) in each molecule of H_2O, so

$$Q = 14N_A q = 14(6.02 \times 10^{23})(10)(1.60 \times 10^{-19}\,\text{C}) = 1.3 \times 10^7\,\text{C} = 13 \text{ MC} .$$

27. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

    (b) Rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

    The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d = (\sqrt{3}/2)a$. The force has magnitude

$$F = k\,\frac{e^2}{d^2} = \frac{ke^2}{(3/4)a^2} = \frac{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\,\text{C})^2}{(3/4)(0.40 \times 10^{-9}\,\text{m})^2} = 1.9 \times 10^{-9} \text{ N} .$$

    Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

28. If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be

$$q_p - |q_e| = 1.6 \times 10^{-25} \text{ C} .$$

Amplified by a factor of $29 \times 3 \times 10^{22}$ as indicated in the problem, this amounts to a deviation from perfect neutrality of

$$\Delta q = \left(29 \times 3 \times 10^{22}\right)\left(1.6 \times 10^{-25}\,\text{C}\right) = 0.14 \text{ C}$$

in a copper penny. Two such pennies, at $r = 1.0$ m, would therefore experience a very large force. Eq. 22-1 gives

$$F = k\frac{(\Delta q)^2}{r^2} = 1.7 \times 10^8 \text{ N} .$$

29. None of the reactions given include a beta decay, so the number of protons, the number of neutrons, and the number of electrons are each conserved. Atomic numbers (numbers of protons and numbers of electrons) and molar masses (combined numbers of protons and neutrons) can be found in Appendix F of the text.

(a) ^{1}H has 1 proton, 1 electron, and 0 neutrons and ^{9}Be has 4 protons, 4 electrons, and $9-4=5$ neutrons, so X has $1+4=5$ protons, $1+4=5$ electrons, and $0+5-1=4$ neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of $5+4=9$ g/mol: ^{9}B.

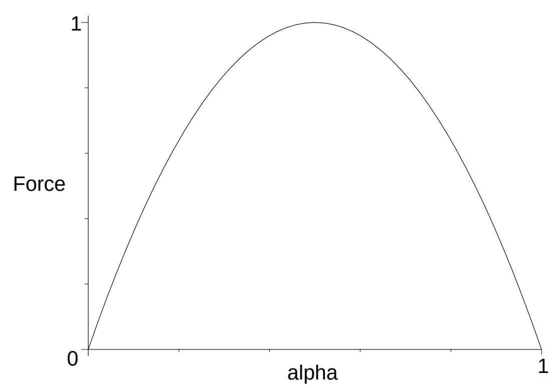
(b) ^{12}C has 6 protons, 6 electrons, and $12-6=6$ neutrons and ^{1}H has 1 proton, 1 electron, and 0 neutrons, so X has $6+1=7$ protons, $6+1=7$ electrons, and $6+0=6$ neutrons. It must be nitrogen with a molar mass of $7+6=13$ g/mol: ^{13}N.

(c) ^{15}N has 7 protons, 7 electrons, and $15-7=8$ neutrons; ^{1}H has 1 proton, 1 electron, and 0 neutrons; and ^{4}He has 2 protons, 2 electrons, and $4-2=2$ neutrons; so X has $7+1-2=6$ protons, 6 electrons, and $8+0-2=6$ neutrons. It must be carbon with a molar mass of $6+6=12$: ^{12}C.

30. (a) The two charges are $q=\alpha Q$ (where α is a pure number presumably less than 1 and greater than zero) and $Q-q=(1-\alpha)Q$. Thus, Eq. 22-4 gives

$$F=\frac{1}{4\pi\varepsilon_0}\frac{(\alpha Q)((1-\alpha)Q)}{d^2}=\frac{Q^2\alpha(1-\alpha)}{4\pi\varepsilon_0\, d^2} .$$

(b) The graph below, of F versus α, has been scaled so that the maximum is 1. In actuality, the maximum value of the force is $F_{\max}=Q^2/16\pi\varepsilon_0\, d^2$.



(c) It is clear that $\alpha=\frac{1}{2}$ gives the maximum value of F.

(d) Seeking the half-height points on the graph is difficult without grid lines or some of the special tracing features found in a variety of modern calculators. It is not difficult to algebraically solve for the half-height points (this involves the use of the quadratic formula). The results are

$$\alpha_1 = \frac{1}{2}\left(1-\frac{1}{\sqrt{2}}\right) \approx 0.15 \quad \text{and}$$
$$\alpha_2 = \frac{1}{2}\left(1+\frac{1}{\sqrt{2}}\right) \approx 0.85 .$$

31. (a) Eq. 22-11 (in absolute value) gives

$$n=\frac{|q|}{e}=\frac{2.00\times10^{-6}\,\mathrm{C}}{1.60\times10^{-19}\,\mathrm{C}}=1.25\times10^{13}\text{ electrons} .$$

(b) Since you have the excess electrons (and electrons are lighter and more mobile than protons) then the electrons "leap" from you to the faucet instead of protons moving from the faucet to you (in the process of neutralizing your body).

(c) Unlike charges attract, and the faucet (which is grounded and is able to gain or lose any number of electrons due to its contact with Earth's large reservoir of mobile charges) becomes positively charged, especially in the region closest to your (negatively charged) hand, just before the spark.

(d) The cat is positively charged (before the spark), and by the reasoning given in part (b) the flow of charge (electrons) is from the faucet to the cat.

(e) If we think of the nose as a conducting sphere, then the side of the sphere closest to the fur is of one sign (of charge) and the side furthest from the fur is of the opposite sign (which, additionally, is oppositely charged from your bare hand which had stroked the cat's fur). The charges in your hand and those of the furthest side of the "sphere" therefore attract each other, and when close enough, manage to neutralize (due to the "jump" made by the electrons) in a painful spark.

32. (a) Using Coulomb's law, we obtain

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.00\,\text{C})^2}{(1.00\,\text{m})^2} = 8.99 \times 10^9\ \text{N}\ .$$

(b) If $r = 1000$ m, then

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{kq^2}{r^2} = \frac{\left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.00\,\text{C})^2}{(1.00 \times 10^3\,\text{m})^2} = 8.99 \times 10^3\ \text{N}\ .$$

33. The unit Ampere is discussed in §22-4. Using i for current, the charge transferred is

$$q = it = \left(2.5 \times 10^4\ \text{A}\right)\left(20 \times 10^{-6}\,\text{s}\right) = 0.50\ \text{C}\ .$$

34. Let the two charges be q_1 and q_2. Then $q_1 + q_2 = Q = 5.0 \times 10^{-5}$ C. We use Eq. 22-1:

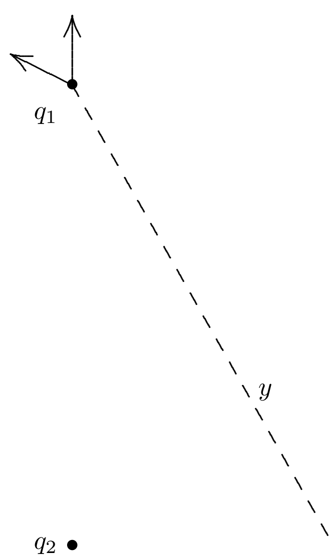
$$1.0\,\text{N} = \frac{\left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right) q_1 q_2}{(2.0\,\text{m})^2}\ .$$

We substitute $q_2 = Q - q_1$ and solve for q_1 using the quadratic formula. The two roots obtained are the values of q_1 and q_2, since it does not matter which is which. We get 1.2×10^{-5} C and 3.8×10^{-5} C.

35. (a) Eq. 22-1 gives

$$F_{12} = k\frac{q_1 q_2}{d^2} = \left(8.99 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(20.0 \times 10^{-6}\ \text{C}\right)^2}{(1.50\,\text{m})^2} = 1.60\ \text{N}\ .$$

(b) A force diagram is shown as well as our choice of y axis (the dashed line).



The y axis is meant to bisect the line between q_2 and q_3 in order to make use of the symmetry in the problem (equilateral triangle of side length d, equal-magnitude charges $q_1 = q_2 = q_3 = q$). We see that the resultant force is along this symmetry axis, and we obtain

$$|F_y| = 2\left(k\frac{q^2}{d^2}\right)\cos 30^\circ = 2.77 \text{ N} .$$

36. (a) Since $q_A = -2Q$ and $q_C = +8Q$, Eq. 22-4 leads to

$$\left|\vec{F}_{AC}\right| = \frac{|(-2Q)(+8Q)|}{4\pi\epsilon_0\, d^2} = \frac{4Q^2}{\pi\epsilon_0\, d^2} .$$

(b) After making contact with each other, both A and B have a charge of

$$\left(\frac{-2Q + (-4Q)}{2}\right) = -3Q .$$

When B is grounded its charge is zero. After making contact with C, which has a charge of $+8Q$, B acquires a charge of $[0 + (-8Q)]/2 = -4Q$, which charge C has as well. Finally, we have $Q_A = -3Q$ and $Q_B = Q_C = -4Q$. Therefore,

$$\left|\vec{F}_{AC}\right| = \frac{|(-3Q)(-4Q)|}{4\pi\epsilon_0\, d^2} = \frac{3Q^2}{\pi\epsilon_0\, d^2} .$$

(c) We also obtain

$$\left|\vec{F}_{BC}\right| = \frac{|(-4Q)(-4Q)|}{4\pi\epsilon_0\, d^2} = \frac{4Q^2}{\pi\epsilon_0\, d^2} .$$

37. The net charge carried by John whose mass is m is roughly

$$\begin{aligned} q &= (0.0001)\frac{mN_AZe}{M} \\ &= (0.0001)\frac{(90\,\text{kg})(6.02\times 10^{23}\,\text{molecules/mol})(18\,\text{electron proton pairs/molecule})\left(1.6\times 10^{-19}\,\text{C}\right)}{0.018\,\text{kg/mol}} \\ &= 8.7\times 10^5 \text{ C} , \end{aligned}$$

and the net charge carried by Mary is half of that. So the electrostatic force between them is estimated to be

$$F \approx k\frac{q(q/2)}{d^2} = \left(8.99\times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(8.7\times 10^5\,\text{C})^2}{2(30\,\text{m})^2} \approx 4\times 10^{18} \text{ N} .$$

38. Letting $kq^2/r^2 = mg$, we get

$$r = q\sqrt{\frac{k}{mg}} = \left(1.60\times 10^{-19}\,\text{C}\right)\sqrt{\frac{8.99\times 10^9\,\frac{\text{Nm}^2}{\text{C}^2}}{(1.67\times 10^{-27}\,\text{kg})\,(9.8\,\text{m/s}^2)}} = 0.119 \text{ m} .$$

39. Coulomb's law gives

$$F = \frac{|q|\cdot|q|}{4\pi\varepsilon_0 r^2} = \frac{k(e/3)^2}{r^2} = \frac{\left(8.99\times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times 10^{-19}\,\text{C})^2}{9(2.6\times 10^{-15}\,\text{m})^2} = 3.8 \text{ N} .$$

40. We are concerned with the charges in the nucleus (not the "orbiting" electrons, if there are any). The nucleus of Helium has 2 protons and that of Thorium has 90.

(a) Eq. 22-1 gives

$$F = k\frac{q^2}{r^2} = \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(2(1.60 \times 10^{-19}\,\text{C})\right)\left(90(1.60 \times 10^{-19}\,\text{C})\right)}{(9.0 \times 10^{-15}\,\text{m})^2} = 5.1 \times 10^2\ \text{N}\ .$$

(b) Estimating the helium nucleus mass as that of 4 protons (actually, that of 2 protons and 2 neutrons, but the neutrons have approximately the same mass), Newton's second law leads to

$$a = \frac{F}{m} = \frac{5.1 \times 10^2\,\text{N}}{4(1.67 \times 10^{-27}\,\text{kg})} = 7.7 \times 10^{28}\ \text{m/s}^2\ .$$

41. Charge $q_1 = -80 \times 10^{-6}$ C is at the origin, and charge $q_2 = +40 \times 10^{-6}$ C is at $x = 0.20$ m. The force on $q_3 = +20 \times 10^{-6}$ C is due to the attractive and repulsive forces from q_1 and q_2, respectively. In symbols, $\vec{F}_{3\,\text{net}} = \vec{F}_{3\,1} + \vec{F}_{3\,2}$, where

$$|\vec{F}_{3\,1}| = k\,\frac{q_3\,|q_1|}{r_{3\,1}^2} \qquad \text{and} \quad |\vec{F}_{3\,2}| = k\,\frac{q_3\,q_2}{r_{3\,2}^2}\ .$$

(a) In this case $r_{3\,1} = 0.40$ m and $r_{3\,2} = 0.20$ m, with $\vec{F}_{3\,1}$ directed towards $-x$ and $\vec{F}_{3\,2}$ directed in the $+x$ direction. Using the value of k in Eq. 22-5, we obtain $\vec{F}_{3\,\text{net}} = 89.9 \approx 90$ N in the $+x$ direction.

(b) In this case $r_{3\,1} = 0.80$ m and $r_{3\,2} = 0.60$ m, with $\vec{F}_{3\,1}$ directed towards $-x$ and $\vec{F}_{3\,2}$ towards $+x$. Now we obtain $\vec{F}_{3\,\text{net}} = 2.5$ N in the $-x$ direction.

(c) Between the locations treated in parts (a) and (b), there must be one where $\vec{F}_{3\,\text{net}} = 0$. Writing $r_{3\,1} = x$ and $r_{3\,2} = x - 0.20$ m, we equate $|\vec{F}_{3\,1}|$ and $|\vec{F}_{3\,2}|$, and after canceling common factors, arrive at

$$\frac{|q_1|}{x^2} = \frac{q_2}{(x - 0.2)^2}\ .$$

This can be further simplified to

$$\frac{(x - 0.2)^2}{x^2} = \frac{q_2}{|q_1|} = \frac{1}{2}\ .$$

Taking the (positive) square root and solving, we obtain $x = 0.68$ m. If one takes the negative root and 'solves', one finds the location where the net force *would* be zero *if* q_1 and q_2 were of like sign (which is not the case here).

42. (a) Charge $Q_1 = +80 \times 10^{-9}$ C is on the y axis at $y = 0.003$ m, and charge $Q_2 = +80 \times 10^{-9}$ C is on the y axis at $y = -0.003$ m. The force on particle 3 (which has a charge of $q = +18 \times 10^{-9}$ C) is due to the vector sum of the repulsive forces from Q_1 and Q_2. In symbols, $\vec{F}_{3\,1} + \vec{F}_{3\,2} = \vec{F}_{3\,\text{net}}$, where

$$|\vec{F}_{3\,1}| = k\,\frac{q_3\,|q_1|}{r_{3\,1}^2} \qquad \text{and} \quad |\vec{F}_{3\,2}| = k\,\frac{q_3\,q_2}{r_{3\,2}^2}\ .$$

Using the Pythagorean theorem, we have $r_{3\,1} = r_{3\,2} = 0.005$ m. In magnitude-angle notation (particularly convenient if one uses a vector capable calculator in polar mode), the indicated vector addition becomes

$$(0.518\ \angle -37^\circ) + (0.518\ \angle\ 37^\circ) = (0.829\ \angle\ 0^\circ)\ .$$

Therefore, the net force is 0.829 N in the $+x$ direction.

(b) Switching the sign of Q_2 amounts to reversing the direction of its force on q. Consequently, we have

$$(0.518\ \angle -37^\circ) + (0.518\ \angle\ -143^\circ) = (0.621\ \angle\ -90^\circ)\ .$$

Therefore, the net force is 0.621 N in the $-y$ direction.

43. (a) For the net force to be in the $+x$ direction, the y components of the individual forces must cancel. The angle of the force exerted by the $q_1 = 40\ \mu$C charge on $q = 20\ \mu$C is 45°, and the angle of force exerted on q by Q is at $-\theta$ where

$$\theta = \tan^{-1}\left(\frac{2.0}{3.0}\right) = 33.7^\circ\ .$$

Therefore, cancellation of y components requires

$$k\,\frac{q_1\,q}{\left(0.02\sqrt{2}\right)^2}\,\sin 45^\circ = k\,\frac{|Q|\,q}{\left(\sqrt{0.03^2+0.02^2}\right)^2}\,\sin\theta$$

from which we obtain $|Q| = 82.9\ \mu$C. Charge Q is "pulling" on q, so (since $q > 0$) we conclude $Q = -82.9\ \mu$C.

(b) Now, we require that the x components cancel, and we note that in this case, the angle of force on q exerted by Q is $+\theta$ (it is repulsive, and Q is positive-valued). Therefore,

$$k\,\frac{q_1\,q}{\left(0.02\sqrt{2}\right)^2}\,\cos 45^\circ = k\,\frac{Q\,q}{\left(\sqrt{0.03^2+0.02^2}\right)^2}\,\cos\theta$$

from which we obtain $Q = 55.2\ \mu$C.

44. We are looking for a charge q which, when placed at the origin, experiences $\vec{F}_{\rm net} = 0$, where

$$\vec{F}_{\rm net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3\ .$$

The magnitude of these individual forces are given by Coulomb's law, Eq. 22-1, and without loss of generality we assume $q > 0$. The charges q_1 ($+6\ \mu$C), q_2 ($-4\ \mu$C), and q_3 (unknown), are located on the $+x$ axis, so that we know $\vec{F}_1$ points towards $-x$, $\vec{F}_2$ points towards $+x$, and $\vec{F}_3$ points towards $-x$ if $q_3 > 0$ and points towards $+x$ if $q_3 < 0$. Therefore, with $r_1 = 8$ m, $r_2 = 16$ m and $r_3 = 24$ m, we have

$$0 = -k\,\frac{q_1\,q}{r_1^2} + k\,\frac{|q_2|\,q}{r_2^2} - k\,\frac{q_3\,q}{r_3^2}\ .$$

Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where q_3 is now understood to be in μC. Thus, we obtain $q_3 = -45\ \mu$C.

45. The magnitude of the net force on the $q = 42\times 10^{-6}$ C charge is

$$k\,\frac{q_1\,q}{0.28^2} + k\,\frac{|q_2|\,q}{0.44^2}$$

where $q_1 = 30\times 10^{-9}$ C and $|q_2| = 40\times 10^{-9}$ C. This yields 0.22 N. Using Newton's second law, we obtain

$$m = \frac{F}{a} = \frac{0.22\,\mathrm{N}}{100\times 10^3\,\mathrm{m/s^2}} = 2.2\times 10^{-6}\ \mathrm{kg}\ .$$

46. The charge dq within a thin shell of thickness dr is $\rho\,A\,dr$ where $A = 4\pi r^2$. Thus, with $\rho = b/r$, we have

$$q = \int dq = 4\pi b\int_{r_1}^{r_2} r\,dr = 2\pi b\left(r_2^2 - r_1^2\right)\ .$$

With $b = 3.0\,\mu\mathrm{C/m^2}$, $r_2 = 0.06$ m and $r_1 = 0.04$ m, we obtain $q = 0.038\ \mu$C.

47. The charge dq within a thin section of the rod (of thickness dx) is $\rho\,A\,dx$ where $A = 4.00 \times 10^{-4}$ m^2 and ρ is the charge per unit volume. The number of (excess) electrons in the rod (of length $L = 2.00$ m) is $N = q/(-e)$ where e is given in Eq. 22-14.

    (a) In the case where $\rho = -4.00 \times 10^{-6}$ C/m^3, we have

    $$N = \frac{q}{-e} = \frac{\rho\,A}{-e}\int_0^L dx = \frac{|\rho|\,A\,L}{e}$$

    which yields $N = 2.00 \times 10^{10}$.

    (b) With $\rho = bx^2$ ($b = -2.00 \times 10^{-6}$ C/m^5) we obtain

    $$N = \frac{b\,A}{-e}\int_0^L x^2\,dx = \frac{|b|\,A\,L^3}{3\,e} = 1.33 \times 10^{10}\ .$$

48. When sphere C touches sphere A, they divide up their total charge ($Q/2$ plus Q) equally between them. Thus, sphere A now has charge $3Q/4$, and the magnitude of the force of attraction between A and B becomes

$$F = k\,\frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{4}\right)}{d^2} = 4.68 \times 10^{-19}\ \text{N}\ .$$

49. In experiment 1, sphere C first touches sphere A, and they divided up their total charge ($Q/2$ plus Q) equally between them. Thus, sphere A and sphere C each acquired charge $3Q/4$. Then, sphere C touches B and those spheres split up their total charge ($3Q/4$ plus $-Q/4$) so that B ends up with charge equal to $Q/4$. The force of repulsion between A and B is therefore

$$F_1 = k\,\frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{4}\right)}{d^2}$$

at the end of experiment 1. Now, in experiment 2, sphere C first touches B which leaves each of them with charge $Q/8$. When C next touches A, sphere A is left with charge $9Q/16$. Consequently, the force of repulsion between A and B is

$$F_2 = k\,\frac{\left(\frac{9Q}{16}\right)\left(\frac{Q}{8}\right)}{d^2}$$

at the end of experiment 2. The ratio is

$$\frac{F_2}{F_1} = \frac{\left(\frac{9}{16}\right)\left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)} = 0.375\ .$$

50. Regarding the forces on q_3 exerted by q_1 and q_2, one must "push" and the other must "pull" in order that the net force is zero; hence, q_1 and q_2 have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k\,\frac{|q_1|\,|q_3|}{(3d)^2} = k\,\frac{|q_2|\,|q_3|}{(2d)^2}$$

which simplifies to

$$\frac{|q_1|}{9} = \frac{|q_2|}{4}\ .$$

Therefore, $q_1 = -\frac{9}{4}q_2$.

51. The individual force magnitudes are found using Eq. 22-1, with SI units (so $a = 0.02$ m) and k as in Eq. 22-5. We use magnitude-angle notation (convenient if ones uses a vector capable calculator in polar mode), listing the forces due to $+4.00q$, $+2.00q$, and $-2.00q$ charges:

$$(4.60 \times 10^{-24}\ \angle\ 180^\circ) + (2.30 \times 10^{-24}\ \angle - 90^\circ) + (1.02 \times 10^{-24}\ \angle - 145^\circ) = (6.16 \times 10^{-24}\ \angle - 152^\circ)$$

Therefore, the net force has magnitude 6.16×10^{-24} N and is at an angle of -152° (or 208° measured counterclockwise from the $+x$ axis).

Chapter 23

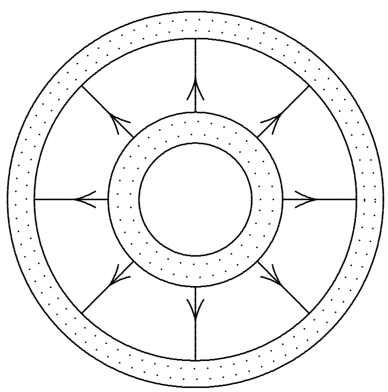
1. (a) We note that the electric field points leftward at both points. Using $\vec{F} = q_0\vec{E}$, and orienting our x axis rightward (so î points right in the figure), we find

$$\vec{F} = \left(+1.6 \times 10^{-19}\,\mathrm{C}\right)\left(-40\ \frac{\mathrm{N}}{\mathrm{C}}\ \hat{\imath}\right) = -6.4 \times 10^{-18}\ \mathrm{N}\ \ \hat{\imath}$$

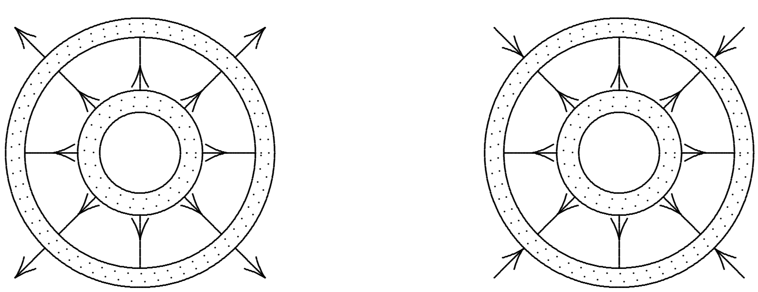
which means the magnitude of the force on the proton is 6.4×10^{-18} N and its direction ($-\hat{\imath}$) is leftward.

(b) As the discussion in §23-2 makes clear, the field strength is proportional to the "crowdedness" of the field lines. It is seen that the lines are twice as crowded at A than at B, so we conclude that $E_A = 2E_B$. Thus, $E_B = 20$ N/C.

2. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge which resides on the larger shell. The following sketch is for $q_1 = q_2$.

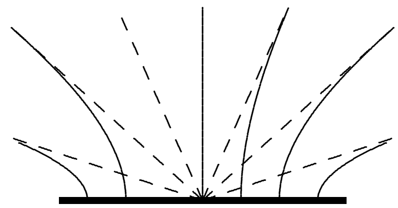


The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



3. The diagram below is an edge view of the disk and shows the field lines above it. Near the disk, the lines are perpendicular to the surface and since the disk is uniformly charged, the lines are uniformly distributed over the surface. Far away from the disk, the lines are like those of a single point charge (the

charge on the disk). Extended back to the disk (along the dotted lines of the diagram) they intersect at the center of the disk.



If the disk is positively charged, the lines are directed outward from the disk. If the disk is negatively charged, they are directed inward toward the disk. A similar set of lines is associated with the region below the disk.

4. We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\text{N/C})(1.00\,\text{m})^2}{8.99 \times 10^9 \,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 1.11 \times 10^{-10}\ \text{C}\ .$$

5. Since the magnitude of the electric field produced by a point charge q is given by $E = |q|/4\pi\varepsilon_0 r^2$, where r is the distance from the charge to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\varepsilon_0 r^2 E = \frac{(0.50\,\text{m})^2(2.0\,\text{N/C})}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 5.6 \times 10^{-11}\ \text{C}\ .$$

6. For concreteness, consider that charge 2 lies 0.15 m east of charge 1, and the point at which we are asked to evaluate their net field is $r = 0.075$ m east of charge 1 and $r = 0.075$ m west of charge 2. The values of charge are $q_1 = -q_2 = 2.0 \times 10^{-7}$ C. The magnitudes and directions of the individual fields are specified:

$$\left|\vec{E}_1\right| = \frac{q_1}{4\pi\varepsilon_0\, r^2} = 3.2 \times 10^5\ \text{N/C} \qquad \text{and} \qquad \vec{E}_1 \text{ points east}$$

$$\left|\vec{E}_2\right| = \frac{|q_2|}{4\pi\varepsilon_0\, r^2} = 3.2 \times 10^5\ \text{N/C} \qquad \text{and} \qquad \vec{E}_2 \text{ points east}$$

Since they point the same direction, the magnitude of the net field is the sum of their amplitudes, $\left|\vec{E}_{\text{net}}\right| = 6.4 \times 10^5$ N/C, and it points east (that is, towards the negative charge).

7. Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

where q is the magnitude of the total charge and R is the sphere radius. The magnitude of the total charge is Ze, so

$$E = \frac{Ze}{4\pi\varepsilon_0 R^2} = \frac{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(94)(1.60 \times 10^{-19}\,\text{C})}{(6.64 \times 10^{-15}\,\text{m})^2} = 3.07 \times 10^{21}\ \text{N/C}\ .$$

The field is normal to the surface and since the charge is positive, it points outward from the surface.

8. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q are unnecessary since these charges are both positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points to the left of q_1 (along the

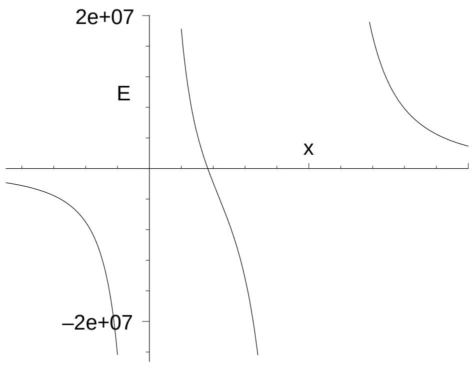
$-x$ axis) both fields point leftward, and at points right of q_2 (at $x > d$) both fields point rightward; in these regions the magnitude of the net field is the sum $\left|\vec{E}_1\right| + \left|\vec{E}_2\right|$. In the region between the charges ($0 < x < d$) $\vec{E}_1$ points rightward and $\vec{E}_2$ points leftward, so the net field in this range is $\vec{E}_{\text{net}} = \left|\vec{E}_1\right| - \left|\vec{E}_2\right|$ in the $\hat{\imath}$ direction. Summarizing, we have

$$\vec{E}_{\text{net}} = \hat{\imath}\,\frac{1}{4\pi\varepsilon_0}\begin{cases} -\frac{q_1}{x^2} - \frac{q_2}{(d+|x|)^2} & \text{for } x < 0 \\ \frac{q_1}{x^2} - \frac{q_2}{(d-x)^2} & \text{for } 0 < x < d \\ \frac{q_1}{x^2} + \frac{q_2}{(x-d)^2} & \text{for } d < x \end{cases} .$$

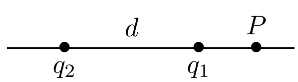
We note that these can be written as a single expression applying to all three regions:

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 x}{|x|^3} + \frac{q_2(x-d)}{|x-d|^3}\right)\hat{\imath} .$$

For $-0.09 \le x \le 0.20$ m with $d = 0.10$ m and charge values as specified in the problem, we find



9. At points between the charges, the individual electric fields are in the same direction and do not cancel. Charge q_2 has a greater magnitude than charge q_1, so a point of zero field must be closer to q_1 than to q_2. It must be to the right of q_1 on the diagram.



We put the origin at q_2 and let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{x^2} - \frac{q_1}{(x-d)^2}\right)$$

where q_1 and q_2 are the magnitudes of the charges. If the field is to vanish,

$$\frac{q_2}{x^2} = \frac{q_1}{(x-d)^2} .$$

We take the square root of both sides to obtain $\sqrt{q_2}/x = \sqrt{q_1}/(x-d)$. The solution for x is

$$x = \left(\frac{\sqrt{q_2}}{\sqrt{q_2} - \sqrt{q_1}}\right) d$$

$$
\begin{aligned}
&= \left(\frac{\sqrt{4.0q_1}}{\sqrt{4.0q_1}-\sqrt{q_1}}\right) d \\
&= \left(\frac{2.0}{2.0-1.0}\right) d = 2.0d \\
&= (2.0)(50\,\text{cm}) = 100\ \text{cm}\ .
\end{aligned}
$$

The point is 50 cm to the right of q_1.

10. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < d$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > d$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\text{net}} = \left|\vec{E}_2\right| - \left|\vec{E}_1\right| \quad \text{in the } \hat{\imath} \text{ direction.}$$

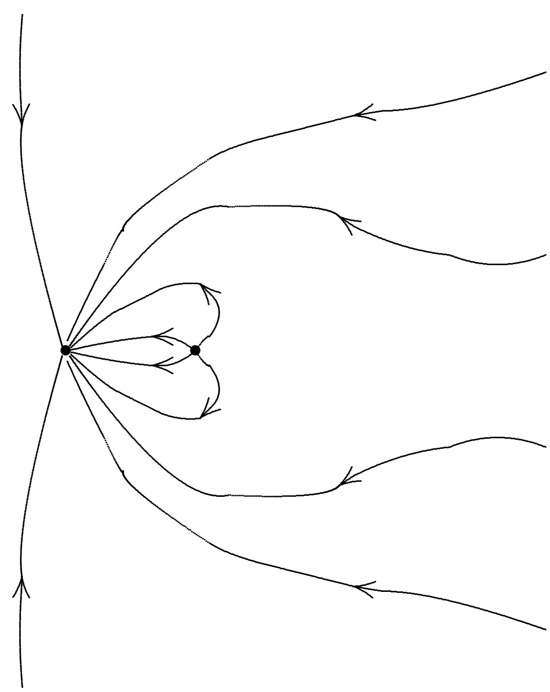
Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\text{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > d$ region:

$$
\begin{aligned}
\left|\vec{E}_1\right| &= \left|\vec{E}_2\right| \\
\frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} &= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-d)^2}
\end{aligned}
$$

which leads to

$$\frac{x-d}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{2}{5}}\ .$$

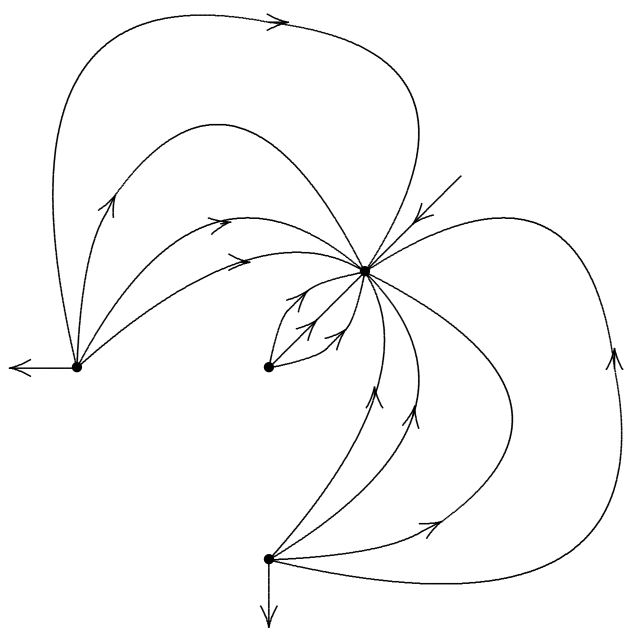
Thus, we obtain $x = \frac{d}{1-\sqrt{2/5}} \approx 2.7d$. A sketch of the field lines is shown below.



11. We place the origin of our coordinate system at point P and orient our y axis in the direction of the $q_4 = -12q$ charge (passing through the $q_3 = +3q$ charge). The x axis is perpendicular to the y axis, and thus passes through the identical $q_1 = q_2 = +5q$ charges. The individual magnitudes $\left|\vec{E}_1\right|$, $\left|\vec{E}_2\right|$, $\left|\vec{E}_3\right|$, and $\left|\vec{E}_4\right|$ are figured from Eq. 23-3, where the absolute value signs for q_1, q_2, and q_3 are unnecessary since those charges are positive (assuming $q > 0$). We note that the contribution from q_1 cancels that of q_2 (that is, $\left|\vec{E}_1\right| = \left|\vec{E}_2\right|$), and the net field (if there is any) should be along the y axis, with magnitude equal to

$$\vec{E}_{\text{net}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{|q_4|}{(2d)^2} - \frac{q_3}{d^2}\right)\hat{\jmath} \quad = \quad \frac{1}{4\pi\varepsilon_0}\left(\frac{12q}{4\,d^2} - \frac{3q}{d^2}\right)\hat{\jmath}$$

which is seen to be zero. A rough sketch of the field lines is shown below:



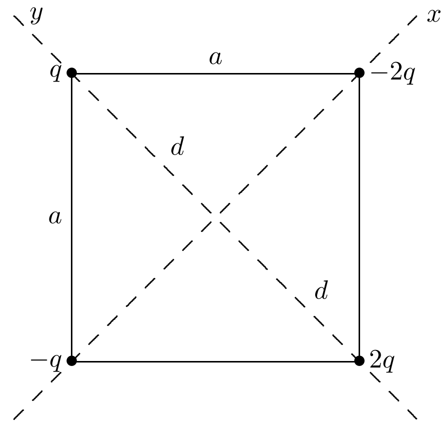
12. By symmetry we see the contributions from the $+q$ charges cancel each other, and we simply use Eq. 23-3 to compute magnitude of the field due to the $+2q$ charge (this field points at 45°, which is clear from the figure in the textbook).

$$\left|\vec{E}_{\text{net}}\right| = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$$

where $r = a/\sqrt{2}$. Thus, we obtain $\left|\vec{E}_{\text{net}}\right| = q/\pi\varepsilon_0 a^2$.

13. We choose the coordinate axes as shown on the diagram below. At the center of the square, the electric fields produced by the charges at the lower left and upper right corners are both along the x axis and each points away from the center and toward the charge that produces it. Since each charge is a distance $d = \sqrt{2}a/2 = a/\sqrt{2}$ away from the center, the net field due to these two charges is

$$\begin{aligned} E_x &= \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2/2} \\ &= \frac{\left(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.0\times10^{-8}\,\text{C}\right)}{(0.050\,\text{m})^2/2} = 7.19\times10^4\ \text{N/C}\ . \end{aligned}$$



At the center of the square, the field produced by the charges at the upper left and lower right corners are both along the y axis and each points away from the charge that produces it. The net field produced at the center by these charges is

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right] = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{a^2/2} = 7.19 \times 10^4 \text{ N/C} .$$

The magnitude of the field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2(7.19 \times 10^4\,\text{N/C})^2} = 1.02 \times 10^5\,\text{N/C}$$

and the angle it makes with the x axis is

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1) = 45^\circ .$$

It is upward in the diagram, from the center of the square toward the center of the upper side.

14. Since both charges are positive (and aligned along the z axis) we have

$$\left|\vec{E}_{\text{net}}\right| = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-d/2)^2} + \frac{q}{(z+d/2)^2}\right] .$$

For $z \gg d$ we have $(z \pm d/2)^{-2} \approx z^{-2}$, so

$$\left|\vec{E}_{\text{net}}\right| \approx \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z^2} + \frac{q}{z^2}\right) = \frac{2q}{4\pi\varepsilon_0 z^2} .$$

15. The magnitude of the dipole moment is given by $p = qd$, where q is the positive charge in the dipole and d is the separation of the charges. For the dipole described in the problem, $p = (1.60 \times 10^{-19}\,\text{C})(4.30 \times 10^{-9}\,\text{m}) = 6.88 \times 10^{-28}\,\text{C}\cdot\text{m}$. The dipole moment is a vector that points from the negative toward the positive charge.

16. From the figure below it is clear that the net electric field at point P points in the $-\hat{\jmath}$ direction. Its magnitude is

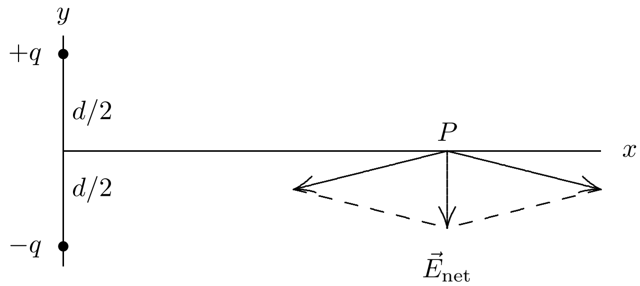
$$\begin{aligned}\left|\vec{E}_{\text{net}}\right| &= 2E_1 \sin\theta = 2\left[k\frac{q}{(d/2)^2 + r^2}\right]\frac{d/2}{\sqrt{(d/2)^2 + r^2}} \\ &= k\frac{qd}{[(d/2)^2 + r^2]^{3/2}}\end{aligned}$$

where we use k for $1/4\pi\varepsilon_0$ for brevity. For $r \gg d$, we write $[(d/2)^2 + r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$\left|\vec{E}_{\rm net}\right| \approx k\frac{qd}{r^3} .$$

Since $\vec{p} = (qd)\hat{\rm j}$,

$$\vec{E}_{\rm net} \approx -k\frac{\vec{p}}{r^3} .$$



17. Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then, the field produced by the right dipole of the pair is $qd/2\pi\varepsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$. Use the binomial expansions $(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$ and $(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$ to obtain

$$E = \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4} .$$

Let $Q = 2qd^2$. Then,

$$E = \frac{3Q}{4\pi\varepsilon_0 z^4} .$$

18. We use Eq. 23-3, assuming both charges are positive.

$$\begin{aligned} E_{\text{left ring}} &= E_{\text{right ring}} \qquad \text{evaluated at } P \\ \frac{q_1 R}{4\pi\varepsilon_0 \left(R^2 + R^2\right)^{3/2}} &= \frac{q_2 (2R)}{4\pi\varepsilon_0 \left((2R)^2 + R^2\right)^{3/2}} \end{aligned}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.51 .$$

19. The electric field at a point on the axis of a uniformly charged ring, a distance z from the ring center, is given by

$$E = \frac{qz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}}$$

where q is the charge on the ring and R is the radius of the ring (see Eq. 23–16). For q positive, the field points upward at points above the ring and downward at points below the ring. We take the positive direction to be upward. Then, the force acting on an electron on the axis is

$$F = -\frac{eqz}{4\pi\varepsilon_0 (z^2 + R^2)^{3/2}} .$$

For small amplitude oscillations $z \ll R$ and z can be neglected in the denominator. Thus,

$$F = -\frac{eqz}{4\pi\varepsilon_0 R^3} .$$

The force is a restoring force: it pulls the electron toward the equilibrium point $z = 0$. Furthermore, the magnitude of the force is proportional to z, just as if the electron were attached to a spring with spring constant $k = eq/4\pi\varepsilon_0 R^3$. The electron moves in simple harmonic motion with an angular frequency given by

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{eq}{4\pi\varepsilon_0 m R^3}}$$

where m is the mass of the electron.

20. From symmetry, we see that the net field at P is twice the field caused by the upper semicircular charge $+q = \lambda \cdot \pi R$ (and that it points downward). Adapting the steps leading to Eq. 23-21, we find

$$\vec{E}_{\text{net}} = 2\left(-\hat{\text{j}}\right)\frac{\lambda}{4\pi\varepsilon_0 R}\Big[\sin\theta\Big]_{-90^\circ}^{90^\circ} = -\frac{q}{\varepsilon_0 \pi^2 R^2}\,\hat{\text{j}}\ .$$

21. Studying Sample Problem 23-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-\theta/2}^{\theta/2} \qquad \text{along the symmetry axis}$$

where $\lambda = q/r\theta$ with θ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$\left|\vec{E}\right| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-\pi/4}^{\pi/4} = \frac{|q|}{\varepsilon_0\pi^2 r^2\sqrt{2}}\ .$$

That produced by the positive quarter-circle points at -45°, and that of the negative quarter-circle points at $+45^\circ$. By symmetry, we conclude that their net field is horizontal (and rightward in the textbook figure) with magnitude

$$E_x = 2\left(\frac{|q|}{\varepsilon_0\pi^2 r^2\sqrt{2}}\right)\cos 45^\circ = \frac{|q|}{\varepsilon_0\pi^2 r^2}\ .$$

22. We find the maximum by differentiating Eq. 23-16 and setting the result equal to zero.

$$\frac{d}{dz}\left(\frac{qz}{4\pi\varepsilon_0\left(z^2+R^2\right)^{3/2}}\right) = \frac{q}{4\pi\varepsilon_0}\frac{R^2-2z^2}{\left(z^2+R^2\right)^{5/2}} = 0$$

which leads to $z = R/\sqrt{2}$.

23. (a) The linear charge density is the charge per unit length of rod. Since the charge is uniformly distributed on the rod, $\lambda = -q/L$.

(b) We position the x axis along the rod with the origin at the left end of the rod, as shown in the diagram. Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\,dx$. The charge dq may be considered to be a point charge. The electric field it produces at point P has only an x component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{(L+a-x)^2}\ .$$

The total electric field produced at P by the whole rod is the integral

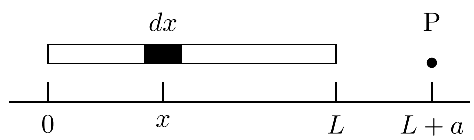
$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2}$$

$$
\begin{aligned}
&= \frac{\lambda}{4\pi\varepsilon_0} \left. \frac{1}{L+a-x} \right|_0^L \\
&= \frac{\lambda}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{L+a} \right) \\
&= \frac{\lambda}{4\pi\varepsilon_0} \frac{L}{a(L+a)} \,.
\end{aligned}
$$

When $-q/L$ is substituted for λ the result is

$$E_x = -\frac{1}{4\pi\varepsilon_0} \frac{q}{a(L+a)} \,.$$

The negative sign indicates that the field is toward the rod.



(c) If a is much larger than L, the quantity $L + a$ in the denominator can be approximated by a and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2} \,.$$

This is the expression for the electric field of a point charge at the origin.

24. We assume $q > 0$. Using the notation $\lambda = q/L$ we note that the (infinitesimal) charge on an element dx of the rod contains charge $dq = \lambda\, dx$. By symmetry, we conclude that all horizontal field components (due to the dq's) cancel and we need only "sum" (integrate) the vertical components. Symmetry also allows us to integrate these contributions over only half the rod ($0 \le x \le L/2$) and then simply double the result. In that regard we note that $\sin\theta = y/r$ where $r = \sqrt{x^2 + y^2}$. Using Eq. 23-3 (with the 2 and $\sin\theta$ factors just discussed) we obtain

$$
\begin{aligned}
\left|\vec{E}\right| &= 2 \int_0^{L/2} \left(\frac{dq}{4\pi\varepsilon_0 r^2} \right) \sin\theta \\
&= \frac{2}{4\pi\varepsilon_0} \int_0^{L/2} \left(\frac{\lambda dx}{x^2+y^2} \right) \left(\frac{y}{\sqrt{x^2+y^2}} \right) \\
&= \frac{\lambda\, y}{2\pi\varepsilon_0} \int_0^{L/2} \frac{dx}{(x^2+y^2)^{3/2}} \\
&= \frac{(q/L)y}{2\pi\varepsilon_0} \left[\frac{x}{y^2\sqrt{x^2+y^2}} \right]_0^{L/2} \\
&= \frac{q}{2\pi\varepsilon_0 L y} \frac{L/2}{\sqrt{(L/2)^2+y^2}} \\
&= \frac{q}{2\pi\varepsilon_0\, y} \frac{1}{\sqrt{L^2+4y^2}}
\end{aligned}
$$

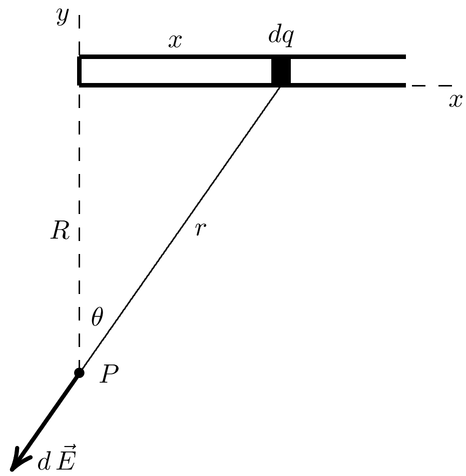
where the integral may be evaluated by elementary means or looked up in Appendix E (item #19 in the list of integrals).

25. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the diagram below. It contains charge $dq = \lambda\, dx$ and is a distance r from P. The magnitude of the field it produces at P is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\ .$$

The x component is $\quad dE_x = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dx}{r^2}\sin\theta$

and the y component is $\quad dE_y = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{\lambda\, dx}{r^2}\cos\theta\ .$



We use θ as the variable of integration and substitute $r = R/\cos\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\theta)\, d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta\, d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}\ .$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.

26. From Eq. 23-26

$$\begin{aligned} E &= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \\ &= \frac{5.3\times 10^{-6}\ \mathrm{C/m^2}}{2\left(8.85\times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)}\left[1 - \frac{12\ \mathrm{cm}}{\sqrt{(12\ \mathrm{cm})^2 + (2.5\ \mathrm{cm})^2}}\right] = 6.3\times 10^3\ \mathrm{N/C}\ . \end{aligned}$$

27. At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 23-26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of

z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} \implies \frac{z}{\sqrt{z^2 + R^2}} = \frac{1}{2} \ .$$

Squaring both sides, then multiplying them by z^2+R^2, we obtain $z^2 = (z^2/4)+(R^2/4)$. Thus, $z^2 = R^2/3$ and $z = R/\sqrt{3}$.

28. Eq. 23-28 gives

$$\vec{E} = \frac{\vec{F}}{q} = \frac{m\vec{a}}{(-e)} = -\left(\frac{m}{e}\right)\vec{a}$$

using Newton's second law. Therefore, with *east* being the î direction,

$$\vec{E} = -\left(\frac{9.11 \times 10^{-31}\,\text{kg}}{1.60 \times 10^{-19}\,\text{C}}\right)\left(1.80 \times 10^9\,\text{m/s}^2\,\hat{\imath}\right) = -0.0102\ \text{N/C}\ \hat{\imath}$$

which means the field has a magnitude of 0.0102 N/C and is directed westward.

29. The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.60 \times 10^{-19}\,\text{C})(2.00 \times 10^4\,\text{N/C})}{9.11 \times 10^{-31}\,\text{kg}} = 3.51 \times 10^{15}\ \text{m/s}^2\ .$$

30. Vertical equilibrium of forces leads to the equality

$$q\left|\vec{E}\right| = mg \implies \left|\vec{E}\right| = \frac{mg}{2e}\ .$$

Using the mass given in the problem, we obtain $\left|\vec{E}\right| = 2.03 \times 10^{-7}$ N/C. Since the force of gravity is downward, then $q\vec{E}$ must point upward. Since $q > 0$ in this situation, this implies $\vec{E}$ must itself point upward.

31. We combine Eq. 23-9 and Eq. 23-28 (in absolute values).

$$F = |q|E = |q|\left(\frac{p}{2\pi\varepsilon_0 z^3}\right) = \frac{2ke\,p}{z^3}$$

where we use Eq. 22-5 in the last step. Thus, we obtain

$$F = \frac{2\left(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\,\text{C}\right)\left(3.6 \times 10^{-29}\,\text{C}\cdot\text{m}\right)}{\left(25 \times 10^{-9}\,\text{m}\right)^3}$$

which yields a force of magnitude 6.6×10^{-15} N. If the dipole is oriented such that $\vec{p}$ is in the $+z$ direction, then $\vec{F}$ points in the $-z$ direction.

32. (a) $F_e = Ee = (3.0 \times 10^6\,\text{N/C})(1.6 \times 10^{-19}\,\text{C}) = 4.8 \times 10^{-13}\,\text{N}$.
    (b) $F_i = Eq_{\text{ion}} = Ee = 4.8 \times 10^{-13}\,\text{N}$.

33. (a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus,

$$E = \frac{F}{q} = \frac{3.0 \times 10^{-6}\,\text{N}}{2.0 \times 10^{-9}\,\text{C}} = 1.5 \times 10^3\ \text{N/C}\ .$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_e = eE = (1.60 \times 10^{-19}\,\mathrm{C})(1.5 \times 10^3\,\mathrm{N/C}) = 2.4 \times 10^{-16}\ \mathrm{N}\ .$$

A proton is positively charged, so the force is in the same direction as the field, upward.

(c) The magnitude of the gravitational force on the proton is

$$F_g = mg = (1.67 \times 10^{-27}\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 1.64 \times 10^{-26}\,\mathrm{N}\ .$$

The force is downward.

(d) The ratio of the forces is

$$\frac{F_e}{F_g} = \frac{2.4 \times 10^{-16}\,\mathrm{N}}{1.64 \times 10^{-26}\,\mathrm{N}} = 1.5 \times 10^{10}\ .$$

34. (a) Since $\vec{E}$ points down and we need an upward electric force (to cancel the downward pull of gravity), then we require the charge of the sphere to be negative. The magnitude of the charge is found by working with the absolute value of Eq. 23-28:

$$|q| = \frac{F}{E} = \frac{mg}{E} = \frac{4.4\,\mathrm{N}}{150\,\mathrm{N/C}} = 0.029\ \mathrm{C}\ .$$

(b) The feasibility of this experiment may be studied by using Eq. 23-3 (using k for $1/4\pi\varepsilon_0$).

$$E = k\frac{|q|}{r^2} \qquad\qquad \text{where} \quad \rho_{\text{sulfur}}\left(\frac{4}{3}\pi r^3\right) = m_{\text{sphere}}$$

Since the mass of the sphere is $4.4/9.8 \approx 0.45$ kg and the density of sulfur is about 2.1×10^3 kg/m^3 (see Appendix F), then we obtain

$$r = \left(\frac{3m_{\text{sphere}}}{4\pi\rho_{\text{sulfur}}}\right)^{1/3} = 0.037\,\mathrm{m} \implies E = k\frac{|q|}{r^2} \approx 2 \times 10^{11}\ \mathrm{N/C}$$

which is much too large a field to maintain in air (see problem #32).

35. (a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus,

$$a = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(2.00 \times 10^4\,\mathrm{N/C})}{1.67 \times 10^{-27}\,\mathrm{kg}} = 1.92 \times 10^{12}\ \mathrm{m/s^2}\ .$$

(b) We assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2(1.92 \times 10^{12}\,\mathrm{m/s^2})(0.0100\,\mathrm{m})} = 1.96 \times 10^5\ \mathrm{m/s}\ .$$

36. (a) The initial direction of motion is taken to be the $+x$ direction (this is also the direction of $\vec{E}$). We use $v_f^2 - v_i^2 = 2a\Delta x$ with $v_f = 0$ and $\vec{a} = \vec{F}/m = -e\vec{E}/m_e$ to solve for distance Δx:

$$\Delta x = \frac{-v_i^2}{2a} = \frac{-m_e v_i^2}{-2eE} = \frac{-(9.11 \times 10^{-31}\,\mathrm{kg})(5.00 \times 10^6\,\mathrm{m/s})^2}{-2(1.60 \times 10^{-19}\,\mathrm{C})(1.00 \times 10^3\,\mathrm{N/C})} = 7.12 \times 10^{-2}\ \mathrm{m}\ .$$

(b) Eq. 2-17 leads to

$$t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_i} = \frac{2(7.12 \times 10^{-2}\,\mathrm{m})}{5.00 \times 10^6\,\mathrm{m/s}} = 2.85 \times 10^{-8}\ \mathrm{s}\ .$$

(c) Using $\Delta v^2 = 2a\Delta x$ with the new value of Δx, we find

$$\begin{aligned}\frac{\Delta K}{K_i} &= \frac{\Delta(\frac{1}{2}m_e v^2)}{\frac{1}{2}m_e v_i^2} = \frac{\Delta v^2}{v_i^2} = \frac{2a\Delta x}{v_i^2} = \frac{-2eE\Delta x}{m_e v_i^2} \\ &= \frac{-2(1.60\times 10^{-19}\,\mathrm{C})(1.00\times 10^3\,\mathrm{N/C})(8.00\times 10^{-3}\,\mathrm{m})}{(9.11\times 10^{-31}\,\mathrm{kg})(5.00\times 10^6\,\mathrm{m/s})^2} = -11.2\%\ .\end{aligned}$$

37. When the drop is in equilibrium, the force of gravity is balanced by the force of the electric field: $mg = qE$, where m is the mass of the drop, q is the charge on the drop, and E is the magnitude of the electric field. The mass of the drop is given by $m = (4\pi/3)r^3\rho$, where r is its radius and ρ is its mass density. Thus,

$$\begin{aligned}q &= \frac{mg}{E} = \frac{4\pi r^3 \rho g}{3E} \\ &= \frac{4\pi(1.64\times 10^{-6}\,\mathrm{m})^3(851\,\mathrm{kg/m^3})(9.8\,\mathrm{m/s^2})}{3(1.92\times 10^5\,\mathrm{N/C})} = 8.0\times 10^{-19}\ \mathrm{C}\end{aligned}$$

and $q/e = (8.0\times 10^{-19}\,\mathrm{C})/(1.60\times 10^{-19}\,\mathrm{C}) = 5$.

38. Our approach (based on Eq. 23-29) consists of several steps. The first is to find an *approximate* value of e by taking differences between all the given data. The smallest difference is between the fifth and sixth values: $18.08\times 10^{-19}\,\mathrm{C} - 16.48\times 10^{-19}\,\mathrm{C} = 1.60\times 10^{-19}\,\mathrm{C}$ which we denote e_{approx}. The goal at this point is to assign integers n using this approximate value of e:

$$\begin{array}{lll}
\text{datum 1} & \dfrac{6.563\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 4.10 & \Longrightarrow\quad n_1 = 4 \\
\text{datum 2} & \dfrac{8.204\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 5.13 & \Longrightarrow\quad n_2 = 5 \\
\text{datum 3} & \dfrac{11.50\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 7.19 & \Longrightarrow\quad n_3 = 7 \\
\text{datum 4} & \dfrac{13.13\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 8.21 & \Longrightarrow\quad n_4 = 8 \\
\text{datum 5} & \dfrac{16.48\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 10.30 & \Longrightarrow\quad n_5 = 10 \\
\text{datum 6} & \dfrac{18.08\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 11.30 & \Longrightarrow\quad n_6 = 11 \\
\text{datum 7} & \dfrac{19.71\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 12.32 & \Longrightarrow\quad n_7 = 12 \\
\text{datum 8} & \dfrac{22.89\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 14.31 & \Longrightarrow\quad n_8 = 14 \\
\text{datum 9} & \dfrac{26.13\times 10^{-19}\,\mathrm{C}}{e_{\mathrm{approx}}} = 16.33 & \Longrightarrow\quad n_9 = 16
\end{array}$$

Next, we construct a new data set $(e_1, e_2, e_3 \ldots)$ by dividing the given data by the respective exact integers n_i (for $i = 1, 2, 3 \ldots$):

$$(e_1, e_2, e_3\ \ldots) = \left(\frac{6.563\times 10^{-19}\,\mathrm{C}}{n_1}, \frac{8.204\times 10^{-19}\,\mathrm{C}}{n_2}, \frac{11.50\times 10^{-19}\,\mathrm{C}}{n_3}\ \ldots\right)$$

which gives (carrying a few more figures than are significant)

$$\left(1.64075\times 10^{-19}\,\mathrm{C},\ 1.6408\times 10^{-19}\,\mathrm{C},\ 1.64286\times 10^{-19}\,\mathrm{C}\ \ldots\right)$$

as the new data set (our experimental values for e). We compute the average and standard deviation of this set, obtaining

$$e_{\text{exptal}} = e_{\text{avg}} \pm \Delta e = (1.641 \pm 0.004) \times 10^{-19}\ \text{C}$$

which does not agree (to within one standard deviation) with the modern accepted value for e. The lower bound on this spread is $e_{\text{avg}} - \Delta e = 1.637 \times 10^{-19}\,\text{C}$ which is still about 2% too high.

39. (a) We use $\Delta x = v_{\text{avg}} t = vt/2$:

$$v = \frac{2\Delta x}{t} = \frac{2(2.0 \times 10^{-2}\,\text{m})}{1.5 \times 10^{-8}\,\text{s}} = 2.7 \times 10^{6}\ \text{m/s}\ .$$

(b) We use $\Delta x = \frac{1}{2}at^2$ and $E = F/e = ma/e$:

$$E = \frac{ma}{e} = \frac{2\Delta x m}{et^2} = \frac{2(2.0 \times 10^{-2}\,\text{m})(9.11 \times 10^{-31}\,\text{kg})}{(1.60 \times 10^{-19}\,\text{C})(1.5 \times 10^{-8}\,\text{s})^2} = 1.0 \times 10^{3}\ \text{N/C}\ .$$

40. We assume there are no forces or force-components along the x direction. We combine Eq. 23-28 with Newton's second law, then use Eq. 4-21 to determine time t followed by Eq. 4-23 to determine the final velocity (with $-g$ replaced by the a_y of this problem); for these purposes, the velocity components *given* in the problem statement are re-labeled as v_{0x} and v_{0y} respectively.

(a) We have $\vec{a} = \frac{q\vec{E}}{m} = -\left(\frac{e}{m}\right)\vec{E}$ which leads to

$$\vec{a} = -\left(\frac{1.60 \times 10^{-19}\,\text{C}}{9.11 \times 10^{-31}\,\text{kg}}\right)\left(120\ \frac{\text{N}}{\text{C}}\right)\hat{\text{j}} = -2.1 \times 10^{13}\,\text{m/s}^2\ \hat{\text{j}}\ .$$

(b) Since $v_x = v_{0x}$ in this problem (that is, $a_x = 0$), we obtain

$$\begin{aligned} t &= \frac{\Delta x}{v_{0x}} = \frac{0.020\,\text{m}}{1.5 \times 10^{5}\,\text{m/s}} = 1.3 \times 10^{-7}\ \text{s} \\ v_y &= v_{0y} + a_y t = 3.0 \times 10^{3}\,\text{m/s} + \left(-2.1 \times 10^{13}\,\text{m/s}^2\right)\left(1.3 \times 10^{-7}\,\text{s}\right) \end{aligned}$$

which leads to $v_y = -2.8 \times 10^{6}$ m/s. Therefore, in unit vector notation (with SI units understood) the final velocity is

$$\vec{v} = 1.5 \times 10^{5}\,\hat{\text{i}} - 2.8 \times 10^{6}\,\hat{\text{j}}\ .$$

41. We take the positive direction to be to the right in the figure. The acceleration of the proton is $a_p = eE/m_p$ and the acceleration of the electron is $a_e = -eE/m_e$, where E is the magnitude of the electric field, m_p is the mass of the proton, and m_e is the mass of the electron. We take the origin to be at the initial position of the proton. Then, the coordinate of the proton at time t is $x = \frac{1}{2}a_p t^2$ and the coordinate of the electron is $x = L + \frac{1}{2}a_e t^2$. They pass each other when their coordinates are the same, or $\frac{1}{2}a_p t^2 = L + \frac{1}{2}a_e t^2$. This means $t^2 = 2L/(a_p - a_e)$ and

$$\begin{aligned} x &= \frac{a_p}{a_p - a_e}L = \frac{eE/m_p}{(eE/m_p) + (eE/m_e)}L = \frac{m_e}{m_e + m_p}L \\ &= \frac{9.11 \times 10^{-31}\,\text{kg}}{9.11 \times 10^{-31}\,\text{kg} + 1.67 \times 10^{-27}\,\text{kg}}(0.050\,\text{m}) \\ &= 2.7 \times 10^{-5}\ \text{m}\ . \end{aligned}$$

42. (a) Using Eq. 23-28, we find

$$\begin{aligned} \vec{F} &= (8.00 \times 10^{-5}\,\text{C})(3.00 \times 10^{3}\,\text{N/C})\hat{\text{i}} + (8.00 \times 10^{-5}\,\text{C})(-600\,\text{N/C})\hat{\text{j}} \\ &= (0.240\,\text{N})\hat{\text{i}} - (0.0480\,\text{N})\hat{\text{j}}\ . \end{aligned}$$

Therefore, the force has magnitude equal to

$$F = \sqrt{(0.240\,\text{N})^2 + (0.0480\,\text{N})^2} = 0.245 \text{ N} ,$$

and makes an angle θ (which, if negative, means clockwise) measured from the $+x$ axis, where

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-0.0480\,\text{N}}{0.240\,\text{N}}\right) = -11.3° .$$

(b) With $m = 0.0100$ kg, the coordinates $(x,\, y)$ at $t = 3.00\,$s are found by combining Newton's second law with the kinematics equations of Chapters 2-4:

$$\begin{aligned} x &= \frac{1}{2}a_x t^2 = \frac{F_x t^2}{2m} = \frac{(0.240)(3.00)^2}{2(0.0100)} = 108 \text{ m} , \\ y &= \frac{1}{2}a_y t^2 = \frac{F_y t^2}{2m} = \frac{(-0.0480)(3.00)^2}{2(0.0100)} = -21.6 \text{ m} . \end{aligned}$$

43. (a) The electric field is upward in the diagram and the charge is negative, so the force of the field on it is downward. The magnitude of the acceleration is $a = eE/m$, where E is the magnitude of the field and m is the mass of the electron. Its numerical value is

$$a = \frac{(1.60 \times 10^{-19}\,\text{C})(2.00 \times 10^3\,\text{N/C})}{9.11 \times 10^{-31}\,\text{kg}} = 3.51 \times 10^{14} \text{ m/s}^2 .$$

We put the origin of a coordinate system at the initial position of the electron. We take the x axis to be horizontal and positive to the right; take the y axis to be vertical and positive toward the top of the page. The kinematic equations are

$$x = v_0 t\cos\theta , \quad y = v_0 t \sin\theta - \frac{1}{2}at^2 , \quad \text{and} \quad v_y = v_0 \sin\theta - at .$$

First, we find the greatest y coordinate attained by the electron. If it is less than d, the electron does not hit the upper plate. If it is greater than d, it will hit the upper plate if the corresponding x coordinate is less than L. The greatest y coordinate occurs when $v_y = 0$. This means $v_0 \sin\theta - at = 0$ or $t = (v_0/a)\sin\theta$ and

$$\begin{aligned} y_{\text{max}} &= \frac{v_0^2 \sin^2\theta}{a} - \frac{1}{2}a\frac{v_0^2\sin^2\theta}{a^2} = \frac{1}{2}\frac{v_0^2\sin^2\theta}{a} \\ &= \frac{(6.00 \times 10^6\,\text{m/s})^2 \sin^2 45°}{2(3.51 \times 10^{14}\,\text{m/s}^2)} = 2.56 \times 10^{-2}\,\text{m} . \end{aligned}$$

Since this is greater than $d = 2.00\,$cm, the electron might hit the upper plate.

(b) Now, we find the x coordinate of the position of the electron when $y = d$. Since

$$v_0 \sin\theta = (6.00 \times 10^6\,\text{m/s})\sin 45° = 4.24 \times 10^6\,\text{m/s}$$

and

$$2ad = 2(3.51 \times 10^{14}\,\text{m/s}^2)(0.0200\,\text{m}) = 1.40 \times 10^{13}\,\text{m}^2/\text{s}^2$$

the solution to $d = v_0 t \sin\theta - \frac{1}{2}at^2$ is

$$\begin{aligned} t &= \frac{v_0\sin\theta - \sqrt{v_0^2\sin^2\theta - 2ad}}{a} \\ &= \frac{4.24 \times 10^6\,\text{m/s} - \sqrt{(4.24 \times 10^6\,\text{m/s})^2 - 1.40 \times 10^{13}\,\text{m}^2/\text{s}^2}}{3.51 \times 10^{14}\,\text{m/s}^2} \\ &= 6.43 \times 10^{-9}\text{ s} . \end{aligned}$$

The negative root was used because we want the *earliest* time for which $y = d$. The x coordinate is

$$\begin{aligned} x &= v_0 t \cos\theta \\ &= (6.00 \times 10^6\,\mathrm{m/s})(6.43 \times 10^{-9}\,\mathrm{s}) \cos 45^\circ = 2.72 \times 10^{-2}\ \mathrm{m}\ . \end{aligned}$$

This is less than L so the electron hits the upper plate at $x = 2.72\,\mathrm{cm}$.

44. (a) The magnitude of the dipole moment is

$$p = qd = \left(1.50 \times 10^{-9}\,\mathrm{C}\right)\left(6.20 \times 10^{-6}\,\mathrm{m}\right) = 9.30 \times 10^{-15}\ \mathrm{C{\cdot}m}\ .$$

(b) Following the solution to part (c) of Sample Problem 23-5, we find

$$U(180^\circ) - U(0) = 2pE = 2\left(9.30 \times 10^{-15}\right)(1100) = 2.05 \times 10^{-11}\ \mathrm{J}\ .$$

45. (a) Eq. 23-33 leads to $\tau = pE \sin 0^\circ = 0$.

(b) With $\theta = 90^\circ$, the equation gives

$$\tau = pE = \left(2(1.6 \times 10^{-19}\,\mathrm{C})(0.78 \times 10^{-9}\,\mathrm{m})\right)\left(3.4 \times 10^6\,\mathrm{N/C}\right) = 8.5 \times 10^{-22}\ \mathrm{N{\cdot}m}\ .$$

(c) Now the equation gives $\tau = pE \sin 180^\circ = 0$.

46. Following the solution to part (c) of Sample Problem 23-5, we find

$$W = U(\theta_0 + \pi) - U(\theta_0) = -pE\left(\cos(\theta_0 + \pi) - \cos(\theta_0)\right) = 2pE\cos\theta_0\ .$$

47. Eq. 23-35 ($\tau = -pE\sin\theta$) captures the sense as well as the magnitude of the effect. That is, this is a restoring torque, trying to bring the tilted dipole back to its aligned equilibrium position. If the amplitude of the motion is small, we may replace $\sin\theta$ with θ in radians. Thus, $\tau \approx -pE\theta$. Since this exhibits a simple negative proportionality to the angle of rotation, the dipole oscillates in simple harmonic motion, like a torsional pendulum with torsion constant $\kappa = pE$. The angular frequency ω is given by

$$\omega^2 = \frac{\kappa}{I} = \frac{pE}{I}$$

where I is the rotational inertia of the dipole. The frequency of oscillation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}\ .$$

48. (a) Using $k = 1/4\pi\varepsilon_0$, we estimate the field at $r = 0.02$ m using Eq. 23-3:

$$E = k\frac{q}{r^2} = \left(8.99 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\right)\frac{45 \times 10^{-12}\,\mathrm{C}}{(0.02\,\mathrm{m})^2} \approx 1 \times 10^3\ \mathrm{N/C}\ .$$

(b) The field described by Eq. 23-3 is nonuniform.

(c) As the positively charged bee approaches the grain, a concentration of negative charge is induced on the closest side of the grain, leading to a force of attraction which makes the grain jump to the bee. Although in physical contact, it is not in electrical contact with the bee, or else it would acquire a net positive charge causing it to be repelled from the bee. As the bee (with grain) approaches the stigma, a concentration of negative charge is induced on the closest side of the stigma which is presumably highly nonuniform. In some configurations, the field from the stigma (acting on the positive side of the grain) will overcome the field from the bee acting on the negative side, and the grain will jump to the stigma.

49. We consider pairs of diametrically opposed charges. The net field due to just the charges in the one o'clock ($-q$) and seven o'clock ($-7q$) positions is clearly equivalent to that of a single $-6q$ charge sitting at the seven o'clock position. Similarly, the net field due to just the charges in the six o'clock ($-6q$) and twelve o'clock ($-12q$) positions is the same as that due to a single $-6q$ charge sitting at the twelve o'clock position. Continuing with this line of reasoning, we see that there are six equal-magnitude electric field vectors pointing at the seven o'clock, eight o'clock ... twelve o'clock positions. Thus, the resultant field of all of these points, by symmetry, is directed toward the position midway between seven and twelve o'clock. Therefore, $\vec{E}_{\text{resultant}}$ points towards the nine-thirty position.

50. (a) From Eq. 23-38 (and the facts that $\hat{\imath} \cdot \hat{\imath} = 1$ and $\hat{\jmath} \cdot \hat{\imath} = 0$), the potential energy is

$$\begin{aligned} U &= -\vec{p} \cdot \vec{E} = -[(3.00\hat{\imath} + 4.00\hat{\jmath})(1.24 \times 10^{-30}\,\text{C}\cdot\text{m})] \cdot [(4000\,\text{N/C})\hat{\imath}] \\ &= -1.49 \times 10^{-26}\ \text{J} . \end{aligned}$$

(b) From Eq. 23-34 (and the facts that $\hat{\imath} \times \hat{\imath} = 0$ and $\hat{\jmath} \times \hat{\imath} = -\hat{k}$), the torque is

$$\begin{aligned} \vec{\tau} &= \vec{p} \times \vec{E} = [(3.00\hat{\imath} + 4.00\hat{\jmath})(1.24 \times 10^{-30}\,\text{C}\cdot\text{m})] \times [(4000\,\text{N/C})\hat{\imath}] \\ &= (-1.98 \times 10^{-26}\,\text{N}\cdot\text{m})\hat{k} . \end{aligned}$$

(c) The work done is

$$\begin{aligned} W &= \Delta U = \Delta(-\vec{p} \cdot \vec{E}) = (\vec{p}_i - \vec{p}_f) \cdot \vec{E} \\ &= [(3.00\hat{\imath} + 4.00\hat{\jmath}) - (-4.00\hat{\imath} + 3.00\hat{\jmath})](1.24 \times 10^{-30}\,\text{C}\cdot\text{m})] \cdot [(4000\,\text{N/C})\hat{\imath}] \\ &= 3.47 \times 10^{-26}\ \text{J} . \end{aligned}$$

51. The point at which we are evaluating the net field is denoted by P. The contributions to the net field caused by the two electrons nearest P (the two electrons on the side of the triangle shared by P) are seen to cancel, so that we only need to compute the field (using Eq. 23-3) caused by the electron at the far corner, at a distance $r = 0.17$ m from P. Using $1/4\pi\varepsilon_0 = k$, we obtain

$$\left|\vec{E}_{\text{net}}\right| = k\frac{e}{r^2} = 4.8 \times 10^{-8}\ \text{N/C} .$$

52. Let q_1 denote the charge at $y = d$ and q_2 denote the charge at $y = -d$. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q are unnecessary since these charges are both positive. The distance from q_1 to a point on the x axis is the same as the distance from q_2 to a point on the x axis: $r = \sqrt{x^2 + d^2}$. By symmetry, the y component of the net field along the x axis is zero. The x component of the net field, evaluated at points on the positive x axis, is

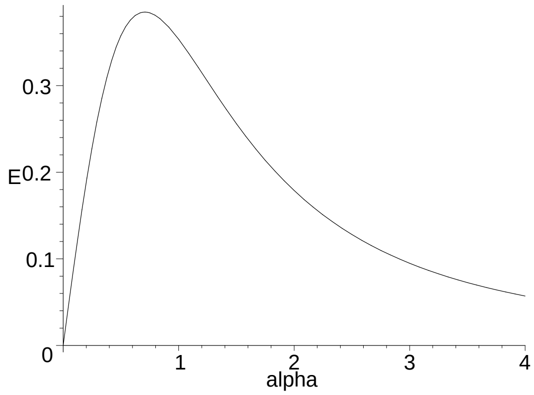
$$E_x = 2\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{q}{x^2 + d^2}\right)\left(\frac{x}{\sqrt{x^2 + d^2}}\right)$$

where the last factor is $\cos\theta = x/r$ with θ being the angle for each individual field as measured from the x axis.

(a) If we simplify the above expression, and plug in $x = \alpha d$, we obtain

$$E_x = \frac{q}{2\pi\varepsilon_0\, d^2}\left(\frac{\alpha}{(\alpha^2 + 1)^{3/2}}\right) .$$

(b) The graph of $E = E_x$ versus α is shown below. For the purposes of graphing, we set $d = 1$ m and $q = 5.56 \times 10^{-11}$ C.



(c) From the graph, we estimate $E_{\max}$ occurs at about $\alpha = 0.7$. More accurate computation shows that the maximum occurs at $\alpha = 1/\sqrt{2}$.

(d) The graph suggests that "half-height" points occur at $\alpha \approx 0.2$ and $\alpha \approx 1.9$. Further numerical exploration leads to the values: $\alpha = 0.2047$ and $\alpha = 1.9864$.

53. (a) We combine Eq. 23-28 (in absolute value) with Newton's second law:

$$a = \frac{|q|E}{m} = \left(\frac{1.60 \times 10^{-19}\,\mathrm{C}}{9.11 \times 10^{-31}\,\mathrm{kg}}\right)\left(1.40 \times 10^{6}\,\frac{\mathrm{N}}{\mathrm{C}}\right) = 2.46 \times 10^{17}\ \mathrm{m/s^2}\ .$$

(b) With $v = \frac{c}{10} = 3.00 \times 10^7$ m/s, we use Eq. 2-11 to find

$$t = \frac{v - v_o}{a} = \frac{3.00 \times 10^7}{2.46 \times 10^{17}} = 1.22 \times 10^{-10}\ \mathrm{s}\ .$$

(c) Eq. 2-16 gives

$$\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{\left(3.00 \times 10^7\right)^2}{2\left(2.46 \times 10^{17}\right)} = 1.83 \times 10^{-3}\ \mathrm{m}\ .$$

54. Studying Sample Problem 23-3, we see that the field evaluated at the center of curvature due to a charged distribution on a circular arc is given by

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-\theta/2}^{\theta/2} \qquad \text{along the symmetry axis}$$

where $\lambda = q/\ell = q/r\theta$ with θ in radians. Here ℓ is the length of the arc, given as $\ell = 4.0$ m. Therefore, $\theta = \ell/r = 4.0/2.0 = 2.0$ rad. Thus, with $q = 20 \times 10^{-9}$ C, we obtain

$$\left|\vec{E}\right| = \frac{q}{\ell}\,\frac{1}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-1.0\,\mathrm{rad}}^{1.0\,\mathrm{rad}} = 38\ \mathrm{N/C}\ .$$

55. A small section of the distribution has charge dq is $\lambda\,dx$, where $\lambda = 9.0 \times 10^{-9}$ C/m. Its contribution to the field at $x_P = 4.0$ m is

$$d\vec{E} = \frac{dq}{4\pi\,\varepsilon_0\,(x - x_P)^2}$$

pointing in the $+x$ direction. Thus, we have

$$\vec{E} = \int_0^{3.0\,\mathrm{m}} \frac{\lambda\,dx}{4\pi\,\varepsilon_0\,(x - x_P)^2}\ \hat{\imath}$$

which becomes, using the substitution $u = x - x_P$,

$$\vec{E} = \frac{\lambda}{4\pi\,\varepsilon_0}\int_{-4.0\,\mathrm{m}}^{-1.0\,\mathrm{m}} \frac{du}{u^2}\,\hat{\imath} \;\; = \frac{\lambda}{4\pi\,\varepsilon_0}\left(\frac{-1}{-1.0\,\mathrm{m}} - \frac{-1}{-4.0\,\mathrm{m}}\right)\hat{\imath}$$

which yields 61 N/C in the $+x$ direction.

56. Let $q_1 = -4Q < 0$ and $q_2 = +2Q > 0$ (where we make the assumption that $Q > 0$). Also, let $d = 2.00$ m, the distance that separates the charges. The individual magnitudes $\left|\vec{E}_1\right|$ and $\left|\vec{E}_2\right|$ are figured from Eq. 23-3, where the absolute value signs for q_2 are unnecessary since this charge is positive. Whether we add the magnitudes or subtract them depends on if $\vec{E}_1$ is in the same, or opposite, direction as $\vec{E}_2$. At points left of q_1 (on the $-x$ axis) the fields point in opposite directions, but there is no possibility of cancellation (zero net field) since $\left|\vec{E}_1\right|$ is everywhere bigger than $\left|\vec{E}_2\right|$ in this region. In the region between the charges ($0 < x < d$) both fields point leftward and there is no possibility of cancellation. At points to the right of q_2 (where $x > d$), $\vec{E}_1$ points leftward and $\vec{E}_2$ points rightward so the net field in this range is

$$\vec{E}_{\mathrm{net}} = \left|\vec{E}_2\right| - \left|\vec{E}_1\right| \quad \text{in the } \hat{\imath} \text{ direction.}$$

Although $|q_1| > q_2$ there is the possibility of $\vec{E}_{\mathrm{net}} = 0$ since these points are closer to q_2 than to q_1. Thus, we look for the zero net field point in the $x > d$ region:

$$\begin{aligned}\left|\vec{E}_1\right| &= \left|\vec{E}_2\right| \\ \frac{1}{4\pi\varepsilon_0}\frac{|q_1|}{x^2} &= \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(x-d)^2}\end{aligned}$$

which leads to

$$\frac{x-d}{x} = \sqrt{\frac{q_2}{|q_1|}} = \sqrt{\frac{1}{2}} \;\; .$$

Therefore, $x = \frac{d\sqrt{2}}{\sqrt{2}-1} = 6.8$ m specifies the position where $\vec{E}_{\mathrm{net}} = 0$.

57. We note that the contributions to the field from the pair of $-2q$ charges exactly cancel, and we are left with the (opposing) contributions from the $4q$ (at $r = 2d$) and $-q$ (at $r = d$) charges. Therefore, using $k = 1/4\pi\varepsilon_0$

$$|\vec{E}_{\mathrm{net}}| = k\,\frac{4q}{(2d)^2} - k\,\frac{q}{d^2} = 0 \; .$$

The net field at P vanishes completely.

58. The field of each charge has magnitude

$$E = k\,\frac{e}{(0.020\,\mathrm{m})^2} = 3.6 \times 10^{-6} \text{ N/C} \; .$$

The directions are indicated in standard format below. We use the magnitude-angle notation (convenient if one is using a vector capable calculator in polar mode) and write (starting with the proton on the left and moving around clockwise) the contributions to $\vec{E}_{\mathrm{net}}$ as follows:

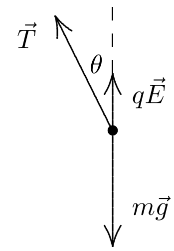
$$(E\angle-20^\circ) + (E\angle\,130^\circ) + (E\angle-100^\circ) + (E\angle-150^\circ) + (E\angle\,0^\circ) \; .$$

This yields $(3.93 \times 10^{-6}\angle-76.4^\circ)$, with the N/C unit understood.

59. Eq. 23-38 gives $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$. We note that $\theta_i = 110^\circ$ and $\theta_f = 70^\circ$. Therefore,

$$\Delta U = -pE\,(\cos 70^\circ - \cos 110^\circ) = -3.3 \times 10^{-21} \text{ J} \; .$$

60. (a) Suppose the pendulum is at the angle θ with the vertical. The force diagram is shown to the right. $\vec{T}$ is the tension in the thread, mg is the magnitude of the force of gravity, and qE is the magnitude of the electric force. The field points upward and the charge is positive, so the force is upward. Taking the angle shown to be positive, then the torque on the sphere about the point where the thread is attached to the upper plate is $\tau = -(mg - qE)\ell \sin\theta$. If $mg > qE$ then the torque is a restoring torque; it tends to pull the pendulum back to its equilibrium position.



If the amplitude of the oscillation is small, $\sin\theta$ can be replaced by θ in radians and the torque is $\tau = -(mg - qE)\ell\theta$. The torque is proportional to the angular displacement and the pendulum moves in simple harmonic motion. Its angular frequency is $\omega = \sqrt{(mg - qE)\ell/I}$, where I is the rotational inertia of the pendulum. Since $I = m\ell^2$ for a simple pendulum,

$$\omega = \sqrt{\frac{(mg - qE)\ell}{m\ell^2}} = \sqrt{\frac{g - qE/m}{\ell}}$$

and the period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g - qE/m}} \ .$$

If $qE > mg$ the torque is not a restoring torque and the pendulum does not oscillate.

(b) The force of the electric field is now downward and the torque on the pendulum is $\tau = -(mg + qE)\ell\theta$ if the angular displacement is small. The period of oscillation is

$$T = 2\pi\sqrt{\frac{\ell}{g + qE/m}} \ .$$

61. (a) Using the density of water ($\rho = 1000\,\text{kg/m}^3$), the weight mg of the spherical drop (of radius $r = 6.0 \times 10^{-7}$ m) is

$$W = \rho V g = \left(1000\,\text{kg/m}^3\right)\left(\frac{4\pi}{3}(6.0 \times 10^{-7}\,\text{m})^3\right)\left(9.8\,\text{m/s}^2\right) = 8.87 \times 10^{-15}\ \text{N} \ .$$

(b) Vertical equilibrium of forces leads to $mg = qE = neE$, which we solve for n, the number of excess electrons:

$$n = \frac{mg}{eE} = \frac{8.87 \times 10^{-15}\,\text{N}}{(1.60 \times 10^{-19}\,\text{C})(462\,\text{N/C})} = 120 \ .$$

62. (a) Let $E = \sigma/2\varepsilon_0 = 3 \times 10^6\,\text{N/C}$. With $\sigma = |q|/A$, this leads to

$$|q| = \pi R^2 \sigma = 2\pi\varepsilon_0 R^2 E = \frac{R^2 E}{2k} = \frac{\left(2.5 \times 10^{-2}\,\text{m}\right)^2 \left(3.0 \times 10^6\,\text{N/C}\right)}{2\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)} = 1.0 \times 10^{-7}\ \text{C} \ .$$

(b) Setting up a simple proportionality (with the areas), the number of atoms is estimated to be

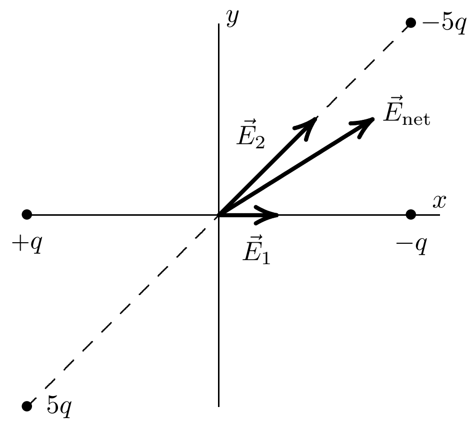
$$N = \frac{\pi(2.5 \times 10^{-2}\,\text{m})^2}{0.015 \times 10^{-18}\,\text{m}^2} = 1.3 \times 10^{17} \ .$$

(c) Therefore, the fraction is

$$\frac{q}{Ne} = \frac{1.0 \times 10^{-7}\,\text{C}}{(1.3 \times 10^{17})(1.6 \times 10^{-19}\,\text{C})} \approx 5 \times 10^{-6} \ .$$

63. On the one hand, the conclusion (that $Q = +1.0\,\mu$C) is clear from symmetry. If a more in-depth justification is desired, one should use Eq. 23-3 for the electric field magnitudes of the three charges (each at the same distance $r = a/\sqrt{3}$ from C) and then find field components along suitably chosen axes, requiring each component-sum to be zero. If the y axis is vertical, then (assuming $Q > 0$) the component-sum along that axis leads to $2kq\sin 30°/r^2 = kQ/r^2$ where q refers to either of the charges at the bottom corners. This yields $Q = 2q\sin 30° = q$ and thus to the conclusion mentioned above.

64. From symmetry, the only two pairs of charges which produce a non-vanishing field $\vec{E}_{\text{net}}$ are: pair 1, which is in the middle of the two vertical sides of the square (the $+q$, $-2q$ pair); and pair 2, the $+5q$, $-5q$ pair. We denote the electric fields produced by each pair as $\vec{E}_1$ and $\vec{E}_2$, respectively. We set up a coordinate system as shown to the right, with the origin at the center of the square. Now,



$$E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{d^2} + \frac{2q}{d^2}\right) = \frac{3q}{4\pi\varepsilon_0 d^2} \qquad \text{and} \qquad E_2 = k\left[\frac{5q}{(\sqrt{2}d)^2} + \frac{5q}{(\sqrt{2}d)^2}\right] = \frac{5q}{4\pi\varepsilon_0 d^2} \ .$$

Therefore, the components of $\vec{E}_{\text{net}}$ are given by

$$\begin{aligned} E_x &= E_{1x} + E_{2x} = E_1 + E_2\cos 45° \\ &= \frac{3q}{4\pi\varepsilon_0 d^2} + \left(\frac{5q}{4\pi\varepsilon_0 d^2}\right)\cos 45° = 6.536\left(\frac{q}{4\pi\varepsilon_0 d^2}\right) \ , \end{aligned}$$

and

$$E_y = E_{1y} + E_{2y} = E_2\sin 45° = \left(\frac{5q}{4\pi\varepsilon_0 d^2}\right)\sin 45° = 3.536\left(\frac{q}{4\pi\varepsilon_0 d^2}\right) \ .$$

Thus, the magnitude of $\vec{E}_{\text{net}}$ is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(6.536)^2 + (3.536)^2}\left(\frac{q}{4\pi\varepsilon_0 d^2}\right) = \frac{7.43q}{4\pi\varepsilon_0 d^2} \ ,$$

and $\vec{E}_{\text{net}}$ makes an angle θ with the positive x axis, where

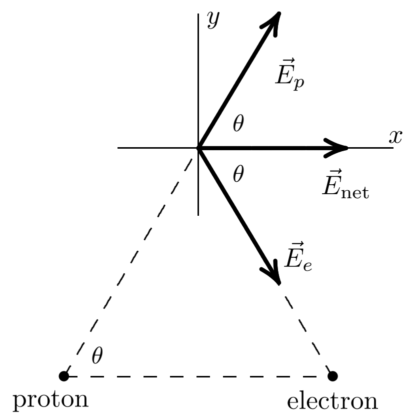
$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{3.536}{6.536}\right) = 28.4° \ .$$

65. We denote the electron with subscript e

and the proton with p. From the figure to the right we see that

$$\left|\vec{E}_e\right| = \left|\vec{E}_p\right| = \frac{e}{4\pi\varepsilon_0 d^2}$$

where $d = 2.0 \times 10^{-6}$ m. We note that the components along the y axis cancel during the vector summation. With $k = 1/4\pi\varepsilon_0$ and $\theta = 60°$, the magnitude of the net electric field is obtained as follows:



$$\begin{aligned}\left|\vec{E}_{\text{net}}\right| &= E_x = 2E_e\cos\theta \\ &= 2\left(\frac{e}{4\pi\varepsilon_0 d^2}\right)\cos\theta = 2k\left[\frac{e}{d^2}\right]\cos\theta \\ &= 2\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(1.6\times10^{-19}\,\text{C})}{(2.0\times10^{-6}\,\text{m})^2}\right]\cos 60° \\ &= 3.6\times10^2\ \text{N/C}\ .\end{aligned}$$

66. (a) Since the two charges in question are of the same sign, the point $x = 2.0$ mm should be located in between them (so that the field vectors point in the opposite direction). Let the coordinate of the second particle be x' ($x' > 0$). Then, the magnitude of the field due to the charge $-q_1$ evaluated at x is given by $E = q_1/4\pi\varepsilon_0 x^2$, while that due to the second charge $-4q_1$ is $E' = 4q_1/4\pi\varepsilon_0(x'-x)^2$. We set the net field equal to zero:

$$\vec{E}_{\text{net}} = 0 \quad\Longrightarrow\quad E = E'$$

so that

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{4q_1}{4\pi\varepsilon_0(x'-x)^2}\ .$$

Thus, we obtain $x' = 3x = 3(2.0\,\text{mm}) = 6.0\,\text{mm}$.

(b) In this case, with the second charge now positive, the electric field vectors produced by both charges are in the negative x direction, when evaluated at $x = 2.0$ mm. Therefore, the net field points in the negative x direction.

67. The distance from Q to P is $5a$, and the distance from q to P is $3a$. Therefore, the magnitudes of the individual electric fields are, using Eq. 23-3 (writing $1/4\pi\varepsilon_0 = k$),

$$|\vec{E}_Q| = \frac{k\,|Q|}{25\,a^2}\ , \qquad |\vec{E}_q| = \frac{k\,|q|}{9\,a^2}\ .$$

We note that $\vec{E}_q$ is along the y axis (directed towards $\pm y$ in accordance with the sign of q), and $\vec{E}_Q$ has x and y components, with $\vec{E}_{Q\,x} = \pm\frac{4}{5}|\vec{E}_Q|$ and $\vec{E}_{Q\,y} = \pm\frac{3}{5}|\vec{E}_Q|$ (signs corresponding to the sign of Q). Consequently, we can write the addition of components in a simple way (basically, by dropping the absolute values):

$$\begin{aligned}\vec{E}_{\text{net}\,x} &= \frac{4\,k\,Q}{125\,a^2} \\ \vec{E}_{\text{net}\,y} &= \frac{3\,k\,Q}{125\,a^2} + \frac{k\,q}{9\,a^2}\end{aligned}$$

(a) Equating $\vec{E}_{\mathrm{net}\,x}$ and $\vec{E}_{\mathrm{net}\,y}$, it is straightforward to solve for the relation between Q and q. We obtain $Q = \frac{125}{9} q \approx 14q$.

(b) We set $\vec{E}_{\mathrm{net}\,y} = 0$ and find the necessary relation between Q and q. We obtain $Q = -\frac{125}{27} q \approx -4.6q$.

68. (a) From the second measurement (at (2.0, 0)) we see that the charge must be somewhere on the x axis. A line passing through (3.0, 3.0) with slope $\tan^{-1} 3/4$ will intersect the x axis at $x = -1.0$. Thus, the location of the particle is specified by the coordinates (in cm): $(-1.0,\ 0)$.

(b) Using $k = 1/4\pi\varepsilon_0$, the field magnitude measured at (2.0, 0) (which is $r = 0.030$ m from the charge) is

$$|\vec{E}| = k\,\frac{q}{r^2} = 100 \text{ N/C} .$$

Therefore, $q = 1.0 \times 10^{-11}$ C.

Chapter 24

1. (a) The mass flux is $wd\rho v = (3.22\,\text{m})(1.04\,\text{m})\left(1000\,\text{kg/m}^3\right)(0.207\,\text{m/s}) = 693\,\text{kg/s}$.

   (b) Since water flows only through area wd, the flux through the larger area is still $693\,\text{kg/s}$.

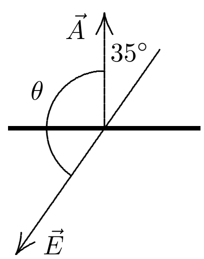
   (c) Now the mass flux is $(wd/2)\rho v = (693\,\text{kg/s})/2 = 347\,\text{kg/s}$.

   (d) Since the water flows through an area $(wd/2)$, the flux is $347\,\text{kg/s}$.

   (e) Now the flux is $(wd\cos\theta)\rho v = (693\,\text{kg/s})(\cos 34°) = 575\,\text{kg/s}$.

2. The vector area $\vec{A}$ and the electric field $\vec{E}$ are shown on the diagram below. The angle θ between them is $180° - 35° = 145°$, so the electric flux through the area is

$$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta = (1800\,\text{N/C})(3.2\times10^{-3}\,\text{m})^2\cos 145° = -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}\ .$$



3. We use $\Phi = \vec{E}\cdot\vec{A}$, where $\vec{A} = A\,\hat{\jmath} = (1.40\,\text{m})^2\,\hat{\jmath}$.

   (a) $\Phi = (6.00\,\text{N/C})\hat{\imath}\cdot(1.40\,\text{m})^2\hat{\jmath} = 0$.

   (b) $\Phi = (-2.00\,\text{N/C})\hat{\jmath}\cdot(1.40\,\text{m})^2\hat{\jmath} = -3.92\,\text{N}\cdot\text{m}^2/\text{C}$.

   (c) $\Phi = [(-3.00\,\text{N/C})\hat{\imath} + (4.00\,\text{N/C})\hat{k}]\cdot(1.40\,\text{m})^2\hat{\jmath} = 0$.

   (d) The total flux of a uniform field through a closed surface is always zero.

4. We use the fact that electric flux relates to the enclosed charge: $\Phi = q_{\text{enclosed}}/\varepsilon_0$.

   (a) A surface which encloses the charges $2q$ and $-2q$, or all four charges.

   (b) A surface which encloses the charges $2q$ and q.

   (c) The maximum amount of negative charge we can enclose by any surface which encloses the charge $2q$ is $-q$, so it is impossible to get a flux of $-2q/\varepsilon_0$.

5. We use Gauss' law: $\varepsilon_0\Phi = q$, where Φ is the total flux through the cube surface and q is the net charge inside the cube. Thus,

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.8\times10^{-6}\,\text{C}}{8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = 2.0\times10^5\ \text{N}\cdot\text{m}^2/\text{C}\ .$$

6. The flux through the flat surface encircled by the rim is given by $\Phi = \pi a^2 E$. Thus, the flux through the netting is $\Phi' = -\Phi = -\pi a^2 E$.

7. (a) Let $A = (1.40\,\mathrm{m})^2$. Then

$$\begin{aligned}\Phi &= (3.00y\,\hat{\mathrm{j}}) \cdot (-A\,\hat{\mathrm{j}})|_{y=0} + (3.00y\,\hat{\mathrm{j}}) \cdot (A\,\hat{\mathrm{j}})|_{y=1.40} \\ &= (3.00)(1.40)(1.40)^2 = 8.23\ \mathrm{N{\cdot}m^2/C}\ .\end{aligned}$$

   (b) The electric field can be re-written as $\vec{E} = 3.00y\hat{\mathrm{j}} + \vec{E}_0$, where $\vec{E}_0 = -4.00\hat{\mathrm{i}} + 6.00\hat{\mathrm{j}}$ is a constant field which does not contribute to the net flux through the cube. Thus Φ is still $8.23\,\mathrm{N{\cdot}m^2/C}$.

   (c) The charge is given by

$$q = \varepsilon_0 \Phi = \left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N{\cdot}m^2}}\right)(8.23\,\mathrm{N{\cdot}m^2/C}) = 7.29 \times 10^{-11}\ \mathrm{C}$$

   in each case.

8. (a) The total surface area bounding the bathroom is

$$A = 2\,(2.5 \times 3.0) + 2\,(3.0 \times 2.0) + 2\,(2.0 \times 2.5) = 37\ \mathrm{m}^2\ .$$

   The absolute value of the total electric flux, with the assumptions stated in the problem, is $|\Phi| = |\sum \vec{E} \cdot \vec{A}| = |\vec{E}|\,A = (600)(37) = 22 \times 10^3\ \mathrm{N{\cdot}m^2/C}$. By Gauss' law, we conclude that the enclosed charge (in absolute value) is $|q_{\mathrm{enc}}| = \varepsilon_0\,|\Phi| = 2.0 \times 10^{-7}$ C. Therefore, with volume $V = 15\ \mathrm{m}^3$, and recognizing that we are dealing with negative charges (see problem), we find the charge density is $q_{\mathrm{enc}}/V = -1.3 \times 10^{-8}\ \mathrm{C/m^3}$.

   (b) We find $(|q_{\mathrm{enc}}|/e)/V = (2.0 \times 10^{-7}/1.6 \times 10^{-19})/15 = 8.2 \times 10^{10}$ excess electrons per cubic meter.

9. Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_ℓ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

$$\begin{aligned}q &= \varepsilon_0\Phi = \varepsilon_0 A(E_\ell - E_u) = (8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2})(100\,\mathrm{m})^2(100\,\mathrm{N/C} - 60.0\,\mathrm{N/C}) \\ &= 3.54 \times 10^{-6}\ \mathrm{C} = 3.54\ \mu\mathrm{C}\ .\end{aligned}$$

10. There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude $(34)(3.0)^2$) and one from the bottom (of magnitude $(20)(3.0)^2$). With "inward" flux conventionally negative, the result is $\Phi = -486\ \mathrm{N{\cdot}m^2/C}$. Gauss' law, then, leads to $q_{\mathrm{enc}} = \varepsilon_0\,\Phi = -4.3 \times 10^{-9}$ C.

11. The total flux through any surface that completely surrounds the point charge is q/ε_0. If we stack identical cubes side by side and directly on top of each other, we will find that eight cubes meet at any corner. Thus, one-eighth of the field lines emanating from the point charge pass through a cube with a corner at the charge, and the total flux through the surface of such a cube is $q/8\varepsilon_0$. Now the field lines are radial, so at each of the three cube faces that meet at the charge, the lines are parallel to the face and the flux through the face is zero. The fluxes through each of the other three faces are the same, so the flux through each of them is one-third of the total. That is, the flux through each of these faces is $(1/3)(q/8\varepsilon_0) = q/24\varepsilon_0$.

12. Using Eq. 24-11, the surface charge density is

$$\sigma = E\varepsilon_0 = \left(2.3 \times 10^5\,\mathrm{N/C}\right)\left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N{\cdot}m^2}}\right) = 2.0 \times 10^{-6}\ \mathrm{C/m^2}\ .$$

13. (a) The charge on the surface of the sphere is the product of the surface charge density σ and the surface area of the sphere (which is $4\pi r^2$, where r is the radius). Thus,

$$q = 4\pi r^2 \sigma = 4\pi \left(\frac{1.2\,\text{m}}{2}\right)^2 \left(8.1 \times 10^{-6}\,\text{C/m}^2\right) = 3.66 \times 10^{-5}\ \text{C} .$$

(b) We choose a Gaussian surface in the form of a sphere, concentric with the conducting sphere and with a slightly larger radius. The flux is given by Gauss' law:

$$\Phi = \frac{q}{\varepsilon_0} = \frac{3.66 \times 10^{-5}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = 4.1 \times 10^6\ \text{N}\cdot\text{m}^2/\text{C} .$$

14. (a) The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

$$\sigma = \frac{q}{\pi D^2} = \frac{2.4 \times 10^{-6}\,\text{C}}{\pi (1.3\,\text{m})^2} = 4.5 \times 10^{-7}\ \text{C/m}^2 .$$

(b) Eq. 24-11 gives

$$E = \frac{\sigma}{\varepsilon_o} = \frac{4.5 \times 10^{-7}\,\text{C/m}^2}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2} = 5.1 \times 10^4\ \text{N/C} .$$

15. (a) Consider a Gaussian surface that is completely within the conductor and surrounds the cavity. Since the electric field is zero everywhere on the surface, the net charge it encloses is zero. The net charge is the sum of the charge q in the cavity and the charge q_w on the cavity wall, so $q + q_w = 0$ and $q_w = -q = -3.0 \times 10^{-6}\,$C.

(b) The net charge Q of the conductor is the sum of the charge on the cavity wall and the charge q_s on the outer surface of the conductor, so $Q = q_w + q_s$ and

$$q_s = Q - q_w = (10 \times 10^{-6}\,\text{C}) - (-3.0 \times 10^{-6}\,\text{C}) = +1.3 \times 10^{-5}\ \text{C} .$$

16. (a) The side surface area A for the drum of diameter D and length h is given by $A = \pi D h$. Thus

$$\begin{aligned} q &= \sigma A = \sigma \pi D h = \pi \varepsilon_0 E D h \\ &= \pi \left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right) \left(2.3 \times 10^5\,\text{N/C}\right) (0.12\,\text{m})(0.42\,\text{m}) \\ &= 3.2 \times 10^{-7}\ \text{C} . \end{aligned}$$

(b) The new charge is

$$\begin{aligned} q' &= q\left(\frac{A'}{A}\right) = q\left(\frac{\pi D' h'}{\pi D h}\right) \\ &= (3.2 \times 10^{-7}\text{C})\left[\frac{(8.0\,\text{cm})(28\,\text{cm})}{(12\,\text{cm})(42\,\text{cm})}\right] = 1.4 \times 10^{-7}\ \text{C} . \end{aligned}$$

17. The magnitude of the electric field produced by a uniformly charged infinite line is $E = \lambda/2\pi\varepsilon_0 r$, where λ is the linear charge density and r is the distance from the line to the point where the field is measured. See Eq. 24-12. Thus,

$$\lambda = 2\pi\varepsilon_0 E r = 2\pi (8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(4.5 \times 10^4\,\text{N/C})(2.0\,\text{m}) = 5.0 \times 10^{-6}\ \text{C/m} .$$

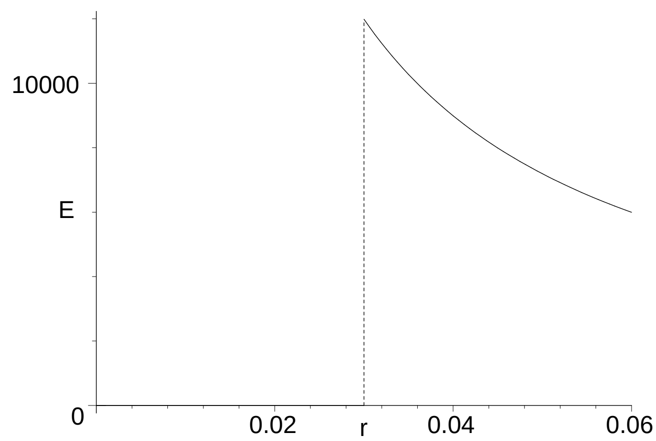
18. We imagine a cylindrical Gaussian surface A of radius r and unit length concentric with the metal tube. Then by symmetry

$$\oint_A \vec{E} \cdot d\vec{A} = 2\pi r E = \frac{q_{\text{enclosed}}}{\varepsilon_0} .$$

(a) For $r > R$, $q_{\text{enclosed}} = \lambda$, so $E(r) = \lambda/2\pi r\varepsilon_0$.

(b) For $r < R$, $q_{\text{enclosed}} = 0$, so $E = 0$. The plot of E vs r is shown below. Here, the maximum value is

$$E_{\max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{(2.0 \times 10^{-8}\,\text{C/m})}{2\pi(0.030\,\text{m})\,(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)} = 1.2 \times 10^4\ \text{N/C}\ .$$



19. We assume the charge density of both the conducting cylinder and the shell are uniform, and we neglect fringing. Symmetry can be used to show that the electric field is radial, both between the cylinder and the shell and outside the shell. It is zero, of course, inside the cylinder and inside the shell.

(a) We take the Gaussian surface to be a cylinder of length L, coaxial with the given cylinders and of larger radius r than either of them. The flux through this surface is $\Phi = 2\pi rLE$, where E is the magnitude of the field at the Gaussian surface. We may ignore any flux through the ends. Now, the charge enclosed by the Gaussian surface is $q - 2q = -q$. Consequently, Gauss' law yields $2\pi r\varepsilon_0 LE = -q$, so

$$E = -\frac{q}{2\pi\varepsilon_0 Lr}\ .$$

The negative sign indicates that the field points inward.

(b) Next, we consider a cylindrical Gaussian surface whose radius places it within the shell itself. The electric field is zero at all points on the surface since any field within a conducting material would lead to current flow (and thus to a situation other than the electrostatic ones being considered here), so the total electric flux through the Gaussian surface is zero and the net charge within it is zero (by Gauss' law). Since the central rod is known to have charge q, then the inner surface of the shell must have charge $-q$. And since the shell is known to have total charge $-2q$, it must therefore have charge $-q$ on its outer surface.

(c) Finally, we consider a cylindrical Gaussian surface whose radius places it between the outside of conducting rod and inside of the shell. Similarly to part (a), the flux through the Gaussian surface is $\Phi = 2\pi rLE$, where E is the field at this Gaussian surface, in the region between the rod and the shell. The charge enclosed by the Gaussian surface is only the charge q on the rod. Therefore, Gauss' law yields

$$2\pi\varepsilon_0 rLE = q \implies E = \frac{q}{2\pi\varepsilon_0 Lr}\ .$$

The positive sign indicates that the field points outward.

20. We denote the radius of the thin cylinder as $R = 0.015$ m. Using Eq. 24-12, the net electric field for $r > R$ is given by

$$E_{\text{net}} = E_{\text{wire}} + E_{\text{cylinder}} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda = -3.6\,\text{nC/m}$ is the linear charge density of the wire and λ' is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

$$q_{\text{cylinder}} = \lambda' L = \sigma(2\pi RL) \implies \lambda' = \sigma(2\pi R) \ .$$

Now, E_{net} outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6\times 10^{-9}\,\text{C/m}}{(2\pi)(0.015\,\text{m})} = 3.8\times 10^{-8}\ \text{C/m}^2 \ .$$

21. We denote the inner and outer cylinders with subscripts i and o, respectively.

    (a) Since $r_i < r = 4.0\,\text{cm} < r_o$,

    $$E(r) = \frac{\lambda_i}{2\pi\varepsilon_0 r} = \frac{5.0\times 10^{-6}\,\text{C/m}}{2\pi\,(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)\,(4.0\times 10^{-2}\,\text{m})} = 2.3\times 10^{6}\ \text{N/C} \ .$$

    $\vec{E}(r)$ points radially outward.

    (b) Since $r > r_o$,

    $$E(r) = \frac{\lambda_i + \lambda_o}{2\pi\varepsilon_0 r} = \frac{5.0\times 10^{-6}\,\text{C/m} - 7.0\times 10^{-6}\,\text{C/m}}{2\pi\,(8.85\times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)\,(8.0\times 10^{-2}\,\text{m})} = -4.5\times 10^{5}\ \text{N/C} \ ,$$

    where the minus sign indicates that $\vec{E}(r)$ points radially inward.

22. To evaluate the field using Gauss' law, we employ a cylindrical surface of area $2\pi\, r\, L$ where L is very large (large enough that contributions from the ends of the cylinder become irrelevant to the calculation). The volume within this surface is $V = \pi\, r^2 L$, or expressed more appropriate to our needs: $dV = 2\pi\, r\, L\, dr$. The charge enclosed is, with $A = 2.5\times 10^{-6}\ \text{C/m}^5$,

    $$q_{\text{enc}} = \int_0^r A\, r^2\, 2\pi\, r\, L\, dr = \frac{\pi}{2}\, A\, L\, r^4 \ .$$

    By Gauss' law, we find $\Phi = |\vec{E}|(2\pi r L) = q_{\text{enc}}/\varepsilon_0$; we thus obtain

    $$\left|\vec{E}\right| = \frac{A\, r^3}{4\,\varepsilon_0} \ .$$

    (a) With $r = 0.030$ m, we find $|\vec{E}| = 1.9$ N/C.

    (b) Once outside the cylinder, Eq. 24-12 is obeyed. To find $\lambda = q/L$ we must find the total charge q. Therefore,

    $$\frac{q}{L} = \frac{1}{L}\int_0^{0.04} A\, r^2\, 2\pi\, r\, L\, dr = 1.0\times 10^{-11}\ \text{C/m} \ .$$

    And the result, for $r = 0.050$ m, is $|\vec{E}| = \lambda/2\pi\varepsilon_0\, r = 3.6$ N/C.

23. The electric field is radially outward from the central wire. We want to find its magnitude in the region between the wire and the cylinder as a function of the distance r from the wire. Since the magnitude of the field at the cylinder wall is known, we take the Gaussian surface to coincide with the wall. Thus, the Gaussian surface is a cylinder with radius R and length L, coaxial with the wire. Only the charge on the wire is actually enclosed by the Gaussian surface; we denote it by q. The area of the Gaussian surface is $2\pi RL$, and the flux through it is $\Phi = 2\pi RLE$. We assume there is no flux through the ends of the cylinder, so this Φ is the total flux. Gauss' law yields $q = 2\pi\varepsilon_0 RLE$. Thus,

    $$q = 2\pi\left(8.85\times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.014\,\text{m})(0.16\,\text{m})\left(2.9\times 10^4\,\text{N/C}\right) = 3.6\times 10^{-9}\ \text{C} \ .$$

24. (a) In Eq. 24-12, $\lambda = q/L$ where q is the net charge enclosed by a cylindrical Gaussian surface of radius r. The field is being measured outside the system (the charged rod coaxial with the neutral cylinder) so that the net enclosed charge is only that which is on the rod. Consequently,

$$|\vec{E}| = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2.0 \times 10^{-9}}{2\pi\varepsilon_0 (0.15)} = 240 \text{ N/C} .$$

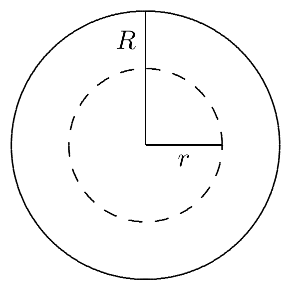
(b) and (c) Since the field is zero inside the conductor (in an electrostatic configuration), then there resides on the inner surface charge $-q$, and on the outer surface, charge $+q$ (where q is the charge on the rod at the center). Therefore, with $r_i = 0.05$ m, the surface density of charge is

$$\sigma_{\text{inner}} = \frac{-q}{2\pi r_i L} = -\frac{\lambda}{2\pi r_i} = -6.4 \times 10^{-9} \text{ C/m}^2$$

for the inner surface. And, with $r_o = 0.10$ m, the surface charge density of the outer surface is

$$\sigma_{\text{outer}} = \frac{+q}{2\pi r_o L} = \frac{\lambda}{2\pi r_o} = +3.2 \times 10^{-9} \text{ C/m}^2 .$$

25. (a) The diagram below shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle). Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ, coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dotted circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell \rho$, where $V = \pi r^2 \ell$ is the volume of the cylinder.



If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to

$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell\rho \implies E = \frac{\rho r}{2\varepsilon_0} .$$

(b) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = 2\pi r\ell E_{\text{ext}}$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell\rho$. In this case, Gauss' law yields

$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell\rho \implies E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r} .$$

26. According to Eq. 24-13 the electric field due to either sheet of charge with surface charge density σ is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

(a) $E = \sigma/\varepsilon_0$, pointing up;

(b) $E = 0$;

(c) and, $E = \sigma/\varepsilon_0$, pointing down.

27. (a) To calculate the electric field at a point very close to the center of a large, uniformly charged conducting plate, we may replace the finite plate with an infinite plate with the same area charge density and take the magnitude of the field to be $E = \sigma/\varepsilon_0$, where σ is the area charge density for the surface just under the point. The charge is distributed uniformly over both sides of the original plate, with half being on the side near the field point. Thus,

$$\sigma = \frac{q}{2A} = \frac{6.0 \times 10^{-6}\,\mathrm{C}}{2(0.080\,\mathrm{m})^2} = 4.69 \times 10^{-4}\ \mathrm{C/m^2}\ .$$

The magnitude of the field is

$$E = \frac{4.69 \times 10^{-4}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2}} = 5.3 \times 10^7\ \mathrm{N/C}\ .$$

The field is normal to the plate and since the charge on the plate is positive, it points away from the plate.

(b) At a point far away from the plate, the electric field is nearly that of a point particle with charge equal to the total charge on the plate. The magnitude of the field is $E = q/4\pi\varepsilon_0 r^2 = kq/r^2$, where r is the distance from the plate. Thus,

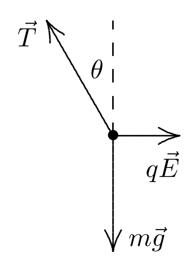
$$E = \frac{\left(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}\right)\left(6.0 \times 10^{-6}\,\mathrm{C}\right)}{(30\,\mathrm{m})^2} = 60\ \mathrm{N/C}\ .$$

28. The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density σ plus a small circular pad of radius R located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. The net electric field $\vec{E}$ is then

$$\begin{aligned}\vec{E} &= \vec{E}_1 + \vec{E}_2 = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathrm{k}} + \frac{(-\sigma)}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{\mathrm{k}} \\ &= \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\,\hat{\mathrm{k}}\end{aligned}$$

where Eq. 23-26 is used for $\vec{E}_2$.

29. The forces acting on the ball are shown in the diagram below. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ $(= 30^\circ)$ with the vertical.



Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$. The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and

$$\begin{aligned}\sigma &= \frac{2\varepsilon_0 mg\tan\theta}{q} \\ &= \frac{2(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.0\times10^{-6}\,\mathrm{kg})(9.8\,\mathrm{m/s^2})\tan 30^\circ}{2.0\times10^{-8}\,\mathrm{C}} \\ &= 5.0\times10^{-9}\ \mathrm{C/m^2}\ .\end{aligned}$$

30. Let $\hat{\imath}$ be a unit vector pointing to the left. We use Eq. 24-13.

   (a) To the left of the plates: $\vec{E} = (\sigma/2\varepsilon_0)\hat{\imath}$ (from the right plate)$+(-\sigma/2\varepsilon_0)\hat{\imath}$ (from the left one)$= 0$.

   (b) To the right of the plates: $\vec{E} = (\sigma/2\varepsilon_0)(-\hat{\imath})$ (from the right plate)$+(-\sigma/2\varepsilon_0)(-\hat{\imath})$ (from the left one)$= 0$.

   (c) Between the plates:

$$\begin{aligned}\vec{E} &= \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\imath} + \left(\frac{-\sigma}{2\varepsilon_0}\right)(-\hat{\imath}) = \left(\frac{\sigma}{\varepsilon_0}\right)\hat{\imath} \\ &= \left(\frac{7.0\times10^{-22}\,\mathrm{C/m^2}}{8.85\times10^{-12}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}}\right)\hat{\imath} = \left(7.9\times10^{-11}\ \mathrm{N/C}\right)\hat{\imath}\ .\end{aligned}$$

31. The charge on the metal plate, which is negative, exerts a force of repulsion on the electron and stops it. First find an expression for the acceleration of the electron, then use kinematics to find the stopping distance. We take the initial direction of motion of the electron to be positive. Then, the electric field is given by $E = \sigma/\varepsilon_0$, where σ is the surface charge density on the plate. The force on the electron is $F = -eE = -e\sigma/\varepsilon_0$ and the acceleration is

$$a = \frac{F}{m} = -\frac{e\sigma}{\varepsilon_0 m}$$

where m is the mass of the electron. The force is constant, so we use constant acceleration kinematics. If v_0 is the initial velocity of the electron, v is the final velocity, and x is the distance traveled between the initial and final positions, then $v^2 - v_0^2 = 2ax$. Set $v = 0$ and replace a with $-e\sigma/\varepsilon_0 m$, then solve for x. We find

$$x = -\frac{v_0^2}{2a} = \frac{\varepsilon_0 m v_0^2}{2e\sigma}\ .$$

Now $\frac{1}{2}mv_0^2$ is the initial kinetic energy K_0, so

$$x = \frac{\varepsilon_0 K_0}{e\sigma}\ .$$

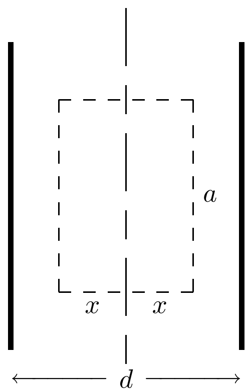
We convert the given value of K_0 to Joules. Since $1.00\,\mathrm{eV} = 1.60\times10^{-19}\,\mathrm{J}$, $100\,\mathrm{eV} = 1.60\times10^{-17}\,\mathrm{J}$. Thus,

$$x = \frac{(8.85\times10^{-12}\,\mathrm{C^2/N\cdot m^2})(1.60\times10^{-17}\,\mathrm{J})}{(1.60\times10^{-19}\,\mathrm{C})(2.0\times10^{-6}\,\mathrm{C/m^2})} = 4.4\times10^{-4}\ \mathrm{m}\ .$$

32. We use the result of part (c) of problem 30 to obtain the surface charge density.

$$E = \sigma/\varepsilon_0 \quad\Longrightarrow\quad \sigma = \varepsilon_0 E = \left(8.85\times10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)(55\,\mathrm{N/C}) = 4.9\times10^{-10}\ \mathrm{C/m^2}\ .$$

33. (a) We use a Gaussian surface in the form of a box with rectangular sides. The cross section is shown with dashed lines in the diagram below. It is centered at the central plane of the slab, so the left and right faces are each a distance x from the central plane. We take the thickness of the rectangular solid to be a, the same as its length, so the left and right faces are squares. The electric field is normal to the left and right faces and is uniform over them. If ρ is positive, it points outward at both faces: toward the left at the left face and toward the right at the right face. Furthermore, the magnitude is the same at both faces. The electric flux through each of these faces is Ea^2. The field is parallel to the other faces of the Gaussian surface and the flux through them is zero. The total flux through the Gaussian surface is $\Phi = 2Ea^2$.



The volume enclosed by the Gaussian surface is $2a^2x$ and the charge contained within it is $q = 2a^2x\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = 2a^2x\rho$. We solve for the magnitude of the electric field:

$$E = \frac{\rho x}{\varepsilon_0} \; .$$

(b) We take a Gaussian surface of the same shape and orientation, but with $x > d/2$, so the left and right faces are outside the slab. The total flux through the surface is again $\Phi = 2Ea^2$ but the charge enclosed is now $q = a^2 d\rho$. Gauss' law yields $2\varepsilon_0 Ea^2 = a^2 d\rho$, so

$$E = \frac{\rho d}{2\varepsilon_0} \; .$$

34. (a) The flux is still $-750\,\mathrm{N\cdot m^2/C}$, since it depends only on the amount of charge enclosed.

(b) We use $\Phi = q/\varepsilon_0$ to obtain the charge q:

$$q = \varepsilon_0 \Phi = \left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)\left(-750\,\mathrm{N\cdot m^2/C}\right) = -6.64 \times 10^{-10}\ \mathrm{C} \; .$$

35. Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus,

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.15\,\mathrm{m})^2(3.0 \times 10^3\,\mathrm{N/C})}{8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}} = 7.5 \times 10^{-9}\,\mathrm{C} \; .$$

The field points inward, toward the sphere center, so the charge is negative: $-7.5 \times 10^{-9}\,\mathrm{C}$.

36. (a) Since $r_1 = 10.0\,\mathrm{cm} < r = 12.0\,\mathrm{cm} < r_2 = 15.0\,\mathrm{cm}$,

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(4.00 \times 10^{-8}\,\mathrm{C})}{(0.120\,\mathrm{m})^2} = 2.50 \times 10^4\ \mathrm{N/C} \; .$$

(b) Since $r_1 < r_2 < r = 20.0\,\mathrm{cm}$,

$$\begin{aligned} E(r) &= \frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2}{r^2} = \frac{(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})(4.00+2.00)(1\times 10^{-8}\,\mathrm{C})}{(0.200\,\mathrm{m})^2} \\ &= 1.35\times 10^4\ \mathrm{N/C}\,. \end{aligned}$$

37. The field is radially outward and takes on equal magnitude-values over the surface of any sphere centered at the atom's center. We take the Gaussian surface to be such a sphere (of radius r). If E is the magnitude of the field, then the total flux through the Gaussian sphere is $\Phi = 4\pi r^2 E$. The charge enclosed by the Gaussian surface is the positive charge at the center of the atom plus that portion of the negative charge within the surface. Since the negative charge is uniformly distributed throughout the large sphere of radius R, we can compute the charge inside the Gaussian sphere using a ratio of volumes. That is, the negative charge inside is $-Zer^3/R^3$. Thus, the total charge enclosed is $Ze - Zer^3/R^3$ for $r \le R$. Gauss' law now leads to

$$4\pi\varepsilon_0 r^2 E = Ze\left(1 - \frac{r^3}{R^3}\right) \implies E = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)\,.$$

38. We interpret the question as referring to the field *just* outside the sphere (that is, at locations roughly equal to the radius r of the sphere). Since the area of a sphere is $A = 4\pi r^2$ and the surface charge density is $\sigma = q/A$ (where we assume q is positive for brevity), then

$$E = \frac{\sigma}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{q}{4\pi r^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

which we recognize as the field of a point charge (see Eq. 23-3).

39. The proton is in uniform circular motion, with the electrical force of the sphere on the proton providing the centripetal force. According to Newton's second law, $F = mv^2/r$, where F is the magnitude of the force, v is the speed of the proton, and r is the radius of its orbit, essentially the same as the radius of the sphere. The magnitude of the force on the proton is $F = eq/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere. Thus,

$$\frac{1}{4\pi\varepsilon_0}\frac{eq}{r^2} = \frac{mv^2}{r}$$

so

$$\begin{aligned} q &= \frac{4\pi\varepsilon_0 mv^2 r}{e} = \frac{(1.67\times 10^{-27}\,\mathrm{kg})(3.00\times 10^5\,\mathrm{m/s})^2(0.0100\,\mathrm{m})}{(8.99\times 10^9\,\mathrm{N\cdot m^2/C^2})(1.60\times 10^{-19}\,\mathrm{C})} \\ &= 1.04\times 10^{-9}\ \mathrm{C}\,. \end{aligned}$$

The force must be inward, toward the center of the sphere, and since the proton is positively charged, the electric field must also be inward. The charge on the sphere is negative: $q = -1.04\times 10^{-9}\,\mathrm{C}$.

40. We imagine a spherical Gaussian surface of radius r centered at the point charge $+q$. From symmetry consideration E is the same throughout the surface, so

$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E = \frac{q_{\mathrm{encl}}}{\varepsilon_0}\,,$$

which gives

$$E(r) = \frac{q_{\mathrm{encl}}}{4\pi\varepsilon_0 r^2}\,,$$

where q_{encl} is the net charge enclosed by the Gaussian surface.

(a) Now $a < r < b$, where $E = 0$. Thus $q_{\mathrm{encl}} = 0$, so the charge on the inner surface of the shell is $q_i = -q$.

(b) The shell as a whole is electrically neutral, so the outer shell must carry a charge of $q_o = +q$.

(c) For $r < a$ $\quad q_{\text{encl}} = +q$, so

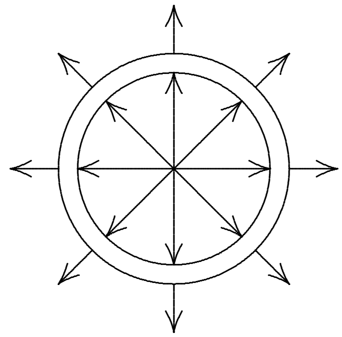
$$E\bigg|_{r<a} = \frac{q}{4\pi\varepsilon_0 r^2} .$$

(d) For $b > r > a$ $\quad E = 0$, since this region is inside the metallic part of the shell.

(e) For $r > b$ $\quad q_{\text{encl}} = +q$, so

$$E\bigg|_{r<a} = \frac{q}{4\pi\varepsilon_0 r^2} .$$

The field lines are sketched to the right.

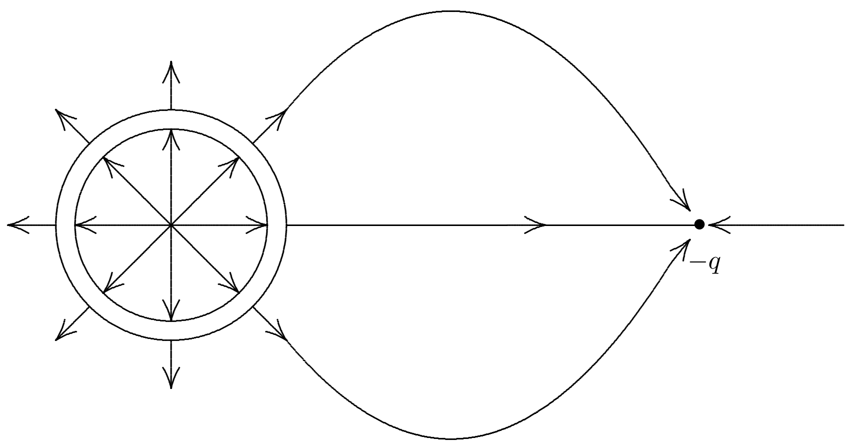


(f) The net charge of the central point charge-inner surface combination is zero. Thus the electric field it produces is also zero.

(g) The outer shell has a spherically symmetric charge distribution with a net charge $+q$. Thus the field it produces for $r > b$ is $E = q/(4\pi\varepsilon_0 r^2)$.

(h) Yes. In fact there will be a distribution of induced charges on the outer shell, as a result of a flow of positive charges toward the side of the surface that is closer to the negative point charge outside the shell.

(i) No. The change in the charge distribution on the outer shell cancels the effect of the negative point charge. The field lines are sketched below.



(j) Yes, there is a force on the $-q$ point charge, as expected from Eq. 22-4.

(k) The field lines around the first charge at the center of the spherical shell is unchanged. The implication, then, is that there is still no net force on that charge.

(l) We assume there is some non-electrical force holding the spherical shell in place, which compensates for the force of the $-q$ point charge exerted on the outside surface charges on the shell. Newton's third law applies to this situation, as far as the $-q$ point charge and the surface charges on the sphere are concerned. There is no direct force between the central $+q$ charge and the external $-q$ point charge, so we would not apply Newton's third law to their interaction.

41. (a) We integrate the volume charge density over the volume and require the result be equal to the total charge:

$$\int dx \int dy \int dz\, \rho = 4\pi \int_0^R dr\, r^2\, \rho = Q\ .$$

Substituting the expression $\rho = \rho_s r/R$ and performing the integration leads to

$$4\pi \left(\frac{\rho_s}{R}\right)\left(\frac{R^4}{4}\right) = Q \quad\Longrightarrow\quad Q = \pi\rho_s R^3\ .$$

(b) At a certain point within the sphere, at some distance r_o from the center, the field (see Eq. 24-8 through Eq. 24-10) is given by Gauss' law:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\rm enc}}{r_o^2}$$

where $q_{\rm enc}$ is given by an integral similar to that worked in part (a):

$$q_{\rm enc} = 4\pi \int_0^{r_o} dr\, r^2\, \rho = 4\pi \left(\frac{\rho_s}{R}\right)\left(\frac{r_o^4}{4}\right)\ .$$

Therefore,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\rho_s r_o^4}{R\, r_o^2}$$

which (using the relation between ρ_s and Q derived in part (a)) becomes

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\pi\left(\frac{Q}{\pi R^3}\right) r_o^2}{R}$$

and simplifies to the desired result (shown in the problem statement) if we change notation $r_o \to r$.

42. (a) We note that the symbol "e" stands for the elementary charge in the manipulations below. From

$$-e = \int_0^\infty \rho(r) 4\pi r^2 dr = \int_0^\infty A \exp\left(-2r/a_0\right) 4\pi r^2\, dr = \pi a_0^3 A$$

we get $A = -e/\pi a_0^3$.

(b) The magnitude of the field is

$$\begin{aligned} E &= \frac{q_{\rm encl}}{4\pi\varepsilon_0 a_0^2} = \frac{1}{4\pi\varepsilon_0 a_0^2}\left(e + \int_0^{a_0} \rho(r) 4\pi r^2 dr\right) \\ &= \frac{e}{4\pi\varepsilon_0 a_0^2}\left(1 - \frac{4}{a_0^3}\int_0^{a_0} \exp(-2r/a_0)\ r^2\, dr\right) \\ &= \frac{5\,e\,\exp(-2)}{4\pi\varepsilon_0\, a_0^2}\ . \end{aligned}$$

We note that $\vec{E}$ points radially outward.

43. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so

$$\oint \vec{E}\cdot d\vec{A} = 4\pi r^2 E$$

where r is the radius of the Gaussian surface.

(a) Here r is less than a and the charge enclosed by the Gaussian surface is $q(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \implies E = \frac{qr}{4\pi\varepsilon_0 a^3} .$$

(b) In this case, r is greater than a but less than b. The charge enclosed by the Gaussian surface is q, so Gauss' law leads to

$$4\pi r^2 E = \frac{q}{\varepsilon_0} \implies E = \frac{q}{4\pi\varepsilon_0 r^2} .$$

(c) The shell is conducting, so the electric field inside it is zero.

(d) For $r > c$, the charge enclosed by the Gaussian surface is zero (charge q is inside the shell cavity and charge $-q$ is on the shell). Gauss' law yields

$$4\pi r^2 E = 0 \implies E = 0 .$$

(e) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E} \cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q + Q_i = 0$ and $Q_i = -q$. Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q$. This means $Q_o = -q - Q_i = -q - (-q) = 0$.

44. The field is zero for $0 \le r \le a$ as a result of Eq. 24-16. Since q_{enc} (for $a \le r \le b$) is related to the volume by

$$q_{\text{enc}} = \rho\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right)$$

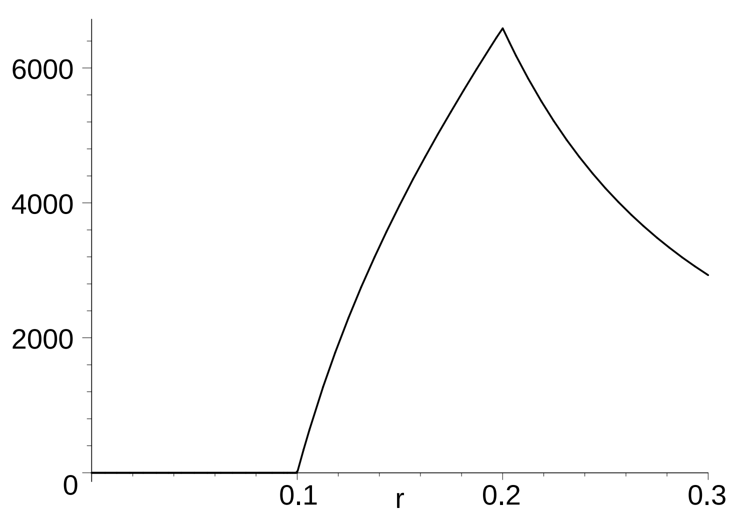
then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} = \frac{\rho}{4\pi\varepsilon_0 r^2}\left(\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3}\right) = \frac{\rho}{3\varepsilon_0}\frac{r^3 - a^3}{r^2}$$

for $a \le r \le b$. And for $r \ge b$ we have $E = q_{\text{total}}/4\pi\varepsilon_0 r^2$ or

$$E = \frac{\rho}{3\varepsilon_0}\frac{b^3 - a^3}{r^2} \qquad r \ge b .$$

This is plotted below for r in meters from 0 to 0.30 m. The peak value of the electric field, reached at $r = b = 0.20$ m, is 6.6×10^3 N/C.



45. To find an expression for the electric field inside the shell in terms of A and the distance from the center of the shell, select A so the field does not depend on the distance. We use a Gaussian surface in the form of a sphere with radius r_g, concentric with the spherical shell and within it ($a < r_g < b$). Gauss' law will be used to find the magnitude of the electric field a distance r_g from the shell center. The charge that is both in the shell and within the Gaussian sphere is given by the integral $q_s = \int \rho \, dV$ over the portion of the shell within the Gaussian surface. Since the charge distribution has spherical symmetry, we may take dV to be the volume of a spherical shell with radius r and infinitesimal thickness dr: $dV = 4\pi r^2 \, dr$. Thus,

$$q_s = 4\pi \int_a^{r_g} \rho r^2 \, dr = 4\pi \int_a^{r_g} \frac{A}{r} r^2 \, dr = 4\pi A \int_a^{r_g} r \, dr = 2\pi A(r_g^2 - a^2) \; .$$

The total charge inside the Gaussian surface is $q + q_s = q + 2\pi A(r_g^2 - a^2)$. The electric field is radial, so the flux through the Gaussian surface is $\Phi = 4\pi r_g^2 E$, where E is the magnitude of the field. Gauss' law yields

$$4\pi\varepsilon_0 E r_g^2 = q + 2\pi A(r_g^2 - a^2) \; .$$

We solve for E:

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_g^2} + 2\pi A - \frac{2\pi A a^2}{r_g^2}\right] \; .$$

For the field to be uniform, the first and last terms in the brackets must cancel. They do if $q - 2\pi A a^2 = 0$ or $A = q/2\pi a^2$.

46. (a) From Gauss' law,

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{encl}}}{r^3}\vec{r} = \frac{1}{4\pi\varepsilon_0}\frac{(4\pi\rho r^3/3)\vec{r}}{r^3} = \frac{\rho\vec{r}}{3\varepsilon_0} \; .$$

(b) The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ which fills the void. By superposition

$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)(\vec{r} - \vec{a})}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0} \; .$$

47. We use

$$E(r) = \frac{q_{\text{encl}}}{4\pi\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\int_0^r \rho(r) 4\pi r^2 \, dr$$

to solve for $\rho(r)$:

$$\rho(r) = \frac{\varepsilon_0}{r^2}\frac{d}{dr}[r^2 E(r)] = \frac{\varepsilon_0}{r^2}\frac{d}{dr}(K r^6) = 6K\varepsilon_0 r^3 \; .$$

48. (a) We consider the radial field produced at points within a uniform cylindrical distribution of charge. The volume enclosed by a Gaussian surface in this case is $L\pi r^2$. Thus, Gauss' law leads to

$$E = \frac{|q_{\text{enc}}|}{\varepsilon_0 A_{\text{cylinder}}} = \frac{|\rho|\left(L\pi r^2\right)}{\varepsilon_0\,(2\pi r L)} = \frac{|\rho| r}{2\varepsilon_0} \; .$$

(b) We note from the above expression that the magnitude of the radial field grows with r.

(c) Since the charged powder is negative, the field points radially inward.

(d) The largest value of r which encloses charged material is $r_{\text{max}} = R$. Therefore, with $|\rho| = 0.0011$ C/m^3 and $R = 0.050$ m, we obtain

$$E_{\text{max}} = \frac{|\rho| R}{2\varepsilon_0} = 3.1 \times 10^6 \text{ N/C} \; .$$

(e) According to condition 1 mentioned in the problem, the field is high enough to produce an electrical discharge (at $r = R$).

49. (a) At A, the only field contribution is from the $+5.00Q$ particle in the hollow (this follows from Gauss' law – it is the only charge enclosed by a Gaussian spherical surface passing through point A, concentric with the shell). Thus, using k for $1/4\pi\varepsilon_0$, we have $\vec{E} = k(5Q)/(0.5)^2 = 20kQ$ directed radially outward.

    (b) Point B is in the conducting material, where the field must be zero in any electrostatic situation.

    (c) Point C is outside the sphere where the net charge at smaller values of radius is $-3.00Q + 5.00Q = 2.00Q$. Therefore, we have $\vec{E} = k(2Q)/(2)^2 = \frac{1}{2}kQ$ directed radially outward.

50. Since the fields involved are uniform, the precise location of P are not relevant. Since the sheets are oppositely charged (though not equally so), the field contributions are additive (since P is between them). Using Eq. 24-13, we obtain

$$\vec{E} = \frac{\sigma_1}{2\varepsilon_0} + \frac{3\sigma_1}{2\varepsilon_0} = \frac{2\sigma_1}{\varepsilon_0}$$

directed towards the negatively charged sheet.

51. (a) We imagine a Gaussian surface A which is just outside the inner surface of the spherical shell. Then $\vec{E}$ is zero everywhere on surface A. Thus

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{(Q' + Q)}{\varepsilon_0} = 0 \; ,$$

where Q' is the charge on the inner surface of the shell. This gives $Q' = -Q$.

    (b) Since $\vec{E}$ remains zero on surface A the result is unchanged.

    (c) Now,

$$\oint_A \vec{E} \cdot d\vec{A} = \frac{(Q' + q + Q)}{\varepsilon_0} = 0 \; ,$$

so $Q' = -(Q + q)$.

    (d) Yes, since $\vec{E}$ remains zero on surface A regardless of where you place the sphere inside the shell.

52. We choose a coordinate system whose origin is at the center of the flat base, such that the base is in the xy plane and the rest of the hemisphere is in the $z > 0$ half space.

    (a) $\Phi = \pi R^2(-\hat{\mathrm{k}}) \cdot E\hat{\mathrm{k}} = -\pi R^2 E$.

    (b) Since the flux through the entire hemisphere is zero, the flux through the curved surface is $\vec{\Phi}_c = -\Phi_{\text{base}} = \pi R^2 E$.

53. Let $\Phi_0 = 10^3\,\mathrm{N{\cdot}m^2/C}$. The net flux through the entire surface of the dice is given by

$$\Phi = \sum_{n=1}^{6} \Phi_n = \sum_{n=1}^{6} (-1)^n n\,\Phi_0 = \Phi_0(-1 + 2 - 3 + 4 - 5 + 6) = 3\Phi_0 \; .$$

Thus, the net charge enclosed is

$$q = \varepsilon_0\Phi = 3\varepsilon_0\Phi_0 = 3\left(8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)\left(10^3\,\mathrm{N{\cdot}m^2/C}\right) = 2.66 \times 10^{-8}\ \mathrm{C} \; .$$

54. We use $\Phi = \int \vec{E} \cdot d\vec{A}$. We note that the side length of the cube is $3.0\,\mathrm{m} - 1.0\,\mathrm{m} = 2.0\,\mathrm{m}$.

    (a) On the top face of the cube $y = 2.0\,\mathrm{m}$ and $d\vec{A} = (dA)\hat{\mathrm{j}}$. So $\vec{E} = 4\hat{\mathrm{i}} - 3((2.0)^2 + 2)\hat{\mathrm{j}} = 4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}$. Thus the flux is

$$\begin{aligned} \Phi &= \int_{\text{top}} \vec{E} \cdot d\vec{A} = \int_{\text{top}} (4\hat{\mathrm{i}} - 18\hat{\mathrm{j}}) \cdot (dA)\hat{\mathrm{j}} \\ &= -18 \int_{\text{top}} dA = (-18)(2.0)^2\,\mathrm{N{\cdot}m^2/C} = -72\ \mathrm{N{\cdot}m^2/C} \; . \end{aligned}$$

(b) On the bottom face of the cube $y = 0$ and $d\vec{A} = (dA)(-\hat{\jmath})$. So $\vec{E} = 4\hat{\imath} - 3(0^2 + 2)\hat{\jmath} = 4\hat{\imath} - 6\hat{\jmath}$. Thus, the flux is

$$\begin{aligned}\Phi &= \int_{\text{bottom}} \vec{E} \cdot d\vec{A} = \int_{\text{bottom}} (4\hat{\imath} - 6\hat{\jmath}) \cdot (dA)(-\hat{\jmath}) \\ &= 6 \int_{\text{bottom}} dA = 6(2.0)^2\,\text{N}\cdot\text{m}^2/\text{C} = +24\ \text{N}\cdot\text{m}^2/\text{C}\ .\end{aligned}$$

(c) On the left face of the cube $d\vec{A} = (dA)(-\hat{\imath})$. So

$$\begin{aligned}\Phi &= \int_{\text{left}} \vec{E} \cdot d\vec{A} = \int_{\text{left}} (4\hat{\imath} + E_y\hat{\jmath}) \cdot (dA)(-\hat{\imath}) \\ &= -4 \int_{\text{bottom}} dA = -4(2.0)^2\,\text{N}\cdot\text{m}^2/\text{C} = -16\ \text{N}\cdot\text{m}^2/\text{C}\ .\end{aligned}$$

(d) On the back face of the cube $d\vec{A} = (dA)(-\hat{k})$. But since $\vec{E}$ has no z component $\vec{E} \cdot d\vec{A} = 0$. Thus, $\Phi = 0$.

(e) We now have to add the flux through all six faces. One can easily verify that the flux through the front face is zero, while that through the right face is the opposite of that through the left one, or $+16\,\text{N}\cdot\text{m}^2/\text{C}$. Thus the net flux through the cube is $\Phi = (-72 + 24 - 16 + 0 + 0 + 16)\,\text{N}\cdot\text{m}^2/\text{C} = -48\ \text{N}\cdot\text{m}^2/\text{C}$.

55. The net enclosed charge q is given by

$$q = \varepsilon_0 \Phi = \left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(-48\,\text{N}\cdot\text{m}^2/\text{C}\right) = -4.2 \times 10^{-10}\ \text{C}\ .$$

56. Since the fields involved are uniform, the precise location of P is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward, and (from Eq. 24-13) is magnitude is

$$\left|\vec{E}\right| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0 \times 10^{-6}}{2 \times 8.85 \times 10^{-12}} = 5.6 \times 10^4\ \text{N/C}\ .$$

57. (a) Outside the sphere, we use Eq. 24-15 and obtain

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2} = 1.5 \times 10^4\,\text{N/C outward}\ .$$

(b) With $q = +6.00 \times 10^{-12}$ C, Eq. 24-20 leads to $\vec{E} = 2.5 \times 10^4$ N/C directed outward.

58. (a) and (b) There is no flux through the sides, so we have two contributions to the flux, one from the $x = 2$ end (with $\Phi_2 = +(2+2)(\pi(0.20)^2) = 0.50\ \text{N}\cdot\text{m}^2/\text{C}$) and one from the $x = 0$ end (with $\Phi_0 = -(2)(\pi(0.20)^2)$). By Gauss' law we have $q_{\text{enc}} = \varepsilon_0\,(\Phi_2 + \Phi_0) = 2.2 \times 10^{-12}$ C.

59. (a) The cube is totally within the spherical volume, so the charge enclosed is $\rho V_{\text{cube}} = (500 \times 10^{-9})(0.040)^3 = 3.2 \times 10^{-11}$ C. By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 3.6\ \text{N}\cdot\text{m}^2/\text{C}$.

(b) Now the sphere is totally contained within the cube (note that the radius of the sphere is less than half the side-length of the cube). Thus, the total charge is $q_{\text{enc}}\rho V_{\text{sphere}} = 4.5 \times 10^{-10}$ C. By Gauss' law, we find $\Phi = q_{\text{enc}}/\varepsilon_0 = 51\ \text{N}\cdot\text{m}^2/\text{C}$.

60. We use $\Phi = q_{\text{enclosed}}/\varepsilon_0$ and the fact that the amount of positive (negative) charges on the left (right) side of the conductor is $q\,(-q)$. Thus, $\Phi_1 = q/\varepsilon_0$, $\Phi_2 = -q/\varepsilon_0$, $\Phi_3 = q/\varepsilon_0$, $\Phi_4 = (q-q)/\varepsilon_0 = 0$, and $\Phi_5 = (q+q-q)/\varepsilon_0 = q/\varepsilon_0$.

61. (a) For $r < R$, $E = 0$ (see Eq. 24-16).

(b) For r slightly greater than R,

$$\begin{aligned} E_R &= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \approx \frac{q}{4\pi\varepsilon_0 R^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(2.0\times10^{-7}\mathrm{C})}{(0.25\,\mathrm{m})^2} \\ &= 2.9\times10^4\ \mathrm{N/C}\ . \end{aligned}$$

(c) For $r > R$,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = E_R\left(\frac{R}{r}\right)^2 = (2.9\times10^4\,\mathrm{N/C})\left(\frac{0.25\,\mathrm{m}}{3.0\,\mathrm{m}}\right)^2 = 200\ \mathrm{N/C}\ .$$

62. The field due to a sheet of charge is given by Eq. 24-13. Both sheets are horizontal (parallel to the xy plane), producing vertical fields (parallel to the z axis). At points above the $z = 0$ sheet (sheet A), its field points upward (towards $+z$); at points above the $z = 2.0$ sheet (sheet B), its field does likewise. However, below the $z = 2.0$ sheet, its field is oriented downward.

(a) The magnitude of the net field in the region between the sheets is

$$\left|\vec{E}\right| = \frac{\sigma_A}{2\varepsilon_0} - \frac{\sigma_B}{2\varepsilon_0} = 2.8\times10^2\ \mathrm{N/C}\ .$$

(b) The magnitude of the net field at points above both sheets is

$$\left|\vec{E}\right| = \frac{\sigma_A}{2\varepsilon_0} + \frac{\sigma_B}{2\varepsilon_0} = 6.2\times10^2\ \mathrm{N/C}\ .$$

63. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with the charge q situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{\mathrm{net}} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus, $\Phi = q/6\varepsilon_0$.

64. (a) At $x = 0.040$ m, the net field has a rightward $(+x)$ contribution (computed using Eq. 24-13) from the charge lying between $x = -0.050$ m and $x = 0.040$ m, and a leftward $(-x)$ contribution (again computed using Eq. 24-13) from the charge in the region from $x = 0.040$ m to $x = 0.050$ m. Thus, since $\sigma = q/A = \rho V/A = \rho\Delta x$ in this situation, we have

$$\left|\vec{E}\right| = \frac{\rho(0.090\ \mathrm{m})}{2\varepsilon_0} - \frac{\rho(0.010\ \mathrm{m})}{2\varepsilon_0} = 5.4\ \mathrm{N/C}\ .$$

(b) In this case, the field contributions from all layers of charge point rightward, and we obtain

$$\left|\vec{E}\right| = \frac{\rho(0.100\ \mathrm{m})}{2\varepsilon_0} = 6.8\ \mathrm{N/C}\ .$$

65. (a) The direction of the electric field at P_1 is away from q_1 and its magnitude is

$$\left|\vec{E}\right| = \frac{q}{4\pi\varepsilon_0 r_1^2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.0\times10^{-7}\mathrm{C})}{(0.015\,\mathrm{m})^2} = 4.0\times10^6\,\mathrm{N/C}\ .$$

(b) $\vec{E} = 0$, since P_2 is inside the metal.

66. We use Eqs. 24-15, 24-16 and the superposition principle.

(a) $E = 0$ in the region inside the shell.

(b) $E = (1/4\pi\varepsilon_0)(q_a/r^2)$.

(c) $E = (1/4\pi\varepsilon_0)(q_a + q_b)/r^2$.

(d) Since $E = 0$ for $r < a$ the charge on the inner surface of the inner shell is always zero. The charge on the outer surface of the inner shell is therefore q_a. Since $E = 0$ inside the metallic outer shell the net charge enclosed in a Gaussian surface that lies in between the inner and outer surfaces of the outer shell is zero. Thus the inner surface of the outer shell must carry a charge $-q_a$, leaving the charge on the outer surface of the outer shell to be $q_b + q_a$.

67. (a) We use $m_e g = eE = e\sigma/\varepsilon_0$ to obtain the surface charge density.

$$\sigma = \frac{m_e g \varepsilon_0}{e} = \frac{(9.11 \times 10^{-31}\,\text{kg})\,(9.8\,\text{m/s}^2)\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{1.60 \times 10^{-19}\,\text{C}} = 4.9 \times 10^{-22}\ \text{C/m}^2\ .$$

(b) Downward (since the electric force exerted on the electron must be upward).

68. (a) In order to have net charge $-10\ \mu$C when $-14\ \mu$C is known to be on the outer surface, then there must be $+4\ \mu$C on the inner surface (since charges reside on the surfaces of a conductor in electrostatic situations).

(b) In order to cancel the electric field inside the conducting material, the contribution from the $+4\ \mu$C on the inner surface must be canceled by that of the charged particle in the hollow. Thus, the particle's charge is $-4\ \mu$C.

Chapter 25

1. (a) An Ampere is a Coulomb per second, so

$$84\,\mathrm{A\cdot h} = \left(84\,\frac{\mathrm{C\cdot h}}{\mathrm{s}}\right)\left(3600\,\frac{\mathrm{s}}{\mathrm{h}}\right) = 3.0\times 10^5\ \mathrm{C}\ .$$

   (b) The change in potential energy is $\Delta U = q\,\Delta V = (3.0\times 10^5\,\mathrm{C})(12\,\mathrm{V}) = 3.6\times 10^6\,\mathrm{J}$.

2. The magnitude is $\Delta U = e\Delta V = 1.2\times 10^9\,\mathrm{eV} = 1.2\,\mathrm{GeV}$.

3. (a) When charge q moves through a potential difference ΔV, its potential energy changes by $\Delta U = q\,\Delta V$. In this case, $\Delta U = (30\,\mathrm{C})(1.0\times 10^9\,\mathrm{V}) = 3.0\times 10^{10}\,\mathrm{J}$.

   (b) We equate the final kinetic energy $\frac{1}{2}mv^2$ of the automobile to the energy released by the lightning, denoted by $U_{\mathrm{lightning}}$.

$$v = \sqrt{\frac{2\,U_{\mathrm{lightning}}}{m}} = \sqrt{\frac{2(3.0\times 10^{10}\,\mathrm{J})}{1000\,\mathrm{kg}}} = 7.7\times 10^3\ \mathrm{m/s}\ .$$

   (c) We equate the energy required to melt mass m of ice to the energy released by the lightning: $\Delta U = mL_F$, where L_F is the heat of fusion for ice. Thus,

$$m = \frac{\Delta U}{L_F} = \frac{3.0\times 10^{10}\,\mathrm{J}}{3.33\times 10^5\,\mathrm{J/kg}} = 9.0\times 10^4\ \mathrm{kg}\ .$$

4. (a) $V_B - V_A = \Delta U/(-e) = (3.94\times 10^{-19}\,\mathrm{J})/(-1.60\times 10^{-19}\,\mathrm{C}) = -2.46\,\mathrm{V}$.

   (b) $V_C - V_A = V_B - V_A = -2.46\,\mathrm{V}$.

   (c) $V_C - V_B = 0$ (Since C and B are on the same equipotential line).

5. The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E\,dx = V_s - Ex\ ,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by $\Delta V = E\Delta x = (\sigma/2\varepsilon_0)\Delta x$. Thus,

$$\Delta x = \frac{2\varepsilon_0\,\Delta V}{\sigma} = \frac{2\left(8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\right)(50\,\mathrm{V})}{0.10\times 10^{-6}\,\mathrm{C/m^2}} = 8.8\times 10^{-3}\ \mathrm{m}\ .$$

6. (a) $E = F/e = (3.9 \times 10^{-15}\,\text{N})/(1.60 \times 10^{-19}\,\text{C}) = 2.4 \times 10^4\,\text{N/C}$.

   (b) $\Delta V = E\Delta s = (2.4 \times 10^4\,\text{N/C})(0.12\,\text{m}) = 2.9 \times 10^3\,\text{V}$.

7. The potential difference between the wire and cylinder is given, not the linear charge density on the wire. We use Gauss' law to find an expression for the electric field a distance r from the center of the wire, between the wire and the cylinder, in terms of the linear charge density. Then integrate with respect to r to find an expression for the potential difference between the wire and cylinder in terms of the linear charge density. We use this result to obtain an expression for the linear charge density in terms of the potential difference and substitute the result into the equation for the electric field. This will give the electric field in terms of the potential difference and will allow you to compute numerical values for the field at the wire and at the cylinder. For the Gaussian surface use a cylinder of radius r and length ℓ, concentric with the wire and cylinder. The electric field is normal to the rounded portion of the cylinder's surface and its magnitude is uniform over that surface. This means the electric flux through the Gaussian surface is given by $2\pi r\ell E$, where E is the magnitude of the electric field. The charge enclosed by the Gaussian surface is $q = \lambda\ell$, where λ is the linear charge density on the wire. Gauss' law yields $2\pi\varepsilon_0 r\ell E = \lambda\ell$. Thus,

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} .$$

Since the field is radial, the difference in the potential V_c of the cylinder and the potential V_w of the wire is

$$\Delta V = V_w - V_c = -\int_{r_c}^{r_w} E\,\mathrm{d}r = \int_{r_w}^{r_c} \frac{\lambda}{2\pi\varepsilon_0 r}\,\mathrm{d}r = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_c}{r_w} ,$$

where r_w is the radius of the wire and r_c is the radius of the cylinder. This means that

$$\lambda = \frac{2\pi\varepsilon_0\,\Delta V}{\ln(r_c/r_w)}$$

and

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\Delta V}{r\ln(r_c/r_w)} .$$

   (a) We substitute r_c for r to obtain the field at the surface of the wire:

$$\begin{aligned} E &= \frac{\Delta V}{r_w\ln(r_c/r_w)} = \frac{850\,\text{V}}{(0.65\times 10^{-6}\,\text{m})\,\ln\left[(1.0\times 10^{-2}\,\text{m})/(0.65\times 10^{-6}\,\text{m})\right]} \\ &= 1.36\times 10^8\ \text{V/m} . \end{aligned}$$

   (b) We substitute r_c for r to find the field at the surface of the cylinder:

$$\begin{aligned} E &= \frac{\Delta V}{r_c\ln(r_c/r_w)} = \frac{850\,\text{V}}{(1.0\times 10^{-2}\,\text{m})\,\ln\left[(1.0\times 10^{-2}\,\text{m})/(0.65\times 10^{-6}\,\text{m})\right]} \\ &= 8.82\times 10^3\ \text{V/m} . \end{aligned}$$

8. (a) The potential as a function of r is

$$V(r) = V(0) - \int_0^r E(r)\,dr = 0 - \int_0^r \frac{qr}{4\pi\varepsilon_0 R^3}\,dr = -\frac{qr^2}{8\pi\varepsilon_0 R^3} .$$

   (b) $\Delta V = V(0) - V(R) = q/8\pi\varepsilon_0 R$.

   (c) Since $\Delta V = V(0) - V(R) > 0$, the potential at the center of the sphere is higher.

9. (a) We use Gauss' law to find expressions for the electric field inside and outside the spherical charge distribution. Since the field is radial the electric potential can be written as an integral of the field along a sphere radius, extended to infinity. Since different expressions for the field apply in different regions the integral must be split into two parts, one from infinity to the surface of the distribution and one from the surface to a point inside. Outside the charge distribution the magnitude of the field is $E = q/4\pi\varepsilon_0 r^2$ and the potential is $V = q/4\pi\varepsilon_0 r$, where r is the distance from the center of the distribution. This is the same as the field and potential of a point charge at the center of the spherical distribution. To find an expression for the magnitude of the field inside the charge distribution, we use a Gaussian surface in the form of a sphere with radius r, concentric with the distribution. The field is normal to the Gaussian surface and its magnitude is uniform over it, so the electric flux through the surface is $4\pi r^2 E$. The charge enclosed is qr^3/R^3. Gauss' law becomes

$$4\pi\varepsilon_0 r^2 E = \frac{qr^3}{R^3} \, ,$$

so

$$E = \frac{qr}{4\pi\varepsilon_0 R^3} \, .$$

If V_s is the potential at the surface of the distribution ($r = R$) then the potential at a point inside, a distance r from the center, is

$$V = V_s - \int_R^r E\, dr = V_s - \frac{q}{4\pi\varepsilon_0 R^3}\int_R^r r\, dr = V_s - \frac{qr^2}{8\pi\varepsilon_0 R^3} + \frac{q}{8\pi\varepsilon_0 R} \, .$$

The potential at the surface can be found by replacing r with R in the expression for the potential at points outside the distribution. It is $V_s = q/4\pi\varepsilon_0 R$. Thus,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{r^2}{2R^3} + \frac{1}{2R}\right] = \frac{q}{8\pi\varepsilon_0 R^3}(3R^2 - r^2) \, .$$

(b) In problem 8 the electric potential was taken to be zero at the center of the sphere. In this problem it is zero at infinity. According to the expression derived in part (a) the potential at the center of the sphere is $V_c = 3q/8\pi\varepsilon_0 R$. Thus $V - V_c = -qr^2/8\pi\varepsilon_0 R^3$. This is the result of problem 8.

(c) The potential difference is

$$\Delta V = V_s - V_c = \frac{2q}{8\pi\varepsilon_0 R} - \frac{3q}{8\pi\varepsilon_0 R} = -\frac{q}{8\pi\varepsilon_0 R} \, .$$

The expression obtained in problem 8 would give this same value.

(d) Only potential differences have physical significance, not the value of the potential at any particular point. The same value can be added to the potential at every point without changing the electric field, for example. Changing the reference point from the center of the distribution to infinity changes the value of the potential at every point but it does not change any potential differences.

10. (a)

$$W = \int_i^f q_0 \vec{E}\cdot d\vec{s} = \frac{q_0\sigma}{2\varepsilon_0}\int_0^z dz = \frac{q_0\sigma z}{2\varepsilon_0} \, .$$

(b) Since $V - V_0 = -W/q_0 = -\sigma z/2\varepsilon_0$,

$$V = V_0 - \frac{\sigma z}{2\varepsilon_0} \, .$$

11. (a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} \, ,$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r. The Gaussian surface is a sphere of radius r, concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)} \, ,$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3}\right)(r^3 - r_1^3)\rho = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \, .$$

Gauss' law yields

$$4\pi\varepsilon_0 r^2 E = Q\left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3}\right) \quad \Longrightarrow \quad E = \frac{Q}{4\pi\varepsilon_0}\frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)} \, .$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$\begin{aligned} V &= V_s - \int_{r_2}^{r} E\,dr = V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\int_{r_2}^{r}\left(r - \frac{r_1^3}{r^2}\right)dr \\ &= V_s - \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2}\right) \, . \end{aligned}$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\varepsilon_0 r_2$. We make this substitution and collect terms to find

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r_2^3 - r_1^3}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right) \, .$$

Since $\rho = 3Q/4\pi(r_2^3 - r_1^3)$ this can also be written

$$V = \frac{\rho}{3\varepsilon_0}\left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r}\right) \, .$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. We put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{3(r_2^2 - r_1^2)}{2(r_2^3 - r_1^3)} \, ,$$

or in terms of the charge density

$$V = \frac{\rho}{2\varepsilon_0}(r_2^2 - r_1^2) \, .$$

(d) The solutions agree at $r = r_1$ and at $r = r_2$.

12. The charge is

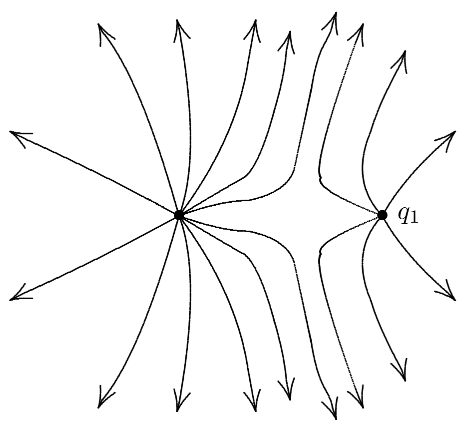
$$q = 4\pi\varepsilon_0 RV = \frac{(10\,\text{m})(-1.0\,\text{V})}{8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = -1.1 \times 10^{-9}\ \text{C} \, .$$

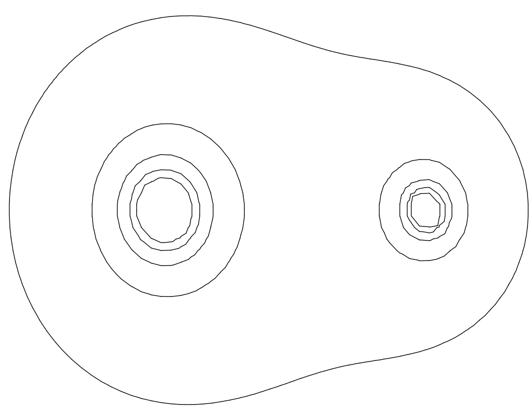
13. (a) The potential difference is

$$
\begin{aligned}
V_A - V_B &= \frac{q}{4\pi\varepsilon_0 r_A} - \frac{q}{4\pi\varepsilon_0 r_B} \\
&= \left(1.0 \times 10^{-6}\,\mathrm{C}\right)\left(8.99 \times 10^{9}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(\frac{1}{2.0\,\mathrm{m}} - \frac{1}{1.0\,\mathrm{m}}\right) = -4500\ \mathrm{V}\ .
\end{aligned}
$$

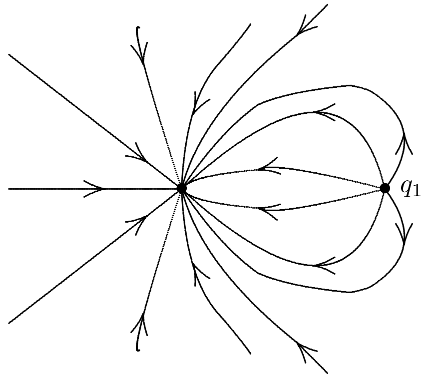
(b) Since $V(r)$ depends only on the magnitude of $\vec{r}$, the result is unchanged.

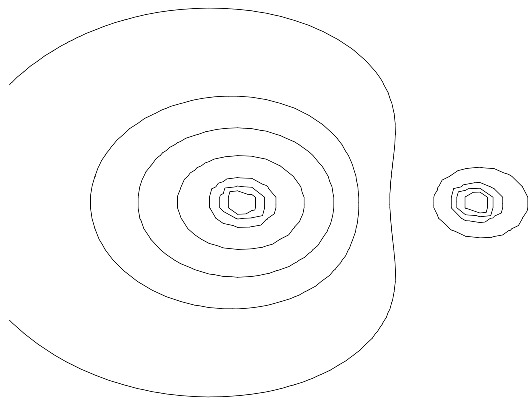
14. In the sketches shown below, the lines with the arrows are field lines and those without are the equipotentials (which become more circular the closer one gets to the individual charges) . In all pictures, q_2 is on the left and q_1 is on the right (which is reversed from the way it is shown in the textbook).

(a) 



(b)





15. First, we observe that $V(x)$ cannot be equal to zero for $x > d$. In fact $V(x)$ is always negative for $x > d$. Now we consider the two remaining regions on the x axis: $x < 0$ and $0 < x < d$. For $x < 0$ the separation between q_1 and a point on the x axis whose coordinate is x is given by $d_1 = -x$; while the corresponding separation for q_2 is $d_2 = d - x$. We set

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{-x} + \frac{-3}{d-x}\right) = 0$$

to obtain $x = -d/2$. Similarly, for $0 < x < d$ we have $d_1 = x$ and $d_2 = d - x$. Let

$$V(x) = k\left(\frac{q_1}{d_1} + \frac{q_2}{d_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{x} + \frac{-3}{d-x}\right) = 0$$

and solve: $x = d/4$.

16. Since according to the problem statement there is a point in between the two charges on the x axis where the net electric field is zero, the fields at that point due to q_1 and q_2 must be directed opposite to each other. This means that q_1 and q_2 must have the same sign (i.e., either both are positive or both negative). Thus, the potentials due to either of them must be of the same sign. Therefore, the net electric potential cannot possibly be zero anywhere except at infinity.

17. (a) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\varepsilon_0 R$. Thus

$$R = \frac{q}{4\pi\varepsilon_0 V} = \frac{(8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})(30 \times 10^{-12}\,\mathrm{C})}{500\,\mathrm{V}} = 5.4 \times 10^{-4}\ \mathrm{m}\ .$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R'} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500\,\text{V}) \quad \approx \quad 790 \text{ V} .$$

18. (a) The charge on the sphere is

$$q = 4\pi\varepsilon_0 VR = \frac{(200\,\text{V})(0.15\,\text{m})}{8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}} = 3.3\times10^{-9} \text{ C} .$$

(b) The (uniform) surface charge density (charge divided by the area of the sphere) is

$$\sigma = \frac{q}{4\pi R^2} = \frac{3.3\times10^{-9}\,\text{C}}{4\pi(0.15\,\text{m})^2} = 1.2\times10^{-8} \text{ C/m}^2 .$$

19. Assume the charge on Earth is distributed with spherical symmetry. If the electric potential is zero at infinity then at the surface of Earth it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on Earth and $R = 6.37\times10^6$ m is the radius of Earth. The magnitude of the electric field at the surface is $E = q/4\pi\varepsilon_0 R^2$, so $V = ER = (100\,\text{V/m})(6.37\times10^6\,\text{m}) = 6.4\times10^8\,\text{V}$.

20. The net electric potential at point P is the sum of those due to the six charges:

$$\begin{aligned}
V_P &= \sum_{i=1}^{6} V_{Pi} = \sum_{i=1}^{6}\frac{q_i}{4\pi\varepsilon_0 r_i} \\
&= \frac{1}{4\pi\varepsilon_0}\left[\frac{5.0q}{\sqrt{d^2+(d/2)^2}} + \frac{-2.0q}{d/2} + \frac{-3.0q}{\sqrt{d^2+(d/2)^2}}\right. \\
&\qquad \left. + \frac{3.0q}{\sqrt{d^2+(d/2)^2}} + \frac{-2.0q}{d/2} + \frac{-5.0q}{\sqrt{d^2+(d/2)^2}}\right] \\
&= \frac{-0.94q}{4\pi\varepsilon_0 d} .
\end{aligned}$$

21. A charge $-5q$ is a distance $2d$ from P, a charge $-5q$ is a distance d from P, and two charges $+5q$ are each a distance d from P, so the electric potential at P is

$$V = \frac{q}{4\pi\varepsilon_0}\left[-\frac{5}{2d} - \frac{5}{d} + \frac{5}{d} + \frac{5}{d}\right] = \frac{5q}{8\pi\varepsilon_0} .$$

The zero of the electric potential was taken to be at infinity.

22. We use Eq. 25-20:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} = \frac{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.47\times3.34\times10^{-30}\,\text{C}\cdot\text{m})}{(52.0\times10^{-9}\,\text{m})^2} = 1.63\times10^{-5} \text{ V} .$$

23. A positive charge q is a distance $r-d$ from P, another positive charge q is a distance r from P, and a negative charge $-q$ is a distance $r+d$ from P. Sum the individual electric potentials created at P to find the total:

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-d} + \frac{1}{r} - \frac{1}{r+d}\right] .$$

We use the binomial theorem to approximate $1/(r-d)$ for r much larger than d:

$$\frac{1}{r-d} = (r-d)^{-1} \approx (r)^{-1} - (r)^{-2}(-d) = \frac{1}{r} + \frac{d}{r^2} .$$

Similarly,

$$\frac{1}{r+d} \approx \frac{1}{r} - \frac{d}{r^2} \ .$$

Only the first two terms of each expansion were retained. Thus,

$$V \approx \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{d}{r^2} + \frac{1}{r} - \frac{1}{r} + \frac{d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} + \frac{2d}{r^2}\right] = \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{2d}{r}\right] \ .$$

24. (a) From Eq. 25-35

$$V = 2\frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L/2 + \sqrt{(L^2/4) + d^2}}{d}\right] \ .$$

(b) The potential at P is $V = 0$ due to superposition.

25. (a) All the charge is the same distance R from C, so the electric potential at C is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} - \frac{6Q}{R}\right] = -\frac{5Q}{4\pi\varepsilon_0 R} \ ,$$

where the zero was taken to be at infinity.

(b) All the charge is the same distance from P. That distance is $\sqrt{R^2 + z^2}$, so the electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\sqrt{R^2+z^2}} - \frac{6Q}{\sqrt{R^2+z^2}}\right] = -\frac{5Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}} \ .$$

26. The potential is

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_{\text{rod}} \frac{dq}{R} = \frac{1}{4\pi\varepsilon_0 R}\int_{\text{rod}} dq = \frac{-Q}{4\pi\varepsilon_0 R} \ .$$

We note that the result is exactly what one would expect for a point-charge $-Q$ at a distance R. This "coincidence" is due, in part, to the fact that V is a scalar quantity.

27. The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P, so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk. We consider a ring of charge with radius r and (infinitesimal) width dr. Its area is $2\pi r\,dr$ and it contains charge $dq = 2\pi\sigma r\,dr$. All the charge in it is a distance $\sqrt{r^2+z^2}$ from P, so the potential it produces at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\sigma r\,dr}{\sqrt{r^2+z^2}} = \frac{\sigma r\,dr}{2\varepsilon_0\sqrt{r^2+z^2}} \ .$$

The total potential at P is

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{r^2+z^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+z^2}\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right] \ .$$

The potential V_{sq} at P due to a single quadrant is

$$V_{sq} = \frac{V}{4} = \frac{\sigma}{8\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right] \ .$$

28. Consider an infinitesimal segment of the rod, located between x and $x+dx$. It has length dx and contains charge $dq = \lambda\,dx$, where $\lambda = Q/L$ is the linear charge density of the rod. Its distance from P_1 is $d+x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dx}{d+x} \ .$$

To find the total potential at P_1, integrate over the rod:

$$V = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{d+x} = \frac{\lambda}{4\pi\varepsilon_0}\ln(d+x)\Big|_0^L = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(1+\frac{L}{d}\right) .$$

29. Consider an infinitesimal segment of the rod, located between x and $x+dx$. It has length dx and contains charge $dq = \lambda\,dx = cx\,dx$. Its distance from P_1 is $d+x$ and the potential it creates at P_1 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{d+x} = \frac{1}{4\pi\varepsilon_0}\frac{cx\,dx}{d+x} .$$

To find the total potential at P_1, integrate over the rod:

$$V = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x\,dx}{d+x} = \frac{c}{4\pi\varepsilon_0}[x - d\,\ln(x+d)]\Big|_0^L = \frac{c}{4\pi\varepsilon_0}\left[L - d\,\ln\left(1+\frac{L}{d}\right)\right] .$$

30. The magnitude of the electric field is given by

$$|E| = \left|-\frac{\Delta V}{\Delta x}\right| = \frac{2(5.0\,\mathrm{V})}{0.015\,\mathrm{m}} = 6.7\times 10^2\ \mathrm{V/m} .$$

At any point in the region between the plates, $\vec{E}$ points away from the positively charged plate, directly towards the negatively charged one.

31. We use Eq. 25-41:

$$\begin{aligned} E_x(x,y) &= -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left((2.0\,\mathrm{V/m^2})x^2 - (3.0\,\mathrm{V/m^2})y^2\right) = -2(2.0\,\mathrm{V/m^2})x ; \\ E_y(x,y) &= -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left((2.0\,\mathrm{V/m^2})x^2 - (3.0\,\mathrm{V/m^2})y^2\right) = 2(3.0\,\mathrm{V/m^2})y . \end{aligned}$$

We evaluate at $x = 3.0\,\mathrm{m}$ and $y = 2.0\,\mathrm{m}$ to obtain the magnitude of $\vec{E}$:

$$E = \sqrt{E_x^2 + E_y^2} = 17\ \mathrm{V/m} .$$

$\vec{E}$ makes an angle θ with the positive x axis, where

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 135^\circ .$$

32. We use Eq. 25-41. This is an ordinary derivative since the potential is a function of only one variable.

$$\begin{aligned} \vec{E} &= -\left(\frac{dV}{dx}\right)\hat{\imath} = -\frac{d}{dx}(1500x^2)\hat{\imath} = (-3000x)\hat{\imath} \\ &= (-3000\,\mathrm{V/m^2})(0.0130\,\mathrm{m})\hat{\imath} = (-39\,\mathrm{V/m})\hat{\imath} . \end{aligned}$$

33. (a) The charge on every part of the ring is the same distance from any point P on the axis. This distance is $r = \sqrt{z^2+R^2}$, where R is the radius of the ring and z is the distance from the center of the ring to P. The electric potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\sqrt{z^2+R^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2+R^2}}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2+R^2}} .$$

(b) The electric field is along the axis and its component is given by

$$\begin{aligned} E &= -\frac{\partial V}{\partial z} = -\frac{q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}(z^2+R^2)^{-1/2} \\ &= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2}\right)(z^2+R^2)^{-3/2}(2z) = \frac{q}{4\pi\varepsilon_0}\frac{z}{(z^2+R^2)^{3/2}} \,. \end{aligned}$$

This agrees with Eq. 23-16.

34. (a) Consider an infinitesimal segment of the rod from x to $x+dx$. Its contribution to the potential at point P_2 is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(x)dx}{\sqrt{x^2+y^2}} = \frac{1}{4\pi\varepsilon_0}\frac{cx}{\sqrt{x^2+y^2}}\,dx \,.$$

Thus,

$$V = \int_{\text{rod}} dV_P = \frac{c}{4\pi\varepsilon_0}\int_0^L \frac{x}{\sqrt{x^2+y^2}}\,dx = \frac{c}{4\pi\varepsilon_0}(\sqrt{L^2+y^2}-y) \,.$$

(b) The y component of the field there is

$$E_y = -\frac{\partial V_P}{\partial y} = -\frac{c}{4\pi\varepsilon_0}\frac{d}{dy}(\sqrt{L^2+y^2}-y) = \frac{c}{4\pi\varepsilon_0}\left(1-\frac{y}{\sqrt{L^2+y^2}}\right) \,.$$

(c) We obtained above the value of the potential at any point P strictly on the y-axis. In order to obtain $E_x(x,y)$ we need to first calculate $V(x,y)$. That is, we must find the potential for an arbitrary point located at $(x,\,y)$. Then $E_x(x,\,y)$ can be obtained from $E_x(x,\,y) = -\partial V(x,\,y)/\partial x$.

35. (a) According to the result of problem 28, the electric potential at a point with coordinate x is given by

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x-L}{x}\right) \,.$$

We differentiate the potential with respect to x to find the x component of the electric field:

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0 L}\frac{\partial}{\partial x}\ln\left(\frac{x-L}{x}\right) = -\frac{Q}{4\pi\varepsilon_0 L}\frac{x}{x-L}\left(\frac{1}{x}-\frac{x-L}{x^2}\right) \\ &= -\frac{Q}{4\pi\varepsilon_0 x(x-L)} \,. \end{aligned}$$

At $x=-d$ we obtain

$$E_x = -\frac{Q}{4\pi\varepsilon_0 d(d+L)} \,.$$

(b) Consider two points an equal infinitesimal distance on either side of P_1, along a line that is perpendicular to the x axis. The difference in the electric potential divided by their separation gives the transverse component of the electric field. Since the two points are situated symmetrically with respect to the rod, their potentials are the same and the potential difference is zero. Thus the transverse component of the electric field is zero.

36. (a) We use Eq. 25-43 with $q_1 = q_2 = -e$ and $r = 2.00\,\text{nm}$:

$$U = k\frac{q_1q_2}{r} = k\frac{e^2}{r} = \frac{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60\times10^{-19}\,\text{C})^2}{2.00\times10^{-9}\,\text{m}} = 1.15\times10^{-19}\ \text{J} \,.$$

(b) Since $U>0$ and $U \propto r^{-1}$ the potential energy U decreases as r increases.

37. We choose the zero of electric potential to be at infinity. The initial electric potential energy U_i of the system before the particles are brought together is therefore zero. After the system is set up the final potential energy is

$$\begin{aligned} U_f &= \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{2}a}\right) \\ &= \frac{2q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}}-2\right) = -\frac{0.21q^2}{\varepsilon_0 a} . \end{aligned}$$

Thus the amount of work required to set up the system is given by $W = \Delta U = U_f - U_i = -0.21q^2/(\varepsilon_0 a)$.

38. The electric potential energy is

$$\begin{aligned} U &= k\sum_{i\neq j}\frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\varepsilon_0 d}\left(q_1q_2+q_1q_3+q_2q_4+q_3q_4+\frac{q_1q_4}{\sqrt{2}}+\frac{q_2q_3}{\sqrt{2}}\right) \\ &= \frac{\left(8.99\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)}{1.3\,\mathrm{m}}\Big[(12)(-24)+(12)(31)+(-24)(17)+(31)(17) \\ &\qquad +\frac{(12)(17)}{\sqrt{2}}+\frac{(-24)(31)}{\sqrt{2}}\Big](10^{-19}\,\mathrm{C})^2 \\ &= -1.2\times10^{-6}\ \mathrm{J} . \end{aligned}$$

39. (a) Let $\ell = 0.15\,\mathrm{m}$ be the length of the rectangle and $w = 0.050\,\mathrm{m}$ be its width. Charge q_1 is a distance ℓ from point A and charge q_2 is a distance w, so the electric potential at A is

$$\begin{aligned} V_A &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{\ell}+\frac{q_2}{w}\right] \\ &= \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\frac{-5.0\times10^{-6}\,\mathrm{C}}{0.15\,\mathrm{m}}+\frac{2.0\times10^{-6}\,\mathrm{C}}{0.050\,\mathrm{m}}\right] \\ &= 6.0\times10^4\ \mathrm{V} . \end{aligned}$$

(b) Charge q_1 is a distance w from point b and charge q_2 is a distance ℓ, so the electric potential at B is

$$\begin{aligned} V_B &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{w}+\frac{q_2}{\ell}\right] \\ &= \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\left[\frac{-5.0\times10^{-6}\,\mathrm{C}}{0.050\,\mathrm{m}}+\frac{2.0\times10^{-6}\,\mathrm{C}}{0.15\,\mathrm{m}}\right] \\ &= -7.8\times10^5\ \mathrm{V} . \end{aligned}$$

(c) Since the kinetic energy is zero at the beginning and end of the trip, the work done by an external agent equals the change in the potential energy of the system. The potential energy is the product of the charge q_3 and the electric potential. If U_A is the potential energy when q_3 is at A and U_B is the potential energy when q_3 is at B, then the work done in moving the charge from B to A is $W = U_A - U_B = q_3(V_A - V_B) = (3.0\times10^{-6}\,\mathrm{C})(6.0\times10^4\,\mathrm{V} + 7.8\times10^5\,\mathrm{V}) = 2.5\,\mathrm{J}$.

(d) The work done by the external agent is positive, so the energy of the three-charge system increases.

(e) and (f) The electrostatic force is conservative, so the work is the same no matter which path is used.

40. The work required is

$$W = \Delta U = \frac{1}{4\pi\varepsilon_0}\left[\frac{(4q)(5q)}{2d}+\frac{(5q)(-2q)}{d}\right] = 0 .$$

41. The particle with charge $-q$ has both potential and kinetic energy, and both of these change when the radius of the orbit is changed. We first find an expression for the total energy in terms of the orbit radius r. Q provides the centripetal force required for $-q$ to move in uniform circular motion. The magnitude of the force is $F = Qq/4\pi\varepsilon_0 r^2$. The acceleration of $-q$ is v^2/r, where v is its speed. Newton's second law yields

$$\frac{Qq}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \implies mv^2 = \frac{Qq}{4\pi\varepsilon_0 r} \ ,$$

and the kinetic energy is $K = \frac{1}{2}mv^2 = Qq/8\pi\varepsilon_0 r$. The potential energy is $U = -Qq/4\pi\varepsilon_0 r$, and the total energy is

$$E = K + U = \frac{Qq}{8\pi\varepsilon_0 r} - \frac{Qq}{4\pi\varepsilon_0 r} = -\frac{Qq}{8\pi\varepsilon_0 r} \ .$$

When the orbit radius is r_1 the energy is $E_1 = -Qq/8\pi\varepsilon_0 r_1$ and when it is r_2 the energy is $E_2 = -Qq/8\pi\varepsilon_0 r_2$. The difference $E_2 - E_1$ is the work W done by an external agent to change the radius:

$$W = E_2 - E_1 = -\frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{Qq}{8\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \ .$$

42. (a) The potential is

$$\begin{aligned} V(r) &= \frac{1}{4\pi\varepsilon_0}\frac{e}{r} \\ &= \frac{\left(8.99 \times 10^9 \, \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(1.60 \times 10^{-19}\,\mathrm{C})}{5.29 \times 10^{-11}\,\mathrm{m}} = 27.2 \ \mathrm{V} \ . \end{aligned}$$

(b) The potential energy is $U = -eV(r) = -27.2\,\mathrm{eV}$.

(c) Since $m_e v^2/r = -e^2/4\pi\varepsilon_0 r^2$,

$$K = \frac{1}{2}mv^2 = -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 r}\right) = -\frac{1}{2}V(r) = \frac{27.2\,\mathrm{eV}}{2} = 13.6 \ \mathrm{eV} \ .$$

(d) The energy required is

$$\Delta E = 0 - [V(r) + K] = 0 - (-27.2\,\mathrm{eV} + 13.6\,\mathrm{eV}) = 13.6 \ \mathrm{eV} \ .$$

43. We use the conservation of energy principle. The initial potential energy is $U_i = q^2/4\pi\varepsilon_0 r_1$, the initial kinetic energy is $K_i = 0$, the final potential energy is $U_f = q^2/4\pi\varepsilon_0 r_2$, and the final kinetic energy is $K_f = \frac{1}{2}mv^2$, where v is the final speed of the particle. Conservation of energy yields

$$\frac{q^2}{4\pi\varepsilon_0 r_1} = \frac{q^2}{4\pi\varepsilon_0 r_2} + \frac{1}{2}mv^2 \ .$$

The solution for v is

$$\begin{aligned} v &= \sqrt{\frac{2q^2}{4\pi\varepsilon_0 m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \\ &= \sqrt{\frac{(8.99 \times 10^9 \mathrm{N\cdot m^2/C^2})(2)(3.1 \times 10^{-6}\,\mathrm{C})^2}{20 \times 10^{-6}\ \mathrm{kg}}\left(\frac{1}{0.90 \times 10^{-3}\,\mathrm{m}} - \frac{1}{2.5 \times 10^{-3}\,\mathrm{m}}\right)} \\ &= 2.5 \times 10^3 \ \mathrm{m/s} \ . \end{aligned}$$

44. Let $r = 1.5\,\mathrm{m}$, $x = 3.0\,\mathrm{m}$, $q_1 = -9.0\,\mathrm{nC}$, and $q_2 = -6.0\,\mathrm{pC}$. The work done by an external agent is given by

$$\begin{aligned} W &= \Delta U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+x^2}}\right) \\ &= (-9.0\times10^{-9}\,\mathrm{C})(-6.0\times10^{-12}\,\mathrm{C})\left(8.99\times10^{9}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\cdot\left[\frac{1}{1.5\,\mathrm{m}} - \frac{1}{\sqrt{(1.5\,\mathrm{m})^2+(3.0\,\mathrm{m})^2}}\right] \\ &= 1.8\times10^{-10}\ \mathrm{J}\ . \end{aligned}$$

45. (a) The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0 d} = \frac{(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2})(5.0\times10^{-6}\,\mathrm{C})^2}{1.00\,\mathrm{m}} = 0.225\ \mathrm{J}$$

relative to the potential energy at infinite separation.

(b) Each sphere repels the other with a force that has magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 d^2} = \frac{(8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2})(5.0\times10^{-6}\,\mathrm{C})^2}{(1.00\,\mathrm{m})^2} = 0.225\ \mathrm{N}\ .$$

According to Newton's second law the acceleration of each sphere is the force divided by the mass of the sphere. Let m_A and m_B be the masses of the spheres. The acceleration of sphere A is

$$a_A = \frac{F}{m_A} = \frac{0.225\,\mathrm{N}}{5.0\times10^{-3}\,\mathrm{kg}} = 45.0\ \mathrm{m/s^2}$$

and the acceleration of sphere B is

$$a_B = \frac{F}{m_B} = \frac{0.225\,\mathrm{N}}{10\times10^{-3}\,\mathrm{kg}} = 22.5\ \mathrm{m/s^2}\ .$$

(c) Energy is conserved. The initial potential energy is $U = 0.225\,\mathrm{J}$, as calculated in part (a). The initial kinetic energy is zero since the spheres start from rest. The final potential energy is zero since the spheres are then far apart. The final kinetic energy is $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, where v_A and v_B are the final velocities. Thus,

$$U = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2\ .$$

Momentum is also conserved, so

$$0 = m_A v_A + m_B v_B\ .$$

These equations may be solved simultaneously for v_A and v_B. Substituting $v_B = -(m_A/m_B)v_A$, from the momentum equation into the energy equation, and collecting terms, we obtain $U = \frac{1}{2}(m_A/m_B)(m_A+m_B)v_A^2$. Thus,

$$\begin{aligned} v_A &= \sqrt{\frac{2Um_B}{m_A(m_A+m_B)}} \\ &= \sqrt{\frac{2(0.225\,\mathrm{J})(10\times10^{-3}\,\mathrm{kg})}{(5.0\times10^{-3}\,\mathrm{kg})(5.0\times10^{-3}\,\mathrm{kg}+10\times10^{-3}\,\mathrm{kg})}} = 7.75\ \mathrm{m/s}\ . \end{aligned}$$

We thus obtain

$$v_B = -\frac{m_A}{m_B}v_A = -\left(\frac{5.0\times10^{-3}\,\mathrm{kg}}{10\times10^{-3}\,\mathrm{kg}}\right)(7.75\,\mathrm{m/s}) = -3.87\ \mathrm{m/s}\ .$$

46. The change in electric potential energy of the electron-shell system as the electron starts from its initial position and just reaches the shell is $\Delta U = (-e)(-V) = eV$. Thus from $\Delta U = K = \frac{1}{2}m_e v_i^2$ we find the initial electron speed to be

$$v_i = \sqrt{\frac{2\Delta U}{m_e}} = \sqrt{\frac{2eV}{m_e}} \ .$$

47. We use conservation of energy, taking the potential energy to be zero when the moving electron is far away from the fixed electrons. The final potential energy is then $U_f = 2e^2/4\pi\varepsilon_0 d$, where d is half the distance between the fixed electrons. The initial kinetic energy is $K_i = \frac{1}{2}mv^2$, where m is the mass of an electron and v is the initial speed of the moving electron. The final kinetic energy is zero. Thus $K_i = U_f$ or $\frac{1}{2}mv^2 = 2e^2/4\pi\varepsilon_0 d$. Hence

$$v = \sqrt{\frac{4e^2}{4\pi\varepsilon_0\, d\, m}} = \sqrt{\frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(4)(1.60\times10^{-19}\,\mathrm{C})^2}{(0.010\,\mathrm{m})(9.11\times10^{-31}\,\mathrm{kg})}} = 3.2\times10^2\ \mathrm{m/s}\ .$$

48. The initial speed v_i of the electron satisfies $K_i = \frac{1}{2}m_e v_i^2 = e\Delta V$, which gives

$$v_i = \sqrt{\frac{2e\Delta V}{m_e}} = \sqrt{\frac{2(1.60\times10^{-19}\,\mathrm{J})(625\,\mathrm{V})}{9.11\times10^{-31}\,\mathrm{kg}}} = 1.48\times10^7\ \mathrm{m/s}\ .$$

49. Let the distance in question be r. The initial kinetic energy of the electron is $K_i = \frac{1}{2}m_e v_i^2$, where $v_i = 3.2\times10^5\,\mathrm{m/s}$. As the speed doubles, K becomes $4K_i$. Thus

$$\Delta U = \frac{-e^2}{4\pi\varepsilon_0 r} = -\Delta K = -(4K_i - K_i) = -3K_i = -\frac{3}{2}m_e v_i^2\ ,$$

or

$$\begin{aligned} r &= \frac{2e^2}{3(4\pi\varepsilon_0)m_e v_i^2} = \frac{2(1.6\times10^{-19}\,\mathrm{C})^2\left(8.99\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)}{3(9.11\times10^{-19}\,\mathrm{kg})(3.2\times10^5\,\mathrm{m/s})^2} \\ &= 1.6\times10^{-9}\ \mathrm{m}\ . \end{aligned}$$

50. Since the electric potential throughout the entire conductor is a constant, the electric potential at its center is also $+400\,\mathrm{V}$.

51. If the electric potential is zero at infinity, then the potential at the surface of the sphere is given by $V = q/4\pi\varepsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\varepsilon_0 rV = \frac{(0.15\,\mathrm{m})(1500\,\mathrm{V})}{8.99\times10^9\,\mathrm{N\cdot m^2/C^2}} = 2.5\times10^{-8}\ \mathrm{C}\ .$$

52. (a) Since the two conductors are connected V_1 and V_2 must be the same.
    (b) Let $V_1 = q_1/4\pi\varepsilon_0 R_1 = V_2 = q_2/4\pi\varepsilon_0 R_2$ and note that $q_1 + q_2 = q$ and $R_2 = 2R_1$. We solve for q_1 and q_2: $q_1 = q/3$, $q_2 = 2q/3$.
    (c) The ratio of surface charge densities is

$$\frac{\sigma_1}{\sigma_2} = \frac{q_1/4\pi R_1^2}{q_2/4\pi R_2^2} = \left(\frac{q_1}{q_2}\right)\left(\frac{R_2}{R_1}\right)^2 = 2\ .$$

53. (a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ $(= 1.0\,\mathrm{m})$ from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1+q_2}{4\pi\varepsilon_0 d/2} = \frac{(8.99\times10^9\,\mathrm{N\cdot m^2/C^2})(1.0\times10^{-8}\,\mathrm{C} - 3.0\times10^{-8}\,\mathrm{C})}{1.0\,\mathrm{m}} = -1.80\times10^2\ \mathrm{V}\ .$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$\begin{aligned} V_1 &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R} + \frac{q_2}{d-R}\right] \\ &= (8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})\left[\frac{1.0 \times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}} - \frac{3.0 \times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}}\right] \\ &= 2.9 \times 10^3\ \mathrm{V}\ . \end{aligned}$$

The potential at the surface of sphere 2 is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{d-R} + \frac{q_2}{R}\right] \\ &= (8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2})\left[\frac{1.0 \times 10^{-8}\,\mathrm{C}}{2.0\,\mathrm{m} - 0.030\,\mathrm{m}} - \frac{3.0 \times 10^{-8}\,\mathrm{C}}{0.030\,\mathrm{m}}\right] \\ &= -8.9 \times 10^3\ \mathrm{V}\ . \end{aligned}$$

54. (a) The magnitude of the electric field is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{(3.0 \times 10^{-8}\,\mathrm{C})\left(8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)}{(0.15\,\mathrm{m})^2} = 1.2 \times 10^4\,\mathrm{N/C}\ .$$

(b) $V = RE = (0.15\,\mathrm{m})(1.2 \times 10^4\,\mathrm{N/C}) = 1.8 \times 10^3\,\mathrm{V}$.

(c) Let the distance be x. Then

$$\Delta V = V(x) - V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R+x} - \frac{1}{R}\right) = -500\,\mathrm{V}\ ,$$

which gives

$$x = \frac{R\Delta V}{-V - \Delta V} = \frac{(0.15\,\mathrm{m})(-500\,\mathrm{V})}{-1800\,\mathrm{V} + 500\,\mathrm{V}} = 5.8 \times 10^{-2}\ \mathrm{m}\ .$$

55. (a) The potential would be

$$\begin{aligned} V_e &= \frac{Q_e}{4\pi\varepsilon_0 R_e} = \frac{4\pi R_e^2 \sigma_e}{4\pi\varepsilon_0 R_e} = 4\pi R_e \sigma_e k \\ &= 4\pi(6.37 \times 10^6\,\mathrm{m})(1.0\,\mathrm{electron/\,m^2})(-1.6 \times 10^{-19}\,\mathrm{C/\,electron})\left(8.99 \times 10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right) \\ &= -0.12\ \mathrm{V}\ . \end{aligned}$$

(b) The electric field is

$$E = \frac{\sigma_e}{\varepsilon_0} = \frac{V_e}{R_e} = -\frac{0.12\,\mathrm{V}}{6.37 \times 10^6\,\mathrm{m}} = -1.8 \times 10^{-8}\ \mathrm{N/C}\ ,$$

where the minus sign indicates that $\vec{E}$ is radially inward.

56. Since the charge distribution is spherically symmetric we may write

$$E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\mathrm{encl}}}{r}\ ,$$

where q_{encl} is the charge enclosed in a sphere of radius r centered at the origin. Also, Eq. 25-18 is implemented in the form: $V(r) - V(r') = \int_r^{r'} E(r)\,dr$. The results are as follows: For $r > R_2 > R_1$

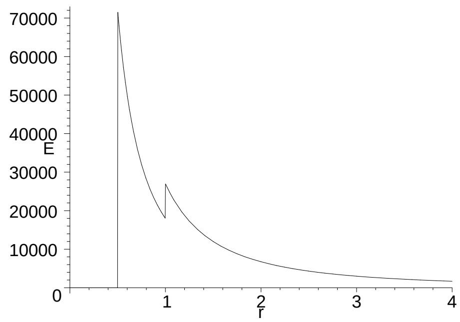
$$V(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r} \quad \text{and} \quad E(r) = \frac{q_1 + q_2}{4\pi\varepsilon_0 r^2} \ .$$

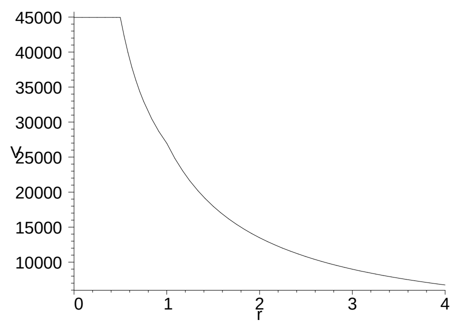
For $R_2 > r > R_1$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{R_2}\right) \quad \text{and} \quad E(r) = \frac{q_1}{4\pi\varepsilon_0 r^2} \ .$$

Finally, for $R_2 > R_1 > r$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{R_1} + \frac{q_2}{R_2}\right) \quad \text{and} \quad E = 0 \ .$$





57. (a) We use Eq. 25-18 to find the potential:

$$\begin{aligned} V_{\text{wall}} - V &= -\int_r^R E\,dr \\ 0 - V &= -\int_r^R \left(\frac{\rho r}{2\varepsilon_0}\right) \\ -V &= -\frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right) \ . \end{aligned}$$

Consequently, $V = \frac{\rho}{4\varepsilon_0}\left(R^2 - r^2\right)$.

(b) The value at $r = 0$ is

$$V_{\text{center}} = \frac{-1.1 \times 10^{-3}\,\text{C/m}^3}{4\,(8.85 \times 10^{-12}\,\text{C/V·m})}\left((0.05\,\text{m})^2 - 0\right) = -7.8 \times 10^4 \ \text{V} \ .$$

58. We treat the system as a superposition of a disk of surface charge density σ and radius R and a smaller, oppositely charged, disk of surface charge density $-\sigma$ and radius r. For each of these, Eq 25-37 applies (for $z > 0$)

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right) + \frac{-\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + r^2} - z\right) \ .$$

This expression does vanish as $r \to \infty$, as the problem requires. Substituting $r = R/5$ and $z = 2R$ and simplifying, we obtain

$$V = \frac{\sigma R}{\varepsilon_0}\left(\frac{5\sqrt{5} - \sqrt{101}}{10}\right) \approx \frac{\sigma R}{\varepsilon_0}(0.113) \ .$$

59. We use $q = 1.37 \times 10^5$ C from Sample Problem 22-7 and $k = 1/4\pi\varepsilon_0$ to find the potential:

$$V = \frac{q}{4\pi\varepsilon_0 R_e} = \frac{(1.37 \times 10^5\,\text{C})\left(8.99 \times 10^9\,\frac{\text{N·m}^2}{\text{C}^2}\right)}{6.37 \times 10^6\,\text{m}} = 1.93 \times 10^8 \ \text{V} \ .$$

60. (a) The potential on the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{(4.0 \times 10^{-6}\,\text{C})\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)}{0.10\,\text{m}} = 3.6 \times 10^5\ \text{V}\ .$$

(b) The field just outside the sphere would be

$$E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{V}{R} = \frac{3.6 \times 10^5\,\text{V}}{0.10\,\text{m}} = 3.6 \times 10^6\ \text{V/m}\ ,$$

which would have exceeded 3.0 MV/m. So this situation cannot occur.

61. If the electric potential is zero at infinity then at the surface of a uniformly charged sphere it is $V = q/4\pi\varepsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. Thus $q = 4\pi\varepsilon_0 RV$ and the number of electrons is

$$N = \frac{|q|}{e} = \frac{4\pi\varepsilon_0 R|V|}{e} = \frac{(1.0 \times 10^{-6}\,\text{m})(400\text{V})}{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\,\text{C})} = 2.8 \times 10^5\ .$$

62. This can be approached more than one way, but the simplest is to observe that the net potential (using Eq. 25-27) due to the $+2q$ and $-2q$ charges is zero at both the initial and final positions of the movable charge ($+5q$). This implies that no work is necessary to effect its change of position, which, in turn, implies there is no resulting change in potential energy of the configuration. Hence, the ratio is unity.

63. We imagine moving all the charges on the surface of the sphere to the center of the the sphere. Using Gauss' law, we see that this would not change the electric field *outside* the sphere. The magnitude of the electric field E of the uniformly charged sphere as a function of r, the distance from the center of the sphere, is thus given by $E(r) = q/(4\pi\varepsilon_0 r^2)$ for $r > R$. Here R is the radius of the sphere. Thus, the potential V at the surface of the sphere (where $r = R$) is given by

$$\begin{aligned} V(R) &= V\Big|_{r=\infty} + \int_R^\infty E(r)\,dr = \int_\infty^R \frac{q}{4\pi\varepsilon_0 r^2}\,dr = \frac{q}{4\pi\varepsilon_0 R} \\ &= \frac{\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.50 \times 10^8\,\text{C})}{0.160\,\text{m}} = 8.43 \times 10^2\ \text{V}\ . \end{aligned}$$

64. We use $E_x = -dV/dx$, where dV/dx is the local slope of the V vs. x curve depicted in Fig. 25-54. The results are: $E_x(ab) = -6.0\,\text{V/m}$, $E_x(bc) = 0$, $E_x(cd) = E_x(de) = 3.0\,\text{V/m}$, $E_x(ef) = 15\,\text{V/m}$, $E_x(fg) = 0$, $E_x(gh) = -3.0\,\text{V/m}$. Since these values are constant during their respective time-intervals, their graph consists of several disconnected line-segments (horizontal) and is not shown here in the interest of saving space.

65. On the dipole axis $\theta = 0$ or π, so $|\cos\theta| = 1$. Therefore, magnitude of the electric field is

$$|E(r)| = \left| -\frac{\partial V}{\partial r} \right| = \frac{p}{4\pi\varepsilon_0}\left|\frac{d}{dr}\left(\frac{1}{r^2}\right)\right| = \frac{p}{2\pi\varepsilon_0 r^3}\ .$$

66. (a) We denote the surface charge density of the disk as σ_1 for $0 < r < R/2$, and as σ_2 for $R/2 < r < R$. Thus the total charge on the disk is given by

$$\begin{aligned} q &= \int_{\text{disk}} dq = \int_0^{R/2} 2\pi\sigma_1 r\,dr + \int_{R/2}^R 2\pi\sigma_2 r\,dr = \frac{\pi}{4}R^2(\sigma_1 + 3\sigma_2) \\ &= \frac{\pi}{4}(2.20 \times 10^{-2}\,\text{m})^2[1.50 \times 10^{-6}\,\text{C/m}^2 + 3(8.00 \times 10^{-7}\,\text{C/m}^2)] \\ &= 1.48 \times 10^{-9}\,\text{C}\ . \end{aligned}$$

(b) We use Eq. 25-36:

$$
\begin{aligned}
V(z) &= \int_{\text{disk}} dV = k\left[\int_0^{R/2} \frac{\sigma_1(2\pi R')dR'}{\sqrt{z^2+R'^2}} + \int_{R/2}^{R} \frac{\sigma_2(2\pi R')dR'}{\sqrt{z^2+R'^2}}\right] \\
&= \frac{\sigma_1}{2\varepsilon_0}\left(\sqrt{z^2+\frac{R^2}{4}} - z\right) + \frac{\sigma_2}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - \sqrt{z^2+\frac{R^2}{4}}\right) .
\end{aligned}
$$

Substituting the numerical values of σ_1, σ_2, R and z, we obtain $V(z) = 7.95 \times 10^2$ V.

67. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

which integrates, using Eq. 25-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f} \frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right) .$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which will be the z axis described in this problem) at a distance $r_f = a$ from the wire. In the "end-view" presented below, the wires and the z axis appear as points as they intersect the xy plane. The potential due to the wire on the left (intersecting the plane at $x = -a$) is

$$V_{\text{negative wire}} = V_o + \frac{(-\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right) ,$$

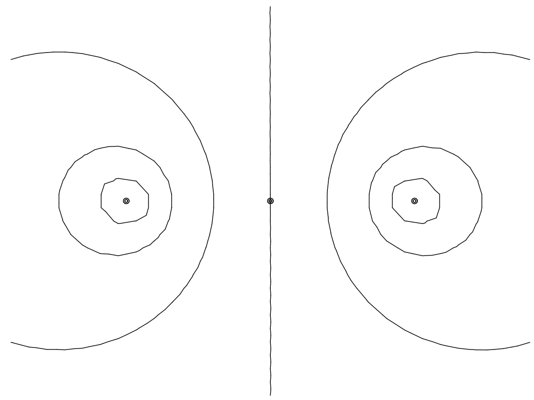
and the potential due to the wire on the right (intersecting the plane at $x = +a$) is

$$V_{\text{positive wire}} = V_o + \frac{(+\lambda)}{2\pi\varepsilon_0}\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) .$$

Since potential is a scalar quantity, the net potential at point P is the addition of $V_{-\lambda}$ and $V_{+\lambda}$ which simplifies to

$$V_{\text{net}} = 2V_o + \frac{\lambda}{2\pi\varepsilon_0}\left(\ln\left(\frac{a}{\sqrt{(x-a)^2+y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2+y^2}}\right)\right) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$$

where we have set the potential along the z axis equal to zero ($V_o = 0$) in the last step (which we are free to do). This is the expression used to obtain the equipotentials shown below. The center dot in the figure is the intersection of the z axis with the xy plane, and the dots on either side are the intersections of the wires with the plane.



68. The potential difference is $\Delta V = E\Delta s = (1.92 \times 10^5\,\mathrm{N/C})(0.0150\,\mathrm{m}) = 2.90 \times 10^3\,\mathrm{V}$.

69. Since the charge distribution on the arc is equidistant from the point where V is evaluated, its contribution is identical to that of a point charge at that distance. We assume $V \to 0$ as $r \to \infty$ and apply Eq. 25-27:

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+4Q}{2R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{R}$$

which simplifies to $Q/4\pi\varepsilon_0 R$.

70. From the previous chapter, we know that the radial field due to an infinite line-source is

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

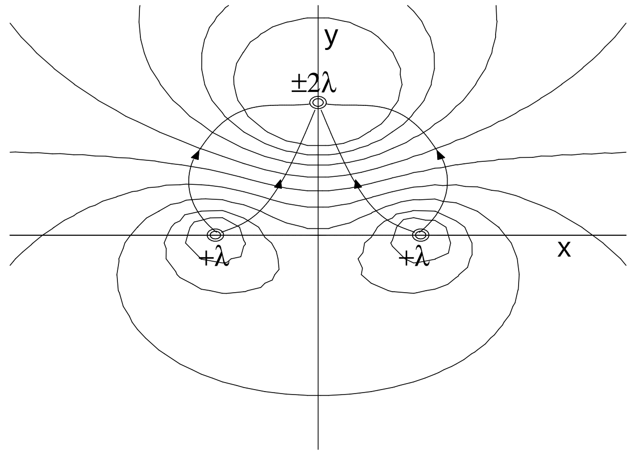
which integrates, using Eq. 25-18, to obtain

$$V_i = V_f + \frac{\lambda}{2\pi\varepsilon_0}\int_{r_i}^{r_f}\frac{dr}{r} = V_f + \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r_f}{r_i}\right) .$$

The subscripts i and f are somewhat arbitrary designations, and we let $V_i = V$ be the potential of some point P at a distance $r_i = r$ from the wire and $V_f = V_o$ be the potential along some reference axis (which intersects the plane of our figure, shown below, at the xy coordinate origin, placed midway between the bottom two line charges – that is, the midpoint of the bottom side of the equilateral triangle) at a distance $r_f = a$ from each of the bottom wires (and a distance $a\sqrt{3}$ from the topmost wire). Thus, each side of the triangle is of length $2a$. Skipping some steps, we arrive at an expression for the net potential created by the three wires (where we have set $V_o = 0$):

$$V_{\text{net}} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{(x^2 + (y - a\sqrt{3})^2)^2}{((x+a)^2 + y^2)((x-a)^2 + y^2)}\right)$$

which forms the basis of our contour plot shown below. On the same plot we have shown four electric field lines, which have been sketched (as opposed to rigorously calculated) and are not meant to be as accurate as the equipotentials. The $\pm 2\lambda$ by the top wire in our figure should be -2λ (the $\pm$ typo is an artifact of our plotting routine).



71. The charges are equidistant from the point where we are evaluating the potential – which is computed using Eq. 25-27 (or its integral equivalent). Eq. 25-27 implicitly assumes $V \to 0$ as $r \to \infty$. Thus, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{+Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{+3Q}{R}$$

which simplifies to $Q/2\pi\varepsilon_0 R$.

72. The radius of the cylinder (0.020 m, the same as r_B) is denoted R, and the field magnitude there (160 N/C) is denoted E_B. The electric field beyond the surface of the sphere follows Eq. 24-12, which expresses inverse proportionality with r:

$$\frac{|\vec{E}|}{E_B} = \frac{R}{r} \qquad \text{for } r \geq R\ .$$

(a) Thus, if $r = r_C = 0.050$ m, we obtain $|\vec{E}| = (160)(0.020)/(0.050) = 64$ N/C.

(b) Integrating the above expression (where the variable to be integrated, r, is now denoted ϱ) gives the potential difference between V_B and V_C.

$$V_B - V_C = \int_R^r \frac{E_B\,R}{\varrho}\,d\varrho = E_B\,R\,\ln\left(\frac{r}{R}\right) = 2.9 \text{ V}\ .$$

(c) The electric field throughout the conducting volume is zero, which implies that the potential there is constant and equal to the value it has on the surface of the charged cylinder: $V_A - V_B = 0$.

73. The net potential (at point A or B) is computed using Eq. 25-27. Thus, using k for $1/4\pi\varepsilon_0$, the difference is

$$\begin{aligned} V_B - V_A &= \left(\frac{kq}{2d} + \frac{k(-5q)}{2d}\right) - \left(\frac{kq}{d} + \frac{k(-5q)}{5d}\right) \\ &= -\frac{4kq}{2d} \end{aligned}$$

which simplifies to $-q/2\pi\varepsilon_0$ in SI units (with $d = 1$ m).

74. Eq. 25-32 applies with $dq = \lambda\,dx = bx\,dx$ (along $0 \leq x \leq 0.20$ m).

(a) Here $r = x > 0$, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{x} = \frac{b(0.20)}{4\pi\varepsilon_0}$$

which yields $V = 36$ V.

(b) Now $r = \sqrt{x^2 + d^2}$ where $d = 0.15$ m, so that

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^{0.20} \frac{bx\,dx}{\sqrt{x^2+d^2}} = \frac{b}{4\pi\varepsilon_0}\left(\sqrt{x^2+d^2}\right)\Big|_0^{0.20}$$

which yields $V = 18$ V.

75. (a) Using Eq. 25-26, we calculate the radius r of the sphere representing the 30 V equipotential surface:

$$r = \frac{q}{4\pi\varepsilon_0 V} = 4.5 \text{ m} .$$

(b) If the potential were a linear function of r then it would have equally spaced equipotentials, but since $V \propto 1/r$ they are spaced more and more widely apart as r increases.

76. We denote $q = 25 \times 10^{-9}$ C, $y = 0.6$ m, $x = 0.8$ m, with $V =$ the net potential (assuming $V \to 0$ as $r \to \infty$). Then,

$$\begin{aligned} V_A &= \frac{1}{4\pi\varepsilon_0}\frac{q}{y} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{x} \\ V_B &= \frac{1}{4\pi\varepsilon_0}\frac{q}{x} + \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{y} \end{aligned}$$

leads to

$$V_B - V_A = \frac{2}{4\pi\varepsilon_0}\frac{q}{x} - \frac{2}{4\pi\varepsilon_0}\frac{q}{y} = \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{x} - \frac{1}{y}\right)$$

which yields $\Delta V = -187 \approx -190$ V.

77. (a) By Eq. 25-18, the change in potential is the negative of the "area" under the curve. Thus, using the area-of-a-triangle formula, we have

$$V - 10 = -\int_0^{x=2} \vec{E}\cdot d\vec{s} = \frac{1}{2}(2)(20)$$

which yields $V = 30$ V.

(b) For any region within $0 < x < 3$ m, $-\int \vec{E}\cdot d\vec{s}$ is positive, but for any region for which $x > 3$ m it is negative. Therefore, $V = V_{\max}$ occurs at $x = 3$ m.

$$V - 10 = -\int_0^{x=3} \vec{E}\cdot d\vec{s} = \frac{1}{2}(3)(20)$$

which yields $V_{\max} = 40$ V.

(c) In view of our result in part (b), we see that now (to find $V = 0$) we are looking for some $X > 3$ m such that the "area" from $x = 3$ m to $x = X$ is 40 V. Using the formula for a triangle ($3 < x < 4$) and a rectangle ($4 < x < X$), we require

$$\frac{1}{2}(1)(20) + (X - 4)(20) = 40 \quad .$$

Therefore, $X = 5.5$ m.

78. In the "inside" region between the plates, the individual fields (given by Eq. 24.13) are in the same direction ($-\hat{\imath}$):

$$\vec{E}_{\text{in}} = -\left(\frac{50 \times 10^{-9}}{2\varepsilon_0} + \frac{25 \times 10^{-9}}{2\varepsilon_0}\right)\hat{\imath} = -4.2 \times 10^3\,\hat{\imath}$$

in SI units (N/C or V/m). And in the "outside" region where $x > 0.5$ m, the individual fields point in opposite directions:

$$\vec{E}_{\text{out}} = -\frac{50 \times 10^{-9}}{2\varepsilon_0}\hat{\imath} + \frac{25 \times 10^{-9}}{2\varepsilon_0}\hat{\imath} = -1.4 \times 10^3\,\hat{\imath}\ .$$

Therefore, by Eq. 25-18, we have

$$\begin{aligned}\Delta V = -\int_0^{0.8} \vec{E} \cdot d\vec{s} &= -\int_0^{0.5} |\vec{E}|_{\text{in}}\, dx - \int_{0.5}^{0.8} |\vec{E}|_{\text{out}}\, dx \\ &= -\left(4.2 \times 10^3\right)(0.5) - \left(1.4 \times 10^3\right)(0.3) \\ &= 2.5 \times 10^3 \text{ V}\ .\end{aligned}$$

79. We connect A to the origin with a line along the y axis, along which there is no change of potential (Eq. 25-18: $\int \vec{E} \cdot d\vec{s} = 0$). Then, we connect the origin to B with a line along the x axis, along which the change in potential is

$$\Delta V = -\int_0^{x=4} \vec{E} \cdot d\vec{s} = -4.00 \int_0^4 x\, dx = -4.00\left(\frac{4^2}{2}\right)$$

which yields $V_B - V_A = -32$ V.

80. (a) The charges are equal and are the same distance from C. We use the Pythagorean theorem to find the distance $r = \sqrt{(d/2)^2 + (d/2)^2} = d/\sqrt{2}$. The electric potential at C is the sum of the potential due to the individual charges but since they produce the same potential, it is twice that of either one:

$$\begin{aligned}V &= \frac{2q}{4\pi\varepsilon_0}\frac{\sqrt{2}}{d} = \frac{2\sqrt{2}q}{4\pi\varepsilon_0 d} \\ &= \frac{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2)\sqrt{2}(2.0 \times 10^{-6}\,\text{C})}{0.020\,\text{m}} = 2.54 \times 10^6 \text{ V}\ .\end{aligned}$$

(b) As you move the charge into position from far away the potential energy changes from zero to qV, where V is the electric potential at the final location of the charge. The change in the potential energy equals the work you must do to bring the charge in:

$$W = qV = \left(2.0 \times 10^{-6}\,\text{C}\right)\left(2.54 \times 10^6\,\text{V}\right) = 5.1 \text{ J}\ .$$

(c) The work calculated in part (b) represents the potential energy of the interactions between the charge brought in from infinity and the other two charges. To find the total potential energy of the three-charge system you must add the potential energy of the interaction between the fixed charges. Their separation is d so this potential energy is $q^2/4\pi\varepsilon_0 d$. The total potential energy is

$$\begin{aligned}U &= W + \frac{q^2}{4\pi\varepsilon_0 d} \\ &= 5.1\,\text{J} + \frac{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.0 \times 10^{-6}\,\text{C})^2}{0.020\,\text{m}} = 6.9 \text{ J}\ .\end{aligned}$$

81. (a) Let the quark-quark separation be r. To "naturally" obtain the eV unit, we only plug in for one of the e values involved in the computation:

$$\begin{aligned} U_{\text{up-up}} &= \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)}{r} = \frac{4ke}{9r}\,e \\ &= \frac{4\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(1.60\times10^{-19}\,\text{C}\right)}{9\left(1.32\times10^{-15}\,\text{m}\right)}\,e \\ &= 4.84\times10^5\,\text{eV} = 0.484\ \text{MeV}\ . \end{aligned}$$

(b) The total consists of all pair-wise terms:

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{\left(\frac{2e}{3}\right)\left(\frac{2e}{3}\right)}{r} + \frac{\left(\frac{-e}{3}\right)\left(\frac{2e}{3}\right)}{r} + \frac{\left(\frac{-e}{3}\right)\left(\frac{2e}{3}\right)}{r}\right] = 0\ .$$

82. (a) At the smallest center-to-center separation r_{min} the initial kinetic energy K_i of the proton is entirely converted to the electric potential energy between the proton and the nucleus. Thus,

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{eq_{\text{lead}}}{r_{\text{min}}} = \frac{82e^2}{4\pi\varepsilon_0 r_{\text{min}}}\ .$$

In solving for r_{min} using the eV unit, we note that a factor of e cancels in the middle line:

$$\begin{aligned} r_{\text{min}} &= \frac{82e^2}{4\pi\varepsilon_0 K_i} = k\frac{82e^2}{4.80\times10^6\,eV} \\ &= \left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{82(1.6\times10^{-19}\,\text{C})}{4.80\times10^6\,\text{V}} \\ &= 2.5\times10^{-14}\,\text{m} = 25\ \text{fm}\ . \end{aligned}$$

It is worth recalling that a volt is a Newton·meter/Coulomb, in making sense of the above manipulations.

(b) An alpha particle has 2 protons (as well as 2 neutrons). Therefore, using r'_{min} for the new separation, we find

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{lead}}}{r'_{\text{min}}} = 2\left(\frac{82e^2}{4\pi\varepsilon_0 r'_{\text{min}}}\right) = \frac{82e^2}{4\pi\varepsilon_0 r_{\text{min}}}$$

which leads to $r'_{\text{min}} = 2r_{\text{min}} = 50\,\text{fm}$.

83. The potential energy of the two-charge system is

$$\begin{aligned} U &= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\right] \\ &= \frac{\left(8.99\times10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(3.0\times10^{-6}\,\text{C})(-4.0\times10^{-6}\,\text{C})}{\sqrt{(3.5+2.0)^2+(0.50-1.5)^2}\,\text{cm}} \end{aligned}$$

$$= -1.9\ \text{J}\ .$$

Thus, $-1.9\,\text{J}$ of work is needed.

84. For a point on the axis of the ring the potential (assuming $V\to0$ as $r\to\infty$) is

$$V = \frac{q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

where $q = 16\times10^{-6}$ C and $R = 0.030$ m. Therefore,

$$V_B - V_A = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{z_B^2+R^2}} - \frac{1}{R}\right)$$

where $z_B = 0.040$ m. The result is -1.92×10^6 V.

85. We apply Eq. 25-41:

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -2yz^2 \\ E_y &= -\frac{\partial V}{\partial y} = -2xz^2 \\ E_z &= -\frac{\partial V}{\partial z} = -4xyz \end{aligned}$$

which, at $(x, y, z) = (3, -2, 4)$, gives $(E_x, E_y, E_z) = (64, -96, 96)$ in SI units. The magnitude of the field is therefore

$$\left|\vec{E}\right| = \sqrt{E_x^2 + E_y^2 + E_z^2} = 150 \text{ V/m} = 150 \text{ N/C} .$$

86. We note that for two points on a circle, separated by angle θ (in radians), the direct-line distance between them is $r = 2R\sin(\theta/2)$. Using this fact, distinguishing between the cases where $N =$ odd and $N =$ even, and counting the pair-wise interactions very carefully, we arrive at the following results for the total potential energies. We use $k = 1/4\pi\varepsilon_0$. For configuration 1 (where all N electrons are on the circle), we have

$$\begin{aligned} U_{1,N=\text{even}} &= \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1} \frac{1}{\sin(j\theta/2)} + \frac{1}{2}\right) \\ U_{1,N=\text{odd}} &= \frac{Nke^2}{2R}\left(\sum_{j=1}^{\frac{N-1}{2}} \frac{1}{\sin(j\theta/2)}\right) \end{aligned}$$

where $\theta = \frac{2\pi}{N}$. For configuration 2, we find

$$\begin{aligned} U_{2,N=\text{even}} &= \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N}{2}-1} \frac{1}{\sin(j\theta'/2)} + 2\right) \\ U_{2,N=\text{odd}} &= \frac{(N-1)ke^2}{2R}\left(\sum_{j=1}^{\frac{N-3}{2}} \frac{1}{\sin(j\theta'/2)} + \frac{5}{2}\right) \end{aligned}$$

where $\theta' = \frac{2\pi}{N-1}$. The results are all of the form

$$U_{1 \text{ or } 2} = \frac{ke^2}{2R} \times \text{a pure number} .$$

In our table, below, we have the results for those "pure numbers" as they depend on N and on which configuration we are considering. The values listed in the U rows are the potential energies divided by $ke^2/2R$.

N	4	5	6	7	8	9	10	11	12	13	14	15
U_1	3.83	6.88	10.96	16.13	22.44	29.92	38.62	48.58	59.81	72.35	86.22	101.5
U_2	4.73	7.83	11.88	16.96	23.13	30.44	39.92	48.62	59.58	71.81	85.35	100.2

We see that the potential energy for configuration 2 is greater than that for configuration 1 for $N < 12$, but for $N \geq 12$ it is configuration 1 that has the greatest potential energy.

(a) Configuration 1 has the smallest U for $2 \leq N \leq 11$, and configuration 2 has the smallest U for $12 \leq N \leq 15$.

(b) $N = 12$ is the smallest value such that $U_2 < U_1$.

(c) For $N = 12$, configuration 2 consists of 11 electrons distributed at equal distances around the circle, and one electron at the center. A specific electron e_0 on the circle is R distance from the one in the center, and is

$$r = 2R\sin\left(\frac{\pi}{11}\right) \quad \approx \quad 0.56R$$

distance away from its nearest neighbors on the circle (of which there are two – one on each side). Beyond the nearest neighbors, the next nearest electron on the circle is

$$r = 2R\sin\left(\frac{2\pi}{11}\right) \quad \approx \quad 1.1R$$

distance away from e_0. Thus, we see that there are only two electrons closer to e_0 than the one in the center.

87. (First problem of **Cluster**)

(a) The field between the plates is uniform; we apply Eq. 25-42 to find the magnitude of the (horizontal) field: $|\vec{E}| = \Delta V/D$ (assuming $\Delta V > 0$). This produces a horizontal acceleration from Eq. 23-1 and Newton's second law (applied along the x axis):

$$a_x = \frac{|\vec{F}_x|}{m} = \frac{q|\vec{E}|}{m} = \frac{q\Delta V}{m\,D}$$

where $q > 0$ has been assumed; the problem indicates that the acceleration is rightward, which constitutes our choice for the $+x$ direction. If we choose upward as the $+y$ direction then $a_y = -g$, and we apply the free-fall equations of Chapter 2 to the y motion while applying the constant (a_x) acceleration equations of Table 2-1 to the x motion. The displacement is defined by $\Delta x = +D/2$ and $\Delta y = -d$, and the initial velocity is zero. Simultaneous solution of

$$\begin{aligned} \Delta x &= v_{0\text{x}}\,t + \frac{1}{2}\,a_x\,t^2 \qquad \text{and} \\ \Delta y &= v_{0\text{y}}\,t + \frac{1}{2}\,a_y\,t^2 \quad , \end{aligned}$$

leads to

$$d = \frac{gD}{2a_x} = \frac{gmD^2}{2q\Delta V}\ .$$

(b) We can continue along the same lines as in part (a) (using Table 2-1) to find v, or we can use energy conservation – which we feel is more instructive. The gain in kinetic energy derives from two potential energy changes: from gravity comes mgd and from electric potential energy comes $q|\vec{E}|\Delta x = q\Delta V/2$. Consequently,

$$\frac{1}{2}mv^2 = mgd + \frac{1}{2}q\Delta V$$

which (upon using the expression for d above) yields

$$v = \sqrt{\frac{mg^2D^2}{q\Delta V} + \frac{q\Delta V}{m}}\ .$$

(c) and (d) Using SI units (so $q = 1.0 \times 10^{-10}$ C, $m = 1.0 \times 10^{-9}$ kg) we plug into our results to obtain $d = 0.049$ m and $v = 1.4$ m/s.

88. (Second problem of **Cluster**)

(a) We argue by symmetry that of the total potential energy in the initial configuration, a third converts into the kinetic energy of each of the particles. And, because the total potential energy consists of three equal contributions

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{d}$$

then any of the particle's final kinetic energy is equal to this U. Therefore, using k for$1/4\pi\varepsilon_0$, we obtain

$$v = \sqrt{\frac{2U}{m}} = |q|\sqrt{\frac{2k}{m\,d}} \quad .$$

(b) In this case, two of the U contributions to the total potential energy are converted into a single kinetic term:

$$v = \sqrt{\frac{2(2U)}{m}} = 2|q|\sqrt{\frac{k}{m\,d}} \quad .$$

(c) Now it is clear that the one remaining U contribution is converted into a particle's kinetic energy:

$$v = \sqrt{\frac{2U}{m}} = |q|\sqrt{\frac{2k}{m\,d}} \quad .$$

(d) This leaves no potential energy to convert into kinetic for the last particle that is released. It maintains zero speed.

89. (Third problem of **Cluster**)

(a) By momentum conservation we see that their final speeds are the same. We use energy conservation (where the "final" subscript refers to when they are infinitely far away from each other):

$$\begin{aligned} U_i &= K_f \\ \frac{1}{4\pi\varepsilon_0}\frac{2Q^2}{D} &= 2\left(\frac{1}{2}mv^2\right) \end{aligned}$$

which (using $k = 1/4\pi\varepsilon_0$) yields

$$v = |Q|\sqrt{\frac{2k}{m\,D}} \quad .$$

(b) As noted above, this result is the same as that of part (a).

(c) We use energy conservation (where the "final" subscript refers to when their surfaces have made contact):

$$\begin{aligned} U_i &= K_f + U_f \\ \frac{1}{4\pi\varepsilon_0}\frac{-2Q^2}{D} &= 2\left(\frac{1}{2}mv^2\right) + \frac{1}{4\pi\varepsilon_0}\frac{-2Q^2}{2r} \end{aligned}$$

which (using $k = 1/4\pi\varepsilon_0$) yields

$$v = |Q|\sqrt{\frac{k}{m\,r} - \frac{2k}{m\,D}} \approx |Q|\sqrt{\frac{k}{m\,r}}$$

since $r \ll D$.

(d) As before, the speeds of the particles are equal (by momentum conservation).

(e) and (f) The collision being elastic means no kinetic energy is lost (or gained), so they are able to return to their original positions (climbing back up that potential "hill") whereupon their potential energy is again U_i and their kinetic energies (hence, speeds) are zero.

90. (Fourth problem of **Cluster**)

(a) At its displaced position, its potential energy (using $k = 1/4\pi\varepsilon_0$) is

$$U_i = k\frac{qQ}{d - x_0} + k\frac{qQ}{d + x_0} \quad = \quad \frac{2kqQd}{d^2 - x_0^2} \quad .$$

And at A, the potential energy is

$$U_A = 2\left(k\frac{qQ}{d}\right) \quad .$$

Setting this difference equal to the kinetic energy of the particle ($\frac{1}{2}mv^2$) and solving for the speed yields

$$v = \sqrt{\frac{2\,(U_i - U_A)}{m}} \quad = \quad 2\,x_0\sqrt{\frac{k\,q\,Q}{m\,d\,(d^2 - x_0^2)}} \quad .$$

(b) It is straightforward to consider small x_0 (more precisely, $x_0/d \ll 1$) in the above expression (so that $d^2 - x_0^2 \approx d^2$). The result is

$$v \approx 2\,\frac{x_0}{d}\sqrt{\frac{k\,q\,Q}{m\,d}} \quad .$$

(c) Plugging in the given values (converted to SI units) yields $v \approx 19$ m/s.

(d) Using the Pythagorean theorem, we now have

$$U_i = 2k\frac{-qQ}{\sqrt{d^2 + x_0^2}} \quad .$$

Therefore, (with U_A in this part equal to the negative of U_A in the previous part)

$$v = \sqrt{\frac{2\,(U_i - U_A)}{m}} \quad = \quad 2\sqrt{\frac{k\,q\,Q}{m}\left(\frac{1}{d} - \frac{1}{\sqrt{d^2 + x_0^2}}\right)} \quad .$$

To simplify, the binomial theorem (Appendix E) is employed:

$$\frac{1}{\sqrt{d^2 + x_0^2}} \quad \approx \quad \frac{1}{d}\left(1 - \frac{1}{2}\frac{x_0^2}{d^2}\right)$$

which leads to

$$v \quad \approx \quad \frac{x_0}{d}\sqrt{\frac{2k\,q\,Q}{m\,d}} \quad .$$

Chapter 26

1. The minimum charge measurable is

$$q_{\min} = CV_{\min} = (50\,\text{pF})(0.15\,\text{V}) = 7.5\ \text{pC}\ .$$

2. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\ \text{pF}\ .$$

(b) The capacitance is independent of q; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\ \text{V}\ .$$

3. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus, $q = (25 \times 10^{-6}\,\text{F})(120\,\text{V}) = 3.0 \times 10^{-3}\,\text{C}$.

4. We verify the units relationship as follows:

$$[\varepsilon_0] = \frac{\text{F}}{\text{m}} = \frac{\text{C}}{\text{V}\cdot\text{m}} = \frac{\text{C}}{(\text{N}\cdot\text{m}/\text{C})\,\text{m}} = \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\ .$$

5. (a) The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus,

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{(8.85 \times 10^{-12}\,\text{F/m})\pi(8.2 \times 10^{-2}\,\text{m})^2}{1.3 \times 10^{-3}\,\text{m}} = 1.4 \times 10^{-10}\,\text{F} = 140\ \text{pF}\ .$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus, $q = (1.4 \times 10^{-10}\,\text{F})(120\,\text{V}) = 1.7 \times 10^{-8}\,\text{C} = 17\,\text{nC}$.

6. We use $C = A\varepsilon_0/d$. Thus

$$d = \frac{A\varepsilon_0}{C} = \frac{(1.00\,\text{m}^2)\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{1.00\,\text{F}} = 8.85 \times 10^{-12}\ \text{m}\ .$$

Since d is much less than the size of an atom ($\sim 10^{-10}\,\text{m}$), this capacitor cannot be constructed.

7. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}(R')^3 = 2\frac{4\pi}{3}R^3 \ ,$$

so

$$R' = 2^{1/3}R \ .$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 \ .$$

8. (a) We use Eq. 26-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\mathrm{mm})(38.0\,\mathrm{mm})}{\left(8.99 \times 10^9 \,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(40.0\,\mathrm{mm} - 38.0\,\mathrm{mm})} = 84.5 \text{ pF} \ .$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b-a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\mathrm{pF})(40.0\,\mathrm{mm} - 38.0\,\mathrm{mm})}{\left(8.85 \times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}\right)} = 191 \text{ cm}^2 \ .$$

9. According to Eq. 26-17 the capacitance of a spherical capacitor is given by

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} \ ,$$

where a and b are the radii of the spheres. If a and b are nearly the same then $4\pi ab$ is nearly the surface area of either sphere. Replace $4\pi ab$ with A and $b-a$ with d to obtain

$$C \approx \frac{\varepsilon_0 A}{d} \ .$$

10. The equivalent capacitance is

$$C_{\mathrm{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4.00\,\mu\mathrm{F} + \frac{(10.0\,\mu\mathrm{F})(5.00\,\mu\mathrm{F})}{10.0\,\mu\mathrm{F} + 5.00\,\mu\mathrm{F}} = 7.33\,\mu\mathrm{F} \ .$$

11. The equivalent capacitance is given by $C_{\mathrm{eq}} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel, $C_{\mathrm{eq}} = NC$, where C is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,\mathrm{C}}{(110\,\mathrm{V})(1.00 \times 10^{-6}\,\mathrm{F})} = 9090 \ .$$

12. The charge that passes through meter A is

$$q = C_{\mathrm{eq}} V = 3CV = 3(25.0\,\mu\mathrm{F})(4200\,\mathrm{V}) = 0.315 \text{ C} \ .$$

13. The equivalent capacitance is

$$C_{\mathrm{eq}} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(10.0\,\mu\mathrm{F} + 5.00\,\mu\mathrm{F})(4.00\,\mu\mathrm{F})}{10.0\,\mu\mathrm{F} + 5.00\,\mu\mathrm{F} + 4.00\,\mu\mathrm{F}} = 3.16\,\mu\mathrm{F} \ .$$

14. (a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{\mathrm{eq}} V}{C_1 + C_2} = \frac{(3.16\,\mu\mathrm{F})(100\,\mathrm{V})}{10.0\,\mu\mathrm{F} + 5.00\,\mu\mathrm{F}} = 21.1 \text{ V} \ .$$

Thus $\Delta V_1 = 100\,\mathrm{V} - 21.1\,\mathrm{V} = 79\,\mathrm{V}$ and $\Delta q_1 = C_1 \Delta V_1 = (10.0\,\mu\mathrm{F})(79\,\mathrm{V}) = 7.9 \times 10^{-4}\,\mathrm{C}$.

15. Let x be the separation of the plates in the lower capacitor. Then the plate separation in the upper capacitor is $a-b-x$. The capacitance of the lower capacitor is $C_\ell = \varepsilon_0 A/x$ and the capacitance of the upper capacitor is $C_u = \varepsilon_0 A/(a-b-x)$, where A is the plate area. Since the two capacitors are in series, the equivalent capacitance is determined from

$$\frac{1}{C_{\rm eq}} = \frac{1}{C_\ell} + \frac{1}{C_u} = \frac{x}{\varepsilon_0 A} + \frac{a-b-x}{\varepsilon_0 A} = \frac{a-b}{\varepsilon_0 A} \ .$$

Thus, the equivalent capacitance is given by $C_{\rm eq} = \varepsilon_0 A/(a-b)$ and is independent of x.

16. (a) The potential difference across C_1 is $V_1 = 10\,\rm V$. Thus, $q_1 = C_1 V_1 = (10\,\mu\rm F)(10\,V) = 1.0\times 10^{-4}\,C$.

(b) Let $C = 10\,\mu\rm F$. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\rm eq} = C + \frac{C_2 C}{C + C_2} = 1.5C \ .$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{eq}} = \frac{CV_1}{C + 1.5C} = \frac{2}{5}V_1 \ .$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2 V_2 = (10\,\mu\rm F)\left(\frac{10\,V}{5}\right) = 2.0\times 10^{-5}\ V \ .$$

17. The charge initially on the charged capacitor is given by $q = C_1 V_0$, where $C_1 = 100\,\rm pF$ is the capacitance and $V_0 = 50\,\rm V$ is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1 V$, where $v = 35\,\rm V$ is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor. Substituting $C_1 V_0$ for q and $C_1 V$ for q_1, we obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V}C_1 = \frac{50\,\rm V - 35\,V}{35\,\rm V}(100\,\rm pF) = 3\ pF \ .$$

18. (a) First, the equivalent capacitance of the two $4.0\,\mu\rm F$ capacitors connected in series is given by $4.0\,\mu\rm F/2 = 2.0\,\mu F$. This combination is then connected in parallel with two other 2.0-μF capacitors (one on each side), resulting in an equivalent capacitance $C = 3(2.0\,\mu\rm F) = 6.0\,\mu F$. This is now seen to be in series with another combination, which consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to $C' = 2(3.0\,\mu\rm F) = 6.0\,\mu F$). Thus, the equivalent capacitance of the circuit is

$$C_{\rm eq} = \frac{CC'}{C + C'} = \frac{(6.0\,\mu\rm F)(6.0\,\mu F)}{6.0\,\mu F + 6.0\,\mu F} = 3.0\,\mu F \ .$$

(b) Let $V = 20\,\rm V$ be the potential difference supplied by the battery. Then $q = C_{\rm eq}V = (3.0\,\mu\rm F)(20\,V) = 6.0\times 10^{-5}\,C$.

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C + C'} = \frac{(6.0\,\mu\rm F)(20\,V)}{6.0\,\mu F + 6.0\,\mu F} = 10\,V \ ,$$

and the charge carried by C_1 is $q_1 = C_1 V_1 = (3.0\,\mu\rm F)(10\,V) = 3.0\times 10^{-5}\ C$.

(d) The potential difference across C_2 is given by $V_2 = V - V_1 = 20\,\text{V} - 10\,\text{V} = 10\,\text{V}$. Consequently, the charge carried by C_2 is $q_2 = C_2V_2 = (2.0\,\mu\text{F})(10\,\text{V}) = 2.0 \times 10^{-5}$ C.

(e) Since this voltage difference V_2 is divided equally between C_3 and the other 4.0-μF capacitors connected in series with it, the voltage difference across C_3 is given by $V_3 = V_2/2 = 10\,\text{V}/2 = 5.0\,\text{V}$. Thus, $q_3 = C_3V_3 = (4.0\,\mu\text{F})(5.0\,\text{V}) = 2.0 \times 10^{-5}$ C.

19. (a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{\text{eq}}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance. The equivalent capacitance is $C_{\text{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}\,\text{F}$. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1V = (1.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 1.0 \times 10^{-4}\,\text{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2V = (3.0 \times 10^{-6}\,\text{F})(100\,\text{V}) = 3.0 \times 10^{-4}\,\text{C}\ ,$$

so the net charge on the combination is $3.0 \times 10^{-4}\,\text{C} - 1.0 \times 10^{-4}\,\text{C} = 2.0 \times 10^{-4}\,\text{C}$. The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4}\,\text{C}}{4.0 \times 10^{-6}\,\text{F}} = 50\,\text{V}\ .$$

(b) The charge on capacitor 1 is now $q_1 = C_1V_{ab} = (1.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 5.0 \times 10^{-5}\,\text{C}$.

(c) The charge on capacitor 2 is now $q_2 = C_2V_{ab} = (3.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 1.5 \times 10^{-4}\,\text{C}$.

20. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1C_3V}{C_1 + C_3} = \frac{(1.0\,\mu\text{F})(3.0\,\mu\text{F})(12\,\text{V})}{1.0\,\mu\text{F} + 3.0\,\mu\text{F}} = 9.0\,\mu\text{C}\ .$$

Also, capacitors 2 and 4 are in series:

$$q_2 = q_4 = \frac{C_2C_4V}{C_2 + C_4} = \frac{(2.0\,\mu\text{F})(4.0\,\mu\text{F})(12\,\text{V})}{2.0\,\mu\text{F} + 4.0\,\mu\text{F}} = 16\,\mu\text{C}\ .$$

(b) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4}V = \frac{(3.0\,\mu\text{F} + 4.0\,\mu\text{F})(12\,\text{V})}{1.0\,\mu\text{F} + 2.0\,\mu\text{F} + 3.0\,\mu\text{F} + 4.0\,\mu\text{F}} = 8.4\,\text{V}\ .$$

Thus, $q_1 = C_1V_1 = (1.0\,\mu\text{F})(8.4\,\text{V}) = 8.4\,\mu\text{C}$, $q_2 = C_2V_1 = (2.0\,\mu\text{F})(8.4\,\text{V}) = 17\,\mu\text{C}$, $q_3 = C_3(V - V_1) = (3.0\,\mu\text{F})(12\,\text{V} - 8.4\,\text{V}) = 11\,\mu\text{C}$, and $q_4 = C_4(V - V_1) = (4.0\,\mu\text{F})(12\,\text{V} - 8.4\,\text{V}) = 14\,\mu\text{C}$.

21. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2C_3}\ .$$

Thus, $C_{\text{eq}} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{\text{eq}}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields

$q_1 + q_2 = q_0 = C_1V_0$, where V_0 is the original potential difference across capacitor 1. Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\rm eq}} \quad \text{and} \quad q_1 + q_2 = C_1V_0$$

for q_1 and q_2, we find

$$q_2 = C_1V_0 - q_1 \qquad \text{and} \quad q_1 = \frac{C_1^2V_0}{C_{\rm eq}+C_1} = \frac{C_1^2V_0}{\dfrac{C_2C_3}{C_2+C_3}+C_1} = \frac{C_1^2(C_2+C_3)V_0}{C_1C_2+C_1C_3+C_2C_3} \ .$$

The charges on capacitors 2 and 3 are

$$q_2 = q_3 = C_1V_0 - q_1 = C_1V_0 - \frac{C_1^2(C_2+C_3)V_0}{C_1C_2+C_1C_3+C_2C_3} = \frac{C_1C_2C_3V_0}{C_1C_2+C_1C_3+C_2C_3} \ .$$

22. Let $\mathcal{V} = 1.00\,\text{m}^3$. Using Eq. 26-23, the energy stored is

$$\begin{aligned} U &= u\mathcal{V} = \frac{1}{2}\varepsilon_0E^2\mathcal{V} \\ &= \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(150\,\text{V/m})^2(1.00\,\text{m}^3) \\ &= 9.96\times10^{-8}\ \text{J} \ . \end{aligned}$$

23. The energy stored by a capacitor is given by $U = \frac{1}{2}CV^2$, where V is the potential difference across its plates. We convert the given value of the energy to Joules. Since a Joule is a watt·second, we multiply by $(10^3\,\text{W/kW})(3600\,\text{s/h})$ to obtain $10\,\text{kW}\cdot\text{h} = 3.6\times10^7\,\text{J}$. Thus,

$$C = \frac{2U}{V^2} = \frac{2(3.6\times10^7\,\text{J})}{(1000\,\text{V})^2} = 72\ \text{F} \ .$$

24. (a) The capacitance is

$$C = \frac{\varepsilon_0A}{d} = \frac{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(40\times10^{-4}\,\text{m}^2)}{1.0\times10^{-3}\,\text{m}} = 3.5\times10^{-11}\,\text{F} = 35\ \text{pF} \ .$$

(b) $q = CV = (35\,\text{pF})(600\,\text{V}) = 2.1\times10^{-8}\,\text{C} = 21\,\text{nC}$.

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(35\,\text{pF})(21\,\text{nC})^2 = 6.3\times10^{-6}\,\text{J} = 6.3\,\mu\text{J}$.

(d) $E = V/d = 600\,\text{V}/1.0\times10^{-3}\,\text{m} = 6.0\times10^5\,\text{V/m}$.

(e) The energy density (energy per unit volume) is

$$u = \frac{U}{Ad} = \frac{6.3\times10^{-6}\,\text{J}}{(40\times10^{-4}\,\text{m}^2)(1.0\times10^{-3}\,\text{m})} = 1.6\ \text{J/m}^3 \ .$$

25. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U = \frac{1}{2}\left(C_1+C_2\right)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\,\text{F} + 4.0\times10^{-6}\,\text{F}\right)(300\,\text{V})^2 = 0.27\ \text{J} \ .$$

26. The total energy stored in the capacitor bank is

$$U = \frac{1}{2}C_{\rm total}V^2 = \frac{1}{2}(2000)(5.00\times10^{-6}\,\text{F})(50000\,\text{V})^2 = 1.3\times10^7\ \text{J} \ .$$

Thus, the cost is

$$\frac{(1.3\times10^7\,\text{J})(3.0\,\text{cent/ kW}\cdot\text{h})}{3.6\times10^6\,\text{J/ kW}\cdot\text{h}} = 10\,\text{cents} \ .$$

27. (a) In the first case $U = q^2/2C$, and in the second case $U = 2(q/2)^2/2C = q^2/4C$. So the energy is now $4.0\,\text{J}/2 = 2.0\,\text{J}$.

    (b) It becomes the thermal energy generated in the wire connecting the capacitors during the discharging process (although a small fraction of it is probably radiated away in the form of radio waves).

28. (a) The potential difference across C_1 (the same as across C_2) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(4.00\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 21.1\ \text{V} .$$

Also, $V_3 = V - V_1 = V - V_2 = 100\,\text{V} - 21.1\,\text{V} = 78.9\,\text{V}$. Thus,

$$\begin{aligned} q_1 &= C_1 V_1 = (10.0\,\mu\text{F})(21.1\,\text{V}) = 2.11 \times 10^{-4}\ \text{C} \\ q_2 &= C_2 V_2 = (5.00\,\mu\text{F})(21.1\,\text{V}) \ = 1.05 \times 10^{-4}\ \text{C} \\ q_3 &= q_1 + q_2 = 2.11 \times 10^{-4}\,\text{C} + 1.05 \times 10^{-4}\,\text{C} = 3.16 \times 10^{-4}\ \text{C} . \end{aligned}$$

    (b) The potential differences were found in the course of solving for the charges in part (a).

    (c) The stored energies are as follows:

$$\begin{aligned} U_1 &= \frac{1}{2}C_1 V_1^2 = \frac{1}{2}(10.0\,\mu\text{F})(21.1\,\text{V})^2 = 2.22 \times 10^{-3}\ \text{J} , \\ U_2 &= \frac{1}{2}C_2 V_2^2 = \frac{1}{2}(5.00\,\mu\text{F})(21.1\,\text{V})^2 = 1.11 \times 10^{-3}\ \text{J} , \\ U_3 &= \frac{1}{2}C_3 V_3^2 = \frac{1}{2}(4.00\,\mu\text{F})(78.9\,\text{V})^2 = 1.25 \times 10^{-2}\ \text{J} . \end{aligned}$$

29. (a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\varepsilon_0 A/d$, the charge is $q = CV = \varepsilon_0 AV/d$. After the plates are pulled apart, their separation is $2d$ and the potential difference is V'. Then $q = \varepsilon_0 AV'/2d$ and

$$V' = \frac{2d}{\varepsilon_0 A}\, q = \frac{2d}{\varepsilon_0 A}\,\frac{\varepsilon_0 A}{d}\, V = 2V .$$

    (b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{\varepsilon_0 AV^2}{2d}$$

and the final energy stored is

$$U_f = \frac{1}{2}\,\frac{\varepsilon_0 A}{2d}\,(V')^2 = \frac{1}{2}\,\frac{\varepsilon_0 A}{2d}\,4V^2 = \frac{\varepsilon_0 AV^2}{d} .$$

This is twice the initial energy.

    (c) The work done to pull the plates apart is the difference in the energy: $W = U_f - U_i = \varepsilon_0 AV^2/2d$.

30. (a) The charge in the Figure is

$$q_3 = C_3 V = (4.00\,\mu\text{F})(100\,\text{V}) = 4.00 \times 10^{-4}\,\text{mC} ,$$

$$q_1 = q_2 = \frac{C_1 C_2 V}{C_1 + C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 3.33 \times 10^{-4}\,\text{C} .$$

    (b) $V_1 = q_1/C_1 = 3.33 \times 10^{-4}\,\text{C}/10.0\,\mu\text{F} = 33.3\,\text{V}$, $V_2 = V - V_1 = 100\,\text{V} - 33.3\,\text{V} = 66.7\,\text{V}$, and $V_3 = V = 100\,\text{V}$.

(c) We use $U_i = \frac{1}{2}C_iV_i^2$, where $i = 1, 2, 3$. The answers are $U_1 = 5.6\,\text{mJ}$, $U_1 = 11\,\text{mJ}$, and $U_1 = 20\,\text{mJ}$.

31. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by

$$E = \frac{q}{2\pi\varepsilon_0 L r}$$

(see Eq. 26-12), and the energy density at that point is given by

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} \ .$$

The energy in the cylinder is the volume integral

$$U_R = \int u\, d\mathcal{V} \ .$$

Now, $d\mathcal{V} = 2\pi r L\, dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2}\, 2\pi r L\, dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{R}{a} \ .$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{b}{a} \ .$$

We want the ratio U_R/U_b to be $1/2$, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln(\sqrt{b/a})$, $\ln(R/a) = \ln(\sqrt{b/a})$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

32. We use $E = q/4\pi\varepsilon_0 R^2 = V/R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{8000\,\text{V}}{0.050\,\text{m}}\right)^2 = 0.11\ \text{J/m}^3.$$

33. The charge is held constant while the plates are being separated, so we write the expression for the stored energy as $U = q^2/2C$, where q is the charge and C is the capacitance. The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/x$, where A is the plate area and x is the plate separation, so

$$U = \frac{q^2 x}{2\varepsilon_0 A} \ .$$

If the plate separation increases by dx, the energy increases by $dU = (q^2/2\varepsilon_0 A)\,dx$. Suppose the agent pulling the plate apart exerts force F. Then the agent does work $F\,dx$ and if the plates begin and end at rest, this must equal the increase in stored energy. Thus,

$$F\,dx = \left(\frac{q^2}{2\varepsilon_0 A}\right)dx$$

and

$$F = \frac{q^2}{2\varepsilon_0 A} \ .$$

The net force on a plate is zero, so this must also be the magnitude of the force one plate exerts on the other. The force can also be computed as the product of the charge q on one plate and the electric field E_1 due to the charge on the other plate. Recall that the field produced by a uniform plane surface of charge is $E_1 = q/2\varepsilon_0 A$. Thus, $F = q^2/2\varepsilon_0 A$.

34. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A/2d$. Thus $C'/C = \kappa/2$ or $\kappa = 2C'/C = 2(2.6\,\text{pF}/1.3\,\text{pF}) = 4.0$.

35. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4 \times 10^{-6}\,\text{J})}{(7.4 \times 10^{-12}\,\text{F})(652\,\text{V})^2} = 4.7 \ .$$

According to Table 26-1, you should use Pyrex.

36. (a) We use $C = \varepsilon_0 A/d$ to solve for d:

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.35\,\text{m}^2)}{50 \times 10^{-12}\,\text{F}} = 6.2 \times 10^{-2}\,\text{m} \ .$$

(b) We use $C \propto \kappa$. The new capacitance is $C' = C(\kappa/\kappa_{\text{air}}) = (50\,\text{pf})(5.6/1.0) = 280\,\text{pF}$.

37. The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)} \ ,$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85 \times 10^{-12}\,\text{F/m})}{\ln\left[(0.60\,\text{mm})/(0.10\,\text{mm})\right]} = 8.1 \times 10^{-11}\,\text{F/m} = 81\ \text{pF/m} \ .$$

38. (a) We use Eq. 26-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\,\text{m})}{2\left(8.99 \times 10^9\,\frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8\,\text{cm}/3.6\,\text{cm})} = 0.73\ \text{nF} \ .$$

(b) The breakdown potential is $(14\,\text{kV/mm})(3.8\,\text{cm} - 3.6\,\text{cm}) = 28\,\text{kV}$.

39. The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Thus,

$$A = \frac{CV}{\kappa\varepsilon_0 E} \ .$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0 \times 10^{-8}\,\text{F})(4.0 \times 10^3\,\text{V})}{2.8(8.85 \times 10^{-12}\,\text{F/m})(18 \times 10^6\,\text{V/m})} = 0.63\ \text{m}^2 \ .$$

40. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus

$$C = C_1 + C_2 = \frac{\varepsilon_0(A/2)\kappa_1}{d} + \frac{\varepsilon_0(A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right) \ .$$

41. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A} ,$$

where A is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A} .$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right] = \frac{qd}{2\varepsilon_0 A}\,\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2} ,$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d}\,\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2} .$$

This expression is exactly the same as the that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

42. Let $C_1 = \varepsilon_0 (A/2)\kappa_1/2d = \varepsilon_0 A \kappa_1/4d$, $C_2 = \varepsilon_0 (A/2)\kappa_2/d = \varepsilon_0 A \kappa_2/2d$, and $C_3 = \varepsilon_0 A \kappa_3/2d$. Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus,

$$\begin{aligned} C &= C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A \kappa_1}{4d} + \frac{(\varepsilon_0 A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2 + \kappa_3/2} \\ &= \frac{\varepsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2 \kappa_3}{\kappa_2 + \kappa_3}\right) . \end{aligned}$$

43. (a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa \varepsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa \varepsilon_0 A/C$ and

$$E = \frac{VC}{\kappa \varepsilon_0 A} = \frac{(50\,\text{V})(100 \times 10^{-12}\,\text{F})}{5.4(8.85 \times 10^{-12}\,\text{F/m})(100 \times 10^{-4}\,\text{m}^2)} = 1.0 \times 10^4\,\text{V/m} .$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\,\text{F})(50\,\text{V}) = 5.0 \times 10^{-9}\,\text{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} ,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 A E \\ &= 5.0 \times 10^{-9}\,\text{C} - (8.85 \times 10^{-12}\,\text{F/m})(100 \times 10^{-4}\,\text{m}^2)(1.0 \times 10^4\,\text{V/m}) \\ &= 4.1 \times 10^{-9}\,\text{C} = 4.1\,\text{nC} . \end{aligned}$$

44. (a) The electric field E_1 in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right) ,$$

and the capacitance is

$$\begin{aligned} C &= \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} \\ &= \frac{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(115\times10^{-4}\,\mathrm{m}^2)(2.61)}{(2.61)(0.0124\,\mathrm{m}-0.00780\,\mathrm{m})+(0.00780\,\mathrm{m})} \\ &= 13.4\ \mathrm{pF}\ . \end{aligned}$$

(b) $q = CV = (13.4\times10^{-12}\,\mathrm{F})(85.5\,\mathrm{V}) = 1.15\,\mathrm{nC}$.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\,\mathrm{C}}{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(115\times10^{-4}\,\mathrm{m}^2)} = 1.13\times10^4\ \mathrm{N/C}\ .$$

(d) Using Eq. 26-32, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^4\,\mathrm{N/C}}{2.61} = 4.33\times10^3\ \mathrm{N/C}\ .$$

45. (a) According to Eq. 26-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\frac{ab}{b-a}\ .$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant κ. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\frac{ab}{b-a}\ .$$

(b) The charge on the positive plate is

$$q = CV = 4\pi\kappa\varepsilon_0\frac{ab}{b-a}V\ .$$

(c) Let the charge on the inner conductor to be $-q$. Immediately adjacent to it is the induced charge q'. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi(\kappa-1)\varepsilon_0\frac{ab}{b-a}V\ .$$

46. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\,\mathrm{C}}{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(1.4\times10^{-6}\,\mathrm{V/m})(100\times10^{-4}\,\mathrm{m}^2)} = 7.2\ .$$

(b) The charge induced is

$$q' = q\left(1-\frac{1}{\kappa}\right) = (8.9\times10^{-7}\,\mathrm{C})\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \mathrm{C}\ .$$

47. Assuming the charge on one plate is $+q$ and the charge on the other plate is $-q$, we find an expression for the electric field in each region, in terms of q, then use the result to find an expression for the potential difference V between the plates. The capacitance is

$$C = \frac{q}{V} .$$

The electric field in the dielectric is $E_d = q/\kappa\varepsilon_0 A$, where κ is the dielectric constant and A is the plate area. Outside the dielectric (but still between the capacitor plates) the field is $E = q/\varepsilon_0 A$. The field is uniform in each region so the potential difference across the plates is

$$V = E_d b + E(d-b) = \frac{qb}{\kappa\varepsilon_0 A} + \frac{q(d-b)}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 A}\,\frac{b + \kappa(d-b)}{\kappa} .$$

The capacitance is

$$C = \frac{q}{V} = \frac{\kappa\varepsilon_0 A}{\kappa(d-b)+b} = \frac{\kappa\varepsilon_0 A}{\kappa d - b(\kappa-1)} .$$

The result does not depend on where the dielectric is located between the plates; it might be touching one plate or it might have a vacuum gap on each side.

For the capacitor of Sample Problem 26-8, $\kappa = 2.61$, $A = 115\,\text{cm}^2 = 115\times10^{-4}\,\text{m}^2$, $d = 1.24\,\text{cm} = 1.24\times10^{-2}\,\text{m}$, and $b = 0.78\,\text{cm} = 0.78\times10^{-2}\,\text{m}$, so

$$\begin{aligned} C &= \frac{2.61(8.85\times10^{-12}\,\text{F/m})(115\times10^{-4}\,\text{m}^2)}{2.61(1.24\times10^{-2}\,\text{m}) - (0.780\times10^{-2}\,\text{m})(2.61-1)} \\ &= 1.34\times10^{-11}\,\text{F} = 13.4\ \text{pF} \end{aligned}$$

in agreement with the result found in the sample problem. If $b = 0$ and $\kappa = 1$, then the expression derived above yields $C = \varepsilon_0 A/d$, the correct expression for a parallel-plate capacitor with no dielectric. If $b = d$, then the derived expression yields $C = \kappa\varepsilon_0 A/d$, the correct expression for a parallel-plate capacitor completely filled with a dielectric.

48. (a) Eq. 26-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\,\text{F}\right)\left(7.0\times10^3\,\text{V}\right)^2 = 4.9\times10^{-3}\ \text{J} .$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

49. (a) With the potential difference equal to 600 V, a capacitance of $2.5\times10^{-10}\,\text{F}$ can only store energy equal to $U = \frac{1}{2}CV^2 = 4.5\times10^{-5}\,\text{J}$.

(b) No, our result from part (a) is only about 20% of that needed to produce a spark.

(c) Considering the charge as a constant, then voltage should be inversely proportional to the capacitance. Therefore, if the capacitance drops by a factor of ten, then we expect the voltage to increase by that same factor: $V_f = 6000$ V.

(d) Now the energy stored is $U' = \frac{1}{2}C_f V_f^2 = 4.5\times10^{-4}\,\text{J}$, a factor of ten greater than the value we obtained in part (a).

(e) Yes, this new value of energy is nearly double that needed for a spark.

50. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\,\text{m})(0.10\,\text{m}) + \pi(0.20\,\text{m})^2 = 0.25\ \text{m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\,\mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge q is induced in the interior of the fluid.

(b) By Eq. 26-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{\left(5.0 \times 10^{-7}\,\mathrm{C}\right)^2}{2\left(35 \times 10^{-12}\,\mathrm{F}\right)} = 3.6 \times 10^{-3}\ \mathrm{J}\ .$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

51. (a) We know from Eq. 26-7 that the magnitude of the electric field is directly proportional to the surface charge density:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{15 \times 10^{-6}\,\mathrm{C/m^2}}{8.85 \times 10^{-12}\,\mathrm{C^2/N{\cdot}m^2}} = 1.7 \times 10^6\ \mathrm{V/m}\ .$$

Regarding the units, it is worth noting that a Volt is equivalent to a N·m/C.

(b) Eq. 26-23 yields

$$u = \frac{1}{2}\varepsilon_0 E^2 = 13\ \mathrm{J/m^3}\ .$$

(c) The energy U is the energy-per-unit-volume multiplied by the (variable) volume of the region between the layers of plastic food wrap. Since the distance between the layers is x, and we use A for the area over which the (say, positive) charge is spread, then that volume is Ax. Thus,

$$U = uAx \qquad \text{where} \qquad u = 13\ \mathrm{J/m^3}\ .$$

(d) The magnitude of force is

$$\left|\vec{F}\right| = \frac{dU}{dx} = uA\ .$$

(e) The force per unit area is

$$\frac{\left|\vec{F}\right|}{A} = u = 13\ \mathrm{N/m^2}\ .$$

Regarding units, it is worth noting that a Joule is equivalent to a N·m, which explains how $\mathrm{J/m^3}$ may be set equal to $\mathrm{N/m^2}$ in the above manipulation. We note, too, that the pressure unit $\mathrm{N/m^2}$ is generally known as a Pascal (Pa).

(f) Combining our steps in parts (a) through (e), we have

$$\begin{aligned} \frac{\left|\vec{F}\right|}{A} &= u = \frac{1}{2}\varepsilon_0 E^2 \\ 6.0\ \mathrm{N/m^2} &= \frac{1}{2}\varepsilon_0\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{\sigma^2}{2\varepsilon_0} \end{aligned}$$

which leads to $\sigma = \sqrt{2(8.85 \times 10^{-12})(6.0)} = 1.0 \times 10^{-5}\,\mathrm{C/m^2}$.

52. (a) We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\mathrm{bat}} = 40\ \mu\mathrm{C}$, and q_1 and q_2 are the charges on C_1 and C_2 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2\ \ .$$

Reducing the right portion of the circuit (the C_3, C_4 parallel pair which are in series with C_2) we have an equivalent capacitance of $C' = 8.0\ \mu\mathrm{F}$ which has charge $q' = q_2$ and potential difference equal to that of C_1. Thus,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q_2}{C'} \end{aligned}$$

which yields $4q_1 = q_2$. Therefore,

$$Q = q_1 + 4q_1$$

leads to $q_1 = 8.0\ \mu\text{C}$ and consequently to $q_2 = 32\,\mu\text{C}$.

(b) From Eq. 26-1, we have $V_2 = (32\,\mu\text{C})(16\,\mu\text{F}) = 2.0$ V.

53. Using Eq. 26-27, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200 \times 10^3\ \text{N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 26-21 and Eq. 26-25 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6 \times 10^{-5}\ \text{J} .$$

54. (a) The potential across capacitor 1 is 10 V, so the charge on it is

$$q_1 = C_1 V_1 = (10\ \mu\text{F})(10\ \text{V}) \ = \ 100\ \mu\text{C} .$$

(b) Reducing the right portion of the circuit produces an equivalence equal to $6.0\,\mu\text{F}$, with 10 V across it. Thus, a charge of $60\,\mu\text{C}$ is on it – and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\,\mu\text{C}}{10\,\mu\text{F}} \ = \ 6.0\ \text{V} ,$$

which leaves $10 - 6 = 4.0$ V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.0 V must be equally divided by C_2 and the capacitor directly below it (in series with it). Therefore, with 2.0 V across capacitor 2, we find

$$q_2 = C_2 V_2 = (10\,\mu\text{F})(2.0\ \text{V}) = 20\,\mu\text{C} .$$

55. (a) We use $q = CV = \varepsilon_0 AV/d$ to solve for A:

$$A = \frac{Cd}{\varepsilon_0} = \frac{(10 \times 10^{-12}\,\text{F})(1.0 \times 10^{-3}\,\text{m})}{\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 1.1 \times 10^{-3}\,\text{m}^2 .$$

(b) Now,

$$C' = C\left(\frac{d}{d'}\right) = (10\,\text{pF})\left(\frac{1.0\,\text{mm}}{0.9\,\text{mm}}\right) = 11\,\text{pF} .$$

(c) The new potential difference is $V' = q/C' = CV/C'$. Thus,

$$\Delta V = V' - V = \frac{(10\,\text{pF})(12\,\text{V})}{11\,\text{pF}} - 12\,\text{V} = 1.2\ \text{V} .$$

In a microphone, mechanical pressure applied to the aluminum foil as a result of sound can cause the capacitance of the foil to change, thereby inducing a variable ΔV in response to the sound signal.

56. (a) Here D is not attached to anything, so that the $6C$ and $4C$ capacitors are in series (equivalent to $2.4C$). This is then in parallel with the $2C$ capacitor, which produces an equivalence of $4.4C$. Finally the $4.4C$ is in series with C and we obtain

$$C_{\text{eq}} = \frac{(C)\,(4.4C)}{C + 4.4C} = 0.82C = 41\,\mu\text{F}$$

where we have used the fact that $C = 50\,\mu\text{F}$.

(b) Now, B is the point which is not attached to anything, so that the $6C$ and $2C$ capacitors are now in series (equivalent to $1.5C$), which is then in parallel with the $4C$ capacitor (and thus equivalent to $5.5C$). The $5.5C$ is then in series with the C capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)\,(5.5C)}{C + 5.5C} = 0.85C = 42\,\mu\text{F} \ .$$

57. In the first case the two capacitors are effectively connected in series, so the output potential difference is $V_{\text{out}} = CV_{\text{in}}/2C = V_{\text{in}}/2 = 50.0\,\text{V}$. In the second case the lower diode acts as a wire so $V_{\text{out}} = 0$.

58. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of $(n-1)$ identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} \ .$$

59. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0 \text{ V} \ .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2 V_2 = (20\,\mu\text{F})\,(2 \text{ V}) = 60 \ \mu\text{C} \ ,$$

which means a total of $90\,\mu$C of charge is on the pair of capacitors C_1 and C_2. This implies there is a total of $90\,\mu$C of charge also on the C_3 and C_4 pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45\,\mu$C. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3 \text{ V} \ .$$

Therefore, $|V_A - V_B| = V_1 + V_3 \ = \ 5.3$ V.

60. (a) The equivalent capacitance is

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.00\,\mu\text{F})(4.00\,\mu\text{F})}{6.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 2.40\,\mu\text{F} \ .$$

(b) $q = C_{\text{eq}} V = (2.40\,\mu\text{F})(200\,\text{V}) = 4.80 \times 10^4\,\text{C}$.

(c) $V_1 = q/C_1 = 4.80 \times 10^4\,\text{C}/2.40\,\mu\text{F} = 120\,\text{V}$, and $V_2 = V - V_1 = 200\,\text{V} - 120\,\text{V} = 80\,\text{V}$.

61. (a) Now $C_{\text{eq}} = C_1 + C_2 = 6.00\,\mu\text{F} + 4.00\,\mu\text{F} = 10.0\,\mu\text{F}$.

(b) $q_1 = C_1 V = (6.00\,\mu\text{F})(200\,\text{V}) = 1.20 \times 10^{-3}\,\text{C}$, $q_2 = C_2 V = (4.00\,\mu\text{F})(200\,\text{V}) = 8.00 \times 10^{-4}\,\text{C}$.

(c) $V_1 = V_2 = 200\,\text{V}$.

62. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors C and 60 μF). We do expect charge to be conserved. Thus, if Q is the charge originally stored on C and q_1, q_2 are the charges on the parallel pair after "setting down," then

$$\begin{aligned} Q &= q_1 + q_2 \\ C(100 \text{ V}) &= C(40 \text{ V}) + (60\,\mu\text{F})\,(40 \text{ V}) \end{aligned}$$

which leads to the solution $C = 40\,\mu$F.

63. (a) Put five such capacitors in series. Then, the equivalent capacitance is $2.0\,\mu\text{F}/5 = 0.40\,\mu\text{F}$. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now $C_{\text{eq}} = 3(0.40\,\mu\text{F}) = 1.2\,\mu\text{F}$. With each capacitor taking a 200-V potential difference the equivalent capacitor can withstand 1000 V.

64. (a) The energy per unit volume is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{32\pi^2\varepsilon_0 r^4} \ .$$

(b) From the expression above $u \propto r^{-4}$. So for $r \to 0 \quad u \to \infty$.

65. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 26-21 or Eq. 26-22, we sum the energies on the capacitors and obtain $U_{\text{total}} = 0.095$ J.

66. (a) Since the field is constant and the capacitors are in parallel (each with 600 V across them) with identical distances ($d = 0.00300$ m) between the plates, then the field in A is equal to the field in B:

$$\left|\vec{E}\right| = \frac{V}{d} = 2.00 \times 10^5 \text{ V/m} \ .$$

(b) See the note in part (a).

(c) For the air-filled capacitor, Eq. 26-4 leads to

$$\sigma = \frac{q}{A} = \varepsilon_0\left|\vec{E}\right| = 1.77 \times 10^{-6} \text{ C/m}^2 \ .$$

(d) For the dielectric-filled capacitor, we use Eq. 26-27:

$$\sigma = \kappa\varepsilon_0\left|\vec{E}\right| \ = \ 4.60 \times 10^{-6} \text{ C/m}^2 \ .$$

(e) Although the discussion in the textbook (§26-8) is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors which have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor B has a relatively large charge but only produces the field that A produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 26-32 and in the material that follows. Adapting Eq. 26-33 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma' = \left(1.77 \times 10^{-6}\right) - \left(4.60 \times 10^{-6}\right) = -2.83 \times 10^{-6} \text{ C/m}^2 \ .$$

67. (a) The equivalent capacitance is $C_{\text{eq}} = C_1C_2/(C_1 + C_2)$. Thus the charge q on each capacitor is

$$q = C_{\text{eq}}V = \frac{C_1C_2V}{C_1 + C_2} = \frac{(2.0\,\mu\text{F})(8.0\,\mu\text{F})(300\text{ V})}{2.0\,\mu\text{F} + 8.0\,\mu\text{F}} = 4.8 \times 10^{-4} \text{ C} \ .$$

The potential differences are: $V_1 = q/C_1 = 4.8 \times 10^{-4}\,\text{C}/2.0\,\mu\text{F} = 240\,\text{V}$, $V_2 = V - V_1 = 300\,\text{V} - 240\,\text{V} = 60\,\text{V}$.

(b) Now we have $q_1'/C_1 = q_2'/C_2 = V'$ (V' being the new potential difference across each capacitor) and $q_1' + q_2' = 2q$. We solve for q_1', q_2' and V:

$$\begin{aligned} q_1' &= \frac{2C_1 q}{C_1 + C_2} = \frac{2(2.0\,\mu\mathrm{F})(4.8\times10^{-4}\,\mathrm{C})}{2.0\,\mu\mathrm{F} + 8.0\,\mu\mathrm{F}} = 1.9\times10^{-4}\ \mathrm{C}\ , \\ q_2' &= 2q - q_1 = 7.7\times10^{-4}\ \mathrm{C}\ , \\ V' &= \frac{q_1'}{C_1} = \frac{1.92\times10^{-4}\,\mathrm{C}}{2.0\,\mu\mathrm{F}} = 96\ \mathrm{V}\ . \end{aligned}$$

(c) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = q_2 = 0$ and $V_1 = V_2 = 0$.

68. We use $U = \frac{1}{2}CV^2$. As V is increased by ΔV, the energy stored in the capacitor increases correspondingly from U to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus, $(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\%\ .$$

69. (a) The voltage across C_1 is 12 V, so the charge is

$$q_1 = C_1 V_1 = 24\,\mu\mathrm{C}\ .$$

(b) We reduce the circuit, starting with C_4 and C_3 (in parallel) which are equivalent to $4\,\mu$F. This is then in series with C_2, resulting in an equivalence equal to $\frac{4}{3}\,\mu$F which would have 12 V across it. The charge on this $\frac{4}{3}\,\mu$F capacitor (and therefore on C_2) is $(\frac{4}{3}\,\mu\mathrm{F})(12\,\mathrm{V}) = 16\,\mu$C. Consequently, the voltage across C_2 is

$$V_2 = \frac{q_2}{C_2} = \frac{16\,\mu\mathrm{C}}{2\,\mu\mathrm{F}} = 8\ \mathrm{V}\ .$$

This leaves $12 - 8 = 4$ V across C_4 (similarly for C_3).

70. (a) The energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(130\times10^{-12}\,\mathrm{F})(56.0\,\mathrm{V})^2 = 2.04\times10^{-7}\,\mathrm{J}\ .$$

(b) No, because we don't know the volume of the space inside the capacitor where the electric field is present.

71. We reduce the circuit, starting with C_1 and C_2 (in series) which are equivalent to $4\,\mu$F. This is then parallel to C_3 and results in a total of $8\,\mu$F, which is now in series with C_4 and can be further reduced. However, the final step in the reduction is not necessary, as we observe that the $8\,\mu$F equivalence from the top 3 capacitors has the same capacitance as C_4 and therefore the same voltage; since they are in series, that voltage is then $12/2 = 6$ V.

72. We use $C = \varepsilon_0 \kappa A/d \propto \kappa/d$. To maximize C we need to choose the material with the greatest value of κ/d. It follows that the mica sheet should be chosen.

73. (a) After reducing the pair of $4\,\mu$F capacitors to a series equivalence of $2\,\mu$F, we have three $2\ \mu$F capacitors in the upper right portion of the circuit all in parallel – and thus equivalent to $6\,\mu$F. In the lower right portion of the circuit are two $3\,\mu$F capacitors in parallel, equivalent also to $6\ \mu$F. These two $6\,\mu$F equivalent-capacitors are then in series, so that the full reduction leads to an equivalence of $3.0\,\mu$F.

(b) With 20 V across the result of part (a), we have a charge equal to $q = CV = (3.0\,\mu\mathrm{F})(20\ \mathrm{V}) = 60\,\mu$C.

74. (a) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d - b)$.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b} \ .$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d-b-d) = -\frac{q^2 b}{2\varepsilon_0 A} \ .$$

Since $W < 0$ the slab is sucked in.

75. (a) $C' = \varepsilon_0 A/(d-b)$, the same as part (a) in problem 74.

(b) Now,

$$\frac{U}{U'} = \frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2} = \frac{C}{C'} = \frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)} = \frac{d-b}{d} \ .$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2}(C'-C)V^2 = \frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b} - \frac{1}{d}\right)V^2 = \frac{\varepsilon_0 AbV^2}{2d(d-b)} \ .$$

Since $W > 0$ the slab must be pushed in.

76. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = 48\,\mu$C, and q_1 and q_3 are the charges on C_1 and C_3 after the switch is thrown to the right (and equilibrium is reached), then

$$Q = q_1 + q_3 \ .$$

We note that $V_{1 \text{ and } 2} = V_3$ because of the parallel arrangement, and $V_1 = \frac{1}{2}V_{1 \text{ and } 2}$ since they are identical capacitors. This leads to

$$\begin{aligned} 2V_1 &= V_3 \\ 2\frac{q_1}{C_1} &= \frac{q_3}{C_3} \\ 2q_1 &= q_3 \end{aligned}$$

where the last step follows from multiplying both sides by $2.00\,\mu$F. Therefore,

$$Q = q_1 + (2q_1)$$

which yields $q_1 = 16\,\mu$C and $q_3 = 32\,\mu$C.

77. (a) Since $u = \frac{1}{2}\kappa\varepsilon_0 E^2$, we select the material with the greatest value of $\kappa E^2_{\max}$, where $E_{\max}$ is its dielectric strength. We therefore choose strontium titanate, with the corresponding minimum volume

$$\mathcal{V}_{\min} = \frac{U}{U_{\max}} = \frac{2U}{\kappa\varepsilon_0 E^2_{\max}} = \frac{2(250\,\text{kJ})}{(310)\left(8.85\times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(8\,\text{kV/mm})^2} = 2.85\ \text{m}^3 \ .$$

(b) We solve for κ' from $U = \frac{1}{2}\kappa'\varepsilon_0 E^2_{\max}\mathcal{V}'_{\min}$:

$$\kappa' = \frac{2U}{\varepsilon_0 \mathcal{V}' E^2_{\max}} = \frac{2(250\,\text{kJ})}{\left(8.85\times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.0870\,\text{m}^3)(8\,\text{kV/mm})^2} = 1.01\times 10^4 \ .$$

78. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85 \times 10^{-12} \,\frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}\right)(0.12\,\mathrm{m}^2)}{1.2 \times 10^{-2}\,\mathrm{m}} = 89 \text{ pF} .$$

(b) Working through Sample Problem 26-6 algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85 \times 10^{-12} \,\frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}\right)(0.12\,\mathrm{m}^2)(4.8)}{(4.8)(1.2-0.40)(10^{-2}\,\mathrm{m}) + (4.0 \times 10^{-3}\,\mathrm{m})} = 120 \text{ pF} .$$

(c) Before the insertion, $q = C_0 V(89\,\mathrm{pF})(120\,\mathrm{V}) = 11\,\mathrm{nC}$. Since the battery is disconnected, q will remain the same after the insertion of the slab.

(d) $E = q/\varepsilon_0 A = 11 \times 10^{-9}\,\mathrm{C}/\left(8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}\right)(0.12\,\mathrm{m}^2) = 10\,\mathrm{kV/m}$.

(e) $E' = E/\kappa = (10\,\mathrm{kV/m})/4.8 = 2.1\,\mathrm{kV/m}$.

(f) $V = E(d-b) + E'b = (10\,\mathrm{kV/m})(0.012\,\mathrm{m} - 0.0040\,\mathrm{m}) + (2.1\,\mathrm{kV/m})(0.40 \times 10^{-3}\,\mathrm{m}) = 88\,\mathrm{V}$.

(g) The work done is

$$\begin{aligned} W_{\text{ext}} &= \Delta U = \frac{q^2}{2}\left(\frac{1}{C} - \frac{1}{C_0}\right) \\ &= \frac{(11 \times 10^{-9}\,\mathrm{C})^2}{2}\left(\frac{1}{89 \times 10^{-12}\,\mathrm{F}} - \frac{1}{120 \times 10^{-12}\,\mathrm{F}}\right) \\ &= -1.7 \times 10^{-7} \text{ J} . \end{aligned}$$

79. (a) Since $u = \frac{1}{2}\kappa\varepsilon_0 E^2$, $E_{\text{slab}} = E_{\text{air}}/\kappa_{\text{slab}}$, and $U = u\mathcal{V}$ (where $\mathcal{V} =$ volume), then the fraction of energy stored in the air gaps is

$$\begin{aligned} \frac{U_{\text{air}}}{U_{\text{total}}} &= \frac{E_{\text{air}}^2 A(d-b)}{E_{\text{air}}^2 A(d-b) + \kappa_{\text{slab}} E_{\text{slab}}^2 Ab} = \frac{1}{1 + \kappa_{\text{slab}}(E_{\text{slab}}/E_{\text{air}})^2[b/(d-b)]} \\ &= \frac{1}{1 + (2.61)(1/2.61)^2[0.780/(1.24-0.780)]} = 0.606 . \end{aligned}$$

(b) The fraction of energy stored in the slab is $1 - 0.606 = 0.394$.

80. (a) The equivalent capacitance of the three capacitors connected in parallel is $C_{\text{eq}} = 3C = 3\varepsilon_0 A/d = \varepsilon_0 A/(d/3)$. Thus, the required spacing is $d/3$.

(b) Now, $C_{\text{eq}} = C/3 = \varepsilon_0 A/3d$, so the spacing should be $3d$.

81. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\text{bat}} = 24\,\mu\mathrm{C}$, and q_1 and q_3 are the charges on C_1 and C_3 after the switch is thrown to the right (and equilibrium is reached), then

$$Q = q_1 + q_3 \quad .$$

We reduce the series pair C_2 and C_3 to $C' = 4/3\,\mu\mathrm{F}$ which has charge $q' = q_3$ and the same voltage that we find across C_1. Therefore,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q_3}{C'} \end{aligned}$$

which leads to $q_1 = 1.5q_3$. Hence,

$$Q = (1.5q_3) + q_3$$

leads to $q_3 = 9.6\,\mu\mathrm{C}$.

82. (First problem of **Cluster**)

(a) We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1 V_{\rm bat} = 400\,\mu$C, and q_1 and q_2 are the charges on C_1 and C_2 after the switch S is closed (and equilibrium is reached), then

$$Q = q_1 + q_2 \quad .$$

After switch S is closed, the capacitor voltages are equal, so that

$$\begin{aligned} V_1 &= V_2 \\ \frac{q_1}{C_1} &= \frac{q_2}{C_2} \end{aligned}$$

which yields $\frac{3}{4}q_1 = q_2$. Therefore,

$$Q = q_1 + \left(\frac{3}{4}\,q_1\right)$$

which gives the result $q_1 = 229\,\mu$C.

(b) The relation $\frac{3}{4}q_1 = q_2$ gives the result $q_2 = 171\,\mu$C.

(c) We apply Eq. 27-1: $V_1 = q_1/C_1 = 5.71$ V.

(d) Similarly, $V_2 = q_2/C_2 = 5.71$ V (which is equal to V_1, of course – since that fact was used in the solution to part (a)).

(e) When C_1 had charge Q and was connected to the battery, the energy stored was $\frac{1}{2}C_1V_{\rm bat}^2 = 2.00 \times 10^{-3}$ J. The energy stored after S is closed is $\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = 1.14 \times 10^{-3}$ J. The *decrease* is therefore 8.6×10^{-4} J.

83. (Second problem of **Cluster**)

(a) The change (from the previous problem) is that the initial charge (before switch S is closed) is $Q + Q'$ where Q is as before but $Q' = C_2(10\text{ V}) = 600\,\mu$C. We assume the polarities of these capacitor charges are the same. With this modification, we follow steps similar to those in the previous solution:

$$\begin{aligned} Q + Q' &= q_1 + q_2 \\ &= q_1 + \left(\frac{3}{4}\,q_1\right) \end{aligned}$$

which yields $q_1 = 571\,\mu$C.

(b) The relation $\frac{3}{4}q_1 = q_2$ gives the result $q_2 = 429\,\mu$C.

(c) We apply Eq. 27-1: $V_1 = q_1/C_1 = 14.3$ V.

(d) Similarly, $V_2 = q_2/C_2 = 14.3$ V.

(e) The initial energy now includes $\frac{1}{2}C_2(20\text{ V})^2$ in addition to the $\frac{1}{2}C_1V_{\rm bat}^2$ computed in the previous case. Thus, the total initial energy is 8.00×10^{-3} J. And the final stored energy is $\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = 7.14 \times 10^{-3}$ J. The *decrease* is therefore 8.6×10^{-4} J, as it was in the previous problem.

84. (Third problem of **Cluster**)

(a) With the series pair C_2 and C_3 reduced to a single $C' = 10\,\mu$F capacitor, this becomes very similar to problem 82. Noting for later use that $q' = q_2 = q_3$, and using notation similar to that used in the solution to problem 82, we have

$$Q = q_1 + q'$$

where $Q = C_1 V_{\text{bat}} = 400\,\mu$C. Also, after switch S is closed,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q'}{C'} \end{aligned}$$

which yields $\frac{1}{4}q_1 = q'$. Therefore,

$$Q = q_1 + \left(\frac{1}{4}q_1\right)$$

which gives the result $q_1 = 320\,\mu$C.

(b) We use $q_2 = q_3 = \frac{1}{4}q_1$ to obtain the result $80\,\mu$C.

(c) See part (b).

(d) (e) and (f) Eq. 26-1 yields

$$V = \frac{q}{C} = \begin{cases} 8.0 \text{ V} & \text{for } C_1 \\ 5.3 \text{ V} & \text{for } C_2 \\ 2.7 \text{ V} & \text{for } C_3 \end{cases}$$

85. (Fourth problem of **Cluster**)

(a) With the parallel pair C_2 and C_3 reduced to a single $C' = 45\,\mu$F capacitor, this becomes very similar to problem 82. Using notation similar to that used in the solution to 82, we have

$$Q = q_1 + q'$$

where $Q = C_1 V_{\text{bat}} = 400\,\mu$C. Also, after switch S is closed,

$$\begin{aligned} V_1 &= V' \\ \frac{q_1}{C_1} &= \frac{q'}{C'} \end{aligned}$$

which yields $\frac{9}{8}q_1 = q'$. Therefore,

$$Q = q_1 + \left(\frac{9}{8}q_1\right)$$

which gives the result $q_1 = 188\,\mu$C.

(b) We find the voltage across capacitor 1 from q_1/C_1 (see below) and (since the capacitors are in parallel) use the fact that $V_1 = V_2 = V_3$ with $q = CV$ to obtain the charges: $q_2 = 71\,\mu$C and $q_3 = 141\,\mu$C.

(c) See part (b).

(d) (e) and (f) The capacitors all have the same voltage. $V = q_1/C_1 = 4.7$ V.

86. (Fifth problem of **Cluster**)

(a) To begin with, the charge on capacitor 1 is $Q_1 = C_1 V_{\text{bat}} = 400\,\mu$C, and the charge on capacitor 2 is $Q_2 = C_2 V_{\text{bat}} = 150\,\mu$C. After the rearrangement and closing of the switch, the total charge in the upper portion of the circuit is $Q_1 - Q_2 = Q = 250\,\mu$C. With notation similar to that in the previous problems,

$$\begin{aligned} Q &= q_1 + q_2 \\ &= C_1 V + C_2 V \end{aligned}$$

which yields $V = 4.55$ V, which, in turn implies $q_1 = C_1 V = 182\,\mu$C and $q_2 = C_2 V = 68\,\mu$C. To achieve this distribution (with $+182\,\mu$C on one upper plate and $+68\,\mu$C on the other upper plate) from the arrangement right before closing the switch (with $+400\,\mu$C on one upper plate and $-150\,\mu$C on the other upper plate), it is necessary for $218\,\mu$C to flow through the switch.

(b) As shown above, $V = 4.55 \text{ V } = V_1 = V_2$.

Chapter 27

1. (a) The charge that passes through any cross section is the product of the current and time. Since $4.0\,\text{min} = (4.0\,\text{min})(60\,\text{s/min}) = 240\,\text{s}$, $q = it = (5.0\,\text{A})(240\,\text{s}) = 1200\,\text{C}$.

   (b) The number of electrons N is given by $q = Ne$, where e is the magnitude of the charge on an electron. Thus $N = q/e = (1200\,\text{C})/(1.60 \times 10^{-19}\,\text{C}) = 7.5 \times 10^{21}$.

2. We adapt the discussion in the text to a moving two-dimensional collection of charges. Using σ for the charge per unit area and w for the belt width, we can see that the transport of charge is expressed in the relationship $i = \sigma v w$, which leads to

$$\sigma = \frac{i}{vw} = \frac{100 \times 10^{-6}\,\text{A}}{(30\,\text{m/s})(50 \times 10^{-2}\,\text{m})} = 6.7 \times 10^{-6}\ \text{C/m}^2\ .$$

3. Suppose the charge on the sphere increases by Δq in time Δt. Then, in that time its potential increases by

$$\Delta V = \frac{\Delta q}{4\pi\varepsilon_0 r}\ ,$$

where r is the radius of the sphere. This means

$$\Delta q = 4\pi\varepsilon_0 r\,\Delta V\ .$$

Now, $\Delta q = (i_{\text{in}} - i_{\text{out}})\,\Delta t$, where i_{in} is the current entering the sphere and i_{out} is the current leaving. Thus,

$$\begin{aligned}\Delta t &= \frac{\Delta q}{i_{\text{in}} - i_{\text{out}}} = \frac{4\pi\varepsilon_0 r\,\Delta V}{i_{\text{in}} - i_{\text{out}}} \\ &= \frac{(0.10\,\text{m})(1000\,\text{V})}{(8.99 \times 10^9\,\text{F/m})(1.0000020\,\text{A} - 1.0000000\,\text{A})} = 5.6 \times 10^{-3}\ \text{s}\ .\end{aligned}$$

4. (a) The magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{i}{\pi d^2/4} = \frac{4(1.2 \times 10^{-10}\,\text{A})}{\pi(2.5 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{-5}\ \text{A/m}^2\ .$$

   (b) The drift speed of the current-carrying electrons is

$$v_d = \frac{J}{ne} = \frac{2.4 \times 10^{-5}\,\text{A/m}^2}{(8.47 \times 10^{28}/\text{m}^3)\,(1.60 \times 10^{-19}\,\text{C})} = 1.8 \times 10^{-15}\ \text{m/s}\ .$$

5. (a) The magnitude of the current density is given by $J = nqv_d$, where n is the number of particles per unit volume, q is the charge on each particle, and v_d is the drift speed of the particles. The particle

concentration is $n = 2.0 \times 10^8/\text{cm}^3 = 2.0 \times 10^{14}\,\text{m}^{-3}$, the charge is $q = 2e = 2(1.60 \times 10^{-19}\,\text{C}) = 3.20 \times 10^{-19}\,\text{C}$, and the drift speed is $1.0 \times 10^5\,\text{m/s}$. Thus,

$$J = (2 \times 10^{14}/\text{m})(3.2 \times 10^{-19}\,\text{C})(1.0 \times 10^5\,\text{m/s}) = 6.4\ \text{A/m}^2\ .$$

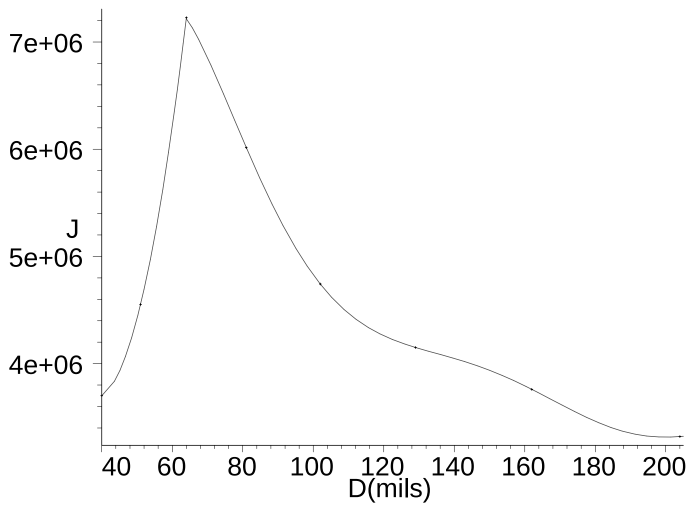
Since the particles are positively charged the current density is in the same direction as their motion, to the north.

(b) The current cannot be calculated unless the cross-sectional area of the beam is known. Then $i = JA$ can be used.

6. We express the magnitude of the current density vector in SI units by converting the diameter values in mils to inches (by dividing by 1000) and then converting to meters (by multiplying by 0.0254) and finally using

$$J = \frac{i}{A} = \frac{i}{\pi R^2} = \frac{4i}{\pi D^2}\ .$$

For example, the gauge 14 wire with $D = 64\,\text{mil} = 0.0016$ m is found to have a (maximum safe) current density of $J = 7.2 \times 10^6\ \text{A/m}^2$. In fact, this is the wire with the largest value of J allowed by the given data. The values of J in SI units are plotted below as a function of their diameters in mils.



7. The cross-sectional area of wire is given by $A = \pi r^2$, where r is its radius (half its thickness). The magnitude of the current density vector is $J = i/A = i/\pi r^2$, so

$$r = \sqrt{\frac{i}{\pi J}} = \sqrt{\frac{0.50\,\text{A}}{\pi(440 \times 10^4\,\text{A/m}^2)}} = 1.9 \times 10^{-4}\,\text{m}\ .$$

The diameter of the wire is therefore $d = 2r = 2(1.9 \times 10^{-4}\,\text{m}) = 3.8 \times 10^{-4}\,\text{m}$.

8. (a) Since $1\,\text{cm}^3 = 10^{-6}\,\text{m}^3$, the magnitude of the current density vector is

$$J = nev = \left(\frac{8.70}{10^{-6}\,\text{m}^3}\right)\left(1.60 \times 10^{-19}\,\text{C}\right)\left(470 \times 10^3\,\text{m/s}\right) = 6.54 \times 10^{-7}\ \text{A/m}^2\ .$$

(b) Although the total surface area of Earth is $4\pi R_E^2$ (that of a sphere), the area to be used in a computation of how many protons in an approximately unidirectional beam (the solar wind) will be captured by Earth is its projected area. In other words, for the beam, the encounter is with a "target" of circular area πR_E^2. The rate of charge transport implied by the influx of protons is

$$i = AJ = \pi R_E^2 J = \pi\left(6.37 \times 10^6\,\text{m}\right)^2\left(6.54 \times 10^{-7}\,\text{A/m}^2\right) = 8.34 \times 10^7\ \text{A}\ .$$

9. (a) The charge that strikes the surface in time Δt is given by $\Delta q = i\,\Delta t$, where i is the current. Since each particle carries charge $2e$, the number of particles that strike the surface is

$$N = \frac{\Delta q}{2e} = \frac{i\,\Delta t}{2e} = \frac{(0.25\times10^{-6}\,\mathrm{A})(3.0\,\mathrm{s})}{2(1.6\times10^{-19}\,\mathrm{C})} = 2.3\times10^{12}\ .$$

(b) Now let N be the number of particles in a length L of the beam. They will all pass through the beam cross section at one end in time $t = L/v$, where v is the particle speed. The current is the charge that moves through the cross section per unit time. That is, $i = 2eN/t = 2eNv/L$. Thus $N = iL/2ev$. To find the particle speed, we note the kinetic energy of a particle is

$$K = 20\,\mathrm{MeV} = (20\times10^{6}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV}) = 3.2\times10^{-12}\ \mathrm{J}\ .$$

Since $K = \frac{1}{2}mv^2$, then the speed is $v = \sqrt{2K/m}$. The mass of an alpha particle is (very nearly) 4 times the mass of a proton, or $m = 4(1.67\times10^{-27}\,\mathrm{kg}) = 6.68\times10^{-27}\,\mathrm{kg}$, so

$$v = \sqrt{\frac{2(3.2\times10^{-12}\,\mathrm{J})}{6.68\times10^{-27}\,\mathrm{kg}}} = 3.1\times10^{7}\,\mathrm{m/s}$$

and

$$N = \frac{iL}{2ev} = \frac{(0.25\times10^{-6})(20\times10^{-2}\,\mathrm{m})}{2(1.60\times10^{-19}\,\mathrm{C})(3.1\times10^{7}\,\mathrm{m/s})} = 5.0\times10^{3}\ .$$

(c) We use conservation of energy, where the initial kinetic energy is zero and the final kinetic energy is $20\,\mathrm{MeV} = 3.2\times10^{-12}\,\mathrm{J}$. We note, too, that the initial potential energy is $U_i = qV = 2eV$, and the final potential energy is zero. Here V is the electric potential through which the particles are accelerated. Consequently,

$$K_f = U_i = 2eV \implies V = \frac{K_f}{2e} = \frac{3.2\times10^{-12}\,\mathrm{J}}{2(1.60\times10^{-19}\mathrm{C})} = 10\times10^{6}\ \mathrm{V}\ .$$

10. (a) The current resulting from this non-uniform current density is

$$i = \int_{\mathrm{cylinder}} J\,dA = \int_0^R J_0\left(1-\frac{r}{R}\right)2\pi r dr = \frac{1}{3}\pi R^2 J_0 = \frac{1}{3}AJ_0\ .$$

(b) In this case,

$$i = \int_{\mathrm{cylinder}} J\,dA = \frac{J_0}{R}\int_0^R r\cdot 2\pi r dr = \frac{2}{3}\pi R^2 J_0 = \frac{2}{3}AJ_0\ .$$

The result is different from that in part (a) because the current density in part (b) is lower near the center of the cylinder (where the area is smaller for the same radial interval) and higher outward, resulting in a higher average current density over the cross section and consequently a greater current than that in part (a).

11. We use $v_d = J/ne = i/Ane$. Thus,

$$\begin{aligned} t &= \frac{L}{v_d} = \frac{L}{i/Ane} = \frac{LAne}{i} \\ &= \frac{(0.85\,\mathrm{m})(0.21\times10^{-4}\,\mathrm{m^2})(8.47\times10^{28}/\,\mathrm{m^3})(1.60\times10^{-19}\,\mathrm{C})}{300\,\mathrm{A}} \\ &= 8.1\times10^{2}\,\mathrm{s} = 13\ \mathrm{min}\ . \end{aligned}$$

12. We find the conductivity of Nichrome (the reciprocal of its resistivity) as follows:

$$\sigma = \frac{1}{\rho} = \frac{L}{RA} = \frac{L}{(V/i)A} = \frac{Li}{VA} = \frac{(1.0\,\mathrm{m})(4.0\,\mathrm{A})}{(2.0\,\mathrm{V})(1.0\times10^{-6}\,\mathrm{m^2})} = 2.0\times10^{6}/\Omega{\cdot}\mathrm{m}\ .$$

13. The resistance of the wire is given by $R = \rho L/A$, where ρ is the resistivity of the material, L is the length of the wire, and A is its cross-sectional area. In this case,

$$A = \pi r^2 = \pi(0.50 \times 10^{-3}\,\mathrm{m})^2 = 7.85 \times 10^{-7}\ \mathrm{m}^2 \ .$$

Thus,

$$\rho = \frac{RA}{L} = \frac{(50 \times 10^{-3}\,\Omega)(7.85 \times 10^{-7}\,\mathrm{m}^2)}{2.0\,\mathrm{m}} = 2.0 \times 10^{-8}\ \Omega\cdot\mathrm{m} \ .$$

14. Since $100\,\mathrm{cm} = 1\,\mathrm{m}$, then $10^4\,\mathrm{cm}^2 = 1\,\mathrm{m}^2$. Thus,

$$R = \frac{\rho L}{A} = \frac{(3.00 \times 10^{-7}\,\Omega\cdot\mathrm{m})(10.0 \times 10^3\,\mathrm{m})}{56.0 \times 10^{-4}\,\mathrm{m}^2} = 0.536\ \Omega \ .$$

15. Since the potential difference V and current i are related by $V = iR$, where R is the resistance of the electrician, the fatal voltage is $V = (50 \times 10^{-3}\,\mathrm{A})(2000\,\Omega) = 100\,\mathrm{V}$.

16. (a) $i = V/R = 23.0\,\mathrm{V}/15.0 \times 10^{-3}\,\Omega = 1.53 \times 10^3\,\mathrm{A}$.

(b) The cross-sectional area is $A = \pi r^2 = \frac{1}{4}\pi D^2$. Thus, the magnitude of the current density vector is

$$J = \frac{i}{A} = \frac{4i}{\pi D^2} = \frac{4(1.53 \times 10^{-3}\,\mathrm{A})}{\pi(6.00 \times 10^{-3}\,\mathrm{m})^2} = 5.41 \times 10^7\ \mathrm{A/m}^2 \ .$$

(c) The resistivity is $\rho = RA/L = (15.0 \times 10^{-3}\,\Omega)(\pi)(6.00 \times 10^{-3}\,\mathrm{m})^2/[4(4.00\,\mathrm{m})] = 10.6 \times 10^{-8}\,\Omega\cdot\mathrm{m}$. The material is platinum.

17. The resistance of the coil is given by $R = \rho L/A$, where L is the length of the wire, ρ is the resistivity of copper, and A is the cross-sectional area of the wire. Since each turn of wire has length $2\pi r$, where r is the radius of the coil, then $L = (250)2\pi r = (250)(2\pi)(0.12\,\mathrm{m}) = 188.5\,\mathrm{m}$. If r_w is the radius of the wire itself, then its cross-sectional area is $A = \pi r_w^2 = \pi(0.65 \times 10^{-3}\,\mathrm{m})^2 = 1.33 \times 10^{-6}\,\mathrm{m}^2$. According to Table 27-1, the resistivity of copper is $1.69 \times 10^{-8}\,\Omega\cdot\mathrm{m}$. Thus,

$$R = \frac{\rho L}{A} = \frac{(1.69 \times 10^{-8}\,\Omega\cdot\mathrm{m})(188.5\,\mathrm{m})}{1.33 \times 10^{-6}\,\mathrm{m}^2} = 2.4\ \Omega \ .$$

18. In Eq. 27-17, we let $\rho = 2\rho_0$ where ρ_0 is the resistivity at $T_0 = 20°\mathrm{C}$:

$$\rho - \rho_0 = 2\rho_0 - \rho_0 = \rho_0\alpha\,(T - T_0) \ ,$$

and solve for the temperature T:

$$T = T_0 + \frac{1}{\alpha} = 20°\mathrm{C} + \frac{1}{4.3 \times 10^{-3}/\mathrm{K}} \ \approx\ 250°\mathrm{C} \ .$$

Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. It is worth nothing that this agrees well with Fig. 27-10.

19. Since the mass and density of the material do not change, the volume remains the same. If L_0 is the original length, L is the new length, A_0 is the original cross-sectional area, and A is the new cross-sectional area, then $L_0A_0 = LA$ and $A = L_0A_0/L = L_0A_0/3L_0 = A_0/3$. The new resistance is

$$R = \frac{\rho L}{A} = \frac{\rho 3L_0}{A_0/3} = 9\frac{\rho L_0}{A_0} = 9R_0 \ ,$$

where R_0 is the original resistance. Thus, $R = 9(6.0\,\Omega) = 54\,\Omega$.

20. The thickness (diameter) of the wire is denoted by D. We use $R \propto L/A$ (Eq. 27-16) and note that $A = \frac{1}{4}\pi D^2 \propto D^2$. The resistance of the second wire is given by

$$R_2 = R\left(\frac{A_1}{A_2}\right)\left(\frac{L_2}{L_1}\right) = R\left(\frac{D_1}{D_2}\right)^2\left(\frac{L_2}{L_1}\right) = R(2)^2\left(\frac{1}{2}\right) = 2R \; .$$

21. The resistance of conductor A is given by

$$R_A = \frac{\rho L}{\pi r_A^2} \; ,$$

where r_A is the radius of the conductor. If r_o is the outside diameter of conductor B and r_i is its inside diameter, then its cross-sectional area is $\pi(r_o^2 - r_i^2)$, and its resistance is

$$R_B = \frac{\rho L}{\pi\left(r_o^2 - r_i^2\right)} \; .$$

The ratio is

$$\frac{R_A}{R_B} = \frac{r_o^2 - r_i^2}{r_A^2} = \frac{(1.0\,\text{mm})^2 - (0.50\,\text{mm})^2}{(0.50\,\text{mm})^2} = 3.0 \; .$$

22. (a) The current in each strand is $i = 0.750\,\text{A}/125 = 6.00 \times 10^{-3}\,\text{A}$.
    (b) The potential difference is $V = iR = (6.00 \times 10^{-3}\,\text{A})(2.65 \times 10^{-6}\,\Omega) = 1.59 \times 10^{-8}\,\text{V}$.
    (c) The resistance is $R_{\text{total}} = 2.65 \times 10^{-6}\,\Omega/125 = 2.12 \times 10^{-8}\,\Omega$.

23. We use $J = E/\rho$, where E is the magnitude of the (uniform) electric field in the wire, J is the magnitude of the current density, and ρ is the resistivity of the material. The electric field is given by $E = V/L$, where V is the potential difference along the wire and L is the length of the wire. Thus $J = V/L\rho$ and

$$\rho = \frac{V}{LJ} = \frac{115\,\text{V}}{(10\,\text{m})\left(1.4 \times 10^4\,\text{A/m}^2\right)} = 8.2 \times 10^{-4}\,\Omega\cdot\text{m} \; .$$

24. (a) $i = V/R = 35.8\,\text{V}/935\,\Omega = 3.83 \times 10^{-2}\,\text{A}$.
    (b) $J = i/A = (3.83 \times 10^{-2}\,\text{A})/(3.50 \times 10^{-4}\,\text{m}^2) = 109\,\text{A/m}^2$.
    (c) $v_d = J/ne = (109\,\text{A/m}^2)/[(5.33 \times 10^{22}/\text{m}^3)(1.60 \times 10^{-19}\,\text{C})] = 1.28 \times 10^{-2}\,\text{m/s}$.
    (d) $E = V/L = 35.8\,\text{V}/0.158\,\text{m} = 227\,\text{V/m}$.

25. The resistance at operating temperature T is $R = V/i = 2.9\,\text{V}/0.30\,\text{A} = 9.67\,\Omega$. Thus, from $R - R_0 = R_0\alpha(T - T_0)$, we find

$$\begin{aligned} T &= T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right) \\ &= 20^\circ\text{C} + \left(\frac{1}{4.5 \times 10^{-3}/\text{K}}\right)\left(\frac{9.67\,\Omega}{1.1\,\Omega} - 1\right) \end{aligned}$$

which yields approximately 1900°C. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of α used in this calculation is not inconsistent with the other units involved. Table 27-1 has been used.

26. We use $J = \sigma E = (n_+ + n_-)ev_d$, which combines Eq. 27-13 and Eq. 27-7.

    (a) The drift velocity is

$$v_d = \frac{\sigma E}{(n_+ + n_-)e} = \frac{(2.70 \times 10^{-14}/\Omega\cdot\text{m})(120\,\text{V/m})}{[(620 + 550)/\text{cm}^3](1.60 \times 10^{-19}\,\text{C})} = 1.73\ \text{cm/s} \; .$$

(b) $J = \sigma E = (2.70 \times 10^{-14}/\,\Omega\cdot\text{m})(120\,\text{V/m}) = 3.24 \times 10^{-12}\,\text{A/m}^2$.

27. (a) Let ΔT be the change in temperature and κ be the coefficient of linear expansion for copper. Then $\Delta L = \kappa L\,\Delta T$ and

$$\frac{\Delta L}{L} = \kappa\,\Delta T = (1.7 \times 10^{-5}/\text{K})(1.0°\text{C}) = 1.7 \times 10^{-5}\ .$$

This is equivalent to 0.0017%. Since a change in Celsius is equivalent to a change on the Kelvin temperature scale, the value of κ used in this calculation is not inconsistent with the other units involved. Incorporating a factor of 2 for the two-dimensional nature of A, the fractional change in area is

$$\frac{\Delta A}{A} = 2\kappa\,\Delta T = 2(1.7 \times 10^{-5}/\text{K})(1.0°\text{C}) = 3.4 \times 10^{-5}$$

which is 0.0034%. For small changes in the resistivity ρ, length L, and area A of a wire, the change in the resistance is given by

$$\Delta R = \frac{\partial R}{\partial \rho}\,\Delta\rho + \frac{\partial R}{\partial L}\,\Delta L + \frac{\partial R}{\partial A}\,\Delta A\ .$$

Since $R = \rho L/A$, $\partial R/\partial\rho = L/A = R/\rho$, $\partial R/\partial L = \rho/A = R/L$, and $\partial R/\partial A = -\rho L/A^2 = -R/A$. Furthermore, $\Delta\rho/\rho = \alpha\,\Delta T$, where α is the temperature coefficient of resistivity for copper ($4.3 \times 10^{-3}/\text{K} = 4.3 \times 10^{-3}/\text{C}°$, according to Table 27-1). Thus,

$$\begin{aligned}\frac{\Delta R}{R} &= \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A}\\ &= (\alpha + \kappa - 2\kappa)\,\Delta T = (\alpha - \kappa)\,\Delta T\\ &= (4.3 \times 10^{-3}/\text{C}° - 1.7 \times 10^{-5}/\text{C}°)\,(1.0\,\text{C}°) = 4.3 \times 10^{-3}\ .\end{aligned}$$

This is 0.43%, which we note (for the purposes of the next part) is primarily determined by the $\Delta\rho/\rho$ term in the above calculation.

(b) The fractional change in resistivity is much larger than the fractional change in length and area. Changes in length and area affect the resistance much less than changes in resistivity.

28. We use $R \propto L/A$. The diameter of a 22-gauge wire is 1/4 that of a 10-gauge wire. Thus from $R = \rho L/A$ we find the resistance of 25 ft of 22-gauge copper wire to be $R = (1.00\,\Omega)(25\,\text{ft}/1000\,\text{ft})(4)^2 = 0.40\,\Omega$.

29. (a) The current i is shown in Fig. 27-22 entering the truncated cone at the left end and leaving at the right. This is our choice of positive x direction. We make the assumption that the current density J at each value of x may be found by taking the ratio i/A where $A = \pi r^2$ is the cone's cross-section area at that particular value of x. The direction of $\vec{J}$ is identical to that shown in the figure for i (our $+x$ direction). Using Eq. 27-11, we then find an expression for the electric field at each value of x, and next find the potential difference V by integrating the field along the x axis, in accordance with the ideas of Chapter 25. Finally, the resistance of the cone is given by $R = V/i$. Thus,

$$J = \frac{i}{\pi r^2} = \frac{E}{\rho}$$

where we must deduce how r depends on x in order to proceed. We note that the radius increases linearly with x, so (with c_1 and c_2 to be determined later) we may write

$$r = c_1 + c_2\,x\ .$$

Choosing the origin at the left end of the truncated cone, the coefficient c_1 is chosen so that $r = a$ (when $x = 0$); therefore, $c_1 = a$. Also, the coefficient c_2 must be chosen so that (at the right end of the truncated cone) we have $r = b$ (when $x = L$); therefore, $c_2 = (b - a)/L$. Our expression, then, becomes

$$r = a + \left(\frac{b-a}{L}\right)\,x\ .$$

Substituting this into our previous statement and solving for the field, we find

$$E = \frac{i\rho}{\pi}\left(a + \frac{b-a}{L}x\right)^{-2} .$$

Consequently, the potential difference between the faces of the cone is

$$\begin{aligned} V &= -\int_0^L E\,dx = -\frac{i\rho}{\pi}\int_0^L \left(a + \frac{b-a}{L}x\right)^{-2} dx \\ &= \frac{i\rho}{\pi}\frac{L}{b-a}\left(a + \frac{b-a}{L}x\right)^{-1}\Bigg|_0^L = \frac{i\rho}{\pi}\frac{L}{b-a}\left(\frac{1}{a} - \frac{1}{b}\right) \\ &= \frac{i\rho}{\pi}\frac{L}{b-a}\frac{b-a}{ab} = \frac{i\rho L}{\pi ab} . \end{aligned}$$

The resistance is therefore

$$R = \frac{V}{i} = \frac{\rho L}{\pi ab} .$$

(b) If $b = a$, then $R = \rho L/\pi a^2 = \rho L/A$, where $A = \pi a^2$ is the cross-sectional area of the cylinder.

30. From Eq. 27-20, $\rho \propto \tau^{-1} \propto v_{\text{eff}}$. The connection with v_{eff} is indicated in part (b) of Sample Problem 27-5, which contains useful insight regarding the problem we are working now. According to Chapter 20, $v_{\text{eff}} \propto \sqrt{T}$. Thus, we may conclude that $\rho \propto \sqrt{T}$.

31. The power dissipated is given by the product of the current and the potential difference:

$$P = iV = (7.0 \times 10^{-3}\,\text{A})(80 \times 10^{3}\,\text{V}) = 560\text{W} .$$

32. Since $P = iV$, $q = it = Pt/V = (7.0\,\text{W})(5.0\,\text{h})(3600\,\text{s/h})/9.0\,\text{V} = 1.4 \times 10^4\,\text{C}$.

33. (a) Electrical energy is converted to heat at a rate given by

$$P = \frac{V^2}{R} ,$$

where V is the potential difference across the heater and R is the resistance of the heater. Thus,

$$P = \frac{(120\,\text{V})^2}{14\,\Omega} = 1.0 \times 10^3\,\text{W} = 1.0\ \text{kW} .$$

(b) The cost is given by

$$(1.0\,\text{kW})(5.0\,\text{h})(5.0\,\text{cents/kW}\cdot\text{h}) = 25\ \text{cents} .$$

34. The resistance is $R = P/i^2 = (100\,\text{W})/(3.00\,\text{A})^2 = 11.1\,\Omega$.

35. The relation $P = V^2/R$ implies $P \propto V^2$. Consequently, the power dissipated in the second case is

$$P = \left(\frac{1.50\,\text{V}}{3.00\,\text{V}}\right)^2 (0.540\,\text{W}) = 0.135\ \text{W} .$$

36. (a) From $P = V^2/R$ we find $R = V^2/P = (120\,\text{V})^2/500\,\text{W} = 28.8\,\Omega$.

(b) Since $i = P/V$, the rate of electron transport is

$$\frac{i}{e} = \frac{P}{eV} = \frac{500\,\text{W}}{(1.60 \times 10^{-19}\,\text{C})(120\,\text{V})} = 2.60 \times 10^{19}/\text{s} .$$

37. (a) The power dissipated, the current in the heater, and the potential difference across the heater are related by $P = iV$. Therefore,

$$i = \frac{P}{V} = \frac{1250\,\mathrm{W}}{115\,\mathrm{V}} = 10.9\ \mathrm{A}\ .$$

(b) Ohm's law states $V = iR$, where R is the resistance of the heater. Thus,

$$R = \frac{V}{i} = \frac{115\,\mathrm{V}}{10.9\,\mathrm{A}} = 10.6\ \Omega\ .$$

(c) The thermal energy E generated by the heater in time $t = 1.0\,\mathrm{h} = 3600\,\mathrm{s}$ is

$$E = Pt = (1250\,\mathrm{W})(3600\,\mathrm{s}) = 4.5 \times 10^6\ \mathrm{J}\ .$$

38. (a) From $P = V^2/R = AV^2/\rho L$, we solve for the length:

$$L = \frac{AV^2}{\rho P} = \frac{(2.60 \times 10^{-6}\,\mathrm{m}^2)(75.0\,\mathrm{V})^2}{(5.00 \times 10^{-7}\,\Omega\cdot\mathrm{m})(5000\,\mathrm{W})} = 5.85\ \mathrm{m}\ .$$

(b) Since $L \propto V^2$ the new length should be

$$L' = L\left(\frac{V'}{V}\right)^2 = (5.85\,\mathrm{m})\left(\frac{100\,\mathrm{V}}{75.0\,\mathrm{V}}\right)^2 = 10.4\ \mathrm{m}\ .$$

39. Let R_H be the resistance at the higher temperature (800°C) and let R_L be the resistance at the lower temperature (200°C). Since the potential difference is the same for the two temperatures, the power dissipated at the lower temperature is $P_L = V^2/R_L$, and the power dissipated at the higher temperature is $P_H = V^2/R_H$, so $P_L = (R_H/R_L)P_H$. Now $R_L = R_H + \alpha R_H\,\Delta T$, where ΔT is the temperature difference $T_L - T_H = -600\,\mathrm{C}^\circ = -600\,\mathrm{K}$. Thus,

$$P_L = \frac{R_H}{R_H + \alpha R_H\,\Delta T}\,P_H = \frac{P_H}{1 + \alpha\,\Delta T} = \frac{500\,\mathrm{W}}{1 + (4.0 \times 10^{-4}/\mathrm{K})(-600\,\mathrm{K})} = 660\ \mathrm{W}\ .$$

40. (a) The monthly cost is $(100\,\mathrm{W})(24\,\mathrm{h/day})(31\,\mathrm{day/month})(6\,\mathrm{cents/kW\cdot h}) = 446\,\mathrm{cents} = \$4.46$, assuming a 31-day month.

(b) $R = V^2/P = (120\,\mathrm{V})^2/100\,\mathrm{W} = 144\,\Omega$.

(c) $i = P/V = 100\,\mathrm{W}/120\,\mathrm{V} = 0.833\,\mathrm{A}$.

41. (a) The charge q that flows past any cross section of the beam in time Δt is given by $q = i\,\Delta t$, and the number of electrons is $N = q/e = (i/e)\,\Delta t$. This is the number of electrons that are accelerated. Thus

$$N = \frac{(0.50\,\mathrm{A})(0.10 \times 10^{-6}\,\mathrm{s})}{1.60 \times 10^{-19}\,\mathrm{C}} = 3.1 \times 10^{11}\ .$$

(b) Over a long time t the total charge is $Q = nqt$, where n is the number of pulses per unit time and q is the charge in one pulse. The average current is given by $i_{\mathrm{avg}} = Q/t = nq$. Now $q = i\,\Delta t = (0.50\,\mathrm{A})(0.10 \times 10^{-6}\,\mathrm{s}) = 5.0 \times 10^{-8}\,\mathrm{C}$, so

$$i_{\mathrm{avg}} = (500/\mathrm{s})(5.0 \times 10^{-8}\,\mathrm{C}) = 2.5 \times 10^{-5}\ \mathrm{A}\ .$$

(c) The accelerating potential difference is $V = K/e$, where K is the final kinetic energy of an electron. Since $K = 50\,\mathrm{MeV}$, the accelerating potential is $V = 50\,\mathrm{kV} = 5.0 \times 10^7\,\mathrm{V}$. During a pulse the power output is

$$P = iV = (0.50\,\mathrm{A})(5.0 \times 10^7\,\mathrm{V}) = 2.5 \times 10^7\ \mathrm{W}\ .$$

This is the peak power. The average power is

$$P_{\mathrm{avg}} = i_{\mathrm{avg}}V = (2.5 \times 10^{-5}\,\mathrm{A})(5.0 \times 10^7\,\mathrm{V}) = 1.3 \times 10^3\ \mathrm{W}\ .$$

42. (a) Since $P = i^2R = J^2A^2R$, the current density is

$$J = \frac{1}{A}\sqrt{\frac{P}{R}} = \frac{1}{A}\sqrt{\frac{P}{\rho L/A}} = \sqrt{\frac{P}{\rho L A}}$$
$$= \sqrt{\frac{1.0\,\mathrm{W}}{\pi(3.5\times10^{-5}\,\Omega\cdot\mathrm{m})(2.0\times10^{-2}\,\mathrm{m})(5.0\times10^{-3}\,\mathrm{m})^2}} = 1.3\times10^{5}\ \mathrm{A/m^2}\ .$$

(b) From $P = iV = JAV$ we get

$$V = \frac{P}{AJ} = \frac{P}{\pi r^2 J} = \frac{1.0\,\mathrm{W}}{\pi(5.0\times10^{-3}\,\mathrm{m})^2(1.3\times10^{5}\,\mathrm{A/m^2})} = 9.4\times10^{-2}\ \mathrm{V}\ .$$

43. (a) Using Table 27-1 and Eq. 27-10 (or Eq. 27-11), we have

$$\left|\vec{E}\right| = \rho\left|\vec{J}\right| = \left(1.69\times10^{-8}\,\Omega\cdot\mathrm{m}\right)\left(\frac{2.0\,\mathrm{A}}{2.0\times10^{-6}\,\mathrm{m^2}}\right) = 1.7\times10^{-2}\ \mathrm{V/m}\ .$$

(b) Using $L = 4.0$ m, the resistance is found from Eq. 27-16: $R = \rho L/A = 0.034\,\Omega$. The rate of thermal energy generation is found from Eq. 27-22: $P = i^2R = 0.14$ W. Assuming a steady rate, the thermal energy generated in 30 minutes is $(0.14\,\mathrm{J/s})(30\times60\,\mathrm{s}) = 2.4\times10^{2}$ J.

44. (a) Current is the transport of charge; here it is being transported "in bulk" due to the volume rate of flow of the powder. From Chapter 15, we recall that the volume rate of flow is the product of the cross-sectional area (of the stream) and the (average) stream velocity. Thus, $i = \rho A v$ where ρ is the charge per unit volume. If the cross-section is that of a circle, then $i = \rho\pi R^2 v$.

(b) Recalling that a Coulomb per second is an Ampere, we obtain

$$i = \left(1.1\times10^{-3}\,\mathrm{C/m^3}\right)\pi(0.050\,\mathrm{m})^2(2.0\,\mathrm{m/s}) = 1.7\times10^{-5}\ \mathrm{A}\ .$$

(c) The motion of charge is not in the same direction as the potential difference computed in problem 57 of Chapter 25. It might be useful to think of (by analogy) Eq. 7-48; there, the scalar (dot) product in $P = \vec{F}\cdot\vec{v}$ makes it clear that $P = 0$ if $\vec{F}\perp\vec{v}$. This suggests that a radial potential difference and an axial flow of charge will not together produce the needed transfer of energy (into the form of a spark).

(d) With the assumption that there is (at least) a voltage equal to that computed in problem 57 of Chapter 25, in the proper direction to enable the transference of energy (into a spark), then we use our result from that problem in Eq. 27-21:

$$P = iV = \left(1.7\times10^{-5}\,\mathrm{A}\right)\left(7.8\times10^{4}\,\mathrm{V}\right) = 1.3\ \mathrm{W}\ .$$

(e) Recalling that a Joule per second is a Watt, we obtain $(1.3\,\mathrm{W})(0.20\,\mathrm{s}) = 0.27\,\mathrm{J}$ for the energy that can be transferred at the exit of the pipe.

(f) This result is greater than the 0.15 J needed for a spark, so we conclude that the spark was likely to have occurred at the exit of the pipe, going into the silo.

45. (a) Since the area of a hemisphere is $2\pi r^2$ then the magnitude of the current density vector is

$$\left|\vec{J}\right| = \frac{i}{A} = \frac{I}{2\pi r^2}\ .$$

(b) Eq. 27-11 yields $\left|\vec{E}\right| = \rho\left|\vec{J}\right| = \rho I/2\pi r^2$.

(c) Eq. 25-18 leads to

$$\Delta V = V_r - V_b = -\int_b^r \vec{E} \cdot d\vec{r} = -\int_b^r \left(\frac{\rho I}{2\pi r^2}\right) dr = \frac{\rho I}{2\pi}\left(\frac{1}{r} - \frac{1}{b}\right) .$$

(d) Using the given values, we obtain $\left|\vec{J}\right| = \frac{100}{2\pi(10)^2} = 0.16\ \text{A/m}^2$.

(e) Also, $\left|\vec{E}\right| = 16$ V/m (or 16 N/C).

(f) With $b = 0.010$ m, the voltage is $\Delta V = -1.6 \times 10^5$ V.

46. (a) Using Eq. 27-11 and Eq. 25-42, we obtain

$$\left|\vec{J}_A\right| = \frac{\left|\vec{E}_A\right|}{\rho} = \frac{|\Delta V_A|}{\rho L} = \frac{40 \times 10^{-6}\,\text{V}}{(100\,\Omega\cdot\text{m})(20\,\text{m})} = 2.0 \times 10^{-8}\ \text{A/m}^2 .$$

(b) Similarly, in region B we find

$$\left|\vec{J}_B\right| = \frac{|\Delta V_B|}{\rho L} = \frac{60 \times 10^{-6}\,\text{V}}{(100\,\Omega\cdot\text{m})(20\,\text{m})} = 3.0 \times 10^{-8}\ \text{A/m}^2 .$$

(c) With $w = 1.0$ m and $d_A = 3.8$ m (so that the cross-section area is $d_A w$) we have (using Eq. 27-5)

$$i_A = \left|\vec{J}_A\right| d_A w = \left(2.0 \times 10^{-8}\,\text{A/m}^2\right)(1.0\,\text{m})(3.8\,\text{m}) = 7.6 \times 10^{-8}\ \text{A} .$$

(d) Assuming $i_A = i_B$ we obtain

$$d_B = \frac{i_B}{\left|\vec{J}_B\right| w} = \frac{7.6 \times 10^{-8}\,\text{A}}{(3.0 \times 10^{-8}\,\text{A/m}^2)\,(1.0\,\text{m})} = 2.5\ \text{m} .$$

(e) We do not show the graph-and-figure here, but describe it briefly. To be meaningful (as a function of x) we would plot $V(x)$ measured relative to $V(0)$ (the voltage at, say, the left edge of the figure, which we are effectively setting equal to 0). From the problem statement, we note that $V(x)$ would grow linearly in region A, increasing by 40 μV for each 20 m distance. Once we reach the transition region (between A and B) we might assume a parabolic shape for $V(x)$ as it changes from the 40 μV-per-20 m slope to a 60 μV-per-20 m slope (which becomes its constant slope once we are into region B, where the function is again linear). The figure goes further than region B, so as we leave region B, we might assume again a parabolic shape for the function as it tends back down toward some lower slope value.

47. (a) It is useful to read the whole problem before considering the sketch here in part (a) (which we do not show, but briefly describe). We find in part (d) and part (f), below, that $J_A > J_B$ which suggests that the streamlines should be closer together in region A than in B (at least for portions of those regions which lie close to the pipe). Associated with this (see part (g)) the sketch of the streamlines should reflect that fact that some of the conduction charge-carriers are entering the pipe walls during the transition from region A to region B.

(b) Eq. 27-16 yields

$$\rho_{\text{pipe}} = R\frac{A}{L} = (6.0\,\Omega)\left(\frac{0.010\,\text{m}^2}{1.0 \times 10^6\,\text{m}}\right) = 6.0 \times 10^{-8}\ \Omega\cdot\text{m} .$$

(c) If the resistance of 1000 km of pipe is 6.0 Ω then the resistance of $L = 1.0$ km of pipe is $R = 6.0\,\text{m}\Omega$. Thus in region A, Ohm's law leads to

$$i_{\text{pipe}} = \frac{V_{ab}}{R} = \frac{8.0\,\text{mV}}{6.0\,\text{m}\Omega} = 1.3\ \text{A} .$$

(d) Using Eq. 27-11 and Eq. 25-42 (in absolute value), we find the magnitude of the current density vector in region A:

$$\left|\vec{J}_{\text{ground}}\right| = \frac{V_{ab}}{\rho_{\text{ground}}L} = \frac{0.0080\,\text{V}}{(500\,\Omega\cdot\text{m})(1000\,\text{m})} = 1.6\times10^{-8}\ \text{A/m}^2\ .$$

(e) Similarly, in region B we obtain

$$i_{\text{pipe}} = \frac{V_{cd}}{R} = \frac{9.5\,\text{mV}}{6.0\,\text{m}\Omega} = 1.6\ \text{A}\ ,$$

(f) and

$$\left|\vec{J}_{\text{ground}}\right| = \frac{V_{cd}}{\rho_{\text{ground}}L} = \frac{0.0095\,\text{V}}{(1000\,\Omega\cdot\text{m})(1000\,\text{m})} = 9.5\times10^{-9}\ \text{A/m}^2\ .$$

(g) These results suggest that the pipe walls, in leaving region A and entering region B, have "absorbed" some of the current, leaving the current density in the nearby ground somewhat "depleted" of the telluric flows.

(h) We assume the transition $B \to A$ is the reverse of that discussed in part (g). Here, some current leaves the pipe walls and joins in the ground-supported telluric flows.

(i) There is no current here, because there is no potential difference along this section of pipe. The reason $V_{gh} = 0$ is best seen using Eq. 27-11 and Eq. 25-18 (and remembering that the scalar dot product gives zero for perpendicular vectors). The arrows shown in the figure for current actually refer, in the technical sense, to the direction of $\vec{J}$. We refer to this as the x direction. The pipe section gh is oriented in what we will refer to as the y direction. Eq. 27-11 implies that $\vec{J}$ and $\vec{E}$ must be in the same direction (x). But a nonzero voltage difference here would require (by Eq. 25-18) $\int \vec{E}\cdot d\vec{s} \neq 0$. But since $d\vec{s} = dy$ for this section of pipe, then $\vec{E}\cdot d\vec{s}$ vanishes identically.

(j) Our discussion in part (j) serves also to motivate the fact that the current in section fg is less than that in section ef by a factor of $\cos 45^\circ = 1/\sqrt{2}$. To see this, one may consider the component of the electric field which would "drive" the current (in the sense of Eq. 27-11) along section fg; it is less than the field responsible for the current in section ef by exactly the factor just mentioned. Thus,

$$i_{fg} = i_{ef}\cos 45^\circ = \frac{1.0\,\text{A}}{\sqrt{2}} = 0.71\ \text{A}\ .$$

(k) The answers to the previous parts indicate that current leaves the pipe at point f and

(l) at point g.

48. (a) We use Eq. 27-16. The new area is $A' = AL/L' = A/2$.

(b) The new resistance is $R' = R(A/A')(L'/L) = 4R$.

49. We use $P = i^2R = i^2\rho L/A$, or $L/A = P/i^2\rho$. So the new values of L and A satisfy

$$\left(\frac{L}{A}\right)_{\text{new}} = \left(\frac{P}{i^2\rho}\right)_{\text{new}} = \frac{30}{4^2}\left(\frac{P}{i^2\rho}\right)_{\text{old}} = \frac{30}{16}\left(\frac{L}{A}\right)_{\text{old}}\ .$$

Consequently, $(L/A)_{\text{new}} = 1.875(L/A)_{\text{old}}$. Note, too, that $(LA)_{\text{new}} = (LA)_{\text{old}}$. We solve the above two equations for L_{new} and A_{new}:

$$\begin{aligned} L_{\text{new}} &= \sqrt{1.875}L_{\text{old}} = 1.369L_{\text{old}} \\ A_{\text{new}} &= \sqrt{1/1.875}A_{\text{old}} = 0.730A_{\text{old}}\ . \end{aligned}$$

50. (a) We denote the copper wire with subscript c and the aluminum wire with subscript a.

$$R = \rho_a\frac{L}{A} = \frac{(2.75\times10^{-8}\,\Omega\cdot\text{m})(1.3\,\text{m})}{(5.2\times10^{-3}\,\text{m})^2} = 1.3\times10^{-3}\ \Omega\ .$$

(b) Let $R = \rho_c L/(\pi d^2/4)$ and solve for the diameter d of the copper wire:

$$d = \sqrt{\frac{4\rho_c L}{\pi R}} = \sqrt{\frac{4(1.69\times10^{-8}\,\Omega\cdot\text{m})(1.3\,\text{m})}{\pi(1.3\times10^{-3}\,\Omega)}} = 4.6\times10^{-3}\ \text{m}\ .$$

51. We use Eq. 27-17: $\rho - \rho_0 = \rho\alpha(T - T_0)$, and solve for T:

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right) = 20°\text{C} + \frac{1}{4.3\times10^{-3}/\text{K}}\left(\frac{58\,\Omega}{50\,\Omega} - 1\right) = 57°\text{C}\ .$$

We are assuming that $\rho/\rho_0 = R/R_0$.

52. Since values from the referred-to graph can only be crudely estimated, we do not present a graph here, but rather indicate a few values. Since $R = V/i$ then we see $R = \infty$ when $i = 0$ (which the graph seems to show throughout the range $-\infty < V < 2$ V) and $V \neq 0$. For voltages values larger than 2 V, the resistance changes rapidly according to the ratio V/i. For instance, $R \approx 3.1/0.002 = 1550\,\Omega$ when $V = 3.1$ V, and $R \approx 3.8/0.006 = 633\,\Omega$ when $V = 3.8$ V

53. (a)

$$V = iR = i\rho\frac{L}{A} = \frac{(12\,\text{A})(1.69\times10^{-8}\,\Omega\cdot\text{m})(4.0\times10^{-2}\,\text{m})}{\pi(5.2\times10^{-3}\,\text{m}/2)^2} = 3.8\times10^{-4}\,\text{V}\ .$$

(b) Since it moves in the direction of the electron drift which is against the direction of the current, its tail is negative compared to its head.

(c) The time of travel relates to the drift speed:

$$\begin{aligned} t &= \frac{L}{v_d} = \frac{lAne}{i} = \frac{\pi L d^2 ne}{4i} \\ &= \frac{\pi(1.0\times10^{-2}\,\text{m})(5.2\times10^{-3}\,\text{m})^2(8.47\times10^{28}/\text{m}^3)(1.60\times10^{-19}\,\text{C})}{4(12\,\text{A})} \\ &= 238\,\text{s} = 3\,\text{min}\,58\,\text{s}\ . \end{aligned}$$

54. Using Eq. 7-48 and Eq. 27-22, the rate of change of mechanical energy of the piston-Earth system, mgv, must be equal to the rate at which heat is generated from the coil: $mgv = i^2R$. Thus

$$v = \frac{i^2R}{mg} = \frac{(0.240\,\text{A})^2(550\,\Omega)}{(12\,\text{kg})\,(9.8\,\text{m/s}^2)} = 0.27\ \text{m/s}\ .$$

55. Eq. 27-21 gives the rate of thermal energy production:

$$P = iV = (10\ \text{A})(120\ \text{V}) = 1.2\ \text{kW}\ .$$

Dividing this into the 180 kJ necessary to cook the three hot-dogs leads to the result $t = 150$ s.

56. We find the drift speed from Eq. 27-7:

$$v_d = \frac{|\vec{J}|}{ne} = 1.5\times10^{-4}\,\text{m/s}\ .$$

At this (average) rate, the time required to travel $L = 5.0$ m is

$$t = \frac{L}{v_d} = 3.4\times10^4\,\text{s}\ .$$

57. (a) $i = (n_h + n_e)e = (2.25\times10^{15}/\text{s} + 3.50\times10^{15}/\text{s})(1.60\times10^{-19}\,\text{C}) = 9.20\times10^{-4}\,\text{A}$.

(b) The magnitude of the current density vector is

$$\left|\vec{J}\right| = \frac{i}{A} = \frac{9.20 \times 10^{-4}\,\text{A}}{\pi(0.165 \times 10^{-3}\,\text{m})^2} = 1.08 \times 10^4 \ \text{A/m}^2 \ .$$

58. (a) Since $\rho = RA/L = \pi R d^2/4L = \pi(1.09 \times 10^{-3}\,\Omega)(5.50 \times 10^{-3}\,\text{m})^2/[4(1.60\,\text{m})] = 1.62 \times 10^{-8}\,\Omega\cdot\text{m}$, the material is silver.

(b) The resistance of the round disk is

$$R = \rho\frac{L}{A} = \frac{4\rho L}{\pi d^2} = \frac{4(1.62 \times 10^{-8}\,\Omega\cdot\text{m})(1.00 \times 10^{-3}\,\text{m})}{\pi(2.00 \times 10^{-2}\,\text{m})^2} = 5.16 \times 10^{-8} \ \Omega \ .$$

59. The horsepower required is

$$P = \frac{iV}{0.80} = \frac{(10\,\text{A})(12\,\text{V})}{(0.80)(746\,\text{W/hp})} = 0.20 \ \text{hp} \ .$$

60. (a) The current is

$$\begin{aligned} i &= \frac{V}{R} = \frac{V}{\rho L/A} = \frac{\pi V d^2}{4\rho L} \\ &= \frac{\pi(1.20\,\text{V})[(0.0400\,\text{in.})(2.54 \times 10^{-2}\,\text{m/in.})]^2}{4(1.69 \times 10^{-8}\,\Omega\cdot\text{m})(33.0\,\text{m})} = 1.74\,\text{A} \ . \end{aligned}$$

(b) The magnitude of the current density vector is

$$\left|\vec{J}\right| = \frac{i}{A} = \frac{4i}{\pi d^2} = \frac{4(1.74\,\text{A})}{\pi[(0.0400\,\text{in.})(2.54 \times 10^{-2}\,\text{m/in.})]^2} = 2.15 \times 10^6 \ \text{A/m}^2.$$

(c) $E = V/L = 1.20\,\text{V}/33.0\,\text{m} = 3.63 \times 10^{-2}\,\text{V/m}$.

(d) $P = Vi = (1.20\,\text{V})(1.74\,\text{A}) = 2.09\,\text{W}$.

61. We use $R/L = \rho/A = 0.150\,\Omega/\text{km}$.

(a) For copper $J = i/A = (60.0\,\text{A})(0.150\,\Omega/\text{km})/(1.69 \times 10^{-8}\,\Omega\cdot\text{m}) = 5.32 \times 10^5\,\text{A/m}^2$; and for aluminum $J = (60.0\,\text{A})(0.150\,\Omega/\text{km})/(2.75 \times 10^{-8}\,\Omega\cdot\text{m}) = 3.27 \times 10^5\,\text{A/m}^2$.

(b) We denote the mass densities as ρ_m. For copper $(m/L)_c = (\rho_m A)_c = (8960\,\text{kg/m}^3)\,(1.69 \times 10^{-8}\,\Omega\cdot\text{m})/(0.150\,\Omega/\text{km}) = 1.01\,\text{kg/m}$; and for aluminum $(m/L)_a = (\rho_m A)_a = (2700\,\text{kg/m}^3)(2.75 \times 10^{-8}\,\Omega\cdot\text{m})/(0.150\,\Omega/\text{km}) = 0.495\,\text{kg/m}$.

62. (a) We use $P = V^2/R \propto V^2$, which gives $\Delta P \propto \Delta V^2 \approx 2V\Delta V$. The percentage change is roughly $\Delta P/P = 2\Delta V/V = 2(110 - 115)/115 = -8.6\%$.

(b) A drop in V causes a drop in P, which in turn lowers the temperature of the resistor in the coil. At a lower temperature R is also decreased. Since $P \propto R^{-1}$ a decrease in R will result in an increase in P, which partially offsets the decrease in P due to the drop in V. Thus, the actual drop in P will be smaller when the temperature dependency of the resistance is taken into consideration.

63. Using $A = \pi r^2$ with $r = 5 \times 10^{-4}$ m with Eq. 27-5 yields

$$|\vec{J}| = \frac{i}{A} = 2.5 \times 10^6\,\text{A/m}^2 \ .$$

Then, with $|\vec{E}| = 5.3$ V/m, Eq. 27-10 leads to

$$\rho = \frac{5.3\,\text{V/m}}{2.5 \times 10^6\,\text{A/m}^2} = 2.1 \times 10^{-6} \ \Omega\cdot\text{m} \ .$$

64. A least squares fit of the data gives $R = \frac{537}{5} + \frac{1111}{1750}T$ with T in degrees Celsius.

(a) At $T = 20°\text{C}$, our expression gives $R = \frac{21017}{175} \approx 120\,\Omega$.

(b) At $T = 0°\text{C}$, our expression gives $R = \frac{537}{5} \approx 107\,\Omega$.

(c) Defining α_R by

$$\alpha_R = \frac{R - R_{20}}{R_{20}\,(T - 20°\text{C})}$$

then we are effectively requiring $\alpha_R R_{20}$ to equal the $\frac{1111}{1750}$ factor in our least squares fit. This implies that $\alpha_R = 1111/210170 = 0.00529/\text{C}°$ if $R_{20} = \frac{21017}{175} \approx 120\,\Omega$ is used as the reference.

(d) Now we define α_R by

$$\alpha_R = \frac{R - R_0}{R_0\,(T - 0°\text{C})}\ ,$$

which means we require $\alpha_R R_0$ to equal the $\frac{1111}{1750}$ factor in our least squares fit. In this case, $\alpha_R = 1111/187950 = 0.00591/\text{C}°$ if $R_0 = \frac{537}{5} \approx 107\,\Omega$ is used as the reference.

(e) Our least squares fit expression predicts $R = 96473/350 \approx 276\,\Omega$ at $T = 265°\text{C}$.

65. The electric field points towards lower values of potential (see Eq. 25-40) so $\vec{E}$ is directed towards point B (which we take to be the $\hat{\imath}$ direction in our calculation). Since the field is considered to be uniform inside the wire, then its magnitude is, by Eq. 25-42,

$$|\vec{E}| = \frac{|\Delta V|}{L} = \frac{50}{200} = 0.25 \text{ V/m}\ .$$

Using Eq. 27-11, with $\rho = 1.7 \times 10^{-8}\,\Omega\cdot\text{m}$, we obtain

$$\vec{E} = \rho\vec{J} \implies \vec{J} = 1.5 \times 10^7\ \hat{\text{ı}}$$

in SI units (A/m^2).

66. Assuming $\vec{J}$ is directed along the wire (with no radial flow) we integrate, starting with Eq. 27-4,

$$i = \int \left|\vec{J}\right|\,dA = \int_{R/2}^{R} kr\,2\pi r\,dr = \frac{2}{3}\,k\pi\left(R^3 - \frac{R^3}{8}\right)$$

where $k = 3.0 \times 10^8$ and SI units understood. Therefore, if $R = 0.00200$ m, we obtain $i = 4.40$ A.

67. (First problem of **Cluster**)

(a) We are told that $r_B = \frac{1}{2}r_A$ and $L_B = 2L_A$. Thus, using Eq. 27-16,

$$R_B = \rho\,\frac{L_B}{\pi r_B^2} = \rho\,\frac{2L_A}{\frac{1}{4}\,\pi r_A^2} = 8R_A = 64\ \Omega\ .$$

(b) The current-densities are assumed uniform.

$$\frac{J_A}{J_B} = \frac{\frac{i}{\pi r_A^2}}{\frac{i}{\pi r_B^2}} = \frac{\frac{i}{\pi r_A^2}}{\frac{i}{\frac{1}{4}\,\pi r_A^2}} = \frac{1}{4}\ .$$

68. (Second problem of **Cluster**)

(a) We use Eq. 27-16 to compute the resistances in SI units:

$$\begin{aligned} R_C &= \rho_C \frac{L_C}{\pi r_C^2} = \left(2 \times 10^{-6}\right) \frac{1}{\pi (0.0005)^2} = 2.5\ \Omega \\ R_D &= \rho_D \frac{L_D}{\pi r_D^2} = \left(1 \times 10^{-6}\right) \frac{1}{\pi (0.00025)^2} = 5.1\ \Omega\ . \end{aligned}$$

The voltages follow from Ohm's law:

$$\begin{aligned} |V_1 - V_2| = V_C &= iR_C = 5.1\ \text{V} \\ |V_2 - V_3| = V_D &= iR_D = 10\ \text{V}\ . \end{aligned}$$

(b) See solution for part (a).

(c) and (d) The power is calculated from Eq. 27-22:

$$P = i^2 R = \begin{cases} 10\ \text{W} & \text{for } R = R_C \\ 20\ \text{W} & \text{for } R = R_D \end{cases}$$

69. (Third problem of **Cluster**)

(a) We use Eq. 27-17 with $\rho = \frac{10}{8}\rho_0$ (we are neglecting any thermal expansion of the material) and $T - T_0 = 100$ K in order to obtain $\alpha = 2.5 \times 10^{-3}$/K. Now with this value of α but $T = 600$ K (so $T - T_0 = 300$ K) we find $\rho = 1.75\rho_0 \to R = 1.75(8.0\,\Omega) = 14\,\Omega$.

(b) We are assuming the wires have unknown but equal length (not the lengths shown in Figure 27-33). With $\alpha_D = 5.0 \times 10^{-3}$/K, we find $\rho = 2.5\rho_0$ for $T - T_0 = 300$ K. With the same assumptions as in part (a), this implies $R = 2.5R_0$ where $R_0 = 16\,\Omega$ (that the resistance of D is twice that of C at 300 K is evident in part (a) of the *previous* solution. Therefore, $R = 2.5(16\,\Omega) = 40\,\Omega$ for wire D at $T = 600$ K.

70. (Fourth problem of **Cluster**)
From Eq. 27-23, we obtain the resistance at temperature T:

$$R = \frac{V^2}{P} = \frac{12^2}{10} = 14.4\ \Omega\ .$$

Thus, the ratio R/R_0 with R_0 representing the resistance at 300 K is 7.2, which we take to equal the ratio of resistivities (ignoring any thermal expansion of the filament). Eq. 27-17, then, leads to

$$\frac{\rho}{\rho_0} = 7.2 = 1 + \alpha\,(T - 300)\quad .$$

Using Table 27-1 ($\alpha = 4.5 \times 10^{-3}$/K) we find $T = 1.7 \times 10^3$ K.

Chapter 28

1. (a) The cost is $(100\,\mathrm{W}\cdot 8.0\,\mathrm{h}/2.0\,\mathrm{W}\cdot\mathrm{h})(\$0.80) = \$320$.
   (b) The cost is $(100\,\mathrm{W}\cdot 8.0\,\mathrm{h}/10^3\,\mathrm{W}\cdot\mathrm{h})(\$0.06) = \$0.048 = 4.8\,\mathrm{cents}$.

2. The chemical energy of the battery is reduced by $\Delta E = q\mathcal{E}$, where q is the charge that passes through in time $\Delta t = 6.0\,\mathrm{min}$, and $\mathcal{E}$ is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and $\Delta E = i\mathcal{E}\,\Delta t = (5.0\,\mathrm{A})(6.0\,\mathrm{V})(6.0\,\mathrm{min})(60\,\mathrm{s/min}) = 1.1\times 10^4\,\mathrm{J}$. We note the conversion of time from minutes to seconds.

3. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\,\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and $\mathcal{E}$ is the emf of the battery, then $\Delta E = q\mathcal{E}$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\mathcal{E}}{P} = \frac{(120\,\mathrm{A}\cdot\mathrm{h})(12\,\mathrm{V})}{100\,\mathrm{W}} = 14.4\,\mathrm{h} = 14\,\mathrm{h}\ 24\,\mathrm{min}\ .$$

4. (a) Since $\mathcal{E}_1 > \mathcal{E}_2$ the current flows counterclockwise.
   (b) Battery 1, since the current flows through it from its negative terminal to the positive one.
   (c) Point B, since the current flows from B to A.

5. (a) Let i be the current in the circuit and take it to be positive if it is to the left in R_1. We use Kirchhoff's loop rule: $\mathcal{E}_1 - iR_2 - iR_1 - \mathcal{E}_2 = 0$. We solve for i:

$$i = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{12\,\mathrm{V} - 6.0\,\mathrm{V}}{4.0\,\Omega + 8.0\,\Omega} = 0.50\ \mathrm{A}\ .$$

   A positive value is obtained, so the current is counterclockwise around the circuit.
   (b) If i is the current in a resistor R, then the power dissipated by that resistor is given by $P = i^2R$. For R_1, $P_1 = (0.50\,\mathrm{A})^2(4.0\,\Omega) = 1.0\,\mathrm{W}$ and for R_2, $P_2 = (0.50\,\mathrm{A})^2(8.0\,\Omega) = 2.0\,\mathrm{W}$.
   (c) If i is the current in a battery with emf $\mathcal{E}$, then the battery supplies energy at the rate $P = i\mathcal{E}$ provided the current and emf are in the same direction. The battery absorbs energy at the rate $P = i\mathcal{E}$ if the current and emf are in opposite directions. For $\mathcal{E}_1$, $P_1 = (0.50\,\mathrm{A})(12\,\mathrm{V}) = 6.0\,\mathrm{W}$ and for $\mathcal{E}_2$, $P_2 = (0.50\,\mathrm{A})(6.0\,\mathrm{V}) = 3.0\,\mathrm{W}$. In battery 1 the current is in the same direction as the emf. Therefore, this battery supplies energy to the circuit; the battery is discharging. The current in battery 2 is opposite the direction of the emf, so this battery absorbs energy from the circuit. It is charging.

6. (a) The energy transferred is

$$U = Pt = \frac{\mathcal{E}^2 t}{r + R} = \frac{(2.0\,\mathrm{V})^2(2.0\,\mathrm{min})(60\,\mathrm{s/min})}{1.0\,\Omega + 5.0\,\Omega} = 80\ \mathrm{J}\ .$$

   (b) The amount of thermal energy generated is

$$U' = i^2Rt = \left(\frac{\mathcal{E}}{r+R}\right)^2 Rt = \left(\frac{2.0\,\mathrm{V}}{1.0\,\Omega + 5.0\,\Omega}\right)^2 (5.0\,\Omega)(2.0\,\mathrm{min})(60\,\mathrm{s/min}) = 67\ \mathrm{J}\ .$$

(c) The difference between U and U', which is equal to 13 J, is the thermal energy that is generated in the battery due to its internal resistance.

7. (a) The potential difference is $V = \mathcal{E} + ir = 12\,\text{V} + (0.040\,\Omega)(50\,\text{A}) = 14\,\text{V}$.

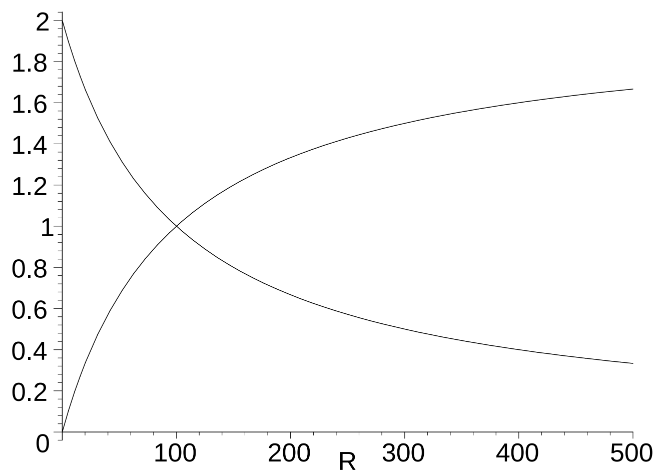
   (b) $P = i^2r = (50\,\text{A})^2(0.040\,\Omega) = 100\,\text{W}$.

   (c) $P' = iV = (50\,\text{A})(12\,\text{V}) = 600\,\text{W}$.

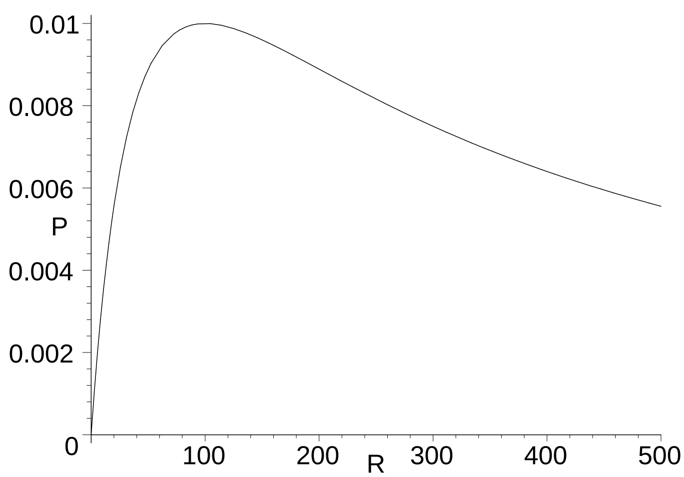
   (d) In this case $V = \mathcal{E} - ir = 12\,\text{V} - (0.040\,\Omega)(50\,\text{A}) = 10\,\text{V}$ and $P = i^2r = 100\,\text{W}$.

8. (a) Below, we graph Eq. 28-4 (scaled by a factor of 100) for $\mathcal{E} = 2.0\,\text{V}$ and $r = 100\,\Omega$ over the range $0 \le R \le 500\,\Omega$. We multiplied the SI output of Eq. 28-4 by 100 so that this graph would not be vanishingly small with the other graph (see part (b)) when they are plotted together.

   (b) In the same graph, we show $V_R = iR$ over the same range. The graph of current i is the one that starts at 2 (which corresponds to 0.02 A in SI units) and the graph of voltage V_R is the one that starts at 0 (when $R = 0$). The value of V_R are in SI units (not scaled by any factor).



   (c) In our final graph, we show the dependence of power $P = iV_R$ (dissipated in resistor R) as a function of R. The units of the vertical axis are Watts. We note that it is maximum when $R = r$.



9. (a) If i is the current and ΔV is the potential difference, then the power absorbed is given by $P = i\,\Delta V$. Thus,

$$\Delta V = \frac{P}{i} = \frac{50\,\mathrm{W}}{1.0\,\mathrm{A}} = 50\ \mathrm{V}\ .$$

Since the energy of the charge decreases, point A is at a higher potential than point B; that is, $V_A - V_B = 50\,\mathrm{V}$.

(b) The end-to-end potential difference is given by $V_A - V_B = +iR + \mathcal{E}$, where $\mathcal{E}$ is the emf of element C and is taken to be positive if it is to the left in the diagram. Thus, $\mathcal{E} = V_A - V_B - iR = 50\,\mathrm{V} - (1.0\,\mathrm{A})(2.0\,\Omega) = 48\,\mathrm{V}$.

(c) A positive value was obtained for $\mathcal{E}$, so it is toward the left. The negative terminal is at B.

10. The current in the circuit is $i = (150\,\mathrm{V} - 50\,\mathrm{V})/(3.0\,\Omega + 2.0\,\Omega) = 20\,\mathrm{A}$. So from $V_Q + 150\,\mathrm{V} - (2.0\,\Omega)i = V_P$, we get $V_Q = 100\,\mathrm{V} + (2.0\,\Omega)(20\,\mathrm{A}) - 150\,\mathrm{V} = -10\,\mathrm{V}$.

11. From $V_a - \mathcal{E}_1 = V_c - ir_1 - iR$ and $i = (\mathcal{E}_1 - \mathcal{E}_2)/(R + r_1 + r_2)$, we get

$$\begin{aligned} V_a - V_c &= \mathcal{E}_1 - i\,(r_1 + R) \\ &= \mathcal{E}_1 - \left(\frac{\mathcal{E}_1 - \mathcal{E}_2}{R + r_1 + r_2}\right)(r_1 + R) \\ &= 4.4\,\mathrm{V} - \left(\frac{4.4\,\mathrm{V} - 2.1\,\mathrm{V}}{5.5\,\Omega + 1.8\,\Omega + 2.3\,\Omega}\right)(2.3\,\Omega + 5.5\,\Omega) \\ &= 2.5\ \mathrm{V}\ . \end{aligned}$$

12. (a) We solve $i = (\mathcal{E}_2 - \mathcal{E}_1)/(r_1 + r_2 + R)$ for R:

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{i} - r_1 - r_2 = \frac{3.0\,\mathrm{V} - 2.0\,\mathrm{V}}{1.0 \times 10^{-3}\,\mathrm{A}} - 3.0\,\Omega - 3.0\,\Omega = 9.9 \times 10^2\ \Omega\ .$$

(b) $P = i^2R = (1.0 \times 10^{-3}\,\mathrm{A})^2(9.9 \times 10^2\,\Omega) = 9.9 \times 10^{-4}\,\mathrm{W}$.

13. Let the emf be V. Then $V = iR = i'(R + R')$, where $i = 5.0\,\mathrm{A}$, $i' = 4.0\,\mathrm{A}$ and $R' = 2.0\,\Omega$. We solve for R:

$$R = \frac{i'R'}{i - i'} = \frac{(4)(2)}{5 - 4} = 8.0\ \Omega\ .$$

14. The internal resistance of the battery is $r = (12\,\mathrm{V} - 11.4\,\mathrm{V})/50\,\mathrm{A} = 0.012\,\Omega < 0.020\,\Omega$, so the battery is OK. The resistance of the cable is $R = 3.0\,\mathrm{V}/50\,\mathrm{A} = 0.060\,\Omega > 0.040\,\Omega$, so the cable is defective.

15. To be as general as possible, we refer to the individual emf's as $\mathcal{E}_1$ and $\mathcal{E}_2$ and wait until the latter steps to equate them ($\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$). The batteries are placed in series in such a way that their voltages add; that is, they do not "oppose" each other. The total resistance in the circuit is therefore $R_{\text{total}} = R + r_1 + r_2$ (where the problem tells us $r_1 > r_2$), and the "net emf" in the circuit is $\mathcal{E}_1 + \mathcal{E}_2$. Since battery 1 has the higher internal resistance, it is the one capable of having a zero terminal voltage, as the computation in part (a) shows.

(a) The current in the circuit is

$$i = \frac{\mathcal{E}_1 + \mathcal{E}_2}{r_1 + r_2 + R}\ ,$$

and the requirement of zero terminal voltage leads to

$$\mathcal{E}_1 = ir_1 \implies R = \frac{\mathcal{E}_2 r_1 - \mathcal{E}_1 r_2}{\mathcal{E}_1}$$

which reduces to $R = r_1 - r_2$ when we set $\mathcal{E}_1 = \mathcal{E}_2$.

(b) As mentioned above, this occurs in battery 1.

16. (a) Let the emf of the solar cell be $\mathcal{E}$ and the output voltage be V. Thus,

$$V = \mathcal{E} - ir = \mathcal{E} - \left(\frac{V}{R}\right) r$$

for both cases. Numerically, we get $0.10\,\text{V} = \mathcal{E} - (0.10\,\text{V}/500\,\Omega)r$ and $0.15\,\text{V} = \mathcal{E} - (0.15\,\text{V}/1000\,\Omega)r$. We solve for $\mathcal{E}$ and r: $\mathcal{E} = 0.30\,\text{V}$, $r = 1000\,\Omega$.

(b) The efficiency is

$$\frac{V^2/R}{P_{\text{received}}} = \frac{0.15\,\text{V}}{(1000\,\Omega)(5.0\,\text{cm}^2)(2.0\times10^{-3}\,\text{W/cm}^2)} = 2.3\times10^{-3}\ .$$

17. (a) Using Eq. 28-4, we take the derivative of the power $P = i^2R$ with respect to R and set the result equal to zero:

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\mathcal{E}^2R}{(R+r)^2}\right) = \frac{\mathcal{E}^2(r-R)}{(R+r)^3} = 0$$

which clearly has the solution $R = r$.

(b) When $R = r$, the power dissipated in the external resistor equals

$$P_{\max} = \left.\frac{\mathcal{E}^2R}{(R+r)^2}\right|_{R=r} = \frac{\mathcal{E}^2}{4r}\ .$$

18. Let the resistances of the two resistors be R_1 and R_2. Note that the smallest value of the possible R_{eq} must be the result of connecting R_1 and R_2 in parallel, while the largest one must be that of connecting them in series. Thus, $R_1R_2/(R_1+R_2) = 3.0\,\Omega$ and $R_1 + R_2 = 16\,\Omega$. So R_1 and R_2 must be $4.0\,\Omega$ and $12\,\Omega$, respectively.

19. The potential difference across each resistor is $V = 25.0\,\text{V}$. Since the resistors are identical, the current in each one is $i = V/R = (25.0\,\text{V})/(18.0\,\Omega) = 1.39\,\text{A}$. The total current through the battery is then $i_{\text{total}} = 4(1.39\,\text{A}) = 5.56\,\text{A}$. One might alternatively use the idea of equivalent resistance; for four identical resistors in parallel the equivalent resistance is given by

$$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R} = \frac{4}{R}\ .$$

When a potential difference of 25.0 V is applied to the equivalent resistor, the current through it is the same as the total current through the four resistors in parallel. Thus $i_{\text{total}} = V/R_{\text{eq}} = 4V/R = 4(25.0\,\text{V})/(18.0\,\Omega) = 5.56\,\text{A}$.

20. We note that two resistors in parallel, say R_1 and R_2, are equivalent to

$$R_{\text{parallel pair}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1R_2}{R_1+R_2}\ .$$

This situation (Figure 28-27) consists of a parallel pair which are then in series with a single $2.50\,\Omega$ resistor. Thus, the situation has an equivalent resistance of

$$R_{\text{eq}} = 2.50\,\Omega + \frac{(4.00\,\Omega)(4.00\,\Omega)}{4.00\,\Omega + 4.00\,\Omega} = 4.50\,\Omega\ .$$

21. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is upward. When the loop rule is applied to the lower loop, the result is

$$\mathcal{E}_2 - i_1 R_1 = 0 \ .$$

and when it is applied to the upper loop, the result is

$$\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3 - i_2 R_2 = 0 \ .$$

The first equation yields

$$i_1 = \frac{\mathcal{E}_2}{R_1} = \frac{5.0\,\mathrm{V}}{100\,\Omega} = 0.050\ \mathrm{A} \ .$$

The second yields

$$i_2 = \frac{\mathcal{E}_1 - \mathcal{E}_2 - \mathcal{E}_3}{R_2} = \frac{6.0\,\mathrm{V} - 5.0\,\mathrm{V} - 4.0\,\mathrm{V}}{50\,\Omega} = -0.060\ \mathrm{A} \ .$$

The negative sign indicates that the current in R_2 is actually downward. If V_b is the potential at point b, then the potential at point a is $V_a = V_b + \mathcal{E}_3 + \mathcal{E}_2$, so $V_a - V_b = \mathcal{E}_3 + \mathcal{E}_2 = 4.0\,\mathrm{V} + 5.0\,\mathrm{V} = 9.0\,\mathrm{V}$.

22.
- S_1, S_2 and S_3 all open: $i_a = 0.00\,\mathrm{A}$.
- S_1 closed, S_2 and S_3 open: $i_a = \mathcal{E}/2R_1 = 120\,\mathrm{V}/40.0\,\Omega = 3.00\,\mathrm{A}$.
- S_2 closed, S_1 and S_3 open: $i_a = \mathcal{E}/(2R_1 + R_2) = 120\,\mathrm{V}/50.0\,\Omega = 2.40\,\mathrm{A}$.
- S_3 closed, S_1 and S_2 open: $i_a = \mathcal{E}/(2R_1 + R_2) = 120\,\mathrm{V}/60.0\,\Omega = 2.00\,\mathrm{A}$.
- S_1 open, S_2 and S_3 closed: $R_{\mathrm{eq}} = R_1 + R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\,\Omega + 10.0\,\Omega + (20.0\,\Omega)(30.0\,\Omega)/(50.0\,\Omega) = 42.0\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/42.0\,\Omega = 2.86\,\mathrm{A}$.
- S_2 open, S_1 and S_3 closed: $R_{\mathrm{eq}} = R_1 + R_1(R_1 + 2R_2)/(2R_1 + 2R_2) = 20.0\,\Omega + (20.0\,\Omega)\times(40.0\,\Omega)/(60.0\,\Omega) = 33.3\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/33.3\,\Omega = 3.60\,\mathrm{A}$.
- S_3 open, S_1 and S_2 closed: $R_{\mathrm{eq}} = R_1 + R_1(R_1 + R_2)/(2R_1 + R_2) = 20.0\,\Omega + (20.0\,\Omega)\times(30.0\,\Omega)/(50.0\,\Omega) = 32.0\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/32.0\,\Omega = 3.75\,\mathrm{A}$.
- S_1, S_2 and S_3 all closed: $R_{\mathrm{eq}} = R_1 + R_1R'/(R_1 + R')$ where $R' = R_2 + R_1(R_1 + R_2)/(2R_1 + R_2) = 22.0\,\Omega$, i.e., $R_{\mathrm{eq}} = 20.0\,\Omega + (20.0\,\Omega)(22.0\,\Omega)/(20.0\,\Omega + 22.0\,\Omega) = 30.5\,\Omega$, so $i_a = \mathcal{E}/R_{\mathrm{eq}} = 120\,\mathrm{V}/30.5\,\Omega = 3.94\,\mathrm{A}$.

23. (a) Let $\mathcal{E}$ be the emf of the battery. When the bulbs are connected in parallel, the potential difference across them is the same and is also the same as the emf of the battery. The power dissipated by bulb 1 is $P_1 = \mathcal{E}^2/R_1$, and the power dissipated by bulb 2 is $P_2 = \mathcal{E}^2/R_2$. Since R_1 is greater than R_2, bulb 2 dissipates energy at a greater rate than bulb 1 and is the brighter of the two.

(b) When the bulbs are connected in series the current in them is the same. The power dissipated by bulb 1 is now $P_1 = i^2R_1$ and the power dissipated by bulb 2 is $P_2 = i^2R_2$. Since R_1 is greater than R_2 greater power is dissipated by bulb 1 than by bulb 2 and bulb 1 is the brighter of the two.

24. The currents i_1, i_2 and i_3 are obtained from Eqs. 28-15 through 28-17:

$$\begin{aligned}
i_1 &= \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2R_3}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(4.0\,\mathrm{V})(10\,\Omega + 5.0\,\Omega) - (1.0\,\mathrm{V})(5.0\,\Omega)}{(10\,\Omega)(10\,\Omega) + (10\,\Omega)(5.0\,\Omega) + (10\,\Omega)(5.0\,\Omega)} \\
&= 0.275\ \mathrm{A} \ , \\
i_2 &= \frac{\mathcal{E}_1R_3 - \mathcal{E}_2(R_1 + R_2)}{R_1R_2 + R_2R_3 + R_1R_3} = \frac{(4.0\,\mathrm{V})(5.0\,\Omega) - (1.0\,\mathrm{V})(10\,\Omega + 5.0\,\Omega)}{(10\,\Omega)(10\,\Omega) + (10\,\Omega)(5.0\,\Omega) + (10\,\Omega)(5.0\,\Omega)} \\
&= 0.025\ \mathrm{A} \ , \\
i_3 &= i_2 - i_1 = 0.025\,\mathrm{A} - 0.275\,\mathrm{A} = -0.250\ \mathrm{A} \ .
\end{aligned}$$

$V_d - V_c$ can now be calculated by taking various paths. Two examples: from $V_d - i_2R_2 = V_c$ we get $V_d - V_c = i_2R_2 = (0.0250\,\mathrm{A})(10\,\Omega) = +0.25\,\mathrm{V}$; from $V_d + i_3R_3 + \mathcal{E}_2 = V_c$ we get $V_d - V_c = -i_3R_3 - \mathcal{E}_2 = -(-0.250\,\mathrm{A})(5.0\,\Omega) - 1.0\,\mathrm{V} = +0.25\,\mathrm{V}$.

25. Let r be the resistance of each of the narrow wires. Since they are in parallel the resistance R of the composite is given by

$$\frac{1}{R} = \frac{9}{r} ,$$

or $R = r/9$. Now $r = 4\rho\ell/\pi d^2$ and $R = 4\rho\ell/\pi D^2$, where ρ is the resistivity of copper. $A = \pi d^2/4$ was used for the cross-sectional area of a single wire, and a similar expression was used for the cross-sectional area of the thick wire. Since the single thick wire is to have the same resistance as the composite,

$$\frac{4\rho\ell}{\pi D^2} = \frac{4\rho\ell}{9\pi d^2} \implies D = 3d .$$

26. (a) $R_{\text{eq}}(FH) = (10.0\,\Omega)(10.0\,\Omega)(5.00\,\Omega)/[(10.0\,\Omega)(10.0\,\Omega) + 2(10.0\,\Omega)(5.00\,\Omega)] = 2.50\,\Omega$.
    (b) $R_{\text{eq}}(FG) = (5.00\,\Omega)R/(R+5.00\,\Omega)$, where $R = 5.00\,\Omega+(5.00\,\Omega)(10.0\,\Omega)/(5.00\,\Omega+10.0\,\Omega) = 8.33\,\Omega$. So $R_{\text{eq}}(FG) = (5.00\,\Omega)(8.33\,\Omega)/(5.00\,\Omega + 8.33\,\Omega) = 3.13\,\Omega$.

27. Let the resistors be divided into groups of n resistors each, with all the resistors in the same group connected in series. Suppose there are m such groups that are connected in parallel with each other. Let R be the resistance of any one of the resistors. Then the equivalent resistance of any group is nR, and R_{eq}, the equivalent resistance of the whole array, satisfies

$$\frac{1}{R_{\text{eq}}} = \sum_{1}^{m} \frac{1}{nR} = \frac{m}{nR} .$$

Since the problem requires $R_{\text{eq}} = 10\,\Omega = R$, we must select $n = m$. Next we make use of Eq. 28-13. We note that the current is the same in every resistor and there are $n \cdot m = n^2$ resistors, so the maximum total power that can be dissipated is $P_{\text{total}} = n^2 P$, where $P = 1.0\,\text{W}$ is the maximum power that can be dissipated by any one of the resistors. The problem demands $P_{\text{total}} \geq 5.0P$, so n^2 must be at least as large as 5.0. Since n must be an integer, the smallest it can be is 3. The least number of resistors is $n^2 = 9$.

28. (a) R_2, R_3 and R_4 are in parallel. By finding a common denominator and simplifying, the equation $1/R = 1/R_2 + 1/R_3 + 1/R_4$ gives an equivalent resistance of

$$R = \frac{R_2R_3R_4}{R_2R_3 + R_2R_4 + R_3R_4} = \frac{(50\,\Omega)(50\,\Omega)(75\,\Omega)}{(50\,\Omega)(50\,\Omega) + (50\,\Omega)(75\,\Omega) + (50\,\Omega)(75\,\Omega)} = 19\,\Omega .$$

Thus, considering the series contribution of resistor R_1, the equivalent resistance for the network is $R_{\text{eq}} = R_1 + R = 100\,\Omega + 19\,\Omega = 1.2 \times 10^2\,\Omega$.

    (b) $i_1 = \mathcal{E}/R_{\text{eq}} = 6.0\,\text{V}/(1.1875 \times 10^2\,\Omega) = 5.1 \times 10^{-2}\,\text{A}$; $i_2 = (\mathcal{E} - V_1)/R_2 = (\mathcal{E} - i_1R_1)/R_2 = [6.0\,\text{V} - (5.05 \times 10^{-2}\,\text{A})(100\,\Omega)]/50\,\Omega = 1.9 \times 10^{-2}\,\text{A}$; $i_3 = (\mathcal{E} - V_1)/R_3 = i_2R_2/R_3 = (1.9 \times 10^{-2}\,\text{A})(50\,\Omega/50\,\Omega) = 1.9\times10^{-2}\,\text{A}$; $i_4 = i_1-i_2-i_3 = 5.0\times10^{-2}\,\text{A}-2(1.895\times10^{-2}\,\text{A}) = 1.2\times10^{-2}\,\text{A}$.

29. (a) The batteries are identical and, because they are connected in parallel, the potential differences across them are the same. This means the currents in them are the same. Let i be the current in either battery and take it to be positive to the left. According to the junction rule the current in R is $2i$ and it is positive to the right. The loop rule applied to either loop containing a battery and R yields

$$\mathcal{E} - ir - 2iR = 0 \implies i = \frac{\mathcal{E}}{r + 2R} .$$

The power dissipated in R is

$$P = (2i)^2R = \frac{4\mathcal{E}^2R}{(r + 2R)^2} .$$

We find the maximum by setting the derivative with respect to R equal to zero. The derivative is

$$\frac{dP}{dR} = \frac{4\mathcal{E}^2}{(r + 2R)^2} - \frac{16\mathcal{E}^2R}{(r + 2R)^3} = \frac{4\mathcal{E}^2(r - 2R)}{(r + 2R)^3} .$$

The derivative vanishes (and P is a maximum) if $R = r/2$.

(b) We substitute $R = r/2$ into $P = 4\mathcal{E}^2 R/(r+2R)^2$ to obtain

$$P_{\max} = \frac{4\mathcal{E}^2(r/2)}{[r+2(r/2)]^2} = \frac{\mathcal{E}^2}{2r} \; .$$

30. (a) By symmetry, when the two batteries are connected in parallel the current i going through either one is the same. So from $\mathcal{E} = ir + (2i)R$ we get $i_R = 2i = 2\mathcal{E}/(r+2R)$. When connected in series $2\mathcal{E} - i_R r - i_R r - i_R R = 0$, or $i_R = 2\mathcal{E}/(2r+R)$.

(b) In series, since $R > r$.

(c) In parallel, since $R < r$.

31. (a) We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is upward. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is to the right. The junction rule produces

$$i_1 + i_2 + i_3 = 0 \; .$$

The loop rule applied to the left-hand loop produces

$$\mathcal{E}_1 - i_3 R_3 + i_1 R_1 = 0$$

and applied to the right-hand loop produces

$$\mathcal{E}_2 - i_2 R_2 + i_1 R_1 = 0 \; .$$

We substitute $i_1 = -i_2 - i_3$, from the first equation, into the other two to obtain

$$\mathcal{E}_1 - i_3 R_3 - i_2 R_1 - i_3 R_1 = 0$$

and

$$\mathcal{E}_2 - i_2 R_2 - i_2 R_1 - i_3 R_1 = 0 \; .$$

The first of these yields

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} \; .$$

Substituting this into the second equation and solving for i_2, we obtain

$$\begin{aligned} i_2 &= \frac{\mathcal{E}_2(R_1+R_3) - \mathcal{E}_1 R_1}{R_1R_2 + R_1R_3 + R_2R_3} \\ &= \frac{(1.00\,\mathrm{V})(5.00\,\Omega + 4.00\,\Omega) - (3.00\,\mathrm{V})(5.00\,\Omega)}{(5.00\,\Omega)(2.00\,\Omega) + (5.00\,\Omega)(4.00\,\Omega) + (2.00\,\Omega)(4.00\,\Omega)} = -0.158\,\mathrm{A} \; . \end{aligned}$$

We substitute into the expression for i_3 to obtain

$$i_3 = \frac{\mathcal{E}_1 - i_2 R_1}{R_1 + R_3} = \frac{3.00\,\mathrm{V} - (-0.158\,\mathrm{A})(5.00\,\Omega)}{5.00\,\Omega + 4.00\,\Omega} = 0.421\,\mathrm{A} \; .$$

Finally,

$$i_1 = -i_2 - i_3 = -(-0.158\,\mathrm{A}) - (0.421\,\mathrm{A}) = -0.263\,\mathrm{A} \; .$$

Note that the current in R_1 is actually downward and the current in R_2 is to the right. The current in R_3 is also to the right. The power dissipated in R_1 is $P_1 = i_1^2 R_1 = (-0.263\,\mathrm{A})^2(5.00\,\Omega) = 0.346\,\mathrm{W}$.

(b) The power dissipated in R_2 is $P_2 = i_2^2 R_2 = (-0.158\,\mathrm{A})^2(2.00\,\Omega) = 0.0499\,\mathrm{W}$.

(c) The power dissipated in R_3 is $P_3 = i_3^2 R_3 = (0.421\,\mathrm{A})^2(4.00\,\Omega) = 0.709\,\mathrm{W}$.

(d) The power supplied by $\mathcal{E}_1$ is $i_3\mathcal{E}_1 = (0.421\,\mathrm{A})(3.00\,\mathrm{V}) = 1.26\,\mathrm{W}$.

(e) The power "supplied" by $\mathcal{E}_2$ is $i_2\mathcal{E}_2 = (-0.158\,\text{A})(1.00\,\text{V}) = -0.158\,\text{W}$. The negative sign indicates that $\mathcal{E}_2$ is actually absorbing energy from the circuit.

32. (a) We use $P = \mathcal{E}^2/R_{\text{eq}}$, where

$$R_{\text{eq}} = 7.00\,\Omega + \frac{(12.0\,\Omega)(4.00\,\Omega)R}{(12.0\,\Omega)(4.0\,\Omega) + (12.0\,\Omega)R + (4.00\,\Omega)R} .$$

Put $P = 60.0\,\text{W}$ and $\mathcal{E} = 24.0\,\text{V}$ and solve for R: $R = 19.5\,\Omega$.

(b) Since $P \propto R_{\text{eq}}$, we must minimize R_{eq}, which means $R = 0$.

(c) Now we must maximize R_{eq}, or set $R = \infty$.

(d) Since $R_{\text{eq, max}} = 7.00\,\Omega + (12.0\,\Omega)(4.00\,\Omega)/(12.0\,\Omega + 4.00\,\Omega) = 10.0\,\Omega$, $P_{\text{min}} = \mathcal{E}^2/R_{\text{eq, max}} = (24.0\,\text{V})^2/10.0\,\Omega = 57.6\,\text{W}$. Since $R_{\text{eq, min}} = 7.00\,\Omega$, $P_{\text{max}} = \mathcal{E}^2/R_{\text{eq, min}} = (24.0\,\text{V})^2/7.00\,\Omega = 82.3\,\text{W}$.

33. (a) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\mathcal{E}_2 = \mathcal{E}_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through $\mathcal{E}_2$ and $\mathcal{E}_3$ are the same: $i_2 = i_3 = i$. Therefore, the current through $\mathcal{E}_1$ is $i_1 = 2i$. Then from $V_b - V_a = \mathcal{E}_2 - iR_2 = \mathcal{E}_1 + (2R_1)(2i)$ we get

$$i = \frac{\mathcal{E}_2 - \mathcal{E}_1}{4R_1 + R_2} = \frac{4.0\,\text{V} - 2.0\,\text{V}}{4(1.0\,\Omega) + 2.0\,\Omega} = 0.33\,\text{A} .$$

Therefore, the current through $\mathcal{E}_1$ is $i_1 = 2i = 0.67\,\text{A}$, flowing downward. The current through $\mathcal{E}_2$ is $0.33\,\text{A}$, flowing upward; the same holds for $\mathcal{E}_3$.

(b) $V_a - V_b = -iR_2 + \mathcal{E}_2 = -(0.333\,\text{A})(2.0\,\Omega) + 4.0\,\text{V} = 3.3\,\text{V}$.

34. The voltage difference across R is $V_R = \mathcal{E}R'/(R' + 2.00\,\Omega)$, where $R' = (5.00\,\Omega R)/(5.00\,\Omega + R)$. Thus,

$$\begin{aligned} P_R &= \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\mathcal{E}R'}{R' + 2.00\,\Omega}\right)^2 = \frac{1}{R}\left(\frac{\mathcal{E}}{1 + 2.00\,\Omega/R'}\right)^2 \\ &= \frac{\mathcal{E}^2}{R}\left[1 + \frac{(2.00\,\Omega)(5.00\,\Omega + R)}{(5.00\,\Omega)R}\right]^{-2} \equiv \frac{\mathcal{E}^2}{f(R)} \end{aligned}$$

where we use the equivalence symbol $\equiv$ to define the expression $f(R)$. To maximize P_R we need to minimize the expression $f(R)$. We set

$$\frac{df(R)}{dR} = -\frac{4.00\,\Omega^2}{R^2} + \frac{49}{25} = 0$$

to obtain $R = \sqrt{(4.00\,\Omega^2)(25)/49} = 1.43\,\Omega$.

35. (a) The copper wire and the aluminum sheath are connected in parallel, so the potential difference is the same for them. Since the potential difference is the product of the current and the resistance, $i_C R_C = i_A R_A$, where i_C is the current in the copper, i_A is the current in the aluminum, R_C is the resistance of the copper, and R_A is the resistance of the aluminum. The resistance of either component is given by $R = \rho L/A$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. The resistance of the copper wire is $R_C = \rho_C L/\pi a^2$, and the resistance of the aluminum sheath is $R_A = \rho_A L/\pi(b^2 - a^2)$. We substitute these expressions into $i_C R_C = i_A R_A$, and cancel the common factors L and π to obtain

$$\frac{i_C \rho_C}{a^2} = \frac{i_A \rho_A}{b^2 - a^2} .$$

We solve this equation simultaneously with $i = i_C + i_A$, where i is the total current. We find

$$i_C = \frac{r_C^2 \rho_C i}{(r_A^2 - r_C^2)\rho_C + r_C^2 \rho_A}$$

and

$$i_A = \frac{(r_A^2 - r_C^2)\rho_C i}{(r_A^2 - r_C^2)\rho_C + r_C^2 \rho_A} \ .$$

The denominators are the same and each has the value

$$\begin{aligned}(b^2 - a^2)\rho_C + a^2\rho_A &= \left[(0.380 \times 10^{-3}\,\mathrm{m})^2 - (0.250 \times 10^{-3}\,\mathrm{m})^2\right](1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) \\ &\quad +(0.250 \times 10^{-3}\,\mathrm{m})^2(2.75 \times 10^{-8}\,\Omega{\cdot}\mathrm{m}) \\ &= 3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3 \ .\end{aligned}$$

Thus,

$$i_C = \frac{(0.250 \times 10^{-3}\,\mathrm{m})^2(2.75 \times 10^{-8}\Omega{\cdot}\mathrm{m})(2.00\,\mathrm{A})}{3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3} = 1.11 \text{ A}$$

and

$$\begin{aligned} i_A &= \frac{\left[(0.380 \times 10^{-3}\,\mathrm{m})^2 - (0.250 \times 10^{-3}\,\mathrm{m})^2\right](1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(2.00\,\mathrm{A})}{3.10 \times 10^{-15}\,\Omega{\cdot}\mathrm{m}^3} \\ &= 0.893\,\mathrm{A} \ .\end{aligned}$$

(b) Consider the copper wire. If V is the potential difference, then the current is given by $V = i_C R_C = i_C \rho_C L/\pi a^2$, so

$$L = \frac{\pi a^2 V}{i_C \rho_C} = \frac{(\pi)(0.250 \times 10^{-3}\,\mathrm{m})^2(12.0\,\mathrm{V})}{(1.11\,\mathrm{A})(1.69 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})} = 126\,\mathrm{m} \ .$$

36. (a) Since $i = \mathcal{E}/(r + R_{\text{ext}})$ and $i_{\max} = \mathcal{E}/r$, we have $R_{\text{ext}} = R(i_{\max}/i - 1)$ where $r = 1.50\,\mathrm{V}/1.00\,\mathrm{mA} = 1.50 \times 10^3\,\Omega$. Thus, $R_{\text{ext}} = (1.5 \times 10^3\,\Omega)(1/0.10 - 1) = 1.35 \times 10^4\,\Omega$;

(b) $R_{\text{ext}} = (1.5 \times 10^3\,\Omega)(1/0.50 - 1) = 1.50 \times 10^3\,\Omega$;

(c) $R_{\text{ext}} = (1.5 \times 10^3\,\Omega)(1/0.90 - 1) = 167\,\Omega$.

(d) Since $r = 20.0\,\Omega + R$, $R = 1.50 \times 10^3\,\Omega - 20.0\,\Omega = 1.48 \times 10^3\,\Omega$.

37. (a) The current in R_1 is given by

$$i_1 = \frac{\mathcal{E}}{R_1 + R_2R_3/(R_2 + R_3)} = \frac{5.0\,\mathrm{V}}{2.0\,\Omega + (4.0\,\Omega)(6.0\,\Omega)/(4.0\,\Omega + 6.0\,\Omega)} = 1.14\,\mathrm{A} \ .$$

Thus

$$i_3 = \frac{\mathcal{E} - V_1}{R_3} = \frac{\mathcal{E} - i_1R_1}{R_3} = \frac{5.0\,\mathrm{V} - (1.14\,\mathrm{A})(2.0\,\Omega)}{6.0\,\Omega} = 0.45\,\mathrm{A} \ .$$

(b) We simply interchange subscripts 1 and 3 in the equation above. Now

$$\begin{aligned} i_3 &= \frac{\mathcal{E}}{R_3 + (R_2R_1/(R_2 + R_1))} \\ &= \frac{5.0\,\mathrm{V}}{6.0\,\Omega + ((2.0\,\Omega)(4.0\,\Omega)/(2.0\,\Omega + 4.0\,\Omega))} \\ &= 0.6818 \text{ A}\end{aligned}$$

and

$$i_1 = \frac{5.0\,\mathrm{V} - (0.6818\,\mathrm{A})(6.0\,\Omega)}{2.0\,\Omega} = 0.45 \text{ A} \ ,$$

the same as before.

38. (a) $\mathcal{E} = V + ir = 12\,\mathrm{V} + (10\,\mathrm{A})(0.050\,\Omega) = 12.5\,\mathrm{V}$.

(b) Now $\mathcal{E} = V' + (i_{\text{motor}} + 8.0\,\text{A})r$, where $V' = i'_A R_{\text{light}} = (8.0\,\text{A})(12\,\text{V}/10\,\text{A}) = 9.6\,\text{V}$. Therefore,

$$i_{\text{motor}} = \frac{\mathcal{E} - V'}{r} - 8.0\,\text{A} = \frac{12.5\,\text{V} - 9.6\,\text{V}}{0.050\,\Omega} - 8.0\,\text{A} = 50\ \text{A}\ .$$

39. The current in R_2 is i. Let i_1 be the current in R_1 and take it to be downward. According to the junction rule the current in the voltmeter is $i - i_1$ and it is downward. We apply the loop rule to the left-hand loop to obtain

$$\mathcal{E} - iR_2 - i_1R_1 - ir = 0\ .$$

We apply the loop rule to the right-hand loop to obtain

$$i_1R_1 - (i - i_1)R_V = 0\ .$$

The second equation yields

$$i = \frac{R_1 + R_V}{R_V}\, i_1\ .$$

We substitute this into the first equation to obtain

$$\mathcal{E} - \frac{(R_2 + r)(R_1 + R_V)}{R_V}\, i_1 + R_1 i_1 = 0\ .$$

This has the solution

$$i_1 = \frac{\mathcal{E} R_V}{(R_2 + r)(R_1 + R_V) + R_1 R_V}\ .$$

The reading on the voltmeter is

$$\begin{aligned} i_1R_1 &= \frac{\mathcal{E} R_V R_1}{(R_2 + r)(R_1 + R_V) + R_1 R_V} \\ &= \frac{(3.0\,\text{V})(5.0\times 10^3\,\Omega)(250\,\Omega)}{(300\,\Omega + 100\,\Omega)(250\,\Omega + 5.0\times 10^3\,\Omega) + (250\,\Omega)(5.0\times 10^3\,\Omega)} = 1.12\,\text{V}\ . \end{aligned}$$

The current in the absence of the voltmeter can be obtained by taking the limit as R_V becomes infinitely large. Then

$$i_1R_1 = \frac{\mathcal{E} R_1}{R_1 + R_2 + r} = \frac{(3.0\,\text{V})(250\,\Omega)}{250\,\Omega + 300\,\Omega + 100\,\Omega} = 1.15\,\text{V}\ .$$

The fractional error is $(1.12 - 1.15)/(1.15) = -0.030$, or -3.0%.

40. The currents in R and R_V are i and $i' - i$, respectively. Since $V = iR = (i' - i)R_V$ we have, by dividing both sides by V, $1 = (i'/V - i/V)R_V = (1/R' - 1/R)R_V$. Thus,

$$\frac{1}{R} = \frac{1}{R'} - \frac{1}{R_V}\ .$$

41. Let the current in the ammeter be i'. We have $V = i'(R + R_A)$, or $R = V/i' - R_A = R' - R_A$, where $R' = V/i'$ is the apparent reading of the resistance.

42. (a) In the first case

$$\begin{aligned} i' &= \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{R_A + R_0 + R_V R/(R + R_V)} \\ &= \frac{12.0\,\text{V}}{3.00\,\Omega + 100\,\Omega + (300\,\Omega)(85.0\,\Omega)/(300\,\Omega + 85.0\,\Omega)} \\ &= 7.09\times 10^{-2}\ \text{A}\ , \end{aligned}$$

and $V = \mathcal{E} - i'(R_A + R_0) = 12.0\,\text{V} - (0.0709\,\text{A})(103.00\,\Omega) = 4.70\,\text{V}$. In the second case $V = \mathcal{E}R'/(R' + R_0)$, where

$$R' = \frac{R_V\,(R + R_A)}{R_V + R + R_A} = \frac{(300\,\Omega)(300\,\Omega + 85.0\,\Omega)}{300\,\Omega + 85.0\,\Omega + 3.00\,\Omega} = 68.0\,\Omega\ .$$

So $V = (12.0\,\text{V})(68.0\,\Omega)/(68.0\,\Omega{+}100\,\Omega) = 4.86\,\text{V}$, and $i' = V(R{+}R_A) = 4.86\,\text{V}/(300\,\Omega{+}85.0\,\Omega) = 5.52 \times 10^{-2}\,\text{A}$.

(b) In the first case $R' = V/i' = 4.70\,\text{V}/(7.09 \times 10^{-2}\,\text{A}) = 66.3\,\Omega$. In the second case $R' = V/i' = 4.86\,\text{V}/(5.52 \times 10^{-2}\,\text{A}) = 88.0\,\Omega$.

43. Let i_1 be the current in R_1 and R_2, and take it to be positive if it is toward point a in R_1. Let i_2 be the current in R_s and R_x, and take it to be positive if it is toward b in R_s. The loop rule yields $(R_1 + R_2)i_1 - (R_x + R_s)i_2 = 0$. Since points a and b are at the same potential, $i_1R_1 = i_2R_s$. The second equation gives $i_2 = i_1R_1/R_s$, which is substituted into the first equation to obtain

$$(R_1 + R_2)i_1 = (R_x + R_s)\,\frac{R_1}{R_s}\,i_1 \quad\Longrightarrow\quad R_x = \frac{R_2R_s}{R_1}\ .$$

44. (a) We use $q = q_0e^{-t/\tau}$, or $t = \tau\ln(q_0/q)$, where $\tau = RC$ is the capacitive time constant. Thus, $t_{1/3} = \tau\ln[q_0/(2q_0/3)] = \tau\ln(3/2) = 0.41\tau$.

(b) $t_{2/3} = \tau\ln[q_0/(q_0/3)] = \tau\ln 3 = 1.1\tau$.

45. During charging, the charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E}(1 - e^{-t/\tau})\ ,$$

where C is the capacitance, $\mathcal{E}$ is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{\text{eq}} = C\mathcal{E}$. We require $q = 0.99q_{\text{eq}} = 0.99C\mathcal{E}$, so

$$0.99 = 1 - e^{-t/\tau}\ .$$

Thus,

$$e^{-t/\tau} = 0.01\ .$$

Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.6$ and $t = 4.6\tau$.

46. (a) $\tau = RC = (1.40 \times 10^6\,\Omega)(1.80 \times 10^{-6}\,\text{F}) = 2.52\,\text{s}$.

(b) $q_o = \mathcal{E}C = (12.0\,\text{V})(1.80\,\mu\,\text{F}) = 21.6\,\mu\text{C}$.

(c) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC\ln\left(\frac{q_0}{q_0 - q}\right) = (2.52\,\text{s})\ln\left(\frac{21.6\,\mu\text{C}}{21.6\,\mu\text{C} - 16.0\,\mu\text{C}}\right) = 3.40\ \text{s}\ .$$

47. (a) The voltage difference V across the capacitor varies with time as $V(t) = \mathcal{E}(1 - e^{-t/RC})$. At $t = 1.30\,\mu\,\text{s}$ we have $V(t) = 5.00\,\text{V}$, so $5.00\,\text{V} = (12.0\,\text{V})(1 - e^{-1.30\,\mu\text{s}/RC})$, which gives $\tau = (1.30\,\mu\,\text{s})/\ln(12/7) = 2.41\,\mu\text{s}$.

(b) $C = \tau/R = 2.41\,\mu\text{s}/15.0\,\text{k}\Omega = 161\,\text{pF}$.

48. The potential difference across the capacitor varies as a function of time t as $V(t) = V_0e^{-t/RC}$. Using $V = V_0/4$ at $t = 2.0\,\text{s}$, we find

$$R = \frac{t}{C\ln(V_0/V)} = \frac{2.0\,\text{s}}{(2.0 \times 10^{-6}\,\text{F})\ln 4} = 7.2 \times 10^5\,\Omega\ .$$

49. (a) The charge on the positive plate of the capacitor is given by

$$q = C\mathcal{E}\left(1 - e^{-t/\tau}\right),$$

where $\mathcal{E}$ is the emf of the battery, C is the capacitance, and τ is the time constant. The value of τ is $\tau = RC = (3.00 \times 10^6\,\Omega)(1.00 \times 10^{-6}\,\mathrm{F}) = 3.00\,\mathrm{s}$. At $t = 1.00\,\mathrm{s}$, $t/\tau = (1.00\,\mathrm{s})/(3.00\,\mathrm{s}) = 0.333$ and the rate at which the charge is increasing is

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{\tau}e^{-t/\tau} = \frac{(1.00 \times 10^{-6})(4.00\,\mathrm{V})}{3.00\,\mathrm{s}}e^{-0.333} = 9.55 \times 10^{-7}\,\mathrm{C/s}.$$

(b) The energy stored in the capacitor is given by

$$U_C = \frac{q^2}{2C}.$$

and its rate of change is

$$\frac{dU_C}{dt} = \frac{q}{C}\frac{dq}{dt}.$$

Now

$$q = C\mathcal{E}\left(1 - e^{-t/\tau}\right) = (1.00 \times 10^{-6})(4.00\,\mathrm{V})(1 - e^{-0.333}) = 1.13 \times 10^{-6}\,\mathrm{C},$$

so

$$\frac{dU_C}{dt} = \left(\frac{1.13 \times 10^{-6}\,\mathrm{C}}{1.00 \times 10^{-6}\,\mathrm{F}}\right)(9.55 \times 10^{-7}\,\mathrm{C/s}) = 1.08 \times 10^{-6}\ \mathrm{W}.$$

(c) The rate at which energy is being dissipated in the resistor is given by $P = i^2R$. The current is $9.55 \times 10^{-7}\mathrm{A}$, so

$$P = (9.55 \times 10^{-7}\mathrm{A})^2(3.00 \times 10^6\,\Omega) = 2.74 \times 10^{-6}\ \mathrm{W}.$$

(d) The rate at which energy is delivered by the battery is

$$i\mathcal{E} = (9.55 \times 10^{-7}\mathrm{A})(4.00\,\mathrm{V}) = 3.82 \times 10^{-6}\,\mathrm{W}.$$

The energy delivered by the battery is either stored in the capacitor or dissipated in the resistor. Conservation of energy requires that $i\mathcal{E} = (q/C)\,(dq/dt) + i^2R$. Except for some round-off error the numerical results support the conservation principle.

50. (a) The charge q on the capacitor as a function of time is $q(t) = (\mathcal{E}C)(1 - e^{-t/RC})$, so the charging current is $i(t) = dq/dt = (\mathcal{E}/R)e^{-t/RC}$. The energy supplied by the emf is then

$$U = \int_0^\infty \mathcal{E}i\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/RC}dt = C\mathcal{E}^2 = 2U_C$$

where $U_C = \frac{1}{2}C\mathcal{E}^2$ is the energy stored in the capacitor.

(b) By directly integrating i^2R we obtain

$$U_R = \int_0^\infty i^2Rdt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2t/RC}\,dt = \frac{1}{2}C\mathcal{E}^2.$$

51. (a) The potential difference V across the plates of a capacitor is related to the charge q on the positive plate by $V = q/C$, where C is capacitance. Since the charge on a discharging capacitor is given by $q = q_0\,e^{-t/\tau}$, this means $V = V_0\,e^{-t/\tau}$ where V_0 is the initial potential difference. We solve for the time constant τ by dividing by V_0 and taking the natural logarithm:

$$\tau = -\frac{t}{\ln(V/V_0)} = -\frac{10.0\,\mathrm{s}}{\ln[(1.00\,\mathrm{V})/(100\,\mathrm{V})]} = 2.17\ \mathrm{s}.$$

(b) At $t = 17.0\,\text{s}$, $t/\tau = (17.0\,\text{s})/(2.17\,\text{s}) = 7.83$, so

$$V = V_0\, e^{-t/\tau} = (100\,\text{V})\, e^{-7.83} = 3.96 \times 10^{-2}\ \text{V}\ .$$

52. The time it takes for the voltage difference across the capacitor to reach V_L is given by $V_L = \mathcal{E}(1-e^{-t/RC})$. We solve for R:

$$R = \frac{t}{C \ln[\mathcal{E}/(\mathcal{E} - V_L)]} = \frac{0.500\,\text{s}}{(0.150 \times 10^{-6}\,\text{F}) \ln[95.0\,\text{V}/(95.0\,\text{V} - 72.0\,\text{V})]} = 2.35 \times 10^{6}\,\Omega$$

where we used $t = 0.500\,\text{s}$ given (implicitly) in the problem.

53. (a) The initial energy stored in a capacitor is given by

$$U_C = \frac{q_0^2}{2C}\ ,$$

where C is the capacitance and q_0 is the initial charge on one plate. Thus

$$q_0 = \sqrt{2CU_C} = \sqrt{2(1.0 \times 10^{-6}\,\text{F})(0.50\,\text{J})} = 1.0 \times 10^{-3}\ \text{C}\ .$$

(b) The charge as a function of time is given by $q = q_0\, e^{-t/\tau}$, where τ is the capacitive time constant. The current is the derivative of the charge

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}\, e^{-t/\tau}\ ,$$

and the initial current is $i_0 = q_0/\tau$. The time constant is $\tau = RC = (1.0 \times 10^{-6}\,\text{F})(1.0 \times 10^{6}\,\Omega) = 1.0\,\text{s}$. Thus $i_0 = (1.0 \times 10^{-3}\,\text{C})/(1.0\,\text{s}) = 1.0 \times 10^{-3}\,\text{A}$.

(c) We substitute $q = q_0\, e^{-t/\tau}$ into $V_C = q/C$ to obtain

$$V_C = \frac{q_0}{C}\, e^{-t/\tau} = \left(\frac{1.0 \times 10^{-3}\,\text{C}}{1.0 \times 10^{-6}\,\text{F}}\right) e^{-t/1.0\,\text{s}} = (1.0 \times 10^{3}\,\text{V})\, e^{-1.0t}\ ,$$

where t is measured in seconds. We substitute $i = (q_0/\tau)\, e^{-t/\tau}$ into $V_R = iR$ to obtain

$$V_R = \frac{q_0 R}{\tau}\, e^{-t/\tau} = \frac{(1.0 \times 10^{-3}\,\text{C})(1.0 \times 10^{6}\,\Omega)}{1.0\,\text{s}}\, e^{-t/1.0\,\text{s}} = (1.0 \times 10^{3}\,\text{V})\, e^{-1.0t}\ ,$$

where t is measured in seconds.

(d) We substitute $i = (q_0/\tau)\, e^{-t/\tau}$ into $P = i^2 R$ to obtain

$$P = \frac{q_0^2 R}{\tau^2}\, e^{-2t/\tau} = \frac{(1.0 \times 10^{-3}\,\text{C})^2(1.0 \times 10^{6}\,\Omega)}{(1.0\,\text{s})^2}\, e^{-2t/1.0\,\text{s}} = (1.0\,\text{W})\, e^{-2.0t}\ ,$$

where t is again measured in seconds.

54. We use the result of problem 48: $R = t/[C \ln(V_0/V)]$. Then, for $t_{\text{min}} = 10.0\,\mu\text{s}$

$$R_{\text{min}} = \frac{10.0\,\mu\text{s}}{(0.220\,\mu\text{F}) \ln(5.00/0.800)} = 24.8\,\Omega.$$

For $t_{\text{max}} = 6.00\,\text{ms}$,

$$R_{\text{max}} = \left(\frac{6.00\,\text{ms}}{10.0\,\mu\text{s}}\right)(24.8\,\Omega) = 1.49 \times 10^{4}\,\Omega\ ,$$

where in the last equation we used $\tau = RC$.

55. (a) At $t = 0$ the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces

$$i_1 = i_2 + i_3 \ ,$$

the loop rule applied to the left-hand loop produces

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0 \ ,$$

and the loop rule applied to the right-hand loop produces

$$i_2 R_2 - i_3 R_3 = 0 \ .$$

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R. The solution to the three simultaneous equations is

$$i_1 = \frac{2\mathcal{E}}{3R} = \frac{2(1.2 \times 10^3\,\mathrm{V})}{3(0.73 \times 10^6\,\Omega)} = 1.1 \times 10^{-3}\,\mathrm{A}$$

and

$$i_2 = i_3 = \frac{\mathcal{E}}{3R} = \frac{1.2 \times 10^3\,\mathrm{V}}{3(0.73 \times 10^6\,\Omega)} = 5.5 \times 10^{-4}\,\mathrm{A} \ .$$

At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields

$$\mathcal{E} - i_1 R_1 - i_1 R_2 = 0 \ .$$

The solution is

$$i_1 = i_2 = \frac{\mathcal{E}}{2R} = \frac{1.2 \times 10^3\,\mathrm{V}}{2(0.73 \times 10^6\,\Omega)} = 8.2 \times 10^{-4}\ \mathrm{A} \ .$$

(b) We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\mathcal{E} - i_1 R - i_2 R = 0 \qquad \text{and} \qquad -\frac{q}{C} - i_3 R + i_2 R = 0 \ .$$

We use the first equation to substitute for i_1 in the second and obtain $\mathcal{E} - 2i_2 R - i_3 R = 0$. Thus $i_2 = (\mathcal{E} - i_3 R)/2R$. We substitute this expression into the third equation above to obtain $-(q/C) - (i_3 R) + (\mathcal{E}/2) - (i_3 R/2) = 0$. Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\mathcal{E}}{2} \ .$$

This is just like the equation for an RC series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\mathcal{E}/2$. The solution is

$$q = \frac{C\mathcal{E}}{2}\left(1 - e^{-2t/3RC}\right) \ .$$

The current in the capacitor branch is

$$i_3 = \frac{dq}{dt} = \frac{\mathcal{E}}{3R}\, e^{-2t/3RC} \ .$$

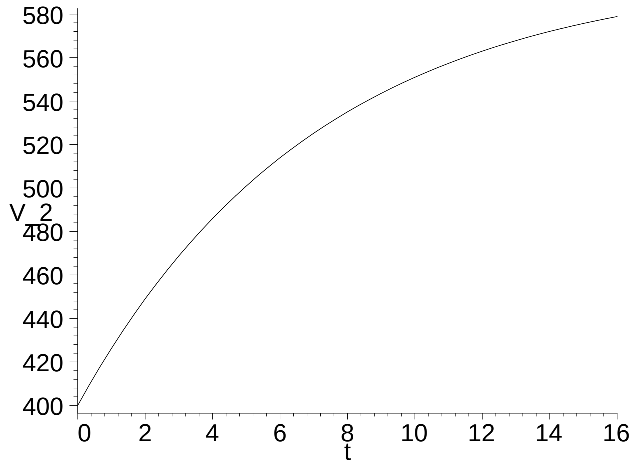
The current in the center branch is

$$\begin{aligned} i_2 &= \frac{\mathcal{E}}{2R} - \frac{i_3}{2} = \frac{\mathcal{E}}{2R} - \frac{\mathcal{E}}{6R}\, e^{-2t/3RC} \\ &= \frac{\mathcal{E}}{6R}\left(3 - e^{-2t/3RC}\right) \end{aligned}$$

and the potential difference across R_2 is

$$V_2 = i_2 R = \frac{\mathcal{E}}{6}\left(3 - e^{-2t/3RC}\right) .$$

This is shown in the following graph.



(c) For $t = 0$, $e^{-2t/3RC}$ is 1 and $V_R = \mathcal{E}/3 = (1.2 \times 10^3\,\text{V})/3 = 400\,\text{V}$. For $t = \infty$, $e^{-2t/3RC}$ is 0 and $V_R = \mathcal{E}/2 = (1.2 \times 20^3\,\text{V})/2 = 600\,\text{V}$.

(d) After "a long time" means after several time constants. Then, the current in the capacitor branch is very small and can be approximated by 0.

56. (a) We found in part (e) of problem 45 in Chapter 27 that the magnitude of the electric field is $E = 16$ V/m. Taking this to be roughly constant over the small distance ($\ell = 0.50$ m) involved here, then we approximate the potential difference between the man's feet as

$$\Delta V \approx E\ell = 8 \text{ V} .$$

(b) The voltage found in part (a) drives a current i through the two feet (each represented by $R_f = 300\,\Omega$) and the torso (represented by $R_t = 1000\,\Omega$). Thus,

$$i = \frac{\Delta V}{2R_f + R_t} = \frac{8\,\text{V}}{2(300\,\Omega) + 1000\,\Omega}$$

which yields $i \approx 5$ mA.

(c) Our value for i is far less than the stated 100 mA minimum required to put the heart into fibrillation.

57. (a) The four tires act as resistors in parallel, with an equivalent value given by

$$\frac{1}{R_{\text{eq}}} = \sum_{n=1}^{4} \frac{1}{R_{\text{tire}}} = \frac{4}{R_{\text{tire}}} \implies R_{\text{eq}} = \frac{R_{\text{tire}}}{4} .$$

Using the stated values ($C = 5.0 \times 10^{-10}\,\text{F}$ and $10^8\,\Omega < R_{\text{tire}} < 10^{11}\,\Omega$) we find the capacitive time constant $\tau = R_{\text{eq}}C$ in the range $0.012\,\text{s} < \tau < 13\,\text{s}$.

(b) Eq. 26-22 leads to

$$U_0 = \frac{1}{2}CV^2 = \frac{1}{2}\left(5.00 \times 10^{-10}\,\text{F}\right)\left(30.0 \times 10^3\,\text{V}\right)^2 = 0.225 \text{ J} .$$

(c) As demonstrated in Sample Problem 28-5, the energy "decays" exponentially according to

$$U = U_0 e^{-2t/\tau} .$$

Solving for the time which gives $U = 0.050$ J, we find

$$t = \frac{\tau}{2} \ln\left(\frac{U_0}{U}\right) = \frac{\tau}{2} \ln\left(\frac{0.225}{0.050}\right)$$

which yields, for the range of time constants found in part (a), values of t in the range $0.094\,\text{s} < t < 9.4\,\text{s}$. To obtain these particular values, we used 3-figure versions of the part (a) results ($0.0125\,\text{s} < \tau < 12.5\,\text{s}$).

(d) The lower range of resistance leads to the smaller times to discharge, which is the more desirable situation. Based on this criterion, low resistance tires are favored.

(e) There are a variety of ways to safely and quickly ground a large charged object. A large metal cable connected to, say, the (metal) building frame and held at the end of, say, a long lucite rod might be used (to touch a part of the car that does not have much paint or grease on it) to make the car safe to handle.

58. (a) In the process described in the problem, no charge is gained or lost. Thus, $q =$ constant. Hence,

$$q = C_1 V_1 = C_2 V_2 \implies V_2 = V_1 \frac{C_1}{C_2} = (200)\left(\frac{150}{10}\right) = 3000 \text{ V} .$$

(b) Eq. 28-36, with $\tau = RC$, describes not only the discharging of q but also of V. Thus,

$$V = V_0 e^{-t/\tau} \implies t = RC \ln\left(\frac{V_0}{V}\right) = \left(300 \times 10^9\,\Omega\right)\left(10 \times 10^{-12}\,\text{F}\right) \ln\left(\frac{3000}{100}\right)$$

which yields $t = 10$ s. This is a longer time than most people are inclined to wait before going on to their next task (such as handling the sensitive electronic equipment).

(c) We solve $V = V_0 e^{-t/RC}$ for R with the new values $V_0 = 1400$ V and $t = 0.30$ s. Thus,

$$R = \frac{t}{C \ln(V_0/V)} = \frac{0.30\,\text{s}}{(10 \times 10^{-12}\,\text{F}) \ln(1400/100)} = 1.1 \times 10^{10}\ \Omega .$$

59. (a) Since $R_{\text{tank}} = 140\,\Omega$, $i = 12\,\text{V}/(10\,\Omega + 140\,\Omega) = 8.0 \times 10^{-2}\,\text{A}$.

(b) Now, $R_{\text{tank}} = (140\,\Omega + 20\,\Omega)/2 = 80\,\Omega$, so $i = 12\,\text{V}/(10\,\Omega + 80\,\Omega) = 0.13\,\text{A}$.

(c) When full, $R_{\text{tank}} = 20\,\Omega$ so $i = 12\,\text{V}/(10\,\Omega + 20\,\Omega) = 0.40\,\text{A}$.

60. (a) The magnitude of the current density vector is

$$\begin{aligned} J_A = J_B &= \frac{i}{A} = \frac{V}{(R_1 + R_2)A} = \frac{4\,\text{V}}{(R_1 + R_2)\pi D^2} \\ &= \frac{4(60.0\,\text{V})}{\pi(0.127\,\Omega + 0.729\,\Omega)(2.60 \times 10^{-3}\,\text{m})^2} \\ &= 1.32 \times 10^7\ \text{A/m}^2 . \end{aligned}$$

(b) $V_A = VR_1/(R_1 + R_2) = (60.0\,\text{V})(0.127\,\Omega)/(0.127\,\Omega + 0.729\,\Omega) = 8.90\,\text{V}$, and $V_B = V - V_A = 60.0\,\text{V} - 8.9\,\text{V} = 51.1\,\text{V}$.

(c) Calculate the resistivity from $R = \rho L/A$ for both materials: $\rho_A = R_A A/L_A = \pi R_A D^2/4L_A = \pi(0.127\,\Omega)(2.60 \times 10^{-3}\,\text{m})^2/[4(40.0\,\text{m})] = 1.69 \times 10^{-8}\,\Omega\cdot\text{m}$. So A is made of copper. Similarly we find $\rho_B = 9.68 \times 10^{-8}\,\Omega\cdot\text{m}$, so B is made of iron.

61. We denote silicon with subscript s and iron with i. Let $T_0 = 20°$. If

$$\begin{aligned} R(T) &= R_s(T) + R_i(T) = R_s(T_0)[1+\alpha(T-T_0)] + R_i(T_0)[1+\alpha_i(T-T_0)] \\ &= (R_s(T)_0\alpha_s + R_i(T_0)\alpha_i) + (\text{temperature independentterms}) \end{aligned}$$

is to be temperature-independent, we must require that $R_s(T_0)\alpha_s + R_i(T_0)\alpha_i = 0$. Also note that $R_s(T_0) + R_i(T_0) = R = 1000\,\Omega$. We solve for $R_s(T_0)$ and $R_i(T_0)$ to obtain

$$R_s(T_0) = \frac{R\alpha_i}{\alpha_i - \alpha_s} = \frac{(1000\,\Omega)(6.5\times10^{-3})}{6.5\times10^{-3} + 70\times10^{-3}} = 85.0\ \Omega\ ,$$

and $R_i(T_0) = 1000\,\Omega - 85.0\,\Omega = 915\,\Omega$.

62. The potential difference across R_2 is

$$V_2 = iR_2 = \frac{\mathcal{E}\,R_2}{R_1+R_2+R_3} = \frac{(12\,\mathrm{V})(4.0\,\Omega)}{3.0\,\Omega + 4.0\,\Omega + 5.0\,\Omega} = 4.0\ \mathrm{V}\ .$$

63. Since $R_{\mathrm{eq}} < R$, the two resistors ($R = 12.0\,\Omega$ and R_x) must be connected in parallel:

$$R_{\mathrm{eq}} = 3.00\,\Omega = \frac{R_xR}{R+R_x} = \frac{R_x(12.0\,\Omega)}{12.0\,\Omega + R_x}\ .$$

We solve for R_x: $R_x = R_{\mathrm{eq}}R/(R - R_{\mathrm{eq}}) = (3.00\,\Omega)(12.0\,\Omega)/(12.0\,\Omega - 3.00\,\Omega) = 4.00\,\Omega$.

64. Consider the lowest branch with the two resistors $R_1 = 3.0\,\Omega$ and $R_2 = 5.0\,\Omega$. The voltage difference across the $5.0\,\Omega$ resistor is

$$V = i_2R_2 = \frac{\mathcal{E}\,R_2}{R_1+R_2} = \frac{(120\,\mathrm{V})(5.0\,\Omega)}{3.0\,\Omega + 5.0\,\Omega} = 7.5\ \mathrm{V}\ .$$

65. When all the batteries are connected in parallel, each supplies a current i; thus, $i_R = Ni$. Then from $\mathcal{E} = ir + i_RR = ir + Nir$, we get $i_R = N\mathcal{E}/[(N+1)r]$. When all the batteries are connected in series, $i_r = i_R$ and $\mathcal{E}_{\mathrm{total}} = N\mathcal{E} = Ni_rr + i_RR = Ni_Rr + i_Rr$, so $i_R = N\mathcal{E}/[(N+1)r]$.

66. (a) They are in parallel and the portions of A and B between the load and their respective sliding contacts have the same potential difference. It is clearly important not to "short" the system (particularly if the load turns out to have very little resistance) by having the sliding contacts too close to the load-ends of A and B to start with. Thus, we suggest putting the contacts roughly in the middle of each. Since $R_1 > R_2$, larger currents generally go through B (depending on the position of the sliding contact) than through A. Therefore, B is analogous to a "coarse" control, as A is to a "fine control." Hence, we recommend adjusting the current roughly with B, and then making fine adjustments with A.

(b) Relatively large percentage changes in A cause only small percentage charges in the resistance of the parallel combination, thus permitting fine adjustment; any change in A causes half as much change in this combination.

67. When connected in series, the rate at which electric energy dissipates is $P_s = \mathcal{E}^2/(R_1+R_2)$. When connected in parallel, the corresponding rate is $P_p = \mathcal{E}^2(R_1+R_2)/R_1R_2$. Letting $P_p/P_s = 5$, we get $(R_1+R_2)^2/R_1R_2 = 5$, where $R_1 = 100\,\Omega$. We solve for R_2: $R_2 = 38\,\Omega$ or $260\,\Omega$.

68. (a) Placing a wire (of resistance r) with current i running directly from point a to point b in Fig. 28-41 divides the top of the picture into a left and a right triangle. If we label the currents through each resistor with the corresponding subscripts (for instance, i_s goes toward the lower right through R_s and i_x goes toward the upper right through R_x), then the currents must be related as follows:

$$\begin{aligned} i_0 &= i_1 + i_s \qquad \text{and} \qquad i_1 = i + i_2 \\ i_s + i &= i_x \qquad \text{and} \qquad i_2 + i_x = i_0 \end{aligned}$$

where the last relation is not independent of the previous three. The loop equations for the two triangles and also for the bottom loop (containing the battery and point b) lead to

$$\begin{aligned} i_s R_s - i_1 R_1 - ir &= 0 \\ i_2 R_2 - i_x R_x - ir &= 0 \\ \mathcal{E} - i_0 R_0 - i_s R_s - i_x R_x &= 0 \ . \end{aligned}$$

We incorporate the current relations from above into these loop equations in order to obtain three well-posed "simultaneous" equations, for three unknown currents (i_s, i_1 and i):

$$\begin{aligned} i_s R_s - i_1 R_1 - ir &= 0 \\ i_1 R_2 - i_s R_x - i\,(r + R_x + R_2) &= 0 \\ \mathcal{E} - i_s\,(R_0 + R_s + R_x) - i_1 R_0 - i R_x &= 0 \end{aligned}$$

The problem statement further specifies $R_1 = R_2 = R$ and $R_0 = 0$, which causes our solution for i to simplify significantly. It becomes

$$i = \frac{\mathcal{E}\,(R_s - R_x)}{2rR_s + 2R_xR_s + R_sR + 2rR_x + R_xR}$$

which is equivalent to the result shown in the problem statement.

(b) Examining the numerator of our final result in part (a), we see that the condition for $i = 0$ is $R_s = R_x$. Since $R_1 = R_2 = R$, this is equivalent to $R_x = R_s R_2 / R_1$, consistent with the result of Problem 43.

69. The voltage across the rightmost resistors is $V_{12} = (1.4 \text{ A})(8.0\ \Omega + 4.0\ \Omega) = 16.8$ V, which is equal to V_{16} (the voltage across the 16 Ω resistor, which has current equal to $V_{16}/(16\ \Omega) = 1.05$ A). By the junction rule, the current in the rightmost 2.0 Ω resistor is $1.05 + 1.4 = 2.45$ A, so its voltage is $V_2 = (2.0\ \Omega)(2.45 \text{ A}) = 4.9$ V. The loop rule tells us the voltage across the 2.0 Ω resistor (the one going "downward" in the figure) is $V_2' = V_2 + V_{16} = 21.7$ V (implying that the current through it is $i_2' = V_2'/(2.0\ \Omega) = 10.85$ A). The junction rule now gives the current in the leftmost 2.0 Ω resistor as $10.85 + 2.45 = 13.3$ A, implying that the voltage across it is $V_2'' = (13.3 \text{ A})(2.0\ \Omega) = 26.6$ V. Therefore, by the loop rule, $\mathcal{E} = V_2'' + V_2' = 48.3$ V.

70. In the steady state situation, the capacitor voltage will equal the voltage across the 15 kΩ resistor:

$$V_0 = (15 \text{ k}\Omega)\left(\frac{20 \text{ V}}{10 \text{ k}\Omega + 15 \text{ k}\Omega}\right) = 12 \text{ V} .$$

Now, multiplying Eq. 28-36 by the capacitance leads to $V = V_0 e^{-t/RC}$ describing the voltage across the capacitor (and across the $R = 15$ kΩ resistor) after the switch is opened (at $t = 0$). Thus, with $t = 0.00400$ s, we obtain

$$V = (12)e^{-0.004/(15000)(0.4\times10^{-6})} = 6.16 \text{ V} .$$

Therefore, using Ohm's law, the current through the 15 kΩ resistor is $6.16/15000 = 4.11 \times 10^{-4}$ A.

71. (a) By symmetry, we see that i_1 is half the current that goes through the battery. The battery current is found by dividing $\mathcal{E}$ by the equivalent resistance of the circuit, which is easily found to be 6.0 Ω. Thus,

$$i_1 = \frac{1}{2}\,i_{\text{bat}} = \frac{1}{2}\,\frac{12 \text{ V}}{6.0\ \Omega} = 1.0 \text{ A}$$

and is clearly downward (in the figure).

(b) We use Eq. 28-14: $P = i_{\text{bat}}\mathcal{E} = 24$ W.

72. The series pair of 2.0 Ω resistors on the right reduce to $R' = 4.0\ \Omega$, and the parallel pair of identical 4.0 Ω resistors on the left reduce to $R'' = 2.0\ \Omega$. The voltage across R' must equal that across R''; thus,

$$\begin{aligned} V' &= V'' \\ i'R' &= i''R'' \\ i' &= \frac{1}{2}i'' \end{aligned}$$

where in the last step we divide by R' and simplify. This relation, plus the junction rule condition $6.0\ \text{A} = i' + i''$ leads to the solution $i'' = 4.0$ A. It is clear by symmetry that $i = \frac{1}{2}i''$, so we conclude $i = 2.0$ A.

73. (a) We reduce the parallel pair of identical 2.0 Ω resistors (on the right side) to $R' = 1.0\ \Omega$, and we reduce the series pair of identical 2.0 Ω resistors (on the upper left side) to $R'' = 4.0\ \Omega$. With R denoting the 2.0 Ω resistor at the bottom (between V_2 and V_1), we now have three resistors in series which are equivalent to

$$R + R' + R'' = 7.0\ \Omega$$

across which the voltage is 7.0 V (by the loop rule, this is 12 V − 5.0 V), implying that the current is 1.0 A (clockwise). Thus, the voltage across R' is (1.0 A)(1.0 Ω) = 1.0 V, which means that (examining the right side of the circuit) the voltage difference between *ground* and V_1 is $12 - 1 = 11$ V. Noting the orientation of the battery, we conclude $V_1 = -11$ V.

(b) The voltage across R'' is (1.0 A)(4.0 Ω) = 4.0 V, which means that (examining the left side of the circuit) the voltage difference between *ground* and V_2 is $5.0 + 4.0 = 9.0$ V. Noting the orientation of the battery, we conclude $V_2 = -9.0$ V. This can be verified by considering the voltage across R and the value we obtained for V_1.

74. (a) From symmetry we see that the current through the top set of batteries (i) is the same as the current through the second set. This implies that the current through the $R = 4.0\,\Omega$ resistor at the bottom is $i_R = 2i$. Thus, with r denoting the internal resistance of each battery (equal to $4.0\,\Omega$) and $\mathcal{E}$ denoting the 20 V emf, we consider one loop equation (the outer loop), proceeding counterclockwise:

$$3\,(\mathcal{E} - ir) - (2i)R = 0\ .$$

This yields $i = 3.0$ A. Consequently, $i_R = 6.0$ A.

(b) The terminal voltage of each battery is $\mathcal{E} - ir = 8.0$ V.

(c) Using Eq. 28-14, we obtain $P = i\mathcal{E} = (3)(20) = 60$ W.

(d) Using Eq. 27-22, we have $P = i^2 r = 36$ W.

75. (a) The work done by the battery relates to the potential energy change:

$$q\Delta V = eV = e(12\,\text{V}) = 12\,\text{eV} = (12\,\text{eV})(1.6\times10^{-19}\,\text{J/eV}) = 1.9\times10^{-18}\ \text{J}\ .$$

(b) $P = iV = neV = (3.4\times10^{18}/\text{s})(1.6\times10^{-19}\,\text{C})(12\,\text{V}) = 6.5\,\text{W}$.

76. (a) We denote $L = 10\,\text{km}$ and $\alpha = 13\,\Omega/\text{km}$. Measured from the east end we have $R_1 = 100\,\Omega = 2\alpha(L-x) + R$, and measured from the west end $R_2 = 200\,\Omega = 2\alpha x + R$. Thus,

$$x = \frac{R_2 - R_1}{4\alpha} + \frac{L}{2} = \frac{200\,\Omega - 100\,\Omega}{4(13\,\Omega/\text{km})} + \frac{10\,\text{km}}{2} = 6.9\ \text{km}\ .$$

(b) Also, we obtain

$$R = \frac{R_1 + R_2}{2} - \alpha L = \frac{100\,\Omega + 200\,\Omega}{2} - (13\,\Omega/\text{km})(10\,\text{km}) = 20\ \Omega\ .$$

77. (a) From $P = V^2/R$ we find $V = \sqrt{PR} = \sqrt{(10\,\text{W})(0.10\,\Omega)} = 1.0\,\text{V}$.

(b) From $i = V/R = (\mathcal{E} - V)/r$ we find

$$r = R\left(\frac{\mathcal{E} - V}{V}\right) = (0.10\,\Omega)\left(\frac{1.5\,\text{V} - 1.0\,\text{V}}{1.0\,\text{V}}\right) = 0.050\ \Omega\ .$$

78. (a) The power delivered by the motor is $P = (2.00\,\text{V})(0.500\,\text{m/s}) = 1.00\,\text{W}$. From $P = i^2 R_{\text{motor}}$ and $\mathcal{E} = i(r + R_{\text{motor}})$ we then find $i^2 r - i\mathcal{E} + P = 0$ (which also follows directly from the conservation of energy principle). We solve for i:

$$i = \frac{\mathcal{E} \pm \sqrt{\mathcal{E}^2 - 4rP}}{2r} = \frac{2.00\,\text{V} \pm \sqrt{(2.00\,\text{V})^2 - 4(0.500\,\Omega)(1.00\,\text{W})}}{2(0.500\,\Omega)}\ .$$

The answer is either $3.41\,\text{A}$ or $0.586\,\text{A}$.

(b) We use $V = \mathcal{E} - ir = 2.00\,\text{V} - i(0.500\,\Omega)$. We substitute the two values of i obtained in part (a) into the above formula to get $V = 0.293\,\text{V}$ or $1.71\,\text{V}$.

(c) The power P delivered by the motor is the same for either solution. Since $P = iV$ we may have a lower i and higher V or, alternatively, a lower V and higher i. One can check that the two sets of solutions for i and V above do yield the same power $P = iV$.

79. Let the power supplied be P_s and that dissipated be P_d. Since $P_d = i^2 R$ and $i = P_s/\mathcal{E}$, we have $P_d = P_s^2/\mathcal{E}^2 R \propto \mathcal{E}^{-2}$. The ratio is then

$$\frac{P_d(\mathcal{E} = 110,000\,\text{V})}{P_d(\mathcal{E} = 110\,\text{V})} = \left(\frac{110\,\text{V}}{110,000\,\text{V}}\right)^2 = 1.0 \times 10^{-6}\ .$$

80. (a) $R_{\text{eq}}(AB) = 20.0\,\Omega/3 = 6.67\,\Omega$ (three $20.0\,\Omega$ resistors in parallel).

(b) $R_{\text{eq}}(AC) = 20.0\,\Omega/3 = 6.67\,\Omega$ (three $20.0\,\Omega$ resistors in parallel).

(c) $R_{\text{eq}}(BC) = 0$ (as B and C are connected by a conducting wire).

81. The maximum power output is $(120\,\text{V})(15\,\text{A}) = 1800\,\text{W}$. Since $1800\,\text{W}/500\,\text{W} = 3.6$, the maximum number of 500 W lamps allowed is 3.

82. The part of R_0 connected in parallel with R is given by $R_1 = R_0 x/L$, where $L = 10\,\text{cm}$. The voltage difference across R is then $V_R = \mathcal{E}R'/R_{\text{eq}}$, where $R' = RR_1/(R + R_1)$ and $R_{\text{eq}} = R_0(1 - x/L) + R'$. Thus

$$P_R = \frac{V_R^2}{R} = \frac{1}{R}\left(\frac{\mathcal{E}RR_1/(R + R_1)}{R_0(1 - x/L) + RR_1/(R + R_1)}\right)^2\ .$$

Algebraic manipulation then leads to

$$P_R = \frac{100R(\mathcal{E}x/R_0)^2}{(100R/R_0 + 10x - x^2)^2}$$

where x is measured in cm.

83. (a) Since $P = \mathcal{E}^2/R_{\text{eq}}$, the higher the power rating the smaller the value of R_{eq}. To achieve this, we can let the low position connect to the larger resistance (R_1), middle position connect to the smaller resistance (R_2), and the high position connect to both of them in parallel.

(b) For $P = 100\,\text{W}$, $R_{\text{eq}} = R_1 = \mathcal{E}^2/P = (120\,\text{V})^2/100\,\text{W} = 144\,\Omega$; for $P = 300\,\text{W}$, $R_{\text{eq}} = R_1R_2/(R_1 + R_2) = (144\,\Omega)R_2/(144\,\Omega + R_2) = (120\,\text{V})^2/300\,\text{W}$. We obtain $R_2 = 72\,\Omega$.

84. Note that there is no voltage drop across the ammeter. Thus, the currents in the bottom resistors are the same, which we call i (so the current through the battery is $2i$ and the voltage drop across each of the bottom resistors is iR). The resistor network can be reduced to an equivalence of

$$R_{\rm eq} = \frac{(2R)(R)}{2R+R} + \frac{(R)(R)}{R+R} = \frac{7}{6}R$$

which means that we can determine the current through the battery (and also through each of the bottom resistors):

$$2i = \frac{\mathcal{E}}{R_{\rm eq}} \implies i = \frac{3\mathcal{E}}{7R} .$$

By the loop rule (going around the left loop, which includes the battery, resistor $2R$ and one of the bottom resistors), we have

$$\mathcal{E} - i_{2R}(2R) - iR = 0 \implies i_{2R} = \frac{\mathcal{E} - iR}{2R} .$$

Substituting $i = 3\mathcal{E}/7R$, this gives $i_{2R} = 2\mathcal{E}/7R$. The difference between i_{2R} and i is the current through the ammeter. Thus,

$$i_{\rm ammeter} = i - i_{2R} = \frac{3\mathcal{E}}{7R} - \frac{2\mathcal{E}}{7R} = \frac{\mathcal{E}}{7R} .$$

85. The current in the ammeter is given by $i_A = \mathcal{E}/(r + R_1 + R_2 + R_A)$. The current in R_1 and R_2 without the ammeter is $i = \mathcal{E}/(r + R_1 + R_2)$. The percent error is then

$$\begin{aligned}\frac{\Delta i}{i} &= \frac{i - i_A}{i} = 1 - \frac{r + R_1 + R_2}{r + R_1 + R_2 + R_A} = \frac{R_A}{r + R_1 + R_2 + R_A} \\ &= \frac{0.10\,\Omega}{2.0\,\Omega + 5.0\,\Omega + 4.0\,\Omega + 0.10\,\Omega} = 0.90\% .\end{aligned}$$

86. When S is open for a long time, the charge on C is $q_i = \mathcal{E}_2 C$. When S is closed for a long time, the current i in R_1 and R_2 is $i = (\mathcal{E}_2 - \mathcal{E}_1)/(R_1 + R_2) = (3.0\,\text{V} - 1.0\,\text{V})/(0.20\,\Omega + 0.40\,\Omega) = 3.33\,\text{A}$. The voltage difference V across the capacitor is then $V = \mathcal{E}_2 - iR_2 = 3.0\,\text{V} - (3.33\,\text{A})(0.40\,\Omega) = 1.67\,\text{V}$. Thus the final charge on C is $q_f = VC$. So the change in the charge on the capacitor is $\Delta q = q_f - q_i = (V - \mathcal{E}_2)C = (1.67\,\text{V} - 3.0\,\text{V})(10\,\mu\text{F}) = -13\,\mu\text{C}$.

87. Requiring no current through the $10.0\,\Omega$ resistor means that 20.0 V will be across R (which has current i_R). The current through the $20.0\,\Omega$ resistor is also i_R, so the loop rule leads to

$$50.0\,\text{V} - 20.0\,\text{V} - i_R\,(20.0\,\Omega) = 0$$

which yields $i_R = 1.5$ A. Therefore,

$$R = \frac{20.0\,\text{V}}{i_R} = 13.3\ \Omega .$$

88. (a) The capacitor is *initially* uncharged, which implies (by the loop rule) that there is zero voltage (at $t = 0$) across the 10 kΩ resistor, and that 30 V is across the 20 kΩ resistor. Therefore, by Ohm's law, $i_{10} = 0$,

(b) and $i_{20} = (30\,\text{V})/(20\ \text{k}\Omega) = 1.5 \times 10^{-3}$ A.

(c) As $t \to \infty$ the current to the capacitor reduces to zero and the 20 kΩ and 10 kΩ resistors behave more like a series pair (having the same current), equivalent to 30 kΩ. The current through them, then, at long times, is $i = (30\,\text{V})/(30\ \text{k}\Omega) = 1.0 \times 10^{-3}$ A.

89. (a) The six resistors to the left of $\mathcal{E}_1 = 16$ V battery can be reduced to a single resistor $R = 8.0\ \Omega$, through which the current must be $i_R = \mathcal{E}_1/R = 2.0$ A. Now, by the loop rule, the current through the 3.0 Ω and 1.0 Ω resistors at the upper right corner is

$$i' = \frac{16.0\,\text{V} - 8.0\,\text{V}}{3.0\ \Omega + 1.0\ \Omega} = 2.0\ \text{A}$$

in a direction that is "backward" relative to the $\mathcal{E}_2 = 8.0$ V battery. Thus, by the junction rule,

$$i_1 = i_R + i' = 4.0\ \text{A}$$

and is upward (that is, in the "forward" direction relative to $\mathcal{E}_1$).

(b) The current i_2 derives from a succession of symmetric splittings of i_R (reversing the procedure of reducing those six resistors to find R in part (a)). We find

$$i_2 = \frac{1}{2}\left(\frac{1}{2}\, i_R\right) = 0.50\ \text{A}$$

and is clearly downward.

(c) Using our conclusions from part (a) in Eq. 28-14, we obtain $P = i_1\mathcal{E}_1 = (4)(16) = 64$ W supplied.

(d) Using results calculated in part (a) in Eq. 28-14, we obtain $P = i'\mathcal{E}_2 = (2)(8) = 16$ W absorbed.

90. We reduce the parallel pair of identical 4.0 Ω resistors to $R' = 2.0\ \Omega$, which has current $i = 2i_1$ going through it. It is in series with a 2.0 Ω resistor, which leads to an equivalence of $R = 4.0\ \Omega$ with current i. We find a path (for use with the loop rule) that goes through this R, the 4.0 V battery, and the 20 V battery, and proceed counterclockwise (assuming i goes rightward through R):

$$20\,\text{V} + 4.0\,\text{V} - iR = 0$$

which leads to $i = 6.0$ A. Consequently, $i_1 = \frac{1}{2}i = 3.0$ A going rightward.

91. With the unit Ω understood, the equivalent resistance for this circuit is

$$R_{\text{eq}} = \frac{20R + 100}{R + 10}\ .$$

Therefore, the power supplied by the battery (equal to the power dissipated in the resistors) is

$$P = \frac{V^2}{R} = V^2\,\frac{R + 10}{20R + 100}$$

where $V = 12$ V. We attempt to extremize the expression by working through the $dP/dR = 0$ condition and do not find a value of R that satisfies it. We note, then, that the function is a monotonically decreasing function of R, with $R = 0$ giving the maximum possible value (since $R < 0$ values are not being allowed). With the value $R = 0$, we obtain $P = 14.4$ W.

92. The resistor by the letter i is above three other resistors; together, these four resistors are equivalent to a resistor $R = 10\ \Omega$ (with current i). As if we were presented with a maze, we find a path through R that passes through any number of batteries (10, it turns out) but no other resistors, which – as in any good maze – winds "all over the place." Some of the ten batteries are opposing each other (particularly the ones along the outside), so that their net emf is only $\mathcal{E} = 40$ V. The current through R is then $i = \mathcal{E}/R = 4.0$ A, and is directed upward in the figure.

93. (First problem of **Cluster**)

(a) R_2 and R_3 are in parallel; their equivalence is in series with R_1. Therefore,

$$R_{\text{eq}} = R_1 + \frac{R_2\,R_3}{R_2 + R_3} = 300\ \Omega\ .$$

(b) The current through the battery is $\mathcal{E}/R_{\text{eq}} = 0.0200$ A, which is also the current through R_1. Hence, the voltage across R_1 is $V_1 = (0.0200 \text{ A})(100\ \Omega) = 2.00$ V.

(c) From the loop rule,
$$\mathcal{E} - V_1 - i_3 R_3 = 0$$
which yields $i_3 = 6.67 \times 10^{-3}$ A.

94. (Second problem of **Cluster**)

(a) The loop rule (proceeding counterclockwise around the right loop) leads to $\mathcal{E}_2 - i_1 R_1 = 0$ (where i_1 was assumed downward). This yields $i_1 = 0.060$ A (downward).

(b) The loop rule (counterclockwise around the left loop) gives
$$(+\mathcal{E}_1) + (+i_1 R_1) + (-i_3 R_3) = 0$$
where i_3 has been assumed leftward. This yields $i_3 = 0.180$ A (leftward).

(c) The junction rule tells us that the current through the 12 V battery is $0.180 + 0.060 = 0.240$ A upward.

95. (Third problem of **Cluster**)

(a) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:
$$\begin{aligned} \mathcal{E}_1 - i_2 R_2 - (i_2 + i_3) R_1 &= 0 \\ \mathcal{E}_2 - i_3 R_3 - (i_2 + i_3) R_1 &= 0 \ . \end{aligned}$$
Solving, we find $i_2 = 0.0109$ A (rightward, as was assumed in writing the equations as we did), $i_3 = 0.0273$ A (leftward), and $i_1 = i_2 + i_3 = 0.0382$ A (downward).

(b) See the results in part (a).

(c) See the results in part (a).

(d) The voltage across R_1 equals V_A: $(0.0382 \text{ A})(100\ \Omega) = +3.82$ V.

96. (Fourth problem of **Cluster**)

(a) The symmetry of the problem allows us to use i_2 as the current in *both* of the R_2 resistors and i_1 for the R_1 resistors. We see from the junction rule that $i_3 = i_1 - i_2$. There are only two independent loop rule equations:
$$\begin{aligned} \mathcal{E} - i_2 R_2 - i_1 R_1 &= 0 \\ \mathcal{E} - 2 i_1 R_1 - (i_1 - i_2) R_3 &= 0 \ . \end{aligned}$$
where in the latter equation, a zigzag path through the bridge has been taken. Solving, we find $i_1 = 0.002625$ A , $i_2 = 0.00225$ A and $i_3 = i_1 - i_2 = 0.000375$ A. Therefore, $V_A - V_B = i_1 R_1 = 5.25$ V.

(b) It follows also that $V_B - V_C = i_3 R_3 = 1.50$ V.

(c) We find $V_C - V_D = i_1 R_1 = 5.25$ V.

(d) Finally, $V_A - V_C = i_2 R_2 = 6.75$ V.

Chapter 29

1. (a) We use Eq. 29-3: $F_B = |q|vB\sin\phi = (+3.2\times10^{-19}\,\mathrm{C})(550\,\mathrm{m/s})(0.045\,\mathrm{T})(\sin 52^\circ) = 6.2\times10^{-18}\,\mathrm{N}$.
   (b) $a = F_B/m = (6.2\times10^{-18}\,\mathrm{N})/(6.6\times10^{-27}\,\mathrm{kg}) = 9.5\times10^{8}\,\mathrm{m/s^2}$.
   (c) Since it is perpendicular to $\vec{v}$, $\vec{F}_B$ does not do any work on the particle. Thus from the work-energy theorem both the kinetic energy and the speed of the particle remain unchanged.

2. (a) The largest value of force occurs if the velocity vector is perpendicular to the field. Using Eq. 29-3,

$$F_{B,\,\max} = |q|vB\sin(90^\circ) = evB = (1.60\times10^{-19}\,\mathrm{C})(7.20\times10^{6}\,\mathrm{m/s})(83.0\times10^{-3}\,\mathrm{T}) = 9.56\times10^{-14}\ \mathrm{N}\ .$$

   The smallest value occurs if they are parallel: $F_{B,\,\min} = |q|vB\sin(0) = 0$.
   (b) By Newton's second law, $a = F_B/m_e = |q|vB\sin\theta/m_e$, so the angle θ between $\vec{v}$ and $\vec{B}$ is

$$\theta = \sin^{-1}\left(\frac{m_e a}{|q|vB}\right) = \sin^{-1}\left[\frac{(9.11\times10^{-31}\,\mathrm{kg})(4.90\times10^{14}\,\mathrm{m/s^2})}{(1.60\times10^{-16}\,\mathrm{C})(7.20\times10^{6}\,\mathrm{m/s})(83.0\times10^{-3}\,\mathrm{T})}\right] = 0.267^\circ\ .$$

3. (a) Eq. 29-3 leads to

$$v = \frac{F_B}{eB\sin\phi} = \frac{6.50\times10^{-17}\,\mathrm{N}}{(1.60\times10^{-19}\,\mathrm{C})(2.60\times10^{-3}\,\mathrm{T})\sin 23.0^\circ} = 4.00\times10^{5}\ \mathrm{m/s}\ .$$

   (b) The kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\,\mathrm{kg})(4.00\times10^{5}\,\mathrm{m/s})^2 = 1.34\times10^{-16}\ \mathrm{J}\ .$$

   This is $(1.34\times10^{-16}\,\mathrm{J})/(1.60\times10^{-19}\,\mathrm{J/eV}) = 835$ eV.

4. (a) The force on the electron is

$$\begin{aligned}
\vec{F}_B &= q\vec{v}\times\vec{B} = q(v_x\hat{\imath} + v_y\hat{\jmath})\times(B_x\hat{\imath} + B_y\vec{j}) \\
&= q(v_xB_y - v_yB_x)\hat{\mathrm{k}} \\
&= (-1.6\times10^{-19}\,\mathrm{C})[(2.0\times10^{6}\,\mathrm{m/s})(-0.15\,\mathrm{T}) - (3.0\times10^{6}\,\mathrm{m/s})(0.030\,\mathrm{T})] \\
&= (6.2\times10^{-14}\ \mathrm{N})\ \hat{\mathrm{k}}\ .
\end{aligned}$$

   Thus, the magnitude of $\vec{F}_B$ is $6.2\times10^{14}\,\mathrm{N}$, and $\vec{F}_B$ points in the positive z direction.
   (b) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\vec{F}_B$ has the same magnitude but points in the negative z direction.

5. (a) The textbook uses "geomagnetic north" to refer to Earth's magnetic pole lying in the northern hemisphere. Thus, the electrons are traveling northward. The vertical component of the magnetic field is downward. The right-hand rule indicates that $\vec{v}\times\vec{B}$ is to the west, but since the electron is negatively charged (and $\vec{F} = q\vec{v}\times\vec{B}$), the magnetic force on it is to the east.

(b) We combine $F = m_e a$ with $F = evB\sin\phi$. Here, $B\sin\phi$ represents the downward component of Earth's field (given in the problem). Thus, $a = evB/m_e$. Now, the electron speed can be found from its kinetic energy. Since $K = \frac{1}{2}mv^2$,

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(12.0\times10^3\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = 6.49\times10^7\ \text{m/s}\ .$$

Therefore,

$$a = \frac{evB}{m_e} = \frac{(1.60\times10^{-19}\,\text{C})(6.49\times10^7\,\text{m/s})(55.0\times10^{-6}\,\text{T})}{9.11\times10^{-31}\,\text{kg}} = 6.27\times10^{14}\ \text{m/s}^2\ .$$

(c) We ignore any vertical deflection of the beam which might arise due to the horizontal component of Earth's field. Technically, then, the electron should follow a circular arc. However, the deflection is so small that many of the technicalities of circular geometry may be ignored, and a calculation along the lines of projectile motion analysis (see Chapter 4) provides an adequate approximation:

$$\Delta x = vt \implies t = \frac{\Delta x}{v} = \frac{0.200\,\text{m}}{6.49\times10^7\,\text{m/s}}$$

which yields a time of $t = 3.08\times10^{-9}$ s. Then, with our y axis oriented eastward,

$$\Delta y = \frac{1}{2}at^2 = \frac{1}{2}\left(6.27\times10^{14}\right)\left(3.08\times10^{-9}\right)^2 = 0.00298\ \text{m}\ .$$

6. (a) The net force on the proton is given by

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.6\times10^{-19}\,\text{C})[(4.0\,\text{V/m})\hat{\text{k}} + (2000\,\text{m/s})\hat{\text{j}}\times(-2.5\,\text{mT})\hat{\text{i}}]\\ &= (1.4\times10^{-18}\ \text{N})\ \hat{\text{k}}\ .\end{aligned}$$

(b) In this case

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.6\times10^{-19}\,\text{C})[(-4.0\,\text{V/m})\hat{\text{k}} + (2000\,\text{m/s})\hat{\text{j}}\times(-2.5\,\text{mT})\hat{\text{i}}]\\ &= (1.6\times10^{-19}\ \text{N})\ \hat{\text{k}}\ .\end{aligned}$$

(c) In the final case,

$$\begin{aligned}\vec{F} &= \vec{F}_E + \vec{F}_B = q\vec{E} + q\vec{v}\times\vec{B}\\ &= (1.6\times10^{-19}\,\text{C})[(4.0\,\text{V/m})\hat{\text{i}} + (2000\,\text{m/s})\hat{\text{j}}\times(-2.5\,\text{mT})\hat{\text{i}}]\\ &= (6.4\times10^{-19}\,\text{N})\,\hat{\text{i}} + (8.0\times10^{-19}\ \text{N})\,\hat{\text{k}}\ .\end{aligned}$$

The magnitude of the force is now

$$\sqrt{F_x^2+F_y^2+F_z^2} = \sqrt{(6.4\times10^{-19}\,\text{N})^2 + 0 + (8.0\times10^{-19}\,\text{N})^2} = 1.0\times10^{-18}\ \text{N}\ .$$

7. (a) Equating the magnitude of the electric force ($F = eE$) with that of the magnetic force (Eq. 29-3), we obtain $B = E/v\sin\phi$. The field is smallest when the $\sin\phi$ factor is at its largest value; that is, when $\phi = 90°$. Now, we use $K = \frac{1}{2}mv^2$ to find the speed:

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.5\times10^3\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = 2.96\times10^7\ \text{m/s}\ .$$

Thus,

$$B = \frac{E}{v} = \frac{10 \times 10^3 \mathrm{V/m}}{2.96 \times 10^7 \mathrm{~m/s}} = 3.4 \times 10^{-4} \mathrm{~T} .$$

The magnetic field must be perpendicular to both the electric field and the velocity of the electron.

(b) A proton will pass undeflected if its velocity is the same as that of the electron. Both the electric and magnetic forces reverse direction, but they still cancel.

8. (a) Letting $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$, we get $vB \sin\phi = E$. We note that (for given values of the fields) this gives a minimum value for speed whenever the $\sin\phi$ factor is at its maximum value (which is 1, corresponding to $\phi = 90°$). So $v_{\min} = E/B = (1.50 \times 10^3 \mathrm{~V/m})/(0.400 \mathrm{~T}) = 3.75 \times 10^3 \mathrm{~m/s}$.

(b) Having noted already that $\vec{v} \perp \vec{B}$, we now point out that $\vec{v} \times \vec{B}$ (which direction is given by the right-hand rule) must be in the direction opposite to $\vec{E}$. Thus, we can use the *left* hand to indicate the arrangement of vectors: if one points the thumb, index finger, and middle finger on the left hand so that all three are mutually perpendicular, then the thumb represents $\vec{v}$, the index finger indicates $\vec{B}$, and the middle finger represents $\vec{E}$.

9. Straight line motion will result from zero net force acting on the system; we ignore gravity. Thus, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = 0$. Note that $\vec{v} \perp \vec{B}$ so $|\vec{v} \times \vec{B}| = vB$. Thus, obtaining the speed from the formula for kinetic energy, we obtain

$$\begin{aligned} B &= \frac{E}{v} = \frac{E}{\sqrt{2m_e K}} \\ &= \frac{100 \mathrm{~V}/(20 \times 10^{-3} \mathrm{~m})}{\sqrt{2(9.11 \times 10^{-31} \mathrm{~kg})(1.0 \times 10^3 \mathrm{~V})(1.60 \times 10^{-19} \mathrm{~C})}} \\ &= 2.7 \times 10^{-4} \mathrm{~T} . \end{aligned}$$

10. We apply $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = m_e\vec{a}$ to solve for $\vec{E}$:

$$\begin{aligned} \vec{E} &= \frac{m_e\vec{a}}{q} + \vec{B} \times \vec{v} \\ &= \frac{(9.11 \times 10^{-31} \mathrm{~kg})(2.00 \times 10^{12} \mathrm{~m/s^2})\hat{\imath}}{-1.60 \times 10^{-19} \mathrm{~C}} + (400 \,\mu\mathrm{T})\hat{\imath} \times [(12.0 \mathrm{~km/s})\hat{\jmath} + (15.0 \mathrm{~km/s})\hat{\mathrm{k}}] \\ &= (-11.4\hat{\imath} - 6.00\hat{\jmath} + 4.80\hat{\mathrm{k}}) \mathrm{~V/m} \quad . \end{aligned}$$

11. Since the total force given by $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$ vanishes, the electric field $\vec{E}$ must be perpendicular to both the particle velocity $\vec{v}$ and the magnetic field $\vec{B}$. The magnetic field is perpendicular to the velocity, so $\vec{v} \times \vec{B}$ has magnitude vB and the magnitude of the electric field is given by $E = vB$. Since the particle has charge e and is accelerated through a potential difference V, $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{2eV/m}$. Thus,

$$E = B\sqrt{\frac{2eV}{m}} = (1.2 \mathrm{T})\sqrt{\frac{2(1.60 \times 10^{-19} \mathrm{~C})(10 \times 10^3 \mathrm{~V})}{(6.0 \mathrm{~u})(1.661 \times 10^{-27} \mathrm{~kg/u})}} = 6.8 \times 10^5 \mathrm{~V/m} .$$

12. We use Eq. 29-12 to solve for V:

$$V = \frac{iB}{nle} = \frac{(23 \mathrm{~A})(0.65 \mathrm{~T})}{(8.47 \times 10^{28}/\mathrm{m}^3)(150 \,\mu\mathrm{m})(1.6 \times 10^{-19} \mathrm{~C})} = 7.4 \times 10^{-6} \mathrm{~V} .$$

13. (a) In Chapter 27, the electric field (called E_C in this problem) which "drives" the current through the resistive material is given by Eq. 27-11, which (in magnitude) reads $E_C = \rho J$. Combining this with Eq. 27-7, we obtain

$$E_C = \rho nev_d .$$

Now, regarding the Hall effect, we use Eq. 29-10 to write $E = v_d B$. Dividing one equation by the other, we get $E/E_c = B/ne\rho$.

(b) Using the value of copper's resistivity given in Chapter 27, we obtain

$$\frac{E}{E_c} = \frac{B}{ne\rho} = \frac{0.65\,\mathrm{T}}{(8.47\times10^{28}/\,\mathrm{m}^3)(1.60\times10^{-19}\,\mathrm{C})(1.69\times10^{-8}\,\Omega\cdot\mathrm{m})} = 2.84\times10^{-3}\ .$$

14. For a free charge q inside the metal strip with velocity $\vec{v}$ we have $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$. We set this force equal to zero and use the relation between (uniform) electric field and potential difference. Thus,

$$v = \frac{E}{B} = \frac{|V_x - V_y|/d_{xy}}{B} = \frac{(3.90\times10^{-9}\,\mathrm{V})}{(1.20\times10^{-3}\,\mathrm{T})(0.850\times10^{-2}\,\mathrm{m})} = 0.382\ \mathrm{m/s}\ .$$

15. From Eq. 29-16, we find

$$B = \frac{m_e v}{er} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(1.3\times10^{6}\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(0.35\,\mathrm{m})} = 2.1\times10^{-5}\ \mathrm{T}\ .$$

16. (a) The accelerating process may be seen as a conversion of potential energy eV into kinetic energy. Since it starts from rest, $\frac{1}{2}m_e v^2 = eV$ and

$$v = \sqrt{\frac{2eV}{m_e}} = \sqrt{\frac{2(1.60\times10^{-19}\,\mathrm{C})(350\,\mathrm{V})}{9.11\times10^{-31}\ \mathrm{kg}}} = 1.11\times10^{7}\ \mathrm{m/s}\ .$$

(b) Eq. 29-16 gives

$$r = \frac{m_e v}{eB} = \frac{(9.11\times10^{-31}\ \mathrm{kg})(1.11\times10^{7}\ \mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(200\times10^{-3}\,\mathrm{T})} = 3.16\times10^{-4}\ \mathrm{m}\ .$$

17. (a) From $K = \frac{1}{2}m_e v^2$ we get

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(1.20\times10^{3}\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{eV/J})}{9.11\times10^{-31}\,\mathrm{kg}}} = 2.05\times10^{7}\ \mathrm{m/s}\ .$$

(b) From $r = m_e v/qB$ we get

$$B = \frac{m_e v}{qr} = \frac{(9.11\times10^{-31}\,\mathrm{kg})(2.05\times10^{7}\,\mathrm{m/s})}{(1.60\times10^{-19}\,\mathrm{C})(25.0\times10^{-2}\,\mathrm{m})} = 4.67\times10^{-4}\ \mathrm{T}\ .$$

(c) The "orbital" frequency is

$$f = \frac{v}{2\pi r} = \frac{2.07\times10^{7}\,\mathrm{m/s}}{2\pi(25.0\times10^{-2}\,\mathrm{m})} = 1.31\times10^{7}\ \mathrm{Hz}\ .$$

(d) $T = 1/f = (1.31\times10^{7}\,\mathrm{Hz})^{-1} = 7.63\times10^{-8}$ s.

18. The period of revolution for the iodine ion is $T = 2\pi r/v = 2\pi m/Bq$, which gives

$$m = \frac{BqT}{2\pi} = \frac{(45.0\times10^{-3}\,\mathrm{T})(1.60\times10^{-19}\,\mathrm{C})(1.29\times10^{-3}\,\mathrm{s})}{(7)(2\pi)(1.66\times10^{-27}\,\mathrm{kg/u})} = 127\ \mathrm{u}\ .$$

19. (a) The frequency of revolution is

$$f = \frac{Bq}{2\pi m_e} = \frac{(35.0\times10^{-6}\,\mathrm{T})(1.60\times10^{-19}\,\mathrm{C})}{2\pi(9.11\times10^{-31}\,\mathrm{kg})} = 9.78\times10^{5}\,\mathrm{Hz}\ .$$

(b) Using Eq. 29-16, we obtain

$$r = \frac{m_e v}{qB} = \frac{\sqrt{2m_e K}}{qB} = \frac{\sqrt{2(9.11\times10^{-31}\,\mathrm{kg})(100\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}}{(1.60\times10^{-19}\,\mathrm{C})(35.0\times10^{-6}\,\mathrm{T})} = 0.964\,\mathrm{m}\ .$$

20. (a) Using Eq. 29-16, we obtain

$$v = \frac{rqB}{m_\alpha} = \frac{2eB}{4.00\,\mathrm{u}} = \frac{2(4.50\times10^{-2}\,\mathrm{m})(1.60\times10^{-19}\,\mathrm{C})(1.20\,\mathrm{T})}{(4.00\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u})} = 2.60\times10^{6}\ \mathrm{m/s}\ .$$

(b) $T = 2\pi r/v = 2\pi(4.50\times10^{-2}\,\mathrm{m})/(2.60\times10^{6}\,\mathrm{m/s}) = 1.09\times10^{-7}\,\mathrm{s}$.

(c) The kinetic energy of the alpha particle is

$$K = \frac{1}{2}m_\alpha v^2 = \frac{(4.00\,\mathrm{u})(1.66\times10^{-27}\,\mathrm{kg/u})(2.60\times10^{6}\,\mathrm{m/s})^2}{2(1.60\times10^{-19}\,\mathrm{J/eV})} = 1.40\times10^{5}\ \mathrm{eV}\ .$$

(d) $\Delta V = K/q = 1.40\times10^{5}\,\mathrm{eV}/2e = 7.00\times10^{4}\,\mathrm{V}$.

21. So that the magnetic field has an effect on the moving electrons, we need a non-negligible component of $\vec{B}$ to be perpendicular to $\vec{v}$ (the electron velocity). It is most efficient, therefore, to orient the magnetic field so it is perpendicular to the plane of the page. The magnetic force on an electron has magnitude $F_B = evB$, and the acceleration of the electron has magnitude $a = v^2/r$. Newton's second law yields $evB = m_e v^2/r$, so the radius of the circle is given by $r = m_e v/eB$ in agreement with Eq. 29-16. The kinetic energy of the electron is $K = \frac{1}{2}m_e v^2$, so $v = \sqrt{2K/m_e}$. Thus,

$$r = \frac{m_e}{eB}\sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2m_e K}{e^2B^2}}\ .$$

This must be less than d, so

$$\sqrt{\frac{2m_e K}{e^2B^2}} \le d$$

or

$$B \ge \sqrt{\frac{2m_e K}{e^2d^2}}\ .$$

If the electrons are to travel as shown in Fig. 29-33, the magnetic field must be out of the page. Then the magnetic force is toward the center of the circular path, as it must be (in order to make the circular motion possible).

22. Let $v_\parallel = v\cos\theta$. The electron will proceed with a uniform speed $v_\parallel$ in the direction of $\vec{B}$ while undergoing uniform circular motion with frequency f in the direction perpendicular to B: $f = eB/2\pi m_e$. The distance d is then

$$\begin{aligned} d &= v_\parallel T = \frac{v_\parallel}{f} = \frac{(v\cos\theta)2\pi m_e}{eB} \\ &= \frac{2\pi(1.5\times10^{7}\,\mathrm{m/s})(9.11\times10^{-31}\,\mathrm{kg})(\cos 10^\circ)}{(1.60\times10^{-19}\,\mathrm{C})(1.0\times10^{-3}\,\mathrm{T})} = 0.53\,\mathrm{m}\ . \end{aligned}$$

23. Referring to the solution of problem 19 part (b), we see that $r = \sqrt{2mK}/qB$ implies $K = (rqB)^2/2m \propto q^2m^{-1}$. Thus,

(a) $K_\alpha = (q_\alpha/q_p)^2(m_p/m_\alpha)K_p = (2)^2(1/4)K_p = K_p = 1.0\,\mathrm{MeV}$;

(b) $K_d = (q_d/q_p)^2(m_p/m_d)K_p = (1)^2(1/2)K_p = 1.0\,\mathrm{MeV}/2 = 0.50\,\mathrm{MeV}$.

24. Referring to the solution of problem 19 part (b), we see that $r = \sqrt{2mK}/qB$ implies the proportionality: $r \propto \sqrt{mK}/qB$. Thus,

$$\begin{aligned} r_\alpha &= \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\frac{q_p}{q_\alpha} r_p = \sqrt{\frac{4.0\,\mathrm{u}}{1.0\,\mathrm{u}}}\frac{e r_p}{2e} = r_p\,; \\ r_d &= \sqrt{\frac{m_d K_d}{m_p K_p}}\frac{q_p}{q_d} r_d = \sqrt{\frac{2.0\,\mathrm{u}}{1.0\,\mathrm{u}}}\frac{e r_d}{e} = \sqrt{2} r_p\ . \end{aligned}$$

25. (a) We solve for B from $m = B^2 q x^2/8V$ (see Sample Problem 29-3):

$$B = \sqrt{\frac{8Vm}{qx^2}}\ .$$

We evaluate this expression using $x = 2.00\,\mathrm{m}$:

$$B = \sqrt{\frac{8(100\times10^3\,\mathrm{V})(3.92\times10^{-25}\,\mathrm{kg})}{(3.20\times10^{-19}\,\mathrm{C})(2.00\,\mathrm{m})^2}} = 0.495\ \mathrm{T}\ .$$

(b) Let N be the number of ions that are separated by the machine per unit time. The current is $i = qN$ and the mass that is separated per unit time is $M = mN$, where m is the mass of a single ion. M has the value

$$M = \frac{100\times10^{-6}\,\mathrm{kg}}{3600\,\mathrm{s}} = 2.78\times10^{-8}\,\mathrm{kg/s}\ .$$

Since $N = M/m$ we have

$$i = \frac{qM}{m} = \frac{(3.20\times10^{-19}\,\mathrm{C})(2.78\times10^{-8}\ \mathrm{kg/s})}{3.92\times10^{-25}\,\mathrm{kg}} = 2.27\times10^{-2}\ \mathrm{A}\ .$$

(c) Each ion deposits energy qV in the cup, so the energy deposited in time Δt is given by

$$E = NqV\,\Delta t = \frac{iqV}{q}\,\Delta t = iV\,\Delta t\ .$$

For $\Delta t = 1.0$h,

$$E = (2.27\times10^{-2}\,\mathrm{A})(100\times10^3\,\mathrm{V})(3600\,\mathrm{s}) = 8.17\times10^6\ \mathrm{J}\ .$$

To obtain the second expression, i/q is substituted for N.

26. The equation of motion for the proton is

$$\begin{aligned} \vec{F} &= q\vec{v}\times\vec{B} = q(v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k})\times B\hat{\imath} = qB(v_z\hat{\jmath} - v_y\hat{k}) \\ &= m_p\vec{a} = m_p\left[\left(\frac{dv_x}{dt}\right)\hat{\imath} + \left(\frac{dv_y}{dt}\right)\hat{\jmath} + \left(\frac{dv_z}{dt}\right)\hat{k}\right]\ . \end{aligned}$$

Thus,

$$\begin{aligned} \frac{dv_x}{dt} &= 0 \\ \frac{dv_y}{dt} &= \omega v_z \\ \frac{dv_z}{dt} &= -\omega v_y\ , \end{aligned}$$

where $\omega = eB/m_p$. The solution is $v_x = v_{0x}$, $v_y = v_{0y}\cos\omega t$ and $v_z = -v_{0y}\sin\omega t$. In summary, we have $\vec{v}(t) = v_{0x}\hat{\imath} + v_{0y}\cos(\omega t)\hat{\jmath} - v_{0y}(\sin\omega t)\hat{k}$.

27. (a) If v is the speed of the positron then $v\sin\phi$ is the component of its velocity in the plane that is perpendicular to the magnetic field. Here ϕ is the angle between the velocity and the field (89°). Newton's second law yields $eBv\sin\phi = m_e(v\sin\phi)^2/r$, where r is the radius of the orbit. Thus $r = (m_e v/eB)\sin\phi$. The period is given by

$$T = \frac{2\pi r}{v\sin\phi} = \frac{2\pi m_e}{eB} = \frac{2\pi(9.11\times10^{-31}\,\text{kg})}{(1.60\times10^{-19}\,\text{C})(0.10\,\text{T})} = 3.6\times10^{-10}\ \text{s}\ .$$

The equation for r is substituted to obtain the second expression for T.

(b) The pitch is the distance traveled along the line of the magnetic field in a time interval of one period. Thus $p = vT\cos\phi$. We use the kinetic energy to find the speed: $K = \frac{1}{2}m_e v^2$ means

$$v = \sqrt{\frac{2K}{m_e}} = \sqrt{\frac{2(2.0\times10^3\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = 2.651\times10^7\ \text{m/s}\ .$$

Thus

$$p = (2.651\times10^7\,\text{m/s})(3.58\times10^{-10}\,\text{s})\cos 89^\circ = 1.7\times10^{-4}\ \text{m}\ .$$

(c) The orbit radius is

$$R = \frac{m_e v\sin\phi}{eB} = \frac{(9.11\times10^{-31}\,\text{kg})\left(2.651\times10^7\,\text{m/s}\right)\sin 89^\circ}{(1.60\times10^{-19}\,\text{C})\,(0.10\text{T})} = 1.5\times10^{-3}\ \text{m}\ .$$

28. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\vec{v}\times\vec{B}$ points leftward, which indeed seems to be the direction it is "pushed"; therefore, $q > 0$ (it is a proton).

(a) Eq. 29-17 becomes

$$\begin{aligned} T &= \frac{2\pi m_\text{p}}{e\,|\vec{B}|} \\ 2\left(130\times10^{-9}\right) &= \frac{2\pi\left(1.67\times10^{-27}\right)}{\left(1.60\times10^{-19}\right)|\vec{B}|} \end{aligned}$$

which yields $|\vec{B}| = 0.252$ T.

(b) Doubling the kinetic energy implies multiplying the speed by $\sqrt{2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $\sqrt{2}$). Thus, $t = T/2 = 130$ ns, again.

29. (a) $-q$, from conservation of charges.

(b) Each of the two particles will move in the same circular path, initially going in the opposite direction. After traveling half of the circular path they will collide. So the time is given by $t = T/2 = \pi m/Bq$ (where Eq. 29-17 has been used).

30. (a) Using Eq. 29-23 and Eq. 29-18, we find

$$f_\text{osc} = \frac{qB}{2\pi m_p} = \frac{(1.60\times10^{-19}\,\text{C})(1.2\,\text{T})}{2\pi(1.67\times10^{-27}\,\text{kg})} = 1.8\times10^7\ \text{Hz}\ .$$

(b) From $r = m_p v/qB = \sqrt{2m_p K}/qB$ we have

$$K = \frac{(rqB)^2}{2m_p} = \frac{[(0.50\,\text{m})(1.60\times10^{-19}\,\text{C})(1.2\,\text{T})]^2}{2(1.67\times10^{-27}\,\text{kg})(1.60\times10^{-19}\,\text{J/eV})} = 1.7\times10^7\ \text{eV}\ .$$

31. We approximate the total distance by the number of revolutions times the circumference of the orbit corresponding to the average energy. This should be a good approximation since the deuteron receives the same energy each revolution and its period does not depend on its energy. The deuteron accelerates twice in each cycle, and each time it receives an energy of $qV = 80 \times 10^3\,\text{eV}$. Since its final energy is 16.6 MeV, the number of revolutions it makes is

$$n = \frac{16.6 \times 10^6\,\text{eV}}{2(80 \times 10^3\,\text{eV})} = 104 .$$

Its average energy during the accelerating process is 8.3 MeV. The radius of the orbit is given by $r = mv/qB$, where v is the deuteron's speed. Since this is given by $v = \sqrt{2K/m}$, the radius is

$$r = \frac{m}{qB}\sqrt{\frac{2K}{m}} = \frac{1}{qB}\sqrt{2Km} .$$

For the average energy

$$r = \frac{\sqrt{2(8.3 \times 10^6\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})(3.34 \times 10^{-27}\,\text{kg})}}{(1.60 \times 10^{-19}\,\text{C})(1.57\,\text{T})} = 0.375\ \text{m} .$$

The total distance traveled is about $n2\pi r = (104)(2\pi)(0.375) = 2.4 \times 10^2\,\text{m}$.

32. (a) Since $K = \frac{1}{2}mv^2 = \frac{1}{2}m(2\pi R f_{\text{osc}})^2 \propto m$,

$$K_p = \left(\frac{m_p}{m_d}\right) K_d = \frac{1}{2}K_d = \frac{1}{2}(17\,\text{MeV}) = 8.5\ \text{MeV} .$$

(b) We require a magnetic field of strength

$$B_p = \frac{1}{2}B_d = \frac{1}{2}(1.6\,\text{T}) = 0.80\ \text{T} .$$

(c) Since $K \propto B^2/m$,

$$K_p' = \left(\frac{m_d}{m_p}\right) K_d = 2K_d = 2(17\,\text{MeV}) = 34\ \text{MeV} .$$

(d) Since $f_{\text{osc}} = Bq/(2\pi m) \propto m^{-1}$,

$$f_{\text{osc},\,d} = \left(\frac{m_d}{m_P}\right) f_{\text{osc},\,p} = 2(12 \times 10^6\,\text{s}^{-1}) = 2.4 \times 10^7\ \text{Hz} .$$

(e) Now,

$$\begin{aligned} K_\alpha &= \left(\frac{m_\alpha}{m_d}\right) K_d = 2K_d = 2(17\,\text{MeV}) = 34\,\text{MeV} , \\ B_\alpha &= \left(\frac{m_\alpha}{m_d}\right)\left(\frac{q_d}{q_\alpha}\right) B_d = 2\left(\frac{1}{2}\right)(1.6\,\text{T}) = 1.6\,\text{T} , \\ K_\alpha' &= K_\alpha = 34\,\text{MeV} \qquad (\text{Since } B_\alpha = B_d = 1.6\,\text{T}) , \end{aligned}$$

and

$$f_{\text{osc},\,\alpha} = \left(\frac{q_\alpha}{a_d}\right)\left(\frac{m_d}{m_\alpha}\right) f_{\text{osc, d}} = 2\left(\frac{2}{4}\right)\left(12 \times 10^6\,\text{s}^{-1}\right) = 1.2 \times 10^7\ \text{Hz} .$$

33. The magnitude of the magnetic force on the wire is given by $F_B = iLB\sin\phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70^\circ$. Thus,

$$F_B = (5000\,\text{A})(100\,\text{m})(60.0 \times 10^{-6}\,\text{T})\sin 70^\circ = 28.2\ \text{N} .$$

We apply the right-hand rule to the vector product $\vec{F}_B = i\vec{L} \times \vec{B}$ to show that the force is to the west.

34. The magnetic force on the (straight) wire is

$$F_B = iBL\sin\theta = (13.0\,\mathrm{A})(1.50\,\mathrm{T})(1.80\,\mathrm{m})(\sin 35.0^\circ) = 20.1\,\mathrm{N}\ .$$

35. The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Applying the right-hand rule reveals that the current must be from left to right. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \implies i = \frac{mg}{LB} = \frac{(0.0130\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(0.620\,\mathrm{m})(0.440\,\mathrm{T})} = 0.467\ \mathrm{A}\ .$$

36. The magnetic force on the wire is

$$\begin{aligned}\vec{F}_B &= i\vec{L}\times\vec{B} = iL\hat{\mathrm{i}}\times(B_y\hat{\mathrm{j}} + B_z\hat{\mathrm{k}}) = iL(-B_z\hat{\mathrm{j}} + B_y\hat{\mathrm{k}})\\ &= (0.50\,\mathrm{A})(0.50\,\mathrm{m})[-(0.010\,\mathrm{T})\hat{\mathrm{j}} + (0.0030\,\mathrm{T})\hat{\mathrm{k}}]\\ &= (-2.5\times10^{-3}\hat{\mathrm{j}} + 0.75\times10^{-3}\hat{\mathrm{k}})\,\mathrm{N}\ .\end{aligned}$$

37. The magnetic force must push horizontally on the rod to overcome the force of friction, but it can be oriented so that it also pulls up on the rod and thereby reduces both the normal force and the force of friction. The forces acting on the rod are: $\vec{F}$, the force of the magnetic field; mg, the magnitude of the (downward) force of gravity; $\vec{N}$, the normal force exerted by the stationary rails upward on the rod; and $\vec{f}$, the (horizontal) force of friction. For definiteness, we assume the rod is on the verge of moving eastward, which means that $\vec{f}$ points westward (and is equal to its maximum possible value $\mu_s N$). Thus, $\vec{F}$ has an eastward component F_x and an upward component F_y, which can be related to the components of the magnetic field once we assume a direction for the current in the rod. Thus, again for definiteness, we assume the current flows northward. Then, by the righthand rule, a downward component (B_d) of $\vec{B}$ will produce the eastward F_x, and a westward component (B_w) will produce the upward F_y. Specifically,

$$F_x = iLB_d \qquad \text{and} \qquad F_y = iLB_w\ .$$

Considering forces along a vertical axis, we find

$$N = mg - F_y = mg - iLB_w$$

so that

$$f = f_{s,\max} = \mu_s\,(mg - iLB_w)\ .$$

It is on the verge of motion, so we set the horizontal acceleration to zero:

$$F_x - f = 0 \implies iLB_d = \mu_s\,(mg - iLB_w)\ .$$

The angle of the field components is adjustable, and we can minimize with respect to it. Defining the angle by $B_w = B\sin\theta$ and $B_d = B\cos\theta$ (which means θ is being measured from a vertical axis) and writing the above expression in these terms, we obtain

$$iLB\cos\theta = \mu_s\,(mg - iLB\sin\theta) \implies B = \frac{\mu_s mg}{iL(\cos\theta + \mu_s\sin\theta)}$$

which we differentiate (with respect to θ) and set the result equal to zero. This provides a determination of the angle:

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31^\circ\ .$$

Consequently,

$$B_{\min} = \frac{0.60(1.0\,\mathrm{kg})(9.8\,\mathrm{m/s^2})}{(50\,\mathrm{A})(1.0\,\mathrm{m})(\cos 31^\circ + 0.60\sin 31^\circ)} = 0.10\ \mathrm{T}\ .$$

38. (a) From $F_B = iLB$ we get

$$i = \frac{F_B}{LB} = \frac{10 \times 10^3\,\mathrm{N}}{(3.0\,\mathrm{m})(10 \times 10^{-6}\,\mathrm{T})} = 3.3 \times 10^8 \text{ A} .$$

(b) $P = i^2R = (3.3 \times 10^8\,\mathrm{A})^2(1.0\,\Omega) = 1.0 \times 10^{17}\,\mathrm{W}$.

(c) It is totally unrealistic because of the huge current and the accompanying high power loss.

39. The applied field has two components: $B_x > 0$ and $B_z > 0$. Considering each straight-segment of the rectangular coil, we note that Eq. 29-26 produces a non-zero force only for the component of $\vec{B}$ which is perpendicular to that segment; we also note that the equation is effectively multiplied by $N = 20$ due to the fact that this is a 20-turn coil. Since we wish to compute the torque about the hinge line, we can ignore the force acting on the straight-segment of the coil which lies along the y axis (forces acting at the axis of rotation produce no torque about that axis). The top and bottom straight-segments experience forces due to Eq. 29-26 (caused by the B_z component), but these forces are (by the right-hand rule) in the $\pm y$ directions and are thus unable to produce a torque about the y axis. Consequently, the torque derives completely from the force exerted on the straight-segment located at $x = 0.050$ m, which has length $L = 0.10$ m and is shown in Figure 29-36 carrying current in the $-y$ direction. Now, the B_z component will produce a force on this straight-segment which points in the $-x$ direction (back towards the hinge) and thus will exert no torque about the hinge. However, the B_x component (which is equal to $B\cos\theta$ where $B = 0.50$ T and $\theta = 30°$) produces a force equal to $NiLB_x$ which points (by the right-hand rule) in the $+z$ direction. Since the action of this force is perpendicular to the plane of the coil, and is located a distance x away from the hinge, then the torque has magnitude

$$\tau = (NiLB_x)\,(x) = NiLxB\cos\theta = (20)(0.10)(0.10)(0.050)(0.50)\cos 30° = 0.0043$$

in SI units (N·m). Since $\vec{\tau} = \vec{r} \times \vec{F}$, the direction of the torque is $-y$. An alternative way to do this problem is through the use of Eq. 29-37. We do not show those details here, but note that the magnetic moment vector (a necessary part of Eq. 29-37) has magnitude

$$|\vec{\mu}| = NiA = (20)(0.10\,\mathrm{A})(0.0050\,\mathrm{m}^2)$$

and points in the $-z$ direction. At this point, Eq. 3-30 may be used to obtain the result for the torque vector.

40. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the $+y$ axis, while the $\ell_x = 120$ cm side runs along the $+x$ axis. The angle made by the hypotenuse (of length 130 cm) is $\theta = \tan^{-1}(50/120) = 22.6°$, relative to the 120 cm side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is $180° - 22.6° = +157°$. Since we are only asked to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis. We take $\vec{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\vec{B}| = 0.0750$ T,

$$B_x = -B\cos\theta = -0.0692 \text{ T} \qquad \text{and} \qquad B_y = B\sin\theta = 0.0288 \text{ T} .$$

(a) Eq. 29-26 produces zero force when $\vec{L} \parallel \vec{B}$ so there is no force exerted on the hypotenuse. On the 50 cm side, the B_x component produces a force $i\ell_yB_x\hat{\mathrm{k}}$, and there is no contribution from the B_y component. Using SI units, the magnitude of the force on the ℓ_y side is therefore

$$(4.00\,\mathrm{A})(0.500\,\mathrm{m})(0.0692\,\mathrm{T}) = 0.138 \text{ N} .$$

On the 120 cm side, the B_y component produces a force $i\ell_xB_y\hat{\mathrm{k}}$, and there is no contribution from the B_x component. Using SI units, the magnitude of the force on the ℓ_x side is also

$$(4.00\,\mathrm{A})(1.20\,\mathrm{m})(0.0288\,\mathrm{T}) = 0.138 \text{ N} .$$

(b) The net force is

$$i\ell_y B_x \hat{\mathrm{k}} + i\ell_x B_y \hat{\mathrm{k}} = 0 \ ,$$

keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\vec{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$ but $B_y < 0$ and a zero net force would still be the result.

41. If N closed loops are formed from the wire of length L, the circumference of each loop is L/N, the radius of each loop is $R = L/2\pi N$, and the area of each loop is $A = \pi R^2 = \pi(L/2\pi N)^2 = L^2/4\pi N^2$. For maximum torque, we orient the plane of the loops parallel to the magnetic field, so the dipole moment is perpendicular to the field. The magnitude of the torque is then

$$\tau = NiAB = (Ni)\left(\frac{L^2}{4\pi N^2}\right) B = \frac{iL^2 B}{4\pi N} \ .$$

To maximize the torque, we take N to have the smallest possible value, 1. Then $\tau = iL^2B/4\pi$.

42. We replace the current loop of arbitrary shape with an assembly of small adjacent rectangular loops filling the same area which was enclosed by the original loop (as nearly as possible). Each rectangular loop carries a current i flowing in the same sense as the original loop. As the sizes of these rectangles shrink to infinitesimally small values, the assembly gives a current distribution equivalent to that of the original loop. The magnitude of the torque $\Delta\vec{\tau}$ exerted by $\vec{B}$ on the nth rectangular loop of area ΔA_n is given by $\Delta\tau_n = NiB\sin\theta\Delta A_n$. Thus, for the whole assembly

$$\tau = \sum_n \Delta\tau_n = NiB \sum_n \Delta A_n = NiAB\sin\theta \ .$$

43. Consider an infinitesimal segment of the loop, of length ds. The magnetic field is perpendicular to the segment, so the magnetic force on it is has magnitude $dF = iB\,ds$. The horizontal component of the force has magnitude $dF_h = (iB\cos\theta)\,ds$ and points inward toward the center of the loop. The vertical component has magnitude $dF_v = (iB\sin\theta)\,ds$ and points upward. Now, we sum the forces on all the segments of the loop. The horizontal component of the total force vanishes, since each segment of wire can be paired with another, diametrically opposite, segment. The horizontal components of these forces are both toward the center of the loop and thus in opposite directions. The vertical component of the total force is

$$F_v = iB\sin\theta \int ds = (iB\sin\theta)2\pi a \ .$$

We note the i, B, and θ have the same value for every segment and so can be factored from the integral.

44. The total magnetic force on the loop L is

$$\vec{F}_B = i\oint_L (d\vec{L} \times \vec{B}) = i(\oint_L d\vec{L}) \times \vec{B} = 0 \ .$$

We note that $\oint_L d\vec{L} = 0$. If $\vec{B}$ is not a constant, however, then the equality

$$\oint_L (d\vec{L} \times \vec{B}) = (\oint_L d\vec{L}) \times \vec{B}$$

is not necessarily valid, so $\vec{F}_B$ is not always zero.

45. (a) The current in the galvanometer should be 1.62 mA when the potential difference across the resistor-galvanometer combination is 1.00 V. The potential difference across the galvanometer alone is $iR_g = (1.62 \times 10^{-3}\,\mathrm{A})(75.3\,\Omega) = 0.122\,\mathrm{V}$, so the resistor must be in series with the galvanometer and the potential difference across it must be $1.00\,\mathrm{V} - 0.122\,\mathrm{V} = 0.878\mathrm{V}$. The resistance should be $R = (0.878\,\mathrm{V})/(1.62 \times 10^{-3}\,\mathrm{A}) = 542\,\Omega$.

(b) The current in the galvanometer should be 1.62 mA when the total current in the resistor and galvanometer combination is 50.0 mA. The resistor should be in parallel with the galvanometer, and the current through it should be $50.0\,\text{mA} - 1.62\,\text{mA} = 48.38\,\text{mA}$. The potential difference across the resistor is the same as that across the galvanometer, 0.122 V, so the resistance should be $R = (0.122\,\text{V})/(48.38 \times 10^{-3}\,\text{A}) = 2.52\,\Omega$.

46. We use $\tau_{\text{max}} = |\vec{\mu} \times \vec{B}|_{\text{max}} = \mu B = i\pi a^2 B$, and note that $i = qf = qv/2\pi a$. So

$$\tau_{\text{max}} = \left(\frac{qv}{2\pi a}\right)\pi a^2 B = \frac{1}{2}qvaB\ .$$

47. We use Eq. 29-37 where $\vec{\mu}$ is the magnetic dipole moment of the wire loop and $\vec{B}$ is the magnetic field, as well as Newton's second law. Since the plane of the loop is parallel to the incline the dipole moment is normal to the incline. The forces acting on the cylinder are the force of gravity mg, acting downward from the center of mass, the normal force of the incline N, acting perpendicularly to the incline through the center of mass, and the force of friction f, acting up the incline at the point of contact. We take the x axis to be positive down the incline. Then the x component of Newton's second law for the center of mass yields

$$mg\sin\theta - f = ma\ .$$

For purposes of calculating the torque, we take the axis of the cylinder to be the axis of rotation. The magnetic field produces a torque with magnitude $\mu B\sin\theta$, and the force of friction produces a torque with magnitude fr, where r is the radius of the cylinder. The first tends to produce an angular acceleration in the counterclockwise direction, and the second tends to produce an angular acceleration in the clockwise direction. Newton's second law for rotation about the center of the cylinder, $\tau = I\alpha$, gives

$$fr - \mu B\sin\theta = I\alpha\ .$$

Since we want the current that holds the cylinder in place, we set $a = 0$ and $\alpha = 0$, and use one equation to eliminate f from the other. The result is $mgr = \mu B$. The loop is rectangular with two sides of length L and two of length $2r$, so its area is $A = 2rL$ and the dipole moment is $\mu = NiA = 2NirL$. Thus, $mgr = 2NirLB$ and

$$i = \frac{mg}{2NLB} = \frac{(0.250\,\text{kg})(9.8\,\text{m/s}^2)}{2(10.0)(0.100\,\text{m})(0.500\,\text{T})} = 2.45\ \text{A}\ .$$

48. From $\mu = NiA = i\pi r^2$ we get

$$i = \frac{\mu}{\pi r^2} = \frac{8.00 \times 10^{22}\,\text{J/T}}{\pi(3500 \times 10^3\,\text{m})^2} = 2.08 \times 10^9\ \text{A}\ .$$

49. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn. Thus

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\,\text{A}\cdot\text{m}^2}{(160)(\pi)(0.0190\,\text{m})^2} = 12.7\ \text{A}\ .$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$$\tau_{\text{max}} = \mu B = \left(2.30\,\text{A}\cdot\text{m}^2\right)\left(35.0 \times 10^{-3}\,\text{T}\right) = 8.05 \times 10^{-2}\ \text{N}\cdot\text{m}\ .$$

50. (a) $\mu = NAi = \pi r^2 i = \pi(0.150\,\text{m})^2(2.60\,\text{A}) = 0.184\,\text{A}\cdot\text{m}^2$.

(b) The torque is

$$\tau = |\vec{\mu} \times \vec{B}| = \mu B\sin\theta = \left(0.184\,\text{A}\cdot\text{m}^2\right)(12.0\,\text{T})\sin 41.0^\circ = 1.45\ \text{N}\cdot\text{m}\ .$$

51. (a) The area of the loop is $A = \frac{1}{2}(30\,\text{cm})(40\,\text{cm}) = 6.0 \times 10^2\,\text{cm}^2$, so

$$\mu = iA = (5.0\,\text{A})\left(6.0 \times 10^{-2}\,\text{m}^2\right) = 0.30\ \text{A}\cdot\text{m}^2\ .$$

(b) The torque on the loop is

$$\tau = \mu B \sin\theta = \left(0.30\,\text{A}\cdot\text{m}^2\right)\left(80 \times 10^3\,\text{T}\right)\sin 90^\circ = 2.4 \times 10^{-2}\ \text{N}\cdot\text{m}\ .$$

52. (a) We use $\vec{\tau} = \vec{\mu} \times \vec{B}$, where $\vec{\mu}$ points into the wall (since the current goes clockwise around the clock). Since $\vec{B}$ points towards the one-hour (or "5-minute") mark, and (by the properties of vector cross products) $\vec{\tau}$ must be perpendicular to it, then (using the right-hand rule) we find $\vec{\tau}$ points at the 20-minute mark. So the time interval is 20 min.

(b) The torque is given by

$$\begin{aligned}\tau &= \left|\vec{\mu} \times \vec{B}\right| = \mu B \sin 90^\circ \\ &= NiAB = \pi N i r^2 B \\ &= 6\pi(2.0\,\text{A})(0.15\,\text{m})^2(70 \times 10^{-3}\,\text{T}) \\ &= 5.9 \times 10^{-2}\ \text{N}\cdot\text{m}\ .\end{aligned}$$

53. (a) The magnitude of the magnetic moment vector is

$$\mu = \sum_n i_n A_n = \pi r_1^2 i_1 + \pi r_2^2 i_2 = \pi(7.00\,\text{A})[(0.300\,\text{m})^2 + (0.200\,\text{m})^2] = 2.86\ \text{A}\cdot\text{m}^2\ .$$

(b) Now,

$$\mu = \pi r_1^2 i_1 - \pi r_2^2 i_2 = \pi(7.00\,\text{A})[(0.300\,\text{m})^2 - (0.200\,\text{m})^2] = 1.10\ \text{A}\cdot\text{m}^2\ .$$

54. Let $a = 30.0\,\text{cm}$, $b = 20.0\,\text{cm}$, and $c = 10.0\,\text{cm}$. From the given hint, we write

$$\begin{aligned}\vec{\mu} &= \vec{\mu}_1 + \vec{\mu}_2 = iab(-\hat{\text{k}}) + iac(\hat{\text{j}}) \\ &= ia(c\hat{\text{j}} - b\hat{\text{k}}) \\ &= (5.00\,\text{A})(0.300\,\text{m})[(0.100\,\text{m})\hat{\text{j}} - (0.200\,\text{m})\hat{\text{k}}] \\ &= (0.150\hat{\text{j}} - 0.300\hat{\text{k}})\ \text{A}\cdot\text{m}^2.\end{aligned}$$

Thus, using the Pythagorean theorem,

$$\mu = \sqrt{(0.150)^2 + (0.300)^2} = 0.335\ \text{A}\cdot\text{m}^2\ ,$$

and $\vec{\mu}$ is in the yz plane at angle θ to the $+y$ direction, where

$$\theta = \tan^{-1}\left(\frac{\mu_y}{\mu_x}\right) = \tan^{-1}\left(\frac{-0.300}{0.150}\right) = -63.4^\circ\ .$$

55. The magnetic dipole moment is $\vec{\mu} = \mu(0.60\,\hat{\text{i}} - 0.80\,\hat{\text{j}})$, where $\mu = NiA = Ni\pi r^2 = 1(0.20\text{A})\pi(0.080\text{m})^2 = 4.02 \times 10^{-4}\,\text{A}\cdot\text{m}^2$. Here i is the current in the loop, N is the number of turns, A is the area of the loop, and r is its radius.

(a) The torque is

$$\begin{aligned}\vec{\tau} &= \vec{\mu} \times \vec{B} = \mu(0.60\,\hat{\text{i}} - 0.80\,\hat{\text{j}}) \times (0.25\,\hat{\text{i}} + 0.30\,\hat{\text{k}}) \\ &= \mu\left[(0.60)(0.30)(\,\hat{\text{i}} \times \hat{\text{k}}) - (0.80)(0.25)(\,\hat{\text{j}} \times \hat{\text{i}}) - (0.80)(0.30)(\,\hat{\text{j}} \times \hat{\text{k}})\right] \\ &= \mu[-0.18\,\hat{\text{j}} + 0.20\,\hat{\text{k}} - 0.24\,\hat{\text{i}}]\ .\end{aligned}$$

Here $\hat{\imath} \times \hat{k} = -\hat{\jmath}$, $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, and $\hat{\jmath} \times \hat{k} = \hat{\imath}$ are used. We also use $\hat{\imath} \times \hat{\imath} = 0$. Now, we substitute the value for μ to obtain

$$\vec{\tau} = \left(-0.97 \times 10^{-4}\,\hat{\imath} - 7.2 \times 10^{-4}\,\hat{\jmath} + 8.0 \times 10^{-4}\,\hat{k}\right) \text{N}\cdot\text{m} .$$

(b) The potential energy of the dipole is given by

$$\begin{aligned} U &= -\vec{\mu}\cdot\vec{B} = -\mu(0.60\,\hat{\imath} - 0.80\,\hat{\jmath})\cdot(0.25\,\hat{\imath} + 0.30\,\hat{k}) \\ &= -\mu(0.60)(0.25) = -0.15\mu = -6.0 \times 10^{-4}\ \text{J} . \end{aligned}$$

Here $\hat{\imath}\cdot\hat{\imath} = 1$, $\hat{\imath}\cdot\hat{k} = 0$, $\hat{\jmath}\cdot\hat{\imath} = 0$, and $\hat{\jmath}\cdot\hat{k} = 0$ are used.

56. The unit vector associated with the current element (of magnitude $d\ell$) is $-\hat{\jmath}$. The (infinitesimal) force on this element is

$$d\vec{F} = i\,d\ell(-\hat{\jmath}) \times (0.3y\hat{\imath} + 0.4y\hat{\jmath})$$

with SI units (and 3 significant figures) understood.

(a) Since $\hat{\jmath} \times \hat{\imath} = -\hat{k}$ and $\hat{\jmath} \times \hat{\jmath} = 0$, we obtain

$$d\vec{F} = 0.3iy\,d\ell\,\hat{k} = \left(6.00 \times 10^{-4}\,\text{N/m}^2\right) y\,d\ell\,\hat{k} .$$

(b) We integrate the force element found in part (a), using the symbol ξ to stand for the coefficient $6.00 \times 10^{-4}\,\text{N/m}^2$, and obtain

$$\vec{F} = \int d\vec{F} = \xi\hat{k} \int_0^{0.25} y\,dy = \xi\hat{k}\left(\frac{0.25^2}{2}\right) = 1.88 \times 10^{-5}\,\text{N}\,\hat{k} .$$

57. Since the velocity is constant, the net force on the proton vanishes. Using Eq. 29-2 and Eq. 23-28, we obtain the requirement (Eq. 29-7) for the proton's speed in terms of the crossed fields:

$$v = \frac{E}{B} \quad \Longrightarrow \quad E = (50\,\text{m/s})(0.0020\,\text{T}) = 0.10\ \text{V/m} .$$

By the right-hand rule, the magnetic force points in the $\hat{k}$ direction. To cancel this, the electric force must be in the $-\hat{k}$ direction. Since $q > 0$ for the proton, we conclude $\vec{E} = -0.10\,\text{V/m}\,\hat{k}$.

58. (a) The kinetic energy gained is due to the potential energy decrease as the dipole swings from a position specified by angle θ to that of being aligned (zero angle) with the field. Thus,

$$K = U_i - U_f = -\mu B\cos\theta - (-\mu B\cos 0^\circ) .$$

Therefore, using SI units, the angle is

$$\theta = \cos^{-1}\left(1 - \frac{K}{\mu B}\right) = \cos^{-1}\left(1 - \frac{0.00080}{(0.020)(0.052)}\right) = 77^\circ .$$

(b) Since we are making the assumption that no energy is dissipated in this process, then the dipole will continue its rotation (similar to a pendulum) until it reaches an angle $\theta = 77^\circ$ on the other side of the alignment axis.

59. Using Eq. 29-2 and Eq. 3-30, we obtain

$$\vec{F} = q\,(v_x B_y - v_y B_x)\,\hat{k} = q\,(v_x(3B_x) - v_y B_x)\,\hat{k}$$

where we use the fact that $B_y = 3B_x$. Since the force (at the instant considered) is $F_z\,\hat{k}$ where $F_z = 6.4 \times 10^{-19}\,\text{N}$, then we are led to the condition

$$q\,(3v_x - v_y)\,B_x = F_z \quad \Longrightarrow \quad B_x = \frac{F_z}{q\,(3v_x - v_y)} .$$

Substituting $V_x = 2.0\,\text{m/s}$, $v_y = 4.0\,\text{m/s}$ and $q = -1.6 \times 10^{-19}\,\text{C}$, we obtain $B_x = -2.0$ T.

60. The current is in the $+\hat{\imath}$ direction. Thus, the $\hat{\imath}$ component of $\vec{B}$ has no effect, and (with x in meters) we evaluate

$$\begin{aligned}\vec{F} &= (3.00\ \text{A}) \int_0^1 \left(-0.600\ \text{T/m}^2\right) x^2\, dx \left(\hat{\imath} \times \hat{\jmath}\right) \\ &= -1.80\hat{\text{k}} \left(\frac{1^3}{3}\right)\ \text{A·T·m} \\ &= -0.600\,\text{N}\ \hat{\text{k}}\ .\end{aligned}$$

61. (a) We seek the electrostatic field established by the separation of charges (brought on by the magnetic force). We use the ideas discussed in §29-4; especially, see SAMPLE PROBLEM 29-2. With Eq. 29-10, we define the magnitude of the electric field as $|\vec{E}| = v|\vec{B}| = (20)(0.03) = 0.6$ V/m. Its direction may be inferred from Figure 29-8; its direction is opposite to that defined by $\vec{v} \times \vec{B}$. In summary,

$$\vec{E} = -0.600\,\text{V/m}\ \hat{\text{k}}$$

which insures that $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ vanishes.

(b) Eq. 29-9 yields $V = (0.6\ \text{V/m})(2\ \text{m}) = 1.20$ V.

62. With the $\vec{B}$ pointing "out of the page," we evaluate the force (using the right-hand rule) at, say, the dot shown on the left edge of the particle's path, where its velocity is down. If the particle were positively charged, then the force at the dot would be toward the left, which is at odds with the figure (showing it being bent towards the right). Therefore, the particle is negatively charged; it is an electron.

(a) Using Eq. 29-3 (with angle ϕ equal to 90°), we obtain

$$v = \frac{|\vec{F}|}{e\,|\vec{B}|} = 4.99 \times 10^6\ \text{m/s}\ .$$

(b) Using either Eq. 29-14 or Eq. 29-16, we find $r = 0.00710$ m.

(c) Using Eq. 29-17 (in either its first or last form) readily yields $T = 8.93 \times 10^{-9}$ s.

63. (a) We are given $\vec{B} = B_x\hat{\imath} = 6 \times 10^{-5}\,\hat{\imath}$ T, so that $\vec{v} \times \vec{B} = -v_y B_x \hat{\text{k}}$ where $v_y = 4 \times 10^4$ m/s. We note that the magnetic force on the electron is $(-e)(-v_y B_x \hat{\text{k}})$ and therefore points in the $+\hat{\text{k}}$ direction, at the instant the electron enters the field-filled region. In these terms, Eq. 29-16 becomes

$$r = \frac{m_e\, v_y}{e\, B_x} = 0.0038\ \text{m}\ .$$

(b) One revolution takes $T = 2\pi r/v_y = 0.60\ \mu$s, and during that time the "drift" of the electron in the x direction (which is the *pitch* of the helix) is $\Delta x = v_x T = 0.019$ m where $v_x = 32 \times 10^3$ m/s.

(c) Returning to our observation of force direction made in part (a), we consider how this is perceived by an observer at some point on the $-x$ axis. As the electron moves away from him, he sees it enter the region with positive v_y (which he might call "upward") but "pushed" in the $+z$ direction (to his right). Hence, he describes the electron's spiral as clockwise.

64. The force associated with the magnetic field must point in the $\hat{\jmath}$ direction in order to cancel the force of gravity in the $-\hat{\jmath}$ direction. By the right-hand rule, $\vec{B}$ points in the $-\hat{\text{k}}$ direction (since $\hat{\imath} \times (-\hat{\text{k}}) = \hat{\jmath}$). Note that the charge is positive; also note that we need to assume $B_y = 0$. The magnitude $|B_z|$ is given by Eq. 29-3 (with $\phi = 90°$). Therefore, with $m = 10 \times 10^{-3}$ kg, $v = 2.0 \times 10^4$ m/s and $q = 80 \times 10^{-6}$ C, we find

$$\vec{B} = B_z\hat{\text{k}} = -\left(\frac{mg}{qv}\right)\hat{\text{k}} = -0.061\hat{\text{k}}$$

in SI units (Tesla).

65. By the right-hand rule, we see that $\vec{v} \times \vec{B}$ points along $-\hat{k}$. From Eq. 29-2 ($\vec{F} = q\vec{v} \times \vec{B}$), we find that for the force to point along $+\hat{k}$, we must have $q < 0$. Now, examining the magnitudes (in SI units) in Eq. 29-3, we find

$$\begin{aligned} |\vec{F}| &= |q|\, v\, |\vec{B}| \sin\phi \\ 0.48 &= |q|(4000)(0.0050)\sin 35^\circ \end{aligned}$$

which yields $|q| = 0.040$ C. In summary, then, $q = -40\,\mathrm{mC}$.

66. (a) Since $K = qV$ we have $K_p = K_d = \frac{1}{2}K_\alpha$ (as $q_\alpha = 2K_d = 2K_p$).

(b) and (c) Since $r = \sqrt{2mK}/qB \propto \sqrt{mK}/q$, we have

$$\begin{aligned} r_d &= \sqrt{\frac{m_d K_d}{m_p K_p}}\,\frac{q_p r_p}{q_d} = \sqrt{\frac{(2.00\,\mathrm{u})K_p}{(1.00\,\mathrm{u})K_p}}\, r_p = 10\sqrt{2}\,\mathrm{cm} = 14 \text{ cm} , \\ r_\alpha &= \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}}\,\frac{q_p r_p}{q_\alpha} = \sqrt{\frac{(4.00\,\mathrm{u})K_\alpha}{(1.00\,\mathrm{u})(K_\alpha/2)}}\,\frac{e r_p}{2e} = 10\sqrt{2}\,\mathrm{cm} = 14 \text{ cm} . \end{aligned}$$

67. (a) The radius of the cyclotron dees should be

$$r = \frac{m_p v}{qB} = \frac{(1.67\times10^{-27}\,\mathrm{kg})(3.00\times10^{8}\,\mathrm{m/s})/10}{(1.60\times10^{-19}\,\mathrm{C})(1.4\,\mathrm{T})} = 0.22 \text{ m} .$$

(b) The frequency should be

$$f_{\mathrm{osc}} = \frac{v}{2\pi r} = \frac{3.00\times10^{7}\,\mathrm{m/s}}{2\pi(0.22\,\mathrm{m})} = 2.1\times10^{7} \text{ Hz} .$$

68. The magnetic force on the wire is $F_B = idB$, pointing to the left. Thus $v = at = (F_B/m)t = idBt/m$, to the left (away from the generator).

69. (a) We use Eq. 29-10: $v_d = E/B = (10\times10^{-6}\,\mathrm{V}/1.0\times10^{-2}\,\mathrm{m})/(1.5\,\mathrm{T}) = 6.7\times10^{-4}\,\mathrm{m/s}$.

(b) We rewrite Eq. 29-12 in terms of the electric field:

$$n = \frac{Bi}{V\ell e} = \frac{Bi}{(Ed)\ell e} = \frac{Bi}{EAe}$$

which we use $A = \ell d$. In this experiment, $A = (0.010\,\mathrm{m})(10\times10^{-6}\,\mathrm{m}) = 1.0\times10^{-7}\,\mathrm{m}^2$. By Eq. 29-10, v_d equals the ratio of the fields (as noted in part (a)), so we are led to

$$\begin{aligned} n &= \frac{Bi}{EAe} = \frac{i}{v_d A e} \\ &= \frac{3.0\,\mathrm{A}}{(6.7\times10^{-4}\,\mathrm{m/s})(1.0\times10^{-7}\,\mathrm{m}^2)(1.6\times10^{-19}\,\mathrm{C})} \\ &= 2.8\times10^{29}/\mathrm{m}^3 . \end{aligned}$$

(c) Since a drawing of an inherently 3-D situation can be misleading, we describe it in terms of horizontal *north, south, east, west* and vertical *up* and *down* directions. We assume $\vec{B}$ points up and the conductor's width of 0.010 m is along an east-west line. We take the current going northward. The conduction electrons experience a westward magnetic force (by the right-hand rule), which results in the west side of the conductor being negative and the east side being positive (with reference to the Hall voltage which becomes established).

70. The fact that the fields are uniform, with the feature that the charge moves in a straight line, implies the speed is constant (if it were not, then the magnetic *force* would vary while the electric force could not – causing it to deviate from straight-line motion). This is then the situation leading to Eq. 29-7, and we find

$$|\vec{E}| = v|\vec{B}| = 500 \text{ V/m} .$$

Its direction (so that $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ vanishes) is downward (in "page" coordinates).

71. (a) We use Eq. 29-2 and Eq. 3-30:

$$\begin{aligned} \vec{F} &= q\vec{v} \times \vec{B} \\ &= (+e)\left((v_yB_z - v_zB_y)\,\hat{\imath} + (v_zB_x - v_xB_z)\,\hat{\jmath} + (v_xB_y - v_yB_x)\,\hat{\mathrm{k}}\right) \\ &= \left(1.60 \times 10^{-19}\right)\Big(((4)(0.008) - (-6)(-0.004))\,\hat{\imath} + \\ &\qquad ((-6)(0.002) - (-2)(0.008))\,\hat{\jmath} + ((-2)(-0.004) - (4)(0.002))\,\hat{\mathrm{k}}\Big) \\ &= \left(1.28 \times 10^{-21}\right)\hat{\imath} + \left(6.41 \times 10^{-22}\right)\hat{\jmath} \end{aligned}$$

with SI units understood.

(b) By definition of the cross product, $\vec{v} \perp \vec{F}$. This is easily verified by taking the dot (scalar) product of $\vec{v}$ with the result of part (a), yielding zero, provided care is taken not to introduce any round-off error.

(c) There are several ways to proceed. It may be worthwhile to note, first, that if B_z were 6.00 mT instead of 8.00 mT then the two vectors would be exactly antiparallel. Hence, the angle θ between $\vec{B}$ and $\vec{v}$ is presumably "close" to 180°. Here, we use Eq. 3-20:

$$\theta = \cos^{-1}\frac{\vec{v}\cdot\vec{B}}{|\vec{v}|\,|\vec{B}|} = \cos^{-1}\frac{-68}{\sqrt{56}\,\sqrt{84}} = 173^\circ .$$

72. (a) From symmetry, we conclude that any x-component of force will vanish (evaluated over the entirety of the bent wire as shown). By the right-hand rule, a field in the $\hat{\mathrm{k}}$ direction produces on each part of the bent wire a y-component of force pointing in the $-\hat{\jmath}$ direction; each of these components has magnitude

$$|F_y| = i\,\ell\,|\vec{B}|\sin 30^\circ = 8 \text{ N} .$$

Therefore, the the force (in Newtons) on the wire shown in the figure is $-16\,\hat{\jmath}$.

(b) The force exerted on the left half of the bent wire points in the $-\hat{\mathrm{k}}$ direction, by the right-hand rule, and the force exerted on the right half of the wire points in the $+\hat{\mathrm{k}}$ direction. It is clear that the magnitude of each force is equal, so that the force (evaluated over the entirety of the bent wire as shown) must necessarily vanish.

73. The contribution to the force by the magnetic field ($\vec{B} = B_x\hat{\imath} = -0.020\hat{\imath}$ T) is given by Eq. 29-2:

$$\begin{aligned} \vec{F}_B &= q\vec{v} \times \vec{B} \\ &= q\left((17000\hat{\imath} \times B_x\hat{\imath}) + \left(-11000\hat{\jmath} \times B_x\hat{\imath}\right) + \left(7000\hat{\mathrm{k}} \times B_x\hat{\imath}\right)\right) \\ &= q\left(-220\hat{\mathrm{k}} - 140\hat{\jmath}\right) \end{aligned}$$

in SI units. And the contribution to the force by the electric field ($\vec{E} = E_y\hat{\jmath} = 300\hat{\jmath}$ V/m) is given by Eq. 23-1: $\vec{F}_E = qE_y\hat{\jmath}$. Using $q = 5.0 \times 10^{-6}$ C, the net force (with the unit newton understood) on the particle is

$$\vec{F} = 0.0008\hat{\jmath} - 0.0011\hat{\mathrm{k}} .$$

74. Letting $B_x = B_y = B_1$ and $B_z = B_2$ and using Eq. 29-2 and Eq. 3-30, we obtain (with SI units understood)

$$\begin{aligned}\vec{F} &= q\vec{v}\times\vec{B}\\ 4\hat{\imath}-20\hat{\jmath}+12\hat{k} &= 2\left((4B_2-6B_1)\,\hat{\imath}+(6B_1-2B_2)\,\hat{\jmath}+(2B_1-4B_1)\,\hat{k}\right)\ .\end{aligned}$$

Equating like components, we find $B_1 = -3$ and $B_2 = -4$. In summary (with the unit Tesla understood), $\vec{B} = -3.0\hat{\imath} - 3.0\hat{\jmath} - 4.0\hat{k}$.

75. (a) We use Eq. 29-16 to calculate r:

$$r = \frac{m_e v}{qB} = \frac{(9.11\times10^{-31}\,\text{kg})(0.10)(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{C})(0.50\,\text{T})} = 3.4\times10^{-4}\ \text{m}\ .$$

(b) The kinetic energy, computed using the formula from Chapter 7, is

$$K = \frac{1}{2}m_e v^2 = \frac{(9.11\times10^{-31}\,\text{kg})(3.0\times10^{7}\,\text{m/s})^2}{2(1.6\times10^{-19}\,\text{J/eV})} = 2.6\times10^{3}\ \text{eV}\ .$$

76. (a) From $m = B^2qx^2/8V$ we have $\Delta m = (B^2q/8V)(2x\Delta x)$. Here $x = \sqrt{8Vm/B^2q}$, which we substitute into the expression for Δm to obtain

$$\Delta m = \left(\frac{B^2q}{8V}\right)2\sqrt{\frac{8mV}{B^2q}}\Delta x = B\sqrt{\frac{mq}{2V}}\,\Delta x\ .$$

(b) The distance between the spots made on the photographic plate is

$$\begin{aligned}\Delta x &= \frac{\Delta m}{B}\sqrt{\frac{2V}{mq}}\\ &= \frac{(37\,\text{u}-35\,\text{u})(1.66\times10^{-27}\,\text{kg/u})}{0.50\,\text{T}}\sqrt{\frac{2(7.3\times10^{3}\,\text{V})}{(36\,\text{u})(1.66\times10^{-27}\,\text{kg/u})(1.60\times10^{-19}\,\text{C})}}\\ &= 8.2\times10^{-3}\ \text{m}\ .\end{aligned}$$

77. (a) Since $\vec{B}$ is uniform,

$$\vec{F}_B = \int_{\text{wire}} i d\vec{L}\times\vec{B} = i\left(\int_{\text{wire}} d\vec{L}\right)\times\vec{B} = i\vec{L}_{ab}\times\vec{B}\ ,$$

where we note that $\int_{\text{wire}} d\vec{L} = \vec{L}_{ab}$, with $\vec{L}_{ab}$ being the displacement vector from a to b.

(b) Now $\vec{L}_{ab} = 0$, so $\vec{F}_B = i\vec{L}_{ab}\times\vec{B} = 0$.

78. We use $d\vec{F}_B = id\vec{L}\times\vec{B}$, where $d\vec{L} = dx\hat{\imath}$ and $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$. Thus,

$$\begin{aligned}\vec{F}_B &= \int id\vec{L}\times\vec{B}\\ &= \int_{x_i}^{x_f} i\,dx\,\hat{\imath}\times(B_x\hat{\imath}+B_y\hat{\jmath}) = i\int_{x_i}^{x_f} B_y\,dx\hat{k}\\ &= (-5.0\,\text{A})\left(\int_{1.0}^{3.0}(8.0x^2\,dx)\,(\text{m}\cdot\text{mT})\right)\hat{k}\\ &= -0.35\ \text{N}\ \hat{k}\ .\end{aligned}$$

Chapter 30

1. (a) The magnitude of the magnetic field due to the current in the wire, at a point a distance r from the wire, is given by

$$B = \frac{\mu_0 i}{2\pi r} .$$

With $r = 20\,\text{ft} = 6.10\,\text{m}$, we find

$$B = \frac{(4\pi \times 10^{-7}\,\text{T·m/A})(100\,\text{A})}{2\pi(6.10\,\text{m})} = 3.3 \times 10^{-6}\,\text{T} = 3.3\,\mu\text{T} .$$

(b) This is about one-sixth the magnitude of the Earth's field. It will affect the compass reading.

2. The current i due to the electron flow is $i = ne = (5.6 \times 10^{14}/\text{s})(1.6 \times 10^{-19}\,\text{C}) = 9.0 \times 10^{-5}\,\text{A}$. Thus,

$$B = \frac{\mu_0 i}{2\pi r} = \frac{(4\pi \times 10^{-7})(9.0 \times 10^{-5})}{2\pi(1.5 \times 10^{-3})} = 1.2 \times 10^{-8}\ \text{T} .$$

3. (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Since $B = \mu_0 i/2\pi r$,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.080\,\text{m})(39 \times 10^{-6}\,\text{T})}{4\pi \times 10^{-7}\,\text{T·m/A}} = 16\ \text{A} .$$

(b) The current must be from west to east to produce a field which is directed southward at points below it.

4. The points must be along a line parallel to the wire and a distance r from it, where r satisfies

$$B_{\text{wire}} = \frac{\mu_0 i}{2\pi r} = B_{\text{ext}} ,$$

or

$$r = \frac{\mu_0 i}{2\pi B_{\text{ext}}} = \frac{(1.26 \times 10^{-6}\,\text{T·m/A})(100\,\text{A})}{2\pi(5.0 \times 10^{-3}\,\text{T})} = 4.0 \times 10^{-3}\ \text{m} .$$

5. We assume the current flows in the $+x$ direction and the particle is at some distance d in the $+y$ direction (away from the wire). Then, the magnetic field at the location of the charge q is

$$\vec{B} = \frac{\mu_0 i}{2\pi d}\ \hat{\text{k}} .$$

Thus,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{\mu_0 i q}{2\pi d}\left(\vec{v} \times \hat{\text{k}}\right) .$$

(a) In this situation, $\vec{v} = v(-\hat{\jmath})$ (where v is the speed and is a positive value). Also, the problem specifies $q > 0$. Thus,

$$\vec{F} = \frac{\mu_0 iqv}{2\pi d}\left((-\hat{\jmath}) \times \hat{\mathrm{k}}\right) = -\frac{\mu_0 iqv}{2\pi d}(\hat{\imath}) ,$$

which tells us that $\vec{F}_q$ has a magnitude of $\mu_0 iqv/2\pi d$ and is in the direction opposite to that of the current flow.

(b) Now the direction $\vec{v}$ is reversed, and we obtain $\vec{F} = +\mu_0 iqv\hat{\imath}/2\pi d$. The magnitude is identical to that found in part (a), but the direction of the force is now in the same direction as that of the current flow.

6. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in Sample Problem 30-1). Also, the fields from the two semi-circular loops cancel at C (by symmetry). Therefore, $B_C = 0$.

7. Each of the semi-infinite straight wires contributes $\mu_0 i/4\pi R$ (Eq. 30-9) to the field at the center of the circle (both contributions pointing "out of the page"). The current in the arc contributes a term given by Eq. 30-11 pointing into the page, and this is able to produce zero total field at that location if

$$\begin{aligned} B_{\text{arc}} &= 2B_{\text{semi infinite}} \\ \frac{\mu_0 i\phi}{4\pi R} &= 2\left(\frac{\mu_0 i}{4\pi R}\right) \end{aligned}$$

which yields $\phi = 2$ rad.

8. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in segments AH and JD do not contribute to the field at point C. Using Eq. 30-11 (with $\phi = \pi$) and the right-hand rule, we find that the current in the semicircular arc HJ contributes $\mu_0 i/4R_1$ (into the page) to the field at C. Also, arc DA contributes $\mu_0 i/4R_2$ (out of the page) to the field there. Thus, the net field at C is

$$\vec{B} = \frac{\mu_0 i}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{into the page .}$$

9. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with P do not contribute to the field at that point. Using Eq. 30-11 (with $\phi = \theta$) and the right-hand rule, we find that the current in the semicircular arc of radius b contributes $\mu_0 i\theta/4\pi b$ (out of the page) to the field at P. Also, the current in the large radius arc contributes $\mu_0 i\theta/4\pi a$ (into the page) to the field there. Thus, the net field at P is

$$\vec{B} = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right) \quad \text{out of the page .}$$

10. (a) Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with C do not contribute to the field at that point.

(b) Eq. 30-11 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at C. The right-hand rule shows that this field is into the page.

(c) The contributions from parts (a) and (b) sum to

$$\vec{B} = \frac{\mu_0 i}{4R} \quad \text{into the page .}$$

11. Our x axis is along the wire with the origin at the midpoint. The current flows in the positive x direction. All segments of the wire produce magnetic fields at P_1 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_1 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx$$

where θ (the angle between the segment and a line drawn from the segment to P_1) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from $x=-L/2$ to $x=L/2$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L/2}^{L/2}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L/2}^{L/2} = \frac{\mu_0 i}{2\pi R}\frac{L}{\sqrt{L^2+4R^2}}\ .$$

If $L \gg R$, then R^2 in the denominator can be ignored and

$$B = \frac{\mu_0 i}{2\pi R}$$

is obtained. This is the field of a long straight wire. For points very close to a finite wire, the field is quite similar to that of an infinitely long wire.

12. The center of a square is a distance $R = a/2$ from the nearest side (each side being of length $L = a$). There are four sides contributing to the field at the center, so the result of problem 11 leads to

$$B_{\text{center}} = 4\left(\frac{\mu_0 i}{2\pi(a/2)}\right)\left(\frac{a}{\sqrt{a^2+4(a/2)^2}}\right) = \frac{2\sqrt{2}\,\mu_0 i}{\pi a}\ .$$

13. Our x axis is along the wire with the origin at the right endpoint, and the current is in the positive x direction. All segments of the wire produce magnetic fields at P_2 that are out of the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P_2 is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dx$$

where θ (the angle between the segment and a line drawn from the segment to P_2) and r (the length of that line) are functions of x. Replacing r with $\sqrt{x^2+R^2}$ and $\sin\theta$ with $R/r = R/\sqrt{x^2+R^2}$, we integrate from $x=-L$ to $x=0$. The total field is

$$B = \frac{\mu_0 iR}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 iR}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}\ .$$

14. We refer to the side of length L as the long side and that of length W as the short side. The center is a distance $W/2$ from the midpoint of each long side, and is a distance $L/2$ from the midpoint of each short side. There are two of each type of side, so the result of problem 11 leads to

$$B = 2\frac{\mu_0 i}{2\pi(W/2)}\frac{L}{\sqrt{L^2+4(W/2)^2}} + 2\frac{\mu_0 i}{2\pi(L/2)}\frac{W}{\sqrt{W^2+4(L/2)^2}}\ .$$

The final form of this expression, shown in the problem statement, derives from finding the common denominator of the above result and adding them, while noting that

$$\frac{L^2+W^2}{\sqrt{W^2+L^2}} = \sqrt{W^2+L^2}\ .$$

15. We imagine the square loop in the yz plane (with its center at the origin) and the evaluation point for the field being along the x axis (as suggested by the notation in the problem). The origin is a distance $a/2$ from each side of the square loop, so the distance from the evaluation point to each side of the square is, by the Pythagorean theorem,

$$R = \sqrt{(a/2)^2+x^2} = \frac{1}{2}\sqrt{a^2+4x^2}\ .$$

Only the x components of the fields (contributed by each side) will contribute to the final result (other components cancel in pairs), so a trigonometric factor of

$$\frac{a/2}{R} = \frac{a}{\sqrt{a^2+4x^2}}$$

multiplies the expression of the field given by the result of problem 11 (for each side of length $L = a$). Since there are four sides, we find

$$B(x) = 4\left(\frac{\mu_0 i}{2\pi R}\right)\left(\frac{a}{\sqrt{a^2+4R^2}}\right)\left(\frac{a}{\sqrt{a^2+4x^2}}\right) = \frac{4\,\mu_0\, i\, a^2}{2\pi\left(\frac{1}{2}\right)\left(\sqrt{a^2+4x^2}\right)^2\sqrt{a^2+4(a/2)^2+4x^2}}$$

which simplifies to the desired result. It is straightforward to set $x = 0$ and see that this reduces to the expression found in problem 12 (noting that $\frac{4}{\sqrt{2}} = 2\sqrt{2}$).

16. Our y axis is along the wire with the origin at the top endpoint, and the current is in the positive y direction. All segments of the wire produce magnetic fields at P that are into the page. According to the Biot-Savart law, the magnitude of the field any (infinitesimal) segment produces at P is given by

$$dB = \frac{\mu_0 i}{4\pi}\frac{\sin\theta}{r^2}\,dy$$

where θ (the angle between the segment and a line drawn from the segment to P) and r (the length of that line) are functions of y. Replacing r with $\sqrt{y^2+a^2}$ and $\sin\theta$ with $a/r = a/\sqrt{y^2+a^2}$, we integrate from $y = -a$ to $y = 0$. The total field is

$$B = \frac{\mu_0 i a}{4\pi}\int_{-a}^{0}\frac{dy}{(y^2+a^2)^{3/2}} = \frac{\mu_0 i a}{4\pi}\frac{1}{a^2}\frac{y}{(y^2+a^2)^{1/2}}\bigg|_{-a}^{0} = \frac{\mu_0 i}{4\pi a}\frac{a}{\sqrt{a^2+a^2}}$$

which simplifies to the desired result (noting that $\frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$).

17. Using the result of problem 12 and Eq. 30-12, we wish to show that

$$\frac{2\sqrt{2}\,\mu_0 i}{\pi a} > \frac{\mu_0 i}{2R} \quad , \quad \text{or} \quad \frac{4\sqrt{2}}{\pi a} > \frac{1}{R} \; ,$$

but to do this we must relate the parameters a and R. If both wires have the same length L then the geometrical relationships $4a = L$ and $2\pi R = L$ provide the necessary connection:

$$4a = 2\pi R \implies a = \frac{\pi R}{2} \; .$$

Thus, our proof consists of the observation that

$$\frac{4\sqrt{2}}{\pi a} = \frac{8\sqrt{2}}{\pi^2 R} > \frac{1}{R} \; ,$$

as one can check numerically (that $8\sqrt{2}/\pi^2 > 1$).

18. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments colinear with P do not contribute to the field at that point. We use the result of problem 16 to evaluate the contributions to the field at P, noting that the nearest wire-segments (each of length a) produce magnetism into the page at P and the further wire-segments (each of length $2a$) produce magnetism pointing out of the page at P. Thus, we find (into the page)

$$B_P = 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi a}\right) - 2\left(\frac{\sqrt{2}\mu_0 i}{8\pi(2a)}\right) = \frac{\sqrt{2}\mu_0 i}{8\pi a} \; .$$

19. Consider a section of the ribbon of thickness dx located a distance x away from point P. The current it carries is $di = i\,dx/w$, and its contribution to B_P is

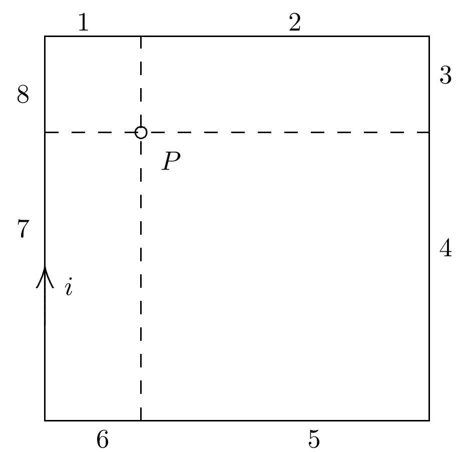
$$dB_P = \frac{\mu_0 di}{2\pi x} = \frac{\mu_0 i dx}{2\pi x w} \ .$$

Thus,

$$B_P = \int d\,B_P = \frac{\mu_0 i}{2\pi w}\int_d^{d+w} \frac{dx}{x} = \frac{\mu_0 i}{2\pi w}\,\ln\!\left(1+\frac{w}{d}\right) \ ,$$

and $\vec{B}_P$ points upward.

20. The two small wire-segments, each of length $a/4$, shown in Fig. 30-39 nearest to point P, are labeled 1 and 8 in the figure below.



Let $\vec{e}$ be a unit vector pointing into the page. We use the results of problems 13 and 16 to calculate B_{P1} through B_{P8}:

$$\begin{aligned}
B_{P1} &= B_{P8} = \frac{\sqrt{2}\mu_0 i}{8\pi(a/4)} = \frac{\sqrt{2}\mu_0 i}{2\pi a} \ , \\
B_{P4} &= B_{P5} = \frac{\sqrt{2}\mu_0 i}{8\pi(3a/4)} = \frac{\sqrt{2}\mu_0 i}{6\pi a} \ , \\
B_{P2} &= B_{P7} = \frac{\mu_0 i}{4\pi(a/4)}\cdot\frac{3a/4}{[(3a/4)^2+(a/4)^2]^{1/2}} = \frac{3\mu_0 i}{\sqrt{10}\pi a} \ ,
\end{aligned}$$

and

$$B_{P3} = B_{P6} = \frac{\mu_0 i}{4\pi(3a/4)}\cdot\frac{a/4}{[(a/4)^2+(3a/4)^2]^{1/2}} = \frac{\mu_0 i}{3\sqrt{10}\pi a} \ .$$

Finally,

$$\begin{aligned}
\vec{B}_P &= \sum_{n=1}^{8} B_{Pn}\vec{e} \\
&= 2\,\frac{\mu_0 i}{\pi a}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)\vec{e} \\
&= \frac{2(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(10\,\mathrm{A})}{\pi(8.0\times10^{-2}\,\mathrm{m})}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{6}+\frac{3}{\sqrt{10}}+\frac{1}{3\sqrt{10}}\right)\vec{e} \\
&= (2.0\times10^{-4}\,\mathrm{T})\,\vec{e} \ ,
\end{aligned}$$

where $\vec{e}$ is a unit vector pointing into the page.

21. (a) If the currents are parallel, the two fields are in opposite directions in the region between the wires. Since the currents are the same, the total field is zero along the line that runs halfway between the wires. There is no possible current for which the field does not vanish.

(b) If the currents are antiparallel, the fields are in the same direction in the region between the wires. At a point halfway between they have the same magnitude, $\mu_0 i/2\pi r$. Thus the total field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\,\mathrm{m})(300 \times 10^{-6}\,\mathrm{T})}{4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}} = 30 \text{ A} .$$

22. Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is $d - r$ away from the wire carrying current $3i$, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi(d-r)} \quad \Longrightarrow \quad r = \frac{d}{4} .$$

23. Using the right-hand rule, we see that the current i_2 carried by wire 2 must be out of the page. Now, $B_{P1} = \mu_0 i_1/2\pi r_1$ where $i_1 = 6.5\,\mathrm{A}$ and $r_1 = 0.75\,\mathrm{cm} + 1.5\,\mathrm{cm} = 2.25\,\mathrm{cm}$, and $B_{P2} = \mu_0 i_2/2\pi r_2$ where $r_2 = 1.5\,\mathrm{cm}$. From $B_{P1} = B_{P2}$ we get

$$i_2 = i_1 \left(\frac{r_2}{r_1}\right) = (6.5\,\mathrm{A}) \left(\frac{1.5\,\mathrm{cm}}{2.25\,\mathrm{cm}}\right) = 4.3 \text{ A} .$$

24. We label these wires 1 through 5, left to right, and use Eq. 30-15 (divided by length). Then,

$$\begin{aligned}
\vec{F}_1 &= \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{d} + \frac{1}{2d} + \frac{1}{3d} + \frac{1}{4d}\right)\hat{\jmath} = \frac{25\mu_0 i^2}{24\pi d}\,\hat{\jmath} \\
&= \frac{(13)(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(3.00\,\mathrm{A})^2(1.00\,\mathrm{m})\hat{\jmath}}{24\pi(8.00 \times 10^{-2}\,\mathrm{m})} \\
&= 4.69 \times 10^{-5}\,\mathrm{N/m}\ \hat{\jmath}\ ;
\end{aligned}$$

$$\vec{F}_2 = \frac{\mu_0 i^2}{2\pi}\left(\frac{1}{2d} + \frac{1}{3d}\right)\hat{\jmath} = \frac{5\mu_0 i^2}{12\pi d}\,\hat{\jmath} = 1.88 \times 10^{-5}\,\mathrm{N/m}\ \hat{\jmath}\ ;$$

$F_3 = 0$ (because of symmetry); $\vec{F}_4 = -\vec{F}_2$; and $\vec{F}_5 = -\vec{F}_1$.

25. Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

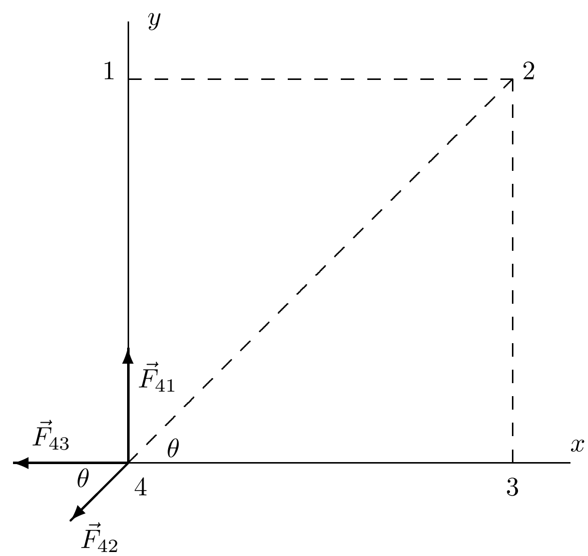
$$\begin{aligned}
B_{\text{total}} &= 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45^\circ = \frac{2\mu_0 i}{\pi a} \\
&= \frac{2(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(20\,\mathrm{A})}{\pi(0.20\,\mathrm{m})} = 8.0 \times 10^{-5} \text{ T} .
\end{aligned}$$

In the calculation $\cos 45^\circ$ was replaced with $1/\sqrt{2}$. The total field points upward.

26. Using Eq. 30-15, the force on, say, wire 1 (the wire at the upper left of the figure) is along the diagonal (pointing towards wire 3 which is at the lower right). Only the forces (or their components) along the diagonal direction contribute. With $\theta = 45°$, we find

$$\begin{aligned} F_1 &= \left|\vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}\right| \\ &= 2F_{12}\cos\theta + F_{13} \\ &= 2\left(\frac{\mu_0 i^2}{2\pi a}\right)\cos 45° + \frac{\mu_0 i^2}{2\sqrt{2}\pi a} \\ &= 0.338\left(\frac{\mu_0 i^2}{a}\right) . \end{aligned}$$

27. We use Eq. 30-15 and the superposition of forces: $\vec{F}_4 = \vec{F}_{14} + \vec{F}_{24} + \vec{F}_{34}$. With $\theta = 45°$, the situation is as shown below:



The components of $\vec{F}_4$ are given by

$$\begin{aligned} F_{4x} &= -F_{43} - F_{42}\cos\theta \\ &= -\frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \cos 45°}{2\sqrt{2}\pi a} \\ &= -\frac{3\mu_0 i^2}{4\pi a} \end{aligned}$$

and

$$\begin{aligned} F_{4y} &= F_{41} - F_{42}\sin\theta \\ &= \frac{\mu_0 i^2}{2\pi a} - \frac{\mu_0 i^2 \sin 45°}{2\sqrt{2}\pi a} \\ &= \frac{\mu_0 i^2}{4\pi a} . \end{aligned}$$

Thus,

$$F_4 = (F_{4x}^2 + F_{4y}^2)^{1/2} = \left[\left(-\frac{3\mu_0 i^2}{4\pi a}\right)^2 + \left(\frac{\mu_0 i^2}{4\pi a}\right)^2\right]^{1/2} = \frac{\sqrt{10}\mu_0 i^2}{4\pi a} ,$$

and $\vec{F}_4$ makes an angle ϕ with the positive x axis, where

$$\phi = \tan^{-1}\left(\frac{F_{4y}}{F_{4x}}\right) = \tan^{-1}\left(-\frac{1}{3}\right) = 162^\circ \ .$$

28. (a) Consider a segment of the projectile between y and $y + dy$. We use Eq. 30-14 to find the magnetic force on the segment, and Eq. 30-9 for the magnetic field of each semi-infinite wire (the top rail referred to as wire 1 and the bottom as wire 2). The current in rail 1 is in the $+\hat{\imath}$ direction, and the current in the rail 2 is in the $-\hat{\imath}$ direction. The field (in the region between the wires) set up by wire 1 is into the paper (the $-\hat{k}$ direction) and that set up by wire 2 is also into the paper. The force element (a function of y) acting on the segment of the projectile (in which the current flows in the $-\hat{\jmath}$ direction) is given below. The coordinate origin is at the bottom of the projectile.

$$\begin{aligned} d\vec{F} &= d\vec{F}_1 + d\vec{F}_2 \\ &= i\,dy(-\hat{\jmath}) \times \vec{B}_1 + \,dy(-\hat{\jmath}) \times \vec{B}_2 \\ &= i[B_1 + B_2]\,\hat{\imath}\,dy \\ &= i\left[\frac{\mu_0 i}{4\pi(2R + w - y)} + \frac{\mu_0 i}{4\pi y}\right]\hat{\imath}\,dy \ . \end{aligned}$$

Thus, the force on the projectile is

$$\vec{F} = \int d\vec{F} = \frac{i^2\mu_0}{4\pi}\int_R^{R+w}\left(\frac{1}{2R + w - y} + \frac{1}{y}\right)dy\,\hat{\imath} \ = \frac{\mu_0 i^2}{2\pi}\ln\left(1 + \frac{w}{R}\right)\hat{\imath} \ .$$

(b) Using the work-energy theorem, we have $\Delta K = \frac{1}{2}mv_f^2 = W_{\text{ext}} = \int \vec{F}\cdot d\vec{s} = FL$. Thus, the final speed of the projectile is

$$\begin{aligned} v_f &= \left(\frac{2W_{\text{ext}}}{m}\right)^{1/2} = \left[\frac{2}{m}\,\frac{\mu_0 i^2}{2\pi}\,\ln\left(1 + \frac{w}{R}\right)L\right]^{1/2} \\ &= \left[\frac{2(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(450 \times 10^3\,\text{A})^2\,\ln(1 + 1.2\,\text{cm}/6.7\,\text{cm})(4.0\,\text{m})}{2\pi(10 \times 10^{-3}\,\text{kg})}\right]^{1/2} \\ &= 2.3 \times 10^3\ \text{m/s} \ . \end{aligned}$$

29. The magnitudes of the forces on the sides of the rectangle which are parallel to the long straight wire (with $i_1 = 30$ A) are computed using Eq. 30-15, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\,\text{sides}} = \int_a^{a+b}\frac{i_2\mu_0 i_1}{2\pi y}\,dy \ .$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$\begin{aligned} F &= \frac{\mu_0 i_1 i_2 L}{2\pi}\left(\frac{1}{a} - \frac{1}{a + d}\right) = \frac{\mu_0 i_1 i_2 b}{2\pi a(a + b)} \\ &= \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(30\,\text{A})(20\,\text{A})(8.0\,\text{cm})(30 \times 10^{-2}\,\text{m})}{2\pi(1.0\,\text{cm} + 8.0\,\text{cm})} \\ &= 3.2 \times 10^{-3}\ \text{N} \ , \end{aligned}$$

and $\vec{F}$ points toward the wire.

30. A close look at the path reveals that only currents 1, 3, 6 and 7 are enclosed. Thus, noting the different current directions described in the problem, we obtain

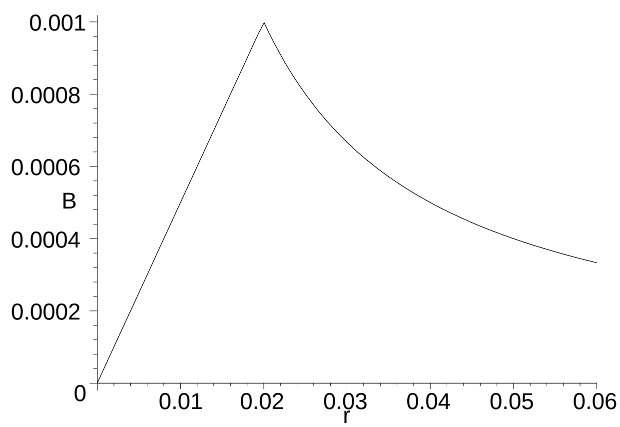
$$\oint \vec{B} \cdot d\vec{s} = \mu_0(7i - 6i + 3i + i) \quad = \quad 5\mu_0 i \ .$$

31. (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(2.0\,\mathrm{A})(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}) = -2.5 \times 10^{-6}\,\mathrm{T{\cdot}m} \ .$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\mathrm{enc}} = 0$.

32. We use Eq. 30-22 for the B-field inside the wire and Eq. 30-19 for that outside the wire. The plot is shown below (with SI units understood).



33. We use Ampere's law. For the dotted loop shown on the diagram $i = 0$. The integral $\int \vec{B}\cdot d\vec{s}$ is zero along the bottom, right, and top sides of the loop. Along the right side the field is zero, along the top and bottom sides the field is perpendicular to $d\vec{s}$. If ℓ is the length of the left edge, then direct integration yields $\oint \vec{B}\cdot d\vec{s} = B\ell$, where B is the magnitude of the field at the left side of the loop. Since neither B nor ℓ is zero, Ampere's law is contradicted. We conclude that the geometry shown for the magnetic field lines is in error. The lines actually bulge outward and their density decreases gradually, not discontinuously as suggested by the figure.

34. We use Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 i$, where the integral is around a closed loop and i is the net current through the loop. For path 1, the result is

$$\begin{aligned}\oint_1 \vec{B} \cdot d\vec{s} &= \mu_0(-5.0\,\mathrm{A} + 3.0\,\mathrm{A}) = (-2.0\,\mathrm{A})(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}) \\ &= -2.5 \times 10^{-6}\ \mathrm{T{\cdot}m} \ .\end{aligned}$$

For path 2, we find

$$\begin{aligned}\oint_2 \vec{B} \cdot d\vec{s} &= \mu_0(-5.0\,\mathrm{A} - 5.0\,\mathrm{A} - 3.0\,\mathrm{A}) = (-13.0\,\mathrm{A})(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}) \\ &= -1.6 \times 10^{-5}\ \mathrm{T{\cdot}m} \ .\end{aligned}$$

35. For $r < a$,

$$B(r) = \frac{\mu_0 i_{\text{enc}}}{2\pi r} = \frac{\mu_0}{2\pi r}\int_0^r J(r)2\pi r\,dr = \frac{\mu_0}{2\pi}\int_0^r J_0\left(\frac{r}{a}\right)2\pi r\,dr = \frac{\mu_0 J_0 r^2}{3a}\ .$$

36. (a) Replacing $i/\pi R^2$ with $J = 100\ \text{A/m}^2$, in Eq. 30-22, we have

$$\left|\vec{B}\right| = \left(\frac{\mu_0\,J}{2}\right)\,r = 1.3\times10^{-7}\ \text{T}$$

where $r = 0.0020$ m.

(b) Similarly, writing $i = J\pi R^2$ in Eq. 30-19 yields

$$\left|\vec{B}\right| = \frac{\mu_0\,J\,R^2}{2r} = 1.4\times10^{-7}\ \text{T}$$

where $r = 0.0040$ m.

37. (a) The the magnetic field at a point within the hole is the sum of the fields due to two current distributions. The first is that of the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder (with the hole). The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but is in the opposite direction. If these two situations are superposed the total current in the region of the hole is zero. Now, a solid cylinder carrying current i, uniformly distributed over a cross section, produces a magnetic field with magnitude

$$B = \frac{\mu_0 i r}{2\pi R^2}$$

a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder. For the cylinder of this problem the current density is

$$J = \frac{i}{A} = \frac{i}{\pi(a^2-b^2)}\ ,$$

where $A = \pi(a^2 - b^2)$ is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$I_1 = JA = \pi J a^2 = \frac{ia^2}{a^2-b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 I_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2(a^2-b^2)} = \frac{\mu_0 i r_1}{2\pi(a^2-b^2)}\ .$$

The current in the cylinder that fills the hole is

$$I_2 = \pi J b^2 = \frac{ib^2}{a^2-b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 I_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2(a^2-b^2)} = \frac{\mu_0 i r_2}{2\pi(a^2-b^2)}\ .$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi(a^2-b^2)}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$ the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2} \ .$$

This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$ the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

(c) Consider a rectangular path with two long sides (side 1 and 2, each with length L) and two short sides (each of length less than b). If side 1 is directly along the axis of the hole, then side 2 would be also parallel to it and also in the hole. To ensure that the short sides do not contribute significantly to the integral in Ampere's law, we might wish to make L *very* long (perhaps longer than the length of the cylinder), or we might appeal to an argument regarding the angle between $\vec{B}$ and the short sides (which is 90° at the axis of the hole). In any case, the integral in Ampere's law reduces to

$$\begin{aligned}
\oint_{\text{rectangle}} \vec{B} \cdot d\vec{s} &= \mu_0 i_{\text{enclosed}} \\
\int_{\text{side 1}} \vec{B} \cdot d\vec{s} + \int_{\text{side 2}} \vec{B} \cdot d\vec{s} &= \mu_0 i_{\text{in hole}} \\
\left(B_{\text{side 1}} - B_{\text{side 2}}\right) L &= 0
\end{aligned}$$

where $B_{\text{side 1}}$ is the field along the axis found in part (a). This shows that the field at off-axis points (where $B_{\text{side 2}}$ is evaluated) is the same as the field at the center of the hole; therefore, the field in the hole is uniform.

38. The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R} \ .$$

For the wire we have $B_{P,\,\text{wire}} > B_{C,\,\text{wire}}$. Thus, for $B_P = B_C = B_{C,\,\text{wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\,\text{wire}} - B_{P,\,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)} \ .$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8$.

39. The "current per unit x-length" may be viewed as current density multiplied by the thickness Δy of the sheet; thus, $\lambda = J\Delta y$. Ampere's law may be (and often is) expressed in terms of the current density vector as follows:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

where the area integral is over the region enclosed by the path relevant to the line integral (and $\vec{J}$ is in the $+z$ direction, out of the paper). With J uniform throughout the sheet, then it clear that the right-hand side of this version of Ampere's law should reduce, in this problem, to $\mu_0 JA = \mu_0 J\Delta y\Delta x = \mu_0\lambda\Delta x$.

(a) Figure 30-52 certainly has the horizontal components of $\vec{B}$ drawn correctly at points P and P' (as reference to Fig. 30-4 will confirm [consider the current elements nearest each of those points]), so the question becomes: is it possible for $\vec{B}$ to have vertical components in the figure? Our focus is on point P. Fig. 30-4 suggests that the current element just to the right of the nearest one (the one directly under point P) will contribute a downward component, but by the same reasoning the current element just to the left of the nearest one should contribute an upward component to the field at P. The current elements are all equivalent, as is reflected in the horizontal-translational symmetry built into this problem; therefore, all vertical components should cancel in pairs. The field at P must be purely horizontal, as drawn.

(b) The path used in evaluating $\oint \vec{B} \cdot d\vec{s}$ is rectangular, of horizontal length Δx (the horizontal sides passing through points P and P' respectively) and vertical size $\delta y > \Delta y$. The vertical sides have no contribution to the integral since $\vec{B}$ is purely horizontal (so the scalar dot product produces zero for those sides), and the horizontal sides contribute two equal terms, as shown below. Ampere's law yields

$$2B\Delta x = \mu_0 \lambda \Delta x \implies B = \frac{1}{2}\mu_0 \lambda .$$

40. It is possible (though tedious) to use Eq. 30-28 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 30-25 for the ideal solenoid (which does not make use of the coil radius) is the preferred method:

$$B = \mu_0 i n = \mu_0 i \left(\frac{N}{\ell}\right)$$

where $i = 3.60$ A, $\ell = 0.950$ m and $N = 1200$. This yields $B = 0.00571$ T.

41. It is possible (though tedious) to use Eq. 30-28 and evaluate the contributions (with the intent to sum them) of all 200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, Eq. 30-25 for the ideal solenoid (which does not make use of the coil diameter) is the preferred method:

$$B = \mu_0 i n = \mu_0 i \left(\frac{N}{\ell}\right)$$

where $i = 0.30$ A, $\ell = 0.25$ m and $N = 200$. This yields $B = 0.0030$ T.

42. We find N, the number of turns of the solenoid, from $B = \mu_0 i n = \mu_0 i N/\ell$: $N = B\ell/\mu_0 i$. Thus, the total length of wire used in making the solenoid is

$$2\pi r N = \frac{2\pi r B \ell}{\mu_0 i} = \frac{2\pi(2.60 \times 10^{-2}\,\text{m})(23.0 \times 10^{-3}\,\text{T})(1.30\,\text{m})}{2(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(18.0\,\text{A})} = 108 \text{ m} .$$

43. (a) We use Eq. 30-26. The inner radius is $r = 15.0$ cm, so the field there is

$$B = \frac{\mu_0 i N}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.150\,\text{m})} = 5.33 \times 10^{-4} \text{ T} .$$

(b) The outer radius is $r = 20.0$ cm. The field there is

$$B = \frac{\mu_0 i N}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(0.800\,\text{A})(500)}{2\pi(0.200\text{m})} = 4.00 \times 10^{-4} \text{ T} .$$

44. (a) The ideal solenoid is long enough (and we are evaluating the field at a point far enough inside) such that the open ends of the solenoid are "out of sight" and the situation displays a horizontal-translational symmetry (assuming the axis of the cylindrical shape of the solenoid is horizontal). A view of a "slice" of, say, the bottom of the solenoid would therefore appear similar to that shown in Fig. 30-52, where point P is in the interior of the solenoid and point P' is outside the coil. Now, Fig. 30-52 differs in at least one respect from our "slice" view of the solenoid in that the field at P' would be zero instead of what is shown in that figure. The field vanishes there because the top of the solenoid (similar to that shown in Fig. 30-52, in "slice" view, but with the currents and field directions reversed) would contribute an equal and opposite field to any exterior point, thus canceling it. For interior points, the top and bottom "slices" each contribute $\frac{1}{2}\mu_0\lambda$ (in the same direction) [this is shown in the solution to problem 39] and thus produce an interior field equal to $B = \mu_0 \lambda$.

(b) Applying Ampere's law to a rectangular path which passes through points P (interior) and P' (exterior) similar to that described in the solution to part (b) of problem 39, we are not surprised to find

$$\oint \vec{B} \cdot d\vec{s} = \left(\vec{B}_P - \vec{B}_{P'}\right) \cdot \hat{\imath} \Delta x = \mu_0 \lambda \Delta x$$

just as we found in part (b) of problem 39 (except that we are now taking the $+x$ direction in the same direction as the field at P, to avoid confusion with signs). The difference with the previous solution is that in 39, $\left(\vec{B}_P - \vec{B}_{P'}\right) \cdot \hat{\imath}$ was equal to $B - (-B) = 2B$, whereas in this case we have $B - 0 = B$. Although the value of B is different in the two problems, we see that the *change* $\left(\vec{B}_P - \vec{B}_{P'}\right) \cdot \hat{\imath}$ is the same: $\mu_0 \lambda$.

45. Consider a circle of radius r, inside the toroid and concentric with it (like either of the loops drawn in Fig. 30-20). The current that passes through the region between this circle and another larger radius circle (well outside the toroid) is Ni, where N is the number of turns and i is the current (note that this region includes a "slice" of the outer rim of the toroid). The current per unit length (of the circle) is $\lambda = Ni/2\pi r$, and $\mu_0 \lambda$ is therefore $\mu_0 Ni/2\pi r$, the magnitude of the magnetic field at the circle (call it B_1). Since the field outside a toroid (call it B_2) is zero, the above result is also the *change* in the magnitude of the field encountered as you move from the circle to the outside (say, to the larger radius circle mentioned above). The equality is not really surprising in light of Ampere's law, particularly if the path used in $\oint \vec{B} \cdot d\vec{s}$ is made to connect the circle in the toroid and the larger radius circle (or portions of each of them, of lengths Δs_1 and Δs_2). The connecting paths (each of size Δr) between the circles can be made perpendicular to the magnetic field lines (so that $\vec{B} \cdot \vec{s} = 0$). In fact, we can keep the connecting paths roughly perpendicular to $\vec{B}$ and manage to have $\Delta s_1 \approx \Delta s_2$ if our Amperian loop is very small (especially if Δr is much smaller than the outer radius of the toroid). Simplifying our notation, the current through the loop is therefore $\Delta s \lambda$, so Ampere's law yields $(B_1 - B_2)\Delta s = \mu_0 \Delta s \lambda$ and $B_2 - B_1 = \mu_0 \lambda$. What this demonstrates is that the change of the magnetic field is $\mu_0 \lambda$ when moving from one point to another (in a direction perpendicular to the field) across a current sheet (as the term is used in problem 39); this principle is useful in any discussion of boundary conditions in electrodynamics applications.

46. The orbital radius for the electron is

$$r = \frac{mv}{eB} = \frac{mv}{e\mu_0 ni}$$

which we solve for i:

$$\begin{aligned} i &= \frac{mv}{e\mu_0 nr} \\ &= \frac{(9.11 \times 10^{-31}\,\mathrm{kg})(0.0460)(3.00 \times 10^8\,\mathrm{m/s})}{(1.60 \times 10^{-19}\,\mathrm{C})(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})(100/0.0100\,\mathrm{m})(2.30 \times 10^{-2}\,\mathrm{m})} \\ &= 0.272 \text{ A} . \end{aligned}$$

47. (a) We denote the $\vec{B}$-fields at point P on the axis due to the solenoid and the wire as $\vec{B}_s$ and $\vec{B}_w$, respectively. Since $\vec{B}_s$ is along the axis of the solenoid and $\vec{B}_w$ is perpendicular to it, $\vec{B}_s \perp \vec{B}_w$, respectively. For the net field $\vec{B}$ to be at 45° with the axis we then must have $B_s = B_w$. Thus,

$$B_s = \mu_0 i_s n = B_w = \frac{\mu_0 i_w}{2\pi d} ,$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\mathrm{A}}{2\pi(20.0 \times 10^{-3}\,\mathrm{A})(10\,\mathrm{turns/cm})} = 4.77 \text{ cm} .$$

(b) The magnetic field strength is

$$\begin{aligned} B &= \sqrt{2}B_s \\ &= \sqrt{2}(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(20.0 \times 10^{-3}\,\mathrm{A})(10\,\mathrm{turns}/0.0100\,\mathrm{m}) \\ &= 3.55 \times 10^{-5}\ \mathrm{T}\ . \end{aligned}$$

48. (a) We set $z = 0$ in Eq. 30-28 (which is equivalent using to Eq. 30-12 multiplied by the number of loops). Thus, $B(0) \propto i/R$. Since case b has two loops,

$$\frac{B_b}{B_a} = \frac{2i/R_b}{i/R_a} = \frac{2R_a}{R_b} = 4\ .$$

(b) The ratio of their magnetic dipole moments is

$$\frac{\mu_b}{\mu_a} = \frac{2iA_b}{iA_a} = \frac{2R_b^2}{R_a^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}\ .$$

49. The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = (200)(0.30\,\mathrm{A})\pi(0.050\mathrm{m})^2 = 0.47\ \mathrm{A\cdot m^2}\ .$$

50. We use Eq. 30-28 and note that the contributions to $\vec{B}_P$ from the two coils are the same. Thus,

$$B_P = \frac{2\mu_0 iR^2N}{2[R^2 + (R/2)^2]^{3/2}} = \frac{8\mu_0 Ni}{5\sqrt{5}R}\ .$$

$\vec{B}_P$ is in the positive x direction.

51. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = Ni\pi R^2 = (300)(4.0\,\mathrm{A})\pi(0.025\,\mathrm{m})^2 = 2.4\ \mathrm{A\cdot m^2}\ .$$

(b) The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\ .$$

We solve for z:

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(2.36\,\mathrm{A\cdot m^2})}{2\pi(5.0 \times 10^{-6}\,\mathrm{T})}\right)^{1/3} = 46\ \mathrm{cm}\ .$$

52. (a) For $x \gg a$, the result of problem 15 reduces to

$$B(x) \approx \frac{4\mu_0 ia^2}{\pi(4x^2)(4x^2)^{1/2}} = \frac{\mu_0(ia^2)}{4\pi x^3}\ ,$$

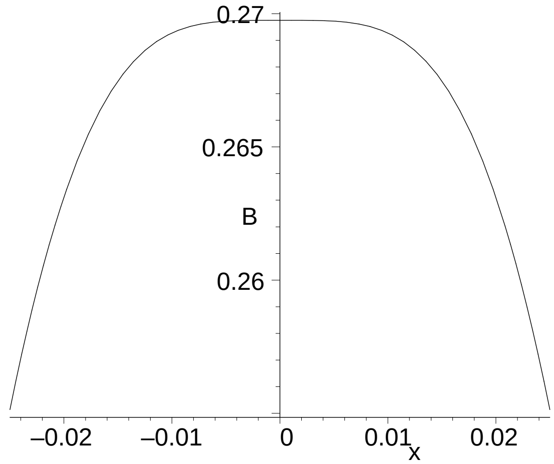
which is indeed the field of a magnetic dipole (see Eq. 30-29).

(b) The magnitude of the magnetic dipole moment is $\mu = ia^2$, by comparison between Eq. 30-29 and the result above.

53. Since the origin is midway between the coils, and the axis is chosen to be x (as opposed to the z used in Eq. 30-28), then the net field of the two coils is

$$B = \frac{\mu_0 N i R^2}{2}\left(\frac{1}{\sqrt{R^2+(R/2-x)^2}^{\,}}+\frac{1}{\sqrt{R^2+(R/2+x)^2}^{\,}}\right)$$

where $i = 50$ A, $N = 300$ and $R = 0.050$ m. The graph of this function (using SI units) is shown below.



54. (a) By imagining that each of the segments bg and cf (which are shown in the figure as having no current) actually has a pair of currents, where both currents are of the same magnitude (i) but opposite direction (so that the pair effectively cancels in the final sum), one can justify the superposition.

(b) The dipole moment of path $abcdefgha$ is

$$\begin{aligned}\vec{\mu} &= \vec{\mu}_{bcfgb}+\vec{\mu}_{abgha}+\vec{\mu}_{cdefc} = (ia^2)(\hat{\jmath}-\hat{\imath}+\hat{\imath}) = ia^2\hat{\jmath} \\ &= (6.0\,\text{A})(0.10\,\text{m})^2\hat{\jmath} = 6.0\times10^{-2}\ \text{A}\cdot\text{m}^2\ \hat{\jmath}\ .\end{aligned}$$

(c) Since both points are far from the cube we can use the dipole approximation. For $(x,\,y,\,z) = (0,\,5.0\,\text{m},\,0)$

$$\begin{aligned}\vec{B}(0,\,5.0\,\text{m},\,0) &\approx \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{y^3} \\ &= \frac{(1.26\times10^{-6}\,\text{T}\cdot\text{m/A})(6.0\times10^{-2}\,\text{m}^2\cdot\ \text{A})\hat{\jmath}}{2\pi(5.0\,\text{m})^3} \\ &= 9.6\times10^{-11}\ \text{T}\ \ \hat{\jmath}\ \ .\end{aligned}$$

For $(x,\,y,\,z) = (5.0\,\text{m},\,0,\,0)$, note that the line joining the end point of interest and the location of the dipole is perpendicular to the axis of the dipole. You can check easily that if an electric dipole is used, the field would be $E \approx (1/4\pi\varepsilon_0)(p/x^3)$, which is half of the magnitude of E for a point on the y axis the same distance from the dipole. By analogy, in our case B is also half the value or $B(0,\,5.0\,\text{m},\,0)$, i.e.,

$$B(5.0\,\text{m},\,0,\,0) = \frac{1}{2}B(0,\,5.0\,\text{m},\,0) = \frac{1}{2}(9.6\times10^{-11}\,\text{T}) = 4.8\times10^{-11}\,\text{T}\ .$$

Just like the electric dipole case, $\vec{B}(5.0\,\text{m},\,0,\,0)$ points in the negative y direction.

55. (a) The magnitude of the magnetic field on the axis of a circular loop, a distance z from the loop center, is given by Eq. 30-28:

$$B = \frac{N\mu_0 i R^2}{2(R^2+z^2)^{3/2}} ,$$

where R is the radius of the loop, N is the number of turns, and i is the current. Both of the loops in the problem have the same radius, the same number of turns, and carry the same current. The currents are in the same sense, and the fields they produce are in the same direction in the region between them. We place the origin at the center of the left-hand loop and let x be the coordinate of a point on the axis between the loops. To calculate the field of the left-hand loop, we set $z = x$ in the equation above. The chosen point on the axis is a distance $s - x$ from the center of the right-hand loop. To calculate the field it produces, we put $z = s - x$ in the equation above. The total field at the point is therefore

$$B = \frac{N\mu_0 i R^2}{2}\left[\frac{1}{(R^2+x^2)^{3/2}} + \frac{1}{(R^2+x^2-2sx+s^2)^{3/2}}\right] .$$

Its derivative with respect to x is

$$\frac{dB}{dx} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3x}{(R^2+x^2)^{5/2}} + \frac{3(x-s)}{(R^2+x^2-2sx+s^2)^{5/2}}\right] .$$

When this is evaluated for $x = s/2$ (the midpoint between the loops) the result is

$$\left.\frac{dB}{dx}\right|_{s/2} = -\frac{N\mu_0 i R^2}{2}\left[\frac{3s/2}{(R^2+s^2/4)^{5/2}} - \frac{3s/2}{(R^2+s^2/4-s^2+s^2)^{5/2}}\right] = 0$$

independently of the value of s.

(b) The second derivative is

$$\begin{aligned}\frac{d^2B}{dx^2} &= \frac{N\mu_0 i R^2}{2}\left[-\frac{3}{(R^2+x^2)^{5/2}} + \frac{15x^2}{(R^2+x^2)^{7/2}}\right. \\ &\qquad \left. -\frac{3}{(R^2+x^2-2sx+s^2)^{5/2}} + \frac{15(x-s)^2}{(R^2+x^2-2sx+s^2)^{7/2}}\right] .\end{aligned}$$

At $x = s/2$,

$$\begin{aligned}\left.\frac{d^2B}{dx^2}\right|_{s/2} &= \frac{N\mu_0 i R^2}{2}\left[-\frac{6}{(R^2+s^2/4)^{5/2}} + \frac{30s^2/4}{(R^2+s^2/4)^{7/2}}\right] \\ &= \frac{N\mu_0 R^2}{2}\left[\frac{-6(R^2+s^2/4)+30s^2/4}{(R^2+s^2/4)^{7/2}}\right] = 3N\mu_0 i R^2\,\frac{s^2-R^2}{(R^2+s^2/4)^{7/2}} .\end{aligned}$$

Clearly, this is zero if $s = R$.

56. (a) By the right-hand rule, $\vec{B}$ points into the paper at P (see Fig. 30-6(c)). To find the magnitude of the field, we use Eq. 30-11 for each semicircle ($\phi = \pi$ rad), and use superposition to obtain the result:

$$B = \frac{\mu_0 i\pi}{4\pi a} + \frac{\mu_0 i\pi}{4\pi b} = \frac{\mu_0 i}{4}\left(\frac{1}{a}+\frac{1}{b}\right) .$$

(b) The direction of $\vec{\mu}$ is the same as the $\vec{B}$ found in part (a): into the paper. The enclosed area is $A = (\pi a^2 + \pi b^2)/2$ which means the magnetic dipole moment has magnitude

$$|\vec{\mu}| = \frac{\pi i}{2}\left(a^2+b^2\right) .$$

57. (a) We denote the large loop and small coil with subscripts 1 and 2, respectively.

$$B_1 = \frac{\mu_0 i_1}{2R_1} = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(15\,\mathrm{A})}{2(0.12\,\mathrm{m})} = 7.9 \times 10^{-5}\ \mathrm{T}\ .$$

(b) The torque has magnitude equal to

$$\begin{aligned}\tau &= \left|\vec{\mu}_2 \times \vec{B}_1\right| = \mu_2 B_1 \sin 90^\circ \\ &= N_2 i_2 A_2 B_1 = \pi N_2 i_2 r_2^2 B_1 \\ &= \pi(50)(1.3\,\mathrm{A})(0.82 \times 10^{-2}\,\mathrm{m})^2(7.9 \times 10^{-5}\,\mathrm{T}) = 1.1 \times 10^{-6}\ \mathrm{N\cdot m}\ .\end{aligned}$$

58. (a) The contribution to B_C from the (infinite) straight segment of the wire is

$$B_{C1} = \frac{\mu_0 i}{2\pi R}\ .$$

The contribution from the circular loop is

$$B_{C2} = \frac{\mu_0 i}{2R}\ .$$

Thus,

$$B_C = B_{C1} + B_{C2} = \frac{\mu_0 i}{2R}\left(1 + \frac{1}{\pi}\right)\ .$$

$\vec{B}_C$ points out of the page.

(b) Now $\vec{B}_{C1} \perp \vec{B}_{C2}$ so

$$B_C = \sqrt{B_{C1}^2 + B_{C2}^2} = \frac{\mu_0 i}{2R}\sqrt{1 + \frac{1}{\pi^2}}\ ,$$

and $\vec{B}_C$ points at an angle (relative to the plane of the paper) equal to

$$\tan^{-1}\left(\frac{B_{C1}}{B_{C2}}\right) = \tan^{-1}\left(\frac{1}{\pi}\right) = 18^\circ\ .$$

59. (a) For the circular path L of radius r concentric with the conductor

$$\oint_L \vec{B}\cdot d\vec{s} = 2\pi r B = \mu_0 i_{\,\mathrm{enc}} = \mu_0\, i\,\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)}\ .$$

Thus,

$$B = \frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{r^2 - b^2}{r}\right)\ .$$

(b) At $r = a$, the magnetic field strength is

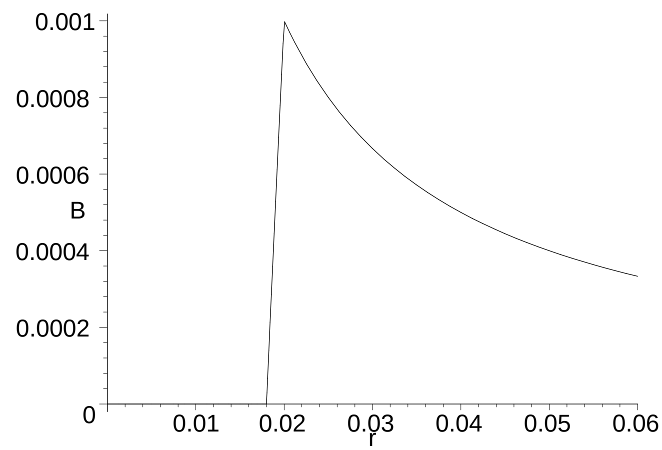
$$\frac{\mu_0 i}{2\pi(a^2 - b^2)}\left(\frac{a^2 - b^2}{a}\right) = \frac{\mu_0 i}{2\pi a}\ .$$

At $r = b$, $B \propto r^2 - b^2 = 0$. Finally, for $b = 0$

$$B = \frac{\mu_0 i}{2\pi a^2}\frac{r^2}{r} = \frac{\mu_0 i r}{2\pi a^2}$$

which agrees with Eq. 30-22.

(c) The field is zero for $r < b$ and is equal to Eq. 30-19 for $r > a$, so this along with the result of part (a) provides a determination of B over the full range of values. The graph (with SI units understood) is shown below.



60. (a) Eq. 30-22 applies for $r < c$. Our sign choice is such that i is positive in the smaller cylinder and negative in the larger one.

$$B = \frac{\mu_0 i r}{2\pi c^2} \qquad \text{for} \quad r \leq c \ .$$

(b) Eq. 30-19 applies in the region between the conductors.

$$B = \frac{\mu_0 i}{2\pi r} \qquad \text{for} \quad c \leq r \leq b \ .$$

(c) Within the larger conductor we have a superposition of the field due to the current in the inner conductor (still obeying Eq. 30-19) plus the field due to the (negative) current in the that part of the outer conductor at radius less than r (see part (a) of problem 59 for more details). The result is

$$B = \frac{\mu_0 i}{2\pi r} - \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - b^2}{a^2 - b^2}\right) \qquad \text{for} \quad b < r \leq a \ .$$

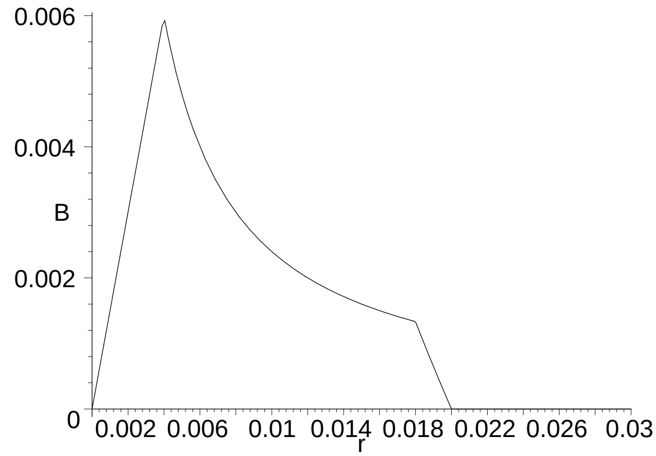
If desired, this expression can be simplified to read

$$B = \frac{\mu_0 i}{2\pi r}\left(\frac{a^2 - r^2}{a^2 - b^2}\right) \ .$$

(d) Outside the coaxial cable, the net current enclosed is zero. So $B = 0$ for $r \geq a$.

(e) We test these expressions for one case. If $a \to \infty$ and $b \to \infty$ (such that $a > b$) then we have the situation described on page 696 of the textbook.

(f) Using SI units, the graph of the field is shown below:



61. (a) We find the field by superposing the results of two semi-infinite wires (Eq. 30-9) and a semicircular arc (Eq. 30-11 with $\phi = \pi$ rad). The direction of $\vec{B}$ is out of the page, as can be checked by referring to Fig. 30-6(c). The magnitude of $\vec{B}$ at point a is therefore

$$B_a = 2\left(\frac{\mu_0 i}{4\pi R}\right) + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi} + \frac{1}{2}\right) .$$

With $i = 10$ A and $R = 0.0050$ m, we obtain $B_a = 1.0 \times 10^{-3}$ T. The direction of this field is out of the page, as Fig. 30-6(c) makes clear.

(b) The last remark in the problem statement implies that treating b as a point midway between two infinite wires is a good approximation. Thus, using Eq. 30-6,

$$B_b = 2\left(\frac{\mu_0 i}{2\pi R}\right) = 8.0 \times 10^{-4} \text{ T} .$$

This field, too, points out of the page.

62. We use $B(x, y, z) = (\mu_0/4\pi)i\,\Delta\vec{s} \times \vec{r}/r^3$, where $\Delta\vec{s} = \Delta s\hat{\text{j}}$ and $\vec{r} = x\hat{\text{i}} + y\hat{\text{j}} + z\vec{k}$. Thus,

$$\vec{B}(x, y, z) = \left(\frac{\mu_0}{4\pi}\right)\frac{i\,\Delta s\hat{\text{j}} \times (x\vec{i} + y\hat{\text{j}} + z\vec{k})}{(x^2 + y^2 + z^2)^{3/2}} = \frac{\mu_0 i\,\Delta s\,(z\hat{\text{i}} - x\hat{\text{k}})}{4\pi(x^2 + y^s + z^2)^{3/2}} .$$

(a) The field on the z axis (at $z = 5.0$ m) is

$$\begin{aligned}\vec{B}(0,\, 0,\, 5.0\,\text{m}) &= \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0 \times 10^{-2}\,\text{m})(5.0\,\text{m})\hat{\text{i}}}{4\pi\,(0^2 + 0^2 + (5.0\,\text{m})^2)^{3/2}} \\ &= 2.4 \times 10^{-10} \text{ T } \hat{\text{i}} .\end{aligned}$$

(b) $\vec{B}(0,\, 6.0\,\text{m},\, 0)$, since $x = z = 0$.

(c) The field in the xy plane, at $(x, y) = (7, 7)$, is

$$\begin{aligned}\vec{B}(7.0\,\text{m},\, 7.0\,\text{m},\, 0) &= \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0 \times 10^{-2}\,\text{m})(-7.0\,\text{m})\hat{\text{k}}}{4\pi\,((7.0\,\text{m})^2 + (7.0\,\text{m})^2 + 0^2)^{3/2}} \\ &= 4.3 \times 10^{-11} \text{ T } \hat{\text{k}} .\end{aligned}$$

(d) The field in the xy plane, at $(x, y) = (-3, -4)$, is

$$\vec{B}(-3.0\,\text{m},\,-4.0\,\text{m},\,0) = \frac{(4\pi\times 10^{-7}\,\text{T}\cdot\text{m/A})(2.0\,\text{A})(3.0\times 10^{-2}\,\text{m})(3.0\,\text{m})\hat{\text{k}}}{4\pi\left((-3.0\,\text{m})^2+(-4.0\,\text{m})^2+0^2\right)^{3/2}} = 1.4\times 10^{-10}\ \text{T}\ \hat{\text{k}}\ .$$

63. (a) Eq. 30-19 applies for each wire, with $r = \sqrt{R^2 + (d/2)^2}$ (by the Pythagorean theorem). The vertical components of the fields cancel, and the two (identical) horizontal components add to yield the final result

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\left(\frac{d/2}{r}\right) = \frac{\mu_0 i d}{2\pi\left(R^2 + (d/2)^2\right)}$$

where $(d/2)/r$ is a trigonometric factor to select the horizontal component. It is clear that this is equivalent to the expression in the problem statement.

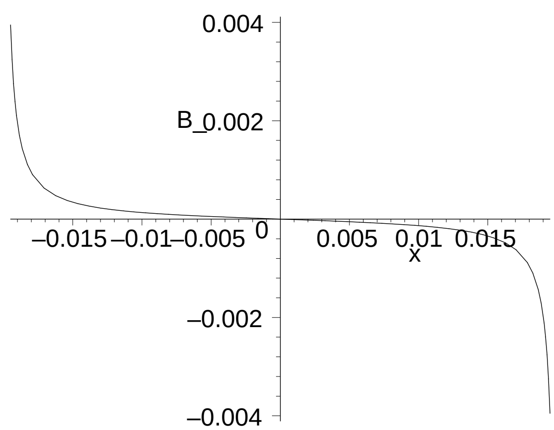
(b) Using the right-hand rule, we find both horizontal components point rightward.

64. (a) The difference between this and Sample Problem 6 is that the current in wire 2 is reversed form what is shown in Fig. 30-59(a). Thus, we replace $i \to -i$ in the expression for $B_2(x)$ and add the fields:

$$B_1(x) + B_2(x) = \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0(-i)}{2\pi(d-x)} = -\frac{\mu_0 i x}{\pi\left(d^2 - x^2\right)}$$

which is equivalent to the desired result.

(b) As remarked in that Sample Problem, this expression does not apply within the wires themselves. If we assume the wires have nearly zero thickness, then the expression applies over nearly all of the range $-0.02 < x < 0.02$ (with SI units understood). To be definite about this issue, we have picked a small wire radius (.005 m) and graphed the field over the range $-.0195 \le x \le 0.0195$.



65. (a) All wires carry parallel currents and attract each other; thus, the "top" wire is pulled downward by the other two:

$$|\vec{F}| = \frac{\mu_0\, L(5.0\text{ A})(3.2\text{ A})}{2\pi(0.10\text{ m})} + \frac{\mu_0\, L(5.0\text{ A})(5.0\text{ A})}{2\pi(0.20\text{ m})}$$

where $L = 3.0$ m. Thus, $|\vec{F}| = 1.7\times 10^{-4}$ N.

(b) Now, the "top" wire is pushed upward by the center wire and pulled downward by the bottom wire:

$$|\vec{F}| = \frac{\mu_0\, L(5.0\text{ A})(3.2\text{ A})}{2\pi(0.10\text{ m})} - \frac{\mu_0\, L(5.0\text{ A})(5.0\text{ A})}{2\pi(0.20\text{ m})}$$

so that $|\vec{F}| = 2.1\times 10^{-5}$ N.

66. With cylindrical symmetry, we have, external to the conductors,

$$|\vec{B}| = \frac{\mu_0\, i_{\rm enc}}{2\pi\, r}$$

which produces $i_{\rm enc} = 25$ mA from the given information. Therefore, the thin wire must carry 5 mA in a direction opposite to the 30 mA carried by the thin conducting surface.

67. The area enclosed by the loop L is $A = \frac{1}{2}(4d)(3d) = 6d^2$. Thus

$$\begin{aligned}\oint_c \vec{B}\cdot d\vec{s} &= \mu_0 i = \mu_0 jA \\ &= (4\pi\times 10^{-7}\,\mathrm{T\cdot m/A})(15\,\mathrm{A/m^2})(6)(0.20\,\mathrm{m})^2 = 4.5\times 10^{-6}\ \mathrm{T{\cdot}m}\ .\end{aligned}$$

68. We refer to the center of the circle (where we are evaluating $\vec{B}$) as C. Recalling the *straight sections* discussion in Sample Problem 30-1, we see that the current in the straight segments which are colinear with C do not contribute to the field there. Eq. 30-11 (with $\phi = \pi/2$ rad) and the right-hand rule indicates that the currents in the two arcs contribute

$$\frac{\mu_0 i(\pi/2)}{4\pi R} - \frac{\mu_0 i(\pi/2)}{4\pi R} = 0$$

to the field at C. Thus, the non-zero contributions come from those straight-segments which are not colinear with C. There are two of these "semi-infinite" segments, one a vertical distance R above C and the other a horizontal distance R to the left of C. Both contribute fields pointing out of the page (see Fig. 30-6(c)). Since the magnitudes of the two contributions (governed by Eq. 30-9) add, then the result is

$$B = 2\left(\frac{\mu_0 i}{4\pi R}\right) = \frac{\mu_0 i}{2\pi R}$$

exactly what one would expect from a single infinite straight wire (see Eq. 30-6). For such a wire to produce such a field (out of the page) with a leftward current requires that the point of evaluating the field be below the wire (again, see Fig. 30-6(c)).

69. Since the radius is $R = 0.0013$ m, then the $i = 50$ A produces

$$B = \frac{\mu_0 i}{2\pi R} = 0.0077\ \mathrm{T}$$

at the edge of the wire. The three equations, Eq. 30-6, Eq. 30-19 and Eq. 30-22, agree at this point.

70. We note that the distance from each wire to P is $r = d/\sqrt{2} = 0.071$ m. In both parts, the current is $i = 100$ A.

(a) With the currents parallel, application of the right-hand rule (to determine each of their contributions to the field at P) reveals that the vertical components cancel and the horizontal components add – yielding the result:

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\cos 45^\circ = 4.0\times 10^{-4}\ \mathrm{T}\ .$$

and directed leftward in the figure.

(b) Now, with the currents antiparallel, application of the right-hand rule shows that the horizontal components cancel and the vertical components add. Thus,

$$B = 2\left(\frac{\mu_0 i}{2\pi r}\right)\sin 45^\circ = 4.0\times 10^{-4}\ \mathrm{T}\ .$$

and directed upward in the figure.

71. (a) As illustrated in Sample Problem 30-1, the radial segments do not contribute to $\vec{B}_P$ and the arc-segments contribute according to Eq. 30-11 (with angle in radians). If $\hat{\mathrm{k}}$ designates the direction "out of the page" then

$$\vec{B} = \frac{\mu_0 (0.40 \text{ A})(\pi \text{ rad})}{4\pi(0.050 \text{ m})}\,\hat{\mathrm{k}} - \frac{\mu_0 (0.80 \text{ A})\left(\frac{2\pi}{3} \text{ rad}\right)}{4\pi(0.040 \text{ m})}\,\hat{\mathrm{k}}$$

which yields $\vec{B} = -1.7 \times 10^{-6}\,\hat{\mathrm{k}}$ T.

(b) Now we have

$$\vec{B} = -\frac{\mu_0 (0.40 \text{ A})(\pi \text{ rad})}{4\pi(0.050 \text{ m})}\,\hat{\mathrm{k}} - \frac{\mu_0 (0.80 \text{ A})\left(\frac{2\pi}{3} \text{ rad}\right)}{4\pi(0.040 \text{ m})}\,\hat{\mathrm{k}}$$

which yields $\vec{B} = -6.7 \times 10^{-6}\,\hat{\mathrm{k}}$ T.

72. (a) We designate the wire along $y = r_A = 0.100$ m wire A and the wire along $y = r_B = 0.050$ m wire B. Using Eq. 30-6, we have

$$\begin{aligned} \vec{B}_{\text{net}} &= \vec{B}_A + \vec{B}_B \\ &= -\frac{\mu_0\, i_A}{2\pi\, r_A}\,\hat{\mathrm{k}} - \frac{\mu_0\, i_B}{2\pi\, r_B}\,\hat{\mathrm{k}} \end{aligned}$$

which yields $\vec{B}_{\text{net}} = 52.0 \times 10^{-6}\hat{\mathrm{k}}$ T.

(b) This will occur for some value $r_B < y < r_A$ such that

$$\frac{\mu_0\, i_A}{2\pi\ (r_A - y)} = \frac{\mu_0\, i_B}{2\pi\ (y - r_B)} \quad .$$

Solving, we find $y = 13/160 \approx 0.081$ m.

(c) We eliminate the $y < r_B$ possibility due to wire B carrying the larger current. We expect a solution in the region $y > r_A$ where

$$\frac{\mu_0\, i_A}{2\pi\ (y - r_A)} = \frac{\mu_0\, i_B}{2\pi\ (y - r_B)} \quad .$$

Solving, we find $y = 7/40 \approx 0.018$ m.

73. (a) The field in this region is entirely due to the long wire (with, presumably, negligible thickness). Using Eq. 30-19,

$$|\vec{B}| = \frac{\mu_0\, i_w}{2\pi r} = 4.8 \times 10^{-3} \text{ T}$$

where $i_w = 24$ A and $r = 0.0010$ m.

(b) Now the field consists of two contributions (which are antiparallel) – from the wire (Eq. 30-19) and from a portion of the conductor (Eq. 30-22 modified for annular area):

$$\begin{aligned} |\vec{B}| &= \frac{\mu_0\, i_w}{2\pi r} - \frac{\mu_0\, i_{\text{enc}}}{2\pi r} \\ &= \frac{\mu_0\, i_w}{2\pi r} - \frac{\mu_0\, i_c}{2\pi r}\left(\frac{\pi r^2 - \pi R_i^2}{\pi R_o^2 - \pi R_i^2}\right) \end{aligned}$$

where $r = 0.0030$ m, $R_i = 0.0020$ m, $R_o = 0.0040$ m and $i_c = 24$ A. Thus, we find $|\vec{B}| = 9.3\times10^{-4}$ T.

(c) Now, in the external region, the individual fields from the two conductors cancel completely (since $i_c = i_w$): $\vec{B} = 0$.

74. In this case $L = 2\pi r$ is roughly the length of the toroid so

$$B = \mu_0 i_0 \left(\frac{N}{2\pi r}\right) = \mu_0 n i_0 \ .$$

This result is expected, since from the perspective of a point inside the toroid the portion of the toroid in the vicinity of the point resembles part of a long solenoid.

75. We take the current ($i = 50$ A) to flow in the $+x$ direction, and the electron to be at a point P which is $r = 0.050$ m above the wire (where "up" is the $+y$ direction). Thus, the field produced by the current points in the $+z$ direction at P. Then, combining Eq. 30-6 with Eq. 29-2, we obtain $\vec{F}_e = (-e\mu_0 i/2\pi r)(\vec{v} \times \hat{\mathrm{k}})$.

(a) The electron is moving down: $\vec{v} = -v\,\hat{\mathrm{j}}$ (where $v = 1.0 \times 10^7$ m/s is the speed) so

$$\vec{F}_e = \frac{-e\mu_0 iv}{2\pi r}\,(-\hat{\mathrm{i}}) = 3.2 \times 10^{-16}\ \mathrm{N}\ \hat{\mathrm{i}}\ .$$

(b) In this case, the electron in the same direction as the current: $\vec{v} = v\,\hat{\mathrm{i}}$ so

$$\vec{F}_e = \frac{-e\mu_0 iv}{2\pi r}\left(-\hat{\mathrm{j}}\right) = 3.2 \times 10^{-16}\ \mathrm{N}\ \hat{\mathrm{j}}\ .$$

(c) Now, $\vec{v} = \pm v\hat{\mathrm{k}}$ so $\vec{F}_e \propto \hat{\mathrm{k}} \times \hat{\mathrm{k}} = 0$.

76. Eq. 30-6 gives

$$i = \frac{2\pi RB}{\mu_0} = \frac{2\pi(0.880\,\mathrm{m})(7.30 \times 10^{-6}\,\mathrm{T})}{4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}} = 32.1\ \mathrm{A}\ .$$

77. For $x > 20$ mm, the field due i_2 is downward and thus subtracts from B_1 and is entirely consistent with the given expression for B_2 (note that it becomes negative when $x > d$). Similarly, for $x < -20$ mm, the field due to i_1 is downward and subtracts from B_2 (which is positive and points upward for all $x < d$). This again is consistent with the expression for B_1 which is seen to become negative for x less than $-d$ (that is, x negative and $|x| > |d|$). We conclude that the given expressions are valid over the whole of the x axis, and their answer (Eq. 30-33) holds for all x (other than at the locations of the wires themselves, where it becomes problematic, as discussed in the Sample Problem).

78. By the right-hand rule, the magnetic field $\vec{B}_1$(evaluated at a) produced by wire 1 (the wire at bottom left) is at $\phi = 150°$ (measured counterclockwise from the $+x$ axis, in the xy plane), and the field produced by wire 2 (the wire at bottom right) is at $\phi = 210°$. By symmetry ($\vec{B}_1 = \vec{B}_2$) we observe that only the x-components survive, yielding

$$\vec{B}_1 + \vec{B}_2 = 2\,\frac{\mu_0\, i}{2\pi\,\ell}\,\cos 150°\,\hat{\mathrm{i}} = -3.46 \times 10^{-5}\,\hat{\mathrm{i}}\ \mathrm{T}$$

where $i = 10$ A, $\ell = 0.10$ m, and Eq. 30-6 has been used. To cancel this, wire b must carry current into the page (that is, the $-\hat{\mathrm{k}}$ direction) of value

$$i_b = \left(3.46 \times 10^{-5}\right)\frac{2\pi\, r}{\mu_0} = 15\ \mathrm{A}$$

where $r = \sqrt{3}\,\ell/2 = 0.087$ m and and Eq. 30-6 has again been used.

79. Using Eq. 30-22 and Eq. 30-19, we have

$$\begin{aligned} |\vec{B}_1| &= \left(\frac{\mu_0\, i}{2\pi\, R^2}\right) r_1 \\ |\vec{B}_2| &= \frac{\mu_0\, i}{2\pi\, r_2} \end{aligned}$$

where $r_1 = 0.0040$ m, $|\vec{B}_1| = 2.8 \times 10^{-4}$ T, $r_2 = 0.010$ m and $|\vec{B}_2| = 2.0 \times 10^{-4}$ T. Point 2 is known to be external to the wire since $|\vec{B}_2| < |\vec{B}_1|$. From the second equation, we find $i = 10$ A. Plugging this into the first equation yields $R = 5.3 \times 10^{-3}$ m.

80. Using a magnifying glass, we see that all but i_2 are directed into the page. Wire 3 is therefore attracted to all but wire 2. Letting $d = 0.50$ m, we find the net force (per meter length) using Eq. 30-15, with positive indicated a rightward force:

$$\frac{|\vec{F}|}{\ell} = \frac{\mu_0 \, i_3}{2\pi}\left(-\frac{i_1}{2d} + \frac{i_2}{d} + \frac{i_4}{d} + \frac{i_5}{2d}\right)$$

which yields $|\vec{F}|/\ell = 8.0 \times 10^{-7}$ N/m.

Chapter 31

1. The magnetic field is normal to the plane of the loop and is uniform over the loop. Thus at any instant the magnetic flux through the loop is given by $\Phi_B = AB = \pi r^2 B$, where $A = \pi r^2$ is the area of the loop. According to Faraday's law the magnitude of the emf in the loop is

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt} = \pi (0.055\,\text{m})^2 (0.16\,\text{T/s}) = 1.5 \times 10^{-3}\ \text{V} .$$

2. The induced emf is

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{dB}{dt} \\ &= -A\frac{d}{dt}(\mu_0 i n) = -A\mu_0 n \frac{d}{dt}(i_0 \sin\omega t) \\ &= -A\mu_0 n i_0 \omega \cos\omega t . \end{aligned}$$

3. (a)

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}(6.0t^2 + 7.0t) = 12t + 7.0 = 12(2.0) + 7.0 = 31\ \text{mV} .$$

   (b) Appealing to Lenz's law (especially Fig. 31-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is from right to left through R.

4. (a) We use $\mathcal{E} = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0\,\text{s}$:

$$\mathcal{E} = -\pi r^2 \frac{dB}{dt} = -\pi (0.12\,\text{m})^2 \left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1 \times 10^{-2}\ \text{V} .$$

   (b) $2.0\,\text{s} < t < 4.0\,\text{s}$: $\mathcal{E} \propto dB/dt = 0$.

   (c) $4.0\,\text{s} < t < 6.0\,\text{s}$:

$$\mathcal{E} = -\pi r^2 \frac{dB}{dt} = -\pi (0.12\,\text{m})^2 \left(\frac{-0.5\,\text{T}}{6.0\,\text{s} - 4.0\,\text{s}}\right) = 1.1 \times 10^{-2}\ \text{V} .$$

5. (a) Table 27-1 gives the resistivity of copper. Thus,

$$R = \rho\frac{L}{A} = (1.68 \times 10^{-8}\,\Omega\cdot\text{m}) \left[\frac{\pi(0.10\,\text{m})}{\pi(2.5 \times 10^{-3})^2/4}\right] = 1.1 \times 10^{-3}\,\Omega .$$

   (b) We use $i = |\mathcal{E}|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{(10\,\text{A})(1.1 \times 10^{-3}\,\Omega)}{\pi (0.05\,\text{m})^2} = 1.4\ \text{T/s} .$$

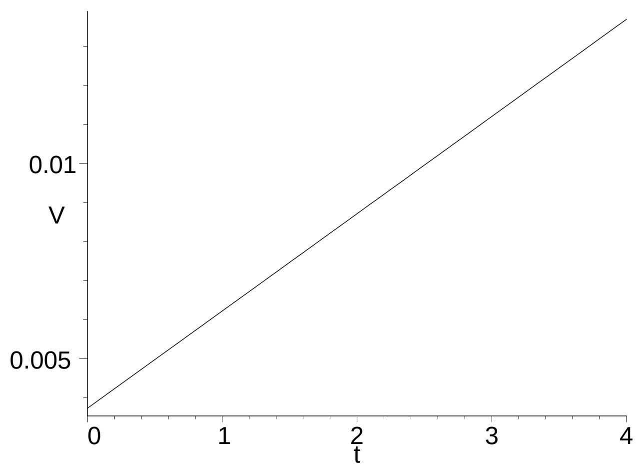
6. (a) Following Sample Problem 31-1, we have

$$\Phi_B = \mu_0 i n A \qquad \text{where} \quad A = \frac{\pi d^2}{4}$$

with $i = 3t + t^2$ (SI units and 2 significant figures understood). The magnitude of the induced emf is therefore

$$\mathcal{E} = N\frac{d\Phi_B}{dt} \approx 0.0012(3 + 2t)$$

where we have used the values specified in Sample Problem 31-1 for all quantities except the current. The plot is shown below.



(b) Using Ohm's law, the induced current is

$$i\Big|_{t=2.0\,\mathrm{s}} = \frac{\mathcal{E}\big|_{t=2.0\,\mathrm{s}}}{R} = \frac{0.0087\,\mathrm{V}}{0.15\,\Omega} = 0.058 \text{ A} .$$

7. The primary difference between this and the situation described in Sample Problem 31-1 is in the quantity A. The area through which there is magnetic flux is not the area of the short coil, in this case, but is the area of the solenoid (there is no field outside an ideal solenoid). Actually, because of the current (which we calculate here) in the short coil, there is a very small amount of field outside the solenoid (caused by that current) – but it may be disregarded in this calculation. The values are as indicated in Sample Problem 31-1 except that $A = \pi D^2/4$ (where $D = 0.032$ m) and $N = 120$ for the short coil. Thus, we find $\Phi_{B,i} = 3.3 \times 10^{-5}$ Wb, and the magnitude of the induced emf is 0.16 V. Ohm's law then yields $0.16\,\mathrm{V}/5.3\,\Omega = 0.030$ A.

8. Using Faraday's law, the induced emf is

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi r B\frac{dr}{dt} \\ &= -2\pi(0.12\,\mathrm{m})(0.800\,\mathrm{T})(-0.750\,\mathrm{m/s}) = 0.452 \text{ V} . \end{aligned}$$

9. (a) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 30-29, with $z = x$ (taken to be much greater than R), gives

$$\vec{B} = \frac{\mu_0 i R^2}{2x^3}\,\hat{\imath}$$

where the $+x$ direction is upward in Fig. 31-36. The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi\mu_0 i r^2 R^2}{2x^3} \; .$$

(b) The emf is given by Faraday's law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4} \; .$$

(c) As the smaller loop moves upward, the flux through it decreases, and we have situation like that shown in Fig. 31-5(b). The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

10. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is 0.

11. (a) Ohm's law combines with Faraday's law to give $i = -\frac{N}{R}\frac{d\Phi_B}{dt}$ where R is the resistance of the coil. In this case, $N = 1$ (it is a single loop), and we integrate to find the charge:

$$\begin{aligned} \int_0^t i\,dt &= -\frac{1}{R}\int_0^t \frac{d\Phi_B}{dt}\,dt \\ q(t) &= -\frac{1}{R}\left(\Phi_B(t) - \Phi_B(0)\right) \end{aligned}$$

which is equivalent to the expression shown in the problem statement. We have used little more than the fundamental theorem of calculus; no particular assumptions have been made about how the integrations should be performed. The result is independent of the way $\vec{B}$ has changed.

(b) If the current is identically zero for over the whole range $0 \to t$ then certainly the left-hand side of our computation, above, gives zero. But the same result can come from the current being in one direction for, say, $0 \to \frac{t}{2}$ and then in the opposite direction for $\frac{t}{2} \to t$ in such a way that $\int_0^t i\,dt = 0$. So a vanishing integral does not necessarily mean the integrand itself is identically zero.

12. (a) Eq. 30-12 gives the field at the center of the large loop with $R = 1.00$ m and current $i(t)$. This is approximately the field throughout the area ($A = 2.00 \times 10^{-4}$ m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$ (where $i_0 = 200$ A and $k = (-200\,\text{A} - 200\,\text{A})/1.00\,\text{s} = -400$ A/s), we find

$$\begin{aligned} B\big|_{t=0} &= \frac{\mu_0 i_0}{2R} = \frac{(4\pi \times 10^{-7}\,\text{H/m})(200\,\text{A})}{2(1.00\,\text{m})} = 1.26 \times 10^{-4}\,\text{T} \; , \\ B\big|_{t=0.500\,\text{s}} &= \frac{(4\pi \times 10^{-7}\,\text{H/m})[200\,\text{A} - (400\,\text{A/s})(0.500\,\text{s})]}{2(1.00\,\text{m})} = 0 \; , \\ B\big|_{t=1.00\,\text{s}} &= \frac{(4\pi \times 10^{-7}\,\text{H/m})[200\,\text{A} - (400\,\text{A/s})(1.00\,\text{s})]}{2(1.00\,\text{m})} = -1.26 \times 10^{-4}\,\text{T} \; . \end{aligned}$$

(b) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right) \\ &= -(2.00 \times 10^{-4}\,\text{m}^2)\left(\frac{-1.26 \times 10^{-4}\,\text{T} - 1.26 \times 10^{-4}\,\text{T}}{1.00\,\text{s}}\right) = 5.04 \times 10^{-8}\,\text{V} \; . \end{aligned}$$

13. From the result of the problem 11,

$$\begin{aligned} q(t) &= \frac{1}{R}[\Phi_B(0) - \Phi_B(t)] = \frac{A}{R}[B(0) - B(t)] \\ &= \frac{1.20 \times 10^{-3}\,\mathrm{m}^2}{13.0\,\Omega}[1.60\,\mathrm{T} - (-1.60\,\mathrm{T})] = 2.95 \times 10^{-2}\ \mathrm{C}\ . \end{aligned}$$

14. We note that $1\,\mathrm{gauss} = 10^{-4}$ T. Adapting the result of the problem 11,

$$\begin{aligned} q(t) &= \frac{N}{R}[BA\cos 20^\circ - (-BA\cos 20^\circ\,)] = \frac{2NBA\cos 20^\circ}{R} \\ &= \frac{2(1000)(0.590 \times 10^{-4}\,\mathrm{T})\pi(0.100\,\mathrm{m})^2(\cos 20^\circ)}{85.0\,\Omega + 140\,\Omega} = 1.55 \times 10^{-5}\,\mathrm{C}\ . \end{aligned}$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

15. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2B/2$, and the induced emf is

$$\mathcal{E}_i = -\frac{\mathrm{d}\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}\ .$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870\,\mathrm{T/s}$. Thus,

$$\mathcal{E}_i = \frac{(2.00\,\mathrm{m})^2}{2}(0.870\,\mathrm{T/s}) = 1.74\ \mathrm{V}\ .$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is $\mathcal{E}+\mathcal{E}_i = 20.0\,\mathrm{V}+1.74\,\mathrm{V} = 21.7\,\mathrm{V}$.

(b) The current is in the sense of the total emf (counterclockwise).

16. (a) Since $\vec{B} = B\,\hat{\imath}$ uniformly, then only the area "projected" onto the yz plane will contribute to the flux (due to the scalar [dot] product). This "projected" area corresponds to one-fourth of a circle. Thus, the magnetic flux Φ_B through the loop is

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \frac{1}{4}\pi r^2 B\ .$$

Thus,

$$\begin{aligned} |\mathcal{E}| &= \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right| \\ &= \frac{1}{4}\pi(0.10\,\mathrm{m})^2(3.0 \times 10^{-3}\,\mathrm{T/s}) = 2.4 \times 10^{-5}\ \mathrm{V}\ . \end{aligned}$$

(b) We have a situation analogous to that shown in Fig. 31-5(a). Thus, the current in segment bc flows from c to b (following Lenz's law).

17. (a) It should be emphasized that the result, given in terms of $\sin(2\pi ft)$, could as easily be given in terms of $\cos(2\pi ft)$ or even $\cos(2\pi ft + \phi)$ where ϕ is a phase constant as discussed in Chapter 16. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in

radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A = ab$, Faraday's law leads to

$$\mathcal{E} = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBab2\pi f\sin(2\pi ft)$$

which is the desired result, shown in the problem statement. The second way this is written ($\mathcal{E}_0\sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\mathcal{E}_0 = 2\pi fNabB$.

(b) We solve $\mathcal{E}_0 = 150\,\text{V} = 2\pi fNabB$ when $f = 60.0$ rev/s and $B = 0.500$ T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $Nab = 0.796\ \text{m}^2$. This means, for instance, that if we wanted the coil to have a square shape and consist of 50 turns, then the side length of the square would be $a = b = 0.126$ m.

18. (a) The rotational frequency (in revolutions per second) is identical to the time-dependent voltage frequency (in cycles per second, or Hertz). This conclusion should not be considered obvious, and the calculation shown in part (b) should serve to reinforce it.

(b) First, we define angle relative to the plane of Fig. 31-41, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0 which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$ then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi ft$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + \frac{\pi a^2}{2}\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\,\frac{d\cos(2\pi ft)}{dt}$$

which yields $\mathcal{E} = B\pi^2a^2f\sin(2\pi ft)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude: $\mathcal{E}_{\max} = B\pi^2a^2f$.

19. First we write $\Phi_B = BA\cos\theta$. We note that the angular position θ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA\cos\theta$ (as opposed to, say, $BA\sin\theta$). Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ if θ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and $f = 1000\,\text{rev/min} \approx 16.7$ rev/s is the frequency). Since the area of the rectangular coil is $A = 0.500 \times 0.300 = 0.150\ \text{m}^2$, Faraday's law leads to

$$\mathcal{E} = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBA2\pi f\sin(2\pi ft)$$

which means it has a voltage amplitude of

$$\mathcal{E}_{\max} = 2\pi fNAB = 2\pi(16.7\,\text{rev/s})(100\,\text{turns})(0.15\,\text{m}^2)(3.5\,\text{T}) = 5.50 \times 10^3\ \text{V}\ .$$

20. The field (due to the current in the straight wire) is out-of-the-page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

21. Consider a (thin) strip of area of height dy and width $\ell = 0.020$ m. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = B\,dA = (4t^2 y)(\ell\,dy)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^\ell \left(4t^2 y \ell\right) dy = 2t^2\ell^3 \ .$$

Thus, Faraday's law yields

$$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 \ .$$

At $t = 2.5$ s, we find the magnitude of the induced emf is 8.0×10^{-5} V. Its "direction" (or "sense") is clockwise, by Lenz's law.

22. (a) First, we observe that a large portion of the figure contributes flux which "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b - a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b - a$ away from it) has exactly the equal-but-opposite flux which cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B\,dA = \int_{b-a}^{a} \left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right) \ .$$

Faraday's law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}
\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi} \ln\left(\frac{a}{b-a}\right)\right] \\
&= -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a}\right)\frac{di}{dt} = -\frac{\mu_0 b}{2\pi} \ln\left(\frac{a}{b-a}\right)\frac{d}{dt}\left(\frac{9}{2}t^2 - 10t\right) \\
&= \frac{-\mu_0 b(9t-10)}{2\pi} \ln\left(\frac{a}{b-a}\right) \ .
\end{aligned}$$

With $a = 0.120$ m and $b = 0.160$ m, then, at $t = 3.00$ s, the magnitude of the emf induced in the rectangular loop is

$$|\mathcal{E}| = \frac{\left(4\pi \times 10^{-7}\right)(0.16)(9(3) - 10)}{2\pi} \ln\left(\frac{0.12}{0.16 - 0.12}\right) = 5.98 \times 10^{-7}\ \text{V} \ .$$

(b) We note that $\frac{di}{dt} > 0$ at $t = 3$ s. The situation is roughly analogous to that shown in Fig. 31-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

23. (a) We refer to the (very large) wire length as L and seek to compute the flux per meter: Φ_B/L. Using the right-hand rule discussed in Chapter 30, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 30-19 and Eq. 30-22. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\text{mm} = 0.020\,\text{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\,(L\,dx) + 2\int_{d/2}^{\ell/2} B\,(L\,dx)$$

where $d = 0.0025$ m is the diameter of each wire. We will use $R = d/2$, and r instead of x in the following steps. Thus, using the equations from Ch. 30 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2\int_0^R \left(\frac{\mu_0 i}{2\pi R^2}r + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr + 2\int_R^{\ell/2}\left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{\ell - R}{\ell}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{\ell - R}{R}\right) \\ &= 0.23 \times 10^{-5}\,\text{T}\cdot\text{m} + 1.08 \times 10^{-5}\,\text{T}\cdot\text{m}\end{aligned}$$

which yields $\Phi_B/L = 1.3 \times 10^{-5}\,\text{T}\cdot\text{m}$ or $1.3 \times 10^{-5}\,\text{Wb/m}$.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wires was computed above to be $0.23 \times 10^{-5}\,\text{T}\cdot\text{m}$. Thus,

$$\frac{0.23 \times 10^{-5}\,\text{T}\cdot\text{m}}{1.3 \times 10^{-5}\,\text{T}\cdot\text{m}} = 0.17 = 17\%\ .$$

(c) What was described in part (a) as a symmetry plane at $x = \ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0 < x < \ell/2$ region is now of opposite sign of the flux in the $\ell/2 < x < \ell$ region which causes the total flux (or, in this case, flux per meter) to be zero.

24. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 30-19). We integrate according to Eq. 31-3, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2}\left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 i a}{2\pi}\ln\left(\frac{r + \frac{b}{2}}{r - \frac{b}{2}}\right)\ .$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $\frac{dr}{dt} = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$i_{\text{loop}} = \left|\frac{\mathcal{E}}{R}\right| = -\frac{\mu_0 i a}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r + \frac{b}{2}}{r - \frac{b}{2}}\right)\right| = \frac{\mu_0 i a b v}{2\pi R\,(r^2 - (b/2)^2)}\ .$$

25. Thermal energy is generated at the rate $P = \mathcal{E}^2/R$ (see Eq. 27-23). Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is $1.69 \times 10^{-8}\,\Omega\cdot\text{m}$ (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire ($d = 0.00100$ m is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire ($L = 0.500$ m) is the circumference of the loop. This enclosed area is used in Faraday's law (where we ignore minus signs in the interest of finding the magnitudes of the quantities):

$$\mathcal{E} = \frac{d\Phi_B}{dt} = A_{\text{loop}}\frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

where the rate of change of the field is $dB/dt = 0.0100$ T/s. Consequently, we obtain

$$P = \frac{\left(\frac{L^2}{4\pi}\frac{dB}{dt}\right)^2}{4\rho L/\pi d^2} = \frac{d^2 L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = 3.68 \times 10^{-6}\ \text{W}\ .$$

26. Noting that $|\Delta B| = B$, we find the thermal energy is

$$P_{\text{thermal}}\Delta t = \frac{\mathcal{E}^2 \Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2 \Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2 \Delta t = \frac{A^2 B^2}{R\Delta t} \ .$$

27. (a) Eq. 31-10 leads to

$$\mathcal{E} = BLv = (0.350\,\mathrm{T})(0.250\,\mathrm{m})(0.550\,\mathrm{m/s}) = 0.0481\ \mathrm{V} \ .$$

(b) By Ohm's law, the induced current is $i = 0.0481\,\mathrm{V}/18.0\,\Omega = 0.00267\,\mathrm{A}$. By Lenz's law, the current is clockwise in Fig. 31-46.

(c) Eq. 27-22 leads to $P = i^2R = 0.000129$ W.

28. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\mathcal{E}|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_tL}{R} \ ,$$

which yields $v_t = mgR/B^2L^2$.

29. (a) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page. Eq. 31-10 leads to

$$\mathcal{E} = BLv = (1.2\,\mathrm{T})(0.10\,\mathrm{m})(5.0\,\mathrm{m/s}) = 0.60\ \mathrm{V} \ .$$

(b) By Ohm's law, the (clockwise) induced current is $i = 0.60\,\mathrm{V}/0.40\,\Omega = 1.5\,\mathrm{A}$.

(c) Eq. 27-22 leads to $P = i^2R = 0.90$ W.

(d) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5\,\mathrm{A})(0.10\,\mathrm{m})(1.2\,\mathrm{T}) = 0.18\ \mathrm{N} \ .$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(e) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent: $P = Fv = (0.18\,\mathrm{N})(5.0\,\mathrm{m/s}) = 0.90$ W, which is the same as our result from part (c).

30. (a) The "height" of the triangular area enclosed by the rails and bar is the same as the distance traveled in time v: $d = vt$, where $v = 5.20\,\mathrm{m/s}$. We also note that the "base" of that triangle (the distance between the intersection points of the bar with the rails) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2 \ .$$

Since the field is a uniform $B = 0.350\,\mathrm{T}$, then the magnitude of the flux (in SI units) is $\Phi_B = BA = (0.350)(5.20)^2t^2 = 9.46t^2$. At $t = 3.00$ s, we obtain $\Phi_B = 85.2\,\mathrm{Wb}$.

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\mathcal{E} = \frac{d\Phi_B}{dt} = 9.46\,\frac{dt^2}{dt} = 18.9t$$

in SI units. At $t = 3.00$ s, this yields $\mathcal{E} = 56.8$ V.

(c) Our calculation in part (b) shows that $n = 1$.

31\. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 30-19, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux $d\Phi_B = (\mu_0 ix/2\pi r)dr$. By Eq. 31-3, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right) .$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\mathcal{E} &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\mathrm{T\cdot m/A})(100\mathrm{A})(5.00\,\mathrm{m/s})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.40\times10^{-4}\ \mathrm{V} .\end{aligned}$$

(b) By Ohm's law, the induced current is $i_\ell = \mathcal{E}/R = (2.40\times10^{-4}\,\mathrm{V})/(0.400\,\Omega) = 6.00\times10^{-4}\,\mathrm{A}$. Since the flux is increasing the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate $P = i_\ell^2 R = (6.00\times10^{-4}\mathrm{A})^2(0.400\,\Omega) = 1.44\times10^{-7}\mathrm{W}$.

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr at a distance r from the long straight wire, is $dF_B = i_\ell B\,dr = (\mu_0 i_\ell i/2\pi r)\,dr$. We integrate to find the magnitude of the total magnetic force on the rod:

$$\begin{aligned}F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\mathrm{T\cdot m/A})(6.00\times10^{-4}\,\mathrm{A})(100\mathrm{A})}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right)\\ &= 2.87\times10^{-8}\ \mathrm{N} .\end{aligned}$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of $2.87\times10^{-8}\,\mathrm{N}$, to the left.

(e) By Eq. 7-48, the external agent does work at the rate $P = Fv = (2.87\times10^{-8}\,\mathrm{N})(5.00\,\mathrm{m/s}) = 1.44\times10^{-7}\,\mathrm{W}$. This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

32\.

$$\begin{aligned}\oint_1 \vec{E}\cdot d\vec{s} &= -\frac{d\vec{\Phi}_{B1}}{dt} = \frac{d}{dt}(B_1A_1) = A_1\frac{dB_1}{dt} = \pi r_1^2\frac{dB_1}{dt}\\ &= \pi(0.200\,\mathrm{m})^2(-8.50\times10^{-3}\,\mathrm{T/s}) = -1.07\times10^{-3}\ \mathrm{V}\end{aligned}$$

$$\begin{aligned}\oint_2 \vec{E}\cdot d\vec{s} &= -\frac{d\vec{\Phi}_{B2}}{dt} = \pi r_2^2\frac{dB_2}{dt}\\ &= \pi(0.300\,\mathrm{m})^2(-8.50\times10^{-3}\,\mathrm{T/s}) = -2.40\times10^{-3}\ \mathrm{V}\end{aligned}$$

$$\oint_3 \vec{E}\cdot d\vec{s} = \oint_1 \vec{E}\cdot d\vec{s} - \oint_2 \vec{E}\cdot d\vec{s} = -1.07\times10^{-3}\,\mathrm{V} - (-2.4\times10^{-3}\,\mathrm{V}) = 1.33\times10^{-3}\ \mathrm{V}$$

33. (a) The point at which we are evaluating the field is inside the solenoid, so Eq. 31-27 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})(0.0220\,\mathrm{m}) = 7.15 \times 10^{-5}\ \mathrm{V/m}\ .$$

(b) Now point at which we are evaluating the field is outside the solenoid and Eq. 31-29 applies. The magnitude of the induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}(6.5 \times 10^{-3}\,\mathrm{T/s})\frac{(0.0600\,\mathrm{m})^2}{(0.0820\,\mathrm{m})} = 1.43 \times 10^{-4}\ \mathrm{V/m}\ .$$

34. The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0)\ ,$$

where $B_0 = (30.0\,\mathrm{T} + 29.6\,\mathrm{T})/2 = 29.8\,\mathrm{T}$ and $B_1 = (30.0\,\mathrm{T} - 29.6\,\mathrm{T})/2 = 0.200\,\mathrm{T}$. Then from Eq. 31-27

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}[B_0 + B_1\sin(\omega t + \phi_0)] = \frac{1}{2}B_1\omega r\cos(\omega t + \phi_0)\ .$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\mathrm{T})(2\pi)(15\,\mathrm{Hz})(1.6 \times 10^{-2}\,\mathrm{m}) = 0.15\ \mathrm{V/m}\ .$$

35. We use Faraday's law in the form $\oint \vec{E}\cdot d\vec{s} = -(d\Phi_B/dt)$, integrating along the dotted path shown in the Figure. At all points on the upper and lower sides the electric field is either perpendicular to the side or else it vanishes. We assume it vanishes at all points on the right side (outside the capacitor). On the left side it is parallel to the side and has constant magnitude. Thus, direct integration yields $\oint \vec{E}\cdot d\vec{s} = EL$, where L is the length of the left side of the rectangle. The magnetic field is zero and remains zero, so $d\Phi_B/dt = 0$. Faraday's law leads to a contradiction: $EL = 0$, but neither E nor L is zero. Therefore, there must be an electric field along the right side of the rectangle.

36. (a) We interpret the question as asking for N multiplied by the flux through one turn:

$$\Phi_{\mathrm{turns}} = N\Phi_B = NBA = NB(\pi r^2) = (30.0)(2.60 \times 10^{-3}\,\mathrm{T})(\pi)(0.100\,\mathrm{m})^2 = 2.45 \times 10^{-3}\ \mathrm{Wb}\ .$$

(b) Eq. 31-35 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45 \times 10^{-3}\,\mathrm{Wb}}{3.80\,\mathrm{A}} = 6.45 \times 10^{-4}\ \mathrm{H}\ .$$

37. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{(8.0 \times 10^{-3}\,\mathrm{H})(5.0 \times 10^{-3}\,\mathrm{A})}{400} = 1.0 \times 10^{-7}\ \mathrm{Wb}\ .$$

38. (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is $B = \mu_0 n\Delta i = \mu_0(N/W)(i/N) = \mu_0 i/W$.

(b) Eq. 31-35 leads to

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2(\mu_0 i/W)}{i} = \frac{\pi\mu_0 R^2}{W}\ .$$

39. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B/ℓ. Using the right-hand rule discussed in Chapter 30, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 30-19 and Eq. 30-22. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\,dA = 2\int_0^a B\,(\ell\,dx) + 2\int_a^{d/2} B\,(\ell\,dx)$$

where $d = 0.0025$ m is diameter of each wire. We will r instead of x in the following steps. Thus, using the equations from Ch. 30 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\pi a^2}r + \frac{\mu_0 i}{2\pi(d-r)}\right) dr + 2\int_a^{d/2}\left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{d-a}{d}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i\ell/\pi \ln((d-a)/a)$. Now, we use Eq. 31-35 (with $N = 1$) to obtain the inductance:

$$L = \frac{\Phi_B}{i} = \frac{\mu_0 \ell}{\pi}\ln\left(\frac{d-a}{a}\right) .$$

40. (a) Speaking anthropomorphically, the coil wants to fight the changes – so if it wants to push current rightward (when the current is already going rightward) then i must be in the process of decreasing.

(b) From Eq. 31-37 (in absolute value) we get

$$L = \left|\frac{\mathcal{E}}{di/dt}\right| = \frac{17\,\text{V}}{2.5\,\text{kA/s}} = 6.8 \times 10^{-4}\ \text{H} .$$

41. Since $\mathcal{E} = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\mathcal{E}}{L} = -\frac{60\,\text{V}}{12\,\text{H}} = -5.0\ \text{A/s} .$$

We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

42. During periods of time when the current is varying linearly with time, Eq. 31-37 (in absolute values) becomes $|\mathcal{E}| = L\left|\frac{\Delta i}{\Delta t}\right|$. For simplicity, we omit the absolute value signs in the following.

(a) For $0 < t < 2$ ms

$$\mathcal{E} = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(7.0\,\text{A} - 0)}{2.0 \times 10^{-3}\,\text{s}} = 1.6 \times 10^4\ \text{V} .$$

(b) For $2\,\text{ms} < t < 5\,\text{ms}$

$$\mathcal{E} = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(5.0\,\text{A} - 7.0\,\text{A})}{(5.0 - 2.0)10^{-3}\,\text{s}} = 3.1 \times 10^3\ \text{V} .$$

(c) For $5\,\text{ms} < t < 6\,\text{ms}$

$$\mathcal{E} = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(0 - 5.0\,\text{A})}{(6.0 - 5.0)10^{-3}\,\text{s}} = 2.3 \times 10^4\ \text{V} .$$

43. (a) Voltage is proportional to inductance (by Eq. 31-37) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ($V_1 + V_2$), then inductances in series must *add* just as was the case for resistances.

(b) To ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §31-12). The requirement is that magnetic field lines from one inductor should not have significant significant presence in any other.

(c) Just as with resistors, $L_{\rm eq} = \sum_{n=1}^{N} L_n$.

44. (a) Voltage is proportional to inductance (by Eq. 31-37) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 28-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt} \; .$$

Thus, although the inductance equation Eq. 31-37 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also applies to inductors. Therefore,

$$\frac{1}{L_{\rm eq}} = \frac{1}{L_1} + \frac{1}{L_2} \; .$$

(b) To ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §31-12). The requirement is that the field of one inductor not have significant influence (or "coupling") in the next.

(c) Just as with resistors, $\frac{1}{L_{\rm eq}} = \sum_{n=1}^{N} \frac{1}{L_n}$.

45. Starting with zero current at $t = 0$ (the moment the switch is closed) the current in the circuit increases according to

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \; ,$$

where $\tau_L = L/R$ is the inductive time constant and $\mathcal{E}$ is the battery emf. To calculate the time at which $i = 0.9990\mathcal{E}/R$, we solve for t:

$$0.9990\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \quad \Longrightarrow \quad \ln(0.0010) = -(t/\tau) \quad \Longrightarrow \quad t = 6.91\tau_L \; .$$

46. The steady state value of the current is also its maximum value, $\mathcal{E}/R$, which we denote as i_m. We are told that $i = i_m/3$ at $t_0 = 5.00\,{\rm s}$. Eq. 31-43 becomes $i = i_m(1 - e^{-t_0/\tau_L})$, which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00\,{\rm s}}{\ln(1 - 1/3)} = 12.3 \text{ s} \; .$$

47. The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R) . We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L} \; .$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0\,{\rm s}}{\ln((10 \times 10^{-3}\,{\rm A})/(1.0\,{\rm A}))} = 0.217 \text{ s} \; .$$

Therefore, $R = L/\tau_L = 10\,{\rm H}/0.217\,{\rm s} = 46\,\Omega$.

48. (a) Immediately after the switch is closed $\mathcal{E} - \mathcal{E}_L = iR$. But $i = 0$ at this instant, so $\mathcal{E}_L = \mathcal{E}$.

(b) $\mathcal{E}_L(t) = \mathcal{E}e^{-t/\tau_L} = \mathcal{E}e^{-2.0\tau_L/\tau_L} = \mathcal{E}e^{-2.0} = 0.135\mathcal{E}$.

(c) From $\mathcal{E}_L(t) = \mathcal{E}e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\mathcal{E}}{\mathcal{E}_L}\right) = \ln 2 \implies t = \tau_L \ln 2 \quad = \quad 0.693\tau_L \ .$$

49. (a) If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \ ,$$

where $\tau_L = L/R$. Our goal is to find the time at which $i = 0.800\mathcal{E}/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \quad \implies \quad e^{-t/\tau_L} = 0.200 \ .$$

Taking the natural logarithm of both sides, we obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30 \times 10^{-6}\,\text{H})}{1.20 \times 10^3\,\Omega} = 8.45 \times 10^{-9} \text{ s} \ .$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\text{V}}{1.20 \times 10^3\,\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37 \times 10^{-3} \text{ A} \ .$$

50. Applying the loop theorem

$$\mathcal{E} - L\left(\frac{di}{dt}\right) = iR \quad ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\begin{aligned}
\mathcal{E} &= L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0 + 5.0t) + (3.0 + 5.0t)R \\
&= (6.0)(5.0) + (3.0 + 5.0t)(4.0) \\
&= (42 + 20t)
\end{aligned}$$

in volts if t is in seconds.

51. Taking the time derivative of both sides of Eq. 31-43, we obtain

$$\begin{aligned}
\frac{di}{dt} &= \frac{d}{dt}\left[\frac{\mathcal{E}}{R}\left(1 - e^{-Rt/\tau_L}\right)\right] = \frac{\mathcal{E}}{L}e^{-RT/L} \\
&= \left(\frac{45.0\,\text{V}}{50.0 \times 10^{-3}\,\text{H}}\right)e^{-(180\,\Omega)(1.20\times 10^{-3}\,\text{s})/50.0\times 10^{-3}\,\text{H}} = 12.0 \text{ A/s} \ .
\end{aligned}$$

52. (a) Our notation is as follows: h is the height of the toroid, a its inner radius, and b its outer radius. Since it has a square cross section, $h = b - a = 0.12\,\text{m} - 0.10\,\text{m} = 0.02\,\text{m}$.. We derive the flux using Eq. 30-26 and the self-inductance using Eq. 31-35:

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi} \ln\left(\frac{b}{a}\right)$$

and $L = N\Phi_B/i = (\mu_0 N^2 h/2\pi)\ln(b/a)$. We note that the formulas for Φ_B and L can also be found in the Supplement for the chapter, in Sample Problem 31-11. Now, since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10\,\text{cm}) \approx 62.8\,\text{cm}$, the number of turns of the toroid is roughly $N \approx 62.8\,\text{cm}/1.0\,\text{mm} = 628$. Thus

$$\begin{aligned} L &= \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) \\ &\approx \frac{\left(4\pi \times 10^{-7}\,\text{H/m}\right)(628)^2(0.02\,\text{m})}{2\pi} \ln\left(\frac{12}{10}\right) \\ &= 2.9 \times 10^{-4}\ \text{H}\ . \end{aligned}$$

(b) Noting that the perimeter of a square is four times its sides, the total length ℓ of the wire is $\ell = (628)4(2.0\,\text{cm}) = 50\,\text{m}$, the resistance of the wire is $R = (50\,\text{m})(0.02\,\Omega/\text{m}) = 1.0\,\Omega$. Thus

$$\tau_L = \frac{L}{R} = \frac{2.9 \times 10^{-4}\,\text{H}}{1.0\,\Omega} = 2.9 \times 10^{-4}\ \text{s}\ .$$

53. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = i_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\ \text{A}\ .$$

(b) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\mathcal{E} - i_1 R_1 - i_2 R_2 = 0 \quad \text{and} \quad \mathcal{E} - i_1 R_1 - (i_1 - i_2) R_3 = 0\ .$$

We solve these simultaneously for i_1 and i_2. The results are

$$\begin{aligned} i_1 &= \frac{\mathcal{E}\,(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ &= \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)} \\ &= 4.55\ \text{A}\ , \end{aligned}$$

and

$$\begin{aligned} i_2 &= \frac{\mathcal{E} R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \\ &= \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)} \\ &= 2.73\ \text{A}\ . \end{aligned}$$

(c) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$). The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is $4.55\,\text{A} - 2.73\,\text{A} = 1.82\,\text{A}$. The current in R_2 is the same as that in R_3 ($1.82\,\text{A}$).

(d) There are no longer any sources of emf in the circuit, so all currents eventually drop to zero.

54. (a) When switch S is just closed (case I), $V_1 = \mathcal{E}$ and $i_1 = \mathcal{E}/R_1 = 10\,\text{V}/5.0\,\Omega = 2.0\,\text{A}$. After a long time (case II) we still have $V_1 = \mathcal{E}$, so $i_1 = 2.0\,\text{A}$.

(b) Case I: since now $\mathcal{E}_L = \mathcal{E}$, $i_2 = 0$; case II: since now $\mathcal{E}_L = 0$, $i_2 = \mathcal{E}/R_2 = 10\,\text{V}/10\,\Omega = 1.0\,\text{A}$.

(c) Case I: $i = i_1 + i_2 = 2.0\,\text{A} + 0 = 2.0\,\text{A}$; case II: $i = i_1 + i_2 = 2.0\,\text{A} + 1.0\,\text{A} = 3.0\,\text{A}$.

(d) Case I: since $\mathcal{E}_L = \mathcal{E}$, $V_2 = \mathcal{E} - \mathcal{E}_L = 0$; case II: since $\mathcal{E}_L = 0$, $V_2 = \mathcal{E} - \mathcal{E}_L = \mathcal{E} = 10\,\mathrm{V}$.

(e) Case I: $\mathcal{E}_L = \mathcal{E} = 10\,\mathrm{V}$; case II: $\mathcal{E}_L = 0$.

(f) Case I: $di_2/dt = \mathcal{E}_L/L = \mathcal{E}/L = 10\,\mathrm{V}/5.0\,\mathrm{H} = 2.0\,\mathrm{A/s}$; case II: $di_2/dt = \mathcal{E}_L/L = 0$.

55. (a) We assume i is from left to right through the closed switch. We let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor, also assumed downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1 R - L(di_2/dt) = 0$. According to the junction rule, $(di_1/dt) = -(di_2/dt)$. We substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1 R = 0 \; .$$

This equation is similar to Eq. 31-48, and its solution is the function given as Eq. 31-49:

$$i_1 = i_0 e^{-Rt/L} \; ,$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L} \qquad \text{and} \qquad i_2 = i - i_1 = i\left(1 - e^{-Rt/L}\right) \; .$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L} \implies e^{-Rt/L} = \frac{1}{2} \; .$$

Taking the natural logarithm of both sides (and using $\ln(1/2) = -\ln 2$) we obtain

$$\left(\frac{Rt}{L}\right) = \ln 2 \implies t = \frac{L}{R}\ln 2 \; .$$

56. Let $U_B(t) = \frac{1}{2}Li^2(t)$. We require the energy at time t to be half of its final value: $U(t) = \frac{1}{2}U_B(t \to \infty) = \frac{1}{4}Li_f^2$. This gives $i(t) = i_f/\sqrt{2}$. But $i(t) = i_f(1 - e^{-t/\tau_L})$, so

$$1 - e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \implies t = -\tau_L \ln\left(1 - \frac{1}{\sqrt{2}}\right) = 1.23\tau_L \; .$$

57. From Eq. 31-51 and Eq. 31-43, the rate at which the energy is being stored in the inductor is

$$\begin{aligned}\frac{dU_B}{dt} &= \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = L\,i\,\frac{di}{dt} \\ &= L\left(\frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)\right)\left(\frac{\mathcal{E}}{R}\frac{1}{\tau_L}\,e^{-t/\tau_L}\right) \\ &= \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)e^{-t/\tau_L}\end{aligned}$$

where $\tau_L = L/R$ has been used. From Eq. 27-22 and Eq. 31-43, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\mathcal{E}^2}{R^2}\left(1 - e^{-t/\tau_L}\right)^2 R = \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)^2 \; .$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)^2 = \frac{\mathcal{E}^2}{R}\left(1 - e^{-t/\tau_L}\right)e^{-t/\tau_L} \implies t = \tau_L \ln 2 = (37.0\,\mathrm{ms})\ln 2 = 25.6\ \mathrm{ms} \; .$$

58. (a) From Eq. 31-51 and Eq. 31-43, the rate at which the energy is being stored in the inductor is

$$\begin{aligned}\frac{dU_B}{dt} &= \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = L\,i\,\frac{di}{dt} \\ &= L\left(\frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\mathcal{E}}{R}\frac{1}{\tau_L}\,e^{-t/\tau_L}\right) \\ &= \frac{\mathcal{E}^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}\ .\end{aligned}$$

Now, $\tau_L = L/R = 2.0\,\text{H}/10\,\Omega = 0.20\,\text{s}$ and $\mathcal{E} = 100\,\text{V}$, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 27-22 and Eq. 31-43, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2R = \frac{\mathcal{E}^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\mathcal{E}^2}{R}\left(1-e^{-t/\tau_L}\right)^2\ .$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2 \text{ W}\ .$$

We note that this could result could alternatively have been found from Eq. 28-14 (with Eq. 31-43).

59. (a) If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\mathcal{E}}{R}\left(1-e^{-t/\tau_L}\right)\ ,$$

where $\mathcal{E}$ is the emf of the battery, R is the resistance, and τ_L is the inductive time constant (L/R). This leads to

$$e^{-t/\tau_L} = 1 - \frac{iR}{\mathcal{E}} \implies -\frac{t}{\tau_L} = \ln\left(1-\frac{iR}{\mathcal{E}}\right)\ .$$

Since

$$\ln\left(1-\frac{iR}{\mathcal{E}}\right) = \ln\left[1-\frac{(2.00\times10^{-3}\,\text{A})(10.0\times10^3\,\Omega)}{50.0\,\text{V}}\right] = -0.5108\ ,$$

the inductive time constant is $\tau_L = t/0.5108 = (5.00 \times 10^{-3}\,\text{s})/0.5108 = 9.79 \times 10^{-3}\,\text{s}$ and the inductance is

$$L = \tau_L R = (9.79\times10^{-3}\,\text{s})(10.0\times10^3\,\Omega) = 97.9 \text{ H}\ .$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\text{H})(2.00\times10^{-3}\,\text{A})^2 = 1.96\times10^{-4}\text{ J}\ .$$

60. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 31-43 for the current):

$$\begin{aligned}\int_0^t P_{\text{battery}}\,dt &= \int_0^t \frac{\mathcal{E}^2}{R}\left(1-e^{-Rt/L}\right)dt = \frac{\mathcal{E}^2}{R}\left[t+\frac{L}{R}\left(e^{-Rt/L}-1\right)\right] \\ &= \frac{(10.0\,\text{V})^2}{6.70\,\Omega}\left[2.00\,\text{s}+\frac{(5.50\,\text{H})\left(e^{-(6.70\,\Omega)(2.00\,\text{s})/5.50\,\text{H}}-1\right)}{6.70\,\Omega}\right] \\ &= 18.7\text{ J}\ .\end{aligned}$$

(b) The energy stored in the magnetic field is given by Eq. 31-51:

$$\begin{aligned} U_B &= \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\mathcal{E}}{R}\right)^2(1-e^{-Rt/L})^2 \\ &= \frac{1}{2}(5.50\,\mathrm{H})\left(\frac{10.0\,\mathrm{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\,\Omega)(2.00\,\mathrm{s})/5.50\,\mathrm{H}}\right]^2 \\ &= 5.10\ \mathrm{J}\ . \end{aligned}$$

(c) The difference of the previous two results gives the amount "lost" in the resistor: $18.7\,\mathrm{J} - 5.10\,\mathrm{J} = 13.6\,\mathrm{J}$.

61. Suppose that the switch had been in position a for a long time so that the current had reached the steady-state value i_0. The energy stored in the inductor is $U_B = \frac{1}{2}Li_0^2$. Now, the switch is thrown to position b at time $t = 0$. Thereafter the current is given by

$$i = i_0 e^{-t/\tau_L}\ ,$$

where τ_L is the inductive time constant, given by $\tau_L = L/R$. The rate at which thermal energy is generated in the resistor is given by

$$P = i^2R = i_0^2Re^{-2t/\tau_L}\ .$$

Over a long time period the energy dissipated is

$$\int_0^\infty P\,dt = i_0^2R\int_0^\infty e^{-2t/\tau_L}\,dt = -\frac{1}{2}i_0^2R\tau_Le^{-2t/\tau_L}\Big|_0^\infty = \frac{1}{2}i_0^2R\tau_L\ .$$

Upon substitution of $\tau_L = L/R$ this becomes $\frac{1}{2}Li_0^2$, the same as the total energy originally stored in the inductor.

62. The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2}Li^2$, where L is its inductance and i is the current. By Eq. 31-56, the energy is also given by $U_B = u_B\mathcal{V}$, where u_B is the average energy density and $\mathcal{V}$ is the volume. Thus

$$i = \sqrt{\frac{2u_B\mathcal{V}}{L}} = \sqrt{\frac{2\left(70.0\,\mathrm{J/m^3}\right)(0.0200\,\mathrm{m^3})}{90.0\times10^{-3}\,\mathrm{H}}} = 5.58\ \mathrm{A}\ .$$

63. (a) At any point the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid $B = \mu_0 ni$, where n, for the solenoid of this problem, is $(950\,\mathrm{turns})/(0.850\,\mathrm{m}) = 1.118\times10^3\,\mathrm{m^{-1}}$. The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0n^2i^2 = \frac{1}{2}\left(4\pi\times10^{-7}\,\mathrm{T{\cdot}m/A}\right)\left(1.118\times10^3\,\mathrm{m^{-1}}\right)^2(6.60\,\mathrm{A})^2 = 34.2\ \mathrm{J/m^3}\ .$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B\mathcal{V}$, where $\mathcal{V}$ is the volume of the solenoid. $\mathcal{V}$ is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = (34.2\,\mathrm{J/m^3})(17.0\times10^{-4}\,\mathrm{m^2})(0.850\,\mathrm{m}) = 4.94\times10^{-2}\ \mathrm{J}\ .$$

64. We use $1\,\mathrm{ly} = 9.46\times10^{15}\,\mathrm{m}$, and use the symbol $\mathcal{V}$ for volume.

$$U_B = \mathcal{V}u_B = \frac{\mathcal{V}B^2}{2\mu_0} = \frac{(9.46\times10^{15}\,\mathrm{m})^3(1\times10^{-10}\,\mathrm{T})^2}{2\,(4\pi\times10^{-7}\,\mathrm{H/m})} = 3\times10^{36}\ \mathrm{J}\ .$$

65. We set $u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2}B^2/\mu_0$ and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\varepsilon_0\mu_0}} = \frac{0.50\,\mathrm{T}}{\sqrt{(8.85\times10^{-12}\,\mathrm{F/m})\,(4\pi\times10^{-7}\,\mathrm{H/m})}} = 1.5\times10^{8}\ \mathrm{V/m}\ .$$

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 30-11, is

$$B = \frac{\mu_0 i}{2R} = \frac{\left(4\pi\times10^{-7}\,\mathrm{H/m}\right)(100\,\mathrm{A})}{2(50\times10^{-3}\,\mathrm{m})} = 1.3\times10^{-3}\ \mathrm{T}\ .$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3\times10^{-3}\,\mathrm{T})^2}{2\,(4\pi\times10^{-7}\,\mathrm{H/m})} = 0.63\ \mathrm{J/m^3}\ .$$

67. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2R}\right)^2 = \frac{\mu_0 i^2}{8R^2} = \frac{\left(4\pi\times10^{-7}\,\mathrm{H/m}\right)(10\,\mathrm{A})^2}{8(2.5\times10^{-3}\,\mathrm{m}/2)^2} = 1.0\ \mathrm{J/m^3}\ .$$

(b) The electric energy density is

$$\begin{aligned} u_E &= \frac{1}{2}\varepsilon_0 E^2 = \frac{\epsilon_0}{2}(\rho J)^2 = \frac{\varepsilon_0}{2}\left(\frac{iR}{\ell}\right)^2 \\ &= \frac{1}{2}(8.85\times10^{-12}\,\mathrm{F/m})\left[(10\,\mathrm{A})(3.3\,\Omega/10^3\,\mathrm{m})\right]^2 \\ &= 4.8\times10^{-15}\ \mathrm{J/m^3}\ . \end{aligned}$$

Here we used $J = i/A$ and $R = \rho\ell/A$ to obtain $\rho J = iR/\ell$.

68. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\,\mathrm{mH})(6.0\,\mathrm{mA})}{100} = 1.5\ \mu\mathrm{Wb}\ ,$$

and the magnitude of the self-induced emf is

$$L_1\frac{di_1}{dt} = (25\,\mathrm{mH})(4.0\,\mathrm{A/s}) = 100\ \mathrm{mV}\ .$$

(b) In coil 2, we find

$$\Phi_{21} = \frac{Mi_1}{N_2} = \frac{(3.0\,\mathrm{mH})(6.0\,\mathrm{mA})}{200} = 90\ \mathrm{nWb}\ ,$$

$$\mathcal{E}_{21} = M\frac{di_1}{dt} = (3.0\,\mathrm{mH})(4.0\,\mathrm{A/s}) = 12\ \mathrm{mV}\ .$$

69. (a) Eq. 31-67 yields

$$M = \frac{\mathcal{E}_1}{|di_2/dt|} = \frac{25.0\,\mathrm{mV}}{15.0\,\mathrm{A/s}} = 1.67\ \mathrm{mH}\ .$$

(b) Eq. 31-62 leads to

$$N_2\Phi_{21} = Mi_1 = (1.67\,\mathrm{mH})(3.60\,\mathrm{A}) = 6.00\ \mathrm{mWb}\ .$$

70. We use $\mathcal{E}_2 = -M\,di_1/dt \approx M|\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\mathcal{E}}{\Delta i_1/\Delta t}\right| = \frac{30\times10^{3}\,\mathrm{V}}{6.0\,\mathrm{A}/(2.5\times10^{-3}\,\mathrm{s})} = 13\ \mathrm{H}\ .$$

71. (a) We assume the current is changing at (nonzero) rate di/dt and calculate the total emf across both coils. First consider the coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus, the induced emf's are

$$\mathcal{E}_1 = -\left(L_1 + M\right)\frac{di}{dt} \quad \text{and} \quad \mathcal{E}_2 = -\left(L_2 + M\right)\frac{di}{dt} \, .$$

Therefore, the total emf across both coils is

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -\left(L_1 + L_2 + 2M\right)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{\text{eq}} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of coil rather than the front (as pictured in the diagram). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\mathcal{E}_1 = -\left(L_1 - M\right)\frac{di}{dt} \, .$$

Similarly, the emf across coil 2 is

$$\mathcal{E}_2 = -\left(L_2 - M\right)\frac{di}{dt} \, .$$

The total emf across both coils is

$$\mathcal{E} = -\left(L_1 + L_2 - 2M\right)\frac{di}{dt} \, .$$

This the same as the emf that would be produced by a single coil with inductance $L_{\text{eq}} = L_1 + L_2 - 2M$.

72. The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N(\mu_0 i_s n\pi R^2)}{i_s} = \mu_0\pi R^2 nN \, .$$

As long as the magnetic field of the solenoid is entirely contained within the cross-section of the coil we have $\Phi_{sc} = B_s A_s = B_s\pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

73. Letting the current in solenoid 1 be i, we calculate the flux linkage in solenoid 2. The mutual inductance, then, is this flux linkage divided by i. The magnetic field inside solenoid 1 is parallel to the axis and has uniform magnitude $B = \mu_0 i n_1$, where n_1 is the number of turns per unit length of the solenoid. The cross-sectional area of the solenoid is πR_1^2. Since $\vec{B}$ is normal to the cross section, the flux here is

$$\Phi = AB = \pi R_1^2 \mu_0 n_1 i \, .$$

Since the magnetic field is zero outside the solenoid, this is also the flux through a cross section of solenoid 2. The number of turns in a length ℓ of solenoid 2 is $N_2 = n_2\ell$, and the flux linkage is

$$N_2\Phi = n_2\ell\pi R_1^2 \mu_0 n_1 i \, .$$

The mutual inductance is

$$M = \frac{N_2\Phi}{i} = \pi R_1^2 \ell\mu_0 n_1 n_2 \, .$$

M does not depend on R_2 because there is no magnetic field in the region between the solenoids. Changing R_2 does not change the flux through solenoid 2, but changing R_1 does.

74. We use the expression for the flux Φ_B over the toroid cross-section derived in our solution to problem 52 obtain the coil-toroid mutual inductance:

$$M_{ct} = \frac{N_c \Phi_{ct}}{i_t} = \frac{N_c}{i_t} \frac{\mu_0 i_t N_t h}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 N_1 N_2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

where $N_t = N_1$ and $N_c = N_2$. We note that the formula for Φ_B can also be found in the Supplement for the chapter, in Sample Problem 31-11.

75. (a) The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}} l \, dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} \, dr = \frac{\mu_0 i l}{2\pi} \ln\left(1 + \frac{b}{a}\right) .$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln\left(1 + \frac{b}{a}\right) .$$

(b) From the formula for M obtained above

$$M = \frac{(100)\left(4\pi \times 10^{-7}\,\text{H/m}\right)(0.30\,\text{m})}{2\pi} \ln\left(1 + \frac{8.0}{1.0}\right) = 1.3 \times 10^{-5}\ \text{H} .$$

76. For $t < 0$, no current goes through L_2, so $i_2 = 0$ and $i_1 = \mathcal{E}/R$. As the switch is opened there will be a very brief sparking across the gap. i_1 drops while i_2 increases, both very quickly. The loop rule can be written as

$$\mathcal{E} - i_1 R - L_1 \frac{di_1}{dt} - i_2 R - L_2 \frac{di_2}{dt} = 0 ,$$

where the initial value of i_1 at $t = 0$ is given by $\mathcal{E}/R$ and that of i_2 at $t = 0$ is 0. We consider the situation shortly after $t = 0$. Since the sparking is very brief, we can reasonably assume that both i_1 and i_2 get equalized quickly, before they can change appreciably from their respective initial values. Here, the loop rule requires that $L_1(di_1/dt)$, which is large and negative, must roughly cancel $L_2(di_2/dt)$, which is large and positive:

$$L_1 \frac{di_1}{dt} \approx -L_2 \frac{di_2}{dt} .$$

Let the common value reached by i_1 and i_2 be i, then

$$\frac{di_1}{dt} \approx \frac{\Delta i_1}{\Delta t} = \frac{i - \mathcal{E}/R}{\Delta t}$$

and

$$\frac{di_2}{dt} \approx \frac{\Delta i_2}{\Delta t} = \frac{i - 0}{\Delta t} .$$

The equations above yield

$$L_1 \left(i - \frac{\mathcal{E}}{R}\right) = -L_2(i - 0) \quad \Longrightarrow \quad i = \frac{\mathcal{E} L_1}{L_2 R_1 + L_1 R_2} = \frac{L_1}{L_1 + L_2} \frac{\mathcal{E}}{R} .$$

77. (a) $i_0 = \mathcal{E}/R = 100\,\text{V}/10\,\Omega = 10\,\text{A}$.

(b) $U_B = \frac{1}{2} L i_0^2 = \frac{1}{2}(2.0\,\text{H})(10\,\text{A})^2 = 100\,\text{J}$.

78. We write $i = i_0 e^{-t/\tau_L}$ and note that $i = 10\%\, i_0$. We solve for t:

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R} \ln\left(\frac{i_0}{i}\right) = \frac{2.00\,\text{H}}{3.00\,\Omega} \ln\left(\frac{i_0}{0.100\, i_0}\right) = 1.54\ \text{s} .$$

79. (a) The energy density at any point is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field. The magnitude of the field inside a toroid, a distance r from the center, is given by Eq. 30-26: $B = \mu_0 iN/2\pi r$, where N is the number of turns and i is the current. Thus

$$u_B = \frac{1}{2\mu_0}\left(\frac{\mu_0 iN}{2\pi r}\right)^2 = \frac{\mu_0 i^2 N^2}{8\pi^2 r^2} .$$

(b) We evaluate the integral $U_B = \int u_B \, d\mathcal{V}$ over the volume of the toroid. A circular strip with radius r, height h, and thickness dr has volume $d\mathcal{V} = 2\pi r h \, dr$, so

$$U_B = \frac{\mu_0 i^2 N^2}{8\pi^2} 2\pi h \int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 N^2 h}{4\pi} \ln\left(\frac{b}{a}\right) .$$

Substituting in the given values, we find

$$\begin{aligned} U_B &= \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(0.500\,\mathrm{A})^2(1250)^2(13 \times 10^{-3}\,\mathrm{m})}{4\pi} \ln\left(\frac{95\,\mathrm{mm}}{52\,\mathrm{mm}}\right) \\ &= 3.06 \times 10^{-4}\ \mathrm{J} . \end{aligned}$$

(c) The inductance is given in Sample Problem 31-11:

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

so the energy is given by

$$U_B = \frac{1}{2} L i^2 = \frac{\mu_0 N^2 i^2 h}{4\pi} \ln\left(\frac{b}{a}\right) .$$

This the exactly the same as the expression found in part (b) and yields the same numerical result.

80. If the solenoid is long and thin, then when it is bent into a toroid $(b-a)/a$ is much less than 1. Therefore,

$$L_{\text{toroid}} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 N^2 h}{2\pi} \ln\left(1 + \frac{b-a}{b}\right) \approx \frac{\mu_0 N^2 h(b-a)}{2\pi b} .$$

Since $A = h(b-a)$ is the cross-sectional area and $l = 2\pi b$ is the length of the toroid, we may rewrite this expression for the toroid self-inductance as

$$\frac{L_{\text{toroid}}}{l} \approx \frac{\mu_0 N^2 A}{l^2} = \mu_0 n^2 A ,$$

which indeed reduces to that of a long solenoid. Note that the approximation $\ln(1+x) \approx x$ is used for very small $|x|$.

81. Using Eq. 31-43

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$$

where $\tau_L = 2.0$ ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - \frac{iR}{\mathcal{E}}}\right) \approx 1.0\ \mathrm{ns} .$$

82. We note that $n = 100\,\text{turns/cm} = 10000\,\text{turns/m}$. The induced emf is

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -A\frac{d}{dt}(\mu_0 n i) = -\mu_0 n \pi r^2 \frac{di}{dt} \\ &= -(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(10000\,\mathrm{turn/m})(\pi)(25 \times 10^{-3}\,\mathrm{m})^2 \left(\frac{0.50\,\mathrm{A} - 1.0\,\mathrm{A}}{10 \times 10^{-3}\,\mathrm{s}}\right) \\ &= 1.2 \times 10^{-3}\ \mathrm{V} . \end{aligned}$$

Note that since $\vec{B}$ only appears inside the solenoid, the area A is be the cross-sectional area of the solenoid, not the (larger) loop.

83. With $\tau_L = L/R = 0.0010$ s, we find the current at $t = 0.0020$ s from Eq. 31-43:

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) = 0.86 \text{ A} .$$

Consequently, the energy stored, from Eq. 31-51, is

$$U_B = \frac{1}{2}Li^2 = 3.7 \times 10^{-3} \text{ J} .$$

84. (a) The magnitude of the average induced emf is

$$\mathcal{E}_{\text{avg}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{BA_i}{t} = \frac{(2.0\,\text{T})(0.20\,\text{m})^2}{0.20\,\text{s}} = -0.40 \text{ V} .$$

(b) The average induced current is

$$i_{\text{avg}} = \frac{\mathcal{E}_{\text{avg}}}{R} = \frac{0.40\,\text{V}}{20 \times 10^{-3}\,\Omega} = 20 \text{ A} .$$

85. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ any current through the battery is also that through the 20 Ω and 10 Ω resistors. Hence,

$$i = \frac{\mathcal{E}}{30\ \Omega} = 0.40 \text{ A}$$

which results in a voltage drop across the 10 Ω resistor equal to $(0.40)(10) = 4.0$ V. The inductor must have this same voltage across it $|\mathcal{E}_L|$, and we use (the absolute value of) Eq. 31-37:

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{4.0}{0.010} = 400 \text{ A/s} .$$

(b) Applying the loop rule to the outer loop, we have

$$\mathcal{E} - (0.50 \text{ A})(20\ \Omega) - |\mathcal{E}_L| = 0 \quad .$$

Therefore, $|\mathcal{E}_L| = 2.0$ V, and Eq. 31-37 leads to

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{2.0}{0.010} = 200 \text{ A/s} .$$

(c) As $t \to \infty$, the inductor has $\mathcal{E}_L = 0$ (since the current is no longer changing). Thus, the loop rule (for the outer loop) leads to

$$\mathcal{E} - i\,(20\ \Omega) - |\mathcal{E}_L| = 0 \quad \Longrightarrow \quad i = 0.60 \text{ A} .$$

86. (a) $L = \Phi/i = 26 \times 10^{-3}\,\text{Wb}/5.5\,\text{A} = 4.7 \times 10^{-3}\,\text{H}$.

(b) We use Eq. 31-43 to solve for t:

$$\begin{aligned} t &= -\tau_L \ln\left(1 - \frac{iR}{\mathcal{E}}\right) = -\frac{L}{R}\ln\left(1 - \frac{iR}{\mathcal{E}}\right) \\ &= -\frac{4.7 \times 10^{-3}\,\text{H}}{0.75\,\Omega}\ln\left[1 - \frac{(2.5\,\text{A})(0.75\,\Omega)}{6.0\,\text{V}}\right] = 2.4 \times 10^{-3} \text{ s} . \end{aligned}$$

87. (a) We use $U_B = \frac{1}{2}Li^2$ to solve for the self-inductance:

$$L = \frac{2U_B}{i^2} = \frac{2(25.0 \times 10^{-3}\,\text{J})}{(60.0 \times 10^{-3}\,\text{A})^2} = 13.9 \text{ H} .$$

(b) Since $U_B \propto i^2$, for U_B to increase by a factor of 4, i must increase by a factor of 2. Therefore, i should be increased to $2(60.0\,\text{mA}) = 120\,\text{mA}$.

88. (a) The self-inductance per meter is

$$\frac{L}{\ell} = \mu_0 n^2 A = \left(4\pi \times 10^{-7}\,\text{H/m}\right)(100\,\text{turns/cm})^2(\pi)(1.6\,\text{cm})^2 = 0.10\ \text{H/m}\ .$$

(b) The induced emf per meter is

$$\frac{\mathcal{E}}{\ell} = \frac{L}{\ell}\frac{di}{dt} = (0.10\,\text{H/m})(13\,\text{A/s}) = 1.3\ \text{V/m}\ .$$

89. (a) The energy needed is

$$U_E = u_E V = \frac{1}{2}\epsilon_0 E^2 V = \frac{1}{2}(8.85 \times 10^{-12}\,\text{F/m})(100\,\text{kV/m})^2(10\,\text{cm})^3 = 4.4 \times 10^{-5}\ \text{J}\ .$$

(b) The energy needed is

$$U_B = u_B V = \frac{1}{2\mu_0}B^2 V = \frac{(1.0\,\text{T})^2}{2\left(4\pi \times 10^{-7}\,\text{H/m}\right)}(10\,\text{cm})^3 = 4.0 \times 10^2\ \text{J}\ .$$

(c) Obviously, since $U_B > U_E$ greater amounts of energy can be stored in the magnetic field.

90. The induced electric field E as a function of r is given by $E(r) = (r/2)(dB/dt)$. So

$$\begin{aligned} a_c &= a_a = \frac{eE}{m} = \frac{er}{2m}\left(\frac{dB}{dt}\right) \\ &= \frac{(1.60 \times 10^{-19}\,\text{C})(5.0 \times 10^{-2}\,\text{m})(10 \times 10^{-3}\,\text{T/s})}{2(9.11 \times 10^{-27}\,\text{kg})} = 4.4 \times 10^7\ \text{m/s}^2\ . \end{aligned}$$

With regard to the directions, $\vec{a}_a$ points to the right and $\vec{a}_c$ points to the left. At point b we have $a_b \propto r_b = 0$.

91. Using Eq. 31-43, we find

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \implies \tau_L = \frac{t}{\ln\left(\frac{1}{1-\frac{iR}{\mathcal{E}}}\right)} = 22.4\ \text{s}\ .$$

Thus, from Eq. 31-44 (the definition of the time constant), we obtain $L = (22.4\,\text{s})(2.0\,\Omega) = 45\ \text{H}$.

92. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf ($\mathcal{E}_{L1}$) of the $L_1 = 0.30$ H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 31-37

$$\frac{di}{dt} = \frac{|\mathcal{E}_{L1}|}{L_1} = \frac{6.0}{0.30} = 20\ \text{A/s}\ .$$

(b) What is being asked for is essentially the current in the battery when the emf's of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0\ \Omega$, we have

$$\mathcal{E} - i\,R_1 - |\mathcal{E}_{L1}| - |\mathcal{E}_{L2}| = 0 \implies i = \frac{6.0\,\text{V}}{R_1} = 0.75\ \text{A}\ .$$

93. The magnetic flux is

$$\Phi_B = \vec{B}\cdot\vec{A} = BA\cos 57^\circ = \left(4.2\times10^{-6}\,\text{T}\right)\left(2.5\,\text{m}^2\right)\cos 57^\circ = 5.7\times10^{-5}\ \text{Wb}\ .$$

94. From the given information, we find

$$\frac{dB}{dt} = \frac{0.030\ \text{T}}{0.015\ \text{s}} = 2.0\ \text{T/s}\ \ .$$

Thus, with $N = 1$ and $\cos 30^\circ = \sqrt{3}/2$, and using Faraday's law with Ohm's law, we have

$$i = \frac{|\mathcal{E}|}{R} = \frac{N\pi r^2}{R}\,\frac{\sqrt{3}}{2}\,\frac{dB}{dt} = 0.021\ \text{A}\ .$$

95. Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\mathcal{E} - L\,di/dt = 0$. So $i = \mathcal{E}t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0\,\text{A}$. Thus,

$$t_0 = \frac{i_0 L}{\mathcal{E}} = \frac{(3.0\,\text{A})(5.0\,\text{H})}{10\,\text{V}} = 1.5\ \text{s}\ .$$

We do not show the graph here; qualitatively, it would be similar to Fig. 31-14.

96. We write (as functions of time) $V_L(t) = \mathcal{E}e^{-t/\tau_l}$. Considering the first two data points, $(V_{L1},\, t_1)$ and $(V_{L2},\, t_2)$, satisfying $V_{Li} = \mathcal{E}e^{-t_i/\tau_L}$ $(i = 1,\, 2)$, we have $V_{L1}/V_{L2} = \mathcal{E}e^{-(t_1-t_2)/\tau_L}$, which gives

$$\tau_L = \frac{t_1 - t_2}{\ln(V_2/V_1)} = \frac{1.0\,\text{ms} - 2.0\,\text{ms}}{\ln(13.8/18.2)} = 3.6\,\text{ms}\ .$$

Therefore, $\mathcal{E} = V_{L1}e^{t_1/\tau_L} = (18.2\,\text{V})e^{1.0\,\text{ms}/3.6\,\text{ms}} = 24\,\text{V}$. One can easily check that the values of τ_L and $\mathcal{E}$ are consistent with the rest of the data points.

97. (a) The energy density is

$$u_B = \frac{B_e^2}{2\mu_0} = \frac{(50\times10^{-6}\,\text{T})^2}{2\,(4\pi\times10^{-7}\,\text{H/m})} = 1.0\times10^{-3}\ \text{J/m}^3\ .$$

(b) The volume of the shell of thickness h is $\mathcal{V} \approx 4\pi R_e^2 h$, where R_e is the radius of the Earth. So

$$U_B \approx \mathcal{V}u_B \approx 4\pi(6.4\times10^6\,\text{m})^2(16\times10^3\,\text{m})(1.0\times10^{-3}\,\text{J/m}^3) = 8.4\times10^{15}\ \text{J}\ .$$

98. (a) $N = 2.0\,\text{m}/2.5\,\text{mm} = 800$.

(b) $L/l = \mu_0 n^2 A = \left(4\pi\times10^{-7}\,\text{H/m}\right)(800/2.0\,\text{m})^2(\pi)(0.040\,\text{m})^2/4 = 2.5\times10^{-4}\,\text{H}$.

99. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem is equivalent to that analyzed in §31-9 with solution Eq. 31-43. The derivative of that solution is

$$\frac{di}{dt} = \frac{\mathcal{E}}{R\,\tau_L}\,e^{-t/\tau_L} = \frac{\mathcal{E}}{L}\,e^{-t/\tau_L}\quad .$$

With $\tau_L = 0.28$ ms (by Eq. 31-44), $L = 0.050$ H and $\mathcal{E} = 45$ V, we obtain $di/dt = 12$ A/s when $t = 1.2$ ms.

100. (a) We apply Newton's second law to the rod

$$m\frac{dv}{dt} = iBL\ ,$$

and integrate to obtain

$$v = \frac{iBLt}{m}\ .$$

The velocity $\vec{v}$ points away from the generator G.

(b) When the current i in the rod becomes zero, the rod will no longer be accelerated by a force $F = iBL$ and will therefore reach a constant terminal velocity. This occurs when $|\mathcal{E}_{\text{induced}}| = \mathcal{E}$. Specifically,

$$|\mathcal{E}_{\text{induced}}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d(BA)}{dt}\right| = B\left|\frac{dA}{dt}\right| = BvL = \mathcal{E} \ .$$

Thus, $\vec{v} = \mathcal{E}/BL$, leftward.

(c) In case (a) electric energy is supplied by the generator and is transferred into the kinetic energy of the rod. In the case considered now the battery initially supplies electric energy to the rod, causing its kinetic energy to increase to a maximum value of $\frac{1}{2}mv^2 = \frac{1}{2}(\mathcal{E}/BL)^2$. Afterwards, there is no further energy transfer from the battery to the rod, and the kinetic energy of the rod remains constant.

101. (a) At $t = 0.50$ s and $t = 1.5$ s, the magnetic field is decreasing at a rate of $3/2$ mT/s, leading to

$$i = \frac{|\mathcal{E}|}{R} = \frac{A\,|dB/dt|}{R} = \frac{(3.0)(3/2)}{9.0} = 0.50 \text{ mA}$$

with a counterclockwise sense (by Lenz's law).

(b) See the results of part (a).

(c) and (d) For $t > 2.0$ s, there is no change in flux and therefore no induced current.

102. The magnetic flux is

$$\begin{aligned}\Phi_B &= BA = \left(\frac{\mu_0 i_0 N}{2\pi r}\right) A \\ &= \frac{\left(4\pi \times 10^{-7}\,\text{H/m}\right)(0.800\,\text{A})(500)(5.00\times10^{-2}\,\text{m})^2}{2\pi(0.150\,\text{m} + 0.0500\,\text{m}/2)} \\ &= 1.15\times10^{-6} \text{ Wb} \ .\end{aligned}$$

103. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ the current through the battery is also zero.

(b) With no current anywhere in the circuit at $t = 0$, the loop rule requires the emf of the inductor $\mathcal{E}_L$ to cancel that of the battery ($\mathcal{E} = 40$ V). Thus, the absolute value of Eq. 31-37 yields

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{40}{0.050} = 800 \text{ A/s} \ .$$

(c) This circuit becomes equivalent to that analyzed in §31-9 when we replace the parallel set of 20000 Ω resistors with $R = 10000\ \Omega$. Now, with $\tau_L = L/R = 5\times10^{-6}$ s, we have $t/\tau_L = 3/5$, and we apply Eq. 31-43:

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-3/5}\right) \approx 1.8\times10^{-3} \text{ A} \ .$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 31-37):

$$\mathcal{E} - i\,R - |\mathcal{E}_L| = 0 \ .$$

Using the values from part (c), we obtain $|\mathcal{E}_L| \approx 22$ V. Then,

$$\frac{di}{dt} = \frac{|\mathcal{E}_L|}{L} = \frac{22}{0.050} \approx 440 \text{ A/s} \ .$$

(e) and (f) As $t \to \infty$, the circuit reaches a steady state condition, so that $di/dt = 0$ and $\mathcal{E}_L = 0$. The loop rule then leads to

$$\mathcal{E} - i\,R - |\mathcal{E}_L| = 0 \implies i = \frac{40}{10000} = 4.0 \times 10^{-3} \text{ A} .$$

104. The magnetic flux Φ_B through the loop is given by $\Phi_B = 2B(\pi r^2/2)(\cos 45°) = \pi r^2 B/\sqrt{2}$. Thus

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right)\\ &= -\frac{\pi(3.7 \times 10^{-2}\,\text{m})^2}{\sqrt{2}}\left(\frac{0 - 76 \times 10^{-3}\,\text{T}}{4.5 \times 10^{-3}\,\text{s}}\right)\\ &= 5.1 \times 10^{-2} \text{ V} .\end{aligned}$$

The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

105. The area enclosed by any turn of the coil is πr^2 where $r = 0.15$ m, and the coil has $N = 50$ turns. Thus, the magnitude of the induced emf, using Eq. 31-7, is

$$|\mathcal{E}| = N\pi r^2 \left|\frac{dB}{dt}\right| = \left(3.53 \text{ m}^2\right)\left|\frac{dB}{dt}\right|$$

where $\left|\frac{dB}{dt}\right| = (0.0126 \text{ T/s})|\cos\omega t|$. Thus, using Ohm's law, we have

$$i = \frac{|\mathcal{E}|}{R} = \frac{(3.53)(0.0126)}{4.0}\,|\cos\omega t| \quad .$$

When $t = 0.020$ s, this yields $i = 0.011$ A.

106. (First problem of **Cluster**)
Combining Ohm's and Faraday's laws, the current magnitude is

$$i = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R}$$

for this "one-loop" circuit, where the version of Faraday's law expressed in Eq. 31-10 (often called "motional emf") has been used. Here, $B = |\vec{B}| = 0.200$ T, $L = 0.300$ m and $v = 12.0$ m/s. Reasoning with Lenz's law, the sense of the induced current is *counterclockwise* (to produce field in its interior out of the page, "fighting" the increasing inward pointed flux due to the applied field).

(a) With $R = 5.00\ \Omega$, this yields $i = 0.144$ A.

(b) With $R = 7.00\ \Omega$, we obtain $i = 0.103$ A.

107. (Second problem of **Cluster**)

(a) With $L = 0.50$ m and $R = 5.00\ \Omega$, we combine Ohm's and Faraday's laws, so that the current magnitude is

$$i = \frac{|\mathcal{E}|}{R} = \frac{BLv}{R} = 0.240 \text{ A} \quad .$$

The direction is counterclockwise, as explained in the solution to the previous problem.

(b) The area in the loop is $A = \frac{1}{2}(L_0 + L)x$ where $x = vt$ and $L_0 = 0.300$ m. But the value of L depends on the distance from the resistor x:

$$\begin{aligned}L &= 30 \text{ cm} + \left(\frac{20 \text{ cm}}{1 \text{ m}}\right)x\\ &= L_0 + 0.200(vt)\end{aligned}$$

where $x = vt$ has been used. Therefore, the area becomes

$$A = L_0\,vt + 0.100\,v^2t^2 \quad .$$

The induced emf is, from Faraday's law,

$$\mathcal{E} = \frac{d\Phi}{dt} = B\,\frac{dA}{dt} = B\left(L_0\,v + 2(0.100)v^2t\right)$$

and the induced current is

$$i = \frac{\mathcal{E}}{R} = 0.144 + 1.152t$$

in SI units and is counterclockwise (for reasons given in previous solution).

108. (Third problem of **Cluster**)

(a) , (b) and (c) The area enclosed by the loop is that of a rectangle with one side (x) expanding. With $B_0 = 0.200$ T and $\xi = 0.050$ T/s (the rate of field increase), we have

$$\begin{aligned}\Phi &= BA = (B_0 + \xi t)\,(Lx) \\ &= B_0Lvt + \xi Lvt^2\end{aligned}$$

where $x = vt$ has been used. Thus, from Faraday's and Ohm's laws, the induced current is

$$i = \frac{\mathcal{E}}{R} = \frac{B_0Lv}{R} + 2\frac{\xi Lv}{R}\,t$$

and is counterclockwise (to produce field in the loop's interior pointing out of the page, "fighting" the increasing inward pointed flux due to the applied field). Therefore, the current at $t = 0$ is $B_0Lv/R = 0.144$ A. And its value at $t = 1.00$ s is $(B_0 + 2\xi)Lv/R = 0.216$ A.

Chapter 32

1. (a) Since the field lines of a bar magnet point towards its South pole, then the $\vec{B}$ arrows in one's sketch should point generally towards the left and also towards the central axis.

   (b) The sign of $\vec{B} \cdot d\vec{A}$ for every $d\vec{A}$ on the side of the paper cylinder is negative.

   (c) No, because Gauss' law for magnetism applies to an *enclosed* surface only. In fact, if we include the top and bottom of the cylinder to form an enclosed surface S then $\oint_s \vec{B} \cdot d\vec{A} = 0$ will be valid, as the flux through the open end of the cylinder near the magnet is positive.

2. We use $\sum_{n=1}^{6} \Phi_{Bn} = 0$ to obtain

$$\Phi_{B6} = -\sum_{n=1}^{5} \Phi_{Bn} = -(-1\,\text{Wb} + 2\,\text{Wb} - 3\,\text{Wb} + 4\,\text{Wb} - 5\,\text{Wb}) = +3 \text{ Wb} .$$

3. We use Gauss' law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$. Now, $\oint \vec{B} \cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C$, where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is $\Phi_1 = -25.0\,\mu\text{Wb}$. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. It value is

$$\Phi_2 = \pi(0.120\,\text{m})^2(1.60 \times 10^{-3}\,\text{T}) = +7.24 \times 10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb} .$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb} .$$

The minus sign indicates that the flux is inward through the curved surface.

4. The flux through Arizona is

$$\Phi = -B_r A = -(43 \times 10^{-6}\,\text{T})(295,000\,\text{km}^2)(10^3\,\text{m/km})^2 = -1.3 \times 10^7 \text{ Wb} ,$$

inward. By Gauss' law this is equal to the negative value of the flux Φ' through the rest of the surface of the Earth. So $\Phi' = 1.3 \times 10^7\,\text{Wb}$, outward.

5. The horizontal component of the Earth's magnetic field is given by $B_h = B\cos\phi_i$, where B is the magnitude of the field and ϕ_i is the inclination angle. Thus

$$B = \frac{B_h}{\cos\phi_i} = \frac{16\,\mu\text{T}}{\cos 73°} = 55\,\mu\text{T} .$$

6. (a) The Pythagorean theorem leads to

$$\begin{aligned} B &= \sqrt{B_h^2 + B_v^2} = \sqrt{\left(\frac{\mu_0\mu}{4\pi r^3}\cos\lambda_m\right)^2 + \left(\frac{\mu_0\mu}{2\pi r^3}\sin\lambda_m\right)^2} \\ &= \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m + 4\sin^2\lambda_m} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\ , \end{aligned}$$

where $\cos^2\lambda_m + \sin^2\lambda_m = 1$ was used.

(b) We use Eq. 3-6:

$$\tan\phi_i = \frac{B_v}{B_h} = \frac{(\mu_0\mu/2\pi r^3)\sin\lambda_m}{(\mu_0\mu/4\pi r^3)\cos\lambda_m} = 2\tan\lambda_m\ .$$

7. (a) At the magnetic equator ($\lambda_m = 0$), the field is

$$B = \frac{\mu_0\mu}{4\pi r^3} = \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(8.00\times10^{22}\,\mathrm{A\cdot m^2}\right)}{4\pi(6.37\times10^6\,\mathrm{m})^3} = 3.10\times10^{-5}\,\mathrm{T}\ ,$$

and $\phi_i = \tan^{-1}(2\tan\lambda_m) = \tan^{-1}(0) = 0$.

(b) At $\lambda_m = 60°$, we find

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = (3.10\times10^{-5})\sqrt{1+3\sin^2 60°} = 5.6\times10^{-5}\,\mathrm{T}\ ,$$

and $\phi_i = \tan^{-1}(2\tan 60°) = 74°$.

(c) At the north magnetic pole ($\lambda_m = 90.0°$), we obtain

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} = (3.1\times10^{-5})\sqrt{1+3(1.00)^2} = 6.20\times10^{-5}\,\mathrm{T}\ ,$$

and $\phi_i = \tan^{-1}(2\tan 90°) = 90°$.

8. (a) At a distance r from the center of the Earth, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m}\ ,$$

where μ is the Earth's dipole moment and λ_m is the magnetic latitude. The ratio of the field magnitudes for two different distances at the same latitude is

$$\frac{B_2}{B_1} = \frac{r_1^3}{r_2^3}\ .$$

With B_1 being the value at the surface and B_2 being half of B_1, we set r_1 equal to the radius R_e of the Earth and r_2 equal to $R_e + h$, where h is altitude at which B is half its value at the surface. Thus,

$$\frac{1}{2} = \frac{R_e^3}{(R_e+h)^3}\ .$$

Taking the cube root of both sides and solving for h, we get

$$h = \left(2^{1/3}-1\right)R_e = \left(2^{1/3}-1\right)(6370\,\mathrm{km}) = 1660\ \mathrm{km}\ .$$

(b) We use the expression for B obtained in problem 6, part (a). For maximum B, we set $\sin\lambda_m = 1$. Also, $r = 6370\,\mathrm{km} - 2900\,\mathrm{km} = 3470\,\mathrm{km}$. Thus,

$$\begin{aligned} B_{\max} &= \frac{\mu_0\mu}{4\pi r^3}\sqrt{1+3\sin^2\lambda_m} \\ &= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(8.00\times10^{22}\,\mathrm{A\cdot m^2}\right)}{4\pi(3.47\times10^6\,\mathrm{m})^3}\sqrt{1+3(1)^2} = 3.83\times10^{-4}\ \mathrm{T}\ . \end{aligned}$$

(c) The angle between the magnetic axis and the rotational axis of the Earth is 11.5°, so $\lambda_m = 90.0° - 11.5° = 78.5°$ at Earth's geographic north pole. Also $r = R_e = 6370\,\text{km}$. Thus,

$$\begin{aligned} B &= \frac{\mu_0 \mu}{4\pi R_E^3}\sqrt{1+3\sin^2\lambda_m} \\ &= \frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)(8.0\times10^{22}\,\text{J/T})\sqrt{1+3\sin^2 78.5°}}{4\pi(6.37\times10^6\,\text{m})^3} = 6.11\times10^{-5}\ \text{T}\ , \end{aligned}$$

and, using the result of part (b) of problem 6,

$$\phi_i = \tan^{-1}(2\tan 78.5°) = 84.2°\ .$$

A plausible explanation to the discrepancy between the calculated and measured values of the Earth's magnetic field is that the formulas we obtained in problem 6 are based on dipole approximation, which does not accurately represent the Earth's actual magnetic field distribution on or near its surface. (Incidentally, the dipole approximation becomes more reliable when we calculate the Earth's magnetic field far from its center.)

9. We use Eq. 32-11: $\mu_{\text{orb},z} = -m_l\mu_B$.

(a) For $m_l = 1$, $\mu_{\text{orb},z} = -(1)\left(9.27\times10^{-24}\,\text{J/T}\right) = -9.27\times10^{-24}\,\text{J/T}$.

(b) For $m_l = -2$, $\mu_{\text{orb},z} = -(-2)\left(9.27\times10^{-24}\,\text{J/T}\right) = 1.85\times10^{-23}\,\text{J/T}$.

10. We use Eq. 32-7 to obtain $\Delta U = -\Delta(\mu_{s,z}B) = -B\Delta\mu_{s,z}$, where $\mu_{s,z} = \pm eh/4\pi m_e = \pm\mu_B$ (see Eqs. 32-4 and 32-5). Thus,

$$\Delta U = -B[\mu_B - (-\mu_B)] = 2\mu_B B = 2\left(9.27\times10^{-24}\,\text{J/T}\right)(0.25\,\text{T}) = 4.6\times10^{-24}\ \text{J}\ .$$

11. (a) Since $m_l = 0$, $L_{\text{orb},z} = m_l h/2\pi = 0$.

(b) Since $m_l = 0$, $\mu_{\text{orb},z} = -m_l\mu_B = 0$.

(c) Since $m_l = 0$, then from Eq. 32-12, $U = -\mu_{\text{orb},z}B_{\text{ext}} = -m_l\mu_B B_{\text{ext}} = 0$.

(d) Regardless of the value of m_l, we find for the spin part

$$U = -\mu_{s,z}B = \pm\mu_B B = \pm\left(9.27\times10^{-24}\,\text{J/T}\right)(35\,\text{mT}) = \pm3.2\times10^{-25}\ \text{J}\ .$$

(e) Now $m_l = -3$, so

$$L_{\text{orb},z} = \frac{m_l h}{2\pi} = \frac{(-3)\left(6.63\times10^{-27}\,\text{J}\cdot\text{s}\right)}{2\pi} = -3.16\times10^{-34}\ \text{J}\cdot\text{s}$$

and

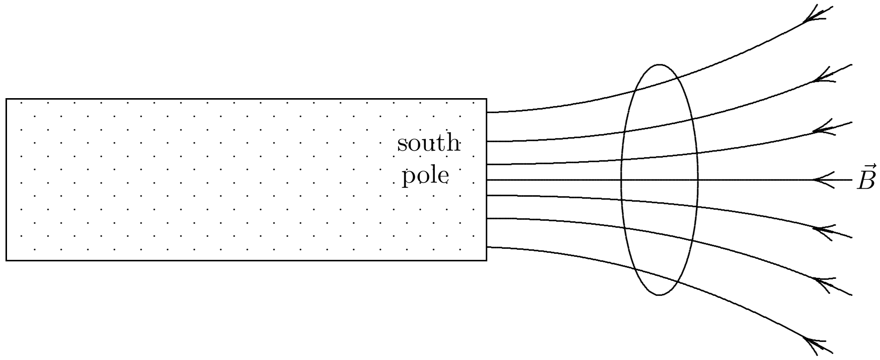
$$\mu_{\text{orb},z} = -m_l\mu_B = -(-3)\left(9.27\times10^{-24}\,\text{J/T}\right) = 2.78\times10^{-23}\ \text{J/T}\ .$$

The potential energy associated with the electron's orbital magnetic moment is now

$$U = -\mu_{\text{orb},z}B_{\text{ext}} = -(2.78\times10^{-23}\,\text{J/T})(35\times10^{-3}\,\text{T}) = -9.73\times10^{-25}\ \text{J}\ ;$$

while the potential energy associated with the electron spin, being independent of m_l, remains the same: $\pm3.2\times10^{-25}\,\text{J}$.

12. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The primary conclusion of §32-6 is two-fold: $\vec{\mu}$ is opposite to $\vec{B}$, and the effect of $\vec{F}$ is to move the material towards regions of smaller $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the left in our sketch.

(c) See our comments in part (b). Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is towards the right in our sketch.

13. An electric field with circular field lines is induced as the magnetic field is turned on. Suppose the magnetic field increases linearly from zero to B in time t. According to Eq. 31-27, the magnitude of the electric field at the orbit is given by

$$E = \left(\frac{r}{2}\right)\frac{dB}{dt} = \left(\frac{r}{2}\right)\frac{B}{t} ,$$

where r is the radius of the orbit. The induced electric field is tangent to the orbit and changes the speed of the electron, the change in speed being given by

$$\Delta v = at = \frac{eE}{m_e}t = \left(\frac{e}{m_e}\right)\left(\frac{r}{2}\right)\left(\frac{B}{t}\right)t = \frac{erB}{2m_e} .$$

The average current associated with the circulating electron is $i = ev/2\pi r$ and the dipole moment is

$$\mu = Ai = \left(\pi r^2\right)\left(\frac{ev}{2\pi r}\right) = \frac{1}{2}evr .$$

The change in the dipole moment is

$$\Delta\mu = \frac{1}{2}er\,\Delta v = \frac{1}{2}er\left(\frac{erB}{2m_e}\right) = \frac{e^2r^2B}{4m_e} .$$

14. Reviewing Sample Problem 32-1 before doing this exercise is helpful. Let

$$K = \frac{3}{2}kT = \left|\vec{\mu}\cdot\vec{B} - (-\vec{\mu}\cdot\vec{B})\right| = 2\mu B$$

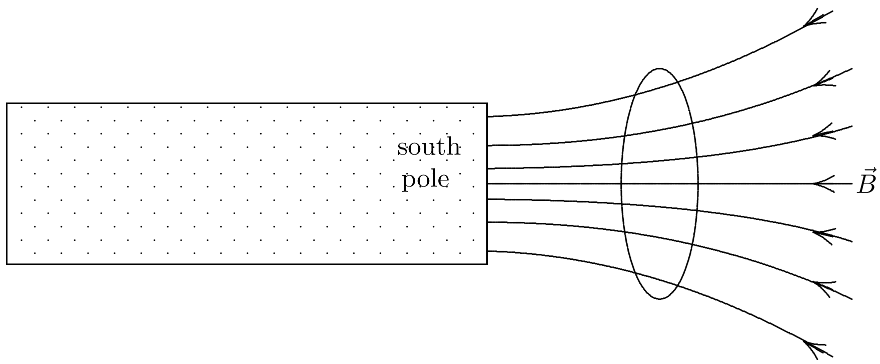
which leads to

$$T = \frac{4\mu B}{3k} = \frac{4(1.0\times10^{-23}\,\mathrm{J/T})(0.50\,\mathrm{T})}{3(1.38\times10^{-23}\,\mathrm{J/K})} = 0.48\ \mathrm{K} .$$

15. The magnetization is the dipole moment per unit volume, so the dipole moment is given by $\mu = M\mathcal{V}$, where M is the magnetization and $\mathcal{V}$ is the volume of the cylinder ($\mathcal{V} = \pi r^2 L$, where r is the radius of the cylinder and L is its length). Thus,

$$\mu = M\pi r^2 L = (5.30\times10^3\,\mathrm{A/m})\pi(0.500\times10^{-2}\,\mathrm{m})^2(5.00\times10^{-2}\,\mathrm{m}) = 2.08\times10^{-2}\ \mathrm{J/T} .$$

16. (a) A sketch of the field lines (due to the presence of the bar magnet) in the vicinity of the loop is shown below:



(b) The textbook, in §32-7, makes it clear that $\vec{\mu}$ is in the same direction as $\vec{B}$, and the effect of $\vec{F}$ is to move the material towards regions of larger $|\vec{B}|$ values. The direction of the magnetic moment vector (of our loop) is toward the right in our sketch.

(c) See our comments in part (b). Since the size of $|\vec{B}|$ relates to the "crowdedness" of the field lines, we see that $\vec{F}$ is towards the left in our sketch.

17. For the measurements carried out, the largest ratio of the magnetic field to the temperature is $(0.50\,\text{T})/(10\,\text{K}) = 0.050\,\text{T/K}$. Look at Fig. 32-9 to see if this is in the region where the magnetization is a linear function of the ratio. It is quite close to the origin, so we conclude that the magnetization obeys Curie's law.

18. (a) From Fig. 32-9 we estimate a slope of $B/T = 0.50\,\text{T/K}$ when $M/M_{\text{max}} = 50\%$. So $B = 0.50\,\text{T} = (0.50\,\text{T/K})(300\,\text{K}) = 150\,\text{T}$.

(b) Similarly, now $B/T \approx 2$ so $B = (2)(300) = 600\,\text{T}$.

(c) Except for very short times and in very small volumes, these values are not attainable in the lab.

19. (a) A charge e traveling with uniform speed v around a circular path of radius r takes time $T = 2\pi r/v$ to complete one orbit, so the average current is

$$i = \frac{e}{T} = \frac{ev}{2\pi r} .$$

The magnitude of the dipole moment is this multiplied by the area of the orbit:

$$\mu = \frac{ev}{2\pi r}\,\pi r^2 = \frac{evr}{2} .$$

Since the magnetic force of with magnitude evB is centripetal, Newton's law yields $evB = m_e v^2/r$, so

$$r = \frac{m_e v}{eB} .$$

Thus,

$$\mu = \frac{1}{2}(ev)\left(\frac{m_e v}{eB}\right) = \left(\frac{1}{B}\right)\left(\frac{1}{2}m_e v^2\right) = \frac{K_e}{B} .$$

The magnetic force $-e\vec{v} \times \vec{B}$ must point toward the center of the circular path. If the magnetic field is directed into the page, for example, the electron will travel clockwise around the circle. Since the electron is negative, the current is in the opposite direction, counterclockwise and, by the right-hand rule for dipole moments, the dipole moment is out of the page. That is, the dipole moment is directed opposite to the magnetic field vector.

(b) We note that the charge canceled in the derivation of $\mu = K_e/B$. Thus, the relation $\mu = K_i/B$ holds for a positive ion. If the magnetic field is directed into the page, the ion travels counterclockwise around a circular orbit and the current is in the same direction. Therefore, the dipole moment is again out of the page, opposite to the magnetic field.

(c) The magnetization is given by $M = \mu_e n_e + \mu_i n_i$, where μ_e is the dipole moment of an electron, n_e is the electron concentration, μ_i is the dipole moment of an ion, and n_i is the ion concentration. Since $n_e = n_i$, we may write n for both concentrations. We substitute $\mu_e = K_e/B$ and $\mu_i = K_i/B$ to obtain

$$M = \frac{n}{B}(K_e + K_i) = \frac{5.3 \times 10^{21}\,\mathrm{m}^{-3}}{1.2\,\mathrm{T}}\left(6.2 \times 10^{-20}\,\mathrm{J} + 7.6 \times 10^{-21}\,\mathrm{J}\right) = 310\ \mathrm{A/m}\ .$$

20. The Curie temperature for iron is 770°C. If x is the depth at which the temperature has this value, then $10°\mathrm{C} + (30°\mathrm{C/km})x = 770°\mathrm{C}$. Therefore,

$$x = \frac{770°\mathrm{C} - 10°\mathrm{C}}{30°\mathrm{C/km}} = 25\ \mathrm{km}\ .$$

21. (a) The field of a dipole along its axis is given by Eq. 30-29:

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}\ ,$$

where μ is the dipole moment and z is the distance from the dipole. Thus,

$$B = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m}/A)(1.5 \times 10^{-23}\,\mathrm{J/T})}{2\pi(10 \times 10^{-9}\,\mathrm{m})} = 3.0 \times 10^{-6}\ \mathrm{T}\ .$$

(b) The energy of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, where ϕ is the angle between the dipole moment and the field. The energy required to turn it end-for-end (from $\phi = 0°$ to $\phi = 180°$) is

$$\Delta U = 2\mu B = 2(1.5 \times 10^{-23}\,\mathrm{J/T})(3.0 \times 10^{-6}\,\mathrm{T}) = 9.0 \times 10^{-29}\ \mathrm{J} = 5.6 \times 10^{-10}\ \mathrm{eV}\ .$$

The mean kinetic energy of translation at room temperature is about 0.04 eV. Thus, if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

22. (a) The number of iron atoms in the iron bar is

$$N = \frac{\left(7.9\,\mathrm{g/cm^3}\right)(5.0\,\mathrm{cm})\left(1.0\,\mathrm{cm^2}\right)}{(55.847\,\mathrm{g/mol})\,/\,(6.022 \times 10^{23}/\mathrm{mol})} = 4.3 \times 10^{23}\ .$$

Thus the dipole moment of the iron bar is

$$\mu = \left(2.1 \times 10^{-23}\,\mathrm{J/T}\right)\left(4.3 \times 10^{23}\right) = 8.9\ \mathrm{A{\cdot}m^2}\ .$$

(b) $\tau = \mu B \sin 90° = (8.9\,\mathrm{A{\cdot}m^2})(1.57\,\mathrm{T}) = 13\,\mathrm{N{\cdot}m}$.

23. The saturation magnetization corresponds to complete alignment of all atomic dipoles and is given by $M_{\mathrm{sat}} = \mu n$, where n is the number of atoms per unit volume and μ is the magnetic dipole moment of an atom. The number of nickel atoms per unit volume is $n = \rho/m$, where ρ is the density of nickel. The mass of a single nickel atom is calculated using $m = M/N_A$, where M is the atomic mass of nickel and N_A is Avogadro's constant. Thus,

$$\begin{aligned} n &= \frac{\rho N_A}{M} = \frac{(8.90\,\mathrm{g/cm^3})(6.02 \times 10^{23}\,\mathrm{atoms/mol})}{58.71\,\mathrm{g/mol}} \\ &= 9.126 \times 10^{22}\,\mathrm{atoms/cm^3} = 9.126 \times 10^{28}\ \mathrm{atoms/m^3}\ . \end{aligned}$$

The dipole moment of a single atom of nickel is

$$\mu = \frac{M_{\rm sat}}{n} = \frac{4.70 \times 10^5\,{\rm A/m}}{9.126 \times 10^{28}\,{\rm m}^3} = 5.15 \times 10^{-24}\ {\rm A{\cdot}m^2}\ .$$

24. From the way the wire is wound it is clear that P_2 is the magnetic north pole while P_1 is the south pole.

    (a) The deflection will be toward P_1 (away from the magnetic north pole).

    (b) As the electromagnet is turned on, the magnetic flux Φ_B through the aluminum changes abruptly, causing a strong induced current which produces a magnetic field opposite to that of the electromagnet. As a result, the aluminum sample will be pushed toward P_1, away from the magnetic north pole of the bar magnet. As Φ_B reaches a constant value, however, the induced current disappears and the aluminum sample, being paramagnetic, will move slightly toward P_2, the magnetic north pole of the electromagnet.

    (c) A magnetic north pole will now be induced on the side of the sample closer to P_1, and a magnetic south pole will appear on the other side. If the magnitude of the field of the electromagnet is larger near P_1, then the sample will move toward P_1.

25. (a) If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\rm total} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}\ .$$

We substitute this into $\mu_{\rm total} = N\mu$ to obtain

$$\mu_{\rm total} = \frac{4\pi\rho R^3\mu}{3m}\ .$$

We solve for R and obtain

$$R = \left(\frac{3m\mu_{\rm total}}{4\pi\rho\mu}\right)^{1/3}\ .$$

The mass of an iron atom is

$$m = 56\,{\rm u} = (56\,{\rm u})(1.66 \times 10^{-27}\,{\rm kg/u}) = 9.30 \times 10^{-26}\ {\rm kg}\ .$$

Therefore,

$$R = \left[\frac{3(9.30 \times 10^{-26}\,{\rm kg})(8.0 \times 10^{22}\,{\rm J/T})}{4\pi(14 \times 10^3\,{\rm kg/m^3})(2.1 \times 10^{-23}\,{\rm J/T})}\right]^{1/3} = 1.8 \times 10^5\ {\rm m}\ .$$

(b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82 \times 10^5\,{\rm m})^3 = 2.53 \times 10^{16}\,{\rm m}^3$$

and the volume of the Earth is

$$V_e = \frac{4\pi}{3}(6.37 \times 10^6\,{\rm m})^3 = 1.08 \times 10^{21}\ {\rm m}^3\ ,$$

so the fraction of the Earth's volume that is occupied by the sphere is

$$\frac{2.53 \times 10^{16}\,{\rm m}^3}{1.08 \times 10^{21}\,{\rm m}^3} = 2.3 \times 10^{-5}\ .$$

26. Let R be the radius of a capacitor plate and r be the distance from axis of the capacitor. For points with $r \le R$, the magnitude of the magnetic field is given by

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} ,$$

and for $r \ge R$, it is

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} .$$

The maximum magnetic field occurs at points for which $r = R$, and its value is given by either of the formulas above:

$$B_{\text{max}} = \frac{\mu_0 \varepsilon_0 R}{2} \frac{dE}{dt} .$$

There are two values of r for which $B = B_{\text{max}}/2$: one less than R and one greater. To find the one that is less than R, we solve

$$\frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = R/2 = (55.0\,\text{mm})/2 = 27.5\,\text{mm}$. To find the one that is greater than R, we solve

$$\frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 R}{4} \frac{dE}{dt}$$

for r. The result is $r = 2R = 2(55.0\,\text{mm}) = 110\,\text{mm}$.

27. We use the result of part (b) in Sample Problem 32-3:

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt} \qquad\qquad (\text{for } r \ge R)$$

to solve for dE/dt:

$$\begin{aligned} \frac{dE}{dt} &= \frac{2Br}{\mu_0 \varepsilon_0 R^2} \\ &= \frac{2(2.0 \times 10^{-7}\ T)(6.0 \times 10^{-3}\,\text{m})}{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})\left(8.85 \times 10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(3.0 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{13}\ \frac{\text{V}}{\text{m}\cdot\text{s}} . \end{aligned}$$

28. (a) Noting that the magnitude of the electric field (assumed uniform) is given by $E = V/d$ (where $d = 5.0$ mm), we use the result of part (a) in Sample Problem 32-3

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2d} \frac{dV}{dt} \qquad\qquad (\text{for } r \le R) .$$

We also use the fact that the time derivative of $\sin(\omega t)$ (where $\omega = 2\pi f = 2\pi(60) \approx 377/\text{s}$ in this problem) is $\omega \cos(\omega t)$. Thus, we find the magnetic field as a function of r (for $r \le R$; note that this neglects "fringing" and related effects at the edges):

$$B = \frac{\mu_0 \varepsilon_0 r}{2d} V_{\text{max}} \omega \cos(\omega t) \implies B_{\text{max}} = \frac{\mu_0 \varepsilon_0 r V_{\text{max}} \omega}{2d}$$

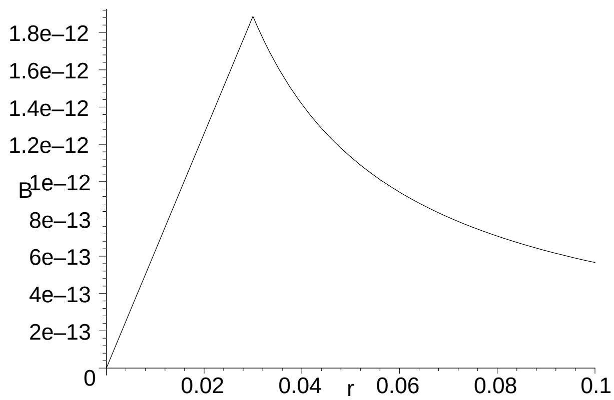
where $V_{\text{max}} = 150$ V. This grows with r until reaching its highest value at $r = R = 30$ mm:

$$\begin{aligned} B_{\text{max}}\Big|_{r=R} &= \frac{\mu_0 \varepsilon_0 R V_{\text{max}} \omega}{2d} \\ &= \frac{(4\pi \times 10^{-7}\,\text{H/m})(8.85 \times 10^{-12}\,\text{F/m})(30 \times 10^{-3}\,\text{m})(150\,\text{V})(377/\text{s})}{2(5.0 \times 10^{-3}\,\text{m})} \\ &= 1.9 \times 10^{-12}\ \text{T} . \end{aligned}$$

(b) For $r \le 0.03\,\mathrm{m}$, we use the $B_{\max} = \frac{\mu_0 \varepsilon_0 r V_{\max}\omega}{2d}$ expression found in part (a) (note the $B \propto r$ dependence), and for $r \ge 0.03\,\mathrm{m}$ we perform a similar calculation starting with the result of part (b)in Sample Problem 32-3:

$$\begin{aligned} B_{\max} &= \left(\frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}\right)_{\max} \\ &= \left(\frac{\mu_0\varepsilon_0 R^2}{2rd}\frac{dV}{dt}\right)_{\max} \\ &= \left(\frac{\mu_0\varepsilon_0 R^2}{2rd} V_{\max}\omega\cos(\omega t)\right)_{\max} \\ &= \frac{\mu_0\varepsilon_0 R^2 V_{\max}\omega}{2rd} \qquad \text{(for } r \ge R\text{)} \end{aligned}$$

(note the $B \propto r^{-1}$ dependence – See also Eqs. 32-40 and 32-41). The plot (with SI units understood) is shown below.



29. The displacement current is given by

$$i_d = \varepsilon_0 A \frac{dE}{dt} ,$$

where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus

$$i_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} .$$

Now, $\varepsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor (not filled with a dielectric), so

$$i_d = C\frac{dV}{dt} .$$

30. Let the area plate be A and the plate separation be d. We use Eq. 32-34:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(AE) = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{dV}{dt}\right) ,$$

or

$$\frac{dV}{dt} = \frac{i_d d}{\varepsilon_0 A} = \frac{i_d}{C} = \frac{1.5\ \mathrm{A}}{2.0 \times 10^{-6}\ \mathrm{F}} = 7.5 \times 10^5\ \mathrm{V/s} .$$

Therefore, we need to change the voltage difference across the capacitor at the rate of 7.5×10^5 V/s.

31. Consider an area A, normal to a uniform electric field $\vec{E}$. The displacement current density is uniform and normal to the area. Its magnitude is given by $J_d = i_d/A$. For this situation

$$i_d = \varepsilon_0 A \frac{dE}{dt} ,$$

so

$$J_d = \frac{1}{A}\varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{dE}{dt} .$$

32. We use Eq. 32-38:

$$i_d = \varepsilon_0 A \frac{dE}{dt} .$$

Note that, in this situation, A is the area over which a changing electric field is present. In this case $r > R$, so $A = \pi R^2$. Thus,

$$\frac{dE}{dt} = \frac{i_d}{\varepsilon_0 A} = \frac{i_d}{\varepsilon_0 \pi R^2} = \frac{2.0\,\mathrm{A}}{\pi\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(0.10\,\mathrm{m})^2} = 7.2\times10^{12}\,\frac{\mathrm{V}}{\mathrm{m\cdot s}} .$$

33. (a) We use $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{enclosed}}$ to find

$$\begin{aligned} B &= \frac{\mu_0 I_{\text{enclosed}}}{2\pi r} = \frac{\mu_0 (J_d \pi r^2)}{2\pi r} = \frac{1}{2}\mu_0 J_d r \\ &= \frac{1}{2}(1.26\times10^{-6}\,\mathrm{H/m})(20\,\mathrm{A/m^2})(50\times10^{-3}\,\mathrm{m}) = 6.3\times10^{-7}\ \mathrm{T} . \end{aligned}$$

(b) From

$$i_d = J_d \pi r^2 = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \pi r^2 \frac{dE}{dt}$$

we get

$$\frac{dE}{dt} = \frac{J_d}{\varepsilon_0} = \frac{20\,\mathrm{A/m^2}}{8.85\times10^{-12}\,\mathrm{F/m}} = 2.3\times10^{12}\,\frac{\mathrm{V}}{\mathrm{m\cdot s}} .$$

34. (a) From Eq. 32-34,

$$\begin{aligned} i_d &= \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left[(4.0\times10^5) - (6.0\times10^4 t)\right] \\ &= -\varepsilon_0 A (6.0\times10^4\,\mathrm{V/m\cdot s}) \\ &= -\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(4.0\times10^{-2}\,\mathrm{m^2})(6.0\times10^4\,\mathrm{V/m\cdot s}) \\ &= -2.1\times10^{-8}\,\mathrm{A} . \end{aligned}$$

(b) If one draws a counterclockwise circular loop s around the plates, then according to Eq. 32-42

$$\oint_s \vec{B}\cdot d\vec{s} = \mu_0 i_d < 0 ,$$

which means that $\vec{B}\cdot d\vec{s} < 0$. Thus $\vec{B}$ must be clockwise.

35. (a) In region a of the graph,

$$\begin{aligned} |i_d| &= \varepsilon_0 \left|\frac{d\Phi_E}{dt}\right| = \varepsilon_0 A \left|\frac{dE}{dt}\right| \\ &= (8.85\times10^{-12}\,\mathrm{F/m})(1.6\,\mathrm{m^2})\left|\frac{4.5\times10^5\,\mathrm{N/C} - 6.0\times10^5\,\mathrm{N/C}}{4.0\times10^{-6}\,\mathrm{s}}\right| = 0.71\ \mathrm{A}. \end{aligned}$$

(b) $i_d \propto dE/dt = 0$.

(c) In region c of the graph,

$$|i_d| = \varepsilon_0 A \left|\frac{dE}{dt}\right| = (8.85 \times 10^{-12}\,\mathrm{F/m})(1.6\,\mathrm{m}^2)\left|\frac{-4.0 \times 10^5\,\mathrm{N/C}}{15 \times 10^{-6}\,\mathrm{s} - 10 \times 10^{-6}\,\mathrm{s}}\right| = 1.1 \text{ A} .$$

36. Using Eq. 32-38, we have

$$\frac{d\left|\vec{E}\right|}{dt} = \frac{i_d}{\varepsilon_0\, A} = 7.2 \times 10^{12}$$

where $A = \pi(0.10)^2$ (fringing is being neglected in §32-10) and SI units are understood.

37. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_d = i = 2.0\,\mathrm{A}$.

(b) The rate of change of the electric field is

$$\frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\left(\varepsilon_0 \frac{d\Phi_E}{dt}\right) = \frac{i_d}{\varepsilon_0 A} = \frac{2.0\,\mathrm{A}}{(8.85 \times 10^{-12}\,\mathrm{F/m})(1.0\,\mathrm{m})^2} = 2.3 \times 10^{11}\ \frac{\mathrm{V}}{\mathrm{m{\cdot}s}} .$$

(c) The displacement current through the indicated path is

$$i_d' = i_d \times \left(\frac{\text{area enclosed by the path}}{\text{area of each plate}}\right) = (2.0\,\mathrm{A})\left(\frac{0.50\,\mathrm{m}}{1.0\,\mathrm{m}}\right)^2 = 0.50 \text{ A} .$$

(d) The integral of the field around the indicated path is

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_d' = (1.26 \times 10^{-6}\,\mathrm{H/m})(0.50\,\mathrm{A}) = 6.3 \times 10^{-7}\ \mathrm{T{\cdot}m} .$$

38. (a) Since $i = i_d$ (Eq. 32-39) then the portion of displacement current enclosed is

$$i_{d,\mathrm{enc}} = i\,\frac{\pi\left(\frac{R}{3}\right)^2}{\pi R^2} = i\,\frac{1}{9} = 1.33 \text{ A} .$$

(b) We see from Sample Problems 32-3 and 32-4 that the maximum field is at $r = R$ and that (in the interior) the field is simply proportional to r. Therefore,

$$\frac{B}{B_{\max}} = \frac{3.00 \text{ mT}}{12.0 \text{ mT}} = \frac{r}{R}$$

which yields $r = R/4$ as a solution. We now look for a solution in the exterior region, where the field is inversely proportional to r (by Eq. 32-41):

$$\frac{B}{B_{\max}} = \frac{3.00 \text{ mT}}{12.0 \text{ mT}} = \frac{R}{r}$$

which yields $r = 4R$ as a solution.

39. (a) Using Eq. 27-10, we find

$$E = \rho J = \frac{\rho i}{A} = \frac{(1.62 \times 10^{-8}\,\Omega{\cdot}\mathrm{m})(100\,\mathrm{A})}{5.00 \times 10^{-6}\,\mathrm{m}^2} = 0.324 \text{ V/m} .$$

(b) The displacement current is

$$\begin{aligned} i_d &= \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{\rho i}{A}\right) = \varepsilon_0 \rho \frac{di}{dt} \\ &= (8.85 \times 10^{-12}\,\mathrm{F})(1.62 \times 10^{-8}\,\Omega)(2000\,\mathrm{A/s}) = 2.87 \times 10^{-16} \text{ A} . \end{aligned}$$

(c) The ratio of fields is

$$\frac{B(\text{ due to } i_d)}{B(\text{ due to } i)} = \frac{\mu_0 i_d/2\pi r}{\mu_0 i/2\pi r} = \frac{i_d}{i} = \frac{2.87 \times 10^{-16}\,\text{A}}{100\,\text{A}} = 2.87 \times 10^{-18} .$$

40. (a) From Sample Problem 32-3 we know that $B \propto r$ for $r \le R$ and $B \propto r^{-1}$ for $r \ge R$. So the maximum value of B occurs at $r = R$, and there are two possible values of r at which the magnetic field is 75% of B_{max}. We denote these two values as r_1 and r_2, where $r_1 < R$ and $r_2 > R$. Then $0.75B_{\text{max}}/B_{\text{max}} = r_1/R$, or $r_1 = 0.75R$; and $0.75B_{\text{max}}/B_{\text{max}} = (r_2/R)^{-1}$, or $r_2 = R/0.75 = 1.3R$.

(b) From Eqs. 32-39 and 32-41,

$$B_{\text{max}} = \frac{\mu_0 i_d}{2\pi R} = \frac{\mu_0 i}{2\pi R} = \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right)(6.0\,\text{A})}{2\pi(0.040\,\text{m})} = 3.0 \times 10^{-5}\ \text{T} .$$

41. (a) At any instant the displacement current i_d in the gap between the plates equals the conduction current i in the wires. Thus $i_{\text{max}} = i_{d\ \text{max}} = 7.60\,\mu\text{A}$.

(b) Since $i_d = \varepsilon_0\,(d\Phi_E/dt)$,

$$\left(\frac{d\Phi_E}{dt}\right)_{\text{max}} = \frac{i_{d\ \text{max}}}{\varepsilon_0} = \frac{7.60 \times 10^{-6}\,\text{A}}{8.85 \times 10^{-12}\ \text{F/m}} = 8.59 \times 10^5\text{V}\cdot\text{m/s} .$$

(c) According to problem 29,

$$i_d = C\frac{dV}{dt} = \frac{\varepsilon_0 A}{d}\,\frac{dV}{dt} .$$

Now the potential difference across the capacitor is the same in magnitude as the emf of the generator, so $V = \mathcal{E}_{\text{m}} \sin\omega t$ and $dV/dt = \omega\mathcal{E}_{\text{m}} \cos\omega t$. Thus,

$$i_d = \frac{\varepsilon_0 A\omega\mathcal{E}_{\text{m}}}{d}\cos\omega t$$

and

$$i_{d\ \text{max}} = \frac{\varepsilon_0 A\omega\mathcal{E}_{\text{m}}}{d} .$$

This means

$$\begin{aligned} d &= \frac{\varepsilon_0 A\omega\mathcal{E}_{\text{m}}}{i_{d\,\text{max}}} = \frac{(8.85 \times 10^{-12}\,\text{F/m})\pi(0.180\text{m})^2(130\,\text{rad/s})(220\text{V})}{7.60 \times 10^{-6}\,\text{A}} \\ &= 3.39 \times 10^{-3}\,\text{m} , \end{aligned}$$

where $A = \pi R^2$ was used.

(d) We use the Ampere-Maxwell law in the form $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_d$, where the path of integration is a circle of radius r between the plates and parallel to them. I_d is the displacement current through the area bounded by the path of integration. Since the displacement current density is uniform between the plates $I_d = (r^2/R^2)i_d$, where i_d is the total displacement current between the plates and R is the plate radius. The field lines are circles centered on the axis of the plates, so $\vec{B}$ is parallel to $d\vec{s}$. The field has constant magnitude around the circular path, so $\oint \vec{B} \cdot d\vec{s} = 2\pi r B$. Thus,

$$2\pi r B = \mu_0 \left(\frac{r^2}{R^2}\right) i_d$$

and

$$B = \frac{\mu_0 i_d r}{2\pi R^2} .$$

The maximum magnetic field is given by

$$B_{\text{max}} = \frac{\mu_0 i_{d\ \text{max}}\, r}{2\pi R^2} = \frac{(4\pi \times 10^{-7}\text{T}\cdot\text{m/A})(7.6 \times 10^{-6}\,\text{A})(0.110\text{m})}{2\pi(0.180\text{m})^2} = 5.16 \times 10^{-12}\ \text{T} .$$

42. From Gauss' law for magnetism, the flux through S_1 is equal to that through S_2, the portion of the xz plane that lies within the cylinder. Here the normal direction of S_2 is $+y$. Therefore,

$$\begin{aligned}\Phi_B(S_1) &= \Phi_B(S_2) = \int_{-r}^{r} B(x)L\,dx \\ &= 2\int_{-r}^{r} B_{\text{left}}(x)L\,dx \\ &= 2\int_{-r}^{r} \frac{\mu_0 i}{2\pi}\frac{1}{2r-x}L\,dx \quad = \quad \frac{\mu_0 iL}{\pi}\ln 3\ .\end{aligned}$$

43. (a) Again from Fig. 32-9, for $M/M_{\text{max}} = 50\%$ we have $B/T = 0.50$. So $T = B/0.50 = 2/0.50 = 4\,\text{K}$.
    (b) Now $B/T = 2.0$, so $T = 2/2.0 = 1\,\text{K}$.

44. (a) For a given value of l, m_l varies from $-l$ to $+l$. Thus, in our case $l = 3$, and the number of different m_l's is $2l + 1 = 2(3) + 1 = 7$. Thus, since $L_{\text{orb},z} \propto m_l$, there are a total of seven different values of $L_{\text{orb},z}$.
    (b) Similarly, since $\mu_{\text{orb},z} \propto m_l$, there are also a total of seven different values of $\mu_{\text{orb},z}$.
    (c) Since $L_{\text{orb},z} = m_l h/2\pi$, the greatest allowed value of $L_{\text{orb},z}$ is given by $|m_l|_{\text{max}}h/2\pi = 3h/2\pi$; while the least allowed value is given by $|m_l|_{\text{min}}h/2\pi = 0$.
    (d) Similar to part (c), since $\mu_{\text{orb},z} = -m_l\mu_B$, the greatest allowed value of $\mu_{\text{orb},z}$ is given by $|m_l|_{\text{max}}\mu_B = 3eh/4\pi m_e$; while the least allowed value is given by $|m_l|_{\text{min}}\mu_B = 0$.
    (e) From Eqs. 32-3 and 32-9 the z component of the net angular momentum of the electron is given by

$$L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = \frac{m_l h}{2\pi} + \frac{m_s h}{2\pi}\ .$$

For the maximum value of $L_{\text{net},z}$ let $m_l = [m_l]_{\text{max}} = 3$ and $m_s = \frac{1}{2}$. Thus

$$[L_{\text{net},z}]_{\text{max}} = \left(3 + \frac{1}{2}\right)\frac{h}{2\pi} = \frac{3.5h}{2\pi}\ .$$

    (f) Since the maximum value of $L_{\text{net},z}$ is given by $[m_J]_{\text{max}}h/2\pi$ with $[m_J]_{\text{max}} = 3.5$ (see the last part above), the number of allowed values for the z component of $L_{\text{net},z}$ is given by $2[m_J]_{\text{max}} + 1 = 2(3.5) + 1 = 8$.

45. (a) We use the result of part (a) in Sample Problem 32-3:

$$B = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt} \qquad\qquad (\text{for } r \le R)\ ,$$

where $r = 0.80R$ and

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{1}{d}\frac{d}{dt}\left(V_0 e^{-t/\tau}\right) = -\frac{V_0}{\tau d}e^{-t/\tau}\ .$$

Here $V_0 = 100\,\text{V}$. Thus

$$\begin{aligned}B(t) &= \left(\frac{\mu_0\varepsilon_0 r}{2}\right)\left(-\frac{V_0}{\tau d}e^{-t/\tau}\right) = -\frac{\mu_0\varepsilon_0 V_0 r}{2\tau d}\,e^{-t/\tau} \\ &= -\frac{\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(100\,\text{V})(0.80)(16\,\text{mm})}{2(12\times10^{-3}\,\text{s})(5.0\,\text{mm})}\,e^{-t/12\,\text{ms}} \\ &= -(1.2\times10^{-13}\,\text{T})\,e^{-t/12\,\text{ms}}\ .\end{aligned}$$

The minus sign here is insignificant.

(b) At time $t = 3\tau$, $B(t) = -(1.2 \times 10^{-13}\,\mathrm{T})e^{-3\tau/\tau} = -5.9 \times 10^{-15}\,\mathrm{T}$.

46. The given value 7.0 mW should be 7.0 mWb. From Eq. 32-1, we have

$$\begin{aligned}(\Phi_B)_{\rm in} &= (\Phi_B)_{\rm out} \\ &= 0.0070\,\mathrm{Wb} + (0.40\,\mathrm{T})(\pi r^2) \\ &= 9.2 \times 10^{-3}\ \mathrm{Wb}\ .\end{aligned}$$

Thus, the magnetic flux at the sides is inward with absolute-value equal to 9.2 mWb.

47. The definition of displacement current is Eq. 32-34, and the formula of greatest convenience here is Eq. 32-41:

$$i_d = \frac{2\pi\, r\, B}{\mu_0} = \frac{2\pi(0.0300\,\mathrm{m})\left(2.00 \times 10^{-6}\,\mathrm{T}\right)}{4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}} = 0.30\ \mathrm{A}\ .$$

48. Ignoring points where the determination of the slope is problematic, we find the interval of largest $\Delta|\vec{E}|/\Delta t$ is $6\,\mu\mathrm{s} < t < 7\,\mu\mathrm{s}$. During that time, we have, from Eq. 32-38,

$$i_d = \varepsilon_0\, A\, \frac{\Delta|\vec{E}|}{\Delta t} = \varepsilon_0 \left(2.0\,\mathrm{m}^2\right)\left(2.0 \times 10^6\,\mathrm{V/m}\right)$$

which yields $i_d = 3.5 \times 10^{-5}$ A.

49. (a) We use the notation $P(\mu)$ for the probability of a dipole being parallel to $\vec{B}$, and $P(-\mu)$ for the probability of a dipole being antiparallel to the field. The magnetization may be thought of as a "weighted average" in terms of these probabilities:

$$M = \frac{N\mu P(\mu) - N\mu P(-\mu)}{P(\mu) + P(-\mu)} = \frac{N\mu\left(e^{\mu B/KT} - e^{-\mu B/KT}\right)}{e^{\mu B/KT} + e^{-\mu B/KT}} = N\mu \tanh\left(\frac{\mu B}{kT}\right)\ .$$

(b) For $\mu B \ll kT$ (that is, $\mu B/kT \ll 1$) we have $e^{\pm\mu B/kT} \approx 1 \pm \mu B/kT$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx \frac{N\mu[(1 + \mu B/kT) - (1 - \mu B/kT)]}{(1 + \mu B/kT) + (1 - \mu B/kT)} = \frac{N\mu^2 B}{kT}\ .$$

(c) For $\mu B \gg kT$ we have $\tanh(\mu B/kT) \approx 1$, so

$$M = N\mu \tanh\left(\frac{\mu B}{kT}\right) \approx N\mu\ .$$

(d) One can easily plot the tanh function using, for instance, a graphical calculator. One can then note the resemblance between such a plot and Fig. 32-9. By adjusting the parameters used in one's plot, the curve in Fig. 32-9 can reliably be fit with a tanh function.

50. (a) From Eq. 22-3,

$$E = \frac{e}{4\pi\varepsilon_0 r^2} = \frac{(1.60 \times 10^{-19}\,\mathrm{C})(8.99 \times 10^9\,\mathrm{N\cdot m^2/\,C^2})}{(5.2 \times 10^{-11}\,\mathrm{m})^2} = 5.3 \times 10^{11}\ \mathrm{N/C}\ .$$

(b) We use Eq. 30-28:

$$B = \frac{\mu_0}{2\pi}\frac{\mu_p}{r^3} = \frac{\left(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A}\right)(1.4 \times 10^{-26}\,\mathrm{J/T})}{2\pi(5.2 \times 10^{-11}\,\mathrm{m})^3} = 2.0 \times 10^{-2}\ \mathrm{T}\ .$$

(c) From Eq. 32-10,

$$\frac{\mu_{\rm orb}}{\mu_p} = \frac{eh/4\pi m_e}{\mu_p} = \frac{\mu_B}{\mu_p} = \frac{9.27 \times 10^{-24}\,\mathrm{J/T}}{1.4 \times 10^{-26}\,\mathrm{J/T}} = 6.6 \times 10^2\ .$$

51. The interacting potential energy between the magnetic dipole of the compass and the Earth's magnetic field is $U = -\vec{\mu}\cdot\vec{B}_e = -\mu B_e \cos\theta$, where θ is the angle between $\vec{\mu}$ and $\vec{B}_e$. For small angle θ

$$U(\theta) = -\mu B_e \cos\theta \quad \approx \quad -\mu B_e\left(1 - \frac{\theta^2}{2}\right) = \frac{1}{2}\kappa\theta^2 - \mu B_e$$

where $\kappa = \mu B_e$. Conservation of energy for the compass then gives

$$\frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\kappa\theta^2 = \text{const.}\ .$$

This is to be compared with the following expression for the mechanical energy of a spring-mass system:

$$\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 = \text{const.}\ ,$$

which yields $\omega = \sqrt{k/m}$. So by analogy, in our case

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{\mu B_e}{I}} = \sqrt{\frac{\mu B_e}{ml^2/12}}\ ,$$

which leads to

$$\mu = \frac{ml^2\omega^2}{12B_e} = \frac{(0.050\,\text{kg})(4.0\times10^{-2}\,\text{m})^2(45\,\text{rad/s})^2}{12(16\times10^{-6}\,\text{T})} = 8.4\times10^2\ \text{J/T}\ .$$

52. Let the area of each circular plate be A and that of the central circular section be a, then

$$\frac{A}{a} = \frac{\pi R^2}{\pi(R/2)^2} = 4\ .$$

Thus, from Eqs. 32-38 and 32-39 the total discharge current is given by $i = i_d = 4(2.0\,\text{A}) = 8.0\,\text{A}$.

53. (a) Using Eq. 32-11, we find $\mu_{\text{orb},z} = -3\mu_B = -2.78\times10^{-23}$ J/T (that these are acceptable units for magnetic moment is seen from Eq. 32-12 or Eq. 32-7; they are equivalent to A·m^2).

(b) Similarly, for $m_\ell = -4$ we obtain $\mu_{\text{orb},z} = 3.71\times10^{-23}$ J/T.

54. (a) Since the field is decreasing, the displacement current (by Eq. 32-38) is downward, which produces (by the right-hand rule) a clockwise sense for the induced magnetic field.

(b) See the solution for part (a).

(c) and (d) We write $\vec{E} = E_z\hat{\text{k}} = (E_0 - \xi t)\hat{\text{k}}$ where $\xi = 60000$(V/m)/s. From Eq. 32-36 (treated in absolute value)

$$i_d = \varepsilon_0\,A\left|\frac{dE_z}{dt}\right| = \varepsilon_0\,A\,\xi$$

which yields $i_d = 2.1\times10^{-8}$ A for all values of t.

55. (a) From $\mu = iA = i\pi R_e^2$ we get

$$i = \frac{\mu}{\pi R_e^2} = \frac{8.0\times10^{22}\,\text{J/T}}{\pi(6.37\times10^6\,\text{m})^2} = 6.3\times10^8\ \text{A}\ .$$

(b) Yes, because far away from the Earth the fields of both the Earth itself and the current loop are dipole fields. If these two dipoles cancel each other out, then the net field will be zero.

(c) No, because the field of the current loop is not that of a magnetic dipole in the region close to the loop.

56. (a) The period of rotation is $T = 2\pi/\omega$ and in this time all the charge passes any fixed point near the ring. The average current is $i = q/T = q\omega/2\pi$ and the magnitude of the magnetic dipole moment is

$$\mu = iA = \frac{q\omega}{2\pi}\,\pi r^2 = \frac{1}{2}q\omega r^2 \ .$$

(b) We curl the fingers of our right hand in the direction of rotation. Since the charge is positive, the thumb points in the direction of the dipole moment. It is the same as the direction of the angular momentum vector of the ring.

57. (a) The potential energy of the atom in association with the presence an external magnetic field $\vec{B}_{\text{ext}}$ is given by Eqs. 32-11 and 32-12:

$$U = -\mu_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}} = -m_l \mu_B B_{\text{ext}} \ .$$

For level E_1 there is no change in energy as a result of the introduction of $\vec{B}_{\text{ext}}$, so $U \propto m_l = 0$, meaning that $m_l = 0$ for this level. For level E_2 the single level splits into a triplet (i.e., three separate ones) in the presence of $\vec{B}_{\text{ext}}$, meaning that there are three different values of m_l. The middle one in the triplet is unshifted from the original value of E_2 so its m_l must be equal to 0.

(b) The other two in the triplet then correspond to $m_l = -1$ and $m_1 = +1$, respectively.

(c) For any pair of adjacent levels in the triplet $|\Delta m_l| = 1$. Thus, the spacing is given by

$$\begin{aligned} \Delta U &= |\Delta(-m_l \mu_B B)| = |\Delta m_l| \mu_B B = \mu_B B \\ &= \left(9.27 \times 10^{-24}\,\text{J/T}\right)(0.50\,\text{T}) = 4.6 \times 10^{-24}\ \text{J} \end{aligned}$$

which is equivalent to 2.9×10^{-5} eV.

58. (a) The magnitude of the toroidal field is given by $B_0 = \mu_0 n i_p$, where n is the number of turns per unit length of toroid and i_p is the current required to produce the field (in the absence of the ferromagnetic material). We use the average radius ($r_{\text{avg}} = 5.5$cm) to calculate n:

$$n = \frac{N}{2\pi r_{\text{avg}}} = \frac{400\,\text{turns}}{2\pi(5.5 \times 10^{-2}\,\text{m})} = 1.16 \times 10^3\,\text{turns/m} \ .$$

Thus,

$$i_p = \frac{B_0}{\mu_0 n} = \frac{0.20 \times 10^{-3}\,\text{T}}{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(1.16 \times 10^3/\text{m})} = 0.14\ \text{A} \ .$$

(b) If Φ is the magnetic flux through the secondary coil, then the magnitude of the emf induced in that coil is $\mathcal{E} = N(d\Phi/dt)$ and the current in the secondary is $i_s = \mathcal{E}/R$, where R is the resistance of the coil. Thus

$$i_s = \left(\frac{N}{R}\right)\frac{d\Phi}{dt} \ .$$

The charge that passes through the secondary when the primary current is turned on is

$$q = \int i_s\,dt = \frac{N}{R}\int \frac{d\Phi}{dt}\,dt = \frac{N}{R}\int_0^\Phi d\Phi = \frac{N\Phi}{R} \ .$$

The magnetic field through the secondary coil has magnitude $B = B_0 + B_M = 801 B_0$, where B_M is the field of the magnetic dipoles in the magnetic material. The total field is perpendicular to the plane of the secondary coil, so the magnetic flux is $\Phi = AB$, where A is the area of the Rowland ring (the field is inside the ring, not in the region between the ring and coil). If r is the radius of the ring's cross section, then $A = \pi r^2$. Thus

$$\Phi = 801\pi r^2 B_0 \ .$$

The radius r is $(6.0\,\text{cm} - 5.0\,\text{cm})/2 = 0.50\,\text{cm}$ and

$$\Phi = 801\pi(0.50 \times 10^{-2}\,\text{m})^2(0.20 \times 10^{-3}\,\text{T}) = 1.26 \times 10^{-5}\,\text{Wb} \ .$$

Consequently,

$$q = \frac{50(1.26 \times 10^{-5}\,\text{Wb})}{8.0\,\Omega} = 7.9 \times 10^{-5}\ \text{C} \ .$$

59. Combining Eq. 32-7 with Eq. 32-2 and Eq. Eq. 32-3, we obtain

$$\Delta U = 2\,\mu_B\,B$$

where μ_B is the Bohr magneton (evaluated in Eq. 32-5). Thus, with $\Delta U = 6.0 \times 10^{-25}$ J, we find $B = |\vec{B}| = 0.032$ T.

60. (a) Using Eq. 32-37 but noting that the capacitor is being *discharged*, we have

$$\frac{d|\vec{E}|}{dt} = -\frac{i}{\varepsilon_0\,A} = -8.8 \times 10^{15}$$

where $A = (0.0080)^2$ and SI units are understood.

(b) Assuming a perfectly uniform field, even so near to an edge (which is consistent with the fact that fringing is neglected in §32-10), we follow part (a) of Sample Problem 32-4 and relate the (absolute value of the) line integral to the portion of displacement current enclosed.

$$\begin{aligned}\left|\oint \vec{B}\cdot d\vec{s}\right| &= \mu_0\,i_{d,\text{enc}}\\ &= \mu_0\,\frac{W\,H}{L^2}\,i\\ &= 5.9 \times 10^{-7}\ \text{Wb/m} \ .\end{aligned}$$

Chapter 33

1. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{(1.60 \times 10^{-6}\,\mathrm{C})^2}{2(140 \times 10^{-6}\,\mathrm{J})} = 9.14 \times 10^{-9}\ \mathrm{F}\ .$$

2. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00 \times 10^{-6}\,\mathrm{C}}{\sqrt{(1.10 \times 10^{-3}\,\mathrm{H})(4.00 \times 10^{-6}\,\mathrm{F})}} = 4.52 \times 10^{-2}\ \mathrm{A}\ .$$

3. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{(2.90 \times 10^{-6}\,\mathrm{C})^2}{2(3.60 \times 10^{-6}\,\mathrm{F})} = 1.17 \times 10^{-6}\ \mathrm{J}\ .$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.168 \times 10^{-6}\,\mathrm{J})}{75 \times 10^{-3}\,\mathrm{H}}} = 5.58 \times 10^{-3}\ \mathrm{A}\ .$$

4. (a) The period is $T = 4(1.50\,\mu\mathrm{s}) = 6.00\,\mu\mathrm{s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{6.00\,\mu\mathrm{s}} = 1.67 \times 10^5\ \mathrm{Hz}\ .$$

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or $3.00\,\mu$s.

5. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 33-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00 \times 10^3\,\mathrm{Hz}} = n(5.00\,\mu\mathrm{s})\ ,$$

where $n = 1, 2, 3, 4, \cdots$.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 33-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + nT = \frac{1}{2}(2n+1)T = \frac{(2n+1)}{2f} = \frac{(2n+1)}{2(2 \times 10^3\,\mathrm{Hz})} = (2n+1)(2.50\,\mu\mathrm{s})\ ,$$

where $n = 0, 1, 2, 3, 4, \cdots$.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 33-1). Later this will repeat every half-period (compare steps c and g in Fig. 33-1). Therefore,

$$t_L = \frac{T}{4} + \frac{nT}{2} = (1 + 2n)\frac{T}{4} = (2n + 1)(1.25\,\mu\text{s}) ,$$

where $n = 0, 1, 2, 3, 4, \cdots$.

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\,\text{N}}{(2.0 \times 10^{-3}\,\text{m})(0.50\,\text{kg})}} = 89 \text{ rad/s} .$$

(b) The period is $1/f$ and $f = \omega/2\pi$. Therefore,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{89\,\text{rad/s}} = 7.0 \times 10^{-2} \text{ s} .$$

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{(89\,\text{rad/s})^2(5.0\,\text{H})} = 2.5 \times 10^{-5} \text{ F} .$$

7. (a) The mass m corresponds to the inductance, so $m = 1.25\,\text{kg}$.

(b) The spring constant k corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175 \times 10^{-6}\,\text{C}\right)^2}{2(5.70 \times 10^{-6}\,\text{J})} = 2.69 \times 10^{-3}\,\text{F}$$

and

$$k = \frac{1}{2.69 \times 10^{-3}\,\text{m/N}} = 372 \text{ N/m} .$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\text{max}} = 175 \times 10^{-6}\,\text{m}$.

(d) The maximum speed v_{max} corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175 \times 10^{-6}\,\text{C}}{\sqrt{(1.25\,\text{H})(2.69 \times 10^{-3}\,\text{F})}} = 3.02 \times 10^{-3} \text{ A} .$$

Consequently, $v_{\text{max}} = 3.02 \times 10^{-3}\,\text{m/s}$.

8. We find the inductance from $f = \omega/2\pi = (2\pi\sqrt{LC})^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10 \times 10^3\,\text{Hz})^2(6.7 \times 10^{-6}\,\text{F})} = 3.8 \times 10^{-5} \text{ H} .$$

9. The time required is $t = T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}$. Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\text{H})(4.0 \times 10^{-6}\,\text{F})}}{4} = 7.0 \times 10^{-4} \text{ s} .$$

10. We apply the loop rule to the entire circuit:

$$\begin{aligned}
\mathcal{E}_{\text{total}} &= \mathcal{E}_{L_1} + \mathcal{E}_{C_1} + \mathcal{E}_{R_1} + \cdots \\
&= \sum_j \left(\mathcal{E}_{L_j} + \mathcal{E}_{C_j} + \mathcal{E}_{R_j}\right) \\
&= \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j\right) \\
&= L\frac{di}{dt} + \frac{q}{C} + iR \qquad \text{where } L = \sum_j L_j \ , \ \frac{1}{C} = \sum_j \frac{1}{C_j} \ , \quad R = \sum_j R_j
\end{aligned}$$

where we require $\mathcal{E}_{\text{total}} = 0$. This is equivalent to the simple LRC circuit shown in Fig. 33-22(b).

11. (a) $Q = CV_{\text{max}} = (1.0 \times 10^{-9}\,\text{F})(3.0\,\text{V}) = 3.0 \times 10^{-9}\,\text{C}$.

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0 \times 10^{-9}\,\text{C}}{\sqrt{(3.0 \times 10^{-3}\,\text{H})(1.0 \times 10^{-9}\,\text{F})}} = 1.7 \times 10^{-3}\ \text{A} \ .$$

(c) When the current is at a maximum, the magnetic field is at maximum:

$$U_{B,\text{max}} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0 \times 10^{-3}\,\text{H})(1.7 \times 10^{-3}\,\text{A})^2 = 4.5 \times 10^{-9}\ \text{J} \ .$$

12. (a) We use $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ to solve for L:

$$\begin{aligned}
L &= \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\text{max}}}{I}\right)^2 \\
&= C\left(\frac{V_{\text{max}}}{I}\right)^2 \\
&= (4.00 \times 10^{-6}\,\text{F})\left(\frac{1.50\,\text{V}}{50.0 \times 10^{-3}\,\text{A}}\right)^2 \\
&= 3.60 \times 10^{-3}\ \text{H} \ .
\end{aligned}$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(3.60 \times 10^{-3}\,\text{H})(4.00 \times 10^{-6}\,\text{F})}} = 1.33 \times 10^3\ \text{Hz} \ .$$

(c) Referring to Fig. 33-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4(1.33 \times 10^3\,\text{Hz})} = 1.88 \times 10^{-4}\ \text{s} \ .$$

13. (a) After the switch is thrown to position b the circuit is an LC circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(54.0 \times 10^{-3}\,\text{H})(6.20 \times 10^{-6}\,\text{F})}} = 275\ \text{Hz} \ .$$

(b) When the switch is thrown, the capacitor is charged to $V = 34.0\,\text{V}$ and the current is zero. Thus, the maximum charge on the capacitor is $Q = VC = (34.0\,\text{V})(6.20 \times 10^{-6}\,\text{F}) = 2.11 \times 10^{-4}\,\text{C}$. The current amplitude is

$$I = \omega Q = 2\pi f Q = 2\pi(275\,\text{Hz})(2.11 \times 10^{-4}\,\text{C}) = 0.365\ \text{A}\ .$$

14. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series. The corresponding oscillation frequencies are:

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0 \times 10^{-2}\,\text{H})(5.0 \times 10^{-6}\,\text{F})}} = 7.1 \times 10^2\ \text{Hz}$$

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0 \times 10^{-2}\,\text{H})(2.0 \times 10^{-6}\,\text{F})}} = 1.1 \times 10^3\ \text{Hz}$$

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1 + C_2)}} = \frac{1}{2\pi\sqrt{(1.0 \times 10^{-2}\,\text{H})(2.0 \times 10^{-6}\,\text{F} + 5.0 \times 10^{-6}\,\text{F})}} = 6.0 \times 10^2\ \text{Hz}$$

$$\begin{aligned} f_4 &= \frac{1}{2\pi\sqrt{LC_1C_2/(C_1 + C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0 \times 10^{-6}\,\text{F} + 5.0 \times 10^{-6}\,\text{F}}{(1.0 \times 10^{-2}\,\text{H})(2.0 \times 10^{-6}\,\text{F})(5.0 \times 10^{-6}\,\text{F})}} \\ &= 1.3 \times 10^3\ \text{Hz} \end{aligned}$$

15. (a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Consequently, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0\ .$$

(b) An additional capacitance C is chosen so the ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96\ .$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads, then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96\ .$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\ \text{pF}\ .$$

We solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency $C = 365\,\text{pF} + 36\,\text{pF} = 401\,\text{pF}$ and $f = 0.54\,\text{MHz}$. Thus

$$L = \frac{1}{(2\pi)^2 C f^2} = \frac{1}{(2\pi)^2(401 \times 10^{-12}\,\text{F})(0.54 \times 10^6\,\text{Hz})^2} = 2.2 \times 10^{-4}\ \text{H}\ .$$

16. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1 - 75.0\%) = 25.0\%$, then

$$\frac{U_E}{U} = \frac{q^2/2C}{Q^2/2C} = 25.0\%$$

which leads to $q = \sqrt{0.250}\,Q = 0.500Q$.

(b) From

$$\frac{U_B}{U} = \frac{Li^2/2}{LI^2/2} = 75.0\% \ ,$$

we find $i = \sqrt{0.750}\, I = 0.866I$.

17. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$\begin{aligned} U &= U_E + U_B = \frac{q^2}{2C} + \frac{i^2 L}{2} \\ &= \frac{(3.80 \times 10^{-6}\,\mathrm{C})^2}{2(7.80 \times 10^{-6}\,\mathrm{F})} + \frac{(9.20 \times 10^{-3}\,\mathrm{A})^2 (25.0 \times 10^{-3}\,\mathrm{H})}{2} = 1.98 \times 10^{-6}\ \mathrm{J}\ . \end{aligned}$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2(7.80 \times 10^{-6}\,\mathrm{F})(1.98 \times 10^{-6}\,\mathrm{J})} = 5.56 \times 10^{-6}\ \mathrm{C}\ .$$

(c) From $U = I^2 L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2(1.98 \times 10^{-6}\,\mathrm{J})}{25.0 \times 10^{-3}\,\mathrm{H}}} = 1.26 \times 10^{-2}\ \mathrm{A}\ .$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80 \times 10^{-6}\,\mathrm{C}}{5.56 \times 10^{-6}\,\mathrm{C}}\right) = \pm 46.9^\circ\ .$$

For $\phi = +46.9^\circ$ the charge on the capacitor is decreasing, for $\phi = -46.9^\circ$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$. We obtain $-\omega Q \sin\phi$, which we wish to be positive. Since $\sin(+46.9^\circ)$ is positive and $\sin(-46.9^\circ)$ is negative, the correct value for increasing charge is $\phi = -46.9^\circ$.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take $\phi = +46.9^\circ$.

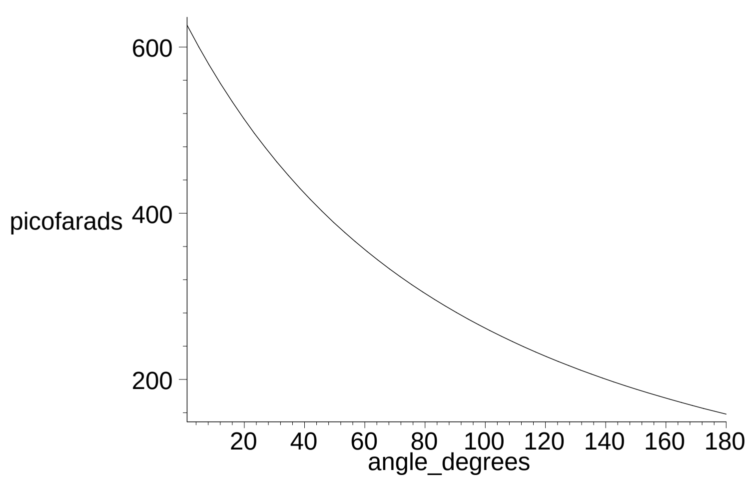
18. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1 + \frac{\theta}{180^\circ}\right) \implies \theta = 180^\circ\left(\frac{f}{f_0} - 1\right)$$

where $f_0 = 2 \times 10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2 L f_0^2 \left(1 + \frac{\theta}{180^\circ}\right)^2} = \frac{81}{400000\pi^2(180^\circ + \theta)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted, below.



19. (a) The charge (as a function of time) is given by $q = Q \sin \omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q \cos \omega t \ ,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC}$,

$$Q = I\sqrt{LC} = (2.00\,\text{A})\sqrt{(3.00 \times 10^{-3}\,\text{H})(2.70 \times 10^{-6}\,\text{F})} = 1.80 \times 10^{-4}\ \text{C} \ .$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2 \omega \sin \omega t \cos \omega t}{C} \ .$$

We use the trigonometric identity $\cos \omega t \sin \omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C} \sin(2\omega t) \ .$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2\,\text{rad}$. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi T}{4(2\pi)} = \frac{T}{8}$$

where T is the period of oscillation. The relationship $\omega = 2\pi/T$ was used.

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\text{max}} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC} \ .$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00 \times 10^{-3}\,\text{H})(2.70 \times 10^{-6}\,\text{F})} = 5.655 \times 10^{-4}\,\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{\text{max}} = \frac{\pi(1.80 \times 10^{-4}\,\text{C})^2}{(5.655 \times 10^{-4}\,\text{s})(2.70 \times 10^{-6}\,\text{F})} = 66.7\ \text{W} \ .$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

20. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\begin{aligned}
\omega' &= \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} \\
&= \frac{1}{\sqrt{(L_1 + L_2)C_1C_2/(C_1 + C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2 + L_2C_2C_1)/(C_1 + C_2)}} \\
&= \frac{1}{\sqrt{L_1C_1}}\,\frac{1}{\sqrt{(C_1 + C_2)/(C_1 + C_2)}} = \omega \ ,
\end{aligned}$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

21. (a) We compare this expression for the current with $i = I\sin(\omega t + \phi_0)$. Setting $(\omega t + \phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56 \times 10^{-4}$ s.

(b) Since $\omega = 2500\,\text{rad/s} = (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2(64.0 \times 10^{-6}\,\text{F})} = 2.50 \times 10^{-3}\ \text{H} .$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50 \times 10^{-3}\,\text{H})(1.60\,\text{A})^2 = 3.20 \times 10^{-3}\ \text{J} .$$

22. (a) The figure implies that the the instantaneous current through the leftmost inductor is the same as that through the rightmost one, which means there is no current through the middle inductor (at any instant). Applying the loop rule to the outer loop (including the rightmost and leftmost inductors), with the current suitably related to the rate of change of charge, we find

$$2L\,\frac{d^2q}{dt^2} + \frac{2}{C}\,q = 0 \implies \omega = \frac{1}{\sqrt{(2L)(C/2)}} = \frac{1}{\sqrt{LC}} .$$

(b) In this case, we see that the middle inductor must have current $2i(t)$ flowing downward, and application of the loop rule to, say, the left loop leads to

$$L\,\frac{d^2q}{dt^2} + L\left(2\,\frac{d^2q}{dt^2}\right) + \frac{1}{C}\,q = 0 \implies \omega = \frac{1}{\sqrt{(3L)(C)}} = \frac{1}{\sqrt{3LC}} .$$

23. The energy needed to charge the $100\,\mu$F capacitor to 300 V is

$$\frac{1}{2}C_2V^2 = \frac{1}{2}(100 \times 10^{-6}\,\text{F})(300\,\text{V})^2 = 4.50\ \text{J} .$$

The energy initially in the $900\,\mu$F capacitor is

$$\frac{1}{2}C_1V^2 = \frac{1}{2}(900 \times 10^{-6}\,\text{F})(100\,\text{V})^2 = 4.50\ \text{J} .$$

All the energy originally in the $900\,\mu$F capacitor must be transferred to the $100\,\mu$F capacitor. The plan is to store it temporarily in the inductor. We do this by leaving switch S_1 open and closing switch S_2. We wait until the $900\,\mu$F capacitor is completely discharged and the current in the circuit is at maximum (this occurs at $t = T_1/4$, one quarter of the relevant period). Since

$$T_1 = 2\pi\sqrt{LC_1} = 2\pi\sqrt{(10.0\,\text{H})(900 \times 10^{-6}\,\text{F})} = 0.596\ \text{s} ,$$

we wait until $t = (0.596\,\text{s})/4 = 0.149$ s. Now, we close switch S_1 while simultaneously opening switch S_2. Next, we wait for one-fourth of the T_2 period to elapse and open switch S_1. The $100\,\mu$F capacitor then has maximum charge, and all the energy resides in it. Since

$$T_2 = 2\pi\sqrt{LC_2} = 2\pi\sqrt{(10.0\,\text{H})(100 \times 10^{-6}\,\text{F})} = 0.199\ \text{s} ,$$

we must keep S_1 closed for $(0.199\,\text{s})/4 = 0.0497$ s. It is helpful to refer to Figure 23-1 to appreciate the emphasis on "quarter-periods" in this solution.

24. (a) Since $T = 2\pi/\omega = 2\pi\sqrt{LC}$, we may rewrite the power on the exponential factor as

$$-\pi R\sqrt{\frac{C}{L}}\frac{t}{T} = -\pi R\sqrt{\frac{C}{L}}\frac{t}{2\pi\sqrt{LC}} = -\frac{Rt}{2L} .$$

Thus $e^{-Rt/2L} = e^{-\pi R\sqrt{C/L}(t/T)}$.

(b) Since $-\pi R\sqrt{C/L}(t/T)$ must be unitless (as is t/T), $R\sqrt{C/L}$ must also be unitless. Thus, the SI unit of $\sqrt{C/L}$ must be Ω^{-1}. In other words, the SI unit for $\sqrt{L/C}$ is Ω.

(c) Since the amplitude of oscillation reduces by a factor of $e^{-\pi R\sqrt{C/L}(T/T)} = e^{-\pi R\sqrt{C/L}}$ after each cycle, the condition is equivalent to $\pi R\sqrt{C/L} \ll 1$, or $R \ll \sqrt{L/C}$.

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$\begin{aligned} t &= 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) \\ &= 50\left(2\pi\sqrt{(220\times10^{-3}\,\mathrm{H})\,(12.0\times10^{-6}\,\mathrm{F})}\right) = 0.5104\ \mathrm{s}\ . \end{aligned}$$

The maximum charge on the capacitor decays according to

$$q_{\max} = Qe^{-Rt/2L}$$

(this is called the *exponentially decaying amplitude* in §33-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 33-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2(220\times10^{-3}\,\mathrm{H})}{0.5104\,\mathrm{s}}\ln(0.99) = 8.66\times10^{-3}\ \Omega\ .$$

26. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 33-25:

$$\begin{aligned} q &= Qe^{-Rt/2L}\cos(\omega' t+\phi) = Qe^{-RNT/2L}\cos(\omega'(2\pi N/\omega')+\phi) \\ &= Qe^{-RN(2\pi\sqrt{L/C})/2L}\cos(2\pi N+\phi) \\ &= Qe^{-N\pi R\sqrt{C/L}}\cos(\phi). \end{aligned}$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q\cos\phi$, where $q_0 = 6.2\,\mu\mathrm{C}$ is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 e^{-N\pi R\sqrt{C/L}}$ and obtain

$$\begin{aligned} q_5 &= (6.2\,\mu\mathrm{C})e^{-5\pi(7.2\,\Omega)\sqrt{0.0000032\,\mathrm{F}/12\,\mathrm{H}}} = 5.85\,\mu\mathrm{C} \\ q_{10} &= (6.2\,\mu\mathrm{C})e^{-10\pi(7.2\,\Omega)\sqrt{0.0000032\,\mathrm{F}/12\,\mathrm{H}}} = 5.52\,\mu\mathrm{C} \\ q_{100} &= (6.2\,\mu\mathrm{C})e^{-100\pi(7.2\,\Omega)\sqrt{0.0000032\,\mathrm{F}/12\,\mathrm{H}}} = 1.93\,\mu\mathrm{C}\ . \end{aligned}$$

27. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 33-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{\max}^2/2C$, where $q_{\max}$ is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{\max}^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \implies q_{\max} = \frac{Q}{\sqrt{2}}\ .$$

Now $q_{\max}$ (referred to as the *exponentially decaying amplitude* in §33-5) is related to Q (and the other parameters of the circuit) by

$$q_{\max} = Qe^{-Rt/2L} \implies \ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}\ .$$

Setting $q_{\max} = Q/\sqrt{2}$, we solve for t:

$$t = -\frac{2L}{R}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R}\ln 2 \ .$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

28. (a) In Eq. 33-25, we set $q = 0$ and $t = 0$ to obtain $0 = Q\cos\phi$. This gives $\phi = \pm\pi/2$ (assuming $Q \neq 0$). It should be noted that other roots are possible (for instance, $\cos(3\pi/2) = 0$) but the $\pm\pi/2$ choices for the phase constant are in some sense the "simplest." We choose $\phi = -\pi/2$ to make the manipulation of signs in the expressions below easier to follow. To simplify the work in part (b), we note that $\cos(\omega' t - \pi/2) = \sin(\omega' t)$.

(b) First, we calculate the time-dependent current $i(t)$ from Eq. 33-25:

$$\begin{aligned} i(t) &= \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-Rt/2L}\sin(\omega' t)\right) \\ &= -\frac{QR}{2L}e^{-Rt/2L}\sin(\omega' t) + Q\omega' e^{-Rt/2L}\cos(\omega' t) \\ &= Qe^{-Rt/2L}\left(-\frac{R\sin(\omega' t)}{2L} + \omega'\cos(\omega' t)\right) \ , \end{aligned}$$

which we evaluate at $t = 0$: $i(0) = Q\omega'$. If we denote $i(0) = I$ as suggested in the problem, then $Q = I/\omega'$. Returning this to Eq. 33-25 leads to

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi) = \left(\frac{I}{\omega'}\right)e^{-Rt/2L}\cos\left(\omega' t - \frac{\pi}{2}\right) = Ie^{-Rt/2L}\frac{\sin(\omega' t)}{\omega'}$$

which answers the question if we interpret "current amplitude" as I. If one, instead, interprets an (exponentially decaying) "current amplitude" to be more appropriately defined as $i_{\max} = i(t)/\cos(\cdots)$ (that is, the current after dividing out its oscillatory behavior), then another step is needed in the $i(t)$ manipulations, above. Using the identity $x\cos\alpha - y\sin\alpha = r\cos(\alpha + \beta)$ where $r = \sqrt{x^2 + y^2}$ and $\tan\beta = y/x$, we can write the current as

$$i(t) = Qe^{-Rt/2L}\left(-\frac{R\sin(\omega' t)}{2L} + \omega'\cos(\omega' t)\right) = Q\sqrt{\omega'^2 + \left(\frac{R}{2L}\right)^2}\, e^{-Rt/2L}\cos(\omega' t + \theta)$$

where $\theta = \tan^{-1}(R/2L\omega')$. Thus, the current amplitude defined in this second way becomes (using Eq. 33-26 for ω')

$$i_{\max} = Q\sqrt{\omega'^2 + \left(\frac{R}{2L}\right)^2}\, e^{-Rt/2L} = Q\omega e^{-Rt/2L} \ .$$

In terms of $i_{\max}$ the expression for charge becomes

$$q = Qe^{-Rt/2L}\sin(\omega' t) = \left(\frac{i_{\max}}{\omega}\right)\sin(\omega' t)$$

which is remarkably similar to our previous "result" in terms of I, except for the fact that ω' in the denominator has now been replaced with ω (and, of course, the exponential has been absorbed into the definition of $i_{\max}$).

29. Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\max 1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q_{\max 1}^2}{2C} = \frac{Q^2}{2C}e^{-Rt/L}$$

where

$$q_{\max 1} = Q\, e^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in §33-5). One period later the charge on the fully charged capacitor is

$$q_{\max 2} = Q\, e^{-R(t+T)/2L} \qquad \text{where} \quad T = \frac{2\pi}{\omega'} \ ,$$

and the energy is

$$U(t+T) = \frac{q_{\max 2}^2}{2C} = \frac{Q^2}{2C}\, e^{-R(t+T)/L} \ .$$

The fractional loss in energy is

$$\frac{|\Delta U|}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L} \ .$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L} + \frac{R^2T^2}{2L^2} + \cdots \ .$$

If we approximate $\omega \approx \omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U} \approx 1 - \left(1 - \frac{RT}{L} + \cdots\right) \approx \frac{RT}{L} = \frac{2\pi R}{\omega L} \ .$$

30. (a) We use $I = \mathcal{E}/X_c = \omega_d C\mathcal{E}$:

$$I = \omega_d C\mathcal{E}_m = 2\pi f_d C\mathcal{E}_m = 2\pi(1.00 \times 10^3\,\text{Hz})(1.50 \times 10^{-6}\,\text{F})(30.0\,\text{V}) = 0.283\ \text{A} \ .$$

(b) $I = 2\pi(8.00 \times 10^3\,\text{Hz})(1.50 \times 10^{-6}\,\text{F})(30.0\,\text{V}) = 2.26\,\text{A}$.

31. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \mathcal{E}_m$. Therefore,

$$I = \frac{V_L}{X_L} = \frac{\mathcal{E}_m}{2\pi f_d L} = \frac{30.0\,\text{V}}{2\pi(1.00 \times 10^3\,\text{Hz})(50.0 \times 10^{-3}\,\text{H})} = 0.0955\ \text{A} \ .$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now $(0.0955\,\text{A})/8 = 0.0119\,\text{A}$.

32. (a) and (b) Regardless of the frequency of the generator, the current through the resistor is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\text{V}}{50\,\Omega} = 0.60\ \text{A} \ .$$

33. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0 \times 10^{-3}\,\text{H})(10 \times 10^{-6}\,\text{F})}} = 650\ \text{Hz} \ .$$

(b) The inductive reactance is $X_L = \omega_d L = 2\pi f_d L = 2\pi(650\,\text{Hz})(6.0 \times 10^{-3}\,\text{H}) = 24\,\Omega$. The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

34. (a) The circuit consists of one generator across one inductor; therefore, $\mathcal{E}_m = V_L$. The current amplitude is

$$I = \frac{\mathcal{E}_m}{X_L} = \frac{\mathcal{E}_m}{\omega_d L} = \frac{25.0\,\text{V}}{(377\,\text{rad/s})(12.7\,\text{H})} = 5.22 \times 10^{-3}\ \text{A}\ .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 31-37 gives $\mathcal{E}_L = 0$ at that instant. Stated another way, since $\mathcal{E}(t)$ and $i(t)$ have a 90° phase difference, then $\mathcal{E}(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\mathcal{E} = -\frac{1}{2}\mathcal{E}_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that $\mathcal{E}$ is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [$n =$ integer]. Consequently, Eq. 33-29 yields (for all values of n)

$$i = I\,\sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = \left(5.22 \times 10^{-3}\,\text{A}\right)\left(\frac{\sqrt{3}}{2}\right) = 4.51 \times 10^{-3}\ \text{A}\ .$$

35. (a) The generator emf is a maximum when $\sin(\omega_d t - \pi/4) = 1$ or $\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi$ [$n =$ integer]. The first time this occurs after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350\,\text{rad/s})} = 6.73 \times 10^{-3}\ \text{s}\ .$$

(b) The current is a maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or $\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi$ [$n =$ integer]. The first time this occurs after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$ (as in part (a), $n = 0$). Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350\,\text{rad/s})} = 1.12 \times 10^{-2}\ \text{s}\ .$$

(c) The current lags the emf by $+\frac{\pi}{2}$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \mathcal{E}_m$. Thus, $\mathcal{E}_m = I\omega_d L$ and

$$L = \frac{\mathcal{E}_m}{I\omega_d} = \frac{30.0\,\text{V}}{(620 \times 10^{-3}\,\text{A})(350\,\text{rad/s})} = 0.138\ \text{H}\ .$$

36. (a) The circuit consists of one generator across one capacitor; therefore, $\mathcal{E}_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\mathcal{E}_m}{X_C} = \omega C\mathcal{E}_m = (377\,\text{rad/s})(4.15 \times 10^{-6}\,\text{F})(25.0\,\text{V}) = 3.91 \times 10^{-2}\,\text{A}\ .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{\max}$), but rather as it passes through the (momentary) states of being uncharged ($q = 0$). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\mathcal{E}(t)$ and current $i(t)$ have a $\phi = -90°$ phase relation, implying $\mathcal{E}(t) = 0$ when $i(t) = I$. The fact that $\phi = -90° = -\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\mathcal{E} = -\frac{1}{2}\mathcal{E}_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that $\mathcal{E}$ is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [$n =$ integer]. Consequently, Eq. 33-29 yields (for all values of n)

$$i = I \sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = \left(3.91 \times 10^{-3}\,\text{A}\right)\left(-\frac{\sqrt{3}}{2}\right) = -3.38 \times 10^{-2}\text{ A} .$$

37. (a) Now $X_C = 0$, while $R = 160\,\Omega$ and $X_L = 86.7\,\Omega$ remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(160\,\Omega)^2 + (86.7\,\Omega)^2} = 182\ \Omega .$$

The current amplitude is now found to be

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\text{V}}{182\,\Omega} = 0.198\text{ A} .$$

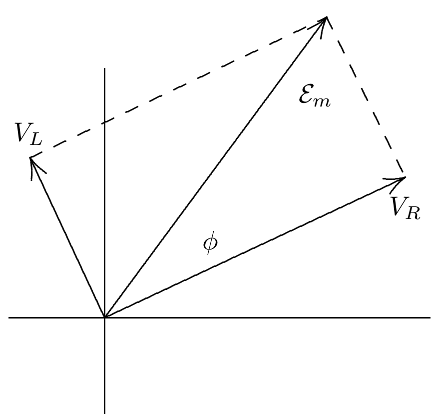
The phase angle is, from Eq. 33-65,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{160\,\Omega}\right) = 28.5^\circ .$$

(b) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.198\,\text{A})(160\,\Omega) &\approx 32\text{ V} \\ V_L &= IX_L = (0.216\,\text{A})(86.7\,\Omega) &\approx 17\text{ V} \end{aligned}$$

This is an inductive circuit, so $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale below.



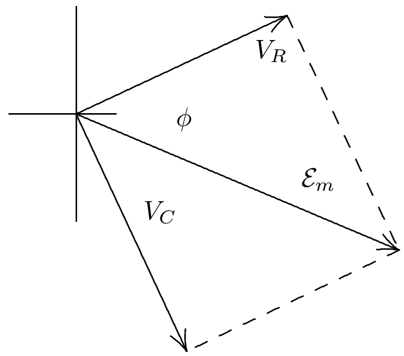
38. (a) Now $X_L = 0$, while $R = 160\,\Omega$ and $X_C = 177\,\Omega$ remain as shown in the Sample Problem. Therefore, the impedance, current amplitude and phase angle are

$$\begin{aligned} Z &= \sqrt{R^2 + X_C^2} = \sqrt{(160\,\Omega)^2 + (177\,\Omega)^2} = 239\ \Omega , \\ I &= \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\text{V}}{239\,\Omega} = 0.151\text{ A} , \\ \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{160\,\Omega}\right) = -47.9^\circ . \end{aligned}$$

(b) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.151\,\mathrm{A})(160\,\Omega) &\approx& \quad 24\ \mathrm{V} \\ V_C &= IX_C = (0.151\,\mathrm{A})(177\,\Omega) &\approx& \quad 27\ \mathrm{V} \end{aligned}$$

The circuit is capacitive, so I leads $\mathcal{E}_m$. The phasor diagram is drawn to scale below.



39. (a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\,\mathrm{Hz})(70.0\times 10^{-6}\,\mathrm{F})} = 37.9\ \Omega\ .$$

The inductive reactance $86.7\,\Omega$ is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(160\,\Omega)^2 + (37.9\,\Omega - 86.7\,\Omega)^2} = 167\ \Omega\ .$$

The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\,\mathrm{V}}{167\,\Omega} = 0.216\ \mathrm{A}\ .$$

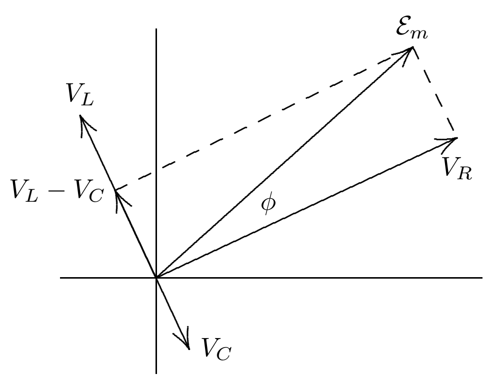
The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{160\,\Omega}\right) = 17.0^\circ\ .$$

(b) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.216\,\mathrm{A})(160\,\Omega) = 34.6\ \mathrm{V} \\ V_L &= IX_L = (0.216\,\mathrm{A})(86.7\,\Omega) = 18.7\ \mathrm{V} \\ V_C &= IX_C = (0.216\,\mathrm{A})(37.9\,\Omega) = 8.19\ \mathrm{V} \end{aligned}$$

Note that $X_L > X_C$, so that $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale below.



40. (a) The resonance frequency f_0 of the circuit is about $(1.50\,\text{kHz} + 1.30\,\text{kHz})/2 = 1.40\,\text{kHz}$. Thus, from $2\pi f_0 = (LC)^{-1/2}$ we get

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 (1.40\times 10^3\,\text{Hz})^2 (5.50\times 10^{-6}\,\text{F})} = 2.35\times 10^{-3}\ \text{H}\ .$$

(b) From the resonance curves shown in the textbook, we see that as R increases the resonance curve gets more spread out, so the two frequencies at which the amplitude is at half-maximum level will move away from each other.

41. The amplitude of the voltage across the inductor in an RLC series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\,\text{H}}{\sqrt{(1.0\,\text{H})(1.0\times 10^{-6}\,\text{F})}} = 1000\ \Omega\ .$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

$$I = \left.\frac{\mathcal{E}_m}{Z}\right|_{\text{resonance}} = \frac{\mathcal{E}_m}{R} = \frac{10\,\text{V}}{10\,\Omega} = 1.0\ \text{A}\ .$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\,\text{A})(1000\,\Omega) = 1000\ \text{V}$$

which is much larger than the amplitude of the generator emf.

42. (a) We note that we obtain the maximum value in Eq. 33-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417\ \text{s}$$

or 4.17 ms. The result is $\mathcal{E}_m \sin(\pi/2) = \mathcal{E}_m \sin(90°) = 36.0\,\text{V}$. We note, for reference in the subsequent parts, that at $t = 4.17\,\text{ms}$, the current is

$$i = I\sin(\omega_d t - \phi) = I\sin(90° - (-29.4°)) = (0.196\,\text{A})\cos(29.4°) = 0.171\ \text{A}$$

using Eq. 33-29 and the results of the Sample Problem.

(b) At $t = 4.17$ ms, Ohm's law directly gives

$$v_R = iR = (I\cos(29.4°))\,R(0.171\,\text{A})(160\,\Omega) = 27.3\ \text{V}\ .$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at $t = 4.17$ ms, we obtain

$$v_C = I\sin(90° - (-29.4°) - 90°)X_C = IX_C\sin(29.4°) = (0.196\,\text{A})(177\,\Omega)\sin(29.4°) = 17.0\ \text{V}\ .$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at $t = 4.17$ ms, we find

$$v_L = I\sin(90° - (-29.4°) + 90°)X_L = -IX_L\sin(29.4°) = -(0.196\,\text{A})(86.7\,\Omega)\sin(29.4°) = -8.3\ \text{V}\ .$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).

43. The resistance of the coil is related to the reactances and the phase constant by Eq. 33-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi\ ,$$

which we solve for R:

$$\begin{aligned} R &= \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) \\ &= \frac{1}{\tan 75^\circ}\left[(2\pi)(930\,\mathrm{Hz})(8.8\times10^{-2}\,\mathrm{H}) - \frac{1}{(2\pi)(930\,\mathrm{Hz})(0.94\times10^{-6}\,\mathrm{F})}\right] \\ &= 89\ \Omega\ . \end{aligned}$$

44. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\,\mathrm{Hz})(24.0\times10^{-6}\,\mathrm{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2} \\ &= \sqrt{(220\,\Omega)^2 + [2\pi(400\,\mathrm{Hz})(150\times10^{-3}\,\mathrm{H}) - 16.6\,\Omega]^2} = 422\ \Omega\ . \end{aligned}$$

(c) The current amplitude is

$$I = \frac{\mathcal{E}_m}{Z} = \frac{220\,\mathrm{V}}{422\,\Omega} = 0.521\ \mathrm{A}\ .$$

(d) Now $X_C \propto C_{\mathrm{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\mathrm{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\,\Omega)^2 + [2\pi(400\,\mathrm{Hz})(150\times10^{-3}\,\mathrm{H}) - 2(16.6\,\Omega)]^2} = 408\,\Omega < 422\,\Omega\ .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

45. (a) For a given amplitude $(E)_m$ of the generator emf, the current amplitude is given by

$$I = \frac{(E)_m}{Z} = \frac{(E)_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}\ .$$

We find the maximum by setting the derivative with respect to ω_d equal to zero:

$$\frac{dI}{d\omega_d} = -(E)_m\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]^{-3/2}\left[\omega_d L - \frac{1}{\omega_d C}\right]\left[L + \frac{1}{\omega_d^2 C}\right]\ .$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$; it does so for $\omega_d = 1/\sqrt{LC} = \omega$. For this circuit,

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00\,\mathrm{H})(20.0\times10^{-6}\,\mathrm{F})}} = 224\ \mathrm{rad/s}\ .$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{(E)_m}{R} = \frac{30.0\,\mathrm{V}}{5.00\,\Omega} = 6.00\ \mathrm{A}\ .$$

(c) We want to find the (positive) values of ω_d for which $I = \frac{(E)_m}{2R}$:

$$\frac{(E)_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{(E)_m}{2R}\ .$$

This may be rearranged to yield

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2 \ .$$

Taking the square root of both sides (acknowledging the two $\pm$ roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3}CR\right) - 1 = 0 \ .$$

Using the quadratic formula, we find the smallest positive solution

$$\begin{aligned}
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{-\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 219 \ \mathrm{rad/s} \ ,
\end{aligned}$$

and the largest positive solution

$$\begin{aligned}
\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
&= \frac{+\sqrt{3}(20.0 \times 10^{-6}\,\mathrm{F})(5.00\,\Omega)}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&\qquad + \frac{\sqrt{3(20.0 \times 10^{-6}\,\mathrm{F})^2(5.00\,\Omega)^2 + 4(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})}}{2(1.00\,\mathrm{H})(20.0 \times 10^{-6}\,\mathrm{F})} \\
&= 228 \ \mathrm{rad/s} \ .
\end{aligned}$$

(d) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\mathrm{rad/s} - 219\,\mathrm{rad/s}}{224\,\mathrm{rad/s}} = 0.04 \ .$$

46. Four possibilities exist: (1) $C_1 = 4.00\,\mu\mathrm{F}$ is used alone; (2) $C_2 = 6.00\,\mu\mathrm{F}$ is used alone; (3) C_1 and C_2 are connected in series; and (4) C_1 and C_2 are connected in parallel. The corresponding resonant frequencies are

$$\begin{aligned}
f_1 &= \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(2.00 \times 10^{-3}\,\mathrm{H})(4.00 \times 10^{-6}\,\mathrm{F})}} = 1.78 \times 10^3 \ \mathrm{Hz} \\
f_2 &= \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(2.00 \times 10^{-3}\,\mathrm{H})(6.00 \times 10^{-6}\,\mathrm{F})}} = 1.45 \times 10^3 \ \mathrm{Hz} \\
f_3 &= \frac{1}{2\pi\sqrt{LC_1C_2/(C_1 + C_2)}} = 2.30 \times 10^3 \ \mathrm{Hz} \\
f_4 &= \frac{1}{2\pi\sqrt{L(C_1 + C_2)}} = 1.13 \times 10^3 \ \mathrm{Hz} \ .
\end{aligned}$$

47. We use the expressions found in Problem 45:

$$\begin{aligned}
\omega_1 &= \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \\
\omega_2 &= \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} \ .
\end{aligned}$$

We also use Eq. 33-4. Thus,

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}} \ .$$

For the data of Problem 45,

$$\frac{\Delta\omega_d}{\omega} = (5.00\,\Omega)\sqrt{\frac{3(20.0\times10^{-6}\,\mathrm{F})}{1.00\,\mathrm{H}}} = 3.87\times10^{-2} \ .$$

This is in agreement with the result of Problem 45. The method of Problem 45, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

48. (a) Since $L_{\mathrm{eq}} = L_1 + L_2$ and $C_{\mathrm{eq}} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\begin{aligned} \omega &= \frac{1}{2\pi\sqrt{L_{\mathrm{eq}}C_{\mathrm{eq}}}} = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C_2+C_3)}} \\ &= \frac{1}{2\pi\sqrt{(1.70\times10^{-3}\,\mathrm{H} + 2.30\times10^{-3}\,\mathrm{H})(4.00\times10^{-6}\,\mathrm{F} + 2.50\times10^{-6}\,\mathrm{F} + 3.50\times10^{-6}\,\mathrm{F})}} \\ &= 796\ \mathrm{Hz}\ . \end{aligned}$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{\mathrm{eq}}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.

49. The average power dissipated in resistance R when the current is alternating is given by $P_{\mathrm{avg}} = I_{\mathrm{rms}}^2 R$, where I_{rms} is the root-mean-square current. Since $I_{\mathrm{rms}} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{\mathrm{avg}} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R$. Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\ \mathrm{A}}{\sqrt{2}} = 1.84\ \mathrm{A}\ .$$

50. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

51. The amplitude (peak) value is

$$V_{\mathrm{max}} = \sqrt{2}\,V_{\mathrm{rms}} = \sqrt{2}(100\,\mathrm{V}) = 141\ \mathrm{V}\ .$$

52. (a) We refer to problem 34, part (c). The power delivered by the generator at this instant is $P = \mathcal{E}(t)i(t) = \mathcal{E}_m \sin(2n\pi - \pi/6) I \sin(\pi/3) = -\mathcal{E}_m I \sin(\pi/6)\sin(\pi/3)$. This is less than zero, so it is taking energy from the rest of the circuit.

(b) We refer to problem 36, part (c). The power delivered by the generator at this instant is $P = \mathcal{E}(t)i(t) = \mathcal{E}_m \sin(2n\pi - \pi/6) I \sin(-2\pi/3) = \mathcal{E}_m I \sin(\pi/6)\sin(2\pi/3)$. Since this is positive, it is supplying energy to the rest of the system.

53. We use $P_{\mathrm{avg}} = I_{\mathrm{rms}}^2 R = \frac{1}{2}I^2R$.

- $P_{\mathrm{avg}} = 0$, since $R = 0$.
- $P_{\mathrm{avg}} = \frac{1}{2}I^2R = \frac{1}{2}(0.600\,\mathrm{A})^2(50\,\Omega) = 9.0\,\mathrm{W}$.
- $P_{\mathrm{avg}} = \frac{1}{2}I^2R = \frac{1}{2}(0.198\,\mathrm{A})^2(160\,\Omega) = 3.14\,\mathrm{W}$.
- $P_{\mathrm{avg}} = \frac{1}{2}I^2R = \frac{1}{2}(0.151\,\mathrm{A})^2(160\,\Omega) = 1.82\,\mathrm{W}$.

54. We start with Eq. 33-76:

$$P_{\text{avg}} = \mathcal{E}_{\text{rms}} I_{\text{rms}} \cos\phi = \mathcal{E}_{\text{rms}} \left(\frac{\mathcal{E}_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right) = \frac{\mathcal{E}_{\text{rms}}^2 R}{Z^2} \ .$$

For a purely resistive circuit, $Z = R$, and this result reduces to Eq. 27-23 (with V replaced with $\mathcal{E}_{\text{rms}}$). This is also the case for a series RLC circuit at resonance. The average rate for dissipating energy is, of course, zero if $R = 0$, as would be the case for a purely inductive circuit.

55. (a) Using Eq. 33-61, the impedance is

$$Z = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\ \Omega \ .$$

(b) We use the result of problem 54:

$$P_{\text{avg}} = \frac{\mathcal{E}_{\text{rms}}^2 R}{Z^2} = \frac{(120\,\text{V})^2(12.0\,\Omega)}{(12.1\,\Omega)^2} = 1.18 \times 10^3\ \text{W} \ .$$

56. The current in the circuit satisfies $i(t) = I\sin(\omega_d t - \phi)$, where

$$\begin{aligned} I &= \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} \\ &= \frac{45.0\,\text{V}}{\sqrt{(16.0\,\Omega)^2 + \{(3000\,\text{rad/s})(9.20\,\text{mH}) - 1/[(3000\,\text{rad/s})(31.2\,\mu\text{F})]\}^2}} \\ &= 1.93\ \text{A} \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right) \\ &= \tan^{-1}\left[\frac{(3000\,\text{rad/s})(9.20\,\text{mH})}{16.0\,\Omega} - \frac{1}{(3000\,\text{rad/s})(16.0\,\Omega)(31.2\,\mu\text{F})}\right] \\ &= 46.5^\circ \ . \end{aligned}$$

(a) The power supplied by the generator is

$$\begin{aligned} P_g &= i(t)\mathcal{E}(t) = I\sin(\omega_d t - \phi)\mathcal{E}_m \sin\omega_d t \\ &= (1.93\,\text{A})(45.0\,\text{V})\sin[(3000\,\text{rad/s})(0.442\,\text{ms})]\sin[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5^\circ] \\ &= 41.4\ \text{W} \ . \end{aligned}$$

(b) The rate at which the energy in the capacitor changes is

$$\begin{aligned} P_c &= -\frac{d}{dt}\left(\frac{q^2}{2C}\right) = -i\frac{q}{C} = -iV_c \\ &= -I\sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin[2(\omega_d t - \phi)] \\ &= -\frac{(1.93\,\text{A})^2}{2(3000\,\text{rad/s})(31.2 \times 10^{-6}\,\text{F})}\sin[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5^\circ)] \\ &= -17.0\ \text{W} \ . \end{aligned}$$

(c) The rate at which the energy in the inductor changes is

$$\begin{aligned}P_i &= \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t-\phi)\frac{d}{dt}[I\sin(\omega_d t-\phi)]\\ &= \frac{1}{2}\omega_d LI^2\sin[2(\omega_d t-\phi)]\\ &= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin[2(3000\,\text{rad/s})(0.442\,\text{ms})-2(46.5^\circ)]\\ &= 44.1\ \text{W}\ .\end{aligned}$$

(d) The rate at which energy is being dissipated by the resistor is

$$\begin{aligned}P_r &= i^2R = I^2R\sin^2(\omega_d t-\phi)\\ &= (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms})-46.5^\circ\right]\\ &= 14.4\,\text{W}\ .\end{aligned}$$

(e) The negative result for P_i means that energy is being taken away from the inductor at this particular time.

(f) $P_i + P_r + P_c = 44.1\text{W} - 17.0\,\text{W} + 14.4\,\text{W} = 41.5\,\text{W} = P_g$.

57. (a) The power factor is $\cos\phi$, where ϕ is the phase constant defined by the expression $i = I\sin(\omega t-\phi)$. Thus, $\phi = -42.0^\circ$ and $\cos\phi = \cos(-42.0^\circ) = 0.743$.

(b) Since $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan\phi = (X_L - X_C)/R$. We have $\tan\phi = \tan(-42.0^\circ) = -0.900$, a negative number. Therefore, $X_L - X_C$ is negative, which leads to $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance X_L would be the same as X_C, $\tan\phi$ would be zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since $\tan\phi$ is negative and finite, neither the capacitive reactance nor the resistance are zero. This means the box must contain a capacitor and a resistor. The inductive reactance may be zero, so there need not be an inductor. If there is an inductor its reactance must be less than that of the capacitor at the operating frequency.

(f) The average power is

$$P_{\text{avg}} = \frac{1}{2}\mathcal{E}_m I\cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\ \text{W}\ .$$

(g) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L and C then the value of the frequency would also be needed to compute the power factor.

58. This circuit contains no reactances, so $\mathcal{E}_{\text{rms}} = I_{\text{rms}}R_{\text{total}}$. Using Eq. 33-71, we find the average dissipated power in resistor R is

$$P_R = I_{\text{rms}}^2 R = \left(\frac{\mathcal{E}_m}{r+R}\right)^2 R\ .$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\mathcal{E}_m^2[(r+R)^2 - 2(r+R)R]}{(r+R)^4} = \frac{\mathcal{E}_{\text{m}}^2(r-R)}{(r+R)^3} = 0 \implies R = r$$

59. We use the result of problem 54:

$$P_{\rm avg} = \frac{(E)_m^2 R}{2Z^2} = \frac{(E)_m^2 R}{2\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]} \ .$$

We use the expression $Z = \sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}$ for the impedance in terms of the angular frequency.

(a) Considered as a function of C, $P_{\rm avg}$ has its largest value when the factor $R^2 + (\omega_d L - 1/\omega_d C)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2(60.0\,{\rm Hz})^2(60.0\times 10^{-3}\,{\rm H})} = 1.17\times 10^{-4}\ {\rm F}\ .$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as C becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\rm avg} = \frac{(E)_m^2}{2R}\ ,$$

so the maximum average power is in the resonant case and is equal to

$$P_{\rm avg} = \frac{(30.0\,{\rm V})^2}{2(5.00\,\Omega)} = 90.0\ {\rm W}\ .$$

On the other hand, the minimum average power is $P_{\rm avg} = 0$ (as it would be for an open switch).

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0\ ,$$

which implies $\phi = 0$. On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90°$.

(e) At maximum power, the power factor is $\cos\phi = \cos 0° = 1$, and at minimum power, it is $\cos\phi = \cos(-90°) = 0$.

60. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{\rm max}/P_{\rm min} = (I/I_{\rm min})^2 = 5$, or

$$\left(\frac{I}{I_{\rm min}}\right)^2 = \left(\frac{\mathcal{E}_m/Z_{\rm min}}{\mathcal{E}_m/Z_{\rm max}}\right)^2 = \left(\frac{Z_{\rm max}}{Z_{\rm min}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{\rm max})^2}}{R}\right)^2 = 5\ .$$

We solve for $L_{\rm max}$:

$$L_{\rm max} = \frac{2R}{\omega} = \frac{2(120\,{\rm V})^2/1000\,{\rm W}}{2\pi(60.0\,{\rm Hz})} = 7.64\times 10^{-2}\ {\rm H}\ .$$

(b) Now we must let

$$\left(\frac{R_{\rm max} + R_{\rm bulb}}{R_{\rm bulb}}\right)^2 = 5\ ,$$

or

$$R_{\rm max} = (\sqrt{5} - 1)R_{\rm bulb} = (\sqrt{5} - 1)\frac{(120\,{\rm V})^2}{1000\,{\rm W}} = 17.8\ \Omega\ .$$

This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

61. (a) The rms current is

$$\begin{aligned} I_{\rm rms} &= \frac{\mathcal{E}_{\rm rms}}{Z} = \frac{\mathcal{E}_{\rm rms}}{\sqrt{R^2 + (2\pi f L - 1/2\pi f C)^2}} \\ &= \frac{75.0\,\mathrm{V}}{\sqrt{(15.0\,\Omega)^2 + \{2\pi(550\,\mathrm{Hz})(25.0\,\mathrm{mH}) - 1/[2\pi(550\,\mathrm{Hz})(4.70\mu\mathrm{F})]\}^2}} \\ &= 2.59\ \mathrm{A}\ . \end{aligned}$$

(b) The various rms voltages are:

$$\begin{aligned} V_{ab} &= I_{\rm rms} R = (2.59\,\mathrm{A})(15.0\,\Omega) = 38.8\ \mathrm{V} \\ V_{bc} &= I_{\rm rms} X_C = \frac{I_{\rm rms}}{2\pi f C} = \frac{2.59\,\mathrm{A}}{2\pi(550\,\mathrm{Hz})(4.70\,\mu\mathrm{F})} = 159\ \mathrm{V} \\ V_{cd} &= I_{\rm rms} X_L = 2\pi I_{\rm rms} f L = 2\pi(2.59\,\mathrm{A})(550\,\mathrm{Hz})(25.0\,\mathrm{mH}) = 224\ \mathrm{V} \\ V_{bd} &= |V_{bc} - V_{cd}| = |159.5\,\mathrm{V} - 223.7\,\mathrm{V}| = 64.2\ \mathrm{V} \\ V_{ad} &= \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,\mathrm{V})^2 + (64.2\,\mathrm{V})^2} = 75.0\ \mathrm{V} \end{aligned}$$

(c) For L and C, the rate is zero since they do not dissipate energy. For R,

$$P_R = \frac{V_{ab}^2}{R} = \frac{(38.8\,\mathrm{V})^2}{15.0\,\Omega} = 100\ \mathrm{W}\ .$$

62. We use Eq. 33-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\mathrm{V})\left(\frac{500}{50}\right) = 1.00\times 10^3\ \mathrm{V}\ .$$

63. (a) The stepped-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\mathrm{V})\left(\frac{10}{500}\right) = 2.4\ \mathrm{V}\ .$$

(b) By Ohm's law, the current in the secondary is

$$I_s = \frac{V_s}{R_s} = \frac{2.4\,\mathrm{V}}{15\,\Omega} = 0.16\ \mathrm{A}\ .$$

We find the primary current from Eq. 33-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\mathrm{A})\left(\frac{10}{500}\right) = 3.2\times 10^{-3}\ \mathrm{A}\ .$$

64. Step up:

- We use T_1T_2 as primary and T_1T_3 as secondary coil: $V_{13}/V_{12} = (800+200)/200 = 5.00$.
- We use T_1T_2 as primary and T_2T_3 as secondary coil: $V_{23}/V_{13} = 800/200 = 4.00$.
- We use T_2T_3 as primary and T_1T_3 as secondary coil: $V_{13}/V_{23} = (800+200)/800 = 1.25$.

Step down: By exchanging the primary and secondary coils in each of the three cases above we get the following possible ratios:

- $1/5.00 = 0.200$
- $1/4.00 = 0.250$

- $1/1.25 = 0.800$

65. The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings. If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = (N_p/N_s)I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R\ .$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\mathcal{E}_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\mathcal{E}_{\text{rms}}}{r + (N_p/N_s)^2 R}$$

where Eq. 33-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\mathcal{E}^2 (N_p/N_s)^2 R}{\left[r + (N_p/N_s)^2 R\right]^2}\ .$$

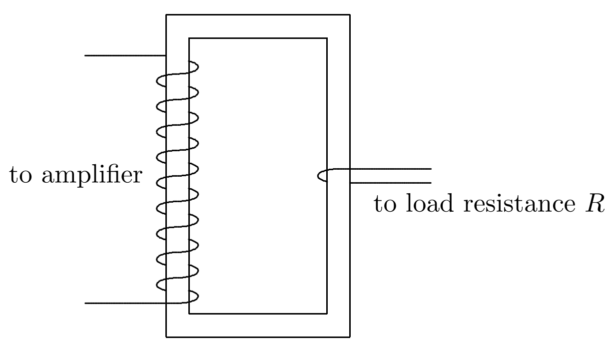
Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\mathcal{E}^2 R x}{(r + xR)^2}\ ,$$

and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\mathcal{E}^2 R(r - xR)}{(r + xR)^3}\ .$$

This is zero for $x = r/R = (1000\,\Omega)/(10\,\Omega) = 100$. We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for $N_p/N_s = \sqrt{x} = 10$. The diagram below is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



66. The effective resistance R_{eff} satisfies $I_{\text{rms}}^2 R_{\text{eff}} = P_{\text{mechanical}}$, or

$$R_{\text{eff}} = \frac{P_{\text{mechanical}}}{I_{\text{rms}}^2} = \frac{(0.100\,\text{hp})(746\,\text{W/hp})}{(0.650\,\text{A})^2} = 177\ \Omega\ .$$

This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I_{\text{rms}}^2 R$ would not give $P_{\text{mechanical}}$ but rather the rate of energy loss due to thermal dissipation.

67. The rms current in the motor is

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z} = \frac{\mathcal{E}_{\text{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\text{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61 \text{ A} .$$

68. We use $nT/2$ to represent the integer number of half-periods specified in the problem. Note that $T = 2\pi/\omega$. We use the calculus-based definition of an average of a function:

$$\begin{aligned} \left[\sin^2(\omega t - \phi)\right]_{\text{avg}} &= \frac{1}{nT/2}\int_0^{\frac{nT}{2}} \sin^2(\omega t - \phi)\,dt \\ &= \frac{2}{nT}\int_0^{\frac{nT}{2}} \frac{1 - \cos(2\omega t - 2\phi)}{2}\,dt \\ &= \frac{2}{nT}\left[\frac{t}{2} - \frac{1}{4\omega}\sin(2\omega t - 2\phi)\right]\Bigg|_0^{\frac{nT}{2}} \\ &= \frac{1}{2} - \frac{1}{2nT\omega}\left[\sin(n\omega T - 2\phi) + \sin 2\phi\right] . \end{aligned}$$

Since $n\omega T = n\omega(2\pi/\omega) = 2n\pi$, we have $\sin(n\omega T - 2\phi) = \sin(2n\pi - 2\phi) = -\sin 2\phi$ so $[\sin(n\omega T - 2\phi) + \sin 2\phi] = 0$. Thus,

$$\left[\sin^2(\omega t - \phi)\right]_{\text{avg}} = \frac{1}{2} .$$

69. (a) The energy stored in the capacitor is given by $U_E = q^2/2C$. Since q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Similarly, the energy stored in the inductor is $U_B = \frac{1}{2}i^2L$. Since i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the generator is

$$P_{\text{avg}}T = (I_{\text{rms}}\mathcal{E}_{\text{rms}}\cos\phi)T = \left(\frac{1}{2}T\right)\mathcal{E}_m I\cos\phi$$

where we substitute $I_{\text{rms}} = I/\sqrt{2}$ and $\mathcal{E}_{\text{rms}} = \mathcal{E}_m/\sqrt{2}$.

(d) The energy dissipated by the resistor is

$$P_{\text{avg,resistor}}\,T = (I_{\text{rms}}V_R)T = I_{\text{rms}}(I_{\text{rms}}R)T = \left(\frac{1}{2}T\right)I^2R .$$

(e) Since $\mathcal{E}_m I\cos\phi = \mathcal{E}_m I(V_R/\mathcal{E}_m) = \mathcal{E}_m I(IR/\mathcal{E}_m) = I^2R$, the two quantities are indeed the same.

70. (a) The rms current in the cable is $I_{\text{rms}} = P/V_t = 250 \times 10^3\,\text{W}/(80 \times 10^3\,\text{V}) = 3.125\,\text{A}$. The rms voltage drop is then $\Delta V = I_{\text{rms}}R = (3.125\,\text{A})(2)(0.30\,\Omega) = 1.9\,\text{V}$, and the rate of energy dissipation is $P_d = I_{\text{rms}}^2R = (3.125\,\text{A})(2)(0.60\,\Omega) = 5.9\,\text{W}$.

(b) Now $I_{\text{rms}} = 250 \times 10^3\,\text{W}/(8.0 \times 10^3\,\text{V}) = 31.25\,\text{A}$, so $\Delta V = (31.25\,\text{A})(0.60\,\Omega) = 19\,\text{V}$ and $P_d = (3.125\,\text{A})^2(0.60\,\Omega) = 5.9 \times 10^2\,\text{W}$.

(c) Now $I_{\text{rms}} = 250 \times 10^3\,\text{W}/(0.80 \times 10^3\,\text{V}) = 312.5\,\text{A}$, so $\Delta V = (312.5\,\text{A})(0.60\,\Omega) = 1.9 \times 10^2\,\text{V}$ and $P_d = (312.5\,\text{A})^2(0.60\,\Omega) = 5.9 \times 10^4\,\text{W}$. Both the rate of energy dissipation and the voltage drop increase as V_t decreases. Therefore, to minimize these effects the best choice among the three V_t's above is $V_t = 80\,\text{kV}$.

71. (a) The impedance is

$$Z = \frac{\mathcal{E}_{\text{m}}}{I} = \frac{125\,\text{V}}{3.20\,\text{A}} = 39.1\ \Omega .$$

(b) From $V_R = IR = \mathcal{E}_m \cos\phi$, we get

$$R = \frac{\mathcal{E}_m \cos\phi}{I} = \frac{(125\,\mathrm{V})\cos(0.982\,\mathrm{rad})}{3.20\,\mathrm{A}} = 21.7\ \Omega\ .$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\,\mathrm{rad})$, we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

72. (a) The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{R}\right) = \tan^{-1}\left(\frac{V_L - V_L/2.00}{V_L/2.00}\right) = \tan^{-1}(1.00) = 45.0^\circ\ .$$

(b) We solve R from $\mathcal{E}_m \cos\phi = IR$:

$$R = \frac{\mathcal{E}_m \cos\phi}{I} = \frac{(30.0\,\mathrm{V})(\cos 45^\circ)}{300\times 10^{-3}\,\mathrm{A}} = 70.7\ \Omega\ .$$

73. (a) We solve L from Eq. 33-4, using the fact that $\omega = 2\pi f$:

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(10.4\times 10^3\,\mathrm{Hz})^2(340\times 10^{-6}\,\mathrm{F})} = 6.89\times 10^{-7}\ \mathrm{H}\ .$$

(b) The total energy may be figured from the inductor (when the current is at maximum):

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(6.89\times 10^{-7}\,\mathrm{H})(7.20\times 10^{-3}\,\mathrm{A})^2 = 1.79\times 10^{-11}\ \mathrm{J}\ .$$

(c) We solve for Q from $U = \frac{1}{2}Q^2/C$:

$$Q = \sqrt{2CU} = \sqrt{2(340\times 10^{-6}\,\mathrm{F})(1.79\times 10^{-11}\,\mathrm{J})} = 1.10\times 10^{-7}\ \mathrm{C}\ .$$

74. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/[4(350\,\mathrm{rad/s})] = 6.73\times 10^{-3}\,\mathrm{s}$.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/[4(350\,\mathrm{rad/s})] = 2.24\times 10^{-3}\,\mathrm{s}$.

(c) Since i leads $\mathcal{E}$ in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \mathcal{E}_m/I$:

$$C = \frac{I}{\mathcal{E}_m\omega} = \frac{6.20\times 10^{-3}\,\mathrm{A}}{(30.0\,\mathrm{V})(350\,\mathrm{rad/s})} = 5.90\times 10^{-5}\ \mathrm{F}\ .$$

75. From the problem statement $2\pi f_0 = (LC)^{-1/2} = 6000\,\mathrm{Hz}$, $Z = \sqrt{R^2 + (2\pi f_1 L - 1/2\pi f_1 C)^2} = 1000\,\Omega$ where $f_1 = 8000\,\mathrm{Hz}$, and $\cos\phi = R/Z = \cos 45^\circ$. We solve these equations for the unknowns.

(a) $R = Z\cos\phi = (1000\,\Omega)\cos 45^\circ = 707\,\Omega$

(b) The self-inductance is

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi(f_1 - f_0^2/f_1)} = \frac{\sqrt{(1000\,\Omega)^2 - (707\,\Omega)^2}}{2\pi[8000\,\mathrm{Hz} - (6000\,\mathrm{Hz})^2/8000\,\mathrm{Hz}]} = 3.22\times 10^{-2}\ \mathrm{H}\ .$$

(c) The capacitance is

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2(6000\,\mathrm{Hz})^2(3.22\times 10^{-2}\,\mathrm{H})} = 2.19\times 10^{-8}\ \mathrm{F}\ .$$

76. (a) From Eq. 33-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L/1.50)}{(V_L/2.00)}\right)$$

which becomes $\tan^{-1} 2/3 = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

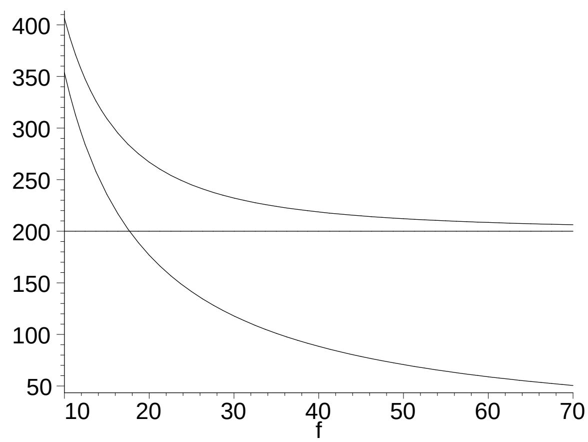
(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 33-60,

$$\mathcal{E}_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$

we find $\mathcal{E}_m = 12.0$ V.

77. (a) With f understood to be in Hertz, the capacitive reactance is $X_C = \left[(2\pi)(45 \times 10^{-6}\,\text{F})f\right]^{-1}$.

(b) The resistance, reactance and impedance are plotted over the range $10 \le f \le 70$ Hz. The horizontal line is R, and the curve that crosses that line is X_C. SI units are understood.

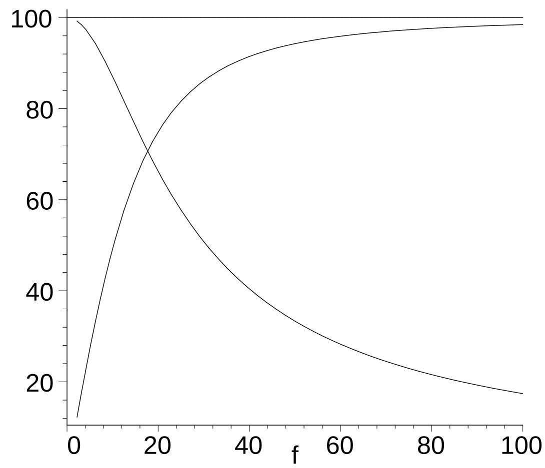


(c) From the graph, we estimate the crossing point to be at about 18 Hz. More careful considerations lead to $f = 17.7$ Hz as the frequency where $X_C = R$.

78. (a) The voltage amplitude for the source is $V_s = 100\,\text{V} = IZ = I\sqrt{R^2 + X_C^2}$, from which we can determine the current at each frequency (the explicit dependence of X_C on frequency is stated in the solution to part (a) of problem 77). This leads to the voltage amplitude across the resistor $V_R = IR$ and the voltage amplitude across the capacitor

$$V_C = IX_C = \left(\frac{V_s}{\sqrt{R^2 + X_C^2}}\right) X_C \quad \text{where} \quad X_C = \frac{1}{2\pi C f}$$

using the values $R = 200\,\Omega$ and $C = 45 \times 10^{-6}$ F given in problem 77. We show, below, the graphs of V_s, V_R and V_C over the range $0 < f < 100$ Hz. The falling curve is V_C and the rising curve is V_R.



(b) The graph indicates that V_C and V_R are equal at roughly 18 Hz. More careful considerations lead to $f = 17.7$ Hz as the frequency for which $V_C = V_R$.

79. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C\tau_L}$.

80. (a) From Eq. 33-25,

$$\frac{dq}{dt} = \frac{d}{dt}\left[Qe^{-Rt/2L}\cos(\omega' t + \phi)\right] = -\frac{RQ}{2L}e^{-Rt/2L}\cos(\omega' t + \phi) - \omega' Qe^{-Rt/2L}\sin(\omega' t + \phi)$$

and

$$\begin{aligned}\frac{d^2q}{dt^2} &= \left(\frac{R}{2L}\right)e^{-Rt/2L}\left[\left(\frac{RQ}{2L}\right)\cos(\omega' t + \phi) - \omega' Q\sin(\omega' t + \phi)\right] \\ &\quad + e^{-Rt/2L}\left[\frac{RQ\omega'}{2L}\sin(\omega' t + \phi) - \omega'^2 Q\cos(\omega' t + \phi)\right].\end{aligned}$$

Substituting these expressions, and Eq. 33-25 itself, into Eq. 33-24, we obtain

$$Qe^{-Rt/2L}\left[-\omega'^2 L - \left(\frac{R}{2L}\right)^2 + \frac{1}{c}\right]\cos(\omega' t + \phi) = 0 .$$

Since this equation is valid at any time t, we must have

$$-\omega'^2 L - \left(\frac{R}{2L}\right)^2 + \frac{1}{C} = 0 \implies \omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2} .$$

(b) The fractional shift in frequency is

$$\begin{aligned}\frac{\Delta f}{f} = \frac{\Delta\omega}{\omega} &= \frac{\omega - \omega'}{\omega} = 1 - \frac{\sqrt{(1/LC) - (R/2L)^2}}{\sqrt{1/LC}} = 1 - \sqrt{1 - \frac{R^2C}{4L}} \\ &= 1 - \sqrt{1 - \frac{(100\,\Omega)^2(7.30\times 10^{-6}\,\mathrm{F})}{4(4.40\,\mathrm{H})}} = 0.00210 = 0.210\% .\end{aligned}$$

81. (a) We find L from $X_L = \omega L = 2\pi f L$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30 \times 10^3 \Omega}{2\pi(45.0 \times 10^{-3}\,\mathrm{H})} = 4.60 \times 10^3 \text{ Hz} .$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi f C)^{-1}$:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(4.60 \times 10^3\,\mathrm{Hz})(1.30 \times 10^3\,\Omega)} = 2.66 \times 10^{-8} \text{ F} .$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus, $X_L = 2(1.30 \times 10^3\,\Omega) = 2.60 \times 10^3\,\Omega$ and $X_C = 1.30 \times 10^3\,\Omega/2 = 6.50 \times 10^2\,\Omega$.

82. (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}\,A\cos(\omega_d t - 60°)$$

where we use $\sin\alpha - \sin\beta = 2\sin[(\alpha-\beta)/2]\cos[(\alpha+\beta)/2]$ and $\sin 60° = \sqrt{3}/2$. Similarly,

$$\Delta V_{13} = A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right) = \sqrt{3}\,A\cos(\omega_d t - 120°)$$

and

$$\Delta V_{23} = A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right) = \sqrt{3}\,A\cos(\omega_d t - 180°) .$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

83. When the switch is open, we have a series LRC circuit involving just the one capacitor near the upper right corner. Eq. 33-65 leads to

$$\frac{\omega_d L - \frac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) \;=\; -\tan 20° .$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is $2C$. In this case, we have

$$\frac{\omega_d L - \frac{1}{2\omega_d C}}{R} = \tan\phi_1 \;=\; \tan 10.0° .$$

Finally, with the switch in position 2, the circuit is simply an LC circuit with current amplitude

$$I_2 = \frac{\mathcal{E}_m}{Z_{LC}} = \frac{\mathcal{E}_m}{\sqrt{\left(\omega_d L - \frac{1}{\omega_d C}\right)^2}} = \frac{\mathcal{E}_m}{\frac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $\frac{1}{\omega_d C} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for L, R and C from the three equations above:

$$\begin{aligned}
R &= \frac{-\mathcal{E}_m}{I_2\tan\phi_o} = \frac{120\,\mathrm{V}}{(2.00\,\mathrm{A})\tan 20.0°} = 165\ \Omega \\
C &= \frac{I_2}{2\omega_d\mathcal{E}_m\left(1 - \frac{\tan\phi_1}{\tan\phi_o}\right)} = \frac{2.00\,\mathrm{A}}{2(2\pi)(60.0\,\mathrm{Hz})(120\,\mathrm{V})\left(1 + \frac{\tan 10.0°}{\tan 20.0°}\right)} = 1.49 \times 10^{-5}\ \mathrm{F} \\
L &= \frac{\mathcal{E}_m}{\omega_d I_2}\left(1 - 2\frac{\tan\phi_1}{\tan\phi_o}\right) = \frac{120\,\mathrm{V}}{2\pi(60.0\,\mathrm{Hz})(2.00\,\mathrm{A})}\left(1 + 2\frac{\tan 10.0°}{\tan 20.0°}\right) 0.313\ \mathrm{H}
\end{aligned}$$

84. (a) Using $X_C = 1/\omega C$ and $V_C = I_C X_C$, we find

$$\omega = \frac{I_C}{CV_C} = 5.77 \times 10^5 \text{ rad/s} .$$

This value is used in the subsequent parts. The period is $T = 2\pi/\omega = 1.09 \times 10^{-5}$ s.

(b) Adapting Eq. 26-22 to the notation of this chapter,

$$U_{E,\max} = \frac{1}{2}CV_C^2 = 4.5 \times 10{-}9 \text{ J} .$$

(c) The discussion in §33-4 shows that $U_{E,\max} = U_{B,\max}$.

(d) We return to Eq. 31-37 (though other, equivalent, approaches could be explored):

$$\frac{di}{dt} = \frac{-\mathcal{E}_L}{L}$$

By the loop rule, $\mathcal{E}_L$ is at its most negative value when the capacitor voltage is at its most positive (V_C). Using this plus the frequency relationship between L and C (Eq. 33-4) leads to

$$\left|\frac{di}{dt}\right|_{\max} = \omega^2 CV_C = 998 \text{ A/s} .$$

(e) Differentiating Eq. 31-51, we have

$$\frac{dU_B}{dt} = Li\frac{di}{dt} .$$

As in the previous part, we use $L = 1/\omega^2 C$. We also use a simple sinusoidal form for the current, $i = I\sin\omega t$:

$$\frac{dU_B}{dt} = \frac{1}{\omega^2 C} I^2 \omega \sin\omega t \cos\omega t$$

where this I is equivalent to the I_C used in part (a). Using a well-known trig identity, we obtain

$$\left(\frac{dU_B}{dt}\right)_{\max} = \frac{I^2}{2\omega^2 C}(\sin 2\omega t)_{\max} = \frac{I^2}{2\omega^2 C}$$

which yields a (maximum) time rate of change (for U_B) equal to 2.60×10^{-3} J/s.

85. (a) At any time, the total energy U in the circuit is the sum of the energy U_E in the capacitor and the energy U_B in the inductor. When $U_E = 0.500U_B$ (at time t), then $U_B = 2.00U_E$ and $U = U_E + U_B = 3.00U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor. Thus, $Q^2/2C = 3.00q^2/2C$ or $q = Q/\sqrt{3.00} = 0.577Q$.

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q\cos\omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos\omega t = 0.557$, or $\omega t = 0.955$ rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$.

86. (a) Eqs. 33-4 and 33-14 lead to

$$Q = \frac{I}{\omega} = I\sqrt{LC} = 1.27 \times 10^{-6} \text{ C} .$$

(b) We choose the phase constant in Eq. 33-12 to be $\phi = -\pi/2$, so that $i_0 = I$ in Eq. 33-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}(\sin\omega t)^2 .$$

Differentiating and using the fact that $2\sin\theta\,\cos\theta = \sin 2\theta$, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\,\omega\,\sin 2\omega t\ .$$

We find the maximum value occurs whenever $\sin 2\omega t = 1$, which leads (with $n =$ *odd* integer) to

$$t = \frac{1}{2\omega}\,\frac{n\,\pi}{2} = \frac{n\,\pi}{4\,\omega} = \frac{n\,\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\,\text{s},\ 2.49\times10^{-4}\,\text{s},\ \ldots\quad .$$

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{Q^2}{2C}\,\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\,\frac{1}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\ \text{J/s}\ .$$

87. (a) We observe that $\omega = 6597$ rad/s, and, consequently, $X_L = 594\ \Omega$ and $X_C = 303\ \Omega$. Since $X_L > X_C$, the phase angle is positive: $\phi = +60°$.

(b) From Eq. 33-65, we obtain

$$R = \frac{X_L - X_C}{\tan\phi} = 168\ \Omega\ \ .$$

(c) Since we are already on the "high side" of resonance, increasing f will only decrease the current further, but *decreasing* f brings us closer to resonance and, consequently, large values of I.

(d) Increasing L increases X_L, but we already have $X_L > X_C$. Thus, if we wish to move closer to resonance (where X_L must equal X_C), we need to *decrease* the value of L.

(e) To change the present condition of $X_C < X_L$ to something closer to $X_C = X_L$ (resonance, large current), we can increase X_C. Since X_C depends inversely on C, this means *decreasing* C.

88. (a) We observe that $\omega_d = 12566$ rad/s. Consequently, $X_L = 754\ \Omega$ and $X_C = 199\ \Omega$. Hence, Eq. 33-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22\ \text{rad}\ .$$

(b) We find the current amplitude from Eq. 33-60:

$$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288\ \text{A}\ .$$

89. From Eq. 33-60, we have

$$\left(\frac{220\ \text{V}}{3.00\ \text{A}}\right)^2 = R^2 + X_L^2 \quad\Longrightarrow\quad X_L = 69.3\ \Omega\ .$$

90. (a) We observe that $\omega = 7540$ rad/s, and, consequently, $X_L = 377\ \Omega$ and $X_C = 15.3\ \Omega$. Therefore, Eq. 33-64 leads to

$$I_{\text{rms}} = \frac{112\ \text{V}}{\sqrt{(35\,\Omega)^2 + (377\,\Omega - 15\,\Omega)^2}} = 0.308\ \text{A}\ .$$

(b) (c) (d) (e) (f) and (g) For the individual elements, we have:

$$\begin{aligned} V_{R,\text{rms}} &= I_{\text{rms}}R = 10.8\ \text{V} \\ V_{C,\text{rms}} &= I_{\text{rms}}X_C = 4.73\ \text{V} \\ V_{L,\text{rms}} &= I_{\text{rms}}X_L = 116\ \text{V} \end{aligned}$$

The capacitor and inductor are not dissipative elements; the only power dissipated (by definition) is in the resistor. If a coil, perhaps referred to as an inductor in building a circuit, is found to have an internal resistance, then the coil (for purposes of analysis) is taken to be an inductor plus a resistor. The power dissipated in the resistive element is $P_{\text{avg}} = (0.308\ \text{A})^2(35\ \Omega) = 3.33$ W.

91. From Eq. 33-4, with $\omega = 2\pi f = 4.49 \times 10^3$ rad/s, we obtain

$$L = \frac{1}{\omega^2 C} = 7.08 \times 10^{-3} \text{ H} .$$

92. (a) From Eq. 33-4, with $\omega = 2\pi f$, we have

$$f = \frac{1}{2\pi\sqrt{LC}} = 7.08 \times 10^{-3} \text{ H} .$$

(b) The maximum current in the oscillator is

$$i_{\text{max}} = I_C = \frac{V_C}{X_C} = \omega C v_{\text{max}} = 4.00 \times 10^{-3} \text{ A} .$$

(c) Using Eq. 31-51, we find the maximum magnetic energy:

$$U_{B,\text{max}} = \frac{1}{2} L i_{\text{max}}^2 = 1.6 \times 10^{-8} \text{ J} .$$

(d) Adapting Eq. 31-37 to the notation of this chapter,

$$v_{\text{max}} = L \left|\frac{di}{dt}\right|_{\text{max}}$$

which yields a (maximum) time rate of change (for i) equal to 2000 A/s.

Chapter 34

1. The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

   (a) We take d to be 150 km = 150×10^3 m. Then,

   $$t = \frac{d}{c} = \frac{150 \times 10^3\,\mathrm{m}}{3.00 \times 10^8\,\mathrm{m/s}} = 5.00 \times 10^{-4}\ \mathrm{s}\ .$$

   (b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is $d = (1.5 \times 10^8\,\mathrm{km}) + 2(3.8 \times 10^5\,\mathrm{km}) = 1.51 \times 10^8\,\mathrm{km} = 1.51 \times 10^{11}\,\mathrm{m}$. The time taken by light to travel this distance is

   $$t = \frac{d}{c} = \frac{1.51 \times 10^{11}\,\mathrm{m}}{3.00 \times 10^8\,\mathrm{m/s}} = 500\,\mathrm{s} = 8.4\ \mathrm{min}\ .$$

   (c) We take d to be $2(1.3 \times 10^9\,\mathrm{km}) = 2.6 \times 10^{12}\,\mathrm{m}$. Then,

   $$t = \frac{d}{c} = \frac{2.6 \times 10^{12}\,\mathrm{m}}{3.00 \times 10^8\,\mathrm{m/s}} = 8.7 \times 10^3\,\mathrm{s} = 2.4\ \mathrm{h}\ .$$

   (d) We take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

   $$t = \frac{d}{c} = \frac{6500\,\mathrm{ly}}{1.00\,\mathrm{ly/y}} = 6500\,\mathrm{y}\ .$$

   The explosion took place in the year $1054 - 6500 = -5446$ or 5446 BCE.

2. (a)

   $$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8\,\mathrm{m/s}}{(1.0 \times 10^5)(6.4 \times 10^6\,\mathrm{m})} = 4.7 \times 10^{-3}\ \mathrm{Hz}\ .$$

   (b)

   $$T = \frac{1}{f} = \frac{1}{4.7 \times 10^{-3}\,\mathrm{Hz}} = 212\,\mathrm{s} = 3\,\mathrm{min}\ 32\,\mathrm{s}\ .$$

3. (a) From Fig. 34-2 we find the wavelengths in question to be about 515 nm and 610 nm.

   (b) Again from Fig. 34-2 the wavelength is about 555 nm. Therefore,

   $$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{555\,\mathrm{nm}} = 5.41 \times 10^{14}\ \mathrm{Hz}\ ,$$

   and the period is $(5.41 \times 10^{14}\,\mathrm{Hz})^{-1} = 1.85 \times 10^{-15}$ s.

4. Since $\Delta\lambda \ll \lambda$, we find Δf is equal to

$$\left|\Delta\left(\frac{c}{\lambda}\right)\right| \approx \frac{c\Delta\lambda}{\lambda^2} = \frac{(3.0\times10^8\,\mathrm{m/s})(0.0100\times10^{-9}\,\mathrm{m})}{(632.8\times10^{-9}\,\mathrm{m})^2} = 7.49\times10^9\ \mathrm{Hz}\ .$$

5. (a) Suppose that at time t_1, the moon is starting a revolution (on the verge of going behind Jupiter, say) and that at this instant, the distance between Jupiter and Earth is ℓ_1. The time of the start of the revolution as seen on Earth is $t_1^* = t_1 + \ell_1/c$. Suppose the moon starts the next revolution at time t_2 and at that instant, the Earth-Jupiter distance is ℓ_2. The start of the revolution as seen on Earth is $t_2^* = t_2 + \ell_2/c$. Now, the actual period of the moon is given by $T = t_2 - t_1$ and the period as measured on Earth is

$$T^* = t_2^* - t_1^* = t_2 - t_1 + \frac{\ell_2}{c} - \frac{\ell_1}{c} = T + \frac{\ell_2 - \ell_1}{c}\ .$$

The period as measured on Earth is longer than the actual period. This is due to the fact that Earth moves during a revolution, and light takes a finite time to travel from Jupiter to Earth. For the situation depicted in Fig. 34-38, light emitted at the end of a revolution travels a longer distance to get to Earth than light emitted at the beginning. Suppose the position of Earth is given by the angle θ, measured from x. Let R be the radius of Earth's orbit and d be the distance from the Sun to Jupiter. The law of cosines, applied to the triangle with the Sun, Earth, and Jupiter at the vertices, yields $\ell^2 = d^2 + R^2 - 2dR\cos\theta$. This expression can be used to calculate ℓ_1 and ℓ_2. Since Earth does not move very far during one revolution of the moon, we may approximate $\ell_2 - \ell_1$ by $(d\ell/dt)T$ and T^* by $T + (d\ell/dt)(T/c)$. Now

$$\frac{d\ell}{dt} = \frac{2Rd\sin\theta}{\sqrt{d^2+R^2-2dR\cos\theta}}\,\frac{d\theta}{dt} = \frac{2vd\sin\theta}{\sqrt{d^2+R^2-2dR\cos\theta}}\ ,$$

where $v = R\,(d\theta/dt)$ is the speed of Earth in its orbit. For $\theta = 0$, $(d\ell/dt) = 0$ and $T^* = T$. Since Earth is then moving perpendicularly to the line from the Sun to Jupiter, its distance from the planet does not change appreciably during one revolution of the moon. On the other hand, when $\theta = 90°$, $d\ell/dt = vd/\sqrt{d^2+R^2}$ and

$$T^* = T\left(1 + \frac{vd}{c\sqrt{d^2+R^2}}\right)\ .$$

The Earth is now moving parallel to the line from the Sun to Jupiter, and its distance from the planet changes during a revolution of the moon.

(b) Our notation is as follows: t is the actual time for the moon to make N revolutions, and t^* is the time for N revolutions to be observed on Earth. Then,

$$t^* = t + \frac{\ell_2 - \ell_1}{c}\ ,$$

where ℓ_1 is the Earth-Jupiter distance at the beginning of the interval and ℓ_2 is the Earth-Jupiter distance at the end. Suppose Earth is at position x at the beginning of the interval, and at y at the end. Then, $\ell_1 = d - R$ and $\ell_2 = \sqrt{d^2+R^2}$. Thus,

$$t^* = t + \frac{\sqrt{d^2+R^2} - (d-R)}{c}\ .$$

A value can be found for t by measuring the observed period of revolution when Earth is at x and multiplying by N. We note that the observed period is the true period when Earth is at x. The time interval as Earth moves from x to y is t^*. The difference is

$$t^* - t = \frac{\sqrt{d^2+R^2} - (d-R)}{c}\ .$$

If the radii of the orbits of Jupiter and Earth are known, the value for $t^* - t$ can be used to compute c. Since Jupiter is much further from the Sun than Earth, $\sqrt{d^2 + R^2}$ may be approximated by d and $t^* - t$ may be approximated by R/c. In this approximation, only the radius of Earth's orbit need be known.

6. The emitted wavelength is

$$\begin{aligned}\lambda &= \frac{c}{f} = 2\pi c\sqrt{LC} \\ &= 2\pi(2.998 \times 10^8\,\mathrm{m/s})\sqrt{(0.253 \times 10^{-6}\,\mathrm{H})(25.0 \times 10^{-12}\,\mathrm{F})} = 4.74\ \mathrm{m}\ .\end{aligned}$$

7. If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. The frequency is the same as the frequency of oscillation of the current in the LC circuit of the generator. That is, $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. Thus

$$\frac{\lambda}{2\pi\sqrt{LC}} = c\ .$$

The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 C c^2} = \frac{(550 \times 10^{-9}\,\mathrm{m})^2}{4\pi^2(17 \times 10^{-12}\,\mathrm{F})(2.998 \times 10^8\,\mathrm{m/s})^2} = 5.00 \times 10^{-21}\ \mathrm{H}\ .$$

This is exceedingly small.

8. The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{3.20 \times 10^{-4}\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 1.07 \times 10^{-12}\ \mathrm{T}\ .$$

9. Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. With SI units understood, we find

$$\begin{aligned}B_y &= B_{\mathrm{m}} \cos\left[\pi \times 10^{15}\left(t - \frac{x}{c}\right)\right] = \frac{2.0\cos[10^{15}\pi(t - x/c)]}{3.0 \times 10^8} \\ &= (6.7 \times 10^{-9})\cos\left[10^{15}\pi\left(t - \frac{x}{c}\right)\right]\end{aligned}$$

10. Using $\vec{S} = (1/\mu_0)\vec{E} \times \vec{B}$, we see that (on the right hand) letting the thumb be in the $\vec{E}$ direction and the index finger be in the $\vec{B}$ direction means that the middle finger (held perpendicular to the other two, making a "triad" of the thumb and two fingers) points in the direction of wave propagation (the direction of $\vec{S}$). Holding the right hand in this manner can facilitate checking the directions in the Figures. A more algebraic approach is to note that $\hat{\jmath} \times \hat{\mathrm{k}} = \hat{\imath}$. This is especially useful for checking Figures 34-6 and 34-7.

11. If P is the power and Δt is the time interval of one pulse, then the energy in a pulse is

$$E = P\,\Delta t = (100 \times 10^{12}\,\mathrm{W})(1.0 \times 10^{-9}\,\mathrm{s}) = 1.0 \times 10^5\ \mathrm{J}\ .$$

12. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0 \times 10^6\,\mathrm{W}}{4\pi[(4.3\,\mathrm{ly})(9.46 \times 10^{15}\,\mathrm{m/ly})]^2} = 4.8 \times 10^{-29}\ \mathrm{W/m^2}\ .$$

13. The region illuminated on the Moon is a circle with radius $R = r\theta/2$, where r is the Earth-Moon distance $(3.82 \times 10^8\,\mathrm{m})$ and θ is the full-angle beam divergence in radians. The area A illuminated is

$$A = \pi R^2 = \frac{\pi r^2\theta^2}{4} = \frac{\pi(3.82 \times 10^8\,\mathrm{m})^2(0.880 \times 10^{-6}\,\mathrm{rad})^2}{4} = 8.88 \times 10^4\ \mathrm{m}^2\ .$$

14. The intensity is the average of the Poynting vector:

$$I = S_{\mathrm{avg}} = \frac{cB_m^2}{2\mu_0} = \frac{(3.0 \times 10^8\,\mathrm{m/s})(1.0 \times 10^{-4}\,\mathrm{T})^2}{2(1.26 \times 10^{-6}\,\mathrm{H/m})^2} = 1.2 \times 10^6\ \mathrm{W/m^2}\ .$$

15. (a) The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{5.00\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 1.67 \times 10^{-8}\ \mathrm{T}\ .$$

(b) The intensity is the average of the Poynting vector:

$$I = S_{\mathrm{avg}} = \frac{E_m^2}{2\mu_0 c} = \frac{(5.00\,\mathrm{V/m})^2}{2(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(2.998 \times 10^8\,\mathrm{m/s})} = 3.31 \times 10^{-2}\ \mathrm{W/m^2}\ .$$

16. We use $I = E_m^2/2\mu_0 c$ to calculate E_{m}:

$$\begin{aligned} E_m &= \sqrt{2\mu_0 I c} = \sqrt{2(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1.40 \times 10^3\,\mathrm{W/m^2})(2.998 \times 10^8\,\mathrm{m/s})} \\ &= 1.03 \times 10^3\ \mathrm{V/m}\ . \end{aligned}$$

The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03 \times 10^4\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 3.43 \times 10^{-6}\ \mathrm{T}\ .$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 6.7 \times 10^{-9}\ \mathrm{T}\ .$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\,\mathrm{V/m})^2}{2(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(2.998 \times 10^8\,\mathrm{m/s})} = 5.3 \times 10^{-3}\ \mathrm{W/m^2}\ .$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\mathrm{avg}} = 4\pi(10\,\mathrm{m})^2(5.3 \times 10^{-3}\,\mathrm{W/m^2}) = 6.7\ \mathrm{W}\ .$$

18. (a) The power received is

$$P_r = (1.0 \times 10^{-12}\,\mathrm{W})\frac{\pi[(1000\,\mathrm{ft})(0.3048\,\mathrm{m/ft})]^2/4}{4\pi(6.37 \times 10^6\,\mathrm{m})^2} = 1.4 \times 10^{-22}\ \mathrm{W}\ .$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi[(2.2 \times 10^4\,\mathrm{ly})(9.46 \times 10^{15}\,\mathrm{m/ly})]^2\left[\frac{1.0 \times 10^{-12}\,\mathrm{W}}{4\pi(6.37 \times 10^6\,\mathrm{m})^2}\right] = 1.1 \times 10^{15}\ \mathrm{W}\ .$$

19. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2/2\mu_0 c$, so

$$\begin{aligned} E_m &= \sqrt{2\mu_0 c I} = \sqrt{2(4\pi \times 10^{-7}\,\text{H/m})(2.998 \times 10^8\,\text{m/s})(10 \times 10^{-6}\,\text{W/m}^2)} \\ &= 8.7 \times 10^{-2}\ \text{V/m} \ . \end{aligned}$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7 \times 10^{-2}\,\text{V/m}}{2.998 \times 10^8\,\text{m/s}} = 2.9 \times 10^{-10}\ \text{T} \ .$$

(c) At a distance r from the transmitter, the intensity is $I = P/4\pi r^2$, where P is the power of the transmitter. Thus

$$P = 4\pi r^2 I = 4\pi(10 \times 10^3\,\text{m})^2(10 \times 10^{-6}\,\text{W/m}^2) = 1.3 \times 10^4\ \text{W} \ .$$

20. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\text{W/m}^2}{2.998 \times 10^8\,\text{m/s}} = 3.3 \times 10^{-8}\ \text{Pa} \ .$$

21. The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity. The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation. Thus

$$p_r = \frac{2P}{Ac} = \frac{2(1.5 \times 10^9\,\text{W})}{(1.00 \times 10^{-6}\,\text{m}^2)(2.998 \times 10^8\,\text{m/s})} = 1.0 \times 10^7\,\text{Pa} = 10\ \text{MPa} \ .$$

22. (a) The radiation pressure produces a force equal to

$$\begin{aligned} F_r &= p_r(\pi R_e^2) = \left(\frac{I}{c}\right)(\pi R_e^2) \\ &= \frac{\pi(1.4 \times 10^3\,\text{W/m}^2)(6.37 \times 10^6\,\text{m})^2}{2.998 \times 10^8\,\text{m/s}} = 6.0 \times 10^8\ \text{N} \ . \end{aligned}$$

(b) The gravitational pull of the Sun on Earth is

$$\begin{aligned} F_{\text{grav}} &= \frac{GM_sM_e}{d_{es}^2} \\ &= \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(2.0 \times 10^{30}\,\text{kg})(5.98 \times 10^{24}\,\text{kg})}{(1.5 \times 10^{11}\,\text{m})^2} \\ &= 3.6 \times 10^{22}\ \text{N} \ , \end{aligned}$$

which is much greater than F_r.

23. Since the surface is perfectly absorbing, the radiation pressure is given by $p_r = I/c$, where I is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance r from it is given by $I = P/4\pi r^2$, where P is the power of the bulb. Thus

$$p_r = \frac{P}{4\pi r^2 c} = \frac{500\,\text{W}}{4\pi(1.5\,\text{m})^2(2.998 \times 10^8\,\text{m/s})} = 5.9 \times 10^{-8}\ \text{Pa} \ .$$

24. (a) We note that the cross section area of the beam is $\pi d^2/4$, where d is the diameter of the spot ($d = 2.00\lambda$). The beam intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{5.00 \times 10^{-3}\,\mathrm{W}}{\pi[(2.00)(633 \times 10^{-9}\,\mathrm{m})]^2/4} = 3.97 \times 10^9\ \mathrm{W/m^2}\ .$$

(b) The radiation pressure is

$$p_r = \frac{I}{c} = \frac{3.97 \times 10^9\,\mathrm{W/m^2}}{2.998 \times 10^8\,\mathrm{m/s}} = 13.2\ \mathrm{Pa}\ .$$

(c) In computing the corresponding force, we can use the power and intensity to eliminate the area (mentioned in part (a)). We obtain

$$F_r = \left(\frac{\pi d^2}{4}\right) p_r = \left(\frac{P}{I}\right) p_r = \frac{(5.00 \times 10^{-3}\,\mathrm{W})(13.2\,\mathrm{Pa})}{3.97 \times 10^9\,\mathrm{W/m^2}} = 1.67 \times 10^{-11}\ \mathrm{N}\ .$$

(d) The acceleration of the sphere is

$$\begin{aligned} a &= \frac{F_r}{m} = \frac{F_r}{\rho(\pi d^3/6)} = \frac{6(1.67 \times 10^{-11}\,\mathrm{N})}{\pi(5.00 \times 10^3\,\mathrm{kg/m^3})[(2.00)(633 \times 10^{-9}\,\mathrm{m})]^3} \\ &= 3.14 \times 10^3\ \mathrm{m/s^2}\ . \end{aligned}$$

25. (a) Since $c = \lambda f$, where λ is the wavelength and f is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{3.0\,\mathrm{m}} = 1.0 \times 10^8\ \mathrm{Hz}\ .$$

(b) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\,\mathrm{V/m}}{2.998 \times 10^8\,\mathrm{m/s}} = 1.00 \times 10^{-6}\ \mathrm{T}\ .$$

$\vec{B}$ must be in the positive z direction when $\vec{E}$ is in the positive y direction in order for $\vec{E} \times \vec{B}$ to be in the positive x direction (the direction of propagation).

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\,\mathrm{m}} = 2.1\,\mathrm{rad/m}\ .$$

The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0 \times 10^8\,\mathrm{Hz}) = 6.3 \times 10^8\ \mathrm{rad/s}\ .$$

(d) The intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\,\mathrm{V/m})^2}{2(4\pi \times 10^{-7}\,\mathrm{H/m})(2.998 \times 10^8\,\mathrm{m/s})} = 119\ \mathrm{W/m^2}\ .$$

(e) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is I/c, so

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\,\mathrm{W/m^2})(2.0\,\mathrm{m^2})}{2.998 \times 10^8\,\mathrm{m/s}} = 8.0 \times 10^{-7}\,\mathrm{N}\ .$$

The radiation pressure is

$$p_r = \frac{dp/dt}{A} = \frac{8.0 \times 10^{-7}\,\mathrm{N}}{2.0\,\mathrm{m^2}} = 4.0 \times 10^{-7}\ \mathrm{Pa}\ .$$

26. The mass of the cylinder is $m = \rho(\pi d_1^2/4)H$, where d_1 is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H d_1^2 g\rho}{4} - \left(\frac{\pi d_1^2}{4}\right)\left(\frac{2I}{c}\right) = 0 \; .$$

We solve for H:

$$\begin{aligned} H &= \frac{2I}{gc\rho} = \left(\frac{2P}{\pi d^2/4}\right)\frac{1}{gc\rho} \\ &= \frac{8(4.60\,\text{W})}{\pi(2.60\times10^{-3}\,\text{m})^2(9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)} \\ &= 4.91\times10^{-7}\ \text{m} \; . \end{aligned}$$

27. Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1-f$. The reflected portion exerts a radiation pressure of

$$p_r = \frac{2fI_0}{c}$$

and the absorbed portion exerts a radiation pressure of

$$p_a = \frac{(1-f)I_0}{c} \, ,$$

where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c} \; .$$

To relate the intensity and energy density, we consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy passes through the end in time $t = \ell/c$, so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc \; .$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is $I = I_0 + fI_0 = (1+f)I_0$, where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c} \; ,$$

the same as radiation pressure.

28. We imagine the bullets (of mass m and speed v each) which will strike a surface of area A of the plane within time t to $t+\Delta t$ to be contained in a cylindrical volume at time t. Since the number of bullets contained in the cylinder is $N = n(Av\Delta t)$ and each bullet changes its momentum by $\Delta p_b = mv$, the rate of change of the total momentum for the bullets that strike the area is

$$F = \frac{\Delta P_{\text{total}}}{\Delta t} = N\frac{p_b}{\Delta t} = \frac{(Av\Delta t)nmv}{\Delta t} = Anmv^2$$

where n is the number density of the bullets (bullets per unit volume). The pressure is then

$$p_r = \frac{F}{A} = nmv^2 = 2nK \; ,$$

where $K = \frac{1}{2}mv^2$. Note that nK is the kinetic energy density. Also note that the relation between energy and momentum for a bullet is quite different from the relation between those quantities for an electromagnetic wave.

29. If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. Since the total momentum of the spaceship and light is conserved, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$. We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10 \times 10^3\,\mathrm{W})(86400\,\mathrm{s})}{(1.5 \times 10^3\,\mathrm{kg})(2.998 \times 10^8\,\mathrm{m/s})} = 1.9 \times 10^{-3}\ \mathrm{m/s}\ .$$

30. We require $F_{\text{grav}} = F_r$ or

$$G\frac{mM_s}{d_{es}^2} = \frac{2IA}{c}\ ,$$

and solve for the area A:

$$\begin{aligned} A &= \frac{cGmM_s}{2Id_{es}^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N{\cdot}m^2/kg^2})(1500\,\mathrm{kg})(1.99 \times 10^{30}\,\mathrm{kg})(2.998 \times 10^8\,\mathrm{m/s})}{2(1.40 \times 10^3\,\mathrm{W/m^2})(1.50 \times 10^{11}\,\mathrm{m})^2} \\ &= 9.5 \times 10^5\ \mathrm{m^2} = 0.95\ \mathrm{km^2}\ . \end{aligned}$$

31. (a) Let r be the radius and ρ be the density of the particle. Since its volume is $(4\pi/3)r^3$, its mass is $m = (4\pi/3)\rho r^3$. Let R be the distance from the Sun to the particle and let M be the mass of the Sun. Then, the gravitational force of attraction of the Sun on the particle has magnitude

$$F_g = \frac{GMm}{R^2} = \frac{4\pi GM\rho r^3}{3R^2}\ .$$

If P is the power output of the Sun, then at the position of the particle, the radiation intensity is $I = P/4\pi R^2$, and since the particle is perfectly absorbing, the radiation pressure on it is

$$p_r = \frac{I}{c} = \frac{P}{4\pi R^2 c}\ .$$

All of the radiation that passes through a circle of radius r and area $A = \pi r^2$, perpendicular to the direction of propagation, is absorbed by the particle, so the force of the radiation on the particle has magnitude

$$F_r = p_r A = \frac{\pi P r^2}{4\pi R^2 c} = \frac{Pr^2}{4R^2 c}\ .$$

The force is radially outward from the Sun. Notice that both the force of gravity and the force of the radiation are inversely proportional to R^2. If one of these forces is larger than the other at some distance from the Sun, then that force is larger at all distances. The two forces depend on the particle radius r differently: F_g is proportional to r^3 and F_r is proportional to r^2. We expect a small radius particle to be blown away by the radiation pressure and a large radius particle with the same density to be pulled inward toward the Sun. The critical value for the radius is the value for which the two forces are equal. Equating the expressions for F_g and F_r, we solve for r:

$$r = \frac{3P}{16\pi GM\rho c}\ .$$

(b) According to Appendix C, $M = 1.99 \times 10^{30}\,\mathrm{kg}$ and $P = 3.90 \times 10^{26}\,\mathrm{W}$. Thus,

$$\begin{aligned} r &= \frac{3(3.90 \times 10^{26}\,\mathrm{W})}{16\pi(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(1.99 \times 10^{30}\,\mathrm{kg})(1.0 \times 10^3\,\mathrm{kg/m^3})(3.00 \times 10^8\,\mathrm{m/s})} \\ &= 5.8 \times 10^{-7}\ \mathrm{m}\ . \end{aligned}$$

32. (a) The discussion in §17-5 regarding the argument of the sine function $(kx + \omega t)$ makes it clear that the wave is traveling in the negative y direction. Thus, $\vec{S}$ points in the $-\hat{\mathrm{j}}$ direction.

(b) Since $\vec{E} \times \vec{B} \propto \vec{S}$ and $\vec{B}$ points in the $\hat{\imath}$ direction, then we may conclude that $\vec{E}$ points in the $-\hat{k}$ direction (recall that $\hat{k} \times \hat{\imath} = \hat{\jmath}$). Therefore, $E_x = E_y = 0$ and $E_z = -cB\sin(kx + \omega t)$.

(c) Since $E_x = E_y = 0$, the wave is polarized along the z axis.

33. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is $I = 5.0\,\mathrm{mW/m^2}$. The intensity and the electric field amplitude are related by $I = E_m^2/2\mu_0 c$, so

$$\begin{aligned} E_m &= \sqrt{2\mu_0 cI} = \sqrt{2(4\pi \times 10^{-7}\,\mathrm{H/m})(3.00 \times 10^8\,\mathrm{m/s})(5.0 \times 10^{-3}\,\mathrm{W/m^2})} \\ &= 1.9\ \mathrm{V/m}\ . \end{aligned}$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0 \times 10^{-3}\,\mathrm{W/m^2}}{3.00 \times 10^8\,\mathrm{m/s}} = 1.7 \times 10^{-11}\ \mathrm{Pa}\ .$$

34. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\begin{aligned} \frac{I_f}{I_0} &= \frac{1}{2}\cos^2(\theta_1 + \theta_2)\cos^2(\theta_2 + \theta_3) \\ &= \frac{1}{2}\cos^2(50^\circ + 50^\circ)\cos^2(50^\circ + 50^\circ) = 4.5 \times 10^{-4}\ . \end{aligned}$$

35. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is $I_1 = \frac{1}{2}I_0$, and the direction of polarization of the transmitted light is $\theta_1 = 40^\circ$ counterclockwise from the y axis in the diagram. The polarizing direction of the second sheet is $\theta_2 = 20^\circ$ clockwise from the y axis, so the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet is $40^\circ + 20^\circ = 60^\circ$. The transmitted intensity is

$$I_2 = I_1 \cos^2 60^\circ = \frac{1}{2}I_0 \cos^2 60^\circ\ ,$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis. The polarizing direction of the third sheet is $\theta_3 = 40^\circ$ counterclockwise from the y axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is $20^\circ + 40^\circ = 60^\circ$. The transmitted intensity is

$$I_3 = I_2 \cos^2 60^\circ = \frac{1}{2}I_0 \cos^4 60^\circ = 3.1 \times 10^{-2}\ .$$

Thus, 3.1% of the light's initial intensity is transmitted.

36. As the unpolarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $\frac{1}{2}I_0$. After passing through the second polarizer, for which the direction of polarization is at an angle θ from that of the first one, the intensity is $I = \frac{1}{2}I_0\cos^2\theta = \frac{1}{3}I_0$. Thus, $\cos^2\theta = 2/3$, which leads to $\theta = 35^\circ$.

37. The angle between the direction of polarization of the light incident on the first polarizing sheet and the polarizing direction of that sheet is $\theta_1 = 70^\circ$. If I_0 is the intensity of the incident light, then the intensity of the light transmitted through the first sheet is

$$I_1 = I_0\cos^2\theta_1 = (43\,\mathrm{W/m^2})\cos^2 70^\circ = 5.03\ \mathrm{W/m^2}\ .$$

The direction of polarization of the transmitted light makes an angle of 70° with the vertical and an angle of $\theta_2 = 20^\circ$ with the horizontal. θ_2 is the angle it makes with the polarizing direction of the second polarizing sheet. Consequently, the transmitted intensity is

$$I_2 = I_1\cos^2\theta_2 = (5.03\,\mathrm{W/m^2})\cos^2 20^\circ = 4.4\ \mathrm{W/m^2}\ .$$

38. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2}I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2}I_0 \cos^2(90° - 70°) = \frac{1}{2}(43\,\mathrm{W/m^2})(\cos^2 20°) = 19\ \mathrm{W/m^2}\ .$$

39. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is fI_0. After transmission, this portion contributes $fI_0 \cos^2\theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1-f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = fI_0 \cos^2\theta + \frac{1}{2}(1-f)I_0\ .$$

As the filter is rotated, $\cos^2\theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\mathrm{min}} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\mathrm{max}} = fI_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0\ .$$

The ratio of I_{max} to I_{min} is

$$\frac{I_{\mathrm{max}}}{I_{\mathrm{min}}} = \frac{1+f}{1-f}\ .$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

40. (a) The fraction of light which is transmitted by the glasses is

$$\frac{I_f}{I_0} = \frac{E_f^2}{E_0^2} = \frac{E_v^2}{E_v^2 + E_h^2} = \frac{E_v^2}{E_v^2 + (2.3E_v)^2} = 0.16\ .$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0} = \frac{E_h^2}{E_v^2 + E_h^2} = \frac{(2.3E_v)^2}{E_v^2 + (2.3E_v)^2} = 0.84\ .$$

41. (a) The rotation cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted. It can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is $I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta$, where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated $90°/n$ in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is $I = I_0 \cos^{2n}(90°/n)$. We want the smallest integer value of n for which this is greater than $0.60I_0$. We start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase n by 1 and try again. We repeat this process, increasing n by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

42. The angle of incidence for the light ray on mirror B is $90° - \theta$. So the outgoing ray r' makes an angle $90° - (90° - \theta) = \theta$ with the vertical direction, and is antiparallel to the incoming one. The angle between i and r' is therefore $180°$.

43. The law of refraction states

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \ .$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48 \ .$$

44. (a) The law of refraction requires that $\sin\theta_1/\sin\theta_2 = n_{\text{water}} = \text{const}$. We can check that this is indeed valid for any given pair of θ_1 and θ_2. For example $\sin 10°/\sin 8° = 1.3$, and $\sin 20°/\sin 15°30' = 1.3$, etc.

(b) $n_{\text{water}} = 1.3$, as shown in part (a).

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by $\tan\theta_1 = w/h$, where h is the height of the tank and w is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{w}{h}\right) = \tan^{-1}\left(\frac{1.10\,\text{m}}{0.850\,\text{m}}\right) = 52.31° \ .$$

The law of refraction yields

$$n_1 = n_2 \frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26 \ ,$$

where the index of refraction of air was taken to be unity.

46. (a) Approximating $n = 1$ for air, we have

$$n_1 \sin\theta_1 = (1)\sin\theta_5 \implies 56.9° = \theta_5$$

and with the more accurate value for n_{air} in Table 34-1, we obtain $56.8°$.

(b) Eq. 34-44 leads to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_4 \sin\theta_4$$

so that

$$\theta_4 = sin^{-1}\left(\frac{n_1}{n_4}\sin\theta_1\right) = 35.3° \ .$$

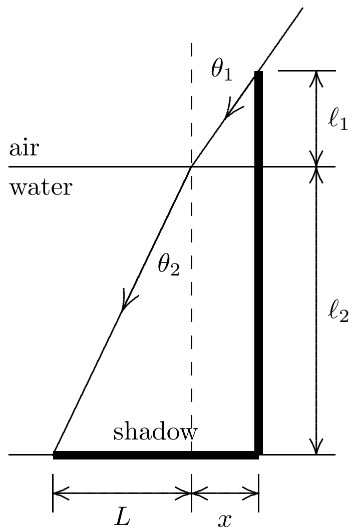
47. Consider a ray that grazes the top of the pole, as shown in the diagram below. Here $\theta_1 = 35°$, $\ell_1 = 0.50\,\text{m}$, and $\ell_2 = 1.50\,\text{m}$. The length of the shadow is $x + L$. x is given by $x = \ell_1 \tan\theta_1 = (0.50\,\text{m})\tan 35° = 0.35\,\text{m}$. According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 34–1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55° \ .$$

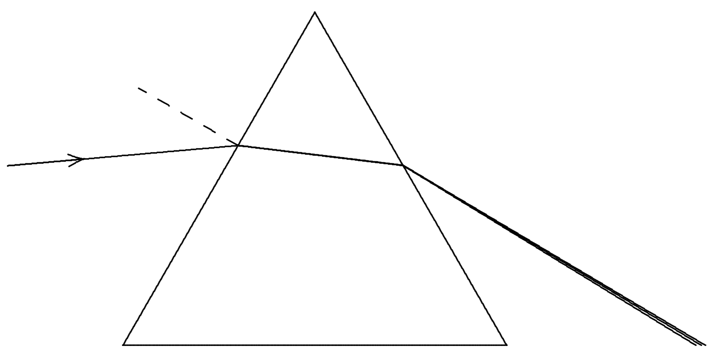
L is given by

$$L = \ell_2 \tan\theta_2 = (1.50\,\text{m})\tan 25.55° = 0.72\,\text{m} \ .$$

The length of the shadow is $0.35\,\text{m} + 0.72\,\text{m} = 1.07\,\text{m}$.



48. We use the law of refraction (assuming $n_{\text{air}} = 1$) and the law of sines to determine the paths of various light rays. The index of refraction for fused quartz can be found in Fig. 34-19. We estimate $n_{\text{blue}} = 1.463$, $n_{\text{y g}} = 1.459$, and $n_{\text{red}} = 1.456$. The light rays as they leave the prism (from the right side of the prism shown below) are very close together; on the scale we used below, the individual rays are difficult to resolve. Measured from the surface of the prism (at the face from which they emerge from the prism) their angles are $\theta_{\text{blue}} = 28.51°, \theta_{\text{y g}} = 28.95°$, and $\theta_{\text{red}} = 29.29°$. The angle between the incident rays (on the left side of the picture) and the dashed line (the axis normal to the left face of the prism) is 35°.

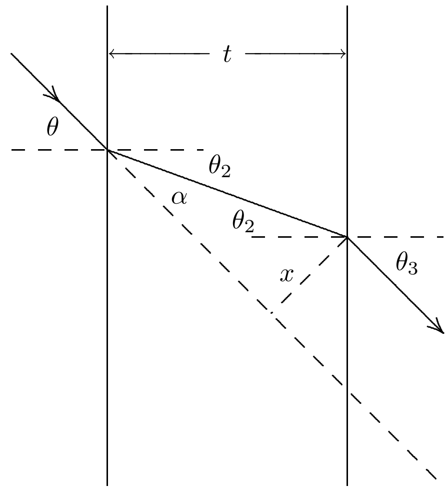


49. Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields $\sin\theta = n\sin\theta_2$. The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then $n\sin\theta_2 = \sin\theta_3$. Thus $\sin\theta_3 = \sin\theta$ and $\theta_3 = \theta$. The emerging ray is parallel to the incident ray. We wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D\cos\theta_2 = t$ and $D = t/\cos\theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and $x = D\sin\alpha = D\sin(\theta - \theta_2)$. Thus

$$x = \frac{t\sin(\theta - \theta_2)}{\cos\theta_2} \, .$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then $\sin\theta \approx \theta$, $\sin\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos\theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n} .$$



50. (a) An incident ray which is normal to the water surface is not refracted, so the angle at which it strikes the first mirror is $\theta_1 = 45°$. According to the law of reflection, the angle of reflection is also 45°. This means the ray is horizontal as it leaves the first mirror, and the angle of incidence at the second mirror is $\theta_2 = 45°$. Since the angle of reflection at the second mirror is also 45° the ray leaves that mirror normal again to the water surface. There is no refraction at the water surface, and the emerging ray is parallel to the incident ray.

(b) We imagine that the incident ray makes an angle θ_1 with the normal to the water surface. The angle of refraction θ_2 is found from $\sin\theta_1 = n \sin\theta_2$, where n is the index of refraction of the water. The normal to the water surface and the normal to the first mirror make an angle of 45°. If the normal to the water surface is continued downward until it meets the normal to the first mirror, the triangle formed has an interior angle of $180° - 45° = 135°$ at the vertex formed by the normal. Since the interior angles of a triangle must sum to 180°, the angle of incidence at the first mirror satisfies $\theta_3 + \theta_2 + 135° = 180°$, so $\theta_3 = 45° - \theta_2$. Using the law of reflection, the angle of reflection at the first mirror is also $45° - \theta_2$. We note that the triangle formed by the ray and the normals to the two mirrors is a right triangle. Consequently, $\theta_3 + \theta_4 + 90° = 180°$ and $\theta_4 = 90° - \theta_3 = 90° - 45° + \theta_2 = 45° + \theta_2$. The angle of reflection at the second mirror is also $45° + \theta_2$. Now, we continue the normal to the water surface downward from the exit point of the ray to the second mirror. It makes an angle of 45° with the mirror. Consider the triangle formed by the second mirror, the ray, and the normal to the water surface. The angle at the intersection of the normal and the mirror is $180° - 45° = 135°$. The angle at the intersection of the ray and the mirror is $90° - \theta_4 = 90° - (45° + \theta_2) = 45° - \theta_2$. The angle at the intersection of the ray and the water surface is θ_5. These three angles must sum to 180°, so $135° + 45° - \theta_2 + \theta_5 = 180°$. This means $\theta_5 = \theta_2$. Finally, we use the law of refraction to find θ_6:

$$\sin\theta_6 = n \sin\theta_5 \implies \sin\theta_6 = n \sin\theta_2 ,$$

since $\theta_5 = \theta_2$. Finally, since $\sin\theta_1 = n \sin\theta_2$, we conclude that $\sin\theta_6 = \sin\theta_1$ and $\theta_6 = \theta_1$. The exiting ray is parallel to the incident ray.

51. We label the light ray's point of entry A, the vertex of the prism B, and the light ray's exit point C. Also, the point in Fig. 34-49 where ψ is defined (at the point of intersection of the extrapolations of the incident and emergent rays) is denoted D. The angle indicated by ADC is the supplement of ψ, so we

denote it $\psi_s = 180° - \psi$. The angle of refraction in the glass is $\theta_2 = \frac{1}{n}\sin\theta$. The angles between the interior ray and the nearby surfaces is the complement of θ_2, so we denote it $\theta_{2c} = 90° - \theta_2$. Now, the angles in the triangle ABC must add to 180°:

$$180° = 2\theta_{2c} + \phi \quad\Longrightarrow\quad \theta_2 = \frac{\phi}{2} \ .$$

Also, the angles in the triangle ADC must add to 180°:

$$180° = 2(\theta - \theta_2) + \psi_s \quad\Longrightarrow\quad \theta = 90° + \theta_2 - \frac{1}{2}\psi_s$$

which simplifies to $\theta = \theta_2 + \frac{1}{2}\psi$. Combining this with our previous result, we find $\theta = \frac{1}{2}(\phi + \psi)$. Thus, the law of refraction yields

$$n = \frac{\sin(\theta)}{\sin(\theta_2)} = \frac{\sin\left(\frac{1}{2}(\phi+\psi)\right)}{\sin\left(\frac{1}{2}\phi\right)} \ .$$

52. The critical angle is

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34° \ .$$

53. Let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is $n\sin\theta_3 \ge 1$. We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields $n\sin\theta_2 = \sin\theta_1$. Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes $1 \le n\sin(90° - \theta_2) = n\cos\theta_2$. Squaring this equation and using $\sin^2\theta_2 + \cos^2\theta_2 = 1$, we obtain $1 \le n^2(1 - \sin^2\theta_2)$. Substituting $\sin\theta_2 = (1/n)\sin\theta_1$ now leads to

$$1 \le n^2\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = n^2 - \sin^2\theta_1 \ .$$

The largest value of n for which this equation is true is the value for which $1 = n^2 - \sin^2\theta_1$. We solve for n:

$$n = \sqrt{1 + \sin^2\theta_1} = \sqrt{1 + \sin^2 45°} = 1.22 \ .$$

54. Reference to Fig. 34-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point a to point f in that figure) is related to the tangent of the angle of incidence. Thus, the diameter D of the circle in question is

$$D = 2h\tan\theta_c = 2h\tan\left[\sin^{-1}\left(\frac{1}{n_w}\right)\right] = 2(80.0\,\text{cm})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = 182 \text{ cm} \ .$$

55. (a) No refraction occurs at the surface ab, so the angle of incidence at surface ac is $90° - \phi$. For total internal reflection at the second surface, $n_g\sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g\cos\phi \ge n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g\cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9° \ .$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0^\circ \ .$$

56. (a) (b) and (c) The index of refraction n for fused quartz is slightly higher on the bluish side of the visible light spectrum (with shorter wavelength). We estimate $n = 1.463$ for blue and $n = 1.456$ for red. Since $\sin\theta_c = 1/n$, the critical angle is slightly smaller for blue than it is for red: $\theta_c = 43.12^\circ$ for blue and $\theta_c = 43.38^\circ$ for red. Thus, at an angle of incidence of, say, $\theta = 43.29^\circ$, the refracted beam would be depleted of blue (and would appear to an outside observer as reddish), and the reflected beam would consequently appear to be bluish (to someone able to observe that beam, the operational details of which are not discussed here).

57. (a) The diagram below shows a cross section, through the center of the cube and parallel to a face. L is the length of a cube edge and S labels the spot. A portion of a ray from the source to a cube face is also shown. Light leaving the source at a small angle θ is refracted at the face and leaves the cube; light leaving at a sufficiently large angle is totally reflected. The light that passes through the cube face forms a circle, the radius r being associated with the critical angle for total internal reflection. If θ_c is that angle, then

$$\sin\theta_c = \frac{1}{n}$$

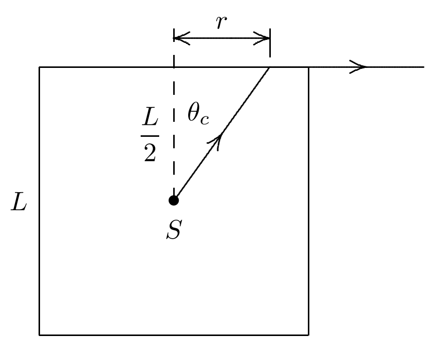
where n is the index of refraction for the glass. As the diagram shows, the radius of the circle is given by $r = (L/2)\tan\theta_c$. Now,

$$\tan\theta_c = \frac{\sin\theta_c}{\cos\theta_c} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} = \frac{1/n}{\sqrt{1-(1/n)^2}} = \frac{1}{\sqrt{n^2-1}}$$

and the radius of the circle is

$$r = \frac{L}{2\sqrt{n^2-1}} = \frac{10\,\text{mm}}{2\sqrt{(1.5)^2-1}} = 4.47\,\text{mm} \ .$$

If an opaque circular disk with this radius is pasted at the center of each cube face, the spot will not be seen (provided internally reflected light can be ignored).



(b) There must be six opaque disks, one for each face. The total area covered by disks is $6\pi r^2$ and the total surface area of the cube is $6L^2$. The fraction of the surface area that must be covered by disks is

$$f = \frac{6\pi r^2}{6L^2} = \frac{\pi r^2}{L^2} = \frac{\pi(4.47\,\text{mm})^2}{(10\,\text{mm})^2} = 0.63 \ .$$

58. (a) We refer to the entry point for the original incident ray as point A (which we take to be on the left side of the prism, as in Fig. 34-49), the prism vertex as point B, and the point where the interior

ray strikes the right surface of the prism as point C. The angle between line AB and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line BC and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point C, yields $n \sin\theta_3 = 1$, so $\sin\theta_3 = 1/n = 1/1.60 = 0.625$ and $\theta_3 = 38.68°$. The interior angles of the triangle ABC must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point A, yields $\sin\theta_1 = n\sin\theta_2 = 1.60 \sin 21.32° = 0.5817$. Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point C. Since the angle of refraction there is the same as the angle of incidence at A, $n\sin\theta_3 = \sin\theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\sin(60° - \theta_2) \implies \sin\theta_1 = n\sin 60° \cos\theta_2 - n\cos 60° \sin\theta_2$$

where the trigonometric identity $\sin(A-B) = \sin A\cos B - \cos A \sin B$ is used. Next, we apply the law of refraction to point A:

$$\sin\theta_1 = n\sin\theta_2 \implies \sin\theta_2 = (1/n)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-(1/n^2)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60° \sqrt{1-(1/n)^2\sin^2\theta_1} - \cos 60° \sin\theta_1$$

or

$$(1+\cos 60°)\sin\theta_1 = \sin 60° \sqrt{n^2 - \sin^2\theta_1}\ .$$

Squaring both sides and solving for $\sin\theta_1$, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = 0.80$$

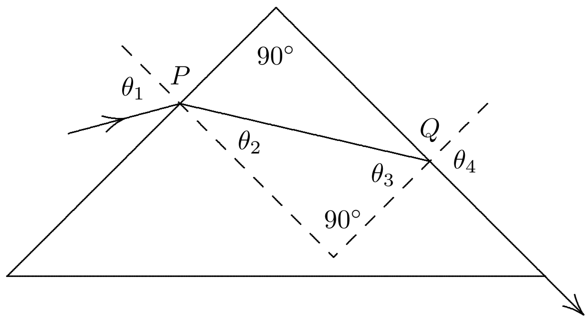
and $\theta_1 = 53.1°$.

59. (a) A ray diagram is shown below. Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is $\theta_4 = 90°$. The law of refraction, applied to the second surface, yields $n\sin\theta_3 = \sin\theta_4 = 1$. As shown in the diagram, the normals to the surfaces at P and Q are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and $\sin\theta_3 = \sin(90° - \theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}$. According to the law of refraction, applied at Q, $n\sqrt{1-\sin^2\theta_2} = 1$. The law of refraction, applied to point P, yields $\sin\theta_1 = n\sin\theta_2$, so $\sin\theta_2 = (\sin\theta_1)/n$ and

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1\ .$$

Squaring both sides and solving for n, we get

$$n = \sqrt{1+\sin^2\theta_1}\ .$$



(b) The greatest possible value of $\sin^2\theta_1$ is 1, so the greatest possible value of n is $n_{\max} = \sqrt{2} = 1.41$.

(c) For a given value of n, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 ($= 90° - \theta_2$). That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at Q.

60. (a) We use Eq. 34-49: $\theta_{\rm B} = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1°$.

(b) Yes, since n_w depends on the wavelength of the light.

61. The angle of incidence θ_B for which reflected light is fully polarized is given by Eq. 34–48 of the text. If n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium, then $\theta_B = \tan^{-1}(n_2/n_1) = \tan^{-1}(1.53/1.33) = 63.8°$.

62. From Fig. 34-19 we find $n_{\max} = 1.470$ for $\lambda = 400\,\text{nm}$ and $n_{\min} = 1.456$ for $\lambda = 700\,\text{nm}$. The corresponding Brewster's angles are $\theta_{\rm B,max} = \tan^{-1} n_{\max} = \tan^{-1}(1.470) = 55.77°$ and $\theta_{\rm B,min} = \tan^{-1}(1.456) = 55.52°$.

63. (a) The Sun is far enough away that we approximate its rays as "parallel" in this Figure. That is, if the sunray makes angle θ from horizontal when the bird is in one position, then it makes the same angle θ when the bird is any other position. Therefore, its shadow on the ground moves as the bird moves: at 15 m/s.

(b) If the bird is in a position, a distance $x > 0$ from the wall, such that its shadow is on the wall at a distance $0 \geq y \geq h$ from the top of the wall, then it is clear from the Figure that $tan\theta = y/x$. Thus,

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = (-15\,\text{m/s})\tan 30° = -8.7\text{ m/s},$$

which means that the distance y (which was measured as a positive number downward from the top of the wall) is shrinking at the rate of 8.7 m/s.

(c) Since $\tan\theta$ grows as $0 \leq \theta < 90°$ increases, then a larger value of $|dy/dt|$ implies a larger value of θ. The Sun is higher in the sky when the hawk glides by.

(d) With $|dy/dt| = 45$ m/s, we find

$$v_{\rm hawk} = \left|\frac{dx}{dt}\right| = \frac{\left|\frac{dy}{dt}\right|}{\tan\theta}$$

so that we obtain $\theta = 72°$ if we assume $v_{\rm hawk} = 15$ m/s.

64. (a) The 63.00 ns arrival times are consistent with the top of the tomb being 31.50 ns (pulse travel time) away from the surface. Since the pulses travel at 10.0 cm/ns in the soil, this travel time corresponds to a distance equal to $315\,\text{cm} = 3.15\,\text{m}$.

(b) We are told that the locations in Fig. 34-54 are 2.0 m apart. Return pulses are registered at stations 2 through 7, but the returns from stations 2 and 7 are not "robust." The tomb's horizontal length is therefore at least 9 m long, and very probably less than 12 m in length.

(c) As demonstrated in part (a), we divide the travel times by 2 to infer depth. Thus, at station 3: the top of the tomb is 31.50 ns (pulse travel time in soil) from the surface; the top stone slab is 1.885 ns thick (pulse travel time in stone); the interior of the tomb is 8.00 ns high (pulse travel time in air); and the bottom stone slab is 1.885 ns thick (pulse travel time in stone). Since the pulse travels at 30 cm/s in the air, the interior of the tomb under station 3 (at the west end of the tomb) is 240 cm = 2.40 m high. At the east end (under, say, station 5), the corresponding time difference is

$$\frac{74.77\,\text{ns} - 66.77\,\text{ns}}{2} = 4.00\ \text{ns}$$

which corresponds to an interior height equal to $(4.00\,\text{ns})(30\,\text{cm/s}) = 120\,\text{cm/s} = 1.20$ m.

65. Since the layers are parallel, the angle of refraction regarding the first surface is the same as the angle of incidence regarding the second surface (as is suggested by the notation in Fig. 34-55). We recall that as part of the derivation of Eq. 34-49 (Brewster's angle), the textbook shows that the refracted angle is the complement of the incident angle:

$$\theta_2 = (\theta_1)_\text{c} = 90° - \theta_1\ .$$

We apply Eq. 34-49 to both refractions, setting up a product:

$$\begin{aligned}\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right) &= (\tan\theta_{\text{B}\,1\to 2})\,(\tan\theta_{\text{B}\,2\to 3}) \\ \frac{n_3}{n_1} &= (\tan\theta_1)\,(\tan\theta_2)\ .\end{aligned}$$

Now, since θ_2 is the complement of θ_1 we have

$$\tan\theta_2 = \tan(\theta_1)_\text{c} = \frac{1}{\tan\theta_1}\ .$$

Therefore, the product of tangents cancel and we obtain $n_3/n_1 = 1$. Consequently, the third medium is air: $n_3 = 1.0$.

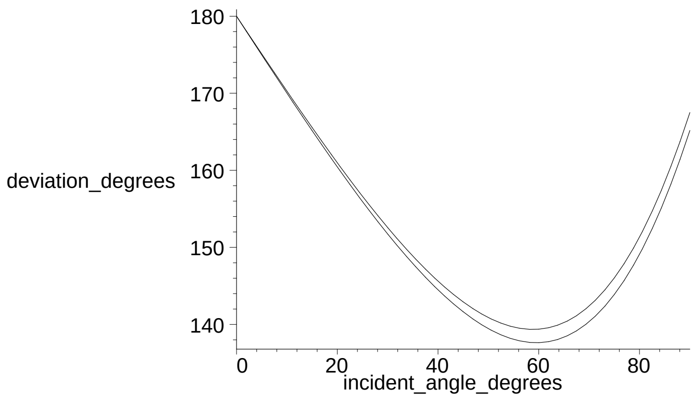
66. In air, light travels at roughly $c = 3.0 \times 10^8$ m/s. Therefore, for $t = 1.0$ ns, we have a distance of

$$d = ct = \left(3.0 \times 10^8\,\text{m/s}\right)\left(1.0 \times 10^{-9}\,\text{s}\right) = 0.30\ \text{m}\ .$$

67. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution to the overall deviation is the reflection. Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after the reflection) is $\delta\theta_2 = 180° - 2\theta_r$. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_\text{dev} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 180° + 2\theta_i - 4\theta_r\ .$$

(b) We substitute $\theta_r = \sin^{-1}(\frac{1}{n}\sin\theta_i)$ into the expression derived in part (a), using the two given values for n. The higher curve is for the blue light.



(c) We can expand the graph and try to estimate the minimum, or search for it with a more sophisticated numerical procedure. We find that the $\theta_{\rm dev}$ minimum for red light is 137.63°, and this occurs at $\theta_i = 59.52°$.

(d) For blue light, we find that the $\theta_{\rm dev}$ minimum is 139.35°, and this occurs at $\theta_i = 59.52°$.

(e) The difference in $\theta_{\rm dev}$ in the previous two parts is 1.72°.

68. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s). Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180° - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\rm dev} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2\,(\theta_i - \theta_r) + k\,(180° - 2\theta_r) = k\,(180°) + 2\theta_i - 2(k+1)\theta_r \ .$$

(b) For $k = 2$ and $n = 1.331$ (given in problem 67), we search for the second-order rainbow angle numerically. We find that the $\theta_{\rm dev}$ minimum for red light is 230.37°, and this occurs at $\theta_i = 71.90°$.

(c) Similarly, we find that the second-order $\theta_{\rm dev}$ minimum for blue light (for which $n = 1.343$) is 233.48°, and this occurs at $\theta_i = 71.52°$.

(d) The difference in $\theta_{\rm dev}$ in the previous two parts is 3.11°.

(e) Setting $k = 3$, we search for the third-order rainbow angle numerically. We find that the $\theta_{\rm dev}$ minimum for red light is 317.53°, and this occurs at $\theta_i = 76.88°$.

(f) Similarly, we find that the third-order $\theta_{\rm dev}$ minimum for blue light is 321.89°, and this occurs at $\theta_i = 76.62°$.

(g) The difference in $\theta_{\rm dev}$ in the previous two parts is 4.37°.

69. Reference to Fig. 34-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point a to point f in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h\tan\theta_c$. For water $n = 1.33$, so Eq. 34-47 gives $\sin\theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h\tan\theta_c = 2(2.00\,\text{m})(\tan 48.75°) = 4.56 \text{ m} \ .$$

70. We apply Eq. 34-42 (twice) to obtain

$$I = I_0\,\cos^2\theta_1\,\cos^2\theta_2$$

where $\theta_1 = 20°$ and $\theta_2 = (20° + \theta)$. Since $I/I_0 = 0.200$, we find $\cos\theta_2 = \sqrt{0.2265}$ which leads to $\theta_2 = 62°$ and consequently to $\theta = 42°$.

71. (a) The electric field amplitude is $E_m = \sqrt{2}\,E_{\text{rms}} = 70.7$ V/m, so that the magnetic field amplitude is $B_m = 2.36 \times 10^{-7}$ T by Eq. 34-5. Since the direction of propagation, $\vec{E}$, and $\vec{B}$ are mutually perpendicular, we infer that the only non-zero component of $\vec{B}$ is B_x, and note that the direction of propagation being along the $-z$ axis means the spatial and temporal parts of the wave function argument are of like sign (see §17-5). Also, from $\lambda = 250$ nm, we find that $f = c/\lambda = 1.20 \times 10^{15}$ Hz, which leads to $\omega = 2\pi f = 7.53 \times 10^{15}$ rad/s. Also, we note that $k = 2\pi/\lambda = 2.51 \times 10^{7}$ m^{-1}. Thus, assuming some "initial condition" (that, say the field is zero, with its derivative positive, at $z = 0$ when $t = 0$), we have

$$B_x = 2.36 \times 10^{-7} \ \sin\left(\left(2.51 \times 10^{7}\right) z + \left(7.53 \times 10^{15}\right) t\right)$$

in SI units.

(b) The exposed area of the triangular chip is $A = \sqrt{3}\,\ell^2/8$, where $\ell = 2.00 \times 10^{-6}$ m. The intensity of the wave is

$$I = \frac{1}{c\mu_0}\,E_{\text{rms}}^2 = 6.64 \text{ W/m}^2 \ .$$

Thus, Eq. 34-33 leads to

$$F = \frac{2IA}{c} = 3.83 \times 10^{-20} \text{ N} \ .$$

72. We follow Sample Problem 34-2 in computing the sunlight intensity at the sail's location.

$$I = \frac{P_S}{4\pi r^2} = \frac{3.9 \times 10^{26} \text{ W}}{4\pi\left(3.0 \times 10^{11} \text{ m}\right)^2} = 345 \text{ W/m}^2$$

With $A = (2.0 \text{ m})^2$, we use Eq. 34-33 to obtain the radiation force:

$$F = \frac{2IA}{c} = 9.2 \times 10^{-6} \text{ N} \ .$$

73. (a) Eq. 34-5 gives $E = cB$, which relates the field values at any instant – and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, $E_{\text{rms}} = cB_{\text{rms}} = 16.8$ V/m. Multiplying by $\sqrt{2}$ yields the electric field amplitude $E_m = 23.7$ V/m.

(b) We use Eq. 34-26:

$$I = \frac{1}{\mu_0\, c}\,E_{\text{rms}}^2 = 0.748 \text{ W/m}^2 \ .$$

74. Consider two wavelengths, λ_1 and λ_2, whose corresponding frequencies are f_1 and f_2. Then $\lambda_1 = C/f_1$ and $\lambda_2 = C/f_2$. If $\lambda_1/\lambda_2 = 10$, then

$$\frac{\lambda_1}{\lambda_2} = \frac{C/f_1}{C/f_2} = \frac{f_2}{f_1} = 10 \ .$$

The spaces are the same on both scales.

75. We take the derivative with respect to x of both sides of Eq. 34-11:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right) = \frac{\partial^2 E}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right) = -\frac{\partial^2 B}{\partial x \partial t} \ .$$

Now we differentiate both sides of Eq. 34-18 with respect to t:

$$\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial x}\right) = -\frac{\partial^2 B}{\partial x \partial t} = \frac{\partial}{\partial t}\left(\varepsilon_0\mu_0\frac{\partial E}{\partial t}\right) = \varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2} \ .$$

Substituting $\partial^2 E/\partial x^2 = -\partial^2 B/\partial x \partial t$ from the first equation above into the second one, we get

$$\varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} ,$$

or

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2 E}{\partial x^2} .$$

Similarly, we differentiate both sides of Eq. 34-11 with respect to t

$$\frac{\partial^2 E}{\partial x \partial t} = -\frac{\partial^2 B}{\partial t^2} ,$$

and differentiate both sides of Eq. 34-18 with respect to x

$$-\frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial x \partial t} .$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2} = c^2 \frac{\partial^2 B}{\partial x^2} .$$

76. The energy density of an electromagnetic wave is given by $u = u_E + u_B$. From the discussion in §34-4, $u_E = u_B = \frac{1}{2}\varepsilon_0 E^2$, so $u = 2u_E = \varepsilon_0 E^2$. Upon averaging over time this becomes

$$u_{\text{avg}} = \varepsilon_0 \overline{E^2} = \varepsilon_0 E_{\text{rms}}^2 .$$

Combining this equation with Eq. 34-26 in the textbook, we obtain

$$I = \frac{1}{c\mu_0} E_{\text{rms}}^2 = \frac{1}{c\mu_0} \frac{u_{\text{avg}}}{\varepsilon_0} = \frac{c^2 u_{\text{avg}}}{c} = c u_{\text{avg}}$$

where $c^2 = 1/\varepsilon_0 \mu_0$ is used.

77. (a) Assuming complete absorption, the radiation pressure is

$$p_r = \frac{I}{c} = \frac{1.4 \times 10^3 \, \text{W/m}^2}{3.0 \times 10^8 \, \text{m/s}} = 4.7 \times 10^{-6} \, \text{N/m}^2 .$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0} = \frac{4.7 \times 10^{-6} \, \text{N/m}^2}{1.0 \times 10^5 \, \text{N/m}^2} = 4.7 \times 10^{-11} .$$

78. (a) Suppose there are a total of N transparent layers ($N = 5$ in our case). We label these layers from left to right with indices 1, 2, ..., N. Let the index of refraction of the air be n_0. We denote the initial angle of incidence of the light ray upon the air-layer boundary as θ_i and the angle of the emerging light ray as θ_f. We note that, since all the boundaries are parallel to each other, the angle of incidence θ_j at the boundary between the j-th and the $(j+1)$-th layers is the same as the angle between the transmitted light ray and the normal in the j-th layer. Thus, for the first boundary (the one between the air and the first layer)

$$\frac{n_1}{n_0} = \frac{\sin\theta_i}{\sin\theta_1} ,$$

for the second boundary

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} ,$$

and so on. Finally, for the last boundary

$$\frac{n_0}{n_N} = \frac{\sin\theta_N}{\sin\theta_f} .$$

Multiplying these equations, we obtain

$$\left(\frac{n_1}{n_0}\right)\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right)\cdots\left(\frac{n_0}{n_N}\right) = \left(\frac{\sin\theta_i}{\sin\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)\left(\frac{\sin\theta_2}{\sin\theta_3}\right)\cdots\left(\frac{\sin\theta_N}{\sin\theta_f}\right) .$$

We see that the L.H.S. of the equation above can be reduced to n_0/n_0 while the R.H.S. is equal to $\sin\theta_i/\sin\theta_f$. Equating these two expressions, we find

$$\sin\theta_f = \left(\frac{n_0}{n_0}\right)\sin\theta_i = \sin\theta_i ,$$

which gives $\theta_i = \theta_f$. So for the two light rays in the problem statement, the angle of the emerging light rays are both the same as their respective incident angles. Thus, $\theta_f = 0$ for ray a and $\theta_f = 20°$ for ray b.

(b) In this case, all we need to do is to change the value of n_0 from 1.0 (for air) to 1.5 (for glass). This does not change the result above. Note that the result of this problem is fairly general. It is independent of the number of layers and the thickness and index of refraction of each layer.

79. We use the result of the problem 51 to solve for ψ. Note that $\phi = 60.0°$ in our case. Thus, from

$$n = \frac{\sin\frac{1}{2}(\psi+\phi)}{\sin\frac{1}{2}\phi} ,$$

we get

$$\sin\frac{1}{2}(\psi+\phi) = n\sin\frac{1}{2}\phi = (1.31)\sin\left(\frac{60.0°}{2}\right) = 0.655 ,$$

which gives $\frac{1}{2}(\psi+\phi) = \sin^{-1}(0.655) = 40.9°$. Thus, $\psi = 2(40.9°) - \phi = 2(40.9°) - 60.0° = 21.8°$.

80. (a) The light that passes through the surface of the lake is within a cone of apex angle $2\theta_c$ making a "circle of light" there; reference to Fig. 34-24 may help in visualizing this (consider revolving that picture about a vertical axis). Since the source is point-like, its energy spreads out with perfect spherical symmetry, until reaching the surface and other boundaries of the lake. The problem asks us to assume there are no partial reflections at the surface, only the total reflections outside the "circle of light." Thus, of the full sphere of light (of area $A_s = 4\pi R^2$) emitted by the source, only a fraction of it – coinciding with the cone of apex angle $2\theta_c$ – enters the air above. If we label the area of that portion of the sphere which reaches the air above as A, then the fraction of the total energy emitted that passes through the surface is

$$frac = \frac{A}{4\pi R^2} \qquad \text{where} \qquad R = \frac{h}{\cos\theta_c}$$

is the distance from the point-source to the edge of the "circle of light." Now, the area A of the spherical cap of height H bounded by that circle is

$$A = 2\pi RH = 2\pi R(R-h)$$

may be looked up in a number of references, or can be derived from $A = 2\pi R^2 \int_0^{\theta_c} \sin\theta\, d\theta$. Consequently,

$$frac = \frac{2\pi R(R-h)}{4\pi R^2} = \frac{1}{2}\left(1 - \frac{h}{R}\right) = \frac{1}{2}(1-\cos\theta_c) .$$

The critical angle is given by $\sin\theta_c = 1/n$, which implies $\cos\theta_c = \sqrt{1-\sin^2\theta_c} = \sqrt{1-1/n^2}$. When this expression is substituted into our result above, we obtain

$$frac = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{n^2}}\right) .$$

(b) For $n = 1.33$,

$$frac = \frac{1}{2}\left(1-\sqrt{1-\frac{1}{(1.33)^2}}\right) = 0.170 .$$

81. We apply Eq. 34-40 (once) and Eq. 34-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2\theta_1 \cos^2\theta_2 .$$

With $\theta_1 = 60° - 20° = 40°$ and $\theta_2 = 40° + 30° = 70°$, this yields $I/I_0 = 0.034$.

82. (a) From $kc = \omega$ where $k = 1.00\times10^6$ m^{-1}, we obtain $\omega = 3.00\times10^{14}$ rad/s. The magnetic field amplitude is, from Eq. 34-5, $B = (5.00\text{ V/m})/c = 1.67\times10^{-8}$ T. From the fact that $-\hat{\text{k}}$ (the direction of propagation), $\vec{E} = E_y\hat{\text{j}}$, and $\vec{B}$ are mutually perpendicular, we conclude that the only non-zero component of $\vec{B}$ is B_x, so that we have (in SI units)

$$B_x = 1.67\times10^{-8}\ \sin\left(\left(1.00\times10^6\right)z + \left(3.00\times10^{14}\right)t\right) .$$

(b) The wavelength is $\lambda = 2\pi/k = 6.28\times10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2.09\times10^{-14}$ s.

(d) The intensity is

$$I = \frac{1}{c\mu_0}\left(\frac{5.00\text{ V/m}}{\sqrt{2}}\right)^2 = 0.0332\text{ W/m}^2 .$$

(e) As noted in part (a), the only nonzero component of $\vec{B}$ is B_x. The magnetic field oscillates along the x axis.

(f) The wavelength found in part (b) places this in the infrared portion of the spectrum.

83. We write $m = \rho\mathcal{V}$ where $\mathcal{V} = 4\pi R^3/3$ is the volume. Plugging this into $F = ma$ and then into Eq. 34-32 (with $A = \pi R^2$, assuming the light is in the form of plane waves), we find

$$\rho\,\frac{4\pi R^3}{3}\,a = \frac{I\pi R^2}{c} .$$

This simplifies to

$$a = \frac{3I}{4\rho c R}$$

which yields $a = 1.5\times10^{-9}$ m/s^2.

84. Since intensity is power divided by area (and the area is spherical in the isotropic case), then the intensity at a distance of $r = 20$ m from the source is

$$I = \frac{P}{4\pi r^2} = 0.040\text{ W/m}^2 .$$

as illustrated in Sample Problem 34-2. Now, in Eq. 34-32 for a totally absorbing area A, we note that the exposed area of the small sphere is that on a flat circle $A = \pi(0.020\,\text{m})^2 = 0.0013$ m^2. Therefore,

$$F = \frac{IA}{c} = \frac{(0.040)(0.0013)}{3\times10^8} = 1.7\times10^{-13}\text{ N} .$$

85. Eq. 34-5 gives $B = E/c$, which relates the field values at any instant – and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, the rms value of the magnetic field is $0.2/3 \times 10^8 = 6.7 \times 10^{-10}$ T, which (upon multiplication by $\sqrt{2}$) yields an amplitude value of magnetic field equal to 9.4×10^{-10} T.

86. (a) From Eq. 34-1,

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2}[E_m \sin(kx - \omega t)] = -\omega^2 E_m \sin(kx - \omega t)\,,$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 \frac{\partial^2}{\partial x^2}[E_{\rm m} \sin(kx - \omega t)] = -k^2 c^2 \sin(kx - \omega t) = -\omega^2 E_m \sin(kx - \omega t)\ .$$

Consequently,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}$$

is satisfied. Analogously, one can show that Eq. 34-2 satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}\ .$$

(b) From $E = E_m f(kx \pm \omega t)$,

$$\frac{\partial^2 E}{\partial t^2} = E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = \omega^2 E_m \left.\frac{d^2 f}{du^2}\right|_{u=kx\pm\omega t}$$

and

$$c^2 \frac{\partial^2 E}{\partial x^2} = c^2 E_m \frac{\partial^2 f(kx \pm \omega t)}{\partial t^2} = c^2 E_m k^2 \left.\frac{d^2 f}{du^2}\right|_{u=kx\pm\omega t}\ .$$

Since $\omega = ck$ the right-hand sides of these two equations are equal. Therefore,

$$\frac{\partial^2 E}{\partial t^2} = c^2 \frac{\partial^2 E}{\partial x^2}\ .$$

Changing E to B and repeating the derivation above shows that $B = B_m f(kx \pm \omega t)$ satisfies

$$\frac{\partial^2 B}{\partial t^2} = c^2 \frac{\partial^2 B}{\partial x^2}\ .$$

87. $\vec{E} \times \vec{B} = \mu_0 \vec{S}$, where $\vec{E} = E\,\hat{k}$ and $\vec{S} = S(-\hat{j})$. One can verify easily that since $\hat{k} \times (-\hat{i}) = -\hat{j}$, $\vec{B}$ has to be in the negative x direction. Also,

$$B = \frac{E}{c} = \frac{100\,\text{V/m}}{3.0 \times 10^8\,\text{m/s}} = 3.3 \times 10^{-7}\ \text{T}\ .$$

88. (a) At $r = 40$ m, the intensity is

$$\begin{aligned} I &= \frac{P}{\pi d^2/4} = \frac{P}{\pi(\theta r)^2/4} \\ &= \frac{4(3.0 \times 10^{-3}\,\text{W})}{\pi[(0.17 \times 10^{-3}\,\text{rad})(40\,\text{m})]^2} \\ &= 83\ \text{W/m}^2\ . \end{aligned}$$

(b) $P' = 4\pi r^2 I = 4\pi(40\,\text{m})^2(83\,\text{W/m}^2) = 1.7 \times 10^6\,\text{W}$.

89. Using Eqs. 34-40 and 34-42, we obtain

$$\frac{I_{\text{final}}}{I_0} = \frac{\left(\frac{1}{2}I_0\right)\left(\cos^2 45°\right)\left(\cos^2 45°\right)}{I_0} = \frac{1}{8} .$$

90. We use the result obtained in problem 51:

$$n = \frac{\sin\frac{1}{2}(\phi+\psi)}{\sin\frac{1}{2}\phi} = \frac{\sin\left[\frac{1}{2}(60.0° + 30.0°)\right]}{\sin\left[\frac{1}{2}(60.0°)\right]} = 1.41 .$$

91. (a) and (b) At the Brewster angle, $\theta_{\text{incident}} + \theta_{\text{refracted}} = \theta_{\text{B}} + 32.0° = 90.0°$, so $\theta_{\text{B}} = 58.0°$ and $n_{\text{glass}} = \tan\theta_{\text{B}} = \tan 58.0° = 1.60$.

92. (a) In the notation of this problem, Eq. 34-47 becomes

$$\theta_c = \sin^{-1}\frac{n_3}{n_2}$$

which yields $n_3 = 1.39$ for $\theta_c = \phi = 60°$.

(b) Applying Eq. 34-44 law to the interface between material 1 and material 2, we have

$$n_2 \sin 30° = n_1 \sin\theta$$

which yields $\theta = 28.1°$.

(c) Decreasing θ will increase ϕ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than θ_c. Therefore, no transmission of light into material 3 can occur.

93. We apply Eq. 34-40 (once) and Eq. 34-42 (twice) to obtain

$$I = \frac{1}{2}\, I_0 \cos^2\theta_1 \cos^2\theta_2 .$$

With $\theta_1 = 110°$ and $\theta_2 = 50°$, this yields $I/I_0 = 0.024$.

94. (a) The wave is traveling in the $-y$ direction (see §17-5 for the significance of the relative sign between the spatial and temporal arguments of the wave function).

(b) Figure 34-5 may help in visualizing this. The direction of propagation (along the y axis) is perpendicular to $\vec{B}$ (presumably along the x axis, since the problem gives B_x and no other component) and both are perpendicular to $\vec{E}$ (which determines the axis of polarization). Thus, the wave is z-polarized.

(c) Since the magnetic field amplitude is $B_m = 4.00\ \mu$T, then (by Eq. 34-5) $E_m = 1199$ V/m. Dividing by $\sqrt{2}$ yields $E_{\text{rms}} = 848$ V/m. Then, Eq. 34-26 gives

$$I = \frac{1}{c\mu_0}\, E_{\text{rms}}^2 = 1.91 \times 10^3\ \text{W/m}^2 .$$

(d) Since $kc = \omega$ (equivalent to $c = f\lambda$), we have

$$k = \frac{2.00 \times 10^{15}}{c} = 6.67 \times 10^6\ \text{m}^{-1} .$$

Summarizing the information gathered so far, we have (with SI units understood)

$$E_z = 1199 \sin\left(\left(6.67 \times 10^6\right) y + \left(2.00 \times 10^{15}\right) t\right) .$$

(e) and (f) Since $\lambda = 2\pi/k = 942$ nm, we see that this is infrared light.

95. From Eq. 34-26, we have $E_{\text{rms}} = \sqrt{\mu_0\, c\, I} = 1941$ V/m, which implies (using Eq. 34-5) that $B_{\text{rms}} = 1941/c = 6.47 \times 10^{-6}$ T. Multiplying by $\sqrt{2}$ yields the magnetic field amplitude $B_m = 9.16 \times 10^{-6}$ T.

96. (a) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8\,\text{m/s}}{0.067 \times 10^{-15}\,\text{m}} = 4.5 \times 10^{24}\ \text{Hz} .$$

(b) In this case, the (very long) wavelength is

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8\,\text{m/s}}{30\,\text{Hz}} = 1.0 \times 10^7\,\text{m}$$

which is about 1.6 Earth radii.

97. The fraction is

$$\frac{\pi R_e^2}{4\pi d_{es}^2} = \frac{1}{4}\left(\frac{6.37 \times 10^6\,\text{m}}{1.50 \times 10^{11}\,\text{m}}\right)^2 = 4.51 \times 10^{-10} .$$

98. (a) When examining Fig. 34-73, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1}\sin\theta \qquad \text{assuming} \quad n_{\text{air}} = 1 .$$

The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp} and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1 - \sin^2\theta'} .$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 34-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1 - \sin^2\theta'} = \sqrt{1 - \left(\frac{1}{n_1}\sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$.

(b) With $n_1 = 1.58$ and $n_2 = 1.53$, we obtain

$$\theta = \sin^{-1}\left(1.58^2 - 1.53^2\right) = 23.2^\circ .$$

99. (a) In our solution here, we assume the reader has looked at our solution for problem 98. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c} .$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is $v_1\cos\theta'$, so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1\cos\theta'} = \frac{n_1 L}{c\sqrt{1 - \left(\frac{1}{n_1}\sin\theta\right)^2}}$$

using results from the previous solution. Plugging in $\sin\theta = \sqrt{n_1^2 - n_2^2}$ and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c(n_2/n_1)} = \frac{n_1^2 L}{n_2 c} .$$

The difference $t_{\text{zig zag}} - t_{\text{direct}}$ readily yields the result shown in the problem statement.

(b) With $n_1 = 1.58$, $n_2 = 1.53$ and $L = 300$ m, we obtain $\Delta t = 52$ ns.

100. (a) The condition (in Eq. 34-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

101. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1 - \left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leads to $\theta = 51.1°$.

(b) Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.

Chapter 35

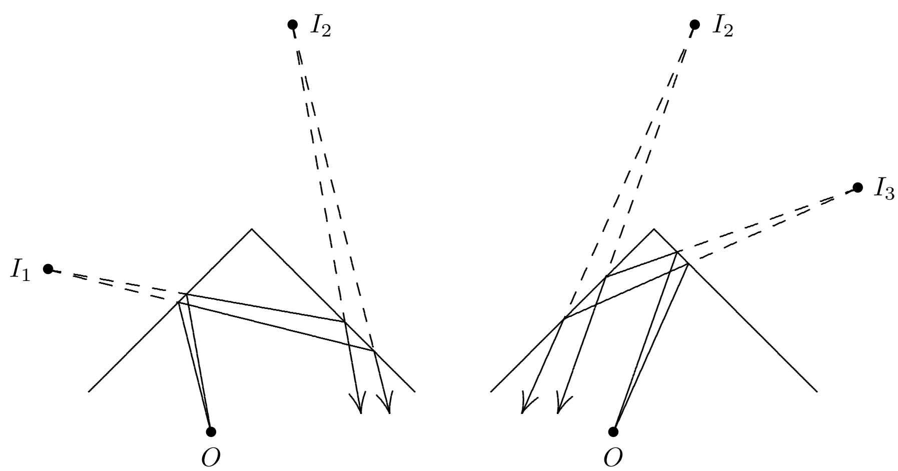
1. The image is 10 cm behind the mirror and you are 30 cm in front of the mirror. You must focus your eyes for a distance of 10 cm + 30 cm = 40 cm.

2. The bird is a distance d_2 in front of the mirror; the plane of its image is that same distance d_2 behind the mirror. The lateral distance between you and the bird is $d_3 = 5.00$ m. We denote the distance from the camera to the mirror as d_1, and we construct a right triangle out of d_3 and the distance between the camera and the image plane $(d_1 + d_2)$. Thus, the focus distance is

$$\begin{aligned} d &= \sqrt{(d_1 + d_2)^2 + d_3^2} \\ &= \sqrt{(4.30\,\text{m} + 3.30\,\text{m})^2 + (5.00\,\text{m})^2} \\ &= 9.10 \text{ m} . \end{aligned}$$

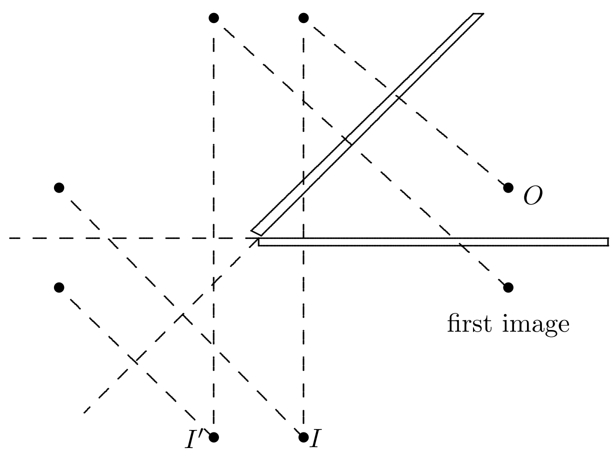
3. (a) There are three images. Two are formed by single reflections from each of the mirrors and the third is formed by successive reflections from both mirrors.

   (b) The positions of the images are shown on the two diagrams below. The diagram on the left below shows the image I_1, formed by reflections from the left-hand mirror. It is the same distance behind the mirror as the object O is in front, and lies on the line perpendicular to the mirror and through the object. Image I_2 is formed by light that is reflected from both mirrors. We may consider I_2 to be the image of I_1 formed by the right-hand mirror, extended. I_2 is the same distance behind the line of the right-hand mirror as I_1 is in front and it is on the line that is perpendicular to the line of the mirror. The diagram on the right, below, shows image I_3, formed by reflections from the right-hand mirror. It is the same distance behind the mirror as the object is in front, and lies on the line perpendicular to the mirror and through the object. As the diagram shows, light that is first reflected from the right-hand mirror and then from the left-hand mirror forms an image at I_2.



4. In each case there is an object and its "first" image in the mirror closest to it (this image is the same distance behind the mirror as the object is in front of it and might be referred to as the object's "twin"). The rest of the "figuring" consists of drawing perpendiculars from these (or imagining doing so) to the mirror-planes and constructing further images.

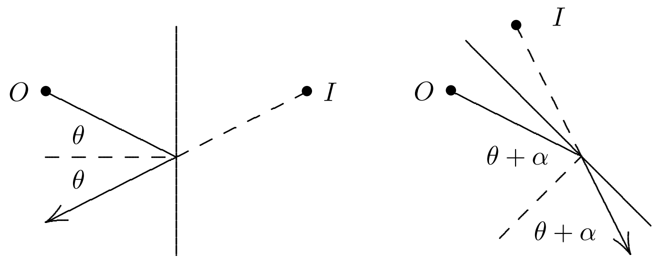
   (a) For $\theta = 45°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. Extending the second mirror plane, we can find two further images of I and I' which are on equal sides of the extension of the first mirror plane. This circumstance implies there are no further images, since these final images are each other's "twins." We show this construction in the figure below. Summarizing, we find $1 + 2 + 2 + 2 = 7$ images in this case.



   (b) For $\theta = 60°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. The images I and I' are each other's "twins" in the sense that they are each other's reflections about the extension of the second mirror plane; there are no further images. Summarizing, we find $1+2+2 = 5$ images in this case.

   (c) For $\theta = 120°$, we have two images I_1' and I_2 behind the extension of the second mirror plane, caused by the object and its "first" image (which we refer to here as I_1). No further images can be constructed from I_1' and I_2, since the method indicated above would place any further possibilities in front of the mirrors. This construction has the disadvantage of deemphasizing the actual ray-tracing, and thus any dependence on where the observer of these images is actually placing his or her eyes. It turns out in this case that the number of images that can be seen ranges from 1 to 3, depending on the locations of both the object and the observer. As an example, if the observer's eye is collinear with I_1 and I_1', then the observer can only see one image (I_1 and not the one behind it). Another observer, close to the second mirror would probably be able to see only I_1 and I_2. However, if that observer moves further back from the vertex of the two mirrors he or she should also be able to see the third image, I_1', which is essentially the "twin" image formed from I_1 relative to the extension of the second mirror plane.

5. Consider a single ray from the source to the mirror and let θ be the angle of incidence. The angle of reflection is also θ and the reflected ray makes an angle of 2θ with the incident ray. Now we rotate the mirror through the angle α so that the angle of incidence increases to $\theta + \alpha$. The reflected ray now makes an angle of $2(\theta + \alpha)$ with the incident ray. The reflected ray has been rotated through an angle of 2α. If the mirror is rotated so the angle of incidence is decreased by α, then the reflected ray makes an angle of $2(\theta - \alpha)$ with the incident ray. Again it has been rotated through 2α. The diagrams below show the situation for $\alpha = 45°$. The ray from the object to the mirror is the same in both cases and the reflected rays are $90°$ apart.



6. When S is barely able to see B the light rays from B must reflect to S off the edge of the mirror. The angle of reflection in this case is 45°, since a line drawn from S to the mirror's edge makes a 45° angle relative to the wall. By the law of reflection, we find

$$\frac{x}{d/2} = \tan 45^\circ \quad \Longrightarrow \quad x = \frac{d}{2} = \frac{3.0\,\mathrm{m}}{2} = 1.5\ \mathrm{m}\ .$$

7. The intensity of light from a point source varies as the inverse of the square of the distance from the source. Before the mirror is in place, the intensity at the center of the screen is given by $I_0 = A/d^2$, where A is a constant of proportionality. After the mirror is in place, the light that goes directly to the screen contributes intensity I_0, as before. Reflected light also reaches the screen. This light appears to come from the image of the source, a distance d behind the mirror and a distance $3d$ from the screen. Its contribution to the intensity at the center of the screen is

$$I_r = \frac{A}{(3d)^2} = \frac{A}{9d^2} = \frac{I_0}{9}\ .$$

The total intensity at the center of the screen is

$$I = I_0 + I_r = I_0 + \frac{I_0}{9} = \frac{10}{9} I_0\ .$$

The ratio of the new intensity to the original intensity is $I/I_0 = 10/9$.

8. We apply the law of refraction, assuming all angles are in radians:

$$\frac{\sin\theta}{\sin\theta'} = \frac{n_w}{n_{\mathrm{air}}}\ ,$$

which in our case reduces to $\theta' \approx \theta/n_w$ (since both θ and θ' are small, and $n_{\mathrm{air}} \approx 1$). We refer to our figure, below. The object O is a vertical distance h_1 above the water, and the water surface is a vertical distance h_2 above the mirror. We are looking for a distance d (treated as a positive number) below the mirror where the image I of the object is formed. In the triangle OAB

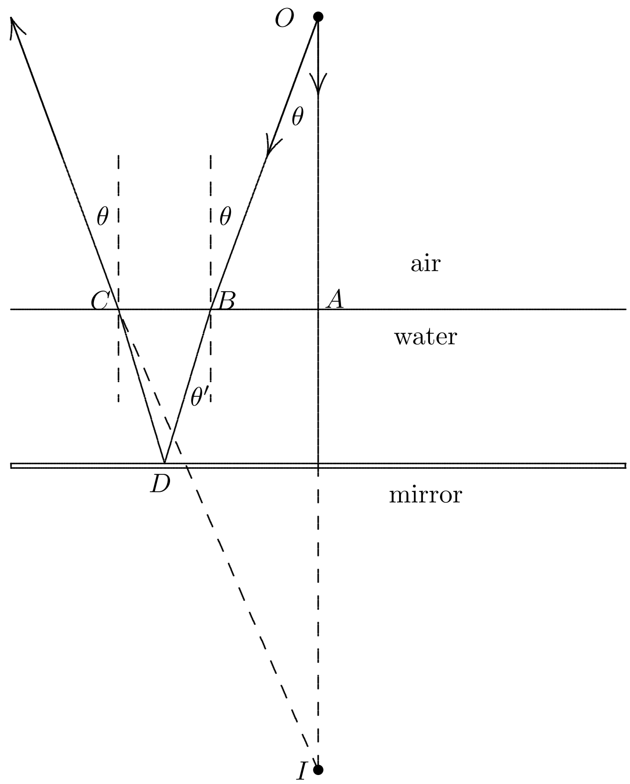
$$|AB| = h_1 \tan\theta \approx h_1\theta\ ,$$

and in the triangle CBD

$$|BC| = 2h_2 \tan\theta' \approx 2h_2\theta' \approx \frac{2h_2\theta}{n_w}\ .$$

Finally, in the triangle ACI, we have $|AI| = d + h_2$. Therefore,

$$\begin{aligned} d &= |AI| - h_2 = \frac{|AC|}{\tan\theta} - h_2 \\ &\approx \frac{|AB| + |BC|}{\theta} - h_2 \\ &= \left(\frac{h_1}{\theta} + \frac{2h_2\theta}{n_w}\right)\frac{1}{\theta} - h_2 = h_1 + \frac{2h_2}{n_w} - h_2 \\ &= 250\,\mathrm{cm} + \frac{2(200\,\mathrm{cm})}{1.33} - 200\,\mathrm{cm} = 351\ \mathrm{cm}\ . \end{aligned}$$



9. We use Eqs. 35-3 and 35-4, and note that $m = -i/p$. Thus,

$$\frac{1}{p} - \frac{1}{pm} = \frac{1}{f} = \frac{2}{r} .$$

We solve for p:

$$p = \frac{r}{2}\left(1 - \frac{1}{m}\right) = \frac{35.0\,\text{cm}}{2}\left(1 - \frac{1}{2.50}\right) = 10.5 \text{ cm} .$$

10. (a) $f = +20\,\text{cm}$ (positive, because the mirror is concave); $r = 2f = 2(+20\,\text{cm}) = +40\,\text{cm}$; $i = (1/f - 1/p)^{-1} = (1/20\,\text{cm} - 1/10\,\text{cm})^{-1} = -20\,\text{cm}$; $m = -i/p = -(-20\,\text{cm}/10\,\text{cm}) = +2.0$. The image is virtual and upright. The ray diagram would be similar to Fig. 35-8(a) in the textbook.

(b) The fact that the magnification is 1 and the image is virtual means that the mirror is flat (plane). Flat mirrors (and flat "lenses" such as a window pane) have $f = \infty$ (or $f = -\infty$ since the sign does not matter in this extreme case), and consequently $r = \infty$ (or $r = -\infty$) by Eq. 35-3. Eq. 35-4 readily yields $i = -10\,\text{cm}$. The magnification being positive implies the image is upright; the answer is "no" (it's not inverted). The ray diagram would be similar to Fig. 35-6(a) in the textbook.

(c) Since $f > 0$, the mirror is concave. Using Eq. 35-3, we obtain $r = 2f = +40\,\text{cm}$. Eq. 35-4 readily yields $i = +60\,\text{cm}$. Substituting this (and the given object distance) into Eq. 35-6 gives $m = -2.0$. Since $i > 0$, the answer is "yes" (the image is real). Since $m < 0$, our answer is "yes" (the image is inverted). The ray diagram would be similar to Fig. 35-8(c) in the textbook.

(d) Since $m < 0$, the image is inverted. With that in mind, we examine the various possibilities in Figs. 35-6, 35-8 and 35-9, and note that an inverted image (for reflections from a single mirror) can only occur if the mirror is concave (and if $p > f$). Next, we find i from Eq. 35-6 (which yields $i = 30\,\text{cm}$) and then use this value (and Eq. 35-4) to compute the focal length; we obtain $f = +20\,\text{cm}$. Then, Eq. 35-3 gives $r = +40\,\text{cm}$. As already noted, $i = +30\,\text{cm}$. Yes, the image is real (since $i > 0$). Yes, the image is inverted (as already noted). The ray diagram would be similar to Figs. 35-9(a) and (b) in the textbook.

(e) Since $r < 0$ then (by Eq. 35-3) $f < 0$, which means the mirror is convex. The focal length is $f = r/2 = -20$ cm. Eq. 35-4 leads to $p = +20$ cm, and Eq. 35-6 gives $m = +0.50$. No, the image is virtual. No, the image is upright. The ray diagram would be similar to Figs. 35-9(c) and (d) in the textbook.

(f) Since $0 < m < 1$, the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 35-6, 35-8 and 35-9, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex. Thus, we must put a minus sign in front of the "20" value given for f. Eq. 35-3 then gives $r = -40$ cm. To solve for i and p we must set up Eq. 35-4 and Eq. 35-6 as a simultaneous set and solve for the two unknowns. The results are $i = -18$ cm and $p = +180$ cm. No, the image is virtual (since $i < 0$). No, the image is upright (as already noted). The ray diagram would be similar to Figs. 35-9(c) and (d) in the textbook.

(g) Knowing the mirror is convex means we must put a minus sign in front of the "40" value given for r. Then, Eq. 35-3 yields $f = r/2 = -20$ cm. The fact that the mirror is convex also means that we need to insert a minus sign in front of the "4.0" value given for i, since the image in this case must be virtual (see Figs. 35-6, 35-8 and 35-9). Eq. 35-4 leads to $p = +5.0$ cm, and Eq. 35-6 gives $m = +0.8$. No, the image is virtual. No, the image is upright. The ray diagram would be similar to Figs. 35-9(c) and (d) in the textbook.

(h) Since the image is inverted, we can scan Figs. 35-6, 35-8 and 35-9 in the textbook and find that the mirror must be concave. This also implies that we must put a minus sign in front of the "0.50" value given for m. To solve for f, we first find $i = +12$ cm from Eq. 35-6 and plug into Eq. 35-4; the result is $f = +8$ cm. Thus, $r = 2f = +16$ cm. Yes, the image is real (since $i > 0$). The ray diagram would be similar to Figs. 35-9(a) and (b) in the textbook.

11. (a) Suppose one end of the object is a distance p from the mirror and the other end is a distance $p+L$. The position i_1 of the image of the first end is given by

$$\frac{1}{p} + \frac{1}{i_1} = \frac{1}{f}$$

where f is the focal length of the mirror. Thus,

$$i_1 = \frac{fp}{p-f} \ .$$

The image of the other end is located at

$$i_2 = \frac{f(p+L)}{p+L-f} \ ,$$

so the length of the image is

$$L' = i_1 - i_2 = \frac{fp}{p-f} - \frac{f(p+L)}{p+L-f} = \frac{f^2 L}{(p-f)(p+L-f)} \ .$$

Since the object is short compared to $p - f$, we may neglect the L in the denominator and write

$$L' = L\left(\frac{f}{p-f}\right)^2 \ .$$

(b) The lateral magnification is $m = -i/p$ and since $i = fp/(p-f)$, this can be written $m = -f/(p-f)$. The longitudinal magnification is

$$m' = \frac{L'}{L} = \left(\frac{f}{p-f}\right)^2 = m^2 \ .$$

12. (a) From Eqs. 35-3 and 35-4, we obtain $i = pf/(p-f) = pr/(2p-r)$. Differentiating both sides with respect to time and using $v_O = -dp/dt$, we find

$$v_I = \frac{di}{dt} = \frac{d}{dt}\left(\frac{pr}{2p-r}\right) = \frac{-rv_O(2p-r)+2v_Opr}{(2p-r)^2} = \left(\frac{r}{2p-r}\right)^2 v_O \ .$$

(b) If $p = 30\,\text{cm}$, we obtain

$$v_I = \left[\frac{15\,\text{cm}}{2(30\,\text{cm})-15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 0.56 \text{ cm/s} \ .$$

(c) If $p = 8.0\,\text{cm}$, we obtain

$$v_I = \left[\frac{15\,\text{cm}}{2(8.0\,\text{cm})-15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 1.1\times 10^3 \text{ cm/s} \ .$$

(d) If $p = 1.0\,\text{cm}$, we obtain

$$v_I = \left[\frac{15\,\text{cm}}{2(1.0\,\text{cm})-15\,\text{cm}}\right]^2 (5.0\,\text{cm/s}) = 6.7 \text{ cm/s} \ .$$

13. (a) We use Eq. 35-8 and note that $n_1 = n_{\text{air}} = 1.00,\ n_2 = n,\ p = \infty$, and $i = 2r$:

$$\frac{1.00}{\infty} + \frac{n}{2r} = \frac{n-1}{r} \ .$$

We solve for the unknown index: $n = 2.00$.

(b) Now $i = r$ so Eq. 35-8 becomes

$$\frac{n}{r} = \frac{n-1}{r} \ ,$$

which is not valid unless $n \to \infty$ or $r \to \infty$. It is impossible to focus at the center of the sphere.

14. We remark that the sign convention for r (for these refracting surfaces) is the opposite of what was used for mirrors. This point is discussed in §35-5.

(a) We use Eq. 35-8:

$$i = n_2\left(\frac{n_2-n_1}{r} - \frac{n_1}{p}\right)^{-1} = 1.5\left(\frac{1.5-1.0}{30\,\text{cm}} - \frac{1.0}{10\,\text{cm}}\right)^{-1} = -18 \text{ cm} \ .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(c) in the textbook.

(b) We manipulate Eq. 35-8 to find r:

$$r = (n_2-n_1)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = (1.5-1.0)\left(\frac{1.0}{10} + \frac{1.5}{-13}\right)^{-1} = -32.5 \text{ cm}$$

which should be rounded to two significant figures. The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(e) in the textbook, but with the object and the image placed closer to the surface.

(c) We manipulate Eq. 35-8 to find p:

$$p = \frac{n_1}{\frac{n_2-n_1}{r} - \frac{n_2}{i}} = \frac{1.0}{\frac{1.5-1.0}{30} - \frac{1.5}{600}} = 71 \text{ cm} \ .$$

The image is real and inverted. The ray diagram would be similar to Fig. 35-10(a) in the textbook.

(d) We manipulate Eq. 35-8 to separate the indices:

$$\begin{aligned} n_2\left(\frac{1}{r}-\frac{1}{i}\right) &= \left(\frac{n_1}{p}+\frac{n_1}{r}\right) \\ n_2\left(\frac{1}{-20}-\frac{1}{-20}\right) &= \left(\frac{1.0}{20}+\frac{1.0}{-20}\right) \\ n_2(0) &= 0 \end{aligned}$$

which is identically satisfied for any choice of n_2. The ray diagram would be similar to Fig. 35-10(d) in the textbook, but with C, O and I together at the same point. The image is virtual and upright.

(e) We manipulate Eq. 35-8 to find r:

$$r = (n_2 - n_1)\left(\frac{n_1}{p}+\frac{n_2}{i}\right)^{-1} = (1.0-1.5)\left(\frac{1.5}{10}+\frac{1.0}{-6.0}\right)^{-1} = 30 \text{ cm} .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(f) in the textbook, but with the object and the image located closer to the surface.

(f) We manipulate Eq. 35-8 to find p:

$$p = \frac{n_1}{\frac{n_2-n_1}{r}-\frac{n_2}{i}} = \frac{1.5}{\frac{1.0-1.5}{-30}-\frac{1.0}{-7.5}} = 10 \text{ cm} .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(d) in the textbook.

(g) We manipulate Eq. 35-8 to find the image distance:

$$i = n_2\left(\frac{n_2-n_1}{r}-\frac{n_1}{p}\right)^{-1} = 1.0\left(\frac{1.0-1.5}{30\,\text{cm}}-\frac{1.5}{70\,\text{cm}}\right)^{-1} = -26 \text{ cm} .$$

The image is virtual and upright. The ray diagram would be similar to Fig. 35-10(f) in the textbook.

(h) We manipulate Eq. 35-8 to separate the indices:

$$\begin{aligned} n_2\left(\frac{1}{r}-\frac{1}{i}\right) &= \left(\frac{n_1}{p}+\frac{n_1}{r}\right) \\ n_2\left(\frac{1}{-30}-\frac{1}{600}\right) &= \left(\frac{1.5}{100}+\frac{1.5}{-30}\right) \\ n_2(-0.035) &= -0.035 \end{aligned}$$

which implies $n_2 = 1.0$. The ray diagram would be similar to Fig. 35-10(b) in the textbook, but with C, O and I together at the same point. The image is real and inverted.

15. The water is medium 1, so $n_1 = n_w$ which we simply write as n. The air is medium 2, for which $n_2 \approx 1$. We refer points where the light rays strike the water surface as A (on the left side of Fig. 35-32) and B (on the right side of the picture). The point midway between A and B (the center point in the picture) is C. The penny P is directly below C, and the location of the "apparent" or Virtual penny is V. We note that the angle $\angle CVB$ (the same as $\angle CVA$) is equal to θ_2, and the angle $\angle CPB$ (the same as $\angle CPA$) is equal to θ_1. The triangles CVB and CPB share a common side, the horizontal distance from C to B (which we refer to as x). Therefore,

$$\tan\theta_2 = \frac{x}{d_\text{a}} \quad \text{and} \quad \tan\theta_1 = \frac{x}{d} .$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\frac{\tan\theta_2}{\tan\theta_1} \approx \frac{\sin\theta_2}{\sin\theta_1}$$

$$\frac{\frac{x}{d_a}}{\frac{x}{d}} \approx \frac{n_1}{n_2}$$
$$\frac{d}{d_a} \approx n$$

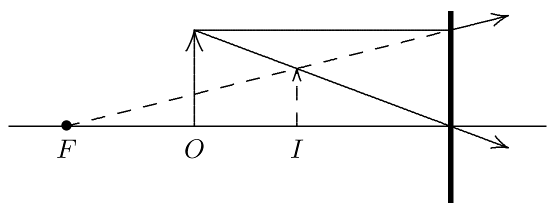
which yields the desired relation: $d_a = d/n$.

16. First, we note that – *relative to the water* – the index of refraction of the carbon tetrachloride should be thought of as $n = 1.46/1.33 = 1.1$ (this notation is chosen to be consistent with problem 15). Now, if the observer were in the water, directly above the 40 mm deep carbon tetrachloride layer, then the apparent depth of the penny as measured below the surface of the carbon tetrachloride is $d_a = 40\,\text{mm}/1.1 = 36.4\,\text{mm}$. This "apparent penny" serves as an "object" for the rays propagating upward through the 20 mm layer of water, where this "object" should be thought of as being $20\,\text{mm} + 36.4\,\text{mm} = 56.4\,\text{mm}$ from the top surface. Using the result of problem 15 again, we find the perceived location of the penny, for a person at the normal viewing position above the water, to be $56.4\,\text{mm}/1.33 = 42\,\text{mm}$ below the water surface.

17. We solve Eq. 35-9 for the image distance i: $i = pf/(p - f)$. The lens is diverging, so its focal length is $f = -30\,\text{cm}$. The object distance is $p = 20\,\text{cm}$. Thus,

$$i = \frac{(20\,\text{cm})(-30\,\text{cm})}{(20\,\text{cm}) - (-30\,\text{cm})} = -12\,\text{cm}\ .$$

The negative sign indicates that the image is virtual and is on the same side of the lens as the object. The ray diagram, drawn to scale, is shown on the right.



18. Let the diameter of the Sun be d_s and that of the image be d_i. Then, Eq. 35-5 leads to

$$\begin{aligned} d_i &= |m|d_s = \left(\frac{i}{p}\right) d_s \approx \left(\frac{f}{p}\right) d_s \\ &= \frac{(20.0 \times 10^{-2}\,\text{m})(2)(6.96 \times 10^8\,\text{m})}{1.50 \times 10^{11}\,\text{m}} \\ &= 1.86 \times 10^{-3}\,\text{m} = 1.86\ \text{mm}\ . \end{aligned}$$

19. We use the lens maker's equation, Eq. 35–10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}\ .$$

We solve for r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60\,\text{mm})}{2} = 45\,\text{mm} .$$

The radii are 45 mm and 90 mm.

20. (a) We use Eq. 35-10:

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]^{-1} = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-20\,\text{cm}}\right)\right]^{-1} = +40 \text{ cm} .$$

(b) From Eq. 35-9,

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{40\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)^{-1} = \infty .$$

21. For a thin lens, $(1/p) + (1/i) = (1/f)$, where p is the object distance, i is the image distance, and f is the focal length. We solve for i:

$$i = \frac{fp}{p-f} .$$

Let $p = f + x$, where x is positive if the object is outside the focal point and negative if it is inside. Then,

$$i = \frac{f(f+x)}{x} .$$

Now let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside. Then,

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

and $xx' = f^2$.

22. We solve Eq. 35-9 for the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \frac{fp}{p-f} .$$

The height of the image is thus

$$h_i = mh_p = \left(\frac{i}{p}\right) h_p = \frac{fh_p}{p-f} = \frac{(75\,\text{mm})(1.80\,\text{m})}{27\,\text{m} - 0.075\,\text{m}} = 5.0 \text{ mm} .$$

23. Using Eq. 35-9 and noting that $p + i = d = 44\,\text{cm}$, we obtain $p^2 - dp + df = 0$. Therefore,

$$p = \frac{1}{2}(d \pm \sqrt{d^2 - 4df}) = 22\,\text{cm} \pm \frac{1}{2}\sqrt{(44\,\text{cm})^2 - 4(44\,\text{cm})(11\,\text{cm})} = 22 \text{ cm} .$$

24. (a) Since this is a converging lens ("C") then $f > 0$, so we should put a plus sign in front of the "10" value given for the focal length. There is not enough information to determine r_1 and r_2. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{20}} = +20 \text{ cm} .$$

There is insufficient information for the determination of n. From Eq. 35-6, $m = -20/20 = -1.0$. The image is real (since $i > 0$) and inverted (since $m < 0$). The ray diagram would be similar to Fig. 35-14(a) in the textbook.

(b) Since $f > 0$, this is a converging lens ("C"). There is not enough information to determine r_1 and r_2. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = -10 \text{ cm} .$$

There is insufficient information for the determination of n. From Eq. 35-6, $m = -(-10)/5 = +2.0$. The image is virtual (since $i < 0$) and upright (since $m > 0$). The ray diagram would be similar to Fig. 35-14(b) in the textbook.

(c) We are told the magnification is positive and greater than 1. Scanning the single-lens-image figures in the textbook (Figs. 35-13, 35-14 and 35-16), we see that such a magnification (which implies an upright image larger than the object) is only possible if the lens is of the converging ("C") type (and if $p < f$). Thus, we should put a plus sign in front of the "10" value given for the focal length. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = -10 \text{ cm} ,$$

which implies the image is virtual. There is insufficient information for the determinations of n, r_1 and r_2. The ray diagram would be similar to Fig. 35-14(b) in the textbook.

(d) We are told the magnification is less than 1, and we note that $p < |f|$). Scanning Figs. 35-13, 35-14 and 35-16, we see that such a magnification (which implies an image smaller than the object) and object position (being fairly close to the lens) are simultaneously possible only if the lens is of the diverging ("D") type. Thus, we should put a minus sign in front of the "10" value given for the focal length. Eq. 35-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-10} - \frac{1}{5}} = -3.3 \text{ cm} ,$$

which implies the image is virtual (and upright). There is insufficient information for the determinations of n, r_1 and r_2. The ray diagram would be similar to Fig. 35-14(c) in the textbook.

(e) Eq. 35-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = +30$ cm. Since $f > 0$, this must be a converging ("C") lens. From Eq. 35-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{30} - \frac{1}{10}} = -15 \text{ cm} .$$

Eq. 35-6 yields $m = -(-15)/10 = +1.5$. Therefore, the image is virtual ($i < 0$) and upright ($m > 0$). The ray diagram would be similar to Fig. 35-14(b) in the textbook.

(f) Eq. 35-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -30$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 35-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-30} - \frac{1}{10}} = -7.5 \text{ cm} .$$

Eq. 35-6 yields $m = -(-7.5)/10 = +0.75$. Therefore, the image is virtual ($i < 0$) and upright ($m > 0$). The ray diagram would be similar to Fig. 35-14(c) in the textbook.

(g) Eq. 35-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -120$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 35-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-120} - \frac{1}{10}} = -9.2 \text{ cm} .$$

Eq. 35-6 yields $m = -(-9.2)/10 = +0.92$. Therefore, the image is virtual ($i < 0$) and upright ($m > 0$). The ray diagram would be similar to Fig. 35-14(c) in the textbook.

(h) We are told the absolute value of the magnification is 0.5 and that the image was upright. Thus, $m = +0.5$. Using Eq. 35-6 and the given value of p, we find $i = -5.0$ cm; it is a virtual image. Eq. 35-9 then yields the focal length: $f = -10$ cm. Therefore, the lens is of the diverging ("D") type. The ray diagram would be similar to Fig. 35-14(c) in the textbook. There is insufficient information for the determinations of n, r_1 and r_2.

(i) Using Eq. 35-6 (which implies the image is inverted) and the given value of p, we find $i = -mp = +5.0$ cm; it is a real image. Eq. 35-9 then yields the focal length: $f = +3.3$ cm. Therefore, the lens is of the converging ("C") type. The ray diagram would be similar to Fig. 35-14(a) in the textbook. There is insufficient information for the determinations of n, r_1 and r_2.

25. For an object in front of a thin lens, the object distance p and the image distance i are related by $(1/p)+(1/i) = (1/f)$, where f is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance x between the object and image is $x = p + i$. We substitute $i = x - p$ into the thin lens equation and solve for x:

$$x = \frac{p^2}{p-f} \ .$$

To find the minimum value of x, we set $dx/dp = 0$ and solve for p. Since

$$\frac{dx}{dp} = \frac{p(p-2f)}{(p-f)^2} \ ,$$

the result is $p = 2f$. The minimum distance is

$$x_{\text{min}} = \frac{p^2}{p-f} = \frac{(2f)^2}{2f-f} = 4f \ .$$

This is a minimum, rather than a maximum, since the image distance i becomes large without bound as the object approaches the focal point.

26. (a) (b) (c) and (d) Our first step is to form the image from the first lens. With $p_1 = 10$ cm and $f_1 = -15$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \quad \Longrightarrow \quad i_1 = -6 \text{ cm} \ .$$

The corresponding magnification is $m_1 = -i_1/p_1 = 0.6$. This image serves the role of "object" for the second lens, with $p_2 = 12 + 6 = 18$ cm, and $f_2 = 12$ cm. Now, Eq. 35-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \quad \Longrightarrow \quad i_2 = 36 \text{ cm}$$

with a corresponding magnification of $m_2 = -i_2/p_2 = -2$, resulting in a net magnification of $m = m_1 m_2 = -1.2$. The fact that m is positive means that the orientation of the final image is inverted with respect to the (original) object. The height of the final image is (in absolute value) $(1.2)(1.0\,\text{cm}) = 1.2$ cm. The fact that i_2 is positive means that the final image is real.

27. Without the diverging lens (lens 2), the real image formed by the converging lens (lens 1) is located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{20\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)^{-1} = 40 \text{ cm}$$

to the right of lens 1. This image now serves as an object for lens 2, with $p_2 = -(40\,\text{cm} - 10\,\text{cm}) = -30\,\text{cm}$. So

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{-15\,\text{cm}} - \frac{1}{-30\,\text{cm}}\right)^{-1} = -30 \text{ cm} \ .$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2. It is virtual (since $i_2 < 0$). The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1$, so the image has the same size and orientation as the object.

28. (a) For the image formed by the first lens

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{10\,\text{cm}} - \frac{1}{20\,\text{cm}}\right)^{-1} = 20 \text{ cm} .$$

For the subsequent image formed by the second lens $p_2 = 30\,\text{cm} - 20\,\text{cm} = 10\,\text{cm}$, so

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{12.5\,\text{cm}} - \frac{1}{10\,\text{cm}}\right)^{-1} = -50 \text{ cm} .$$

Thus, the final image is 50 cm to the left of the second lens, which means that it coincides with the object. The magnification is

$$m = \left(\frac{i_1}{p_1}\right)\left(\frac{i_2}{p_2}\right) = \left(\frac{20\,\text{cm}}{20\,\text{cm}}\right)\left(\frac{-50\,\text{cm}}{10\,\text{cm}}\right) = -5.0 ,$$

which means that the final image is five times larger than the original object.

(b) The ray diagram would be very similar to Fig. 35-17 in the textbook, except that the final image would be directly underneath the original object.

(c) and (d) It is virtual and inverted.

29. We place an object far away from the composite lens and find the image distance i. Since the image is at a focal point, $i = f$, where f equals the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$. The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thicknesses of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive. In the thin lens equation, we replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1} + \frac{1}{f} = \frac{1}{f_2}$$

or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1 + f_2}{f_1 f_2} .$$

Thus,

$$f = \frac{f_1 f_2}{f_1 + f_2} .$$

30. (a) A convex (converging) lens, since a real image is formed.

(b) Since $i = d - p$ and $i/p = 1/2$,

$$p = \frac{2d}{3} = \frac{2(40.0\,\text{cm})}{3} = 26.7 \text{ cm} .$$

(c) The focal length is

$$f = \left(\frac{1}{i} + \frac{1}{p}\right)^{-1} = \left(\frac{1}{d/3} + \frac{1}{2d/3}\right)^{-1} = \frac{2d}{9} = \frac{2(40.0\,\text{cm})}{9} = 8.89\,\text{cm} .$$

31. (a) If the object distance is x, then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D - x} = \frac{1}{f} .$$

We multiply each term in the equation by $fx(D - x)$ and obtain $x^2 - Dx + Df = 0$. Solving for x, we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D - 4f)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D - 4f)}}{2} .$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D - 4f)} .$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1} .$$

Now $x_1 = \frac{1}{2}(D - d)$, where $d = \sqrt{D(D - f)}$, so

$$|m_1| = \frac{D - (D - d)/2}{(D - d)/2} = \frac{D + d}{D - d} .$$

Similarly, when the object is at x_2, the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D - x_2}{x_2} = \frac{D - (D + d)/2}{(D + d)/2} = \frac{D - d}{D + d} .$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D - d)/(D + d)}{(D + d)/(D - d)} = \left(\frac{D - d}{D + d}\right)^2 .$$

32. The minimum diameter of the eyepiece is given by

$$d_{\text{ey}} = \frac{d_{\text{ob}}}{m_\theta} = \frac{75\,\text{mm}}{36} = 2.1 \text{ mm} .$$

33. (a) If L is the distance between the lenses, then according to Fig. 35-17, the tube length is $s = L - f_{\text{ob}} - f_{\text{ey}} = 25.0\,\text{cm} - 4.00\,\text{cm} - 8.00\,\text{cm} = 13.0\,\text{cm}$.

(b) We solve $(1/p) + (1/i) = (1/f_{\text{ob}})$ for p. The image distance is $i = f_{\text{ob}} + s = 4.00\,\text{cm} + 13.0\,\text{cm} = 17.0\,\text{cm}$, so

$$p = \frac{i f_{\text{ob}}}{i - f_{\text{ob}}} = \frac{(17.0\,\text{cm})(4.00\,\text{cm})}{17.0\,\text{cm} - 4.00\,\text{cm}} = 5.23 \text{ cm} .$$

(c) The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{17.0\,\text{cm}}{5.23\,\text{cm}} = -3.25 .$$

(d) The angular magnification of the eyepiece is

$$m_\theta = \frac{25\,\text{cm}}{f_{\text{ey}}} = \frac{25\,\text{cm}}{8.00\,\text{cm}} = 3.13 .$$

(e) The overall magnification of the microscope is

$$M = mm_\theta = (-3.25)(3.13) = -10.2 \; .$$

34. (a) Without the magnifier, $\theta = h/P_n$ (see Fig. 35-16). With the magnifier, letting $p = P_n$ and $i = -|i| = -P_n$, we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n} \; .$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25\,\text{cm}}{f} \; .$$

(b) Now $i = -|i| \to -\infty$, so $1/p + 1/i = 1/p = 1/f$ and

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25\,\text{cm}}{f} \; .$$

(c) For $f = 10\,\text{cm}$,

$$m_\theta = 1 + \frac{25\,\text{cm}}{10\,\text{cm}} = 3.5 \text{ (case (a))} \quad \text{and} \quad \frac{25\,\text{cm}}{10\,\text{cm}} = 2.5 \text{ (case (b))} \; .$$

35. (a) When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. We set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus, $i = f = 2.50\,\text{cm}$. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change. If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f'} \; .$$

We substitute $i = f$ and solve for f':

$$f' = \frac{pf}{f+p} = \frac{(40.0\,\text{cm})(2.50\,\text{cm})}{40.0\,\text{cm} + 2.50\,\text{cm}} = 2.35 \text{ cm} \; .$$

(b) Consider the lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 35-34, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of both radii.

36. Refer to Fig. 35-17. For the intermediate image $p = 10\,\text{mm}$ and $i = (f_{\text{ob}} + s + f_{\text{ey}}) - f_{\text{ey}} = 300\,\text{m} - 50\,\text{mm} = 250\,\text{mm}$, so

$$\frac{1}{f_{\text{ob}}} = \frac{1}{i} + \frac{1}{p} = \frac{1}{250\,\text{mm}} + \frac{1}{10\,\text{mm}} \implies f_{\text{ob}} = 9.62 \text{ mm} \; ,$$

and $s = (f_{\text{ob}} + s + f_{\text{ey}}) - f_{\text{ob}} - f_{\text{ey}} = 300\,\text{mm} - 9.62\,\text{mm} - 50\,\text{mm} = 240\,\text{mm}$. Then from Eq. 35-14,

$$M = -\frac{s}{f_{\text{ob}}}\frac{25\,\text{cm}}{f_{\text{ey}}} = -\left(\frac{240\,\text{mm}}{9.62\,\text{mm}}\right)\left(\frac{150\,\text{mm}}{50\,\text{mm}}\right) = -125 \; .$$

37. (a) Now, the lens-film distance is

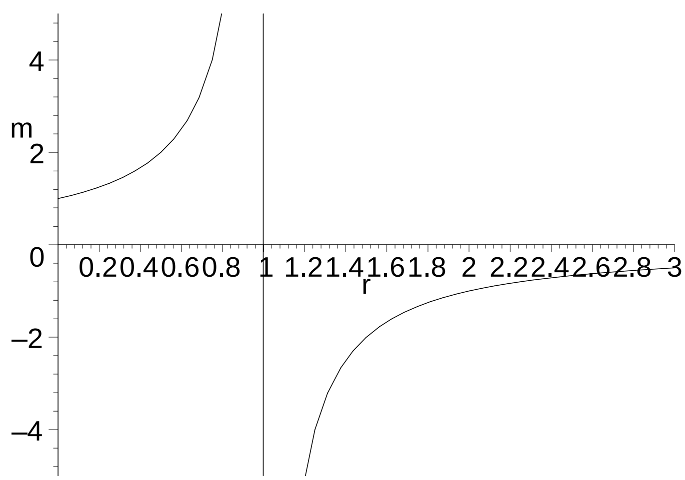
$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{5.0\,\text{cm}} - \frac{1}{100\,\text{cm}}\right)^{-1} = 5.3 \text{ cm} .$$

(b) The change in the lens-film distance is $5.3\,\text{cm} - 5.0\,\text{cm} = 0.30\,\text{cm}$.

38. We combine Eq. 35-4 and Eq. 35-6 and arrive at

$$m = -\frac{pf/(p-f)}{p} = \frac{1}{1-r} \qquad \text{where} \quad r = \frac{p}{f}$$

We emphasize that this r (for ratio) is not the radius of curvature. The magnification as a function of r is graphed below:



39. (a) The discussion in the textbook of the refracting telescope (a subsection of §35-7) applies to the Newtonian arrangement if we replace the objective lens of Fig. 35-18 with an objective mirror (with the light incident on it from the right). This might suggest that the incident light would be blocked by the person's head in Fig. 35-18, which is why Newton added the mirror M' in his design (to move the head and eyepiece out of the way of the incoming light). The beauty of the idea of characterizing both lenses and mirrors by focal lengths is that it is easy, in a case like this, to simply carry over the results of the objective-lens telescope to the objective-mirror telescope, so long as we replace a positive f device with another positive f device. Thus, the converging lens serving as the objective of Fig. 35-18 must be replaced (as Newton has done in Fig. 35-44) with a concave mirror. With this change of language, the discussion in the textbook leading up to Eq. 35-15 applies equally as well to the Newtonian telescope: $m_\theta = -f_{\text{ob}}/f_{\text{ey}}$.

(b) A meter stick (held perpendicular to the line of sight) at a distance of 2000 m subtends an angle of

$$\theta_{\text{stick}} \approx \frac{1\,\text{m}}{2000\,\text{m}} = 0.0005 \text{ rad} .$$

Multiplying this by the mirror focal length gives $(16.8\,\text{m})(0.0005) = 8.4$ mm for the size of the image.

(c) With $r = 10$ m, Eq. 35-3 gives $f_{\text{ob}} = 5$ m. Plugging this into (the absolute value of) Eq. 35-15 leads to $f_{\text{ey}} = 5/200 = 2.5$ cm.

40. (a) The "object" for the mirror which results in that box-image is equally in front of the mirror (4 cm). This object is actually the first image formed by the system (produced by the first transmission through the lens); in those terms, it corresponds to $i_1 = 10 - 4 = 6$ cm. Thus, with $f_1 = 2$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \quad \Longrightarrow \quad p_1 = 3.00 \text{ cm} .$$

(b) The previously mentioned box-image (4 cm behind the mirror) serves as an "object" (at $p_3 = 14$ cm) for the return trip of light through the lens ($f_3 = f_1 = 2$ cm). This time, Eq. 35-9 leads to

$$\frac{1}{p_3} + \frac{1}{i_3} = \frac{1}{f_3} \quad \Longrightarrow \quad i_3 = 2.33 \text{ cm} .$$

41. (a) In this case $m > +1$ and we know we are dealing with a converging lens (producing a virtual image), so that our result for focal length should be positive. Since $|p + i| = 20$ cm and $i = -2p$, we find $p = 20$ cm and $i = -40$ cm. Substituting these into Eq. 35-9,

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

leads to $f = +40$ cm, which is positive as we expected.

(b) In this case $0 < m < 1$ and we know we are dealing with a diverging lens (producing a virtual image), so that our result for focal length should be negative. Since $|p + i| = 20$ cm and $i = -p/2$, we find $p = 40$ cm and $i = -20$ cm. Substituting these into Eq. 35-9 leads to $f = -40$ cm, which is negative as we expected.

42. (a) The first image is figured using Eq. 35-8, with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 8/5$.

$$\frac{1}{p} + \frac{8}{5i} = \frac{1.6 - 1}{r}$$

For a "flat lens" $r = \infty$, so we obtain $i = -8p/5 = -64/5$ (with the unit cm understood) for that object at $p = 10$ cm. Relative to the second surface, this image is at a distance of $3 + 64/5 = 79/5$. This serves as an object in order to find the final image, using Eq. 35-8 again (and $r = \infty$) but with $n_1 = 8/5$ and $n_2 = 4/3$.

$$\frac{8}{5p'} + \frac{4}{3i'} = 0$$

which produces (for $p' = 79/5$) $i' = -5p/6 = -79/6 \approx -13.2$. This means the observer appears $13.2 + 6.8 = 20$ cm from the fish.

(b) It is straightforward to "reverse" the above reasoning, the result being that the final fish-image is 7.0 cm to the right of the air-wall interface, and thus 15 cm from the observer.

43. (a) (b) and (c) Since $m = +0.250$, we have $i = -0.25p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex and that $f < 0$; in fact, $f = -2.00$ cm. Substituting $i = -p/4$ into Eq. 35-4 produces

$$\frac{1}{p} - \frac{4}{p} = -\frac{3}{p} = \frac{1}{f}$$

Therefore, we find $p = 6.00$ cm and $i = -1.50$ cm.

44. (a) A parallel ray of light focuses at the focal point behind the lens. In the case of farsightedness we need to bring the focal point closer. That is, we need to reduce the focal length. From problem 29, we know that we need to use a converging lens of certain focal length $f_1 > 0$ which, when combined with the eye of focal length f_2, gives $f = f_1 f_2/(f_1 + f_2) < f_2$. Similarly, we see that in the case of nearsightedness we need to do a similar computation but with a diverging ($f_1 < 0$) lens.

(b) In this case, the unaided eyes are able to accommodate rays of light coming from distant (and medium-range) sources, but not from close ones. The person (not wearing glasses) is able to see far (not near), so the person is farsighted.

(c) The bifocal glasses can provide suitable corrections for different types of visual defects that prove a hindrance in different situations, such as reading (difficult for the farsighted individual) and viewing a distant object (difficult for a nearsighted individual).

45. (a) We use Eq. 35-10, with the conventions for signs discussed in §35-5 and §35-6.

(b) For the bi-convex (or double convex) case, we have

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right]^{-1} = \left[(1.5-1)\left(\frac{1}{40\,\text{cm}} - \frac{1}{-40\,\text{cm}}\right)\right]^{-1} = 40 \text{ cm} .$$

Since $f > 0$ the lens forms a real image of the Sun.

(c) For the planar convex lens, we find

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-40\,\text{cm}}\right)\right]^{-1} = 80 \text{ cm} ,$$

and the image formed is real (since $f > 0$).

(d) Now

$$f = \left[(1.5-1)\left(\frac{1}{40\,\text{cm}} - \frac{1}{60\,\text{cm}}\right)\right]^{-1} = 240 \text{ cm} ,$$

and the image formed is real (since $f > 0$).

(e) For the bi-concave lens, the focal length is

$$f = \left[(1.5-1)\left(\frac{1}{-40\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -40 \text{ cm} ,$$

and the image formed is virtual (since $f < 0$).

(f) In this case,

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -80 \text{ cm} ,$$

and the image formed is virtual (since $f < 0$).

(g) Now

$$f = \left[(1.5-1)\left(\frac{1}{60\,\text{cm}} - \frac{1}{40\,\text{cm}}\right)\right]^{-1} = -240 \text{ cm} ,$$

and the image formed is virtual (since $f < 0$).

46. Of course, the shortest possible path between A and B is the straight line path which does not go to the mirror at all. In this problem, we are concerned with only those paths which do strike the mirror. The problem statement suggests that we turn our attention to the mirror-image point of A (call it A') and requests that we construct a proof without calculus. We can see that the length of any line segment AP drawn from A to the mirror (at point P on the mirror surface) is the same as the length of its "mirror segment" $A'P$ drawn from A' to that point P. Thus, the total length of the light path from A to P to B is the same as the total length of segments drawn from A' to P to B. Now, we dismissed (in the first sentence of this solution) the possibility of a straight line path directly from A to B because it does not strike the mirror. However, we *can* construct a straight line path from A' to B which does intersect the mirror surface! Any other pair of segments ($A'P$ and PB) would give greater total length than the straight path (with $A'P$ and PB collinear), so if the straight path $A'B$ obeys the law of reflection,

then we have our proof. Now, since $A'P$ is the mirror-twin of AP, then they both approach the mirror surface with the same angle α (one from the front side and the other from the back side). And since $A'P$ is collinear with PB, then PB also makes the same angle α with respect to the mirror surface (by vertex angles). If AP and PB are each α degrees away from the front of the mirror, then they are each θ degrees (where θ is the complement of α) measured from the normal axis. Thus, the law of reflection is consistent with the concept of the shortest light path.

47. (a) (b) and (c) Our first step is to form the image from the first lens. With $p_1 = 4$ cm and $f_1 = -4$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \implies i_1 = -2 \text{ cm} .$$

The corresponding magnification is $m_1 = -i_1/p_1 = 1/2$. This image serves the role of "object" for the second lens, with $p_2 = 10 + 2 = 12$ cm, and $f_2 = -4$ cm. Now, Eq. 35-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \implies i_2 = -3.00 \text{ cm}$$

with a corresponding magnification of $m_2 = -i_2/p_2 = 1/4$, resulting in a net magnification of $m = m_1 m_2 = 1/8$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. The fact that i_2 is negative means that the final image is virtual.

48. (a) (b) (c) and (d) Our first step is to form the image from the first lens. With $p_1 = 3$ cm and $f_1 = +4$ cm, Eq. 35-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \implies i_1 = -12 \text{ cm} .$$

The corresponding magnification is $m_1 = -i_1/p_1 = 4$. This image serves the role of "object" for the second lens, with $p_2 = 8 + 12 = 20$ cm, and $f_2 = -4$ cm. Now, Eq. 35-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \implies i_2 = -3.33 \text{ cm}$$

with a corresponding magnification of $m_2 = -i_2/p_2 = 1/6$, resulting in a net magnification of $m = m_1 m_2 = 2/3$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. The fact that i_2 is negative means that the final image is virtual (and therefore to the left of the second lens).

49. Since $0 < m < 1$, we conclude the lens is of the diverging type (so $f = -40$ cm). Thus, substituting $i = -3p/10$ into Eq. 35-9 produces

$$\frac{1}{p} - \frac{10}{3p} = -\frac{7}{3p} = \frac{1}{f} .$$

Therefore, we find $p = 93.3$ cm and $i = -28.0$ cm.

50. (a) We use Eq. 35-8 (and Fig. 35-10(b) is useful), with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 1.5$.

$$\frac{1}{p} + \frac{1.5}{i} = \frac{1.5 - 1}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 35-8 (so that $r = +6.0$ cm), we obtain $i = -90$ cm for objects at $p = 10$ cm. Thus, the object and image are 80 cm apart.

(b) The image distance i is negative with increasing magnitude as p increases from very small values to some value p_0 at which point $i \to -\infty$. Since $1/(-\infty) = 0$, the above equation yields

$$\frac{1}{p_0} = \frac{1.5 - 1}{r} \implies p_0 = 2r.$$

Thus, the range for producing virtual images is $0 < p \leq 12$ cm.

51. (a) Since $m = +0.200$, we have $i = -0.2p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex (and that $f = -40.0$ cm).

(b) Substituting $i = -p/5$ into Eq. 35-4 produces

$$\frac{1}{p} - \frac{5}{p} = -\frac{4}{p} = \frac{1}{f} \ .$$

Therefore, we find $p = 160$ cm.

52. (a) First, the lens forms a real image of the object located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1$$

to the right of the lens, or at $p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2$ in front of the mirror. The subsequent image formed by the mirror is located at a distance

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{f_2} - \frac{1}{2f_2}\right)^{-1} = 2f_2$$

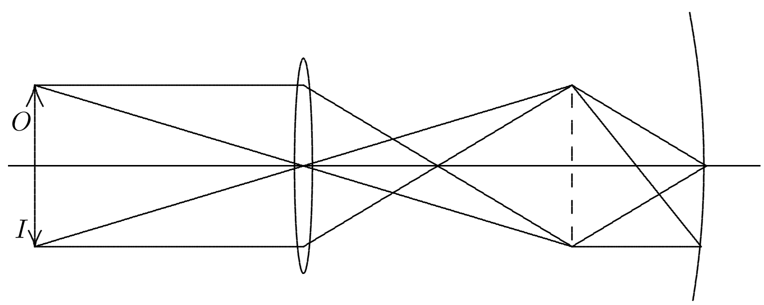
to the left of the mirror, or at $p'_1 = 2(f_1 + f_2) - 2f_2 = 2f_1$ to the right of the lens. The final image formed by the lens is that at a distance i'_1 to the left of the lens, where

$$i'_1 = \left(\frac{1}{f_1} - \frac{1}{p'_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1 \ .$$

This turns out to be the same as the location of the original object. The final image is real and inverted. The lateral magnification is

$$m = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right)\left(-\frac{i'_1}{p'_1}\right) = \left(-\frac{2f_1}{2f_1}\right)\left(-\frac{2f_2}{2f_2}\right)\left(-\frac{2f_1}{2f_1}\right) = -1.0 \ .$$

(b) The ray diagram is shown below. We set the ratio $f_2/f_1 = 1/2$ for the purposes of this sketch. The intermediate images are not shown explicitly, but they are both located on the plane indicated by the dashed line.



53. From Eq. 35-10, if

$$f \propto \left(\frac{1}{r_1} - \frac{1}{r_2}\right)^{-1} = \frac{r_1 r_2}{r_2 - r_1}$$

is positive (that is, if $r_2 > r_1$), then the lens is converging. Otherwise it is diverging.

(a) Converging, since $r_2 \to \infty$ and r_1 is finite (so $r_2 > r_1$).

(b) Diverging, since $r_1 \to \infty$ and r_2 is finite (so $r_2 < r_1$).

(c) Converging, since $r_2 > r_1$.

(d) Diverging, since $r_2 < r_1$.

54. We refer to Fig. 35-2 in the textbook. Consider the two light rays, r and r', which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to θ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray r strikes the mirror at point A and ray r' strikes the mirror at B, the distance between A and B (call it x) is

$$x = 2d_o \tan\theta$$

where d_o is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance d_o behind the mirror; see I in Fig. 35-2). One side of the triangle follows the extended normal axis (which would reach from I to the middle of the pupil), and the hypotenuse is along the extension of ray r (after reflection). The distance from the pupil to I is $d_{\text{ey}} + d_o$, and the small angle in this triangle is again θ. Thus,

$$\tan\theta = \frac{R}{d_{\text{ey}} + d_o}$$

where R is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{\text{ey}} + d_o} = 2(100\,\text{mm})\frac{2.5\,\text{mm}}{300\,\text{mm} + 100\,\text{mm}}$$

which yields $x = 1.67$ mm. Now, x serves as the diameter of a circular area A on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4}\pi x^2 = \frac{\pi}{4}(1.67\,\text{mm})^2 = 2.2\ \text{mm}^2 .$$

55. The sphere (of radius 0.35 m) is a convex mirror with focal length $f = -0.175$ m. We adopt the approximation that the rays are close enough to the central axis for Eq. 35-4 to be applicable. We also take the "1.0 m in front of ... [the] sphere" to mean $p = 1.0$ m (measured from the front surface as opposed to being measured from the center-point of the sphere).

(a) The equation $1/p + 1/i = 1/f$ yields $i = -0.15$ m, which means the image is 15 cm from the front surface, appearing to be *inside* the sphere.

(b) and (c) The lateral magnification is $m = -i/p$ which yields $m = 0.15$. Therefore, the image distance is $(0.15)(2.0\,\text{m}) = 0.30$ m; that this is a positive value implies the image is erect (upright).

56. (a) The mirror has focal length $f = 12$ cm. With $m = +3$, we have $i = -3p$. We substitute this into Eq. 35-4:

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{1}{-3p} &= \frac{1}{12} \\ \frac{2}{3p} &= \frac{1}{12} \end{aligned}$$

with the unit cm understood. Consequently, we find $p = 2(12)/3 = 8.0$ cm.

(b) With $m = -3$, we have $i = +3p$, which we substitute into Eq. 35-4:

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{1}{3p} &= \frac{1}{12} \\ \frac{4}{3p} &= \frac{1}{12} \end{aligned}$$

with the unit cm understood. Consequently, we find $p = 4(12)/3 = 16$ cm.

(c) With $m = -1/3$, we have $i = p/3$. Thus, Eq. 35-4 leads to

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{3}{p} &= \frac{1}{12} \\ \frac{4}{p} &= \frac{1}{12} \end{aligned}$$

with the unit cm understood. Consequently, we find $p = 4(12) = 48$ cm.

57. Since $m = -2$ and $p = 4$ cm, then $i = 8$ cm (and is real). Eq. 35-9 is

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

and leads to $f = 2.67$ cm (which is positive, as it must be for a converging lens).

58. We use Eq. 35-8 (and Fig. 35-10(d) is useful), with $n_1 = 1.6$ and $n_2 = 1$ (using the rounded-off value for air).

$$\frac{1.6}{p} + \frac{1}{i} = \frac{1 - 1.6}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 35-8 (so that $r = -5.0$ cm), we obtain $i = -2.4$ cm for objects at $p = 3.0$ cm. Returning to Fig. 35-52 (and noting the location of the observer), we conclude that the tabletop seems 7.4 cm away.

59. The fact that it is inverted implies $m < 0$. Therefore, with $m = -1/2$, we have $i = p/2$, which we substitute into Eq. 35-4:

$$\begin{aligned} \frac{1}{p} + \frac{1}{i} &= \frac{1}{f} \\ \frac{1}{p} + \frac{2}{p} &= \frac{1}{f} \\ \frac{3}{30.0} &= \frac{1}{f} \end{aligned}$$

with the unit cm understood. Consequently, we find $f = 30/3 = 10.0$ cm. The fact that $f > 0$ implies the mirror is concave.

60. (a) Suppose that the lens is placed to the left of the mirror. The image formed by the converging lens is located at a distance

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{0.50\,\text{m}} - \frac{1}{1.0\,\text{m}}\right)^{-1} = 1.0 \text{ m}$$

to the right of the lens, or $2.0\,\text{m} - 1.0\,\text{m} = 1.0\,\text{m}$ in front of the mirror. The image formed by the mirror for this real image is then at $1.0\,\text{m}$ to the right of the the mirror, or $2.0\,\text{m} + 1.0\,\text{m} = 3.0\,\text{m}$ to the right of the lens. This image then results in another image formed by the lens, located at a distance

$$i' = \left(\frac{1}{f} - \frac{1}{p'}\right)^{-1} = \left(\frac{1}{0.50\,\text{m}} - \frac{1}{3.0\,\text{m}}\right)^{-1} = 6.0 \text{ m}$$

to the left of the lens (that is, 2.6 cm from the mirror).

(b) The final image is real since $i' > 0$.

(c) It also has the same orientation as the object, as one can verify by drawing a ray diagram or finding the product of the magnifications (see the next part, which shows $m > 0$).

(d) The lateral magnification is

$$m = \left(-\frac{i}{p}\right)\left(-\frac{i'}{p'}\right) = \left(-\frac{1.0\,\text{m}}{1.0\,\text{m}}\right)\left(-\frac{0.60\,\text{m}}{3.0\,\text{m}}\right) = +0.20 \ .$$

61. (a) Parallel rays are bent by positive-f lenses to their focal points F_1, and rays that come from the focal point positions F_2 in front of positive-f lenses are made to emerge parallel. The key, then, to this type of beam expander is to have the rear focal point F_1 of the first lens coincide with the front focal point F_2 of the second lens. Since the triangles that meet at the coincident focal point are similar (they share the same angle; they are vertex angles), then $W_2/f_2 = W_1/f_1$ follows immediately.

(b) The previous argument can be adapted to the first lens in the expanding pair being of the diverging type, by ensuring that the front focal point of the first lens coincides with the front focal point of the second lens. The distance between the lenses in this case is $f_2 - |f_1|$ (where we assume $f_2 > |f_1|$), which we can write as $f_2 + f_1$ just as in part (a).

62. The area is proportional to W^2, so the result of problem 61 plus the definition of intensity (power P divided by area) leads to

$$\frac{I_2}{I_1} = \frac{P/W_2^2}{P/W_1^2} = \frac{W_1^2}{W_2^2} = \frac{f_1^2}{f_2^2} \ .$$

63. (a) Virtual, since the image is formed by plane mirrors.

(b) Same. One can easily verify this by locating, for example, the images of two points, one at the head of the penguin and the other at its feet.

(c) Same, since the image formed by any plane mirror retains the original shape and size of an object.

(d) The image of the penguin formed by the top mirror is located a distance D above the top mirror, or $L + D$ above the bottom one. Therefore, the final image of the penguin, formed by the bottom mirror, is a distance $L + D$ from the bottom mirror.

64. In the closest mirror, the "first" image I_1 is 10 cm behind the mirror and therefore 20 cm from the object O. There are images from both O and I_1 in the more distant mirror: an image I_2 which is 30 cm behind that mirror (since O is 30 cm in front of it), and an image I_3 which is 50 cm behind the mirror (since I_1 is 50 cm in front of it). We note that I_2 is 60 cm from O, and I_3 is 80 cm from O. Returning to the closer mirror, we find images of I_2 and I_3, as follows: an image I_4 which is 70 cm behind the mirror (since I_2 is 70 cm in front of it) and an image I_5 which is 90 cm behind the mirror (since I_3 is 90 cm in front of it). The distances (measured from O) for I_4 and I_5 are 80 cm and 100 cm, respectively.

Chapter 36

1. (a) The frequency of yellow sodium light is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{589 \times 10^{-9}\,\mathrm{m}} = 5.09 \times 10^{14}\ \mathrm{Hz}\ .$$

(b) When traveling through the glass, its wavelength is

$$\lambda_n = \frac{\lambda}{n} = \frac{589\,\mathrm{nm}}{1.52} = 388\ \mathrm{nm}\ .$$

(c) The light speed when traveling through the glass is

$$v = f\lambda_n = (5.09 \times 10^{14}\,\mathrm{Hz})(388 \times 10^{-9}\,\mathrm{m}) = 1.97 \times 10^8\ \mathrm{m/s}\ .$$

2. Comparing the light speeds in sapphire and diamond, we obtain

$$\begin{aligned} \Delta v &= v_s - v_d = c\left(\frac{1}{n_s} - \frac{1}{n_d}\right) \\ &= (2.998 \times 10^8\,\mathrm{m/s})\left(\frac{1}{1.77} - \frac{1}{2.42}\right) = 4.55 \times 10^7\ \mathrm{m/s}\ . \end{aligned}$$

3. The index of refraction is found from Eq. 36-3:

$$n = \frac{c}{v} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{1.92 \times 10^8\,\mathrm{m/s}} = 1.56\ .$$

4. The index of refraction of fused quartz at $\lambda = 550\,\mathrm{nm}$ is about 1.459, obtained from Fig. 34-19. Thus, from Eq. 36-3, we find

$$v = \frac{c}{n} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{1.459} = 2.06 \times 10^8\ \mathrm{m/s}\ .$$

5. Applying the law of refraction, we obtain $\sin\theta/\sin 30° = v_s/v_d$. Consequently,

$$\theta = \sin^{-1}\left(\frac{v_s \sin 30°}{v_d}\right) = \sin^{-1}\left[\frac{(3.0\,\mathrm{m/s})\sin 30°}{4.0\,\mathrm{m/s}}\right] = 22°\ .$$

The angle of incidence is gradually reduced due to refraction, such as shown in the calculation above (from 30° to 22°). Eventually after many refractions, θ will be virtually zero. This is why most waves come in normal to a shore.

6. (a) The time t_2 it takes for pulse 2 to travel through the plastic is

$$t_2 = \frac{L}{c/1.55} + \frac{L}{c/1.70} + \frac{L}{c/1.60} + \frac{L}{c/1.45} = \frac{6.30L}{c}\ .$$

Similarly for pulse 1:

$$t_1 = \frac{2L}{c/1.59} + \frac{L}{c/1.65} + \frac{L}{c/1.50} = \frac{6.33L}{c} \ .$$

Thus, pulse 2 travels through the plastic in less time.

(b) The time difference (as a multiple of L/c) is

$$\Delta t = t_2 - t_1 = \frac{6.33L}{c} - \frac{6.30L}{c} = \frac{0.03L}{c} \ .$$

7. (a) We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1 L - \omega t$, where k_1 $(= 2\pi/\lambda_1)$ is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2 L - \omega t$, where k_2 $(= 2\pi/\lambda_2)$ is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L \ .$$

Now, $\lambda_1 = \lambda_{\text{air}}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{\text{air}}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is $\phi_1 - \phi_2 = (2\pi/\lambda_{\text{air}})(n_1 - n_2)L$. The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{\text{air}}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9}\,\text{m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6}\ \text{m} \ .$$

(b) 5.65 rad is less than 2π rad = 6.28 rad, the phase difference for completely constructive interference, and greater than π rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

8. (a) Eq. 36-11 (in absolute value) yields

$$\frac{L}{\lambda}\,|n_2 - n_1| = \frac{(8.50 \times 10^{-6}\,\text{m})}{500 \times 10^{-9}\,\text{m}}(1.60 - 1.50) = 1.70 \ .$$

(b) Similarly,

$$\frac{L}{\lambda}\,|n_2 - n_1| = \frac{(8.50 \times 10^{-6}\,\text{m})}{500 \times 10^{-9}\,\text{m}}(1.72 - 1.62) = 1.70 \ .$$

(c) In this case, we obtain

$$\frac{L}{\lambda}\,|n_2 - n_1| = \frac{(3.25 \times 10^{-6}\,\text{m})}{500 \times 10^{-9}\,\text{m}}(1.79 - 1.59) = 1.30 \ .$$

(d) Since their phase differences were identical, the brightness should be the same for (a) and (b). Now, the phase difference in (c) differs from an integer by 0.30, which is also true for (a) and (b). Thus, their effective phase differences are equal, and the brightness in case (c) should be the same as that in (a) and (b).

9. (a) We choose a horizontal x axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 36-11 (with L in that equation replaced with L_2). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 36-11 but with L replaced with $L_1 - L_2$ and n_2 replaced with 1 (since the top ray in Fig. 36-28 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences and letting all lengths be in μm (so $\lambda = 0.600$), we have

$$\frac{L_2}{\lambda}\,(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}\,(1 - n_1) = \frac{3.50}{0.600}\,(1.60 - 1.40) + \frac{4.00 - 3.50}{0.600}\,(1 - 1.40) = 0.833 \ .$$

(b) Since the answer in part (a) is closer to an integer than to a half-integer, then the interference is more nearly constructive than destructive.

10. (a) We wish to set Eq. 36-11 equal to $\frac{1}{2}$, since a half-wavelength phase difference is equivalent to a π radians difference. Thus,

$$L_{\min} = \frac{\lambda}{2\,(n_2 - n_1)} = \frac{620\,\text{nm}}{2(1.65 - 1.45)} = 1550\,\text{nm} = 1.55\ \mu\text{m}\ .$$

(b) Since a phase difference of $\frac{3}{2}$ (wavelengths) is effectively the same as what we required in part (a), then

$$L = \frac{3\lambda}{2\,(n_2 - n_1)} = 3L_{\min} = 3(1.55\mu\text{m}) = 4.65\ \mu\text{m}\ .$$

11. (a) We use Eq. 36-14 with $m = 3$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{2(550\times10^{-9}\,\text{m})}{7.70\times10^{-6}\,\text{m}}\right] = 0.216\ \text{rad}\ .$$

(b) $\theta = (0.216)(180^\circ/\pi) = 12.4^\circ$.

12. Here we refer to phase difference in radians (as opposed to wavelengths or degrees). For the first dark fringe $\phi_1 = \pm\pi$, and for the second one $\phi_2 = \pm 3\pi$, etc. For the mth one $\phi_m = \pm(2m+1)\pi$.

13. The condition for a maximum in the two-slit interference pattern is $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m+1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by $\Delta y = D\,\Delta\theta = \lambda D/d$. Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\ \text{mm}\ .$$

14. (a) For the maximum adjacent to the central one, we set $m = 1$ in Eq. 36-14 and obtain

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)\bigg|_{m=1} = \sin^{-1}\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\ \text{rad}\ .$$

(b) Since $y_1 = D\tan\theta_1$ (see Fig. 36-8(a)), we obtain $y_1 = (500\,\text{mm})\tan(0.010\,\text{rad}) = 5.0\,\text{mm}$. The separation is $\Delta y = y_1 - y_0 = y_1 - 0 = 5.0\,\text{mm}$.

15. The angular positions of the maxima of a two-slit interference pattern are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be approximated by θ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$. Let λ' be the wavelength for which the angular separation is 10.0% greater. Then, $1.10\lambda/d = \lambda'/d$ or $\lambda' = 1.10\lambda = 1.10(589\,\text{nm}) = 648\,\text{nm}$.

16. In Sample Problem 36-2, an experimentally useful relation is derived: $\Delta y = \lambda D/d$. Dividing both sides by D, this becomes $\Delta\theta = \lambda/d$ with θ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20^\circ}{1.33} = 0.15^\circ\ .$$

17. Interference maxima occur at angles θ such that $d\sin\theta = m\lambda$, where m is an integer. Since $d = 2.0\,\text{m}$ and $\lambda = 0.50\,\text{m}$, this means that $\sin\theta = 0.25m$. We want all values of m (positive and negative) for which $|0.25m| \le 1$. These are -4, -3, -2, -1, 0, $+1$, $+2$, $+3$, and $+4$. For each of these except -4 and $+4$, there are two different values for θ. A single value of θ ($-90°$) is associated with $m = -4$ and a single value ($+90°$) is associated with $m = +4$. There are sixteen different angles in all and, therefore, sixteen maxima.

18. Initially, source A leads source B by $90°$, which is equivalent to $1/4$ wavelength. However, source A also lags behind source B since r_A is longer than r_B by $100\,\text{m}$, which is $100\,\text{m}/400\,\text{m} = 1/4$ wavelength. So the net phase difference between A and B at the detector is zero.

19. The maxima of a two-slit interference pattern are at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin\theta$ may be replaced by θ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is $\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$, and their separation on a screen a distance D away is

$$\begin{aligned}\Delta y &= D\tan\Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1) \\ &= \left[\frac{3(1.0\,\text{m})}{5.0\times10^{-3}\,\text{m}}\right](600\times10^{-9}\,\text{m} - 480\times10^{-9}\,\text{m}) = 7.2\times10^{-5}\ \text{m}\ .\end{aligned}$$

The small angle approximation $\tan\Delta\theta \approx \Delta\theta$ (in radians) is made.

20. Let the distance in question be x. The path difference (between rays originating from S_1 and S_2 and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda\ ,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \cdots$. After some algebraic steps, we solve for the distance in terms of m:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}\ .$$

To obtain the largest value of x, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda\ .$$

21. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness x is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where λ_m is the wavelength in the mica and n is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for λ_m. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

$$x = \frac{7\lambda}{n-1} = \frac{7(550\times10^{-9}\,\text{m})}{1.58-1} = 6.64\times10^{-6}\ \text{m}\ .$$

22. (a) We use $\Delta y = D\lambda/d$ (see Sample Problem 36-2). Because of the placement of the mirror in the problem $D = 2(20.0\,\text{m}) = 40.0\,\text{m}$, which we express in millimeters in the calculation below:

$$d = \frac{D\lambda}{\Delta y} = \frac{(4.00\times 10^4\,\text{mm})(632.8\times 10^{-6}\,\text{mm})}{100\,\text{mm}} = 0.253\ \text{mm}\ .$$

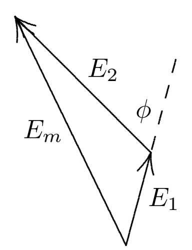
(b) In this case the interference pattern will be shifted. At the location of the original central maximum, the effective phase difference is now $\frac{1}{2}$ wavelength, so there is now a minimum instead of a maximum.

23. The phasor diagram is shown below. Here $E_1 = 1.00$, $E_2 = 2.00$, and $\phi = 60°$. The resultant amplitude E_m is given by the trigonometric law of cosines:

$$E_m^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(180° - \phi)\ .$$

Thus,

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65\ .$$



24. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ (see Sample Problem 36-3) and add them as vectors:

$$\begin{aligned} y_h &= 10\cos 0° + 8.0\cos 30° = 16.9 \\ y_v &= 10\sin 0° + 8.0\sin 30° = 4.0 \end{aligned}$$

so that

$$\begin{aligned} y_R &= \sqrt{y_h^2 + y_v^2} = 17.4 \\ \beta &= \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3°\ . \end{aligned}$$

Thus, $y = y_1 + y_2 = y_R\sin(\omega t + \beta) = 17.4\sin(\omega t + 13.3°)$.

25. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ (see Sample Problem 36-3) and add them as vectors:

$$\begin{aligned} y_h &= 10\cos 0° + 15\cos 30° + 5.0\cos(-45°) = 26.5 \\ y_v &= 10\sin 0° + 15\sin 30° + 5.0\sin(-45°) = 4.0 \end{aligned}$$

so that

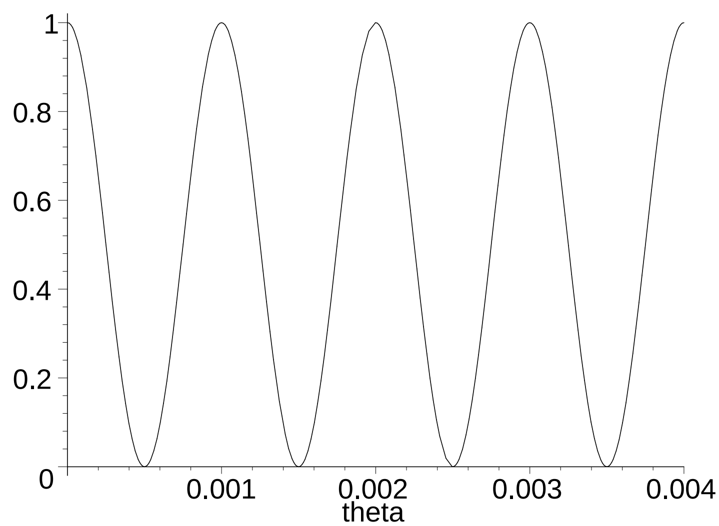
$$\begin{aligned} y_R &= \sqrt{y_h^2 + y_v^2} = 26.8 \\ \beta &= \tan^{-1}\left(\frac{y_v}{y_h}\right) = 8.5°\ . \end{aligned}$$

Thus, $y = y_1 + y_2 + y_3 = y_R\sin(\omega t + \beta) = 26.8\sin(\omega t + 8.5°)$.

26. Fig. 36-9 in the textbook is plotted versus the phase difference (in radians), whereas this problem requests that we plot the intensity versus the physical angle θ (defined in Fig. 36-8). The values given in the problem imply $d\lambda = 1000$. Combining this with Eq. 36-22 and Eq. 36-21, we solve for the (normalized) intensity:

$$\frac{I}{4I_0} = \cos^2(1000\pi \sin\theta) \ .$$

This is plotted over $0 \leq \theta \leq 0.0040$ rad:



27. (a) To get to the detector, the wave from S_1 travels a distance x and the wave from S_2 travels a distance $\sqrt{d^2 + x^2}$. The phase difference (in terms of wavelengths) between the two waves is

$$\sqrt{d^2 + x^2} - x = m\lambda \qquad m = 0, 1, 2, \ldots$$

where we are requiring constructive interference. The solution is

$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda} \ .$$

The largest value of m that produces a positive value for x is $m = 3$. This corresponds to the maximum that is nearest S_1, at

$$x = \frac{(4.00\,\text{m})^2 - 9(1.00\,\text{m})^2}{(2)(3)(1.00\,\text{m})} = 1.17 \text{ m} \ .$$

For the next maximum, $m = 2$ and $x = 3.00\,\text{m}$. For the third maximum, $m = 1$ and $x = 7.50\,\text{m}$.

(b) Minima in intensity occur where the phase difference is π rad; the intensity at a minimum, however, is not zero because the amplitudes of the waves are different. Although the amplitudes are the same at the sources, the waves travel different distances to get to the points of minimum intensity and each amplitude decreases in inverse proportion to the distance traveled.

28. Setting $I = 2I_0$ in Eq. 36-21 and solving for the smallest (in absolute value) two roots for $\phi/2$, we find

$$\phi = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pm\frac{\pi}{2} \text{ rad} \ .$$

Now, for small θ in radians, Eq. 36-22 becomes $\phi = 2\pi d\theta/\lambda$. This leads to two corresponding angle values:

$$\theta = \pm\frac{\lambda}{4d} \ .$$

The difference between these two values is $\Delta\theta = \frac{\lambda}{4d} - \left(-\frac{\lambda}{4d}\right) = \frac{\lambda}{2d}$.

29. We take the electric field of one wave, at the screen, to be

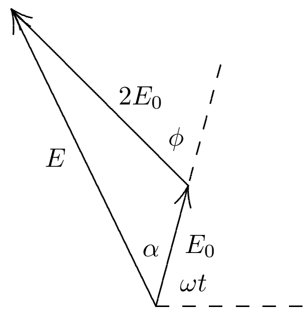
$$E_1 = E_0 \sin(\omega t)$$

and the electric field of the other to be

$$E_2 = 2E_0 \sin(\omega t + \phi) \ ,$$

where the phase difference is given by

$$\phi = \left(\frac{2\pi d}{\lambda}\right) \sin\theta \ .$$



Here d is the center-to-center slit separation and λ is the wavelength. The resultant wave can be written $E = E_1 + E_2 = E\sin(\omega t + \alpha)$, where α is a phase constant. The phasor diagram is shown above. The resultant amplitude E is given by the trigonometric law of cosines:

$$E^2 = E_0^2 + (2E_0)^2 - 4E_0^2\cos(180^\circ - \phi) = E_0^2(5 + 4\cos\phi) \ .$$

The intensity is given by $I = I_0(5 + 4\cos\phi)$, where I_0 is the intensity that would be produced by the first wave if the second were not present. Since $\cos\phi = 2\cos^2(\phi/2) - 1$, this may also be written $I = I_0\left[1 + 8\cos^2(\phi/2)\right]$.

30. The fact that wave W_2 reflects two additional times has no substantive effect on the calculations, since two reflections amount to a $2(\lambda/2) = \lambda$ phase difference, which is effectively not a phase difference at all. The substantive difference between W_2 and W_1 is the extra distance $2L$ traveled by W_2.

(a) For wave W_2 to be a half-wavelength "behind" wave W_1, we require $2L = \lambda/2$, or $L = \lambda/4 = 155$ nm using the wavelength value given in the problem.

(b) Destructive interference will again appear if W_2 is $\frac{3}{2}\lambda$ "behind" the other wave. In this case, $2L' = 3\lambda/2$, and the difference is

$$L' - L = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} = 310 \text{ nm} \ .$$

31. The wave reflected from the front surface suffers a phase change of π rad since it is incident in air on a medium of higher index of refraction. The phase of the wave reflected from the back surface does not change on reflection since the medium beyond the soap film is air and has a lower index of refraction than the film. If L is the thickness of the film, this wave travels a distance $2L$ farther than the wave reflected from the front surface. The phase difference of the two waves is $2L(2\pi/\lambda_f) - \pi$, where λ_f is the wavelength in the film. If λ is the wavelength in vacuum and n is the index of refraction of the soap film, then $\lambda_f = \lambda/n$ and the phase difference is

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2(1.33)(1.21 \times 10^{-6}\,\text{m})\left(\frac{2\pi}{585 \times 10^{-9}\,\text{m}}\right) - \pi = 10\pi \text{ rad} \ .$$

Since the phase difference is an even multiple of π, the interference is completely constructive.

32. In contrast to the initial conditions of problem 30, we now consider waves W_2 and W_1 with an initial effective phase difference (in wavelengths) equal to $\frac{1}{2}$, and seek positions of the sliver which cause the wave to constructively interfere (which corresponds to an integer-valued phase difference in wavelengths). Thus, the extra distance $2L$ traveled by W_2 must amount to $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, and so on. We may write this requirement succinctly as

$$L = \frac{2m+1}{4}\lambda \qquad \text{where} \;\; m = 0, 1, 2, \ldots \;\; .$$

33. For constructive interference, we use Eq. 36-34: $2n_2L = (m + 1/2)\lambda$. For the two smallest values of L, let $m = 0$ and 1:

$$\begin{aligned} L_0 &= \frac{\lambda/2}{2n_2} = \frac{624\,\text{nm}}{4(1.33)} = 117\,\text{nm} = 0.117\,\mu\text{m} \\ L_1 &= \frac{(1+1/2)\lambda}{2n_2} = \frac{3\lambda}{2n_2} = 3L_0 = 3(0.1173\mu\text{m}) = 0.352\,\mu\text{m} \; . \end{aligned}$$

34. We use the formula obtained in Sample Problem 36-5:

$$L_{\text{min}} = \frac{\lambda}{4n_2} = \frac{\lambda}{4(1.25)} = 0.200\lambda \; .$$

35. Light reflected from the front surface of the coating suffers a phase change of π rad while light reflected from the back surface does not change phase. If L is the thickness of the coating, light reflected from the back surface travels a distance $2L$ farther than light reflected from the front surface. The difference in phase of the two waves is $2L(2\pi/\lambda_c) - \pi$, where λ_c is the wavelength in the coating. If λ is the wavelength in vacuum, then $\lambda_c = \lambda/n$, where n is the index of refraction of the coating. Thus, the phase difference is $2nL(2\pi/\lambda) - \pi$. For fully constructive interference, this should be a multiple of 2π. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2m\pi$$

for L. Here m is an integer. The solution is

$$L = \frac{(2m+1)\lambda}{4n} \; .$$

To find the smallest coating thickness, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{560 \times 10^{-9}\,\text{m}}{4(2.00)} = 7.00 \times 10^{-8}\;\text{m} \; .$$

36. Let the thicknesses (which appear in Fig. 36-31 as different heights h) of the structure be $h = kL$, where k is a pure number. In section (b), for example, $k = 2$. Using Eq. 36-34, the condition for constructive interference becomes

$$2h = 2(kL) = \frac{(m+1/2)\lambda}{n_2} \qquad \text{where} \;\; m = 0, 1, 2, \ldots$$

which leads to

$$k = \frac{(m+1/2)\lambda}{2n_2L} = \frac{(m+1/2)(600\,\text{nm})}{2(1.50)(4.00 \times 10^3\,\text{nm})} = \frac{2m+1}{40} \; ,$$

or $40k - 1 = 2m$. This means that $40k - 1$ would have to be an even integer. One can check that none of the given values of k (1, 2, $\frac{1}{2}$, 3, $\frac{1}{10}$) will satisfy this condition. Therefore, none of the sections provides the right thickness for constructive interference.

37. For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad. Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for L. Here m is an integer. The result is

$$L = \frac{(2m+1)\lambda}{4n} .$$

To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{600 \times 10^{-9}\,\text{m}}{4(1.25)} = 1.2 \times 10^{-7}\ \text{m} .$$

38. Eqs. 36-14 and 36-16 treat the interference of reflections, and here we are concerned with interference of the transmitted light. Maxima in the reflections should, reasonably enough, correspond to minima in the transmissions, and vice versa. So we might expect to apply those equations to this case if we switch the designations "maxima" and "minima," *if* we are careful with the phase shifts that occur at the points of reflection (which depend on the relative values of n). Now, if the expression $2L = m\lambda/n_2$ is to give the condition for constructive interference for the transmitted light, then the situation should be similar to that which led in the textbook to Eqs. 36-14 and 36-16; namely, the thin film should be surrounded by two higher-index or two lower-index media. Such is the case for Fig. 36-32(a) and Fig. 36-32(c), but not for the others.

39. The situation is analogous to that treated in Sample Problem 36-5, in the sense that the incident light is in a low index medium, the thin film has somewhat higher $n = n_2$, and the last layer has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. The value of L which corresponds to no reflection corresponds, reasonably enough, to the value which gives maximum transmission of light (into the highest index medium – which in this problem is the water).

(a) If $2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}$ (Eq. 36-34) gives zero reflection in this type of system, then we might reasonably expect that its counterpart, Eq. 36-35, gives maximum reflection here. A more careful analysis such as that given in §36-7 bears this out. We disregard the $m = 0$ value (corresponding to $L = 0$) since there is *some* oil on the water. Thus, for $m = 1, 2, \ldots$ maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\,,\ 552\,\text{nm}\,,\ 368\,\text{nm}\ \ldots$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) As remarked above, maximum transmission into the water occurs for wavelengths given by

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \implies \lambda = \frac{4n_2L}{2m+1}$$

which yields $\lambda = 2208\,\text{nm}\,,\ 736\,\text{nm}\,,\ 442\,\text{nm}\ \ldots$ for the different values of m. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

40. The situation is analogous to that treated in Sample Problem 36-5, in the sense that the incident light is in a low index medium, the thin film of oil has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. With $\lambda = 500$ nm and $n_2 = 1.30$, the possible answers for L are

$$L = 96\,\text{nm}\,,\ 288\,\text{nm}\,,\ 481\,\text{nm}\,,\ 673\,\text{nm}\,,\ 865\,\text{nm}\,,\ \ldots$$

And, with $\lambda = 700$ nm and the same value of n_2, the possible answers for L are

$$L = 135\,\text{nm}\,,\ 404\,\text{nm}\,,\ 673\,\text{nm}\,,\ 942\,\text{nm}\,,\ \ldots$$

The lowest number these lists have in common is $L = 673$ nm.

41. Light reflected from the upper oil surface (in contact with air) changes phase by π rad. Light reflected from the lower surface (in contact with glass) changes phase by π rad if the index of refraction of the oil is less than that of the glass and does not change phase if the index of refraction of the oil is greater than that of the glass.

- First, suppose the index of refraction of the oil is greater than the index of refraction of the glass. The condition for fully destructive interference is $2n_o d = m\lambda$, where d is the thickness of the oil film, n_o is the index of refraction of the oil, λ is the wavelength in vacuum, and m is an integer. For the shorter wavelength, $2n_o d = m_1\lambda_1$ and for the longer, $2n_o d = m_2\lambda_2$. Since λ_1 is less than λ_2, m_1 is greater than m_2, and since fully destructive interference does not occur for any wavelengths between, $m_1 = m_2 + 1$. Solving $(m_2 + 1)\lambda_1 = m_2\lambda_2$ for m_2, we obtain

$$m_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} = \frac{500\,\text{nm}}{700\,\text{nm} - 500\,\text{nm}} = 2.50\ .$$

  Since m_2 must be an integer, the oil cannot have an index of refraction that is greater than that of the glass.

- Now suppose the index of refraction of the oil is less than that of the glass. The condition for fully destructive interference is then $2n_o d = (2m+1)\lambda$. For the shorter wavelength, $2m_o d = (2m_1+1)\lambda_1$, and for the longer, $2n_o d = (2m_2+1)\lambda_2$. Again, $m_1 = m_2 + 1$, so $(2m_2+3)\lambda_1 = (2m_2+1)\lambda_2$. This means the value of m_2 is

$$m_2 = \frac{3\lambda_1 - \lambda_2}{2(\lambda_2 - \lambda_1)} = \frac{3(500\,\text{nm}) - 700\,\text{nm}}{2(700\,\text{nm} - 500\,\text{nm})} = 2.00\ .$$

  This is an integer. Thus, the index of refraction of the oil is less than that of the glass.

42. We solve Eq. 36-34 with $n_2 = 1.33$ and $\lambda = 600$ nm for $m = 1, 2, 3, \ldots$:

$$L = 113\,\text{nm}\,,\ 338\,\text{nm}\,,\ 564\,\text{nm}\,,\ 789\,\text{nm}\,,\ \ldots$$

And, we similarly solve Eq. 36-35 with the same n_2 and $\lambda = 450$ nm:

$$L = 0\,,\ 169\,\text{nm}\,,\ 338\,\text{nm}\,,\ 508\,\text{nm}\,,\ 677\,\text{nm}\,,\ \ldots$$

The lowest number these lists have in common is $L = 338$ nm.

43. Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by π rad. At a place where the thickness of the air film is L, the condition

for fully constructive interference is $2L = (m + \frac{1}{2})\lambda$, where λ (= 683 nm) is the wavelength and m is an integer. This is satisfied for $m = 140$:

$$L = \frac{(m + \frac{1}{2})\lambda}{2} = \frac{(140.5)(683 \times 10^{-9}\,\text{m})}{2} = 4.80 \times 10^{-5}\,\text{m} = 0.048 \text{ mm} .$$

At the thin end of the air film, there is a bright fringe. It is associated with $m = 0$. There are, therefore, 140 bright fringes in all.

44. (a) At the left end, the plates touch, so $L = 0$ there, which is clearly consistent with Eq. 36-35 (the destructive interference or "dark fringe" equation) for $m = 0$.

    (b) Eq. 36-35 shows a simple proportionality between L and λ. So as we slowly increase L (from zero – its value in part (a)), the smallest nonzero value of L for which the equation (which specifies destructive interference) is satisfied occurs for the lowest possible value of λ. Wavelengths for blue light are the shortest of the visible portion of the spectrum.

45. Assume the wedge-shaped film is in air, so the wave reflected from one surface undergoes a phase change of π rad while the wave reflected from the other surface does not. At a place where the film thickness is L, the condition for fully constructive interference is $2nL = (m + \frac{1}{2})\lambda$, where n is the index of refraction of the film, λ is the wavelength in vacuum, and m is an integer. The ends of the film are bright. Suppose the end where the film is narrow has thickness L_1 and the bright fringe there corresponds to $m = m_1$. Suppose the end where the film is thick has thickness L_2 and the bright fringe there corresponds to $m = m_2$. Since there are ten bright fringes, $m_2 = m_1 + 9$. Subtract $2nL_1 = (m_1 + \frac{1}{2})\lambda$ from $2nL_2 = (m_1 + 9 + \frac{1}{2})\lambda$ to obtain $2n\,\Delta L = 9\lambda$, where $\Delta L = L_2 - L_1$ is the change in the film thickness over its length. Thus,

$$\Delta L = \frac{9\lambda}{2n} = \frac{9(630 \times 10^{-9}\,\text{m})}{2(1.50)} = 1.89 \times 10^{-6} \text{ m} .$$

46. The situation is analogous to that treated in Sample Problem 36-5, in the sense that the incident light is in a low index medium, the thin film of acetone has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \qquad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. This is the same as Eq. 36-34 which was developed for the opposite situation (constructive interference) regarding a thin film surrounded on both sides by air (a very different context than the one in this problem). By analogy, we expect Eq. 36-35 to apply in this problem to reflection *maxima*. A more careful analysis such as that given in §36-7 bears this out. Thus, using Eq. 36-35 with $n_2 = 1.25$ and $\lambda = 700$ nm yields

$$L = 0\,,\ 280\,\text{nm}\,,\ 560\,\text{nm}\,,\ 840\,\text{nm}\,,\ 1120\,\text{nm}\,,\ \ldots$$

for the first several m values. And the equation shown above (equivalent to Eq. 36-34) gives, with $\lambda = 600$ nm,

$$L = 120\,\text{nm}\,,\ 360\,\text{nm}\,,\ 600\,\text{nm}\,,\ 840\,\text{nm}\,,\ 1080\,\text{nm}\,,\ \ldots$$

for the first several m values. The lowest number these lists have in common is $L = 840$ nm.

47. We use Eq. 36-34:

$$\begin{aligned} L_{16} &= \left(16 + \frac{1}{2}\right)\frac{\lambda}{2n_2} \\ L_6 &= \left(6 + \frac{1}{2}\right)\frac{\lambda}{2n_2} \end{aligned}$$

The difference between these, using the fact that $n_2 = n_{\text{air}} = 1.0$, is

$$L_{16} - L_6 = (10)\frac{480\,\text{nm}}{2(1.0)} = 2400 \text{ nm} .$$

48. We apply Eq. 36-25 to both scenarios: $m = 4001$ and $n_2 = n_{\text{air}}$, and $m = 4000$ and $n_2 = n_{\text{vacuum}} = 1.00000$:

$$2L = (4001)\frac{\lambda}{n_{\text{air}}} \qquad \text{and} \qquad 2L = (4000)\frac{\lambda}{1.00000} .$$

Since the $2L$ factor is the same in both cases, we set the right hand sides of these expressions equal to each other and cancel the wavelength. Finally, we obtain

$$n_{\text{air}} = (1.00000)\frac{4001}{4000} = 1.00025 .$$

We remark that this same result can be obtained starting with Eq. 36-41 (which is developed in the textbook for a somewhat different situation) and using Eq. 36-40 to eliminate the $2L/\lambda$ term.

49. Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d, the condition for a maximum in intensity is $2d = (m + \frac{1}{2})\lambda$, where λ is the wavelength in air and m is an integer. Thus, $d = (2m+1)\lambda/4$. As the geometry of Fig. 36-34 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus, $(2m+1)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4} .$$

Next, we square both sides, rearrange to solve for r^2, then take the square root. We get

$$r = \sqrt{\frac{(2m+1)R\lambda}{2} - \frac{(2m+1)^2\lambda^2}{16}} .$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m+1)R\lambda}{2}} .$$

50. (a) We find m from the last formula obtained in problem 49:

$$m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{(10 \times 10^{-3}\,\text{m})^2}{(5.0\,\text{m})(589 \times 10^{-9}\,\text{m})} - \frac{1}{2}$$

which (rounding down) yields $m = 33$. Since the first bright fringe corresponds to $m = 0$, $m = 33$ corresponds to the thirty-fourth bright fringe.

(b) We now replace λ by $\lambda_n = \lambda/n_w$. Thus,

$$m_n = \frac{r^2}{R\lambda_n} - \frac{1}{2} = \frac{n_w r^2}{R\lambda} - \frac{1}{2} = \frac{(1.33)(10 \times 10^{-3}\,\text{m})^2}{(5.0\,\text{m})(589 \times 10^{-9}\,\text{m})} - \frac{1}{2} = 45 .$$

This corresponds to the forty-sixth bright fringe (see remark at the end of our solution in part (a)).

51. We solve for m using the formula $r = \sqrt{(2m+1)R\lambda/2}$ obtained in problem 49 and find $m = r^2/R\lambda - 1/2$. Now, when m is changed to $m + 20$, r becomes r', so $m + 20 = r'^2/R\lambda - 1/2$. Taking the difference between the two equations above, we eliminate m and find

$$R = \frac{r'^2 - r^2}{20\lambda} = \frac{(0.368\,\text{cm})^2 - (0.162\,\text{cm})^2}{20(546 \times 10^{-7}\,\text{cm})} = 100 \text{ cm} .$$

52. (a) The binomial theorem (Appendix E) allows us to write

$$\sqrt{k(1+x)} = \sqrt{k}\left(1 + \frac{x}{2} + \frac{x^2}{8} + \frac{3x^3}{48} + \cdots\right) \approx \sqrt{k} + \frac{x}{2}\sqrt{k}$$

for $x \ll 1$. Thus, the end result from the solution of problem 49 yields

$$r_m = \sqrt{R\lambda m\left(1 + \frac{1}{2m}\right)} \approx \sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}$$

and

$$r_{m+1} = \sqrt{R\lambda m\left(1 + \frac{3}{2m}\right)} \approx \sqrt{R\lambda m} + \frac{3}{4m}\sqrt{R\lambda m}$$

for very large values of m. Subtracting these, we obtain

$$\Delta r = \frac{3}{4m}\sqrt{R\lambda m} - \frac{1}{4m}\sqrt{R\lambda m} = \frac{1}{2}\sqrt{\frac{R\lambda}{m}} .$$

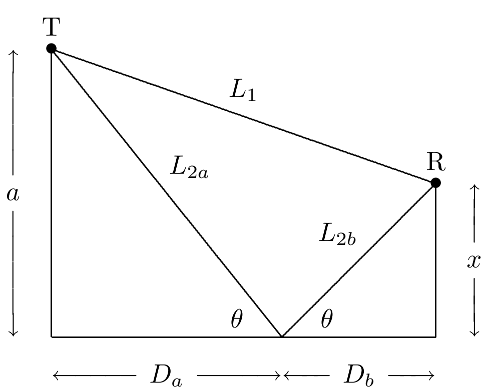
(b) We take the differential of the area: $dA = d(\pi r^2) = 2\pi r\, dr$, and replace dr with Δr in anticipation of using the result from part (a). Thus, the area between adjacent rings for large values of m is

$$2\pi r_m(\Delta r) \approx 2\pi\left(\sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right) \approx 2\pi\left(\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right)$$

which simplifies to the desired result ($\pi\lambda R$).

53. The wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2. Since the index of refraction of water is greater than that of air this last wave suffers a phase change on reflection of half a wavelength. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength. Consider the diagram below. The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$. The right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line leads to $D_b = x/\tan\theta$. Since $D_a + D_b = D$,

$$\tan\theta = \frac{a+x}{D} .$$



We use the identity $\sin^2\theta = \tan^2\theta/(1 + \tan^2\theta)$ to show that $\sin\theta = (a+x)/\sqrt{D^2 + (a+x)^2}$. This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2 + (a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2 + (a+x)^2}}{a+x} \ .$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2+(a+x)^2}}{a+x} = \sqrt{D^2+(a+x)^2} \ .$$

Using the binomial theorem, with D^2 large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a+x)^2/2D$. The distance traveled by the direct wave is $L_1 = \sqrt{D^2 + (a-x)^2}$. Using the binomial theorem, we approximate this expression: $L_1 \approx D + (a-x)^2/2D$. Thus,

$$L_2 - L_1 \approx D + \frac{a^2 + 2ax + x^2}{2D} - D - \frac{a^2 - 2ax + x^2}{2D} = \frac{2ax}{D} \ .$$

Setting this equal to $(m + \frac{1}{2})\lambda$, where m is zero or a positive integer, we find $x = (m + \frac{1}{2})(D/2a)\lambda$.

54. According to Eq. 36-41, the number of fringes shifted (ΔN) due to the insertion of the film of thickness L is $\Delta N = (2L/\lambda)(n-1)$. Therefore,

$$L = \frac{\lambda\Delta N}{2(n-1)} = \frac{(589\,\text{nm})(7.0)}{2(1.40-1)} = 5.2\,\mu\text{m} \ .$$

55. A shift of one fringe corresponds to a change in the optical path length of one wavelength. When the mirror moves a distance d the path length changes by $2d$ since the light traverses the mirror arm twice. Let N be the number of fringes shifted. Then, $2d = N\lambda$ and

$$\lambda = \frac{2d}{N} = \frac{2(0.233 \times 10^{-3}\,\text{m})}{792} = 5.88 \times 10^{-7}\,\text{m} = 588 \text{ nm} \ .$$

56. We denote the two wavelengths as λ and λ', respectively. We apply Eq. 36-40 to both wavelengths and take the difference:

$$N' - N = \frac{2L}{\lambda'} - \frac{2L}{\lambda} = 2L\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) \ .$$

We now require $N' - N = 1$ and solve for L:

$$\begin{aligned} L &= \frac{1}{2}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)^{-1} \\ &= \frac{1}{2}\left(\frac{1}{589.10\,\text{nm}} - \frac{1}{589.59\,\text{nm}}\right)^{-1} \\ &= 3.54 \times 10^5\,\text{nm} = 354\,\mu\text{m} \ . \end{aligned}$$

57. Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it, and let ϕ_2 be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If λ is the wavelength in vacuum, then the wavelength in air is λ/n, where n is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n-1)L}{\lambda}$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated,

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi$$

and

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500 \times 10^{-9}\,\text{m})}{2(5.0 \times 10^{-2}\,\text{m})} = 1.00030 \ .$$

58. Let the position of the mirror measured from the point at which $d_1 = d_2$ be x. We assume the beam-splitting mechanism is such that the two waves interfere constructively for $x = 0$ (with some beam-splitters, this would not be the case). We can adapt Eq. 36-22 to this situation by incorporating a factor of 2 (since the interferometer utilizes directly reflected light in contrast to the double-slit experiment) and eliminating the $\sin\theta$ factor. Thus, the phase difference between the two light paths is $\Delta\phi = 2(2\pi x/\lambda) = 4\pi x/\lambda$. Then from Eq. 36-21 (writing $4I_0$ as I_m) we find

$$I = I_m \cos^2\left(\frac{\Delta\phi}{2}\right) = I_m \cos^2\left(\frac{2\pi x}{\lambda}\right) .$$

59. (a) To get to the detector, the wave from S_1 travels a distance x and the wave from S_2 travels a distance $\sqrt{d^2 + x^2}$. The phase difference (in terms of wavelengths) between the two waves is

$$\sqrt{d^2 + x^2} - x = m\lambda \qquad m = 0, 1, 2, \ldots$$

where we are requiring constructive interference. The solution is

$$x = \frac{d^2 - m^2\lambda^2}{2m\lambda} .$$

We see that setting $m = 0$ in this expression produces $x = \infty$; hence, the phase difference between the waves when P is very far away is 0.

(b) The result of part (a) implies that the waves constructively interfere at P.

(c) As is particularly evident from our results in part (d), the phase difference increases as x decreases.

(d) We can use our formula from part (a) for the 0.5λ, 1.50λ, etc differences by allowing m in our formula to take on half-integer values. The half- integer values, though, correspond to destructive interference. Using the values $\lambda = 0.500\,\mu$m and $d = 2.00\,\mu$m, we find $x = 7.88\,\mu$m for $m = \frac{1}{2}$, $x = 3.75\,\mu$m for $m = 1$, $x = 2.29\,\mu$m for $m = \frac{3}{2}$, $x = 1.50\,\mu$m for $m = 2$, and $x = 0.975\,\mu$m for $m = \frac{5}{2}$.

60. (a) In a reference frame fixed on Earth, the ether travels leftward with speed v. Thus, the speed of the light beam in this reference frame is $c - v$ as the beam travels rightward from M to M_1, and $c + v$ as it travels back from M_1 to M. The total time for the round trip is therefore given by

$$t_1 = \frac{d_1}{c - v} + \frac{d_1}{c + v} = \frac{2cd_1}{c^2 - v^2} .$$

(b) In a reference frame fixed on the ether, the mirrors travel rightward with speed v, while the speed of the light beam remains c. Thus, in this reference frame, the total distance the beam has to travel is given by

$$d_2{}' = 2\sqrt{d_2^2 + \left[v\left(\frac{t_2}{2}\right)\right]^2}$$

[see Fig. 36-37(h)-(j)]. Thus,

$$t_2 = \frac{d_2{}'}{c} = \frac{2}{c}\sqrt{d_2^2 + \left[v\left(\frac{t_2}{2}\right)\right]^2} ,$$

which we solve for t_2:

$$t_2 = \frac{2d_2}{\sqrt{c^2 - v^2}} .$$

(c) We use the binomial expansion (Appendix E)

$$(1+x)^n = 1 + nx + \cdots \approx 1 + nx \qquad (|x| \ll 1) \ .$$

In our case let $x = v/c \ll 1$, then

$$L_1 = \frac{2c^2 d_1}{c^2 - v^2} = 2d_1 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1} \approx 2d_1 \left[1 + \left(\frac{v}{c}\right)^2\right] ,$$

and

$$L_2 = \frac{2cd_2}{\sqrt{c^2 - v^2}} = 2d_2 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} \approx 2d_2 \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right] .$$

Thus, if $d_1 = d_2 = d$ then

$$\Delta L = L_1 - L_2 \approx 2d \left[1 + \left(\frac{v}{c}\right)^2\right] - 2d \left[1 + \frac{1}{2}\left(\frac{v}{c}\right)^2\right] = \frac{dv^2}{c^2} .$$

(d) In terms of the wavelength, the phase difference is given by

$$\frac{\Delta L}{\lambda} = \frac{dv^2}{\lambda c^2} .$$

(e) We now must reverse the indices 1 and 2, so the new phase difference is

$$\frac{-\Delta L}{\lambda} = -\frac{dv^2}{\lambda c^2} .$$

The shift in phase difference between these two cases is

$$\text{shift} = \left(\frac{\Delta L}{\lambda}\right) - \left(-\frac{\Delta L}{\lambda}\right) = \frac{2dv^2}{\lambda c^2} .$$

(f) Assume that v is about the same as the orbital speed of the Earth, so that $v \approx 29.8\,\text{km/s}$ (see Appendix C). Thus,

$$\text{shift} = \frac{2dv^2}{\lambda c^2} = \frac{2(10\,\text{m})(29.8 \times 10^3\,\text{m/s})^2}{(500 \times 10^{-9}\,\text{m})(2.998 \times 10^8\,\text{m/s})^2} = 0.40 .$$

61. (a) Every time one more destructive (constructive) fringe appears the increase in thickness of the air gap is $\lambda/2$. Now that there are 6 more destructive fringes in addition to the one at point A, the thickness at B is $t_B = 6(\lambda/2) = 3(600\,\text{nm}) = 1.80\,\mu\text{m}$.

(b) We must now replace λ by $\lambda' = \lambda/n_w$. Since t_B is unchanged $t_B = N(\lambda'/2) = N(\lambda/2n_w)$, or

$$N = \frac{2t_B n_w}{\lambda} = \frac{2(3\lambda)n_w}{\lambda} = 6n_w = 6(1.33) = 8 .$$

62. We adapt Eq. 36-21 to the non-reflective coating on a glass lens: $I = I_{\text{max}} \cos^2(\phi/2)$, where $\phi = (2\pi/\lambda)(2n_2 L) + \pi$. At $\lambda = 450\,\text{nm}$

$$\begin{aligned}\frac{I}{I_{\text{max}}} &= \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{2\pi n_2 L}{\lambda} + \frac{\pi}{2}\right) \\ &= \cos^2\left[\frac{2\pi(1.38)(99.6\,\text{nm})}{450\,\text{nm}} + \frac{\pi}{2}\right] = 0.883 ,\end{aligned}$$

and at $\lambda = 650\,\text{nm}$

$$\frac{I}{I_{\text{max}}} = \cos^2\left[\frac{2\pi(1.38)(99.6\,\text{nm})}{650\,\text{nm}} + \frac{\pi}{2}\right] = 0.942 .$$

63. For the fifth maximum $y_5 = D\sin\theta_5 = D(5\lambda/d)$, and for the seventh minimum $y'_7 = D\sin\theta'_7 = D[(6 + 1/2)\lambda/d]$. Thus,

$$\begin{aligned}\Delta y &= y'_7 - y_5 = D\left[\frac{(6+1/2)\lambda}{d}\right] - D\left(\frac{5\lambda}{d}\right) = \frac{3\lambda D}{2d} \\ &= \frac{3(546\times10^{-9}\,\text{m})(20\times10^{-2}\,\text{m})}{2(0.10\times10^{-3}\,\text{m})} \\ &= 1.6\times10^{-3}\,\text{m} = 1.6\ \text{mm}\ .\end{aligned}$$

64. Let the $m = 10$ bright fringe on the screen be a distance y from the central maximum. Then from Fig. 36-8(a)

$$r_1 - r_2 = \sqrt{(y+d/2)^2 + D^2} - \sqrt{(y-d/2)^2 + D^2} = 10\lambda\ ,$$

from which we may solve for y. To the order of $(d/D)^2$ we find

$$y = y_0 + \frac{y(y^2 + d^2/4)}{2D^2}\ ,$$

where $y_0 = 10D\lambda/d$. Thus, we find the percent error as follows:

$$\frac{y_0(y_0^2 + d^2/4)}{2y_0D^2} = \frac{1}{2}\left(\frac{10\lambda}{D}\right)^2 + \frac{1}{8}\left(\frac{d}{D}\right)^2 = \frac{1}{2}\left(\frac{5.89\,\mu\text{m}}{2000\,\mu\text{m}}\right)^2 + \frac{1}{8}\left(\frac{2.0\,\text{mm}}{40\,\text{mm}}\right)^2$$

which yields 0.03%.

65. $v_{\min} = c/n = (2.998\times10^8\,\text{m/s})/1.54 = 1.95\times10^8\,\text{m/s}$.

66. With phasor techniques, this amounts to a vector addition problem $\vec{R} = \vec{A}+\vec{B}+\vec{C}$ where (in magnitude-angle notation) $\vec{A} = (10\ \angle\ 0°)$, $\vec{B} = (5\ \angle\ 45°)$, and $\vec{C} = (5\ \angle -45°)$, where the magnitudes are understood to be in μV/m. We obtain the resultant (especially efficient on a vector capable calculator in polar mode):

$$\vec{R} = (10\ \angle\ 0°) + (5\ \angle\ 45°) + (5\ \angle -45°) = (17.1\ \angle\ 0°)$$

which leads to

$$E_R = (17.1\ \mu\text{V/m})\sin(\omega t)$$

where $\omega = 2.0\times10^{14}\,\text{rad/s}$.

67. (a) and (b) Dividing Eq. 36-12 by the wavelength, we obtain

$$N = \frac{\Delta L}{\lambda} = \frac{d}{\lambda}\sin\theta = 39.6$$

wavelengths. This is close to a half-integer value (destructive interference), so that the correct response is "intermediate illumination but closer to darkness."

68. To explore one quadrant of the circle, we look for angles where Eq. 36-14 is satisfied.

$$\theta = \sin^{-1}\frac{m\lambda}{d} \qquad \text{for } m = 0, 1, 2...$$

where $m\lambda/d$ cannot exceed unity. For $m = 1...7$ we have solutions that are "mirrored" in every other quadrant; so there are $4\times7 = 28$ of these. The solutions at $m = 0$ and $m = 8$ are "special" in that they have twins (at 180° and 270°, respectively) and their multiplicity is 2, not 4. Thus, we have $28+2(2) = 32$ points of maxima.

69. In this case the path traveled by ray no. 2 is longer than that of ray no. 1 by $2L/\cos\theta_r$, instead of $2L$. Here $\sin\theta_i/\sin\theta_r = n_2$, or $\theta_r = \sin^{-1}(\sin\theta_i/n_2)$. So if we replace $2L$ by $2L/\cos\theta_r$ in Eqs. 36-34 and 36-35, we obtain

$$\frac{2n_2L}{\cos\theta_r} = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0,\ 1,\ 2,\cdots$$

for the maxima, and

$$\frac{2n_2L}{cos\theta_r} = m\lambda \qquad m = 0,\ 1,\ 2,\cdots$$

for the minima.

70. (a) and (b) Straightforward application of Eq. 36-3 and $v = \Delta x/\Delta t$ yields the result: pistol 1 with a time equal to 42.03×10^{-12} s; pistol 2 with a time equal to 42.3×10^{-12} s; pistol 3 with a time equal to 43.2×10^{-12} s; and, pistol 4 with a time equal to 41.96×10^{-12} s. We see that the blast from pistol 1 arrives first.

71. We use Eq. 36-34 for constructive interference: $2n_2L = (m + 1/2)\lambda$, or

$$\lambda = \frac{2n_2L}{m+1/2} = \frac{2(1.50)(410\,\text{nm})}{m+1/2} = \frac{1230\,\text{nm}}{m+1/2}\,,$$

where $m = 0, 1, 2, \cdots$. The only value of m which, when substituted into the equation above, would yield a wavelength which falls within the visible light range is $m = 1$. Therefore,

$$\lambda = \frac{1230\,\text{nm}}{1+1/2} = 492\ \text{nm}\ .$$

72. For the first maximum $m = 0$ and for the tenth one $m = 9$. The separation is $\Delta y = (D\lambda/d)\Delta m = 9D\lambda/d$. We solve for the wavelength:

$$\lambda = \frac{d\Delta y}{9D} = \frac{(0.15 \times 10^{-3}\,\text{m})(18 \times 10^{-3}\,\text{m})}{9(50 \times 10^{-2}\,\text{m})} = 6.0 \times 10^{-7}\,\text{m} = 600\ \text{nm}\ .$$

73. In the case of a distant screen the angle θ is close to zero so $\sin\theta \approx \theta$. Thus from Eq. 36-14,

$$\Delta\theta \approx \Delta\sin\theta = \Delta\left(\frac{m\lambda}{d}\right) = \frac{\lambda}{d}\Delta m = \frac{\lambda}{d}\,,$$

or $d \approx \lambda/\Delta\theta = 589 \times 10^{-9}\,\text{m}/0.018\,\text{rad} = 3.3 \times 10^{-5}\,\text{m} = 33\,\mu\text{m}$.

74. Using the relations of §36-7, we find that the (vertical) change between the center of one dark band and the next is

$$\Delta y = \lambda 2 = 2.5 \times 10^{-4}\ \text{mm}\ .$$

Thus, with the (horizontal) separation of dark bands given by $\Delta x = 1.2$ mm, we have

$$\theta \approx \tan\theta = \frac{\Delta y}{\Delta x} = 2.08 \times 10^{-4}\ \text{rad}\ .$$

Converting this angle into degrees, we arrive at $\theta = 0.012°$.

75. (a) A path length difference of $\lambda/2$ produces the first dark band, of $3\lambda/2$ produces the second dark band, and so on. Therefore, the fourth dark band corresponds to a path length difference of $7\lambda/2 = 1750$ nm.

(b) In the small angle approximation (which we assume holds here), the fringes are equally spaced, so that if Δy denotes the distance from one maximum to the next, then the distance from the middle of the pattern to the fourth dark band must be $16.8\,\text{mm} = 3.5\Delta y$. Therefore, we obtain $\Delta y = 16.8/3.5 = 4.8$ mm.

76. (a) With $\lambda = 0.5\,\mu$m, Eq. 36-14 leads to

$$\theta = \sin^{-1}\frac{(3)(0.5\,\mu\text{m})}{2.00\,\mu\text{m}} = 48.6^\circ \ .$$

(b) Decreasing the frequency means increasing the wavelength – which implies y increases. Qualitatively, this is easily seen with Eq. 36-17. One should exercise caution in appealing to Eq. 36-17 here, due to the fact the small angle approximation is not justified in this problem. The new wavelength is $0.5/0.9 = 0.556\,\mu$m, which produces a new angle of

$$\theta = \sin^{-1}\frac{(3)(0.556\,\mu\text{m})}{2.00\,\mu\text{m}} = 56.4^\circ \ .$$

Using $y = D\tan\theta$ for the old and new angles, and subtracting, we find

$$\Delta y = D\,(\tan 56.4^\circ - \tan 48.6^\circ) = 1.49 \text{ m} \ .$$

77. (a) Following Sample Problem 36-1, we have

$$N_2 - N_1 = \frac{L}{\lambda}\,(n_2 - n_1) = 1.87$$

which represents a meaningful difference of 0.87 wavelength.

(b) The result in part (a) is closer to 1 wavelength (constructive interference) than it is to $\frac{1}{2}$ wavelength (destructive interference) so the latter choice applies.

(c) This would insert a $\pm\frac{1}{2}$ wavelength into the previous result – resulting in a meaningful difference (between the two rays) equal to $0.87 - 0.50 = 0.37$ wavelength, which is closer to the destructive interference condition. Thus, there is intermediate illumination but closer to darkness.

78. (a) Straightforward application of Eq. 36-3 and $v = \Delta x/\Delta t$ yields the result: film 1 with a traversal time equal to 4.0×10^{-15} s.

(b) Use of Eq. 36-9 leads to the number of wavelengths:

$$N = \frac{L_1 n_1 + L_2 n_2 + L_3 n_3}{\lambda} = 7.5 \ .$$

79. (a) In this case, we are dealing with the situation that leads in the textbook to Eq. 36-35 for minima in reflected light from a thin film. The smallest non-zero answer, then, is for $m = 1$: $L = \lambda/2n_2$.

(b) Now, we are dealing with a situation exactly like that treated in Sample Problem 36-5, where the relation $L = \lambda/4n_2$ is derived.

(c) The indices bear the same relation here as in part (b), but we are looking now for the "opposite" result (maximum reflection instead of maximum transmission). We adapt the treatment in Sample Problem 36-5 by requiring $2L = m\lambda/n_2$ instead of $(m + \frac{1}{2})\lambda/2$. The smallest nonzero result in this case is for $m = 1$: $L = \lambda/2n_2$.

80. (a) Since $n_2 > n_3$, this case has no π-phase shift, and the condition for constructive interference is $m\lambda = 2Ln_2$. We solve for L:

$$L = \frac{m\lambda}{2n_2} = \frac{m(525\,\text{nm})}{2(1.55)} = (169\,\text{nm})m \ .$$

For the minimum value of L, let $m = 1$ to obtain $L_{\min} = 169\,\text{nm}$.

(b) The light of wavelength λ (other than 525 nm) that would also be preferentially transmitted satisfies $m'\lambda = 2n_2L$, or

$$\lambda = \frac{2n_2L}{m'} = \frac{2(1.55)(169\,\text{nm})}{m'} = \frac{525\,\text{nm}}{m'}\ .$$

Here $m' = 2, 3, 4, \ldots$ (note that $m' = 1$ corresponds to the $\lambda = 525$ nm light, so it should not be included here). Since the minimum value of m' is 2, one can easily verify that no m' will give a value of λ which falls into the visible light range. So no other parts of the visible spectrum will be preferentially transmitted. They are, in fact, reflected.

(c) For a sharp reduction of transmission let

$$\lambda = \frac{2n_2L}{m'+1/2} = \frac{525\,\text{nm}}{m'+1/2}\ ,$$

where $m' = 0, 1, 2, 3, \cdots$. In the visible light range $m' = 1$ and $\lambda = 350$ nm. This corresponds to the blue-violet light.

81. We adapt the result of problem 21. Now, the phase difference in radians is

$$\frac{2\pi t}{\lambda}(n_2 - n_1) = 2m\pi\ .$$

The problem implies $m = 5$, so the thickness is

$$t = \frac{m\lambda}{n_2 - n_1} = \frac{5(480\,\text{nm})}{1.7 - 1.4} = 8.0 \times 10^3\,\text{nm} = 8.0\,\mu\text{m}\ .$$

82. In Sample Problem 36-2, the relation $\Delta y = \lambda D/d$ is derived. Thus, to prevent Δy from changing, then (since $\Delta y \propto D/d$) we need to double D if d is doubled.

83. (a) In this case, the film has a smaller index material on one side (air) and a larger index material on the other (glass), and we are dealing (in part (a)) with strongly transmitted light, so the condition is given by Eq. 36-35 (which would give dark *reflection* in this scenario)

$$L = \frac{\lambda}{2n_2}\left(m + \frac{1}{2}\right) = 110\ \text{nm}$$

for $n_2 = 1.25$ and $m = 0$.

(b) Now, we are dealing with strongly reflected light, so the condition is given by Eq. 36-34 (which would give no *transmission* in this scenario)

$$L = \frac{m\lambda}{2n_2} = 220\ \text{nm}$$

for $n_2 = 1.25$ and $m = 1$ (the $m = 0$ option is excluded in the problem statement).

84. We infer from Sample Problem 36-2, that (with angle in radians)

$$\Delta\theta = \frac{\lambda}{d}$$

for adjacent fringes. With the wavelength change ($\lambda' = \lambda/n$ by Eq. 36-8), this equation becomes

$$\Delta\theta' = \frac{\lambda'}{d}\ .$$

Dividing one equation by the other, the requirement of *radians* can now be relaxed and we obtain

$$\frac{\Delta\theta'}{\Delta\theta} = \frac{\lambda'}{\lambda} = \frac{1}{n}\ .$$

Therefore, with $n = 1.33$ and $\Delta\theta = 0.30°$, we find $\Delta\theta' = 0.23°$.

85. Using Eq. 36-16 with the small-angle approximation (illustrated in Sample Problem 36-2), we arrive at

$$y = \frac{\left(m + \frac{1}{2}\right)\lambda D}{d}$$

for the position of the $(m+1)^{\text{th}}$ dark band (a simple way to get this is by averaging the expressions in Eq. 36-17 and Eq. 36-18). Thus, with $m = 1$, $y = 0.012$ m and $d = 800\lambda$, we find $D = 6.4$ m.

86. (a) The path length difference between Rays 1 and 2 is $7d - 2d = 5d$. For this to correspond to a half-wavelength requires $5d = \lambda/2$, so that $d = 50.0$ nm.

(b) The above requirement becomes $5d = \lambda/2n$ in the presence of the solution, with $n = 1.38$. Therefore, $d = 36.2$ nm.

87. (a) The path length difference is $0.5\,\mu$m $= 500$ nm, which is represents $500/400 = 1.25$ wavelengths – that is, a meaningful difference of 0.25 wavelengths. In angular measure, this corresponds to a phase difference of $(0.25)2\pi = \pi/2$ radians.

(b) When a difference of index of refraction is involved, the approach used in Eq. 36-9 is quite useful. In this approach, we count the wavelengths between S_1 and the origin

$$N_1 = \frac{Ln}{\lambda} + \frac{L'n'}{\lambda}$$

where $n = 1$ (rounding off the index of air), $L = 5.0\,\mu$m, $n' = 1.5$ and $L' = 1.5\,\mu$m. This yields $N_1 = 18.125$ wavelengths. The number of wavelengths between S_2 and the origin is (with $L_2 = 6.0\,\mu$m) given by

$$N_2 = \frac{L_2 n}{\lambda} = 15.000\ .$$

Thus, $N_1 - N_2 = 3.125$ wavelengths, which gives us a meaningful difference of 0.125 wavelength and which "converts" to a phase of $\pi/4$ radian.

Chapter 37

1. The condition for a minimum of a single-slit diffraction pattern is

$$a \sin\theta = m\lambda$$

where a is the slit width, λ is the wavelength, and m is an integer. The angle θ is measured from the forward direction, so for the situation described in the problem, it is $0.60°$ for $m = 1$. Thus

$$a = \frac{m\lambda}{\sin\theta} = \frac{633 \times 10^{-9}\ \text{m}}{\sin 0.60°} = 6.04 \times 10^{-5}\ \text{m} .$$

2. (a) $\theta = \sin^{-1}(1.50\,\text{cm}/2.00\,\text{m}) = 0.430°$.

   (b) For the mth diffraction minimum $a \sin\theta = m\lambda$. We solve for the slit width:

$$a = \frac{m\lambda}{\sin\theta} = \frac{2(441\,\text{nm})}{\sin 0.430°} = 0.118\ \text{mm} .$$

3. (a) The condition for a minimum in a single-slit diffraction pattern is given by $a \sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. For $\lambda = \lambda_a$ and $m = 1$, the angle θ is the same as for $\lambda = \lambda_b$ and $m = 2$. Thus $\lambda_a = 2\lambda_b$.

   (b) Let m_a be the integer associated with a minimum in the pattern produced by light with wavelength λ_a, and let m_b be the integer associated with a minimum in the pattern produced by light with wavelength λ_b. A minimum in one pattern coincides with a minimum in the other if they occur at the same angle. This means $m_a\lambda_a = m_b\lambda_b$. Since $\lambda_a = 2\lambda_b$, the minima coincide if $2m_a = m_b$. Consequently, every other minimum of the λ_b pattern coincides with a minimum of the λ_a pattern.

4. (a) We use Eq. 37-3 to calculate the separation between the first ($m_1 = 1$) and fifth ($m_2 = 5$) minima:

$$\Delta y = D\Delta \sin\theta = D\Delta\left(\frac{m\lambda}{a}\right) = \frac{D\lambda}{a}\Delta m = \frac{D\lambda}{a}(m_2 - m_1) .$$

   Solving for the slit width, we obtain

$$a = \frac{D\lambda(m_2 - m_1)}{\Delta y} = \frac{(400\,\text{mm})(550 \times 10^{-6}\,\text{mm})(5-1)}{0.35\,\text{mm}} = 2.5\ \text{mm} .$$

   (b) For $m = 1$,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(550 \times 10^{-6}\,\text{mm})}{2.5\,\text{mm}} = 2.2 \times 10^{-4} .$$

   The angle is $\theta = \sin^{-1}(2.2 \times 10^{-4}) = 2.2 \times 10^{-4}\,\text{rad}$.

5. (a) A plane wave is incident on the lens so it is brought to focus in the focal plane of the lens, a distance of 70 cm from the lens.

(b) Waves leaving the lens at an angle θ to the forward direction interfere to produce an intensity minimum if $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. The distance on the screen from the center of the pattern to the minimum is given by $y = D\tan\theta$, where D is the distance from the lens to the screen. For the conditions of this problem,

$$\sin\theta = \frac{m\lambda}{a} = \frac{(1)(590\times10^{-9}\,\mathrm{m})}{0.40\times10^{-3}\,\mathrm{m}} = 1.475\times10^{-3} \ .$$

This means $\theta = 1.475\times10^{-3}$ rad and $y = (70\times10^{-2}\,\mathrm{m})\tan(1.475\times10^{-3}\,\mathrm{rad}) = 1.03\times10^{-3}$ m.

6. Let the first minimum be a distance y from the central axis which is perpendicular to the speaker. Then $\sin\theta = y/(D^2+y^2)^{1/2} = m\lambda/a = \lambda/a$ (for $m=1$). Therefore,

$$\begin{aligned} y &= \frac{D}{\sqrt{(a/\lambda)^2-1}} = \frac{D}{\sqrt{(af/v_s)^2-1}} \\ &= \frac{100\,\mathrm{m}}{\sqrt{[(0.300\,\mathrm{m})(3000\,\mathrm{Hz})/(343\,\mathrm{m/s})]^2-1}} = 41.2\ \mathrm{m}\ . \end{aligned}$$

7. The condition for a minimum of intensity in a single-slit diffraction pattern is $a\sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. To find the angular position of the first minimum to one side of the central maximum, we set $m=1$:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{589\times10^{-9}\,\mathrm{m}}{1.00\times10^{-3}\,\mathrm{m}}\right) = 5.89\times10^{-4}\ \mathrm{rad}\ .$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is

$$y_1 = D\tan\theta_1 = (3.00\,\mathrm{m})\tan(5.89\times10^{-4}\,\mathrm{rad}) = 1.767\times10^{-3}\ \mathrm{m}\ .$$

To find the second minimum, we set $m=2$:

$$\theta_2 = \sin^{-1}\left(\frac{2(589\times10^{-9}\,\mathrm{m})}{1.00\times10^{-3}\,\mathrm{m}}\right) = 1.178\times10^{-3}\ \mathrm{rad}\ .$$

The distance from the center of the pattern to this second minimum is $y_2 = D\tan\theta_2 = (3.00\,\mathrm{m})\tan(1.178\times10^{-3}\,\mathrm{rad}) = 3.534\times10^{-3}\,\mathrm{m}$. The separation of the two minima is $\Delta y = y_2 - y_1 = 3.534\,\mathrm{mm} - 1.767\,\mathrm{mm} = 1.77\,\mathrm{mm}$.

8. We note that $\mathrm{nm} = 10^{-9}\,\mathrm{m} = 10^{-6}\,\mathrm{mm}$. From Eq. 37-4,

$$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x\sin\theta) = \left(\frac{2\pi}{589\times10^{-6}\,\mathrm{mm}}\right)\left(\frac{0.10\,\mathrm{mm}}{2}\right)\sin 30^\circ = 266.7\ \mathrm{rad}\ .$$

This is equivalent to $266.7 - 84\pi = 2.8\,\mathrm{rad} = 160^\circ$.

9. We imagine dividing the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor. Then, at the central maximum in the diffraction pattern, we would add the N phasors, all in the same direction and each with the same amplitude. We would find that the intensity there is proportional to N^2. If we double the slit width, we need $2N$ phasors if they are each to have the amplitude of the phasors we used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

10. (a) $\theta = \sin^{-1}(0.011\,\mathrm{cm}/3.5\,\mathrm{m}) = 0.18^\circ$.

(b) We use Eq. 37-6:

$$\alpha = \left(\frac{\pi a}{\lambda}\right)\sin\theta = \frac{\pi(0.025\,\text{mm})\sin 0.18^\circ}{538\times 10^{-6}\,\text{mm}} = 0.46\text{ rad}\ .$$

(c) Making sure our calculator is in radian mode, Eq. 37-5 yields

$$\frac{I(\theta)}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0.93\ .$$

11. (a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where $\alpha = (\pi a/\lambda)\sin\theta$, a is the slit width and λ is the wavelength. The angle θ is measured from the forward direction. We require $I = I_m/2$, so

$$\sin^2\alpha = \frac{1}{2}\alpha^2\ .$$

(b) We evaluate $\sin^2\alpha$ and $\alpha^2/2$ for $\alpha = 1.39$ rad and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, we could also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda)\sin\theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right)\ .$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right)\ .$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right)\ .$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5^\circ\ ,$$

for $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1^\circ\ ,$$

and for $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06^\circ\ .$$

12. Consider Huygens' explanation of diffraction phenomena. When A is in place only the Huygens' wavelets that pass through the hole get to point P. Suppose they produce a resultant electric field E_A. When B is in place, the light that was blocked by A gets to P and the light that passed through the hole in A is blocked. Suppose the electric field at P is now $\vec{E}_B$. The sum $\vec{E}_A + \vec{E}_B$ is the resultant of all waves that get to P when neither A nor B are present. Since P is in the geometric shadow, this is zero. Thus $\vec{E}_A = -\vec{E}_B$, and since the intensity is proportional to the square of the electric field, the intensity at P is the same when A is present as when B is present.

13. (a) The intensity for a single-slit diffraction pattern is given by

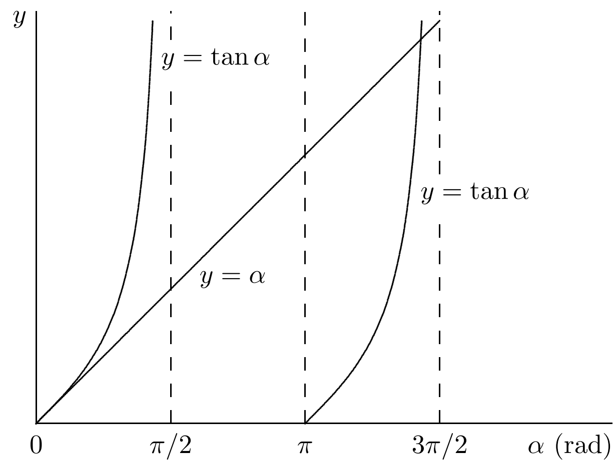
$$I = I_m \frac{\sin^2\alpha}{\alpha^2}$$

where α is described in the text (see Eq. 37-6). To locate the extrema, we set the derivative of I with respect to α equal to zero and solve for α. The derivative is

$$\frac{dI}{d\alpha} = 2I_m \frac{\sin\alpha}{\alpha^3} \left(\alpha\cos\alpha - \sin\alpha\right) \ .$$

The derivative vanishes if $\alpha \neq 0$ but $\sin\alpha = 0$. This yields $\alpha = m\pi$, where m is a nonzero integer. These are the intensity minima: $I = 0$ for $\alpha = m\pi$. The derivative also vanishes for $\alpha\cos\alpha - \sin\alpha = 0$. This condition can be written $\tan\alpha = \alpha$. These implicitly locate the maxima.

(b) The values of α that satisfy $\tan\alpha = \alpha$ can be found by trial and error on a pocket calculator or computer. Each of them is slightly less than one of the values $(m + \frac{1}{2})\pi$ rad, so we start with these values. The first few are 0, 4.4934, 7.7252, 10.9041, 14.0662, and 17.2207. They can also be found graphically. As in the diagram below, we plot $y = \tan\alpha$ and $y = \alpha$ on the same graph. The intersections of the line with the $\tan\alpha$ curves are the solutions. The first two solutions listed above are shown on the diagram.



(c) We write $\alpha = (m + \frac{1}{2})\pi$ for the maxima. For the central maximum, $\alpha = 0$ and $m = -\frac{1}{2}$. For the next, $\alpha = 4.4934$ and $m = 0.930$. For the next, $\alpha = 7.7252$ and $m = 1.959$.

14. We use Eq. 37-12 with $\theta = 2.5^\circ/2 = 1.25^\circ$. Thus,

$$d = \frac{1.22\lambda}{\sin\theta} = \frac{1.22(550\,\text{nm})}{\sin 1.25^\circ} = 31\,\mu\text{m} \ .$$

15. (a) We use the Rayleigh criteria. Thus, the angular separation (in radians) of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. For the headlights of this problem,

$$\theta_R = \frac{1.22(550\times10^{-9}\,\text{m})}{5.0\times10^{-3}\,\text{m}} = 1.34\times10^{-4}\ \text{rad} \ .$$

(b) If L is the distance from the headlights to the eye when the headlights are just resolvable and D is the separation of the headlights, then $D = L\theta_R$, where the small angle approximation is made. This is valid for θ_R in radians. Thus,

$$L = \frac{D}{\theta_R} = \frac{1.4\,\text{m}}{1.34\times10^{-4}\,\text{rad}} = 1.0\times10^4\,\text{m} = 10\ \text{km} \ .$$

16. (a) We use Eq. 37-14:

$$\theta_R = 1.22\,\frac{\lambda}{d} = \frac{(1.22)(540\times10^{-6}\,\text{mm})}{5.0\,\text{mm}} = 1.3\times10^{-4}\ \text{rad} \ .$$

(b) The linear separation is $D = L\theta_R = (160 \times 10^3\,\text{m})(1.3 \times 10^{-4}\,\text{rad}) = 21\,\text{m}$.

17. Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the minimum separation is

$$D = L\theta_R = L\left(1.22\,\frac{\lambda}{d}\right) = \left(3.82 \times 10^8\,\text{m}\right)\frac{(1.22)(550 \times 10^{-9}\,\text{m})}{5.1\,\text{m}} = 50\,\text{m} \ .$$

18. Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the maximum distance is

$$L = \frac{D}{\theta_R} = \frac{D}{1.22\lambda/d} = \frac{(5.0 \times 10^{-3}\,\text{m})(4.0 \times 10^{-3}\,\text{m})}{1.22(550 \times 10^{-9}\,\text{m})} = 30\ \text{m} \ .$$

19. (a) We use the Rayleigh criteria. If L is the distance from the observer to the objects, then the smallest separation D they can have and still be resolvable is $D = L\theta_R$, where θ_R is measured in radians. The small angle approximation is made. Thus,

$$D = \frac{1.22L\lambda}{d} = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.1 \times 10^7\,\text{m} = 1.1 \times 10^4\ \text{km} \ .$$

This distance is greater than the diameter of Mars; therefore, one part of the planet's surface cannot be resolved from another part.

(b) Now $d = 5.1\,\text{m}$ and

$$D = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.1\,\text{m}} = 1.1 \times 10^4\,\text{m} = 11\ \text{km} \ .$$

20. Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the minimum separation is

$$D = L\theta_R = L\left(\frac{1.22\lambda}{d}\right) = \frac{(6.2 \times 10^3\,\text{m})(1.22)(1.6 \times 10^{-2}\,\text{m})}{2.3\,\text{m}} = 53\ \text{m} \ .$$

21. Eq. 37-14 gives $\theta_R = 1.22\lambda/d$, where in our case $\theta_R \approx D/L$, with $D = 60\,\mu\text{m}$ being the size of the object your eyes must resolve, and L being the maximum viewing distance in question. If $d = 3.00\,\text{mm} = 3000\,\mu\text{m}$ is the diameter of your pupil, then

$$L = \frac{Dd}{1.22\lambda} = \frac{(60\,\mu\text{m})(3000\,\mu\text{m})}{1.22(0.55\,\mu\text{m})} = 2.7 \times 10^5\ \mu\text{m} = 27\ \text{cm} \ .$$

22. Since we are considering the *diameter* of the central diffraction maximum, then we are working with *twice* the Rayleigh angle. Using notation similar to that in Sample Problem 37-6 (which is in the textbook supplement), we have $2(1.22\lambda/d) = D/L$. Therefore,

$$d = 2\frac{1.22\lambda L}{D} = 2\frac{(1.22)(500 \times 10^{-9}\,\text{m})(3.54 \times 10^5\,\text{m})}{9.1\,\text{m}} = 0.047\ \text{m} \ .$$

23. (a) The first minimum in the diffraction pattern is at an angular position θ, measured from the center of the pattern, such that $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\text{m/s}}{220 \times 10^9\,\text{Hz}} = 1.36 \times 10^{-3}\ \text{m} \ .$$

Thus

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(1.36 \times 10^{-3}\,\text{m})}{55.0 \times 10^{-2}\,\text{m}}\right) = 3.02 \times 10^{-3}\ \text{rad} \ .$$

The angular width of the central maximum is twice this, or $6.04 \times 10^{-3}\,\text{rad}$ $(0.346°)$.

(b) Now $\lambda = 1.6\,\text{cm}$ and $d = 2.3\,\text{m}$, so

$$\theta = \sin^{-1}\left(\frac{1.22(1.6\times 10^{-2}\,\text{m})}{2.3\,\text{m}}\right) = 8.5\times 10^{-3}\ \text{rad}\ .$$

The angular width of the central maximum is $1.7\times 10^{-2}\,\text{rad}$ $(0.97°)$.

24. (a) Since $\theta = 1.22\lambda/d$, the larger the wavelength the larger the radius of the first minimum (and second maximum, etc). Therefore, the white pattern is outlined by red lights (with longer wavelength than blue lights).

(b) The diameter of a water drop is

$$d = \frac{1.22\lambda}{\theta} \approx \frac{1.22(7\times 10^{-7}\,\text{m})}{1.5(0.50°)(\pi/180°)/2} = 1.3\times 10^{-4}\ \text{m}\ .$$

25. (a) Using Eq. 37-14, the angular separation is

$$\theta_{\text{R}} = \frac{1.22\lambda}{d} = \frac{(1.22)(550\times 10^{-9}\,\text{m})}{0.76\,\text{m}} = 8.8\times 10^{-7}\ \text{rad}\ .$$

(b) Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the distance between the stars is

$$D = L\theta_{\text{R}} = \frac{(10\,\text{ly})(9.46\times 10^{12}\,\text{km/ly})(0.18)\pi}{(3600)(180)} = 8.4\times 10^{7}\ \text{km}\ .$$

(c) The diameter of the first dark ring is

$$d = 2\theta_{\text{R}}L = \frac{2(0.18)(\pi)(14\,\text{m})}{(3600)(180)} = 2.5\times 10^{-5}\,\text{m} = 0.025\ \text{mm}\ .$$

26. We denote the Earth-Moon separation as L. The energy of the beam of light which is projected onto the moon is concentrated in a circular spot of diameter d_1, where $d_1/L = 2\theta_{\text{R}} = 2(1.22\lambda/d_0)$, with d_0 the diameter of the mirror on Earth. The fraction of energy picked up by the reflector of diameter d_2 on the Moon is then $\eta' = (d_2/d_1)^2$. This reflected light, upon reaching the Earth, has a circular cross section of diameter d_3 satisfying $d_3/L = 2\theta_{\text{R}} = 2(1.22\lambda/d_2)$. The fraction of the reflected energy that is picked up by the telescope is then $\eta'' = (d_0/d_3)^2$. Consequently, the fraction of the original energy picked up by the detector is

$$\begin{aligned}\eta &= \eta'\eta'' = \left(\frac{d_0}{d_3}\right)^2\left(\frac{d_2}{d_1}\right)^2 = \left[\frac{d_0 d_2}{(2.44\lambda d_{em}/d_0)(2.44\lambda d_{em}/d_2)}\right]^2 = \left(\frac{d_0 d_2}{2.44\lambda d_{em}}\right)^4 \\ &= \left[\frac{(2.6\,\text{m})(0.10\,\text{m})}{2.44(0.69\times 10^{-6}\,\text{m})(3.82\times 10^{8}\,\text{m})}\right]^4 \approx 4\times 10^{-13}\ .\end{aligned}$$

27. Bright interference fringes occur at angles θ given by $d\sin\theta = m\lambda$, where m is an integer. For the slits of this problem, $d = 11a/2$, so $a\sin\theta = 2m\lambda/11$ (see Sample Problem 37-4). The first minimum of the diffraction pattern occurs at the angle θ_1 given by $a\sin\theta_1 = \lambda$, and the second occurs at the angle θ_2 given by $a\sin\theta_2 = 2\lambda$, where a is the slit width. We should count the values of m for which $\theta_1 < \theta < \theta_2$, or, equivalently, the values of m for which $\sin\theta_1 < \sin\theta < \sin\theta_2$. This means $1 < (2m/11) < 2$. The values are $m = 6$, 7, 8, 9, and 10. There are five bright fringes in all.

28. In a manner similar to that discussed in Sample Problem 37-4, we find the number is $2(d/a) - 1 = 2(2a/a) - 1 = 3$.

29. (a) In a manner similar to that discussed in Sample Problem 37-4, we find the ratio should be $d/a = 4$. Our reasoning is, briefly, as follows: we let the location of the fourth bright fringe coincide with the first minimum of diffraction pattern, and then set $\sin\theta = 4\lambda/d = \lambda/a$ (so $d = 4a$).

(b) Any bright fringe which happens to be at the same location with a diffraction minimum will vanish. Thus, if we let $\sin\theta = m_1\lambda/d = m_2\lambda/a = m_1\lambda/4a = m_2\lambda/a$, or $m_1 = 4m_2$ where $m_2 = 1, 2, 3, \cdots$. The fringes missing are the 4th, 8th, 12th, and so on. Hence, every fourth fringe is missing.

30. The angular location of the mth bright fringe is given by $d\sin\theta = m\lambda$, so the linear separation between two adjacent fringes is

$$\Delta y = \Delta(D\sin\theta) = \Delta\left(\frac{D_m\lambda}{d}\right) = \frac{D\lambda}{d}\Delta m = \frac{D\lambda}{d} \ .$$

31. (a) The angular positions θ of the bright interference fringes are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a\sin\theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so we should count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or, equivalently, the number of values of m for which $-\sin\theta_1 < \sin\theta < +\sin\theta_1$. This means $-1/a < m/d < 1/a$ or $-d/a < m < +d/a$. Now $d/a = (0.150 \times 10^{-3}\,\text{m})/(30.0 \times 10^{-6}\,\text{m}) = 5.00$, so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are nine fringes.

(b) The intensity at the screen is given by

$$I = I_m\,\left(\cos^2\beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2$$

where $\alpha = (\pi a/\lambda)\sin\theta$, $\beta = (\pi d/\lambda)\sin\theta$, and I_m is the intensity at the center of the pattern. For the third bright interference fringe, $d\sin\theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2\beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi}\right)^2 = 0.255 \ .$$

The intensity ratio is $I/I_m = 0.255$.

32. (a) The first minimum of the diffraction pattern is at $5.00°$, so

$$a = \frac{\lambda}{\sin\theta} = \frac{0.440\,\mu\text{m}}{\sin 5.00°} = 5.05\,\mu\text{m} \ .$$

(b) Since the fourth bright fringe is missing, $d = 4a = 4(5.05\,\mu\text{m}) = 20.2\,\mu\text{m}$.

(c) For the $m = 1$ bright fringe,

$$\alpha = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi(5.05\,\mu\text{m})\sin 1.25°}{0.440\,\mu\text{m}} = 0.787 \text{ rad} \ .$$

Consequently, the intensity of the $m = 1$ fringe is

$$I = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2 = (7.0\,\text{mW/cm}^2)\left(\frac{\sin 0.787\,\text{rad}}{0.787}\right)^2 = 5.7 \text{ mW/cm}^2 \ ,$$

which agrees with Fig. 37-36. Similarly for $m = 2$, the intensity is $I = 2.9\,\text{mW/cm}^2$, also in agreement with Fig. 37-36.

33. (a) $d = 20.0\,\text{mm}/6000 = 0.00333\,\text{mm} = 3.33\,\mu\text{m}$.

(b) Let $d\sin\theta = m\lambda\,(m = 0, \pm1, \pm2, \cdots)$. We find $\theta = 0$ for $m = 0$, and

$$\theta = \sin^{-1}(\pm\lambda/d) = \sin^{-1}\left(\pm\frac{0.589\,\mu\text{m}}{3.30\,\mu\text{m}}\right) = \pm10.2^\circ$$

for $m = \pm1$. Similarly, we find $\pm20.7^\circ$ for $m = \pm2$, $\pm32.2^\circ$ for $m = \pm3$, $\pm45^\circ$ for $m = \pm4$, and $\pm62.2^\circ$ for $m = \pm5$. Since $|m|\lambda/d > 1$ for $|m| \geq 6$, these are all the maxima.

34. The angular location of the mth order diffraction maximum is given by $m\lambda = d\sin\theta$. To be able to observe the fifth-order maximum, we must let $\sin\theta|_{m=5} = 5\lambda/d < 1$, or

$$\lambda < \frac{d}{5} = \frac{1.00\,\text{nm}/315}{5} = 635\ \text{nm}\ .$$

Therefore, all wavelengths shorter than 635 nm can be used.

35. The ruling separation is $d = 1/(400\,\text{mm}^{-1}) = 2.5\times10^{-3}\,\text{mm}$. Diffraction lines occur at angles θ such that $d\sin\theta = m\lambda$, where λ is the wavelength and m is an integer. Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. We take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90°. That is, find the greatest integer value of m for which $m\lambda < d$. Since $d/\lambda = (2.5\times10^{-6}\,\text{m})/(700\times10^{-9}\,\text{m}) = 3.57$, that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

36. We use Eq. 37-22 for diffraction maxima: $d\sin\theta = m\lambda$. In our case, since the angle between the $m = 1$ and $m = -1$ maxima is 26°, the angle θ corresponding to $m = 1$ is $\theta = 26^\circ/2 = 13^\circ$. We solve for the grating spacing:

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(550\,\text{nm})}{\sin 13^\circ} = 2.4\,\mu\text{m}\ .$$

37. (a) Maxima of a diffraction grating pattern occur at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The two lines are adjacent, so their order numbers differ by unity. Let m be the order number for the line with $\sin\theta = 0.2$ and $m+1$ be the order number for the line with $\sin\theta = 0.3$. Then, $0.2d = m\lambda$ and $0.3d = (m+1)\lambda$. We subtract the first equation from the second to obtain $0.1d = \lambda$, or $d = \lambda/0.1 = (600\times10^{-9}\,\text{m})/0.1 = 6.0\times10^{-6}\,\text{m}$.

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a\sin\theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a\sin\theta = \lambda$. It is also given by $d\sin\theta = 4\lambda$, so $a = d/4 = (6.0\times10^{-6}\,\text{m})/4 = 1.5\times10^{-6}\,\text{m}$.

(c) First, we set $\theta = 90^\circ$ and find the largest value of m for which $m\lambda < d\sin\theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since $d/\lambda = (6.0\times10^{-6}\,\text{m})/(600\times10^{-9}\,\text{m}) = 10.0$, the highest order seen is the $m = 9$ order. The fourth and eighth orders are missing, so the observable orders are $m = 0$, 1, 2, 3, 5, 6, 7, and 9.

38. (a) For the maximum with the greatest value of $m\,(= M)$ we have $M\lambda = a\sin\theta < d$, so $M < d/\lambda = 900\,\text{nm}/600\,\text{nm} = 1.5$, or $M = 1$. Thus three maxima can be seen, with $m = 0, \pm1$.

(b) From Eq. 37-25

$$\begin{aligned}\Delta\theta_{\text{hw}} &= \frac{\lambda}{Nd\cos\theta} = \frac{d\sin\theta}{Nd\cos\theta} = \frac{\tan\theta}{N} = \frac{1}{N}\tan\left[\sin^{-1}\left(\frac{\lambda}{d}\right)\right] \\ &= \frac{1}{1000}\tan\left[\sin^{-1}\left(\frac{600\,\text{nm}}{900\,\text{nm}}\right)\right] = 0.051^\circ\ .\end{aligned}$$

39. The angular positions of the first-order diffraction lines are given by $d\sin\theta = \lambda$. Let λ_1 be the shorter wavelength (430 nm) and θ be the angular position of the line associated with it. Let λ_2 be the longer wavelength (680 nm), and let $\theta+\Delta\theta$ be the angular position of the line associated with it. Here $\Delta\theta = 20°$. Then, $d\sin\theta = \lambda_1$ and $d\sin(\theta+\Delta\theta) = \lambda_2$. We write $\sin(\theta+\Delta\theta)$ as $\sin\theta\cos\Delta\theta + \cos\theta\sin\Delta\theta$, then use the equation for the first line to replace $\sin\theta$ with λ_1/d, and $\cos\theta$ with $\sqrt{1-\lambda_1^2/d^2}$. After multiplying by d, we obtain

$$\lambda_1\cos\Delta\theta + \sqrt{d^2-\lambda_1^2}\sin\Delta\theta = \lambda_2 \ .$$

Solving for d, we find

$$\begin{aligned} d &= \sqrt{\frac{(\lambda_2-\lambda_1\cos\Delta\theta)^2+(\lambda_1\sin\Delta\theta)^2}{\sin^2\Delta\theta}} \\ &= \sqrt{\frac{[(680\,\text{nm})-(430\,\text{nm})\cos 20°]^2+[(430\,\text{nm})\sin 20°]^2}{\sin^2 20°}} \\ &= 914\,\text{nm} = 9.14\times10^{-4}\ \text{mm} \ . \end{aligned}$$

There are $1/d = 1/(9.14\times10^{-4}\,\text{mm}) = 1090$ rulings per mm.

40. We use Eq. 37-22. For $m = \pm1$

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.73\mu\text{m})\sin(\pm17.6°)}{\pm1} = 523\ \text{nm} \ ,$$

and for $m = \pm2$

$$\lambda = \frac{(1.73\mu\text{m})\sin(\pm37.3°)}{\pm2} = 524\,\text{nm} \ .$$

Similarly, we may compute the values of λ corresponding to the angles for $m = \pm3$. The average value of these λ's is 523 nm.

41. Consider two of the rays shown in Fig. 37-37, one just above the other. The extra distance traveled by the lower one may be found by drawing perpendiculars from where the top ray changes direction (point P) to the incident and diffracted paths of the lower one. Where these perpendiculars intersect the lower ray's paths are here referred to as points A and C. Where the bottom ray changes direction is point B. We note that angle $\angle APB$ is the same as ψ, and angle BPC is the same as θ (see Fig. 37-37). The difference in path lengths between the two adjacent light rays is $\Delta x = |AB| + |BC| = d\sin\psi + d\sin\theta$. The condition for bright fringes to occur is therefore

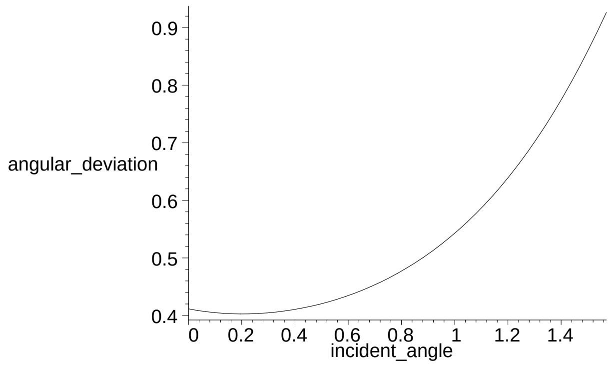
$$\Delta x = d\,(\sin\psi + \sin\theta) = m\lambda$$

where $m = 0, 1, 2, \cdots$. If we set $\psi = 0$ then this reduces to Eq. 37-22.

42. Referring to problem 41, we note that the angular deviation of a diffracted ray (the angle between the forward extrapolation of the incident ray and its diffracted ray) is $\psi+\theta$. For $m = 1$, this becomes

$$\psi+\theta = \psi + \sin^{-1}\left(\frac{\lambda}{d} - \sin\psi\right)$$

where the ratio $\lambda/d = 0.40$ using the values given in the problem statement. The graph of this is shown below (with radians used along both axes).



43. The derivation is similar to that used to obtain Eq. 37-24. At the first minimum beyond the mth principal maximum, two waves from adjacent slits have a phase difference of $\Delta\phi = 2\pi m + (2\pi/N)$, where N is the number of slits. This implies a difference in path length of $\Delta L = (\Delta\phi/2\pi)\lambda = m\lambda + (\lambda/N)$. If θ_m is the angular position of the mth maximum, then the difference in path length is also given by $\Delta L = d\sin(\theta_m + \Delta\theta)$. Thus $d\sin(\theta_m + \Delta\theta) = m\lambda + (\lambda/N)$. We use the trigonometric identity $\sin(\theta_m + \Delta\theta) = \sin\theta_m\cos\Delta\theta + \cos\theta_m\sin\Delta\theta$. Since $\Delta\theta$ is small, we may approximate $\sin\Delta\theta$ by $\Delta\theta$ in radians and $\cos\Delta\theta$ by unity. Thus $d\sin\theta_m + d\,\Delta\theta\cos\theta_m = m\lambda + (\lambda/N)$. We use the condition $d\sin\theta_m = m\lambda$ to obtain $d\,\Delta\theta\cos\theta_m = \lambda/N$ and

$$\Delta\theta = \frac{\lambda}{Nd\cos\theta_m} .$$

44. At the point on the screen where we find the inner edge of the hole, we have $\tan\theta = 5.0\,\text{cm}/30\,\text{cm}$, which gives $\theta = 9.46°$. We note that d for the grating is equal to $1.0\,\text{mm}/350 = 1.0\times10^6\,\text{nm}/350$. From $m\lambda = d\sin\theta$, we find

$$m = \frac{d\sin\theta}{\lambda} = \frac{\left(\frac{1.0\times10^6\,\text{nm}}{350}\right)(0.1644)}{\lambda} = \frac{470\,\text{nm}}{\lambda} .$$

Since for white light $\lambda > 400\,\text{nm}$, the only integer m allowed here is $m = 1$. Thus, at one edge of the hole, $\lambda = 470\,\text{nm}$. However, at the other edge, we have $\tan\theta' = 6.0\,\text{cm}/30\,\text{cm}$, which gives $\theta' = 11.31°$. This leads to

$$\lambda' = d\sin\theta' = \left(\frac{1.0\times10^6\,\text{nm}}{350}\right)\sin 11.31° = 560\ \text{nm} .$$

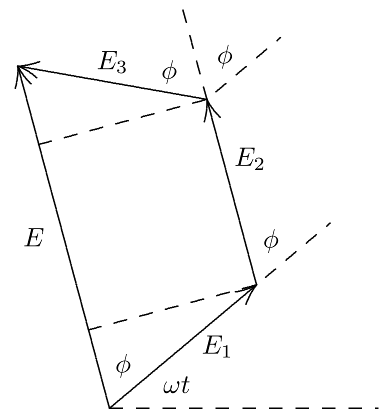
Consequently, the range of wavelength is from 470 to 560 nm.

45. Since the slit width is much less than the wavelength of the light, the central peak of the single-slit diffraction pattern is spread across the screen and the diffraction envelope can be ignored. Consider three waves, one from each slit. Since the slits are evenly spaced, the phase difference for waves from the first and second slits is the same as the phase difference for waves from the second and third slits. The electric fields of the waves at the screen can be written $E_1 = E_0\sin(\omega t)$, $E_2 = E_0\sin(\omega t + \phi)$, and $E_3 = E_0\sin(\omega t + 2\phi)$, where $\phi = (2\pi d/\lambda)\sin\theta$. Here d is the separation of adjacent slits and λ is the wavelength. The phasor diagram is shown below. It yields

$$E = E_0\cos\phi + E_0 + E_0\cos\phi = E_0(1 + 2\cos\phi)$$

for the amplitude of the resultant wave. Since the intensity of a wave is proportional to the square of the electric field, we may write $I = AE_0^2(1 + 2\cos\phi)^2$, where A is a constant of proportionality. If I_m is the intensity at the center of the pattern, for which $\phi = 0$, then $I_m = 9AE_0^2$. We take A to be $I_m/9E_0^2$ and obtain

$$I = \frac{I_m}{9}\,(1 + 2\cos\phi)^2 = \frac{I_m}{9}\,\left(1 + 4\cos\phi + 4\cos^2\phi\right) .$$



46. Letting $R = \lambda/\Delta\lambda = Nm$, we solve for N:

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(589.6\,\text{nm} + 589.0\,\text{nm})/2}{2(589.6\,\text{nm} - 589.0\,\text{nm})} = 491 \ .$$

47. If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{\text{avg}}/\Delta\lambda$. The text shows this is Nm, where N is the number of rulings in the grating and m is the order of the lines. Thus $\lambda_{\text{avg}}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{\text{avg}}}{m\,\Delta\lambda} = \frac{656.3\,\text{nm}}{(1)(0.18\,\text{nm})} = 3650\,\text{rulings} \ .$$

48. (a) We find $\Delta\lambda$ from $R = \lambda/\Delta\lambda = Nm$:

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{500\,\text{nm}}{(600/\text{mm})(5.0\,\text{mm})(3)} = 0.056\,\text{nm} = 56 \text{ pm} \ .$$

(b) Since $\sin\theta = m_{\text{max}}\lambda/d < 1$,

$$m_{\text{max}} < \frac{d}{\lambda} = \frac{1}{(600/\text{mm})(500 \times 10^{-6}\,\text{mm})} = 3.3 \ .$$

Therefore, $m_{\text{max}} = 3$. No higher orders of maxima can be seen.

49. The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ. The angular position and wavelength are related by $\mathbf{d}\sin\theta = m\lambda$, where $\mathbf{d}$ is the slit separation (which we made boldfaced in order not to confuse it with the d used in the derivative, below) and m is an integer. We differentiate this expression with respect to θ to obtain

$$\frac{d\theta}{d\lambda}\,\mathbf{d}\,\cos\theta = m \ ,$$

or

$$D = \frac{d\theta}{d\lambda} = \frac{m}{\mathbf{d}\cos\theta} \ .$$

Now $m = (\mathbf{d}/\lambda)\sin\theta$, so

$$D = \frac{\mathbf{d}\sin\theta}{\mathbf{d}\lambda\cos\theta} = \frac{\tan\theta}{\lambda} \ .$$

50. (a) From $d\sin\theta = m\lambda$ we find

$$d = \frac{m\lambda_{\text{avg}}}{\sin\theta} = \frac{3(589.3\,\text{nm})}{\sin 10^\circ} = 1.0 \times 10^4\,\text{nm} = 10\,\mu\text{m} \ .$$

(b) The total width of the ruling is

$$L = Nd = \left(\frac{R}{m}\right) d = \frac{\lambda_{\rm avg} d}{m\Delta\lambda} = \frac{(589.3\,{\rm nm})(10\,\mu{\rm m})}{3(589.59\,{\rm nm} - 589.00\,{\rm nm})} = 3.3\times 10^3 \mu{\rm m} = 3.3~{\rm mm}~.$$

51. (a) Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm, the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\,\Delta\lambda + \lambda\,\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f}\,\Delta f = -\frac{\lambda^2}{c}\,\Delta f$$

where $f = c/\lambda$ is used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength. We may interpret Δf as the range of frequencies that can be resolved and take it to be positive. Then,

$$\frac{\lambda^2}{c}\,\Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}~.$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are separated by $(N-1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N-1)d\sin\theta$ and the time difference is

$$\Delta t = \frac{(N-1)d\sin\theta}{c}~.$$

If N is large, this may be approximated by $\Delta t = (Nd/c)\sin\theta$. The lens does not affect the travel time.

(c) Substituting the expressions we derived for Δt and Δf, we obtain

$$\Delta f\,\Delta t = \left(\frac{c}{Nm\lambda}\right)\left(\frac{Nd\sin\theta}{c}\right) = \frac{d\sin\theta}{m\lambda} = 1~.$$

The condition $d\sin\theta = m\lambda$ for a diffraction line is used to obtain the last result.

52. (a) From the expression for the half-width $\Delta\theta_{\rm hw}$ (given by Eq. 37-25) and that for the resolving power R (given by Eq. 37-29), we find the product of $\Delta\theta_{\rm hw}$ and R to be

$$\Delta\theta_{\rm hw}R = \left(\frac{\lambda}{Nd\cos\theta}\right) Nm = \frac{m\lambda}{d\cos\theta} = \frac{d\sin\theta}{d\cos\theta} = \tan\theta\,,$$

where we used $m\lambda = d\sin\theta$ (see Eq. 37-22).

(b) For first order $m = 1$, so the corresponding angle θ_1 satisfies $d\sin\theta_1 = m\lambda = \lambda$. Thus the product in question is given by

$$\begin{aligned}\tan\theta_1 &= \frac{\sin\theta_1}{\cos\theta_1} = \frac{\sin\theta_1}{\sqrt{1-\sin^2\theta_1}}\\ &= \frac{1}{\sqrt{(1/\sin\theta_1)^2 - 1}} = \frac{1}{\sqrt{(d/\lambda)^2 - 1}}\\ &= \frac{1}{\sqrt{(900\,{\rm nm}/600\,{\rm nm})^2 - 1}} = 0.89~.\end{aligned}$$

53. Bragg's law gives the condition for a diffraction maximum:

$$2d\sin\theta = m\lambda$$

where d is the spacing of the crystal planes and λ is the wavelength. The angle θ is measured from the surfaces of the planes. For a second-order reflection $m = 2$, so

$$d = \frac{m\lambda}{2\sin\theta} = \frac{2(0.12 \times 10^{-9}\,\mathrm{m})}{2\sin 28^\circ} = 2.56 \times 10^{-10}\,\mathrm{m} = 256\ \mathrm{pm}\ .$$

54. We use Eq. 37-31. From the peak on the left at angle 0.75° (estimated from Fig. 37-38), we have

$$\lambda_1 = 2d\sin\theta_1 = 2(0.94\,\mathrm{nm})\sin(0.75^\circ) = 0.025\,\mathrm{nm} = 25\ \mathrm{pm}\ .$$

This estimation should be viewed as reliable to within ± 2 pm. We now consider the next peak:

$$\lambda_2 = 2d\sin\theta_2 = 2(0.94\,\mathrm{nm})\sin 1.15^\circ = 0.038\,\mathrm{nm} = 38\ \mathrm{pm}\ .$$

One can check that the third peak from the left is the second-order one for λ_1.

55. The x ray wavelength is $\lambda = 2d\sin\theta = 2(39.8\,\mathrm{pm})\sin 30.0^\circ = 39.8\,\mathrm{pm}$.

56. (a) For the first beam $2d\sin\theta_1 = \lambda_A$ and for the second one $2d\sin\theta_2 = 3\lambda_B$. The values of d and λ_A can then be determined:

$$d = \frac{3\lambda_B}{2\sin\theta_2} = \frac{3(97\,\mathrm{pm})}{2\sin 60^\circ} = 1.7 \times 10^2\ \mathrm{pm}\ .$$

(b)

$$\lambda_A = 2d\sin\theta_1 = 2(1.7 \times 10^2\,\mathrm{pm})(\sin 23^\circ) = 1.3 \times 10^2\ \mathrm{pm}\ .$$

57. There are two unknowns, the x-ray wavelength λ and the plane separation d, so data for scattering at two angles from the same planes should suffice. The observations obey Bragg's law, so

$$2d\sin\theta_1 = m_1\lambda$$

and

$$2d\sin\theta_2 = m_2\lambda\ .$$

However, these cannot be solved for the unknowns. For example, we can use the first equation to eliminate λ from the second. We obtain

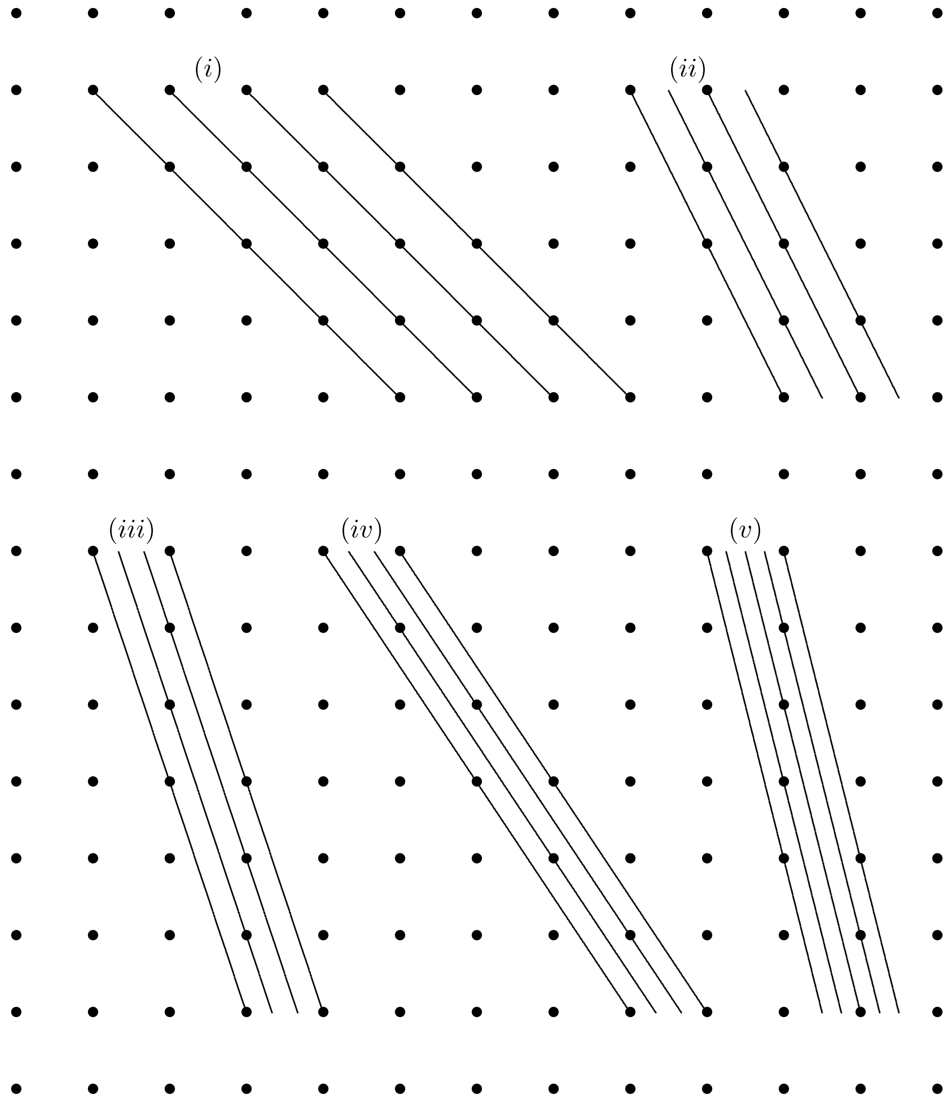
$$m_2\sin\theta_1 = m_1\sin\theta_2\ ,$$

an equation that does not contain either of the unknowns.

58. The angle of incidence on the reflection planes is $\theta = 63.8^\circ - 45.0^\circ = 18.8^\circ$, and the plane-plane separation is $d = a_0/\sqrt{2}$. Thus, using $2d\sin\theta = \lambda$, we get

$$a_0 = \sqrt{2}d = \frac{\sqrt{2}\lambda}{2\sin\theta} = \frac{0.260\,\mathrm{nm}}{\sqrt{2}\sin 18.8^\circ} = 0.570\ \mathrm{nm}\ .$$

59. (a) The sets of planes with the next five smaller interplanar spacings (after a_0) are shown in the diagram below.



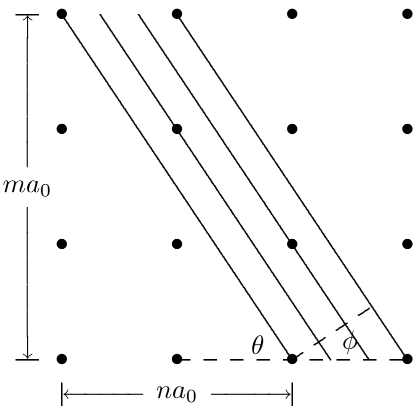
In terms of a_0, the spacings are:

$$\begin{aligned}
(i):&\quad a_0/\sqrt{2} = 0.7071a_0\\
(ii):&\quad a_0/\sqrt{5} = 0.4472a_0\\
(iii):&\quad a_0/\sqrt{10} = 0.3162a_0\\
(iv):&\quad a_0/\sqrt{13} = 0.2774a_0\\
(v):&\quad a_0/\sqrt{17} = 0.2425a_0
\end{aligned}$$

(b) Since a crystal plane passes through lattice points, its slope can be written as the ratio of two integers. Consider a set of planes with slope m/n, as shown in the diagram below. The first and last planes shown pass through adjacent lattice points along a horizontal line and there are $m-1$ planes between. If h is the separation of the first and last planes, then the interplanar spacing is $d = h/m$. If the planes make the angle θ with the horizontal, then the normal to the planes (shown dotted) makes the angle $\phi = 90^\circ - \theta$. The distance h is given by $h = a_0 \cos\phi$ and the interplanar spacing is $d = h/m = (a_0/m)\cos\phi$. Since $\tan\theta = m/n$, $\tan\phi = n/m$ and

$\cos\phi = 1/\sqrt{1+\tan^2\phi} = m/\sqrt{n^2+m^2}$. Thus,

$$d = \frac{h}{m} = \frac{a_0\cos\phi}{m} = \frac{a_0}{\sqrt{n^2+m^2}} \ .$$



60. The wavelengths satisfy $m\lambda = 2d\sin\theta = 2(275\,\text{pm})(\sin 45°) = 389\,\text{pm}$. In the range of wavelengths given, the allowed values of m are $m = 3$, 4, with the corresponding wavelengths being $389\,\text{pm}/3 = 130\,\text{pm}$ and $389\,\text{pm}/4 = 97.2\,\text{pm}$, respectively.

61. We want the reflections to obey the Bragg condition $2d\sin\theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. We solve for θ:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{(0.125\times10^{-9}\,\text{m})m}{2(0.252\times10^{-9}\,\text{m})}\right) = 0.2480m \ .$$

For $m = 1$ this gives $\theta = 14.4°$. The crystal should be turned $45° - 14.4° = 30.6°$ clockwise. For $m = 2$ it gives $\theta = 29.7°$. The crystal should be turned $45° - 29.7° = 15.3°$ clockwise. For $m = 3$ it gives $\theta = 48.1°$. The crystal should be turned $48.1° - 45° = 3.1°$ counterclockwise. For $m = 4$ it gives $\theta = 82.8°$. The crystal should be turned $82.8° - 45° = 37.8°$ counterclockwise. There are no intensity maxima for $m > 4$ as one can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4.

62. (a) Eq. 37-3 and Eq. 37-12 imply smaller angles for diffraction for smaller wavelengths. This suggests that diffraction effects in general would decrease.

    (b) Using Eq. 37-3 with $m = 1$ and solving for 2θ (the angular width of the central diffraction maximum), we find

$$2\theta = 2\sin^{-1}\left(\frac{\lambda}{a}\right) = 2\sin^{-1}\left(\frac{0.50\,\text{m}}{5.0\,\text{m}}\right) = 11° \ .$$

    (c) A similar calculation yields 0.23° for $\lambda = 0.010$ m.

63. (a) Using the notation of Sample Problem 37-6 (which is in the textbook supplement), the minimum separation is

$$D = L\theta_\text{R} = L\left(\frac{1.22\lambda}{d}\right) = \frac{(400\times10^3\,\text{m})(1.22)(550\times10^{-9}\,\text{m})}{(0.005\,\text{m})} \approx 50\ \text{m} \ .$$

    (b) The Rayleigh criterion suggests that the astronaut will not be able to discern the Great Wall (see the result of part (a)).

    (c) The signs of intelligent life would probably be, at most, ambiguous on the sunlit half of the planet. However, while passing over the half of the planet on the opposite side from the Sun, the astronaut would be able to notice the effects of artificial lighting.

64. Consider two light rays crossing each other at the middle of the lens (see Fig. 37-42(c)). The rays come from opposite sides of the circular dot of diameter D, a distance L from the eyes, so we are using the same notation found in Sample Problem 37-6 (which is in the textbook supplement). Those two rays reach the retina a distance L' behind the lens, striking two points there which are a distance D' apart. Therefore,

$$\frac{D}{L} = \frac{D'}{L'}$$

where $D = 2\,\text{mm}$ and $L' = 20\,\text{mm}$. If we estimate $L \approx 450\,\text{mm}$, we find $D' \approx 0.09\,\text{mm}$. Turning our attention to Fig. 37-42(d), we see

$$\theta = \tan^{-1}\left(\frac{\frac{1}{2}D'}{x}\right)$$

which we wish to set equal to the angle in Eq. 37-12. We could use the small angle approximation $\sin\theta \approx \tan\theta$ to relate these directly, or we could be "exact" – as we show below:

$$\text{If} \quad \tan\phi = \frac{b}{a}\ , \quad \text{then} \quad \sin\phi = \frac{b}{\sqrt{a^2+b^2}} \ .$$

Therefore, this "exact" use of Eq. 37-12 leads to

$$1.22\frac{\lambda}{d} = \sin\theta = \frac{\frac{1}{2}D'}{\sqrt{x^2 + (D'/2)^2}}$$

where $\lambda = 550 \times 10^{-6}\,\text{mm}$ and $1\,\text{mm} \le x \le 15\,\text{mm}$. Using the value of D' found above, this leads to a range of d values: $0.015\,\text{mm} \le d \le 0.23\,\text{mm}$.

65. Using the same notation found in Sample Problem 37-6,

$$\frac{D}{L} = \theta_{\text{R}} = 1.22\frac{\lambda}{d}$$

where we will assume a "typical" wavelength for visible light: $\lambda \approx 550 \times 10^{-9}\,\text{m}$.

(a) With $L = 400 \times 10^3$ m and $D = 0.85$ m, the above relation leads to $d = 0.32$ m.

(b) Now with $D = 0.10$ m, the above relation leads to $d = 2.7$ m.

(c) The military satellites do not use Hubble Telescope-sized apertures. A great deal of very sophisticated optical filtering and digital signal processing techniques go into the final product, for which there is not space for us to describe here.

66. Assuming all $N = 2000$ lines are uniformly illuminated, we have

$$\frac{\lambda_{\text{av}}}{\Delta\lambda} = Nm$$

from Eq. 37-28 and Eq. 37-29. With $\lambda_{\text{av}} = 600$ nm and $m = 2$, we find $\Delta\lambda = 0.15$ nm.

67. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1}\frac{\lambda}{a} \ .$$

The maxima in the double-slit pattern are located at

$$\theta_m = \sin^{-1}\frac{m\lambda}{d} \ ,$$

so that our range specification becomes

$$-\sin^{-1}\frac{\lambda}{a} \ < \ \sin^{-1}\frac{m\lambda}{d} < \ +\sin^{-1}\frac{\lambda}{a} \ ,$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} < \frac{m\lambda}{d} < +\frac{\lambda}{a} \ .$$

Rewriting this as $-d/a < m < +d/a$, we find $-6 < m < +6$, or, since m is an integer, $-5 \le m \le +5$. Thus, we find eleven values of m that satisfy this requirement.

68. Employing Eq. 37-3, we find (with $m = 3$ and all lengths in μm)

$$\theta = \sin^{-1}\frac{m\lambda}{a} = \sin^{-1}\frac{(3)(0.5)}{2}$$

which yields $\theta = 48.6°$. Now, we use the experimental geometry ($\tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern) to find

$$y = D\tan\theta = 2.27 \text{ m} \ .$$

69. (a) From $R = \lambda/\Delta\lambda = Nm$ we find

$$N = \frac{\lambda}{m\Delta\lambda} = \frac{(415.496\,\text{nm} + 415.487\,\text{nm})/2}{2(415.96\,\text{nm} - 415.487\,\text{nm})} = 23100 \ .$$

(b) We note that $d = (4.0 \times 10^7\,\text{nm})/23100 = 1732\,\text{nm}$. The maxima are found at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(2)(415.5\,\text{nm})}{1732\,\text{nm}}\right] = 28.7° \ .$$

70. We use Eq. 37-31. For smallest value of θ, we let $m = 1$. Thus,

$$\theta_{\text{min}} = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left[\frac{(1)(30\,\text{pm})}{2(0.30 \times 10^3\,\text{pm})}\right] = 2.9° \ .$$

71. (a) We use Eq. 37-12:

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left[\frac{1.22(v_s/f)}{d}\right] \\ &= \sin^{-1}\left[\frac{(1.22)(1450\,\text{m/s})}{(25 \times 10^3\,\text{Hz})(0.60\,\text{m})}\right] = 6.8° \ .\end{aligned}$$

(b) Now $f = 1.0 \times 10^3$ Hz so

$$\frac{1.22\lambda}{d} = \frac{(1.22)(1450\,\text{m/s})}{(1.0 \times 10^3\,\text{Hz})(0.60\,\text{m})} = 2.9 > 1 \ .$$

Since $\sin\theta$ cannot exceed 1 there is no minimum.

72. From Eq. 37-3,

$$\frac{a}{\lambda} = \frac{m}{\sin\theta} = \frac{1}{\sin 45.0°} = 1.41 \ .$$

73. (a) Use of Eq. 37-22 for the limit-wavelengths ($\lambda_1 = 700$ nm and $\lambda_2 = 550$ nm) leads to the condition

$$m_1\lambda_1 \ge m_2\lambda_2$$

for $m_1 + 1 = m_2$ (the low end of a high-order spectrum is what is overlapping with the high end of the next-lower-order spectrum). Assuming equality in the above equation, we can solve for "m_1" (realizing it might not be an integer) and obtain $m_1 \approx 4$ where we have rounded *up*. It is the fourth order spectrum that is the lowest-order spectrum to overlap with the next higher spectrum.

(b) The problem specifies $d = 1/200$ using the mm unit, and we note there are no refraction angles greater than 90°. We concentrate on the largest wavelength $\lambda = 700$ nm $= 7 \times 10^{-4}$ mm and solve Eq. 37-22 for "$m_{\rm max}$" (realizing it might not be an integer):

$$m_{\rm max} = \frac{d \sin 90^\circ}{\lambda} = \frac{1}{(200)\,(7 \times 10^{-4})} \approx 7$$

where we have rounded down. There are no values of m (for the appearance of the full spectrum) greater than $m = 7$.

74. The central diffraction envelope spans the range $-\theta_1 < \theta < +\theta_1$ where

$$\theta_1 = \sin^{-1} \frac{\lambda}{a} \ .$$

The maxima in the double-slit pattern are at

$$\theta_m = \sin^{-1} \frac{m\lambda}{d} \ ,$$

so that our range specification becomes

$$-\sin^{-1} \frac{\lambda}{a} \ < \ \sin^{-1} \frac{m\lambda}{d} < \ +\sin^{-1} \frac{\lambda}{a} \ ,$$

which we change (since sine is a monotonically increasing function in the fourth and first quadrants, where all these angles lie) to

$$-\frac{\lambda}{a} \ < \ \frac{m\lambda}{d} \ < \ +\frac{\lambda}{a} \ .$$

Rewriting this as $-d/a < m < +d/a$ we arrive at the result $m_{\rm max} < d/a \le m_{\rm max} + 1$. Due to the symmetry of the pattern, the multiplicity of the m values is $2m_{\rm max} + 1 = 17$ so that $m_{\rm max} = 8$, and the result becomes

$$8 < \frac{d}{a} \le 9$$

where these numbers are as accurate as the experiment allows (that is, "9" means "9.000" if our measurements are that good).

75. As a slit is narrowed, the pattern spreads outward, so the question about "minimum width" suggests that we are looking at the lowest possible values of m (the label for the minimum produced by light $\lambda = 600$ nm) and m' (the label for the minimum produced by light $\lambda' = 500$ nm). Since the angles are the same, then Eq. 37-3 leads to

$$m\lambda = m'\lambda'$$

which leads to the choices $m = 5$ and $m' = 6$. We find the slit width from Eq. 37-3:

$$a = \frac{m\lambda}{\sin\theta} \approx \frac{m\lambda}{\theta}$$

which yields $a = 3.0$ mm.

76. (a) We note that $d = (76 \times 10^6\,\text{nm})/40000 = 1900\,\text{nm}$. For the first order maxima $\lambda = d \sin\theta$, which leads to

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{589\,\text{nm}}{1900\,\text{nm}}\right) = 18^\circ \ .$$

Now, substituting $m = d\sin\theta/\lambda$ into Eq. 37-27 leads to $D = \tan\theta/\lambda = \tan 18^\circ/589\,\text{nm} = 5.5 \times 10^{-4}\,\text{rad/nm} = 0.032^\circ/\text{nm}$. Similarly for $m = 2$ and $m = 3$, we have $\theta = 38^\circ$ and 68°, and the corresponding values of dispersion are $0.076^\circ/\text{nm}$ and $0.24^\circ/\text{nm}$, respectively.

(b) $R = Nm = 40000\,m = 40000$ (for $m = 1$); 80000 (for $m = 2$); and, $120,000$ (for $m = 3$).

77. Letting $d \sin\theta = (L/N)\sin\theta = m\lambda$, we get

$$\lambda = \frac{(L/N)\sin\theta}{m} = \frac{(1.0 \times 10^7\,\mathrm{nm})(\sin 30^\circ)}{(1)(10000)} = 500 \text{ nm} .$$

78. (a) Using the notation of Sample Problem 37-6,

$$L = \frac{D}{1.22\lambda/d} = \frac{2(50 \times 10^{-6}\,\mathrm{m})(1.5 \times 10^{-3}\,\mathrm{m})}{1.22(650 \times 10^{-9}\,\mathrm{m})} = 0.19 \text{ m} .$$

(b) The wavelength of the blue light is shorter so $L_{\max} \propto \lambda^{-1}$ will be larger.

79. From $y = m\lambda D/a$ we get

$$\Delta y = \Delta\left(\frac{m\lambda D}{a}\right) = \frac{\lambda D}{a}\Delta m = \frac{(632.8\,\mathrm{nm})(2.60)}{1.37\,\mathrm{mm}}[10 - (-10)] = 24.0 \text{ mm} .$$

80. For $\lambda = 0.10$ nm, we have scattering for order m, and for $\lambda' = 0.075$ nm, we have scattering for order m'. From Eq. 37-31, we see that we must require

$$m\lambda = m'\lambda'$$

which suggests (looking for the smallest integer solutions) that $m = 3$ and $m' = 4$. Returning with this result and with $d = 0.25$ nm to Eq. 37-31, we obtain

$$\theta = \sin^{-1}\frac{m\lambda}{2d} = 37^\circ .$$

Studying Figure 37-26, we conclude that the angle between incident and scattered beams is $180^\circ - 2\theta = 106^\circ$.

81. (a) We express all lengths in mm, and since $1/d = 180$, we write Eq. 37-22 as

$$\theta = \sin^{-1}\left(\frac{1}{d}m\lambda\right) = \sin^{-1}(180)(2)\lambda$$

where $\lambda_1 = 4 \times 10^{-4}$ and $\lambda_2 = 5 \times 10^{-4}$ (in mm). Thus, $\Delta\theta = \theta_2 - \theta_1 = 2.1^\circ$.

(b) Use of Eq. 37-22 for each wavelength leads to the condition

$$m_1\lambda_1 = m_2\lambda_2$$

for which the smallest possible choices are $m_1 = 5$ and $m_2 = 4$. Returning to Eq. 37-22, then, we find

$$\theta = \sin^{-1}\left(\frac{1}{d}m_1\lambda_1\right) = 21^\circ .$$

(c) There are no refraction angles greater than 90°, so we can solve for "$m_{\max}$" (realizing it might not be an integer):

$$m_{\max} = \frac{d\sin 90^\circ}{\lambda_2} = 11$$

where we have rounded down. There are no values of m (for light of wavelength λ_2) greater than $m = 11$.

82. Following Sample Problem 37-6, we use Eq. 37-35:

$$L = \frac{Dd}{1.22\lambda} = 164 \text{ m} .$$

83. (a) Employing Eq. 37-3 with the small angle approximation ($\sin\theta \approx \tan\theta = y/D$ where y locates the minimum relative to the middle of the pattern), we find (with $m = 1$ and all lengths in mm)

$$D = \frac{ya}{m\lambda} = \frac{(0.9)(0.4)}{4.5 \times 10^{-4}} = 800$$

which places the screen 80 cm away from the slit.

(b) The above equation gives for the value of y (for $m = 3$)

$$y = \frac{(3)\lambda D}{a} = 2.7 \text{ mm} .$$

Subtracting this from the first minimum position $y = 0.9$ mm, we find the result $\Delta y = 1.8$ mm.

84. (a) We require that $\sin\theta = m\lambda_{1,2}/d \leq \sin 30^\circ$, where $m = 1, 2$ and $\lambda_1 = 500\,\text{nm}$. This gives

$$d \geq \frac{2\lambda_s}{\sin 30^\circ} = \frac{2(600\,\text{nm})}{\sin 30^\circ} = 2400\,\text{nm} .$$

For a grating of given total width L we have $N = L/d \propto d^{-1}$, so we need to minimize d to maximize $R = mN \propto d^{-1}$. Thus we choose $d = 2400\,\text{nm}$.

(b) Let the third-order maximum for $\lambda_2 = 600\,\text{nm}$ be the first minimum for the single-slit diffraction profile. This requires that $d\sin\theta = 3\lambda_2 = a\sin\theta$, or $a = d/3 = 2400\,\text{nm}/3 = 800\,\text{nm}$.

(c) Letting $\sin\theta = m_{\text{max}}\lambda_2/d \leq 1$, we obtain

$$m_{\text{max}} \leq \frac{d}{\lambda_2} = \frac{2400\,\text{nm}}{800\,\text{nm}} = 3 .$$

Since the third order is missing the only maxima present are the ones with $m = 0$, 1 and 2.

85. (a) Letting $d\sin\theta = m\lambda$, we solve for λ:

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.0\,\text{mm}/200)(\sin 30^\circ)}{m} = \frac{2500\,\text{nm}}{m}$$

where $m = 1, 2, 3\cdots$. In the visible light range m can assume the following values: $m_1 = 4$, $m_2 = 5$ and $m_3 = 6$. The corresponding wavelengths are $\lambda_1 = 2500\,\text{nm}/4 = 625\,\text{nm}$, $\lambda_2 = 2500\,\text{nm}/5 = 500\,\text{nm}$, and $\lambda_3 = 2500\,\text{nm}/6 = 416\,\text{nm}$.

(b) The colors are orange (for $\lambda_1 = 625\,\text{nm}$), blue-green (for $\lambda_2 = 500\,\text{nm}$), and violet (for $\lambda_3 = 416\,\text{nm}$).

86. Using the notation of Sample Problem 37-6,

$$L = \frac{D}{\theta_{\text{R}}} = \frac{D}{1.22\lambda/d} = \frac{(5.0 \times 10^{-2}\,\text{m})(4.0 \times 10^{-3}\,\text{m})}{1.22(0.10 \times 10^{-9}\,\text{m})} = 1.6 \times 10^6\,\text{m} = 1600 \text{ km} .$$

87. The condition for a minimum in a single-slit diffraction pattern is given by Eq. 37-3, which we solve for the wavelength:

$$\lambda = \frac{a\sin\theta}{m} = \frac{(0.022\,\text{mm})\sin 1.8^\circ}{1} = 6.9 \times 10^{-4}\,\text{mm} = 690 \text{ nm} .$$

Chapter 38

1. (a) The time an electron with a horizontal component of velocity v takes to travel a horizontal distance L is

$$t = \frac{L}{v} = \frac{20 \times 10^{-2}\,\mathrm{m}}{(0.992)(2.998 \times 10^{8}\,\mathrm{m/s})} = 6.72 \times 10^{-10}\ \mathrm{s}\ .$$

(b) During this time, it falls a vertical distance

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\,\mathrm{m/s^2})(6.72 \times 10^{-10}\,\mathrm{s})^2 = 2.2 \times 10^{-18}\ \mathrm{m}\ .$$

This distance is much less than the radius of a proton. We can conclude that for particles traveling near the speed of light in a laboratory, Earth may be considered an approximately inertial frame.

2. (a) The speed parameter β is v/c. Thus,

$$\beta = \frac{(3\,\mathrm{cm/y})(0.01\,\mathrm{m/cm})(1\,\mathrm{y}/3.15 \times 10^{7}\,\mathrm{s})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 3 \times 10^{-18}\ .$$

(b) For the highway speed limit, we find

$$\beta = \frac{(90\,\mathrm{km/h})(1000\,\mathrm{m/km})(1\,\mathrm{h}/3600\,\mathrm{s})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 8.3 \times 10^{-8}\ .$$

(c) Mach 2.5 corresponds to

$$\beta = \frac{(1200\,\mathrm{km/h})(1000\,\mathrm{m/km})(1\,\mathrm{h}/3600\,\mathrm{s})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 1.1 \times 10^{-6}\ .$$

(d) We refer to Table 14-2:

$$\beta = \frac{(11.2\,\mathrm{km/s})(1000\,\mathrm{m/km})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 3.7 \times 10^{-5}\ .$$

(e) For the quasar recession speed, we obtain

$$\beta = \frac{(3.0 \times 10^{4}\,\mathrm{km/s})(1000\,\mathrm{m/km})}{3.0 \times 10^{8}\,\mathrm{m/s}} = 0.10\ .$$

3. From the time dilation equation $\Delta t = \gamma \Delta t_0$ (where Δt_0 is the proper time interval, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$), we obtain

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}\ .$$

The proper time interval is measured by a clock at rest relative to the muon. Specifically, $\Delta t_0 = 2.2\,\mu\text{s}$. We are also told that Earth observers (measuring the decays of moving muons) find $\Delta t = 16\,\mu\text{s}$. Therefore,

$$\beta = \sqrt{1 - \left(\frac{2.2\,\mu\text{s}}{16\,\mu\text{s}}\right)^2} = 0.9905\ .$$

The muon speed is $v = \beta c = 0.9905(2.998 \times 10^8\,\text{m/s}) = 2.97 \times 10^8\,\text{m/s}$.

4. (a) We find β from $\gamma = 1/\sqrt{1-\beta^2}$:

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.01)^2}} = 0.140371 \ \approx\ 0.140\ .$$

(b) Similarly, $\beta = \sqrt{1 - (10.0)^{-2}} = 0.994987 \approx 0.9950$.

(c) In this case, $\beta = \sqrt{1 - (100)^{-2}} = 0.999\,950$.

(d) This last case might prove problematic for some calculators. The result is $\beta = \sqrt{1 - (1000)^{-2}} = 0.999\,999\,50$. The discussion in Sample Problem 38-7 dealing with large γ values may prove helpful for those whose calculators do not yield this answer.

5. In the laboratory, it travels a distance $d = 0.00105\,\text{m} = vt$, where $v = 0.992c$ and t is the time measured on the laboratory clocks. We can use Eq. 38-7 to relate t to the proper lifetime of the particle t_0:

$$t = \frac{t_0}{\sqrt{1 - (v/c)^2}} \quad\Longrightarrow\quad t_0 = t\sqrt{1 - \left(\frac{v}{c}\right)^2} = \frac{d}{0.992c}\sqrt{1 - 0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13}$ s.

6. (a) The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval Δt_0) and 1000 years according to the clocks on Earth which measure Δt. We solve Eq. 38-7 for v and then plug in:

$$\begin{aligned} v &= c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} \\ &= (299792458\,\text{m/s})\sqrt{1 - \left(\frac{1\,\text{y}}{1000\,\text{y}}\right)^2} \\ &= 299792308\ \text{m/s} \end{aligned}$$

which may also be expressed as $v = c\sqrt{1 - (1000)^{-2}} = 0.999\,999\,50c$. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield this answer.

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question which has occasionally precipitated debates among professional physicists.

7. The length L of the rod, as measured in a frame in which it is moving with speed v parallel to its length, is related to its rest length L_0 by $L = L_0/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Since γ must be greater than 1, L is less than L_0. For this problem, $L_0 = 1.70\,\text{m}$ and $\beta = 0.630$, so $L = (1.70\,\text{m})\sqrt{1 - (0.630)^2} = 1.32\,\text{m}$.

8. The contracted length of the tube would be

$$L = L_0\sqrt{1-\beta^2} = (3.00\,\text{m})\sqrt{1-0.999987^2} = 0.0153 \text{ m} .$$

9. Only the "component" of the length in the x direction contracts, so its y component stays

$$\ell'_y = \ell_y = \ell \sin 30^\circ = 0.5000 \text{ m}$$

while its x component becomes

$$\ell'_x = \ell_x\sqrt{1-\beta^2} = \ell \cos 30^\circ \sqrt{1-0.90^2} = 0.3775 \text{ m} .$$

Therefore, using the Pythagorean theorem, the length measured from S' is

$$\ell' = \sqrt{\left(\ell'_x\right)^2 + \left(\ell'_y\right)^2} = 0.626 \text{ m} .$$

10. (a) We solve Eq. 38-13 for v and then plug in:

$$\begin{aligned} v &= c\sqrt{1-\left(\frac{L}{L_0}\right)^2} \\ &= (299792458\,\text{m/s})\sqrt{1-\left(\frac{1}{2}\right)^2} \\ &= 259627884 \text{ m/s} \end{aligned}$$

which may also be expressed as $v = 0.8660254c$.

(b) The Lorentz factor in this case is $\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 2$ "exactly."

11. (a) The rest length $L_0 = 130\,\text{m}$ of the spaceship and its length L as measured by the timing station are related by Eq. 38-13. Therefore, $L = (130\,\text{m})\sqrt{1-(0.740)^2} = 87.4\,\text{m}$.

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4\,\text{m}}{(0.740)(3.00\times 10^8\,\text{m/s})} = 3.94\times 10^{-7} \text{ s} .$$

12. (a) According solely to the principles of Special Relativity, yes. If the person moves fast enough, then the time dilation argument will allow for his proper travel time to be much less than that measured from the Earth. Stated differently, length contraction can make that travel distance seem much shorter to the traveler than to our Earth-based estimations. This does not include important considerations such as fuel requirements, stresses to the human body (due to the accelerations, primarily), and so on.

(b) Let $d = 23000\,\text{ly} = 23000\,c\,\text{y}$, which would give the distance in meters if we included a conversion factor for years $\rightarrow$ seconds. With $\Delta t_0 = 30\,\text{y}$ and $\Delta t = d/v$ (see Eq. 38-10), we wish to solve for v from Eq. 38-7. Our first step is as follows:

$$\begin{aligned} \Delta t &= \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \\ \frac{d}{v} &= \frac{\Delta t_0}{\sqrt{1-(v/c)^2}} \\ \frac{23000\,c\,\text{y}}{v} &= \frac{30\,\text{y}}{\sqrt{1-(v/c)^2}} , \end{aligned}$$

at which point we can cancel the unit year and manipulate the equation to solve for the speed. After a couple of algebraic steps, we obtain

$$\begin{aligned} v &= \frac{c}{\sqrt{1+\left(\frac{30}{23000}\right)^2}} \\ &= \frac{299792458\,\mathrm{m/s}}{\sqrt{1+0.000017013}} \\ &= 299792203\ \mathrm{m/s} \end{aligned}$$

which may also be expressed as $v = 0.999\,999\,15c$. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield this answer.

13. (a) The speed of the traveler is $v = 0.99c$, which may be equivalently expressed as $0.99\,\mathrm{ly/y}$. Let d be the distance traveled. Then, the time for the trip as measured in the frame of Earth is $\Delta t = d/v = (26\,\mathrm{ly})/(0.99\,\mathrm{ly/y}) = 26.3\,\mathrm{y}$.

(b) The signal, presumed to be a radio wave, travels with speed c and so takes $26.0\,\mathrm{y}$ to reach Earth. The total time elapsed, in the frame of Earth, is $26.3\,\mathrm{y} + 26.0\,\mathrm{y} = 52.3\,\mathrm{y}$.

(c) The proper time interval is measured by a clock in the spaceship, so $\Delta t_0 = \Delta t/\gamma$. Now $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.99)^2} = 7.09$. Thus, $\Delta t_0 = (26.3\,\mathrm{y})/(7.09) = 3.7\,\mathrm{y}$.

14. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 38-20 to apply without additional terms. In part (a), we apply these equations directly with $v = +0.400c = 1.199 \times 10^8\,\mathrm{m/s}$, and in part (b) we simply change $v \to -v$ and recalculate the primed values.

(a) The position coordinate measured in the S' frame is

$$\begin{aligned} x' &= \gamma(x - vt) = \frac{x - vt}{\sqrt{1-\beta^2}} \\ &= \frac{3.00 \times 10^8\,\mathrm{m} - (1.199 \times 10^8\,\mathrm{m/s})(2.50\,\mathrm{s})}{\sqrt{1-(0.400)^2}} \\ &= 2.7 \times 10^5\ \mathrm{m/s} \ \approx \ 0\ , \end{aligned}$$

where we conclude that the numerical result (2.7×10^5 or 2.3×10^5 depending on how precise a value of v is used) is not meaningful (in the significant figures sense) and should be set equal to zero (that is, it is "consistent with zero" in view of the statistical uncertainties involved). The time coordinate measured in the S' frame is

$$\begin{aligned} t' &= \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - \frac{\beta x}{c}}{\sqrt{1-\beta^2}} \\ &= \frac{2.50\,\mathrm{s} - \frac{(0.400)(3.00\times10^3\,\mathrm{m})}{2.998\times10^8\,\mathrm{m/s}}}{\sqrt{1-(0.400)^2}} \\ &= 2.29\ \mathrm{s}\ . \end{aligned}$$

(b) Now, we obtain

$$x' = \frac{x + vt}{\sqrt{1-\beta^2}} = \frac{3.00 \times 10^8\,\mathrm{m} + (1.199 \times 10^8\,\mathrm{m/s})(2.50\,\mathrm{s})}{\sqrt{1-(0.400)^2}} = 6.54 \times 10^8\ \mathrm{m}\ ,$$

and

$$t' = \gamma\left(t + \frac{vx}{c^2}\right) = \frac{2.50\,\mathrm{s} + \frac{(0.400)(3.00\times10^8\,\mathrm{m})}{2.998\times10^8\,\mathrm{m/s}}}{\sqrt{1-(0.400)^2}} = 3.16\ \mathrm{s}\ .$$

15. The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma(x - vt) \qquad \text{and} \qquad t' = \gamma(t - \beta x/c)$$

where $\beta = v/c = 0.950$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.20256$. Thus,

$$\begin{aligned} x' &= (3.20256)\left(100 \times 10^3\,\text{m} - (0.950)(2.998 \times 10^8\,\text{m/s})(200 \times 10^{-6}\,\text{s})\right) \\ &= 1.38 \times 10^5\,\text{m} = 138\text{ km} \end{aligned}$$

and

$$t' = (3.20256)\left[200 \times 10^{-6}\,\text{s} - \frac{(0.950)(100 \times 10^3\,\text{m})}{2.998 \times 10^8\,\text{m/s}}\right] = -3.74 \times 10^{-4}\,\text{s} = -374\,\mu\text{s}\ .$$

16. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 38-20 to apply without additional terms. We label the event coordinates with subscripts: $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (3000, 4.0 \times 10^{-6})$ with SI units understood. Of course, we expect $(x_1', t_1') = (0, 0)$, and this may be verified using Eq. 38-20. We now compute (x_2', t_2'), assuming $v = +0.60c = +1.799 \times 10^8$ m/s (the sign of v is not made clear in the problem statement, but the Figure referred to, Fig. 38-9, shows the motion in the positive x direction).

$$\begin{aligned} x_2' &= \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3000 - \left(1.799 \times 10^8\right)\left(4.0 \times 10^{-6}\right)}{\sqrt{1-(0.60)^2}} = 2.85 \times 10^3 \\ t_2' &= \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{4.0 \times 10^{-6} - (0.60)(3000)/(2.998 \times 10^8)}{\sqrt{1-(0.60)^2}} = -2.5 \times 10^{-6} \end{aligned}$$

The two events in frame S occur in the order: first 1, then 2. However, in frame S' where $t_2' < 0$, they occur in the reverse order: first 2, then 1. We note that the distances $x_2 - x_1$ and $x_2' - x_1'$ are larger than how far light can travel during the respective times ($c(t_2 - t_1) = 1.2$ km and $c|t_2' - t_1'| \approx 750$ m), so that no inconsistencies arise as a result of the order reversal (that is, no signal from event 1 could arrive at event 2 or vice versa).

17. (a) We take the flashbulbs to be at rest in frame S, and let frame S' be the rest frame of the second observer. Clocks in neither frame measure the proper time interval between the flashes, so the full Lorentz transformation (Eq. 38-20) must be used. Let t_s be the time and x_s be the coordinate of the small flash, as measured in frame S. Then, the time of the small flash, as measured in frame S', is

$$t_s' = \gamma\left(t_s - \frac{\beta x_s}{c}\right)$$

where $\beta = v/c = 0.250$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.250)^2} = 1.0328$. Similarly, let t_b be the time and x_b be the coordinate of the big flash, as measured in frame S. Then, the time of the big flash, as measured in frame S', is

$$t_b' = \gamma\left(t_b - \frac{\beta x_b}{c}\right)\ .$$

Subtracting the second Lorentz transformation equation from the first and recognizing that $t_s = t_b$ (since the flashes are simultaneous in S), we find

$$\Delta t' = -\frac{\gamma\beta(x_s - x_b)}{c} = -\frac{(1.0328)(0.250)(30 \times 10^3\,\text{m})}{3.00 \times 10^8\,\text{m/s}} = -2.58 \times 10^{-5}\ \text{s}$$

where $\Delta t' = t_s' - t_b'$.

(b) Since $\Delta t'$ is negative, t_b' is greater than t_s'. The small flash occurs first in S'.

18. (a) In frame S, our coordinates are such that $x_1 = +1200$ m for the big flash, and $x_2 = 1200 - 720 = 480$ m for the small flash (which occurred later). Thus, $\Delta x = x_2 - x_1 = -720$ m. If we set $\Delta x' = 0$ in Eq. 38-24, we find

$$0 = \gamma\left(\Delta x - v\Delta t\right) = \gamma\left(-720\,\mathrm{m} - v(5.00 \times 10^{-6}\,\mathrm{s})\right)$$

which yields $v = -1.44 \times 10^8$ m/s. Therefore, frame S' must be moving in the $-x$ direction with a speed of $0.480c$.

(b) Eq. 38-27 leads to

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(5.00 \times 10^{-6}\,\mathrm{s} - \frac{(-1.44 \times 10^8\,\mathrm{m/s})(-720\,\mathrm{m})}{(2.998 \times 10^8\,\mathrm{m/s})^2}\right)$$

which turns out to be positive (regardless of the specific value of γ). Thus, the order of the flashes is the same in the S' frame as it is in the S frame (where Δt is also positive). Thus, the big flash occurs first, and the small flash occurs later.

(c) Finishing the computation begun in part (b), we obtain

$$\Delta t' = \frac{5.00 \times 10^{-6}\,\mathrm{s} - \frac{(-1.44\times10^8\,\mathrm{m/s})(-720\,\mathrm{m})}{(2.998\times10^8\,\mathrm{m/s})^2}}{\sqrt{1 - 0.480^2}} = 4.39 \times 10^{-6}\ \mathrm{s}\ .$$

19. (a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25\ .$$

(b) In the unprimed frame, the time for the clock to travel from the origin to $x = 180$ m is

$$t = \frac{x}{v} = \frac{180\,\mathrm{m}}{(0.600)(3.00 \times 10^8\,\mathrm{m/s})} = 1.00 \times 10^{-6}\ \mathrm{s}\ .$$

The proper time interval between the two events (at the origin and at $x = 180$ m) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00 \times 10^{-6}\,\mathrm{s}}{1.25} = 8.00 \times 10^{-7}\ \mathrm{s}\ .$$

20. We refer to the solution of problem 18. We wish to adjust Δt so that

$$\Delta x' = 0 = \gamma\left(-720\,\mathrm{m} - v\Delta t\right)$$

in the limiting case of $|v| \to c$. Thus,

$$\Delta t = \frac{720\,\mathrm{m}}{2.998 \times 10^8\,\mathrm{m/s}} = 2.40 \times 10^{-6}\ \mathrm{s}\ .$$

21. We assume S' is moving in the $+x$ direction. With $u' = +0.40c$ and $v = +0.60c$, Eq. 38-28 yields

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.40c + 0.60c}{1 + (0.40c)(+0.60c)/c^2} = 0.81c\ .$$

22. (a) We use Eq. 38-28:

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{0.47c + 0.62c}{1 + (0.47)(0.62)} = 0.84c\ ,$$

in the direction of increasing x (since $v > 0$). The classical theory predicts that $v = 0.47c + 0.62c = 1.1c > c$.

(b) Now $v' = -0.47c$ so

$$v = \frac{v' + u}{1 + uv'/c^2} = \frac{-0.47c + 0.62c}{1 + (-0.47)(0.62)} = 0.21c \ ,$$

again in the direction of increasing x. By contrast, the classical prediction is $v = 0.62c - 0.47c = 0.15c$.

23. (a) One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If Joe sees Fred moving at 20 m/s eastward away from him (Joe), then Fred should see Joe moving at 20 m/s westward away from him (Fred). Similarly, if we see Galaxy A moving away from us at $0.35c$ then an observer in Galaxy A should see our galaxy move away from him at $0.35c$.

(b) We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. Using the notation of Eq. 38-28, the problem indicates $v = +0.35c$ (velocity of Galaxy A relative to Earth) and $u = -0.35c$ (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{(-0.35c) - 0.35c}{1 - (-0.35)(0.35)} = -0.62c$$

or $u' = -1.87 \times 10^8$ m/s.

24. Using the notation of Eq. 38-28 and taking "away" (from us) as the positive direction, the problem indicates $v = +0.4c$ and $u = +0.8c$ (with 3 significant figures understood). We solve for the velocity of Q_2 relative to Q_1:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.8c - 0.4c}{1 - (0.8)(0.4)} = 0.588c$$

or $u' = 1.76 \times 10^8$ m/s in a direction away from Earth.

25. Using the notation of Eq. 38-28 and taking the micrometeorite motion as the positive direction, the problem indicates $v = -0.82c$ (spaceship velocity) and $u = +0.82c$ (micrometeorite velocity). We solve for the velocity of the micrometeorite relative to the spaceship:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.82c - (-0.82c)}{1 - (0.82)(-0.82)} = 0.98c$$

or 2.94×10^8 m/s. Using Eq. 38-10, we conclude that observers on the ship measure a transit time for the micrometeorite (as it passes along the length of the ship) equal to

$$\Delta t = \frac{d}{u'} = \frac{350\,\text{m}}{2.94 \times 10^8\,\text{m/s}} = 1.2 \times 10^{-6}\ \text{s} \ .$$

26. (a) In the messenger's rest system (called S_m), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m/c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c \ .$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma_{v'}} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781\ \text{ly} \ .$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\,\text{ly}}{0.625c} = 1.25\ \text{y} \ .$$

(b) In the armada's rest frame (called S_a), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a/c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c \ .$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.6 \text{ y} \ .$$

(c) Measured in system S, the length of the armada is

$$L = \frac{L_0}{\gamma} = 1.0\,\text{ly}\sqrt{1 - (0.80)^2} = 0.60 \text{ ly} \ ,$$

so the length of the trip is

$$t = \frac{L}{v_m - v_a} = \frac{0.60\,\text{ly}}{0.95c - 0.80c} = 4.0 \text{ y} \ .$$

27. The spaceship is moving away from Earth, so the frequency received is given directly by Eq. 38-30. Thus,

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}} = (100\,\text{MHz})\sqrt{\frac{1-0.9000}{1+0.9000}} = 22.9 \text{ MHz} \ .$$

28. (a) Eq. 38-33 leads to

$$v = \frac{\Delta\lambda}{\lambda}c = \frac{12\,\text{nm}}{513\,\text{nm}}\left(2.998 \times 10^8\,\text{m/s}\right) = 7.0 \times 10^6 \text{ m/s} \ .$$

(b) The line is shifted to a larger wavelength, which means shorter frequency. Recalling Eq. 38-30 and the discussion that follows it, this means galaxy NGC is moving away from Earth.

29. Eq. 38-33 leads to a recessional speed of

$$v = \frac{\Delta\lambda}{\lambda}c = (0.004)\left(3.0 \times 10^8\,\text{m/s}\right) = 1 \times 10^6 \text{ m/s} \ .$$

30. We obtain

$$v = \frac{\Delta\lambda}{\lambda}c = \left(\frac{620 - 540}{620}\right)c = 0.13c = 3.9 \times 10^6 \text{ m/s} \ .$$

31. The frequency received is given by

$$\begin{aligned} f &= f_0\sqrt{\frac{1-\beta}{1+\beta}} \\ \frac{c}{\lambda} &= \frac{c}{\lambda_0}\sqrt{\frac{1-0.20}{1+0.20}} \end{aligned}$$

which implies

$$\lambda = (450\,\text{nm})\sqrt{\frac{1+0.20}{1-0.20}} = 550 \text{ nm} \ .$$

This is in the yellow-green portion of the visible spectrum.

32. (a) The work-kinetic energy theorem applies as well to Einsteinian physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K = m_e c^2(\gamma - 1)$ (Eq. 38-49) and $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$ (Table 38-3), and obtain

$$W = m_e c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (511\,\text{keV})\left[\frac{1}{\sqrt{1-(0.50)^2}} - 1\right] = 79 \text{ keV} \ .$$

(b)

$$W = (0.511\,\text{MeV})\left(\frac{1}{\sqrt{1-(0.990)^2}} - 1\right) = 3.11\ \text{MeV}\ .$$

(c)

$$W = (0.511\,\text{MeV})\left(\frac{1}{\sqrt{1-(0.9990)^2}} - 1\right) = 10.9\ \text{MeV}\ .$$

33. (a) Using $K = m_ec^2(\gamma - 1)$ (Eq. 38-49) and $m_ec^2 = 511\,\text{keV} = 0.511\,\text{MeV}$ (Table 38-3), we obtain

$$\gamma = \frac{K}{m_ec^2} + 1 = \frac{1.00\,\text{keV}}{511\,\text{keV}} + 1 = 1.00196\ .$$

Therefore, the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{1.00196^2}} = 0.0625\ .$$

(b) We could first find β and then find γ, as illustrated here: With $K = 1.00\,\text{MeV}$, we find

$$\beta = \sqrt{1 - \left(\frac{1.00\,\text{MeV}}{0.511\,\text{MeV}} + 1\right)^{-2}} = 0.941$$

and $\gamma = 1/\sqrt{1-\beta^2} = 2.96$.

(c) Finally, $K = 1000\,\text{MeV}$, so

$$\beta = \sqrt{1 - \left(\frac{1000\,\text{MeV}}{0.511\,\text{MeV}} + 1\right)^{-2}} = 0.999\,999\,87$$

and $\gamma = 1000\,\text{MeV}/0.511\,\text{MeV} + 1 = 1.96 \times 10^3$. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield these answers.

34. From Eq. 38-49, $\gamma = (K/mc^2) + 1$, and from Eq. 38-8, the speed parameter is $\beta = \sqrt{1 - (1/\gamma)^2}$.

(a) Table 38-3 gives $m_ec^2 = 511\,\text{keV} = 0.511\,\text{MeV}$, so the Lorentz factor is

$$\gamma = \frac{10.0\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 20.57\ ,$$

and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(20.57)^2}} = 0.9988\ .$$

(b) Table 38-3 gives $m_pc^2 = 938\,\text{MeV}$, so the Lorentz factor is $\gamma = 1 + 10.0\,\text{MeV}/938\,\text{MeV} = 1.01$, and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{1.01^2}} = 0.145\ .$$

(c) If we refer to the data shown in problem 36, we find $m_\alpha = 4.0026$ u, which (using Eq. 38-43) implies $m_\alpha c^2 = 3728$ MeV. This leads to $\gamma = 10/3728 + 1 = 1.0027$. And, being careful not to do any unnecessary rounding off in the intermediate steps, we find $\beta = 0.073$. We remark that the mass value used in our solution is not exactly the alpha particle mass (it's the helium-4 atomic mass), but this slight difference does not introduce significant error in this computation.

35. From Eq. 38-49, $\gamma = (K/mc^2) + 1$, and from Eq. 38-8, the speed parameter is $\beta = \sqrt{1 - (1/\gamma)^2}$. Table 38-3 gives $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$, so the Lorentz factor is

$$\gamma = \frac{100\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 197\ ,$$

and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(197)^2}} = 0.999987\ .$$

Thus, the speed of the electron is $0.999987c$, or 99.9987% of the speed of light. The discussion in Sample Problem 38-7 dealing with these sorts of values may prove helpful for those whose calculators do not yield this answer.

36. The mass change is

$$\Delta M = (4.002603\,\text{u} + 15.994915\,\text{u}) - (1.007825\,\text{u} + 18.998405\,\text{u}) = -0.008712\ \text{u}\ .$$

Using Eq. 38-47 and Eq. 38-43, this leads to

$$Q = -\Delta M\, c^2 = -(-0.008712\,\text{u})(931.5\,\text{MeV/u}) = 8.12\ \text{MeV}\ .$$

37. Since the rest energy E_0 and the mass m of the quasar are related by $E_0 = mc^2$, the rate P of energy radiation and the rate of mass loss are related by $P = dE_0/dt = (dm/dt)c^2$. Thus,

$$\frac{dm}{dt} = \frac{P}{c^2} = \frac{1 \times 10^{41}\,\text{W}}{(2.998 \times 10^8\,\text{m/s})^2} = 1.11 \times 10^{24}\ \text{kg/s}\ .$$

Since a solar mass is 2.0×10^{30} kg and a year is 3.156×10^7 s,

$$\frac{dm}{dt} = (1.11 \times 10^{24}\,\text{kg/s}) \left(\frac{3.156 \times 10^7\,\text{s/y}}{2.0 \times 10^{30}\,\text{kg/smu}} \right) \approx 18\ \text{smu/y}\ .$$

38. (a) The work-kinetic energy theorem applies as well to Einsteinian physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K$ where $K = m_e c^2(\gamma - 1)$ (Eq. 38-49), and $m_e c^2 = 511\,\text{keV} = 0.511\,\text{MeV}$ (Table 38-3). Noting that $\Delta K = m_e c^2(\gamma_f - \gamma_i)$, we obtain

$$W = m_e c^2 \left(\frac{1}{\sqrt{1 - \beta_f^2}} - \frac{1}{\sqrt{1 - \beta_i^2}} \right) = (511\,\text{keV}) \left(\frac{1}{\sqrt{1 - (0.19)^2}} - \frac{1}{\sqrt{1 - (0.18)^2}} \right) = 0.996\ \text{keV}\ .$$

(b) Similarly,

$$W = (511\,\text{keV}) \left(\frac{1}{\sqrt{1 - (0.99)^2}} - \frac{1}{\sqrt{1 - (0.98)^2}} \right) = 1055\ \text{keV}\ .$$

We see the dramatic increase in difficulty in trying to accelerate a particle when its initial speed is very close to the speed of light.

39. (a) We set Eq. 38-38 equal to mc, as required by the problem, and solve for the speed. Thus,

$$\frac{mv}{\sqrt{1 - v^2/c^2}} = mc$$

leads to $v = c/\sqrt{2} = 0.707c$.

(b) Substituting $v = \sqrt{2}c$ into the definition of γ, we obtain

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-(1/2)}} = \sqrt{2} \approx 1.41 \, .$$

(c) The kinetic energy is

$$K = (\gamma - 1)mc^2 = (\sqrt{2} - 1)mc^2 = 0.414mc^2 \, .$$

40. (a) We set Eq. 38-49 equal to $2mc^2$, as required by the problem, and solve for the speed. Thus,

$$mc^2 \left(\frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} - 1 \right) = 2mc^2$$

leads to $v = \frac{2\sqrt{2}}{3}c \approx 0.943c$.

(b) We now set Eq. 38-45 equal to $2mc^2$ and solve for the speed. In this case,

$$\frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = 2mc^2$$

leads to $v = \frac{\sqrt{3}}{2}c \approx 0.866c$.

41. We set Eq. 38-52 equal to $(3mc^2)^2$, as required by the problem, and solve for the speed. Thus,

$$(pc)^2 + (mc^2)^2 = 9(mc^2)^2$$

leads to $p = mc\sqrt{8}$.

42. (a) Squaring Eq. 38-44 gives

$$E^2 = \left(mc^2\right)^2 + 2mc^2K + K^2$$

which we set equal to Eq. 38-52. Thus,

$$\left(mc^2\right)^2 + 2mc^2K + K^2 = (pc)^2 + \left(mc^2\right)^2 \implies m = \frac{(pc)^2 - K^2}{2Kc^2} \, .$$

(b) At low speeds, the pre-Einsteinian expressions $p = mv$ and $K = \frac{1}{2}mv^2$ apply. We note that $pc \gg K$ at low speeds since $c \gg v$ in this regime. Thus,

$$m \to \frac{(mvc)^2 - \left(\frac{1}{2}mv^2\right)^2}{2\left(\frac{1}{2}mv^2\right)c^2} \approx \frac{(mvc)^2}{2\left(\frac{1}{2}mv^2\right)c^2} = m \, .$$

(c) Here, $pc = 121\,\text{MeV}$, so

$$m = \frac{121^2 - 55^2}{2(55)c^2} = 105.6\ \text{MeV/c}^2 \, .$$

Now, the mass of the electron (see Table 38-3) is $m_e = 0.511\,\text{MeV/c}^2$, so our result is roughly 207 times bigger than an electron mass.

43. The energy equivalent of one tablet is $mc^2 = (320 \times 10^{-6}\,\text{kg})(3.00 \times 10^8\,\text{m/s})^2 = 2.88 \times 10^{13}\,\text{J}$. This provides the same energy as $(2.88 \times 10^{13}\,\text{J})/(3.65 \times 10^7\,\text{J/L}) = 7.89 \times 10^5\,\text{L}$ of gasoline. The distance the car can go is $d = (7.89 \times 10^5\,\text{L})(12.75\,\text{km/L}) = 1.01 \times 10^7\,\text{km}$. This is roughly 250 times larger than the circumference of Earth (see Appendix C).

44. (a) The proper lifetime Δt_0 is $2.20\,\mu$s, and the lifetime measured by clocks in the laboratory (through which the muon is moving at high speed) is $\Delta t = 6.90\,\mu$s. We use Eq. 38-7 to solve for the speed:

$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = 0.9478c$$

or $v = 2.84 \times 10^8$ m/s.

(b) From the answer to part (a), we find $\gamma = 3.136$. Thus, with $m_\mu c^2 = 207 m_e c^2 = 105.8$ MeV (see Table 38-3), Eq. 38-49 yields

$$K = m_\mu c^2(\gamma - 1) = 226 \text{ MeV} .$$

(c) We write $m_\mu c = 105.8$ MeV$/c$ and apply Eq. 38-38:

$$p = \gamma m_\mu v = \gamma m_\mu c\beta = (3.136)(105.8\,\text{MeV}/c)(0.9478) = 314\ \text{MeV}/c$$

which can also be expressed in SI units ($p = 1.7 \times 10^{-19}$ kg·m/s).

45. The distance traveled by the pion in the frame of Earth is (using Eq. 38-12) $d = v\,\Delta t$. The proper lifetime Δt_0 is related to Δt by the time-dilation formula: $\Delta t = \gamma \Delta t_0$. To use this equation, we must first find the Lorentz factor γ (using Eq. 38-45). Since the total energy of the pion is given by $E = 1.35 \times 10^5$ MeV and its mc^2 value is 139.6 MeV, then

$$\gamma = \frac{E}{mc^2} = \frac{1.35 \times 10^5\,\text{MeV}}{139.6\,\text{MeV}} = 967.05 .$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma \Delta t_0 = (967.1)(35.0 \times 10^{-9}\,\text{s}) = 3.385 \times 10^{-5}\ \text{s} ,$$

and the distance it travels is

$$d \approx c\Delta t = (2.998 \times 10^8\,\text{m/s})(3.385 \times 10^{-5}\,\text{s}) = 1.015 \times 10^4\,\text{m} = 10.15\ \text{km}$$

where we have approximated its speed as c (note: its speed can be found by solving Eq. 38-8, which gives $v = 0.9999995c$; this more precise value for v would not significantly alter our final result). Thus, the altitude at which the pion decays is 120 km − 10.15 km = 110 km.

46. The q in the denominator is to be interpreted as $|q|$ (so that the orbital radius r is a positive number). We interpret the given 10.0 MeV to be the kinetic energy of the electron. In order to make use of the mc^2 value for the electron given in Table 38-3 (511 keV = 0.511 MeV) we write the classical kinetic energy formula as

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}\left(mc^2\right)\left(\frac{v^2}{c^2}\right) = \frac{1}{2}\left(mc^2\right)\beta^2 .$$

(a) If $K_{\text{classical}} = 10.0$ MeV, then

$$\beta = \sqrt{\frac{2K_{\text{classical}}}{mc^2}} = \sqrt{\frac{2(10.0\,\text{MeV})}{0.511\,\text{MeV}}} = 6.256 ,$$

which, of course, is impossible (see the Ultimate Speed subsection of §38-2). If we use this value anyway, then the classical orbital radius formula yields

$$\begin{aligned} r &= \frac{mv}{|q|B} = \frac{m\beta c}{eB} \\ &= \frac{\left(9.11 \times 10^{-31}\,\text{kg}\right)(6.256)\left(2.998 \times 10^8\,\text{m/s}\right)}{\left(1.6 \times 10^{-19}\,\text{C}\right)(2.20\,\text{T})} \\ &= 4.85 \times 10^{-3}\ \text{m} . \end{aligned}$$

If, however, we use the correct value for β (calculated in the next part) then the classical radius formula would give about 0.77 mm.

(b) Before using the relativistically correct orbital radius formula, we must compute β in a relativistically correct way:

$$K = mc^2(\gamma - 1) \implies \gamma = \frac{10.0\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 20.57$$

which implies (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99882 .$$

Therefore,

$$\begin{aligned} r &= \frac{\gamma m v}{|q|B} = \frac{\gamma m \beta c}{eB} \\ &= \frac{(20.57)\left(9.11\times10^{-31}\,\text{kg}\right)(0.99882)\left(2.998\times10^{8}\,\text{m/s}\right)}{(1.6\times10^{-19}\,\text{C})\,(2.20\,\text{T})} \\ &= 1.59\times10^{-2}\ \text{m} . \end{aligned}$$

(c) The period is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(0.0159\,\text{m})}{(0.99882)\,(2.998\times10^{8}\,\text{m/s})} = 3.34\times10^{-10}\ \text{s} .$$

Whereas the purely classical result gives a period which is independent of speed, this is no longer true in the relativistic case (due to the γ factor in the equation).

47. The radius r of the path is given in problem 46 as $r = \gamma mvqB$. Thus,

$$\begin{aligned} m &= \frac{qBr\sqrt{1-\beta^2}}{v} \\ &= \frac{2(1.60\times10^{-19}\,\text{C})(1.00\,\text{T})(6.28\,\text{m})\sqrt{1-(0.710)^2}}{(0.710)(3.00\times10^{8}\,\text{m/s})} \\ &= 6.64\times10^{-27}\ \text{kg} . \end{aligned}$$

Since $1.00\,\text{u} = 1.66\times10^{-27}\,\text{kg}$, the mass is $m = 4.00\,\text{u}$. The nuclear particle contains four nucleons. Since there must be two protons to provide the charge $2e$, the nuclear particle is a helium nucleus (usually referred to as an alpha particle) with two protons and two neutrons.

48. We interpret the given $10\,\text{GeV} = 10000\,\text{MeV}$ to be the kinetic energy of the proton. Using Table 38-3 and Eq. 38-49, we find

$$\gamma = \frac{K}{m_p c^2} + 1 = \frac{10000\,\text{MeV}}{938\,\text{MeV}} + 1 = 11.66 ,$$

and (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9963 .$$

Therefore, using the equation introduced in problem 46, we obtain

$$\begin{aligned} r &= \frac{\gamma m v}{qB} = \frac{\gamma m_p \beta c}{eB} \\ &= \frac{(11.66)\left(1.67\times10^{-27}\,\text{kg}\right)(0.9963)\left(2.998\times10^{8}\,\text{m/s}\right)}{(1.6\times10^{-19}\,\text{C})\,(55\times10^{-6}\,\text{T})} \\ &= 6.6\times10^{5}\ \text{m} . \end{aligned}$$

49. We interpret the given $2.50\,\text{MeV} = 2500\,\text{keV}$ to be the kinetic energy of the electron. Using Table 38-3 and Eq. 38-49, we find

$$\gamma = \frac{K}{m_e c^2} + 1 = \frac{2500\,\text{keV}}{511\,\text{keV}} + 1 = 5.892 ,$$

and (from Eq. 38-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.9855 \; .$$

Therefore, using the equation introduced in problem 46 (with "q" interpreted as $|q|$), we obtain

$$\begin{aligned} B &= \frac{\gamma\, m_e\, v}{|q|\, r} = \frac{\gamma m_e \beta c}{er} \\ &= \frac{(5.892)\left(9.11 \times 10^{-31}\,\text{kg}\right)(0.9855)\left(2.998 \times 10^8\,\text{m/s}\right)}{(1.6 \times 10^{-19}\,\text{C})\,(0.030\,\text{m})} \\ &= 0.33\ \text{T} \; . \end{aligned}$$

50. (a) Using Table 38-3 and Eq. 38-49 (or, to be more precise, the value given at the end of the problem statement), we find

$$\gamma = \frac{K}{m_p c^2} + 1 = \frac{500 \times 10^3\,\text{MeV}}{938.3\,\text{MeV}} + 1 = 533.88 \; .$$

(b) From Eq. 38-8, we obtain

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99999825 \; .$$

The discussion in Sample Problem 38-7 dealing with large γ values may prove helpful for those whose calculators do not yield this answer.

(c) To make use of the precise $m_p c^2$ value given here, we rewrite the expression introduced in problem 46 (as applied to the proton) as follows:

$$r = \frac{\gamma m v}{qB} = \frac{\gamma\left(mc^2\right)\left(\frac{v}{c^2}\right)}{eB} = \frac{\gamma\left(mc^2\right)\beta}{ecB} \; .$$

Therefore, the magnitude of the magnetic field is

$$\begin{aligned} B &= \frac{\gamma\left(mc^2\right)\beta}{ecr} \\ &= \frac{(533.88)(938.3\,\text{MeV})(0.99999825)}{ec(750\,\text{m})} \\ &= \frac{667.92 \times 10^6\,\text{V/m}}{c} \end{aligned}$$

where we note the cancellation of the "e" in MeV with the e in the denominator. After substituting $c = 2.998 \times 10^8$ m/s, we obtain $B = 2.23$ T.

51. (a) Before looking at our solution to part (a) (which uses momentum conservation), it might be advisable to look at our solution (and accompanying remarks) for part (b) (where a very different approach is used). Since momentum is a vector, its conservation involves two equations (along the original direction of alpha particle motion, the x direction, as well as along the final proton direction of motion, the y direction). The problem states that all speeds are much less than the speed of light, which allows us to use the classical formulas for kinetic energy and momentum ($K = \frac{1}{2}mv^2$ and $\vec{p} = m\vec{v}$, respectively). Along the x and y axes, momentum conservation gives (for the components of $\vec{v}_{\text{oxy}}$):

$$\begin{aligned} m_\alpha v_\alpha &= m_{\text{oxy}} v_{\text{oxy},x} \implies v_{\text{oxy},x} = \frac{m_\alpha}{m_{\text{oxy}}}\, v_\alpha \approx \frac{4}{17}\, v_\alpha \\ 0 &= m_{\text{oxy}} v_{\text{oxy},y} + m_p v_p \implies v_{\text{oxy},y} = -\frac{m_p}{m_{\text{oxy}}}\, v_p \approx -\frac{1}{17}\, v_p \; . \end{aligned}$$

To complete these determinations, we need values (inferred from the kinetic energies given in the problem) for the initial speed of the alpha particle (v_α) and the final speed of the proton (v_p). One way to do this is to rewrite the classical kinetic energy expression as $K = \frac{1}{2}(mc^2)\beta^2$ and solve for β (using Table 38-3 and/or Eq. 38-43). Thus, for the proton, we obtain

$$\beta_p = \sqrt{\frac{2K_p}{m_p c^2}} = \sqrt{\frac{2(4.44\,\text{MeV})}{938\,\text{MeV}}} = 0.0973 \ .$$

This is almost 10% the speed of light, so one might worry that the relativistic expression (Eq. 38-49) should be used. If one does so, one finds $\beta_p = 0.969$, which is reasonably close to our previous result based on the classical formula. For the alpha particle, we write $m_\alpha c^2 = (4.0026\,\text{u})(931.5\,\text{MeV/u}) =$ 3728 MeV (which is actually an overestimate due to the use of the "atomic mass" value in our calculation, but this does not cause significant error in our result), and obtain

$$\beta_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha c^2}} = \sqrt{\frac{2(7.70\,\text{MeV})}{3728\,\text{MeV}}} = 0.064 \ .$$

Returning to our oxygen nucleus velocity components, we are now able to conclude:

$$\begin{aligned} v_{\text{oxy},x} &\approx \frac{4}{17}\,v_\alpha & \Longrightarrow \quad \beta_{\text{oxy},x} &\approx \frac{4}{17}\,\beta_\alpha = \frac{4}{17}(0.064) = 0.015 \\ |v_{\text{oxy},y}| &\approx \frac{1}{17}\,v_p & \Longrightarrow \quad \beta_{\text{oxy},y} &\approx \frac{1}{17}\,\beta_p = \frac{1}{17}(0.097) = 0.0057 \end{aligned}$$

Consequently, with $m_{\text{oxy}}c^2 \approx (17\,\text{u})(931.5\,\text{MeV/u}) = 1.58 \times 10^4$ MeV, we obtain

$$K_{\text{oxy}} = \frac{1}{2}\left(m_{\text{oxy}}c^2\right)\left(\beta^2_{\text{oxy},x} + \beta^2_{\text{oxy},y}\right) = \frac{1}{2}\left(1.58 \times 10^4\,\text{MeV}\right)\left(0.015^2 + 0.0057^2\right) \approx 2.0 \text{ MeV} \ .$$

(b) Using Eq. 38-47 and Eq. 38-43,

$$Q = -(1.007825\,\text{u} + 16.99914\,\text{u} - 4.00260\,\text{u} - 14.00307\,\text{u})c^2 = -(0.001295\,\text{u})(931.5\,\text{MeV/u})$$

which yields $Q = -1.206$ MeV. Incidentally, this provides an alternate way to obtain the answer (and a more accurate one at that!) to part (a). Eq. 38-46 leads to

$$K_{\text{oxy}} = K_\alpha + Q - K_p = 7.70\,\text{MeV} - 1.206\,\text{MeV} - 4.44\,\text{MeV} = 2.05 \text{ MeV} \ .$$

This approach to finding K_{oxy} avoids the many computational steps and approximations made in part (a).

52. (a) From the length contraction equation, the length L'_c of the car according to Garageman is

$$L'_c = \frac{L_c}{\gamma} = L_c\sqrt{1-\beta^2} = (30.5\,\text{m})\sqrt{1-(0.9980)^2} = 1.93 \text{ m} \ .$$

(b) Since the x_g axis is fixed to the garage $x_{g2} = L_g = 6.00\,\text{m}$. As for t_{g2}, note from Fig. 38-21(b) that, at $t_g = t_{g1} = 0$ the coordinate of the front bumper of the limo in the x_g frame is L'_c, meaning that the front of the limo is still a distance $L_g - L'_c$ from the back door of the garage. Since the limo travels at a speed v, the time it takes for the front of the limo to reach the back door of the garage is given by

$$\Delta t_g = t_{g2} - t_{g1} = \frac{L_g - L'_c}{v} = \frac{6.00\,\text{m} - 1.93\,\text{m}}{0.9980(2.998 \times 10^8\,\text{m/s})} = 1.36 \times 10^{-8} \text{ s} \ .$$

Thus $t_{g2} = t_{g1} + \Delta t_g = 0 + 1.36 \times 10^{-8}\,\text{s} = 1.36 \times 10^{-8}\,\text{s}$.

(c) The limo is inside the garage between times t_{g1} and t_{g2}, so the time duration is $t_{g2} - t_{g1} = 1.36 \times 10^{-8}$ s.

(d) Again from Eq. 38-13, the length L'_g of the garage according to Carman is

$$L'_g = \frac{L_g}{\gamma} = L_g\sqrt{1-\beta^2} = (6.00\,\text{m})\sqrt{1-(0.9980)^2} = 0.379 \text{ m} .$$

(e) Again, since the x_c axis is fixed to the limo $x_{c2} = L_c = 30.5\,\text{m}$. Now, from the two diagrams described in part (h) below, we know that at $t_c = t_{c2}$ (when event 2 takes place), the distance between the rear bumper of the limo and the back door of the garage is given by $L_c - L'_g$. Since the garage travels at a speed v, the front door of the garage will reach the rear bumper of the limo a time Δt_c later, where Δt_c satisfies

$$\Delta t_c = t_{c1} - t_{c2} = \frac{L_c - L'_g}{v} = \frac{30.5\,\text{m} - 0.379\,\text{m}}{0.9980(2.998 \times 10^8\,\text{m/s})} = 1.01 \times 10^{-7} \text{ s} .$$

Thus $t_{c2} = t_{c1} - \Delta t_c = 0 - 1.01 \times 10^{-7}\,\text{s} = -1.01 \times 10^{-7}\,\text{s}$.

(f) From Carman's point of view, the answer is clearly no.

(g) Event 2 occurs first according to Carman, since $t_{c2} < t_{c1}$.

(h) We describe the essential features of the two pictures. For event 2, the front of the limo coincides with the back door, and the garage itself seems very short (perhaps failing to reach as far as the front window of the limo). For event 1, the rear of the car coincides with the front door and the front of the limo has traveled a significant distance beyond the back door. In this picture, as in the other, the garage seems very short compared to the limo.

(i) Both Carman and Garageman are correct in their respective reference frames. But, in a sense, Carman should lose the bet since he dropped his physics course before reaching the Theory of Special Relativity!

53. (a) The spatial separation between the two bursts is vt. We project this length onto the direction perpendicular to the light rays headed to Earth and obtain $D_{\text{app}} = vt\sin\theta$.

(b) Burst 1 is emitted a time t ahead of burst 2. Also, burst 1 has to travel an extra distance L more than burst 2 before reaching the Earth, where $L = vt\cos\theta$ (see Fig. 38-22); this requires an additional time $t' = L/c$. Thus, the apparent time is given by

$$T_{\text{app}} = t - t' = t - \frac{vt\cos\theta}{c} = t\left[1 - \left(\frac{v}{c}\right)\cos\theta\right] .$$

(c) We obtain

$$V_{\text{app}} = \frac{D_{\text{app}}}{T_{\text{app}}} = \left[\frac{(v/c)\sin\theta}{1-(v/c)\cos\theta}\right]c = \left[\frac{(0.980)\sin 30.0^\circ}{1-(0.980)\cos 30.0^\circ}\right]c = 3.24\,c .$$

54. (a) The strategy is to find the γ factor from $E = 14.24 \times 10^{-9}$ J and $m_pc^2 = 1.5033 \times 10^{-10}$ J and from that find the contracted length. From the energy relation (Eq. 38-45), we obtain

$$\gamma = \frac{E}{mc^2} = 94.73 .$$

Consequently, Eq. 38-13 yields

$$L = \frac{L_0}{\gamma} = 0.222\,\text{cm} = 2.22 \times 10^{-3} \text{ m} .$$

(b) and (c) From the γ factor, we find the speed:

$$v = c\sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = 0.99994c \ .$$

Therefore, the trip (according to the proton) took $\Delta t_0 = 2.22 \times 10^{-3}/0.99994c = 7.40 \times 10^{-12}$ s. Finally, the time dilation formula (Eq. 38-7) leads to

$$\Delta t = \gamma \Delta t_0 = 7.01 \times 10^{-10} \text{ s}$$

which can be checked using $\Delta t = L_0/v$ in our frame of reference.

55. Since it has two protons, its kinetic energy is 600 MeV. With the given value $mc^2 = 3727$ MeV, we use Eq. 38-37:

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{600^2 + 2(600)(3727)}$$

which yields $p = 2198$ MeV/c.

56. For the purposes of using Eq. 38-28, we choose our frame to be the primed frame and note that, as a consequence, $v = -0.800c\,\hat{\imath}$ for the velocity of us relative to Bullwinkle.

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.990c\hat{\imath} - 0.800c\hat{\imath}}{1 - (0.990)(0.800)} = 0.913c\,\hat{\imath} \ .$$

57. (a) We compute

$$\gamma = \frac{1}{\sqrt{1 - (0.9990)^2}} = 22.4$$

Now, the length contraction formula (Eq. 38-13) yields

$$L = \frac{2.50 \text{ m}}{\gamma} = 0.112 \text{ m} \ .$$

(b) (c) and (d) We assume our spacetime coordinate origins coincide and use the Lorentz transformations (Eq. 38-20, but with primes and non-primes swapped, and $v \to -v$). Lengths are in meters and time is in nanoseconds (so that $c = 0.2998$ in these units).

$$\begin{aligned} x_\alpha &= \gamma\left(4.0 + (0.9990c)(40)\right) = 357 \\ t_\alpha &= \gamma\left(40 + (0.9990c)(4.0)/c^2\right) = 1193 \\ x_\beta &= \gamma\left(-4.0 + (0.9990c)(80)\right) = 446 \\ t_\beta &= \gamma\left(80 + (0.9990c)(-4.0)/c^2\right) = 1491 \end{aligned}$$

Thus, our reckoning of the distance between events is $x_\beta - x_\alpha = 89.0$ m. We note that event alpha took place first (smallest value of t) and that the time-separation is $t_\alpha - t_\beta = 298$ ns.

58. Using Eq. 38-10,

$$v = \frac{d}{t} = \frac{6.0\,\text{ly}}{2.0\,\text{y} + 6.0\,\text{y}} = \frac{(6.0c)(1.0\,\text{y})}{2.0\,\text{y} + 6.0\,\text{y}} = 0.75c.$$

59. To illustrate the technique, we derive Eq. 1$'$ from Eqs. 1 and 2 (in Table 38-2). We multiply Eq. 2 by speed v and subtract it from Eq. 1:

$$\Delta x - v\Delta t = \gamma\left(\Delta x' + v\Delta t'\right) - v\gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) = \gamma\Delta x'\left(1 - \frac{v^2}{c^2}\right)$$

We note that $\gamma\left(1 - v^2/c^2\right) = 1/\gamma$ (using Eq. 38-8), so that if we multiply the above equation by γ we obtain Eq. 1$'$:

$$\gamma\left(\Delta x - v\Delta t\right) = \gamma\left(\gamma\Delta x'\left(1 - \frac{v^2}{c^2}\right)\right) = \Delta x'$$

60. (a) $v_r = 2v = 2(27000\,\text{km/h}) = 54000\,\text{km/h}$.

(b) We can express c in these units by multiplying by 3.6: $c = 1.08 \times 10^9$ km/h. The correct formula for v_r is $v_r = 2v/(1 + v^2/c^2)$, so the fractional error is

$$1 - \frac{1}{1 + v^2/c^2} = 1 - \frac{1}{1 + [(27000\,\text{km/h})/(1.08 \times 10^9\,\text{km/h})]^2} = 6.3 \times 10^{-10} .$$

The discussion in Sample Problem 38-7 dealing with numerical considerations may prove helpful for those whose calculators do not yield this answer.

61. (a) We assume the electron starts from rest. The classical formula for kinetic energy is Eq. 38-48, so if $v = c$ then this (for an electron) would be $\frac{1}{2}mc^2 = \frac{1}{2}(511\,\text{keV}) = 255.5$ keV (using Table 38-3). Setting this equal to the potential energy loss (which is responsible for its acceleration), we find (using Eq. 25-7)

$$V = \frac{255.5\,\text{keV}}{|q|} = \frac{255\,\text{keV}}{e} = 255.5\ \text{kV} .$$

(b) Setting this amount of potential energy loss ($|\Delta U| = 255.5$ keV) equal to the correct relativistic kinetic energy, we obtain (using Eq. 38-49)

$$mc^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) = |\Delta U| \implies v = c\sqrt{1 + \left(\frac{1}{1 - \Delta U/mc^2}\right)^2}$$

which yields $v = 0.745c = 2.23 \times 10^8$ m/s.

62. (a) $\Delta E = \Delta mc^2 = (3.0\,\text{kg})(0.0010)(2.998 \times 10^8\,\text{m/s})^2 = 2.7 \times 10^{14}\,\text{J}$.

(b) The mass of TNT is

$$m_{\text{TNT}} = \frac{(2.7 \times 10^{14}\,\text{J})(0.227\,\text{kg/mol})}{3.4 \times 10^6\,\text{J}} = 1.8 \times 10^7\ \text{kg} .$$

(c) The fraction of mass converted in the TNT case is

$$\frac{\Delta m_{\text{TNT}}}{m_{\text{TNT}}} = \frac{(3.0\,\text{kg})(0.0010)}{1.8 \times 10^7\,\text{kg}} = 1.6 \times 10^{-9} ,$$

Therefore, the fraction is $0.0010/1.6 \times 10^{-9} = 6.0 \times 10^6$.

63. (a) Eq. 38-33 yields

$$v = \frac{\Delta\lambda}{\lambda}c = \left(\frac{462 - 434}{434}\right)c = 0.065c$$

or $v = 1.93 \times 10^7$ m/s.

(b) Since it is shifted "towards the red" (towards longer wavelengths) then the galaxy is moving away from us (receding).

64. When $\beta = 0.9860$, we have $\gamma = 5.9972$, and when $\beta = 0.9850$, we have $\gamma = 5.7953$. Thus, $\Delta\gamma = 0.202$ and the change in kinetic energy (equal to the work) becomes (using Eq. 38-49)

$$W = \Delta K = mc^2\Delta\gamma = 189\ \text{MeV}$$

where $mc^2 = 938$ MeV has been used (see Table 38-3).

65. Using $m_p = 1.672623 \times 10^{-27}$ kg in Eq. 38-45 yields

$$\gamma = \frac{E}{m_p c^2} = \frac{14.242 \times 10^{-9}\,\mathrm{J}}{1.50328 \times 10^{-10}\,\mathrm{J}} = 94.740 \ .$$

Solving for the speed , we obtain

$$v = c\sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = 0.99994c \ .$$

66. (a) According to ship observers, the duration of proton flight is $\Delta t' = (760\,\mathrm{m})/0.980c = 2.59\ \mu\mathrm{s}$ (assuming it travels the entire length of the ship).

(b) To transform to our point of view, we use Eq. 2 in Table 38-2. Thus, with $\Delta x' = -750$ m, we have

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 0.57\ \mu\mathrm{s} \ .$$

(c) and (d) For the ship observers, firing the proton from back to front makes no difference, and $\Delta t' = 2.59\ \mu$s as before. For us, the fact that now $\Delta x' = +750$ m is a significant change.

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 16.0\ \mu\mathrm{s} \ .$$

67. (a) Our lab-based measurement of its lifetime is figured simply from $t = L/v = 7.99 \times 10^{-13}$ s. Use of the time-dilation relation (Eq. 38-7) leads to

$$\Delta t_0 = \left(7.99 \times 10^{-13}\,\mathrm{s}\right)\sqrt{1 - (0.960)^2} = 2.24 \times 10^{-13}\,\mathrm{s} \ .$$

(b) The length contraction formula can be used, or we can use the simple speed-distance relation (from the point of view of the particle, who watches the lab and all its meter sticks rushing past him at $0.960c$ until he expires): $L = v\Delta t_0 = 6.44 \times 10^{-5}$ m.

68. Using Appendix C, we find that the contraction is

$$\begin{aligned} |\Delta L| &= L_0 - L = L_0\left(1 - \frac{1}{\gamma}\right) = L_0(1 - \sqrt{1-\beta^2}) \\ &= 2(6.370 \times 10^6\,\mathrm{m})\left(1 - \sqrt{1 - \left(\frac{3.0 \times 10^4\,\mathrm{m/s}}{2.998 \times 10^8\,\mathrm{m/s}}\right)^2}\right) \\ &= 0.064\ \mathrm{m} \ . \end{aligned}$$

The discussion in Sample Problem 38-7 dealing with numerical considerations may prove helpful for those whose calculators do not yield this answer.

69. The speed of the spaceship after the first increment is $v_1 = 0.5c$. After the second one, it becomes

$$v_2 = \frac{v' + v_1}{1 + v'v_1/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)^2/c^2} = 0.80c \ ,$$

and after the third one, the speed is

$$v_3 = \frac{v' + v_2}{1 + v'v_2/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)(0.80c)/c^2} = 0.929c \ .$$

Continuing with this process, we get $v_4 = 0.976c$, $v_5 = 0.992c$, $v_6 = 0.997c$ and $v_7 = 0.999c$. Thus, seven increments are needed.

70. We use the transverse Doppler shift formula, Eq. 38-34: $f = f_0\sqrt{1-\beta^2}$, or

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\sqrt{1-\beta^2} \ .$$

We solve for $\lambda - \lambda_0$:

$$\lambda - \lambda_0 = \lambda_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (589.00\,\text{mm})\left[\frac{1}{\sqrt{1-(0.100)^2}} - 1\right] = +2.97 \text{ nm} \ .$$

71. The mean lifetime of a pion measured by observers on the Earth is $\Delta t = \gamma\Delta t_0$, so the distance it can travel (using Eq. 38-12) is

$$d = v\Delta t = \gamma v\Delta t_0 = \frac{(0.99)(2.998\times10^8\,\text{m/s})(26\times10^{-9}\,\text{s})}{\sqrt{1-(0.99)^2}} = 55 \text{ m} \ .$$

72. (a) For a proton (using Table 38-3), our results are:

$$E = \gamma m_p c^2 = \frac{938\,\text{MeV}}{\sqrt{1-(0.990)^2}} = 6.65 \text{ GeV}$$

$$K = E - m_p c^2 = 6.65\,\text{GeV} - 938\,\text{MeV} = 5.71 \text{ GeV}$$

$$p = \gamma m_p v = \gamma(m_p c^2)\beta/c = \frac{(938\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 6.59 \text{ GeV}/c$$

(b) For an electron:

$$E = \gamma m_e c^2 = \frac{0.511\,\text{MeV}}{\sqrt{1-(0.990)^2}} = 3.62 \text{ MeV}$$

$$K = E - m_e c^2 = 3.625\,\text{MeV} - 0.511\,\text{MeV} = 3.11 \text{ MeV}$$

$$p = \gamma m_e v = \gamma(m_e c^2)\beta/c = \frac{(0.511\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 3.59 \text{ MeV}/c$$

73. The strategy is to find the speed from $E = 1533$ MeV and $mc^2 = 0.511$ MeV (see Table 38-3) and from that find the time. From the energy relation (Eq. 38-45), we obtain

$$v = c\sqrt{1-\left(\frac{mc^2}{E}\right)^2} = 0.99999994c \ \approx \ c$$

so that we conclude it took the electron 26 y to reach us. In order to transform to its own "clock" it's useful to compute γ directly from Eq. 38-45:

$$\gamma = \frac{E}{mc^2} = 3000$$

though if one is careful one can also get this result from $\gamma = 1/\sqrt{1-(v/c)^2}$. Then, Eq. 38-7 leads to

$$\Delta t_0 = \frac{26 \text{ y}}{\gamma} = 0.0087\,\text{y}$$

so that the electron "concludes" the distance he traveled is 0.0087 light-years (stated differently, the Earth, which is rushing towards him at very nearly the speed of light, seemed to start its journey from a distance of 0.0087 light-years away).

74. (a) Using Eq. 38-7, we expect the dilated time intervals to be

$$\tau = \gamma\tau_0 = \frac{\tau_0}{\sqrt{1-(v/c)^2}} .$$

(b) We rewrite Eq. 38-30 using the fact that period is the reciprocal of frequency ($f_R = \tau_R^{-1}$ and $f_0 = \tau_0^{-1}$):

$$\tau_R = \frac{1}{f_R} = \left(f_0\sqrt{\frac{1-\beta}{1+\beta}}\right)^{-1} = \tau_0\sqrt{\frac{1+\beta}{1-\beta}} = \tau_0\sqrt{\frac{c+v}{c-v}} .$$

(c) The Doppler shift combines two physical effects: the time dilation of the moving source *and* the travel-time differences involved in periodic emission (like a sine wave or a series of pulses) from a traveling source to a "stationary" receiver). To isolate the purely time-dilation effect, it's useful to consider "local" measurements (say, comparing the readings on a moving clock to those of two of your clocks, spaced some distance apart, such that the moving clock and each of your clocks can make a close-comparison of readings at the moment of passage).

75. We use the relative velocity formula (Eq. 38-28) with the primed measurements being those of the scout ship. We note that $v = -0.900c$ since the velocity of the scout ship relative to the cruiser is opposite to that of the cruiser relative to the scout ship.

$$u = \frac{u'+v}{1+u'v/c^2} = \frac{0.980c - 0.900c}{1-(0.980)(0.900)} = 0.678c .$$

76. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

where $\Delta t_0 = 120$ y. This yields $\Delta t = 2684$ y.

77. (a) The relative contraction is

$$\begin{aligned}
\frac{|\Delta L|}{L_0} &= \frac{L_0(1-\gamma^{-1})}{L_0} = 1-\sqrt{1-\beta^2} \\
&\approx 1-\left(1-\frac{1}{2}\beta^2\right) = \frac{1}{2}\beta^2 \\
&= \frac{1}{2}\left(\frac{630\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2 \\
&= 2.21\times10^{-12} .
\end{aligned}$$

(b) Letting $|\Delta t - \Delta t_0| = \Delta t_0(\gamma-1) = \tau = 1.00\,\mu$s, we solve for Δt_0:

$$\begin{aligned}
\Delta t_0 &= \frac{\tau}{\gamma-1} = \frac{\tau}{(1-\beta^2)^{-1/2}-1} \approx \frac{\tau}{1+\frac{1}{2}\beta^2-1} = \frac{2\tau}{\beta^2} \\
&= \frac{2(1.00\times10^{-6}\,\text{s})(1\,\text{d}/86400\,\text{s})}{[(630\,\text{m/s})/(2.998\times10^8\,\text{m/s})]^2} \\
&= 5.25\ \text{d} .
\end{aligned}$$

78. Let the reference frame be S in which the particle (approaching the South Pole) is at rest, and let the frame that is fixed on Earth be S'. Then $v = 0.60c$ and $u' = 0.80c$ (calling "downwards" [in the sense of Fig. 38-31] positive). The relative speed is now the speed of the other particle as measured in S:

$$u = \frac{u'+v}{1+u'v/c^2} = \frac{0.80c+0.60c}{1+(0.80c)(0.60c)/c^2} = 0.95c .$$

79. We refer to the particle in the first sentence of the problem statement as particle 2. Since the total momentum of the two particles is zero in S', it must be that the velocities of these two particles are equal in magnitude and opposite in direction in S'. Letting the velocity of the S' frame be v relative to S, then the particle which is at rest in S must have a velocity of $u'_1 = -v$ as measured in S', while the velocity of the other particle is given by solving Eq. 38-28 for u':

$$u'_2 = \frac{u_2 - v}{1 - u_2 v/c^2} = \frac{\left(\frac{c}{2}\right) - v}{1 - \left(\frac{c}{2}\right)\left(\frac{v}{c^2}\right)} \ .$$

Letting $u'_2 = -u'_1 = v$, we obtain

$$\frac{\left(\frac{c}{2}\right) - v}{1 - \left(\frac{c}{2}\right)\left(\frac{v}{c^2}\right)} = v \quad \Longrightarrow \quad v = c(2 \pm \sqrt{3}) \ \approx \ 0.27c$$

where the quadratic formula has been used (with the smaller of the two roots chosen so that $v \le c$).

80. From Eq. 28-37, we have

$$Q = -\Delta M c^2 = -\left(3(4.00151\,\mathrm{u}) - 11.99671\,\mathrm{u}\right)c^2 = -(0.00782\,\mathrm{u})(931.5\,\mathrm{MeV/u}) = -7.28\ \mathrm{MeV}\ .$$

Thus, it takes a minimum of 7.28 MeV supplied to the system to cause this reaction. We note that the masses given in this problem are strictly for the nuclei involved; they are not the "atomic" masses which are quoted in several of the other problems in this chapter.

81. We use Eq. 38-51 with $mc^2 = 0.511$ MeV (see Table 38-3):

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(2.00)^2 + 2(2.00)(0.511)}$$

This readily yields $p = 2.46$ MeV/c.

Chapter 39

1. Eq. 39-3 gives $h = 4.14 \times 10^{-15}\,\text{eV}\cdot\text{s}$, but the metric prefix which stands for 10^{-15} is femto (f). Thus, $h = 4.14\,\text{eV}\cdot\text{fs}$.

2. Let $E = 1240\,\text{eV}\cdot\text{nm}/\lambda_{\text{min}} = 0.6\,\text{eV}$ to get $\lambda = 2.1 \times 10^3\,\text{nm} = 2.1\,\mu\text{m}$. It is in the infrared region.

3. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}\,\text{J}\cdot\text{s}$ and $c = 2.998 \times 10^8\,\text{m/s}$,

$$hc = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^8\,\text{m/s})}{(1.602 \times 10^{-19}\,\text{J/eV})(10^{-9}\,\text{m/nm})} = 1240\,\text{eV}\cdot\text{nm} .$$

   Thus,

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda} .$$

4. From the result of problem 3,

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{589\,\text{nm}} = 2.11\ \text{eV} .$$

5. Let R be the rate of photon emission (number of photons emitted per unit time) of the Sun and let E be the energy of a single photon. Then the power output of the Sun is given by $P = RE$. Now $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and

$$R = \frac{\lambda P}{hc} = \frac{(550\,\text{nm})(3.9 \times 10^{26}\,\text{W})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^8\,\text{m/s})} = 1.0 \times 10^{45}\ \text{photons/s} .$$

6. We denote the diameter of the laser beam as d. The cross-sectional area of the beam is $A = \pi d^2/4$. From the formula obtained in problem 5, the rate is given by

$$\begin{aligned}\frac{R}{A} &= \frac{\lambda P}{hc(\pi d^2/4)} \\ &= \frac{4(633\,\text{nm})(5.0 \times 10^{-3}\,\text{W})}{\pi(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^8\,\text{m/s})(3.5 \times 10^{-3}\,\text{m})^2} \\ &= 1.7 \times 10^{21}\ \frac{\text{photons}}{\text{m}^2 \cdot \text{s}} .\end{aligned}$$

7. Using the result of problem 3,

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{21 \times 10^7\,\text{nm}} = 5.9 \times 10^{-6}\,\text{eV} = 5.9\,\mu\text{eV} .$$

8. Let

$$\frac{1}{2}m_e v^2 = E_{\text{photon}} = \frac{hc}{\lambda}$$

and solve for v:

$$\begin{aligned} v &= \sqrt{\frac{2hc}{\lambda m_e}} = \sqrt{\frac{2hc}{\lambda m_e c^2}\,c^2} = c\sqrt{\frac{2hc}{\lambda (m_e c^2)}} \\ &= (2.998 \times 10^8\,\text{m/s})\sqrt{\frac{2(1240\,\text{eV}\cdot\text{nm})}{(590\,\text{nm})(511 \times 10^3\,\text{eV})}} = 8.6 \times 10^5 \text{ m/s} . \end{aligned}$$

Since $v \ll c$, the non-relativistic formula $K = \frac{1}{2}mv^2$ may be used. The result of problem 3 and the $m_e c^2$ value of Table 38-3 are used in our calculation.

9. Since $\lambda = (1,650,763.73)^{-1}\,\text{m} = 6.0578021 \times 10^{-7}\,\text{m} = 605.78021\,\text{nm}$, the energy is (using the result of problem 3)

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{605.78021\,\text{nm}} = 2.047 \text{ eV} .$$

10. Following Sample Problem 39-1, we have

$$P = \frac{Rhc}{\lambda} = \frac{(100/\text{s})\left(6.63 \times 10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998 \times 10^8\,\text{m/s}\right)}{550 \times 10^{-9}\,\text{m}} = 3.6 \times 10^{-17} \text{ W} .$$

11. The total energy emitted by the bulb is $E = 0.93Pt$, where $P = 60\,\text{W}$ and $t = 730\,\text{h} = (730\,\text{h})(3600\,\text{s/h}) = 2.628 \times 10^6\,\text{s}$. The energy of each photon emitted is $E_{\text{ph}} = hc/\lambda$. Therefore, the number of photons emitted is

$$N = \frac{E}{E_{\text{ph}}} = \frac{0.93Pt}{hc/\lambda} = \frac{(0.93)(60\,\text{W})(2.628 \times 10^6\,\text{s})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})\,(2.998 \times 10^8\,\text{m/s})\,/(630 \times 10^{-9}\,\text{m})} = 4.7 \times 10^{26} .$$

12. The rate at which photons are emitted from the argon laser source is given by $R = P/E_{\text{ph}}$, where $P = 1.5\,\text{W}$ is the power of the laser beam and $E_{\text{ph}} = hc/\lambda$ is the energy of each photon of wavelength λ. Since $\alpha = 84\%$ of the energy of the laser beam falls within the central disk, the rate of photon absorption of the central disk is

$$\begin{aligned} R' &= \alpha R = \frac{\alpha P}{hc/\lambda} = \frac{(0.84)(1.5\,\text{W})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})\,(2.998 \times 10^8\,\text{m/s})\,/(515 \times 10^{-9}\,\text{m})} \\ &= 3.3 \times 10^{18} \text{ photons/s} . \end{aligned}$$

13. (a) Let R be the rate of photon emission (number of photons emitted per unit time) and let E be the energy of a single photon. Then, the power output of a lamp is given by $P = RE$ if all the power goes into photon production. Now, $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and $R = \lambda P/hc$. The lamp emitting light with the longer wavelength (the 700 nm lamp) emits more photons per unit time. The energy of each photon is less, so it must emit photons at a greater rate.

    (b) Let R be the rate of photon production for the 700 nm lamp. Then,

$$R = \frac{\lambda P}{hc} = \frac{(700\,\text{nm})(400\,\text{J/s})}{(1.60 \times 10^{-19}\,\text{J/eV})(1240\,\text{eV}\cdot\text{nm})} = 1.41 \times 10^{21} \text{ photon/s} .$$

    The result $hc = 1240\,\text{eV}\cdot\text{nm}$ developed in Exercise 3 is used.

14. (a) The rate at which solar energy strikes the panel is

$$P = (1.39\,\text{kW/m}^2)(2.60\,\text{m}^2) = 3.61 \text{ kW} .$$

(b) The rate at which solar photons are absorbed by the panel is

$$R = \frac{P}{E_{\text{ph}}} = \frac{3.61 \times 10^3 \text{ W}}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})/(550 \times 10^{-9} \text{ m})} = 1.00 \times 10^{22}/\text{s} .$$

(c) The time in question is given by

$$t = \frac{N_A}{R} = \frac{6.02 \times 10^{23}}{1.00 \times 10^{22}/\text{s}} = 60.2 \text{ s} .$$

15. (a) We assume all the power results in photon production at the wavelength $\lambda = 589\,\text{nm}$. Let R be the rate of photon production and E be the energy of a single photon. Then, $P = RE = Rhc/\lambda$, where $E = hf$ and $f = c/\lambda$ are used. Here h is the Planck constant, f is the frequency of the emitted light, and λ is its wavelength. Thus,

$$R = \frac{\lambda P}{hc} = \frac{(589 \times 10^{-9} \text{ m})(100 \text{ W})}{(6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})} = 2.96 \times 10^{20} \text{ photon/s} .$$

(b) Let I be the photon flux a distance r from the source. Since photons are emitted uniformly in all directions, $R = 4\pi r^2 I$ and

$$r = \sqrt{\frac{R}{4\pi I}} = \sqrt{\frac{2.96 \times 10^{20} \text{ photon/s}}{4\pi(1.00 \times 10^4 \text{ photon/m}^2 \cdot \text{s})}} = 4.85 \times 10^7 \text{ m} .$$

(c) The photon flux is

$$I = \frac{R}{4\pi r^2} = \frac{2.96 \times 10^{20} \text{ photon/s}}{4\pi(2.00 \text{ m})^2} = 5.89 \times 10^{18} \, \frac{\text{photon}}{\text{m}^2 \cdot \text{s}} .$$

16. (a) Since $E_{\text{ph}} = h/\lambda = 1240\,\text{eV}\cdot\text{nm}/680\,\text{nm} = 1.82\,\text{eV} < \Phi = 2.28\,\text{eV}$, there is no photoelectric emission. The result of problem 3 is used in our calculation.

(b) The cutoff wavelength is the longest wavelength of photons which will cause photoelectric emission. In sodium, this is given by $E_{\text{ph}} = hc/\lambda_{\text{max}} = \Phi$, or $\lambda_{\text{max}} = hc/\Phi = (1240\,\text{eV}\cdot\text{nm})/2.28\,\text{eV} = 544\,\text{nm}$. This corresponds to the color green.

17. The energy of the most energetic photon in the visible light range (with wavelength of about 400 nm) is about $E = (1240\,\text{eV}\cdot\text{nm}/400\,\text{nm}) = 3.1\,\text{eV}$ (using the result of problem 3). Consequently, barium and lithium can be used, since their work functions are both lower than 3.1 eV.

18. (a) For $\lambda = 565\,\text{nm}$

$$hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{565 \text{ nm}} = 2.20 \text{ eV} .$$

Since $\Phi_{\text{potassium}} > hf > \Phi_{\text{cesium}}$, the photoelectric effect can occur in cesium but not in potassium at this wavelength. The result of problem 3 is used in our calculation.

(b) Now $\lambda = 518\,\text{nm}$ so

$$hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{518 \text{ m}} = 2.40 \text{ eV} .$$

This is greater than both Φ_{cesium} and $\Phi_{\text{potassium}}$, so the photoelectric effect can now occur for both metals.

19. The energy of an incident photon is $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the electromagnetic radiation, and λ is its wavelength. The kinetic energy of the most energetic electron emitted is $K_m = E - \Phi = (hc/\lambda) - \Phi$, where Φ is the work function for sodium. The stopping potential V_0 is related to the maximum kinetic energy by $eV_0 = K_m$, so $eV_0 = (hc/\lambda) - \Phi$ and

$$\lambda = \frac{hc}{eV_0 + \Phi} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.0 \text{ eV} + 2.2 \text{ eV}} = 170 \text{ nm} .$$

Here $eV_0 = 5.0\,\text{eV}$ and $hc = 1240\,\text{eV}\cdot\text{nm}$ are used. See problem 3.

20. We use Eq. 39-5 to find the maximum kinetic energy of the ejected electrons:

$$K_{\max} = hf - \Phi = (4.14\times10^{-15}\,\text{eV}\cdot\text{s})(3.0\times10^{15}\,\text{Hz}) - 2.3\,\text{eV} = 10\ \text{eV}\ .$$

21. The speed v of the electron satisfies $K_{\max} = \frac{1}{2}m_e v^2 = \frac{1}{2}(m_e c^2)(v/c)^2 = E_{\text{photon}} - \Phi$. Using Table 38-3, we find

$$v = c\sqrt{\frac{2(E_{\text{photon}}-\Phi)}{m_e c^2}} = (2.998\times10^8\,\text{m/s})\sqrt{\frac{2(5.80\,\text{eV}-4.50\,\text{eV})}{511\times10^3\,\text{eV}}} = 6.76\times10^5\ \text{m/s}\ .$$

22. (a) We use Eq. 39-6:

$$V_{\text{stop}} = \frac{hf-\Phi}{e} = \frac{hc/\lambda-\Phi}{e} = \frac{(1240\,\text{eV}\cdot\text{nm}/400\,\text{nm}) - 1.8\,\text{eV}}{e} = 1.3\ \text{V}\ .$$

(b) We use the formula obtained in the solution of problem 21:

$$\begin{aligned} v &= \sqrt{\frac{2(E_{\text{photon}}-\Phi)}{m_e}} = \sqrt{\frac{2eV_{\text{stop}}}{m_e}} = c\sqrt{\frac{2eV_{\text{stop}}}{m_e c^2}} \\ &= (2.998\times10^8\,\text{m/s})\sqrt{\frac{2e(1.3\,\text{V})}{511\times10^3\,\text{eV}}} \\ &= 6.8\times10^5\ \text{m/s}\ . \end{aligned}$$

23. (a) The kinetic energy K_m of the fastest electron emitted is given by $K_m = hf - \Phi = (hc/\lambda) - \Phi$, where Φ is the work function of aluminum, f is the frequency of the incident radiation, and λ is its wavelength. The relationship $f = c/\lambda$ was used to obtain the second form. Thus,

$$K_m = \frac{1240\,\text{eV}\cdot\text{nm}}{200\,\text{nm}} - 4.20\,\text{eV} = 2.00\ \text{eV}$$

where the result of Exercise 3 is used.

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_0 is given by $K_m = eV_0$, so $V_0 = K_m/e = (2.00\,\text{eV})/e = 2.00\,\text{V}$.

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus $hc/\lambda = \Phi$ or $\lambda = hc/\Phi = (1240\,\text{eV}\cdot\text{nm})/(4.2\,\text{eV}) = 295\,\text{nm}$. If the wavelength is longer, the photon energy is less and a photon does not have sufficient energy to knock even the most energetic electron out of the aluminum sample.

24. We use Eq. 39-6 and the result of problem 3:

$$K_{\max} = E_{\text{photon}} - \Phi = \frac{hc}{\lambda} - \frac{hc}{\lambda_{\max}} = \frac{1240\,\text{eV}\cdot\text{nm}}{254\,\text{nm}} - \frac{1240\,\text{eV}\cdot\text{nm}}{325\,\text{nm}} = 1.07\ \text{eV}\ .$$

25. To find the longest possible wavelength $\lambda_{\max}$ (corresponding to the lowest possible energy) of a photon which can produce a photoelectric effect in platinum, we set $K_{\max} = 0$ in Eq. 39-5 and use $hf = hc/\lambda$. Thus $hc/\lambda_{\max} = \Phi$. We solve for $\lambda_{\max}$:

$$\lambda_{\max} = \frac{hc}{\Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.32\,\text{eV}} = 233\ \text{nm}\ .$$

26. (a) For the first and second case (labeled 1 and 2) we have $eV_{01} = hc/\lambda_1 - \Phi$ and $eV_{02} = hc/\lambda_2 - \Phi$, from which h and Φ can be determined. Thus,

$$\begin{aligned} h &= \frac{e(V_1-V_2)}{c(\lambda_1^{-1}-\lambda_2^{-1})} = \frac{1.85\,\text{eV}-0.820\,\text{eV}}{(3.00\times10^{17}\,\text{nm/s})[(300\,\text{nm})^{-1}-(400\,\text{nm})^{-1}]} \\ &= 4.12\times10^{-15}\ \text{eV}\cdot\text{s}\ . \end{aligned}$$

(b) The work function is

$$\Phi = \frac{3(V_2\lambda_2 - V_1\lambda_1)}{\lambda_1 - \lambda_2} = \frac{(0.820\,\mathrm{eV})(400\,\mathrm{nm}) - (1.85\,\mathrm{eV})(300\,\mathrm{nm})}{300\,\mathrm{nm} - 400\,\mathrm{nm}} = 2.27 \text{ eV} .$$

(c) Let $\Phi = hc/\lambda_{\max}$ to obtain

$$\lambda_{\max} = \frac{hc}{\Phi} = \frac{1240\,\mathrm{eV\cdot nm}}{2.27\,\mathrm{eV}} = 545 \text{ nm} .$$

27. (a) We use the photoelectric effect equation (Eq. 39-5) in the form $hc/\lambda = \Phi + K_m$. The work function depends only on the material and the condition of the surface, and not on the wavelength of the incident light. Let λ_1 be the first wavelength described and λ_2 be the second. Let $K_{m1} = 0.710\,\mathrm{eV}$ be the maximum kinetic energy of electrons ejected by light with the first wavelength, and $K_{m2} = 1.43\,\mathrm{eV}$ be the maximum kinetic energy of electrons ejected by light with the second wavelength. Then,

$$\frac{hc}{\lambda_1} = \Phi + K_{m1} \quad \text{and} \quad \frac{hc}{\lambda_2} = \Phi + K_{m2} .$$

The first equation yields $\Phi = (hc/\lambda_1) - K_{m1}$. When this is used to substitute for Φ in the second equation, the result is $(hc/\lambda_2) = (hc/\lambda_1) - K_{m1} + K_{m2}$. The solution for λ_2 is

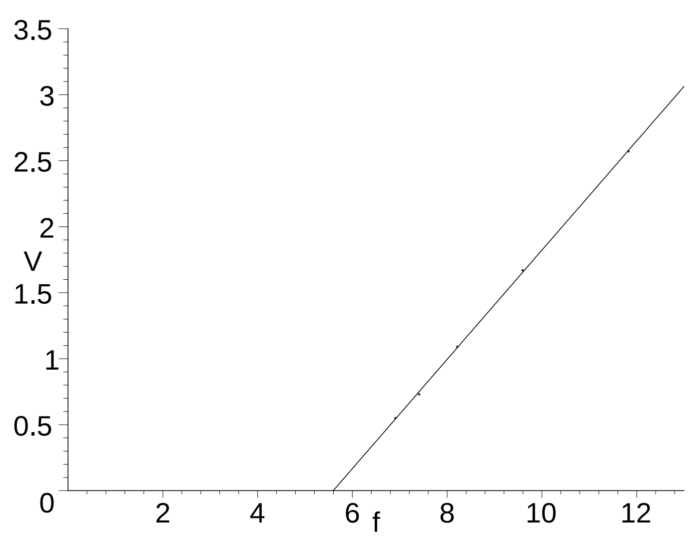
$$\begin{aligned}\lambda_2 &= \frac{hc\lambda_1}{hc + \lambda_1(K_{m2} - K_{m1})} \\ &= \frac{(1240\,\mathrm{eV\cdot nm})(491\,\mathrm{nm})}{1240\,\mathrm{eV\cdot nm} + (491\,\mathrm{nm})(1.43\,\mathrm{eV} - 0.710\,\mathrm{eV})} \\ &= 382 \text{ nm} .\end{aligned}$$

Here $hc = 1240\,\mathrm{eV\cdot nm}$, calculated in Exercise 3, is used.

(b) The first equation displayed above yields

$$\Phi = \frac{hc}{\lambda_1} - K_{m1} = \frac{1240\,\mathrm{eV\cdot nm}}{491\,\mathrm{nm}} - 0.710\,\mathrm{eV} = 1.82 \text{ eV} .$$

28. (a) We calculate frequencies from the wavelengths (expressed in SI units) using Eq. 39-1. Our plot of the points and the line which gives the least squares fit to the data is shown below. The vertical axis is in volts and the horizontal axis, when multiplied by 10^{14}, gives the frequencies in Hertz.



From our least squares fit procedure, we determine the slope to be $4.14 \times 10^{-15}\,\mathrm{V{\cdot}s}$, which is in very good agreement with the value given in Eq. 39-3 (once it has been multiplied by e).

(b) Our least squares fit procedure can also determine the y-intercept for that line. The y-intercept is the negative of the photoelectric work function. In this way, we find $\Phi = 2.31\,\mathrm{eV}$.

29. Using the result of problem 3, the number of photons emitted from the laser per unit time is

$$R = \frac{P}{E_{\mathrm{ph}}} = \frac{2.00 \times 10^{-3}\,\mathrm{W}}{(1240\,\mathrm{eV{\cdot}nm}/600\,\mathrm{nm})(1.60 \times 10^{-19}\,\mathrm{J/eV})} = 6.05 \times 10^{15}/\mathrm{s}\ ,$$

of which $(1.0 \times 10^{-16})(6.05 \times 10^{15}/\mathrm{s}) = 0.605/\mathrm{s}$ actually cause photoelectric emissions. Thus the current is $i = (0.605/\mathrm{s})(1.60 \times 10^{-19}\,\mathrm{C}) = 9.68 \times 10^{-20}\,\mathrm{A}$.

30. (a) Find the speed v of the electron from $r = m_e v/eB$: $v = rBe/m_e$. Thus

$$\begin{aligned} K_{\max} &= \frac{1}{2}m_e v^2 = \frac{1}{2}m_e\left(\frac{rBe}{m_e}\right)^2 = \frac{(rB)^2e^2}{2m_e} \\ &= \frac{(1.88 \times 10^{-4}\,\mathrm{T{\cdot}m})^2(1.60 \times 10^{-19}\,\mathrm{C})^2}{2(9.11 \times 10^{-31}\,\mathrm{kg})(1.60 \times 10^{-19}\,\mathrm{J/eV})} \\ &= 3.10\ \mathrm{keV}\ . \end{aligned}$$

(b) Using the result of problem 3, the work done is

$$W = E_{\mathrm{photon}} - K_{\max} = \frac{1240\,\mathrm{eV{\cdot}nm}}{71 \times 10^{-3}\,\mathrm{nm}} - 3.10\,\mathrm{keV} = 14\ \mathrm{keV}\ .$$

31. (a) When a photon scatters from an electron initially at rest, the change in wavelength is given by $\Delta\lambda = (h/mc)(1 - \cos\phi)$, where m is the mass of an electron and ϕ is the scattering angle. Now, $h/mc = 2.43 \times 10^{-12}\,\mathrm{m} = 2.43\,\mathrm{pm}$, so $\Delta\lambda = (2.43\,\mathrm{pm})(1 - \cos 30^\circ) = 0.326\,\mathrm{pm}$. The final wavelength is $\lambda' = \lambda + \Delta\lambda = 2.4\,\mathrm{pm} + 0.326\,\mathrm{pm} = 2.73\,\mathrm{pm}$.

(b) Now, $\Delta\lambda = (2.43\,\mathrm{pm})(1 - \cos 120^\circ) = 3.645\,\mathrm{pm}$ and $\lambda' = 2.4\,\mathrm{pm} + 3.645\,\mathrm{pm} = 6.05\,\mathrm{pm}$.

32. (a) The rest energy of an electron is given by $E = m_e c^2$. Thus the momentum of the photon in question is given by

$$\begin{aligned} p &= \frac{E}{c} = \frac{m_e c^2}{c} = m_e c \\ &= (9.11 \times 10^{-31}\,\mathrm{kg})(2.998 \times 10^8\,\mathrm{m/s}) \\ &= 2.73 \times 10^{-22}\ \mathrm{kg{\cdot}m/s}\ . \end{aligned}$$

We may also express the momentum in terms of MeV/c: $p = m_e c^2/c = 0.511\,\mathrm{MeV}/c$.

(b) From Eq. 39-7,

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\mathrm{J{\cdot}s}}{2.73 \times 10^{-22}\,\mathrm{kg{\cdot}m/s}} = 2.43 \times 10^{-12}\,\mathrm{m} = 2.43\ \mathrm{pm}\ .$$

(c) Using Eq. 39-1,

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{2.43 \times 10^{-12}\,\mathrm{m}} = 1.24 \times 10^{20}\ \mathrm{Hz}\ .$$

33. (a) The x-ray frequency is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^8\,\mathrm{m/s}}{35.0 \times 10^{-12}\,\mathrm{m}} = 8.57 \times 10^{18}\ \mathrm{Hz}\ .$$

(b) The x-ray photon energy is

$$E = hf = (4.14 \times 10^{-15}\,\text{eV·s})(8.57 \times 10^{18}\,\text{Hz}) = 3.55 \times 10^{4}\ \text{eV}\ .$$

(c) From Eq. 39-7,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\,\text{J·s}}{35.0 \times 10^{-12}\,\text{m}} = 1.89 \times 10^{-23}\ \text{kg·m/s}\ .$$

34. (a) Eq. 39-11 yields

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\phi) = (2.43\,\text{pm})(1 - \cos 180^\circ) = +4.86\ \text{pm}\ .$$

(b) Using the result of problem 3, the change in photon energy is

$$\Delta E = \frac{hc}{\lambda'} - \frac{hc}{\lambda} = (1240\,\text{eV·nm})\left(\frac{1}{0.01\,\text{nm} + 4.86\,\text{pm}} - \frac{1}{0.01\,\text{nm}}\right) = -41\ \text{keV}\ .$$

(c) From conservation of energy, $\Delta K = -\Delta E = 41\,\text{keV}$.

(d) The electron will move straight ahead after the collision, since it has acquired some of the forward linear momentum from the photon.

35. With no loss of generality, we assume the electron is initially at rest (which simply means we are analyzing the collision from its initial rest frame). If the photon gave all its momentum and energy to the (free) electron, then the momentum and the kinetic energy of the electron would become

$$p = \frac{hf}{c} \qquad \text{and} \qquad K = hf\ ,$$

respectively. Plugging these expressions into Eq. 38-51 (with m referring to the mass of the electron) leads to

$$\begin{aligned} (pc)^2 &= K^2 + 2Kmc^2 \\ (hf)^2 &= (hf)^2 + 2hfmc^2 \end{aligned}$$

which is clearly impossible, since the last term ($2hfmc^2$) is not zero. We have shown that considering total momentum and energy absorption of a photon by a free electron leads to an inconsistency in the mathematics, and thus cannot be expected to happen in nature.

36. (a) Using the result of problem 3, we find

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{nm·eV}}{0.511\,\text{MeV}} = 2.43 \times 10^{-3}\,\text{nm} = 2.43\ \text{pm}\ .$$

(b) Now, Eq. 39-11 leads to

$$\begin{aligned} \lambda' &= \lambda + \Delta\lambda = \lambda + \frac{h}{m_e c}(1 - \cos\phi) \\ &= 2.43\,\text{pm} + (2.43\,\text{pm})(1 - \cos 90.0^\circ) = 4.86\ \text{pm}\ . \end{aligned}$$

(c) The scattered photons have energy equal to

$$E' = E\left(\frac{\lambda}{\lambda'}\right) = (0.511\,\text{MeV})\left(\frac{2.43\,\text{pm}}{4.86\,\text{pm}}\right) = 0.255\ \text{MeV}\ .$$

37. (a) Since the mass of an electron is $m = 9.109 \times 10^{-31}$ kg, its Compton wavelength is

$$\lambda_C = \frac{h}{mc} = \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{(9.109 \times 10^{-31}\,\text{kg})(2.998 \times 10^8\,\text{m/s})} = 2.426 \times 10^{-12}\,\text{m} = 2.43\ \text{pm}\ .$$

(b) Since the mass of a proton is $m = 1.673 \times 10^{-27}$ kg, its Compton wavelength is

$$\lambda_C = \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{(1.673 \times 10^{-27}\,\text{kg})(2.998 \times 10^8\,\text{m/s})} = 1.321 \times 10^{-15}\,\text{m} = 1.32\ \text{fm}\ .$$

(c) We use the formula developed in Exercise 3: $E = (1240\,\text{eV}\cdot\text{nm})/\lambda$, where E is the energy and λ is the wavelength. Thus for the electron, $E = (1240\,\text{eV}\cdot\text{nm})/(2.426 \times 10^{-3}\,\text{nm}) = 5.11 \times 10^5\,\text{eV} = 0.511\,\text{MeV}$.

(d) For the proton, $E = (1240\,\text{eV}\cdot\text{nm})/(1.321 \times 10^{-6}\,\text{nm}) = 9.39 \times 10^8\,\text{eV} = 939\,\text{MeV}$.

38. The $(1 - \cos\phi)$ factor in Eq. 39-11 is largest when $\phi = 180°$. Thus, using Table 38-3, we obtain

$$\Delta\lambda_{\text{max}} = \frac{hc}{m_p c^2}(1 - \cos 180°) = \frac{1240\,\text{MeV}\cdot\text{fm}}{938\,\text{MeV}}(1 - (-1)) = 2.6\ \text{fm}$$

where we have extended the result of problem 3 somewhat by noting that $hc = 1240\,\text{eV}\cdot\text{nm}$ can equivalently be written as $1240\,\text{MeV}\cdot\text{fm}$.

39. If E is the original energy of the photon and E' is the energy after scattering, then the fractional energy loss is

$$frac = \frac{E - E'}{E}\ .$$

Sample Problem 39-4 shows that this is

$$frac = \frac{\Delta\lambda}{\lambda + \Delta\lambda}\ .$$

Thus

$$\frac{\Delta\lambda}{\lambda} = \frac{frac}{1 - frac} = \frac{0.75}{1 - 0.75} = 3\ .$$

A 300% increase in the wavelength leads to a 75% decrease in the energy of the photon.

40. (a) The fractional change is

$$\begin{aligned}\frac{\Delta E}{E} &= \frac{\Delta(hc/\lambda)}{hc/\lambda} = \lambda\Delta\left(\frac{1}{\lambda}\right) = \lambda\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = \frac{\lambda}{\lambda'} - 1 = \frac{\lambda}{\lambda + \Delta\lambda} - 1 \\ &= -\frac{1}{\lambda/\Delta\lambda + 1} = -\frac{1}{(\lambda/\lambda_C)(1 - \cos\phi)^{-1} + 1}\ .\end{aligned}$$

If $\lambda = 3.0\,\text{cm} = 3.0 \times 10^{10}\,\text{pm}$ and $\phi = 90°$, the result is

$$\frac{\Delta E}{E} = -\frac{1}{(3.0 \times 10^{10}\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -8.1 \times 10^{-11}\ .$$

(b) Now $\lambda = 500\,\text{nm} = 5.00 \times 10^5\,\text{pm}$ and $\phi = 90°$, so

$$\frac{\Delta E}{E} = -\frac{1}{(5.00 \times 10^5\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -4.9 \times 10^{-6}\ .$$

(c) With$\lambda = 25\,\text{pm}$ and $\phi = 90°$, we find

$$\frac{\Delta E}{E} = -\frac{1}{(25\,\text{pm}/2.43\,\text{pm})(1 - \cos 90°)^{-1} + 1} = -8.9 \times 10^{-2}\ .$$

(d) In this case, $\lambda = hc/E = 1240\,\text{nm}\cdot\text{eV}/1.0\,\text{MeV} = 1.24 \times 10^{-3}\,\text{nm} = 1.24\,\text{pm}$, so

$$\frac{\Delta E}{E} = -\frac{1}{(1.24\,\text{pm}/2.43\,\text{pm})(1-\cos 90^\circ)^{-1}+1} = -0.66\ .$$

(e) From the calculation above, we see that the shorter the wavelength the greater the fractional energy change for the photon as a result of the Compton scattering. Since $\Delta E/E$ is virtually zero for microwave and visible light, the Compton effect is significant only in the x-ray to gamma ray range of the electromagnetic spectrum.

41. The difference between the electron-photon scattering process in this problem and the one studied in the text (the Compton shift, see Eq. 39-11) is that the electron is in motion relative with speed v to the laboratory frame. To utilize the result in Eq. 39-11, shift to a new reference frame in which the electron is at rest before the scattering. Denote the quantities measured in this new frame with a prime ($'$), and apply Eq. 39-11 to yield

$$\Delta\lambda' = \lambda' - \lambda'_0 = \frac{h}{m_e c}(1-\cos\pi) = \frac{2h}{m_e c}\ ,$$

where we note that $\phi = \pi$ (since the photon is scattered back in the direction of incidence). Now, from the Doppler shift formula (Eq. 38-25) the frequency f'_0 of the photon prior to the scattering in the new reference frame satisfies

$$f'_0 = \frac{c}{\lambda'_0} = f_0\sqrt{\frac{1+\beta}{1-\beta}}\,,$$

where $\beta = v/c$. Also, as we switch back from the new reference frame to the original one after the scattering

$$f = f'\sqrt{\frac{1-\beta}{1+\beta}} = \frac{c}{\lambda'}\sqrt{\frac{1-\beta}{1+\beta}}\ .$$

We solve the two Doppler-shift equations above for λ' and λ'_0 and substitute the results into the Compton shift formula for $\Delta\lambda'$:

$$\Delta\lambda' = \frac{1}{f}\sqrt{\frac{1-\beta}{1+\beta}} - \frac{1}{f_0}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{2h}{m_e c^2}\ .$$

Some simple algebra then leads to

$$E = hf = hf_0\left(1 + \frac{2h}{m_e c^2}\sqrt{\frac{1+\beta}{1-\beta}}\right)^{-1}\ .$$

42. From Sample Problem 39-4, we have

$$\begin{aligned}\frac{\Delta E}{E} &= \frac{\Delta\lambda}{\lambda+\Delta\lambda}\\ &= \frac{(h/mc)(1-\cos\phi)}{\lambda'}\\ &= \frac{hf'}{mc^2}(1-\cos\phi)\end{aligned}$$

where we use the fact that $\lambda + \Delta\lambda = \lambda' = c/f'$.

43. (a) From Eq. 39-11, $\Delta\lambda = (h/m_e c)(1-\cos\phi)$. In this case $\phi = 180^\circ$ (so $\cos\phi = -1$), and the change in wavelength for the photon is given by $\Delta\lambda = 2h/m_e c$. The energy E' of the scattered photon (whose initial energy is $E = hc/\lambda$) is then

$$\begin{aligned}E' &= \frac{hc}{\lambda+\Delta\lambda} = \frac{E}{1+\Delta\lambda/\lambda} = \frac{E}{1+(2h/m_e c)(E/hc)} = \frac{E}{1+2E/m_e c^2}\\ &= \frac{50.0\,\text{keV}}{1+2(50.0\,\text{keV})/0.511\,\text{MeV}} = 41.8\,\text{keV}\ .\end{aligned}$$

(b) From conservation of energy the kinetic energy K of the electron is given by $K = E - E' = 50.0\,\mathrm{keV} - 41.8\,\mathrm{keV} = 8.2\,\mathrm{keV}$.

44. (a) From Eq. 39-11

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = (2.43\,\mathrm{pm})(1-\cos 90^\circ) = 2.43\,\mathrm{pm}\ .$$

(b) The fractional shift should be interpreted as $\Delta\lambda$ divided by the original wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{2.425\,\mathrm{pm}}{590\,\mathrm{nm}} = 4.11\times 10^{-6}\ .$$

(c) The change in energy for a photon with $\lambda = 590\,\mathrm{nm}$ is given by

$$\begin{aligned}\Delta E_{\mathrm{ph}} &= \Delta\left(\frac{hc}{\lambda}\right) \approx -\frac{hc\Delta\lambda}{\lambda^2}\\ &= -\frac{(4.14\times 10^{-15}\,\mathrm{eV\cdot s})(2.998\times 10^8\,\mathrm{m/s})(2.43\,\mathrm{pm})}{(590\,\mathrm{nm})^2}\\ &= -8.67\times 10^{-6}\ \mathrm{eV}\ .\end{aligned}$$

For an x ray photon of energy $E_{\mathrm{ph}} = 50\,\mathrm{keV}$, $\Delta\lambda$ remains the same (2.43 pm), since it is independent of E_{ph}. The fractional change in wavelength is now

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\lambda}{hc/E_{\mathrm{ph}}} = \frac{(50\times 10^3\,\mathrm{eV})(2.43\,\mathrm{pm})}{(4.14\times 10^{-15}\,\mathrm{eV\cdot s})(2.998\times 10^8\,\mathrm{m/s})} = 9.78\times 10^{-2}\ ,$$

and the change in photon energy is now

$$\Delta E_{\mathrm{ph}} = hc\left(\frac{1}{\lambda+\Delta\lambda} - \frac{1}{\lambda}\right) = -\left(\frac{hc}{\lambda}\right)\frac{\Delta\lambda}{\lambda+\Delta\lambda} = -E_{\mathrm{ph}}\left(\frac{\alpha}{1+\alpha}\right)$$

where $\alpha = \Delta\lambda/\lambda$. We substitute $E_{\mathrm{ph}} = 50\,\mathrm{keV}$ and $\alpha = 9.78\times 10^{-2}$ to obtain $\Delta E_{\mathrm{ph}} = -4.45\,\mathrm{keV}$. (Note that in this case $\alpha \approx 0.1$ is not close enough to zero so the approximation $\Delta E_{\mathrm{ph}} \approx hc\Delta\lambda/\lambda^2$ is not as accurate as in the first case, in which $\alpha = 4.12\times 10^{-6}$. In fact if one were to use this approximation here, one would get $\Delta E_{\mathrm{ph}} \approx -4.89\,\mathrm{keV}$, which does not amount to a satisfactory approximation.)

45. The initial wavelength of the photon is (using the result of problem 3)

$$\lambda = \frac{hc}{E} = \frac{1240\,\mathrm{eV\cdot nm}}{17500\,\mathrm{eV}} = 0.07086\ \mathrm{nm}$$

or 70.86 pm. The maximum Compton shift occurs for $\phi = 180^\circ$, in which case Eq. 39-11 (applied to an electron) yields

$$\Delta\lambda = \left(\frac{hc}{m_e c^2}\right)(1-\cos 180^\circ) = \left(\frac{1240\,\mathrm{eV\cdot nm}}{511\times 10^3\,\mathrm{eV}}\right)(1-(-1)) = 0.00485\ \mathrm{nm}$$

where Table 38-3 is used. Therefore, the new photon wavelength is $\lambda' = 0.07086\,\mathrm{nm} + 0.00485\,\mathrm{nm} = 0.0757\,\mathrm{nm}$. Consequently, the new photon energy is

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\mathrm{eV\cdot nm}}{0.0757\,\mathrm{nm}} = 1.64\times 10^4\,\mathrm{eV} = 16.4\ \mathrm{keV}\ .$$

By energy conservation, then, the kinetic energy of the electron must equal $E'-E = 17.5\,\mathrm{keV} - 16.4\,\mathrm{keV} = 1.1\,\mathrm{keV}$.

46. We rewrite Eq. 39-9 as

$$\frac{h}{m\lambda} - \frac{h}{m\lambda'}\cos\phi = \frac{v}{\sqrt{1-(v/c)^2}}\cos\theta \ ,$$

and Eq. 39-10 as

$$\frac{h}{m\lambda'}\sin\phi = \frac{v}{\sqrt{1-(v/c)^2}}\sin\theta \ .$$

We square both equations and add up the two sides:

$$\left(\frac{h}{m}\right)^2\left[\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\phi\right)^2 + \left(\frac{1}{\lambda'}\sin\phi\right)^2\right] = \frac{v^2}{1-(v/c)^2} \ ,$$

where we use $\sin^2\theta + \cos^2\theta = 1$ to eliminate θ. Now the right-hand side can be written as

$$\frac{v^2}{1-(v/c)^2} = -c^2\left[1 - \frac{1}{1-(v/c)^2}\right] \ ,$$

so

$$\frac{1}{1-(v/c)^2} = \left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\phi\right)^2 + \left(\frac{1}{\lambda'}\sin\phi\right)^2\right] + 1 \ .$$

Now we rewrite Eq. 39-8 as

$$\frac{h}{mc}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + 1 = \frac{1}{\sqrt{1-(v/c)^2}} \ .$$

If we square this, then it can be directly compared with the previous equation we obtained for $[1-(v/c)^2]^{-1}$. This yields

$$\left[\frac{h}{mc}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) + 1\right]^2 = \left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\phi\right)^2 + \left(\frac{1}{\lambda'}\sin\phi\right)^2\right] + 1 \ .$$

We have so far eliminated θ and v. Working out the squares on both sides and noting that $\sin^2\phi + \cos^2\phi = 1$, we get

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1-\cos\phi) \ .$$

47. The magnitude of the fractional energy change for the photon is given by

$$\left|\frac{\Delta E_{\rm ph}}{E_{\rm ph}}\right| = \left|\frac{\Delta(hc/\lambda)}{hc/\lambda}\right| = \left|\lambda\,\Delta\left(\frac{1}{\lambda}\right)\right| = \lambda\left(\frac{1}{\lambda} - \frac{1}{\lambda+\Delta\lambda}\right) = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \beta$$

where $\beta = 0.10$. Thus $\Delta\lambda = \lambda\beta/(1-\beta)$. We substitute this expression for $\Delta\lambda$ in Eq. 39-11 and solve for $\cos\phi$:

$$\begin{aligned} \cos\phi &= 1 - \frac{mc}{h}\Delta\lambda = 1 - \frac{mc\lambda\beta}{h(1-\beta)} = 1 - \frac{\beta(mc^2)}{(1-\beta)E_{\rm ph}} \\ &= 1 - \frac{(0.10)(511\,\text{keV})}{(1-0.10)(200\,\text{keV})} = 0.716 \ . \end{aligned}$$

This leads to an angle of $\phi = 44°$.

48. Referring to Sample Problem 39-4, we see that the fractional change in photon energy is

$$\frac{E-E'}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \frac{h/mc(1-\cos\phi)}{(hc/E) + (h/mc(1-\cos\phi))} \ .$$

Energy conservation demands that $E - E' = K$, the kinetic energy of the electron. In the maximal case, $\phi = 180°$, and we find

$$\frac{K}{E} = \frac{h/mc(1 - \cos 180°)}{(hc/E) + (h/mc(1 - \cos 180°))} = \frac{h/mc(2)}{(hc/E) + (h/mc(2))} \ .$$

Multiplying both sides by E and simplifying the fraction on the right-hand side leads to

$$K = E\left(\frac{2/mc}{c/E + 2/mc}\right) = \frac{E^2}{mc^2/2 + E} \ .$$

49. We substitute the classical relationship between momentum p and velocity v, $v = p/m$ into the classical definition of kinetic energy, $K = \frac{1}{2}mv^2$, to obtain $K = p^2/2m$. Here m is the mass of an electron. Thus $p = \sqrt{2mK}$. The relationship between the momentum and the de Broglie wavelength λ is $\lambda = h/p$, where h is the Planck constant. Thus,

$$\lambda = \frac{h}{\sqrt{2mK}} \ .$$

If K is given in electron volts, then

$$\begin{aligned} \lambda &= \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109 \times 10^{-31}\,\text{kg})(1.602 \times 10^{-19}\,\text{J/eV})K}} = \frac{1.226 \times 10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}} \\ &= \frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}} \ . \end{aligned}$$

50. The de Broglie wavelength for the bullet is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{(40 \times 10^{-3}\,\text{kg})(1000\,\text{m/s})} = 1.7 \times 10^{-35}\ \text{m} \ .$$

51. We start with the result of Exercise 49: $\lambda = h/\sqrt{2mK}$. Replacing K with eV, where V is the accelerating potential and e is the fundamental charge, we obtain

$$\begin{aligned} \lambda &= \frac{h}{\sqrt{2meV}} = \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109 \times 10^{-31}\,\text{kg})(1.602 \times 10^{-19}\,\text{C})(25.0 \times 10^{3}\,\text{V})}} \\ &= 7.75 \times 10^{-12}\,\text{m} = 7.75\ \text{pm} \ . \end{aligned}$$

52. (a) Using Table 38-3 and the result of problem 3, we obtain

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{hc}{\sqrt{2m_ec^2K}} = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(511000\,\text{eV})(1000\,\text{eV})}} = 0.039\ \text{nm} \ .$$

(b) A photon's de Broglie wavelength is equal to its familiar wave-relationship value. Using the result of problem 3,

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{keV}} = 1.24\,\text{nm} \ .$$

(c) The neutron mass may be found in Appendix B. Using the conversion from electronvolts to Joules, we obtain

$$\lambda = \frac{h}{\sqrt{2m_nK}} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(1.675 \times 10^{-27}\,\text{kg})(1.6 \times 10^{-16}\,\text{J})}} = 9.1 \times 10^{-13}\ \text{m} \ .$$

53. We use the result of Exercise 49: $\lambda = (1.226\,\text{nm}\cdot\text{eV}^{1/2})/\sqrt{K}$, where K is the kinetic energy. Thus

$$K = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\lambda}\right)^2 = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{590\,\text{nm}}\right)^2 = 4.32 \times 10^{-6}\ \text{eV}\ .$$

54. (a) We solve v from $\lambda = h/p = h/(m_p v)$:

$$v = \frac{h}{m_p\lambda} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{(1.675 \times 10^{-27}\,\text{kg})(0.100 \times 10^{-12}\,\text{m})} = 3.96 \times 10^6\ \text{m/s}\ .$$

(b) We set $eV = K = \frac{1}{2}m_p v^2$ and solve for the voltage:

$$V = \frac{m_p v^2}{2e} = \frac{(1.67 \times 10^{-27}\,\text{kg})(3.96 \times 10^6\,\text{m/s})^2}{2(1.60 \times 10^{-19}\,\text{C})} = 8.18 \times 10^3\ \text{V}\ .$$

55. (a) The average kinetic energy is

$$K = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23}\,\text{J/K})(300\,\text{K}) = 6.21 \times 10^{-21}\,\text{J} = 3.88 \times 10^{-2}\ \text{eV}\ .$$

(b) The de Broglie wavelength is

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2m_n K}} \\ &= \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(1.675 \times 10^{-27}\,\text{kg})(6.21 \times 10^{-21}\,\text{J})}} \\ &= 1.5 \times 10^{-10}\ \text{m}\ .\end{aligned}$$

56. (a) and (b) The momenta of the electron and the photon are the same:

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{0.20 \times 10^{-9}\,\text{m}} = 3.3 \times 10^{-24}\ \text{kg}\cdot\text{m/s}\ .$$

The kinetic energy of the electron is

$$K_e = \frac{p^2}{2m_e} = \frac{(3.3 \times 10^{-24}\,\text{kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31}\,\text{kg})} = 6.0 \times 10^{-18}\,\text{J} = 38\ \text{eV}\ ,$$

while that for the photon is

$$K_{\text{ph}} = pc = (3.3 \times 10^{-24}\,\text{kg}\cdot\text{m/s})(2.998 \times 10^8\,\text{m/s}) = 9.9 \times 10^{-16}\,\text{J} = 6.2\ \text{keV}\ .$$

57. (a) The momentum of the photon is given by $p = E/c$, where E is its energy. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{eV}} = 1240\ \text{nm}\ .$$

See Exercise 3. The momentum of the electron is given by $p = \sqrt{2mK}$, where K is its kinetic energy and m is its mass. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}\ .$$

According to Exercise 49, if K is in electron volts, this is

$$\lambda = \frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}} = \frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{1.00\,\text{eV}}} = 1.23\ \text{nm}\ .$$

(b) For the photon,

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\times 10^9\,\text{eV}} = 1.24\times 10^{-6}\,\text{nm} .$$

Relativity theory must be used to calculate the wavelength for the electron. According to Eq. 38-51, the momentum p and kinetic energy K are related by $(pc)^2 = K^2 + 2Kmc^2$. Thus,

$$\begin{aligned} pc &= \sqrt{K^2 + 2Kmc^2} \\ &= \sqrt{(1.00\times 10^9\,\text{eV})^2 + 2(1.00\times 10^9\,\text{eV})(0.511\times 10^6\,\text{eV})} \\ &= 1.00\times 10^9\ \text{eV} . \end{aligned}$$

The wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\times 10^9\,\text{eV}} = 1.24\times 10^{-6}\ \text{nm} .$$

58. (a) The average de Broglie wavelength is

$$\begin{aligned} \lambda_{\text{avg}} &= \frac{h}{p_{\text{avg}}} = \frac{h}{\sqrt{2mK_{\text{avg}}}} = \frac{h}{\sqrt{2m(3kT/2)}} = \frac{hc}{\sqrt{2(mc^2)kT}} \\ &= \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{3(4)(938\,\text{MeV})(8.62\times 10^{-5}\,\text{eV/K})(300\,\text{K})}} \\ &= 7.3\times 10^{-11}\,\text{m} = 73\ \text{pm} . \end{aligned}$$

(b) The average separation is

$$\begin{aligned} d_{\text{avg}} &= \frac{1}{\sqrt[3]{n}} = \frac{1}{\sqrt[3]{p/kT}} \\ &= \sqrt[3]{\frac{(1.38\times 10^{-23}\,\text{J/K})(300\,\text{K})}{1.01\times 10^5\,\text{Pa}}} = 3.4\,\text{nm} . \end{aligned}$$

(c) Yes, since $\lambda_{\text{avg}} \ll d_{\text{avg}}$.

59. (a) The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus $K = (1.60\times 10^{-19}\,\text{C})(300\,\text{V}) = 4.80\times 10^{-17}\,\text{J}$. The mass of a single sodium atom is, from Appendix F, $m = (22.9898\,\text{g/mol})/(6.02\times 10^{23}\,\text{atom/mol}) = 3.819\times 10^{-23}\,\text{g} = 3.819\times 10^{-26}\,\text{kg}$. Thus, the momentum of an ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819\times 10^{-26}\,\text{kg})(4.80\times 10^{-17}\,\text{J})} = 1.91\times 10^{-21}\ \text{kg}\cdot\text{m/s} .$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times 10^{-34}\,\text{J}\cdot\text{s}}{1.91\times 10^{-21}\,\text{kg}\cdot\text{m/s}} = 3.47\times 10^{-13}\ \text{m} .$$

60. (a) We use the result of problem 3:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\,\text{nm}} = 1.24\,\text{keV}$$

and for the electron

$$K = \frac{p^2}{2m_e} = \frac{(h/\lambda)^2}{2m_e} = \frac{(hc/\lambda)^2}{2m_ec^2} = \frac{1}{2(0.511\,\text{MeV})}\left(\frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{nm}}\right)^2 = 1.50\ \text{eV} .$$

(b) In this case, we find

$$E_{\text{photon}} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00 \times 10^{-6}\,\text{nm}} = 1.24 \times 10^9\,\text{eV} = 1.24\ \text{GeV}\ ,$$

and for the electron (recognizing that $1240\,\text{eV}\cdot\text{nm} = 1240\,\text{MeV}\cdot\text{fm}$)

$$\begin{aligned} K &= \sqrt{p^2c^2 + (m_ec^2)^2} - m_ec^2 = \sqrt{(hc/\lambda)^2 + (m_ec^2)^2} - m_ec^2 \\ &= \sqrt{\left(\frac{1240\,\text{MeV}\cdot\text{fm}}{1.00\,\text{fm}}\right)^2 + (0.511\,\text{MeV})^2} - 0.511\,\text{MeV} \\ &= 1.24 \times 10^3\,\text{MeV} = 1.24\ \text{GeV}\ . \end{aligned}$$

We note that at short λ (large K) the kinetic energy of the electron, calculated with the relativistic formula, is about the same as that of the photon. This is expected since now $K \approx E \approx pc$ for the electron, which is the same as $E = pc$ for the photon.

61. We need to use the relativistic formula $p = \sqrt{(E/c)^2 - m_e^2c^2} \approx E/c \approx K/c$ (since $E \gg m_ec^2$). So

$$\lambda = \frac{h}{p} \approx \frac{hc}{K} = \frac{1240\,\text{eV}\cdot\text{nm}}{50 \times 10^9\,\text{eV}} = 2.5 \times 10^{-8}\ \text{nm}\ ,$$

which is about 200 times smaller than the radius of an average nucleus.

62. (a) Since $K = 7.5\,\text{MeV} \ll m_\alpha c^2 = 4(932\,\text{MeV})$, we may use the non-relativistic formula $p = \sqrt{2m_\alpha K}$. Using Eq. 38-43 (and recognizing that $1240\,\text{eV}\cdot\text{nm} = 1240\,\text{MeV}\cdot\text{fm}$), we obtain

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2m_\alpha c^2 K}} = \frac{1240\,\text{MeV}\cdot\text{fm}}{\sqrt{2(4\,\text{u})(931.5\,\text{MeV/u})(7.5\,\text{MeV})}} = 5.2\ \text{fm}\ .$$

(b) Since $\lambda = 5.2\,\text{fm} \ll 30\,\text{fm}$, to a fairly good approximation, the wave nature of the α particle does not need to be taken into consideration.

63. The wavelength associated with the unknown particle is $\lambda_p = h/p_p = h/(m_pv_p)$, where p_p is its momentum, m_p is its mass, and v_p is its speed. The classical relationship $p_p = m_pv_p$ was used. Similarly, the wavelength associated with the electron is $\lambda_e = h/(m_ev_e)$, where m_e is its mass and v_e is its speed. The ratio of the wavelengths is $\lambda_p/\lambda_e = (m_ev_e)/(m_pv_p)$, so

$$m_p = \frac{v_e\lambda_e}{v_p\lambda_p}\,m_e = \frac{9.109 \times 10^{-31}\,\text{kg}}{3(1.813 \times 10^{-4})} = 1.675 \times 10^{-27}\ \text{kg}\ .$$

According to Appendix B, this is the mass of a neutron.

64. (a) Setting $\lambda = h/p = h/\sqrt{(E/c)^2 - m_e^2c^2}$, we solve for $K = E - m_ec^2$:

$$\begin{aligned} K &= \sqrt{\left(\frac{hc}{\lambda}\right)^2 + m_e^2c^4} - m_ec^2 \\ &= \sqrt{\left(\frac{1240\,\text{eV}\cdot\text{nm}}{10 \times 10^{-3}\,\text{nm}}\right)^2 + (0.511\,\text{MeV})^2} - 0.511\,\text{MeV} \\ &= 0.015\,\text{MeV} = 15\ \text{keV}\ . \end{aligned}$$

(b) Using the result of problem 3,

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{10 \times 10^{-3}\,\text{nm}} = 1.2 \times 10^5\,\text{eV} = 120\ \text{keV}\ .$$

(c) The electron microscope is more suitable, as the required energy of the electrons is much less than that of the photons.

65. The same resolution requires the same wavelength, and since the wavelength and particle momentum are related by $p = h/\lambda$, we see that the same particle momentum is required. The momentum of a 100 keV photon is $p = E/c = (100 \times 10^3\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})/(3.00 \times 10^8\,\text{m/s}) = 5.33 \times 10^{-23}\,\text{kg}\cdot\text{m/s}$. This is also the magnitude of the momentum of the electron. The kinetic energy of the electron is

$$K = \frac{p^2}{2m} = \frac{(5.33 \times 10^{-23}\,\text{kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31}\,\text{kg})} = 1.56 \times 10^{-15}\ \text{J} .$$

The accelerating potential is

$$V = \frac{K}{e} = \frac{1.56 \times 10^{-15}\,\text{J}}{1.60 \times 10^{-19}\,\text{C}} = 9.76 \times 10^3\ \text{V} .$$

66. (a)

$$\begin{aligned} nn^* &= (a+ib)(a+ib)^* = (a+ib)(a^* + i^*b^*) = (a+ib)(a-ib) \\ &= a^2 + iba - iab + (ib)(-ib) = a^2 + b^2 , \end{aligned}$$

which is always real since both a and b are real.

(b)

$$\begin{aligned} |nm| &= |(a+ib)(c+id)| \\ &= |ac + iad + ibc + (-i)^2 bd| \\ &= |(ac - bd) + i(ad + bc)| \\ &= \sqrt{(ac-bd)^2 + (ad+bc)^2} \\ &= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2} . \end{aligned}$$

But

$$\begin{aligned} |n|\,|m| &= |a+ib|\,|c+id| = \sqrt{a^2+b^2}\,\sqrt{c^2+d^2} \\ &= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2} , \end{aligned}$$

so $|nm| = |n|\,|m|$.

67. We plug Eq. 39-17 into Eq. 39-16, and note that

$$\frac{d\psi}{dx} = \frac{d}{dx}\left(Ae^{ikx} + Be^{-ikx}\right) = ikAe^{ikx} - ikBe^{-ikx}.$$

Also,

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left(ikAe^{ikx} - ikBe^{-ikx}\right) = -k^2Ae^{ikx} - k^2Be^{ikx} .$$

Thus,

$$\frac{d^2\psi}{dx^2} + k^2\psi = -k^2Ae^{ikx} - k^2Be^{ikx} + k^2\left(Ae^{ikx} + Be^{-ikx}\right) = 0 .$$

68. (a) We use Euler's formula $e^{i\phi} = \cos\phi + i\sin\phi$ to re-write $\psi(x)$ as

$$\begin{aligned} \psi(x) &= \psi_0 e^{ikx} = \psi_0(\cos kx + i\sin kx) \\ &= (\psi_0 \cos kx) + i(\psi_0 \sin kx) = a + ib , \end{aligned}$$

where $a = \psi_0 \cos kx$ and $b = \psi_0 \sin kx$ are both real quantities.

(b)

$$\begin{aligned}\psi(x,t) &= \psi(x)e^{-i\omega t} = \psi_0 e^{ikx}\, e^{-i\omega t} = \psi_0 e^{i(kx-\omega t)} \\ &= \left[\psi_0 \cos(kx-\omega t)\right] + i\left[\psi_0 \sin(kx-\omega t)\right] \ .\end{aligned}$$

69. The angular wave number k is related to the wavelength λ by $k = 2\pi/\lambda$ and the wavelength is related to the particle momentum p by $\lambda = h/p$, so $k = 2\pi p/h$. Now, the kinetic energy K and the momentum are related by $K = p^2/2m$, where m is the mass of the particle. Thus $p = \sqrt{2mK}$ and

$$k = \frac{2\pi\sqrt{2mK}}{h} \ .$$

70. We note that $|e^{ikx}|^2 = (e^{ikx})^*(e^{ikx}) = e^{-ikx}e^{ikx} = 1$. Referring to Eq. 39-14, we see therefore that $|\psi|^2 = |\Psi|^2$.

71. For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = 0 \ .$$

We substitute $\psi = \psi_0 e^{ikx}$. The second derivative is $d^2\psi/dx^2 = -k^2\psi_0 e^{ikx} = -k^2\psi$. The result is

$$-k^2\psi + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = 0 \ .$$

Solving for k, we obtain

$$k = \sqrt{\frac{8\pi^2 m}{h^2}\left[E - U_0\right]} = \frac{2\pi}{h}\sqrt{2m\left[E - U_0\right]} \ .$$

72. The wave function is now given by

$$\Psi(x,t) = \psi_0\, e^{-i(kx+\omega t)} \ .$$

This function describes a plane matter wave traveling in the negative x direction. An example of the actual particles that fit this description is a free electron with linear momentum $\vec{p} = -(hk/2\pi)\hat{\imath}$ and kinetic energy $K = p^2/2m_e = h^2k^2/8\pi^2 m_e$.

73. (a) The wave function is now given by

$$\Psi(x,t) = \psi_0\left[e^{i(kx-\omega t)} + e^{-i(kx+\omega t)}\right] = \psi_0\, e^{-i\omega t}\left(e^{ikx} + e^{-ikx}\right) \ .$$

Thus

$$\begin{aligned}\left|\Psi(x,t)\right|^2 &= \left|\psi_0\, e^{-i\omega t}\left(e^{ikx} + e^{-ikx}\right)\right|^2 \\ &= \left|\psi_0\, e^{-i\omega t}\right|^2 \left|e^{ikx} + e^{-ikx}\right|^2 \\ &= \psi_0^2\left|e^{ikx} + e^{-ikx}\right|^2 \\ &= \psi_0^2\left|(\cos kx + i\sin kx) + (\cos kx - i\sin kx)\right|^2 \\ &= 4\psi_0^2(\cos kx)^2 \\ &= 2\psi_0^2(1 + \cos 2kx) \ .\end{aligned}$$

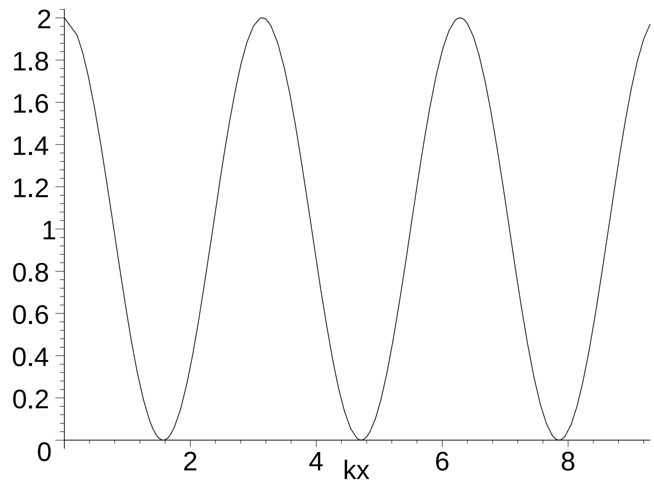
(b) Consider two plane matter waves, each with the same amplitude $\psi_0/\sqrt{2}$ and traveling in opposite directions along the x axis. The combined wave Ψ is a standing wave:

$$\begin{aligned}\Psi(x,t) &= \psi_0\, e^{i(kx-\omega t)} + \psi_0\, e^{-i(kx+\omega t)} = \psi_0\left(e^{ikx} + e^{-ikx}\right)e^{-i\omega t} \\ &= (2\psi_0 \cos kx)\, e^{-i\omega t} \ .\end{aligned}$$

Thus, the squared amplitude of the matter wave is

$$|\Psi(x,t)|^2 = (2\psi_0 \cos kx)^2 \left|e^{-i\omega t}\right|^2 = 2\psi_0^2(1+\cos 2kx) \ ,$$

which is shown below.



(c) We set $|\Psi(x,t)|^2 = 2\psi_0^2(1+\cos 2kx) = 0$ to obtain $\cos(2kx) = -1$. This gives

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = (2n+1)\pi \ , \qquad (n = 0, 1, 2, 3, \ldots)$$

We solve for x:

$$x = \frac{1}{4}(2n+1)\lambda \ .$$

(d) The most probable positions for finding the particle are where $|\Psi(x,t)| \propto (1+\cos 2kx)$ reaches its maximum. Thus $\cos 2kx = 1$, or

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = 2n\pi \ , \qquad (n = 0, 1, 2, 3, \ldots)$$

We solve for x:

$$x = \frac{1}{2}n\lambda \ .$$

74. (a) Since $p_x = p_y = 0$, $\Delta p_x = \Delta p_y = 0$. Thus from Eq. 39-20 both Δx and Δy are infinite. It is therefore impossible to assign a y or z coordinate to the position of an electron.

(b) Since it is independent of y and z the wave function $\Psi(x)$ should describe a plane wave that extends infinitely in both the y and z directions. Also from Fig. 39-11 we see that $|\Psi(x)|^2$ extends infinitely along the x axis. Thus the matter wave described by $\Psi(x)$ extends throughout the entire three-dimensional space.

75. The uncertainty in the momentum is $\Delta p = m\,\Delta v = (0.50\,\text{kg})(1.0\,\text{m/s}) = 0.50\,\text{kg}\cdot\text{m/s}$, where Δv is the uncertainty in the velocity. Solving the uncertainty relationship $\Delta x\,\Delta p \geq \hbar$ for the minimum uncertainty in the coordinate x, we obtain

$$\Delta x = \frac{\hbar}{\Delta p} = \frac{0.60\,\text{J}\cdot\text{s}}{2\pi(0.50\,\text{kg}\cdot\text{m/s})} = 0.19 \text{ m} \ .$$

76. If the momentum is measured at the same time as the position, then

$$\Delta p \approx \frac{\hbar}{\Delta x} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{2\pi(50\,\text{pm})} = 2.1 \times 10^{-24}\ \text{kg}\cdot\text{m/s}\ .$$

77. We use the uncertainty relationship $\Delta x\,\Delta p \geq \hbar$. Letting $\Delta x = \lambda$, the de Broglie wavelength, we solve for the minimum uncertainty in p:

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi\lambda} = \frac{p}{2\pi}$$

where the de Broglie relationship $p = h/\lambda$ is used. We use $1/2\pi = 0.080$ to obtain $\Delta p = 0.080p$. We would expect the measured value of the momentum to lie between $0.92p$ and $1.08p$. Measured values of zero, $0.5p$, and $2p$ would all be surprising.

78. (a) Using the result of problem 3,

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{10.0 \times 10^{-3}\,\text{nm}} = 124\,\text{keV}\ .$$

(b) The kinetic energy gained by the electron is equal to the energy decrease of the photon:

$$\begin{aligned}\Delta E &= \Delta\left(\frac{hc}{\lambda}\right) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda + \Delta\lambda}\right) = \left(\frac{hc}{\lambda}\right)\left(\frac{\Delta\lambda}{\lambda + \Delta\lambda}\right) = \frac{E}{1 + \frac{\lambda}{\Delta\lambda}} \\ &= \frac{E}{1 + \frac{\lambda}{\lambda_C(1-\cos\phi)}} = \frac{124\,\text{keV}}{1 + \frac{10.0\,\text{pm}}{(2.43\,\text{pm})(1-\cos 180^\circ)}} \\ &= 40.5\ \text{keV}\ .\end{aligned}$$

(c) It is impossible to "view" an atomic electron with such a high-energy photon, because with the energy imparted to the electron the photon would have knocked the electron out of its orbit.

79. (a) The transmission coefficient T for a particle of mass m and energy E that is incident on a barrier of height U and width L is given by

$$T = e^{-2kL}\ ,$$

where

$$k = \sqrt{\frac{8\pi^2 m(U - E)}{h^2}}\ .$$

For the proton,

$$\begin{aligned}k &= \sqrt{\frac{8\pi^2(1.6726 \times 10^{-27}\,\text{kg})(10\,\text{MeV} - 3.0\,\text{MeV})(1.6022 \times 10^{-13}\,\text{J/MeV})}{(6.6261 \times 10^{-34}\,\text{J}\cdot\text{s})^2}} \\ &= 5.8082 \times 10^{14}\ \text{m}^{-1}\ ,\end{aligned}$$

$kL = (5.8082 \times 10^{14}\,\text{m}^{-1})(10 \times 10^{-15}\,\text{m}) = 5.8082$, and

$$T = e^{-2\times 5.8082} = 9.02 \times 10^{-6}\ .$$

The value of k was computed to a greater number of significant digits than usual because an exponential is quite sensitive to the value of the exponent. The mass of a deuteron is $2.0141\,\text{u} = 3.3454 \times 10^{-27}\,\text{kg}$, so

$$\begin{aligned}k &= \sqrt{\frac{8\pi^2(3.3454 \times 10^{-27}\,\text{kg})(10\,\text{MeV} - 3.0\,\text{MeV})(1.6022 \times 10^{-13}\,\text{J/MeV})}{(6.6261 \times 10^{-34}\,\text{J}\cdot\text{s})^2}} \\ &= 8.2143 \times 10^{14}\ \text{m}^{-1}\ ,\end{aligned}$$

$kL = (8.2143 \times 10^{14}\,\text{m}^{-1})(10 \times 10^{-15}\,\text{m}) = 8.2143$, and

$$T = e^{-2\times 8.2143} = 7.33 \times 10^{-8}\ .$$

(b) Mechanical energy is conserved. Before the particles reach the barrier, each of them has a kinetic energy of 3.0 MeV and a potential energy of zero. After passing through the barrier, each again has a potential energy of zero, so each has a kinetic energy of 3.0 MeV.

(c) Energy is also conserved for the reflection process. After reflection, each particle has a potential energy of zero, so each has a kinetic energy of 3.0 MeV.

80. Letting

$$T \approx e^{-2kL} = \exp\left(-2L\sqrt{\frac{8\pi^2 m(U-E)}{h^2}}\right) \ ,$$

and using the result of Exercise 3 in Chapter 39, we solve for E:

$$\begin{aligned} E &= U - \frac{1}{2m}\left(\frac{h\ln T}{4\pi L}\right)^2 \\ &= 6.0\,\text{eV} - \frac{1}{2(0.511\,\text{MeV})}\left[\frac{(1240\,\text{eV}\cdot\text{nm})(\ln 0.001)}{4\pi(0.70\,\text{nm})}\right]^2 \\ &= 5.1\ \text{eV}\ . \end{aligned}$$

81. (a) If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U and width L is given by

$$T = e^{-2kL} \ ,$$

where

$$k = \sqrt{\frac{8\pi^2 m(U-E)}{h^2}} \ .$$

If the change ΔU in U is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU}\,\Delta U = -2LT\,\frac{dk}{dU}\,\Delta U \ .$$

Now,

$$\frac{dk}{dU} = \frac{1}{2\sqrt{U-E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U-E)}\sqrt{\frac{8\pi^2 m(U-E)}{h^2}} = \frac{k}{2(U-E)} \ .$$

Thus,

$$\Delta T = -LTk\,\frac{\Delta U}{U-E} \ .$$

For the data of Sample Problem 39-7, $2kL = 10.0$, so $kL = 5.0$ and

$$\frac{\Delta T}{T} = -kL\frac{\Delta U}{U-E} = -(5.0)\frac{(0.010)(6.8\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = -0.20 \ .$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\,\Delta L = -2ke^{-2kL}\,\Delta L = -2kT\,\Delta L$$

and

$$\frac{\Delta T}{T} = -2k\,\Delta L = -2(6.67\times10^{9}\,\text{m}^{-1})(0.010)(750\times10^{-12}\,\text{m}) = -0.10 \ .$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\,\Delta E = -2Le^{-2kL}\frac{dk}{dE}\,\Delta E = -2LT\frac{dk}{dE}\,\Delta E\ .$$

Now, $dk/dE = -dk/dU = -k/2(U-E)$, so

$$\frac{\Delta T}{T} = kL\frac{\Delta E}{U-E} = (5.0)\frac{(0.010)(5.1\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = 0.15\ .$$

There is a 15% increase in the transmission coefficient.

82. (a) The rate at which incident protons arrive at the barrier is $n = 1.0\,\text{kA}/1.60\times10^{-19}\,\text{C} = 6.25\times10^{23}/\text{s}$. Letting $nTt = 1$, we find the waiting time t:

$$\begin{aligned} t &= (nT)^{-1} = \frac{1}{n}\exp\left(2L\sqrt{\frac{8\pi^2 m_p(U-E)}{h^2}}\right) \\ &= \left(\frac{1}{6.25\times10^{23}/\text{s}}\right)\exp\left(\frac{2\pi(0.70\,\text{nm})}{1240\,\text{eV}\cdot\text{nm}}\sqrt{8(938\,\text{MeV})(6.0\,\text{eV}-5.0\,\text{eV})}\right) \\ &= 3.37\times10^{111}\,\text{s} \ \approx\ 10^{104}\ \text{y}\ , \end{aligned}$$

which is much longer than the age of the universe.

(b) Replacing the mass of the proton with that of the electron, we obtain the corresponding waiting time for an electron:

$$\begin{aligned} t &= (nT)^{-1} = \frac{1}{n}\exp\left[2L\sqrt{\frac{8\pi^2 m_e(U-E)}{h^2}}\right] \\ &= \left(\frac{1}{6.25\times10^{23}/\text{s}}\right)\exp\left[\frac{2\pi(0.70\,\text{nm})}{1240\,\text{eV}\cdot\text{nm}}\sqrt{8(0.511\,\text{MeV})(6.0\,\text{eV}-5.0\,\text{eV})}\right] \\ &= 2.1\times10^{-19}\ \text{s}\ . \end{aligned}$$

The enormous difference between the two waiting times is the result of the difference between the masses of the two kinds of particles.

83. The kinetic energy of the car of mass m moving at speed v is given by $E = \frac{1}{2}mv^2$, while the potential barrier it has to tunnel through is $U = mgh$, where $h = 24\,\text{m}$. According to Eq. 39-21 and 39-22 the tunneling probability is given by $T \approx e^{-2kL}$, where

$$\begin{aligned} k &= \sqrt{\frac{8\pi^2 m(U-E)}{h^2}} = \sqrt{\frac{8\pi^2 m(mgh - \frac{1}{2}mv^2)}{h^2}} \\ &= \frac{2\pi(1500\,\text{kg})}{6.63\times10^{-34}\,\text{J}\cdot\text{s}}\sqrt{2\left[(9.8\,\text{m/s}^2)(24\,\text{m}) - \frac{1}{2}(20\,\text{m/s})^2\right]} \\ &= 1.2\times10^{38}\ \text{m}^{-1}\ . \end{aligned}$$

Thus, $2kL = 2(1.2\times10^{38}\,\text{m}^{-1})(30\,\text{m}) = 7.2\times10^{39}$. One can see that $T \approx e^{-2kL}$ is essentially zero.

Chapter 40

1. (a) This is computed in part (a) of Sample Problem 40-1.

   (b) With $m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8mL^2}\right) n^2 = \left(\frac{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})^2}{8m_p(100\ \times 10^{12}\,\mathrm{m})^2}\right)(1)^2 = 3.29 \times 10^{-21}\,\mathrm{J} = 0.0206\ \mathrm{eV}\ .$$

2. According to Eq. 40-4 $E_n \propto L^{-2}$. As a consequence, the new energy level E'_n satisfies

$$\frac{E'_n}{E_n} = \left(\frac{L'}{L}\right)^{-2} = \left(\frac{L}{L'}\right)^2 = \frac{1}{2}\ ,$$

   which gives $L' = \sqrt{2}L$. Thus, the width of the potential well must be multiplied by a factor of $\sqrt{2}$.

3. To estimate the energy, we use Eq. 40-4, with $n = 1$, L equal to the atomic diameter, and m equal to the mass of an electron:

$$E = n^2\,\frac{h^2}{8mL^2} = \frac{(1)^2(6.63 \times 10^{-34}\,\mathrm{J \cdot s})^2}{8(9.11 \times 10^{-31}\,\mathrm{kg})(1.4 \times 10^{-14}\,\mathrm{m})^2} = 3.07 \times 10^{-10}\,\mathrm{J} = 1920\ \mathrm{MeV}\ .$$

4. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by writing Eq. 40-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2}\ .$$

   For $n = 3$, we set this expression equal to 4.7 eV and solve for L:

$$L = \frac{n(hc)}{\sqrt{8(mc^2)E_n}} = \frac{3(1240\,\mathrm{eV \cdot nm})}{\sqrt{8(511 \times 10^3\,\mathrm{eV})(4.7\,\mathrm{eV})}} = 0.85\ \mathrm{nm}\ .$$

5. With $m_p = 1.67 \times 10^{-27}$ kg, we obtain

$$E_1 = \left(\frac{h^2}{8mL^2}\right) n^2 = \left(\frac{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})^2}{8m_p(100\ \times 10^{12}\,\mathrm{m})^2}\right)(1)^2 = 3.29 \times 10^{-21}\,\mathrm{J} = 0.0206\ \mathrm{eV}\ .$$

   Alternatively, we can use the mc^2 value for a proton from Table 38-3 (938×10^6 eV) and the $hc = 1240\,\mathrm{eV \cdot nm}$ value developed in problem 3 of Chapter 39 by writing Eq. 40-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(m_pc^2)L^2}\ .$$

   This alternative approach is perhaps easier to plug into, but it is recommended that both approaches be tried to find which is most convenient.

6. Since $E_n \propto L^{-2}$ in Eq. 40-4, we see that if L is doubled, then E_1 becomes $(2.6\,\text{eV})(2)^{-2} = 0.65\,\text{eV}$.

7. We can use the mc^2 value for an electron from Table 38-3 ($511 \times 10^3\,\text{eV}$) and the hc value developed in problem 3 of Chapter 39 by writing Eq. 40-4 as

$$E_n = \frac{n^2 h^2}{8mL^2} = \frac{n^2 (hc)^2}{8(mc^2)L^2} \; .$$

The energy to be absorbed is therefore

$$\begin{aligned} \Delta E &= E_4 - E_1 = \frac{(4^2 - 1^2)h^2}{8m_e L^2} = \frac{15(hc)^2}{8(m_e c^2)L^2} \\ &= \frac{15(1240\,\text{eV}\cdot\text{nm})^2}{8(511 \times 10^3\,\text{eV})(0.250\,\text{nm})^2} = 90.3\text{ eV} \; . \end{aligned}$$

8. (a) Let the quantum numbers of the pair in question be n and $n+1$, respectively. We note that

$$E_{n+1} - E_n = \frac{(n+1)^2 h^2}{8mL^2} - \frac{n^2 h^2}{8mL^2} = \frac{(2n+1)h^2}{8mL^2}$$

Therefore, $E_{n+1} - E_n = (2n+1)E_1$. Now

$$E_{n+1} - E_n = E_5 = 5^2 E_1 = 25E_1 = (2n+1)E_1 \; ,$$

which leads to $2n + 1 = 25$, or $n = 12$.

(b) Now let

$$E_{n+1} - E_n = E_6 = 6^2 E_1 = 36E_1 = (2n+1)E_1 \; ,$$

which gives $2n+1 = 36$, or $n = 17.5$. This is not an integer, so it is impossible to find the pair that fits the requirement.

9. From Eq. 40-4

$$E_{n+2} - E_n = \left(\frac{h^2}{8mL^2}\right)(n+2)^2 - \left(\frac{h^2}{8mL^2}\right)n^2 = \left(\frac{h^2}{2mL^2}\right)(n+1) \; .$$

10. (a) Let the quantum numbers of the pair in question be n and $n+1$, respectively. Then $E_{n+1} - E_n = E_1(n+1)^2 - E_1 n^2 = (2n+1)E_1$. Letting

$$E_{n+1} - E_n = (2n+1)E_1 = 3(E_4 - E_3) = 3(4^2 E_1 - 3^2 E_1) = 21E_1 \; ,$$

we get $2n + 1 = 21$, or $n = 10$.

(b) Now letting

$$E_{n+1} - E_n = (2n+1)E_1 = 2(E_4 - E_3) = 2(4^2 E_1 - 3^2 E_1) = 14E_1 \; ,$$

we get $2n + 1 = 14$, which does not have an integer-valued solution. So it is impossible to find the pair of energy levels that fits the requirement.

11. The energy levels are given by $E_n = n^2 h^2 / 8mL^2$, where h is the Planck constant, m is the mass of an electron, and L is the width of the well. The frequency of the light that will excite the electron from the state with quantum number n_i to the state with quantum number n_f is $f = \Delta E / h = (h/8mL^2)(n_f^2 - n_i^2)$ and the wavelength of the light is

$$\lambda = \frac{c}{f} = \frac{8mL^2 c}{h(n_f^2 - n_i^2)} \; .$$

We evaluate this expression for $n_i = 1$ and $n_f = 2$, 3, 4, and 5, in turn. We use $h = 6.626 \times 10^{-34}\,\text{J}\cdot\text{s}$, $m = 9.109 \times 10^{-31}\,\text{kg}$, and $L = 250 \times 10^{-12}\,\text{m}$, and obtain $6.87 \times 10^{-8}\,\text{m}$ for $n_f = 2$, $2.58 \times 10^{-8}\,\text{m}$ for $n_f = 3$, $1.37 \times 10^{-8}\,\text{m}$ for $n_f = 4$, and $8.59 \times 10^{-9}\,\text{m}$ for $n_f = 5$.

12. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by rewriting Eq. 40-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2} \ .$$

(a) The first excited state is characterized by $n = 2$, and the third by $n' = 4$. Thus,

$$\begin{aligned}\Delta E &= \frac{(hc)^2}{8(mc^2)L^2}\left(n'^2 - n^2\right) \\ &= \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511 \times 10^3\,\text{eV})(0.250\,\text{nm})^2}\left(4^2 - 2^2\right) \\ &= (6.02\,\text{eV})(16 - 4)\end{aligned}$$

which yields $\Delta E = 72.2\,\text{eV}$.

(b) Now that the electron is in the $n' = 4$ level, it can "drop" to a lower level (n'') in a variety of ways. Each of these drops is presumed to cause a photon to be emitted of wavelength

$$\lambda = \frac{hc}{E_{n'} - E_{n''}} = \frac{8(mc^2)L^2}{hc\left(n'^2 - n''^2\right)} \ .$$

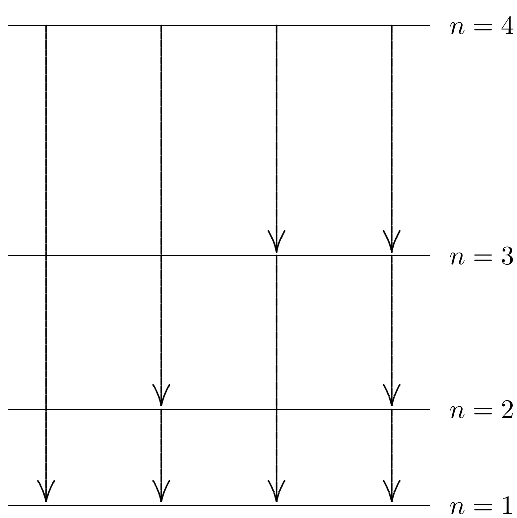
For example, for the transition $n' = 4$ to $n'' = 3$, the photon emitted would have wavelength

$$\lambda = \frac{8(511 \times 10^3\,\text{eV})(0.250\,\text{nm})^2}{(1240\,\text{eV}\cdot\text{nm})\left(4^2 - 3^2\right)} = 29.4 \text{ nm} \ ,$$

and once it is then in level $n'' = 3$ it might fall to level $n''' = 2$ emitting another photon. Calculating in this way all the possible photons emitted during the de-excitation of this system, we find $\lambda_{4\to1} = 13.7\,\text{nm}$, $\lambda_{4\to2} = 17.2\,\text{nm}$, $\lambda_{3\to1} = 25.8\,\text{nm}$, $\lambda_{4\to3} = 29.4\,\text{nm}$, $\lambda_{3\to2} = 41.2\,\text{nm}$, and $\lambda_{2\to1} = 68.7\,\text{nm}$.

(c) A system making the $4 \to 1$ transition will make no further transitions unless it is re-excited. If it makes the $4 \to 2$ transition, then that is likely to followed by the $2 \to 1$ transition. However, if it makes the $4 \to 3$ transition, then it could make either the $3 \to 1$ transition or the pair of transitions: $3 \to 2$ and $2 \to 1$.

(d) The possible transitions are shown below. The energy levels are not drawn to scale.



13. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by writing Eq. 40-4 as

$$E_n = \frac{n^2h^2}{8mL^2} = \frac{n^2(hc)^2}{8(mc^2)L^2} \ .$$

(a) With $L = 3.0 \times 10^9$ nm, the energy difference is

$$E_2 - E_1 = \frac{1240^2}{8(511 \times 10^3)(3.0 \times 10^9)^2}\left(2^2 - 1^2\right) = 1.3 \times 10^{-19} \text{ eV} \ .$$

(b) Since $(n+1)^2 - n^2 = 2n + 1$, we have

$$\Delta E = E_{n+1} - E_n = \frac{h^2}{8mL^2}(2n+1) = \frac{(hc)^2}{8(mc^2)L^2}(2n+1) \ .$$

Setting this equal to 1.0 eV, we solve for n:

$$\begin{aligned} n &= \frac{4(mc^2)L^2\Delta E}{(hc)^2} - \frac{1}{2} \\ &= \frac{4(511 \times 10^3 \text{ eV})(3.0 \times 10^9 \text{ nm})^2(1.0 \text{ eV})}{(1240 \text{ eV}\cdot\text{nm})^2} - \frac{1}{2} \\ &\approx 12 \times 10^{18} \ . \end{aligned}$$

(c) At this value of n, the energy is

$$E_n = \frac{1240^2}{8(511 \times 10^3)(3.0 \times 10^9)^2}\left(6 \times 10^{18}\right)^2 \approx 6 \times 10^{18} \text{ eV} \ .$$

(d) Since

$$\frac{E_n}{mc^2} = \frac{6 \times 10^{18} \text{ eV}}{511 \times 10^3 \text{ eV}} \gg 1 \ ,$$

the energy is indeed in the relativistic range.

14. (a) With Eq. 40-11, we compare the ψ_1^2 and ψ_2^2 graphs in Fig. 40-6. The former has a maximum at the center and the latter is zero there. Thus, the excitation of the system described in this problem implies the electron has become much less likely to be detected near the middle of the well.

(b) Examining the $0 \le x \le 25$ pm regions of those two graphs, we conclude that the excited state electron is somewhat more likely to be "near" (not "at") a well wall. Eq. 40-13 supports this conclusion in the sense that there is more "area" under the curve of ψ_2^2 in the $0 \le x \le 25$ pm region than under the ψ_1^2 curve for that region.

15. (a) The allowed energy values are given by $E_n = n^2h^2/8mL^2$. The difference in energy between the state n and the state $n+1$ is

$$\Delta E_{\text{adj}} = E_{n+1} - E_n = \left[(n+1)^2 - n^2\right]\frac{h^2}{8mL^2} = \frac{(2n+1)h^2}{8mL^2}$$

and

$$\frac{\Delta E_{\text{adj}}}{E} = \left[\frac{(2n+1)h^2}{8mL^2}\right]\left(\frac{8mL^2}{n^2h^2}\right) = \frac{2n+1}{n^2} \ .$$

As n becomes large, $2n + 1 \longrightarrow 2n$ and $(2n+1)/n^2 \longrightarrow 2n/n^2 = 2/n$.

(b) As $n \longrightarrow \infty$, ΔE_{adj} and E do not approach 0, but $\Delta E_{\text{adj}}/E$ does.

(c) See part (b).

(d) See part (b).

(e) $\Delta E_{\text{adj}}/E$ is a better measure than either ΔE_{adj} or E alone of the extent to which the quantum result is approximated by the classical result.

16. We follow Sample Problem 40-3 in the presentation of this solution. The integration result quoted below is discussed in a little more detail in that Sample Problem. We note that the arguments of the sine functions used below are in radians.

(a) The probability of detecting the particle in the region $0 \le x \le \frac{L}{4}$ is

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_0^{\pi/4} \sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)_0^{\pi/4} = 0.091 \ .$$

(b) As expected from symmetry,

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{\pi} \sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)_{\pi/4}^{\pi} = 0.091 \ .$$

(c) For the region $\frac{L}{4} \le x \le \frac{3L}{4}$, we obtain

$$\left(\frac{2}{L}\right)\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4} \sin^2 y\,dy = \frac{2}{\pi}\left(\frac{y}{2} - \frac{\sin 2y}{4}\right)_{\pi/4}^{3\pi/4} = 0.82$$

which we could also have gotten by subtracting the results of part (a) and (b) from 1; that is, $1 - 2(0.091) = 0.82$.

17. The probability that the electron is found in any interval is given by $P = \int |\psi|^2\,dx$, where the integral is over the interval. If the interval width Δx is small, the probability can be approximated by $P = |\psi|^2\,\Delta x$, where the wave function is evaluated for the center of the interval, say. For an electron trapped in an infinite well of width L, the ground state probability density is

$$|\psi|^2 = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right) \ ,$$

so

$$P = \left(\frac{2\,\Delta x}{L}\right)\sin^2\left(\frac{\pi x}{L}\right) \ .$$

(a) We take $L = 100\,\text{pm}$, $x = 25\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(25\,\text{pm})}{100\,\text{pm}}\right] = 0.050 \ .$$

(b) We take $L = 100\,\text{pm}$, $x = 50\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(50\,\text{pm})}{100\,\text{pm}}\right] = 0.10 \ .$$

(c) We take $L = 100\,\text{pm}$, $x = 90\,\text{pm}$, and $\Delta x = 5.0\,\text{pm}$. Then,

$$P = \left[\frac{2(5.0\,\text{pm})}{100\,\text{pm}}\right]\sin^2\left[\frac{\pi(90\,\text{pm})}{100\,\text{pm}}\right] = 0.0095 \ .$$

18. (a) We recall that a derivative with respect to a dimensional quantity carries the (reciprocal) units of that quantity. Thus, the first term in Eq. 40-18 has dimensions of ψ multiplied by dimensions of x^{-2}. The second term contains no derivatives, does contain ψ, and involves several other factors that (as we show below) turn out to have dimensions of x^{-2}:

$$\frac{8\pi^2 m}{h^2}\left[E - U(x)\right] \implies \frac{\text{kg}}{(\text{J}\cdot\text{s})^2}[\text{J}]$$

assuming SI units. Recalling from Eq. 7-9 that J$=$ kg$\cdot$m^2/s^2, then we see the above is indeed in units of m^{-2} (which means dimensions of x^{-2}).

(b) In one-dimensional Quantum Physics, the wavefunction has units of m$^{-1/2}$ as Sample Problem 40-2 shows. Thus, since each term in Eq. 40-18 has units of ψ multiplied by units of x^{-2}, then those units are m$^{-1/2}\cdot$m^{-2} = m$^{-2.5}$.

19. According to Fig. 40-9, the electron's initial energy is 109 eV. After the additional energy is absorbed, the total energy of the electron is 109 eV + 400 eV = 509 eV. Since it is in the region $x > L$, its potential energy is 450 eV (see Section 40-5), so its kinetic energy must be 509 eV − 450 eV = 59 eV.

20. From Fig. 40-9, we see that the sum of the kinetic and potential energies in that particular finite well is 280 eV. The potential energy is zero in the region $0 < x < L$. If the kinetic energy of the electron is detected while it is in that region (which is the only region where this is likely to happen), we should find $K = 280$ eV.

21. (a) and (b) Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = 0\ ,$$

where $E - U_0 < 0$. If $\psi^2(x) = Ce^{-2kx}$, then $\psi(x) = C'e^{-kx}$, where C' is another constant satisfying $C'^2 = C$. Thus $d^2\psi/dx^2 = 4k^2C'e^{-kx} = 4k^2\psi$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = k^2\psi + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi\ .$$

This is zero provided that

$$k^2 = \frac{8\pi^2 m}{h^2}\left[U_0 - E\right]\ .$$

The quantity on the right-hand side is positive, so k is real and the proposed function satisfies Schrödinger's equation. If k is negative, however, the proposed function would be physically unrealistic. It would increase exponentially with x. Since the integral of the probability density over the entire x axis must be finite, ψ diverging as $x \to \infty$ would be unacceptable. Therefore, we choose

$$k = \frac{2\pi}{h}\sqrt{2m\left(U_0 - E\right)} > 0\ .$$

22. (a) and (b) In the region $0 < x < L$, $U_0 = 0$, so Schrödinger's equation for the region is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

where $E > 0$. If $\psi^2(x) = B\sin^2 kx$, then $\psi(x) = B'\sin kx$, where B' is another constant satisfying $B'^2 = B$. Thus $d^2\psi/dx^2 = -k^2B'\sin kx = -k^2\psi(x)$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = -k^2\psi + \frac{8\pi^2 m}{h^2}E\psi\ .$$

This is zero provided that

$$k^2 = \frac{8\pi^2 mE}{h^2} \ .$$

The quantity on the right-hand side is positive, so k is real and the proposed function satisfies Schrödinger's equation. In this case, there exists no physical restriction as to the sign of k. It can assume either positive or negative values. Thus

$$k = \pm \frac{2\pi}{h}\sqrt{2mE} \ .$$

23. Schrödinger's equation for the region $x > L$ is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = 0 \ .$$

If $\psi = De^{2kx}$, then $d^2\psi/dx^2 = 4k^2De^{2kx} = 4k^2\psi$ and

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi = 4k^2\psi + \frac{8\pi^2 m}{h^2}\left[E - U_0\right]\psi \ .$$

This is zero provided

$$k = \frac{\pi}{h}\sqrt{2m\left(U_0 - E\right)} \ .$$

The proposed function satisfies Schrödinger's equation provided k has this value. Since U_0 is greater than E in the region $x > L$, the quantity under the radical is positive. This means k is real. If k is positive, however, the proposed function is physically unrealistic. It increases exponentially with x and becomes large without bound. The integral of the probability density over the entire x axis must be unity. This is impossible if ψ is the proposed function.

24. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by writing Eq. 40-20 as

$$E_{nx,ny} = \frac{2h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) = \frac{(hc)^2}{8(mc^2)}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) \ .$$

For $n_x = n_y = 1$, we obtain

$$E_{1,1} = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511 \times 10^3\,\text{eV})}\left(\frac{1}{(0.800\,\text{nm})^2} + \frac{1}{(1.600\,\text{nm})^2}\right) = 0.73\ \text{eV} \ .$$

25. We can use the mc^2 value for an electron from Table 38-3 (511×10^3 eV) and the hc value developed in problem 3 of Chapter 39 by writing Eq. 40-21 as

$$E_{nx,ny,nz} = \frac{2h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \frac{(hc)^2}{8(mc^2)}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) \ .$$

For $n_x = n_y = n_z = 1$, we obtain

$$E_{1,1} = \frac{(1240\,\text{eV}\cdot\text{nm})^2}{8(511 \times 10^3\,\text{eV})}\left(\frac{1}{(0.800\,\text{nm})^2} + \frac{1}{(1.600\,\text{nm})^2} + \frac{1}{(0.400\,\text{nm})^2}\right) = 3.1\ \text{eV} \ .$$

26. We are looking for the values of the ratio

$$\frac{E_{nx,ny}}{h^2/8mL^2} = L^2\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) = \left(n_x^2 + \frac{1}{4}\,n_y^2\right)$$

and the corresponding differences.

(a) For $n_x = n_y = 1$, the ratio becomes $1 + \frac{1}{4} = 1.25$.

(b) For $n_x = 1$ and $n_y = 2$, the ratio becomes $1 + \frac{1}{4}(4) = 2.00$. One can check (by computing other (n_x, n_y) values) that this is the next to lowest energy in the system.

(c) The lowest set of states that are degenerate are $(n_x, n_y) = (1, 4)$ and $(2, 2)$. Both of these states have that ratio equal to $1 + \frac{1}{4}(16) = 5.00$.

(d) For $n_x = 1$ and $n_y = 3$, the ratio becomes $1 + \frac{1}{4}(9) = 3.25$. One can check (by computing other (n_x, n_y) values) that this is the lowest energy greater than that computed in part (b). The next higher energy comes from $(n_x, n_y) = (2, 1)$ for which the ratio is $4 + \frac{1}{4}(1) = 4.25$. The difference between these two values is $4.25 - 3.25 = 1.00$.

27. The energy levels are given by

$$E_{n_x,n_y} = \frac{h^2}{8m}\left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right] = \frac{h^2}{8mL^2}\left[n_x^2 + \frac{n_y^2}{4}\right]$$

where the substitutions $L_x = L$ and $L_y = 2L$ were made. In units of $h^2/8mL^2$, the energy levels are given by $n_x^2 + n_y^2/4$. The lowest five levels are $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, $E_{2,1} = 4.25$, and $E_{2,2} = E_{1,4} = 5.00$. It is clear that there are no other possible values for the energy less than 5. The frequency of the light emitted or absorbed when the electron goes from an initial state i to a final state f is $f = (E_f - E_i)/h$, and in units of $h/8mL^2$ is simply the difference in the values of $n_x^2 + n_y^2/4$ for the two states. The possible frequencies are 0.75 (1,2 $\longrightarrow$ 1,1), 2.00 (1,3 $\longrightarrow$ 1,1), 3.00 (2,1 $\longrightarrow$ 1,1), 3.75 (2,2 $\longrightarrow$ 1,1), 1.25 (1,3 $\longrightarrow$ 1,2), 2.25 (2,1 $\longrightarrow$ 1,2), 3.00 (2,2 $\longrightarrow$ 1,2), 1.00 (2,1 $\longrightarrow$ 1,3), 1.75 (2,2 $\longrightarrow$ 1,3), 0.75 (2,2 $\longrightarrow$ 2,1), all in units of $h/8mL^2$.

28. We are looking for the values of the ratio

$$\frac{E_{n_x,n_y,n_z}}{h^2/8mL^2} = L^2\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right) = \left(n_x^2 + n_y^2 + n_z^2\right)$$

and the corresponding differences.

(a) For $n_x = n_y = n_z = 1$, the ratio becomes $1 + 1 + 1 = 3.00$.

(b) For $n_x = n_y = 2$ and $n_z = 1$, the ratio becomes $4 + 4 + 1 = 9.00$. One can check (by computing other (n_x, n_y, n_z) values) that this is the third lowest energy in the system. One can also check that this same ratio is obtained for $(n_x, n_y, n_z) = (2, 1, 2)$ and $(1, 2, 2)$.

(c) For $n_x = n_y = 1$ and $n_z = 3$, the ratio becomes $1 + 1 + 9 = 11.00$. One can check (by computing other (n_x, n_y, n_z) values) that this is three "steps" up from the lowest energy in the system. One can also check that this same ratio is obtained for $(n_x, n_y, n_z) = (1, 3, 1)$ and $(3, 1, 1)$. If we take the difference between this and the result of part (b), we obtain $11.00 - 9.00 = 2.00$.

(d) For $n_x = n_y = 1$ and $n_z = 2$, the ratio becomes $1 + 1 + 4 = 6.00$. One can check (by computing other (n_x, n_y, n_z) values) that this is the next to the lowest energy in the system. One can also check that this same ratio is obtained for $(n_x, n_y, n_z) = (2, 1, 1)$ and $(1, 2, 1)$. Thus, three states (three arrangements of (n_x, n_y, n_z) values) have this energy.

(e) For $n_x = 1$, $n_y = 2$ and $n_z = 3$, the ratio becomes $1 + 4 + 9 = 14.00$. One can check (by computing other (n_x, n_y, n_z) values) that this is five "steps" up from the lowest energy in the system. One can also check that this same ratio is obtained for $(n_x, n_y, n_z) = (1, 3, 2), (2, 3, 1), (2, 1, 3), (3, 1, 2)$ and $(3, 2, 1)$. Thus, six states (six arrangements of (n_x, n_y, n_z) values) have this energy.

29. The ratios computed in problem 28 can be related to the frequencies emitted using $f = \Delta E/h$, where each level E is equal to one of those ratios multiplied by $h^2/8mL^2$. This effectively involves no more

than a cancellation of one of the factors of h. Thus, for a transition from the second excited state (see part (b) of problem 28) to the ground state (treated in part (a) of that problem), we find

$$f = (9.00 - 3.00)\left(\frac{h}{8mL^2}\right) = (6.00)\left(\frac{h}{8mL^2}\right) .$$

In the following, we omit the $h/8mL^2$ factors. For a transition between the fourth excited state and the ground state, we have $f = 12.00 - 3.00 = 9.00$. For a transition between the third excited state and the ground state, we have $f = 11.00 - 3.00 = 8.00$. For a transition between the third excited state and the first excited state, we have $f = 11.00 - 6.00 = 5.00$. For a transition between the fourth excited state and the third excited state, we have $f = 12.00 - 11.00 = 1.00$. For a transition between the third excited state and the second excited state, we have $f = 11.00 - 9.00 = 2.00$. For a transition between the second excited state and the first excited state, we have $f = 9.00 - 6.00 = 3.00$, which also results from some other transitions.

30. For $n = 1$

$$\begin{aligned} E_1 &= -\frac{m_e e^4}{8\varepsilon_0^2 h^2} \\ &= -\frac{(9.11\times10^{-31}\,\text{kg})(1.6\times10^{-19}\,\text{C})^4}{8(8.85\times10^{-12}\,\text{F/m})^2(6.63\times10^{-34}\,\text{J}\cdot\text{s})^2(1.60\times10^{-19}\,\text{J/eV})} \\ &= -13.6\text{ eV} . \end{aligned}$$

31. From Eq. 40-6,

$$\Delta E = hf = (4.14\times10^{-15}\,\text{eV}\cdot\text{s})(6.2\times10^{14}\,\text{Hz}) = 2.6\text{ eV} .$$

32. The difference between the energy absorbed and the energy emitted is

$$E_{\text{photon absorbed}} - E_{\text{photon emitted}} = \frac{hc}{\lambda_{\text{absorbed}}} - \frac{hc}{\lambda_{\text{emitted}}} .$$

Thus, using the result of problem 3 in Chapter 39, the net energy absorbed is

$$hc\Delta\left(\frac{1}{\lambda}\right) = (1240\,\text{eV}\cdot\text{nm})\left(\frac{1}{375\,\text{nm}} - \frac{1}{580\,\text{nm}}\right) = 1.17\text{ eV} .$$

33. The energy E of the photon emitted when a hydrogen atom jumps from a state with principal quantum number u to a state with principal quantum number ℓ is given by

$$E = A\left(\frac{1}{\ell^2} - \frac{1}{u^2}\right)$$

where $A = 13.6\,\text{eV}$. The frequency f of the electromagnetic wave is given by $f = E/h$ and the wavelength is given by $\lambda = c/f$. Thus,

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E}{hc} = \frac{A}{hc}\left(\frac{1}{\ell^2} - \frac{1}{u^2}\right) .$$

The shortest wavelength occurs at the series limit, for which $u = \infty$. For the Balmer series, $\ell = 2$ and the shortest wavelength is $\lambda_B = 4hc/A$. For the Lyman series, $\ell = 1$ and the shortest wavelength is $\lambda_L = hc/A$. The ratio is $\lambda_B/\lambda_L = 4$.

34. (a) The energy level corresponding to the probability density distribution shown in Fig. 40-20 is the $n = 2$ level. Its energy is given by

$$E_2 = -\frac{13.6\,\text{eV}}{2^2} = -3.4\text{ eV} .$$

(b) As the electron is removed from the hydrogen atom the final energy of the proton-electron system is zero. Therefore, one needs to supply at least 3.4 eV of energy to the system in order to bring its energy up from $E_2 = -3.4\,\text{eV}$ to zero. (If more energy is supplied, then the electron will retain some kinetic energy after it is removed from the atom.)

35. (a) Since energy is conserved, the energy E of the photon is given by $E = E_i - E_f$, where E_i is the initial energy of the hydrogen atom and E_f is the final energy. The electron energy is given by $(-13.6\,\text{eV})/n^2$, where n is the principal quantum number. Thus,

$$E = E_i - E_f = \frac{-13.6\,\text{eV}}{(3)^2} - \frac{-13.6\,\text{eV}}{(1)^2} = 12.1\ \text{eV}\ .$$

(b) The photon momentum is given by

$$p = \frac{E}{c} = \frac{(12.1\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{3.00 \times 10^{8}\,\text{m/s}} = 6.45 \times 10^{-27}\ \text{kg}\cdot\text{m/s}\ .$$

(c) Using the result of problem 3 in Chapter 39, the wavelength is

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{12.1\,\text{eV}} = 102\ \text{nm}\ .$$

36. (a) The "home-base" energy level for the Balmer series is $n = 2$. Thus the transition with the least energetic photon is the one from the $n = 3$ level to the $n = 2$ level. The energy difference for this transition is

$$\Delta E = E_3 - E_2 = -(13.6\,\text{eV})\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.889\,\text{eV}\ .$$

Using the result of problem 3 in Chapter 39, the corresponding wavelength is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.889\,\text{eV}} = 658\ \text{nm}\ .$$

(b) For the series limit, the energy difference is

$$\Delta E = E_\infty - E_2 = -(13.6\,\text{eV})\left(\frac{1}{\infty^2} - \frac{1}{2^2}\right) = 3.40\ \text{eV}\ .$$

The corresponding wavelength is then

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\text{eV}\cdot\text{nm}}{3.40\,\text{eV}} = 366\ \text{nm}\ .$$

37. If kinetic energy is not conserved, some of the neutron's initial kinetic energy is used to excite the hydrogen atom. The least energy that the hydrogen atom can accept is the difference between the first excited state ($n = 2$) and the ground state ($n = 1$). Since the energy of a state with principal quantum number n is $-(13.6\,\text{eV})/n^2$, the smallest excitation energy is $13.6\,\text{eV} - (13.6\,\text{eV})/(2)^2 = 10.2\,\text{eV}$. The neutron does not have sufficient kinetic energy to excite the hydrogen atom, so the hydrogen atom is left in its ground state and all the initial kinetic energy of the neutron ends up as the final kinetic energies of the neutron and atom. The collision must be elastic.

38. (a) We use Eq. 40-25. At $r = a$

$$\psi^2(r) = \left(\frac{1}{\sqrt{\pi}a^{3/2}}\,e^{-a/a}\right)^2 = \frac{1}{\pi a^3}\,e^{-2} = \frac{1}{\pi(5.29 \times 10^{-2}\,\text{nm})^3}\,e^{-2} = 291\ \text{nm}^{-3}\ .$$

(b) We use Eq. 40-31. At $r = a$

$$P(r) = \frac{4}{a^3}\, a^2 e^{-2a/a} = \frac{4e^{-2}}{a} = \frac{4e^{-2}}{5.29 \times 10^{-2}\,\text{nm}} = 10.2\ \text{nm}^{-1}\ .$$

39. (a) We use Eq. 40-31. At $r = 0$, $P(r) \propto r^2 = 0$.

(b) At $r = a$

$$P(r) = \frac{4}{a^3}\, a^2 e^{-2a/a} = \frac{4e^{-2}}{a} = \frac{4e^{-2}}{5.29 \times 10^{-2}\,\text{nm}} = 10.2\ \text{nm}^{-1}\ .$$

(c) At $r = 2a$

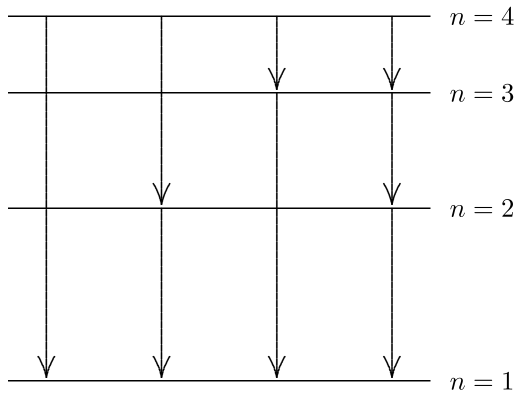
$$P(r) = \frac{4}{a^3}\,(2a)^2 e^{-4a/a} = \frac{16e^{-4}}{a} = \frac{16e^{-4}}{5.29 \times 10^{-2}\,\text{nm}} = 5.54\ \text{nm}^{-1}\ .$$

40. (a) $\Delta E = -(13.6\,\text{eV})(4^{-2} - 1^{-2}) = 12.8\,\text{eV}$.

(b) The values of the photon energies are:

$$\begin{aligned}
E_{4\to1} &= \Delta E_{\text{part (a)}} = 12.8\ \text{eV} \\
E_{3\to1} &= -(13.6\,\text{eV})(3^{-2} - 1^{-2}) = 12.1\ \text{eV} \\
E_{2\to1} &= -(13.6\,\text{eV})(2^{-2} - 1^{-2}) = 10.2\ \text{eV} \\
E_{4\to2} &= -(13.6\,\text{eV})(4^{-2} - 2^{-2}) = 2.55\ \text{eV} \\
E_{3\to2} &= -(13.6\,\text{eV})(3^{-2} - 2^{-2}) = 1.89\ \text{eV} \\
E_{4\to3} &= -(13.6\,\text{eV})(4^{-2} - 3^{-2}) = 0.66\ \text{eV}
\end{aligned}$$

The various photon energies correspond to the transitions between energy levels indicated below. The levels are not drawn to scale.



41. (a) We take the electrostatic potential energy to be zero when the electron and proton are far removed from each other. Then, the final energy of the atom is zero and the work done in pulling it apart is $W = -E_i$, where E_i is the energy of the initial state. The energy of the initial state is given by $E_i = (-13.6\,\text{eV})/n^2$, where n is the principal quantum number of the state. For the ground state, $n = 1$ and $W = 13.6\,\text{eV}$.

(b) For the state with $n = 2$, $W = (13.6\,\text{eV})/(2)^2 = 3.40\,\text{eV}$.

42. Conservation of linear momentum of the atom-photon system requires that

$$p_{\text{recoil}} = p_{\text{photon}} \quad\Longrightarrow\quad m_p v_{\text{recoil}} = \frac{hf}{c}$$

where we use Eq. 39-7 for the photon and use the classical momentum formula for the atom (since we expect its speed to be much less than c). Thus, from Eq. 40-6 and Table 38-3,

$$\begin{aligned} v_{\text{recoil}} &= \frac{\Delta E}{m_p c} = \frac{E_4 - E_1}{(m_p c^2)/c} \\ &= \frac{(-13.6\,\text{eV})\left(4^{-2} - 1^{-2}\right)}{(938 \times 10^6\,\text{eV})/(2.998 \times 10^8\,\text{m/s})} \\ &= 4.1\ \text{m/s}\ . \end{aligned}$$

43. (a) and (b) Using Eq. 40-6 and the result of problem 3 in Chapter 39, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{486.1\,\text{nm}} = 2.55\ \text{eV}\ .$$

Referring to Fig. 40-16, we see that this must be one of the Balmer series transitions (this fact could also be found from Fig. 40-17). Therefore, $n_{\text{low}} = 2$, but what precisely is n_{high}?

$$\begin{aligned} E_{\text{high}} &= E_{\text{low}} + \Delta E \\ -\frac{13.6\,\text{eV}}{n^2} &= -\frac{13.6\,\text{eV}}{2^2} + 2.55\,\text{eV} \end{aligned}$$

which yields $n = 4$. Thus, the transition is from the $n = 4$ to the $n = 2$ state.

44. (a) The calculation is shown in Sample Problem 40-6. The difference in the values obtained in parts (a) and (b) of that Sample Problem is $122\,\text{nm} - 91.4\,\text{nm} \approx 31\,\text{nm}$.

(b) Fig. 40-17 shows that the width of the Balmer series is $656.3\,\text{nm} - 364.6\,\text{nm} \approx 292\,\text{nm}$. This can be confirmed with a calculation very much like the one shown in Sample Problem 40-6, but with the longest wavelength arising from the $3 \to 2$ transition, and the series limit obtained from the $\infty \to 2$ transition.

(c) We use Eq. 39-1. For the Lyman series,

$$\Delta f = \frac{2.998 \times 10^8\,\text{m/s}}{91.4 \times 10^{-9}\,\text{m}} - \frac{2.998 \times 10^8\,\text{m/s}}{122 \times 10^{-9}\,\text{m}} = 8.2 \times 10^{14}\ \text{Hz}$$

or 8.2×10^2 THz. For the Balmer series,

$$\Delta f = \frac{2.998 \times 10^8\,\text{m/s}}{364.6 \times 10^{-9}\,\text{m}} - \frac{2.998 \times 10^8\,\text{m/s}}{656.3 \times 10^{-9}\,\text{m}} = 3.65 \times 10^{14}\ \text{Hz}$$

which is equivalent to 365 THz.

45. Letting $a = 5.292 \times 10^{-11}$ m be the Bohr radius, the potential energy becomes

$$U = -\frac{e^2}{4\pi\varepsilon_0 a} = \frac{(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.602 \times 10^{-19}\,\text{C})^2}{5.292 \times 10^{-11}\,\text{m}} = -4.36 \times 10^{-18}\,\text{J} = -27.2\ \text{eV}\ .$$

The kinetic energy is $K = E - U = (-13.6\,\text{eV}) - (-27.2\,\text{eV}) = 13.6\,\text{eV}$.

46. (a) and (b) Using Eq. 40-6 and the result of problem 3 in Chapter 39, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{121.6\,\text{nm}} = 10.2\ \text{eV}\ .$$

Referring to Fig. 40-16, we see that this must be one of the Lyman series transitions. Therefore, $n_{\text{low}} = 1$, but what precisely is n_{high}?

$$\begin{aligned} E_{\text{high}} &= E_{\text{low}} + \Delta E \\ -\frac{13.6\,\text{eV}}{n^2} &= -\frac{13.6\,\text{eV}}{1^2} + 10.2\,\text{eV} \end{aligned}$$

which yields $n = 2$ (this is confirmed by the calculation found from Sample Problem 40-6). Thus, the transition is from the $n = 2$ to the $n = 1$ state.

47. (a) Since $E_2 = -0.85\,\text{eV}$ and $E_1 = -13.6\,\text{eV} + 10.2\,\text{eV} = -3.4\,\text{eV}$, the photon energy is $E_{\text{photon}} = E_2 - E_1 = -0.85\,\text{eV} - (-3.4\,\text{eV}) = 2.6\,\text{eV}$.

(b) From

$$E_2 - E_1 = (-13.6\,\text{eV})\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right) = 2.6 \text{ eV}$$

we obtain

$$\frac{1}{n_2^2} - \frac{1}{n_1^2} = -\frac{2.6\,\text{eV}}{13.6\,\text{eV}} \approx -\frac{3}{16} = \frac{1}{4^2} - \frac{1}{2^2} .$$

Thus, $n_2 = 4$ and $n_1 = 2$. So the transition is from the $n = 4$ state to the $n = 2$ state. One can easily verify this by inspecting the energy level diagram of Fig. 40-16.

48. The wavelength λ of the photon emitted in a transition belonging to the Balmer series satisfies

$$E_{\text{ph}} = \frac{hc}{\lambda} = E_n - E_2 = -(13.6\,\text{eV})\left(\frac{1}{n^2} - \frac{1}{2^2}\right) \quad \text{where } n = 3,\ 4,\ 5,\ \ldots$$

Using the result of problem 3 in Chapter 39, we find

$$\lambda = \frac{4hcn^2}{(13.6\,\text{eV})(n^2-4)} = \frac{4(1240\,\text{eV}\cdot\text{nm})}{13.6\,\text{eV}}\left(\frac{n^2}{n^2-4}\right) .$$

Plugging in the various values of n, we obtain these values of the wavelength: $\lambda = 656\,\text{nm}$ (for $n = 3$), $\lambda = 486\,\text{nm}$ (for $n = 4$), $\lambda = 434\,\text{nm}$ (for $n = 5$), $\lambda = 410\,\text{nm}$ (for $n = 6$), $\lambda = 397\,\text{nm}$ (for $n = 7$), $\lambda = 389\,\text{nm}$ (for $n = 8$), etc. Finally for $n = \infty$, $\lambda = 365\,\text{nm}$. These values agree well with the data found in Fig. 40-17. [One can also find λ beyond three significant figures by using the more accurate values for m_e, e and h listed in Appendix B when calculating E_n in Eq. 40-24. Another factor that contributes to the error is the motion of the atomic nucleus. It can be shown that this effect can be accounted for by replacing the mass of the electron m_e by $m_e m_p/(m_p + m_e)$ in Eq. 40-24, where m_p is the mass of the proton. Since $m_p \gg m_e$, this is not a major effect.]

49. According to Sample Problem 40-8, the probability the electron in the ground state of a hydrogen atom can be found inside a sphere of radius r is given by

$$p(r) = 1 - e^{-2x}\left(1 + 2x + 2x^2\right)$$

where $x = r/a$ and a is the Bohr radius. We want $r = a$, so $x = 1$ and

$$p(a) = 1 - e^{-2}\,(1 + 2 + 2) = 1 - 5e^{-2} = 0.323 .$$

The probability that the electron can be found outside this sphere is $1 - 0.323 = 0.677$. It can be found outside about 68% of the time.

50. Using Eq. 40-6 and the result of problem 3 in Chapter 39, we find

$$\Delta E = E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{102.6\,\text{nm}} = 12.09 \text{ eV} .$$

Referring to Fig. 40-16, we see that this must be one of the Lyman series transitions. Therefore, $n_{\text{low}} = 1$, but what precisely is n_{high}?

$$\begin{aligned} E_{\text{high}} &= E_{\text{low}} + \Delta E \\ -\frac{13.6\,\text{eV}}{n^2} &= -\frac{13.6\,\text{eV}}{1^2} + 12.09\,\text{eV} \end{aligned}$$

which yields $n = 3$. Thus, the transition is from the $n = 3$ to the $n = 1$ state.

51. The proposed wave function is

$$\psi = \frac{1}{\sqrt{\pi}a^{3/2}} e^{-r/a}$$

where a is the Bohr radius. Substituting this into the right side of Schrödinger's equation, our goal is to show that the result is zero. The derivative is

$$\frac{d\psi}{dr} = -\frac{1}{\sqrt{\pi}a^{5/2}} e^{-r/a}$$

so

$$r^2 \frac{d\psi}{dr} = -\frac{r^2}{\sqrt{\pi}a^{5/2}} e^{-r/a}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\psi}{dr}\right) = \frac{1}{\sqrt{\pi}a^{5/2}}\left[-\frac{2}{r}+\frac{1}{a}\right] e^{-r/a} = \frac{1}{a}\left[-\frac{2}{r}+\frac{1}{a}\right]\psi \ .$$

The energy of the ground state is given by $E = -me^4/8\varepsilon_0^2 h^2$, and the Bohr radius is given by $a = h^2\varepsilon_0/\pi m e^2$, so $E = -e^2/8\pi\varepsilon_0 a$. The potential energy is given by $U = -e^2/4\pi\varepsilon_0 r$, so

$$\begin{aligned}
\frac{8\pi^2 m}{h^2}[E-U]\psi &= \frac{8\pi^2 m}{h^2}\left[-\frac{e^2}{8\pi\varepsilon_0 a}+\frac{e^2}{4\pi\varepsilon_0 r}\right]\psi = \frac{8\pi^2 m}{h^2}\frac{e^2}{8\pi\varepsilon_0}\left[-\frac{1}{a}+\frac{2}{r}\right]\psi \\
&= \frac{\pi m e^2}{h^2\varepsilon_0}\left[-\frac{1}{a}+\frac{2}{r}\right]\psi = \frac{1}{a}\left[-\frac{1}{a}+\frac{2}{r}\right]\psi \ .
\end{aligned}$$

The two terms in Schrödinger's equation cancel, and the proposed function ψ satisfies that equation.

52. From Sample Problem 40-8, we know that the probability of finding the electron in the ground state of the hydrogen atom inside a sphere of radius r is given by

$$p(r) = 1 - e^{-2x}\left(1+2x+2x^2\right)$$

where $x = r/a$. Thus the probability of finding the electron between the two shells indicated in this problem is given by

$$\begin{aligned}
p(a<r<2a) &= p(2a) - p(a) \\
&= \left[1-e^{-2x}\left(1+2x+2x^2\right)\right]_{x=2} - \left[1-e^{-2x}\left(1+2x+2x^2\right)\right]_{x=1} \\
&= 0.44 \ .
\end{aligned}$$

53. The radial probability function for the ground state of hydrogen is $P(r) = (4r^2/a^3)e^{-2r/a}$, where a is the Bohr radius. (See Eq. 40-31.) We want to evaluate the integral $\int_0^\infty P(r)\,dr$. Eq. 15 in the integral table of Appendix E is an integral of this form. We set $n = 2$ and replace a in the given formula with $2/a$ and x with r. Then

$$\int_0^\infty P(r)\,dr = \frac{4}{a^3}\int_0^\infty r^2 e^{-2r/a}\,dr = \frac{4}{a^3}\frac{2}{(2/a)^3} = 1 \ .$$

54. (a) The allowed values of l for a given n are $0, 1, 2, \ldots, n-1$. Thus there are n different values of l.

(b) The allowed values of m_l for a given l are $-l, -l+1, \cdots, l$. Thus there are $2l+1$ different values of m_l.

(c) According to part (a) above, for a given n there are n different values of l. Also, each of these l's can have $2l+1$ different values of m_l [see part (b) above]. Thus, the total number of m_l's is

$$\sum_{l=0}^{n-1}(2l+1) = n^2 \ .$$

55. Since Δr is small, we may calculate the probability using $p = P(r)\,\Delta r$, where $P(r)$ is the radial probability density. The radial probability density for the ground state of hydrogen is given by Eq. 40-31:

$$P(r) = \left(\frac{4r^2}{a^3}\right) e^{-2r/a}$$

where a is the Bohr radius.

(a) Here, $r = 0.500a$ and $\Delta r = 0.010a$. Then,

$$p = \left(\frac{4r^2\,\Delta r}{a^3}\right) e^{-2r/a} = 4(0.500)^2(0.010)\,e^{-1} = 3.68 \times 10^{-3} \ .$$

(b) We set $r = 1.00a$ and $\Delta r = 0.010a$. Then,

$$p = \left(\frac{4r^2\,\Delta r}{a^3}\right) e^{-2r/a} = 4(1.00)^2(0.010)\,e^{-2} = 5.41 \times 10^{-3} \ .$$

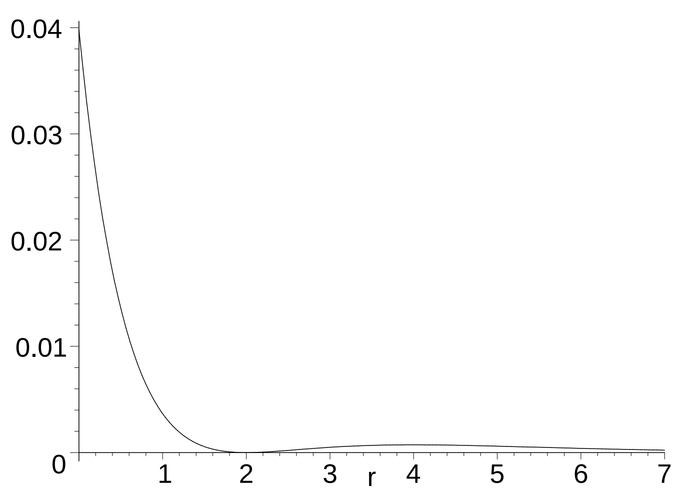
56. According to Fig. 40-23, the quantum number n in question satisfies $r = n^2 a$. Letting $r = 1.0\,\text{mm}$, we solve for n:

$$n = \sqrt{\frac{r}{a}} = \sqrt{\frac{1.0 \times 10^{-3}\,\text{m}}{5.29 \times 10^{-11}\,\text{m}}} \approx 4.3 \times 10^3 \ .$$

57. The radial probability function for the ground state of hydrogen is $P(r) = (4r^2/a^3)e^{-2r/a}$, where a is the Bohr radius. (See Eq. 40-31.) The integral table of Appendix E may be used to evaluate the integral $r_{\text{avg}} = \int_0^\infty rP(r)\,dr$. Setting $n = 3$ and replacing a in the given formula with $2/a$ (and x with r), we obtain

$$r_{\text{avg}} = \int_0^\infty rP(r)\,dr = \frac{4}{a^3}\int_0^\infty r^3 e^{-2r/a}\,dr = \frac{4}{a^3}\,\frac{6}{(2/a)^4} = 1.5a \ .$$

58. (a) The plot shown below for $|\psi_{200}(r)|^2$ is to be compared with the dot plot of Fig. 40-20. We note that the horizontal axis of our graph is labeled "r," but it is actually r/a (that is, it is in units of the parameter a). Now, in the plot below there is a high central peak between $r = 0$ and $r \sim 2a$, corresponding to the densely dotted region around the center of the dot plot of Fig. 40-20. Outside this peak is a region of near-zero values centered at $r = 2a$, where $\psi_{200} = 0$. This is represented in the dot plot by the empty ring surrounding the central peak. Further outside is a broader, flatter, low peak which reaches its maximum value at $r = 4a$. This corresponds to the outer ring with near-uniform dot density which is lower than that of the central peak.



(b) The extrema of $\psi^2(r)$ for $0 < r < \infty$ may be found by squaring the given function, differentiating with respect to r, and setting the result equal to zero:

$$-\frac{1}{32}\,\frac{(r-2a)(r-4a)}{a^6\pi}\,e^{-r/a} = 0$$

which has roots at $r = 2a$ and $r = 4a$. We can verify directly from the plot above that $r = 4a$ is indeed a local maximum of $\psi^2_{200}(r)$. As discussed in part (a), the other root ($r = 2a$) is a local minimum.

(c) Using Eq. 40-30 and Eq. 40-28, the radial probability is

$$P_{200}(r) = 4\pi r^2\psi^2_{200}(r) = \frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}\ .$$

(d) Let $x = r/a$. Then

$$\begin{aligned}\int_0^\infty P_{200}(r)\,dr &= \int_0^\infty \frac{r^2}{8a^3}\left(2-\frac{r}{a}\right)^2 e^{-r/a}\,dr \\ &= \frac{1}{8}\int_0^\infty x^2(2-x)^2e^{-x}\,dx \\ &= \int_0^\infty (x^4-4x^3+4x^2)e^{-x}\,dx \\ &= \frac{1}{8}\left[4! - 4(3!) + 4(2!)\right] \\ &= 1\end{aligned}$$

where the integral formula

$$\int_0^\infty x^n\,e^{-x}\,dx = n!$$

is used.

59. (a) ψ_{210} is real. Squaring it, we obtain the probability density:

$$|\psi_{210}|^2 = \frac{r^2}{32\pi a^5}e^{-r/a}\cos^2\theta\ .$$

Each of the other functions is multiplied by its complex conjugate, obtained by replacing i with $-i$ in the function. Since $e^{i\phi}e^{-i\phi} = e^0 = 1$, the result is the square of the function without the exponential factor:

$$|\psi_{21+1}|^2 = \frac{r^2}{64\pi a^5}e^{-r/a}\sin^2\theta$$

and

$$|\psi_{21-1}|^2 = \frac{r^2}{64\pi a^5}e^{-r/a}\sin^2\theta\ .$$

The last two functions lead to the same probability density.

(b) The total probability density for the three states is the sum:

$$\begin{aligned}|\psi_{210}|^2 + |\psi_{21+1}|^2 + |\psi_{21-1}|^2 &= \frac{r^2}{32\pi a^5}e^{-r/a}\left[\cos^2\theta + \frac{1}{2}\sin^2\theta + \frac{1}{2}\sin^2\theta\right] \\ &= \frac{r^2}{32\pi a^5}e^{-r/a}\ .\end{aligned}$$

The trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. We note that the total probability density does not depend on θ or ϕ; it is spherically symmetric.

Chapter 41

1. One way to think of the units of h is that, because of the equation $E = hf$ and the fact that f is in cycles/second, then the "explicit" units for h should be J·s/cycle. Then, since 2π rad/cycle is a conversion factor for cycles → radians, $\hbar = h/2\pi$ can be thought of as the Planck constant expressed in terms of radians instead of cycles. Using the precise values stated in Appendix B,

$$\begin{aligned}\hbar &= \frac{h}{2\pi} = \frac{6.62606876 \times 10^{-34}\,\text{J·s}}{2\pi} = 1.05457 \times 10^{-34}\,\text{J·s} \\ &= \frac{1.05457 \times 10^{-34}\,\text{J·s}}{1.6021765 \times 10^{-19}\,\text{J/eV}} = 6.582 \times 10^{-16}\ \text{eV·s} .\end{aligned}$$

2. For a given quantum number l there are $(2l + 1)$ different values of m_l. For each given m_l the electron can also have two different spin orientations. Thus, the total number of electron states for a given l is given by $N_l = 2(2l + 1)$.

   (a) Now $l = 3$, so $N_l = 2(2 \times 3 + 1) = 14$.

   (b) In this case, $l = 1$, which means $N_l = 2(2 \times 1 + 1) = 6$.

   (c) Here $l = 1$, so $N_l = 2(2 \times 1 + 1) = 6$.

   (d) Now $l = 0$, so $N_l = 2(2 \times 0 + 1) = 2$.

3. (a) For a given value of the principal quantum number n, the orbital quantum number ℓ ranges from 0 to $n - 1$. For $n = 3$, there are three possible values: 0, 1, and 2.

   (b) For a given value of ℓ, the magnetic quantum number m_ℓ ranges from $-\ell$ to $+\ell$. For $\ell = 1$, there are three possible values: -1, 0, and $+1$.

4. (a) We use Eq. 41-2:

$$L = \sqrt{l(l+1)}\,\hbar = \sqrt{3(3+1)}\,(1.055 \times 10^{-34}\,\text{J·s}) = 3.653 \times 10^{-34}\ \text{J·s} .$$

   (b) We use Eq. 41-7: $L_z = m_l\hbar$. For the maximum value of L_z set $m_l = l$. Thus

$$[L_z]_{\max} = l\hbar = 3(1.055 \times 10^{-34}\,\text{J·s}) = 3.165 \times 10^{-34}\ \text{J·s} .$$

5. For a given quantum number n there are n possible values of l, ranging from 0 to $n - 1$. For each l the number of possible electron states is $N_l = 2(2l + 1)$ (see problem 2). Thus, the total number of possible electron states for a given n is

$$N_n = \sum_{l=0}^{n-1} N_l = 2\sum_{l=0}^{n-1}(2l + 1) = 2n^2 .$$

   (a) In this case $n = 4$, which implies $N_n = 2(4^2) = 32$.

(b) Now $n = 1$, so $N_n = 2(1^2) = 2$.

(c) Here $n = 3$, and we obtain $N_n = 2(3^2) = 18$.

(d) Finally, $n = 2 \;\rightarrow\; N_n = 2(2^2) = 8$.

6. Using Table 41-1, we find for $n = 4$ and $l = 3$: $m_l = +3, +2, +1, 0, -1, -2, -3$ and $m_s = \pm\frac{1}{2}$.

7. The principal quantum number n must be greater than 3. The magnetic quantum number m_ℓ can have any of the values -3, -2, -1, 0, $+1$, $+2$, or $+3$. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.

8. Using Table 41-1, we find $l = [m_l]_{\max} = 4$ and $n = l_{\max} + 1 \geq l + 1 = 5$. And, as usual, $m_s = \pm\frac{1}{2}$.

9. The principal quantum number n must be greater than 3. The magnetic quantum number m_l can have any of the values -3, -2, -1, 0, $+1$, $+2$, or $+3$. The spin quantum number can have either of the values $-\frac{1}{2}$ or $+\frac{1}{2}$.

10. For a given quantum number n there are n possible values of l, ranging from 0 to $n - 1$. For each l the number of possible electron states is $N_l = 2(2l + 1)$ (see problem 2). Thus the total number of possible electron states for a given n is

$$N_n = \sum_{l=0}^{n-1} N_l = 2\sum_{l=0}^{n-1}(2l + 1) = 2n^2 \ .$$

Thus, in this problem, the total number of electron states is $N_n = 2n^2 = 2(5)^2 = 50$.

11. (a) For $\ell = 3$, the magnitude of the orbital angular momentum is $L = \sqrt{\ell(\ell + 1)}\hbar = \sqrt{3(3 + 1)}\hbar = \sqrt{12}\hbar$.

(b) The magnitude of the orbital dipole moment is $\mu_{\text{orb}} = \sqrt{\ell(\ell + 1)}\mu_B = \sqrt{12}\mu_B$.

(c) We use $L_z = m_\ell\hbar$ to calculate the z component of the orbital angular momentum, $\mu_z = -m_\ell\mu_B$ to calculate the z component of the orbital magnetic dipole moment, and $\cos\theta = m_\ell/\sqrt{\ell(\ell + 1)}$ to calculate the angle between the orbital angular momentum vector and the z axis. For $\ell = 3$, the magnetic quantum number m_ℓ can take on the values -3, -2, -1, 0, $+1$, $+2$, $+3$. Results are tabulated below.

m_ℓ	L_z	$\mu_{\text{orb},\ z}$	θ
-3	$-3\hbar$	$+3\mu_B$	150.0°
-2	$-2\hbar$	$+2\mu_B$	125°
-1	$-\hbar$	$+\mu_B$	107°
0	0	0	90.0°
1	$+\hbar$	$-\mu_B$	73.2°
2	$2\hbar$	$-2\mu_B$	54.7°
3	$3\hbar$	$-3\mu_B$	30.0°

12. (a) For $n = 3$ there are 3 possible values of l: 0, 1, and 2.

(b) We interpret this as asking for the number of distinct values for m_l (this ignores the multiplicity of any particular value). For each l there are $2l+1$ possible values of m_l. Thus the number of possible m_l's for $l=2$ is $(2l+1)=5$. Examining the $l=1$ and $l=0$ cases cannot lead to any new (distinct) values for m_l, so the answer is 5.

(c) Regardless of the values of n, l and m_l, for an electron there are always two possible values of m_s: $\pm\frac{1}{2}$.

(d) The population in the $n=3$ shell is equal to the number of electron states in the shell, or $2n^2 = 2(3^2) = 18$.

(e) Each subshell has its own value of l. Since there are three different values of l for $n=3$, there are three subshells in the $n=3$ shell.

13. Since $L^2 = L_x^2 + L_y^2 + L_z^2$, $\sqrt{L_x^2 + L_y^2} = \sqrt{L^2 - L_z^2}$. Replacing L^2 with $\ell(\ell+1)\hbar^2$ and L_z with $m_\ell\hbar$, we obtain

$$\sqrt{L_x^2 + L_y^2} = \hbar\sqrt{\ell(\ell+1) - m_\ell^2}\ .$$

For a given value of ℓ, the greatest that m_ℓ can be is ℓ, so the smallest that $\sqrt{L_x^2 + L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1) - \ell^2} = \hbar\sqrt{\ell}$. The smallest possible magnitude of m_ℓ is zero, so the largest $\sqrt{L_x^2 + L_y^2}$ can be is $\hbar\sqrt{\ell(\ell+1)}$. Thus,

$$\hbar\sqrt{\ell} \le \sqrt{L_x^2 + L_y^2} \le \hbar\sqrt{\ell(\ell+1)}\ .$$

14. (a) The value of l satisfies $\sqrt{l(l+1)}\hbar \approx \sqrt{l^2}\hbar = l\hbar = L$, so $l \simeq L/\hbar \simeq 3\times 10^{74}$.

(b) The number is $2l+1 \approx 2(3\times 10^{74}) = 6\times 10^{74}$.

(c) Since

$$\cos\theta_{\min} = \frac{m_{l\,\max}\hbar}{\sqrt{l(l+1)}\hbar} = \frac{l}{\sqrt{l(l+1)}} \approx 1 - \frac{1}{2l} = 1 - \frac{1}{2(3\times 10^{74})}$$

or $\cos\theta_{\min} \simeq 1 - \theta_{\min}^2/2 \approx 1 - 10^{-74}/6$, we have $\theta_{\min} \simeq \sqrt{10^{-74}/3} = 6\times 10^{-38}$ rad. The correspondence principle requires that all the quantum effects vanish as $\hbar \to 0$. In this case $\hbar/L$ is extremely small so the quantization effects are barely existent, with $\theta_{\min} \simeq 10^{-38}$ rad $\simeq 0$.

15. The magnitude of the spin angular momentum is $S = \sqrt{s(s+1)}\,\hbar = (\sqrt{3}/2)\hbar$, where $s = \frac{1}{2}$ is used. The z component is either $S_z = \hbar/2$ or $-\hbar/2$. If $S_z = +\hbar/2$, the angle θ between the spin angular momentum vector and the positive z axis is

$$\theta = \cos^{-1}\left(\frac{S_z}{S}\right) = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.7^\circ\ .$$

If $S_z = -\hbar/2$, the angle is $\theta = 180^\circ - 54.7^\circ = 125.3^\circ$.

16. (a) From Fig. 41-10 and Eq. 41-18,

$$\Delta E = 2\mu_B B = \frac{2(9.27\times 10^{-24}\ \mathrm{J/T})(0.50\ \mathrm{T})}{1.60\times 10^{-19}\ \mathrm{J/eV}} = 58\ \mu\mathrm{eV}\ .$$

(b) From $\Delta E = hf$ we get

$$f = \frac{\Delta E}{h} = \frac{9.27\times 10^{-24}\ \mathrm{J}}{6.63\times 10^{-34}\ \mathrm{J\cdot s}} = 1.4\times 10^{10}\ \mathrm{Hz} = 14\ \mathrm{GHz}\ .$$

(c) The wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998\times 10^8\ \mathrm{m/s}}{1.4\times 10^{10}\ \mathrm{Hz}} = 2.1\ \mathrm{cm}\ ,$$

which is in the short radio wave region.

17. The acceleration is

$$a = \frac{F}{M} = \frac{(\mu \cos\theta)\,(dB/dz)}{M},$$

where M is the mass of a silver atom, μ is its magnetic dipole moment, B is the magnetic field, and θ is the angle between the dipole moment and the magnetic field. We take the moment and the field to be parallel ($\cos\theta = 1$) and use the data given in Sample Problem 41-1 to obtain

$$a = \frac{(9.27 \times 10^{-24}\,\mathrm{J/T})(1.4 \times 10^{3}\,\mathrm{T/m})}{1.8 \times 10^{-25}\,\mathrm{kg}} = 7.21 \times 10^{4}\ \mathrm{m/s^2}\ .$$

18. (a) From Eq. 41-19,

$$F = \mu_B \left|\frac{dB}{dz}\right| = (9.27 \times 10^{-24}\,\mathrm{J/T})(1.6 \times 10^{2}\,\mathrm{T/m}) = 1.5 \times 10^{-21}\ \mathrm{N}\ .$$

(b) The vertical displacement is

$$\begin{aligned}\Delta x &= \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{l}{v}\right)^2 \\ &= \frac{1}{2}\left(\frac{1.5 \times 10^{-21}\,\mathrm{N}}{1.67 \times 10^{-27}\,\mathrm{kg}}\right)\left(\frac{0.80\,\mathrm{m}}{1.2 \times 10^{5}\,\mathrm{m/s}}\right)^2 \\ &= 2.0 \times 10^{-5}\ \mathrm{m}\ .\end{aligned}$$

19. The energy of a magnetic dipole in an external magnetic field $\vec{B}$ is $U = -\vec{\mu} \cdot \vec{B} = -\mu_z B$, where $\vec{\mu}$ is the magnetic dipole moment and μ_z is its component along the field. The energy required to change the moment direction from parallel to antiparallel is $\Delta E = \Delta U = 2\mu_z B$. Since the z component of the spin magnetic moment of an electron is the Bohr magneton μ_B, $\Delta E = 2\mu_B B = 2(9.274 \times 10^{-24}\,\mathrm{J/T})(0.200\,\mathrm{T}) = 3.71 \times 10^{-24}\,\mathrm{J}$. The photon wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\,\mathrm{J\cdot s})(3.00 \times 10^{8}\,\mathrm{m/s})}{3.71 \times 10^{-24}\,\mathrm{J}} = 5.36 \times 10^{-2}\ \mathrm{m}\ .$$

20. We let $\Delta E = 2\mu_B B_{\mathrm{eff}}$ (based on Fig. 41-10 and Eq. 41-18) and solve for B_{eff}:

$$B_{\mathrm{eff}} = \frac{\Delta E}{2\mu_B} = \frac{hc}{2\lambda\mu_B} = \frac{1240\,\mathrm{nm\cdot eV}}{2(21 \times 10^{-7}\,\mathrm{nm})(5.788 \times 10^{-5}\,\mathrm{eV/T})} = 51\ \mathrm{mT}\ .$$

21. (a) Using the result of problem 3 in Chapter 39,

$$\Delta E = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = (1240\,\mathrm{eV\cdot nm})\left(\frac{1}{588.995\,\mathrm{nm}} - \frac{1}{589.592\,\mathrm{nm}}\right) = 2.13\ \mathrm{meV}\ .$$

(b) From $\Delta E = 2\mu_B B$ (see Fig. 41-10 and Eq. 41-18), we get

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.13 \times 10^{-3}\,\mathrm{eV}}{2(5.788 \times 10^{-5}\,\mathrm{eV/T})} = 18\ \mathrm{T}\ .$$

22. The total magnetic field, $B = B_{\mathrm{local}} + B_{\mathrm{ext}}$, satisfies $\Delta E = hf = 2\mu B$ (see Eq. 41-22). Thus,

$$B_{\mathrm{local}} = \frac{hf}{2\mu} - B_{\mathrm{ext}} = \frac{(6.63 \times 10^{-34}\mathrm{J\cdot s})(34 \times 10^{6}\,\mathrm{Hz})}{2(1.41 \times 10^{-26}\,\mathrm{J/T})} - 0.78\,\mathrm{T} = 19\ \mathrm{mT}\ .$$

23. Because of the Pauli principle (and the requirement that we construct a state of lowest possible total energy), two electrons fill the $n = 1, 2, 3$ levels and one electron occupies the $n = 4$ level. Thus, using Eq. 40-4,

$$\begin{aligned} E_{\text{ground}} &= 2E_1 + 2E_2 + 2E_3 + E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 8 + 18 + 16)\left(\frac{h^2}{8mL^2}\right) = 44\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

24. Using Eq. 40-20 (see also problem 27 in Chapter 40) we find that the lowest four levels of the rectangular corral (with this specific "aspect ratio") are non-degenerate, with energies $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, and $E_{2,1} = 4.25$ (all of these understood to be in "units" of $h^2/8mL^2$). Therefore, obeying the Pauli principle, we have

$$E_{\text{ground}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,1} = 2(1.25) + 2(2.00) + 2(3.25) + 4.25$$

which means (putting the "unit" factor back in) that the lowest possible energy of the system is $E_{\text{ground}} = 17.25(h^2/8mL^2)$.

25. (a) Promoting one of the electrons (described in problem 23) to a not-fully occupied higher level, we find that the configuration with the least total energy greater than that of the ground state has the $n = 1$ and 2 levels still filled, but now has only one electron in the $n = 3$ level; the remaining two electrons are in the $n = 4$ level. Thus,

$$\begin{aligned} E_{\text{first excited}} &= 2E_1 + 2E_2 + E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + \left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 8 + 9 + 32)\left(\frac{h^2}{8mL^2}\right) = 51\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

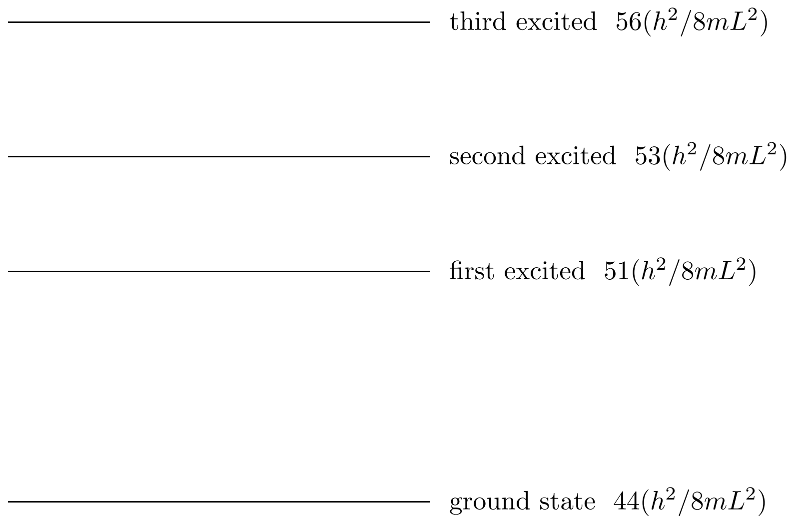
(b) Now, the configuration which provides the next higher total energy, above that found in part (a), has the bottom three levels filled (just as in the ground state configuration) and has the seventh electron occupying the $n = 5$ level:

$$\begin{aligned} E_{\text{second excited}} &= 2E_1 + 2E_2 + 2E_3 + E_5 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + 2\left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + \left(\frac{h^2}{8mL^2}\right)(5)^2 \\ &= (2 + 8 + 18 + 25)\left(\frac{h^2}{8mL^2}\right) = 53\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

(c) The third excited state has the $n = 1, 3, 4$ levels filled, and the $n = 2$ level half-filled:

$$\begin{aligned} E_{\text{third excited}} &= 2E_1 + E_2 + 2E_3 + 2E_4 \\ &= 2\left(\frac{h^2}{8mL^2}\right)(1)^2 + \left(\frac{h^2}{8mL^2}\right)(2)^2 + 2\left(\frac{h^2}{8mL^2}\right)(3)^2 + 2\left(\frac{h^2}{8mL^2}\right)(4)^2 \\ &= (2 + 4 + 18 + 32)\left(\frac{h^2}{8mL^2}\right) = 56\left(\frac{h^2}{8mL^2}\right) . \end{aligned}$$

(d) The energy states of this problem and problem 23 are suggested in the sketch below:



26. (a) Using Eq. 40-20 (see also problem 27 in Chapter 40) we find that the lowest five levels of the rectangular corral (with this specific "aspect ratio") have energies $E_{1,1} = 1.25$, $E_{1,2} = 2.00$, $E_{1,3} = 3.25$, $E_{2,1} = 4.25$, and $E_{2,2} = 5.00$ (all of these understood to be in "units" of $h^2/8mL^2$). It should be noted that the energy level we denote $E_{2,2}$ actually corresponds to two energy levels ($E_{2,2}$ and $E_{1,4}$; they are degenerate), but that will not affect our calculations in this problem. The configuration which provides the lowest system energy higher than that of the ground state has the first three levels filled, the fourth one empty, and the fifth one half-filled:

$$E_{\text{first excited}} = 2E_{1,1} + 2E_{1,2} + 2E_{1,3} + E_{2,2} = 2(1.25) + 2(2.00) + 2(3.25) + 5.00$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 18.00(h^2/8mL^2)$.

(b) The configuration which provides the next higher system energy has the first two levels filled, the third one half-filled, and the fourth one filled:

$$E_{\text{second excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + 2E_{2,1} = 2(1.25) + 2(2.00) + 3.25 + 2(4.25)$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 18.25(h^2/8mL^2)$.

(c) Now, the configuration which provides the *next* higher system energy has the first two levels filled, with the next three levels half-filled:

$$E_{\text{third excited}} = 2E_{1,1} + 2E_{1,2} + E_{1,3} + E_{2,1} + E_{2,2} = 2(1.25) + 2(2.00) + 3.25 + 4.25 + 5.00$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 19.00(h^2/8mL^2)$.

(d) The energy states of this problem and problem 24 are suggested in the sketch below:

third excited $19.00(h^2/8mL^2)$

second excited $18.25(h^2/8mL^2)$

first excited $18.00(h^2/8mL^2)$

ground state $17.25(h^2/8mL^2)$

27. In terms of the quantum numbers n_x, n_y, and n_z, the single-particle energy levels are given by

$$E_{n_x,n_y,n_z} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) .$$

The lowest single-particle level corresponds to $n_x = 1$, $n_y = 1$, and $n_z = 1$ and is $E_{1,1,1} = 3(h^2/8mL^2)$. There are two electrons with this energy, one with spin up and one with spin down. The next lowest single-particle level is three-fold degenerate in the three integer quantum numbers. The energy is $E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2)$. Each of these states can be occupied by a spin up and a spin down electron, so six electrons in all can occupy the states. This completes the assignment of the eight electrons to single-particle states. The ground state energy of the system is $E_{\text{gr}} = (2)(3)(h^2/8mL^2) + (6)(6)(h^2/8mL^2) = 42(h^2/8mL^2)$.

28. We use the results of problem 28 in Chapter 40. The Pauli principle requires that no more than two electrons be in the lowest energy level (at $E_{1,1,1} = 3(h^2/8mL^2)$), but – due to their degeneracies – as many as six electrons can be in the next three levels ($E' = E_{1,1,2} = E_{1,2,1} = E_{2,1,1} = 6(h^2/8mL^2)$, $E'' = E_{1,2,2} = E_{2,2,1} = E_{2,1,2} = 9(h^2/8mL^2)$, and $E''' = E_{1,1,3} = E_{1,3,1} = E_{3,1,1} = 11(h^2/8mL^2)$). Using Eq. 40-21, the level above those can only hold two electrons: $E_{2,2,2} = (2^2 + 2^2 + 2^2)(h^2/8mL^2) = 12(h^2/8mL^2)$. And the next higher level can hold as much as twelve electrons (see part (e) of problem 28 in Chapter 40) and has energy $E'''' = 14(h^2/8mL^2)$.

(a) The configuration which provides the lowest system energy higher than that of the ground state has the first level filled, the second one with one vacancy, and the third one with one occupant:

$$E_{\text{first excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 9$$

which means (putting the "unit" factor back in) the energy of the first excited state is $E_{\text{first excited}} = 45(h^2/8mL^2)$.

(b) The configuration which provides the next higher system energy has the first level filled, the second one with one vacancy, the third one empty, and the fourth one with one occupant:

$$E_{\text{second excited}} = 2E_{1,1,1} + 5E' + E'' = 2(3) + 5(6) + 11$$

which means (putting the "unit" factor back in) the energy of the second excited state is $E_{\text{second excited}} = 47(h^2/8mL^2)$.

(c) Now, there are a couple of configurations which provides the *next* higher system energy. One has the first level filled, the second one with one vacancy, the third and fourth ones empty, and the fifth one with one occupant:

$$E_{\text{third excited}} = 2E_{1,1,1} + 5E' + E''' = 2(3) + 5(6) + 12$$

which means (putting the "unit" factor back in) the energy of the third excited state is $E_{\text{third excited}} = 48(h^2/8mL^2)$. The other configuration with this same total energy has the first level filled, the second one with two vacancies, and the third one with one occupant.

(d) The energy states of this problem and problem 27 are suggested in the sketch below:

third excited $48(h^2/8mL^2)$

second excited $47(h^2/8mL^2)$

first excited $45(h^2/8mL^2)$

ground state $42(h^2/8mL^2)$

29. For a given shell with quantum number n the total number of available electron states is $2n^2$. Thus, for the first four shells ($n = 1$ through 4) the number of available states are 2, 8, 18, and 32 (see Appendix G). Since $2 + 8 + 18 + 32 = 60 < 63$, according to the "logical" sequence the first four shells would be completely filled in an europium atom, leaving $63 - 60 = 3$ electrons to partially occupy the $n = 5$ shell. Two of these three electrons would fill up the $5s$ subshell, leaving only one remaining electron in the only partially filled subshell (the $5p$ subshell). In chemical reactions this electron would have the tendency to be transferred to another element, leaving the remaining 62 electrons in chemically stable, completely filled subshells. This situation is very similar to the case of sodium, which also has only one electron in a partially filled shell (the $3s$ shell).

30. The first three shells ($n = 1$ through 3), which can accommodate a total of $2 + 8 + 18 = 28$ electrons, are completely filled. For selenium ($Z = 34$) there are still $34 - 28 = 6$ electrons left. Two of them go to the $4s$ subshell, leaving the remaining four in the highest occupied subshell, the $4p$ subshell. Similarly, for bromine ($Z = 35$) the highest occupied subshell is also the $4p$ subshell, which contains five electrons; and for krypton ($Z = 36$) the highest occupied subshell is also the $4p$ subshell, which now accommodates six electrons.

31. Without the spin degree of freedom the number of available electron states for each shell would be reduced by half. So the values of Z for the noble gas elements would become half of what they are now: $Z = 1, 5, 9, 18, 27$, and 43. Of this set of numbers, the only one which coincides with one of the familiar noble gas atomic numbers ($Z = 2, 10, 18, 36, 54$, and 86) is 18. Thus, argon would be the only one that would remain "noble."

32. When a helium atom is in its ground state, both of its electrons are in the $1s$ state. Thus, for each of the electrons, $n = 1$, $l = 0$, and $m_l = 0$. One of the electrons is spin up ($m_s = +\frac{1}{2}$), while the other is spin down ($m_s = -\frac{1}{2}$).

33. (a) All states with principal quantum number $n = 1$ are filled. The next lowest states have $n = 2$. The orbital quantum number can have the values $\ell = 0$ or 1 and of these, the $\ell = 0$ states have the lowest energy. The magnetic quantum number must be $m_\ell = 0$ since this is the only possibility if $\ell = 0$. The spin quantum number can have either of the values $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. Since there is no external magnetic field, the energies of these two states are the same. Therefore, in the ground state, the quantum numbers of the third electron are either $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = -\frac{1}{2}$ or $n = 2$, $\ell = 0$, $m_\ell = 0$, $m_s = +\frac{1}{2}$.

(b) The next lowest state in energy is an $n = 2$, $\ell = 1$ state. All $n = 3$ states are higher in energy. The magnetic quantum number can be $m_\ell = -1$, 0, or $+1$; the spin quantum number can be $m_s = -\frac{1}{2}$ or $+\frac{1}{2}$. If both external and internal magnetic fields can be neglected, all these states have the same energy.

34. (a) The number of different m_l's is $2l+1=3$, and the number of different m_s's is 2. Thus, the number of combinations is $N=(3\times 2)^2/2=18$.

(b) There are six states disallowed by the exclusion principle, in which both electrons share the quantum numbers

$$(n,l,m_l,m_s)=\left(2,1,1,\frac{1}{2}\right),\left(2,1,1,-\frac{1}{2}\right),\left(2,1,0,\frac{1}{2}\right),\left(2,1,0,-\frac{1}{2}\right),\left(2,1,-1,\frac{1}{2}\right),\left(2,1,-1,-\frac{1}{2}\right).$$

35. For a given value of the principal quantum number n, there are n possible values of the orbital quantum number ℓ, ranging from 0 to $n-1$. For any value of ℓ, there are $2\ell+1$ possible values of the magnetic quantum number m_ℓ, ranging from $-\ell$ to $+\ell$. Finally, for each set of values of ℓ and m_ℓ, there are two states, one corresponding to the spin quantum number $m_s=-\frac{1}{2}$ and the other corresponding to $m_s=+\frac{1}{2}$. Hence, the total number of states with principal quantum number n is

$$N=2\sum_0^{n-1}(2\ell+1)\ .$$

Now

$$\sum_0^{n-1}2\ell=2\sum_0^{n-1}\ell=2\frac{n}{2}(n-1)=n(n-1)\,,$$

since there are n terms in the sum and the average term is $(n-1)/2$. Furthermore,

$$\sum_0^{n-1}1=n\ .$$

Thus $N=2\left[n(n-1)+n\right]=2n^2$.

36. The kinetic energy gained by the electron is eV, where V is the accelerating potential difference. A photon with the minimum wavelength (which, because of $E=hc/\lambda$, corresponds to maximum photon energy) is produced when all of the electron's kinetic energy goes to a single photon in an event of the kind depicted in Fig. 41-15. Thus, using the result of problem 3 in Chapter 39,

$$eV=\frac{hc}{\lambda_{\min}}=\frac{1240\,\text{eV}\cdot\text{nm}}{0.10\,\text{nm}}=1.24\times 10^4\ \text{eV}\ .$$

Therefore, the accelerating potential difference is $V=1.24\times 10^4\,\text{V}=12.4\,\text{kV}$.

37. We use $eV=hc/\lambda_{\min}$ (see Eq. 41-23 and Eq. 39-4):

$$h=\frac{eV\lambda_{\min}}{c}=\frac{(1.60\times 10^{-19}\,\text{C})(40.0\times 10^3\,\text{eV})(31.1\times 10^{-12}\,\text{m})}{2.998\times 10^8\,\text{m/s}}=6.63\times 10^{-34}\ \text{J}\cdot\text{s}\ .$$

38. Letting $eV=hc/\lambda_{\min}$ (see Eq. 41-23 and Eq. 39-4), we get

$$\lambda_{\min}=\frac{hc}{eV}=\frac{1240\,\text{nm}\cdot\text{eV}}{eV}=\frac{1240\,\text{pm}\cdot\text{keV}}{eV}=\frac{1240\,\text{pm}}{V}$$

where V is measured in kV.

39. The initial kinetic energy of the electron is 50.0 keV. After the first collision, the kinetic energy is 25 keV; after the second, it is 12.5 keV; and after the third, it is zero. The energy of the photon produced in the first collision is 50.0 keV − 25.0 keV = 25.0 keV. The wavelength associated with this photon is

$$\lambda=\frac{1240\,\text{eV}\cdot\text{nm}}{25.0\times 10^3\,\text{eV}}=4.96\times 10^{-2}\,\text{nm}=49.6\,\text{pm}$$

where the result of Exercise 3 of Chapter 39 is used. The energies of the photons produced in the second and third collisions are each 12.5 keV and their wavelengths are

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{12.5\times10^3\,\text{eV}} = 9.92\times10^{-2}\,\text{nm} = 99.2\,\text{pm}\ .$$

40. (a) and (b) Let the wavelength of the two photons be λ_1 and $\lambda_2 = \lambda_1 + \Delta\lambda$. Then,

$$eV = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_1 + \Delta\lambda}\ ,$$

or

$$\lambda_1 = \frac{-(\Delta\lambda/\lambda_0 - 2) \pm \sqrt{(\Delta\lambda/\lambda_0)^2 + 4}}{2/\Delta\lambda}\ .$$

Here, $\Delta\lambda = 130\,\text{pm}$ and $\lambda_0 = hc/eV = 1240\,\text{keV}\cdot\text{pm}/20\,\text{keV} = 62\,\text{pm}$. The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). We choose the plus sign in the expression for λ_1 (since $\lambda_1 > 0$) and obtain

$$\lambda_1 = \frac{-(130\,\text{pm}/62\,\text{pm} - 2) + \sqrt{(130\,\text{pm}/62\,\text{pm})^2 + 4}}{2/62\,\text{pm}} = 87\ \text{pm}\ ,$$

and

$$\lambda_2 = \lambda_1 + \Delta\lambda = 87\,\text{pm} + 130\,\text{pm} = 2.2\times10^2\ \text{pm}\ .$$

The energy of the electron after its first deceleration is

$$K = K_i - \frac{hc}{\lambda_1} = 20\,\text{keV} - \frac{1240\,\text{keV}\cdot\text{pm}}{87\,\text{pm}} = 5.7\ \text{keV}.$$

The energies of the two photons are

$$E_1 = \frac{hc}{\lambda_1} = \frac{1240\,\text{keV}\cdot\text{pm}}{87\,\text{pm}} = 14\ \text{keV}$$

and

$$E_2 = \frac{hc}{\lambda_2} = \frac{1240\,\text{keV}\cdot\text{pm}}{130\,\text{pm}} = 5.7\ \text{keV}\ .$$

41. Suppose an electron with total energy E and momentum $\mathbf{p}$ spontaneously changes into a photon. If energy is conserved, the energy of the photon is E and its momentum has magnitude E/c. Now the energy and momentum of the electron are related by $E^2 = (pc)^2 + (mc^2)^2$, so $pc = \sqrt{E^2 - (mc^2)^2}$. Since the electron has non-zero mass, E/c and p cannot have the same value. Hence, momentum cannot be conserved. A third particle must participate in the interaction, primarily to conserve momentum. It does, however, carry off some energy.

42. (a) We use $eV = hc/\lambda_{\text{min}}$ (see Eq. 41-23 and Eq. 39-4). The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$).

$$\lambda_{\text{min}} = \frac{hc}{eV} = \frac{1240\,\text{keV}\cdot\text{pm}}{50.0\,\text{keV}} = 24.8\ \text{pm}\ .$$

(b) and (c) The values of λ for the K_α and K_β lines do not depend on the external potential and are therefore unchanged.

43. (a) The cut-off wavelength λ_{min} is characteristic of the incident electrons, not of the target material. This wavelength is the wavelength of a photon with energy equal to the kinetic energy of an incident electron. According to the result of Exercise 3 of Chapter 39,

$$\lambda_{\text{min}} = \frac{1240\,\text{eV}\cdot\text{nm}}{35\times10^3\,\text{eV}} = 3.54\times10^{-2}\,\text{nm} = 35.4\ \text{pm}\ .$$

(b) A K_α photon results when an electron in a target atom jumps from the L-shell to the K-shell. The energy of this photon is $25.51\,\text{keV} - 3.56\,\text{keV} = 21.95\,\text{keV}$ and its wavelength is $\lambda_{K\alpha} = (1240\,\text{eV}\cdot\text{nm})/(21.95\times10^3\,\text{eV}) = 5.65\times10^{-2}\,\text{nm} = 56.5\,\text{pm}$.

(c) A K_β photon results when an electron in a target atom jumps from the M-shell to the K-shell. The energy of this photon is $25.51\,\text{keV} - 0.53\,\text{keV} = 24.98\,\text{keV}$ and its wavelength is $\lambda_{K\beta} = (1240\,\text{eV}\cdot\text{nm})/(24.98\times10^3\,\text{eV}) = 4.96\times10^{-2}\,\text{nm} = 49.6\,\text{pm}$.

44. The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). For the K_α line from iron

$$\Delta E = \frac{hc}{\lambda} = \frac{1240\,\text{keV}\cdot\text{pm}}{193\,\text{pm}} = 6.4\ \text{keV}\ .$$

We remark that for the hydrogen atom the corresponding energy difference is

$$\Delta E_{12} = -(13.6\,\text{eV})\left(\frac{1}{2^2} - \frac{1}{1^1}\right) = 10\ \text{eV}.$$

That this difference is much greater in iron is due to the fact that its atomic nucleus contains 26 protons, exerting a much greater force on the K- and L-shell electrons than that provided by the single proton in hydrogen.

45. Since the frequency of an x-ray emission is proportional to $(Z-1)^2$, where Z is the atomic number of the target atom, the ratio of the wavelength λ_{Nb} for the K_α line of niobium to the wavelength λ_{Ga} for the K_α line of gallium is given by $\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (Z_{\text{Ga}}-1)^2/(Z_{\text{Nb}}-1)^2$, where Z_{Nb} is the atomic number of niobium (41) and Z_{Ga} is the atomic number of gallium (31). Thus $\lambda_{\text{Nb}}/\lambda_{\text{Ga}} = (30)^2/(40)^2 = 9/16$.

46. The result of problem 3 in Chapter 39 is adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). The energy difference $E_L - E_M$ for the x-ray atomic energy levels of molybdenum is

$$\Delta E = E_L - E_M = \frac{hc}{\lambda_L} - \frac{hc}{\lambda_M} = \frac{1240\,\text{keV}\cdot\text{pm}}{63.0\,\text{pm}} - \frac{1240\,\text{keV}\cdot\text{pm}}{71.0\,\text{pm}} = 2.2\ \text{keV}\ .$$

47. From the data given in the problem, we calculate frequencies (using Eq. 39-1), take their square roots, look up the atomic numbers (see Appendix F), and do a least-squares fit to find the slope: the result is 5.02×10^7 with the odd-sounding unit of a square root of a Hertz. We remark that the least squares procedure also returns a value for the y-intercept of this statistically determined "best-fit" line; that result is negative and would appear on a graph like Fig. 41-17 to be at about -0.06 on the vertical axis. Also, we can estimate the slope of the Moseley line shown in Fig. 41-17:

$$\frac{(1.95-0.50)10^9\,\text{Hz}^{1/2}}{40-11} \approx 5.0\times10^7\,\text{Hz}^{1/2}\ .$$

These are in agreement with the discussion in §41-10.

48. (a) From Fig. 41-14 we estimate the wavelengths corresponding to the K_α and K_β lines to be $\lambda_\alpha = 70.0\,\text{pm}$ and $\lambda_\beta = 63.0\,\text{pm}$, respectively. Using the result of problem 3 in Chapter 39, adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$),

$$E_\alpha = \frac{hc}{\lambda_\alpha} = \frac{1240\,\text{keV}\cdot\text{pm}}{70.0\,\text{pm}} = 17.7\ \text{keV}\ ,$$

and $E_\beta = (1240\,\text{keV}\cdot\text{nm})/(63.0\,\text{pm}) = 19.7\,\text{keV}$.

(b) Both Zr and Nb can be used, since $E_\alpha < 18.00\,\text{eV} < E_\beta$ and $E_\alpha < 18.99\,\text{eV} < E_\beta$. According to the hint given in the problem statement, Zr is the better choice.

49. (a) An electron must be removed from the K-shell, so that an electron from a higher energy shell can drop. This requires an energy of $69.5\,\text{keV}$. The accelerating potential must be at least $69.5\,\text{kV}$.

(b) After it is accelerated, the kinetic energy of the bombarding electron is 69.5 keV. The energy of a photon associated with the minimum wavelength is 69.5 keV, so its wavelength is

$$\lambda_{\min} = \frac{1240\,\text{eV}\cdot\text{nm}}{69.5 \times 10^3\,\text{eV}} = 1.78 \times 10^{-2}\,\text{nm} = 17.8\,\text{pm}\ .$$

(c) The energy of a photon associated with the K_α line is $69.5\,\text{keV} - 11.3\,\text{keV} = 58.2\,\text{keV}$ and its wavelength is $\lambda_{\text{K}\alpha} = (1240\,\text{eV}\cdot\text{nm})/(58.2 \times 10^3\,\text{eV}) = 2.13 \times 10^{-2}\,\text{nm} = 21.3\,\text{pm}$. The energy of a photon associated with the K_β line is $69.5\,\text{keV} - 2.30\,\text{keV} = 67.2\,\text{keV}$ and its wavelength is $\lambda_{\text{K}\beta} = (1240\,\text{eV}\cdot\text{nm})/(67.2 \times 10^3\,\text{eV}) = 1.85 \times 10^{-2}\,\text{nm} = 18.5\,\text{pm}$. The result of Exercise 3 of Chapter 39 is used.

50. We use Eq. 37-31, Eq. 40-6, and the result of problem 3 in Chapter 39, adapted to these units ($hc = 1240\,\text{eV}\cdot\text{nm} = 1240\,\text{keV}\cdot\text{pm}$). Letting $2d\sin\theta = m\lambda = mhc/\Delta E$, where $\theta = 74.1°$, we solve for d:

$$d = \frac{mhc}{2\Delta E\sin\theta} = \frac{(1)(1240\,\text{keV}\cdot\text{nm})}{2(8.979\,\text{keV} - 0.951\,\text{keV})(\sin 74.1°)} = 80.3\ \text{pm}\ .$$

51. (a) According to Eq. 41-26, $f \propto (Z-1)^2$, so the ratio of energies is (using Eq. 39-2) $f/f' = [(Z-1)/(Z'-1)]^2$.

(b) We refer to Appendix F. Applying the formula from part (a) to $Z = 92$ and $Z' = 13$, we obtain

$$\frac{E}{E'} = \frac{f}{f'} = \left(\frac{Z-1}{Z'-1}\right)^2 = \left(\frac{92-1}{13-1}\right)^2 = 57.5\ .$$

(c) Applying this to $Z = 92$ and $Z' = 3$, we obtain

$$\frac{E}{E'} = \left(\frac{92-1}{3-1}\right)^2 = 2070\ .$$

52. (a) The transition is from $n = 2$ to $n = 1$, so Eq. 41-26 combined with Eq. 41-24 yields

$$f = \left(\frac{m_e e^4}{8\varepsilon_0^2 h^3}\right)\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2$$

so that the constant in Eq. 41-27 is

$$C = \sqrt{\frac{3m_e e^4}{32\varepsilon_0^2 h^3}} = 4.9673 \times 10^7\ \text{Hz}^{1/2}$$

using the values in the next-to-last column in the Table in Appendix B (but note that the power of ten is given in the middle column).

(b) We are asked to compare the results of Eq. 41-27 (squared, then multiplied by the accurate values of h/e found in Appendix B to convert to x ray energies) with those in the table of K_α energies (in eV) given at the end of the problem. We look up the corresponding atomic numbers in Appendix F. An example is shown below (for Nitrogen):

$$E_{\text{theory}} = \frac{h}{e}C^2(Z-1)^2 = \frac{6.6260688 \times 10^{-34}\,\text{J}\cdot\text{s}}{1.6021765 \times 10^{-19}\,\text{J/eV}}\left(4.9673 \times 10^7\ \text{Hz}^{1/2}\right)^2(7-1)^2 = 367.35\ \text{eV}$$

which is 6.4% lower than the experimental value of 392.4 eV. Progressing through the list, from Lithium to Magnesium , we find all the theoretical values are lower than the experimental ones by these percentages: 24.8%, 15.4%, 10.9%, 7.9%, 6.4%, 4.7%, 3.5%, 2.6%, 2.0%, and 1.5%.

(c) The trend is clear from the list given above: the agreement between theory and experiment becomes better as Z increases. One might argue that the most questionable step in §41-10 is the replacement $e^4 \to (Z-1)^2 e^4$ and ask why this could not equally well be $e^4 \to (Z-.9)^2 e^4$ or $e^4 \to (Z-.8)^2 e^4$? For large Z, these subtleties would not matter so much as they do for small Z, since $Z - \xi \approx Z$ for $Z \gg \xi$.

53. (a) The length of the pulse's wave train is given by $L = c\Delta t = (2.998 \times 10^8\,\text{m/s})(10 \times 10^{-15}\,\text{s}) = 3.0 \times 10^{-6}\,\text{m}$. Thus, the number of wavelengths contained in the pulse is

$$N = \frac{L}{\lambda} = \frac{3.0 \times 10^{-6}\,\text{m}}{500 \times 10^{-9}\,\text{m}} = 6.0 \; .$$

(b) We solve for X from $10\,\text{fm}/1\,\text{m} = 1\,\text{s}/X$:

$$X = \frac{(1\,\text{s})(1\,\text{m})}{10 \times 10^{-15}\,\text{m}} = \frac{1\,\text{s}}{(10 \times 10^{-15})(3.15 \times 10^7\,\text{s/y})} = 3.2 \times 10^6\text{y} \; .$$

54. According to Sample Problem 41-6, $N_x/N_0 = 1.3 \times 10^{-38}$. Let the number of moles of the lasing material needed be n; then $N_0 = nN_A$, where N_A is the Avogadro constant. Also $N_x = 10$. We solve for n:

$$n = \frac{N_x}{(1.3 \times 10^{-38})\,N_A} = \frac{10}{(1.3 \times 10^{-38})(6.02 \times 10^{23})} = 1.3 \times 10^{15} \text{ mol} \; .$$

55. The number of atoms in a state with energy E is proportional to $e^{-E/kT}$, where T is the temperature on the Kelvin scale and k is the Boltzmann constant. Thus the ratio of the number of atoms in the thirteenth excited state to the number in the eleventh excited state is

$$\frac{n_{13}}{n_{11}} = e^{-\Delta E/kT} \, ,$$

where ΔE is the difference in the energies: $\Delta E = E_{13} - E_{11} = 2(1.2\,\text{eV}) = 2.4\,\text{eV}$. For the given temperature, $kT = (8.62 \times 10^{-2}\,\text{eV/K})(2000\,\text{K}) = 0.1724\,\text{eV}$. Hence,

$$\frac{n_{13}}{n_{11}} = e^{-2.4/0.1724} = 9.0 \times 10^{-7} \; .$$

56. (a) The distance from the Earth to the Moon is $d_{em} = 3.82 \times 10^8\,\text{m}$ (see Appendix C). Thus, the time required is given by

$$t = \frac{2d_{em}}{c} = \frac{2(3.82 \times 10^8\,\text{m})}{2.998 \times 10^8\,\text{m/s}} = 2.55 \text{ s} \; .$$

(b) We denote the uncertainty in time measurement as δt and let $2\delta d_{es} = 15\,\text{cm}$. Then, since $d_{em} \propto t$, $\delta t/t = \delta d_{em}/d_{em}$. We solve for δt:

$$\delta t = \frac{t\delta d_{em}}{d_{em}} = \frac{(2.55\,\text{s})(0.15\,\text{m})}{2(3.82 \times 10^8\,\text{m})} = 5.0 \times 10^{-10} \text{ s} \; .$$

57. From Eq. 41-29, $N_2/N_1 = e^{-(E_2-E_1)/kT}$. We solve for T:

$$T = \frac{E_2 - E_1}{k\ln(N_1/N_2)} = \frac{3.2\,\text{eV}}{(1.38 \times 10^{-23}\,\text{J/K})\ln(2.5 \times 10^{15}/6.1 \times 10^{13})} = 10000 \text{ K} \; .$$

58. Consider two levels, labeled 1 and 2, with $E_2 > E_1$. Since $T = -|T| < 0$,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-|E_2-E_1|/(-k|T|)} = e^{|E_2-E_1|/k|T|} > 1 \; .$$

Thus, $N_2 > N_1$; this is population inversion. We solve for T:

$$T = -|T| = -\frac{E_2 - E_1}{k\ln(N_2/N_1)} = -\frac{2.26\,\text{eV}}{(8.62 \times 10^{-5}\,\text{eV/K})\ln(1 + 0.100)} = -2.75 \times 10^5 \text{ K} \; .$$

59. (a) If t is the time interval over which the pulse is emitted, the length of the pulse is $L = ct = (3.00 \times 10^8\,\text{m/s})(1.20 \times 10^{-11}\,\text{s}) = 3.60 \times 10^{-3}\,\text{m}$.

(b) If E_p is the energy of the pulse, E is the energy of a single photon in the pulse, and N is the number of photons in the pulse, then $E_p = NE$. The energy of the pulse is $E_p = (0.150\,\text{J})/(1.602 \times 10^{-19}\,\text{J/eV}) = 9.36 \times 10^{17}\,\text{eV}$ and the energy of a single photon is $E = (1240\,\text{eV}\cdot\text{nm})/(694.4\,\text{nm}) = 1.786\,\text{eV}$. Hence,

$$N = \frac{E_p}{E} = \frac{9.36 \times 10^{17}\,\text{eV}}{1.786\,\text{eV}} = 5.24 \times 10^{17}\,\text{photons}\ .$$

60. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$\begin{aligned} R &= \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \\ &= \frac{(2.3 \times 10^{-3}\,\text{W})(632.8 \times 10^{-9}\,\text{m})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^8\,\text{m/s})} \\ &= 7.3 \times 10^{15}\ \text{s}^{-1}\ . \end{aligned}$$

61. The Moon is a distance $R = 3.82 \times 10^8\,\text{m}$ from Earth (see Appendix C). We note that the "cone" of light has apex angle equal to 2θ. If we make the small angle approximation (equivalent to using Eq. 37-14), then the diameter D of the spot on the Moon is

$$\begin{aligned} D &= 2R\theta = 2R\left(\frac{1.22\lambda}{d}\right) \\ &= \frac{2(3.82 \times 10^8\,\text{m})(1.22)(600 \times 10^{-9}\,\text{m})}{0.12\,\text{m}} \\ &= 4.7 \times 10^3\,\text{m} = 4.7\ \text{km}\ . \end{aligned}$$

62. Let the range of frequency of the microwave be Δf. Then the number of channels that could be accommodated is

$$N = \frac{\Delta f}{10\,\text{MHz}} = \frac{(2.998 \times 10^8\,\text{m/s})[(450\,\text{nm})^{-1} - (650\,\text{nm})^{-1}]}{10\,\text{MHz}} = 2.1 \times 10^7\ .$$

The higher frequencies of visible light would allow many more channels to be carried compared with using the microwave.

63. Let the power of the laser beam be P and the energy of each photon emitted be E. Then, the rate of photon emission is

$$\begin{aligned} R &= \frac{P}{E} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \\ &= \frac{(5.0 \times 10^{-3}\,\text{W})(0.80 \times 10^{-6}\,\text{m})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^8\,\text{m/s})} \\ &= 2.0 \times 10^{16}\ \text{s}^{-1}\ . \end{aligned}$$

64. For the nth harmonic of the standing wave of wavelength λ in the cavity of width L we have $n\lambda = 2L$, so $n\Delta\lambda + \lambda\Delta n = 0$. Let $\Delta n = \pm 1$ and use $\lambda = 2L/n$ to obtain

$$|\Delta\lambda| = \frac{\lambda|\Delta n|}{n} = \frac{\lambda}{n} = \lambda\left(\frac{\lambda}{2L}\right) = \frac{(533\,\text{nm})^2}{2(8.0 \times 10^7\,\text{nm})} = 1.8 \times 10^{-12}\,\text{m} = 1.8\ \text{pm}\ .$$

65. (a) If both mirrors are perfectly reflecting, there is a node at each end of the crystal. With one end partially silvered, there is a node very close to that end. We assume nodes at both ends, so there are an integer number of half-wavelengths in the length of the crystal. The wavelength in the crystal is $\lambda_c = \lambda/n$, where λ is the wavelength in a vacuum and n is the index of refraction of ruby. Thus $N(\lambda/2n) = L$, where N is the number of standing wave nodes, so

$$N = \frac{2nL}{\lambda} = \frac{2(1.75)(0.0600\,\text{m})}{694 \times 10^{-9}\,\text{m}} = 3.03 \times 10^5 \ .$$

(b) Since $\lambda = c/f$, where f is the frequency, $N = 2nLf/c$ and $\Delta N = (2nL/c)\,\Delta f$. Hence,

$$\Delta f = \frac{c\,\Delta N}{2nL} = \frac{(2.998 \times 10^8\,\text{m/s})(1)}{2(1.75)(0.0600\,\text{m})} = 1.43 \times 10^9\,\text{Hz} \ .$$

(c) The speed of light in the crystal is c/n and the round-trip distance is $2L$, so the round-trip travel time is $2nL/c$. This is the same as the reciprocal of the change in frequency.

(d) The frequency is $f = c/\lambda = (2.998 \times 10^8\,\text{m/s})/(694 \times 10^{-9}\,\text{m}) = 4.32 \times 10^{14}\,\text{Hz}$ and the fractional change in the frequency is $\Delta f/f = (1.43 \times 10^9\,\text{Hz})/(4.32 \times 10^{14}\,\text{Hz}) = 3.31 \times 10^{-6}$.

66. (a) We denote the upper level as level 1 and the lower one as level 2. From $N_1/N_2 = e^{-(E_1-E_2)/kT}$ we get (using the result of problem 3 in Chapter 39)

$$\begin{aligned} N_1 &= N_2 e^{-(E_1-E_2)/kT} = N_2 e^{-hc/\lambda kT} \\ &= (4.0 \times 10^{20})e^{-(1240\,\text{eV}\cdot\text{nm})/[(580\,\text{nm})(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})]} \\ &= 5.0 \times 10^{-16} \ll 1 \ , \end{aligned}$$

so practically no electron occupies the upper level.

(b) With $N_1 = 3.0 \times 10^{20}$ atoms emitting photons and $N_2 = 1.0 \times 10^{20}$ atoms absorbing photons, then the net energy output is

$$\begin{aligned} E &= (N_1 - N_2)\,E_{\text{photon}} = (N_1 - N_2)\,\frac{hc}{\lambda} \\ &= \left(2.0 \times 10^{20}\right) \frac{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^8\,\text{m/s})}{580 \times 10^{-9}\,\text{m}} \\ &= 68\ \text{J} \ . \end{aligned}$$

67. (a) The intensity at the target is given by $I = P/A$, where P is the power output of the source and A is the area of the beam at the target. We want to compute I and compare the result with $10^8\,\text{W/m}^2$. The beam spreads because diffraction occurs at the aperture of the laser. Consider the part of the beam that is within the central diffraction maximum. The angular position of the edge is given by $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture (see Exercise 61). At the target, a distance D away, the radius of the beam is $r = D\tan\theta$. Since θ is small, we may approximate both $\sin\theta$ and $\tan\theta$ by θ, in radians. Then, $r = D\theta = 1.22D\lambda/d$ and

$$\begin{aligned} I &= \frac{P}{\pi r^2} = \frac{Pd^2}{\pi(1.22D\lambda)^2} \\ &= \frac{(5.0 \times 10^6\,\text{W})(4.0\,\text{m})^2}{\pi\left[1.22(3000 \times 10^3\,\text{m})(3.0 \times 10^{-6}\,\text{m})\right]^2} \\ &= 2.1 \times 10^5\ \text{W/m}^2 \ , \end{aligned}$$

not great enough to destroy the missile.

(b) We solve for the wavelength in terms of the intensity and substitute $I = 1.0 \times 10^8\,\mathrm{W/m^2}$:

$$\begin{aligned}\lambda &= \frac{d}{1.22D}\sqrt{\frac{P}{\pi I}} = \frac{4.0\,\mathrm{m}}{1.22(3000 \times 10^3\,\mathrm{m})}\sqrt{\frac{5.0 \times 10^6\,\mathrm{W}}{\pi(1.0 \times 10^8\,\mathrm{W/m^2})}} \\ &= 1.4 \times 10^{-7}\,\mathrm{m} = 140\ \mathrm{nm}\ .\end{aligned}$$

68. (a) The radius of the central disk is

$$R = \frac{1.22 f\lambda}{d} = \frac{(1.22)(3.50\,\mathrm{cm})(515\,\mathrm{nm})}{3.00\,\mathrm{mm}} = 7.33\,\mu\mathrm{m}\ .$$

(b) The average power flux density in the incident beam is

$$\frac{P}{\pi d^2/4} = \frac{4(5.00\,\mathrm{W})}{\pi(3.00\,\mathrm{mm})^2} = 707\ \mathrm{kW/m^2}\ .$$

(c) The average power flux density in the central disk is

$$\frac{(0.84)P}{\pi R^2} = \frac{(0.84)(5.00\,\mathrm{W})}{\pi(7.33\,\mu\mathrm{m})^2} = 24.9\ \mathrm{GW/m^2}\ .$$

69. (a) In the lasing action the molecules are excited from energy level E_0 to energy level E_2. Thus the wavelength λ of the sunlight that causes this excitation satisfies

$$\Delta E = E_2 - E_0 = \frac{hc}{\lambda}\ ,$$

which gives (using the result of problem 3 in Chapter 39)

$$\lambda = \frac{hc}{E_2 - E_0} = \frac{1240\,\mathrm{eV{\cdot}nm}}{0.289\,\mathrm{eV} - 0} = 4.29 \times 10^3\,\mathrm{nm} = 4.29\,\mu\mathrm{m}\ .$$

(b) Lasing occurs as electrons jump down from the higher energy level E_2 to the lower level E_1. Thus the lasing wavelength λ' satisfies

$$\Delta E' = E_2 - E_1 = \frac{hc}{\lambda'}\ ,$$

which gives

$$\lambda' = \frac{hc}{E_2 - E_1} = \frac{1240\,\mathrm{eV{\cdot}nm}}{0.289\,\mathrm{eV} - 0.165\,\mathrm{eV}} = 1.00 \times 10^4\,\mathrm{nm} = 10.0\,\mu\mathrm{m}\ .$$

(c) Both λ and λ' belong to the infrared region of the electromagnetic spectrum.

70. (a) The energy difference between the two states 1 and 2 was equal to the energy of the photon emitted. Since the photon frequency was $f = 1666\,\mathrm{MHz}$, its energy was given by $hf = (4.14 \times 10^{-15}\,\mathrm{eV\cdot s})(1666\,\mathrm{MHz}) = 6.90 \times 10^{-6}\,\mathrm{eV}$. Thus,

$$E_2 - E_1 = hf = 6.9 \times 10^{-6}\,\mathrm{eV} = 6.9\,\mu\mathrm{eV}\ .$$

(b) The emission was in the *radio* region of the electromagnetic spectrum.

Chapter 42

1. The number of atoms per unit volume is given by $n = d/M$, where d is the mass density of copper and M is the mass of a single copper atom. Since each atom contributes one conduction electron, n is also the number of conduction electrons per unit volume. Since the molar mass of copper is $A = 63.54\,\text{g/mol}$, $M = A/N_A = (63.54\,\text{g/mol})/(6.022 \times 10^{23}\,\text{mol}^{-1}) = 1.055 \times 10^{-22}\,\text{g}$. Thus,

$$n = \frac{8.96\,\text{g/cm}^3}{1.055 \times 10^{-22}\,\text{g}} = 8.49 \times 10^{22}\,\text{cm}^{-3} = 8.49 \times 10^{28}\ \text{m}^{-3}\ .$$

2. We compute $\left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \approx 0.121$.

3. We use the ideal gas law in the form of Eq. 20-9:

$$p = nkT = (8.43 \times 10^{28}\,\text{m}^{-3})(1.38 \times 10^{-23}\,\text{J/K})(300\,\text{K}) = 3.49 \times 10^{8}\,\text{Pa} = 3490\ \text{atm}\ .$$

4. We note that $n = 8.43 \times 10^{28}\,\text{m}^{-3} = 84.3\,\text{nm}^{-3}$. From Eq. 42-9,

$$E_\text{F} = \frac{0.121(hc)^2}{m_e c^2} n^{2/3} = \frac{0.121(1240\,\text{eV}\cdot\text{nm})^2}{511 \times 10^3\,\text{eV}} (84.3\,\text{nm}^{-3})^{2/3} = 7.0\ \text{eV}$$

where the result of problem 3 in Chapter 39 is used.

5. (a) For copper, Eq. 42-10 leads to

$$\frac{d\rho}{dT} = [\rho\alpha]_\text{Cu} = (2 \times 10^{-8}\,\Omega\cdot\text{m})(4 \times 10^{-3}\,\text{K}^{-1}) = 8 \times 10^{-11}\ \Omega\cdot\text{m/K}\ .$$

(b) For silicon,

$$\frac{d\rho}{dT} = [\rho\alpha]_\text{Si} = (3 \times 10^{3}\,\Omega\cdot\text{m})(-70 \times 10^{-3}\,\text{K}^{-1}) = -2.1 \times 10^{2}\ \Omega\cdot\text{m/K}\ .$$

6. We note that there is one conduction electron per atom and that the molar mass of gold is $197\,\text{g/mol}$. Therefore, combining Eqs. 42-2, 42-3 and 42-4 leads to

$$n = \frac{(19.3\,\text{g/cm}^3)(10^6\,\text{cm}^3/\text{m}^3)}{(197\,\text{g/mol})/(6.02 \times 10^{23}\,\text{mol}^{-1})} = 5.90 \times 10^{28}\ \text{m}^{-3}\ .$$

7. (a) Eq. 42-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$n(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi(9.109 \times 10^{-31}\,\mathrm{kg})^{3/2}}{(6.626 \times 10^{-34}\,\mathrm{J{\cdot}s})^3} = 1.062 \times 10^{56}\,\mathrm{kg}^{3/2}/\mathrm{J}^3 \cdot \mathrm{s}^3 \ .$$

Now, $1\,\mathrm{J} = 1\,\mathrm{kg{\cdot}m^2/s^2}$ (think of the equation for kinetic energy $K = \frac{1}{2}mv^2$), so $1\,\mathrm{kg} = 1\,\mathrm{J{\cdot}s^2{\cdot}m^{-2}}$. Thus, the units of C can be written $(\mathrm{J{\cdot}s^2})^{3/2}{\cdot}(\mathrm{m}^{-2})^{3/2}{\cdot}\mathrm{J}^{-3}{\cdot}\mathrm{s}^{-3} = \mathrm{J}^{-3/2}{\cdot}\mathrm{m}^{-3}$. This means

$$C = (1.062 \times 10^{56}\,\mathrm{J}^{-3/2}{\cdot}\mathrm{m}^{-3})(1.602 \times 10^{-19}\,\mathrm{J/eV})^{3/2} = 6.81 \times 10^{27}\,\mathrm{m}^{-3}{\cdot}\mathrm{eV}^{-3/2} \ .$$

(b) If $E = 5.00\,\mathrm{eV}$, then

$$n(E) = (6.81 \times 10^{27}\,\mathrm{m}^{-3}{\cdot}\mathrm{eV}^{-3/2})(5.00\,\mathrm{eV})^{1/2} = 1.52 \times 10^{28}\,\mathrm{eV}^{-1}{\cdot}\mathrm{m}^{-3} \ .$$

8. We equate E_F with $\frac{1}{2}m_e v_\mathrm{F}^2$ and write our expressions in such a way that we can make use of the electron mc^2 value found in Table 38-3:

$$v_\mathrm{F} = \sqrt{\frac{2E_\mathrm{F}}{m}} = c\sqrt{\frac{2E_\mathrm{F}}{mc^2}} = (3.0 \times 10^5\,\mathrm{km/s})\sqrt{\frac{2(7.0\,\mathrm{eV})}{5.11 \times 10^5\,\mathrm{eV}}} = 1.6 \times 10^3\ \mathrm{km/s} \ .$$

9. (a) At absolute temperature $T = 0$, the probability is zero that any state with energy above the Fermi energy is occupied.

(b) The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy. Now, $E - E_F = 0.062\,\mathrm{eV}$ and $(E - E_F)/kT = (0.062\,\mathrm{eV})/(8.62 \times 10^{-5}\,\mathrm{eV/K})(320\,\mathrm{K}) = 2.248$, so

$$P(E) = \frac{1}{e^{2.248} + 1} = 0.0956 \ .$$

See Appendix B or Sample Problem 42-1 for the value of k.

10. We use the result of problem 7:

$$n(E) = CE^{1/2} = \left[6.81 \times 10^{27}\,\mathrm{m}^{-3}{\cdot}(\mathrm{eV})^{-2/3}\right](8.0\,\mathrm{eV})^{1/2} = 1.9 \times 10^{28}\,\mathrm{m}^{-3}{\cdot}\mathrm{eV}^{-1} \ .$$

This is consistent with Fig.42-5.

11. According to Eq. 42-9, the Fermi energy is given by

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3}$$

where n is the number of conduction electrons per unit volume, m is the mass of an electron, and h is the Planck constant. This can be written $E_F = An^{2/3}$, where

$$A = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{(6.626 \times 10^{-34}\,\mathrm{J{\cdot}s})^2}{9.109 \times 10^{-31}\,\mathrm{kg}} = 5.842 \times 10^{-38}\ \mathrm{J^2{\cdot}s^2/kg} \ .$$

Since $1\,\mathrm{J} = 1\,\mathrm{kg{\cdot}m^2/s^2}$, the units of A can be taken to be $\mathrm{m^2{\cdot}J}$. Dividing by $1.602 \times 10^{-19}\,\mathrm{J/eV}$, we obtain $A = 3.65 \times 10^{-19}\,\mathrm{m^2{\cdot}\,eV}$.

12. We reproduce the calculation of Exercise 6: Combining Eqs. 42-2, 42-3 and 42-4, the number density of conduction electrons in gold is

$$n = \frac{(19.3\,\mathrm{g/cm^3})(6.02\times10^{23}/\mathrm{mol})}{(197\,\mathrm{g/mol})} = 5.90\times10^{22}\,\mathrm{cm^{-3}} = 59.0\ \mathrm{nm^{-3}}\ .$$

Now, using the result of Exercise 3 in Chapter 39, Eq. 42-9 leads to

$$E_\mathrm{F} = \frac{0.121(hc)^2}{(m_ec^2)}n^{2/3} = \frac{0.121(1240\,\mathrm{eV{\cdot}nm})^2}{511\times10^3\,\mathrm{eV}}(59.0\,\mathrm{nm^{-3}})^{2/3} = 5.52\ \mathrm{eV}\ .$$

13. Let $E_1 = 63\,\mathrm{meV} + E_\mathrm{F}$ and $E_2 = -63\,\mathrm{meV} + E_\mathrm{F}$. Then according to Eq. 42-6,

$$P_1 = \frac{1}{e^{(E_1-E_\mathrm{F})/kT}+1} = \frac{1}{e^x+1}$$

where $x = (E_1 - E_\mathrm{F})/kT$. We solve for e^x:

$$e^x = \frac{1}{P_1} - 1 = \frac{1}{0.090} - 1 = \frac{91}{9}\ .$$

Thus,

$$P_2 = \frac{1}{e^{(E_2-E_\mathrm{F})/kT}+1} = \frac{1}{e^{-(E_1-E_\mathrm{F})/kT}+1} = \frac{1}{e^{-x}+1} == \frac{1}{(91/9)^{-1}+1} = 0.91\ ,$$

where we use $E_2 - E_\mathrm{F} = -63\,\mathrm{meV} = E_F - E_1 = -(E_1 - E_\mathrm{F})$.

14. (a) Eq. 42-6 leads to

$$\begin{aligned} E &= E_\mathrm{F} + kT\ \ln(P^{-1}-1) \\ &= 7.0\,\mathrm{eV} + (8.62\times10^{-5}\,\mathrm{eV/K})(1000\,\mathrm{K})\ \ln\left(\frac{1}{0.90}-1\right) \\ &= 6.8\ \mathrm{eV}\ . \end{aligned}$$

(b) $n(E) = CE^{1/2} = (6.81\times10^{27}\,\mathrm{m^{-3}{\cdot}eV^{-3/2}})(6.8\,\mathrm{eV})^{1/2} = 1.77\times10^{28}\ \mathrm{m^{-3}{\cdot}eV^{-1}}$.

(c) $n_0(E) = P(E)n(E) = (0.90)(1.77\times10^{28}\,\mathrm{m^{-3}{\cdot}eV^{-1}}) = 1.6\times10^{28}\,\mathrm{m^{-3}{\cdot}eV^{-1}}$.

15. The Fermi-Dirac occupation probability is given by $P_\mathrm{FD} = 1/\left(e^{\Delta E/kT}+1\right)$, and the Boltzmann occupation probability is given by $P_\mathrm{B} = e^{-\Delta E/kT}$. Let f be the fractional difference. Then

$$f = \frac{P_\mathrm{B} - P_\mathrm{FD}}{P_\mathrm{B}} = \frac{e^{-\Delta E/kT} - \dfrac{1}{e^{\Delta E/kT}+1}}{e^{-\Delta E/kT}}\ .$$

Using a common denominator and a little algebra yields

$$f = \frac{e^{-\Delta E/kT}}{e^{-\Delta E/kT}+1}\ .$$

The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1-f}\ .$$

We take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k\ln\left(\dfrac{f}{1-f}\right)}\ .$$

(a) Letting f equal 0.01, we evaluate the expression for T:

$$T = \frac{(1.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln\left(\frac{0.010}{1-0.010}\right)} = 2.5\times10^{3}\ \text{K}\ .$$

(b) We set f equal to 0.10 and evaluate the expression for T:

$$T = \frac{(1.00\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{(1.38\times10^{-23}\,\text{J/K})\ln\left(\frac{0.10}{1-0.10}\right)} = 5.3\times10^{3}\ \text{K}\ .$$

16. According to Eq. 42-6,

$$P(E_{\text{F}}+\Delta E) = \frac{1}{e^{(E_{\text{F}}+\Delta E-E_{\text{F}})/kT}+1} = \frac{1}{e^{\Delta E/kT}+1} = \frac{1}{e^{x}+1}$$

where $x = \Delta E/kT$. Also,

$$P(E_{\text{F}}-\Delta E) = \frac{1}{e^{(E_{\text{F}}-\Delta E-E_{\text{F}})/kT}+1} = \frac{1}{e^{-\Delta E/kT}+1} = \frac{1}{e^{-x}+1}\ .$$

Thus,

$$P(E_{\text{F}}+\Delta E)+P(E_{\text{F}}-\Delta E) = \frac{1}{e^{x}+1}+\frac{1}{e^{-x}+1} = \frac{e^{x}+1+e^{-x}+1}{(e^{-x}+1)(e^{x}+1)} = 1\ .$$

A special case of this general result can be found in problem 13, where $\Delta E = 63\,\text{meV}$ and $P(E_{\text{F}} + 63\,\text{meV}) + P(E_{\text{F}} - 63\,\text{meV}) = 0.090 + 0.91 = 1.0$.

17. (a) The volume per cubic meter of sodium occupied by the sodium ions is

$$V_{\text{Na}} = \frac{(971\,\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(98\times10^{-12}\,\text{m})^3}{(23\,\text{g/mol})} = 0.100\ \text{m}^3\ ,$$

so the fraction available for conduction electrons is $1-(V_{\text{Na}}/1.00\,\text{m}^3) = 1-0.100 = 0.900$.

(b) For copper,

$$V_{\text{Cu}} = \frac{(8960\,\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(135\times10^{-12}\,\text{m})^3}{63.5\,\text{g/mol}} = 0.876\ \text{m}^{-3}\ .$$

Thus, the fraction is $1-(V_{\text{Cu}}/1.00\,\text{m}^3) = 1-0.876 = 0.124$.

(c) Sodium, because the electrons occupy a greater portion of the space available.

18. We use $N_0 = N(E)P(E) = CE^{1/2}\left[e^{(E-E_{\text{F}})/kT}+1\right]^{-1}$, where C is given in problem 7(a). At $E = 4.00\,\text{eV}$,

$$\begin{aligned} n_0 &= \frac{\left(6.8\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}\right)(4.00\,\text{eV})^{1/2}}{e^{(4.00\,\text{eV}-7.00\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})]}+1} \\ &= 1.36\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}\ , \end{aligned}$$

and at $E = 6.75\,\text{eV}$,

$$\begin{aligned} n_0 &= \frac{\left(6.8\times10^{27}\,\text{m}^{-3}\cdot(\text{eV})^{-3/2}\right)(6.75\,\text{eV})^{1/2}}{e^{(6.75\,\text{eV}-7.00\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(1000\,\text{K})]}+1} \\ &= 1.67\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}\ . \end{aligned}$$

Similarly at $E = 7.00$, 7.25 and 9.00 eV, the values of $n_0(E)$ are $9.0\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-1}$, $9.5\times10^{26}\,\text{m}^{-3}\cdot\text{eV}^{-1}$ and $1.7\times10^{18}\,\text{m}^{-3}\cdot\text{eV}^{-1}$, respectively. We note that the latter value is effectively zero (relative to the other results).

19. (a) The ideal gas law in the form of Eq. 20-9 leads to $p = NkT/V = nkT$. Thus, we solve for the molecules per cubic meter:

$$n = \frac{p}{kT} = \frac{(1.0\,\text{atm})(1.0 \times 10^5\,\text{Pa/atm})}{(1.38 \times 10^{-23}\,\text{J/K})(273\,\text{K})} = 2.7 \times 10^{25}\ \text{m}^{-3} \ .$$

(b) Combining Eqs. 42-2, 42-3 and 42-4 leads to the conduction electrons per cubic meter in copper:

$$n = \frac{8.96 \times 10^3\,\text{kg/m}^3}{(63.54)(1.67 \times 10^{-27}\,\text{kg})} = 8.43 \times 10^{28}\ \text{m}^{-3} \ .$$

(c) The ratio is $(8.43 \times 10^{28}\,\text{m}^{-3})/(2.7 \times 10^{25}\,\text{m}^{-3}) = 3.1 \times 10^3$.

(d) We use $d_{\text{avg}} = n^{-1/3}$. For case (a), $d_{\text{avg}} = (2.7 \times 10^{25}\,\text{m}^{-3})^{-1/3}$ which equals 3.3 nm. For case (b), $d_{\text{avg}} = (8.43 \times 10^{28}\,\text{m}^{-3})^{-1/3} = 0.23\,\text{nm}$.

20. The molar mass of carbon is $m = 12.01115\,\text{g/mol}$ and the mass of the Earth is $M_e = 5.98 \times 10^{24}\,\text{kg}$. Thus, the number of carbon atoms in a diamond as massive as the Earth is $N = (M_e/m)N_{\text{A}}$, where N_{A} is the Avogadro constant. From the result of Sample Problem 42-1, the probability in question is given by

$$\begin{aligned} P &= Ne^{-E_g/kT} = \left(\frac{M_e}{m}\right) N_{\text{A}} e^{-E_g/kT} \\ &= \left(\frac{5.98 \times 10^{24}\,\text{kg}}{12.01115\,\text{g/mol}}\right)(6.02 \times 10^{23}/\text{mol})(3 \times 10^{-93}) = 9 \times 10^{-43} \ . \end{aligned}$$

21. (a) We evaluate $P(E) = 1/\left(e^{(E-E_F)/kT} + 1\right)$ for the given value of E, using

$$kT = \frac{(1.381 \times 10^{-23}\,\text{J/K})(273\,\text{K})}{1.602 \times 10^{-19}\,\text{J/eV}} = 0.02353\ \text{eV} \ .$$

For $E = 4.4\,\text{eV}$, $(E - E_F)/kT = (4.4\,\text{eV} - 5.5\,\text{eV})/(0.02353\,\text{eV}) = -46.25$ and

$$P(E) = \frac{1}{e^{-46.25} + 1} = 1.00 \ .$$

Similarly, for $E = 5.4\,\text{eV}$, $P(E) = 0.986$, for $E = 5.5\,\text{eV}$, $P(E) = 0.500$, for $E = 5.6\,\text{eV}$, $P(E) = 0.0141$, and for $E = 6.4\,\text{eV}$, $P(E) = 2.57 \times 10^{-17}$.

(b) Solving $P = 1/\left(e^{\Delta E/kT} + 1\right)$ for $e^{\Delta E/kT}$, we get

$$e^{\Delta E/kT} = \frac{1}{P} - 1 \ .$$

Now, we take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k \ln\left(\frac{1}{P} - 1\right)} = \frac{(5.6\,\text{eV} - 5.5\,\text{eV})(1.602 \times 10^{-19}\,\text{J/eV})}{(1.381 \times 10^{-23}\,\text{J/K}) \ln\left(\frac{1}{0.16} - 1\right)} = 699\ \text{K} \ .$$

22. The probability P_h that a state is occupied by a hole is the same as the probability the state is *unoccupied* by an electron. Since the total probability that a state is either occupied or unoccupied is 1, we have $P_h + P = 1$. Thus,

$$P_h = 1 - \frac{1}{e^{(E-E_{\text{F}})/kT} + 1} = \frac{e^{(E-E_{\text{F}})/kT}}{1 + e^{(E-E_{\text{F}})/kT}} = \frac{1}{e^{-(E-E_{\text{F}})/kT} + 1} \ .$$

23. Let N be the number of atoms per unit volume and n be the number of free electrons per unit volume. Then, the number of free electrons per atom is n/N. We use the result of Exercise 11 to find n: $E_F = An^{2/3}$, where $A = 3.65 \times 10^{-19}\,\mathrm{m}^2 \cdot \mathrm{eV}$. Thus,

$$n = \left(\frac{E_F}{A}\right)^{3/2} = \left(\frac{11.6\,\mathrm{eV}}{3.65 \times 10^{-19}\,\mathrm{m}^2 \cdot \mathrm{eV}}\right)^{3/2} = 1.79 \times 10^{29}\,\mathrm{m}^{-3} \ .$$

If M is the mass of a single aluminum atom and d is the mass density of aluminum, then $N = d/M$. Now, $M = (27.0\,\mathrm{g/mol})/(6.022 \times 10^{23}\,\mathrm{mol}^{-1}) = 4.48 \times 10^{-23}\,\mathrm{g}$, so $N = (2.70\,\mathrm{g/cm}^3)/(4.48 \times 10^{-23}\,\mathrm{g}) = 6.03 \times 10^{22}\,\mathrm{cm}^{-3} = 6.03 \times 10^{28}\,\mathrm{m}^{-3}$. Thus, the number of free electrons per atom is

$$\frac{n}{N} = \frac{1.79 \times 10^{29}\,\mathrm{m}^{-3}}{6.03 \times 10^{28}\,\mathrm{m}^{-3}} = 2.97 \ .$$

24. Let the energy of the state in question be an amount ΔE above the Fermi energy E_F. Then, Eq. 42-6 gives the occupancy probability of the state as

$$P = \frac{1}{e^{(E_\mathrm{F}+\Delta E - E_\mathrm{F})/kT} + 1} = \frac{1}{e^{\Delta E/kT} + 1} \ .$$

We solve for ΔE to obtain

$$\Delta E = kT \,\ln\left(\frac{1}{P} - 1\right) = (1.38 \times 10^{23}\,\mathrm{J/K})(300\,\mathrm{K})\,\ln\left(\frac{1}{0.10} - 1\right) = 9.1 \times 10^{-21}\ \mathrm{J} \ ,$$

which is equivalent to $5.7 \times 10^{-2}\,\mathrm{eV} = 57\,\mathrm{meV}$.

25. (a) According to Appendix F the molar mass of silver is $107.870\,\mathrm{g/mol}$ and the density is $10.49\,\mathrm{g/cm}^3$. The mass of a silver atom is

$$\frac{107.870 \times 10^{-3}\,\mathrm{kg/mol}}{6.022 \times 10^{23}\,\mathrm{mol}^{-1}} = 1.791 \times 10^{-25}\,\mathrm{kg} \ .$$

We note that silver is monovalent, so there is one valence electron per atom (see Eq. 42-2). Thus, Eqs. 42-4 and 42-3 lead to

$$n = \frac{\rho}{M} = \frac{10.49 \times 10^3\,\mathrm{kg/m}^3}{1.791 \times 10^{25}\,\mathrm{kg}} = 5.86 \times 10^{28}\ \mathrm{m}^{-3} \ .$$

(b) The Fermi energy is

$$\begin{aligned} E_F &= \frac{0.121h^2}{m} n^{2/3} = \frac{(0.121)(6.626 \times 10^{-34}\,\mathrm{J{\cdot}s})^2}{9.109 \times 10^{-31}\,\mathrm{kg}} (5.86 \times 10^{28}\,\mathrm{m}^{-3})^{2/3} \\ &= 8.80 \times 10^{-19}\,\mathrm{J} = 5.49\ \mathrm{eV} \ . \end{aligned}$$

(c) Since $E_F = \frac{1}{2}mv_F^2$,

$$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2(8.80 \times 10^{-19}\,\mathrm{J})}{9.109 \times 10^{-31}\,\mathrm{kg}}} = 1.39 \times 10^6\ \mathrm{m/s} \ .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{mv_F} = \frac{6.626 \times 10^{-34}\,\mathrm{J{\cdot}s}}{(9.109 \times 10^{-31}\,\mathrm{kg})(1.39 \times 10^6\,\mathrm{m/s})} = 5.23 \times 10^{-10}\ \mathrm{m} \ .$$

26. (a) Combining Eqs. 42-2, 42-3 and 42-4 leads to the conduction electrons per cubic meter in zinc:

$$n = \frac{2(7.133\,\mathrm{g/cm^3})}{(65.37\,\mathrm{g/mol})/(6.02\times10^{23}/\mathrm{mol})} = 1.31\times10^{23}\,\mathrm{cm^{-3}} = 1.31\times10^{29}\ \mathrm{m^{-3}}\ .$$

(b) From Eq. 42-9,

$$E_\mathrm{F} = \frac{0.121h^2}{m_e}n^{2/3} = \frac{0.121(6.63\times10^{-34}\,\mathrm{J\cdot s})^2(1.31\times10^{29}\,\mathrm{m^{-3}})^{2/3}}{(9.11\times10^{-31}\,\mathrm{kg})(1.60\times10^{-19}\,\mathrm{J/eV})} = 9.43\ \mathrm{eV}\ .$$

(c) Equating the Fermi energy to $\frac{1}{2}m_e v_F^2$, we find (using the $m_e c^2$ value in Table 38-3)

$$v_\mathrm{F} = \sqrt{\frac{2E_\mathrm{F}c^2}{m_e c^2}} = \sqrt{\frac{2(9.43\,\mathrm{eV})(2.998\times10^8\,\mathrm{m/s})^2}{511\times10^3\,\mathrm{eV}}} = 1.82\times10^6\ \mathrm{m/s}\ .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v_\mathrm{F}} = \frac{6.63\times10^{-34}\,\mathrm{J\cdot s}}{(9.11\times10^{-31}\,\mathrm{kg})(1.82\times10^6\,\mathrm{m/s})} = 0.40\ \mathrm{nm}\ .$$

27. (a) Setting $E = E_F$ (see Eq. 42-9), Eq. 42-5 becomes

$$N(E_F) = \frac{8\pi m\sqrt{2m}}{h^3}\left(\frac{3}{16\pi\sqrt{2}}\right)^{1/3}\frac{h}{\sqrt{m}}\,n^{1/3}\ .$$

Noting that $16\sqrt{2} = 2^4 2^{1/2} = 2^{9/2}$ so that the cube root of this is $2^{3/2} = 2\sqrt{2}$, we are able to simplify the above expression and obtain

$$N(E_F) = \frac{4m}{h^2}\sqrt[3]{3\pi^2 n}$$

which is equivalent to the result shown in the problem statement. Since the desired numerical answer uses eV units, we multiply numerator and denominator of our result by c^2 and make use of the mc^2 value for an electron in Table 38-3 as well as the hc value found in problem 3 of Chapter 39:

$$N(E_F) = \left(\frac{4mc^2}{(hc)^2}\sqrt[3]{3\pi^2}\right)n^{1/3} = \left(\frac{4(511\times10^3\,\mathrm{eV})}{(1240\,\mathrm{eV\cdot nm})^2}\sqrt[3]{3\pi^2}\right)n^{1/3} = \left(4.11\ \mathrm{nm^{-2}}\cdot\mathrm{eV^{-1}}\right)n^{1/3}$$

which is equivalent to the value indicated in the problem statement.

(b) Since there are 10^{27} cubic nanometers in a cubic meter, then the result of problem 1 may be written

$$n = 8.49\times10^{28}\,\mathrm{m^{-3}} = 84.9\ \mathrm{nm^{-3}}\ .$$

The cube root of this is $n^{1/3} \approx 4.4/\mathrm{nm}$. Hence, the expression in part (a) leads to

$$N(E_F) = \left(4.11\,\mathrm{nm^{-2}}\cdot\mathrm{eV^{-1}}\right)\left(4.4\,\mathrm{nm^{-1}}\right) = 18\ \mathrm{nm^{-3}}\cdot\mathrm{eV^{-1}}\ .$$

If we multiply this by $10^{27}\,\mathrm{m^3/nm^3}$, we see this compares very well with the curve in Fig. 42-5 evaluated at 7.0 eV.

28. (a) The derivative of $P(E)$ is

$$\left(\frac{-1}{\left(e^{(E-E_F)/kT}+1\right)^2}\right)\frac{d}{dE}\,e^{(E-E_F)/kT} = \left(\frac{-1}{\left(e^{(E-E_F)/kT}+1\right)^2}\right)\frac{1}{kT}\,e^{(E-E_F)/kT}\ .$$

Evaluating this at $E = E_F$ we readily obtain the desired result.

(b) The equation of a line may be written $y = m(x - x_o)$ where m is the slope (here: equal to $-1/kT$, from part (a)) and x_o is the x-intercept (which is what we are asked to solve for). It is clear that $P(E_F) = 2$, so our equation of the line, evaluated at $x = E_F$, becomes $2 = (-1/kT)(E_F - x_o)$, which leads to $x_o = E_F + 2kT$.

29. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n}\int_0^\infty EN(E)P(E)\,dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n}\int_0^{E_F} E^{3/2}\,dE = \frac{2C}{5n}E_F^{5/2} .$$

Now

$$n = \int_0^\infty N(E)P(E)\,dE = C\int_0^{E_F} E^{1/2}\,dE = \frac{2C}{3}E_F^{3/2} .$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}} = \left(\frac{2C}{5}\right)E_F^{5/2}\left(\frac{3}{2CE_F^{3/2}}\right) = \frac{3}{5}E_F .$$

30. Let the volume be $\mathcal{V} = 1.0 \times 10^{-6}\,\text{m}^3$. Then,

$$\begin{aligned} K_{\text{total}} &= NE_{\text{avg}} = n\mathcal{V}E_{\text{avg}} \\ &= (8.43 \times 10^{28}\,\text{m}^{-3})(1.0 \times 10^{-6}\,\text{m}^3)\left(\frac{3}{5}\right)(7.0\,\text{eV})(1.6 \times 10^{-19}\,\text{J/eV}) \\ &= 5.7 \times 10^4\,\text{J} = 57\text{ kJ} . \end{aligned}$$

31. (a) Using Eq. 42-4, the energy released would be

$$\begin{aligned} E &= NE_{\text{avg}} \\ &= \frac{(3.1\,\text{g})}{(63.54\,\text{g/mol})/(6.02 \times 10^{23}/\text{mol})}\left(\frac{3}{5}\right)(7.0\,\text{eV})(1.6 \times 10^{-19}\,\text{J/eV}) \\ &= 1.98 \times 10^4\,\text{J} \approx 20\text{ kJ} . \end{aligned}$$

(b) Keeping in mind that a Watt is a Joule per second, we have

$$\frac{1.98 \times 10^4\,\text{J}}{100\,\text{J/s}} = 198\text{ s} .$$

32. (a) At $T = 300\,\text{K}$

$$f = \frac{3kT}{2E_F} = \frac{3(8.62 \times 10^{-5}\,\text{eV/K})(300\,\text{K})}{2(7.0\,\text{eV})} = 5.5 \times 10^{-3} .$$

(b) At $T = 1000\,\text{K}$,

$$f = \frac{3kT}{2E_F} = \frac{3(8.62 \times 10^{-5}\,\text{eV/K})(1000\,\text{K})}{2(7.0\,\text{eV})} = 1.8 \times 10^{-2} .$$

(c) Many calculators and most math software packages (here we use MAPLE) have built-in numerical integration routines. Setting up ratios of integrals of Eq. 42-7 and canceling common factors, we obtain

$$frac = \frac{\int_{E_F}^{\infty} \sqrt{E}/(e^{(E-E_F)/kT}+1)\,dE}{\int_0^{\infty} \sqrt{E}/(e^{(E-E_F)/kT}+1)\,dE}$$

where $k = 8.62\times10^{-5}\,\text{eV/K}$. We use the Fermi energy value for copper ($E_F = 7.0\,\text{eV}$) and evaluate this for $T = 300\,\text{K}$ and $T = 1000\,\text{K}$; we find $frac = 0.00385$ and $frac = 0.0129$, respectively.

33. The fraction f of electrons with energies greater than the Fermi energy is (approximately) given in Problem 42-32:

$$f = \frac{3kT/2}{E_F}$$

where T is the temperature on the Kelvin scale, k is the Boltzmann constant, and E_F is the Fermi energy. We solve for T:

$$T = \frac{2fE_F}{3k} = \frac{2(0.013)(4.7\,\text{eV})}{3(8.62\times10^{-5}\,\text{eV/K})} = 4.7\times10^2\ \text{K}\ .$$

It should be noted that the numerical approach, discussed briefly in part (c) of problem 32, would lead to a value closer to $T = 6.5\times10^2$ K.

34. If we use the approximate formula discussed in problem 32, we obtain

$$frac = \frac{3(8.62\times10^{-5}\,\text{eV/K})(961+273\,\text{K})}{2(5.5\,\text{eV})} \approx\ 0.03\ .$$

The numerical approach is briefly discussed in part (c) of problem 32. Although the problem does not ask for it here, we remark that numerical integration leads to a fraction closer to 0.02.

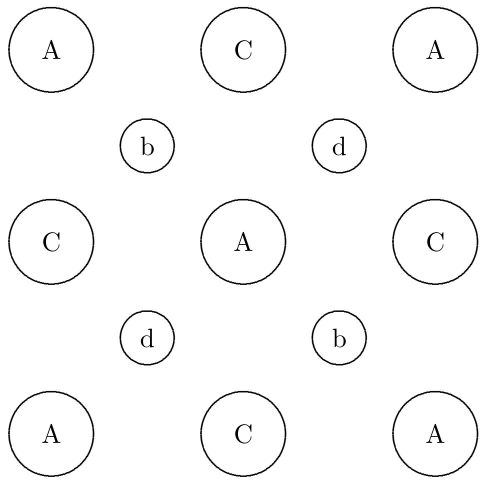
35. (a) Since the electron jumps from the conduction band to the valence band, the energy of the photon equals the energy gap between those two bands. The photon energy is given by $hf = hc/\lambda$, where f is the frequency of the electromagnetic wave and λ is its wavelength. Thus, $E_g = hc/\lambda$ and

$$\lambda = \frac{hc}{E_g} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^{8}\,\text{m/s})}{(5.5\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})} = 2.26\times10^{-7}\,\text{m} = 226\ \text{nm}\ .$$

Photons from other transitions have a greater energy, so their waves have shorter wavelengths.

(b) These photons are in the ultraviolet portion of the electromagnetic spectrum.

36. Each Arsenic atom is connected (by covalent bonding) to four Gallium atoms, and each Gallium atom is similarly connected to four Arsenic atoms. The "depth" of their very non-trivial lattice structure is, of course, not evident in a flattened-out representation such as shown for Silicon in Fig. 42-9. Still we try to convey some sense of this (in the $[1,0,0]$ view shown below – for those who might be familiar with Miller indices) by using letters to indicate the depth: A for the closest atoms (to the observer), b for the next layer deep, C for further into the page, d for the last layer seen, and E (not shown) for the atoms that are at the deepest layer (and are behind the A's) needed for our description of the structure. The capital letters are used for the Gallium atoms, and the small letters for the Arsenic. Consider the Arsenic atom (with the letter b) near the upper left; it has covalent bonds with the two A's and the two C's near it. Now consider the Arsenic atom (with the letter d) near the upper right; it has covalent bonds with the two C's which are near it and with the two E's (which are behind the A's which are near it).



(a) The 3p, 3d and 4s subshells of both Arsenic and Gallium are filled. They both have partially filled 4p subshells. An isolated, neutral Arsenic atom has three electrons in the 4p subshell, and an isolated, neutral Gallium atom has one electron in the 4p subshell. To supply the total of eight shared electrons (for the four bonds connected to each ion in the lattice), not only the electrons from 4p must be shared but also the electrons from 4s. The core of the Arsenic ion has charge $q = +5e$ (due to the "loss" of the three 4p and two 4s electrons), and the charge of the Gallium ion has charge $q = +3e$ (due to the "loss" of its single 4p and two 4s electrons).

(b) As remarked in part (a), there are two electrons shared in each of the covalent bonds. This is the same situation that one finds for Silicon (see Fig. 42-9).

37. The description in the problem statement implies that an atom is at the centerpoint C of the regular tetrahedron, since its four *neighbors* are at the four vertices. The side length for the tetrahedron is given as $a = 388$ pm. Since each face is an equilateral triangle, the "altitude" of each of those triangles (which is not to be confused with the altitude of the tetrahedron itself) is $h' = \frac{1}{2}a\sqrt{3}$ (this is generally referred to as the "slant height" in the solid geometry literature). At a certain location along the line segment representing "slant height" of each face is the center C' of the face. Imagine this line segment starting at atom A and ending at the midpoint of one of the sides. Knowing that this line segment bisects the $60°$ angle of the equilateral face, then it is easy to see that C' is a distance $AC' = a/\sqrt{3}$. If we draw a line from C' all the way to farthest point on the tetrahedron (this will land on an atom we label B), then this new line is the altitude h of the tetrahedron. Using the Pythagorean theorem,

$$h = \sqrt{a^2 - (AC')^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = a\sqrt{\frac{2}{3}} \, .$$

Now we include coordinates: imagine atom B is on the $+y$ axis at $y_b = h = a\sqrt{2/3}$, and atom A is on the $+x$ axis at $x_a = AC' = a/\sqrt{3}$. Then point C' is the origin. The tetrahedron centerpoint C is on the y axis at some value y_c which we find as follows: C must be equidistant from A and B, so

$$\begin{aligned} y_b - y_c &= \sqrt{x_a^2 + y_c^2} \\ a\sqrt{\frac{2}{3}} - y_c &= \sqrt{\left(\frac{a}{\sqrt{3}}\right)^2 + y_c^2} \end{aligned}$$

which yields $y_c = a/2\sqrt{6}$.

(a) In unit vector notation, using the information found above, we express the vector starting at C and going to A as

$$\vec{r}_{ac} = x_a\,\hat{\imath} + (-y_c)\,\hat{\jmath} = \frac{a}{\sqrt{3}}\,\hat{\imath} - \frac{a}{2\sqrt{6}}\,\hat{\jmath} \, .$$

Similarly, the vector starting at C and going to B is $\vec{r}_{bc} = (y_b - y_c)\hat{\mathrm{j}} = \frac{a}{2}\sqrt{3/2}\,\hat{\mathrm{j}}$. Therefore, using Eq. 3-20,

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{ac}\cdot\vec{r}_{bc}}{|\vec{r}_{ac}|\,|\vec{r}_{bc}|}\right) = \cos^{-1}\left(-\frac{1}{3}\right)$$

which yields $\theta = 109.5°$ for the angle between adjacent bonds.

(b) The length of vector $\vec{r}_{bc}$ (which is, of course, the same as the length of $\vec{r}_{ac}$) is

$$|\vec{r}_{bc}| = \frac{a}{2}\sqrt{\frac{3}{2}} = \frac{388\,\text{pm}}{2}\sqrt{\frac{3}{2}} = 237.6\ \text{pm}\ .$$

We note that in the solid geometry literature, the distance $\frac{a}{2}\sqrt{\frac{3}{2}}$ is known as the circumradius of the regular tetrahedron.

38. (a) At the bottom of the conduction band $E = 0.67\,\text{eV}$. Also $E_\text{F} = 0.67\,\text{eV}/2 = 0.335\,\text{eV}$. So the probability that the bottom of the conduction band is occupied is

$$\begin{aligned} P(E) &= \frac{1}{e^{(E-E_\text{F})/kT}+1} = \frac{1}{e^{(0.67\,\text{eV}-0.335\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(290\,\text{K})]}+1} \\ &= 1.5\times10^{-6}\ . \end{aligned}$$

(b) At the top of the valence band $E = 0$, so the probability that the state is *unoccupied* is given by

$$\begin{aligned} 1-P(E) &= 1-\frac{1}{e^{(E-E_\text{F})/kT}+1} = \frac{1}{e^{-(E-E_\text{F})/kT}+1} \\ &= \frac{1}{e^{-(0-0.335\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(290\,\text{K})]}+1} \\ &= 1.5\times10^{-6}\ . \end{aligned}$$

39. (a) The number of electrons in the valence band is

$$N_\text{ev} = N_v P(E_v) = \frac{N_v}{e^{(E_v-E_\text{F})/kT}+1}\ .$$

Since there are a total of N_v states in the valence band, the number of holes in the valence band is

$$\begin{aligned} N_\text{hv} &= N_v - N_\text{ev} = N_v\left[1-\frac{1}{e^{(E_v-E_\text{F})/kT}+1}\right] \\ &= \frac{N_v}{e^{-(E_v-E_\text{F})/kT}+1}\ . \end{aligned}$$

Now, the number of electrons in the conduction band is

$$N_\text{ec} = N_c P(E_c) = \frac{N_c}{e^{(E_c-E_\text{F})/kT}+1}\ ,$$

Hence, from $N_\text{ev} = N_\text{hc}$, we get

$$\frac{N_v}{e^{-(E_v-E_\text{F})/kT}+1} = \frac{N_c}{e^{(E_c-E_\text{F})/kT}+1}\ .$$

(b) In this case, $e^{(E_c-E_\text{F})/kT} \gg 1$ and $e^{-(E_v-E_\text{F})/kT} \gg 1$. Thus, from the result of part (a),

$$\frac{N_c}{e^{(E_c-E_\text{F})/kT}} \approx \frac{N_v}{e^{-(E_v-E_\text{F})/kT}}\ ,$$

or $e^{(E_v-E_c+2E_\text{F})/kT} \approx N_v/N_c$. We solve for E_F:

$$E_\text{F} \approx \frac{1}{2}(E_c+E_v)+\frac{1}{2}kT\ln\left(\frac{N_v}{N_c}\right)\ .$$

40. (a) n-type, since each phosphorous atom has one more valence electron than a silicon atom.

(b) The added charge carrier density is $n_\text{P} = 10^{-7} n_\text{Si} = 10^{-7}(5 \times 10^{28}\,\text{m}^{-3}) = 5 \times 10^{21}\ \text{m}^{-3}$.

(c) The ratio is $(5 \times 10^{21}\,\text{m}^{-3})/[2(5 \times 10^{15}\,\text{m}^{-3})] = 5 \times 10^5$. Here the factor of 2 in the denominator reflects the contribution to the charge carrier density from *both* the electrons in the conduction band *and* the holes in the valence band.

41. Sample Problem 42-6 gives the fraction of silicon atoms that must be replaced by phosphorus atoms. We find the number the silicon atoms in 1.0 g, then the number that must be replaced, and finally the mass of the replacement phosphorus atoms. The molar mass of silicon is 28.086 g/mol, so the mass of one silicon atom is $(28.086\,\text{g/mol})/(6.022 \times 10^{23}\,\text{mol}^{-1}) = 4.66 \times 10^{-23}\,\text{g}$ and the number of atoms in 1.0 g is $(1.0\,\text{g})/(4.66 \times 10^{-23}\,\text{g}) = 2.14 \times 10^{22}$. According to Sample Problem 42-6 one of every 5×10^6 silicon atoms is replaced with a phosphorus atom. This means there will be $(2.14 \times 10^{22})/(5 \times 10^6) = 4.29 \times 10^{15}$ phosphorus atoms in 1.0 g of silicon. The molar mass of phosphorus is 30.9758 g/mol so the mass of a phosphorus atom is $(30.9758\,\text{g/mol})/(6.022 \times 10^{-23}\,\text{mol}^{-1}) = 5.14 \times 10^{-23}\,\text{g}$. The mass of phosphorus that must be added to 1.0 g of silicon is $(4.29 \times 10^{15})(5.14 \times 10^{-23}\,\text{g}) = 2.2 \times 10^{-7}\,\text{g}$.

42. (a) Measured from the top of the valence band, the energy of the donor state is $E = 1.11\,\text{eV} - 0.11\,\text{eV} = 1.0\,\text{eV}$. We solve E_F from Eq. 42-6:

$$\begin{aligned} E_\text{F} &= E - kT \ln\left[P^{-1} - 1\right]) \\ &= 1.0\,\text{eV} - (8.62 \times 10^{-5}\,\text{eV/K})(300\,\text{K}) \ln\left[(5.00 \times 10^{-5})^{-1} - 1\right] \\ &= 0.744\ \text{eV}\ . \end{aligned}$$

(b) Now $E = 1.11\,\text{eV}$, so

$$\begin{aligned} P(E) &= \frac{1}{e^{(E-E_\text{F})/kT} + 1} = \frac{1}{e^{(1.11\,\text{eV} - 0.744\,\text{eV})/[(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})]} + 1} \\ &= 7.13 \times 10^{-7}\ . \end{aligned}$$

43. (a) The probability that a state with energy E is occupied is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where E_F is the Fermi energy, T is the temperature on the Kelvin scale, and k is the Boltzmann constant. If energies are measured from the top of the valence band, then the energy associated with a state at the bottom of the conduction band is $E = 1.11\,\text{eV}$. Furthermore, $kT = (8.62 \times 10^{-5}\,\text{eV/K})(300\,\text{K}) = 0.02586\,\text{eV}$. For pure silicon, $E_F = 0.555\,\text{eV}$ and $(E - E_F)/kT = (0.555\,\text{eV})/(0.02586\,\text{eV}) = 21.46$. Thus,

$$P(E) = \frac{1}{e^{21.46} + 1} = 4.79 \times 10^{-10}\ .$$

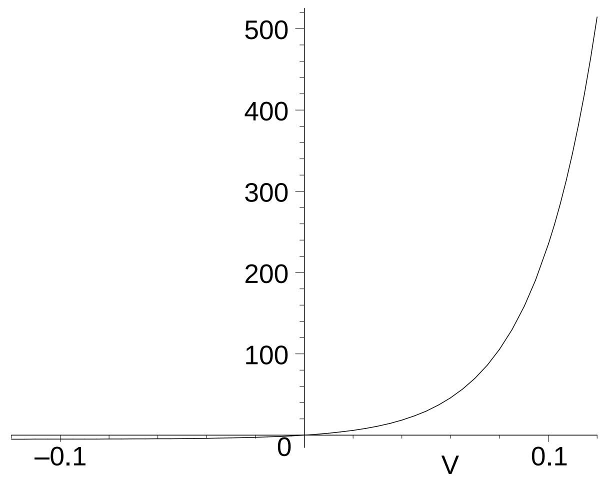
For the doped semiconductor, $(E - E_F)/kT = (0.11\,\text{eV})/(0.02586\,\text{eV}) = 4.254$ and

$$P(E) = \frac{1}{e^{4.254} + 1} = 1.40 \times 10^{-2}\ .$$

(b) The energy of the donor state, relative to the top of the valence band, is $1.11\,\text{eV} - 0.15\,\text{eV} = 0.96\,\text{eV}$. The Fermi energy is $1.11\,\text{eV} - 0.11\,\text{eV} = 1.00\,\text{eV}$. Hence, $(E - E_F)/kT = (0.96\,\text{eV} - 1.00\,\text{eV})/(0.02586\,\text{eV}) = -1.547$ and

$$P(E) = \frac{1}{e^{-1.547} + 1} = 0.824\ .$$

44. (a) The vertical axis in the graph below is the current in nanoamperes:



(b) The ratio is

$$\frac{i|_{v=+0.50\,\mathrm{V}}}{i|_{v=-0.50\,\mathrm{V}}} = \frac{i_0[e^{+0.50\,\mathrm{eV}/[(8.62\times10^{-5}\,\mathrm{eV/K})(300\,\mathrm{K})]}-1]}{i_0[e^{-0.50\,\mathrm{eV}/[(8.62\times10^{-5}\,\mathrm{eV/K})(300\,\mathrm{K})]}-1]} = 2.5\times10^{8}\ .$$

45. The energy received by each electron is exactly the difference in energy between the bottom of the conduction band and the top of the valence band (1.1 eV). The number of electrons that can be excited across the gap by a single 662-keV photon is $N = (662 \times 10^3\,\mathrm{eV})/(1.1\,\mathrm{eV}) = 6.0 \times 10^5$. Since each electron that jumps the gap leaves a hole behind, this is also the number of electron-hole pairs that can be created.

46. Since (using the result of problem 3 in Chapter 39)

$$E_{\mathrm{photon}} = \frac{hc}{\lambda} = \frac{1240\,\mathrm{eV{\cdot}nm}}{140\,\mathrm{nm}} = 8.86\,\mathrm{eV} > 7.6\,\mathrm{eV}\ ,$$

the light will be absorbed by the KCI crystal. Thus, the crystal is opaque to this light.

47. The valence band is essentially filled and the conduction band is essentially empty. If an electron in the valence band is to absorb a photon, the energy it receives must be sufficient to excite it across the band gap. Photons with energies less than the gap width are not absorbed and the semiconductor is transparent to this radiation. Photons with energies greater than the gap width are absorbed and the semiconductor is opaque to this radiation. Thus, the width of the band gap is the same as the energy of a photon associated with a wavelength of 295 nm. We use the result of Exercise 3 of Chapter 39 to obtain

$$E_{\mathrm{gap}} = \frac{1240\,\mathrm{eV{\cdot}nm}}{\lambda} = \frac{1240\,\mathrm{eV{\cdot}nm}}{295\,\mathrm{nm}} = 4.20\ \mathrm{eV}\ .$$

48. We denote the maximum dimension (side length) of each transistor as $\ell_{\max}$, the size of the chip as A, and the number of transistors on the chip as N. Then $A = N\ell_{\max}^2$. Therefore,

$$\ell_{\max} = \sqrt{\frac{A}{N}} = \sqrt{\frac{(1.0\,\mathrm{in.}\times0.875\,\mathrm{in.})(2.54\times10^{-2}\,\mathrm{m/in.})^2}{3.5\times10^{6}}} = 1.3\times10^{-5}\,\mathrm{m} = 13\,\mu\mathrm{m}\ .$$

49. (a) According to Chapter 26, the capacitance is $C = \kappa\varepsilon_0 A/d$. In our case $\kappa = 4.5$, $A = (0.50\,\mu\mathrm{m})^2$, and $d = 0.20\,\mu\mathrm{m}$, so

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(4.5)(8.85\times10^{-12}\,\mathrm{F/m})(0.50\,\mu\mathrm{m})^2}{0.20\,\mu\mathrm{m}} = 5.0\times10^{-17}\ \mathrm{F}\ .$$

(b) Let the number of elementary charges in question be N. Then, the total amount of charges that appear in the gate is $q = Ne$. Thus, $q = Ne = CV$, which gives

$$N = \frac{CV}{e} = \frac{(5.0 \times 10^{-17}\,\mathrm{F})(1.0\,\mathrm{V})}{1.6 \times 10^{-19}\,\mathrm{C}} = 3.1 \times 10^{2}\ .$$

Chapter 43

1. In order for the α particle to penetrate the gold nucleus, the separation between the centers of mass of the two particles must be no greater than $r = r_{\text{Cu}} + r_\alpha = 6.23\,\text{fm} + 1.80\,\text{fm} = 8.03\,\text{fm}$. Thus, the minimum energy K_α is given by

$$\begin{aligned} K_\alpha &= U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\text{Au}}}{r} = \frac{kq_\alpha q_{\text{Au}}}{r} \\ &= \frac{(8.99\times10^9\,\text{V}\cdot\text{m/C})(2e)(79)(1.60\times10^{-19}\,\text{C})}{8.03\times10^{-15}\,\text{m}} = 28.3\times10^6\ \text{eV}\ . \end{aligned}$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to $1.60\times10^{-19}\,\text{C}$, but was instead carried through to become part of the final units.

2. Our calculation is similar to that shown in Sample Problem 43-1. We set $K = 5.30\,\text{MeV} = U = (1/4\pi\varepsilon_0)(q_\alpha q_{\text{Cu}}/r_{\text{min}})$ and solve for the closest separation, r_{min} :

$$\begin{aligned} r_{\text{min}} &= \frac{q_\alpha q_{\text{Cu}}}{4\pi\varepsilon_0 K} = \frac{kq_\alpha q_{\text{Cu}}}{4\pi\varepsilon_0 K} \\ &= \frac{(2e)(29)(1.60\times10^{-19}\,\text{C})(8.99\times10^9\,\text{V}\cdot\text{m/C})}{5.30\times10^6\,\text{eV}} \\ &= 1.58\times10^{-14}\,\text{m} = 15.8\ \text{fm}\ . \end{aligned}$$

We note that the factor of e in $q_\alpha = 2e$ was not set equal to $1.60\times10^{-19}\,\text{C}$, but was instead allowed to cancel the "e" in the non-SI energy unit, electronvolt.

3. The conservation laws of (classical kinetic) energy and (linear) momentum determine the outcome of the collision. The results are given in Chapter 10, Eqs. 10-30 and 10-31. The final speed of the α particle is

$$v_{\alpha f} = \frac{m_\alpha - m_{\text{Au}}}{m_\alpha + m_{\text{Au}}}\,v_{\alpha i}\ ,$$

and that of the recoiling gold nucleus is

$$v_{\text{Au},f} = \frac{2m_\alpha}{m_\alpha + m_{\text{Au}}}\,v_{\alpha i}\ .$$

(a) Therefore, the kinetic energy of the recoiling nucleus is

$$\begin{aligned} K_{\text{Au},f} &= \frac{1}{2}m_{\text{Au}}v_{\text{Au},f}^2 \\ &= \frac{1}{2}m_{\text{Au}}\left(\frac{2m_\alpha}{m_\alpha + m_{\text{Au}}}\right)^2 v_{\alpha i}^2 = K_{\alpha i}\frac{4m_{\text{Au}}m_\alpha}{(m_\alpha + m_{\text{Au}})^2} \\ &= (5.00\,\text{MeV})\frac{4(197\,\text{u})(4.00\,\text{u})}{(4.00\,\text{u} + 197\,\text{u})^2} \\ &= 0.390\ \text{MeV}\ . \end{aligned}$$

(b) The final kinetic energy of the alpha particle is

$$\begin{aligned} K_{\alpha f} &= \frac{1}{2} m_\alpha v_{\alpha f}^2 \\ &= \frac{1}{2} m_\alpha \left(\frac{m_\alpha - m_{\rm Au}}{m_\alpha + m_{\rm Au}} \right)^2 v_{\alpha i}^2 = K_{\alpha i} \left(\frac{m_\alpha - m_{\rm Au}}{m_\alpha + m_{\rm Au}} \right)^2 \\ &= (5.00\,{\rm MeV}) \left(\frac{4.00\,{\rm u} - 197\,{\rm u}}{4.00\,{\rm u} + 197\,{\rm u}} \right)^2 \\ &= 4.61 \text{ MeV} . \end{aligned}$$

We note that $K_{\alpha f} + K_{{\rm Au},f} = K_{\alpha i}$ is indeed satisfied.

4. We solve for A from Eq. 43-3:
$$A = \left(\frac{r}{r_0} \right)^3 = \left(\frac{3.6\,{\rm fm}}{1.2\,{\rm fm}} \right)^3 = 27 .$$

5. We locate a nuclide from Table 43-1 by finding the coordinate (N, Z) of the corresponding point in Fig. 43-4. It is clear that all the nuclides listed in Table 43-1 are stable except the last two, ^{227}Ac and ^{239}Pu.

6. We note that the mean density and mean radius for the Sun are given in Appendix C. Since $\rho = M/V$ where $V \propto r^3$, we get $r \propto \rho^{-1/3}$. Thus, the new radius would be
$$r = R_s \left(\frac{\rho_s}{\rho} \right)^{1/3} = (6.96 \times 10^8\,{\rm m}) \left(\frac{1410\,{\rm kg/m^3}}{2 \times 10^{17}\,{\rm kg/m^3}} \right)^{1/3} = 1.3 \times 10^4 \text{ m} .$$

7. (a) 6 protons, since $Z = 6$ for carbon (see Appendix F).
   (b) 8 neutrons, since $A - Z = 14 - 6 = 8$ (see Eq. 43-1).

8. The problem with Web-based services is that there are no guarantees of accuracy or that the webpage addresses will not change from the time this solution is written to the time someone reads this. Still, it is worth mentioning that a very accessible website for a wide variety of periodic table and isotope-related information is http://www.webelements.com. Two websites aimed more towards the nuclear professional are http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf, which are where some of the information mentioned below was obtained.

   (a) According to Appendix F, the atomic number 60 corresponds to the element Neodymium (Nd). The first website mentioned above gives ^{142}Nd, ^{143}Nd, ^{144}Nd, ^{145}Nd, ^{146}Nd, ^{148}Nd, and ^{150}Nd in its list of naturally occurring isotopes. Two of these, ^{144}Nd and ^{150}Nd, are not perfectly stable, but their half-lives are much longer than the age of the universe (detailed information on their half-lives, modes of decay, etc are available at the last two websites referred to, above).

   (b) In this list, we are asked to put the nuclides which contain 60 neutrons and which are recognized to exist but not stable nuclei (this is why, for example, ^{108}Cd is not included here). Although the problem does not ask for it, we include the half-lives of the nuclides in our list, though it must be admitted that not all reference sources agree on those values (we picked the ones we regarded as "most reliable"). Thus, we have ^{97}Rb (0.2 s), ^{98}Sr (0.7 s), ^{99}Y (2 s), ^{100}Zr (7 s), ^{101}Nb (7 s), ^{102}Mo (11 minutes), ^{103}Tc (54 s), ^{105}Rh (35 hours), ^{109}In (4 hours), ^{110}Sn (4 hours), ^{111}Sb (75 s), ^{112}Te (2 minutes), ^{113}I (7 s), ^{114}Xe (10 s), ^{115}Cs (1.4 s), and ^{116}Ba (1.4 s).

   (c) We would include in this list: ^{60}Zn, ^{60}Cu, ^{60}Ni, ^{60}Co, ^{60}Fe, ^{60}Mn, ^{60}Cr, and ^{60}V.

9. Although we haven't drawn the requested lines in the following table, we can indicate their slopes: lines of constant A would have $-45°$ slopes, and those of constant $N - Z$ would have $45°$. As an example of the latter, the $N - Z = 20$ line (which is one of "eighteen-neutron excess") would pass through Cd-114 at the

lower left corner up through Te-122 at the upper right corner. The first column corresponds to $N = 66$, and the bottom row to $Z = 48$. The last column corresponds to $N = 70$, and the top row to $Z = 52$. Much of the information below (regarding values of $T_{1/2}$ particularly) was obtained from the websites http://nucleardata.nuclear.lu.se/nucleardata and http://www.nndc.bnl.gov/nndc/ensdf (we refer the reader to the remarks we made in the solution to problem 8).

^{118}Te 6.0 days	^{119}Te 16.0 h	^{120}Te 0.1%	^{121}Te 19.4 days	^{122}Te 2.6%
^{117}Sb 2.8 h	^{118}Sb 3.6 min	^{119}Sb 38.2 s	^{120}Sb 15.9 min	^{121}Sb 57.2%
^{116}Sn 14.5%	^{117}Sn 7.7%	^{118}Sn 24.2%	^{119}Sn 8.6%	^{120}Sn 32.6%
^{115}In 95.7%	^{116}In 14.1 s	^{117}In 43.2 min	^{118}In 5.0 s	^{119}In 2.4 min
^{114}Cd 28.7%	^{115}Cd 53.5 h	^{116}Cd 7.5%	^{117}Cd 2.5 h	^{118}Cd 50.3 min

10. (a) The atomic number $Z = 39$ corresponds to the element Yttrium (see Appendix F and/or Appendix G), and $Z = 53$ corresponds to Iodine.
   (b) A detailed listing of stable nuclides (such as the website http://nucleardata.nuclear.lu.se/nucleardata) shows that the stable isotope of Iodine has 74 neutrons, and that the stable isotope of Yttrium has 50 neutrons (this can also be inferred from the Molar Mass values listed in Appendix F).
   (c) The number of neutrons left over is $235 - 127 - 89 = 19$.

11. (a) For ^{239}Pu, $Q = 94e$ and $R = 6.64\,\text{fm}$. Including a conversion factor for J → eV, we obtain

$$\begin{aligned} U &= \frac{3Q^2}{20\pi\varepsilon_0 r} = \frac{3[94(1.60\times10^{-19}\,\text{C})]^2(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)}{5(6.64\times10^{-15}\,\text{m})}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) \\ &= 1.15\times10^9\,\text{eV} = 1.15\ \text{GeV}\ . \end{aligned}$$

   (b) Since $Z = 94$ and $A = 239$, the electrostatic potential per nucleon is $1.15\,\text{GeV}/239 = 4.81\,\text{MeV/nucleon}$, and per proton is $1.15\,\text{GeV}/94 = 12.2\,\text{MeV/proton}$. These are of the same order of magnitude as the binding energy per nucleon.
   (c) The binding energy is significantly reduced by the electrostatic repulsion among the protons.

12. (a) For ^{55}Mn the mass density is

$$\rho_m = \frac{M}{V} = \frac{0.055\,\text{kg/mol}}{(4\pi/3)[(1.2\times10^{-15}\,\text{m})(55)^{1/3}]^3(6.02\times10^{23}/\text{mol})} = 2.3\times10^{17}\ \text{kg/m}^3\ ,$$

and for ^{209}Bi

$$\rho_m = \frac{M}{V} = \frac{0.209\,\text{kg/mol}}{(4\pi/3)[(1.2\times10^{-15}\,\text{m})(209)^{1/3}]^3(6.02\times10^{23}/\text{mol})} = 2.3\times10^{17}\ \text{kg/m}^3\ .$$

(b) For ^{55}Mn the charge density is

$$\rho_q = \frac{Ze}{V} = \frac{(25)(1.6\times10^{-19}\,\text{C})}{(4\pi/3)[(1.2\times10^{-15}\,\text{m})(55)^{1/3}]^3} = 1.0\times10^{25}\ \text{C/m}^3\ ,$$

and for ^{209}Bi

$$\rho_q = \frac{Ze}{V} = \frac{(83)(1.6\times10^{-19}\,\text{C})}{(4\pi/3)[(1.2\times10^{-15}\,\text{m})(209)^{1/3}]^3} = 8.8\times10^{24}\ \text{C/m}^3\ .$$

(c) Since $V \propto r^3 = (r_0A^{1/3})^3 \propto A$, we expect $\rho_m \propto A/V \propto A/A \approx$ const. for all nuclides, while $\rho_q \propto Z/V \propto Z/A$ should gradually decrease since $A > 2Z$ for large nuclides.

13. The binding energy is given by $\Delta E_{\text{be}} = [Zm_H + (A-Z)m_n - M_{\text{Pu}}]\,c^2$, where Z is the atomic number (number of protons), A is the mass number (number of nucleons), m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and M_{Pu} is the mass of a $^{239}_{94}$Pu atom. In principle, nuclear masses should be used, but the mass of the Z electrons included in ZM_H is canceled by the mass of the Z electrons included in M_{Pu}, so the result is the same. First, we calculate the mass difference in atomic mass units: $\Delta m = (94)(1.00783\,\text{u}) + (239-94)(1.00867\,\text{u}) - (239.05216\,\text{u}) = 1.94101\,\text{u}$. Since 1 u is equivalent to 931.5 MeV, $\Delta E_{\text{be}} = (1.94101\,\text{u})(931.5\,\text{MeV/u}) = 1808\,\text{MeV}$. Since there are 239 nucleons, the binding energy per nucleon is $\Delta E_{\text{ben}} = E/A = (1808\,\text{MeV})/239 = 7.56\,\text{MeV}$.

14. (a) The mass number A is the number of nucleons in an atomic nucleus. Since $m_p \approx m_n$ the mass of the nucleus is approximately Am_p. Also, the mass of the electrons is negligible since it is much less than that of the nucleus. So $M \approx Am_p$.

(b) For ^{1}H, the approximate formula gives $M \approx Am_p = (1)(1.007276\,\text{u}) = 1.007276\,\text{u}$. The actual mass is (see Table 47-1) 1.007825 u. The percent error committed is then $\delta = (1.007825\,\text{u} - 1.007276\,\text{u})/1.007825\,\text{u} = 0.054\%$. Similarly, $\delta = 0.50\%$ for ^{7}Li, 0.81% for ^{31}P, 0.83% for ^{81}Br, 0.81% for ^{120}Sn, 0.78% for ^{157}Gd, 0.74% for ^{197}Au, 0.72% for ^{272}Ac, and 0.71% for ^{239}Pu.

(c) No. In a typical nucleus the binding energy per nucleon is several MeV, which is a bit less than 1% of the nucleon mass times c^2. This is comparable with the percent error calculated in part (b), so we need to use a more accurate method to calculate the nuclear mass.

15. (a) The de Broglie wavelength is given by $\lambda = h/p$, where p is the magnitude of the momentum. The kinetic energy K and momentum are related by Eq. 38-51, which yields

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(200\,\text{MeV})^2 + 2(200\,\text{MeV})(0.511\,\text{MeV})} = 200.5\,\text{MeV}\ .$$

Thus,

$$\lambda = \frac{hc}{pc} = \frac{1240\,\text{eV}\cdot\text{nm}}{200.5\times10^6\,\text{eV}} = 6.18\times10^{-6}\,\text{nm} = 6.18\ \text{fm}\ .$$

(b) The diameter of a copper nucleus, for example, is about 8.6 fm, just a little larger than the de Broglie wavelength of a 200-MeV electron. To resolve detail, the wavelength should be smaller than the target, ideally a tenth of the diameter or less. 200-MeV electrons are perhaps at the lower limit in energy for useful probes.

16. We take the speed to be constant, and apply the classical kinetic energy formula:

$$\begin{aligned} t &= \frac{d}{v} = \frac{d}{\sqrt{2K/m}} = 2r\sqrt{\frac{m_n}{2K}} = \frac{r}{c}\sqrt{\frac{2mc^2}{K}} \\ &\approx \frac{(1.2\times10^{-15}\,\text{m})(100)^{1/3}}{3.0\times10^8\,\text{m/s}}\sqrt{\frac{2(938\,\text{MeV})}{5\,\text{MeV}}} \\ &\approx 10^{-22}\ \text{s}\ . \end{aligned}$$

17. We note that $hc = 1240\,\text{MeV}\cdot\text{fm}$ (see problem 3 of Chapter 39), and that the classical kinetic energy $\frac{1}{2}mv^2$ can be written directly in terms of the classical momentum $p = mv$ (see below). Letting $p \simeq \Delta p \simeq h/\Delta x \simeq h/r$, we get

$$E = \frac{p^2}{2m} \simeq \frac{(hc)^2}{2(mc^2)r^2} = \frac{(1240\,\text{MeV}\cdot\text{fm})^2}{2(938\,\text{MeV})[(1.2\,\text{fm})(100)^{1/3}]^2} \simeq 30\ \text{MeV}\ .$$

18. (a) In terms of the original value of u, the newly defined u is greater by a factor of 1.007825. So the mass of ^{1}H would be 1.000000 u, the mass of ^{12}C would be $(12.000000/1.007825)\text{u} = 11.90683\,\text{u}$, and the mass of ^{238}U would be $(238.050785/1.007825)\text{u} = 236.2025\,\text{u}$.

    (b) Defining the mass of ^{1}H to be exactly 1 does not result in any overall simplification.

19. (a) Since the nuclear force has a short range, any nucleon interacts only with its nearest neighbors, not with more distant nucleons in the nucleus. Let N be the number of neighbors that interact with any nucleon. It is independent of the number A of nucleons in the nucleus. The number of interactions in a nucleus is approximately NA, so the energy associated with the strong nuclear force is proportional to NA and, therefore, proportional to A itself.

    (b) Each proton in a nucleus interacts electrically with every other proton. The number of pairs of protons is $Z(Z-1)/2$, where Z is the number of protons. The Coulomb energy is, therefore, proportional to $Z(Z-1)$.

    (c) As A increases, Z increases at a slightly slower rate but Z^2 increases at a faster rate than A and the energy associated with Coulomb interactions increases faster than the energy associated with strong nuclear interactions.

20. (a) The first step is to add energy to produce $^4\text{He} \to p + {}^3\text{H}$, which – to make the electrons "balance" – may be rewritten as $^4\text{He} \to {}^1\text{H} + {}^3\text{H}$. The energy needed is $\Delta E_1 = (m_{^3\text{H}} + m_{^1\text{H}} - m_{^4\text{He}})c^2 = (3.01605\,\text{u} + 1.00783\,\text{u} - 4.00260\,\text{u})(931.5\,\text{MeV/u}) = 19.8\,\text{MeV}$. The second step is to add energy to produce $^3\text{H} \to n + {}^2\text{H}$. The energy needed is $\Delta E_2 = (m_{^2\text{H}} + m_n - m_{^3\text{H}})c^2 = (2.01410\,\text{u} + 1.00867\,\text{u} - 3.01605\,\text{u})(931.5\,\text{MeV/u}) = 6.26\,\text{MeV}$. The third step: $^2\text{H} \to p + n$, which – to make the electrons "balance" – may be rewritten as $^2\text{H} \to {}^1\text{H} + n$. The work required is $\Delta E_3 = (m_{^1\text{H}} + m_n - m_{^2\text{H}})c^2 = (1.00783\,\text{u} + 1.00867\,\text{u} - 2.01410\,\text{u})(931.5\,\text{MeV/u}) = 2.23\,\text{MeV}$.

    (b) The total binding energy is $\Delta E_{\text{be}} = \Delta E_1 + \Delta E_2 + \Delta E_3 = 19.8\,\text{MeV} + 6.26\,\text{MeV} + 2.23\,\text{MeV} = 28.3\,\text{MeV}$.

    (c) The binding energy per nucleon is $\Delta E_{\text{ben}} = \Delta E_{\text{be}}/A = 28.3\,\text{MeV}/4 = 7.07\,\text{MeV}$.

21. Let f_{24} be the abundance of ^{24}Mg, let f_{25} be the abundance of ^{25}Mg, and let f_{26} be the abundance of ^{26}Mg. Then, the entry in the periodic table for Mg is $24.312 = 23.98504 f_{24} + 24.98584 f_{25} + 25.98259 f_{26}$. Since there are only three isotopes, $f_{24} + f_{25} + f_{26} = 1$. We solve for f_{25} and f_{26}. The second equation gives $f_{26} = 1 - f_{24} - f_{25}$. We substitute this expression and $f_{24} = 0.7899$ into the first equation to obtain $24.312 = (23.98504)(0.7899) + 24.98584 f_{25} + 25.98259 - (25.98259)(0.7899) - 25.98259 f_{25}$. The solution is $f_{25} = 0.09303$. Then, $f_{26} = 1 - 0.7899 - 0.09303 = 0.1171$. 78.99% of naturally occurring magnesium is ^{24}Mg, 9.30% is ^{25}Mg, and 11.71% is ^{26}Mg.

22. (a) Table 43-1 gives the atomic mass of ^{1}H as $m = 1.007825\,\text{u}$. Therefore, the *mass excess* for ^{1}H is $\Delta = (1.007825\,\text{u} - 1.000000\,\text{u})(931.5\,\text{MeV/u}) = +7.29\,\text{MeV}$.

    (b) The mass of the neutron is given in Sample Problem 43-3. Thus, for the neutron, $\Delta = (1.008665\,\text{u} - 1.000000\,\text{u})(931.5\,\text{MeV/u}) = +8.07\,\text{MeV}$.

    (c) Appealing again to Table 43-1, we obtain, for ^{120}Sn, $\Delta = (119.902199\,\text{u} - 120.000000\,\text{u})(931.5\,\text{MeV/u}) = -91.10\,\text{MeV}$.

23. We first "separate" all the nucleons in one copper nucleus (which amounts to simply calculating the nuclear binding energy) and then figure the number of nuclei in the penny (so that we can multiply the

two numbers and obtain the result). To begin, we note that (using Eq. 43-1 with Appendix F and/or G) the copper-63 nucleus has 29 protons and 34 neutrons. We use the more accurate values given in Sample Problem 43-3:

$$\Delta E_{\text{be}} = (29(1.007825\,\text{u}) + 34(1.008665\,\text{u}) - 62.92960\,\text{u})\,(931.5\,\text{MeV/u}) = 551.4 \text{ MeV} .$$

To figure the number of nuclei (or, equivalently, the number of atoms), we adapt Eq. 43-20:

$$N_{\text{Cu}} = \left(\frac{3.0\,\text{g}}{62.92960\,\text{g/mol}}\right)\left(6.02\times10^{23}\,\text{atoms/mol}\right) \approx 2.9\times10^{22} \text{ atoms} .$$

Therefore, the total energy needed is

$$N_{\text{Cu}}\Delta E_{\text{be}} = (551.4\,\text{MeV})\left(2.9\times10^{22}\right) = 1.6\times10^{25} \text{ MeV} .$$

24. It should be noted that when the problem statement says the "masses of the proton and the deuteron are ..." they are actually referring to the corresponding atomic masses (given to very high precision). That is, the given masses include the "orbital" electrons. As in many computations in this chapter, this circumstance (of implicitly including electron masses in what should be a purely nuclear calculation) does not cause extra difficulty in the calculation (see remarks in Sample Problems 43-4, 43-6, and 43-7). Setting the gamma ray energy equal to ΔE_{be}, we solve for the neutron mass (with each term understood to be in u units):

$$\begin{aligned} m_{\text{n}} &= M_{\text{d}} - m_{\text{H}} + \frac{E_\gamma}{c^2} \\ &= 2.0141019 - 1.007825035 + \frac{2.2233}{931.502} \\ &= 1.0062769 + 0.0023868 \end{aligned}$$

which yields $m_{\text{n}} = 1.0086637\,\text{u}$, where the last digit (7) is uncertain to within roughly ±2 (but this depends on what precisely the uncertainties are in the given data).

25. If a nucleus contains Z protons and N neutrons, its binding energy is $\Delta E_{\text{be}} = (Zm_H + Nm_n - m)c^2$, where m_H is the mass of a hydrogen atom, m_n is the mass of a neutron, and m is the mass of the atom containing the nucleus of interest. If the masses are given in atomic mass units, then mass excesses are defined by $\Delta_H = (m_H - 1)c^2$, $\Delta_n = (m_n - 1)c^2$, and $\Delta = (m - A)c^2$. This means $m_Hc^2 = \Delta_H + c^2$, $m_nc^2 = \Delta_n + c^2$, and $mc^2 = \Delta + Ac^2$. Thus $E = (Z\Delta_H + N\Delta_n - \Delta) + (Z + N - A)c^2 = Z\Delta_H + N\Delta_n - \Delta$, where $A = Z + N$ is used. For ${}^{197}_{79}\text{Au}$, $Z = 79$ and $N = 197 - 79 = 118$. Hence,

$$\Delta E_{\text{be}} = (79)(7.29\,\text{MeV}) + (118)(8.07\,\text{MeV}) - (-31.2\,\text{MeV}) = 1560\,\text{MeV} .$$

This means the binding energy per nucleon is $\Delta E_{\text{ben}} = (1560\,\text{MeV})/197 = 7.92\,\text{MeV}$.

26. (a) Since $60\,\text{y} = 2(30\,\text{y}) = 2T_{1/2}$, the fraction left is $2^{-2} = 1/4$.

    (b) Since $90\,\text{y} = 3(30\,\text{y}) = 3T_{1/2}$, the fraction that remains is $2^{-3} = 1/8$.

27. By the definition of half-life, the same has reduced to $\frac{1}{2}$ its initial amount after 140 d. Thus, reducing it to $\frac{1}{4} = (\frac{1}{2})^2$ of its initial number requires that two half-lives have passed: $t = 2T_{1/2} = 280\,\text{d}$.

28. We note that $t = 24\,\text{h}$ is four times $T_{1/2} = 6.5\,\text{h}$. Thus, it has reduced by half, four-fold:

$$\left(\frac{1}{2}\right)^4\left(48\times10^{19}\right) = 3\times10^{19} .$$

29. (a) The decay rate is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. Initially, $R = R_0 = \lambda N_0$, where N_0 is the number of undecayed nuclei at that time. One must find values for both N_0 and λ. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(78\,\text{h}) = 8.89 \times 10^{-3}\,\text{h}^{-1}$. If M is the mass of the sample and m is the mass of a single atom of gallium, then $N_0 = M/m$. Now, $m = (67\,\text{u})(1.661 \times 10^{-24}\,\text{g/u}) = 1.113 \times 10^{-22}\,\text{g}$ and $N_0 = (3.4\,\text{g})/(1.113 \times 10^{-22}\,\text{g}) = 3.05 \times 10^{22}$. Thus $R_0 = (8.89 \times 10^{-3}\,\text{h}^{-1})(3.05 \times 10^{22}) = 2.71 \times 10^{20}\,\text{h}^{-1} = 7.53 \times 10^{16}\,\text{s}^{-1}$.

(b) The decay rate at any time t is given by

$$R = R_0\, e^{-\lambda t}$$

where R_0 is the decay rate at $t = 0$. At $t = 48\,\text{h}$, $\lambda t = (8.89 \times 10^{-3}\,\text{h}^{-1})(48\,\text{h}) = 0.427$ and

$$R = (7.53 \times 10^{16}\,\text{s}^{-1})\, e^{-0.427} = 4.91 \times 10^{16}\,\text{s}^{-1} \ .$$

30. (a) Replacing differentials with deltas in Eq. 43-11, we use the fact that $\Delta N = -12$ during $\Delta t = 1.0$ s to obtain

$$\frac{\Delta N}{N} = -\lambda \Delta t \quad \Longrightarrow \quad \lambda = 4.8 \times 10^{-18}/\text{s}$$

where $N = 2.5 \times 10^{18}$, mentioned at the second paragraph of §43-3, is used.

(b) Eq. 43-17 yields $T_{1/2} = \ln 2/\lambda = 1.4 \times 10^{17}\,\text{s}$, or about 4.6 billion years.

31. (a) The half-life $T_{1/2}$ and the disintegration constant are related by $T_{1/2} = (\ln 2)/\lambda$, so $T_{1/2} = (\ln 2)/(0.0108\,\text{h}^{-1}) = 64.2\,\text{h}$.

(b) At time t, the number of undecayed nuclei remaining is given by

$$N = N_0\, e^{-\lambda t} = N_0\, e^{-(\ln 2)t/T_{1/2}} \ .$$

We substitute $t = 3T_{1/2}$ to obtain

$$\frac{N}{N_0} = e^{-3 \ln 2} = 0.125 \ .$$

In each half-life, the number of undecayed nuclei is reduced by half. At the end of one half-life, $N = N_0/2$, at the end of two half-lives, $N = N_0/4$, and at the end of three half-lives, $N = N_0/8 = 0.125 N_0$.

(c) We use

$$N = N_0\, e^{-\lambda t} \ .$$

$10.0\,d$ is 240 h, so $\lambda t = (0.0108\,\text{h}^{-1})(240\,\text{h}) = 2.592$ and

$$\frac{N}{N_0} = e^{-2.592} = 0.0749 \ .$$

32. (a) We adapt Eq. 43-20:

$$N_{\text{Pu}} = \left(\frac{0.002\,\text{g}}{239\,\text{g/mol}}\right)\left(6.02 \times 10^{23}\,\text{nuclei/mol}\right) \approx 5 \times 10^{18}\ \text{nuclei} \ .$$

(b) Eq. 43-19 leads to

$$R = \frac{N \ln 2}{T_{1/2}} = \frac{5 \times 10^{18} \ln 2}{2.41 \times 10^{4}\,\text{y}} = 1.4 \times 10^{14}/\text{y}$$

which is equivalent to $4.6 \times 10^6/\text{s} = 4.6 \times 10^6$ Bq (the unit becquerel is defined in §43-3).

33. The rate of decay is given by $R = \lambda N$, where λ is the disintegration constant and N is the number of undecayed nuclei. In terms of the half-life $T_{1/2}$, the disintegration constant is $\lambda = (\ln 2)/T_{1/2}$, so

$$\begin{aligned} N &= \frac{R}{\lambda} = \frac{RT_{1/2}}{\ln 2} = \frac{(6000\,\text{Ci})(3.7\times10^{10}\,\text{s}^{-1}/\text{Ci})(5.27\,\text{y})(3.16\times10^{7}\,\text{s/y})}{\ln 2} \\ &= 5.33\times10^{22}\ \text{nuclei}\ . \end{aligned}$$

34. Using Eq. 43-14 and Eq. 43-17 (and the fact that mass is proportional to the number of atoms), the amount decayed is

$$\begin{aligned} |\Delta m| &= m|_{t_f=16.0\,\text{h}} - m|_{t_i=14.0\,\text{h}} \\ &= m_0(1-e^{-t_i\ln 2/T_{1/2}}) - m_0(1-e^{-t_f\ln 2/T_{1/2}}) \\ &= m_0(e^{-t_f\ln 2/T_{1/2}} - e^{-t_i\ln 2/T_{1/2}}) \\ &= (5.50\,\text{g})\left[e^{-(16.0\,\text{h}/12.7\,\text{h})\ln 2} - e^{-(14.0\,\text{h}/12.7\,\text{h})\ln 2}\right] \\ &= 0.256\ \text{g}\ . \end{aligned}$$

35. (a) We assume that the chlorine in the sample had the naturally occurring isotopic mixture, so the average mass number was 35.453, as given in Appendix F. Then, the mass of ^{226}Ra was

$$m = \frac{226}{226+2(35.453)}(0.10\,\text{g}) = 76.1\times10^{-3}\ \text{g}\ .$$

The mass of a ^{226}Ra nucleus is $(226\,\text{u})(1.661\times10^{-24}\,\text{g/u}) = 3.75\times10^{-22}\,\text{g}$, so the number of ^{226}Ra nuclei present was $N = (76.1\times10^{-3}\,\text{g})/(3.75\times10^{-22}\,\text{g}) = 2.03\times10^{20}$.

(b) The decay rate is given by $R = N\lambda = (N\ln 2)/T_{1/2}$, where λ is the disintegration constant, $T_{1/2}$ is the half-life, and N is the number of nuclei. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. Thus,

$$R = \frac{(2.03\times10^{20})\ln 2}{(1600\,\text{y})(3.156\times10^{7}\,\text{s/y})} = 2.79\times10^{9}\ \text{s}^{-1}\ .$$

36. (a) We use $R = R_0e^{-\lambda t}$ to find t:

$$t = \frac{1}{\lambda}\ln\frac{R_0}{R} = \frac{T_{1/2}}{\ln 2}\ln\frac{R_0}{R} = \frac{14.28\,\text{d}}{\ln 2}\ln\frac{3050}{170} = 59.5\ \text{d}\ .$$

(b) The required factor is

$$\frac{R_0}{R} = e^{\lambda t} = e^{t\ln 2/T_{1/2}} = e^{(3.48\,\text{d}/14.28\,\text{d})\ln 2} = 1.18\ .$$

37. We label the two isotopes with subscripts 1 (for ^{32}P) and 2 (for ^{33}P). Initially, 10% of the decays come from ^{33}P, which implies that the initial rate $R_{02} = 9R_{01}$. Using Eq. 43-16, this means

$$R_{01} = \lambda_1 N_{01} = \frac{1}{9}R_{02} = \frac{1}{9}\lambda_2 N_{02}\ .$$

At time t, we have $R_1 = R_{01}e^{-\lambda_1 t}$ and $R_2 = R_{02}e^{-\lambda_2 t}$. We seek the value of t for which $R_1 = 9R_2$ (which means 90% of the decays arise from ^{33}P). We divide equations to obtain $(R_{01}/R_{02})e^{-(\lambda_1-\lambda_2)t} = 9$, and solve for t:

$$\begin{aligned} t &= \frac{1}{\lambda_1-\lambda_2}\ln\left(\frac{R_{01}}{9R_{02}}\right) = \frac{\ln(R_{01}/9R_{02})}{\ln 2/T_{1/2\,1} - \ln 2/T_{1/2\,2}} \\ &= \frac{\ln[(1/9)^2]}{\ln 2[(14.3\,\text{d})^{-1} - (25.3\,\text{d})^{-1}]} = 209\ \text{d}\ . \end{aligned}$$

38. We have one alpha particle (helium nucleus) produced for every plutonium nucleus that decays. To find the number that have decayed, we use Eq. 43-14, Eq. 43-17, and adapt Eq. 43-20:

$$N_0 - N = N_0 \left(1 - e^{-t \ln 2/T_{1/2}}\right) = N_A \frac{12.0\,\text{g/mol}}{239\,\text{g/mol}} \left(1 - e^{-20000 \ln 2/24100}\right)$$

where N_A is the Avogadro constant. This yields 1.32×10^{22} alpha particles produced. In terms of the amount of helium gas produced (assuming the α particles slow down and capture the appropriate number of electrons), this corresponds to

$$m_{\text{He}} = \left(\frac{1.32 \times 10^{22}}{6.02 \times 10^{23}/\text{mol}}\right)(4.0\,\text{g/mol}) = 87.9 \times 10^{-3}\ \text{g}\ .$$

39. The number N of undecayed nuclei present at any time and the rate of decay R at that time are related by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$, so $R = (N \ln 2)/T_{1/2}$ and $T_{1/2} = (N \ln 2)/R$. Since 15.0% by mass of the sample is ^{147}Sm, the number of ^{147}Sm nuclei present in the sample is

$$N = \frac{(0.150)(1.00\,\text{g})}{(147\,\text{u})(1.661 \times 10^{-24}\,\text{g/u})} = 6.143 \times 10^{20}\ .$$

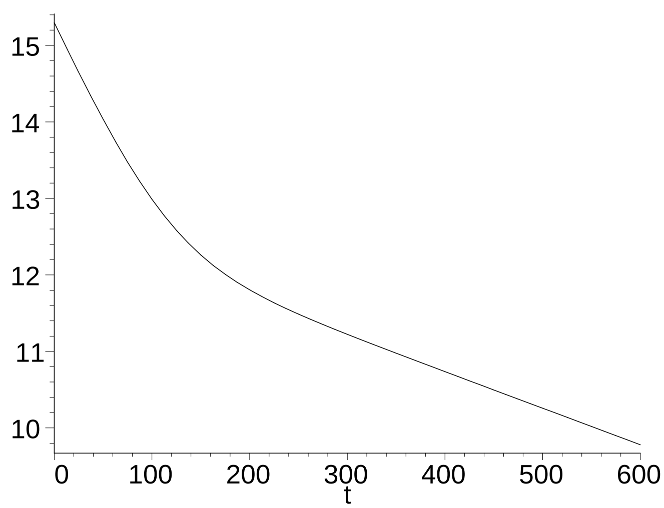
Thus

$$T_{1/2} = \frac{(6.143 \times 10^{20}) \ln 2}{120\,\text{s}^{-1}} = 3.55 \times 10^{18}\,\text{s} = 1.12 \times 10^{11}\ \text{y}\ .$$

40. We note that $2.42\,\text{min} = 145.2\,\text{s}$. We are asked to plot (with SI units understood)

$$\ln R = \ln\left(R_0 e^{-\lambda t} + R_0' e^{-\lambda' t}\right)$$

where $R_0 = 3.1 \times 10^5$, $R_0' = 4.1 \times 10^6$, $\lambda = \ln 2/145.2$ and $\lambda' = \ln 2/24.6$. Our plot is shown below.



We note that the magnitude of the slope for small t is λ' (the disintegration constant for ^{110}Ag), and for large t is λ (the disintegration constant for ^{108}Ag).

41. If N is the number of undecayed nuclei present at time t, then

$$\frac{dN}{dt} = R - \lambda N$$

where R is the rate of production by the cyclotron and λ is the disintegration constant. The second term gives the rate of decay. Rearrange the equation slightly and integrate:

$$\int_{N_0}^{N} \frac{dN}{R - \lambda N} = \int_0^t dt$$

where N_0 is the number of undecayed nuclei present at time $t = 0$. This yields

$$-\frac{1}{\lambda} \ln \frac{R - \lambda N}{R - \lambda N_0} = t \; .$$

We solve for N:

$$N = \frac{R}{\lambda} + \left(N_0 - \frac{R}{\lambda} \right) e^{-\lambda t} \; .$$

After many half-lives, the exponential is small and the second term can be neglected. Then, $N = R/\lambda$, regardless of the initial value N_0. At times that are long compared to the half-life, the rate of production equals the rate of decay and N is a constant.

42. Combining Eqs. 43-19 and 43-20, we obtain

$$M_{\text{sam}} = N \frac{M_{\text{K}}}{N_{\text{A}}} = \left(\frac{R T_{1/2}}{\ln 2} \right) \left(\frac{40\,\text{g/mol}}{6.02 \times 10^{23}/\text{mol}} \right)$$

which gives 0.66 g for the mass of the sample once we plug in 1.7×10^5/s for the decay rate and $1.28 \times 10^9\,\text{y} = 4.04 \times 10^{16}\,\text{s}$ for the half-life.

43. (a) The sample is in secular equilibrium with the source and the decay rate equals the production rate. Let R be the rate of production of ^{56}Mn and let λ be the disintegration constant. According to the result of problem 41, $R = \lambda N$ after a long time has passed. Now, $\lambda N = 8.88 \times 10^{10}\,\text{s}^{-1}$, so $R = 8.88 \times 10^{10}\,\text{s}^{-1}$.

 (b) They decay at the same rate as they are produced, $8.88 \times 10^{10}\,\text{s}^{-1}$.

 (c) We use $N = R/\lambda$. If $T_{1/2}$ is the half-life, then the disintegration constant is $\lambda = (\ln 2)/T_{1/2} = (\ln 2)/(2.58\,\text{h}) = 0.269\,\text{h}^{-1} = 7.46 \times 10^{-5}\,\text{s}^{-1}$, so $N = (8.88 \times 10^{10}\,\text{s}^{-1})/(7.46 \times 10^{-5}\,\text{s}^{-1}) = 1.19 \times 10^{15}$.

 (d) The mass of a ^{56}Mn nucleus is $(56\,\text{u})(1.661 \times 10^{-24}\,\text{g/u}) = 9.30 \times 10^{-23}\,\text{g}$ and the total mass of ^{56}Mn in the sample at the end of the bombardment is $Nm = (1.19 \times 10^{15})(9.30 \times 10^{-23}\,\text{g}) = 1.11 \times 10^{-7}\,\text{g}$.

44. (a) The rate at which Radium-226 is decaying is

$$R = \lambda N = \left(\frac{\ln 2}{T_{1/2}} \right) \left(\frac{M}{m} \right) = \frac{(\ln 2)(1.00\,\text{mg})(6.02 \times 10^{23}/\text{mol})}{(1600\,\text{y})(3.15 \times 10^7\,\text{s/y})(226\,\text{g/mol})} = 3.66 \times 10^7\ \text{s}^{-1} \; .$$

 (b) Since $1600\,\text{y} \gg 3.82\,\text{d}$ the time required is $t \gg 3.82\,\text{d}$.

 (c) It is decaying at the same rate as it is produced, or $R = 3.66 \times 10^7\,\text{s}^{-1}$.

 (d) From $R_{\text{Ra}} = R_{\text{Rn}}$ and $R = \lambda N = (\ln 2/T_{1/2})(M/m)$, we get

$$\begin{aligned} M_{\text{Rn}} &= \left(\frac{T_{1/2\,\text{Rn}}}{T_{1/2\,\text{Ra}}} \right) \left(\frac{m_{\text{Rn}}}{m_{\text{Ra}}} \right) M_{\text{Ra}} \\ &= \frac{(3.82\,\text{d})(1.00 \times 10^{-3}\,\text{g})(222\,\text{u})}{(1600\,\text{y})(365\,\text{d/y})(226\,\text{u})} \\ &= 6.42 \times 10^{-9}\ \text{g} \; . \end{aligned}$$

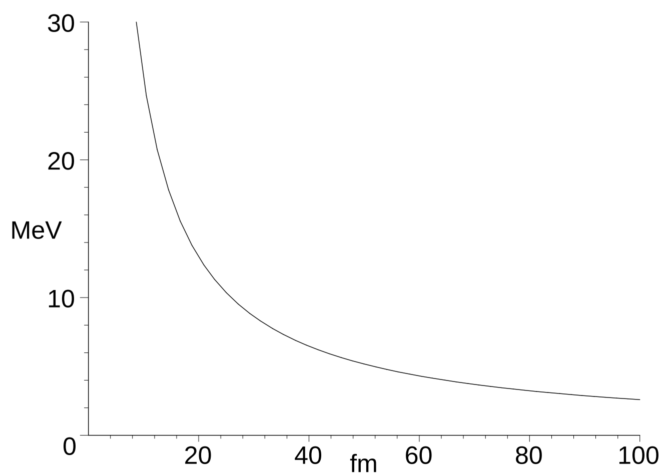
45. Since the spreading is assumed uniform, the count rate $R = 74{,}000/\text{s}$ is given by $R = \lambda N = \lambda(M/m)(a/A)$, where $M = 400\,\text{g}$, m is the mass of the ^{90}Sr nucleus, $A = 2000\,\text{km}^2$, and a is the area in question. We solve for a:

$$\begin{aligned} a &= A\left(\frac{m}{M}\right)\left(\frac{R}{\lambda}\right) = \frac{AmRT_{1/2}}{M\ln 2} \\ &= \frac{(2000\times 10^6\,\text{m}^2)(90\,\text{g/mol})(29\,\text{y})(3.15\times 10^7\,\text{s/y})(74{,}000/\text{s})}{(400\,\text{g})(6.02\times 10^{23}/\text{mol})(\ln 2)} \\ &= 7.3\times 10^{-2}\,\text{m}^2 = 730\ \text{cm}^2\ . \end{aligned}$$

46. Eq. 25-43 gives the electrostatic potential energy between two uniformly charged spherical charges (in this case $q_1 = 2e$ and $q_2 = 90e$) with r being the distance between their centers. Assuming the "uniformly charged spheres" condition is met in this instance, we write the equation in such a way that we can make use of $k = 1/4\pi\varepsilon_0$ and the electronvolt unit:

$$U = k\frac{(2e)(90e)}{r} = \left(8.99\times 10^9\,\frac{\text{V}\cdot\text{m}}{\text{C}}\right)\frac{\left(3.2\times 10^{-19}\,\text{C}\right)(90e)}{r} = \frac{2.59\times 10^{-7}}{r}\ \text{eV}$$

with r understood to be in meters. It is convenient to write this for r in femtometers, in which case $U = 259/r$ MeV. This is shown plotted below.



47. The fraction of undecayed nuclei remaining after time t is given by

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$$

where λ is the disintegration constant and $T_{1/2}$ $(= (\ln 2)/\lambda)$ is the half-life. The time for half the original ^{238}U nuclei to decay is 4.5×10^9 y. For ^{244}Pu at that time,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times 10^9\,\text{y})}{8.2\times 10^7\,\text{y}} = 38.0$$

and

$$\frac{N}{N_0} = e^{-38.0} = 3.1\times 10^{-17}\ .$$

For ^{248}Cm at that time,

$$\frac{(\ln 2)t}{T_{1/2}} = \frac{(\ln 2)(4.5\times 10^9\,\text{y})}{3.4\times 10^5\,\text{y}} = 9170$$

and

$$\frac{N}{N_0} = e^{-9170} = 3.31 \times 10^{-3983} .$$

For any reasonably sized sample this is less than one nucleus and may be taken to be zero. A standard calculator probably cannot evaluate e^{-9170} directly. Our recommendation is to treat it as $(e^{-91.70})^{100}$.

48. (a) The nuclear reaction is written as ${}^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}$. The energy released is

$$\begin{aligned} \Delta E_1 &= (m_{\text{U}} - m_{\text{He}} - m_{\text{Th}})c^2 \\ &= (238.05079\,\text{u} - 4.00260\,\text{u} - 234.04363\,\text{u})(931.5\,\text{MeV/u}) \\ &= 4.25\ \text{MeV} . \end{aligned}$$

(b) The reaction series consists of ${}^{238}\text{U} \rightarrow {}^{237}\text{U} + n$, followed by

$$\begin{aligned} {}^{237}\text{U} \rightarrow\ & {}^{236}\text{Pa} + p \\ & {}^{236}\text{Pa} \rightarrow {}^{235}\text{Pa} + n \\ & {}^{235}\text{Pa} \rightarrow {}^{234}\text{Th} + p \end{aligned}$$

The net energy released is then

$$\begin{aligned} \Delta E_2 &= (m_{^{238}\text{U}} - m_{^{237}\text{U}} - m_n)c^2 + (m_{^{237}\text{U}} - m_{^{236}\text{Pa}} - m_p)c^2 \\ &\quad + (m_{^{236}\text{Pa}} - m_{^{235}\text{Pa}} - m_n)c^2 + (m_{^{235}\text{Pa}} - m_{^{234}\text{Th}} - m_p)c^2 \\ &= (m_{^{238}\text{U}} - 2m_n - 2m_p - m_{^{234}\text{Th}})c^2 \\ &= [238.05079\,\text{u} - 2(1.00867\,\text{u}) - 2(1.00783\,\text{u}) - 234.04363\,\text{u}](931.5\,\text{MeV/u}) \\ &= -24.1\ \text{MeV} . \end{aligned}$$

(c) This leads us to conclude that the binding energy of the α particle is

$$\left|(2m_n + 2m_p - m_{\text{He}})c^2\right| = |-24.1\,\text{MeV} - 4.25\,\text{MeV}| = 28.3\ \text{MeV} .$$

49. Energy and momentum are conserved. We assume the residual thorium nucleus is in its ground state. Let K_α be the kinetic energy of the alpha particle and K_{Th} be the kinetic energy of the thorium nucleus. Then, $Q = K_\alpha + K_{\text{Th}}$. We assume the uranium nucleus is initially at rest. Then, conservation of momentum yields $0 = p_\alpha + p_{\text{Th}}$, where p_α is the momentum of the alpha particle and p_{Th} is the momentum of the thorium nucleus. Both particles travel slowly enough that the classical relationship between momentum and energy can be used. Thus $K_{\text{Th}} = p_{\text{Th}}^2/2m_{\text{Th}}$, where m_{Th} is the mass of the thorium nucleus. We substitute $p_{\text{Th}} = -p_\alpha$ and use $K_\alpha = p_\alpha^2/2m_\alpha$ to obtain $K_{\text{Th}} = (m_\alpha/m_{\text{Th}})K_\alpha$. Consequently,

$$Q = K_\alpha + \frac{m_\alpha}{m_{\text{Th}}}K_\alpha = \left(1 + \frac{m_\alpha}{m_{\text{Th}}}\right)K_\alpha = \left(1 + \frac{4.00\,\text{u}}{234\,\text{u}}\right)(4.196\,\text{MeV}) = 4.27\ \text{MeV} .$$

50. (a) The disintegration energy for uranium-235 “decaying” into thorium-232 is

$$\begin{aligned} Q_3 &= \left(m_{^{235}\text{U}} - m_{^{232}\text{Th}} - m_{^{3}\text{He}}\right)c^2 \\ &= (235.0439\,\text{u} - 232.0381\,\text{u} - 3.0160\,\text{u})(931.5\,\text{MeV/u}) \\ &= -9.50\ \text{MeV} . \end{aligned}$$

(b) Similarly, the disintegration energy for uranium-235 decaying into thorium-231 is

$$\begin{aligned} Q_4 &= \left(m_{^{235}\text{U}} - m_{^{231}\text{Th}} - m_{^{4}\text{He}}\right)c^2 \\ &= (235.0439\,\text{u} - 231.0363\,\text{u} - 4.0026\,\text{u})(931.5\,\text{MeV/u}) \\ &= 4.66\ \text{MeV} . \end{aligned}$$

(c) Finally, the considered transmutation of uranium-235 into thorium-230 has a Q-value of

$$\begin{aligned} Q_5 &= (m_{^{235}\mathrm{U}} - m_{^{230}\mathrm{Th}} - m_{^5\mathrm{He}})\,c^2 \\ &= (235.0439\,\mathrm{u} - 230.0331\,\mathrm{u} - 5.0122\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= -1.30\ \mathrm{MeV}\ . \end{aligned}$$

Only the second decay process (the α decay) is spontaneous, as it releases energy.

51. (a) For the first reaction

$$\begin{aligned} Q_1 &= (m_{\mathrm{Ra}} - m_{\mathrm{Pb}} - m_{\mathrm{C}})c^2 \\ &= (223.01850\,\mathrm{u} - 208.98107\,\mathrm{u} - 14.00324\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= 31.8\ \mathrm{MeV}\ , \end{aligned}$$

and for the second one

$$\begin{aligned} Q_2 &= (m_{\mathrm{Ra}} - m_{\mathrm{Rn}} - m_{\mathrm{He}})c^2 \\ &= (223.01850\,\mathrm{u} - 219.00948\,\mathrm{u} - 4.00260\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= 5.98\ \mathrm{MeV}\ . \end{aligned}$$

(b) From $U \propto q_1 q_2/r$, we get

$$U_1 \approx U_2\left(\frac{q_{\mathrm{Pb}}\,q_C}{q_{\mathrm{Rn}}\,q_{\mathrm{He}}}\right) = (30.0\,\mathrm{MeV})\frac{(82e)(6.0e)}{(86e)(2.0e)} = 86\ \mathrm{MeV}\ .$$

52. (a) The mass number A of a radionuclide changes by 4 in an α decay and is unchanged in a β decay. If the mass numbers of two radionuclides are given by $4n+k$ and $4n'+k$ (where $k = 0, 1, 2, 3$), then the heavier one can decay into the lighter one by a series of α (and β) decays, as their mass numbers differ by only an integer times 4. If $A = 4n+k$, then after α-decaying for m times, its mass number becomes $A = 4n + k - 4m = 4(n-m) + k$, still in the same chain.

(b) $235 = 58\times 4 + 3 = 4n_1 + 3,\ 236 = 59\times 4 = 4n_2,\ 238 = 59\times 4 + 2 = 4n_2 + 2,\ 239 = 59\times 4 + 3 = 4n_2 + 3,\ 240 = 60\times 4 = 4n_3,\ 245 = 61\times 4 + 1 = 4n_4 + 1,\ 246 = 61\times 4 + 2 = 4n_4 + 2,\ 249 = 62\times 4 + 1 = 4n_5 + 1,\ 253 = 63\times 4 + 1 = 4n_6 + 1.$

53. Let ${}^A_Z X$ represent the unknown nuclide. The reaction equation is

$${}^A_Z X + {}^1_0\mathrm{n} \to {}^{\;0}_{-1}\mathrm{e} + 2\,{}^4_2\mathrm{He}\ .$$

Conservation of charge yields $Z + 0 = -1 + 4$ or $Z = 3$. Conservation of mass number yields $A + 1 = 0 + 8$ or $A = 7$. According to the periodic table in Appendix G (also see Appendix F), lithium has atomic number 3, so the nuclide must be ${}^7_3\mathrm{Li}$.

54. (a) We recall that $mc^2 = 0.511$ MeV from Table 38-3, and note that the result of problem 3 in Chapter 39 can be written as $hc = 1240\,\mathrm{MeV\cdot fm}$. Using Eq. 38-51 and Eq. 39-13, we obtain

$$\begin{aligned} \lambda &= \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} \\ &= \frac{1240\,\mathrm{MeV\cdot fm}}{\sqrt{(1.0\,\mathrm{MeV})^2 + 2(1.0\,\mathrm{MeV})(0.511\,\mathrm{MeV})}} = 9.0\times 10^2\ \mathrm{fm}\ . \end{aligned}$$

(b) $r = r_0 A^{1/3} = (1.2\,\mathrm{fm})(150)^{1/3} = 6.4\,\mathrm{fm}$.

(c) Since $\lambda \gg r$ the electron cannot be confined in the nuclide. We recall from Chapters 40 and 41, that at least $\lambda/2$ was needed in any particular direction, to support a standing wave in an "infinite well." A finite well is able to support *slightly* less than $\lambda/2$ (as one can infer from the ground state wavefunction in Fig. 40-8), but in the present case λ/r is far too big to be supported.

(d) A strong case can be made on the basis of the remarks in part (c), above.

55. Let $M_{\rm Cs}$ be the mass of one atom of ${}^{137}_{55}$Cs and $M_{\rm Ba}$ be the mass of one atom of ${}^{137}_{56}$Ba. To obtain the nuclear masses, we must subtract the mass of 55 electrons from $M_{\rm Cs}$ and the mass of 56 electrons from $M_{\rm Ba}$. The energy released is $Q = [(M_{\rm Cs} - 55m) - (M_{\rm Ba} - 56m) - m]\,c^2$, where m is the mass of an electron. Once cancellations have been made, $Q = (M_{\rm Cs} - M_{\rm Ba})c^2$ is obtained. Therefore,

$$Q = [136.9071\,\text{u} - 136.9058\,\text{u}]\,c^2 = (0.0013\,\text{u})c^2 = (0.0013\,\text{u})(931.5\,\text{MeV/u}) = 1.21\ \text{MeV}\ .$$

56. Assuming the neutrino has negligible mass, then

$$\Delta m\, c^2 = (\mathbf{m}_{\rm Ti} - \mathbf{m}_{\rm V} - m_e)\, c^2\ .$$

Now, since Vanadium has 23 electrons (see Appendix F and/or G) and Titanium has 22 electrons, we can add and subtract $22m_e$ to the above expression and obtain

$$\Delta m\, c^2 = (\mathbf{m}_{\rm Ti} + 22m_e - \mathbf{m}_{\rm V} - 23m_e)\, c^2 = (m_{\rm Ti} - m_{\rm V}\,)\, c^2\ .$$

We note that our final expression for Δmc^2 involves the *atomic* masses, and that this assumes (due to the way they are usually tabulated) the atoms are in the ground states (which is certainly not the case here, as we discuss below). The question now is: do we set $Q = -\Delta mc^2$ as in Sample Problem 43-7? The answer is "no." The atom is left in an excited (high energy) state due to the fact that an electron was captured from the lowest shell (where the absolute value of the energy, E_K, is quite large for large Z – see Eq. 41-25). To a very good approximation, the energy of the K-shell electron in Vanadium is equal to that in Titanium (where there is now a "vacancy" that must be filled by a readjustment of the whole electron cloud), and we write $Q = -\Delta mc^2 - E_K$ so that Eq. 43-27 still holds. Thus,

$$Q = (m_{\rm V} - m_{\rm Ti}\,)\, c^2 - E_K\ .$$

57. The decay scheme is $\text{n} \rightarrow \text{p} + \text{e}^- + \nu$. The electron kinetic energy is a maximum if no neutrino is emitted. Then, $K_{\max} = (m_n - m_p - m_e)c^2$, where m_n is the mass of a neutron, m_p is the mass of a proton, and m_e is the mass of an electron. Since $m_p + m_e = m_H$, where m_H is the mass of a hydrogen atom, this can be written $K_{\max} = (m_n - m_H)c^2$. Hence, $K_{\max} = (840 \times 10^{-6}\,\text{u})c^2 = (840 \times 10^{-6}\,\text{u})(931.5\,\text{MeV/u}) = 0.783\,\text{MeV}$.

58. We obtain

$$\begin{aligned} Q &= (m_{\rm V} - m_{\rm Ti}\,)\, c^2 - E_K \\ &= (48.94852\,\text{u} - 48.94787\,\text{u})\,(931.5\,\text{MeV/u}) - 0.00547\,\text{MeV} \\ &= 0.600\ \text{MeV}\ . \end{aligned}$$

59. (a) Since the positron has the same mass as an electron, and the neutrino has negligible mass, then

$$\Delta m\, c^2 = (\mathbf{m}_{\rm B} + m_e - \mathbf{m}_{\rm C})\, c^2\ .$$

Now, since Carbon has 6 electrons (see Appendix F and/or G) and Boron has 5 electrons, we can add and subtract $6m_e$ to the above expression and obtain

$$\Delta m\, c^2 = (\mathbf{m}_{\rm B} + 7m_e - \mathbf{m}_{\rm C} - 6m_e)\, c^2 = (m_{\rm B} + 2m_e - m_{\rm C}\,)\, c^2\ .$$

We note that our final expression for Δmc^2 involves the *atomic* masses, as well an "extra" term corresponding to two electron masses. From Eq. 38-47 and Table 38-3, we obtain

$$Q = (m_{\rm C} - m_{\rm B} - 2m_e)\, c^2 = (m_{\rm C} - m_{\rm B})\, c^2 - 2(0.511\,\text{MeV})\ .$$

(b) The disintegration energy for the positron decay of Carbon-11 is

$$Q = (11.011434\,\text{u} - 11.009305\,\text{u})\,(931.5\,\text{MeV/u}) - 1.022\,\text{MeV} = 0.961\ \text{MeV}\ .$$

60. (a) The rate of heat production is

$$\begin{aligned}
\frac{dE}{dt} &= \sum_{i=1}^{3} R_i Q_i = \sum_{i=1}^{3} \lambda_1 N_i Q_i = \sum_{i=1}^{3} \left(\frac{\ln 2}{T_{1/2_i}}\right) \frac{(1.00\,\text{kg}) f_i}{m_i} Q_i \\
&= \frac{(1.00\,\text{kg})(\ln 2)(1.60 \times 10^{-13}\,\text{J/MeV})}{(3.15 \times 10^7\,\text{s/y})(1.661 \times 10^{-27}\,\text{kg/u})} \left[\frac{(4 \times 10^{-6})(51.7\,\text{MeV})}{(238\,\text{u})(4.47 \times 10^9\,\text{y})}\right. \\
&\qquad \left. + \frac{(13 \times 10^{-6})(42.7\,\text{MeV})}{(232\,\text{u})(1.41 \times 10^{10}\,\text{y})} + \frac{(4 \times 10^{-6})(1.31\,\text{MeV})}{(40\,\text{u})(1.28 \times 10^9\,\text{y})}\right] \\
&= 1.0 \times 10^{-9}\ \text{W}\ .
\end{aligned}$$

(b) The contribution to heating, due to radioactivity, is $P = (2.7 \times 10^{22}\,\text{kg})(1.0 \times 10^{-9}\,\text{W/kg}) = 2.7 \times 10^{13}\,\text{W}$, which is very small compared to what is received from the Sun.

61. Since the electron has the maximum possible kinetic energy, no neutrino is emitted. Since momentum is conserved, the momentum of the electron and the momentum of the residual sulfur nucleus are equal in magnitude and opposite in direction. If p_e is the momentum of the electron and p_S is the momentum of the sulfur nucleus, then $p_S = -p_e$. The kinetic energy K_S of the sulfur nucleus is $K_S = p_S^2/2M_S = p_e^2/2M_S$, where M_S is the mass of the sulfur nucleus. Now, the electron's kinetic energy K_e is related to its momentum by the relativistic equation $(p_e c)^2 = K_e^2 + 2K_e mc^2$, where m is the mass of an electron. See Eq. 38-51. Thus,

$$\begin{aligned}
K_S &= \frac{(p_e c)^2}{2M_S c^2} = \frac{K_e^2 + 2K_e mc^2}{2M_S c^2} = \frac{(1.71\,\text{MeV})^2 + 2(1.71\,\text{MeV})(0.511\,\text{MeV})}{2(32\,\text{u})(931.5\,\text{MeV/u})} \\
&= 7.83 \times 10^{-5}\,\text{MeV} = 78.3\ \text{eV}
\end{aligned}$$

where $mc^2 = 0.511\,\text{MeV}$ is used (see Table 38-3).

62. We solve for t from $R = R_0 e^{-\lambda t}$:

$$t = \frac{1}{\lambda} \ln \frac{R_0}{R} = \left(\frac{5730\,\text{y}}{\ln 2}\right) \ln\left[\left(\frac{15.3}{63.0}\right)\left(\frac{5.00}{1.00}\right)\right] = 1.61 \times 10^3\ \text{y}\ .$$

63. (a) The mass of a ^{238}U atom is $(238\,\text{u})(1.661 \times 10^{-24}\,\text{g/u}) = 3.95 \times 10^{-22}\,\text{g}$, so the number of uranium atoms in the rock is $N_U = (4.20 \times 10^{-3}\,\text{g})/(3.95 \times 10^{-22}\,\text{g}) = 1.06 \times 10^{19}$. The mass of a ^{206}Pb atom is $(206\,\text{u})(1.661 \times 10^{-24}\,\text{g}) = 3.42 \times 10^{-22}\,\text{g}$, so the number of lead atoms in the rock is $N_{\text{Pb}} = (2.135 \times 10^{-3}\,\text{g})/(3.42 \times 10^{-22}\,\text{g}) = 6.24 \times 10^{18}$.

(b) If no lead was lost, there was originally one uranium atom for each lead atom formed by decay, in addition to the uranium atoms that did not yet decay. Thus, the original number of uranium atoms was $N_{U0} = N_U + N_{\text{Pb}} = 1.06 \times 10^{19} + 6.24 \times 10^{18} = 1.68 \times 10^{19}$.

(c) We use

$$N_U = N_{U0}\, e^{-\lambda t}$$

where λ is the disintegration constant for the decay. It is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$. Thus

$$t = -\frac{1}{\lambda} \ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{T_{1/2}}{\ln 2} \ln\left(\frac{N_U}{N_{U0}}\right) = -\frac{4.47 \times 10^9\,\text{y}}{\ln 2} \ln\left(\frac{1.06 \times 10^{19}}{1.68 \times 10^{19}}\right) = 2.97 \times 10^9\ \text{y}\ .$$

64. The original amount of ^{238}U the rock contains is given by

$$m_0 = me^{\lambda t} = (3.70\,\text{mg})\,e^{(\ln 2)(260\times 10^6\,\text{y})/(4.47\times 10^9\,\text{y})} = 3.85 \text{ mg} .$$

Thus, the amount of lead produced is

$$m' = (m_0 - m)\left(\frac{m_{206}}{m_{238}}\right) = (3.85\,\text{mg} - 3.70\,\text{mg})\left(\frac{206}{238}\right) = 0.132 \text{ mg} .$$

65. We can find the age t of the rock from the masses of ^{238}U and ^{206}Pb. The initial mass of ^{238}U is

$$m_{\text{U}_0} = m_\text{U} + \frac{238}{206} m_\text{Pb} .$$

Therefore, $m_\text{U} = m_{\text{U}_0} e^{-\lambda_\text{U} t} = (m_\text{U} + m_{^{238}\text{Pb}}/206)e^{-(t\ln 2)/T_{1/2\text{U}}}$. We solve for t:

$$\begin{aligned} t &= \frac{T_{1/2\text{U}}}{\ln 2} \ln\left(\frac{m_\text{U} + (238/206)m_\text{Pb}}{m_\text{U}}\right) \\ &= \frac{4.47\times 10^9\text{ y}}{\ln 2} \ln\left[1 + \left(\frac{238}{206}\right)\left(\frac{0.15\,\text{mg}}{0.86\,\text{mg}}\right)\right] \\ &= 1.18\times 10^9 \text{ y} . \end{aligned}$$

For the β decay of ^{40}K, the initial mass of ^{40}K is

$$m_{\text{K}_0} = m_\text{K} + (40/40)m_\text{Ar} = m_\text{K} + m_\text{Ar} ,$$

so

$$m_\text{K} = m_{\text{K}_0} e^{-\lambda_\text{K} t} = (m_\text{K} + m_\text{Ar})e^{-\lambda_\text{K} t} .$$

We solve for m_K:

$$\begin{aligned} m_\text{K} &= \frac{m_\text{Ar} e^{-\lambda_\text{K} t}}{1 - e^{-\lambda_\text{K} t}} = \frac{m_\text{Ar}}{e^{\lambda_\text{K} t} - 1} \\ &= \frac{1.6\,\text{mg}}{e^{(\ln 2)(1.18\times 10^9\,\text{y})/(1.25\times 10^9\,\text{y})} - 1} = 1.7 \text{ mg} . \end{aligned}$$

66. The becquerel (Bq) and curie (Ci) are defined in §43-3. Thus, $R = 8700/60 = 145$ Bq, and

$$R = \frac{145\,\text{Bq}}{3.7\times 10^{10}\,\text{Bq/Ci}} = 3.92\times 10^{-9} \text{ Ci} .$$

67. The decay rate R is related to the number of nuclei N by $R = \lambda N$, where λ is the disintegration constant. The disintegration constant is related to the half-life $T_{1/2}$ by $\lambda = (\ln 2)/T_{1/2}$, so $N = R/\lambda = RT_{1/2}/\ln 2$. Since $1\,\text{Ci} = 3.7\times 10^{10}$ disintegrations/s,

$$N = \frac{(250\,\text{Ci})(3.7\times 10^{10}\,\text{s}^{-1}/\text{Ci})(2.7\,\text{d})(8.64\times 10^4\,\text{s/d})}{\ln 2} = 3.11\times 10^{18} .$$

The mass of a ^{198}Au atom is $M = (198\,\text{u})(1.661\times 10^{-24}\,\text{g/u}) = 3.29\times 10^{-22}\,\text{g}$, so the mass required is $NM = (3.11\times 10^{18})(3.29\times 10^{-22}\,\text{g}) = 1.02\times 10^{-3}\,\text{g} = 1.02\,\text{mg}$.

68. The annual dose equivalent is $(20\,\text{h})(52\,\text{week/y})(7.0\,\mu\text{Sv/h}) = 7.3\,\text{mSv}$.

69. The dose equivalent is the product of the absorbed dose and the RBE factor, so the absorbed dose is (dose equivalent)/(RBE) $= (250\times 10^{-6}\,\text{Sv})/(0.85) = 2.94\times 10^{-4}\,\text{Gy}$. But $1\,\text{Gy} = 1\,\text{J/kg}$, so the absorbed dose is

$$\left(2.94\times 10^{-4}\,\text{Gy}\right)\left(1\,\frac{\text{J}}{\text{kg}\cdot\text{Gy}}\right) = 2.94\times 10^{-4} \text{ J/kg} .$$

To obtain the total energy received, we multiply this by the mass receiving the energy: $E = (2.94\times 10^{-4}\,\text{J/kg})(44\,\text{kg}) = 1.29\times 10^{-2}\,\text{J}$.

70. (a) Using Eq. 43-31, the energy absorbed is

$$(2.4 \times 10^{-4}\,\text{Gy})(75\,\text{kg}) = 18\,\text{mJ} .$$

(b) The dose equivalent is

$$(2.4 \times 10^{-4}\,\text{Gy})(12) = 2.9 \times 10^{-3}\,\text{Sv} = 0.29 \text{ rem}$$

where Eq. 43-32 is used in the last step.

71. (a) Adapting Eq. 43-20, we find

$$N_0 = \frac{(2.5 \times 10^{-3}\,\text{g})(6.02 \times 10^{23}/\text{mol})}{239\,\text{g/mol}} = 6.3 \times 10^{18} .$$

(b) From Eq. 43-14 and Eq. 43-17,

$$\begin{aligned} |\Delta N| &= N_0 \left[1 - e^{-t\ln 2/T_{1/2}}\right] \\ &= (6.3 \times 10^{18}) \left[1 - e^{-(12\,\text{h})\ln 2/(24{,}100\,\text{y})(8760\,\text{h/y})}\right] \\ &= 2.5 \times 10^{11} . \end{aligned}$$

(c) The energy absorbed by the body is

$$(0.95)E_\alpha|\Delta N| = (0.95)\,(5.2\,\text{MeV})\,\left(2.5 \times 10^{11}\right)\,\left(1.6 \times 10^{-13}\,\text{J/MeV}\right) = 0.20 \text{ J} .$$

(d) On a per unit mass basis, the previous result becomes (according to Eq. 43-31)

$$\frac{0.20\,\text{mJ}}{85\,\text{kg}} = 2.3 \times 10^{-3}\,\text{J/kg} = 2.3 \text{ mGy} .$$

(e) Using Eq. 43-32, $(2.3\,\text{mGy})(13) = 30\,\text{mSv}$.

72. From Eq. 20-24, we obtain

$$T = \frac{2}{3}\left(\frac{K_{\text{avg}}}{k}\right) = \frac{2}{3}\left(\frac{5.00 \times 10^6\,\text{eV}}{8.62 \times 10^{-5}\,\text{eV/K}}\right) = 3.9 \times 10^{10} \text{ K} .$$

73. (a) Following Sample Problem 43-10, we compute

$$\Delta E \approx \frac{\hbar}{t_{\text{avg}}} = \frac{(4.14 \times 10^{-15}\,\text{eV}\cdot\text{fs})/2\pi}{1.0 \times 10^{-22}\,\text{s}} = 6.6 \times 10^6 \text{ eV} .$$

(b) In order to fully distribute the energy in a fairly large nucleus, and create a "compound nucleus" equilibrium configuration, about 10^{-15} s is typically required. A reaction state that exists no more than about 10^{-22} s does not qualify as a compound nucleus.

74. (a) We compare both the proton numbers (atomic numbers, which can be found in Appendix F and/or G) and the neutron numbers (see Eq. 43-1) with the magic nucleon numbers (special values of either Z or N) listed in §43-8. We find that ^{18}O, ^{60}Ni, ^{92}Mo, ^{144}Sm, and ^{207}Pb each have a filled shell for either the protons or the neutrons (two of these, ^{18}O and ^{92}Mo, are explicitly discussed in that section).

(b) Consider ^{40}K, which has $Z = 19$ protons (which is one less than the magic number 20). It has $N = 21$ neutrons, so it has one neutron outside a closed shell for neutrons, and thus qualifies for this list. Others in this list include ^{91}Zr, ^{121}Sb, and ^{143}Nd.

(c) Consider ^{13}C, which has $Z = 6$ and $N = 13 - 6 = 7$ neutrons. Since 8 is a magic number, then ^{13}C has a vacancy in an otherwise filled shell for neutrons. Similar arguments lead to inclusion of ^{40}K, ^{49}Ti, ^{205}Tl, and ^{207}Pb in this list.

75. A generalized formation reaction can be written X + x $\to$ Y, where X is the target nucleus, x is the incident light particle, and Y is the excited compound nucleus (^{20}Ne). We assume X is initially at rest. Then, conservation of energy yields

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + K_Y + E_Y$$

where m_X, m_x, and m_Y are masses, K_x and K_Y are kinetic energies, and E_Y is the excitation energy of Y. Conservation of momentum yields

$$p_x = p_Y \ .$$

Now, $K_Y = p_Y^2/2m_Y = p_x^2/2m_Y = (m_x/m_Y)K_x$, so

$$m_X c^2 + m_x c^2 + K_x = m_Y c^2 + (m_x/m_Y)K_x + E_Y$$

and

$$K_x = \frac{m_Y}{m_Y - m_x}\left[(m_Y - m_X - m_x)c^2 + E_Y\right] \ .$$

(a) Let x represent the alpha particle and X represent the ^{16}O nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\text{u} - 15.99491\,\text{u} - 4.00260\,\text{u})(931.5\,\text{MeV/u}) = -4.722\,\text{MeV}$ and

$$K_\alpha = \frac{19.99244\,\text{u}}{19.99244\,\text{u} - 4.00260\,\text{u}}(-4.722\,\text{MeV} + 25.0\,\text{MeV}) = 25.35\ \text{MeV} \ .$$

(b) Let x represent the proton and X represent the ^{19}F nucleus. Then, $(m_Y - m_X - m_x)c^2 = (19.99244\,\text{u} - 18.99841\,\text{u} - 1.00783\,\text{u})(931.5\,\text{MeV/u}) = -12.85\,\text{MeV}$ and

$$K_\alpha = \frac{19.99244\,\text{u}}{19.99244\,\text{u} - 1.00783\,\text{u}}(-12.85\,\text{MeV} + 25.0\,\text{MeV}) = 12.80\ \text{MeV} \ .$$

(c) Let x represent the photon and X represent the ^{20}Ne nucleus. Since the mass of the photon is zero, we must rewrite the conservation of energy equation: if E_γ is the energy of the photon, then $E_\gamma + m_X c^2 = m_Y c^2 + K_Y + E_Y$. Since $m_X = m_Y$, this equation becomes $E_\gamma = K_Y + E_Y$. Since the momentum and energy of a photon are related by $p_\gamma = E_\gamma/c$, the conservation of momentum equation becomes $E_\gamma/c = p_Y$. The kinetic energy of the compound nucleus is $K_Y = p_Y^2/2m_Y = E_\gamma^2/2m_Y c^2$. We substitute this result into the conservation of energy equation to obtain

$$E_\gamma = \frac{E_\gamma^2}{2m_Y c^2} + E_Y \ .$$

This quadratic equation has the solutions

$$E_\gamma = m_Y c^2 \pm \sqrt{(m_Y c^2)^2 - 2m_Y c^2 E_Y} \ .$$

If the problem is solved using the relativistic relationship between the energy and momentum of the compound nucleus, only one solution would be obtained, the one corresponding to the negative sign above. Since $m_Y c^2 = (19.99244\,\text{u})(931.5\,\text{MeV/u}) = 1.862 \times 10^4\,\text{MeV}$,

$$\begin{aligned} E_\gamma &= (1.862 \times 10^4\,\text{MeV}) - \sqrt{(1.862 \times 10^4\,\text{MeV})^2 - 2(1.862 \times 10^4\,\text{MeV})(25.0\,\text{MeV})} \\ &= 25.0\ \text{MeV} \ . \end{aligned}$$

The kinetic energy of the compound nucleus is very small; essentially all of the photon energy goes to excite the nucleus.

76. (a) From the decay series, we know that N_{210}, the amount of ^{210}Pb nuclei, changes because of two decays: the decay from ^{226}Ra into ^{210}Pb at the rate $R_{226} = \lambda_{226} N_{226}$, and the decay from ^{210}Pb into ^{206}Pb at the rate $R_{210} = \lambda_{210} N_{210}$. The first of these decays causes N_{210} to increase while the second one causes it to decrease. Thus,

$$\frac{dN_{210}}{dt} = R_{226} - R_{210} = \lambda_{226} N_{226} - \lambda_{210} N_{210} \ .$$

(b) We set $dN_{210}/dt = R_{226} - R_{210} = 0$ to obtain $R_{226}/R_{210} = 1$.

(c) From $R_{226} = \lambda_{226} N_{226} = R_{210} = \lambda_{210} N_{210}$, we obtain

$$\frac{N_{226}}{N_{210}} = \frac{\lambda_{210}}{\lambda_{226}} = \frac{T_{1/2_{226}}}{T_{1/2_{210}}} = \frac{1.60 \times 10^3\,\mathrm{y}}{22.6\,\mathrm{y}} = 70.8 \ .$$

(d) Since only 1.00% of the ^{226}Ra remains, the ratio R_{226}/R_{210} is 0.00100 of that of the equilibrium state computed in part (b). Thus the ratio is $(0.0100)(1) = 0.0100$.

(e) This is similar to part (d) above. Since only 1.00% of the ^{226}Ra remains, the ratio N_{226}/N_{210} is 1.00% of that of the equilibrium state computed in part (c), or $(0.0100)(70.8) = 0.708$.

(f) Since the actual value of N_{226}/N_{210} is 0.09, which much closer to 0.0100 than to 1, the sample of the lead pigment cannot be 300 years old. So *Emmaus* is not a *Vermeer*.

77. Using Eq. 43-14 with Eq. 43-17, we find the fraction remaining:

$$\frac{N}{N_0} = e^{-t \ln 2/T_{1/2}} = e^{-30 \ln 2/29} \ = \ 0.49 \ .$$

78. Using Eq. 43-15 with Eq. 43-17, we find the initial activity:

$$R_0 = R e^{t \ln 2/T_{1/2}} = \left(7.4 \times 10^8\,\mathrm{Bq}\right) e^{24 \ln 2/83.61} \ = \ 9.0 \times 10^8\ \mathrm{Bq} \ .$$

79. (a) Molybdenum beta decays into Technetium:

$$^{99}_{42}\mathrm{Mo} \ \rightarrow \ ^{99}_{43}\mathrm{Tc} + e^- + \nu$$

(b) Each decay corresponds to a photon produced when the Technetium nucleus de-excites [note that the de-excitation half-life is much less than the beta decay half-life]. Thus, the gamma rate is the same as the decay rate: $8.2 \times 10^7/\mathrm{s}$.

(c) Eq. 43-19 leads to

$$N = \frac{R T_{1/2}}{\ln 2} = \frac{(38/\mathrm{s})(6.0\,\mathrm{h})(3600\,\mathrm{s/h})}{\ln 2} = 1.2 \times 10^6 \ .$$

80. (a) Assuming a "target" area of one square meter, we establish a ratio:

$$\frac{\text{rate through you}}{\text{total rate upward}} = \frac{1\,\mathrm{m}^2}{(2.6 \times 10^5\,\mathrm{km}^2)(1000\,\mathrm{m/km})^2} = 3.8 \times 10^{-12} \ .$$

The SI unit becquerel is equivalent to a disintegration per second. With half the beta-decay electrons moving upward, we find

$$\text{rate through you} = \frac{1}{2}\left(1 \times 10^{16}/\mathrm{s}\right)\left(3.8 \times 10^{-12}\right) = 1.9 \times 10^4/\mathrm{s}$$

which implies (converting s $\rightarrow$ h) the rate of electrons you would intercept is $R_0 = 7 \times 10^7/\mathrm{h}$.

(b) Let D indicate the current year (2000, 2001, etc) Combining Eq. 43-15 and Eq. 43-17, we find

$$R = R_0 e^{-t \ln 2/T_{1/2}} = \left(7 \times 10^7/\mathrm{h}\right) e^{-(D-1996) \ln 2/(30.2\,\mathrm{y})} \ .$$

81. Eq. 43-19 leads to

$$\begin{aligned} R &= \frac{\ln 2}{T_{1/2}} N \\ &= \frac{\ln 2}{30.2\,\mathrm{y}} \left(\frac{M_{\mathrm{sam}}}{m_{\mathrm{atom}}} \right) \\ &= \frac{\ln 2}{9.53 \times 10^8\,\mathrm{s}} \left(\frac{0.0010\,\mathrm{kg}}{137 \times 1.661 \times 10^{-27}\,\mathrm{kg}} \right) \\ &= 3.2 \times 10^{12}\ \mathrm{Bq} \ = \ 86\ \mathrm{Ci}\ . \end{aligned}$$

82. The lines that lead toward the lower left are alpha decays, involving an atomic number change of $\Delta Z_\alpha = -2$ and a mass number change of $\Delta A_\alpha = -4$. The short horizontal lines toward the right are beta decays (involving electrons, not positrons) in which case A stays the same but the change in atomic number is $\Delta Z_\beta = +1$. Fig. 43-16 shows three alpha decays and two beta decays; thus,

$$Z_f = Z_i + 3\Delta Z_\alpha + 2\Delta Z_\beta \qquad \text{and} \qquad A_f = A_i + 3\Delta A_\alpha \ .$$

Referring to Appendix F or G, we find $Z_i = 93$ for Neptunium, so $Z_f = 93 + 3(-2) + 2(1) = 89$, which indicates the element Actinium. We are given $A_i = 237$, so $A_f = 237 + 3(-4) = 225$. Therefore, the final isotope is ^{225}Ac.

83. We note that every Calcium-40 atom and Krypton-40 atom found now in the sample was once one of the original number of Potassium atoms. Thus, using Eq. 43-13 and Eq. 43-17, we find

$$\begin{aligned} \ln\left(\frac{N_{\mathrm{K}}}{N_{\mathrm{K}} + N_{\mathrm{Ar}} + N_{\mathrm{Ca}}} \right) &= -\lambda t \\ \ln\left(\frac{1}{1+1+8.54} \right) &= -\frac{\ln 2}{T_{1/2}} t \end{aligned}$$

which (with $T_{1/2} = 1.26 \times 10^9\,\mathrm{y}$) yields $t = 4.3 \times 10^9\,\mathrm{y}$.

84. We note that 3.82 days is 330048 s, and that a becquerel is a disintegration per second (see §43-3). From Eq. 34-19, we have

$$\frac{N}{\mathcal{V}} = \frac{R}{\mathcal{V}} \frac{T_{1/2}}{\ln 2} = \left(1.55 \times 10^5\, \frac{\mathrm{Bq}}{\mathrm{m}^3} \right) \frac{330048\,\mathrm{s}}{\ln 2} = 7.4 \times 10^{10}\ \frac{\mathrm{atoms}}{\mathrm{m}^3}$$

where we have divided by volume $\mathcal{V}$. We estimate $\mathcal{V}$ (the volume breathed in $48\,\mathrm{h} = 2880\,\mathrm{min}$) as follows:

$$\left(2\, \frac{\mathrm{Liters}}{\mathrm{breath}} \right) \left(\frac{1\,\mathrm{m}^3}{1000\,\mathrm{L}} \right) \left(40\, \frac{\mathrm{breaths}}{\mathrm{min}} \right) (2880\,\mathrm{min})$$

which yields $\mathcal{V} \approx 200\,\mathrm{m}^3$. Thus, the order of magnitude of N is

$$\left(\frac{N}{\mathcal{V}} \right) (\mathcal{V}) \approx \left(7 \times 10^{10}\, \frac{\mathrm{atoms}}{\mathrm{m}^3} \right) (200\,\mathrm{m}^3) \ \approx \ 10^{13}\ \mathrm{atoms}\ .$$

85. Kinetic energy (we use the classical formula since v is much less than c) is converted into potential energy (see Eq. 25-43). From Appendix F or G, we find $Z = 3$ for Lithium and $Z = 90$ for Thorium; the charges on those nuclei are therefore $3e$ and $90e$, respectively. We manipulate the terms so that one of the factors of e cancels the "e" in the kinetic energy unit MeV, and the other factor of e is set equal to its SI value $1.6 \times 10^{-19}\,\mathrm{C}$. We note that $k = 1/4\pi\varepsilon_0$ can be written as $8.99 \times 10^9\,\mathrm{V{\cdot}m/C}$. Thus, from energy conservation, we have

$$K = U \quad \Longrightarrow \quad r = \frac{kq_1q_2}{K} = \frac{\left(8.99 \times 10^9\, \frac{\mathrm{V\cdot m}}{\mathrm{C}}\right) \left(3 \times 1.6 \times 10^{-19}\,\mathrm{C}\right) (90e)}{3.00 \times 10^6\,\mathrm{eV}}$$

which yields $r = 1.3 \times 10^{-13}\,\mathrm{m}$ (or about 130 fm).

86. Form Appendix F and/or G, we find $Z = 107$ for Bohrium, so this isotope has $N = A - Z = 262 - 107 = 155$ neutrons. Thus,

$$\Delta E_{\text{ben}} = \frac{(Zm_{\text{H}} + Nm_n - m_{\text{Bh}})\,c^2}{A} = \frac{((107)(1.007825\,\text{u}) + (155)(1.008665\,\text{u}) - 262.1231\,\text{u})\,(931.5\,\text{MeV/u})}{262}$$

which yields 7.3 MeV per nucleon.

87. Since R is proportional to N (see Eq. 43-16) then $N/N_0 = R/R_0$. Combining Eq. 43-13 and Eq.43-17 leads to

$$t = -\frac{T_{1/2}}{\ln 2}\ln\left(\frac{R}{R_0}\right) = -\frac{5730\,\text{y}}{\ln 2}\ln(0.020) = 3.2 \times 10^4 \text{ y} .$$

88. Adapting Eq. 43-20, we have

$$N_{\text{Kr}} = \frac{M_{\text{sam}}}{M_{\text{Kr}}}N_A = \left(\frac{20 \times 10^{-9}\,\text{g}}{92\,\text{g/mol}}\right)\left(6.02 \times 10^{23}\,\text{atoms/mol}\right) = 1.3 \times 10^{14} \text{ atoms} .$$

Consequently, Eq. 43-19 leads to

$$R = \frac{N\,\ln 2}{T_{1/2}} = \frac{\left(1.3 \times 10^{14}\right)\ln 2}{1.84\,\text{s}} = 4.9 \times 10^{13} \text{ Bq} .$$

Chapter 44

1. (a) The mass of a single atom of ^{235}U is $(235\,\text{u})(1.661 \times 10^{-27}\,\text{kg/u}) = 3.90 \times 10^{-25}\,\text{kg}$, so the number of atoms in 1.0 kg is $(1.0\,\text{kg})/(3.90 \times 10^{-25}\,\text{kg}) = 2.56 \times 10^{24}$. An alternate approach (but essentially the same once the connection between the "u" unit and N_A is made) would be to adapt Eq. 43-20.

   (b) The energy released by N fission events is given by $E = NQ$, where Q is the energy released in each event. For 1.0 kg of ^{235}U, $E = (2.56 \times 10^{24})(200 \times 10^{6}\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV}) = 8.19 \times 10^{13}\,\text{J}$.

   (c) If P is the power requirement of the lamp, then $t = E/P = (8.19 \times 10^{13}\,\text{J})/(100\,\text{W}) = 8.19 \times 10^{11}\,\text{s} = 2.6 \times 10^{4}\,\text{y}$. The conversion factor $3.156 \times 10^{7}\,\text{s/y}$ is used to obtain the last result.

2. We note that the sum of superscripts (mass numbers A) must balance, as well as the sum of Z values (where reference to Appendix F or G is helpful). A neutron has $Z = 0$ and $A = 1$. Uranium has $Z = 92$.

   - Since xenon has $Z = 54$, then "Y" must have $Z = 92 - 54 = 38$, which indicates the element Strontium. The mass number of "Y" is $235 + 1 - 140 - 1 = 95$, so "Y" is ^{95}Sr.
   - Iodine has $Z = 53$, so "Y" has $Z = 92 - 53 = 39$, corresponding to the element Yttrium (the symbol for which, coincidentally, is Y). Since $235 + 1 - 139 - 2 = 95$, then the unknown isotope is ^{95}Y.
   - The atomic number of Zirconium is $Z = 40$. Thus, $92 - 40 - 2 = 52$, which means that "X" has $Z = 52$ (Tellurium). The mass number of "X" is $235 + 1 - 100 - 2 = 134$, so we obtain ^{134}Te.
   - Examining the mass numbers, we find $b = 235 + 1 - 141 - 92 = 3$.

3. If R is the fission rate, then the power output is $P = RQ$, where Q is the energy released in each fission event. Hence, $R = P/Q = (1.0\,\text{W})/(200 \times 10^{6}\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV}) = 3.12 \times 10^{10}$ fissions/s.

4. Adapting Eq. 43-20, there are

$$N_\text{Pu} = \frac{M_\text{sam}}{M_\text{Pu}} N_\text{A} = \left(\frac{1000\,\text{g}}{239\,\text{g/mol}}\right)\left(6.02 \times 10^{23}/\text{mol}\right) = 2.5 \times 10^{24}$$

   plutonium nuclei in the sample. If they all fission (each releasing 180 MeV), then the total energy release is 4.5×10^{26} MeV.

5. At $T = 300\,\text{K}$, the average kinetic energy of the neutrons is (using Eq. 20-24)

$$K_\text{avg} = \frac{3}{2}kT = \frac{3}{2}(8.62 \times 10^{-5}\,\text{eV/K})(300\,\text{K}) \approx 0.04\ \text{eV}\ .$$

6. We consider the process $^{98}\text{Mo} \to {}^{49}\text{Sc} + {}^{49}\text{Sc}$. The disintegration energy is $Q = (m_\text{Mo} - 2m_\text{Sc})c^2 = [97.90541\,\text{u} - 2(48.95002\,\text{u})](931.5\,\text{MeV/u}) = +5.00\,\text{MeV}$. The fact that it is positive does not necessarily mean we should expect to find a great deal of Molybdenum nuclei spontaneously fissioning; the energy barrier (see Fig. 44-3) is presumably higher and/or broader for Molybdenum than for Uranium.

7. If M_Cr is the mass of a ^{52}Cr nucleus and M_Mg is the mass of a ^{26}Mg nucleus, then the disintegration energy is $Q = (M_\text{Cr} - 2M_\text{Mg})\,c^2 = [51.94051\,\text{u} - 2(25.98259\,\text{u})]\,(931.5\,\text{MeV/u}) = -23.0\,\text{MeV}$.

8. (a) Using Eq. 43-19 and adapting Eq. 43-20 to this sample, the number of fission-events per second is

$$\begin{aligned} R_{\text{fission}} &= \frac{N \ln 2}{T_{1/2\,\text{fission}}} = \frac{M_{\text{sam}} N_A \ln 2}{M_{\text{U}} T_{1/2\,\text{fission}}} \\ &= \frac{(1.0\,\text{g})(6.02 \times 10^{23}/\text{mol}) \ln 2}{(235\,\text{g/mol})(3.0 \times 10^{17}\,\text{y})(365\,\text{d/y})} = 16 \text{ fissions/day} . \end{aligned}$$

(b) Since $R \propto \frac{1}{T_{1/2}}$ (see Eq. 43-19), the ratio of rates is

$$\frac{R_\alpha}{R_{\text{fission}}} = \frac{T_{1/2\,\text{fission}}}{T_{1/2_\alpha}} = \frac{3.0 \times 10^{17}\,\text{y}}{7.0 \times 10^{8}\,\text{y}} = 4.3 \times 10^8 .$$

9. The energy released is

$$\begin{aligned} Q &= (m_{\text{U}} + m_n - m_{\text{Cs}} - m_{\text{Rb}} - 2m_n)c^2 \\ &= (235.04392\,\text{u} - 1.00867\,\text{u} - 140.91963\,\text{u} - 92.92157\,\text{u})(931.5\,\text{MeV/u}) \\ &= 181 \text{ MeV} . \end{aligned}$$

10. First, we figure out the mass of U-235 in the sample (assuming "3.0%" refers to the proportion by weight as opposed to proportion by number of atoms):

$$\begin{aligned} M_{\text{U–235}} &= (3.0\%) M_{\text{sam}} \left(\frac{(97\%) m_{238} + (3.0\%) m_{235}}{(97\%) m_{238} + (3.0\%) m_{235} + 2m_{16}} \right) \\ &= (0.030)(1000\,\text{g}) \left(\frac{0.97(238) + 0.030(235)}{0.97(238) + 0.030(235) + 2(16.0)} \right) = 26.4 \text{ g} . \end{aligned}$$

Next, this uses some of the ideas illustrated in Sample Problem 43-5; our notation is similar to that used in that example. The number of ^{235}U nuclei is

$$N_{235} = \frac{(26.4\,\text{g})(6.02 \times 10^{23}/\text{mol})}{235\,\text{g/mol}} = 6.77 \times 10^{22} .$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 44-6) is

$$N_{235} Q_{\text{fission}} = \left(6.77 \times 10^{22}\right)(200\,\text{MeV}) = 1.35 \times 10^{25}\,\text{MeV} = 2.17 \times 10^{12} \text{ J} .$$

Keeping in mind that a Watt is a Joule per second, the time that this much energy can keep a 100-W lamp burning is found to be

$$t = \frac{2.17 \times 10^{12}\,\text{J}}{100\,\text{W}} = 2.17 \times 10^{10}\,\text{s} \approx 690 \text{ y} .$$

If we had instead used the $Q = 208\,\text{MeV}$ value from Sample Problem 44-1, then our result would have been 715 y, which perhaps suggests that our result is meaningful to just one significant figure ("roughly 700 years").

11. (a) If X represents the unknown fragment, then the reaction can be written

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{83}_{32}\text{Ge} + ^{A}_{Z}X$$

where A is the mass number and Z is the atomic number of the fragment. Conservation of charge yields $92+0 = 32+Z$, so $Z = 60$. Conservation of mass number yields $235+1 = 83+A$, so $A = 153$. Looking in Appendix F or G for nuclides with $Z = 60$, we find that the unknown fragment is $^{153}_{60}$Nd.

(b) We neglect the small kinetic energy and momentum carried by the neutron that triggers the fission event. Then, $Q = K_{\text{Ge}} + K_{\text{Nd}}$, where K_{Ge} is the kinetic energy of the germanium nucleus and K_{Nd} is the kinetic energy of the neodymium nucleus. Conservation of momentum yields $\vec{p}_{\text{Ge}} + \vec{p}_{\text{Nd}} = 0$. Now, we can write the classical formula for kinetic energy in terms of the magnitude of the momentum vector:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that $K_{\text{Nd}} = (m_{\text{Ge}}/m_{\text{Nd}})K_{\text{Ge}}$. Thus, the energy equation becomes

$$Q = K_{\text{Ge}} + \frac{M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}} = \frac{M_{\text{Nd}} + M_{\text{Ge}}}{M_{\text{Nd}}}K_{\text{Ge}}$$

and

$$K_{\text{Ge}} = \frac{M_{\text{Nd}}}{M_{\text{Nd}} + M_{\text{Ge}}}Q = \frac{153\,\text{u}}{153\,\text{u} + 83\,\text{u}}(170\,\text{MeV}) = 110\text{ MeV} .$$

Similarly,

$$K_{\text{Nd}} = \frac{M_{\text{Ge}}}{M_{\text{Nd}} + M_{\text{Ge}}}Q = \frac{83\,\text{u}}{153\,\text{u} + 83\,\text{u}}(170\,\text{MeV}) = 60\text{ MeV} .$$

(c) The initial speed of the germanium nucleus is

$$v_{\text{Ge}} = \sqrt{\frac{2K_{\text{Ge}}}{M_{\text{Ge}}}} = \sqrt{\frac{2(110 \times 10^6\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{(83\,\text{u})(1.661 \times 10^{-27}\,\text{kg/u})}} = 1.60 \times 10^7\text{ m/s} .$$

The initial speed of the neodymium nucleus is

$$v_{\text{Nd}} = \sqrt{\frac{2K_{\text{Nd}}}{M_{\text{Nd}}}} = \sqrt{\frac{2(60 \times 10^6\,\text{eV})(1.60 \times 10^{-19}\,\text{J/eV})}{(153\,\text{u})(1.661 \times 10^{-27}\,\text{kg/u})}} = 8.69 \times 10^6\text{ m/s} .$$

12. (a) Consider the process ${}^{239}\text{U} + \text{n} \to {}^{140}\text{Ce} + {}^{99}\text{Ru} + \text{Ne}$. We have $Z_f - Z_i = Z_{\text{Ce}} + Z_{\text{Ru}} - Z_{\text{U}} = 58 + 44 - 92 = 10$. Thus the number of beta-decay events is 10.

(b) Using Table 38-3, the energy released in this fission process is

$$\begin{aligned} Q &= (m_{\text{U}} + m_n - m_{\text{Ce}} - m_{\text{Ru}} - 10m_e)c^2 \\ &= (238.05079\,\text{u} + 1.00867\,\text{u} - 139.90543\,\text{u} - 98.90594\,\text{u})(931.5\,\text{MeV/u}) - 10(0.511\,\text{MeV}) \\ &= 226\text{ MeV} . \end{aligned}$$

13. (a) The electrostatic potential energy is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Z_{\text{Xe}}Z_{\text{Sr}}e^2}{r_{\text{Xe}} + r_{\text{Sr}}}$$

where Z_{Xe} is the atomic number of xenon, Z_{Sr} is the atomic number of strontium, r_{Xe} is the radius of a xenon nucleus, and r_{Sr} is the radius of a strontium nucleus. Atomic numbers can be found either in Appendix F or Appendix G. The radii are given by $r = (1.2\,\text{fm})A^{1/3}$, where A is the mass number, also found in Appendix F. Thus, $r_{\text{Xe}} = (1.2\,\text{fm})(140)^{1/3} = 6.23\,\text{fm} = 6.23 \times 10^{-15}\,\text{m}$ and $r_{\text{Sr}} = (1.2\,\text{fm})(96)^{1/3} = 5.49\,\text{fm} = 5.49 \times 10^{-15}\,\text{m}$. Hence, the potential energy is

$$U = (8.99 \times 10^9\,\text{V}\cdot\text{m/C})\frac{(54)(38)(1.60 \times 10^{-19}\,\text{C})^2}{6.23 \times 10^{-15}\,\text{m} + 5.49 \times 10^{-15}\,\text{m}} = 4.08 \times 10^{-11}\,\text{J} = 251\text{ MeV} .$$

(b) The energy released in a typical fission event is about 200 MeV, roughly the same as the electrostatic potential energy when the fragments are touching. The energy appears as kinetic energy of the fragments and neutrons produced by fission.

14. (a) The surface area a of a nucleus is given by $a \simeq 4\pi R^2 \simeq 4\pi[R_0 A^{1/3}]^2 \propto A^{2/3}$. Thus, the fractional change in surface area is

$$\frac{\Delta a}{a_i} = \frac{a_f - a_i}{a_i} = \frac{(140)^{2/3} + (96)^{2/3}}{(236)^{2/3}} - 1 = +0.25 \ .$$

(b) Since $V \propto R^3 \propto (A^{1/3})^3 = A$, we have

$$\frac{\Delta V}{V} = \frac{V_f}{V_i} - 1 = \frac{140 + 96}{236} - 1 = 0 \ .$$

(c) The fractional change in potential energy is

$$\begin{aligned} \frac{\Delta U}{U} &= \frac{U_f}{U_i} - 1 \\ &= \frac{Q_{\mathrm{Xe}}^2/R_{\mathrm{Xe}} + Q_{\mathrm{Sr}}^2/R_{\mathrm{Sr}}}{Q_{\mathrm{U}}^2/R_{\mathrm{U}}} - 1 \\ &= \frac{(54)^2(140)^{-1/3} + (38)^2(96)^{-1/3}}{(92)^2(236)^{-1/3}} - 1 \ = \ -0.36 \ . \end{aligned}$$

15. If P is the power output, then the energy E produced in the time interval Δt $(= 3\,\mathrm{y})$ is $E = P\,\Delta t = (200 \times 10^6\,\mathrm{W})(3\,\mathrm{y})(3.156 \times 10^7\,\mathrm{s/y}) = 1.89 \times 10^{16}\,\mathrm{J}$, or $(1.89 \times 10^{16}\,\mathrm{J})/(1.60 \times 10^{-19}\,\mathrm{J/eV}) = 1.18 \times 10^{35}\,\mathrm{eV} = 1.18 \times 10^{29}\,\mathrm{MeV}$. At 200 MeV per event, this means $(1.18 \times 10^{29})/200 = 5.90 \times 10^{26}$ fission events occurred. This must be half the number of fissionable nuclei originally available. Thus, there were $2(5.90 \times 10^{26}) = 1.18 \times 10^{27}$ nuclei. The mass of a ^{235}U nucleus is $(235\,\mathrm{u})(1.661 \times 10^{-27}\,\mathrm{kg/u}) = 3.90 \times 10^{-25}\,\mathrm{kg}$, so the total mass of ^{235}U originally present was $(1.18 \times 10^{27})(3.90 \times 10^{-25}\,\mathrm{kg}) = 462\,\mathrm{kg}$.

16. In Sample Problem 44-2, it is noted that the rate of consumption of U-235 by (nonfission) neutron capture is one-fourth as big as the rate of the rate of neutron-induced fission events. Consequently, the mass of ^{235}U should be larger than that computed in problem 15 by 25%: $(1.25)(462\,\mathrm{kg}) = 5.8 \times 10^2\,\mathrm{kg}$. If appeal is to made to other sources (other than Sample Problem 44-2), then it might be possible to argue for a factor other than 1.25 (we found others in our brief search) and thus to a somewhat different result.

17. When a neutron is captured by ^{237}Np it gains 5.0 MeV, more than enough to offset the 4.2 MeV required for ^{238}Np to fission. Consequently, ^{237}Np is fissionable by thermal neutrons.

18. (a) Using the result of problem 4, the TNT equivalent is

$$\frac{(2.50\,\mathrm{kg})(4.54 \times 10^{26}\,\mathrm{MeV/kg})}{2.6 \times 10^{28}\,\mathrm{MeV}/10^6\,\mathrm{ton}} = 4.4 \times 10^4\,\mathrm{ton} = 44\,\mathrm{kton} \ .$$

(b) Assuming that this is a fairly inefficiently designed bomb, then much of the remaining 92.5 kg is probably "wasted" and was included perhaps to make sure the bomb did not "fizzle." There is also an argument for having more than just the critical mass based on the short assembly-time of the material during the implosion, but this so-called "super-critical mass," as generally quoted, is much less than 92.5 kg, and does not necessarily have to be purely Plutonium.

19. If R is the decay rate then the power output is $P = RQ$, where Q is the energy produced by each alpha decay. Now $R = \lambda N = N \ln 2/T_{1/2}$, where λ is the disintegration constant and $T_{1/2}$ is the half-life. The relationship $\lambda = (\ln 2)/T_{1/2}$ is used. If M is the total mass of material and m is the mass of a single ^{238}Pu nucleus, then

$$N = \frac{M}{m} = \frac{1.00\,\mathrm{kg}}{(238\,\mathrm{u})(1.661 \times 10^{-27}\,\mathrm{kg/u})} = 2.53 \times 10^{24} \ .$$

Thus,

$$P = \frac{NQ\,\ln 2}{T_{1/2}} = \frac{(2.53 \times 10^{24})(5.50 \times 10^6\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV})(\ln 2)}{(87.7\,\mathrm{y})(3.156 \times 10^7\,\mathrm{s/y})} = 558\,\mathrm{W} \ .$$

20. (a) We solve Q_{eff} from $P = RQ_{\text{eff}}$:

$$\begin{aligned} Q_{\text{eff}} &= \frac{P}{R} = \frac{P}{N\lambda} = \frac{mPT_{1/2}}{M\ln 2} \\ &= \frac{(90.0\,\text{u})(1.66\times10^{-27}\,\text{kg/u})(0.93\,\text{W})(29\,\text{y})(3.15\times10^{7}\,\text{s/y})}{(1.00\times10^{-3}\,\text{kg})(\ln 2)(1.60\times10^{-13}\,\text{J/MeV})} \\ &= 1.2\ \text{MeV}\ . \end{aligned}$$

(b) The amount of ^{90}Sr needed is

$$M = \frac{150\,\text{W}}{(0.050)(0.93\,\text{W/g})} = 3.2\ \text{kg}\ .$$

21. Since Plutonium has $Z = 94$ and Uranium has $Z = 92$, we see that (to conserve charge) two electrons must be emitted so that the nucleus can gain a $+2e$ charge. In the beta decay processes described in Chapter 43, electrons and neutrinos are emitted. The reaction series is as follows:

$$\begin{aligned} {}^{238}\text{U} + \text{n} \rightarrow\ \ {}^{239}\text{Np} &\ + {}^{239}\text{U} + e + \nu \\ {}^{239}\text{Np} &\ \rightarrow {}^{239}\text{Pu} + e + \nu \end{aligned}$$

22. After each time interval t_{gen} the number of nuclides in the chain reaction gets multiplied by k. The number of such time intervals that has gone by at time t is t/t_{gen}. For example, if the multiplication factor is 5 and there were 12 nuclei involved in the reaction to start with, then after one interval 60 nuclei are involved. And after another interval 300 nuclei are involved. Thus, the number of nuclides engaged in the chain reaction at time t is $N(t) = N_0 k^{t/t_{\text{gen}}}$. Since $P \propto N$ we have

$$P(t) = P_0 k^{t/t_{\text{gen}}}\ .$$

23. (a) The energy yield of the bomb is $E = (66\times10^{-3}\,\text{megaton})(2.6\times10^{28}\,\text{MeV/megaton}) = 1.72\times10^{27}\,\text{MeV}$. At 200 MeV per fission event, $(1.72\times10^{27}\,\text{MeV})/(200\,\text{MeV}) = 8.58\times10^{24}$ fission events take place. Since only 4.0% of the ^{235}U nuclei originally present undergo fission, there must have been $(8.58\times10^{24})/(0.040) = 2.14\times10^{26}$ nuclei originally present. The mass of ^{235}U originally present was $(2.14\times10^{26})(235\,\text{u})(1.661\times10^{-27}\,\text{kg/u}) = 83.7\,\text{kg}$.

(b) Two fragments are produced in each fission event, so the total number of fragments is $2(8.58\times10^{24}) = 1.72\times10^{25}$.

(c) One neutron produced in a fission event is used to trigger the next fission event, so the average number of neutrons released to the environment in each event is 1.5. The total number released is $(8.58\times10^{24})(1.5) = 1.29\times10^{25}$.

24. We recall Eq. 44-6: $Q \approx 200\,\text{MeV} = 3.2\times10^{-11}$ J. It is important to bear in mind that Watts multiplied by seconds give Joules. From $E = Pt_{\text{gen}} = NQ$ we get the number of free neutrons:

$$N = \frac{Pt_{\text{gen}}}{Q} = \frac{(500\times10^{6}\,\text{W})(1.0\times10^{-3}\,\text{s})}{3.2\times10^{-11}\,\text{J}} = 1.6\times10^{16}\ .$$

25. Let P_0 be the initial power output, P be the final power output, k be the multiplication factor, t be the time for the power reduction, and t_{gen} be the neutron generation time. Then, according to the result of Problem 22,

$$P = P_0\, k^{t/t_{\text{gen}}}\ .$$

We divide by P_0, take the natural logarithm of both sides of the equation and solve for $\ln k$:

$$\ln k = \frac{t_{\text{gen}}}{t}\ \ln\frac{P}{P_0} = \frac{1.3\times10^{-3}\,\text{s}}{2.6\,\text{s}}\ \ln\frac{350\,\text{MW}}{1200\,\text{MW}} = -0.0006161\ .$$

Hence, $k = e^{-0.0006161} = 0.99938$.

26. We use the formula from problem 22:

$$\begin{aligned} P(t) &= P_0 k^{t/t_{\text{gen}}} \\ &= (400\,\text{MW})(1.0003)^{(5.00\,\text{min})(60\,\text{s/min})/(0.00300\,\text{s})} \\ &= 8.03 \times 10^3\ \text{MW}\ . \end{aligned}$$

27. (a) Let v_{ni} be the initial velocity of the neutron, v_{nf} be its final velocity, and v_f be the final velocity of the target nucleus. Then, since the target nucleus is initially at rest, conservation of momentum yields $m_n v_{ni} = m_n v_{nf} + m v_f$ and conservation of energy yields $\frac{1}{2}m_n v_{ni}^2 = \frac{1}{2}m_n v_{nf}^2 + \frac{1}{2}m v_f^2$. We solve these two equations simultaneously for v_f. This can be done, for example, by using the conservation of momentum equation to obtain an expression for v_{nf} in terms of v_f and substituting the expression into the conservation of energy equation. We solve the resulting equation for v_f. We obtain $v_f = 2m_n v_{ni}/(m + m_n)$. The energy lost by the neutron is the same as the energy gained by the target nucleus, so

$$\Delta K = \frac{1}{2}mv_f^2 = \frac{1}{2}\frac{4m_n^2 m}{(m+m_n)^2}\, v_{ni}^2\ .$$

The initial kinetic energy of the neutron is $K = \frac{1}{2}m_n v_{ni}^2$, so

$$\frac{\Delta K}{K} = \frac{4m_n m}{(m+m_n)^2}\ .$$

(b) The mass of a neutron is 1.0 u and the mass of a hydrogen atom is also 1.0 u. (Atomic masses can be found in Appendix G.) Thus,

$$\frac{\Delta K}{K} = \frac{4(1.0\,\text{u})(1.0\,\text{u})}{(1.0\,\text{u} + 1.0\,\text{u})^2} = 1.0\ .$$

Similarly, the mass of a deuterium atom is 2.0 u, so $(\Delta K)/K = 4(1.0\,\text{u})(2.0\,\text{u})/(2.0\,\text{u}+1.0\,\text{u})^2 = 0.89$. The mass of a carbon atom is 12 u, so $(\Delta K)/K = 4(1.0\,\text{u})(12\,\text{u})/(12\,\text{u}+1.0\,\text{u})^2 = 0.28$. The mass of a lead atom is 207 u, so $(\Delta K)/K = 4(1.0\,\text{u})(207\,\text{u})/(207\,\text{u}+1.0\,\text{u})^2 = 0.019$.

(c) During each collision, the energy of the neutron is reduced by the factor $1 - 0.89 = 0.11$. If E_i is the initial energy, then the energy after n collisions is given by $E = (0.11)^n E_i$. We take the natural logarithm of both sides and solve for n. The result is

$$n = \frac{\ln(E/E_i)}{\ln 0.11} = \frac{\ln(0.025\,\text{eV}/1.00\,\text{eV})}{\ln 0.11} = 7.9\ .$$

The energy first falls below 0.025 eV on the eighth collision.

28. Our approach is the same as that shown in Sample Problem 44-3. We have

$$\frac{N_5(t)}{N_8(t)} = \frac{N_5(0)}{N_8(0)}\, e^{-(\lambda_5-\lambda_8)t}\ ,$$

or

$$\begin{aligned} t &= \frac{1}{\lambda_8 - \lambda_5}\ln\left[\left(\frac{N_5(t)}{N_8(t)}\right)\left(\frac{N_8(0)}{N_5(0)}\right)\right] \\ &= \frac{1}{(1.55 - 9.85)10^{-10}\,\text{y}^{-1}}\ln\left[(0.0072)(0.15)^{-1}\right] \;=\; 3.6\times 10^9\ \text{y}\ . \end{aligned}$$

29. (a) $P_{\text{avg}} = (15\times 10^9\,\text{W}\cdot\text{y})/(200,000\,\text{y}) = 7.5\times 10^4\,\text{W} = 75\,\text{kW}$.

(b) Using the result of Eq. 44-6, we obtain

$$\begin{aligned} M &= \frac{m_{\mathrm{U}} E_{\mathrm{total}}}{Q} \\ &= \frac{(235\,\mathrm{u})(1.66\times 10^{-27}\,\mathrm{kg/u})(15\times 10^{9}\,\mathrm{W\cdot y})(3.15\times 10^{7}\,\mathrm{s/y})}{(200\,\mathrm{MeV})(1.6\times 10^{-13}\,\mathrm{J/MeV})} \\ &= 5.8\times 10^{3}\ \mathrm{kg}\ . \end{aligned}$$

30. The nuclei of ^{238}U can capture neutrons and beta-decay. With large amount of neutrons available due to the fission of ^{235}U, the probability for this process is substantially increased, resulting in a much higher decay rate for ^{238}U and causing the depletion of ^{238}U (and relative enrichment of ^{235}U).

31. Let t be the present time and $t = 0$ be the time when the ratio of ^{235}U to ^{238}U was 3.0%. Let N_{235} be the number of ^{235}U nuclei present in a sample now and $N_{235,\,0}$ be the number present at $t = 0$. Let N_{238} be the number of ^{238}U nuclei present in the sample now and $N_{238,\,0}$ be the number present at $t = 0$. The law of radioactive decay holds for each specie, so

$$N_{235} = N_{235,\,0}\, e^{-\lambda_{235} t}$$

and

$$N_{238} = N_{238,\,0}\, e^{-\lambda_{238} t}\ .$$

Dividing the first equation by the second, we obtain

$$r = r_0\, e^{-(\lambda_{235}-\lambda_{238})t}$$

where $r = N_{235}/N_{238}$ $(= 0.0072)$ and $r_0 = N_{235,\,0}/N_{238,\,0}$ $(= 0.030)$. We solve for t:

$$t = -\frac{1}{\lambda_{235}-\lambda_{238}}\ \ln\frac{r}{r_0}\ .$$

Now we use $\lambda_{235} = (\ln 2)/T_{1/2_{235}}$ and $\lambda_{238} = (\ln 2)/T_{1/2_{238}}$ to obtain

$$t = -\frac{T_{1/2_{235}} T_{1/2_{238}}}{(T_{1/2_{238}} - T_{1/2_{235}})\ln 2}\ \ln\frac{r}{r_0} = -\frac{(7.0\times 10^{8}\,\mathrm{y})(4.5\times 10^{9}\,\mathrm{y})}{(4.5\times 10^{9}\,\mathrm{y} - 7.0\times 10^{8}\,\mathrm{y})\ln 2}\ \ln\frac{0.0072}{0.030} = 1.71\times 10^{9}\ \mathrm{y}\ .$$

32. (a) Fig. 43-9 shows the barrier height to be about 30 MeV.

    (b) The potential barrier height listed in Table 44-2 is roughly 5 MeV. There is some model-dependence involved in arriving at this estimate, and other values can be found in the literature (6 MeV is frequently cited).

33. The height of the Coulomb barrier is taken to be the value of the kinetic energy K each deuteron must initially have if they are to come to rest when their surfaces touch (see Sample Problem 44-4). If r is the radius of a deuteron, conservation of energy yields

$$2K = \frac{1}{4\pi\varepsilon_0}\,\frac{e^2}{2r}\ ,$$

so

$$K = \frac{1}{4\pi\varepsilon_0}\,\frac{e^2}{4r} = (8.99\times 10^{9}\,\mathrm{V\cdot m/C})\frac{(1.60\times 10^{-19}\,\mathrm{C})^2}{4(2.1\times 10^{-15}\,\mathrm{m})} = 2.74\times 10^{-14}\,\mathrm{J} = 170\ \mathrm{keV}\ .$$

34. We are given the energy release per fusion ($Q = 3.27\,\mathrm{MeV} = 5.24\times 10^{-13}\,\mathrm{J}$) and that a pair of deuterium atoms are consumed in each fusion event. To find how many pairs of deuterium atoms are in the sample, we adapt Eq. 43-20:

$$N_{d\,\mathrm{pairs}} = \frac{M_{\mathrm{sam}}}{2M_d}\,N_{\mathrm{A}} = \left(\frac{1000\,\mathrm{g}}{2(2.0\,\mathrm{g/mol})}\right)(6.02\times 10^{23}/\mathrm{mol}) = 1.5\times 10^{26}\ .$$

Multiplying this by Q gives the total energy released: 7.9×10^{13} J. Keeping in mind that a Watt is a Joule per second, we have

$$t = \frac{7.9 \times 10^{13}\,\mathrm{J}}{100\,\mathrm{W}} = 7.9 \times 10^{11}\,\mathrm{s} = 2.5 \times 10^4\ \mathrm{y}\ .$$

35. (a) Our calculation is identical to that in Sample Problem 44-4 except that we are now using R appropriate to two deuterons coming into "contact," as opposed to the $R = 1.0$ fm value used in the Sample Problem. If we use $R = 2.1$ fm for the deuterons (this is the value given in problem 33), then our K is simply the K calculated in Sample Problem 44-4, divided by 2.1:

$$K_{d+d} = \frac{K_{p+p}}{2.1} = \frac{360\,\mathrm{keV}}{2.1} \approx 170\ \mathrm{keV}\ .$$

Consequently, the voltage needed to accelerate each deuteron from rest to that value of K is 170 kV.

(b) Not all deuterons that are accelerated towards each other will come into "contact" and not all of those that do so will undergo nuclear fusion. Thus, a great many deuterons must be repeatedly encountering other deuterons in order to produce a macroscopic energy release. An accelerator needs a fairly good vacuum in its beam pipe, and a very large number flux is either impractical and/or very expensive. Regarding expense, there are other factors that have dissuaded researchers from using accelerators to build a controlled fusion "reactor," but those factors may become less important in the future – making the feasibility of accelerator "add-on's" to magnetic and inertial confinement schemes more cost-effective.

36. Our calculation is very similar to that in Sample Problem 44-4 except that we are now using R appropriate to two Lithium-7 nuclei coming into "contact," as opposed to the $R = 1.0$ fm value used in the Sample Problem. If we use

$$R = r = r_0 A^{1/3} = (1.2\,\mathrm{fm})\sqrt[3]{7} = 2.3\ \mathrm{fm}$$

and $q = Ze = 3e$, then our K is given by (see Sample Problem 44-4)

$$K = \frac{Z^2 e^2}{16\pi\varepsilon_0 r} = \frac{3^2(1.6 \times 10^{-19}\,\mathrm{C})^2}{16\pi(8.85 \times 10^{-12}\,\mathrm{F/m})(2.3 \times 10^{15}\,\mathrm{m})}$$

which yields 2.3×10^{-13} J $= 1.4$ MeV. We interpret this as the answer to the problem, though the term "Coulomb barrier height" as used here may be open to other interpretations.

37. From the expression for $n(K)$ given we may write $n(K) \propto K^{1/2} e^{-K/kT}$. Thus, with $k = 8.62 \times 10^{-5}$ eV/K $= 8.62 \times 10^{-8}$ keV/K, we have

$$\begin{aligned}\frac{n(K)}{n(K_{\mathrm{avg}})} &= \left(\frac{K}{K_{\mathrm{avg}}}\right)^{1/2} e^{-(K-K_{\mathrm{avg}})/kT} \\ &= \left(\frac{5.00\,\mathrm{keV}}{1.94\,\mathrm{keV}}\right)^{1/2} e^{-(5.00\,\mathrm{keV}-1.94\,\mathrm{keV})/[(8.62\times10^{-8}\,\mathrm{keV/K})(1.50\times10^{7}\,\mathrm{K})]} \\ &= 0.151\ .\end{aligned}$$

38. (a) Rather than use $P(v)$ as it is written in Eq. 20-27, we use the more convenient nK expression given in problem 37 of this chapter [44]. The $n(K)$ expression can be derived from Eq. 20-27, but we do not show that derivation here. To find the most probable energy, we take the derivative of $n(K)$ and set the result equal to zero:

$$\left.\frac{dn(K)}{dK}\right|_{K=K_p} = \frac{1.13n}{(kT)^{3/2}}\left(\frac{1}{2K^{1/2}} - \frac{K^{3/2}}{kT}\right) e^{-K/kT}\Bigg|_{K=K_p} = 0\,,$$

which gives $K_p = \frac{1}{2}kT$. Specifically, for $T = 1.5 \times 10^7$ K we find

$$K_p = \frac{1}{2}kT = \frac{1}{2}(8.62 \times 10^{-5}\,\text{eV/K})(1.5 \times 10^7\,\text{K}) = 6.5 \times 10^2\ \text{eV}$$

or 0.65 keV, in good agreement with Fig. 44-10.

(b) Eq. 20-35 gives the most probable speed in terms of the molar mass M, and indicates its derivation (see also Sample Problem 20-6). Since the mass m of the particle is related to M by the Avogadro constant, then

$$v_p = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2RT}{m\,N_A}} = \sqrt{\frac{2kT}{m}}$$

using Eq. 20-7. With $T = 1.5 \times 10^7$ K and $m = 1.67 \times 10^{-27}$ kg, this yields $v_p = 5.0 \times 10^5$ m/s.

(c) The corresponding kinetic energy is

$$K_{v,p} = \frac{1}{2}mv_p^2 = \frac{1}{2}m\left(\sqrt{\frac{2kT}{m}}\right)^2 = kT$$

which is twice as large as that found in part (a). Thus, at $T = 1.5 \times 10^7$ K we have $K_{v,p} = 1.3$ keV, which is indicated in Fig. 44-10 by a single vertical line.

39. If M_{He} is the mass of an atom of helium and M_{C} is the mass of an atom of carbon, then the energy released in a single fusion event is

$$Q = [3M_{\text{He}} - M_{\text{C}}]\,c^2 = [3(4.0026\,\text{u}) - (12.0000\,\text{u})]\,(931.5\,\text{MeV/u}) = 7.27\,\text{MeV}\ .$$

Note that $3M_{\text{He}}$ contains the mass of six electrons and so does M_{C}. The electron masses cancel and the mass difference calculated is the same as the mass difference of the nuclei.

40. In Fig. 44-11, let $Q_1 = 0.42$ MeV, $Q_2 = 1.02$ MeV, $Q_3 = 5.49$ MeV and $Q_4 = 12.86$ MeV. For the overall proton-proton cycle

$$\begin{aligned} Q &= 2Q_1 + 2Q_2 + 2Q_3 + Q_4 \\ &= 2(0.42\,\text{MeV} + 1.02\,\text{MeV} + 5.49\,\text{MeV}) + 12.86\,\text{MeV} = 26.7\ \text{MeV}\ . \end{aligned}$$

41. (a) From $\rho_{\text{H}} = 0.35\rho = n_p m_p$, we get the proton number density n_p:

$$n_p = \frac{0.35\rho}{m_p} = \frac{(0.35)(1.5 \times 10^5\,\text{kg/m}^3)}{1.67 \times 10^{-27}\,\text{kg}} = 3.14 \times 10^{31}\ \text{m}^{-3}\ .$$

(b) From Chapter 20 (see Eq. 20-9), we have

$$\frac{N}{V} = \frac{p}{kT} = \frac{1.01 \times 10^5\,\text{Pa}}{(1.38 \times 10^{-23}\,\text{J/K})(273\,\text{K})} = 2.68 \times 10^{25}\ \text{m}^{-3}$$

for an ideal gas under "standard conditions." Thus,

$$\frac{n_p}{(N/V)} = \frac{3.14 \times 10^{31}\,\text{m}^{-3}}{2.44 \times 10^{25}\,\text{m}^{-3}} = 1.2 \times 10^6\ .$$

42. We assume the neutrino has negligible mass. The photons, of course, are also taken to have zero mass.

$$\begin{aligned} Q_1 &= (2m_p - m_2 - m_e)c^2 = [2(m_1 - m_e) - (m_2 - m_e) - m_e]c^2 \\ &= [2(1.007825\,\text{u}) - 2.014102\,\text{u} - 2(0.0005486\,\text{u})](931.5\,\text{MeV/u}) \\ &= 0.42\ \text{MeV} \end{aligned}$$

$$\begin{aligned} Q_2 &= (m_2 + m_p - m_3)c^2 = (m_2 + m_p - m_3)c^2 \\ &= (2.014102\,\mathrm{u}) + 1.007825\,\mathrm{u} - 3.016029\,\mathrm{u})(931.5\,\mathrm{MeV/u}) \\ &= 5.49\ \mathrm{MeV} \end{aligned}$$

$$\begin{aligned} Q_3 &= (2m_3 - m_4 - 2m_p)c^2 = (2m_3 - m_4 - 2m_p)c^2 \\ &= [2(3.016029\,\mathrm{u}) - 4.002603\,\mathrm{u} - 2(1.007825\,\mathrm{u})](931.5\,\mathrm{MeV/u}) \\ &= 12.86\ \mathrm{MeV}\ . \end{aligned}$$

43. (a) Let M be the mass of the Sun at time t and E be the energy radiated to that time. Then, the power output is $P = dE/dt = (dM/dt)c^2$, where $E = Mc^2$ is used. At the present time,

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9 \times 10^{26}\,\mathrm{W}}{(2.998 \times 10^8\,\mathrm{m/s})^2} = 4.33 \times 10^9\ \mathrm{kg/s}\ .$$

(b) We assume the rate of mass loss remained constant. Then, the total mass loss is $\Delta M = (dM/dt)\,\Delta t = (4.33 \times 10^9\,\mathrm{kg/s})(4.5 \times 10^9\,\mathrm{y})(3.156 \times 10^7\,\mathrm{s/y}) = 6.15 \times 10^{26}\,\mathrm{kg}$. The fraction lost is

$$\frac{\Delta M}{M + \Delta M} = \frac{6.15 \times 10^{26}\,\mathrm{kg}}{2.0 \times 10^{30}\,\mathrm{kg} + 6.15 \times 10^{26}\,\mathrm{kg}} = 3.07 \times 10^{-4}\ .$$

44. (a) We are given the energy release per fusion (calculated in §44-7: $Q = 26.7\,\mathrm{MeV} = 4.28 \times 10^{-12}\,\mathrm{J}$) and that four protons are consumed in each fusion event. To find how many sets of four protons are in the sample, we adapt Eq. 43-20:

$$N_{4p} = \frac{M_{\mathrm{sam}}}{4M_H}\,N_{\mathrm{A}} = \left(\frac{1000\,\mathrm{g}}{4(1.0\,\mathrm{g/mol})}\right)\left(6.02 \times 10^{23}/\mathrm{mol}\right) = 1.5 \times 10^{26}\ .$$

Multiplying this by Q gives the total energy released: 6.4×10^{14} J. It is not required that the answer be in SI units; we could have used MeV throughout (in which case the answer is 4.0×10^{27} MeV).

(b) The number of ^{235}U nuclei is

$$N_{235} = \left(\frac{1000\,\mathrm{g}}{235\,\mathrm{g/mol}}\right)\left(6.02 \times 10^{23}/\mathrm{mol}\right) = 2.56 \times 10^{24}\ .$$

If all the U-235 nuclei fission, the energy release (using the result of Eq. 44-6) is

$$N_{235}Q_{\mathrm{fission}} = \left(2.56 \times 10^{22}\right)(200\,\mathrm{MeV}) = 5.1 \times 10^{26}\,\mathrm{MeV} = 8.2 \times 10^{13}\ \mathrm{J}\ .$$

We see that the fusion process (with regard to a unit mass of fuel) produces a larger amount of energy (despite the fact that the Q value per event is smaller).

45. (a) Since two neutrinos are produced per proton-proton cycle (see Eq. 44-10 or Fig. 44-11), the rate of neutrino production R_ν satisfies

$$R_\nu = \frac{2P}{Q} = \frac{2(3.9 \times 10^{26}\,\mathrm{W})}{(26.7\,\mathrm{MeV})(1.6 \times 10^{-13}\,\mathrm{J/MeV})} = 1.8 \times 10^{38}\ \mathrm{s}^{-1}\ .$$

(b) Let d_{es} be the Earth to Sun distance, and R be the radius of Earth (see Appendix C). Earth represents a small cross section in the "sky" as viewed by a fictitious observer on the Sun. The rate of neutrinos intercepted by that area (very small, relative to the area of the full "sky") is

$$R_{\nu,\mathrm{Earth}} = R_\nu\left(\frac{\pi R_e^2}{4\pi d_{es}^2}\right) = \frac{(1.8 \times 10^{38}\,\mathrm{s}^{-1})}{4}\left(\frac{6.4 \times 10^6\,\mathrm{m}}{1.5 \times 10^{11}\,\mathrm{m}}\right)^2 = 8.2 \times 10^{28}\ \mathrm{s}^{-1}\ .$$

46. (a) The products of the carbon cycle are $2e^+ + 2\nu + {}^4$He, the same as that of the proton-proton cycle (see Eq. 44-10). The difference in the number of photons is not significant.

(b) $Q_{\text{carbon}} = Q_1 + Q_2 + \cdots + Q_6 = (1.95+1.19+7.55+7.30+1.73+4.97)\text{MeV} = 24.7\,\text{MeV}$, which is the same as that for the proton-proton cycle (once we subtract out the electron-positron annihilations; see Fig. 44-11): $Q_{p-p} = 26.7\,\text{MeV} - 2(1.02\,\text{MeV}) = 24.7\,\text{MeV}$.

47. (a) The mass of a carbon atom is $(12.0\,\text{u})(1.661\times10^{-27}\,\text{kg/u}) = 1.99\times10^{-26}\,\text{kg}$, so the number of carbon atoms in 1.00 kg of carbon is $(1.00\,\text{kg})/(1.99\times10^{-26}\,\text{kg}) = 5.02\times10^{25}$. The heat of combustion per atom is $(3.3\times10^7\,\text{J/kg})/(5.02\times10^{25}\,\text{atom/kg}) = 6.58\times10^{-19}\,\text{J/atom}$. This is 4.11 eV/atom.

(b) In each combustion event, two oxygen atoms combine with one carbon atom, so the total mass involved is $2(16.0\,\text{u}) + (12.0\,\text{u}) = 44\,\text{u}$. This is $(44\,\text{u})(1.661\times10^{-27}\,\text{kg/u}) = 7.31\times10^{-26}\,\text{kg}$. Each combustion event produces 6.58×10^{-19} J so the energy produced per unit mass of reactants is $(6.58\times10^{-19}\,\text{J})/(7.31\times10^{-26}\,\text{kg}) = 9.00\times10^6\,\text{J/kg}$.

(c) If the Sun were composed of the appropriate mixture of carbon and oxygen, the number of combustion events that could occur before the Sun burns out would be $(2.0\times10^{30}\,\text{kg})/(7.31\times10^{-26}\,\text{kg}) = 2.74\times10^{55}$. The total energy released would be $E = (2.74\times10^{55})(6.58\times10^{-19}\,\text{J}) = 1.80\times10^{37}\,\text{J}$. If P is the power output of the Sun, the burn time would be

$$t = \frac{E}{P} = \frac{1.80\times10^{37}\,\text{J}}{3.9\times10^{26}\,\text{W}} = 4.62\times10^{10}\,\text{s} = 1460\,\text{y}\ .$$

48. The mass of the hydrogen in the Sun's core is $m_{\text{H}} = 0.35(\frac{1}{8}M_{\text{Sun}})$. The time it takes for the hydrogen to be entirely consumed is

$$t = \frac{M_{\text{H}}}{dm/dt} = \frac{(0.35)(\frac{1}{8})(2.0\times10^{30}\,\text{kg})}{(6.2\times10^{11}\,\text{kg/s})(3.15\times10^7\,\text{s/y})} = 5\times10^9\ \text{y}\ .$$

49. Since the mass of a helium atom is $(4.00\,\text{u})(1.661\times10^{-27}\,\text{kg/u}) = 6.64\times10^{-27}\,\text{kg}$, the number of helium nuclei originally in the star is $(4.6\times10^{32}\,\text{kg})/(6.64\times10^{-27}\,\text{kg}) = 6.92\times10^{58}$. Since each fusion event requires three helium nuclei, the number of fusion events that can take place is $N = 6.92\times10^{58}/3 = 2.31\times10^{58}$. If Q is the energy released in each event and t is the conversion time, then the power output is $P = NQ/t$ and

$$t = \frac{NQ}{P} = \frac{(2.31\times10^{58})(7.27\times10^6\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{5.3\times10^{30}\,\text{W}} = 5.07\times10^{15}\,\text{s} = 1.6\times10^8\ \text{y}\ .$$

50. (a) From $E = NQ = (M_{\text{sam}}/4m_p)Q$ we get the energy per kilogram of hydrogen consumed:

$$\frac{E}{M_{\text{sam}}} = \frac{Q}{4m_p} = \frac{(26.2\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})}{4(1.67\times10^{-27}\,\text{kg})} = 6.3\times10^{14}\ \text{J/kg}\ .$$

(b) Keeping in mind that a Watt is a Joule per second, the rate is

$$\frac{dm}{dt} = \frac{3.9\times10^{26}\,\text{W}}{6.3\times10^{14}\,\text{J/kg}} = 6.2\times10^{11}\ \text{kg/s}\ .$$

This agrees with the computation shown in Sample Problem 44-5.

(c) From the Einstein relation $E = Mc^2$ we get $P = dE/dt = c^2 dM/dt$, or

$$\frac{dM}{dt} = \frac{P}{c^2} = \frac{3.9\times10^{26}\,\text{W}}{(3.0\times10^8\,\text{m/s})^2} = 4.3\times10^9\ \text{kg/s}\ .$$

This finding, that $\frac{dm}{dt} > \frac{dM}{dt}$, is in large part due to the fact that, as the protons are consumed, their mass is mostly turned into alpha particles (helium), which remain in the Sun.

(d) The time to lose 0.10% of its total mass is

$$t = \frac{0.0010M}{dM/dt} = \frac{(0.0010)(2.0 \times 10^{30}\,\mathrm{kg})}{(4.3 \times 10^9\,\mathrm{kg/s})(3.15 \times 10^7\,\mathrm{s/y})} = 1.5 \times 10^{10}\ \mathrm{y}\ .$$

51. (a) $Q = (5m_{^2\mathrm{H}} - m_{^3\mathrm{He}} - m_{^4\mathrm{He}} - m_{^1\mathrm{H}} - 2m_n)c^2 = [5(2.014102\,\mathrm{u}) - 3.016029\,\mathrm{u} - 4.002603\,\mathrm{u} - 1.007825\,\mathrm{u} - 2(1.008665\,\mathrm{u})](931.5\,\mathrm{MeV/u}) = 24.9\,\mathrm{MeV}$.

(b) Assuming 30.0% of the deuterium undergoes fusion, the total energy released is

$$E = NQ = \left(\frac{0.300M}{5m_{^2\mathrm{H}}}\right) Q\ .$$

Thus, the rating is

$$\begin{aligned} R &= \frac{E}{2.6 \times 10^{28}\,\mathrm{MeV/megaton\ TNT}} \\ &= \frac{(0.300)(500\,\mathrm{kg})(24.9\,\mathrm{MeV})}{5(2.0\,\mathrm{u})(1.66 \times 10^{-27}\,\mathrm{kg/u})(2.6 \times 10^{28}\,\mathrm{MeV/megaton\ TNT})} \\ &= 8.65\,\mathrm{megaton\ TNT}\ . \end{aligned}$$

52. In Eq. 44-13,

$$\begin{aligned} Q &= (2m_{^2\mathrm{H}} - m_{^3\mathrm{He}} - m_n)c^2 \\ &= [2(2.014102\,\mathrm{u}) - 3.016049\,\mathrm{u} - 1.008665\,\mathrm{u}](931.5\,\mathrm{MeV/u}) \\ &= 3.27\ \mathrm{MeV}\ . \end{aligned}$$

In Eq. 44-14,

$$\begin{aligned} Q &= (2m_{^2\mathrm{H}} - m_{^3\mathrm{H}} - m_{^1\mathrm{H}})c^2 \\ &= [2(2.014102\,\mathrm{u}) - 3.016049\,\mathrm{u} - 1.007825\,\mathrm{u}](931.5\,\mathrm{MeV/u}) \\ &= 4.03\ \mathrm{MeV}\ . \end{aligned}$$

Finally, in Eq. 44-15,

$$\begin{aligned} Q &= (m_{^2\mathrm{H}} + m_{^3\mathrm{H}} - m_{^4\mathrm{He}} - m_n)c^2 \\ &= [2.014102\,\mathrm{u} + 3.016049\,\mathrm{u} - 4.002603\,\mathrm{u} - 1.008665\,\mathrm{u}](931.5\,\mathrm{MeV/u}) \\ &= 17.59\ \mathrm{MeV}\ . \end{aligned}$$

53. Since 1.00 L of water has a mass of 1.00 kg, the mass of the heavy water in 1.00 L is $0.0150 \times 10^{-2}\,\mathrm{kg} = 1.50 \times 10^{-4}\,\mathrm{kg}$. Since a heavy water molecule contains one oxygen atom, one hydrogen atom and one deuterium atom, its mass is $(16.0\,\mathrm{u} + 1.00\,\mathrm{u} + 2.00\,\mathrm{u}) = 19.0\,\mathrm{u}$ or $(19.0\,\mathrm{u})(1.661 \times 10^{-27}\,\mathrm{kg/u}) = 3.16 \times 10^{-26}\,\mathrm{kg}$. The number of heavy water molecules in a liter of water is $(1.50 \times 10^{-4}\,\mathrm{kg})/(3.16 \times 10^{-26}\,\mathrm{kg}) = 4.75 \times 10^{21}$. Since each fusion event requires two deuterium nuclei, the number of fusion events that can occur is $N = 4.75 \times 10^{21}/2 = 2.38 \times 10^{21}$. Each event releases energy $Q = (3.27 \times 10^6\,\mathrm{eV})(1.60 \times 10^{-19}\,\mathrm{J/eV}) = 5.23 \times 10^{-13}\,\mathrm{J}$. Since all events take place in a day, which is $8.64 \times 10^4\,\mathrm{s}$, the power output is

$$P = \frac{NQ}{t} = \frac{(2.38 \times 10^{21})(5.23 \times 10^{-13}\,\mathrm{J})}{8.64 \times 10^4\,\mathrm{s}} = 1.44 \times 10^4\,\mathrm{W} = 14.4\ \mathrm{kW}\ .$$

54. Conservation of energy gives $Q = K_\alpha + K_\mathrm{n}$, and conservation of linear momentum (due to the assumption of negligible initial velocities) gives $|p_\alpha| = |p_\mathrm{n}|$. We can write the classical formula for kinetic energy in terms of momentum:

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

which implies that $K_n = (m_\alpha/m_n)K_\alpha$. Consequently, conservation of energy and momentum allows us to solve for kinetic energy of the alpha particle which results from the fusion:

$$K_\alpha = \frac{Q}{1 + \frac{m_\alpha}{m_n}} = \frac{17.59\,\text{MeV}}{1 + \frac{4.0015\,\text{u}}{1.008665\text{u}}} = 3.541 \text{ MeV}$$

where we have found the mass of the alpha particle by subtracting two electron masses from the ^{4}He mass (quoted several times in this and the previous chapter). Then, $K_n = Q - K_\alpha$ yields 14.05 MeV for the neutron kinetic energy.

Chapter 45

1. Using Table 45-1, the difference in mass between the muon and the pion is

$$\Delta m = \left(139.6\,\frac{\text{MeV}}{c^2} - 105.7\,\frac{\text{MeV}}{c^2}\right) = \frac{(33.9\,\text{MeV})(1.60\times10^{-13}\,\text{J/MeV})}{(2.998\times10^{8}\,\text{m/s})^2} = 6.03\times10^{-29}\ \text{kg}\ .$$

2. We establish a ratio, using Eq. 22-4 and Eq. 14-1:

$$\begin{aligned}\frac{F_{\text{gravity}}}{F_{\text{electric}}} &= \frac{Gm_e^2/r^2}{ke^2/r^2} = \frac{4\pi\varepsilon_0 G m_e^2}{e^2} \\ &= \frac{(6.67\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{C}^2)(9.11\times10^{-31}\,\text{kg})^2}{(9.0\times10^{9}\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\,\text{C})^2} \\ &= 2.4\times10^{-43}\ .\end{aligned}$$

Since $F_{\text{gravity}} \ll F_{\text{electric}}$, we can neglect the gravitational force acting between particles in a bubble chamber.

3. Conservation of momentum requires that the gamma ray particles move in opposite directions with momenta of the same magnitude. Since the magnitude p of the momentum of a gamma ray particle is related to its energy by $p = E/c$, the particles have the same energy E. Conservation of energy yields $m_\pi c^2 = 2E$, where m_π is the mass of a neutral pion. According to Table 45-4, the rest energy of a neutral pion is $m_\pi c^2 = 135.0\,\text{MeV}$. Hence, $E = (135.0\,\text{MeV})/2 = 67.5\,\text{MeV}$. We use the result of Exercise 3 of Chapter 39 to obtain the wavelength of the gamma rays:

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{67.5\times10^{6}\,\text{eV}} = 1.84\times10^{-5}\,\text{nm} = 18.4\ \text{fm}\ .$$

4. By charge conservation, it is clear that reversing the sign of the pion means we must reverse the sign of the muon. In effect, we are replacing the charged particles by their antiparticles. Less obvious is the fact that we should now put a "bar" over the neutrino (something we should also have done for some of the reactions and decays discussed in the previous two chapters, except that we had not yet learned about antiparticles). To understand the "bar" we refer the reader to the discussion in §45-4. The decay of the negative pion is $\pi^- \to \mu^- + \bar{\nu}$. A subscript can be added to the antineutrino to clarify what "type" it is, as discussed in §45-4.

5. The energy released would be twice the rest energy of Earth, or $E = 2mc^2 = 2(5.98\times10^{24}\,\text{kg})(2.998\times10^{8}\,\text{m/s})^2 = 1.08\times10^{42}\,\text{J}$. The mass of Earth can be found in Appendix C.

6. Since the density of water is $\rho = 1000\,\text{kg/m}^3 = 1\,\text{kg/L}$, then the total mass of the pool is $\rho\mathcal{V} = 4.32\times10^{5}\,\text{kg}$, where $\mathcal{V}$ is the given volume. Now, the fraction of that mass made up by the protons is 10/18 (by counting the protons versus total nucleons in a water molecule). Consequently, if we ignore

the effects of neutron decay (neutrons can beta decay into protons) in the interest of making an order-of-magnitude calculation, then the number of particles susceptible to decay via this $T_{1/2} = 10^{32}$ y half-life is

$$N = \frac{\frac{10}{18} M_{\text{pool}}}{m_p} = \frac{\frac{10}{18}(4.32 \times 10^5\,\text{kg})}{1.67 \times 10^{-27}\,\text{kg}} = 1.44 \times 10^{32} \ .$$

Using Eq. 43-19, we obtain

$$R = \frac{N \ln 2}{T_{1/2}} = \frac{\left(1.44 \times 10^{32}\right) \ln 2}{10^{32}\,\text{y}} \ \approx \ 1 \text{ decay/y} \ .$$

7. From Eq. 38-45, the Lorentz factor would be

$$\gamma = \frac{E}{mc^2} = \frac{1.5 \times 10^6\,\text{eV}}{20\,\text{eV}} = 75000 \ .$$

Solving Eq. 38-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \quad \Longrightarrow \quad v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

which implies that the difference between v and c is

$$c - v = c\left(1 - \sqrt{1 - \frac{1}{\gamma^2}}\right) \ \approx \ c\left(1 - \left(1 - \frac{1}{2\gamma^2} + \cdots\right)\right)$$

where we use the binomial expansion (see Appendix E) in the last step. Therefore,

$$c - v \approx c\left(\frac{1}{2\gamma^2}\right) = (299792458\,\text{m/s})\left(\frac{1}{2(75000)^2}\right) = 0.0266 \text{ m/s} \ .$$

8. From Eq. 38-49, the Lorentz factor is

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{80\,\text{MeV}}{135\,\text{MeV}} = 1.59 \ .$$

Solving Eq. 38-8 for the speed, we find

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \quad \Longrightarrow \quad v = c\sqrt{1 - \frac{1}{\gamma^2}}$$

which yields $v = 0.778c$ or $v = 2.33 \times 10^8$ m/s. Now, in the reference frame of the laboratory, the lifetime of the pion is not the given τ value but is "dilated." Using Eq. 38-9, the time in the lab is

$$t = \gamma\tau = (1.59)\left(8.3 \times 10^{-17}\,\text{s}\right) = 1.3 \times 10^{-16} \text{ s} \ .$$

Finally, using Eq. 38-10, we find the distance in the lab to be

$$x = vt = \left(2.33 \times 10^8\,\text{m/s}\right)\left(1.3 \times 10^{-16}\,\text{s}\right) = 3.1 \times 10^{-8} \text{ m} \ .$$

9. Table 45-4 gives the rest energy of each pion as 139.6 MeV. The magnitude of the momentum of each pion is $p_\pi = (358.3\,\text{MeV})/c$. We use the relativistic relationship between energy and momentum (Eq. 38-52) to find the total energy of each pion:

$$E_\pi = \sqrt{(p_\pi c)^2 + (m_\pi c^2)^2} = \sqrt{(358.3\,\text{MeV})^2 + (139.6\,\text{MeV})^2} = 384.5 \text{ MeV} \ .$$

Conservation of energy yields $m_\rho c^2 = 2E_\pi = 2(384.5\,\text{MeV}) = 769\,\text{MeV}$.

10. (a) In SI units, $K = (2200\,\text{MeV})(1.6\times10^{-13}\,\text{J/MeV}) = 3.52\times10^{-10}\,\text{J}$. Similarly, $mc^2 = 2.85\times10^{-10}\,\text{J}$ for the positive tau. Eq. 38-51 leads to the relativistic momentum:

$$p = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{1}{2.998\times10^8}\sqrt{\left(3.52\times10^{-10}\right)^2 + 2\left(3.52\times10^{-10}\right)\left(2.85\times10^{-10}\right)}$$

which yields $p = 1.90\times10^{-18}\,\text{kg}\cdot\text{m/s}$.

(b) According to problem 46 in Chapter 38, the radius should be calculated with the relativistic momentum:

$$r = \frac{\gamma mv}{|q|B} = \frac{p}{eB}$$

where we use the fact that the positive tau has charge $e = 1.6\times10^{-19}\,\text{C}$. With $B = 1.20\,\text{T}$, this yields $r = 9.9\,\text{m}$.

11. (a) Conservation of energy gives $Q = K_2 + K_3 = E_1 - E_2 - E_3$ where E refers here to the *rest* energies (mc^2) instead of the total energies of the particles. Writing this as $K_2 + E_2 - E_1 = -(K_3 + E_3)$ and squaring both sides yields

$$K_2^2 + 2K_2E_2 - 2K_2E_1 + (E_1 - E_2)^2 = K_3^2 + 2K_3E_3 + E_3^2 \ .$$

Next, conservation of linear momentum (in a reference frame where particle 1 was at rest) gives $|p_2| = |p_3|$ (which implies $(p_2c)^2 = (p_3c)^2$). Therefore, Eq. 38-51 leads to

$$K_2^2 + 2K_2E_2 = K_3^2 + 2K_3E_3$$

which we subtract from the above expression to obtain

$$-2K_2E_1 + (E_1 - E_2)^2 = E_3^2 \ .$$

This is now straightforward to solve for K_2 and yields the result stated in the problem.

(b) Setting $E_3 = 0$ in

$$K_2 = \frac{1}{2E_1}\left[(E_1 - E_2)^2 - E_3^2\right]$$

and using the rest energy values given in Table 45-1 readily gives the same result for K_μ as computed in Sample Problem 45-1.

12. (a) Eq. 45-14 conserves charge since both the proton and the positron have $q = +e$ (and the neutrino is uncharged).

(b) Energy conservation is not violated since $m_pc^2 > m_ec^2 + m_\nu c^2$.

(c) We are free to view the decay from the rest frame of the proton. Both the positron and the neutrino are able to carry momentum, and so long as they travel in opposite directions with appropriate values of p (so that $\sum \vec{p} = 0$) then linear momentum is conserved.

(d) If we examine the spin angular momenta, there does seem to be a violation of angular momentum conservation (Eq. 45-14 shows a spin-one-half particle decaying into two spin-one-half particles).

13. (a) The conservation laws considered so far are associated with energy, momentum, angular momentum, charge, baryon number, and the three lepton numbers. The rest energy of the muon is 105.7 MeV, the rest energy of the electron is 0.511 MeV, and the rest energy of the neutrino is zero. Thus, the total rest energy before the decay is greater than the total rest energy after. The excess energy can be carried away as the kinetic energies of the decay products and energy can be conserved. Momentum is conserved if the electron and neutrino move away from the decay in opposite directions with equal magnitudes of momenta. Since the orbital angular momentum is zero, we consider only spin angular momentum. All the particles have spin $\hbar/2$. The total angular momentum after the decay must be either $\hbar$ (if the spins are aligned) or zero (if the spins are antialigned). Since the spin before the

decay is $\hbar/2$, angular momentum cannot be conserved. The muon has charge $-e$, the electron has charge $-e$, and the neutrino has charge zero, so the total charge before the decay is $-e$ and the total charge after is $-e$. Charge is conserved. All particles have baryon number zero, so baryon number is conserved. The muon lepton number of the muon is $+1$, the muon lepton number of the muon neutrino is $+1$, and the muon lepton number of the electron is 0. Muon lepton number is conserved. The electron lepton numbers of the muon and muon neutrino are 0 and the electron lepton number of the electron is $+1$. Electron lepton number is not conserved. The laws of conservation of angular momentum and electron lepton number are not obeyed and this decay does not occur..

(b) We analyze the decay in the same way. We find that only charge is not conserved.

(c) Here we find that energy and muon lepton number cannot be conserved.

14. (a) Noting that there are two positive pions created (so, in effect, its decay products are doubled), then we count up the electrons, positrons and neutrinos: $2\mathrm{e}^+ + \mathrm{e}^- + 5\nu + 4\bar{\nu}$.

(b) The final products are all leptons, so the baryon number of A_2^+ is zero. Both the pion and rho meson have integer-valued spins, so A_2^+ is a meson (and a boson).

15. For purposes of deducing the properties of the antineutron, one may cancel a proton from each side of the reaction and write the equivalent reaction as

$$\pi^+ \to \mathrm{p} + \overline{\mathrm{n}}\ .$$

Particle properties can be found in Tables 45-3 and 45-4. The pion and proton each have charge $+e$, so the antineutron must be neutral. The pion has baryon number zero (it is a meson) and the proton has baryon number $+1$, so the baryon number of the antineutron must be -1. The pion and the proton each have strangeness zero, so the strangeness of the antineutron must also be zero. In summary, $q = 0$, $B = -1$, and $S = 0$ for the antineutron.

16. (a) Referring to Tables 45-3 and 45-4, we find the strangeness of K^0 is $+1$, while it is zero for both π^+ and π^-. Consequently, strangeness is not conserved in this decay; $\mathrm{K}^0 \to \pi^+ + \pi^-$ does not proceed via the strong interaction.

(b) The strangeness of each side is -1, which implies that the decay is governed by the strong interaction.

(c) The strangeness or Λ^0 is -1 while that of $\mathrm{p}+\pi^-$ is zero, so the decay is not via the strong interaction.

(d) The strangeness of each side is -1; it proceeds via the strong interaction.

17. (a) See the solution to Exercise 13 for the quantities to be considered, adding strangeness to the list. The lambda has a rest energy of 1115.6 MeV, the proton has a rest energy of 938.3 MeV, and the kaon has a rest energy of 493.7 MeV. The rest energy before the decay is less than the total rest energy after, so energy cannot be conserved. Momentum can be conserved. The lambda and proton each have spin $\hbar/2$ and the kaon has spin zero, so angular momentum can be conserved. The lambda has charge zero, the proton has charge $+e$, and the kaon has charge $-e$, so charge is conserved. The lambda and proton each have baryon number $+1$, and the kaon has baryon number zero, so baryon number is conserved. The lambda and kaon each have strangeness -1 and the proton has strangeness zero, so strangeness is conserved. Only energy cannot be conserved.

(b) The omega has a rest energy of 1680 MeV, the sigma has a rest energy of 1197.3 MeV, and the pion has a rest energy of 135 MeV. The rest energy before the decay is greater than the total rest energy after, so energy can be conserved. Momentum can be conserved. The omega and sigma each have spin $\hbar/2$ and the pion has spin zero, so angular momentum can be conserved. The omega has charge $-e$, the sigma has charge $-e$, and the pion has charge zero, so charge is conserved. The omega and sigma have baryon number $+1$ and the pion has baryon number 0, so baryon number is conserved. The omega has strangeness -3, the sigma has strangeness -1, and the pion has strangeness zero, so strangeness is not conserved.

(c) The kaon and proton can bring kinetic energy to the reaction, so energy can be conserved even though the total rest energy after the collision is greater than the total rest energy before. Momentum can be conserved. The proton and lambda each have spin $\hbar/2$ and the kaon and pion each have spin zero, so angular momentum can be conserved. The kaon has charge $-e$, the proton has charge $+e$, the lambda has charge zero, and the pion has charge $+e$, so charge is not conserved. The proton and lambda each have baryon number $+1$, and the kaon and pion each have baryon number zero; baryon number is conserved. The kaon has strangeness -1, the proton and pion each have strangeness zero, and the lambda has strangeness -1, so strangeness is conserved. Only charge is not conserved.

18. (a) From Eq. 38-47,

$$\begin{aligned} Q &= -\Delta m\, c^2 = (m_{\Sigma^+} + m_{K^+} - m_{\pi^+} - m_p)c^2 \\ &= 1189.4\,\text{MeV} + 493.7\,\text{MeV} - 139.6\,\text{MeV} - 938.3\,\text{MeV} \\ &= 605 \text{ MeV} . \end{aligned}$$

(b) Similarly,

$$\begin{aligned} Q &= -\Delta m\, c^2 = (m_{\Lambda^0} + m_{\pi^0} - m_{K^-} - m_p)c^2 \\ &= 1115.6\,\text{MeV} + 135.0\,\text{MeV} - 493.7\,\text{MeV} - 938.3\,\text{MeV} \\ &= -181 \text{ MeV} . \end{aligned}$$

19. Conservation of energy (see Eq. 38-44) leads to

$$\begin{aligned} K_f &= -\Delta m\, c^2 + K_i = (m_{\Sigma^-} - m_{\pi^-} - m_n)c^2 + K_i \\ &= 1197.3\,\text{MeV} - 139.6\,\text{MeV} - 939.6\,\text{MeV} + 220\,\text{MeV} \\ &= 338 \text{ MeV} . \end{aligned}$$

20. The formula for T_z as it is usually written to include strange baryons is $T_z = q - (S + B)/2$. Also, we interpret the symbol q in the T_z formula in terms of elementary charge units; this is how q is listed in Table 45-3. In terms of charge q as we have used it in previous chapters, the formula is $T_z = \frac{q}{e} - \frac{1}{2}(B+S)$. For instance, $T_z = +\frac{1}{2}$ for the proton (and the neutral Xi) and $T_z = -\frac{1}{2}$ for the neutron (and the negative Xi). The baryon number B is $+1$ for all the particles in Fig. 45-4(a). Rather than use a sloping axis as in Fig. 45-4 (there it is done for the q values), one reproduces (if one uses the "corrected" formula for T_z mentioned above) exactly the same pattern using regular rectangular axes (T_z values along the horizontal axis and Y values along the vertical) with the neutral lambda and sigma particles situated at the origin.

21. (a) As far as the conservation laws are concerned, we may cancel a proton from each side of the reaction equation and write the reaction as $\text{p} \rightarrow \Lambda^0 + x$. Since the proton and the lambda each have a spin angular momentum of $\hbar/2$, the spin angular momentum of x must be either zero or $\hbar$. Since the proton has charge $+e$ and the lambda is neutral, x must have charge $+e$. Since the proton and the lambda each have a baryon number of $+1$, the baryon number of x is zero. Since the strangeness of the proton is zero and the strangeness of the lambda is -1, the strangeness of x is $+1$. We take the unknown particle to be a spin zero meson with a charge of $+e$ and a strangeness of $+1$. Look at Table 45-4 to identify it as a K^+ particle.

(b) Similar analysis tells us that x is a spin-$\frac{1}{2}$ antibaryon ($B = -1$) with charge and strangeness both zero. Inspection of Table 45-3 reveals it is an antineutron.

(c) Here x is a spin-0 (or spin-1) meson with charge zero and strangeness -1. According to Table 45-4, it could be a $\overline{K}^0$ particle.

22. (a) From Eq. 38-47,

$$\begin{aligned} Q &= -\Delta m\, c^2 = (m_{\Lambda^0} - m_p - m_{\pi^-})c^2 \\ &= 1115.6\,\text{MeV} - 938.3\,\text{MeV} - 139.6\,\text{MeV} = 37.7\ \text{MeV}\ . \end{aligned}$$

(b) We use the formula obtained in problem 11 (where it should be emphasized that E is used to mean the rest energy, not the total energy):

$$\begin{aligned} K_p &= \frac{1}{2E_\Lambda}\left[(E_\Lambda - E_p)^2 - E_\pi^2\right] \\ &= \frac{(1115.6\,\text{MeV} - 938.3\,\text{MeV})^2 - (139.6\,\text{MeV})^2}{2(1115.6\,\text{MeV})} = 5.35\ \text{MeV}\ . \end{aligned}$$

(c) By conservation of energy,

$$K_{\pi^-} = Q - K_p = 37.7\,\text{MeV} - 5.35\,\text{MeV} = 32.4\ \text{MeV}\ .$$

23. (a) We indicate the antiparticle nature of each quark with a "bar" over it. Thus, $\bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$ represents an antiproton.

(b) Similarly, $\bar{\text{u}}\bar{\text{d}}\bar{\text{d}}$ represents an antineutron.

24. (a) The combination ddu has a total charge of $(-\frac{1}{3} - \frac{1}{3} + \frac{2}{3}) = 0$, and a total strangeness of zero. From Table 45-3, we find it to be a neutron (n).

(b) For the combination uus, we have $Q = +\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = 1$ and $S = 0 + 0 - 1 = 1$. This is the Σ^+ particle.

(c) For the quark composition ssd, we have $Q = -\frac{1}{3} - \frac{1}{3} - \frac{1}{3} = -1$ and $S = -1 - 1 + 0 = -2$. This is a Ξ^-.

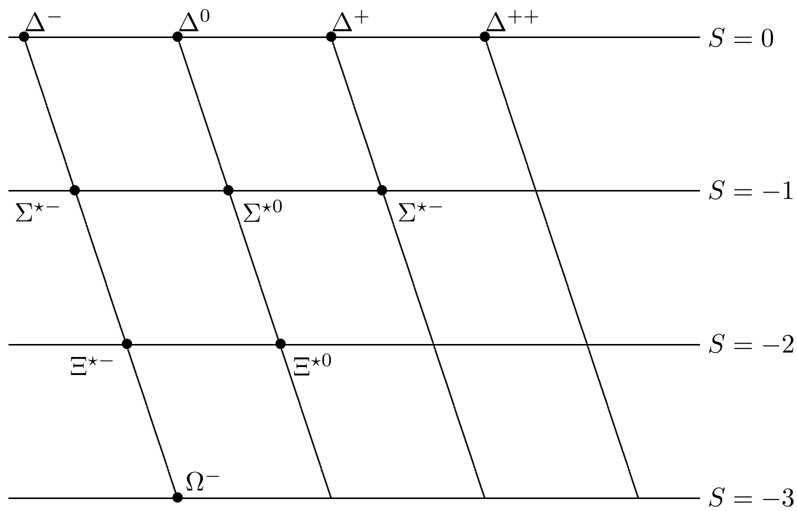
25. (a) Looking at the first three lines of Table 45-5, since the particle is a baryon, we determine that it must consist of three quarks. To obtain a strangeness of -2, two of them must be s quarks. Each of these has a charge of $-e/3$, so the sum of their charges is $-2e/3$. To obtain a total charge of e, the charge on the third quark must be $5e/3$. There is no quark with this charge, so the particle cannot be constructed. In fact, such a particle has never been observed.

(b) Again the particle consists of three quarks (and no antiquarks). To obtain a strangeness of zero, none of them may be s quarks. We must find a combination of three u and d quarks with a total charge of $2e$. The only such combination consists of three u quarks.

26. (a) Using Table 45-3, we find $q = 0$ and $S = -1$ for this particle (also, $B = 1$, since that is true for all particles in that table). From Table 45-5, we see it must therefore contain a strange quark (which has charge $-1/3$), so the other two quarks must have charges to add to zero. Assuming the others are among the lighter quarks (none of them being an antiquark, since $B = 1$), then the quark composition is $\bar{\text{u}}\bar{\text{s}}\bar{\text{d}}$.

(b) The reasoning is very similar to that of part (a). The main difference is that this particle must have two strange quarks. Its quark combination turns out to be $\bar{\text{u}}\bar{\text{s}}\bar{\text{s}}$.

27. If we were to use regular rectangular axes, then this would appear as a right triangle. Using the sloping q axis as the problem suggests, it is similar to an "upside down" equilateral triangle as we show below. The leftmost slanted line is for the -1 charge, and the rightmost slanted line is for the $+2$ charge.



28. Since only the strange quark (s) has non-zero strangeness, in order to obtain $S = -1$ we need to combine s with some non-strange antiquark (which would have the negative of the quantum numbers listed in Table 45-5). The difficulty is that the charge of the strange quark is $-1/3$, which means that (to obtain a total charge of $+1$) the antiquark would have to have a charge of $+\frac{4}{3}$. Clearly, there are no such antiquarks in our list. Thus, a meson with $S = -1$ and $q = +1$ cannot be formed with the quarks/antiquarks of Table 45-5. Similarly, one can show that, since no quark has $q = -\frac{4}{3}$, there cannot be a meson with $S = +1$ and $q = -1$.

29. From $\gamma = 1 + K/mc^2$ (see Eq. 38-49) and $v = \beta c = c\sqrt{1-\gamma^{-2}}$ (see Eq. 38-8), we get

$$v = c\sqrt{1 - \left(1 + \frac{K}{mc^2}\right)^{-2}} .$$

Therefore, for the Σ^{*0} particle,

$$v = (2.9979 \times 10^8 \,\mathrm{m/s})\sqrt{1 - \left(1 + \frac{1000\,\mathrm{MeV}}{1385\,\mathrm{MeV}}\right)^{-2}} = 2.4406 \times 10^8 \text{ m/s} ,$$

and for Σ^0,

$$v' = (2.9979 \times 10^8 \,\mathrm{m/s})\sqrt{1 - \left(1 + \frac{1000\,\mathrm{MeV}}{1192.5\,\mathrm{MeV}}\right)^{-2}} = 2.5157 \times 10^8 \text{ m/s} .$$

Thus Σ^0 moves faster than Σ^{*0} by

$$\Delta v = v' - v = (2.5157 - 2.4406)(10^8 \,\mathrm{m/s}) = 7.51 \times 10^6 \text{ m/s} .$$

30. Letting $v = Hr = c$, we obtain

$$r = \frac{c}{H} = \frac{3.0 \times 10^8 \,\mathrm{m/s}}{0.0193 \,\mathrm{m/s{\cdot}ly}} = 1.6 \times 10^{10} \text{ ly} .$$

31. We apply Eq. 38-33 for the Doppler shift in wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

where v is the recessional speed of the galaxy. We use Hubble's law to find the recessional speed: $v = Hr$, where r is the distance to the galaxy and H is the Hubble constant $(19.3 \times 10^{-3} \frac{\text{m}}{\text{s·ly}})$. Thus, $v = [19.3 \times 10^{-3} \frac{\text{m}}{\text{s·ly}}](2.40 \times 10^8 \text{ ly}) = 4.63 \times 10^6 \text{ m/s}$ and

$$\Delta\lambda = \frac{v}{c}\lambda = \left(\frac{4.63 \times 10^6 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)(656.3 \text{ nm}) = 10.1 \text{ nm} .$$

Since the galaxy is receding, the observed wavelength is longer than the wavelength in the rest frame of the galaxy. Its value is $656.3 \text{ nm} + 10.1 \text{ nm} = 666.4 \text{ nm}$.

32. First, we find the speed of the receding galaxy from Eq. 38-30:

$$\begin{aligned}\beta &= \frac{1-(f/f_0)^2}{1+(f/f_0)^2} = \frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2} \\ &= \frac{1-(590.0 \text{ nm}/602.0 \text{ nm})^2}{1+(590.0 \text{ nm}/602.0 \text{ nm})^2} = 0.02013\end{aligned}$$

where we use $f = c/\lambda$ and $f_0 = c/\lambda_0$. Then from Eq. 45-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{(0.02013)(2.998 \times 10^8 \text{ m/s})}{19.3 \text{ mm/s·ly}} = 3.13 \times 10^8 \text{ ly} .$$

(Note: if one uses the classical Doppler shift formula instead of the relativistic version in Eq. 38-30, one obtains $r = 31.7 \times 10^8$ ly, which is reasonably close to the value we obtained above. This is to be expected since $\beta \approx 0.02 \ll 1$.)

33. (a) Letting $v(r) = Hr \le v_e = \sqrt{2GM/r}$, we get $M/r^3 \ge H^2/2G$. Thus,

$$\rho = \frac{M}{4\pi r^3/3} = \frac{3}{4\pi}\frac{M}{r^3} \ge \frac{3H^2}{8\pi G} .$$

(b) The density being expressed in H-atoms/m^3 is equivalent to expressing it in terms of $\rho_0 = m_{\text{H}}/\text{m}^3 = 1.67 \times 10^{-27} \text{ kg/m}^3$. Thus,

$$\begin{aligned}\rho &= \frac{3H^2}{8\pi G\rho_0}\left(\text{H atoms/m}^3\right) = \frac{3(0.0193 \text{ m/s·ly})^2(1.00 \text{ ly}/9.460 \times 10^{15} \text{ m})^2(\text{H atoms/m}^3)}{8\pi(6.67 \times 10^{-11} \text{ m}^3/\text{kg·s}^2)(1.67 \times 10^{-27} \text{ kg/m}^3)} \\ &= 4.5 \text{ H atoms/m}^3 .\end{aligned}$$

34. (a) From $f = c/\lambda$ and Eq. 38-30, we get

$$\lambda_0 = \lambda\sqrt{\frac{1-\beta}{1+\beta}} = (\lambda_0 + \Delta\lambda)\sqrt{\frac{1-\beta}{1+\beta}} .$$

Dividing both sides by λ_0 leads to

$$1 = (1+z)\sqrt{\frac{1-\beta}{1+\beta}} .$$

We solve for β:

$$\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{z^2 + 2z}{z^2 + 2z + 2} .$$

(b) Now $z = 4.43$, so

$$\beta = \frac{(4.43)^2 + 2(4.43)}{(4.43)^2 + 2(4.43) + 2} = 0.934 .$$

(c) From Eq. 45-19,

$$r = \frac{v}{H} = \frac{\beta c}{H} = \frac{(0.943)(3.0 \times 10^8\,\mathrm{m/s})}{0.0193\,\mathrm{m/s{\cdot}ly}} = 1.5 \times 10^{10}\ \mathrm{ly}\ .$$

35. (a) From Eq. 41-29, we know that $N_2/N_1 = e^{-\Delta E/kT}$. We solve for ΔE:

$$\begin{aligned}\Delta E &= kT\,\ln\frac{N_1}{N_2} = (8.62 \times 10^{-5}\,\mathrm{eV/K})(2.7\,\mathrm{K})\,\ln\left(\frac{1-0.25}{0.25}\right) \\ &= 2.56 \times 10^{-4}\,\mathrm{eV} = 256\,\mu\mathrm{eV}\ .\end{aligned}$$

(b) Using the result of problem 3 in Chapter 39,

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,\mathrm{eV{\cdot}nm}}{2.56 \times 10^{-4}\,\mathrm{eV}} = 4.84 \times 10^6\,\mathrm{nm} = 4.84\ \mathrm{mm}\ .$$

36. From $F_{\mathrm{grav}} = GMm/r^2 = mv^2/r$ we find $M \propto v^2$. Thus, the mass of the Sun would be

$$M_s' = \left(\frac{v_{\mathrm{Mercury}}}{v_{\mathrm{Pluto}}}\right)^2 M_s = \left(\frac{47.9\,\mathrm{km/s}}{4.74\,\mathrm{km/s}}\right)^2 M_s = 102 M_s\ .$$

37. (a) The mass M within Earth's orbit is used to calculate the gravitational force on Earth. If r is the radius of the orbit, R is the radius of the new Sun, and M_S is the mass of the Sun, then

$$M = \left(\frac{r}{R}\right)^3 M_S = \left(\frac{1.50 \times 10^{11}\,\mathrm{m}}{5.90 \times 10^{12}\,\mathrm{m}}\right)^3 (1.99 \times 10^{30}\,\mathrm{kg}) = 3.27 \times 10^{25}\,\mathrm{kg}\ .$$

The gravitational force on Earth is given by GMm/r^2, where m is the mass of Earth and G is the universal gravitational constant. Since the centripetal acceleration is given by v^2/r, where v is the speed of Earth, $GMm/r^2 = mv^2/r$ and

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\mathrm{m^3/s^2\cdot kg})(3.27 \times 10^{25}\,\mathrm{kg})}{1.50 \times 10^{11}\,\mathrm{m}}} = 1.21 \times 10^2\ \mathrm{m/s}\ .$$

(b) The period of revolution is

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.50 \times 10^{11}\,\mathrm{m})}{1.21 \times 10^2\,\mathrm{m/s}} = 7.82 \times 10^9\,\mathrm{s} = 248\ \mathrm{y}\ .$$

38. (a) The mass of the portion of the galaxy within the radius r from its center is given by $M' = (r/R)^3 M$. Thus, from $GM'm/r^2 = mv^2/r$ (where m is the mass of the star) we get

$$v = \sqrt{\frac{GM'}{r}} = \sqrt{\frac{GM}{r}\left(\frac{r}{R}\right)^3} = r\sqrt{\frac{GM}{R^3}}\ .$$

(b) In the case where $M' = M$, we have

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM}} = \frac{2\pi r^{3/2}}{\sqrt{GM}}\ .$$

39. (a) We substitute $\lambda = (2898\,\mu\mathrm{m{\cdot}K})/T$ into the result of Exercise 3 of Chapter 39: $E = (1240\,\mathrm{eV{\cdot}nm})/\lambda$. First, we convert units: $2898\,\mu\mathrm{m{\cdot}K} = 2.898 \times 10^6\,\mathrm{nm{\cdot}K}$ and $1240\,\mathrm{eV{\cdot}nm} = 1.240 \times 10^{-3}\,\mathrm{MeV{\cdot}nm}$. Hence,

$$E = \frac{(1.240 \times 10^{-3}\,\mathrm{MeV{\cdot}nm})T}{2.898 \times 10^6\,\mathrm{nm{\cdot}K}} = (4.28 \times 10^{-10}\ \mathrm{MeV/K})T\ .$$

(b) The minimum energy required to create an electron-positron pair is twice the rest energy of an electron, or $2(0.511\,\text{MeV}) = 1.022\,\text{MeV}$. Hence,

$$T = \frac{E}{4.28 \times 10^{-10}\,\text{MeV/K}} = \frac{1.022\,\text{MeV}}{4.28 \times 10^{-10}\,\text{MeV/K}} = 2.39 \times 10^{9}\ \text{K} .$$

40. (a) For the universal microwave background, Wien's law leads to

$$T = \frac{2898\,\mu\text{m}\cdot\text{K}}{\lambda_{\text{max}}} = \frac{2.898\,\text{mm}\cdot\text{K}}{1.1\,\text{mm}} = 2.6\ \text{K} .$$

(b) At "decoupling" (when the universe became approximately "transparent"),

$$\lambda_{\text{max}} = \frac{2898\,\mu\text{m}\cdot\text{K}}{T} = \frac{2898\,\mu\text{m}\cdot\text{K}}{10^5\,\text{K}} = 29\ \text{nm} .$$

41. (a) We use the relativistic relationship between speed and momentum:

$$p = \gamma m v = \frac{mv}{\sqrt{1-(v/c)^2}} ,$$

which we solve for the speed v:

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(\frac{pc}{mc^2}\right)^2 + 1}} .$$

For an antiproton $mc^2 = 938.3\,\text{MeV}$ and $pc = 1.19\,\text{GeV} = 1190\,\text{MeV}$, so

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/938.3\,\text{MeV})^2 + 1}} = 0.785c .$$

For the negative pion $mc^2 = 193.6\,\text{MeV}$, and pc is the same. Therefore,

$$v = c\sqrt{1 - \frac{1}{(1190\,\text{MeV}/193.6\,\text{MeV})^2 + 1}} = 0.993c .$$

(b) See part (a).

(c) Since the speed of the antiprotons is about $0.78c$ but not over $0.79c$, an antiproton will trigger C1.

(d) Since the speed of the negative pions exceeds $0.79c$, a negative pion will trigger C2.

(e) and (f) We use $\Delta t = d/v$, where $d = 12\,\text{m}$. For an antiproton

$$\Delta t = \frac{12\,\text{m}}{0.785(2.998 \times 10^8\,\text{m/s})} = 5.1 \times 10^{-8}\,\text{s} = 51\ \text{ns} ,$$

and for a negative pion

$$\Delta t = \frac{12\,\text{m}}{0.993(2.998 \times 10^8\,\text{m/s})} = 4.0 \times 10^{-8}\,\text{s} = 40\ \text{ns} .$$

42. We note from track 1, and the quantum numbers of the original particle (A), that positively charged particles move in counterclockwise curved paths, and – by inference – negatively charged ones move along clockwise arcs. This immediately shows that tracks 1, 2, 4, 6, and 7 belong to positively charged particles, and tracks 5, 8 and 9 belong to negatively charged ones. Looking at the fictitious particles

in the table (and noting that each appears in the cloud chamber once [or not at all]), we see that this observation (about charged particle motion) greatly narrows the possibilities:

$$\begin{aligned} \text{tracks } 2,4,6,7 &\leftrightarrow \text{particles } C,F,H,J \\ \text{tracks } 5,8,9 &\leftrightarrow \text{particles } D,E,G \end{aligned}$$

This tells us, too, that the particle that does not appear at all is either B or I (since only one neutral particle "appears"). By charge conservation, tracks 2, 4 and 6 are made by particles with a single unit of positive charge (note that track 5 is made by one with a single unit of negative charge), which implies (by elimination) that track 7 is made by particle H. This is confirmed by examining charge conservation at the end-point of track 6. Having exhausted the charge-related information, we turn now to the fictitious quantum numbers. Consider the vertex where tracks 2, 3 and 4 meet (the Whimsy number is listed here as a subscript):

$$\begin{aligned} \text{tracks } 2,4 &\leftrightarrow \text{particles } C_2, F_0, J_{-6} \\ \text{tracks } 3 &\leftrightarrow \text{particle } B_4 \text{ or } I_6 \end{aligned}$$

The requirement that the Whimsy quantum number of the particle making track 4 must equal the sum of the Whimsy values for the particles making tracks 2 and 3 places a powerful constraint (see the subscripts above). A fairly quick trial and error procedure leads to the assignments: particle F makes track 4, and particles J and I make tracks 2 and 3, respectively. Particle B, then, is irrelevant to this set of events. By elimination, the particle making track 6 (the only positively charged particle not yet assigned) must be C. At the vertex defined by

$$A \rightarrow F + C + (\text{track } 5)_- \quad ,$$

where the charge of that particle is indicated by the subscript, we see that Cuteness number conservation requires that the particle making track 5 has Cuteness $= -1$, so this must be particle G. We have only one decision remaining:

$$\text{tracks } 8,9 \leftrightarrow \text{particles } D,E$$

Re-reading the problem, one finds that the particle making track 8 must be particle D since it is the one with seriousness $= 0$. Consequently, the particle making track 9 must be E.

43. (a) During the time interval Δt, the light emitted from galaxy A has traveled a distance $c\Delta t$. Meanwhile, the distance between Earth and the galaxy has expanded from r to $r' = r + r\alpha\Delta t$. Let $c\Delta t = r' = r + r\alpha\Delta t$, which leads to

$$\Delta t = \frac{r}{c - r\alpha} \ .$$

(b) The detected wavelength λ' is longer then λ by $\lambda\alpha\Delta t$ due to the expansion of the universe: $\lambda' = \lambda + \lambda\alpha\Delta t$. Thus,

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \alpha\Delta t = \frac{\alpha r}{c - \alpha r} \ .$$

(c) We use the binomial expansion formula (see Appendix E):

$$(1 \pm x)^n = 1 \pm \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \qquad (x^2 < 1)$$

to obtain

$$\begin{aligned} \frac{\Delta\lambda}{\lambda} &= \frac{\alpha r}{c - \alpha r} = \frac{\alpha r}{c}\left(1 - \frac{\alpha r}{c}\right)^{-1} \\ &= \frac{\alpha r}{c}\left[1 + \frac{-1}{1!}\left(-\frac{\alpha r}{c}\right) + \frac{(-1)(-2)}{2!}\left(-\frac{\alpha r}{c}\right)^2 + \cdots\right] \\ &\approx \frac{\alpha r}{c} + \left(\frac{\alpha r}{c}\right)^2 + \left(\frac{\alpha r}{c}\right)^3 \ . \end{aligned}$$

(d) When only the first term in the expansion for $\Delta\lambda/\lambda$ is retained we have

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\alpha r}{c} \ .$$

(e) We set

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{Hr}{c}$$

and compare with the result of part (d) to obtain $\alpha = H$.

(f) We use the formula $\Delta\lambda/\lambda = \alpha r/(c - \alpha r)$ to solve for r:

$$r = \frac{c(\Delta\lambda/\lambda)}{\alpha(1+\Delta\lambda/\lambda)} = \frac{(2.998\times10^8\,\text{m/s})(0.050)}{(0.0193\,\text{m/s}\cdot\text{ly})(1+0.050)} = 7.4\times10^8 \text{ ly} \ .$$

(g) From the result of part (a),

$$\Delta t = \frac{r}{c-\alpha r} = \frac{(7.4\times10^8\,\text{ly})(9.46\times10^{15}\,\text{m/ly})}{2.998\times10^8\,\text{m/s} - (0.0193\,\text{m/s}\cdot\text{ly})(7.4\times10^8\,\text{ly})} = 2.5\times10^{16} \text{ s} \ ,$$

which is equivalent to 7.8×10^8 y.

(h) Letting $r = c\Delta t$, we solve for Δt:

$$\Delta t = \frac{r}{c} = \frac{7.4\times10^8\,\text{ly}}{c} = 7.4\times10^8 \text{ y} \ .$$

(i) The distance is given by

$$r = c\Delta t = c(7.8\times10^8\,\text{y}) = 7.8\times10^8 \text{ ly} \ .$$

(j) From the result of part (f),

$$r_\text{B} = \frac{c(\Delta\lambda/\lambda)}{\alpha(1+\Delta\lambda/\lambda)} = \frac{(2.998\times10^8\,\text{m/s})(0.080)}{(0.0193\,\text{mm/s}\cdot\text{ly})(1+0.080)} = 1.15\times10^9 \text{ ly} \ .$$

(k) From the formula obtained in part (a),

$$\Delta t_\text{B} = \frac{r_\text{B}}{c - r_\text{B}\alpha} = \frac{(1.15\times10^9\,\text{ly})(9.46\times10^{15}\,\text{m/ly})}{2.998\times10^8\,\text{m/s} - (1.15\times10^9\,\text{ly})(0.0193\,\text{m/s}\cdot\text{ly})} = 3.9\times10^{16} \text{ s} \ ,$$

which is equivalent to 1.2×10^9 y.

(l) At the present time, the separation between the two galaxies A and B is given by $r_\text{now} = c\Delta t_\text{B} - c\Delta t_\text{A}$. Since $r_\text{now} = r_\text{then} + r_\text{then}\alpha\Delta t$, we get

$$r_\text{then} = \frac{r_\text{now}}{1+\alpha\Delta t} = 4.4\times10^8 \text{ ly} \ .$$

44. Assuming the line passes through the origin, its slope is $0.40c/(5.3\times10^9\,\text{ly})$. Then,

$$T = \frac{1}{H} = \frac{1}{\text{slope}} = \frac{5.3\times10^9\,\text{ly}}{0.40c} = \frac{5.3\times10^9\,\text{y}}{0.40} \approx 13\times10^9 \text{ y} \ .$$