



## Management Science

Publication details, including instructions for authors and subscription information:  
<http://pubsonline.informs.org>

### Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations

G rard P. Cachon, Martin A. Lariviere,

To cite this article:

G rard P. Cachon, Martin A. Lariviere, (2005) Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations. Management Science 51(1):30-44. <http://dx.doi.org/10.1287/mnsc.1040.0215>

Full terms and conditions of use: <http://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact [permissions@informs.org](mailto:permissions@informs.org).

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

  2005 INFORMS

Please scroll down for article—it is on subsequent pages



INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

# Supply Chain Coordination with Revenue-Sharing Contracts: Strengths and Limitations

Gérard P. Cachon

The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104, cachon@wharton.upenn.edu

Martin A. Lariviere

Kellogg School of Management, Northwestern University, Evanston, Illinois 60208, m-lariviere@kellogg.nwu.edu

Under a revenue-sharing contract, a retailer pays a supplier a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generates. Such contracts have become more prevalent in the videocassette rental industry relative to the more conventional wholesale price contract. This paper studies revenue-sharing contracts in a general supply chain model with revenues determined by each retailer's purchase quantity and price. Demand can be deterministic or stochastic and revenue is generated either from rentals or outright sales. Our model includes the case of a supplier selling to a classical fixed-price newsvendor or a price-setting newsvendor. We demonstrate that revenue sharing coordinates a supply chain with a single retailer (i.e., the retailer chooses optimal price and quantity) and arbitrarily allocates the supply chain's profit. We compare revenue sharing to a number of other supply chain contracts (e.g., buy-back contracts, price-discount contracts, quantity-flexibility contracts, sales-rebate contracts, franchise contracts, and quantity discounts). We find that revenue sharing is equivalent to buybacks in the newsvendor case and equivalent to price discounts in the price-setting newsvendor case. Revenue sharing also coordinates a supply chain with retailers competing in quantities, e.g., Cournot competitors or competing newsvendors with fixed prices. Despite its numerous merits, we identify several limitations of revenue sharing to (at least partially) explain why it is not prevalent in all industries. In particular, we characterize cases in which revenue sharing provides only a small improvement over the administratively cheaper wholesale price contract. Additionally, revenue sharing does not coordinate a supply chain with demand that depends on costly retail effort. We develop a variation on revenue sharing for this setting.

**Key words:** game theory; bargaining; newsvendor; inventory competition; sales effort; Cournot

**History:** Accepted by Fangruo Chen and Stefanos A. Zenios, special issue editors; received June 28, 2000. This paper was with the authors 14 months for 3 revisions.

## 1. Introduction

The videocassette retailer faces a challenging capacity problem. The peak popularity of a rental title lasts only a few weeks, but the cost of a tape has traditionally been high relative to the price of a rental. In a conventional sales agreement, the retailer purchases each tape from his supplier for about \$65 and collects about \$3 per rental. Hence, a tape earns a profit only after 22 rentals. Because the demand for a tape typically starts high and tapers quickly, a retailer cannot justify purchasing enough tapes to cover the initial peak demand entirely.

At Blockbuster Inc., a large video retailer, the poor availability of new-release videos was consistently a major customer complaint (McCollum 1998, Shapiro 1998a). Seeking a solution to this problem, in 1998 Blockbuster agreed to pay its suppliers a portion (probably in the range of 30% to 45%) of its rental income in exchange for a reduction in the initial

price per tape from \$65 to \$8.<sup>1</sup> If Blockbuster kept half of the rental income, the break-even point for a tape would drop to approximately six rentals, thereby allowing Blockbuster to purchase many more tapes.

The introduction of revenue sharing coincided with a significant improvement in performance at Blockbuster: Warren and Peers (2002) report that Blockbuster's market share of video rentals increased from 24% in 1997 to 40% in 2002. Not surprisingly, this has led to litigation against Blockbuster and the movie studios, alleging that revenue-sharing contracts have hurt competition in the industry. To date, these have been unsuccessful (*Wall Street Journal* 2002). Indeed, evidence shows that the new terms of trade helped

<sup>1</sup> Blockbuster's terms are not public. Rentrak, a distributor, offers the following: The studio gets 45% of the revenue, Rentrak 10%, and the retailer 45% (www.renttrak.com). Because Blockbuster deals directly with the studios, its terms should be at least as generous.

the industry in aggregate: Mortimer (2000) estimates revenue sharing increased the industry's total profit by 7%.

This paper studies how revenue sharing alters the performance of a supply chain. While inspired by the video rental industry, our model encompasses many settings. It applies to any industry and any link between two levels in a supply chain (e.g., supplier-manufacturer or manufacturer-distributor). It does not matter whether the asset produced at the upstream level is rented at the downstream level (as in the video industry) or sold outright (as in the book industry), or whether demand is stochastic or deterministic. To be specific, our base model has a supplier selling to a single retailer. The retailer makes two decisions that determine the total revenue generated over a single selling period: the number of units to purchase from a supplier and the retail price. The marketing literature often assumes the revenue function is derived from a deterministic demand curve (see Lilien et al. 1992), whereas the operations literature often assumes it is derived from stochastic demand with a fixed retail price, i.e., a newsvendor model (see Tsay et al. 1998). Our formulation includes both of those revenue functions. We show that revenue sharing coordinates this supply chain; i.e., the retailer chooses supply chain optimal actions (quantity and price) and the supply chain's profit can be arbitrarily divided between the firms. Further, a single revenue-sharing contract can coordinate a supply chain with multiple noncompeting retailers even if the retailers have different revenue functions.

Several alternative contracts have been shown to coordinate this supply chain when the revenue function is generated from a fixed-price newsvendor model: buy-back contracts (Pasternack 1985), quantity-flexibility (QF) contracts (Tsay 1999), and sales-rebate contracts (Taylor 2002). In fact, we show that revenue sharing and buy-back contracts are equivalent in this setting in the strongest sense: For any buy-back contract there exists a revenue-sharing contract that generates the same cash flows for *any* realization of demand. The comparable result does not hold between revenue sharing and the other two contracts. However, revenue sharing and buybacks are not equivalent with a price-setting newsvendor. While revenue sharing also coordinates that supply chain, neither buybacks nor quantity flexibility nor sales rebates are able to do so. Bernstein and Federgruen (2005) study price-discount contracts and demonstrate that those contracts do coordinate the price-setting newsvendor. A price-discount contract has a wholesale price and a buy-back rate, just like a traditional buy-back contract, but coordination is achieved because both of the contract terms are conditional on the chosen retail price. To be specific, they are linear

in the chosen retail price. While the description and implementation of a price-discount contract differs from revenue sharing, we show that revenue sharing and price-discount contracts in the price-setting newsvendor model are equivalent, again, in the sense that they generate the same cash flows for any realization of demand.

We next extend our base model to include quantity-competing retailers; i.e., each retailer's revenue depends on its quantity as well as the other retailers' quantities. This framework includes Cournot competitors or competing newsvendors (as in Lippman and McCardle 1997). It has been observed in similar settings that simple wholesale price contracts can coordinate this system (van Ryzin and Mahajan 2000, Bernstein and Federgruen 2003), but the coordinating wholesale price vector only allows one split of channel profit. We show that revenue sharing again allows coordination while supporting alternative profit allocations.

Our results suggest that revenue-sharing contracts are very effective in a wide range of supply chains. However, they must have some limitations, otherwise we would expect to observe revenue sharing in every industry. We identify three. First, revenue sharing generally does not coordinate competing retailers when each retailer's revenue depends on its quantity, its price, and the actions of the other retailers, e.g., competing price-setting newsvendors with each retailer's demand depending on the vector of retail prices.<sup>2</sup> For this setting, more complex contracts are needed, e.g., additional parameters or nonlinear components. Bernstein and Federgruen (2005) show that a nonlinear version of the price-discount contract does coordinate this setting.

A second limitation of revenue sharing, which is probably more significant than the first, is the administrative burden it imposes on the firms. Under revenue sharing, the supplier must monitor the retailer's revenues to verify that they are split appropriately. The gains from coordination may not always cover these costs. To explore this idea, we study the performance of the supply chain under a wholesale price contract, which clearly has a lower administrative cost than revenue sharing. We demonstrate that there is considerable variation in supply chain performance under a wholesale price contract and conclude that revenue sharing's administrative burden may explain why it is not implemented in some settings.

Finally, revenue sharing does not coordinate a supply chain when noncontractable and costly retailer effort influences demand. Nevertheless, we show that the supplier may still choose to implement revenue

<sup>2</sup> An exception is the case of perfect competition. See Dana and Spier (2001) for details.

sharing if the impact of effort is sufficiently small. Several coordinating contracts have been offered for this setting. The franchise literature suggests selling at marginal cost and charging franchise fees. For the newsvendor problem, Taylor (2002) suggests a sales-rebate contract combined with a buy-back contract. We offer a variation on revenue sharing, which acts like a quantity discount, to coordinate this supply chain.

The next section outlines our model. Section 3 studies how revenue sharing coordinates the supply chain and compares revenue sharing to other contracts. Section 4 considers multiple competing retailers, and §5 studies wholesale price contracts. Section 6 investigates revenue sharing when costly retail effort increases demand. The final section discusses our results and concludes.

## 2. The Model

Consider a supply chain with two risk-neutral firms, a supplier and a retailer. The retailer makes two decisions: the quantity of an asset,  $q \geq 0$ , it purchases from the supplier at the start of the selling season, and its price,  $p$ . We take the sales period as exogenously specified. For a model in which the sales period is endogeneously specified, see Gerchak et al. (2001). Only the retailer generates revenue in this supply chain. Let  $R(q, p) + vq$  be the retailer's total revenue over the sales period given its decisions, where  $v < 0$  is possible.  $R(q, p)$  only includes the revenue impact that can be directly attributed to the  $q$  purchased units. The long-run revenue impact of poor availability (e.g., a goodwill penalty for lost sales in a newsvendor model) is not included in our model. It can be shown that revenue sharing coordinates the fixed-price newsvendor with goodwill penalty costs and arbitrarily divides profits, but with the price-setting newsvendor, coordination can only be achieved for a single division of profit. The supplier's production cost is  $c_s q$ ; the retailer's cost, not including any payment to the supplier, is  $c_r q$ . Assume  $c_j \geq 0$  for  $j = r, s$ , and let  $c = c_s + c_r$ .

Before the retailer chooses  $q$  and  $p$ , the supplier and the retailer agree to a revenue-sharing contract with two parameters. The first is the wholesale price the retailer pays per unit,  $w$ . The second,  $\phi$ , is the retailer's share of revenue generated from each unit. The supplier's share is  $1 - \phi$ .<sup>3</sup> A conventional wholesale price contract is a revenue-sharing contract with  $\phi = 1$ . We assume the same revenue share is applied to all units. Pasternack (2002) considers a contract that allows for outright sales to the retailer on some units

and revenue sharing on other units. That additional degree of freedom is not needed in our model.

To summarize, the firms' profit functions are

$$\pi_r(q, p) = \phi R(q, p) - (c_r + w - \phi v)q, \quad (1)$$

$$\pi_s(q, p) = (1 - \phi)R(q, p) - (c_s - w - (1 - \phi)v)q, \quad (2)$$

and the supply chain's profit function is

$$\Pi(q, p) = \pi_r(q, p) + \pi_s(q, p) = R(q, p) - (c - v)q.$$

Note that salvage revenues are shared. Rentrak, a distributor in the video rental industry, offers contracts with such a provision. For a fixed retail price, revenue sharing achieves supply chain coordination even if only  $R(q, p)$  is shared. However, when  $p$  is a decision variable, then sharing  $vq$  is necessary to achieve coordination with arbitrary profit division.

This model is general enough to encompass several situations. If the retailer rents the asset,  $R(q, p)$  is interpreted as the rental revenues generated during the season and  $vq$  is the salvage revenue generated at the end of the season. If the retailer sells the asset,  $R(q, p)$  is the revenue generated from sales in addition to the certain salvage revenue  $qv$ . To explain, let  $S(q, p)$  be expected unit sales. Expected sales revenue is then  $pS(q, p)$  and expected salvage revenue is  $v(q - S(q, p))$ . Total revenue is  $(p - v)S(q, p) + vq$ , which conforms to our model when  $R(q, p) = (p - v)S(q, p)$ . In either the rental or the outright sales case,  $R(q, p)$  can be derived from a deterministic or stochastic demand function. The newsvendor model is an example of the latter. In that model, stochastic demand,  $D(p)$ , occurs in a single selling season, and let  $F(x, p) = \Pr(D(p) \leq x)$ . The expected unit sales function is  $S(q, p)$ , where

$$S(x, p) = E[(D(p) - x)^+] = x - \int_0^x F(x, p) dx.$$

## 3. Supply Chain Coordination

This section first considers supply chain coordination with revenue-sharing contracts and then compares and contrasts revenue sharing to several other contracts: buy-back, price-discount, quantity-flexibility, sales-rebate, franchise, and quantity discount.

### 3.1. Revenue-Sharing Contracts

Let  $\{q^0, p^0\}$  be a quantity-price pair that maximizes  $\Pi(q, p)$ . We assume that  $\Pi(q, p)$  is upper semicontinuous in  $q$  and  $p$ , so  $\{q^0, p^0\}$  exists, but it need not be unique. Revenue-sharing contracts achieve supply chain coordination by making the retailer's profit function an affine transformation of the supply chain's profit function; hence,  $\{q^0, p^0\}$  maximizes  $\pi_r(q, p)$ .

<sup>3</sup> Holmstrom (1988) demonstrates that in some cases it is advantageous if the revenue shares do not sum to one, but that constraint is reasonable in this setting.



**THEOREM 1.** Consider the set of revenue-sharing contracts with

$$w = \phi c - c_r, \quad (3)$$

and  $\phi \in (0, 1]$ . With those contracts, the firms' profit functions are

$$\pi_r(q, p) = \phi \Pi(q, p), \quad (4)$$

and

$$\pi_s(q, p) = (1 - \phi) \Pi(q, p).$$

Furthermore,  $\{q^0, p^0\}$  is the retailer's optimal quantity and price; i.e., those contracts coordinate the supply chain.

**PROOF.** Given the profit function (4), it follows that  $\{q^0, p^0\}$  maximizes the retailer's profit when  $\phi > 0$ . To obtain (4), substitute  $w = \phi c - c_r$  into (1) and simplify. The supplier's profit function follows from (4) and  $\pi_s(q, p) = \pi_r(q, p) - \Pi(q, p)$ ;  $\phi \leq 1$  ensures  $\pi_s(q, p) \geq 0$ .  $\square$

The theorem indicates that  $\phi$  is the retailer's share of the supply chain's profit in addition to its share of revenue. Thus, revenue-sharing contracts coordinate the supply chain and arbitrarily allocate profit. The particular profit split chosen probably depends on the firms' relative bargaining power. As the retailer's bargaining position becomes stronger, one would anticipate  $\phi$  increases. As a proxy for bargaining power, each firm may have an outside opportunity profit,  $\pi_i > 0$ , that the firm requires to engage in the relationship; i.e.,  $\pi_i(q, p) \geq \pi_i$  is required to gain firm  $i$ 's participation. It is possible to satisfy both firms' requirements when  $\pi_r + \pi_s < \Pi(q^0, p^0)$ , but the feasible range for  $\phi$  will be more limited.

Extreme  $\phi$  values raise two other issues. First, the retailer's profit function becomes quite flat as  $\phi \rightarrow 0$ ; while  $q^0$  remains optimal for the retailer, a deviation from  $q^0$  imposes little penalty on the retailer. Second, from (3), the coordinating wholesale price is actually negative when  $\phi < c_r/c$ . Essentially, if the retailer's share of the channel's cost is high, the retailer is already in a low-margin business before the supplier takes a slice of revenue. If the supplier wants to claim a large portion of revenue, she must subsidize the retailer's acquisition of product. If one wishes to rule out negative wholesale price, then a positive retailer cost establishes a floor on retailer profit under coordinating contracts.

The theorem also shows that coordination requires a wholesale price below the supplier's cost of production  $c_s$ . The supplier loses money in selling the product and only makes money by participating in the retailer's revenue. Selling below cost is necessary because revenue sharing systematically drops the retailer's marginal revenue curve below the integrated supply chain's. To have marginal revenue equal marginal cost at the desired point, the retailer's

marginal cost must also be less than the integrated system's.

Given that the set of coordinating contracts is independent of the revenue function, it follows immediately that a single revenue-sharing contract can coordinate the actions of multiple retailers with different revenue functions as long as each retailer's revenue is independent of the other retailer's actions (i.e., they do not compete) and they have the same marginal cost,  $c_r$ . Section 4 considers revenue sharing with competing retailers.

### 3.2. Other Contracts

Pasternack (1985) was the first to identify that buy-back contracts coordinate the fixed-price newsvendor. With that contract the supplier charges a wholesale price  $w_b$  per unit and pays the retailer  $b$  per unit the retailer salvages. The retailer still collects the  $v$  salvage revenue per unit. (To accommodate the retailer actually returning units to the supplier, and the supplier salvages each unit for  $v$ , just increase  $b$  to  $b + v$ .) Recall that  $S(q, p) = R(q, p)/(p - v)$ , so the retailer's profit function is

$$\begin{aligned} \pi_r(q, p, b, w_b) &= \frac{p}{p - v} R(q, p) + (b + v) \left( q - \frac{R(q, p)}{p - v} \right) - (c_r + w_b)q \\ &= \left( 1 - \frac{b}{p - v} \right) R(q, p) - (c_r + w_b - b - v)q. \end{aligned} \quad (5)$$

The supplier's profit function is

$$\pi_s(q, p, b, w_b) = \frac{b}{p - v} R(q, p) - (c_s - w_b + b)q.$$

With a fixed retail price, buybacks and revenue sharing are equivalent in a very strong sense.

**THEOREM 2.** In the newsvendor setting with a fixed retail price, for any coordinating revenue-sharing contract,  $\{\phi, w\}$ , there exists a unique buy-back contract,  $\{b, w_b\}$ , that generates the same profit for each firm for any realization of demand:

$$b = (1 - \phi)(p - v), \quad (6)$$

$$w_b = (1 - \phi)p + \phi c - c_r. \quad (7)$$

**PROOF.** Let  $r(q, p)$  be realized revenue and replace  $R(q, p)$  with  $r(q, p)$  in both  $\pi_r(q, p, b, w_b)$  and  $\pi_s(q, p)$ . The profit functions are the same if  $1 - b/(p - v) = \phi$  and  $c_r + w_b - b - v = c_r + w - \phi v$ . Rearranging terms and substituting  $w = \phi c - c_r$  from Theorem 1 yields (6) and (7). The analogous procedure confirms the result for the supplier.  $\square$

As can be seen from (5), under a buy-back contract the retailer pays the supplier  $b$  per unit sold and  $w_b - b$  per unit purchased. Consequently, with the fixed-price

newsvendor the supplier can implement revenue sharing either by requiring a percentage of realized revenue or by demanding a fixed payment per unit sold (as in Pasternack 2002). Dana and Spier (2001) note that this is also true in their model with perfect competition. However, one cannot coordinate a bilateral monopoly in which the retailer sets both the stocking level and retail price using revenue sharing based on a fixed payment per unit sold. In that case, Marvel and Peck (1995) and Bernstein and Federgruen (2005) demonstrate that buy-back contracts coordinate the price-setting newsvendor only if the supplier earns zero profit. The problem is apparent in (6) and (7): Unlike revenue sharing, the coordinating buy-back parameters depend on the retail price.

Buybacks would coordinate the price-setting newsvendor if the supplier could commit to adjust the buyback and wholesale price in response to any price chosen by the retailer. To be specific, according to Theorem 2, coordination is achieved if the supplier announces that the buy-back rate will be  $b(p) = (1 - \phi)(p - v)$  and the wholesale price will be  $w(p) = (1 - \phi)p + \phi c - c_r$ ; i.e., the buy-back rate and wholesale price are adjusted linearly in the retailer's price. That is precisely the contract studied by Bernstein and Federgruen (2005), which they call a price-discount contract.<sup>4</sup> Thus, their contract is equivalent to revenue sharing in the strongest sense; i.e., for any revenue-sharing contract there exists a unique price-discount contract that generates the same profits for both firms, no matter the realization of demand. Furthermore, the two approaches are equally costly to administer in the newsvendor setting: With the price-discount contract the retailer must report its retail price and its leftover inventory, which, in combination with the retailer's order quantity, yields the retailer's sales, sales revenue, and salvage revenue. Hence, the information collected to implement a price-discount contract yields the information needed to implement revenue sharing. However, it is not clear how to interpret a price-discount contract when revenues are generated from rental fees because then the retailer ends the season with the same number of units as it begins the season.

Although revenue sharing, buybacks, and price-discount contracts are linked, the same is not true for all coordinating supply chain contracts. Consider the QF contract of Tsay and Lovejoy (1999) and Tsay

(1999) with a fixed retail price. Here, the retailer purchases  $q$  units for  $w_\Delta$  per unit at the start of the season and may return up to  $\Delta q$  units at the end of the season for a full refund,  $\Delta \in [0, 1)$ . Units that are not returned can be salvaged for  $v$  per unit. The retailer's expected profit is

$$\begin{aligned}\pi_r(q, p, w_\Delta, \Delta) &= R(q, p) - (c_r + w_\Delta - v)q \\ &\quad + (w_\Delta - v)(\Delta q - (S(q, p) - S((1 - \Delta)q, p))).\end{aligned}$$

For a fixed retail price, the key condition for coordination is

$$\frac{\partial \pi_r(q^0, p, w_\Delta, \Delta)}{\partial q} = \frac{\partial \Pi(q^0, p)}{\partial q},$$

which occurs when

$$w_\Delta = v + \frac{c_s - v}{1 - F(q^0) + (1 - \Delta)F((1 - \Delta)q^0)}.$$

QF contracts also arbitrarily allocate profit (Tsay 1999).<sup>5</sup> Nevertheless, there are several differences between the QF contract and the revenue-sharing contract. With revenue sharing, the ratio of the retailer's marginal profit to the supply chain's marginal profit is held constant for all  $q$  (both marginals are with respect to  $q$ ). However, that does not hold with the QF contract because  $S(q, p) - S((1 - \Delta)q, p)$  is not included in the supply chain's profit function. Consequently, the two contracts do not result in the same realized division of profit for all outcomes of demand. Further, coordinating QF contracts are not independent of the retailer's demand distribution.

While the QF contract coordinates the fixed-price newsvendor, it is less effective with a price-setting newsvendor. For price to be coordinated as well, we need at least

$$\frac{\partial \pi_r(q^0, p, w_\Delta, \Delta)}{\partial p} = \frac{\partial R(q^0, p)}{\partial p} + \int_{(1-\Delta)q^0}^{q^0} \frac{\partial F(x, p)}{\partial p} dx = 0, \quad (8)$$

but given that

$$\frac{\partial \Pi(q^0, p)}{\partial p} = \frac{\partial R(q^0, p)}{\partial p} = 0,$$

we see (8) is achieved only with  $\Delta = 0$ . With  $\Delta = 0$ , coordination of  $q$  requires  $w_\Delta = c_s$ . So the only coordinating QF contract for a price-setting newsvendor has the supplier pricing at marginal cost and earning zero profit.

The sales-rebate contract (Krishnan et al. 2001, Taylor 2002) also coordinates a newsvendor supply

<sup>4</sup> Bernstein and Federgruen (2005) have the supplier earning the salvage revenue and  $c_r = 0$ . Their coordinating contract is  $b'(p) = \alpha(p - v) + v$  and  $w'(p) = \alpha p + (1 - \alpha)c$ , where  $0 < \alpha < 1$ . Let  $\alpha = 1 - \phi$ , in which case  $b'(p) = (1 - \phi)(p - v) + v = b(p) + v$ ; i.e., the retailer's total revenue from each unit salvaged is the same with either contract. The wholesale prices are clearly the same given  $c_r = 0$ .

<sup>5</sup> Tsay (1999) and Taylor (2002) do not include  $c_r$ , but we conjecture that their results continue to hold in our setting.

chain with a fixed price. The supplier charges the retailer a per-unit wholesale price  $w$  but gives the retailer a rebate  $r > 0$  per unit sold above a fixed threshold  $t$ , and the retailer continues to salvage leftover units for  $v$  per unit:

$$\pi_r(q, p, w_s, r, t) = R(q, p) - (c_r + w_s - v)q + r(S(t, p) - S(q, p)).$$

The retailer's marginal profit is

$$\frac{\partial \pi_r(q, p, w_s, r, t)}{\partial q} = \begin{cases} \frac{\partial R(q, p)}{\partial q} - (c_r + w_s - v), & q < t, \\ (1+r) \frac{\partial R(q, p)}{\partial q} - (c_r + w_s - v), & q \geq t. \end{cases}$$

Thus, for  $q \geq t$ , the sales-rebate contract acts like a revenue-sharing contract in the sense that one contract parameter,  $r$ , modifies the retailer's marginal revenue and the second contract parameter,  $w_s$ , modifies the retailer's marginal cost. However, unlike the revenue-sharing contract, which modifies the retailer's marginal revenue for all  $q$ , the sales-rebate contract does not modify the retailer's marginal revenue for  $q < t$ . Due to the inclusion of this absolute threshold, the retailer's profit function may not be unimodal even if the supply chain's profit function is unimodal. Furthermore, for the supplier to earn a positive profit, we must have  $q^0 > t$ .<sup>6</sup>

As with the QF contract, the sales-rebate contract struggles with the price-setting newsvendor. To coordinate price and generate a positive profit for the supplier (i.e.,  $q^0 > t$ ), we must at least have

$$0 = \frac{\partial \pi_r(q^0, p, w_s, r, t)}{\partial p} = \frac{\partial R(q^0, p)}{\partial p} - r \int_t^{q^0} \frac{\partial F(q^0, p)}{\partial p}, \quad (9)$$

but given that

$$\frac{\partial \Pi(q^0, p)}{\partial p} = \frac{\partial R(q^0, p)}{\partial p} = 0,$$

we see (9) is achieved only with  $t = q^0$ . In that case  $w_s = c_s$  is needed to coordinate the quantity decision, so the supplier earns zero profit.<sup>7</sup>

<sup>6</sup> If  $q^0 \leq t$ , then the coordinating contract has  $c_r + w_s - v = c - v$ , which implies  $w_s = c_s$ .

<sup>7</sup> Krishnan et al. (2001) and Taylor (2002) include a buyback with the sales-rebate contract to coordinate the newsvendor with a fixed price but effort-dependent demand. Incorporating a buyback may also allow for the coordination of the price-setting newsvendor: The rebate induces the retailer to price too low (in an effort to generate sales above the rebate threshold), but a buyback induces the retailer to price too high, so it is possible to counteract the deleterious effects of the rebate on price. We leave to future research the confirmation of this hypothesis.

With a two-part tariff the supplier charges a per-unit wholesale price,  $w_2$ , and a fixed fee,  $F$ . Coordination is achieved with marginal cost pricing,  $w_2 = c_s$ , because then the retailer's profit is  $\Pi(q, p) - F$ . The fixed fee serves to allocate profit between the supplier and the retailer. Because two-part tariffs achieve the same results as a revenue-sharing contract in the single-retailer model, we do not provide an explanation for why one contractual form would be favored over another. However, Dana and Spier (2001) find that revenue sharing is more effective when the supplier sells to perfectly competitive retailers.<sup>8</sup> Furthermore, while a single two-part tariff can coordinate multiple noncompeting retailers, it cannot guarantee an arbitrary allocation of profit; the fixed fee would be dictated by the retailer earning the smallest profit.

A franchise contract combines revenue sharing with a two-part tariff: The supplier charges a fixed fee, a per-unit wholesale price, and a revenue share per transaction, which is usually called a royalty rate. As a result, a franchise contract enjoys the capabilities of both revenue sharing and two-part tariffs. We discuss further the relationship between revenue sharing and franchise contracts in §6.

The final contract we consider is a quantity discount contract: The supplier charges the retailer  $w(q)$  per unit purchased, where  $w(q)$  is a decreasing function:

$$\pi_r(q, p) = R(q, p) - (c_r - v + w(q))q.$$

It too can coordinate the supply chain and arbitrarily allocate profit.

**THEOREM 3.** Let

$$w(q) = (1 - \chi) \frac{R(q, p^0)}{q} + \chi c + (1 - \chi)v - c_r$$

for  $\chi \in (0, 1]$ .  $\{q^0, p^0\}$  maximizes the retailer's profit and

$$\pi_r(q, p^0) = \chi \Pi(q, p^0).$$

**PROOF.** Substitute  $w(q)$  into the retailer's profit function:

$$\pi_r(q, p) = R(q, p) - (1 - \chi)R(q, p^0) - \chi(c - v)q.$$

Because the retailer retains all revenue, the retailer's optimal price for any given  $q$  equals the supply chain's optimal price. Thus,  $\{q^0, p^0\}$  is optimal if  $q^0$  is optimal given  $p^0$ , which clearly holds for  $\chi > 0$ :

$$\pi_r(q, p^0) = \chi[R(q, p^0) - (c - v)q] = \chi \Pi(q, p^0).$$

<sup>8</sup> In their model, revenue sharing achieves coordination, but a two-part tariff does not. See Hart and Tirole (1990) and O'Brien and Shaffer (1992) for additional discussion on the challenges of implementing two-part tariffs with competing retailers.

The upper bound on  $\chi$  ensures that the supplier earns a nonnegative profit.  $\square$

For the fixed price  $p^o$ , the quantity discount achieves coordination because, as with revenue sharing, the retailer's expected profit is proportional to the supply chain's expected profit. However, there are some differences between these contracts. Under revenue sharing, the retailer's profit is proportional to the supply chain's profit even when the retailer sets a nonoptimal price, but not so with the quantity discount. Furthermore, realized profits are not the same with the two contracts because with the quantity discount the retailer pays a fraction of the supply chain's expected revenue (given  $p^o$ ), whereas with revenue sharing the retailer pays a fraction of realized revenue. In other words, with a quantity discount the supplier earns the same profit no matter what the realization of demand, whereas with revenue sharing the supplier bears some demand risk. Furthermore, because the revenue function is included in  $w(q)$ , a single quantity discount schedule can coordinate multiple independent retailers only if they have identical revenue functions.

To summarize, there are many contracts that coordinate the fixed-price newsvendor, including two that are identical to revenue sharing in the sense that they can generate the same profits for both firms with any realization of revenue: buy-back contracts and price-discount contracts. With a price-setting retailer, coordination is achieved with revenue sharing, its equivalent price-discount contract, two-part tariffs (and its equivalent franchise contract), and quantity discounts. The latter two have some disadvantages relative to revenue sharing with multiple noncompeting retailers.

#### 4. Competing Retailers

This section considers revenue-sharing contracts with  $n$  competing retailers. We consider a simpler market than that examined above. We assume the revenue earned by retailer  $i$  (for  $i = 1, \dots, n$ ) depends on a single action by each retailer, which we take to be the stocking quantity. We shall show that revenue sharing can coordinate such systems and has some flexibility to shift profit between players. At the end of this section, we will discuss competition in which retailers must choose stocking quantities and prices. For the special case of perfect competition, Dana and Spier (2001) show that revenue sharing can coordinate the supply chain. We will show that this fails to be true in an oligopoly.

Denote the vector of stocking levels as  $\bar{q} = \{q_1, \dots, q_n\}$  and expected revenue at retailer  $i$  as  $R_i(\bar{q})$ . For simplicity we incorporate all salvage revenue into  $R_i(\bar{q})$  (i.e., the  $vq$  term is now incorporated into the revenue function). Possible examples of  $R_i(\bar{q})$

include competing newsvendors with a fixed retail price (Parlar 1988, Lippman and McCardle 1997) and Cournot competition (Tirole 1988, Tyagi 1999) with deterministic linear demand, e.g.,

$$R_i(\bar{q}) = q_i \left( 1 - q_i - \gamma \sum_{j \neq i} q_j \right) \quad (10)$$

for  $0 \leq \gamma < 1$ . In the case of a one-for-one relationship between the stocking quantity and the retail price, as with deterministic demand, then it is straightforward to consider the case with revenue determined by the vector of prices. We later discuss revenue functions of the form  $R_i(q_i, \bar{p})$  and  $R(\bar{q}, \bar{p})$ ; i.e., retailers choose stocking quantities and prices without a one-to-one relationship between them.

Assume  $R_i(\bar{q})$  is differentiable in all arguments and firm  $i$ 's marginal revenue is decreasing in  $q_i$ ; i.e.,  $R_i^i(\bar{q}) < 0$ , where  $R_i^i(\bar{q}) = \partial R_i(\bar{q}) / \partial q_i$ . In addition, inventory at retailer  $i$  and  $j$  are substitutes; i.e.,  $\partial^2 R_i / \partial q_i \partial q_j \leq 0$  for all  $j \neq i$ . Finally, there exists a finite  $q_i^e$  such that  $R_i^i(\bar{q}_i) < \varepsilon$  for all  $q_i \geq q_i^e$ ; i.e., marginal revenue at firm  $i$  drops below any positive number at a finite quantity level no matter what the stocking quantities are at the other retailers.

Let  $c_s$  be the supplier's production cost per unit and let  $c_{ri}$  be retailer  $i$ 's incremental cost. Let  $c_i = c_s + c_{ri}$ . Let  $\Pi_i(\bar{q})$  be the supply chain's profit earned at location  $i$ ,

$$\Pi_i(\bar{q}) = R_i(\bar{q}) - c_i q_i,$$

and let  $\Pi(\bar{q})$  be the supply chain's total profit,

$$\Pi(\bar{q}) = \sum_{i=1}^n \Pi_i(\bar{q}).$$

Denote a system optimal vector of quantities as  $\bar{q}^o = \{q_1^o, \dots, q_n^o\}$ . Because  $\Pi(\bar{q})$  is differentiable,  $\bar{q}^o$  satisfies the following system of first-order conditions:

$$R_i^i(\bar{q}) + \sum_{j \neq i} R_j^i(\bar{q}) = c_i, \quad i = 1, \dots, n.$$

In a decentralized system the supplier offers each retailer a revenue-sharing contract,  $\{\phi_i, w_i\}$ , and the terms of those contracts can differ across retailers.<sup>9</sup> Retailer  $i$  sets  $q_i$  to maximize his own profit given the contractual terms without coordinating his decision with other retailers.

A rational retailer never chooses a quantity that pushes his marginal revenue below his acquisition cost. Hence, the game is unchanged if we restrict retailer  $i$ 's quantity choice to the interval  $[0, q_i^{\varepsilon_i}]$ , where  $\varepsilon_i = c_i$ . Combined with our earlier assumptions, this assures the existence of a pure-strategy

<sup>9</sup> See Bhattacharyya and Lafontaine (1995) for a discussion on the prevalence of relatively uniform contractual terms in practice.



Nash equilibrium,  $\bar{q}^*$  (see Theorem 1.2 of Fudenberg and Tirole 1991). The next theorem demonstrates that supply chain coordination, i.e.,  $\bar{q}^* = \bar{q}^o$ , is possible via revenue-sharing contracts.

**THEOREM 4.** *The following revenue-sharing contracts coordinate the supply chain with multiple competing retailers and revenue functions  $R_i(\bar{q})$ :*

$$w_i = \phi_i(c_i - \xi_i^o) - c_{ri},$$

where  $\xi_i^o = \xi_i(\bar{q}^o)$ ,

$$\xi_i(\bar{q}) = \sum_{j \neq i}^n R_j^i(\bar{q}),$$

and

$$0 < \phi_i \leq \frac{\Pi_i(\bar{q}^o)}{\Pi_i(\bar{q}^o) + q_i^o \xi_i^o} \quad \text{for } i = 1, \dots, n.$$

The firms' optimal profits are

$$\begin{aligned} \pi_i(\bar{q}^o) &= \phi_i(\Pi_i(\bar{q}^o) + q_i^o \xi_i^o), \\ \pi_s(\bar{q}^o) &= \sum_{i=1}^n (1 - \phi_i)(\Pi_i(\bar{q}^o) + q_i^o \xi_i^o) - q_i^o \xi_i^o. \end{aligned}$$

**PROOF.** The first-order condition for the supply chain has

$$\frac{\partial \Pi(\bar{q}^o)}{\partial q_i} = R_i^i(\bar{q}^o) - (c_i - \xi_i(\bar{q}^o)) = 0.$$

Because  $-\xi_i(\bar{q})$  is increasing in  $q_i$  and  $R_i^i(\bar{q})$  is decreasing in  $q_i$ , there is a unique optimal  $q_i$  for any  $\bar{q}_{-i} = \{q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_n\}$ . Given the above and the contract parameters,

$$\begin{aligned} \frac{\partial \pi_i(q_i)}{\partial q_i} &= \phi_i R_i^i(\bar{q}) - (w_i + c_{ri}) \\ &= \phi_i [R_i^i(\bar{q}) - (c_i - \xi_i^o)]. \end{aligned}$$

Because  $\phi_i > 0$ ,  $\xi_i^o < 0$ ,  $R_i^i(\bar{q})$  is decreasing in  $q_i$ , and  $\lim_{q_i \rightarrow \infty} R_i^i(\bar{q}) \leq c_i$ , there is a unique positive  $q_i^*$  that satisfies  $\partial \pi_i(q_i^*) / \partial q_i = 0$ ; otherwise,  $q_i^* = 0$ . In either case,  $q_i^* = q_i^o$ . The profit functions follow from straightforward algebra. The retailer's profit function is increasing in  $\phi$  if  $\Pi_i(\bar{q}^o) + q_i^o \xi_i^o > 0$ . From the supply chain first-order condition,

$$q_i^o \xi_i^o = q_i^o [c_i - R_i^i(\bar{q}^o)],$$

which implies

$$\Pi_i(\bar{q}^o) + q_i^o \xi_i^o = R_i^i(\bar{q}^o) - q_i^o R_i^i(\bar{q}^o) > 0.$$

Thus,  $\pi_i(\bar{q}^o)$  ( $\pi_s(\bar{q}^o)$ ) is increasing (decreasing) in  $\phi_i$ . The bounds on  $\phi_i$  ensure that all players earn a non-negative profit.  $\square$

The coordinating wholesale price in this model is the same as in the single-retailer case with the addition of the  $-\phi_i \xi_i^o$  term, which is the revenue externality retailer  $i$  imposes on the others at the optimal solution. Because that term is fixed, the retailers' profits are no longer an affine transformation of the supply chain's profit for all  $\bar{q}$ , so the additional structure on the revenue function is required. In the single-retailer case,  $\phi_i$  ranges from zero to one and can be interpreted as the retailer's share of the supply chain profit. Here, the range on each  $\phi_i$  exceeds one, so that interpretation is no longer entirely suitable. Nevertheless, the retailer's profit is increasing, and the supplier's profit is decreasing in  $\phi_i$ .

Note that a wholesale price contract,  $\phi_i = 1$ , is included in the set of coordinating revenue-sharing contracts.<sup>10</sup> Bernstein and Federgruen (2003) present a related model with a similar result. To support additional allocations of profit they use lump sum transfers. In our case, alternative profit allocations can be achieved by varying  $\phi_i$ .

While coordination via revenue sharing is possible with competing retailers and  $R_i(\bar{q})$  revenue functions, in general the supplier must offer each retailer different contractual terms. (One of the two parameters,  $\phi$  or  $w$ , could be identical across retailers.) In fact, it is an open question whether there exists a simple legal contract with common terms that coordinates heterogeneous competing retailers. A single revenue-sharing contract only coordinates the supply chain when the retailers have identical costs and each retailer imposes the same revenue externality on the other retailers (i.e., for all  $i \neq j$ ,  $\xi_i^o = \xi_j^o$ ). That condition may hold (approximately) when there are a large number of similar retailers. If the supplier is legally obligated to offer identical contractual terms and the supplier sells to retailers that differ in the abovementioned ways, revenue-sharing contracts may improve performance but cannot guarantee supply chain coordination. Offering one set of terms to heterogeneous retailers may by default favor some retailers over others. Intuitively, this may explain why some small retailers feel that revenue sharing has put them at a disadvantage to Blockbuster (Warren and Peers 2002).

In the single-retailer case, we demonstrated that revenue sharing coordinates the quantity and price-setting retailer. However, revenue sharing stumbles with  $R_i(q_i, \bar{p})$  or  $R_i(\bar{q}, \bar{p})$  revenue functions. Consider the latter revenue function. At an optimal solution we have

$$\frac{\partial \Pi(\bar{q}^o, \bar{p}^o)}{\partial p_i} = \frac{\partial R_i(\bar{q}^o, \bar{p}^o)}{\partial p_i} + \sum_{j \neq i}^n \frac{\partial R_j(\bar{q}^o, \bar{p}^o)}{\partial p_i} = 0,$$

<sup>10</sup> In Dana and Spier's (2001) setting with perfectly competitive price-setting retailers, a wholesale price does not coordinate the supply chain; revenue sharing is required.

but, for  $\phi_i > 0$ ,

$$\frac{\partial \pi_i(\bar{q}^o, \bar{p}^o)}{\partial p_i} = \phi_i \frac{\partial R_i(\bar{q}^o, \bar{p}^o)}{\partial p_i} < 0.$$

Hence, the supply chain optimal price is higher than the retailer's optimal price. Furthermore,  $\phi_i$  has no power to force the retailer to a higher price. Revenue sharing could accommodate the externalities among retailers with  $R_i(\bar{q})$  revenue functions because each retailer's externality could be introduced into the retailer's profit function via the wholesale price. However, the wholesale price is not a consideration when setting the retail price with  $R_i(\bar{q}, \bar{p})$ , and the single parameter  $\phi_i$  is insufficient to do the job. In contrast to our result for oligopolistic competition, Dana and Spier (2001) find that revenue sharing does coordinate perfectly competitive price-setting newsvendors (i.e., each retailer earns zero profit in equilibrium). Bernstein and Federgruen (2005) show that a nonlinear form of the price-discount contract coordinates competing retailers with  $R_i(q_i, \bar{p})$  revenue functions (i.e., price competing newsvendors). Unfortunately, nonlinear contracts are more complex to administer than linear contracts.

## 5. Revenue-Sharing vs. Wholesale Price Contracts

With revenue sharing the supplier must be able to ex post verify the retailer's revenue. We have supposed that monitoring is costless, but this need not be so. At a minimum, the channel would incur the cost of linking the supplier's and retailer's information systems. More likely, the supplier would have to monitor closely how the downstream firm manages the assets it has purchased. In general, a supplier must balance the costs of running revenue sharing with the profit sacrificed by using a noncoordinating contract. The simplest such contract is the wholesale price contract. Selling the product outright would then be the only way for the supplier to earn a profit. We now consider supply chain performance with that contract in both single and multiretailer settings. For simplicity we work with a single revenue function (i.e., revenue from regular sales/rentals is combined with salvage revenue) and assume marginal revenue is decreasing in  $q$ . Furthermore, we assume a fixed retail price. (The single retailer chooses the supply chain optimal price for any quantity with a wholesale price contract.)

### 5.1. The Single-Retailer Case

In the single-retailer setting under a wholesale price contract, the retailer's optimal quantity is the unique solution to

$$R'(q) - w = 0 \quad (11)$$

if  $R'(0) > w$ ; otherwise the optimal order quantity is zero. Because  $R'(q)$  is strictly decreasing, from (11) there exists a function  $w(q) = R'(q)$  such that  $q = \arg \max \pi_r(q | w(q))$ . The supplier's profit can then be expressed as  $\pi_s(q)$ ,

$$\pi_s(q) = q(w(q) - c) = q(R'(q) - c),$$

and

$$\pi'_s(q) = w(q) - c + qw'(q) = R'(q) + qR''(q) - c. \quad (12)$$

The supplier's profit function is unimodal in  $q$  if  $R'(q) + qR''(q)$  is decreasing in  $q$ . This is equivalent to assuming that the elasticity of the retailer's order decreases in  $q$ , so successive percentage decreases in the wholesale price bring about smaller and smaller increases in sales. For tractability, we assume that this condition holds.

Let  $q_s^*$  be the supplier's optimal quantity to induce.  $q_s^*$  is the solution to  $\pi'_s(q) = 0$  and  $w(q_s^*)$  is the supplier's optimal wholesale price contract. Because  $qR''(q) < 0$ , the supply chain performance is not optimal; i.e.,  $q_s^* < q^o$ . Further, because  $w(q)$  is decreasing and  $R'(q^o) = c$ , the optimal wholesale price is greater than marginal cost, which is in sharp contrast to the optimal wholesale price under a revenue-sharing contract.

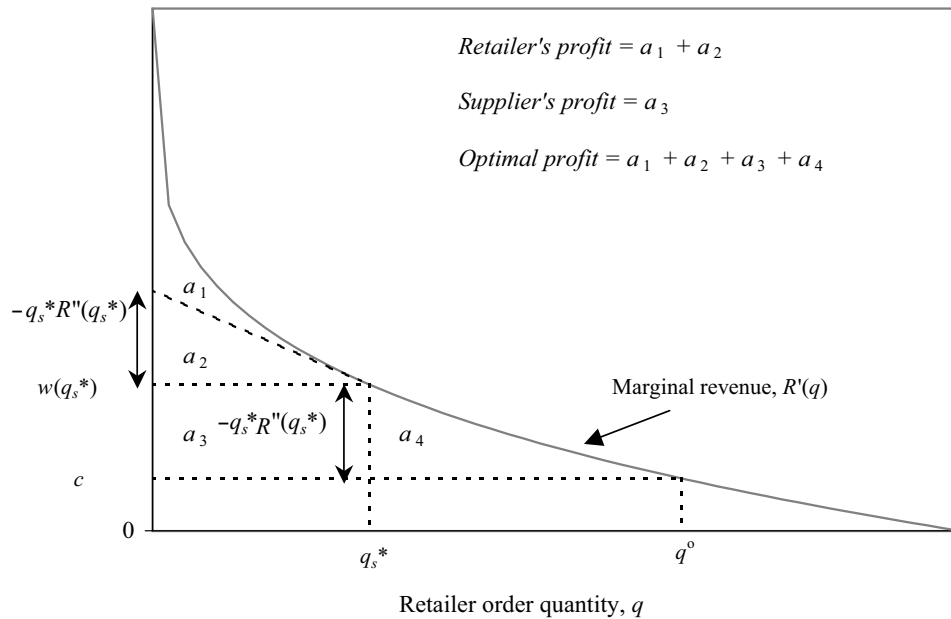
The supplier can evaluate any contract in terms of her share of the supply chain's maximum profit,  $\pi_s(q_s^*)/\Pi(q^o)$ . That share can be divided into two terms, the efficiency of the contract (the percentage of the optimal profit achieved under that contract) and the supplier's profit share of actual supply chain profit:

$$\frac{\pi_s(q_s^*)}{\Pi(q^o)} = \left( \frac{\pi_s(q_s^*) + \pi_r(q_s^*)}{\Pi(q^o)} \right) \left( \frac{\pi_s(q_s^*)}{\pi_s(q_s^*) + \pi_r(q_s^*)} \right).$$

Hence, a wholesale price contract is attractive to the supplier if its efficiency and her profit share are close to one.

Because the optimal wholesale price is  $w(q_s^*) = c - q_s^*R''(q_s^*)$ , the curvature of the marginal revenue curve  $R'(q)$  plays an important role in determining the contract's efficiency and profit share. This is shown in Figure 1. At the optimal solution  $R'(q_s^*) - c = -q_s^*R''(q_s^*)$ , because  $R'(q_s^*) = w$ . Thus, in the optimal solution  $-q_s^*R''(q_s^*)$ , which is the height of the triangle label  $a_2$ , equals the height of the rectangle labeled  $a_3$ . (The triangle  $a_2$  is formed by the tangent of the marginal revenue curve at  $q_s^*$ .) The supplier's profit equals the area of the rectangle  $a_3$ ,  $q_s^*(w(q_s^*) - c)$ . The triangle  $a_2$  is an approximation for the retailer's profit. It underestimates the retailer's earnings if  $R'(q)$  is convex and it overestimates the retailer's profit if  $R'(q)$  is concave. Because the area of the triangle is half of

Figure 1 Optimal Wholesale Price Only Contract with  $\theta = 1/4$ ; Efficiency = 74%



the area of the rectangle, the supplier's profit share is less (more) than two-thirds if the marginal revenue is convex (concave).<sup>11</sup>

Turning to system efficiency, the loss in supply chain profit is

$$\int_{q_s^*}^{q^0} (R'(z) - c) dz.$$

The corresponding region is labeled  $a_4$  in the diagram. An approximation for this loss is the triangle formed by dropping the tangent to  $R'(q_s^*)$  from  $q_s^*$  down to where it crosses the horizontal at  $c$ . This happens at  $2q_s^*$ . The area of the resulting triangle is again equal to half of the supplier's profit. It is less than the area of  $a_4$  if  $R'(q)$  is convex, but greater if  $R'(q)$  is concave. It is straightforward to see that this also implies  $q_s^* > q^0/2$  ( $< q^0/2$ ) when marginal revenue is concave (convex). Consequently, coordinating the system increases total profit by more (less) than 50% of the supplier's profit if marginal revenue is convex (concave). It increases by exactly 50% of the supplier's profit if marginal revenue is linear.

Interestingly, Rentrak, a videocassette distributor, claims a retailer should quadruple his order quantity when switching from conventional wholesale price contracts to revenue sharing (www.renttrak.com). If we assume optimal contracts are implemented, then the marginal revenue curve in that industry must be quite convex, and efficiency could be substantially

lower than 75%. (Recall that for a linear marginal revenue curve  $2q_s^* = q^0$ .) In such a setting, revenue sharing could significantly increase the profit of both firms in the supply chain.

To illustrate these results, suppose  $R'(q) = 1 - q^\theta$  for  $\theta > 0$  and  $q \in [0, 1]$ . Such a marginal revenue curve results if the supply chain faces a deterministic inverse-demand curve  $P(q) = 1 - q^\theta/(\theta + 1)$ . Note that the marginal revenue curve is convex for  $\theta < 1$ , linear for  $\theta = 1$ , and concave for  $\theta > 1$ . Furthermore, it satisfies our assumption that  $R'(q) + qR''(q)$  is decreasing, which guarantees a unique optimal contract for the supplier. Figures 1 and 2 are drawn with that revenue function.

The optimal quantity for the supplier to induce under a wholesale price contract is

$$q_s^* = \left( \frac{1-c}{1+\theta} \right)^{1/\theta} = \frac{q^0}{(1+\theta)^{1/\theta}},$$

where  $q^0$  is the optimal quantity for an integrated channel. The resulting profits are

$$\pi_r(q_s^*) = \left( \frac{\theta}{1+\theta} \right) \left( \frac{1-c}{1+\theta} \right)^{(1+\theta)/\theta},$$

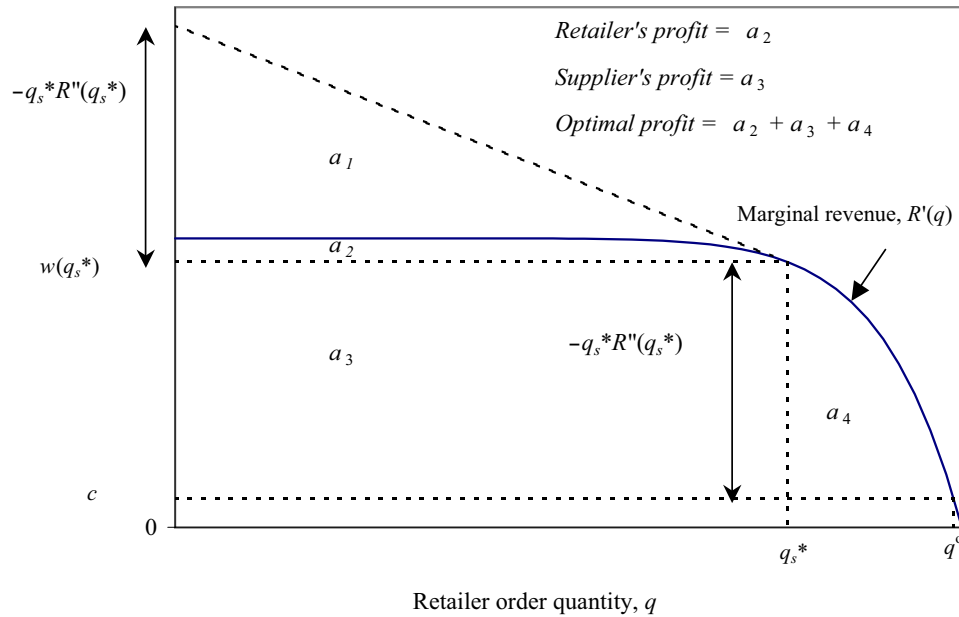
$$\pi_s(q_s^*) = \theta \left( \frac{1-c}{1+\theta} \right)^{(1+\theta)/\theta},$$

$$\Pi(q_l) = \left( \frac{\theta}{1+\theta} \right) (1-c)^{(1+\theta)/\theta}.$$

We see that the profit share is  $(1+\theta)/(2+\theta)$  and the efficiency is

$$\frac{\pi_s(q_s^*) + \pi_r(q_s^*)}{\Pi(q_l)} = \frac{2+\theta}{(1+\theta)^{(1+\theta)/\theta}}.$$

<sup>11</sup> Bresnahan and Reiss (1985) present a similar analysis of profits shares for a deterministic demand curve.

Figure 2 Optimal Wholesale Price Only Contract with  $\theta = 10$ ; Efficiency = 86%

Efficiency is an increasing function of  $\theta$ ; i.e., efficiency improves as the marginal revenue curve becomes more concave. As  $\theta \rightarrow 0$ , efficiency approaches  $2/e \approx 0.73$ ; and as  $\theta \rightarrow \infty$ , efficiency approaches one and the system is coordinated in the limit. However, it approaches coordination rather slowly. For example, with  $\theta = 10$ , which is displayed in Figure 2, efficiency is 86% even though the marginal revenue curve is quite concave. What changes much more quickly is the profit share. At  $\theta = 10$ , the supplier now captures 91.7% of the supply chain's profit.

To summarize, the potential profit gain from coordination in a supply chain with a single retailer depends on the shape of the marginal revenue curve. A convex marginal revenue curve generally leads to worse performance; the decentralized system stocks less than half of the integrated system quantity, and efficiency is frequently less than 75%. Supply chain efficiency is generally higher when the marginal revenue curve is concave.

## 5.2. The Multiple-Retailer Case

This section explores how supply chain efficiency varies with the level of competition among retailers when the supplier offers them a wholesale price contract. Suppose there are  $n$  symmetric retailers and the revenue function in market  $i$  for  $i = 1, \dots, n$  is as given in (10). This structure allows two measures of competition, the parameter  $\gamma$  and number of retailers  $n$ , with an increase in either implying more intense competition.

For a fixed  $\gamma$  and  $n$ , there is a unique equilibrium such that  $q_i^* = (1 - w)/(2 + \gamma(n - 1))$ . The integrated

channel, in contrast, has  $q_i^0 = (1 - c)/(2 + 2\gamma(n - 1))$ .  $R_j^i(\bar{q}) = -\beta q_j$  for all  $j \neq i$ , so the system is coordinated if the supplier charges

$$w^0 = c + \frac{\gamma(n - 1)(1 - c)}{2 + 2\gamma(n - 1)}.$$

One can show that  $w^0$  is increasing  $\gamma$  and  $n$ . As competition increases by either measure, a higher wholesale price is required to moderate competition. Because the total amount the centralized channel sells for a given  $\gamma$  is increasing in  $n$ , a greater number of retailers in the system thus shifts more profit to the supplier if she were to price at  $w^0$ .

The supplier, however, will not price at  $w^0$ . From her perspective,  $w^0$  is too low. Somewhat remarkably, her optimal wholesale price  $w^* = (1 + c)/2$  is independent of both  $\gamma$  and  $n$  (Tyagi 1999). The gap between  $w^0$  and  $w^*$  is  $(1 - c)/(2 + 2\gamma(n - 1))$  and drops to zero as  $n$  gets large. Indeed, if  $\gamma$  is close to one, the difference between the two wholesale prices is quite small for even low values of  $n$ . This suggests that the supply chain may not suffer much loss when the supplier prices to maximize her own profit.

Channel efficiency when the supplier charges  $w^*$  is  $1 - (2 + \gamma(n - 1))^{-2}$ . If  $\gamma$  equals zero, the system reduces to  $n$  independent linear markets and the efficiency is 75%. If  $\gamma > 0$ , efficiency improves rapidly as the number of retailers increases. For example, if  $\gamma$  equals  $1/3$ , efficiency is over 85% with just three retailers, while five retailers brings efficiency over 90%. Double  $\gamma$  to  $2/3$ , and efficiency with three and five retailers is 91% and 95.4%, respectively. Tyagi (1999) shows that for essentially any demand structure the



supplier's profit always increases as more Cournot competitors are added, but does not consider the efficiency of the supply chain. van Ryzin and Mahajan (2000) do consider system efficiency for an inventory problem in which stocking levels of substitute products are set by distinct firms. Similarly, they find efficiency improves rapidly as the number of competitors increases.

Contrasting this example with that of the single-retailer case suggests that competition in the retail market may have a greater impact on supply chain efficiency under a wholesale price contract than the nature of the revenue function. Thus, revenue sharing should be less attractive to the supplier when several competitors serve the market. This is particularly true if there are limited economies of scale in administering revenue sharing so that each retailer added to the system requires a significant additional administrative cost.

## 6. Retailer Effort and Revenue Sharing

In our single-retailer model, revenue depends on the retailer's order quantity and price. However, there are other actions the retailer could take to influence revenue, e.g., advertising, service quality, and store presentation. This section considers how these costly actions influence coordination with revenue-sharing contracts.

Suppose the retailer's expected revenue is  $R(q, e)$ , where  $e$  is a measure of the retailer's effort. (Note that we now fix the retail price and incorporate salvage revenues into a single revenue function.)  $R(q, e)$  is differentiable and strictly increasing in  $e$ . Assume  $\lim_{e \rightarrow \infty} \partial R(q, e) / \partial e = 0$ , so there is some large effort level that fails to incrementally increase revenue. The retailer chooses both  $q$  and  $e$  after observing the terms of the revenue-sharing contract. The retailer incurs a cost  $g(e)$  to choose effort level  $e$ , but no incremental purchase cost,  $c_r = 0$ .  $g(e)$  is increasing, differentiable, and convex with  $g(0) = 0$ .

$\Pi(q, e)$  is the integrated channel's profit function,

$$\Pi(q, e) = R(q, e) - g(e) - qc.$$

Let  $\{q^o, e^o\}$  be an optimal solution.  $e^o$  must satisfy

$$\frac{\partial \Pi(q^o, e^o)}{\partial e} = \frac{\partial R(q^o, e^o)}{\partial e} - g'(e^o) = 0. \quad (13)$$

The retailer's profit function is

$$\pi_r(q, e) = \phi R(q, e) - g(e) - qw.$$

From (13),

$$\frac{\partial \pi_r(q^o, e^o)}{\partial e} = \phi \frac{\partial R(q^o, e^o)}{\partial e} - g'(e^o) < 0.$$

Revenue sharing coordinates the effort decision only if  $\phi = 1$ , but then the retailer's quantity decision is only coordinated if the supplier sells at marginal cost,  $w = c$ .<sup>12</sup>

In the price-dependent newsvendor setting, revenue is also determined by two retailer actions. However, revenue sharing coordinates that supply chain because the retailer does not bear any other cost associated with changing  $p$ , whereas in the effort case the cost of effort,  $g(e)$ , falls entirely on the retailer. Coordinating effort is possible if the supplier could assume part of the effort cost but the retailer then has every reason to misrepresent the true cost incurred. Corbett and DeCroix (2001) make a similar argument.

The franchise literature also demonstrates that revenue sharing (i.e., a royalty rate) reduces a risk-neutral retailer's incentive to incur costly effort (see Mathewson and Winter 1985, Lal 1990, Desai 1997).<sup>13</sup> As a result, the recommended contract is a fixed franchise fee with marginal cost pricing and no revenue sharing. However, as mentioned earlier, there can be situations in which a fixed fee is difficult to implement. In those cases, the supplier may need to choose between a revenue-sharing contract and a wholesale price contract. Relative to a revenue-sharing contract, the wholesale price contract leaves the retailer with the incentive to exert effort, but limits the supplier's ability to extract rents. The supplier's preferred contract depends on which effect dominates, as demonstrated by the following simple model.

Suppose the retailer faces the following inverse-demand curve

$$P(q, e) = 1 - q + 2\tau e,$$

where  $\tau \in [0, 1]$  is a constant parameter. (The upper bound ensures joint concavity of the retailer's profit

<sup>12</sup> Holmstrom (1988) and Atkinson et al. (1988) also consider coordination among agents with revenue sharing and noncontractable effort, but in their models there is no comparable quantity action; i.e., an action by one player (in this case the retailer's quantity) that increases the cost of another player (in this case the supplier's production cost).

<sup>13</sup> Early work on franchise contracts (e.g., Stiglitz 1974) highlights the trade-off between risk avoidance and incentives. The retailer's risk decreases as his share of revenue decreases, but so does his incentive to exert revenue-enhancing effort. See Gaynor and Gertler (1995) for more recent modeling and empirical work in this vein. In a risk-neutral setting, Lal (1990) demonstrates that the supplier may offer revenue sharing when the supplier can engage in costly revenue-enhancing effort (e.g., national brand advertising): Revenue sharing induces the supplier to engage in such effort. Our model does not consider that motivation for revenue sharing. Other work suggests that revenue sharing is used when a franchisor has private information (e.g., the quality of the franchise format) that she wishes to credibly communicate to franchisees (e.g., Gallini and Lutz 1992, Desai and Srinivasan 1995). That motivation for revenue sharing is also not present in our model.

function.) Retail effort has a greater impact on demand as  $\tau$  increases. For a given quantity  $q$ , the marginal change in the quantity clearing price with respect to retail effort is increasing in  $\tau$ . Expected revenue is  $R(q, e) = qP(q, e)$ . Let  $g(e) = e^2$ .

The retailer's profit function is  $\pi_r(q, e) = \phi R(q, e) - e^2 - qw$ . Let  $e(q)$  be the retailer's unique optimal effort,  $e(q) = \phi\tau q$ , and let  $q(w, \phi)$  be the retailer's optimal quantity conditional on  $e(q)$ ,

$$q(w, \phi) = \frac{\phi - w}{2(\phi - \phi^2\tau^2)},$$

assuming  $w < \phi$ , otherwise  $q(w, \phi) = 0$ . The supplier's profit function is then

$$\begin{aligned}\pi_s(w, \phi) &= (1 - \phi)R(q(w, \phi), e(q(w, \phi))) \\ &\quad + q(w, \phi)(w - c).\end{aligned}$$

Because there exists an optimal wholesale price,  $w(\phi)$ , for each  $\phi$ , the supplier's profit function simplifies to

$$\pi_s(w(\phi), \phi) = \frac{(1 - c)^2}{4(1 + \phi(1 - 2\tau^2))}.$$

The supplier's profit is increasing in  $\phi$  if  $\tau > 1/\sqrt{2}$ , otherwise the supplier's profit is decreasing in  $\phi$ . Consequently, the supplier's optimal contract is a wholesale price contract ( $\phi = 1$ ) if  $\tau > 1/\sqrt{2}$ , but is a revenue-sharing contract with  $\phi = 0$  otherwise. Hence, only if the effort effect dominates ( $\tau > 1/\sqrt{2}$ ) does the supplier seek to minimize the distortion in the retailer's effort decision by offering a wholesale price contract. If the impact of effort is small, the supplier prefers to use revenue sharing to extract a large share of supply chain profit even though that profit is less than optimal.

Our discussion so far leaves open the question of whether there is a coordinating contract without fixed payments. In the newsvendor setting, Taylor (2002) shows that the combination of a sales-rebate contract with a buy-back contract coordinates the supply chain. The sales rebate gives the retailer too much incentive to exert effort, but the buyback reduces the retailer's incentive, thereby providing the needed balance. However, that contract requires four parameters ( $w, b, t, r$ ). There is a simpler alternative, which is a quantity discount contract related to revenue sharing.

**THEOREM 5.** Suppose the supplier charges the retailer  $w(q)$  per unit, where  $w(q) = (1 - \chi)R(q, e^0)/q + \chi c$  for  $\chi \in (0, 1]$ , and the retailer retains all revenue.  $\{q^0, e^0\}$  maximizes the retailer's expected profit, which is

$$\pi_r(q^0, e^0) = \chi\Pi(q^0, e^0) - (1 - \chi)g(e^0).$$

**PROOF.** With this contract,

$$\pi_r(q, e) = R(q, e) - g(e) - (1 - \chi)R(q, e^0) - \chi cq.$$

Because the retailer retains all revenue and incurs all effort costs,  $e^0$  is optimal for the retailer. The retailer's profit function is then simplified to

$$\begin{aligned}\pi_r(q, e^0) &= \chi R(q, e^0) - g(e^0) - \chi cq \\ &= \chi\Pi(q, e^0) - (1 - \chi)g(e^0).\end{aligned}$$

$q^0$  maximizes the above for  $\chi > 0$ . The upper bound on  $\chi$  ensures that the supplier earns a positive profit.  $\square$

As with the quantity-discount contract discussed in §3, the quantity-discount contract charges the retailer a fixed percentage of *expected* revenue, whereas a revenue-sharing contract charges a fixed percentage of *realized* revenue. The distinction, while subtle, is critical because the retailer retains all realized revenues. Because the retailer bears the full cost of effort, but also receives the full benefit of effort, the retailer's effort decision is coordinated. As before, this contract depends on the retailer's marginal revenue curve, so a single contract cannot coordinate multiple independent retailers with heterogeneous demands. Nevertheless, this variation on revenue sharing does coordinate the supply chain with just two parameters, and arbitrarily divides profits.

## 7. Discussion

Our analysis demonstrates that revenue sharing is a very attractive contract. Given a single supplier and retailer, it coordinates the supply chain and arbitrarily divides the resulting profits for any reasonable revenue function that depends on the retailer's purchase quantity and price. The supplier sells at a wholesale price below marginal cost, but her participation in the retailer's revenue more than offsets the loss on sales. We have shown that the widely studied buy-back contract of Pasternack (1985) is a special case of our proportional revenue-sharing contract when the retail price is fixed. However, revenue sharing coordinates a broader array of supply chains than do buybacks. In particular, revenue sharing continues to coordinate a newsvendor with price-dependent demand, which buybacks cannot.

With so much going for it, one might argue that revenue sharing should be ubiquitous. We present some reasons why it is not. First, while revenue sharing coordinates retailers that compete on quantity, it does not coordinate retailers that compete on price and quantity. Second, there are cases in which the gains from revenue sharing over a simpler wholesale price contract may not cover revenue sharing's additional administrative expense. In particular, revenue

sharing's incremental improvement over the wholesale price contract diminishes as the revenue function becomes more concave or as retail competition intensifies.

We also demonstrate that revenue sharing may not be attractive if the retailer's actions influence demand. Specifically, we assume that the retailer can increase demand by exerting costly effort, and that this effort is noncontractable. Because revenue-sharing contracts reduce the retailer's incentive to undertake effort relative to a wholesale price contract, the supplier may prefer offering a wholesale price contract. In other words, while revenue-sharing contracts are effective at coordinating the retailer's purchase quantity and pricing decisions, they work against the coordination of the retailer's effort decision. When demand is sufficiently influenced by retail effort, revenue-sharing contracts should be avoided. However, a variation on revenue sharing, best described as a quantity discount, does coordinate the supply chain with effort-dependent demand and allocates rents without the use of fixed fees.

Other factors beyond those we have considered may influence the decision to offer revenue sharing. For example, a retailer may carry substitute or complementary products from other suppliers. If one supplier offers revenue sharing and the other does not in the substitute case, the retailer could be predisposed to favor the supplier that allows the retailer to keep all revenue by, for example, recommending the product to undecided consumers. In the case of complements (say, personal computers and printers), the retailer may discount the product offered under revenue sharing to spur sales of the other product. Here, revenue sharing may result in a product being used as a loss leader. We leave these issues to future research.

We began this paper with a discussion of the videocassette rental industry, so we close with it as well. Our model suggests that, in a wholesale price contract, the optimal wholesale price should be set above marginal cost, but with revenue sharing the wholesale price should be set below marginal cost. Consistent with that result, the wholesale price in the video industry fell from \$65 per tape to \$8 per tape when revenue sharing was introduced. A wholesale price of \$8 is plausibly below marginal cost (production, royalties, transportation, handling, etc.), so the industry may have adopted a channel-coordinating contract.

The adoption of revenue sharing in the video industry is also consistent with the limitations we identified for revenue sharing. The first limitation is that administrative costs should be sufficiently low. Almost all video stores have systems of computers and bar codes to track each tape rental, so it should not be difficult for the suppliers to monitor and verify revenues. Further, it is unlikely that retail effort has a sufficient

impact on demand. In a video rental store, the retailer merely displays boxes of available tapes from which customers make their selections. Unlike home appliance or automobile retailing (to name just two examples), customers do not make their video selection after substantial consultation with a retail salesperson (which requires effort). Hence, we feel that the video rental supply chain is particularly suited for revenue sharing. Although there are limits to these contracts, we suspect that other industries have yet to discover the virtues of revenue sharing (see Cachon and Lariviere 2001).

### Acknowledgments

The authors thank Fangruo Chen, Karen Donohue, Jim Dana, Steve Gilbert, Steve Graves, and Lawrence Robinson for their helpful comments, as well as the seminar participants at Columbia University, Harvard University, Lehigh University, Purdue University, University of Michigan, the Conference on Incentives in Operations Management held at Stanford University and the University of Washington. They are also grateful for the assistance from the anonymous referees and the associate editor.

### References

- Atkinson, S., L. Stanley, J. Tschirhart. 1988. Revenue sharing as an incentive in an agency problem: An example from the National Football League. *RAND J. Econom.* **19**(1) 27–43.
- Bernstein, F., A. Federgruen. 2003. Pricing and replenishment strategies in a distribution system with competing retailers. *Oper. Res.* **51**(3) 409–426.
- Bernstein, F., A. Federgruen. 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Management Sci.* **51**(1) 18–29.
- Bhattacharyya, S., F. Lafontaine. 1995. Double-sided moral hazard and the nature of share contracts. *RAND J. Econom.* **26**(4) 761–781.
- Bresnahan, T. F., P. C. Reiss. 1985. Dealer and manufacturer margins. *RAND J. Econom.* **16**(2) 253–268.
- Cachon, G., M. Lariviere. 2001. Turning the supply chain into a revenue chain. *Harvard Bus. Rev.* (March) 20–21.
- Corbett, C., G. DeCroix. 2001. Shared savings contracts in supply chains. *Management Sci.* **47**(7) 881–893.
- Dana, J., K. Spier. 2001. Revenue sharing and vertical control in the video rental industry. *J. Indust. Econom.* **59**(3) 223–245.
- Desai, P. 1997. Advertising fee in business-format franchising. *Management Sci.* **43**(10) 1401–1419.
- Desai, P., K. Srinivasan. 1995. Demand signalling under unobservable effort in franchising. *Management Sci.* **41** 1608–1623.
- Fudenberg, D., J. Tirole. 1991. *Game Theory*. MIT Press, Cambridge, MA.
- Gallini, N., N. Lutz. 1992. Dual distribution and royalty fees in franchising. *J. Law, Econom., Organ.* **8** 471–501.
- Gaynor, M., P. Gertler. 1995. Moral hazard and risk spreading in partnerships. *RAND J. Econom.* **26**(4) 591–613.
- Gerchak, Y., R. Cho, S. Ray. 2001. Coordination and dynamic shelf-space management of video movie rentals. Working paper, University of Waterloo, Waterloo, Ontario, Canada.
- Hart, O., J. Tirole. 1990. Vertical integration and market foreclosure. *Brookings Papers* 205–286.

- Holmstrom, B. 1988. Moral hazard in teams. *RAND J. Econom.* **13**(2) 324–340.
- Krishnan, H., R. Kapuscinski, D. Butz. 2001. Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management Sci.* **50**(1) 48–63.
- Lal, R. 1990. Improving channel coordination through franchising. *Marketing Sci.* **9** 299–318.
- Lilien, G., P. Kotler, K. S. Moorthy. 1992. *Marketing Models*. Prentice-Hall, Englewood Cliffs, NJ.
- Lippman, S., K. McCardle. 1997. The competitive newsboy. *Oper. Res.* **45** 54–65.
- Marvel, H., J. Peck. 1995. Demand uncertainty and returns policies. *Internat. Econom. Rev.* **36**(3) 691–714.
- Mathewson, G. F., R. Winter. 1985. The economics of franchise contracts. *J. Law Econom.* **28** 503–526.
- McCollum, C. 1998. Video stores put stock in change. *Morning Star* (October 2) 6c. Wilmington, NC.
- Mortimer, J. H. 2000. The effects of revenue-sharing contracts on welfare in vertically separated markets: Evidence from the video rental industry. Working paper, University of California at Los Angeles, Los Angeles, CA.
- O'Brien, D., G. Shaffer. 1992. Vertical control with bilateral contracts. *RAND J. Econom.* **23** 299–308.
- Parlar, M. 1988. Game theoretic analysis of the substitutable product inventory problem with random demands. *Naval Res. Logist.* **35** 397–409.
- Pasternack, B. 1985. Optimal pricing and returns policies for perishable commodities. *Marketing Sci.* **4** 166–176.
- Pasternack, B. 2002. Using revenue sharing to achieve channel coordination for a newsboy type inventory model. J. Geunes, P. Pardalos, H. E. Romeijn, eds. *Supply Chain Management: Models, Applications and Research*. Kluwer Academic Publishers.
- Shapiro, E. 1998a. Movies: Blockbuster seeks a new deal with Hollywood. *Wall Street Journal* (March 25) B1.
- Shapiro, E. 1998b. Blockbuster's return is on fast forward, says CEO with big plans for rentals. *Wall Street Journal* (April 7) A3.
- Stiglitz, J. 1974. Incentives and risk sharing in sharecropping. *Rev. Econom. Stud.* **41**(2) 219–256.
- Taylor, T. 2002. Supply chain coordination under channel rebates with sales effort effects. *Management Sci.* **48**(8) 992–1007.
- Tirole, J. 1988. *The Theory of Industrial Organization*. MIT Press, Cambridge, MA.
- Tsay, A. 1999. Quantity-flexibility contract and supplier-customer incentives. *Management Sci.* **45** 1339–1358.
- Tsay, A., W. Lovejoy. 1999. Supply chain control with quantity flexibility. *Manufacturing Service Oper. Management* **1** 89–111.
- Tsay, A., S. Nahmias, N. Agrawal, eds. 1998. Modeling supply chain contracts: A review. *Quantitative Models for Supply Chain Management*. Kluwer, Boston, MA.
- Tyagi, R. 1999. On the effects of downstream entry. *Management Sci.* **45** 59–73.
- van Ryzin, G., S. Mahajan. 2000. Supply chain coordination under horizontal competition. Working paper, Columbia University, New York.
- Wall Street Journal*. 2002. Judge tosses out suit against Blockbuster. (June 28) C5.
- Warren, A., M. Peers. 2002. Video retailers have day in court—Plaintiffs say supply deals between Blockbuster Inc. and studios violate laws. *Wall Street Journal* (June 13) B10.