

Foundations of Technical Analysis: Computational Algorithms, Statistical Inference, and Empirical Implementation

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ABSTRACT

Technical analysis, also known as “charting,” has been a part of financial practice for many decades, but this discipline has not received the same level of academic scrutiny and acceptance as more traditional approaches such as fundamental analysis. One of the main obstacles is the highly subjective nature of technical analysis—the presence of geometric shapes in historical price charts is often in the eyes of the beholder. In this paper, we propose a systematic and automatic approach to technical pattern recognition using nonparametric kernel regression, and we apply this method to a large number of U.S. stocks from 1962 to 1996 to evaluate the effectiveness of technical analysis. By comparing the unconditional empirical distribution of daily stock returns to the conditional distribution—conditioned on specific technical indicators such as head-and-shoulders or double-bottoms—we find that over the 31-year sample period, several technical indicators do provide incremental information and may have some practical value.

ONE OF THE GREATEST GULFS between academic finance and industry practice is the separation that exists between technical analysts and their academic critics. In contrast to fundamental analysis, which was quick to be adopted by the scholars of modern quantitative finance, technical analysis has been an orphan from the very start. It has been argued that the difference between fundamental analysis and technical analysis is not unlike the difference between astronomy and astrology. Among some circles, technical analysis is known as “voodoo finance.” And in his influential book *A Random Walk down Wall Street*, Burton Malkiel (1996) concludes that “[u]nder scientific scrutiny, chart-reading must share a pedestal with alchemy.”

However, several academic studies suggest that despite its jargon and methods, technical analysis may well be an effective means for extracting useful information from market prices. For example, in rejecting the Random Walk

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Hypothesis for weekly U.S. stock indexes, Lo and MacKinlay (1988, 1999) have shown that past prices may be used to forecast future returns to some degree, a fact that all technical analysts take for granted. Studies by Tabell and Tabell (1964), Treynor and Ferguson (1985), Brown and Jennings (1989), Jegadeesh and Titman (1993), Blume, Easley, and O'Hara (1994), Chan, Jegadeesh, and Lakonishok (1996), Lo and MacKinlay (1997), Grundy and Martin (1998), and Rouwenhorst (1998) have also provided indirect support for technical analysis, and more direct support has been given by Pruitt and White (1988), Neftci (1991), Brock, Lakonishok, and LeBaron (1992), Neely, Weller, and Dittmar (1997), Neely and Weller (1998), Chang and Osler (1994), Osler and Chang (1995), and Allen and Karjalainen (1999).

One explanation for this state of controversy and confusion is the unique and sometimes impenetrable jargon used by technical analysts, some of which has developed into a standard lexicon that can be translated. But there are many "homegrown" variations, each with its own patois, which can often frustrate the uninitiated. Campbell, Lo, and MacKinlay (1997, 43–44) provide a striking example of the linguistic barriers between technical analysts and academic finance by contrasting this statement:

The presence of clearly identified support and resistance levels, coupled with a one-third retracement parameter when prices lie between them, suggests the presence of strong buying and selling opportunities in the near-term.

with this one:

The magnitudes and decay pattern of the first twelve autocorrelations and the statistical significance of the Box-Pierce Q -statistic suggest the presence of a high-frequency predictable component in stock returns.

Despite the fact that both statements have the same meaning—that past prices contain information for predicting future returns—most readers find one statement plausible and the other puzzling or, worse, offensive.

These linguistic barriers underscore an important difference between technical analysis and quantitative finance: technical analysis is primarily *visual*, whereas quantitative finance is primarily algebraic and numerical. Therefore, technical analysis employs the tools of geometry and pattern recognition, and quantitative finance employs the tools of mathematical analysis and probability and statistics. In the wake of recent breakthroughs in financial engineering, computer technology, and numerical algorithms, it is no wonder that quantitative finance has overtaken technical analysis in popularity—the principles of portfolio optimization are far easier to program into a computer than the basic tenets of technical analysis. Nevertheless, technical analysis has survived through the years, perhaps because its visual mode of analysis is more conducive to human cognition, and because pattern recognition is one of the few repetitive activities for which computers do not have an absolute advantage (yet).

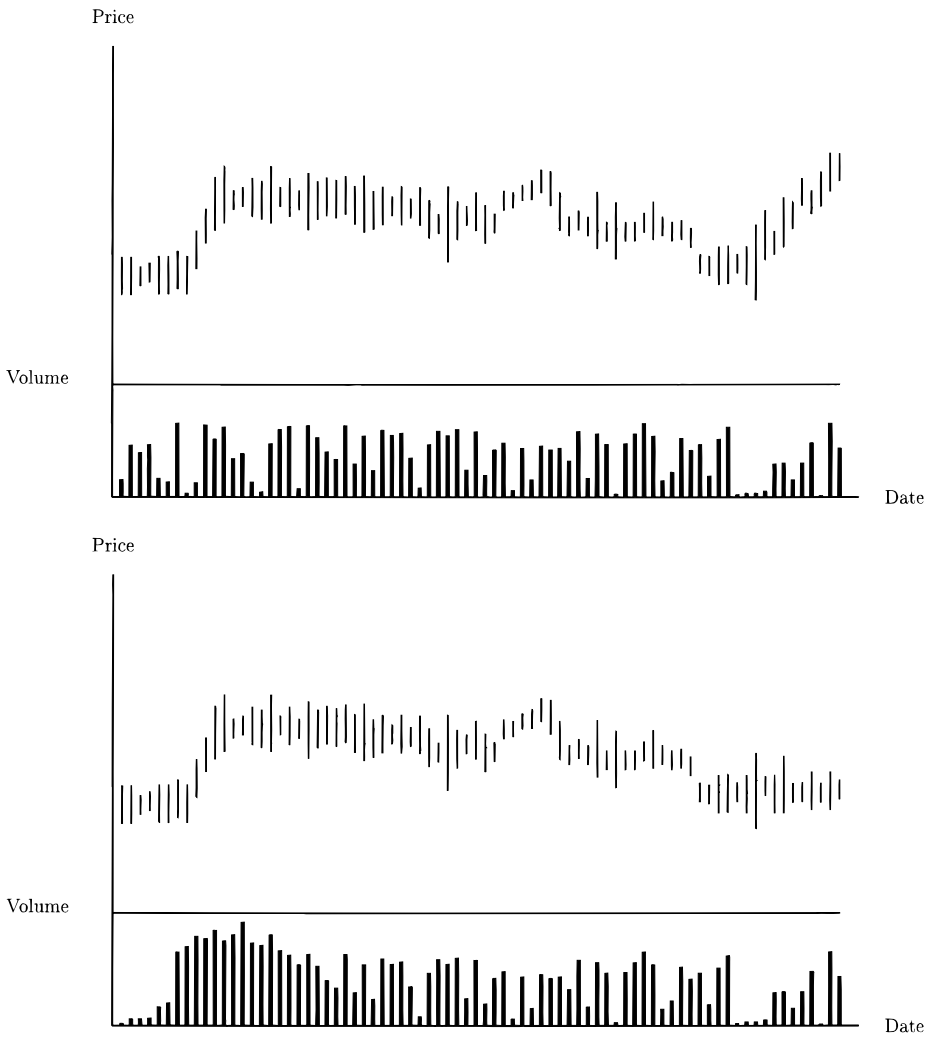


Figure 1. Two hypothetical price/volume charts.

Indeed, it is difficult to dispute the potential value of price/volume charts when confronted with the visual evidence. For example, compare the two hypothetical price charts given in Figure 1. Despite the fact that the two price series are identical over the first half of the sample, the volume patterns differ, and this seems to be informative. In particular, the lower chart, which shows high volume accompanying a positive price trend, suggests that there may be more information content in the trend, e.g., broader participation among investors. The fact that the joint distribution of prices and volume contains important information is hardly controversial among academics. Why, then, is the value of a visual depiction of that joint distribution so hotly contested?

In this paper, we hope to bridge this gulf between technical analysis and quantitative finance by developing a systematic and scientific approach to the practice of technical analysis and by employing the now-standard methods of empirical analysis to gauge the efficacy of technical indicators over time and across securities. In doing so, our goal is not only to develop a lingua franca with which disciples of both disciplines can engage in productive dialogue but also to extend the reach of technical analysis by augmenting its tool kit with some modern techniques in pattern recognition.

The general goal of technical analysis is to identify regularities in the time series of prices by extracting nonlinear patterns from noisy data. Implicit in this goal is the recognition that some price movements are significant—they contribute to the formation of a specific pattern—and others are merely random fluctuations to be ignored. In many cases, the human eye can perform this “signal extraction” quickly and accurately, and until recently, computer algorithms could not. However, a class of statistical estimators, called *smoothing estimators*, is ideally suited to this task because they extract nonlinear relations $\hat{m}(\cdot)$ by “averaging out” the noise. Therefore, we propose using these estimators to mimic and, in some cases, sharpen the skills of a trained technical analyst in identifying certain patterns in historical price series.

In Section I, we provide a brief review of smoothing estimators and describe in detail the specific smoothing estimator we use in our analysis: kernel regression. Our algorithm for automating technical analysis is described in Section II. We apply this algorithm to the daily returns of several hundred U.S. stocks from 1962 to 1996 and report the results in Section III. To check the accuracy of our statistical inferences, we perform several Monte Carlo simulation experiments and the results are given in Section IV. We conclude in Section V.

I. Smoothing Estimators and Kernel Regression

The starting point for any study of technical analysis is the recognition that prices evolve in a nonlinear fashion over time and that the nonlinearities contain certain regularities or patterns. To capture such regularities quantitatively, we begin by asserting that prices $\{P_t\}$ satisfy the following expression:

$$P_t = m(X_t) + \epsilon_t, \quad t = 1, \dots, T, \quad (1)$$

where $m(X_t)$ is an arbitrary fixed but unknown nonlinear function of a state variable X_t and $\{\epsilon_t\}$ is white noise.

For the purposes of pattern recognition in which our goal is to construct a smooth function $\hat{m}(\cdot)$ to approximate the time series of prices $\{p_t\}$, we set the state variable equal to time, $X_t = t$. However, to keep our notation consistent with that of the kernel regression literature, we will continue to use X_t in our exposition.

When prices are expressed as equation (1), it is apparent that geometric patterns can emerge from a visual inspection of historical price series—prices are the sum of the nonlinear pattern $m(X_t)$ and white noise—and

that such patterns may provide useful information about the unknown function $m(\cdot)$ to be estimated. But just how useful is this information?

To answer this question empirically and systematically, we must first develop a method for automating the identification of technical indicators; that is, we require a pattern-recognition algorithm. Once such an algorithm is developed, it can be applied to a large number of securities over many time periods to determine the efficacy of various technical indicators. Moreover, quantitative comparisons of the performance of several indicators can be conducted, and the statistical significance of such performance can be assessed through Monte Carlo simulation and bootstrap techniques.¹

In Section I.A, we provide a brief review of a general class of pattern-recognition techniques known as *smoothing estimators*, and in Section I.B we describe in some detail a particular method called *nonparametric kernel regression* on which our algorithm is based. Kernel regression estimators are calibrated by a *bandwidth* parameter, and we discuss how the bandwidth is selected in Section I.C.

A. Smoothing Estimators

One of the most common methods for estimating nonlinear relations such as equation (1) is *smoothing*, in which observational errors are reduced by averaging the data in sophisticated ways. Kernel regression, orthogonal series expansion, projection pursuit, nearest-neighbor estimators, average derivative estimators, splines, and neural networks are all examples of smoothing estimators. In addition to possessing certain statistical optimality properties, smoothing estimators are motivated by their close correspondence to the way human cognition extracts regularities from noisy data.² Therefore, they are ideal for our purposes.

To provide some intuition for how averaging can recover nonlinear relations such as the function $m(\cdot)$ in equation (1), suppose we wish to estimate $m(\cdot)$ at a particular date t_0 when $X_{t_0} = x_0$. Now suppose that for this one observation, X_{t_0} , we can obtain *repeated* independent observations of the price P_{t_0} , say $P_{t_0}^1 = p_1, \dots, P_{t_0}^n = p_n$ (note that these are n independent realizations of the price at the *same* date t_0 , clearly an impossibility in practice, but let us continue this thought experiment for a few more steps). Then a natural estimator of the function $m(\cdot)$ at the point x_0 is

$$\hat{m}(x_0) = \frac{1}{n} \sum_{i=1}^n p_i = \frac{1}{n} \sum_{i=1}^n [m(x_0) + \epsilon_t^i] \tag{2}$$

$$= m(x_0) + \frac{1}{n} \sum_{i=1}^n \epsilon_t^i, \tag{3}$$

¹A similar approach has been proposed by Chang and Osler (1994) and Osler and Chang (1995) for the case of foreign-currency trading rules based on a head-and-shoulders pattern. They develop an algorithm for automatically detecting geometric patterns in price or exchange data by looking at properly defined local extrema.

² See, for example, Beymer and Poggio (1996), Poggio and Beymer (1996), and Riesenhuber and Poggio (1997).

and by the Law of Large Numbers, the second term in equation (3) becomes negligible for large n .

Of course, if $\{P_t\}$ is a time series, we do not have the luxury of repeated observations for a given X_t . However, if we assume that the function $m(\cdot)$ is sufficiently smooth, then for time-series observations X_t near the value x_0 , the corresponding values of P_t should be close to $m(x_0)$. In other words, if $m(\cdot)$ is sufficiently smooth, then in a small neighborhood around x_0 , $m(x_0)$ will be nearly constant and may be estimated by taking an average of the P_t s that correspond to those X_t s near x_0 . The closer the X_t s are to the value x_0 , the closer an average of corresponding P_t s will be to $m(x_0)$. This argues for a *weighted* average of the P_t s, where the weights decline as the X_t s get farther away from x_0 . This weighted-average or “local averaging” procedure of estimating $m(x)$ is the essence of smoothing.

More formally, for any arbitrary x , a smoothing estimator of $m(x)$ may be expressed as

$$\hat{m}(x) \equiv \frac{1}{T} \sum_{t=1}^T \omega_t(x) P_t, \quad (4)$$

where the weights $\{\omega_t(x)\}$ are large for those P_t s paired with X_t s near x , and small for those P_t s with X_t s far from x . To implement such a procedure, we must define what we mean by “near” and “far.” If we choose too large a neighborhood around x to compute the average, the weighted average will be too smooth and will not exhibit the genuine nonlinearities of $m(\cdot)$. If we choose too small a neighborhood around x , the weighted average will be too variable, reflecting noise as well as the variations in $m(\cdot)$. Therefore, the weights $\{\omega_t(x)\}$ must be chosen carefully to balance these two considerations.

B. Kernel Regression

For the *kernel regression* estimator, the weight function $\omega_t(x)$ is constructed from a probability density function $K(x)$, also called a *kernel*:³

$$K(x) \geq 0, \quad \int K(u) du = 1. \quad (5)$$

By rescaling the kernel with respect to a parameter $h > 0$, we can change its spread; that is, let

$$K_h(u) \equiv \frac{1}{h} K(u/h), \quad \int K_h(u) du = 1 \quad (6)$$

³ Despite the fact that $K(x)$ is a probability density function, it plays no probabilistic role in the subsequent analysis—it is merely a convenient method for computing a weighted average and does *not* imply, for example, that X is distributed according to $K(x)$ (which would be a parametric assumption).

and define the weight function to be used in the weighted average (equation (4)) as

$$\omega_{t,h}(x) \equiv K_h(x - X_t)/g_h(x), \tag{7}$$

$$g_h(x) \equiv \frac{1}{T} \sum_{t=1}^T K_h(x - X_t). \tag{8}$$

If h is very small, the averaging will be done with respect to a rather small neighborhood around each of the X_t s. If h is very large, the averaging will be over larger neighborhoods of the X_t s. Therefore, controlling the degree of averaging amounts to adjusting the smoothing parameter h , also known as the *bandwidth*. Choosing the appropriate bandwidth is an important aspect of any local-averaging technique and is discussed more fully in Section II.C.

Substituting equation (8) into equation (4) yields the *Nadaraya–Watson* kernel estimator $\hat{m}_h(x)$ of $m(x)$:

$$\hat{m}_h(x) = \frac{1}{T} \sum_{t=1}^T \omega_{t,h}(x) Y_t = \frac{\sum_{t=1}^T K_h(x - X_t) Y_t}{\sum_{t=1}^T K_h(x - X_t)}. \tag{9}$$

Under certain regularity conditions on the shape of the kernel K and the magnitudes and behavior of the weights as the sample size grows, it may be shown that $\hat{m}_h(x)$ converges to $m(x)$ asymptotically in several ways (see Härdle (1990) for further details). This convergence property holds for a wide class of kernels, but for the remainder of this paper we shall use the most popular choice of kernel, the Gaussian kernel:

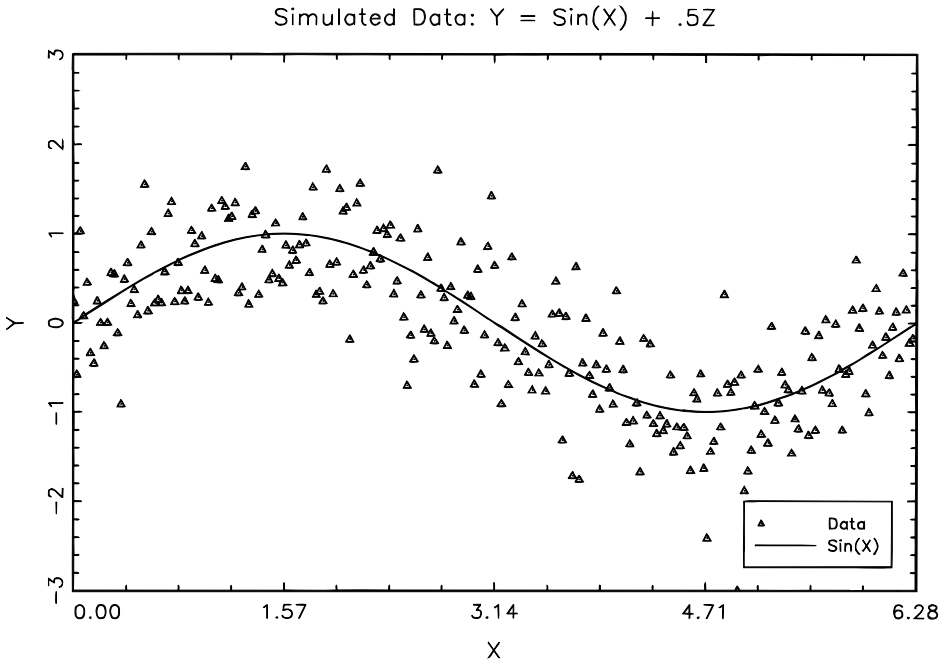
$$K_h(x) = \frac{1}{h\sqrt{2\pi}} e^{-x^2/2h^2} \tag{10}$$

C. Selecting the Bandwidth

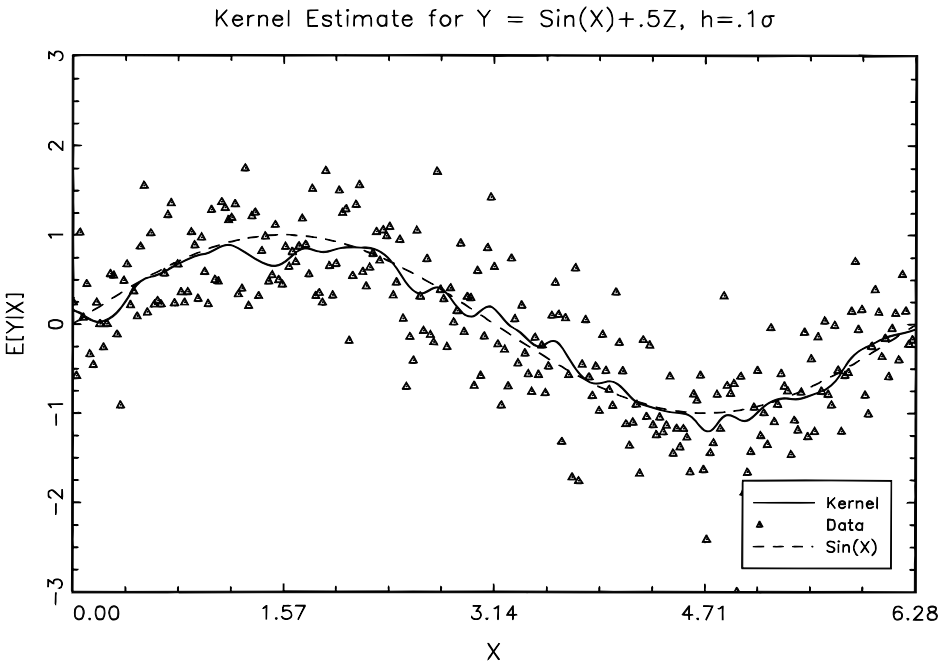
Selecting the appropriate bandwidth h in equation (9) is clearly central to the success of $\hat{m}_h(\cdot)$ in approximating $m(\cdot)$ —too little averaging yields a function that is too choppy, and too much averaging yields a function that is too smooth. To illustrate these two extremes, Figure 2 displays the Nadaraya–Watson kernel estimator applied to 500 data points generated from the relation:

$$Y_t = \text{Sin}(X_t) + 0.5\epsilon Z_t, \quad \epsilon Z_t \sim \mathcal{N}(0,1), \tag{11}$$

where X_t is evenly spaced in the interval $[0, 2\pi]$. Panel 2 (a) plots the raw data and the function to be approximated.

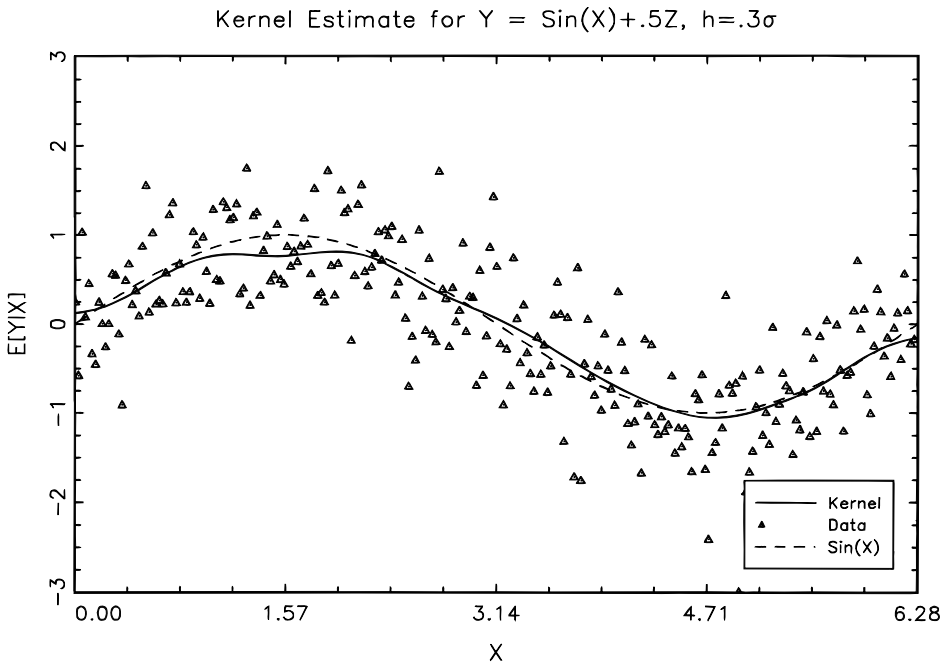


(a)

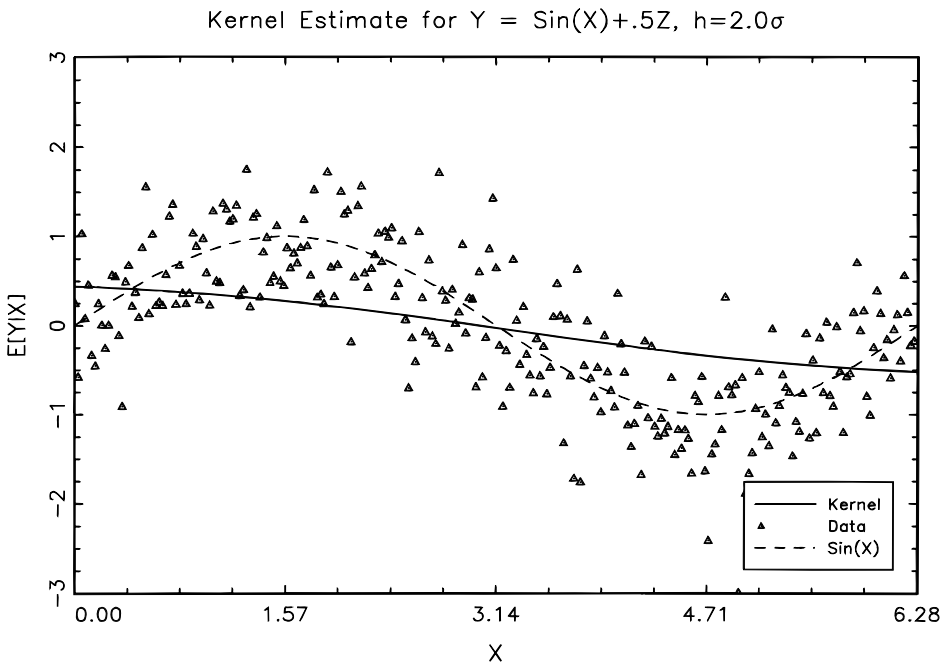


(b)

Figure 2. Illustration of bandwidth selection for kernel regression.



(c)



(d)

Figure 2. Continued

Kernel estimators for three different bandwidths are plotted as solid lines in Panels 2(b)–(c). The bandwidth in 2(b) is clearly too small; the function is too variable, fitting the “noise” $0.5\epsilon Z_t$ and also the “signal” $\text{Sin}(\cdot)$. Increasing the bandwidth slightly yields a much more accurate approximation to $\text{Sin}(\cdot)$ as Panel 2(c) illustrates. However, Panel 2(d) shows that if the bandwidth is increased beyond some point, there is too much averaging and information is lost.

There are several methods for automating the choice of bandwidth h in equation (9), but the most popular is the *cross-validation* method in which h is chosen to minimize the cross-validation function

$$\text{CV}(h) = \frac{1}{T} \sum_{t=1}^T (P_t - \hat{m}_{h,t})^2, \quad (12)$$

where

$$\hat{m}_{h,t} \equiv \frac{1}{T} \sum_{\tau \neq t} \omega_{\tau,h} Y_{\tau}. \quad (13)$$

The estimator $\hat{m}_{h,t}$ is the kernel regression estimator applied to the price history $\{P_{\tau}\}$ with the t th observation omitted, and the summands in equation (12) are the squared errors of the $\hat{m}_{h,t}$ s, each evaluated at the omitted observation. For a given bandwidth parameter h , the cross-validation function is a measure of the ability of the kernel regression estimator to fit each observation P_t when that observation is not used to construct the kernel estimator. By selecting the bandwidth that minimizes this function, we obtain a kernel estimator that satisfies certain optimality properties, for example, minimum asymptotic mean-squared error.⁴

Interestingly, the bandwidths obtained from minimizing the cross-validation function are generally too large for our application to technical analysis—when we presented several professional technical analysts with plots of cross-validation-fitted functions $\hat{m}_h(\cdot)$, they all concluded that the fitted functions were too smooth. In other words, the cross-validation-determined bandwidth places too much weight on prices far away from any given time t , inducing too much averaging and discarding valuable information in local price movements. Through trial and error, and by polling professional technical analysts, we have found that an acceptable solution to this problem is to use a bandwidth of $0.3 \times h^*$, where h^* minimizes $\text{CV}(h)$.⁵ Admittedly, this is an ad hoc approach, and it remains an important challenge for future research to develop a more rigorous procedure.

⁴ However, there are other bandwidth-selection methods that yield the same asymptotic optimality properties but that have different implications for the finite-sample properties of kernel estimators. See Härdle (1990) for further discussion.

⁵ Specifically, we produced fitted curves for various bandwidths and compared their extrema to the original price series visually to see if we were fitting more “noise” than “signal,” and we asked several professional technical analysts to do the same. Through this informal process, we settled on the bandwidth of $0.3 \times h^*$ and used it for the remainder of our analysis. This procedure was followed before we performed the statistical analysis of Section III, and we made no revision to the choice of bandwidth afterward.

Another promising direction for future research is to consider alternatives to kernel regression. Although kernel regression is useful for its simplicity and intuitive appeal, kernel estimators suffer from a number of well-known deficiencies, for instance, boundary bias, lack of local variability in the degree of smoothing, and so on. A popular alternative that overcomes these particular deficiencies is *local polynomial regression* in which local averaging of polynomials is performed to obtain an estimator of $m(x)$.⁶ Such alternatives may yield important improvements in the pattern-recognition algorithm described in Section II.

II. Automating Technical Analysis

Armed with a mathematical representation $\hat{m}(\cdot)$ of $\{P_t\}$ with which geometric properties can be characterized in an objective manner, we can now construct an algorithm for automating the detection of technical patterns. Specifically, our algorithm contains three steps:

1. Define each technical pattern in terms of its geometric properties, for example, local extrema (maxima and minima).
2. Construct a kernel estimator $\hat{m}(\cdot)$ of a given time series of prices so that its extrema can be determined numerically.
3. Analyze $\hat{m}(\cdot)$ for occurrences of each technical pattern.

The last two steps are rather straightforward applications of kernel regression. The first step is likely to be the most controversial because it is here that the skills and judgment of a professional technical analyst come into play. Although we will argue in Section II.A that most technical indicators can be characterized by specific *sequences* of local extrema, technical analysts may argue that these are poor approximations to the kinds of patterns that trained human analysts can identify.

While pattern-recognition techniques have been successful in automating a number of tasks previously considered to be uniquely human endeavors—fingerprint identification, handwriting analysis, face recognition, and so on—nevertheless it is possible that no algorithm can completely capture the skills of an experienced technical analyst. We acknowledge that any automated procedure for pattern recognition may miss some of the more subtle nuances that human cognition is capable of discerning, but whether an algorithm is a poor approximation to human judgment can only be determined by investigating the approximation errors empirically. As long as an algorithm can provide a reasonable approximation to *some* of the cognitive abilities of a human analyst, we can use such an algorithm to investigate the empirical performance of those aspects of technical analysis for which the algorithm is a good approximation. Moreover, if technical analysis is an art form that can

⁶ See Simonoff (1996) for a discussion of the problems with kernel estimators and alternatives such as local polynomial regression.

be taught, then surely its basic precepts can be quantified and automated to some degree. And as increasingly sophisticated pattern-recognition techniques are developed, a larger fraction of the art will become a science.

More important, from a practical perspective, there may be significant benefits to developing an algorithmic approach to technical analysis because of the leverage that technology can provide. As with many other successful technologies, the automation of technical pattern recognition may not replace the skills of a technical analyst but can amplify them considerably.

In Section II.A, we propose definitions of 10 technical patterns based on their extrema. In Section II.B, we describe a specific algorithm to identify technical patterns based on the local extrema of price series using kernel regression estimators, and we provide specific examples of the algorithm at work in Section II.C.

A. Definitions of Technical Patterns

We focus on five pairs of technical patterns that are among the most popular patterns of traditional technical analysis (see, e.g., Edwards and Magee (1966, Chaps. VII–X)): head-and-shoulders (HS) and inverse head-and-shoulders (IHS), broadening tops (BTOP) and bottoms (BBOT), triangle tops (TTOP) and bottoms (TBOT), rectangle tops (RTOP) and bottoms (RBOT), and double tops (DTOP) and bottoms (DBOT). There are many other technical indicators that may be easier to detect algorithmically—moving averages, support and resistance levels, and oscillators, for example—but because we wish to illustrate the power of smoothing techniques in automating technical analysis, we focus on precisely those patterns that are most difficult to quantify analytically.

Consider the systematic component $m(\cdot)$ of a price history $\{P_t\}$ and suppose we have identified n local extrema, that is, the local maxima and minima, of $\{P_t\}$. Denote by E_1, E_2, \dots, E_n the n extrema and $t_1^*, t_2^*, \dots, t_n^*$ the dates on which these extrema occur. Then we have the following definitions.

Definition 1 (Head-and-Shoulders) Head-and-shoulders (HS) and inverted head-and-shoulders (IHS) patterns are characterized by a sequence of five consecutive local extrema E_1, \dots, E_5 such that

$$\text{HS} \equiv \begin{cases} E_1 \text{ is a maximum} \\ E_3 > E_1, E_3 > E_5 \\ E_1 \text{ and } E_5 \text{ are within 1.5 percent of their average} \\ E_2 \text{ and } E_4 \text{ are within 1.5 percent of their average,} \end{cases}$$

$$\text{IHS} \equiv \begin{cases} E_1 \text{ is a minimum} \\ E_3 < E_1, E_3 < E_5 \\ E_1 \text{ and } E_5 \text{ are within 1.5 percent of their average} \\ E_2 \text{ and } E_4 \text{ are within 1.5 percent of their average.} \end{cases}$$

Observe that only five consecutive extrema are required to identify a head-and-shoulders pattern. This follows from the formalization of the geometry of a head-and-shoulders pattern: three peaks, with the middle peak higher than the other two. Because consecutive extrema must alternate between maxima and minima for smooth functions,⁷ the three-peaks pattern corresponds to a sequence of five local extrema: maximum, minimum, highest maximum, minimum, and maximum. The inverse head-and-shoulders is simply the mirror image of the head-and-shoulders, with the initial local extrema a minimum.

Because broadening, rectangle, and triangle patterns can begin on either a local maximum or minimum, we allow for both of these possibilities in our definitions by distinguishing between broadening tops and bottoms.

Definition 2 (Broadening) Broadening tops (BTOP) and bottoms (BBOT) are characterized by a sequence of five consecutive local extrema E_1, \dots, E_5 such that

$$\text{BTOP} \equiv \begin{cases} E_1 \text{ is a maximum} \\ E_1 < E_3 < E_5 \\ E_2 > E_4 \end{cases}, \quad \text{BBOT} \equiv \begin{cases} E_1 \text{ is a minimum} \\ E_1 > E_3 > E_5 \\ E_2 < E_4 \end{cases}.$$

Definitions for triangle and rectangle patterns follow naturally.

Definition 3 (Triangle) Triangle tops (TTOP) and bottoms (TBOT) are characterized by a sequence of five consecutive local extrema E_1, \dots, E_5 such that

$$\text{TTOP} \equiv \begin{cases} E_1 \text{ is a maximum} \\ E_1 > E_3 > E_5 \\ E_2 < E_4 \end{cases}, \quad \text{TBOT} \equiv \begin{cases} E_1 \text{ is a minimum} \\ E_1 < E_3 < E_5 \\ E_2 > E_4 \end{cases}.$$

Definition 4 (Rectangle) Rectangle tops (RTOP) and bottoms (RBOT) are characterized by a sequence of five consecutive local extrema E_1, \dots, E_5 such that

$$\text{RTOP} \equiv \begin{cases} E_1 \text{ is a maximum} \\ \text{tops are within 0.75 percent of their average} \\ \text{bottoms are within 0.75 percent of their average} \\ \text{lowest top} > \text{highest bottom,} \end{cases}$$

⁷ After all, for two consecutive maxima to be local maxima, there must be a local minimum in between and vice versa for two consecutive minima.

$$\text{RBOT} \equiv \begin{cases} E_1 \text{ is a minimum} \\ \text{tops are within 0.75 percent of their average} \\ \text{bottoms are within 0.75 percent of their average} \\ \text{lowest top} > \text{highest bottom.} \end{cases}$$

The definition for double tops and bottoms is slightly more involved. Consider first the double top. Starting at a local maximum E_1 , we locate the highest local maximum E_a occurring after E_1 in the set of all local extrema in the sample. We require that the two tops, E_1 and E_a , be within 1.5 percent of their average. Finally, following Edwards and Magee (1966), we require that the two tops occur at least a month, or 22 trading days, apart. Therefore, we have the following definition.

Definition 5 (Double Top and Bottom) Double tops (DTOP) and bottoms (DBOT) are characterized by an initial local extremum E_1 and subsequent local extrema E_a and E_b such that

$$E_a \equiv \sup\{P_{t_k}^* : t_k^* > t_1^*, k = 2, \dots, n\}$$

$$E_b \equiv \inf\{P_{t_k}^* : t_k^* > t_1^*, k = 2, \dots, n\}$$

and

$$\text{DTOP} \equiv \begin{cases} E_1 \text{ is a maximum} \\ E_1 \text{ and } E_a \text{ are within 1.5 percent of their average} \\ t_a^* - t_1^* > 22 \end{cases}$$

$$\text{DBOT} \equiv \begin{cases} E_1 \text{ is a minimum} \\ E_1 \text{ and } E_b \text{ are within 1.5 percent of their average} \\ t_a^* - t_1^* > 22 \end{cases}$$

B. The Identification Algorithm

Our algorithm begins with a sample of prices $\{P_1, \dots, P_T\}$ for which we fit kernel regressions, one for each subsample or *window* from t to $t + l + d - 1$, where t varies from 1 to $T - l - d + 1$, and l and d are fixed parameters whose purpose is explained below. In the empirical analysis of Section III, we set $l = 35$ and $d = 3$; hence each window consists of 38 trading days.

The motivation for fitting kernel regressions to rolling windows of data is to narrow our focus to patterns that are completed within the span of the window— $l + d$ trading days in our case. If we fit a single kernel regression to the entire dataset, many patterns of various durations may emerge, and without imposing some additional structure on the nature of the patterns, it

is virtually impossible to distinguish signal from noise in this case. Therefore, our algorithm fixes the length of the window at $l + d$, but kernel regressions are estimated on a rolling basis and we search for patterns in each window.

Of course, for any fixed window, we can only find patterns that are completed within $l + d$ trading days. Without further structure on the systematic component of prices $m(\cdot)$, this is a restriction that any empirical analysis must contend with.⁸ We choose a shorter window length of $l = 35$ trading days to focus on short-horizon patterns that may be more relevant for active equity traders, and we leave the analysis of longer-horizon patterns to future research.

The parameter d controls for the fact that in practice we do not observe a realization of a given pattern as soon as it has completed. Instead, we assume that there may be a lag between the pattern completion and the time of pattern detection. To account for this lag, we require that the final extremum that completes a pattern occurs on day $t + l - 1$; hence d is the number of days following the completion of a pattern that must pass before the pattern is detected. This will become more important in Section III when we compute conditional returns, conditioned on the realization of each pattern. In particular, we compute postpattern returns starting from the end of trading day $t + l + d$, that is, one day after the pattern has completed. For example, if we determine that a head-and-shoulder pattern has completed on day $t + l - 1$ (having used prices from time t through time $t + l + d - 1$), we compute the conditional one-day gross return as $Z_1 \equiv Y_{t+l+d+1}/Y_{t+l+d}$. Hence we do *not* use any forward information in computing returns conditional on pattern completion. In other words, the lag d ensures that we are computing our conditional returns completely out-of-sample and without any “look-ahead” bias.

Within each window, we estimate a kernel regression using the prices in that window, hence:

$$\hat{m}_h(\tau) = \frac{\sum_{s=t}^{t+l+d-1} K_h(\tau - s) P_s}{\sum_{s=t}^{t+l+d-1} K_h(\tau - s)}, \quad t = 1, \dots, T - l - d + 1, \tag{14}$$

where $K_h(z)$ is given in equation (10) and h is the bandwidth parameter (see Sec. II.C). It is clear that $\hat{m}_h(\tau)$ is a differentiable function of τ .

Once the function $\hat{m}_h(\tau)$ has been computed, its local extrema can be readily identified by finding times τ such that $\text{Sgn}(\hat{m}'_h(\tau)) = -\text{Sgn}(\hat{m}'_h(\tau + 1))$, where \hat{m}'_h denotes the derivative of \hat{m}_h with respect to τ and $\text{Sgn}(\cdot)$ is the signum function. If the signs of $\hat{m}'_h(\tau)$ and $\hat{m}'_h(\tau + 1)$ are $+1$ and -1 , respectively, then

⁸ If we are willing to place additional restrictions on $m(\cdot)$, for example, linearity, we can obtain considerably more accurate inferences even for partially completed patterns in any fixed window.

we have found a local maximum, and if they are -1 and $+1$, respectively, then we have found a local minimum. Once such a time τ has been identified, we proceed to identify a maximum or minimum in the original price series $\{P_t\}$ in the range $[t - 1, t + 1]$, and the extrema in the original price series are used to determine whether or not a pattern has occurred according to the definitions of Section II.A.

If $\hat{m}'_h(\tau) = 0$ for a given τ , which occurs if closing prices stay the same for several consecutive days, we need to check whether the price we have found is a local minimum or maximum. We look for the date s such that $s = \inf\{s > \tau : \hat{m}'_h(s) \neq 0\}$. We then apply the same method as discussed above, except here we compare $\text{Sgn}(\hat{m}'_h(\tau - 1))$ and $\text{Sgn}(\hat{m}'_h(s))$.

One useful consequence of this algorithm is that the series of extrema that it identifies contains alternating minima and maxima. That is, if the k th extremum is a maximum, then it is always the case that the $(k + 1)$ th extremum is a minimum and vice versa.

An important advantage of using this kernel regression approach to identify patterns is the fact that it ignores extrema that are "too local." For example, a simpler alternative is to identify local extrema from the raw price data directly, that is, identify a price P_t as a local maximum if $P_{t-1} < P_t$ and $P_t > P_{t+1}$ and vice versa for a local minimum. The problem with this approach is that it identifies too many extrema and also yields patterns that are not visually consistent with the kind of patterns that technical analysts find compelling.

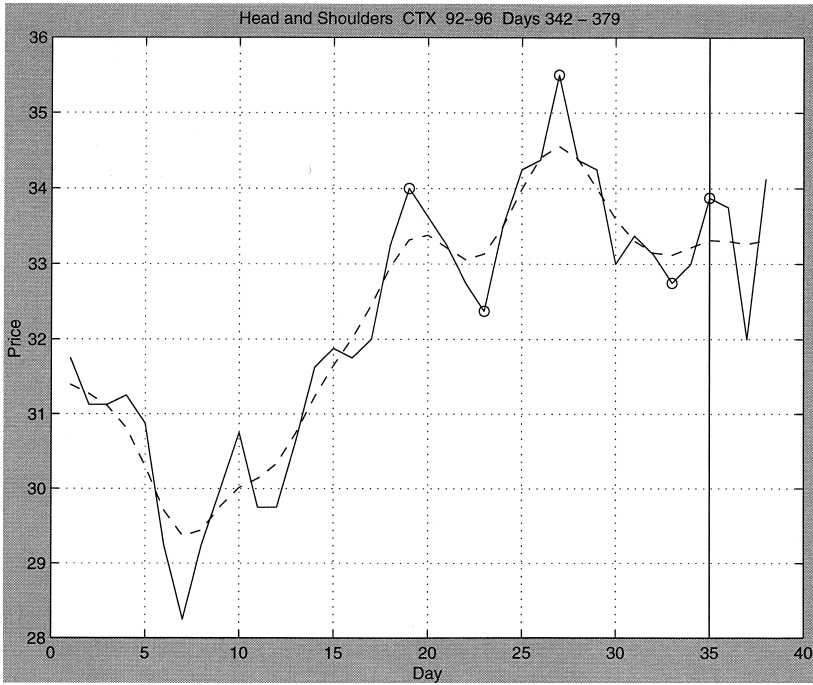
Once we have identified all of the local extrema in the window $[t, t + l + d - 1]$, we can proceed to check for the presence of the various technical patterns using the definitions of Section II.A. This procedure is then repeated for the next window $[t + 1, t + l + d]$ and continues until the end of the sample is reached at the window $[T - l - d + 1, T]$.

C. Empirical Examples

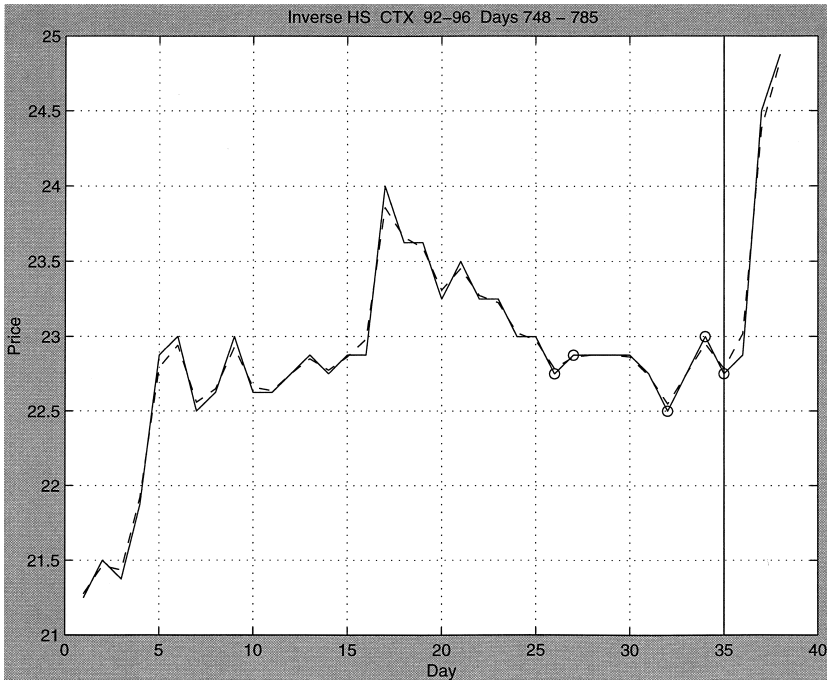
To see how our algorithm performs in practice, we apply it to the daily returns of a single security, CTX, during the five-year period from 1992 to 1996. Figures 3–7 plot occurrences of the five pairs of patterns defined in Section II.A that were identified by our algorithm. Note that there were no rectangle bottoms detected for CTX during this period, so for completeness we substituted a rectangle bottom for CDO stock that occurred during the same period.

In each of these graphs, the solid lines are the raw prices, the dashed lines are the kernel estimators $\hat{m}_h(\cdot)$, the circles indicate the local extrema, and the vertical line marks date $t + l - 1$, the day that the final extremum occurs to complete the pattern.

Casual inspection by several professional technical analysts seems to confirm the ability of our automated procedure to match human judgment in identifying the five pairs of patterns in Section II.A. Of course, this is merely anecdotal evidence and not meant to be conclusive—we provide these figures simply to illustrate the output of a technical pattern-recognition algorithm based on kernel regression.

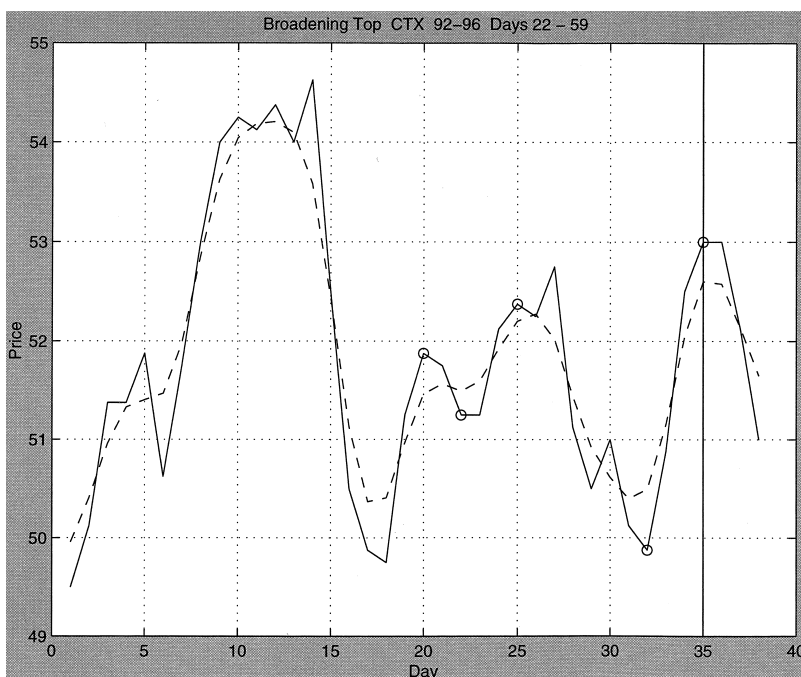


(a) Head-and-Shoulders

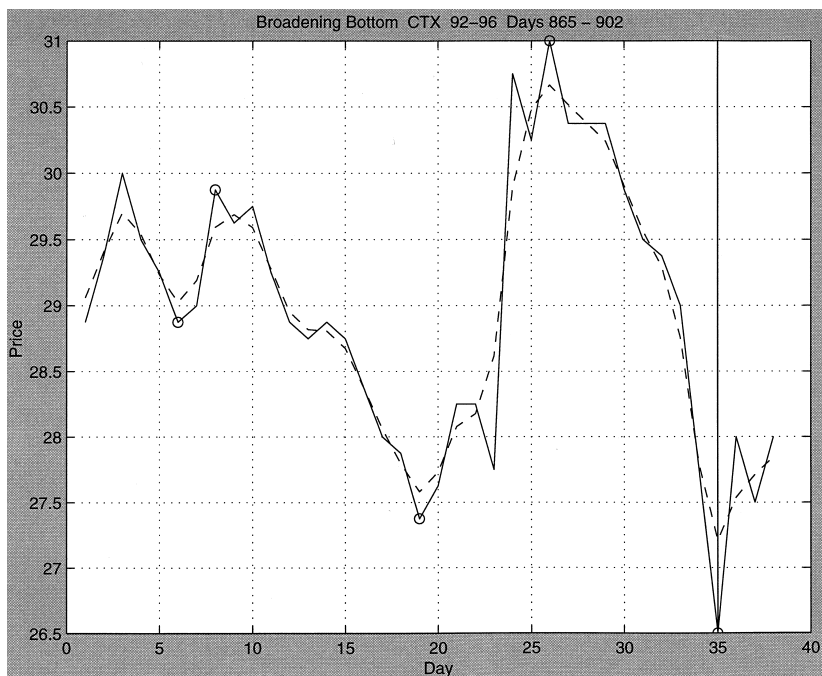


(b) Inverse Head-and-Shoulders

Figure 3. Head-and-shoulders and inverse head-and-shoulders.

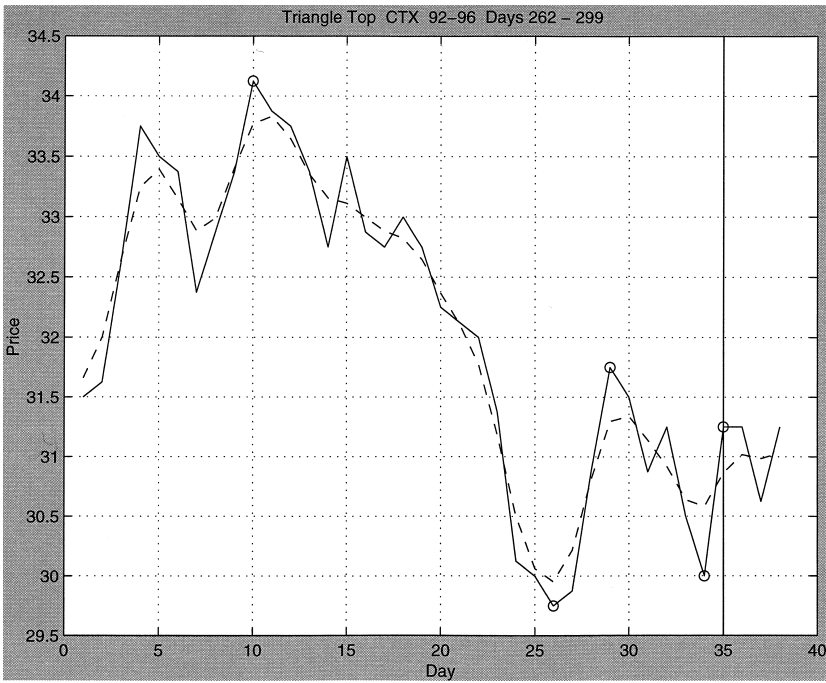


(a) Broadening Top

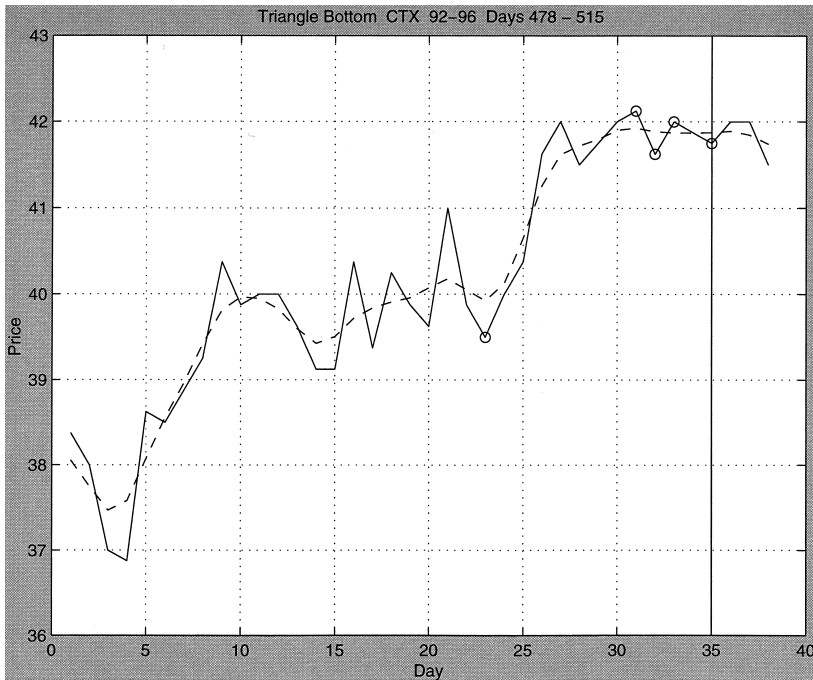


(b) Broadening Bottom

Figure 4. Broadening tops and bottoms.

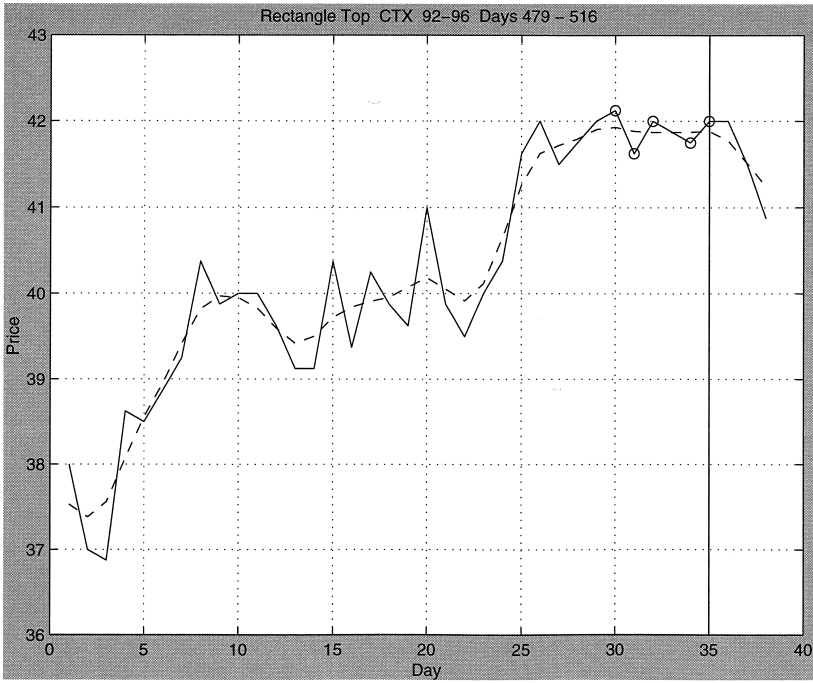


(a) Triangle Top

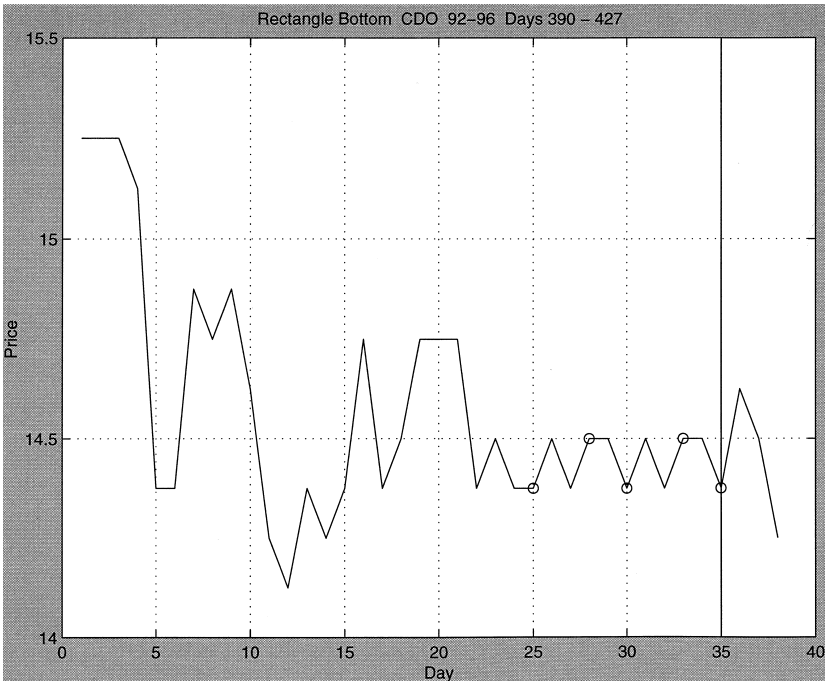


(b) Triangle Bottom

Figure 5. Triangle tops and bottoms.

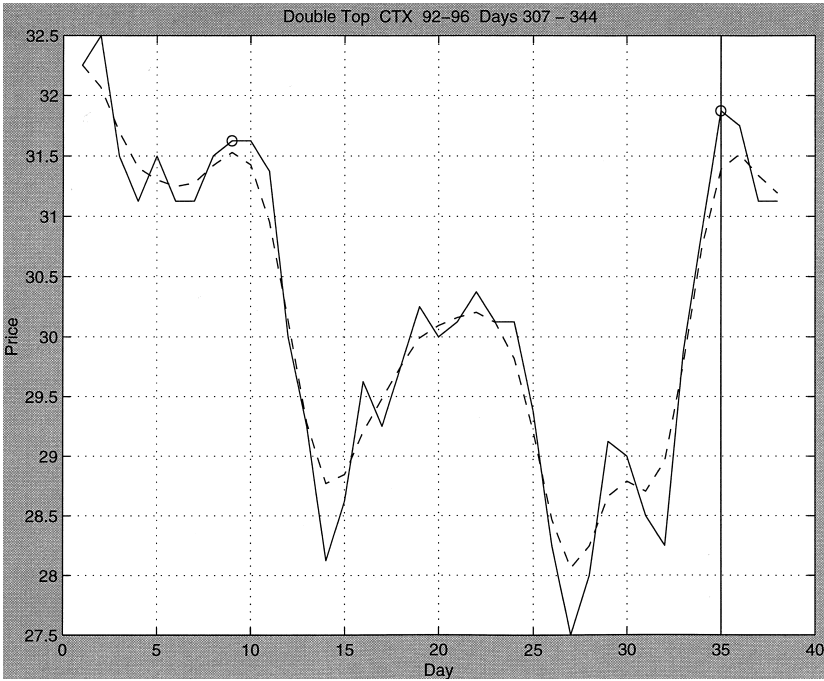


(a) Rectangle Top

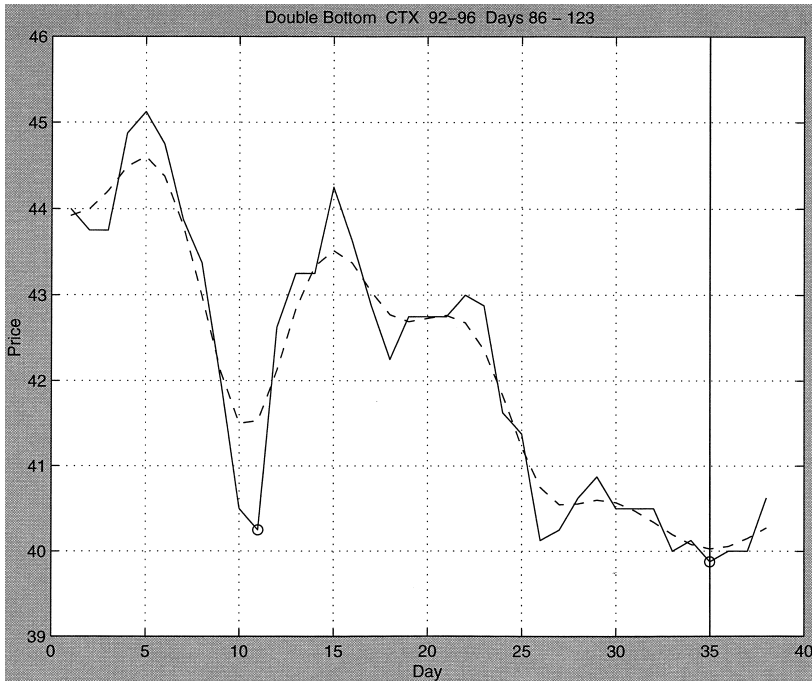


(b) Rectangle Bottom

Figure 6. Rectangle tops and bottoms.



(a) Double Top



(b) Double Bottom

Figure 7. Double tops and bottoms.

III. Is Technical Analysis Informative?

Although there have been many tests of technical analysis over the years, most of these tests have focused on the profitability of technical trading rules.⁹ Although some of these studies do find that technical indicators can generate statistically significant trading profits, but they beg the question of whether or not such profits are merely the equilibrium rents that accrue to investors willing to bear the risks associated with such strategies. Without specifying a fully articulated dynamic general equilibrium asset-pricing model, it is impossible to determine the economic source of trading profits.

Instead, we propose a more fundamental test in this section, one that attempts to gauge the information content in the technical patterns of Section II.A by comparing the unconditional empirical distribution of returns with the corresponding conditional empirical distribution, conditioned on the occurrence of a technical pattern. If technical patterns are informative, conditioning on them should alter the empirical distribution of returns; if the information contained in such patterns has already been incorporated into returns, the conditional and unconditional distribution of returns should be close. Although this is a weaker test of the effectiveness of technical analysis—informativeness does not guarantee a profitable trading strategy—it is, nevertheless, a natural first step in a quantitative assessment of technical analysis.

To measure the distance between the two distributions, we propose two goodness-of-fit measures in Section III.A. We apply these diagnostics to the daily returns of individual stocks from 1962 to 1996 using a procedure described in Sections III.B to III.D, and the results are reported in Sections III.E and III.F.

A. *Goodness-of-Fit Tests*

A simple diagnostic to test the informativeness of the 10 technical patterns is to compare the quantiles of the conditional returns with their unconditional counterparts. If conditioning on these technical patterns provides no incremental information, the quantiles of the conditional returns should be similar to those of unconditional returns. In particular, we compute the

⁹ For example, Chang and Osler (1994) and Osler and Chang (1995) propose an algorithm for automatically detecting head-and-shoulders patterns in foreign exchange data by looking at properly defined local extrema. To assess the efficacy of a head-and-shoulders trading rule, they take a stand on a class of trading strategies and compute the profitability of these across a sample of exchange rates against the U.S. dollar. The null return distribution is computed by a bootstrap that samples returns randomly from the original data so as to induce temporal independence in the bootstrapped time series. By comparing the actual returns from trading strategies to the bootstrapped distribution, the authors find that for two of the six currencies in their sample (the yen and the Deutsche mark), trading strategies based on a head-and-shoulders pattern can lead to statistically significant profits. See, also, Neftci and Policano (1984), Pruitt and White (1988), and Brock et al. (1992).

deciles of unconditional returns and tabulate the relative frequency $\hat{\delta}_j$ of conditional returns falling into decile j of the unconditional returns, $j = 1, \dots, 10$:

$$\hat{\delta}_j \equiv \frac{\text{number of conditional returns in decile } j}{\text{total number of conditional returns}}. \tag{15}$$

Under the null hypothesis that the returns are independently and identically distributed (IID) and the conditional and unconditional distributions are identical, the asymptotic distributions of $\hat{\delta}_j$ and the corresponding goodness-of-fit test statistic Q are given by

$$\sqrt{n}(\hat{\delta}_j - 0.10) \stackrel{a}{\sim} \mathcal{N}(0, 0.10(1 - 0.10)), \tag{16}$$

$$Q \equiv \sum_{j=1}^{10} \frac{(n_j - 0.10n)^2}{0.10n} \stackrel{a}{\sim} \chi_9^2, \tag{17}$$

where n_j is the number of observations that fall in decile j and n is the total number of observations (see, e.g., DeGroot (1986)).

Another comparison of the conditional and unconditional distributions of returns is provided by the Kolmogorov–Smirnov test. Denote by $\{Z_{1t}\}_{t=1}^{n_1}$ and $\{Z_{2t}\}_{t=1}^{n_2}$ two samples that are each IID with cumulative distribution functions $F_1(z)$ and $F_2(z)$, respectively. The Kolmogorov–Smirnov statistic is designed to test the null hypothesis that $F_1 = F_2$ and is based on the empirical cumulative distribution functions \hat{F}_i of both samples:

$$\hat{F}_i(z) \equiv \frac{1}{n_i} \sum_{k=1}^{n_i} \mathbf{1}(Z_{ik} \leq z), \quad i = 1, 2, \tag{18}$$

where $\mathbf{1}(\cdot)$ is the indicator function. The statistic is given by the expression

$$\gamma_{n_1, n_2} = \left(\frac{n_1 n_2}{n_1 + n_2} \right)^{1/2} \sup_{-\infty < z < \infty} |\hat{F}_1(z) - \hat{F}_2(z)|. \tag{19}$$

Under the null hypothesis $F_1 = F_2$, the statistic γ_{n_1, n_2} should be small. Moreover, Smirnov (1939a, 1939b) derives the limiting distribution of the statistic to be

$$\lim_{\min(n_1, n_2) \rightarrow \infty} \text{Prob}(\gamma_{n_1, n_2} \leq x) = \sum_{k=-\infty}^{\infty} (-1)^k \exp(-2k^2 x^2), \quad x > 0. \tag{20}$$

An approximate α -level test of the null hypothesis can be performed by computing the statistic and rejecting the null if it exceeds the upper 100 α th percentile for the null distribution given by equation (20) (see Hollander and Wolfe (1973, Table A.23), Csáki (1984), and Press et al. (1986, Chap. 13.5)).

Note that the sampling distributions of both the goodness-of-fit and Kolmogorov–Smirnov statistics are derived under the assumption that returns are IID, which is not plausible for financial data. We attempt to address this problem by normalizing the returns of each security, that is, by subtracting its mean and dividing by its standard deviation (see Sec. III.C), but this does not eliminate the dependence or heterogeneity. We hope to extend our analysis to the more general non-IID case in future research.

B. The Data and Sampling Procedure

We apply the goodness-of-fit and Kolmogorov–Smirnov tests to the daily returns of individual NYSE/AMEX and Nasdaq stocks from 1962 to 1996 using data from the Center for Research in Securities Prices (CRSP). To ameliorate the effects of nonstationarities induced by changing market structure and institutions, we split the data into NYSE/AMEX stocks and Nasdaq stocks and into seven five-year periods: 1962 to 1966, 1967 to 1971, and so on. To obtain a broad cross section of securities, in each five-year subperiod, we randomly select 10 stocks from each of five market-capitalization quintiles (using mean market capitalization over the subperiod), with the further restriction that at least 75 percent of the price observations must be nonmissing during the subperiod.¹⁰ This procedure yields a sample of 50 stocks for each subperiod across seven subperiods (note that we sample with replacement; hence there may be names in common across subperiods).

As a check on the robustness of our inferences, we perform this sampling procedure twice to construct two samples, and we apply our empirical analysis to both. Although we report results only from the first sample to conserve space, the results of the second sample are qualitatively consistent with the first and are available upon request.

C. Computing Conditional Returns

For each stock in each subperiod, we apply the procedure outlined in Section II to identify all occurrences of the 10 patterns defined in Section II.A. For each pattern detected, we compute the one-day continuously compounded return d days after the pattern has completed. Specifically, consider a window of prices $\{P_t\}$ from t to $t + l + d - 1$ and suppose that the

¹⁰ If the first price observation of a stock is missing, we set it equal to the first nonmissing price in the series. If the t th price observation is missing, we set it equal to the first nonmissing price prior to t .

identified pattern p is completed at $t + l - 1$. Then we take the conditional return R^p as $\log(1 + R_{t+l+d+1})$. Therefore, for each stock, we have 10 sets of such conditional returns, each conditioned on one of the 10 patterns of Section II.A.

For each stock, we construct a sample of *unconditional* continuously compounded returns using nonoverlapping intervals of length τ , and we compare the empirical distribution functions of these returns with those of the conditional returns. To facilitate such comparisons, we standardize all returns—both conditional and unconditional—by subtracting means and dividing by standard deviations, hence:

$$X_{it} = \frac{R_{it} - \text{Mean}[R_{it}]}{\text{SD}[R_{it}]}, \quad (21)$$

where the means and standard deviations are computed for each individual stock within each subperiod. Therefore, by construction, each normalized return series has zero mean and unit variance.

Finally, to increase the power of our goodness-of-fit tests, we combine the normalized returns of all 50 stocks within each subperiod; hence for each subperiod we have two samples—unconditional and conditional returns—and from these we compute two empirical distribution functions that we compare using our diagnostic test statistics.

D. Conditioning on Volume

Given the prominent role that volume plays in technical analysis, we also construct returns conditioned on increasing or decreasing volume. Specifically, for each stock in each subperiod, we compute its average share turnover during the first and second halves of each subperiod, τ_1 and τ_2 , respectively. If $\tau_1 > 1.2 \times \tau_2$, we categorize this as a “decreasing volume” event; if $\tau_2 > 1.2 \times \tau_1$, we categorize this as an “increasing volume” event. If neither of these conditions holds, then neither event is considered to have occurred.

Using these events, we can construct conditional returns conditioned on two pieces of information: the occurrence of a technical pattern and the occurrence of increasing or decreasing volume. Therefore, we shall compare the empirical distribution of unconditional returns with three conditional-return distributions: the distribution of returns conditioned on technical patterns, the distribution conditioned on technical patterns and increasing volume, and the distribution conditioned on technical patterns and decreasing volume.

Of course, other conditioning variables can easily be incorporated into this procedure, though the “curse of dimensionality” imposes certain practical limits on the ability to estimate multivariate conditional distributions nonparametrically.

E. Summary Statistics

In Tables I and II, we report frequency counts for the number of patterns detected over the entire 1962 to 1996 sample, and within each subperiod and each market-capitalization quintile, for the 10 patterns defined in Section II.A. Table I contains results for the NYSE/AMEX stocks, and Table II contains corresponding results for Nasdaq stocks.

Table I shows that the most common patterns across all stocks and over the entire sample period are double tops and bottoms (see the row labeled "Entire"), with over 2,000 occurrences of each. The second most common patterns are the head-and-shoulders and inverted head-and-shoulders, with over 1,600 occurrences of each. These total counts correspond roughly to four to six occurrences of each of these patterns for each stock during each five-year subperiod (divide the total number of occurrences by 7×50), not an unreasonable frequency from the point of view of professional technical analysts. Table I shows that most of the 10 patterns are more frequent for larger stocks than for smaller ones and that they are relatively evenly distributed over the five-year subperiods. When volume trend is considered jointly with the occurrences of the 10 patterns, Table I shows that the frequency of patterns is not evenly distributed between increasing (the row labeled " $\tau(\nearrow)$ ") and decreasing (the row labeled " $\tau(\searrow)$ ") volume-trend cases. For example, for the entire sample of stocks over the 1962 to 1996 sample period, there are 143 occurrences of a broadening top with decreasing volume trend but 409 occurrences of a broadening top with increasing volume trend.

For purposes of comparison, Table I also reports frequency counts for the number of patterns detected in a sample of simulated geometric Brownian motion, calibrated to match the mean and standard deviation of each stock in each five-year subperiod.¹¹ The entries in the row labeled "Sim. GBM" show that the random walk model yields very different implications for the frequency counts of several technical patterns. For example, the simulated sample has only 577 head-and-shoulders and 578 inverted-head-and-shoulders patterns, whereas the actual data have considerably more, 1,611 and 1,654, respectively. On the other hand, for broadening tops and bottoms, the simulated sample contains many more occurrences than the actual data, 1,227 and 1,028, compared to 725 and 748, respectively. The number of triangles is roughly comparable across the two samples, but for rectangles and

¹¹ In particular, let the price process satisfy

$$dP(t) = \mu P(t)dt + \sigma P(t)dW(t),$$

where $W(t)$ is a standard Brownian motion. To generate simulated prices for a single security in a given period, we estimate the security's drift and diffusion coefficients by maximum likelihood and then simulate prices using the estimated parameter values. An independent price series is simulated for each of the 350 securities in both the NYSE/AMEX and the Nasdaq samples. Finally, we use our pattern-recognition algorithm to detect the occurrence of each of the 10 patterns in the simulated price series.

double tops and bottoms, the differences are dramatic. Of course, the simulated sample is only one realization of geometric Brownian motion, so it is difficult to draw general conclusions about the relative frequencies. Nevertheless, these simulations point to important differences between the data and IID lognormal returns.

To develop further intuition for these patterns, Figures 8 and 9 display the cross-sectional and time-series distribution of each of the 10 patterns for the NYSE/AMEX and Nasdaq samples, respectively. Each symbol represents a pattern detected by our algorithm, the vertical axis is divided into the five size quintiles, the horizontal axis is calendar time, and alternating symbols (diamonds and asterisks) represent distinct subperiods. These graphs show that the distribution of patterns is not clustered in time or among a subset of securities.

Table II provides the same frequency counts for Nasdaq stocks, and despite the fact that we have the same number of stocks in this sample (50 per subperiod over seven subperiods), there are considerably fewer patterns detected than in the NYSE/AMEX case. For example, the Nasdaq sample yields only 919 head-and-shoulders patterns, whereas the NYSE/AMEX sample contains 1,611. Not surprisingly, the frequency counts for the sample of simulated geometric Brownian motion are similar to those in Table I.

Tables III and IV report summary statistics—means, standard deviations, skewness, and excess kurtosis—of unconditional and conditional normalized returns of NYSE/AMEX and Nasdaq stocks, respectively. These statistics show considerable variation in the different return populations. For example, in Table III the first four moments of normalized raw returns are 0.000, 1.000, 0.345, and 8.122, respectively. The same four moments of post-BTOP returns are -0.005 , 1.035 , -1.151 , and 16.701 , respectively, and those of post-DTOP returns are 0.017 , 0.910 , 0.206 , and 3.386 , respectively. The differences in these statistics among the 10 conditional return populations, and the differences between the conditional and unconditional return populations, suggest that conditioning on the 10 technical indicators does have some effect on the distribution of returns.

F. Empirical Results

Tables V and VI report the results of the goodness-of-fit test (equations (16) and (17)) for our sample of NYSE and AMEX (Table V) and Nasdaq (Table VI) stocks, respectively, from 1962 to 1996 for each of the 10 technical patterns. Table V shows that in the NYSE/AMEX sample, the relative frequencies of the conditional returns are significantly different from those of the unconditional returns for seven of the 10 patterns considered. The three exceptions are the conditional returns from the BBOT, TTOP, and DBOT patterns, for which the p -values of the test statistics Q are 5.1 percent, 21.2 percent, and 16.6 percent, respectively. These results yield mixed support for the overall efficacy of technical indicators. However, the results of Table VI tell a different story: there is overwhelming significance for all 10 indicators

Table I

Frequency counts for 10 technical indicators detected among NYSE/AMEX stocks from 1962 to 1996, in five-year subperiods, in size quintiles, and in a sample of simulated geometric Brownian motion. In each five-year subperiod, 10 stocks per quintile are selected at random among stocks with at least 80% nonmissing prices, and each stock's price history is scanned for any occurrence of the following 10 technical indicators within the subperiod: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT). The "Sample" column indicates whether the frequency counts are conditioned on decreasing volume trend ($\tau(\searrow)$), increasing volume trend ($\tau(\nearrow)$), unconditional ("Entire"), or for a sample of simulated geometric Brownian motion with parameters calibrated to match the data ("Sim. GBM").

| Sample | Raw | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|--------|---------|------|------|------|------|------|------|------|------|------|------|
| Entire | 423,556 | 1611 | 1654 | 725 | 748 | 1294 | 1193 | 1482 | 1616 | 2076 | 2075 |
| | 423,556 | 577 | 578 | 1227 | 1028 | 1049 | 1176 | 122 | 113 | 535 | 574 |
| | — | 655 | 593 | 143 | 220 | 666 | 710 | 582 | 637 | 691 | 974 |
| | — | 553 | 614 | 409 | 337 | 300 | 222 | 523 | 552 | 776 | 533 |
| Entire | 84,363 | 182 | 181 | 78 | 97 | 203 | 159 | 265 | 320 | 261 | 271 |
| | 84,363 | 82 | 99 | 279 | 256 | 269 | 295 | 18 | 16 | 129 | 127 |
| | — | 90 | 81 | 13 | 42 | 122 | 119 | 113 | 131 | 78 | 161 |
| | — | 58 | 76 | 51 | 37 | 41 | 22 | 99 | 120 | 124 | 64 |
| Entire | 83,986 | 309 | 321 | 146 | 150 | 255 | 228 | 299 | 322 | 372 | 420 |
| | 83,986 | 108 | 105 | 291 | 251 | 261 | 278 | 20 | 17 | 106 | 126 |
| | — | 133 | 126 | 25 | 48 | 135 | 147 | 130 | 149 | 113 | 211 |
| | — | 112 | 126 | 90 | 63 | 55 | 39 | 104 | 110 | 153 | 107 |
| Entire | 84,420 | 361 | 388 | 145 | 161 | 291 | 247 | 334 | 399 | 458 | 443 |
| | 84,420 | 122 | 120 | 268 | 222 | 212 | 249 | 24 | 31 | 115 | 125 |
| | — | 152 | 131 | 20 | 49 | 151 | 149 | 130 | 160 | 154 | 215 |
| | — | 125 | 146 | 83 | 66 | 67 | 44 | 121 | 142 | 179 | 106 |
| Entire | 84,780 | 332 | 317 | 176 | 173 | 262 | 255 | 259 | 264 | 424 | 420 |
| | 84,780 | 143 | 127 | 249 | 210 | 183 | 210 | 35 | 24 | 116 | 122 |
| | — | 131 | 115 | 36 | 42 | 138 | 145 | 85 | 97 | 144 | 184 |
| | — | 110 | 126 | 103 | 89 | 56 | 55 | 102 | 96 | 147 | 118 |
| Entire | 86,007 | 427 | 447 | 180 | 167 | 283 | 304 | 325 | 311 | 561 | 521 |
| | 86,007 | 122 | 127 | 140 | 89 | 124 | 144 | 25 | 25 | 69 | 74 |
| | — | 149 | 140 | 49 | 39 | 120 | 150 | 124 | 100 | 202 | 203 |
| | — | 148 | 140 | 82 | 82 | 81 | 62 | 97 | 84 | 173 | 138 |

All Stocks, 1962 to 1996
 Smallest Quintile, 1962 to 1996
 2nd Quintile, 1962 to 1996
 3rd Quintile, 1962 to 1996
 4th Quintile, 1962 to 1996
 Largest Quintile, 1962 to 1996

| | | | | | | | | | | | |
|--------------------------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| All Stocks, 1962 to 1966 | | | | | | | | | | | |
| Entire | 55,254 | 276 | 278 | 85 | 103 | 179 | 165 | 316 | 354 | 356 | 352 |
| Sim. GBM | 55,254 | 56 | 58 | 144 | 126 | 129 | 139 | 9 | 16 | 60 | 68 |
| $\tau(\mathcal{S})$ | — | 104 | 88 | 26 | 29 | 93 | 109 | 130 | 141 | 113 | 188 |
| $\tau(\mathcal{L})$ | — | 96 | 112 | 44 | 39 | 37 | 25 | 130 | 122 | 137 | 88 |
| All Stocks, 1967 to 1971 | | | | | | | | | | | |
| Entire | 60,299 | 179 | 175 | 112 | 134 | 227 | 172 | 115 | 117 | 239 | 258 |
| Sim. GBM | 60,299 | 92 | 70 | 167 | 148 | 150 | 180 | 19 | 16 | 84 | 77 |
| $\tau(\mathcal{S})$ | — | 68 | 64 | 16 | 45 | 126 | 111 | 42 | 39 | 80 | 143 |
| $\tau(\mathcal{L})$ | — | 71 | 69 | 68 | 57 | 47 | 29 | 41 | 41 | 87 | 53 |
| All Stocks, 1972 to 1976 | | | | | | | | | | | |
| Entire | 59,915 | 152 | 162 | 82 | 93 | 165 | 136 | 171 | 182 | 218 | 223 |
| Sim. GBM | 59,915 | 75 | 85 | 183 | 154 | 156 | 178 | 16 | 10 | 70 | 71 |
| $\tau(\mathcal{S})$ | — | 64 | 55 | 16 | 23 | 88 | 78 | 60 | 64 | 53 | 97 |
| $\tau(\mathcal{L})$ | — | 54 | 62 | 42 | 50 | 32 | 21 | 61 | 67 | 80 | 59 |
| All Stocks, 1977 to 1981 | | | | | | | | | | | |
| Entire | 62,133 | 223 | 206 | 134 | 110 | 188 | 167 | 146 | 182 | 274 | 290 |
| Sim. GBM | 62,133 | 83 | 88 | 245 | 200 | 188 | 210 | 18 | 12 | 90 | 115 |
| $\tau(\mathcal{S})$ | — | 114 | 61 | 24 | 39 | 100 | 97 | 54 | 60 | 82 | 140 |
| $\tau(\mathcal{L})$ | — | 56 | 93 | 78 | 44 | 35 | 36 | 53 | 71 | 113 | 76 |
| All Stocks, 1982 to 1986 | | | | | | | | | | | |
| Entire | 61,984 | 242 | 256 | 106 | 108 | 182 | 190 | 182 | 207 | 313 | 299 |
| Sim. GBM | 61,984 | 115 | 120 | 188 | 144 | 152 | 169 | 31 | 23 | 99 | 87 |
| $\tau(\mathcal{S})$ | — | 101 | 104 | 28 | 30 | 93 | 104 | 70 | 95 | 109 | 124 |
| $\tau(\mathcal{L})$ | — | 89 | 94 | 51 | 62 | 46 | 40 | 73 | 68 | 116 | 85 |
| All Stocks, 1987 to 1991 | | | | | | | | | | | |
| Entire | 61,780 | 240 | 241 | 104 | 98 | 180 | 169 | 260 | 259 | 287 | 285 |
| Sim. GBM | 61,780 | 68 | 79 | 168 | 132 | 131 | 150 | 11 | 10 | 76 | 68 |
| $\tau(\mathcal{S})$ | — | 95 | 89 | 16 | 30 | 86 | 101 | 103 | 102 | 105 | 137 |
| $\tau(\mathcal{L})$ | — | 81 | 79 | 68 | 43 | 53 | 36 | 73 | 87 | 100 | 68 |
| All Stocks, 1992 to 1996 | | | | | | | | | | | |
| Entire | 62,191 | 299 | 336 | 102 | 102 | 173 | 194 | 292 | 315 | 389 | 368 |
| Sim. GBM | 62,191 | 88 | 78 | 132 | 124 | 143 | 150 | 18 | 26 | 56 | 88 |
| $\tau(\mathcal{S})$ | — | 109 | 132 | 17 | 109 | 80 | 110 | 123 | 136 | 149 | 145 |
| $\tau(\mathcal{L})$ | — | 106 | 105 | 58 | 42 | 50 | 35 | 92 | 96 | 143 | 104 |

Table II

Frequency counts for 10 technical indicators detected among Nasdaq stocks from 1962 to 1996, in five-year subperiods, in size quintiles, and in a sample of simulated geometric Brownian motion. In each five-year subperiod, 10 stocks per quintile are selected at random among stocks with at least 80% nonmissing prices, and each stock's price history is scanned for any occurrence of the following 10 technical indicators within the subperiod: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT). The "Sample" column indicates whether the frequency counts are conditioned on decreasing volume trend ($\tau(\downarrow)$), increasing volume trend ($\tau(\uparrow)$), unconditional ("Entire"), or for a sample of simulated geometric Brownian motion with parameters calibrated to match the data ("Sim. GBM").

| Sample | Raw | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|--------------------|---------|-----|-----|------|------|------|------|------|------|------|------|
| Entire | 411,010 | 919 | 817 | 414 | 508 | 850 | 789 | 1134 | 1320 | 1208 | 1147 |
| Sim. GBM | 411,010 | 434 | 447 | 1297 | 1139 | 1169 | 1309 | 96 | 91 | 567 | 579 |
| $\tau(\downarrow)$ | — | 408 | 268 | 69 | 133 | 429 | 460 | 488 | 550 | 339 | 580 |
| $\tau(\uparrow)$ | — | 284 | 325 | 234 | 209 | 185 | 125 | 391 | 461 | 474 | 229 |
| Entire | 81,754 | 84 | 64 | 41 | 73 | 111 | 93 | 165 | 218 | 113 | 125 |
| Sim. GBM | 81,754 | 85 | 84 | 341 | 289 | 334 | 367 | 11 | 12 | 140 | 125 |
| $\tau(\downarrow)$ | — | 36 | 25 | 6 | 20 | 56 | 59 | 77 | 102 | 31 | 81 |
| $\tau(\uparrow)$ | — | 31 | 23 | 31 | 30 | 24 | 15 | 59 | 85 | 46 | 17 |
| Entire | 81,336 | 191 | 138 | 68 | 88 | 161 | 148 | 242 | 305 | 219 | 176 |
| Sim. GBM | 81,336 | 67 | 84 | 243 | 225 | 219 | 229 | 24 | 12 | 99 | 124 |
| $\tau(\downarrow)$ | — | 94 | 51 | 11 | 28 | 86 | 109 | 111 | 131 | 69 | 101 |
| $\tau(\uparrow)$ | — | 66 | 57 | 46 | 38 | 45 | 22 | 85 | 120 | 90 | 42 |
| Entire | 81,772 | 224 | 186 | 105 | 121 | 183 | 155 | 235 | 244 | 279 | 267 |
| Sim. GBM | 81,772 | 69 | 86 | 227 | 210 | 214 | 239 | 15 | 14 | 105 | 100 |
| $\tau(\downarrow)$ | — | 108 | 66 | 23 | 35 | 87 | 91 | 90 | 84 | 78 | 145 |
| $\tau(\uparrow)$ | — | 71 | 79 | 56 | 49 | 39 | 29 | 84 | 86 | 122 | 58 |
| Entire | 82,727 | 212 | 214 | 92 | 116 | 187 | 179 | 296 | 303 | 289 | 297 |
| Sim. GBM | 82,727 | 104 | 92 | 242 | 219 | 209 | 255 | 23 | 26 | 115 | 97 |
| $\tau(\downarrow)$ | — | 88 | 68 | 12 | 26 | 101 | 101 | 127 | 141 | 77 | 143 |
| $\tau(\uparrow)$ | — | 62 | 83 | 57 | 56 | 34 | 22 | 104 | 93 | 118 | 66 |
| Entire | 83,421 | 208 | 215 | 108 | 110 | 208 | 214 | 196 | 250 | 308 | 282 |
| Sim. GBM | 83,421 | 109 | 101 | 244 | 196 | 193 | 219 | 23 | 27 | 108 | 133 |
| $\tau(\downarrow)$ | — | 82 | 58 | 17 | 24 | 99 | 100 | 83 | 92 | 84 | 110 |
| $\tau(\uparrow)$ | — | 54 | 83 | 44 | 36 | 43 | 37 | 59 | 77 | 98 | 46 |

All Stocks, 1962 to 1996

Smallest Quintile, 1962 to 1996

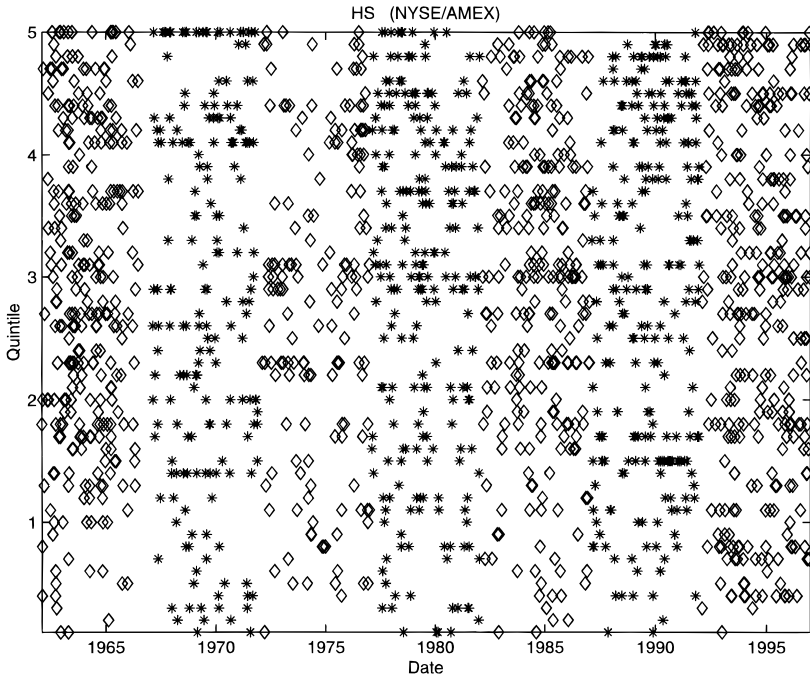
2nd Quintile, 1962 to 1996

3rd Quintile, 1962 to 1996

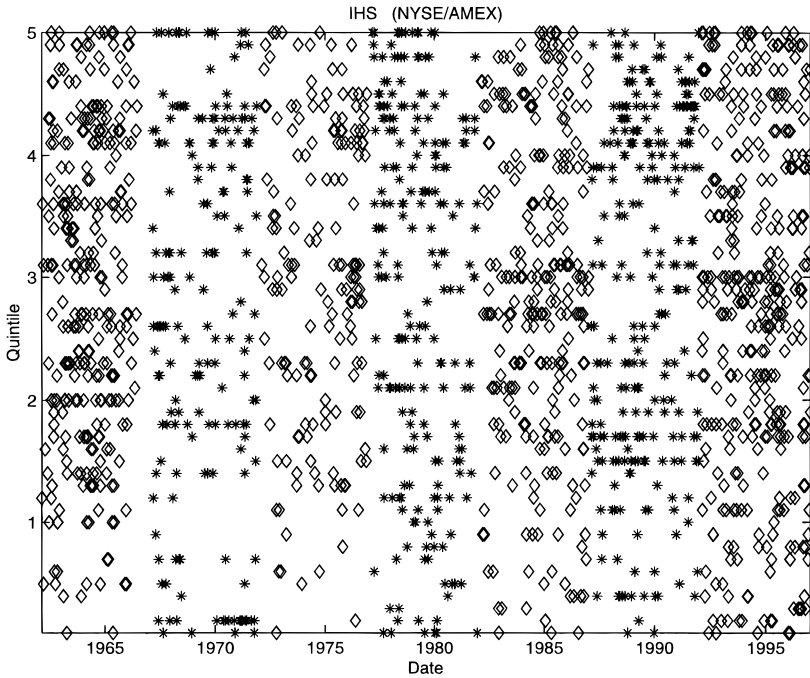
4th Quintile, 1962 to 1996

Largest Quintile, 1962 to 1996

| | | | | | | | | | | | | | |
|---------------------|--------|-----|-----|-----|--------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| Entire | 55,969 | 274 | 268 | 72 | All Stocks, 1962 to 1966 | | | | 144 | 288 | 329 | 326 | 342 |
| Sim. GBM | 55,969 | 69 | 63 | 163 | 99 | 182 | 137 | 149 | 24 | 22 | 22 | 77 | 90 |
| $\tau(\mathcal{S})$ | — | 129 | 99 | 10 | 123 | 104 | 98 | 115 | 115 | 136 | 96 | 96 | 210 |
| $\tau(\mathcal{Z})$ | — | 83 | 103 | 48 | 51 | 37 | 23 | 101 | 101 | 116 | 144 | 144 | 64 |
| Entire | 60,563 | 115 | 120 | 104 | All Stocks, 1967 to 1971 | | | | 171 | 65 | 83 | 196 | 200 |
| Sim. GBM | 60,563 | 58 | 61 | 194 | 184 | 181 | 188 | 9 | 9 | 8 | 90 | 90 | 83 |
| $\tau(\mathcal{S})$ | — | 61 | 29 | 15 | 40 | 127 | 123 | 26 | 39 | 123 | 49 | 26 | 137 |
| $\tau(\mathcal{Z})$ | — | 24 | 57 | 71 | 51 | 45 | 19 | 25 | 25 | 16 | 86 | 86 | 17 |
| Entire | 51,446 | 34 | 30 | 14 | All Stocks, 1972 to 1976 | | | | 28 | 51 | 55 | 55 | 58 |
| Sim. GBM | 51,446 | 32 | 37 | 115 | 113 | 107 | 110 | 5 | 5 | 6 | 46 | 46 | 46 |
| $\tau(\mathcal{S})$ | — | 5 | 4 | 0 | 4 | 5 | 7 | 12 | 12 | 8 | 3 | 3 | 8 |
| $\tau(\mathcal{Z})$ | — | 8 | 7 | 1 | 2 | 2 | 0 | 5 | 5 | 12 | 8 | 8 | 3 |
| Entire | 61,972 | 56 | 53 | 41 | All Stocks, 1977 to 1981 | | | | 73 | 57 | 65 | 89 | 96 |
| Sim. GBM | 61,972 | 90 | 84 | 236 | 165 | 176 | 212 | 19 | 19 | 19 | 110 | 110 | 98 |
| $\tau(\mathcal{S})$ | — | 7 | 7 | 1 | 2 | 4 | 8 | 12 | 12 | 12 | 7 | 7 | 9 |
| $\tau(\mathcal{Z})$ | — | 6 | 6 | 5 | 1 | 4 | 0 | 5 | 5 | 8 | 7 | 7 | 6 |
| Entire | 61,110 | 71 | 64 | 46 | All Stocks, 1982 to 1986 | | | | 107 | 109 | 115 | 120 | 97 |
| Sim. GBM | 61,110 | 86 | 90 | 162 | 44 | 97 | 174 | 23 | 23 | 21 | 97 | 97 | 98 |
| $\tau(\mathcal{S})$ | — | 37 | 19 | 8 | 14 | 46 | 58 | 45 | 45 | 52 | 40 | 40 | 48 |
| $\tau(\mathcal{Z})$ | — | 21 | 25 | 24 | 18 | 26 | 22 | 42 | 42 | 42 | 38 | 38 | 24 |
| Entire | 60,862 | 158 | 120 | 50 | All Stocks, 1987 to 1991 | | | | 109 | 265 | 312 | 177 | 155 |
| Sim. GBM | 60,862 | 59 | 57 | 229 | 187 | 205 | 244 | 7 | 7 | 7 | 79 | 79 | 88 |
| $\tau(\mathcal{S})$ | — | 79 | 46 | 11 | 19 | 73 | 69 | 130 | 130 | 140 | 50 | 50 | 69 |
| $\tau(\mathcal{Z})$ | — | 58 | 56 | 33 | 30 | 26 | 28 | 100 | 100 | 122 | 89 | 89 | 55 |
| Entire | 59,088 | 211 | 162 | 87 | All Stocks, 1992 to 1996 | | | | 157 | 299 | 361 | 245 | 199 |
| Sim. GBM | 59,088 | 40 | 55 | 198 | 115 | 143 | 232 | 9 | 9 | 8 | 68 | 68 | 76 |
| $\tau(\mathcal{S})$ | — | 90 | 64 | 24 | 31 | 70 | 97 | 148 | 148 | 163 | 94 | 94 | 99 |
| $\tau(\mathcal{Z})$ | — | 84 | 71 | 52 | 56 | 45 | 33 | 113 | 113 | 145 | 102 | 102 | 60 |

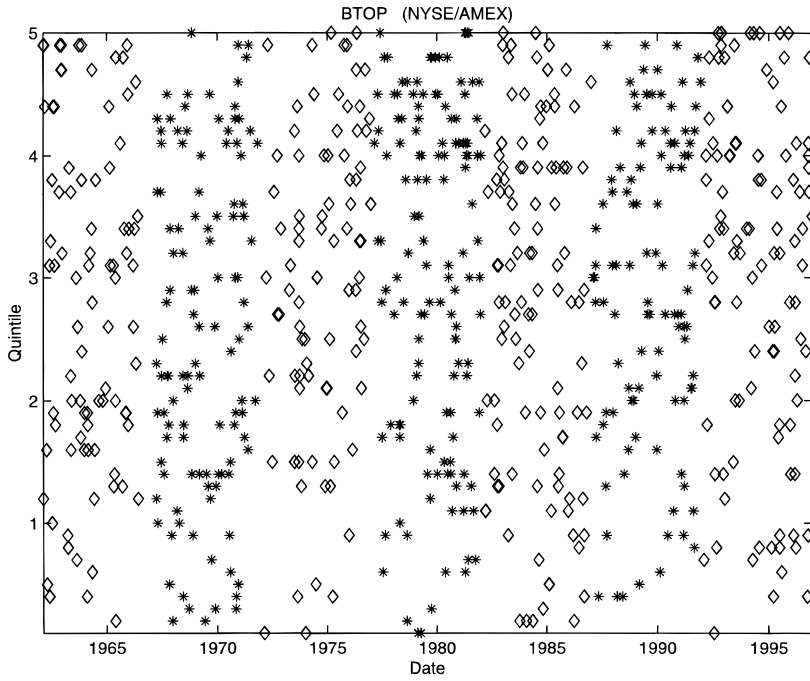


(a)

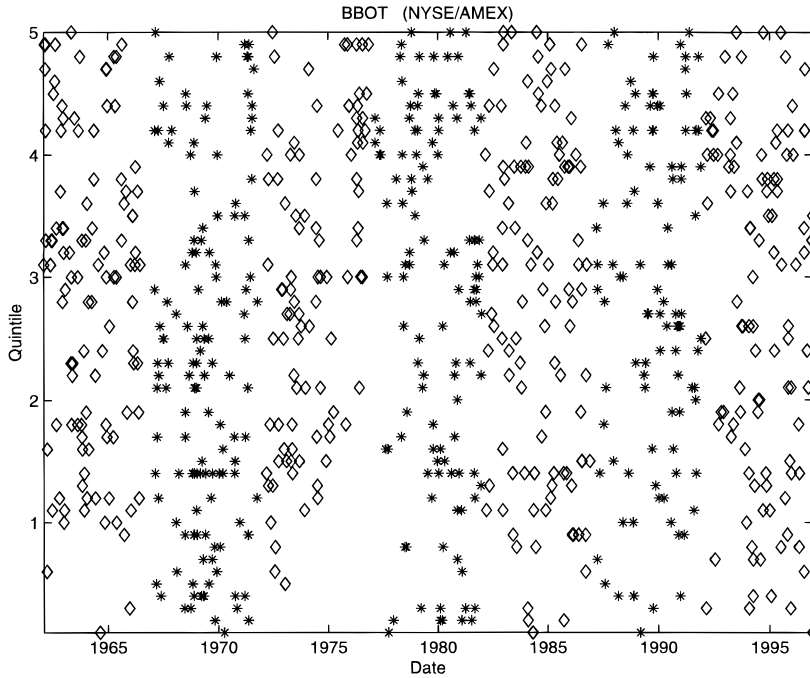


(b)

Figure 8. Distribution of patterns in NYSE/AMEX sample.



(c)



(d)

Figure 8. Continued

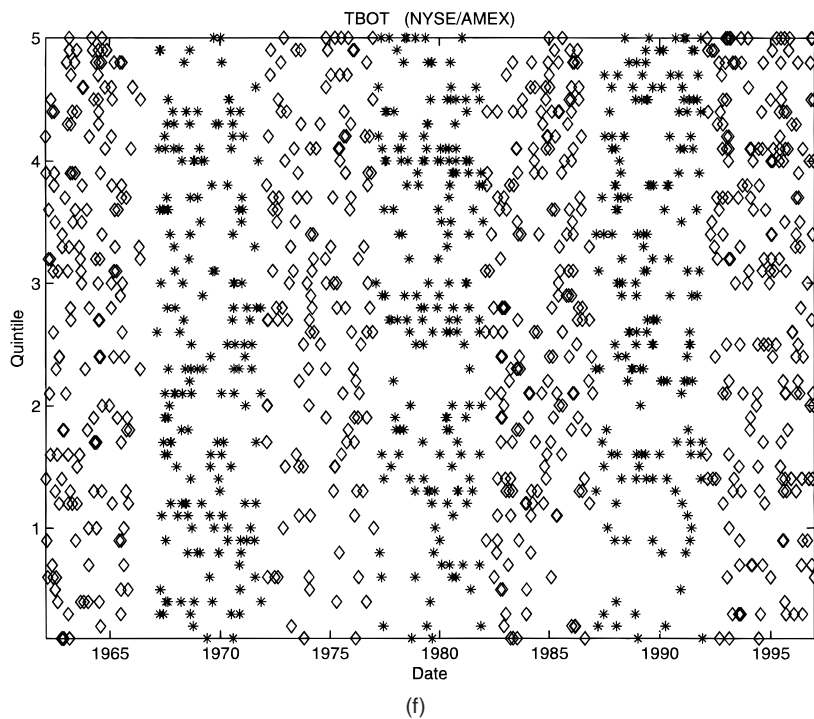
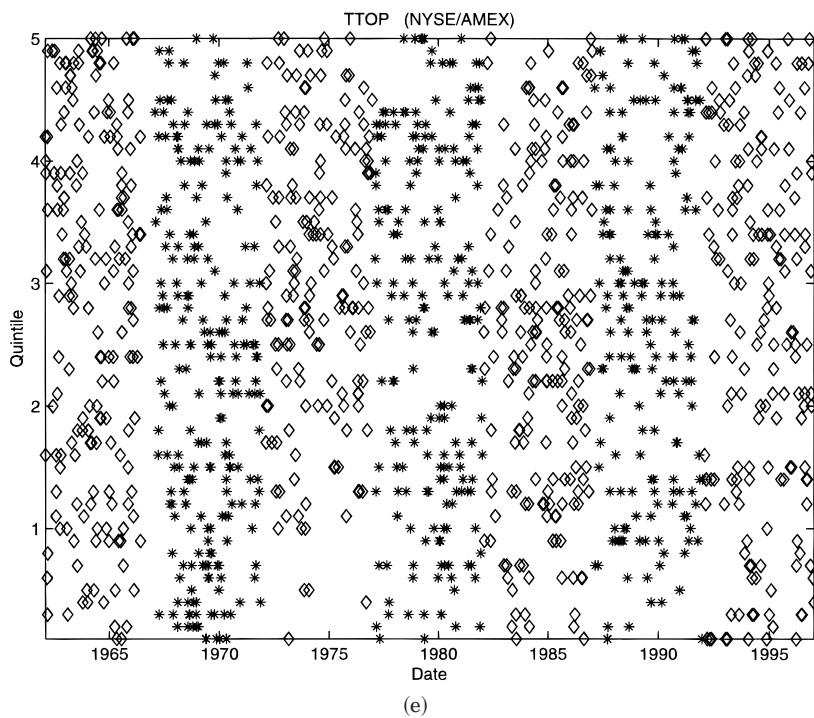
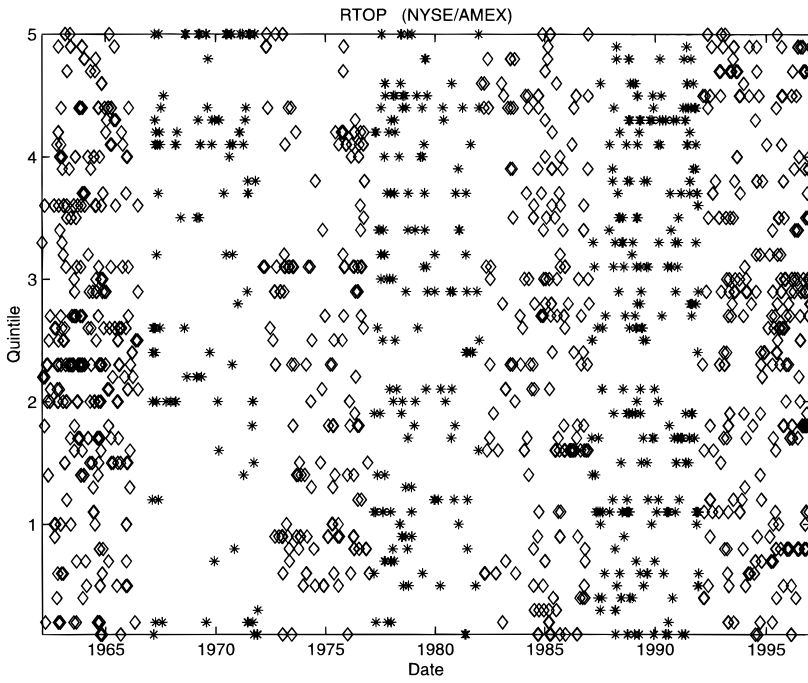
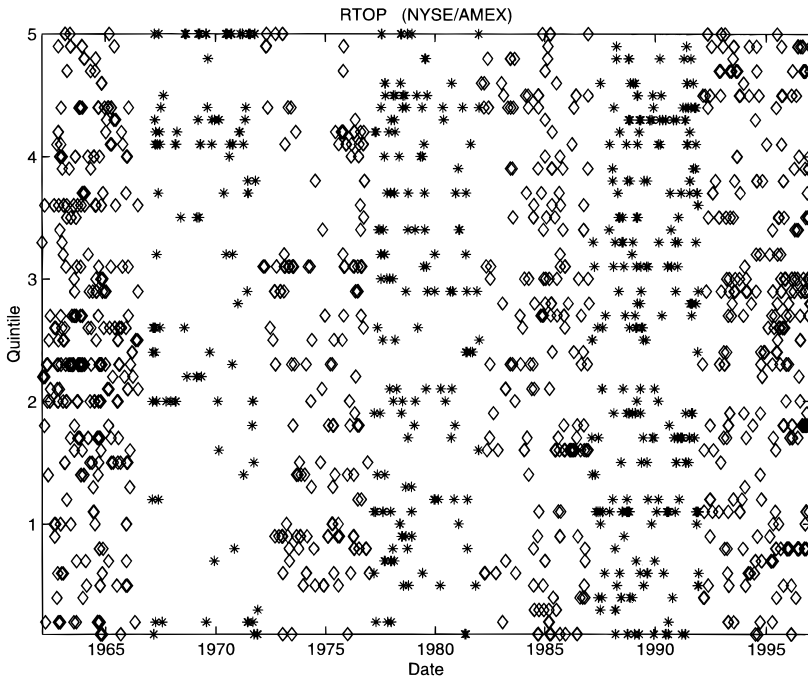


Figure 8. Continued

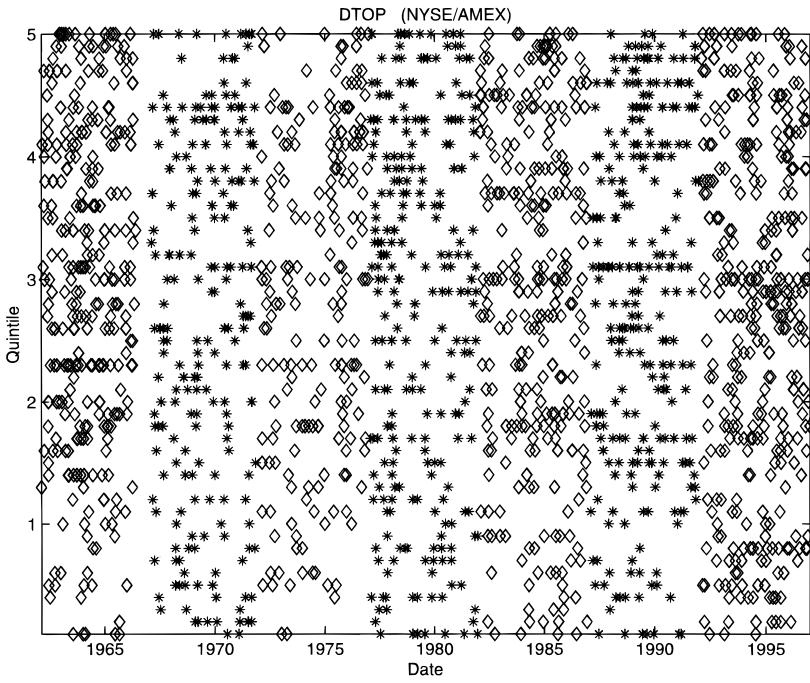


(g)

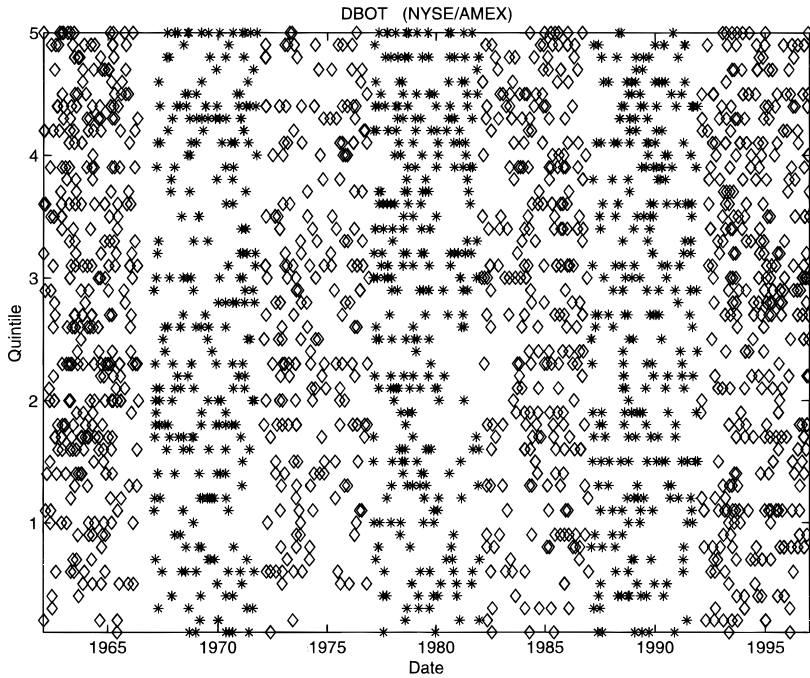


(h)

Figure 8. Continued

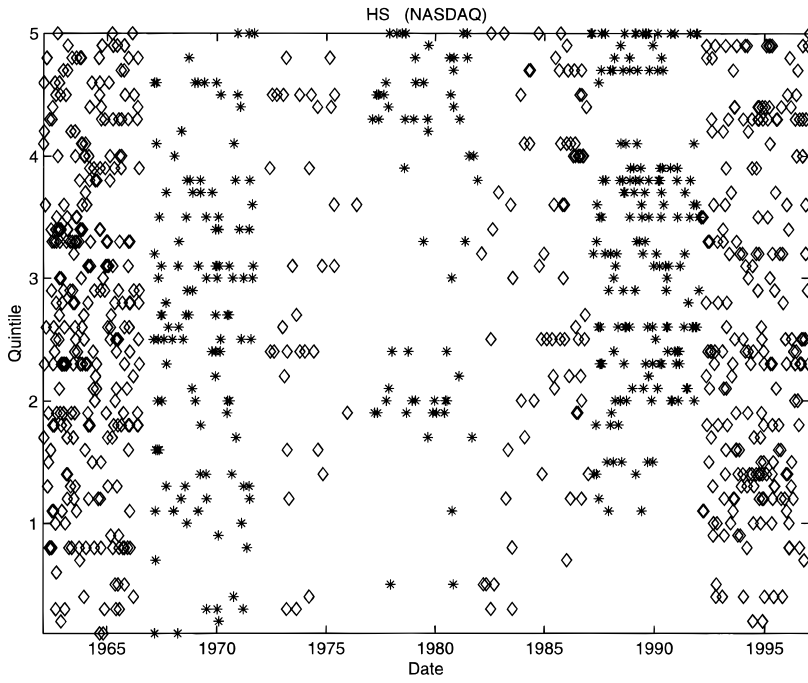


(i)

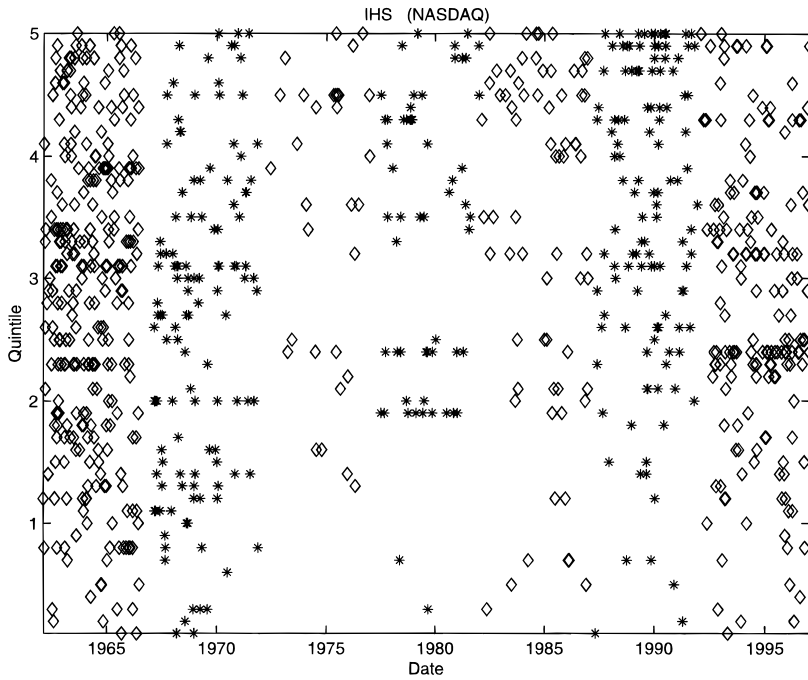


(j)

Figure 8. Continued

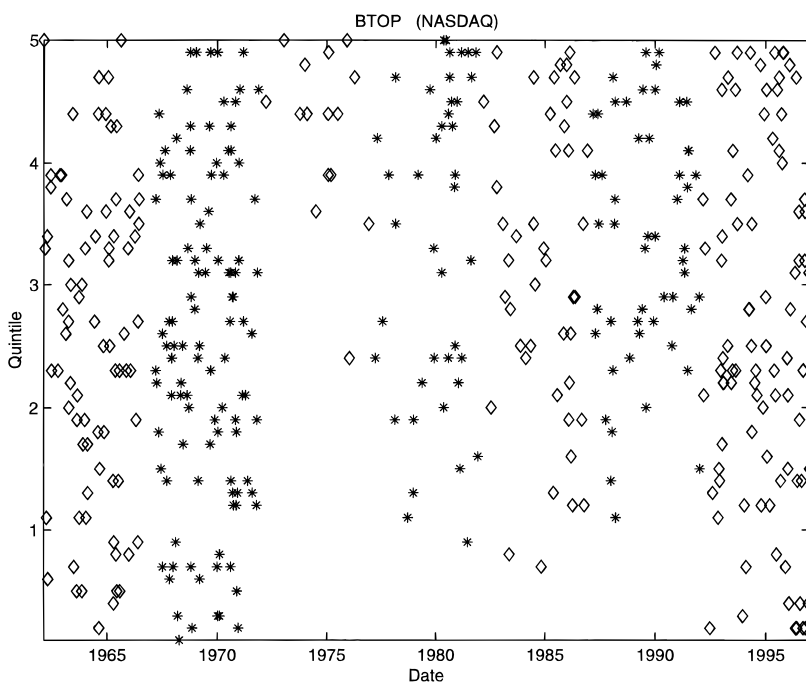


(a)

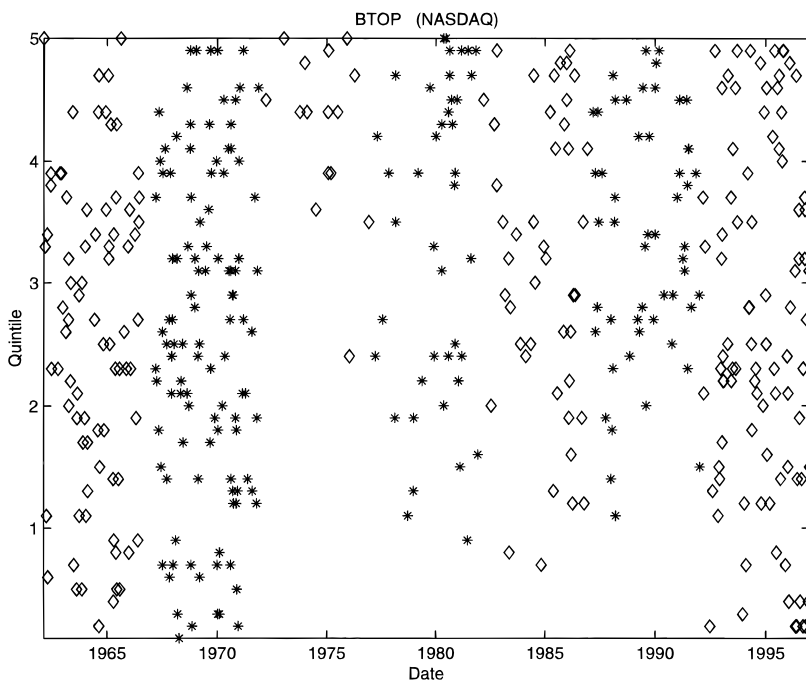


(b)

Figure 9. Distribution of patterns in Nasdaq sample.

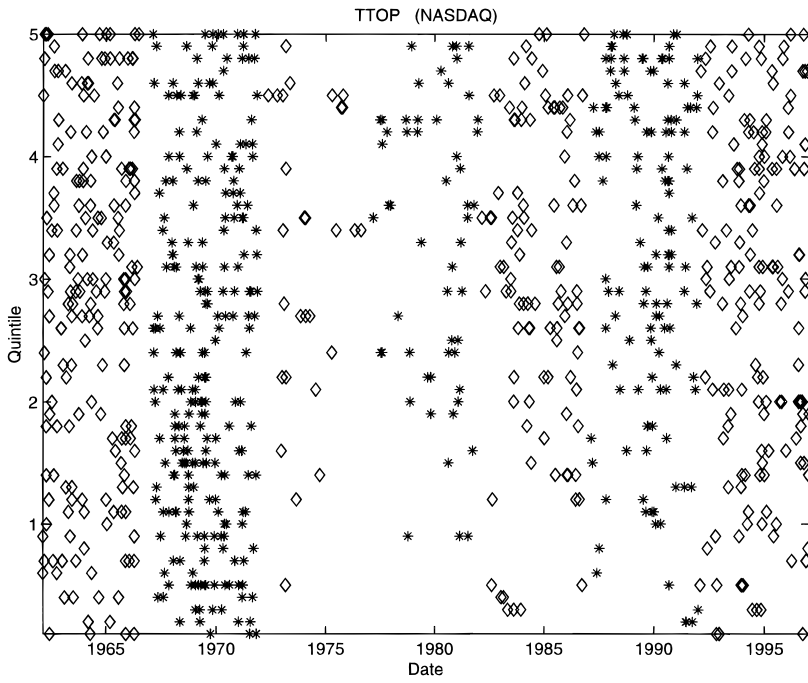


(c)

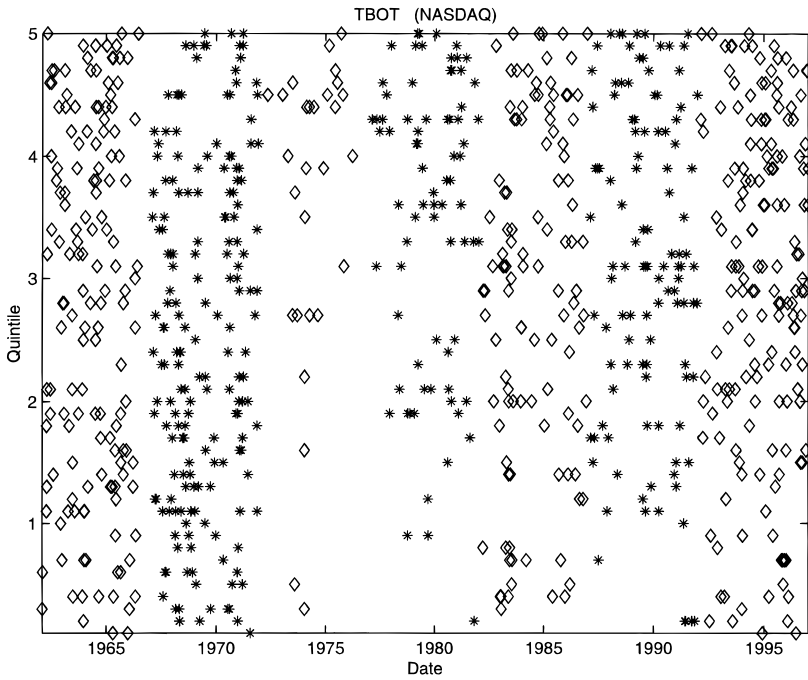


(d)

Figure 9. Continued

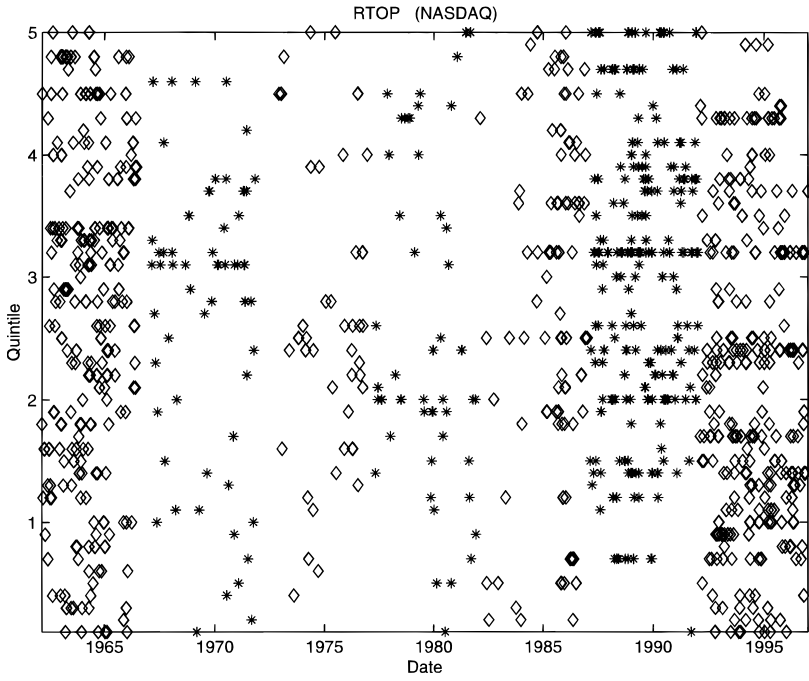


(e)

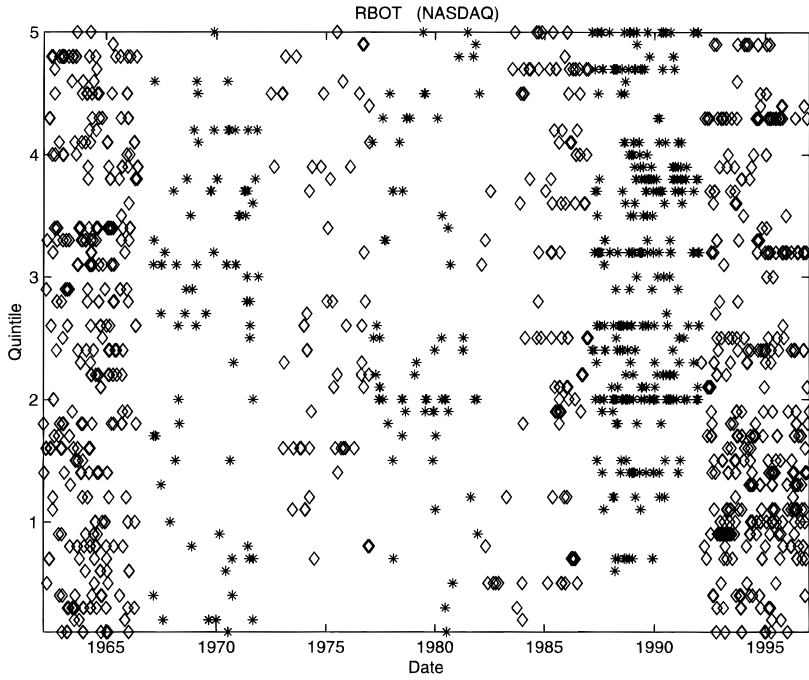


(f)

Figure 9. Continued

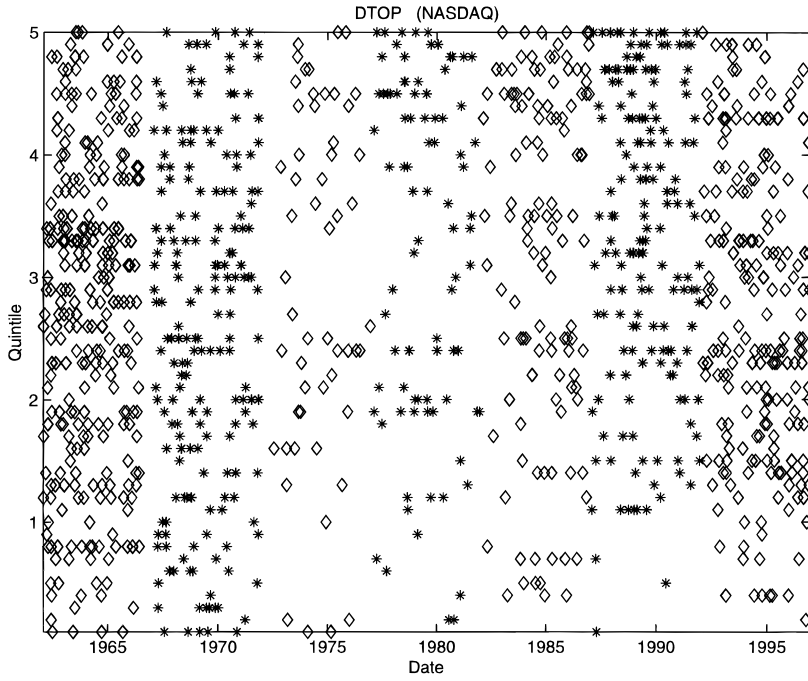


(g)

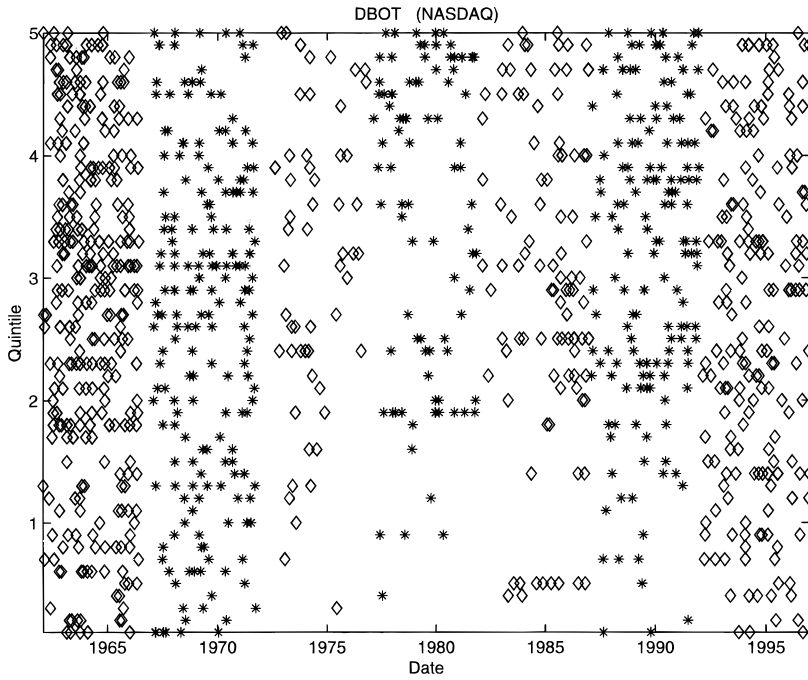


(h)

Figure 9. Continued



(i)



(j)

Figure 9. Continued

Table III

Summary statistics (mean, standard deviation, skewness, and excess kurtosis) of raw and conditional one-day normalized returns of NYSE/AMEX stocks from 1962 to 1996, in five-year subperiods, and in size quintiles. Conditional returns are defined as the daily return three days following the conclusion of an occurrence of one of 10 technical indicators: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT). All returns have been normalized by subtraction of their means and division by their standard deviations.

| Moment | Raw | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|--------|--------|--------|--------|--------|---------------------------------|--------|--------|--------|--------|--------|--------|
| Mean | -0.000 | -0.038 | 0.040 | -0.005 | All Stocks, 1962 to 1996 | | | | | | |
| S.D. | 1.000 | 0.867 | 0.937 | 1.035 | -0.062 | 0.021 | -0.009 | 0.009 | 0.014 | 0.017 | -0.001 |
| Skew. | 0.345 | 0.135 | 0.660 | -1.151 | 0.979 | 0.955 | 0.959 | 0.865 | 0.883 | 0.910 | 0.959 |
| Kurt. | 8.122 | 2.428 | 4.527 | 16.701 | 0.090 | 0.137 | 0.643 | -0.420 | 0.110 | 0.206 | 0.460 |
| Mean | -0.000 | -0.014 | 0.036 | -0.093 | Smallest Quintile, 1962 to 1996 | | | | | | |
| S.D. | 1.000 | 0.854 | 1.002 | 0.940 | -0.188 | 0.036 | -0.020 | 0.037 | -0.093 | 0.043 | -0.055 |
| Skew. | 0.697 | 0.802 | 1.337 | -1.771 | 0.850 | 0.937 | 1.157 | 0.833 | 0.986 | 0.950 | 0.962 |
| Kurt. | 10.873 | 3.870 | 7.143 | 6.701 | -0.367 | 0.861 | 2.592 | -0.187 | 0.445 | 0.511 | 0.002 |
| Mean | -0.000 | -0.069 | 0.144 | 0.061 | 2nd Quintile, 1962 to 1996 | | | | | | |
| S.D. | 1.000 | 0.772 | 1.031 | 1.278 | -0.113 | 0.003 | 0.035 | 0.018 | 0.019 | 0.067 | -0.011 |
| Skew. | 0.392 | 0.223 | 1.128 | -3.296 | 1.004 | 0.913 | 0.965 | 0.979 | 0.868 | 0.776 | 1.069 |
| Kurt. | 7.836 | 0.657 | 6.734 | 32.750 | 0.485 | -0.529 | 0.166 | -1.375 | 0.452 | 0.392 | 1.728 |
| Mean | -0.000 | -0.048 | -0.043 | -0.076 | 3rd Quintile, 1962 to 1996 | | | | | | |
| S.D. | 1.000 | 0.888 | 0.894 | 0.894 | -0.056 | 0.036 | 0.012 | 0.075 | 0.028 | -0.039 | -0.034 |
| Skew. | 0.246 | -0.465 | 0.107 | -0.023 | 0.925 | 0.973 | 0.796 | 0.798 | 0.892 | 0.956 | 1.026 |
| Kurt. | 7.466 | 3.239 | 1.612 | 1.024 | 0.233 | 0.538 | 0.166 | 0.678 | -0.618 | 0.013 | -0.242 |
| Mean | -0.000 | -0.012 | 0.022 | 0.115 | 4th Quintile, 1962 to 1996 | | | | | | |
| S.D. | 1.000 | 0.964 | 0.903 | 0.990 | -0.028 | 0.022 | -0.014 | -0.113 | 0.065 | 0.015 | -0.006 |
| Skew. | 0.222 | 0.055 | 0.592 | 0.458 | 1.093 | 0.986 | 0.959 | 0.854 | 0.821 | 0.858 | 0.992 |
| Kurt. | 6.452 | 1.444 | 1.745 | 1.251 | 0.537 | -0.217 | -0.456 | -0.415 | 0.820 | 0.550 | -0.062 |
| Mean | -0.000 | -0.038 | 0.054 | -0.081 | Largest Quintile, 1962 to 1996 | | | | | | |
| S.D. | 1.000 | 0.843 | 0.927 | 0.997 | -0.042 | 0.010 | -0.049 | 0.009 | 0.060 | 0.018 | 0.067 |
| Skew. | 0.174 | 0.182 | 0.438 | 0.470 | 0.951 | 0.964 | 0.965 | 0.850 | 0.820 | 0.971 | 0.941 |
| Kurt. | 7.992 | 2.621 | 3.465 | 3.275 | -1.099 | 0.089 | 0.357 | -0.167 | -0.140 | 0.011 | 0.511 |
| | | | | | 6.603 | 2.107 | 2.509 | 0.816 | 3.179 | 3.498 | 5.035 |

| | | | | | | | | | | |
|-------|--------|--------|--------|--------|--------------------------|--------|--------|--------|--------|--------|
| Mean | -0.000 | 0.070 | 0.090 | 0.159 | All Stocks, 1962 to 1966 | -0.039 | -0.041 | 0.019 | -0.071 | -0.100 |
| S.D. | 1.000 | 0.797 | 0.925 | 0.825 | 0.079 | -0.033 | 0.061 | 0.814 | 0.859 | 0.962 |
| Skew. | 0.563 | 0.159 | 0.462 | 0.363 | 1.085 | 1.068 | 1.151 | -0.341 | -0.427 | -0.876 |
| Kurt. | 9.161 | 0.612 | 1.728 | 0.657 | 5.063 | 2.674 | 4.826 | 1.400 | 3.416 | 5.622 |
| Mean | -0.000 | -0.044 | 0.079 | -0.035 | All Stocks, 1967 to 1971 | 0.057 | -0.101 | 0.110 | 0.093 | 0.079 |
| S.D. | 1.000 | 0.809 | 0.944 | 0.793 | -0.056 | 0.025 | 0.886 | 0.863 | 1.083 | 0.835 |
| Skew. | 0.342 | 0.754 | 0.666 | 0.304 | 0.850 | 0.885 | 0.697 | 0.395 | 1.360 | 0.701 |
| Kurt. | 5.810 | 3.684 | 2.725 | 0.706 | 0.085 | 0.650 | 1.659 | 3.254 | 4.487 | 1.853 |
| Mean | -0.000 | -0.035 | 0.043 | 0.101 | All Stocks, 1972 to 1976 | -0.010 | -0.025 | -0.003 | -0.051 | -0.108 |
| S.D. | 1.000 | 1.015 | 0.810 | 0.985 | -0.138 | -0.045 | 0.922 | 0.754 | 0.914 | 0.903 |
| Skew. | 0.316 | -0.334 | 0.717 | -0.699 | 0.918 | 0.945 | 0.676 | 0.199 | 0.056 | -0.366 |
| Kurt. | 6.520 | 2.286 | 1.565 | 6.562 | 0.272 | -1.014 | 4.912 | 2.337 | 3.520 | 5.047 |
| Mean | -0.000 | -0.138 | -0.040 | 0.076 | All Stocks, 1977 to 1981 | -0.050 | -0.004 | 0.026 | 0.042 | 0.178 |
| S.D. | 1.000 | 0.786 | 0.863 | 1.015 | -0.114 | 0.135 | 1.011 | 0.956 | 0.827 | 1.095 |
| Skew. | 0.466 | -0.304 | 0.052 | 1.599 | 0.989 | 1.041 | 0.110 | 0.534 | 0.761 | 2.214 |
| Kurt. | 6.419 | 1.132 | 1.048 | 4.961 | -0.033 | 0.776 | 0.989 | 2.184 | 2.369 | 15.290 |
| Mean | -0.000 | -0.099 | -0.007 | 0.011 | All Stocks, 1982 to 1986 | -0.067 | 0.050 | 0.005 | 0.011 | -0.013 |
| S.D. | 1.000 | 0.883 | 1.002 | 1.109 | 0.095 | -0.114 | 0.801 | 0.934 | 0.850 | 1.026 |
| Skew. | 0.460 | 0.464 | 0.441 | 0.372 | 0.956 | 0.924 | -1.249 | 0.467 | 0.528 | 0.867 |
| Kurt. | 6.799 | 2.280 | 6.128 | 2.566 | -0.165 | 0.473 | 5.278 | 4.294 | 1.515 | 7.400 |
| Mean | -0.000 | -0.037 | 0.033 | -0.091 | All Stocks, 1987 to 1991 | 0.003 | 0.040 | -0.020 | -0.022 | -0.017 |
| S.D. | 1.000 | 0.848 | 0.895 | 0.955 | -0.040 | 0.053 | 0.981 | 0.833 | 0.873 | 1.052 |
| Skew. | -0.018 | -0.526 | 0.272 | 0.108 | 0.818 | 0.857 | -1.216 | 0.124 | -1.184 | -0.368 |
| Kurt. | 13.478 | 3.835 | 4.395 | 2.247 | 0.231 | 0.165 | 9.586 | 3.973 | 4.808 | 4.297 |
| Mean | -0.000 | -0.014 | 0.069 | -0.231 | All Stocks, 1992 to 1996 | 0.041 | 0.082 | 0.011 | 0.102 | -0.016 |
| S.D. | 1.000 | 1.021 | 1.005 | 1.406 | -0.272 | 0.122 | 1.078 | 0.996 | 0.960 | 1.035 |
| Skew. | 0.308 | 0.545 | 1.305 | -0.502 | 1.187 | 0.953 | 2.460 | -0.129 | -0.091 | 0.379 |
| Kurt. | 8.683 | 2.249 | 6.684 | 27.022 | 3.947 | 1.235 | 12.883 | 6.399 | 1.507 | 3.358 |

Table IV

Summary statistics (mean, standard deviation, skewness, and excess kurtosis) of raw and conditional one-day normalized returns of Nasdaq stocks from 1962 to 1996, in five-year subperiods, and in size quintiles. Conditional returns are defined as the daily return three days following the conclusion of an occurrence of one of 10 technical indicators: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT). All returns have been normalized by subtraction of their means and division by their standard deviations.

| Moment | Raw | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Mean | 0.000 | -0.016 | 0.042 | -0.009 | 0.009 | 0.017 | 0.052 | 0.043 | 0.043 | 0.003 | -0.035 |
| S.D. | 1.000 | 0.907 | 0.994 | 0.960 | 0.995 | 0.984 | 0.948 | 0.929 | 0.948 | 0.933 | 0.880 |
| Skew. | 0.608 | -0.017 | 1.290 | 0.397 | 0.586 | 0.895 | 0.716 | 0.755 | 0.755 | 0.405 | -0.104 |
| Kurt. | 12.728 | 3.039 | 8.774 | 3.246 | 2.783 | 6.692 | 3.844 | 5.173 | 4.368 | 4.150 | 2.052 |
| Mean | -0.000 | 0.018 | -0.032 | 0.087 | -0.153 | 0.059 | 0.108 | 0.136 | 0.136 | 0.040 | 0.043 |
| S.D. | 1.000 | 0.845 | 1.319 | 0.874 | 0.894 | 1.113 | 1.044 | 1.187 | 0.982 | 0.773 | 0.906 |
| Skew. | 0.754 | 0.325 | 1.756 | -0.239 | -0.109 | 2.727 | 2.300 | 1.741 | 0.199 | 0.126 | -0.368 |
| Kurt. | 15.859 | 1.096 | 4.221 | 1.490 | 0.571 | 14.270 | 10.594 | 8.670 | 1.918 | 0.127 | 0.730 |
| Mean | -0.000 | -0.064 | 0.076 | -0.109 | -0.093 | -0.085 | -0.038 | -0.066 | -0.015 | 0.039 | -0.034 |
| S.D. | 1.000 | 0.848 | 0.991 | 1.106 | 1.026 | 0.805 | 0.997 | 0.898 | 0.897 | 1.119 | 0.821 |
| Skew. | 0.844 | 0.406 | 1.892 | -0.122 | 0.635 | 0.036 | 0.455 | -0.579 | 0.416 | 1.196 | 0.190 |
| Kurt. | 16.738 | 2.127 | 11.561 | 2.496 | 3.458 | 0.689 | 1.332 | 2.699 | 3.871 | 3.910 | 0.777 |
| Mean | -0.000 | 0.033 | 0.028 | 0.078 | 0.210 | -0.030 | 0.068 | 0.117 | 0.210 | -0.109 | -0.075 |
| S.D. | 1.000 | 0.933 | 0.906 | 0.931 | 0.971 | 0.825 | 1.002 | 0.992 | 0.970 | 0.997 | 0.973 |
| Skew. | 0.698 | 0.223 | 0.529 | 0.656 | 0.326 | 0.539 | 0.442 | 0.885 | 0.820 | -0.163 | 0.123 |
| Kurt. | 12.161 | 1.520 | 1.526 | 1.003 | 0.430 | 1.673 | 1.038 | 2.908 | 4.915 | 5.266 | 2.573 |
| Mean | 0.000 | -0.079 | 0.037 | -0.006 | -0.044 | -0.080 | 0.007 | 0.084 | 0.044 | 0.038 | -0.048 |
| S.D. | 1.000 | 0.911 | 0.957 | 0.992 | 0.975 | 1.076 | 0.824 | 0.890 | 0.851 | 0.857 | 0.819 |
| Skew. | 0.655 | -0.456 | 2.671 | -0.174 | 0.385 | 0.554 | 0.717 | 0.290 | 1.034 | 0.154 | -0.149 |
| Kurt. | 11.043 | 2.525 | 19.593 | 2.163 | 1.601 | 7.723 | 3.930 | 1.555 | 2.982 | 2.807 | 2.139 |
| Mean | 0.000 | 0.026 | 0.058 | -0.070 | 0.031 | 0.052 | -0.013 | 0.001 | -0.024 | 0.032 | -0.018 |
| S.D. | 1.000 | 0.952 | 1.002 | 0.895 | 1.060 | 1.076 | 0.871 | 0.794 | 0.958 | 0.844 | 0.877 |
| Skew. | 0.100 | -0.266 | -0.144 | 1.699 | 1.225 | 0.409 | 0.025 | 0.105 | 1.300 | 0.315 | -0.363 |
| Kurt. | 7.976 | 5.807 | 4.367 | 8.371 | 5.778 | 1.970 | 2.696 | 1.336 | 7.503 | 2.091 | 2.241 |

| | | | | | | | | | | |
|-------|--------|--------|--------|--------|--------------------------|--------|--------|--------|--------|--------|
| Mean | -0.000 | 0.116 | 0.041 | 0.099 | All Stocks, 1962 to 1966 | -0.066 | 0.100 | 0.010 | 0.096 | 0.027 |
| S.D. | 1.000 | 0.912 | 0.949 | 0.989 | 0.090 | 0.839 | 0.925 | 0.873 | 1.039 | 0.840 |
| Skew. | 0.575 | 0.711 | 1.794 | 0.252 | 1.039 | 0.247 | 2.016 | 1.021 | 0.533 | -0.351 |
| Kurt. | 6.555 | 1.538 | 9.115 | 2.560 | 1.258 | 1.324 | 13.653 | 5.603 | 6.277 | 2.243 |
| Mean | -0.000 | -0.127 | 0.114 | 0.121 | 6.445 | 0.077 | 0.154 | 0.136 | -0.000 | 0.006 |
| S.D. | 1.000 | 0.864 | 0.805 | 0.995 | All Stocks, 1967 to 1971 | 0.955 | 1.016 | 1.118 | 0.882 | 0.930 |
| Skew. | 0.734 | -0.097 | 1.080 | 0.574 | 0.016 | 0.843 | 0.810 | 1.925 | 0.465 | 0.431 |
| Kurt. | 5.194 | 1.060 | 2.509 | 0.380 | 1.013 | 1.908 | 1.712 | 5.815 | 1.585 | 2.476 |
| Mean | 0.000 | 0.014 | 0.089 | -0.403 | 2.928 | -0.422 | -0.076 | 0.108 | -0.004 | -0.163 |
| S.D. | 1.000 | 0.575 | 0.908 | 0.569 | All Stocks, 1972 to 1976 | 0.830 | 0.886 | 0.910 | 0.924 | 0.564 |
| Skew. | 0.466 | -0.281 | 0.973 | -1.176 | -0.034 | -1.503 | 2.047 | 2.047 | -0.551 | -0.791 |
| Kurt. | 17.228 | 2.194 | 1.828 | 0.077 | 0.803 | 2.137 | 13.320 | 9.510 | 1.434 | 2.010 |
| Mean | -0.000 | 0.025 | -0.212 | -0.112 | 0.046 | 0.086 | 0.055 | 0.177 | 0.081 | 0.040 |
| S.D. | 1.000 | 0.769 | 1.025 | 1.091 | 0.587 | 0.834 | 1.036 | 1.047 | 0.986 | 0.880 |
| Skew. | 1.092 | 0.230 | -1.516 | -0.731 | -0.056 | 0.249 | 2.391 | 2.571 | 1.520 | -0.291 |
| Kurt. | 20.043 | 1.618 | 4.397 | 3.766 | 0.368 | 4.722 | 9.137 | 10.961 | 7.127 | 3.682 |
| Mean | 0.000 | -0.147 | 0.204 | -0.137 | 0.460 | -0.022 | -0.028 | 0.116 | -0.224 | -0.052 |
| S.D. | 1.000 | 1.073 | 1.442 | 0.804 | All Stocks, 1977 to 1981 | 1.158 | 0.910 | 0.830 | 0.868 | 1.082 |
| Skew. | 1.267 | -1.400 | 2.192 | 0.001 | -0.001 | 1.690 | -0.120 | 0.048 | 0.001 | -0.091 |
| Kurt. | 21.789 | 4.899 | 10.530 | 0.863 | 1.040 | 7.086 | 0.780 | 0.444 | 1.174 | 0.818 |
| Mean | 0.000 | 0.012 | 0.120 | -0.080 | 0.732 | 0.038 | 0.098 | 0.049 | -0.048 | -0.122 |
| S.D. | 1.000 | 0.907 | 1.136 | 0.925 | All Stocks, 1982 to 1986 | 0.878 | 0.936 | 1.000 | 0.772 | 0.860 |
| Skew. | 0.104 | -0.326 | 0.976 | -0.342 | -0.001 | 1.002 | 0.233 | 0.023 | -0.105 | -0.375 |
| Kurt. | 12.688 | 3.922 | 5.183 | 1.839 | 1.040 | 2.768 | 1.038 | 2.350 | 0.313 | 2.598 |
| Mean | 0.000 | -0.119 | -0.058 | -0.033 | 0.734 | 0.086 | -0.006 | -0.011 | 0.003 | -0.105 |
| S.D. | 1.000 | 0.854 | 0.964 | 1.106 | All Stocks, 1987 to 1991 | 0.901 | 0.973 | 0.879 | 0.932 | 0.875 |
| Skew. | -0.036 | 0.079 | -0.015 | 1.399 | -0.031 | 0.150 | 0.283 | 0.236 | 0.039 | -0.097 |
| Kurt. | 5.377 | 2.818 | -0.059 | 7.584 | -0.052 | 1.040 | 1.266 | 1.445 | 1.583 | 0.205 |

Table V

Goodness-of-fit diagnostics for the conditional one-day normalized returns, conditional on 10 technical indicators, for a sample of 350 NYSE/AMEX stocks from 1962 to 1996 (10 stocks per size-quintile with at least 80% nonmissing prices are randomly chosen in each five-year subperiod, yielding 50 stocks per subperiod over seven subperiods). For each pattern, the percentage of conditional returns that falls within each of the 10 unconditional return deciles is tabulated. If conditioning on the pattern provides no information, the expected percentage falling in each decile is 10%. Asymptotic z -statistics for this null hypothesis are reported in parentheses, and the χ^2 goodness-of-fitness test statistic Q is reported in the last column with the p -value in parentheses below the statistic. The 10 technical indicators are as follows: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT).

| Pattern | Decile: | | | | | | | | | | Q (p -Value) |
|---------|----------------|----------------|-----------------|----------------|----------------|----------------|-----------------|-----------------|----------------|-----------------|----------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| HS | 8.9 (-1.49) | 10.4 (0.56) | 11.2 (1.49) | 11.7 (2.16) | 12.2 (2.73) | 7.9 (-3.05) | 9.2 (-1.04) | 10.4 (0.48) | 10.8 (1.04) | 7.1 (-4.46) | 39.31 (0.000) |
| IHS | 8.6 (-2.05) | 9.7 (-0.36) | 9.4 (-0.88) | 11.2 (1.60) | 13.7 (4.34) | 7.7 (-3.44) | 9.1 (-1.32) | 11.1 (1.38) | 9.6 (-0.62) | 10.0 (-0.03) | 40.95 (0.000) |
| BTOP | 9.4 (-0.57) | 10.6 (0.54) | 10.6 (0.54) | 11.9 (1.55) | 8.7 (-1.25) | 6.6 (-3.66) | 9.2 (-0.71) | 13.7 (2.87) | 9.2 (-0.71) | 10.1 (0.06) | 23.40 (0.005) |
| BBOT | 11.5 (1.28) | 9.9 (-0.10) | 13.0 (2.42) | 11.1 (0.95) | 7.8 (-2.30) | 9.2 (-0.73) | 8.3 (-1.70) | 9.0 (-1.00) | 10.7 (0.62) | 9.6 (-0.35) | 16.87 (0.051) |
| TTOP | 7.8 (-2.94) | 10.4 (0.42) | 10.9 (1.03) | 11.3 (1.46) | 9.0 (-1.30) | 9.9 (-0.13) | 10.0 (-0.04) | 10.7 (0.77) | 10.5 (0.60) | 9.7 (-0.41) | 12.03 (0.212) |
| TBOT | 8.9 (-1.35) | 10.6 (0.72) | 10.9 (0.99) | 12.2 (2.36) | 9.2 (-0.93) | 8.7 (-1.57) | 9.3 (-0.83) | 11.6 (1.69) | 8.7 (-1.57) | 9.8 (-0.22) | 17.12 (0.047) |
| RTOP | 8.4 (-2.27) | 9.9 (-0.10) | 9.2 (-1.10) | 10.5 (0.58) | 12.5 (2.89) | 10.1 (0.16) | 10.0 (-0.02) | 10.0 (-0.02) | 11.4 (1.70) | 8.1 (-2.69) | 22.72 (0.007) |
| RBOT | 8.6 (-2.01) | 9.6 (-0.56) | 7.8 (-3.30) | 10.5 (0.60) | 12.9 (3.45) | 10.8 (1.07) | 11.6 (1.98) | 9.3 (-0.99) | 10.3 (0.44) | 8.7 (-1.91) | 33.94 (0.000) |
| DTOP | 8.2 (-2.92) | 10.9 (1.36) | 9.6 (-0.64) | 12.4 (3.29) | 11.8 (2.61) | 7.5 (-4.39) | 8.2 (-2.92) | 11.3 (1.83) | 10.3 (0.46) | 9.7 (-0.41) | 50.97 (0.000) |
| DBOT | 9.7 (-0.48) | 9.9 (-0.18) | 10.0 (-0.04) | 10.9 (1.37) | 11.4 (1.97) | 8.5 (-2.40) | 9.2 (-1.33) | 10.0 (0.04) | 10.7 (0.96) | 9.8 (-0.33) | 12.92 (0.166) |

Table VI

Goodness-of-fit diagnostics for the conditional one-day normalized returns, conditional on 10 technical indicators, for a sample of 350 Nasdaq stocks from 1962 to 1996 (10 stocks per size-quintile with at least 80% nonmissing prices are randomly chosen in each five-year subperiod, yielding 50 stocks per subperiod over seven subperiods). For each pattern, the percentage of conditional returns that falls within each of the 10 unconditional return deciles is tabulated. If conditioning on the pattern provides no information, the expected percentage falling in each decile is 10%. Asymptotic z -statistics for this null hypothesis are reported in parentheses, and the χ^2 goodness-of-fitness test statistic Q is reported in the last column with the p -value in parentheses below the statistic. The 10 technical indicators are as follows: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT).

| Pattern | Decile: | | | | | | | | | | Q (p -Value) |
|---------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| HS | 10.8 (0.76) | 10.8 (0.76) | 13.7 (3.27) | 8.6 (-1.52) | 8.5 (-1.65) | 6.0 (-5.13) | 6.0 (-5.13) | 12.5 (2.30) | 13.5 (3.10) | 9.7 (-0.32) | 64.41 (0.000) |
| IHS | 9.4 (-0.56) | 14.1 (3.35) | 12.5 (2.15) | 8.0 (-2.16) | 7.7 (-2.45) | 4.8 (-7.01) | 6.4 (-4.26) | 13.5 (2.90) | 12.5 (2.15) | 11.3 (1.14) | 75.84 (0.000) |
| BTOP | 11.6 (1.01) | 12.3 (1.44) | 12.8 (1.71) | 7.7 (-1.73) | 8.2 (-1.32) | 6.8 (-2.62) | 4.3 (-5.64) | 13.3 (1.97) | 12.1 (1.30) | 10.9 (0.57) | 34.12 (0.000) |
| BBOT | 11.4 (1.00) | 11.4 (1.00) | 14.8 (3.03) | 5.9 (-3.91) | 6.7 (-2.98) | 9.6 (-0.27) | 5.7 (-4.17) | 11.4 (1.00) | 9.8 (-0.12) | 13.2 (2.12) | 43.26 (0.000) |
| TTOP | 10.7 (0.67) | 12.1 (1.89) | 16.2 (4.93) | 6.2 (-4.54) | 7.9 (-2.29) | 8.7 (-1.34) | 4.0 (-8.93) | 12.5 (2.18) | 11.4 (1.29) | 10.2 (0.23) | 92.09 (0.000) |
| TBOT | 9.9 (-0.11) | 11.3 (1.14) | 15.6 (4.33) | 7.9 (-2.24) | 7.7 (-2.39) | 5.7 (-5.20) | 5.3 (-5.85) | 14.6 (3.64) | 12.0 (1.76) | 10.0 (0.01) | 85.26 (0.000) |
| RTOP | 11.2 (1.28) | 10.8 (0.92) | 8.8 (-1.40) | 8.3 (-2.09) | 10.2 (0.25) | 7.1 (-3.87) | 7.7 (-2.95) | 9.3 (-0.75) | 15.3 (4.92) | 11.3 (1.37) | 57.08 (0.000) |
| RBOT | 8.9 (-1.35) | 12.3 (2.52) | 8.9 (-1.35) | 8.9 (-1.45) | 11.6 (1.81) | 8.9 (-1.35) | 7.0 (-4.19) | 9.5 (-0.66) | 13.6 (3.85) | 10.3 (0.36) | 45.79 (0.000) |
| DTOP | 11.0 (1.12) | 12.6 (2.71) | 11.7 (1.81) | 9.0 (-1.18) | 9.2 (-0.98) | 5.5 (-6.76) | 5.8 (-6.26) | 11.6 (1.73) | 12.3 (2.39) | 11.3 (1.47) | 71.29 (0.000) |
| DBOT | 10.9 (0.98) | 11.5 (1.60) | 13.1 (3.09) | 8.0 (-2.47) | 8.1 (-2.35) | 7.1 (-3.75) | 7.6 (-3.09) | 11.5 (1.60) | 12.8 (2.85) | 9.3 (-0.78) | 51.23 (0.000) |

in the Nasdaq sample, with p -values that are zero to three significant digits and test statistics Q that range from 34.12 to 92.09. In contrast, the test statistics in Table V range from 12.03 to 50.97.

One possible explanation for the difference between the NYSE/AMEX and Nasdaq samples is a difference in the power of the test because of different sample sizes. If the NYSE/AMEX sample contained fewer conditional returns, that is, fewer patterns, the corresponding test statistics might be subject to greater sampling variation and lower power. However, this explanation can be ruled out from the frequency counts of Tables I and II—the number of patterns in the NYSE/AMEX sample is considerably larger than those of the Nasdaq sample for all 10 patterns. Tables V and VI seem to suggest important differences in the informativeness of technical indicators for NYSE/AMEX and Nasdaq stocks.

Table VII and VIII report the results of the Kolmogorov–Smirnov test (equation (19)) of the equality of the conditional and unconditional return distributions for NYSE/AMEX (Table VII) and Nasdaq (Table VIII) stocks, respectively, from 1962 to 1996, in five-year subperiods and in market-capitalization quintiles. Recall that conditional returns are defined as the one-day return starting three days following the conclusion of an occurrence of a pattern. The p -values are with respect to the asymptotic distribution of the Kolmogorov–Smirnov test statistic given in equation (20). Table VII shows that for NYSE/AMEX stocks, five of the 10 patterns—HS, BBOT, RTOP, RBOT, and DTOPT—yield statistically significant test statistics, with p -values ranging from 0.000 for RBOT to 0.021 for DTOPT patterns. However, for the other five patterns, the p -values range from 0.104 for IHS to 0.393 for TTOP, which implies an inability to distinguish between the conditional and unconditional distributions of normalized returns.

When we also condition on declining volume trend, the statistical significance declines for most patterns, but the statistical significance of TBOT patterns increases. In contrast, conditioning on increasing volume trend yields an increase in the statistical significance of BTOP patterns. This difference may suggest an important role for volume trend in TBOT and BTOP patterns. The difference between the increasing and decreasing volume-trend conditional distributions is statistically insignificant for almost all the patterns (the sole exception is the TBOT pattern). This drop in statistical significance may be due to a lack of power of the Kolmogorov–Smirnov test given the relatively small sample sizes of these conditional returns (see Table I for frequency counts).

Table VIII reports corresponding results for the Nasdaq sample, and as in Table VI, in contrast to the NYSE/AMEX results, here all the patterns are statistically significant at the 5 percent level. This is especially significant because the the Nasdaq sample exhibits far fewer patterns than the NYSE/AMEX sample (see Tables I and II), and hence the Kolmogorov–Smirnov test is likely to have lower power in this case.

As with the NYSE/AMEX sample, volume trend seems to provide little incremental information for the Nasdaq sample except in one case: increasing volume and BTOP. And except for the TTOP pattern, the Kolmogorov–

Smirnov test still cannot distinguish between the decreasing and increasing volume-trend conditional distributions, as the last pair of rows of Table VIII's first panel indicates.

IV. Monte Carlo Analysis

Tables IX and X contain bootstrap percentiles for the Kolmogorov–Smirnov test of the equality of conditional and unconditional one-day return distributions for NYSE/AMEX and Nasdaq stocks, respectively, from 1962 to 1996, for five-year subperiods, and for market-capitalization quintiles, under the null hypothesis of equality. For each of the two sets of market data, two sample sizes, m_1 and m_2 , have been chosen to span the range of frequency counts of patterns reported in Tables I and II. For each sample size m_i , we resample one-day normalized returns (with replacement) to obtain a bootstrap sample of m_i observations, compute the Kolmogorov–Smirnov test statistic (against the entire sample of one-day normalized returns), and repeat this procedure 1,000 times. The percentiles of the asymptotic distribution are also reported for comparison in the column labeled “ Δ ”.

Tables IX and X show that for a broad range of sample sizes and across size quintiles, subperiod, and exchanges, the bootstrap distribution of the Kolmogorov–Smirnov statistic is well approximated by its asymptotic distribution, equation (20).

V. Conclusion

In this paper, we have proposed a new approach to evaluating the efficacy of technical analysis. Based on smoothing techniques such as nonparametric kernel regression, our approach incorporates the essence of technical analysis: to identify regularities in the time series of prices by extracting nonlinear patterns from noisy data. Although human judgment is still superior to most computational algorithms in the area of visual pattern recognition, recent advances in statistical learning theory have had successful applications in fingerprint identification, handwriting analysis, and face recognition. Technical analysis may well be the next frontier for such methods.

We find that certain technical patterns, when applied to many stocks over many time periods, do provide incremental information, especially for Nasdaq stocks. Although this does not necessarily imply that technical analysis can be used to generate “excess” trading profits, it does raise the possibility that technical analysis can add value to the investment process.

Moreover, our methods suggest that technical analysis can be improved by using automated algorithms such as ours and that traditional patterns such as head-and-shoulders and rectangles, although sometimes effective, need not be optimal. In particular, it may be possible to determine “optimal patterns” for detecting certain types of phenomena in financial time series, for example, an optimal shape for detecting stochastic volatility or changes in regime. Moreover, patterns that are optimal for detecting statistical anomalies need not be optimal for trading profits, and vice versa. Such consider-

Table VII

Kolmogorov-Smirnov test of the equality of conditional and unconditional one-day return distributions for NYSE/AMEX stocks from 1962 to 1996, in five-year subperiods, and in size quintiles. Conditional returns are defined as the daily return three days following the conclusion of an occurrence of one of 10 technical indicators: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT). All returns have been normalized by subtraction of their means and division by their standard deviations. p -values are with respect to the asymptotic distribution of the Kolmogorov-Smirnov test statistic. The symbols " $\tau(\sphericalangle)$ " and " $\tau(\sphericalcap)$ " indicate that the conditional distribution is also conditioned on decreasing and increasing volume trend, respectively.

| Statistic | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|--------------------------------|---------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| γ | 1.89 | 1.22 | 1.15 | 1.76 | 0.90 | 1.09 | 1.84 | 2.45 | 1.51 | 1.06 |
| p -value | 0.002 | 0.104 | 0.139 | 0.004 | 0.393 | 0.185 | 0.002 | 0.000 | 0.021 | 0.215 |
| $\gamma \tau(\sphericalcap)$ | 1.49 | 0.95 | 0.44 | 0.62 | 0.73 | 1.33 | 1.37 | 1.77 | 0.96 | 0.78 |
| p -value | 0.024 | 0.327 | 0.989 | 0.839 | 0.657 | 0.059 | 0.047 | 0.004 | 0.319 | 0.579 |
| $\gamma \tau(\sphericalangle)$ | 0.72 | 1.05 | 1.33 | 1.59 | 0.92 | 1.29 | 1.13 | 1.24 | 0.74 | 0.84 |
| p -value | 0.671 | 0.220 | 0.059 | 0.013 | 0.368 | 0.073 | 0.156 | 0.090 | 0.638 | 0.481 |
| γ Diff. | 0.88 | 0.54 | 0.59 | 0.94 | 0.75 | 1.37 | 0.79 | 1.20 | 0.82 | 0.71 |
| p -value | 0.418 | 0.935 | 0.879 | 0.342 | 0.628 | 0.046 | 0.557 | 0.111 | 0.512 | 0.698 |
| | Smallest Quintile, 1962 to 1996 | | | | | | | | | |
| γ | 0.59 | 1.19 | 0.72 | 1.20 | 0.98 | 1.43 | 1.09 | 1.19 | 0.84 | 0.78 |
| p -value | 0.872 | 0.116 | 0.679 | 0.114 | 0.290 | 0.033 | 0.188 | 0.120 | 0.485 | 0.583 |
| $\gamma \tau(\sphericalcap)$ | 0.67 | 0.80 | 1.16 | 0.69 | 1.00 | 1.46 | 1.31 | 0.94 | 1.12 | 0.73 |
| p -value | 0.765 | 0.540 | 0.136 | 0.723 | 0.271 | 0.029 | 0.065 | 0.339 | 0.165 | 0.663 |
| $\gamma \tau(\sphericalangle)$ | 0.43 | 0.95 | 0.67 | 1.03 | 0.47 | 0.88 | 0.51 | 0.93 | 0.94 | 0.58 |
| p -value | 0.994 | 0.325 | 0.756 | 0.236 | 0.981 | 0.423 | 0.959 | 0.356 | 0.342 | 0.892 |
| γ Diff. | 0.52 | 0.48 | 1.14 | 0.68 | 0.48 | 0.98 | 0.98 | 0.79 | 1.16 | 0.62 |
| p -value | 0.951 | 0.974 | 0.151 | 0.741 | 0.976 | 0.291 | 0.294 | 0.552 | 0.133 | 0.840 |
| | 2nd Quintile, 1962 to 1996 | | | | | | | | | |
| γ | 1.82 | 1.63 | 0.93 | 0.92 | 0.82 | 0.84 | 0.88 | 1.29 | 1.46 | 0.84 |
| p -value | 0.003 | 0.010 | 0.353 | 0.365 | 0.505 | 0.485 | 0.417 | 0.073 | 0.029 | 0.478 |
| $\gamma \tau(\sphericalcap)$ | 1.62 | 1.03 | 0.88 | 0.42 | 0.91 | 0.90 | 0.71 | 0.86 | 1.50 | 0.97 |
| p -value | 0.010 | 0.242 | 0.427 | 0.994 | 0.378 | 0.394 | 0.703 | 0.443 | 0.022 | 0.298 |
| $\gamma \tau(\sphericalangle)$ | 1.06 | 1.63 | 0.96 | 0.83 | 0.89 | 0.98 | 1.19 | 1.15 | 0.96 | 0.99 |
| p -value | 0.213 | 0.010 | 0.317 | 0.497 | 0.407 | 0.289 | 0.119 | 0.141 | 0.317 | 0.286 |
| γ Diff. | 0.78 | 0.94 | 1.04 | 0.71 | 1.22 | 0.92 | 0.99 | 0.79 | 1.18 | 0.68 |
| p -value | 0.576 | 0.334 | 0.228 | 0.687 | 0.102 | 0.361 | 0.276 | 0.564 | 0.126 | 0.745 |

| | | | | | | | | | | |
|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| γ | 0.83 | 1.56 | 1.00 | 1.28 | 0.57 | 1.03 | 1.96 | 1.50 | 1.55 | 1.14 |
| p -value | 0.502 | 0.016 | 0.266 | 0.074 | 0.903 | 0.243 | 0.001 | 0.023 | 0.016 | 0.150 |
| $\gamma \tau(\searrow)$ | 0.95 | 0.94 | 0.66 | 0.76 | 0.61 | 0.82 | 1.45 | 1.61 | 1.17 | 1.01 |
| p -value | 0.326 | 0.346 | 0.775 | 0.613 | 0.854 | 0.520 | 0.031 | 0.012 | 0.131 | 0.258 |
| $\gamma \tau(\nearrow)$ | 1.05 | 1.43 | 0.93 | 1.14 | 0.63 | 0.80 | 0.93 | 0.78 | 0.59 | 0.86 |
| p -value | 0.223 | 0.033 | 0.350 | 0.147 | 0.826 | 0.544 | 0.354 | 0.578 | 0.878 | 0.450 |
| γ Diff. | 1.02 | 1.14 | 0.45 | 0.48 | 0.50 | 0.89 | 0.66 | 0.91 | 0.72 | 1.15 |
| p -value | 0.246 | 0.148 | 0.986 | 0.974 | 0.964 | 0.413 | 0.774 | 0.383 | 0.670 | 0.143 |
| 4th Quintile, 1962 to 1996 | | | | | | | | | | |
| γ | 0.72 | 0.61 | 1.29 | 0.84 | 0.61 | 0.84 | 1.37 | 1.37 | 0.72 | 0.53 |
| p -value | 0.683 | 0.852 | 0.071 | 0.479 | 0.855 | 0.480 | 0.048 | 0.047 | 0.682 | 0.943 |
| $\gamma \tau(\searrow)$ | 1.01 | 0.95 | 0.83 | 0.96 | 0.78 | 0.84 | 1.34 | 0.72 | 0.62 | 1.01 |
| p -value | 0.255 | 0.330 | 0.504 | 0.311 | 0.585 | 0.487 | 0.056 | 0.680 | 0.841 | 0.258 |
| $\gamma \tau(\nearrow)$ | 0.93 | 0.66 | 1.29 | 0.96 | 1.16 | 0.69 | 0.64 | 1.16 | 0.69 | 0.85 |
| p -value | 0.349 | 0.772 | 0.072 | 0.316 | 0.137 | 0.731 | 0.810 | 0.136 | 0.720 | 0.468 |
| γ Diff. | 1.10 | 0.97 | 0.64 | 1.16 | 1.31 | 0.78 | 0.64 | 0.92 | 0.66 | 1.10 |
| p -value | 0.175 | 0.301 | 0.804 | 0.138 | 0.065 | 0.571 | 0.806 | 0.363 | 0.780 | 0.176 |
| Largest Quintile, 1962 to 1996 | | | | | | | | | | |
| γ | 1.25 | 1.16 | 0.98 | 0.48 | 0.50 | 0.80 | 0.94 | 1.76 | 0.90 | 1.28 |
| p -value | 0.088 | 0.136 | 0.287 | 0.977 | 0.964 | 0.544 | 0.346 | 0.004 | 0.395 | 0.077 |
| $\gamma \tau(\searrow)$ | 1.12 | 0.90 | 0.57 | 0.78 | 0.64 | 1.17 | 0.91 | 0.87 | 0.64 | 1.20 |
| p -value | 0.164 | 0.386 | 0.906 | 0.580 | 0.806 | 0.127 | 0.379 | 0.442 | 0.802 | 0.114 |
| $\gamma \tau(\nearrow)$ | 0.81 | 0.93 | 0.83 | 0.61 | 0.69 | 0.81 | 0.73 | 0.87 | 0.46 | 0.88 |
| p -value | 0.522 | 0.350 | 0.495 | 0.854 | 0.729 | 0.532 | 0.661 | 0.432 | 0.982 | 0.418 |
| γ Diff. | 0.71 | 0.54 | 0.59 | 0.64 | 0.76 | 1.21 | 0.85 | 1.11 | 0.54 | 0.79 |
| p -value | 0.699 | 0.934 | 0.874 | 0.800 | 0.607 | 0.110 | 0.467 | 0.170 | 0.929 | 0.552 |
| All Stocks, 1962 to 1966 | | | | | | | | | | |
| γ | 1.29 | 1.67 | 1.07 | 0.72 | 0.75 | 1.32 | 1.20 | 1.53 | 2.04 | 1.73 |
| p -value | 0.072 | 0.007 | 0.202 | 0.671 | 0.634 | 0.062 | 0.112 | 0.018 | 0.001 | 0.005 |
| $\gamma \tau(\searrow)$ | 0.83 | 1.01 | 1.04 | 0.80 | 0.63 | 1.80 | 0.66 | 1.84 | 1.03 | 1.54 |
| p -value | 0.499 | 0.260 | 0.232 | 0.539 | 0.826 | 0.003 | 0.771 | 0.002 | 0.244 | 0.017 |
| $\gamma \tau(\nearrow)$ | 1.13 | 1.13 | 0.84 | 0.84 | 0.58 | 1.40 | 1.12 | 0.83 | 1.09 | 1.16 |
| p -value | 0.156 | 0.153 | 0.480 | 0.475 | 0.894 | 0.040 | 0.163 | 0.492 | 0.183 | 0.135 |
| γ Diff. | 0.65 | 0.71 | 0.75 | 0.76 | 0.60 | 1.90 | 0.68 | 1.35 | 0.73 | 0.83 |
| p -value | 0.799 | 0.691 | 0.629 | 0.615 | 0.863 | 0.001 | 0.741 | 0.052 | 0.657 | 0.503 |

continued

Table VII—Continued

| Statistic | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|-------------------------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | All Stocks, 1967 to 1971 | | | | | | | | | |
| γ | 1.10 | 0.96 | 0.60 | 0.65 | 0.98 | 0.76 | 1.29 | 1.65 | 0.87 | 1.22 |
| p -value | 0.177 | 0.317 | 0.867 | 0.797 | 0.292 | 0.606 | 0.071 | 0.009 | 0.436 | 0.101 |
| $\gamma \tau(\searrow)$ | 1.02 | 0.80 | 0.53 | 0.85 | 0.97 | 0.77 | 0.71 | 1.42 | 0.97 | 1.06 |
| p -value | 0.248 | 0.551 | 0.943 | 0.464 | 0.303 | 0.590 | 0.700 | 0.035 | 0.300 | 0.214 |
| $\gamma \tau(\nearrow)$ | 1.08 | 0.86 | 0.68 | 0.91 | 1.11 | 0.82 | 0.79 | 0.73 | 0.71 | 0.96 |
| p -value | 0.190 | 0.454 | 0.750 | 0.373 | 0.169 | 0.508 | 0.554 | 0.660 | 0.699 | 0.315 |
| γ Diff. | 1.36 | 0.51 | 0.53 | 0.76 | 0.68 | 0.71 | 0.71 | 0.98 | 1.06 | 1.12 |
| p -value | 0.049 | 0.956 | 0.942 | 0.616 | 0.751 | 0.699 | 0.701 | 0.290 | 0.210 | 0.163 |
| | All Stocks, 1972 to 1976 | | | | | | | | | |
| γ | 0.47 | 0.75 | 0.87 | 1.56 | 1.21 | 0.75 | 0.87 | 0.94 | 1.64 | 1.20 |
| p -value | 0.980 | 0.620 | 0.441 | 0.015 | 0.106 | 0.627 | 0.441 | 0.341 | 0.009 | 0.113 |
| $\gamma \tau(\searrow)$ | 0.80 | 0.40 | 0.50 | 1.24 | 1.21 | 0.65 | 1.26 | 0.63 | 0.70 | 1.39 |
| p -value | 0.539 | 0.998 | 0.966 | 0.093 | 0.106 | 0.794 | 0.084 | 0.821 | 0.718 | 0.041 |
| $\gamma \tau(\nearrow)$ | 0.49 | 0.78 | 0.94 | 1.21 | 1.12 | 1.03 | 0.81 | 0.95 | 0.84 | 0.70 |
| p -value | 0.970 | 0.577 | 0.340 | 0.108 | 0.159 | 0.244 | 0.521 | 0.331 | 0.485 | 0.719 |
| γ Diff. | 0.55 | 0.56 | 0.51 | 0.95 | 0.81 | 1.11 | 1.15 | 0.62 | 0.67 | 1.31 |
| p -value | 0.925 | 0.915 | 0.960 | 0.333 | 0.525 | 0.170 | 0.141 | 0.836 | 0.767 | 0.065 |
| | All Stocks, 1977 to 1981 | | | | | | | | | |
| γ | 1.16 | 0.73 | 0.76 | 1.16 | 0.82 | 1.14 | 1.01 | 0.87 | 0.86 | 1.79 |
| p -value | 0.138 | 0.665 | 0.617 | 0.136 | 0.506 | 0.147 | 0.263 | 0.428 | 0.449 | 0.003 |
| $\gamma \tau(\searrow)$ | 1.04 | 0.73 | 1.00 | 1.31 | 1.10 | 1.32 | 0.83 | 0.80 | 1.20 | 1.81 |
| p -value | 0.228 | 0.654 | 0.274 | 0.065 | 0.176 | 0.062 | 0.494 | 0.550 | 0.113 | 0.003 |
| $\gamma \tau(\nearrow)$ | 0.75 | 0.84 | 0.88 | 0.65 | 0.67 | 0.76 | 1.51 | 1.41 | 0.86 | 0.99 |
| p -value | 0.623 | 0.476 | 0.426 | 0.799 | 0.754 | 0.602 | 0.020 | 0.450 | 0.450 | 0.280 |
| γ Diff. | 0.67 | 0.94 | 0.88 | 0.70 | 0.65 | 0.70 | 1.11 | 1.29 | 1.16 | 0.70 |
| p -value | 0.767 | 0.335 | 0.423 | 0.708 | 0.785 | 0.716 | 0.172 | 0.073 | 0.137 | 0.713 |

| | | | | | | | | | | |
|-------------------------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | All Stocks, 1982 to 1986 | | | | | | | | | |
| γ | 1.57 | 0.99 | 0.59 | 1.46 | 1.47 | 1.04 | 0.87 | 0.68 | 0.76 | 0.90 |
| p-value | 0.015 | 0.276 | 0.883 | 0.029 | 0.027 | 0.232 | 0.431 | 0.742 | 0.617 | 0.387 |
| $\gamma \tau(\searrow)$ | 1.17 | 0.68 | 0.44 | 1.30 | 1.53 | 1.21 | 1.08 | 0.93 | 0.84 | 0.88 |
| p-value | 0.129 | 0.741 | 0.991 | 0.070 | 0.018 | 0.106 | 0.190 | 0.356 | 0.478 | 0.421 |
| $\gamma \tau(\nearrow)$ | 0.81 | 1.03 | 0.74 | 0.62 | 0.83 | 1.23 | 0.77 | 0.79 | 0.63 | 0.81 |
| p-value | 0.533 | 0.243 | 0.640 | 0.831 | 0.499 | 0.097 | 0.597 | 0.564 | 0.821 | 0.528 |
| γ Diff. | 0.51 | 0.79 | 0.70 | 0.81 | 0.74 | 1.21 | 0.73 | 0.75 | 0.93 | 0.74 |
| p-value | 0.961 | 0.567 | 0.717 | 0.532 | 0.643 | 0.107 | 0.657 | 0.623 | 0.352 | 0.642 |
| | All Stocks, 1987 to 1991 | | | | | | | | | |
| γ | 1.36 | 1.53 | 1.05 | 0.67 | 0.75 | 0.86 | 0.60 | 1.09 | 1.20 | 0.67 |
| p-value | 0.048 | 0.019 | 0.219 | 0.756 | 0.627 | 0.456 | 0.862 | 0.185 | 0.111 | 0.764 |
| $\gamma \tau(\searrow)$ | 0.52 | 1.16 | 1.25 | 0.72 | 1.03 | 0.81 | 0.81 | 0.61 | 1.07 | 0.68 |
| p-value | 0.953 | 0.135 | 0.087 | 0.673 | 0.235 | 0.522 | 0.527 | 0.848 | 0.201 | 0.751 |
| $\gamma \tau(\nearrow)$ | 1.72 | 1.03 | 0.64 | 1.37 | 0.74 | 1.10 | 1.04 | 1.20 | 1.02 | 1.32 |
| p-value | 0.006 | 0.241 | 0.813 | 0.046 | 0.639 | 0.181 | 0.232 | 0.111 | 0.250 | 0.062 |
| γ Diff. | 1.11 | 1.29 | 1.07 | 1.06 | 0.67 | 0.93 | 0.89 | 0.74 | 0.84 | 1.17 |
| p-value | 0.168 | 0.072 | 0.201 | 0.215 | 0.753 | 0.357 | 0.403 | 0.638 | 0.483 | 0.129 |
| | All Stocks, 1992 to 1996 | | | | | | | | | |
| γ | 1.50 | 1.31 | 1.05 | 1.89 | 1.27 | 0.94 | 1.23 | 0.66 | 1.72 | 1.54 |
| p-value | 0.022 | 0.066 | 0.222 | 0.002 | 0.078 | 0.343 | 0.095 | 0.782 | 0.005 | 0.018 |
| $\gamma \tau(\searrow)$ | 0.87 | 1.05 | 0.60 | 0.89 | 1.11 | 1.03 | 0.90 | 0.65 | 0.99 | 1.12 |
| p-value | 0.443 | 0.218 | 0.858 | 0.404 | 0.174 | 0.242 | 0.390 | 0.787 | 0.283 | 0.165 |
| $\gamma \tau(\nearrow)$ | 0.72 | 0.66 | 0.75 | 1.42 | 1.02 | 0.58 | 0.61 | 0.64 | 1.36 | 0.93 |
| p-value | 0.670 | 0.778 | 0.624 | 0.036 | 0.246 | 0.895 | 0.854 | 0.813 | 0.048 | 0.357 |
| γ Diff. | 0.58 | 0.88 | 0.50 | 0.49 | 0.43 | 0.81 | 0.60 | 0.46 | 0.96 | 0.99 |
| p-value | 0.887 | 0.422 | 0.966 | 0.971 | 0.993 | 0.528 | 0.858 | 0.984 | 0.314 | 0.282 |

Table VIII

Kolmogorov–Smirnov test of the equality of conditional and unconditional one-day return distributions for Nasdaq stocks from 1962 to 1996, in five-year subperiods, and in size quintiles. Conditional returns are defined as the daily return three days following the conclusion of an occurrence of one of 10 technical indicators: head-and-shoulders (HS), inverted head-and-shoulders (IHS), broadening top (BTOP), broadening bottom (BBOT), triangle top (TTOP), triangle bottom (TBOT), rectangle top (RTOP), rectangle bottom (RBOT), double top (DTOP), and double bottom (DBOT). All returns have been normalized by subtraction of their means and division by their standard deviations. p -values are with respect to the asymptotic distribution of the Kolmogorov–Smirnov test statistic. The symbols “ $\tau(\nearrow)$ ” and “ $\tau(\searrow)$ ” indicate that the conditional distribution is also conditioned on decreasing and increasing volume trend, respectively.

| Statistic | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|-------------------------|----------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| γ | 2.31 | 2.68 | 1.60 | 1.84 | 2.81 | 2.34 | 2.69 | 1.90 | 2.29 | 2.06 |
| p -value | 0.000 | 0.000 | 0.012 | 0.002 | 0.000 | 0.000 | 0.000 | 0.001 | 0.000 | 0.000 |
| $\gamma \tau(\searrow)$ | 1.86 | 1.53 | 1.35 | 0.99 | 1.97 | 1.95 | 2.16 | 1.73 | 1.38 | 1.94 |
| p -value | 0.002 | 0.019 | 0.052 | 0.281 | 0.001 | 0.001 | 0.000 | 0.005 | 0.045 | 0.001 |
| $\gamma \tau(\nearrow)$ | 1.59 | 2.10 | 1.82 | 1.59 | 1.89 | 1.18 | 1.57 | 1.22 | 2.15 | 1.46 |
| p -value | 0.013 | 0.000 | 0.003 | 0.013 | 0.002 | 0.126 | 0.014 | 0.102 | 0.000 | 0.028 |
| γ Diff. | 1.08 | 0.86 | 1.10 | 0.80 | 1.73 | 0.74 | 0.91 | 0.75 | 0.76 | 1.52 |
| p -value | 0.195 | 0.450 | 0.175 | 0.542 | 0.005 | 0.637 | 0.379 | 0.621 | 0.619 | 0.020 |
| | All Stocks, 1962 to 1996 | | | | | | | | | |
| γ | 1.51 | 2.16 | 1.72 | 1.68 | 1.22 | 1.55 | 2.13 | 1.70 | 1.74 | 1.98 |
| p -value | 0.021 | 0.000 | 0.006 | 0.007 | 0.101 | 0.016 | 0.000 | 0.006 | 0.005 | 0.001 |
| $\gamma \tau(\searrow)$ | 1.16 | 1.30 | 0.85 | 1.14 | 1.25 | 1.62 | 1.43 | 1.05 | 1.08 | 1.95 |
| p -value | 0.139 | 0.070 | 0.463 | 0.150 | 0.089 | 0.010 | 0.033 | 0.216 | 0.191 | 0.001 |
| $\gamma \tau(\nearrow)$ | 0.85 | 1.73 | 1.61 | 2.00 | 1.34 | 0.79 | 1.58 | 1.52 | 1.47 | 1.20 |
| p -value | 0.462 | 0.005 | 0.012 | 0.001 | 0.055 | 0.553 | 0.014 | 0.019 | 0.026 | 0.115 |
| γ Diff. | 1.04 | 0.95 | 0.83 | 1.44 | 1.39 | 0.78 | 0.95 | 0.73 | 0.94 | 1.09 |
| p -value | 0.227 | 0.334 | 0.493 | 0.031 | 0.042 | 0.574 | 0.326 | 0.654 | 0.338 | 0.184 |
| | 2nd Quintile, 1962 to 1996 | | | | | | | | | |
| γ | 1.55 | 1.46 | 0.94 | 1.44 | 1.24 | 1.08 | 1.20 | 1.10 | 1.90 | 1.27 |
| p -value | 0.016 | 0.029 | 0.341 | 0.031 | 0.095 | 0.192 | 0.113 | 0.175 | 0.001 | 0.078 |
| $\gamma \tau(\searrow)$ | 1.11 | 1.13 | 1.08 | 0.92 | 1.23 | 0.79 | 1.34 | 1.19 | 1.09 | 1.61 |
| p -value | 0.173 | 0.157 | 0.192 | 0.371 | 0.097 | 0.557 | 0.055 | 0.117 | 0.185 | 0.011 |
| $\gamma \tau(\nearrow)$ | 1.37 | 0.87 | 0.73 | 0.97 | 1.38 | 1.29 | 1.12 | 0.91 | 1.12 | 0.94 |
| p -value | 0.048 | 0.439 | 0.665 | 0.309 | 0.044 | 0.073 | 0.162 | 0.381 | 0.165 | 0.343 |
| γ Diff. | 1.23 | 0.62 | 0.97 | 0.69 | 1.02 | 1.05 | 1.09 | 0.78 | 0.58 | 0.51 |
| p -value | 0.095 | 0.835 | 0.309 | 0.733 | 0.248 | 0.224 | 0.183 | 0.579 | 0.894 | 0.955 |

| | | | | | | | | | |
|-------------------------|-------|-------|-------|-------|--------------------------------|-------|-------|-------|-------|
| γ | 1.25 | 1.72 | 0.82 | 1.71 | 3rd Quintile, 1962 to 1996 | 1.52 | 1.84 | 1.86 | 1.82 |
| p -value | 0.005 | 0.005 | 0.510 | 0.006 | 0.038 | 0.020 | 0.002 | 0.002 | 0.003 |
| $\gamma \tau(\searrow)$ | 0.93 | 1.08 | 0.54 | 1.23 | 1.06 | 1.02 | 1.47 | 1.38 | 0.88 |
| p -value | 0.348 | 0.194 | 0.930 | 0.097 | 0.213 | 0.245 | 0.026 | 0.044 | 0.423 |
| $\gamma \tau(\nearrow)$ | 0.59 | 1.14 | 0.97 | 1.37 | 0.75 | 1.01 | 1.34 | 1.37 | 1.78 |
| p -value | 0.873 | 0.146 | 0.309 | 0.047 | 0.633 | 0.262 | 0.054 | 0.047 | 0.003 |
| γ Diff. | 0.61 | 0.89 | 0.58 | 0.46 | 0.61 | 0.89 | 0.38 | 0.60 | 1.09 |
| p -value | 0.852 | 0.405 | 0.890 | 0.984 | 0.844 | 0.404 | 0.999 | 0.864 | 0.188 |
| | | | | | 4th Quintile, 1962 to 1996 | | | | |
| γ | 1.04 | 0.82 | 1.20 | 0.98 | 1.30 | 1.25 | 0.79 | 0.94 | 0.66 |
| p -value | 0.233 | 0.510 | 0.111 | 0.298 | 0.067 | 0.087 | 0.002 | 0.341 | 0.779 |
| $\gamma \tau(\searrow)$ | 0.81 | 0.54 | 0.57 | 1.05 | 0.92 | 1.06 | 1.23 | 1.53 | 0.87 |
| p -value | 0.528 | 0.935 | 0.897 | 0.217 | 0.367 | 0.215 | 0.097 | 0.019 | 0.431 |
| $\gamma \tau(\nearrow)$ | 0.97 | 1.04 | 1.29 | 0.53 | 2.25 | 0.71 | 1.05 | 1.20 | 0.97 |
| p -value | 0.306 | 0.229 | 0.071 | 0.938 | 0.000 | 0.696 | 0.219 | 0.114 | 0.309 |
| γ Diff. | 1.17 | 0.89 | 0.98 | 0.97 | 1.86 | 0.62 | 0.93 | 1.31 | 0.92 |
| p -value | 0.128 | 0.400 | 0.292 | 0.301 | 0.002 | 0.843 | 0.352 | 0.065 | 0.371 |
| | | | | | Largest Quintile, 1962 to 1996 | | | | |
| γ | 1.08 | 1.01 | 1.03 | 0.66 | 0.92 | 0.68 | 1.16 | 1.14 | 0.67 |
| p -value | 0.190 | 0.255 | 0.242 | 0.778 | 0.360 | 0.742 | 0.462 | 0.150 | 0.756 |
| $\gamma \tau(\searrow)$ | 1.03 | 0.54 | 0.93 | 0.47 | 0.77 | 0.76 | 0.85 | 0.85 | 1.14 |
| p -value | 0.237 | 0.931 | 0.356 | 0.981 | 0.587 | 0.612 | 0.468 | 0.465 | 0.149 |
| $\gamma \tau(\nearrow)$ | 1.18 | 1.39 | 0.50 | 0.93 | 0.88 | 1.25 | 0.77 | 1.13 | 1.12 |
| p -value | 0.123 | 0.041 | 0.967 | 0.358 | 0.415 | 0.089 | 0.597 | 0.292 | 0.160 |
| γ Diff. | 0.94 | 1.25 | 0.73 | 0.84 | 0.76 | 1.11 | 0.73 | 0.86 | 0.77 |
| p -value | 0.342 | 0.090 | 0.668 | 0.476 | 0.617 | 0.169 | 0.662 | 0.454 | 0.598 |
| | | | | | All Stocks, 1962 to 1966 | | | | |
| γ | 1.01 | 0.84 | 1.08 | 0.82 | 0.71 | 0.70 | 1.59 | 1.12 | 1.10 |
| p -value | 0.261 | 0.481 | 0.193 | 0.508 | 0.697 | 0.718 | 0.013 | 0.166 | 0.175 |
| $\gamma \tau(\searrow)$ | 0.95 | 0.65 | 0.41 | 1.05 | 0.51 | 1.13 | 0.79 | 0.93 | 1.21 |
| p -value | 0.322 | 0.798 | 0.997 | 0.224 | 0.956 | 0.155 | 0.556 | 0.350 | 0.108 |
| $\gamma \tau(\nearrow)$ | 0.77 | 0.96 | 0.83 | 0.73 | 1.35 | 0.49 | 1.17 | 1.18 | 1.15 |
| p -value | 0.586 | 0.314 | 0.489 | 0.663 | 0.052 | 0.972 | 0.130 | 0.121 | 0.140 |
| γ Diff. | 1.10 | 0.67 | 0.32 | 0.69 | 1.29 | 0.58 | 0.80 | 0.98 | 1.06 |
| p -value | 0.174 | 0.761 | 1.000 | 0.735 | 0.071 | 0.892 | 0.551 | 0.298 | 0.208 |

continued

Table VIII—Continued

| Statistic | HS | IHS | BTOP | BBOT | TTOP | TBOT | RTOP | RBOT | DTOP | DBOT |
|-------------------------|-------|-------|--------|--------------------------|--------|--------|-------|-------|-------|-------|
| | | | | All Stocks, 1967 to 1971 | | | | | | |
| γ | 0.75 | 1.10 | 1.00 | 0.74 | 1.27 | 1.35 | 1.16 | 0.74 | 0.74 | 1.21 |
| p -value | 0.636 | 0.175 | 0.273 | 0.637 | 0.079 | 0.052 | 0.136 | 0.642 | 0.638 | 0.107 |
| $\gamma \tau(\searrow)$ | 1.03 | 0.52 | 0.70 | 0.87 | 1.24 | 1.33 | 1.29 | 0.83 | 0.72 | 1.45 |
| p -value | 0.241 | 0.947 | 0.714 | 0.438 | 0.092 | 0.058 | 0.072 | 0.490 | 0.684 | 0.031 |
| $\gamma \tau(\nearrow)$ | 1.05 | 1.08 | 1.12 | 0.64 | 0.79 | 0.65 | 0.55 | 0.53 | 0.75 | 0.69 |
| p -value | 0.217 | 0.192 | 0.165 | 0.810 | 0.566 | 0.797 | 0.923 | 0.941 | 0.631 | 0.723 |
| γ Diff. | 1.24 | 0.89 | 0.66 | 0.78 | 1.07 | 0.88 | 0.88 | 0.40 | 0.91 | 0.76 |
| p -value | 0.093 | 0.413 | 0.770 | 0.585 | 0.203 | 0.418 | 0.423 | 0.997 | 0.385 | 0.602 |
| | | | | All Stocks, 1972 to 1976 | | | | | | |
| γ | 0.82 | 1.28 | 1.84 | 1.13 | 1.45 | 1.53 | 1.31 | 0.96 | 0.85 | 1.76 |
| p -value | 0.509 | 0.077 | 0.002 | 0.156 | 0.029 | 0.019 | 0.064 | 0.314 | 0.464 | 0.004 |
| $\gamma \tau(\searrow)$ | 0.59 | 0.73 | -99.00 | 0.91 | 1.39 | 0.73 | 1.37 | 0.98 | 1.22 | 0.94 |
| p -value | 0.875 | 0.669 | 0.000 | 0.376 | 0.042 | 0.654 | 0.046 | 0.292 | 0.100 | 0.344 |
| $\gamma \tau(\nearrow)$ | 0.65 | 0.73 | -99.00 | -99.00 | -99.00 | -99.00 | 0.59 | 0.76 | 0.78 | 0.65 |
| p -value | 0.800 | 0.653 | 0.000 | 0.000 | 0.000 | 0.000 | 0.878 | 0.611 | 0.573 | 0.798 |
| γ Diff. | 0.48 | 0.57 | -99.00 | -99.00 | -99.00 | -99.00 | 0.63 | 0.55 | 0.92 | 0.37 |
| p -value | 0.974 | 0.902 | 0.000 | 0.000 | 0.000 | 0.000 | 0.828 | 0.925 | 0.362 | 0.999 |
| | | | | All Stocks, 1977 to 1981 | | | | | | |
| γ | 1.35 | 1.40 | 1.03 | 1.02 | 1.55 | 2.07 | 0.74 | 0.62 | 0.92 | 1.28 |
| p -value | 0.053 | 0.039 | 0.236 | 0.249 | 0.016 | 0.000 | 0.636 | 0.842 | 0.369 | 0.077 |
| $\gamma \tau(\searrow)$ | 1.19 | 1.47 | -99.00 | -99.00 | 0.96 | 0.98 | 0.86 | 0.79 | 0.81 | 0.68 |
| p -value | 0.117 | 0.027 | 0.000 | 0.000 | 0.317 | 0.290 | 0.453 | 0.554 | 0.522 | 0.748 |
| $\gamma \tau(\nearrow)$ | 0.69 | 0.94 | 0.80 | -99.00 | 1.46 | -99.00 | 0.56 | 0.82 | 1.06 | 0.94 |
| p -value | 0.728 | 0.341 | 0.542 | 0.000 | 0.028 | 0.000 | 0.918 | 0.514 | 0.207 | 0.336 |
| γ Diff. | 0.73 | 0.90 | -99.00 | -99.00 | 0.35 | -99.00 | 0.44 | 0.37 | 0.80 | 0.53 |
| p -value | 0.665 | 0.395 | 0.000 | 0.000 | 1.000 | 0.000 | 0.991 | 0.999 | 0.541 | 0.944 |

| | | | | | | | | | | |
|---------------------------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | All Stocks, 1982 to 1986 | | | | | | | | | |
| γ | 1.66 | 1.59 | 1.17 | 0.73 | 1.46 | 1.69 | 1.04 | 1.24 | 2.44 | 1.27 |
| p -value | 0.008 | 0.013 | 0.129 | 0.654 | 0.028 | 0.006 | 0.232 | 0.093 | 0.000 | 0.078 |
| $\gamma \tau(\backslash)$ | 1.65 | 1.10 | 0.46 | 0.74 | 0.95 | 1.47 | 0.83 | 1.18 | 1.20 | 0.59 |
| p -value | 0.009 | 0.176 | 0.984 | 0.641 | 0.330 | 0.027 | 0.503 | 0.121 | 0.112 | 0.873 |
| $\gamma \tau(\nearrow)$ | 1.13 | 1.31 | 0.86 | 0.42 | 1.17 | 1.04 | 0.97 | 1.13 | 1.68 | 0.89 |
| p -value | 0.153 | 0.065 | 0.445 | 0.995 | 0.129 | 0.231 | 0.302 | 0.155 | 0.007 | 0.405 |
| γ Diff. | 0.67 | 0.39 | 0.51 | 0.42 | 0.85 | 0.43 | 0.41 | 0.67 | 0.66 | 0.75 |
| p -value | 0.755 | 0.998 | 0.957 | 0.994 | 0.462 | 0.993 | 0.996 | 0.766 | 0.782 | 0.627 |
| | All Stocks, 1987 to 1991 | | | | | | | | | |
| γ | 1.24 | 1.29 | 0.91 | 0.88 | 1.28 | 1.41 | 2.01 | 1.49 | 1.55 | 1.53 |
| p -value | 0.091 | 0.070 | 0.384 | 0.421 | 0.074 | 0.039 | 0.001 | 0.024 | 0.017 | 0.019 |
| $\gamma \tau(\backslash)$ | 1.05 | 1.00 | 1.00 | 0.78 | 1.68 | 0.92 | 1.67 | 1.25 | 0.61 | 0.86 |
| p -value | 0.221 | 0.266 | 0.274 | 0.580 | 0.007 | 0.369 | 0.008 | 0.087 | 0.849 | 0.448 |
| $\gamma \tau(\nearrow)$ | 1.23 | 1.26 | 1.06 | 1.32 | 0.65 | 1.27 | 1.10 | 1.26 | 1.67 | 1.81 |
| p -value | 0.099 | 0.084 | 0.208 | 0.060 | 0.787 | 0.078 | 0.176 | 0.085 | 0.007 | 0.003 |
| γ Diff. | 0.80 | 0.91 | 1.22 | 1.28 | 1.22 | 0.92 | 0.87 | 0.81 | 1.07 | 1.05 |
| p -value | 0.552 | 0.375 | 0.103 | 0.075 | 0.102 | 0.360 | 0.431 | 0.520 | 0.202 | 0.217 |
| | All Stocks, 1992 to 1996 | | | | | | | | | |
| γ | 1.21 | 1.61 | 0.84 | 0.90 | 0.97 | 0.91 | 1.60 | 1.51 | 1.13 | 1.00 |
| p -value | 0.108 | 0.011 | 0.476 | 0.394 | 0.299 | 0.379 | 0.012 | 0.021 | 0.156 | 0.265 |
| $\gamma \tau(\backslash)$ | 0.68 | 1.02 | 0.81 | 0.78 | 0.81 | 0.93 | 0.79 | 1.07 | 0.94 | 0.64 |
| p -value | 0.752 | 0.246 | 0.530 | 0.578 | 0.532 | 0.357 | 0.558 | 0.201 | 0.340 | 0.814 |
| $\gamma \tau(\nearrow)$ | 1.56 | 0.85 | 0.71 | 1.00 | 1.10 | 1.04 | 1.43 | 0.93 | 0.90 | 1.44 |
| p -value | 0.015 | 0.470 | 0.688 | 0.275 | 0.180 | 0.231 | 0.034 | 0.352 | 0.392 | 0.031 |
| γ Diff. | 1.45 | 0.59 | 0.94 | 0.62 | 1.15 | 1.14 | 0.64 | 0.52 | 0.59 | 1.35 |
| p -value | 0.030 | 0.879 | 0.346 | 0.840 | 0.139 | 0.148 | 0.814 | 0.953 | 0.874 | 0.052 |

Table IX

Bootstrap percentiles for the Kolmogorov–Smirnov test of the equality of conditional and unconditional one-day return distributions for NYSE/AMEX and Nasdaq stocks from 1962 to 1996, and for size quintiles, under the null hypothesis of equality. For each of the two sets of market data, two sample sizes, m_1 and m_2 , have been chosen to span the range of frequency counts of patterns reported in Table I. For each sample size m_i , we resample one-day normalized returns (with replacement) to obtain a bootstrap sample of m_i observations, compute the Kolmogorov–Smirnov test statistic (against the entire sample of one-day normalized returns), and repeat this procedure 1,000 times. The percentiles of the asymptotic distribution are also reported for comparison.

| Percentile | NYSE/AMEX Sample | | | | | Nasdaq Sample | | | | |
|---------------------------------|------------------|------------------|-------|------------------|----------|---------------|------------------|-------|------------------|----------|
| | m_1 | $\Delta_{m_1,n}$ | m_2 | $\Delta_{m_2,n}$ | Δ | m_1 | $\Delta_{m_1,n}$ | m_2 | $\Delta_{m_2,n}$ | Δ |
| All Stocks, 1962 to 1996 | | | | | | | | | | |
| 0.01 | 2076 | 0.433 | 725 | 0.435 | 0.441 | 1320 | 0.430 | 414 | 0.438 | 0.441 |
| 0.05 | 2076 | 0.515 | 725 | 0.535 | 0.520 | 1320 | 0.514 | 414 | 0.522 | 0.520 |
| 0.10 | 2076 | 0.568 | 725 | 0.590 | 0.571 | 1320 | 0.573 | 414 | 0.566 | 0.571 |
| 0.50 | 2076 | 0.827 | 725 | 0.836 | 0.828 | 1320 | 0.840 | 414 | 0.826 | 0.828 |
| 0.90 | 2076 | 1.219 | 725 | 1.237 | 1.224 | 1320 | 1.244 | 414 | 1.229 | 1.224 |
| 0.95 | 2076 | 1.385 | 725 | 1.395 | 1.358 | 1320 | 1.373 | 414 | 1.340 | 1.358 |
| 0.99 | 2076 | 1.608 | 725 | 1.611 | 1.628 | 1320 | 1.645 | 414 | 1.600 | 1.628 |
| Smallest Quintile, 1962 to 1996 | | | | | | | | | | |
| 0.01 | 320 | 0.456 | 78 | 0.406 | 0.441 | 218 | 0.459 | 41 | 0.436 | 0.441 |
| 0.05 | 320 | 0.535 | 78 | 0.502 | 0.520 | 218 | 0.533 | 41 | 0.498 | 0.520 |
| 0.10 | 320 | 0.586 | 78 | 0.559 | 0.571 | 218 | 0.590 | 41 | 0.543 | 0.571 |
| 0.50 | 320 | 0.848 | 78 | 0.814 | 0.828 | 218 | 0.847 | 41 | 0.801 | 0.828 |
| 0.90 | 320 | 1.231 | 78 | 1.204 | 1.224 | 218 | 1.229 | 41 | 1.216 | 1.224 |
| 0.95 | 320 | 1.357 | 78 | 1.330 | 1.358 | 218 | 1.381 | 41 | 1.332 | 1.358 |
| 0.99 | 320 | 1.661 | 78 | 1.590 | 1.628 | 218 | 1.708 | 41 | 1.571 | 1.628 |
| 2nd Quintile, 1962 to 1996 | | | | | | | | | | |
| 0.01 | 420 | 0.445 | 146 | 0.428 | 0.441 | 305 | 0.458 | 68 | 0.426 | 0.441 |
| 0.05 | 420 | 0.530 | 146 | 0.505 | 0.520 | 305 | 0.557 | 68 | 0.501 | 0.520 |
| 0.10 | 420 | 0.580 | 146 | 0.553 | 0.571 | 305 | 0.610 | 68 | 0.559 | 0.571 |
| 0.50 | 420 | 0.831 | 146 | 0.823 | 0.828 | 305 | 0.862 | 68 | 0.804 | 0.828 |
| 0.90 | 420 | 1.197 | 146 | 1.210 | 1.224 | 305 | 1.265 | 68 | 1.210 | 1.224 |
| 0.95 | 420 | 1.349 | 146 | 1.343 | 1.358 | 305 | 1.407 | 68 | 1.409 | 1.358 |
| 0.99 | 420 | 1.634 | 146 | 1.626 | 1.628 | 305 | 1.686 | 68 | 1.614 | 1.628 |
| 3rd Quintile, 1962 to 1996 | | | | | | | | | | |
| 0.01 | 458 | 0.442 | 145 | 0.458 | 0.441 | 279 | 0.464 | 105 | 0.425 | 0.441 |
| 0.05 | 458 | 0.516 | 145 | 0.508 | 0.520 | 279 | 0.539 | 105 | 0.525 | 0.520 |
| 0.10 | 458 | 0.559 | 145 | 0.557 | 0.571 | 279 | 0.586 | 105 | 0.570 | 0.571 |
| 0.50 | 458 | 0.838 | 145 | 0.835 | 0.828 | 279 | 0.832 | 105 | 0.818 | 0.828 |
| 0.90 | 458 | 1.216 | 145 | 1.251 | 1.224 | 279 | 1.220 | 105 | 1.233 | 1.224 |
| 0.95 | 458 | 1.406 | 145 | 1.397 | 1.358 | 279 | 1.357 | 105 | 1.355 | 1.358 |
| 0.99 | 458 | 1.660 | 145 | 1.661 | 1.628 | 279 | 1.606 | 105 | 1.638 | 1.628 |
| 4th Quintile, 1962 to 1996 | | | | | | | | | | |
| 0.01 | 424 | 0.429 | 173 | 0.418 | 0.441 | 303 | 0.454 | 92 | 0.446 | 0.441 |
| 0.05 | 424 | 0.506 | 173 | 0.516 | 0.520 | 303 | 0.526 | 92 | 0.506 | 0.520 |
| 0.10 | 424 | 0.552 | 173 | 0.559 | 0.571 | 303 | 0.563 | 92 | 0.554 | 0.571 |
| 0.50 | 424 | 0.823 | 173 | 0.815 | 0.828 | 303 | 0.840 | 92 | 0.818 | 0.828 |
| 0.90 | 424 | 1.197 | 173 | 1.183 | 1.224 | 303 | 1.217 | 92 | 1.178 | 1.224 |
| 0.95 | 424 | 1.336 | 173 | 1.313 | 1.358 | 303 | 1.350 | 92 | 1.327 | 1.358 |
| 0.99 | 424 | 1.664 | 173 | 1.592 | 1.628 | 303 | 1.659 | 92 | 1.606 | 1.628 |
| Largest Quintile, 1962 to 1996 | | | | | | | | | | |
| 0.01 | 561 | 0.421 | 167 | 0.425 | 0.441 | 308 | 0.441 | 108 | 0.429 | 0.441 |
| 0.05 | 561 | 0.509 | 167 | 0.500 | 0.520 | 308 | 0.520 | 108 | 0.508 | 0.520 |
| 0.10 | 561 | 0.557 | 167 | 0.554 | 0.571 | 308 | 0.573 | 108 | 0.558 | 0.571 |
| 0.50 | 561 | 0.830 | 167 | 0.817 | 0.828 | 308 | 0.842 | 108 | 0.816 | 0.828 |
| 0.90 | 561 | 1.218 | 167 | 1.202 | 1.224 | 308 | 1.231 | 108 | 1.226 | 1.224 |
| 0.95 | 561 | 1.369 | 167 | 1.308 | 1.358 | 308 | 1.408 | 108 | 1.357 | 1.358 |
| 0.99 | 561 | 1.565 | 167 | 1.615 | 1.628 | 308 | 1.724 | 108 | 1.630 | 1.628 |

Table X

Bootstrap percentiles for the Kolmogorov–Smirnov test of the equality of conditional and unconditional one-day return distributions for NYSE/AMEX and Nasdaq stocks from 1962 to 1996, for five-year subperiods, under the null hypothesis of equality. For each of the two sets of market data, two sample sizes, m_1 and m_2 , have been chosen to span the range of frequency counts of patterns reported in Table I. For each sample size m_i , we resample one-day normalized returns (with replacement) to obtain a bootstrap sample of m_i observations, compute the Kolmogorov–Smirnov test statistic (against the entire sample of one-day normalized returns), and repeat this procedure 1,000 times. The percentiles of the asymptotic distribution are also reported for comparison.

| Percentile | NYSE/AMEX Sample | | | | | Nasdaq Sample | | | | |
|------------|--------------------------|------------------|-------|------------------|----------|---------------|------------------|-------|------------------|----------|
| | m_1 | $\Delta_{m_1,n}$ | m_2 | $\Delta_{m_2,n}$ | Δ | m_1 | $\Delta_{m_1,n}$ | m_2 | $\Delta_{m_2,n}$ | Δ |
| | All Stocks, 1962 to 1966 | | | | | | | | | |
| 0.01 | 356 | 0.431 | 85 | 0.427 | 0.441 | 342 | 0.460 | 72 | 0.417 | 0.441 |
| 0.05 | 356 | 0.516 | 85 | 0.509 | 0.520 | 342 | 0.539 | 72 | 0.501 | 0.520 |
| 0.10 | 356 | 0.576 | 85 | 0.559 | 0.571 | 342 | 0.589 | 72 | 0.565 | 0.571 |
| 0.50 | 356 | 0.827 | 85 | 0.813 | 0.828 | 342 | 0.849 | 72 | 0.802 | 0.828 |
| 0.90 | 356 | 1.233 | 85 | 1.221 | 1.224 | 342 | 1.242 | 72 | 1.192 | 1.224 |
| 0.95 | 356 | 1.359 | 85 | 1.363 | 1.358 | 342 | 1.384 | 72 | 1.339 | 1.358 |
| 0.99 | 356 | 1.635 | 85 | 1.711 | 1.628 | 342 | 1.582 | 72 | 1.684 | 1.628 |
| | All Stocks, 1967 to 1971 | | | | | | | | | |
| 0.01 | 258 | 0.432 | 112 | 0.423 | 0.441 | 227 | 0.435 | 65 | 0.424 | 0.441 |
| 0.05 | 258 | 0.522 | 112 | 0.508 | 0.520 | 227 | 0.512 | 65 | 0.498 | 0.520 |
| 0.10 | 258 | 0.588 | 112 | 0.562 | 0.571 | 227 | 0.571 | 65 | 0.546 | 0.571 |
| 0.50 | 258 | 0.841 | 112 | 0.819 | 0.828 | 227 | 0.811 | 65 | 0.812 | 0.828 |
| 0.90 | 258 | 1.194 | 112 | 1.253 | 1.224 | 227 | 1.179 | 65 | 1.219 | 1.224 |
| 0.95 | 258 | 1.315 | 112 | 1.385 | 1.358 | 227 | 1.346 | 65 | 1.357 | 1.358 |
| 0.99 | 258 | 1.703 | 112 | 1.563 | 1.628 | 227 | 1.625 | 65 | 1.669 | 1.628 |
| | All Stocks, 1972 to 1976 | | | | | | | | | |
| 0.01 | 223 | 0.439 | 82 | 0.440 | 0.441 | 58 | 0.433 | 25 | 0.405 | 0.441 |
| 0.05 | 223 | 0.518 | 82 | 0.503 | 0.520 | 58 | 0.495 | 25 | 0.479 | 0.520 |
| 0.10 | 223 | 0.588 | 82 | 0.554 | 0.571 | 58 | 0.542 | 25 | 0.526 | 0.571 |
| 0.50 | 223 | 0.854 | 82 | 0.798 | 0.828 | 58 | 0.793 | 25 | 0.783 | 0.828 |
| 0.90 | 223 | 1.249 | 82 | 1.208 | 1.224 | 58 | 1.168 | 25 | 1.203 | 1.224 |
| 0.95 | 223 | 1.406 | 82 | 1.364 | 1.358 | 58 | 1.272 | 25 | 1.345 | 1.358 |
| 0.99 | 223 | 1.685 | 82 | 1.635 | 1.628 | 58 | 1.618 | 25 | 1.616 | 1.628 |
| | All Stocks, 1977 to 1981 | | | | | | | | | |
| 0.01 | 290 | 0.426 | 110 | 0.435 | 0.441 | 96 | 0.430 | 36 | 0.417 | 0.441 |
| 0.05 | 290 | 0.519 | 110 | 0.504 | 0.520 | 96 | 0.504 | 36 | 0.485 | 0.520 |
| 0.10 | 290 | 0.573 | 110 | 0.555 | 0.571 | 96 | 0.570 | 36 | 0.542 | 0.571 |
| 0.50 | 290 | 0.841 | 110 | 0.793 | 0.828 | 96 | 0.821 | 36 | 0.810 | 0.828 |
| 0.90 | 290 | 1.262 | 110 | 1.184 | 1.224 | 96 | 1.197 | 36 | 1.201 | 1.224 |
| 0.95 | 290 | 1.383 | 110 | 1.342 | 1.358 | 96 | 1.352 | 36 | 1.371 | 1.358 |
| 0.99 | 290 | 1.598 | 110 | 1.645 | 1.628 | 96 | 1.540 | 36 | 1.545 | 1.628 |
| | All Stocks, 1982 to 1986 | | | | | | | | | |
| 0.01 | 313 | 0.462 | 106 | 0.437 | 0.441 | 120 | 0.448 | 44 | 0.417 | 0.441 |
| 0.05 | 313 | 0.542 | 106 | 0.506 | 0.520 | 120 | 0.514 | 44 | 0.499 | 0.520 |
| 0.10 | 313 | 0.585 | 106 | 0.559 | 0.571 | 120 | 0.579 | 44 | 0.555 | 0.571 |
| 0.50 | 313 | 0.844 | 106 | 0.819 | 0.828 | 120 | 0.825 | 44 | 0.802 | 0.828 |
| 0.90 | 313 | 1.266 | 106 | 1.220 | 1.224 | 120 | 1.253 | 44 | 1.197 | 1.224 |
| 0.95 | 313 | 1.397 | 106 | 1.369 | 1.358 | 120 | 1.366 | 44 | 1.337 | 1.358 |
| 0.99 | 313 | 1.727 | 106 | 1.615 | 1.628 | 120 | 1.692 | 44 | 1.631 | 1.628 |
| | All Stocks, 1987 to 1991 | | | | | | | | | |
| 0.01 | 287 | 0.443 | 98 | 0.449 | 0.441 | 312 | 0.455 | 50 | 0.432 | 0.441 |
| 0.05 | 287 | 0.513 | 98 | 0.522 | 0.520 | 312 | 0.542 | 50 | 0.517 | 0.520 |
| 0.10 | 287 | 0.565 | 98 | 0.566 | 0.571 | 312 | 0.610 | 50 | 0.563 | 0.571 |
| 0.50 | 287 | 0.837 | 98 | 0.813 | 0.828 | 312 | 0.878 | 50 | 0.814 | 0.828 |
| 0.90 | 287 | 1.200 | 98 | 1.217 | 1.224 | 312 | 1.319 | 50 | 1.216 | 1.224 |
| 0.95 | 287 | 1.336 | 98 | 1.348 | 1.358 | 312 | 1.457 | 50 | 1.323 | 1.358 |
| 0.99 | 287 | 1.626 | 98 | 1.563 | 1.628 | 312 | 1.701 | 50 | 1.648 | 1.628 |
| | All Stocks, 1992 to 1996 | | | | | | | | | |
| 0.01 | 389 | 0.438 | 102 | 0.432 | 0.441 | 361 | 0.447 | 87 | 0.428 | 0.441 |
| 0.05 | 389 | 0.522 | 102 | 0.506 | 0.520 | 361 | 0.518 | 87 | 0.492 | 0.520 |
| 0.10 | 389 | 0.567 | 102 | 0.558 | 0.571 | 361 | 0.559 | 87 | 0.550 | 0.571 |
| 0.50 | 389 | 0.824 | 102 | 0.818 | 0.828 | 361 | 0.817 | 87 | 0.799 | 0.828 |
| 0.90 | 389 | 1.220 | 102 | 1.213 | 1.224 | 361 | 1.226 | 87 | 1.216 | 1.224 |
| 0.95 | 389 | 1.321 | 102 | 1.310 | 1.358 | 361 | 1.353 | 87 | 1.341 | 1.358 |
| 0.99 | 389 | 1.580 | 102 | 1.616 | 1.628 | 361 | 1.617 | 87 | 1.572 | 1.628 |

ations may lead to an entirely new branch of technical analysis, one based on selecting pattern-recognition algorithms to optimize specific objective functions. We hope to explore these issues more fully in future research.

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Discussion

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Academics have long been skeptical about the usefulness of technical trading strategies. The literature that evaluates the performance of such trading strategies has found mixed results. This literature has generally focused on evaluating simple technical trading rules such as filter rules and moving average rules that are fairly straightforward to define and implement. Lo, Mamaysky, and Wang (hereafter LMW) move this literature forward by evaluating more complicated trading strategies used by chartists that are hard to define and implement objectively.

Broadly, the primary objectives of LMW are to automate the process of identifying patterns in stock prices and evaluate the usefulness of trading strategies based on various patterns. They start with quantitative definitions of 10 patterns that are commonly used by chartists. They then smooth the price data using kernel regressions. They identify various patterns in the smoothed prices based on their definitions of these patterns. Their algorithms allow them to recognize patterns objectively on the computer rather

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