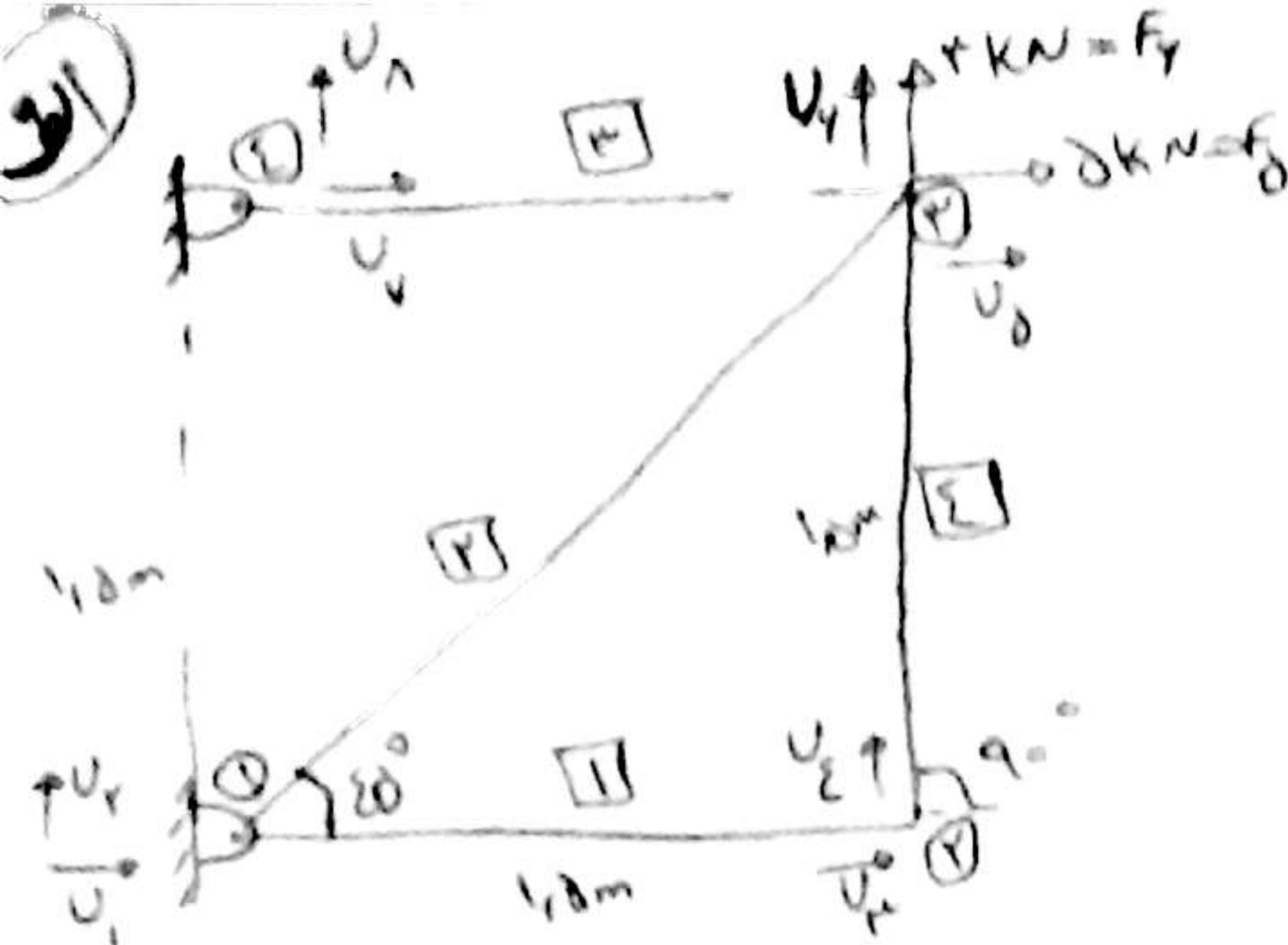
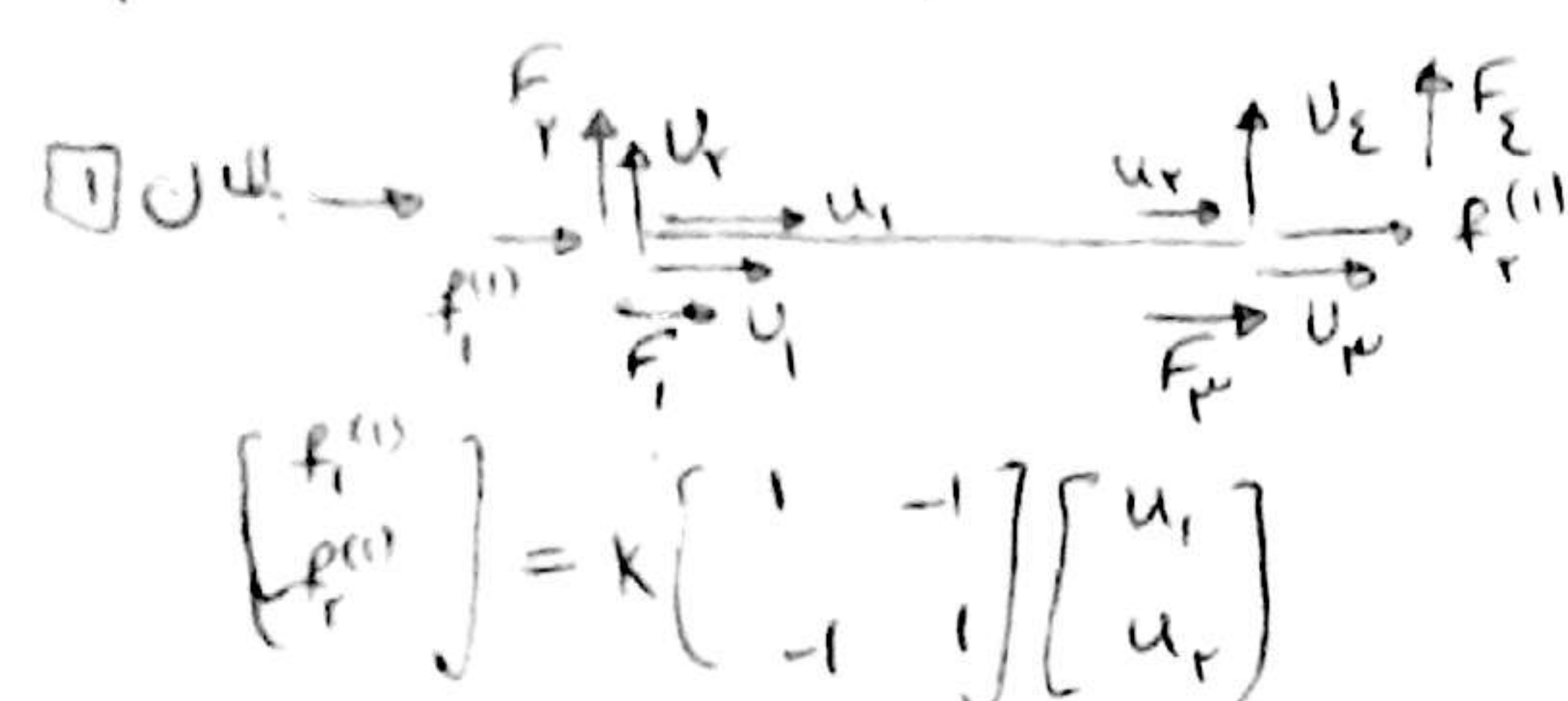




$A = 18 \times 18 \text{ mm}^2$   
 $E = 49 \text{ GPa} = 49 \times 10^3 \text{ MPa} = 49 \times 10^3 \frac{\text{N}}{\text{mm}^2}$   
 $L_y = 1.0 \text{ m}$   
 $L_x = 1.0 \text{ m}$   
 $L_z = 1.0 \text{ m}$   
 $F_y = 8 \text{ kN}$   
 $F_x = 1 \text{ kN}$

$\frac{EA}{L} = \frac{\frac{\text{N}}{\text{mm}^2} \times \text{mm}^2}{\text{m}} = \frac{\text{N}}{\text{m}}$   
 $F \rightarrow \text{N}$   
 $L \rightarrow \text{m}$   
 $9 \text{ k} \rightarrow \frac{\text{N}}{\text{m}}$

11/25



$u_1 = U_1 \cos 0 + U_y \sin 0 = U_1$   
 $u_y = U_y \cos 0 + U_z \sin 0 = U_y$

$\cos 90 = 0, \sin 90 = 1$   
 $\cos 0 = 1, \sin 0 = 0$

$F_1 = f_1^{(1)}$   
 $F_y = f_y^{(1)}$   
 $F_z = 0$   
 $F_x = 0$

$K = \frac{EA}{L} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1.0} & 0 & -\frac{1}{1.0} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{1.0} & 0 & \frac{1}{1.0} \end{pmatrix}$

$K^{(1)} = K \begin{pmatrix} c^x & sc & -c^x & -sc \\ sc & s^x & -sc & -s^x \\ -c^x & -sc & c^x & sc \\ -sc & -s^x & sc & s^x \end{pmatrix}$

$\cos 45 = \sin 45 = \frac{\sqrt{2}}{2}$   
 $\cos^2 45 = \sin^2 45 = \cos 45 \sin 45 = \frac{1}{2}$

$K^{(1)}, K^{(2)} = EA \begin{pmatrix} \frac{1}{1.0} & 0 & -\frac{1}{1.0} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{1.0} & 0 & \frac{1}{1.0} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$K^{(2)} = EA \begin{pmatrix} \frac{1}{1.0} & \frac{1}{1.0} & -\frac{1}{1.0} & -\frac{1}{1.0} \\ \frac{1}{1.0} & \frac{1}{1.0} & -\frac{1}{1.0} & -\frac{1}{1.0} \\ -\frac{1}{1.0} & -\frac{1}{1.0} & \frac{1}{1.0} & \frac{1}{1.0} \\ -\frac{1}{1.0} & -\frac{1}{1.0} & \frac{1}{1.0} & \frac{1}{1.0} \end{pmatrix}$

$K_{11} = K_{11}^{(1)} + K_{11}^{(2)}$   
 $K_{12} = K_{12}^{(1)} + K_{12}^{(2)}$   
 $K_{13} = K_{13}^{(1)}$   
 $K_{14} = K_{14}^{(1)}$   
 $K_{15} = K_{15}^{(1)}$   
 $K_{16} = K_{16}^{(1)}$   
 $K_{17} = K_{17}^{(1)}$   
 $K_{18} = K_{18}^{(1)}$   
 $K_{19} = 0$   
 $K_{20} = 0$

$K_{21} = K_{21}^{(1)} + K_{21}^{(2)}$   
 $K_{22} = K_{22}^{(1)}$   
 $K_{23} = K_{23}^{(1)}$   
 $K_{24} = K_{24}^{(1)}$   
 $K_{25} = K_{25}^{(1)}$   
 $K_{26} = K_{26}^{(1)}$   
 $K_{27} = K_{27}^{(1)}$   
 $K_{28} = K_{28}^{(1)}$   
 $K_{29} = 0$

$K_{31} = K_{31}^{(1)} + K_{31}^{(2)}$   
 $K_{32} = K_{32}^{(1)}$   
 $K_{33} = K_{33}^{(1)}$   
 $K_{34} = K_{34}^{(1)}$   
 $K_{35} = K_{35}^{(1)}$   
 $K_{36} = K_{36}^{(1)}$   
 $K_{37} = K_{37}^{(1)}$   
 $K_{38} = K_{38}^{(1)}$   
 $K_{39} = 0$

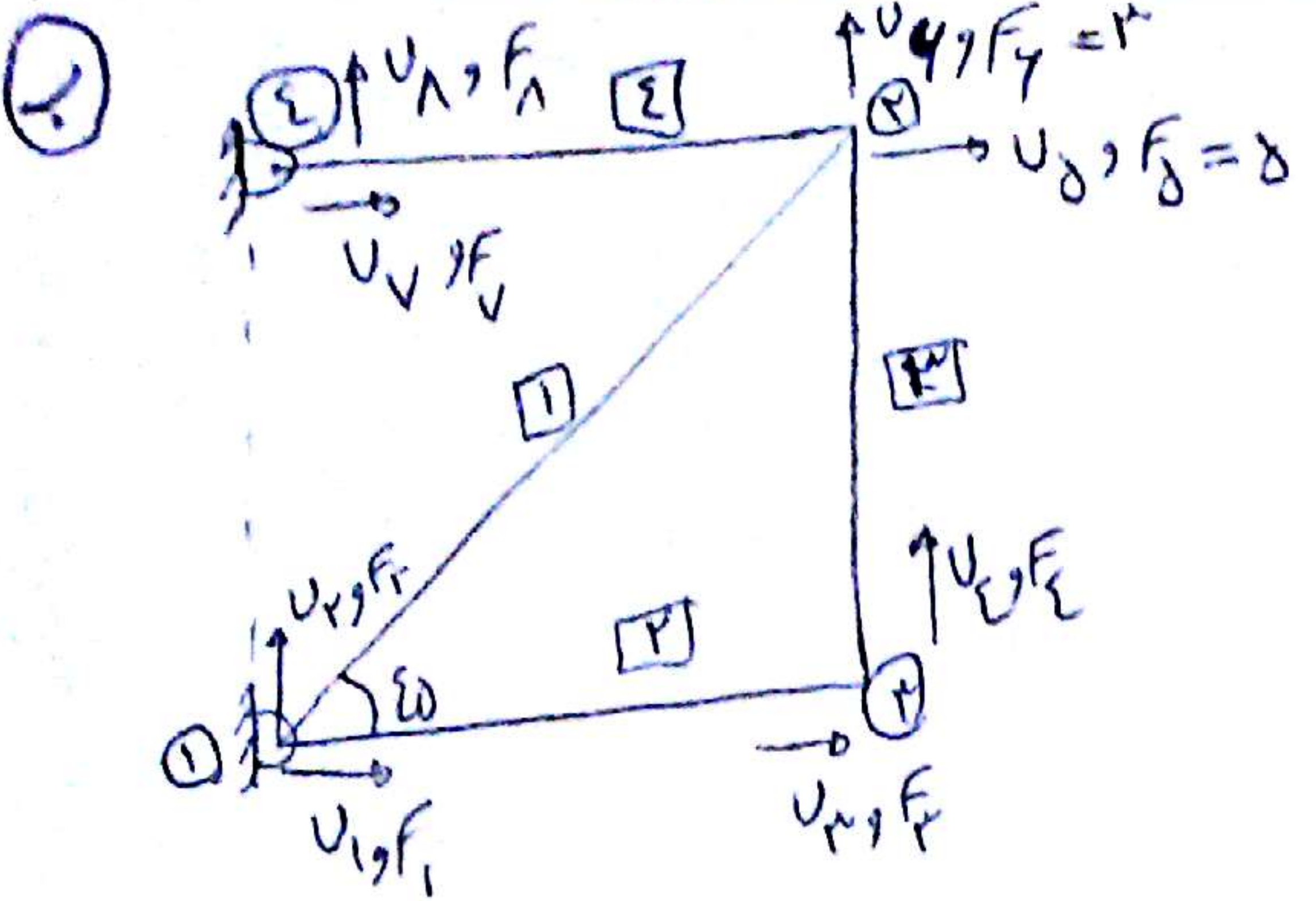
$K_{41} = K_{41}^{(1)} + K_{41}^{(2)}$   
 $K_{42} = K_{42}^{(1)}$   
 $K_{43} = K_{43}^{(1)}$   
 $K_{44} = K_{44}^{(1)}$   
 $K_{45} = K_{45}^{(1)}$   
 $K_{46} = K_{46}^{(1)}$   
 $K_{47} = K_{47}^{(1)}$   
 $K_{48} = K_{48}^{(1)}$   
 $K_{49} = 0$

$K_{51} = K_{51}^{(1)} + K_{51}^{(2)}$   
 $K_{52} = K_{52}^{(1)}$   
 $K_{53} = K_{53}^{(1)}$   
 $K_{54} = K_{54}^{(1)}$   
 $K_{55} = K_{55}^{(1)}$   
 $K_{56} = K_{56}^{(1)}$   
 $K_{57} = K_{57}^{(1)}$   
 $K_{58} = K_{58}^{(1)}$   
 $K_{59} = 0$

$K_{61} = K_{61}^{(1)} + K_{61}^{(2)}$   
 $K_{62} = K_{62}^{(1)}$   
 $K_{63} = K_{63}^{(1)}$   
 $K_{64} = K_{64}^{(1)}$   
 $K_{65} = K_{65}^{(1)}$   
 $K_{66} = K_{66}^{(1)}$   
 $K_{67} = K_{67}^{(1)}$   
 $K_{68} = K_{68}^{(1)}$   
 $K_{69} = 0$

| Σ U U Z | Σ U U Z | Σ U U Z | Σ U U Z | Σ U U Z |
|---------|---------|---------|---------|---------|
| 0       | 0       | 1       | 1       | 1       |
| 0       | 0       | 2       | 2       | 2       |
| 1       | 0       | 0       | Σ       | Σ       |
| 2       | 0       | 0       | 0       | 0       |
| Σ       | 2       | Σ       | 0       | 0       |
| 0       | Σ       | 0       | 0       | 0       |
| 0       | Σ       | 0       | 0       | 0       |





$$K^{(2)} = \frac{AE}{1.8} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\theta = 180^\circ$

$$K^{(1)} = \frac{AE}{1.8} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$\theta = 45^\circ$   
 $\sin = \cos = \frac{\sqrt{2}}{2}$   
 $\lambda = \sin \cos = \sin^2 = \cos^2$

$$K^{(r)} = \frac{AE}{1.8} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\theta = 0^\circ$

$$K^{(r)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$\theta = 90^\circ$   
 $\cos = 0$   
 $\sin = 1$

$$K_{11} = K_{11}^I + K_{11}^r$$

$$K_{12} = K_{12}^I + K_{12}^r$$

$$K_{13} = K_{13}^r$$

$$K_{14} = K_{14}^r$$

$$K_{15} = K_{15}^r$$

$$K_{16} = K_{16}^r$$

$$K_{17} = K_{17}^r$$

$$K_{18} = K_{18}^r$$

$$K_{19} = K_{19}^r$$

$$K_{10} = K_{10}^r$$

$$K_{11} = K_{11}^r$$

$$K_{22} = K_{22}^r + K_{22}^r$$

$$K_{23} = K_{23}^r$$

$$K_{24} = K_{24}^r$$

$$K_{25} = K_{25}^r$$

$$K_{26} = K_{26}^r$$

$$K_{27} = K_{27}^r$$

$$K_{28} = K_{28}^r$$

$$K_{29} = K_{29}^r$$

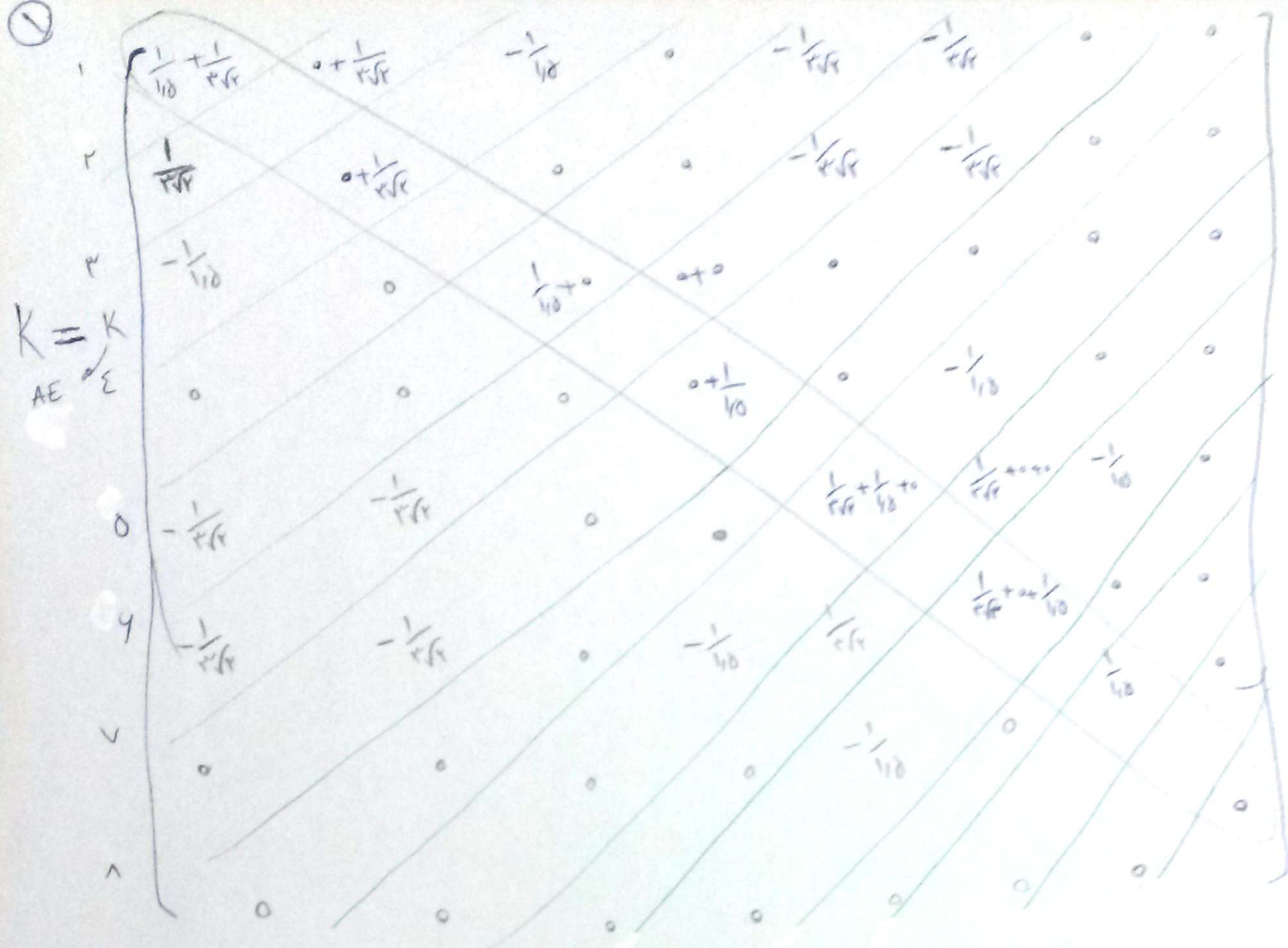
$$K_{20} = K_{20}^r$$

$$K_{21} = K_{21}^r$$

| Σ U d E | Σ U d E | Σ U d E | Σ U d E | Σ U d E |
|---------|---------|---------|---------|---------|
| 0       | 0       | 1       | 1       | 1       |
| 0       | 0       | 2       | 2       | 2       |
| 0       | 1       | 3       | 0       | 3       |
| 0       | 2       | Σ       | 0       | Σ       |
| 1       | 3       | 0       | 2       | 0       |
| 2       | Σ       | 0       | Σ       | 2       |
| 3       | 0       | 0       | 0       | 3       |
| Σ       | 0       | 0       | 0       | Σ       |

|                          |                       |    |    |                          |                          |    |   |
|--------------------------|-----------------------|----|----|--------------------------|--------------------------|----|---|
| $\frac{1}{\sqrt{2}} + 1$ | $\frac{1}{\sqrt{2}}$  | -1 | 0  | $\frac{1}{\sqrt{2}}$     | $-\frac{1}{\sqrt{2}}$    | 0  | 0 |
| $\frac{1}{\sqrt{2}}$     | $\frac{1}{\sqrt{2}}$  | 0  | 0  | $-\frac{1}{\sqrt{2}}$    | $-\frac{1}{\sqrt{2}}$    | 0  | 0 |
| -1                       | 0                     | 1  | 0  | 0                        | 0                        | 0  | 0 |
| 0                        | 0                     | 0  | 1  | 0                        | -1                       | 0  | 0 |
| $-\frac{1}{\sqrt{2}}$    | $-\frac{1}{\sqrt{2}}$ | 0  | 0  | $\frac{1}{\sqrt{2}} + 1$ | $\frac{1}{\sqrt{2}}$     | -1 | 0 |
| $-\frac{1}{\sqrt{2}}$    | $-\frac{1}{\sqrt{2}}$ | 0  | -1 | $\frac{1}{\sqrt{2}}$     | $\frac{1}{\sqrt{2}} + 1$ | 0  | 0 |
| 0                        | 0                     | 0  | 0  | -1                       | 0                        | 1  | 0 |
| 0                        | 0                     | 0  | 0  | 0                        | 0                        | 0  | 0 |





$$KU = F$$

$$K \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{bmatrix}$$

$$\rightarrow K \begin{bmatrix} U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}$$

$$\left(\frac{1}{r} + \frac{1}{10}\right)U_5 + \frac{1}{r}U_6 = 0$$

$$-\frac{1}{10}U_3 + \frac{1}{r}U_5 + \left(\frac{1}{r} + \frac{1}{10}\right)U_6 = r$$

$$\textcircled{V} -\frac{1}{10}U_5 + \frac{1}{10}U_6 = F_5$$

$$\textcircled{A} 0 = F_1$$

$$\textcircled{P} \frac{1}{10}U_2 = F_2$$

$$\frac{1}{10}U_2 - \frac{1}{r}U_5 - \frac{1}{r}U_6 = F_1$$

①

②

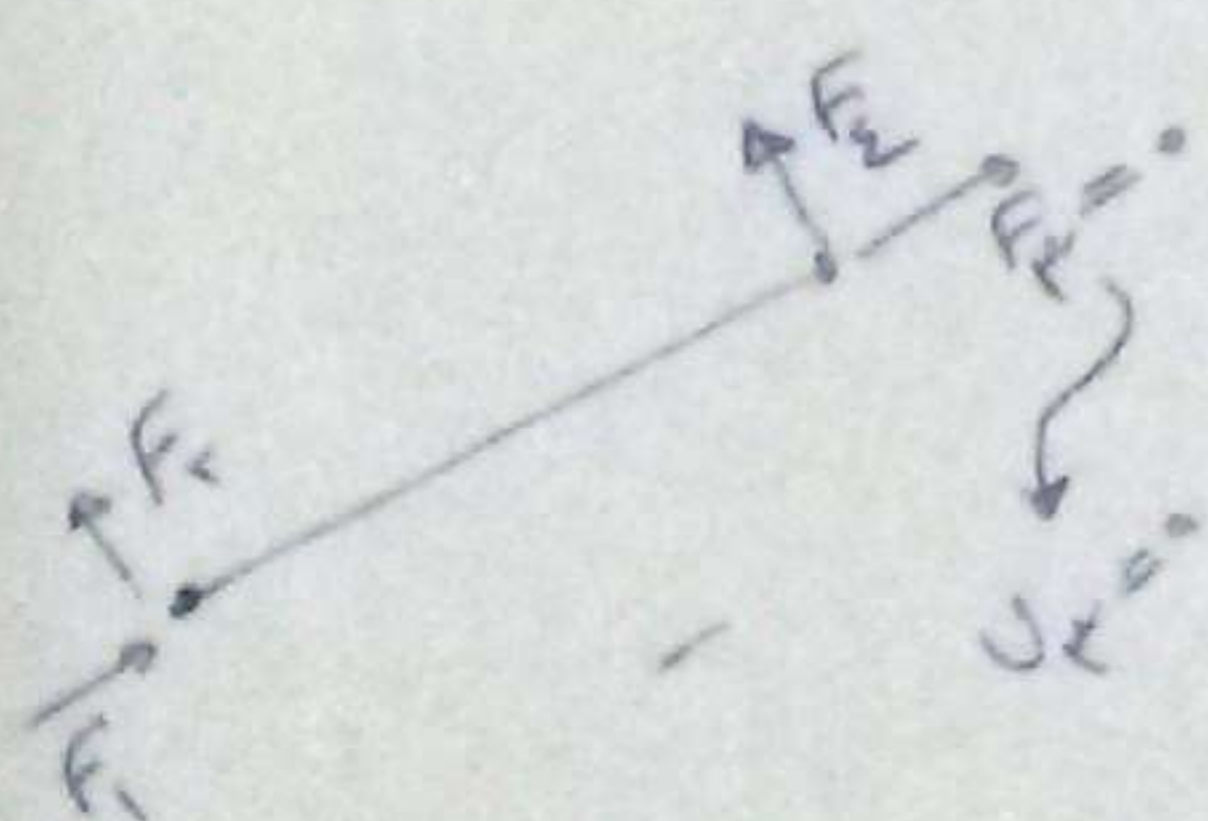
$$-\frac{1}{r}U_5 - \frac{1}{r}U_6 = F_2$$

③

$$\frac{1}{10}U_2 - \frac{1}{10}U_6 = F_2$$

$$\textcircled{B} \left(\frac{1}{r} + \frac{1}{10}\right)U_5 + \frac{1}{r}U_6 - \frac{1}{10}U_6 = F_5$$

$$\textcircled{Q} -\frac{1}{10}U_2 - \frac{1}{r}U_5 + \left(\frac{1}{r} + \frac{1}{10}\right)U_6 = F_4$$





②

$$K = EA \begin{bmatrix} \frac{1}{10} & 0 & 0 & 0 \\ 0 & \frac{1}{10} & 0 & -\frac{1}{10} \\ 0 & 0 & \frac{1}{\sqrt{r}} + \frac{1}{10} & \frac{1}{\sqrt{r}} \\ 0 & -\frac{1}{10} & \frac{1}{\sqrt{r}} & \frac{1}{\sqrt{r}} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} U_r \\ U_z \\ U_\delta \\ U_\gamma \end{bmatrix} = \begin{bmatrix} F_r = 0 \\ F_z = 0 \\ \delta \\ r \end{bmatrix}$$

$$\frac{1}{m} U \quad KN$$

$$\frac{N}{m}(U) = KN$$

$$\frac{1}{10} U_z - \frac{1}{10} U_\gamma = 0 \rightarrow U_z = U_\gamma$$

$$\begin{aligned} \frac{1}{10} U &= 1000 \\ 49 \times \frac{1}{49} U &= 1000 \\ U &= \frac{r}{49} \end{aligned}$$

$$\left(\frac{1}{\sqrt{r}} + \frac{1}{10}\right) U_\delta + \frac{1}{\sqrt{r}} U_\gamma = \delta \rightarrow \frac{1}{\sqrt{r}} U_\delta + \frac{1}{10} U_\delta + \frac{1}{\sqrt{r}} U_\gamma = \delta$$

$$\cancel{-\frac{1}{10} U_\gamma} + \frac{1}{\sqrt{r}} U_\delta + \frac{1}{\sqrt{r}} U_\gamma + \cancel{\frac{1}{10} U_\gamma} = r \rightarrow \frac{1}{\sqrt{r}} U_\delta + \frac{1}{\sqrt{r}} U_\gamma = r$$

$$\Rightarrow \frac{1}{10} U_\delta = r \rightarrow U_\delta = r \times 10 = r \text{ mm}$$

$$\frac{1}{\sqrt{r}} U_\gamma = r - \frac{1}{\sqrt{r}} \times r = r - \frac{1}{\sqrt{r}} = \frac{r\sqrt{r}-1}{\sqrt{r}}$$

$$U_\gamma = \frac{\frac{r\sqrt{r}-1}{\sqrt{r}}}{\frac{1}{\sqrt{r}}} = \frac{r\sqrt{r}(r\sqrt{r}-1)}{\sqrt{r}} = 9\sqrt{r} - r = 9,4 \text{ mm}$$

II) 1) 1)

$$\begin{Bmatrix} u_r^{(1)} \\ u_z^{(1)} \end{Bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \end{bmatrix}_{2 \times 2} \begin{Bmatrix} U_r \\ U_z \\ U_\delta \\ U_\gamma \end{Bmatrix}_{2 \times 1} = \begin{Bmatrix} 0 \\ U_r \end{Bmatrix}$$

$$\sigma^{(1)} = E \begin{bmatrix} -\frac{1}{10} & \frac{1}{10} \end{bmatrix}_{1 \times 2} \begin{Bmatrix} U_r \\ U_\gamma \end{Bmatrix}_{2 \times 1} = 49 \times 10^6 \begin{bmatrix} 0 \\ 10 U_r \end{bmatrix} = 0$$

II) 2) 1)

$$\begin{Bmatrix} u_r^{(1)} \\ u_\gamma^{(1)} \end{Bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \end{bmatrix} \begin{Bmatrix} U_\delta \\ U_\gamma \\ U_\delta \\ U_\gamma \end{Bmatrix} = \begin{Bmatrix} U_\delta \\ U_\gamma \end{Bmatrix}$$

$$\sigma^{(1)} = E \begin{bmatrix} -\frac{1}{10} & \frac{1}{10} \end{bmatrix} \begin{Bmatrix} U_\delta \\ U_\gamma \end{Bmatrix} = 49 \times 10^6 \begin{bmatrix} -\frac{1}{10} \times \frac{r}{49} \\ \frac{1}{10} \times \frac{r}{49} \end{bmatrix} = -\frac{r \times 10^6}{10} = -1000 \text{ MPa}$$

II) 2) 2)

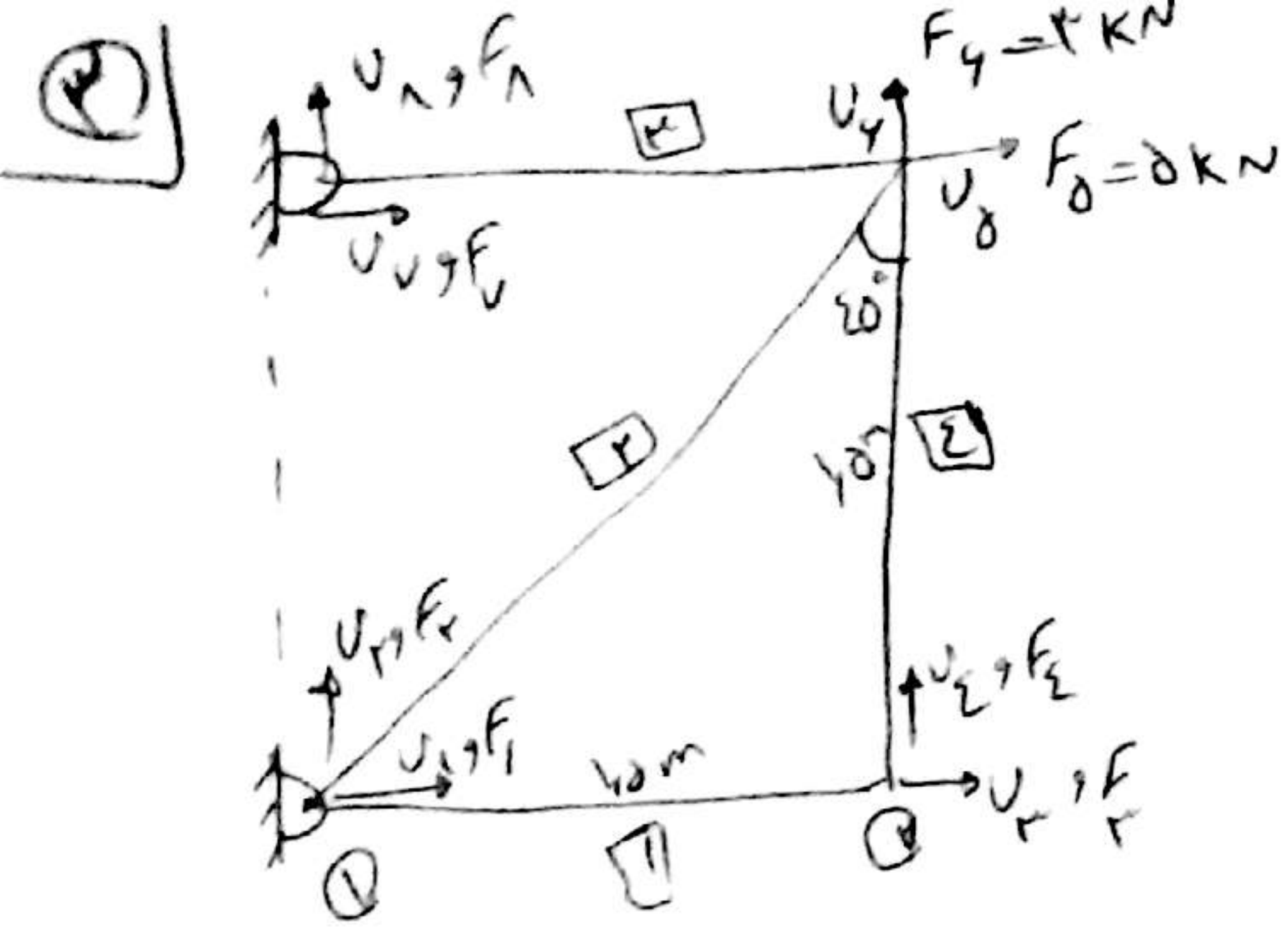
$$\begin{Bmatrix} u_z^{(1)} \\ u_\gamma^{(1)} \end{Bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \end{bmatrix} \begin{Bmatrix} U_r \\ U_z \\ U_\delta \\ U_\gamma \end{Bmatrix}_{2 \times 1} = \begin{Bmatrix} U_z \\ U_\gamma \end{Bmatrix} = \begin{Bmatrix} \frac{9,4}{49} \\ \frac{9,4}{49} \end{Bmatrix}$$

$$\sigma^{(2)} = E \begin{bmatrix} -\frac{1}{10} & \frac{1}{10} \end{bmatrix} \begin{Bmatrix} \frac{9,4}{49} \\ \frac{9,4}{49} \end{Bmatrix} = 0$$

$$\begin{Bmatrix} u_r^{(1)} \\ u_\gamma^{(1)} \end{Bmatrix} = \begin{bmatrix} \frac{r}{49} & \frac{r}{49} & 0 & 0 \\ 0 & 0 & -\frac{r}{49} & -\frac{r}{49} \end{bmatrix} \begin{Bmatrix} U_r \\ U_z \\ U_\delta \\ U_\gamma \end{Bmatrix}_{2 \times 1} = \begin{Bmatrix} \frac{r}{49} U_r + \frac{r}{49} U_\gamma \\ -\frac{r}{49} (U_\delta + U_\gamma) \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\frac{r}{49} \left(\frac{12,4}{49}\right) \end{Bmatrix}$$

$$\sigma^{(2)} = 49 \times 10^6 \times \frac{12,4}{49 \times 49} = \frac{12,4}{49} \times 10^6 = 2,5 \times 10^6$$





$$A = 18 \times 18 \text{ mm}^2$$

$$E = 79 \text{ GPa} = 79 \times 10^9 \text{ MPa} = 79 \times 10^9 \frac{\text{N}}{\text{mm}^2}$$

$$L_r = 1.8 \text{ m}$$

$$L_1 = L_2 = L_3 = 1.8 \text{ m}$$

$$K = \frac{EA}{L}$$

$$L = 1.8$$

$$\cos^2 = \frac{1}{4}$$

$$\cos = \frac{1}{2}$$

$$\sin = \frac{\sqrt{3}}{2}$$

$$\sin \cos = \frac{1}{4}$$

$$\sin^2 = \frac{3}{4}$$

$$\cos = 0$$

$$\sin = 1$$

$$\sin \cos = 0$$

$$K_r = K = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_r = \frac{EA}{L} \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$K_\Sigma = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$K_{11} = K_{21}^{(1)} \quad K_{12} = 0, K_{13} = 0, K_{14} = 0, K_{15} = 0, K_{16} = 0, K_{17} = 0, K_{18} = 0, K_{19} = 0, K_{10} = 0, K_{11} = 0, K_{12} = 0, K_{13} = 0, K_{14} = 0, K_{15} = 0, K_{16} = 0, K_{17} = 0, K_{18} = 0, K_{19} = 0$$

$$K_{21} = K_{12}^{(1)} \quad K_{22} = K_{11}^{(1)} + K_{12}^{(1)} + K_{13}^{(1)} + K_{14}^{(1)} + K_{15}^{(1)} + K_{16}^{(1)} + K_{17}^{(1)} + K_{18}^{(1)} + K_{19}^{(1)}$$

$$K_{31} = K_{13}^{(1)} \quad K_{32} = K_{14}^{(1)} + K_{15}^{(1)} + K_{16}^{(1)} + K_{17}^{(1)} + K_{18}^{(1)} + K_{19}^{(1)}$$

$$K_{41} = K_{14}^{(1)} \quad K_{42} = K_{15}^{(1)} + K_{16}^{(1)} + K_{17}^{(1)} + K_{18}^{(1)} + K_{19}^{(1)}$$

$$K_{51} = K_{15}^{(1)} \quad K_{52} = K_{16}^{(1)} + K_{17}^{(1)} + K_{18}^{(1)} + K_{19}^{(1)}$$

$$K_{61} = K_{16}^{(1)} \quad K_{62} = K_{17}^{(1)} + K_{18}^{(1)} + K_{19}^{(1)}$$

$$K_{71} = K_{17}^{(1)} \quad K_{72} = K_{18}^{(1)} + K_{19}^{(1)}$$

$$K_{81} = K_{18}^{(1)} \quad K_{82} = K_{19}^{(1)}$$

$$K_{91} = K_{19}^{(1)} \quad K_{92} = 0$$

$$K_{101} = 0 \quad K_{102} = 0$$

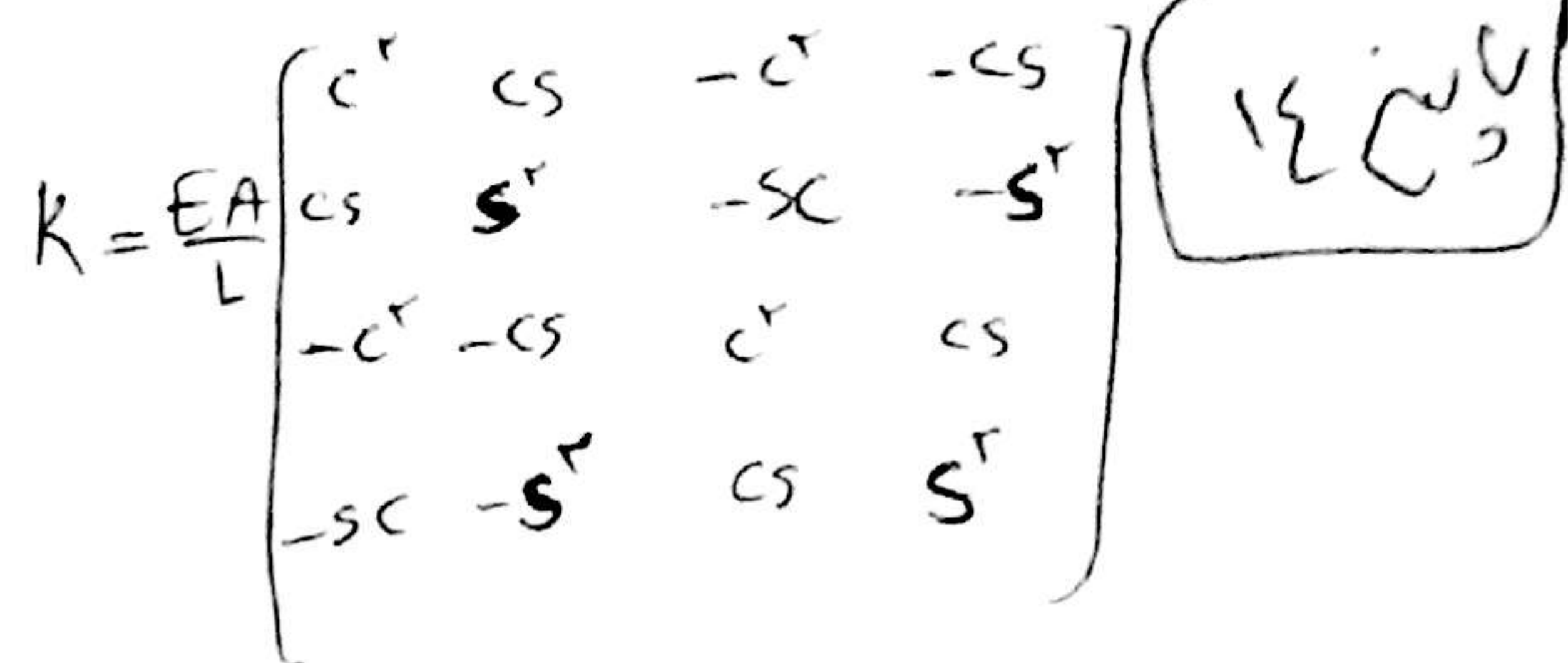
$$K_{111} = 0 \quad K_{112} = 0$$

$$K_{121} = 0 \quad K_{122} = 0$$

$$K_{131} = 0 \quad K_{132} = 0$$

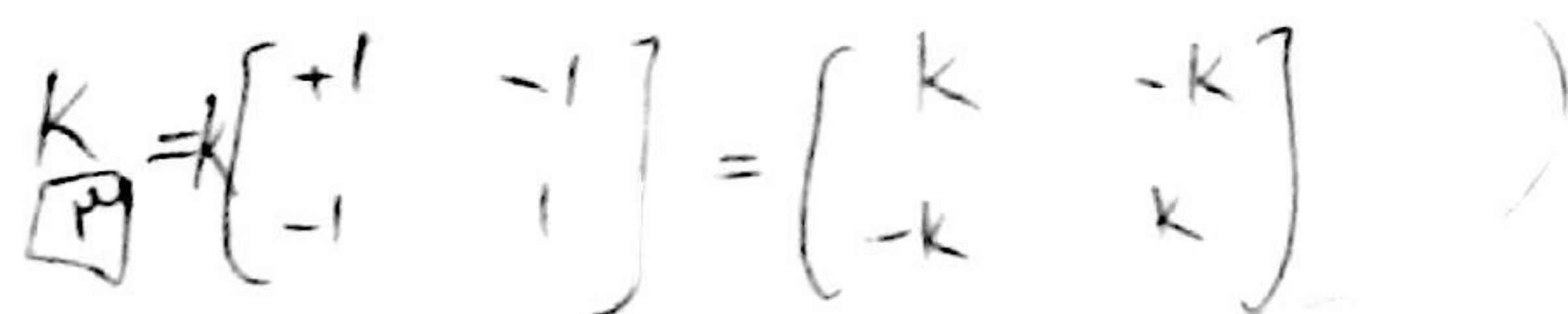
| $\Sigma$ | $\Sigma$ | $\Sigma$ | $\Sigma$ | $\Sigma$ |
|----------|----------|----------|----------|----------|
| 0        | 0        | 1        | 1        | 1        |
| 0        | 0        | 2        | 2        | 2        |
| 1        | 0        | 3        | 3        | 3        |
| 2        | 0        | 4        | 4        | 4        |
| 3        | 1        | 5        | 5        | 5        |
| 4        | 2        | 6        | 6        | 6        |
| 5        | 3        | 7        | 7        | 7        |
| 6        | 4        | 8        | 8        | 8        |
| 7        | 5        | 9        | 9        | 9        |
| 8        | 6        | 10       | 10       | 10       |
| 9        | 7        | 11       | 11       | 11       |
| 10       | 8        | 12       | 12       | 12       |
| 11       | 9        | 13       | 13       | 13       |
| 12       | 10       | 14       | 14       | 14       |
| 13       | 11       | 15       | 15       | 15       |
| 14       | 12       | 16       | 16       | 16       |
| 15       | 13       | 17       | 17       | 17       |
| 16       | 14       | 18       | 18       | 18       |
| 17       | 15       | 19       | 19       | 19       |
| 18       | 16       | 20       | 20       | 20       |
| 19       | 17       | 21       | 21       | 21       |
| 20       | 18       | 22       | 22       | 22       |
| 21       | 19       | 23       | 23       | 23       |
| 22       | 20       | 24       | 24       | 24       |
| 23       | 21       | 25       | 25       | 25       |
| 24       | 22       | 26       | 26       | 26       |
| 25       | 23       | 27       | 27       | 27       |
| 26       | 24       | 28       | 28       | 28       |
| 27       | 25       | 29       | 29       | 29       |
| 28       | 26       | 30       | 30       | 30       |
| 29       | 27       | 31       | 31       | 31       |
| 30       | 28       | 32       | 32       | 32       |
| 31       | 29       | 33       | 33       | 33       |
| 32       | 30       | 34       | 34       | 34       |
| 33       | 31       | 35       | 35       | 35       |
| 34       | 32       | 36       | 36       | 36       |
| 35       | 33       | 37       | 37       | 37       |
| 36       | 34       | 38       | 38       | 38       |
| 37       | 35       | 39       | 39       | 39       |
| 38       | 36       | 40       | 40       | 40       |
| 39       | 37       | 41       | 41       | 41       |
| 40       | 38       | 42       | 42       | 42       |
| 41       | 39       | 43       | 43       | 43       |
| 42       | 40       | 44       | 44       | 44       |
| 43       | 41       | 45       | 45       | 45       |
| 44       | 42       | 46       | 46       | 46       |
| 45       | 43       | 47       | 47       | 47       |
| 46       | 44       | 48       | 48       | 48       |
| 47       | 45       | 49       | 49       | 49       |
| 48       | 46       | 50       | 50       | 50       |
| 49       | 47       | 51       | 51       | 51       |
| 50       | 48       | 52       | 52       | 52       |
| 51       | 49       | 53       | 53       | 53       |
| 52       | 50       | 54       | 54       | 54       |
| 53       | 51       | 55       | 55       | 55       |
| 54       | 52       | 56       | 56       | 56       |
| 55       | 53       | 57       | 57       | 57       |
| 56       | 54       | 58       | 58       | 58       |
| 57       | 55       | 59       | 59       | 59       |
| 58       | 56       | 60       | 60       | 60       |
| 59       | 57       | 61       | 61       | 61       |
| 60       | 58       | 62       | 62       | 62       |
| 61       | 59       | 63       | 63       | 63       |
| 62       | 60       | 64       | 64       | 64       |
| 63       | 61       | 65       | 65       | 65       |
| 64       | 62       | 66       | 66       | 66       |
| 65       | 63       | 67       | 67       | 67       |
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| 68       | 66       | 70       | 70       | 70       |
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| 81       | 79       | 83       | 83       | 83       |
| 82       | 80       | 84       | 84       | 84       |
| 83       | 81       | 85       | 85       | 85       |
| 84       | 82       | 86       | 86       | 86       |
| 85       | 83       | 87       | 87       | 87       |
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| 87       | 85       | 89       | 89       | 89       |
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| 89       | 87       | 91       | 91       | 91       |
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| 91       | 89       | 93       | 93       | 93       |
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| 288      | 286      | 290      | 290      | 290      |
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| 290      | 288      | 292      | 292      | 292      |
| 291      | 289      | 293      | 293</    |          |





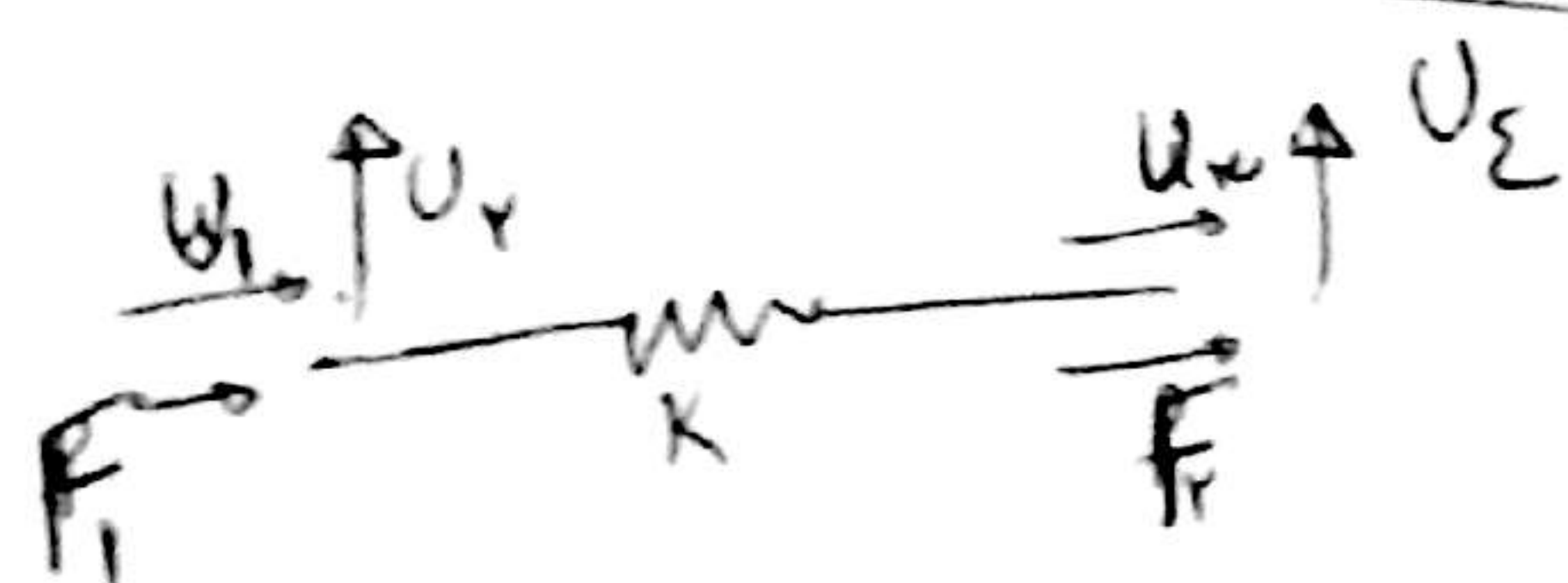
$$K = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K_{\square} = \frac{EA}{\delta} \begin{bmatrix} \frac{14}{r_0} & \frac{14}{r_0} & -\frac{14}{r_0} & -\frac{14}{r_0} \\ \frac{14}{r_0} & \frac{9}{r_0} & -\frac{14}{r_0} & -\frac{9}{r_0} \\ -\frac{14}{r_0} & -\frac{14}{r_0} & \frac{14}{r_0} & \frac{14}{r_0} \\ -\frac{14}{r_0} & -\frac{9}{r_0} & \frac{14}{r_0} & \frac{9}{r_0} \end{bmatrix}$$



| $\square Z$ | $\square Z$ | $\square Z$ | $\square Z$ |
|-------------|-------------|-------------|-------------|
| 0           | 0           | 1           | 1           |
| 0           | 0           | 2           | 2           |
| 0           | 1           | 3           | 3           |
| 1           | 2           | 2           | 2           |
| 0           | 3           | 0           | 0           |
| 0           | 2           | 0           | 0           |
| 0           | 0           | 0           | 0           |

$$\begin{array}{l|l|l} K_{\theta\theta} = K_{rr}^{(r)} & K_{\theta\theta} = K_{\theta\theta}^{(r)} & K_{\theta\theta} = K_{\theta\theta}^{(r)} \\ K_{\theta\gamma} = K_{r\theta}^{(r)} & K_{\theta\gamma} = K_{\theta\gamma}^{(r)} & K_{\theta\theta} = K_{\theta\theta}^{(r)} \\ K_{\theta v} = 0 & K_{\theta v} = 0 & K_{\theta\theta} = K_{\theta\theta}^{(r)} \end{array}$$



$$Ku = f \rightarrow \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} u_1 \\ u_r \end{bmatrix} = \begin{bmatrix} F_1 \\ F_r \end{bmatrix} \rightarrow \begin{aligned} ku_1 - ku_r &= F_1 \\ -ku_1 + ku_r &= F_r \end{aligned}$$

$$K \rightarrow K \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{cases} KV_L - KV_C = F_L \quad \checkmark \\ -KV_L + KV_C = F_C \quad \checkmark \end{cases}$$



$$\textcircled{5} \quad -\frac{EA}{2000} U_{\mu} = F_1 \Rightarrow F_1 = -2391,23 \text{ N}$$

$$F_y = 0$$

$$F_{\mu} = 18000 \cos \delta_0 = 9421,11 \text{ N}$$

$$F_{\xi} = 18000 \sin \delta_0 = 11290,44 \text{ N}$$

$$EA \left( \left( \frac{-14}{20 \times 2000} \right) U_{\mu} + \left( \frac{-12}{20 \times 2000} \right) U_{\xi} \right) = F_0 \Rightarrow F_0 = -8124,11 \text{ N}$$

$$EA \left( \left( \frac{-12}{20 \times 2000} \right) U_{\mu} - \left( \frac{9}{20 \times 2000} \right) U_{\xi} \right) = F_y \Rightarrow F_y = -2840,49 \text{ N}$$

$$F_v = -K U_{\xi} = -90 \text{ N}$$

$$W = \int_0^{\delta} P d\delta = \frac{1}{2} P \delta$$

$$U = \frac{1}{2} \frac{AE}{L} \delta^2$$

$$\textcircled{3} \text{ برای اولی } \delta = \frac{PL}{AE} = \frac{8 \times 10^3 \times 1,8 \times 10^3}{18 \times 18 \times 49 \times 10^3} = 0,1213 \text{ mm} \Rightarrow W = \frac{1}{2} \times 8 \times 10^3 \times 0,1213 \times 10^{-3} = 1,2 \text{ J}$$

$$U = \frac{1 \times 18 \times 18 \times 49 \times 10^3}{2 \times 1,8 \times 10^3} \times (0,1213)^2 = 2,2 \text{ N.m (J)}$$

$$\textcircled{4} \text{ برای دومی } \delta = \frac{3 \times 10^3 \times 1,8 \times 10^3}{18 \times 18 \times 49 \times 10^3} = 0,121 \text{ mm} \Rightarrow W = \frac{1}{2} \times 3 \times 10^3 \times 0,121 \times 10^{-3} = 0,18 \text{ J}$$

$$U = \frac{1}{2} \left( \frac{18 \times 18 \times 49 \times 10^3}{1,8 \times 10^3} \right) \times (0,121)^2 = 0,19 \text{ J}$$

$$\textcircled{5} \text{ برای سومی } \delta = \frac{PL}{AE} \text{ , } P = 8 \times 10^3 \cos \theta + 3 \times 10^3 \sin \theta \xrightarrow{\theta=28^\circ} P = 2,11 \times 10^3 \text{ N}$$

$$\delta = \frac{2,11 \times 10^3 \times 1,8 \sqrt{2} \times 10^3}{18 \times 18 \times 49 \times 10^3} = 0,400 \Rightarrow W = \frac{1}{2} \times 2,11 \times 10^3 \times 0,400 \times 10^{-3} = 1,8 \text{ J}$$

$$U = \frac{1}{2} \left( \frac{18 \times 18 \times 49 \times 10^3}{1,8 \sqrt{2} \times 10^3} \right) \times (0,400)^2 = 1,0 \text{ J}$$



④

$$E = 1.0 \times 10^5 \text{ MPa} \left( \frac{\text{N}}{\text{mm}^2} \right)$$

$$A = \frac{\pi d^2}{4} \text{ (mm}^2\text{)}$$

$$K = 80 \frac{\text{N}}{\text{mm}}$$

$$d = 10 \text{ mm}$$

$$EA \left( \left( \frac{1}{L} + \frac{14}{20 \times 10^3} \right) \times U_1 + \left( \frac{14}{20 \times 10^3} \right) U_2 \right) = 18000 \cos 30^\circ$$

$$EA \left( \left( \frac{14}{20 \times 10^3} \right) U_1 + \left( \frac{1}{L} + \frac{9}{20 \times 10^3} + \frac{K}{EA} \right) U_2 \right) = 18000 \sin 30^\circ$$

$$EA = 20144 \text{ N}$$

$$EA \begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L} + \frac{14}{20 \times 10^3} & \frac{14}{20 \times 10^3} & 0 & 0 \\ 0 & 0 & \frac{14}{20 \times 10^3} & \frac{1}{L} + \frac{9}{20 \times 10^3} + \frac{K}{EA} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{14}{20 \times 10^3} & \frac{14}{20 \times 10^3} \\ 0 & 0 & 0 & 0 & \frac{14}{20 \times 10^3} & -\frac{9}{20 \times 10^3} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \\ U_8 \\ U_9 \\ U_{10} \\ U_{11} \\ U_{12} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ 18000 \cos 30^\circ \\ 18000 \sin 30^\circ \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \\ F_{11} \\ F_{12} \end{bmatrix}$$

$$EA \begin{bmatrix} 20144 & 9144 \\ 9144 & 20144 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 18000 \cos 30^\circ \\ 18000 \sin 30^\circ \end{bmatrix}$$

$$\begin{bmatrix} 9000 & 10911.8 \\ 10911.8 & 9000 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 15000 \\ 11250 \end{bmatrix} \text{ mm} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$



⑤

$$\begin{bmatrix} \delta_0 & -\delta_0 & 0 & 0 \\ -\delta_0 & 1.0 & -\delta_0 & 0 \\ 0 & -\delta_0 & 1.0 & -1.0 \\ 0 & 0 & -1.0 & 1.0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_r \\ U_\mu \\ U_\Sigma \end{bmatrix} = \begin{bmatrix} \Sigma_0 \\ F_r \\ \Sigma_0 \\ \Sigma_0 \end{bmatrix}$$

⑤

$$\delta_0 U_1 - \delta_0 U_r = \Sigma_0 \Rightarrow \delta_0 U_1 = \Sigma_0 + \delta_0 \left( \frac{1}{r} \right) = \Sigma_0 + 1.0 \delta = 0.8 \delta \Rightarrow U_1 = \frac{0.8 \delta}{\delta_0} = 1.1$$

$$-\delta_0 U_r + 1.0 U_\mu - \delta_0 U_\Sigma = \Sigma_0 \Rightarrow 1.0 U_\mu - \delta_0 U_\Sigma = \Sigma_0 + \delta_0 = 0.8 \delta$$

$$-1.0 U_\mu + 1.0 U_\Sigma = \Sigma_0 \Rightarrow -1.0 U_\Sigma = -\Sigma_0 + 1.0 U_\mu$$

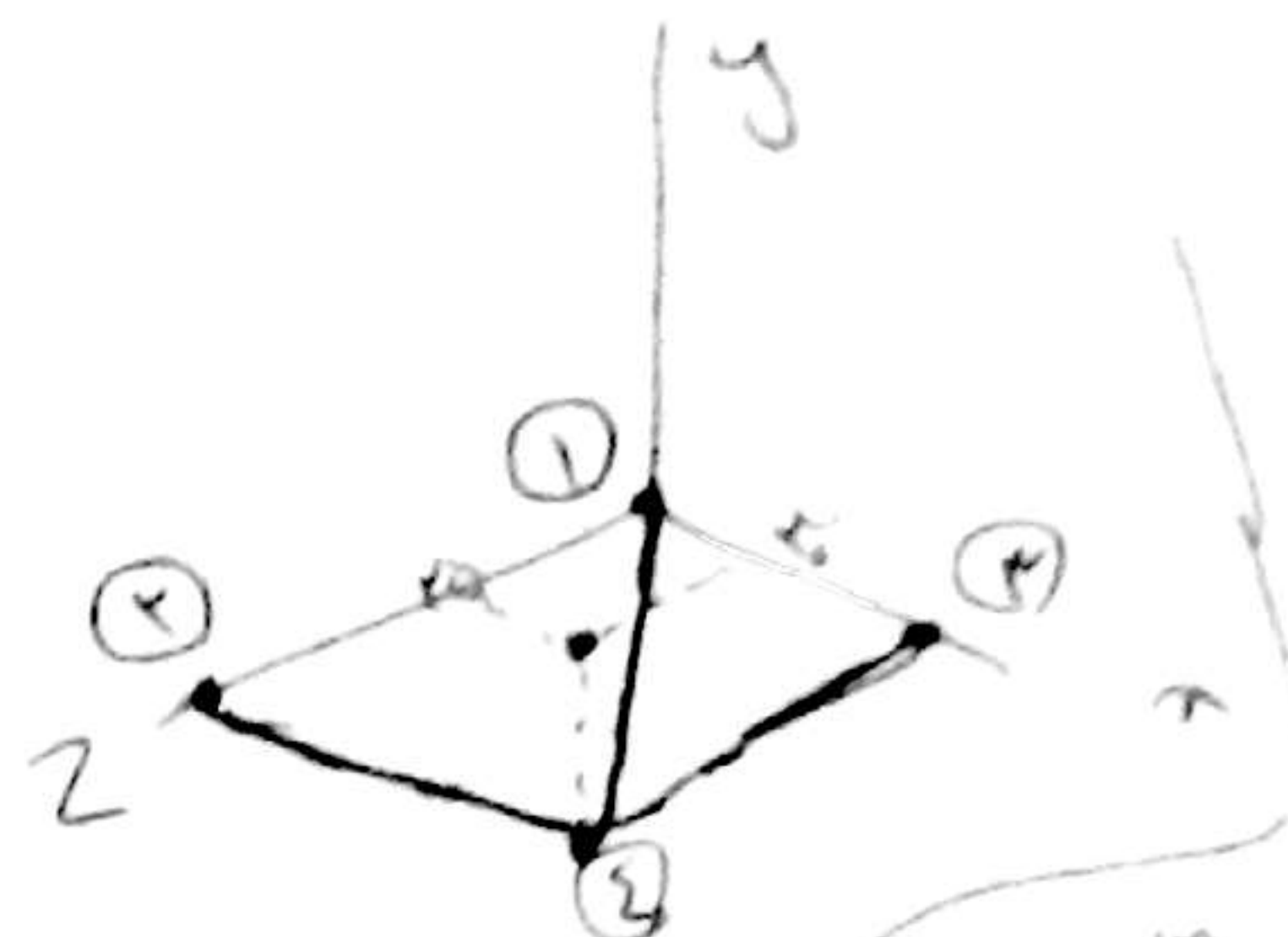
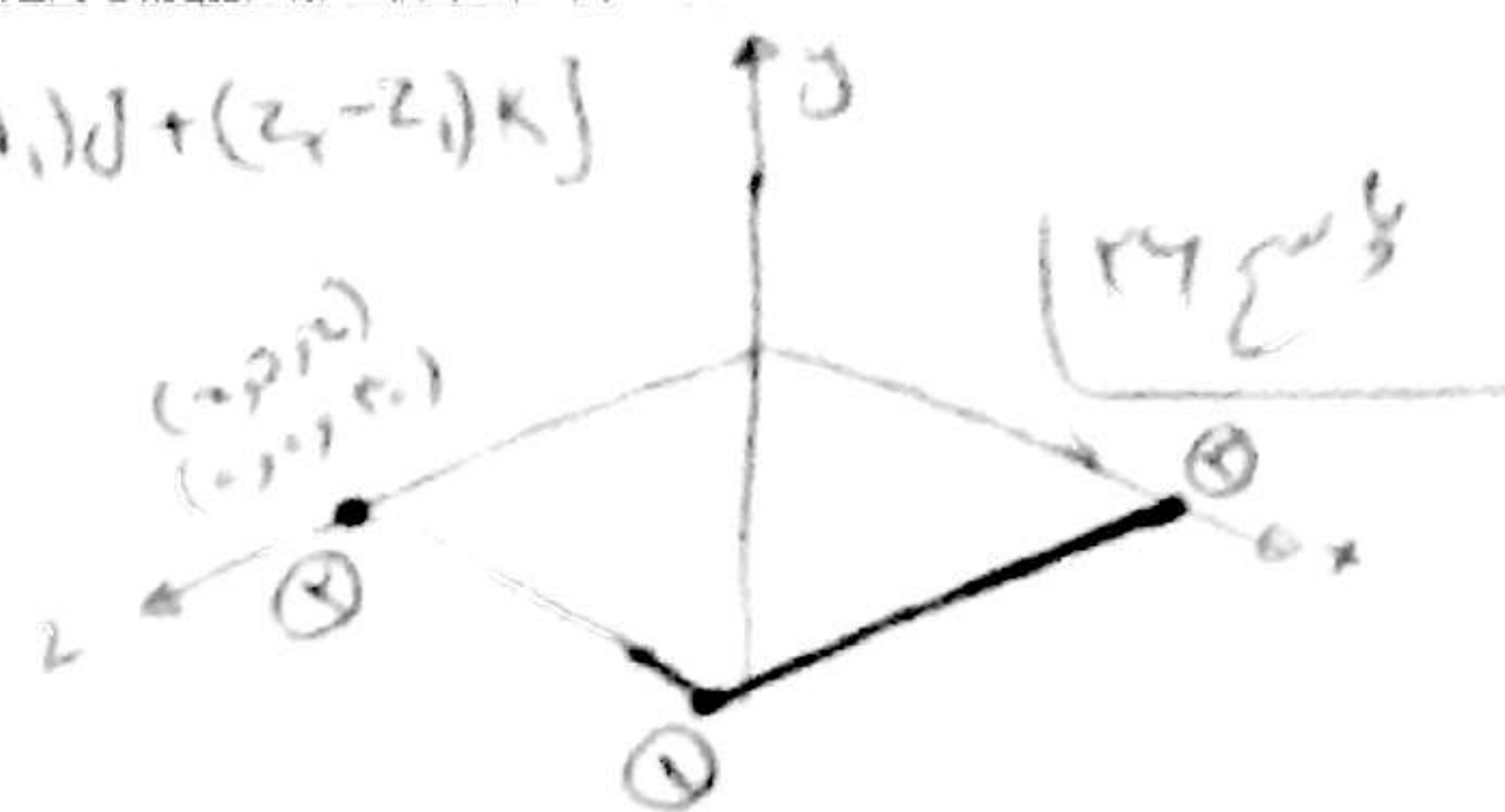
$$1.0 U_\mu - \Sigma_0 + 1.0 U_\mu = 0.8 \delta \Rightarrow 1.0 U_\mu = 0.8 \delta \Rightarrow U_\mu = 1.0 \delta$$

$$1.0 U_\Sigma = \Sigma_0 + 1.0 U_\mu = \Sigma_0 + 1.0 \delta \Rightarrow U_\Sigma = 1.4 \delta$$

$$1.0 U_\Sigma = 0.8 \delta + 1.0 \delta \Rightarrow U_\Sigma = 1.8 \delta$$

$$-\delta_0 U_1 + 1.0 U_r - \delta_0 U_\mu = F_r \Rightarrow F_r = -\delta_0 \times 1.1 + 1.0 \times 0.8 \delta - \delta_0 \times 1.0 = -0.3 \delta$$



$$\lambda = \frac{1}{L} \{ (x_1 - x_1)I + (y_1 - y_1)J + (z_1 - z_1)K \}$$


$$A = 1,8 \text{ m}$$

$$K_{44} = K_{44}^{(1)}, K_{91} = 0, K_{911} = 0$$

$$K_{g14} = 0$$

$$K_{9,12} = 0$$


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$$K_{1,6} = K_{5,2}^{(1)}, K_{1,11} = K_{5,8}^{(1)}$$

$$K_{\text{air}} = K_{\text{ev}}$$

$$K_{III} = K_{00}^{(r)}, K_{IIIr} = K_{01}^{(r)}$$

$$K_{11} = K_{44}$$

$$L_Q = \sqrt{10^2 + 20^2 + 30^2} = 44.72 \Omega$$

| $\Sigma U d Z$ | $\Sigma U d Z$ | $\Sigma U d Z$ | $\Sigma U d Z$ |
|----------------|----------------|----------------|----------------|
| 0              | 0              | 1              | 1              |
| 0              | 0              | 2              | 2              |
| 0              | 0              | 3              | 3              |
| 1              | 1              | Σ              | Σ              |
| 2              | 2              | 0              | 0              |
| 3              | 3              | 4              | 4              |
|                |                |                | √              |
| 4              | Σ              | 0              | 1              |
|                | 0              | 0              | 9              |
| 0              | 9              | 0              | 1              |
| 0              | 0              | 0              | 11             |
| Σ              | 0              | 0              | 14             |
| 0              | 0              | 0              |                |
| 9              | 0              |                |                |

$$K_{\Lambda\Lambda} = K_{\Lambda\Lambda}^{(4)}$$

$$K_{\Lambda 9} = K_{07}^{(Y)}$$

$$K_{Na} = 0$$

$$K_{\text{ли}} = 0$$

$$K_{AlY} = 0$$

K. V. (1)

$$N_{\text{eff}} = N_{\text{eff}}$$

$$K_{10} = K_{20}$$

$$K_{29} = K_{29}^{(1)}$$

$$K_{WV} = K_{WV}^{(1)}$$

$$k_{eq} = k_{eq}^{(1)}$$

$$K_{eq} = K_{eq}^{(1)}$$

$$K_{ss} = K_{ss}^{(1)} + K_{ss}^{(2)} + K_{ss}^{(3)}$$

$$K_{\text{eq}} = K_{\text{eq}}^{(1)} + K_{\text{eq}}^{(r)} + K_{\text{eq}}^{(r)}$$

$$K_{29} = K_{29}^{(1)} + K_{1r}^{(1)} + K_{1r}^{(n)}$$

$$K_{12} = K_{21} \quad K_{01} = K_{10}$$

$$K_{11} = K_{10} \quad K_{09} = K_{14}$$

$$K_{210} = K_{11} \quad K_{211} = K_{11}$$

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$$K_{10} = K_{10}^{(n)}, K_{11} = K_{11}^{(n)}$$

$$K_{00} = K_{00}^{(1)} + K_{rr}^{(r)} + K_{rr}^{(a)}$$

$$K_{OY} = K_{OY}^{(1)} + K_{Yr}^{(r)} + K_{rY}^{(r)}$$

$$K_{Yr} = K^{(r)} = K = K$$

$$K_{yy} = K_{yy}^{(1)} + K_{yy}^{(2)} + K_{yy}^{(3)}$$

$$K_{4V} = K_{r2}^{(1)}, K_{4A} = K_{r0}^{(1)}, K_{4B} = K_{r1}^{(1)}$$

$$K_{y1} = K_{yE} \text{ and } K_{y11} = K_{y0} \quad K_v$$

$$K_{V9} = K_{V1} \cdot 9K_{V10} = 0.9K_{V11} =$$

1

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(r)

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$$y_9 = K_{xy}^{(1)}$$

$$v = K \frac{(r)}{r^2}$$

$\log K_{VII} = 0$



$$9) \lambda_1 = \frac{1}{L_1} [(x_r - x_1)I + (y_r - y_1)J + (z_r - z_1)K]$$

$$① (x_0, -x_0, x_0)$$

$$② (0, 0, 0)$$

$$1) \cos \theta_x = \frac{x_r - x_1}{L_1} = \frac{x_0}{2.114V} = 0.41$$

$$\cos \theta_y = \frac{y_r - y_1}{L_1} = \frac{-x_0}{2.114V} = -0.20$$

$$\cos \theta_z = \frac{z_r - z_1}{L_1} = \frac{x_0}{2.114V} = 0.10V$$

$$2) \begin{cases} \cos \theta_x = \frac{x_0 - x_0}{2.114V} = \frac{1.0}{2.114V} = 0.47 \rightarrow x^r = 0.9 \\ \cos \theta_y = \frac{y_0}{2.114V} = 0.17 \rightarrow y^r = 0.37 \\ \cos \theta_z = \frac{-x_0}{2.114V} = -0.47 \rightarrow z^r = 0.10 \end{cases}$$

$$② (x_0, 0, 0)$$

$$① (x_0, -x_0, x_0)$$

$$② \begin{cases} x_y = 0.118 \\ x_z = 0.122 \\ y_z = 0.119 \end{cases}$$

$$3) \begin{cases} \cos \theta_x = \frac{-x_0}{2.114V} = 0.47 \rightarrow x^r = 0.9 \\ \cos \theta_y = \frac{x_0}{2.114V} = 0.47 \rightarrow y^r = 0.9 \\ \cos \theta_z = \frac{0}{2.114V} = 0.17 \rightarrow z^r = 0.1 \end{cases} \rightarrow \begin{cases} x_y = 0.48 \\ x_z = 0.1 \\ y_z = 0.10V \end{cases}$$

$$② (0, 0, x_0) \quad ① (x_0, -x_0, x_0)$$

$$K = \frac{AE}{L} \begin{bmatrix} c_x^r & c_x c_y & c_x c_z & -c_x^r & -c_x c_y & c_x c_z \\ c_x c_y & c_y^r & c_y c_z & -c_x c_y & -c_y^r & -c_y c_z \\ c_x c_z & c_y c_z & c_z^r & -c_x c_z & -c_y c_z & -c_z^r \\ -c_x^r & -c_x c_y & -c_x c_z & c_x^r & c_x c_y & c_x c_z \\ -c_x c_y & -c_y^r & -c_y c_z & c_x c_y & c_y^r & c_y c_z \\ -c_x c_z & -c_y c_z & -c_z^r & c_x c_z & c_y c_z & c_z^r \end{bmatrix}$$

$$K_{II} = \frac{EA}{L} \begin{bmatrix} 0.47 & 0.17 & 0.118 & -0.48 & -0.17 & 0.118 \\ 0.17 & 0.17 & 0.120 & -0.48 & -0.17 & 0.120 \\ 0.118 & 0.120 & 0.122 & 0.118 & 0.120 & 0.122 \\ -0.48 & -0.48 & 0.118 & 0.48 & 0.17 & 0.118 \\ 0.17 & 0.17 & 0.120 & 0.17 & 0.17 & 0.120 \\ 0.118 & 0.120 & 0.122 & 0.118 & 0.120 & 0.122 \end{bmatrix}$$



(10)

$$K_{\square} = \frac{EA}{L} \begin{bmatrix} 0.09 & 0.18 & 0.22 & -0.09 & -0.18 & -0.22 \\ 0.18 & 0.34 & 0.19 & -0.18 & -0.34 & -0.19 \\ 0.22 & 0.19 & 0.52 & -0.22 & -0.19 & -0.52 \\ -0.09 & -0.18 & -0.22 & 0.09 & 0.18 & 0.22 \\ -0.18 & -0.34 & -0.19 & 0.18 & 0.34 & 0.19 \\ -0.22 & -0.19 & -0.52 & 0.22 & 0.19 & 0.52 \end{bmatrix}$$

$$K_{\square} = \frac{EA}{L} \begin{bmatrix} 0.47 & 0.20 & 0.1 & -0.47 & -0.20 & -0.1 \\ 0.20 & 0.1 & 0.07 & -0.20 & -0.1 & -0.07 \\ 0.1 & 0.07 & 0.01 & -0.1 & -0.07 & -0.01 \\ -0.47 & -0.20 & -0.1 & 0.47 & 0.20 & 0.1 \\ -0.20 & -0.1 & -0.07 & 0.20 & 0.1 & 0.07 \\ -0.1 & -0.07 & -0.01 & 0.1 & 0.07 & 0.01 \end{bmatrix}$$

$$\frac{EA}{L} \begin{bmatrix} \frac{0.24}{L_1} & \frac{0.2}{L_1} & \frac{0.21}{L_1} & -\frac{0.24}{L_1} & -\frac{0.2}{L_1} & -\frac{0.21}{L_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{0.2}{L_1} & \frac{0.20}{L_1} & -\frac{0.24}{L_1} & -\frac{0.2}{L_1} & -\frac{0.20}{L_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{0.22}{L_1} & -\frac{0.21}{L_1} & -\frac{0.20}{L_1} & -\frac{0.22}{L_1} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{EA}{L}$

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①



(U<sub>1</sub>, U<sub>0</sub>, U<sub>2</sub>)  $\Sigma$  20, 5, 6, 6

$$\begin{bmatrix} 0.1021 & 0.1022 & 0.1023 \\ 0.1022 & 0.1023 & 0.1024 \\ 0.1023 & -2.11 \times 10^{-2} & 0.1024 \end{bmatrix} \begin{bmatrix} U_1 \\ U_0 \\ U_2 \end{bmatrix} = \begin{bmatrix} F_1=0 \\ F_2=F_3 \\ F_4=0 \end{bmatrix}$$

$$[A]^{-1}[F]=[U]$$

$$A_{ij} = (-1)^{i+j} |M_{ji}|$$

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \rightarrow A_{ij} = (-1)^{i+j} \begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

$$\Rightarrow A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \rightarrow A^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$M^{-1} = \frac{1}{|M|} A^T$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 0.1023 & 0.1024 \\ -2.11 \times 10^{-2} & 0.1024 \end{vmatrix} = 4.142 \times 10^{-2}$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 0.1022 & 0.1023 \\ 0.1023 & 0.1024 \end{vmatrix} = -1.9 \times 10^{-2}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 0.1022 & 0.1023 \\ 0.1023 & 0.1024 \end{vmatrix} = -1.9 \times 10^{-2}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 0.1021 & 0.1023 \\ 0.1023 & 0.1024 \end{vmatrix} = 2.9 \times 10^{-2}$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 0.1021 & 0.1023 \\ 0.1023 & 0.1024 \end{vmatrix} = 1.9 \times 10^{-2}$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 0.1021 & 0.1023 \\ 0.1023 & 0.1024 \end{vmatrix} = -1.9 \times 10^{-2}$$

$$|A| = -8.1234 \times 10^{-4}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 0.1021 & 0.1022 \\ -2.11 \times 10^{-2} & 0.1024 \end{vmatrix} = 4.94 \times 10^{-2}$$

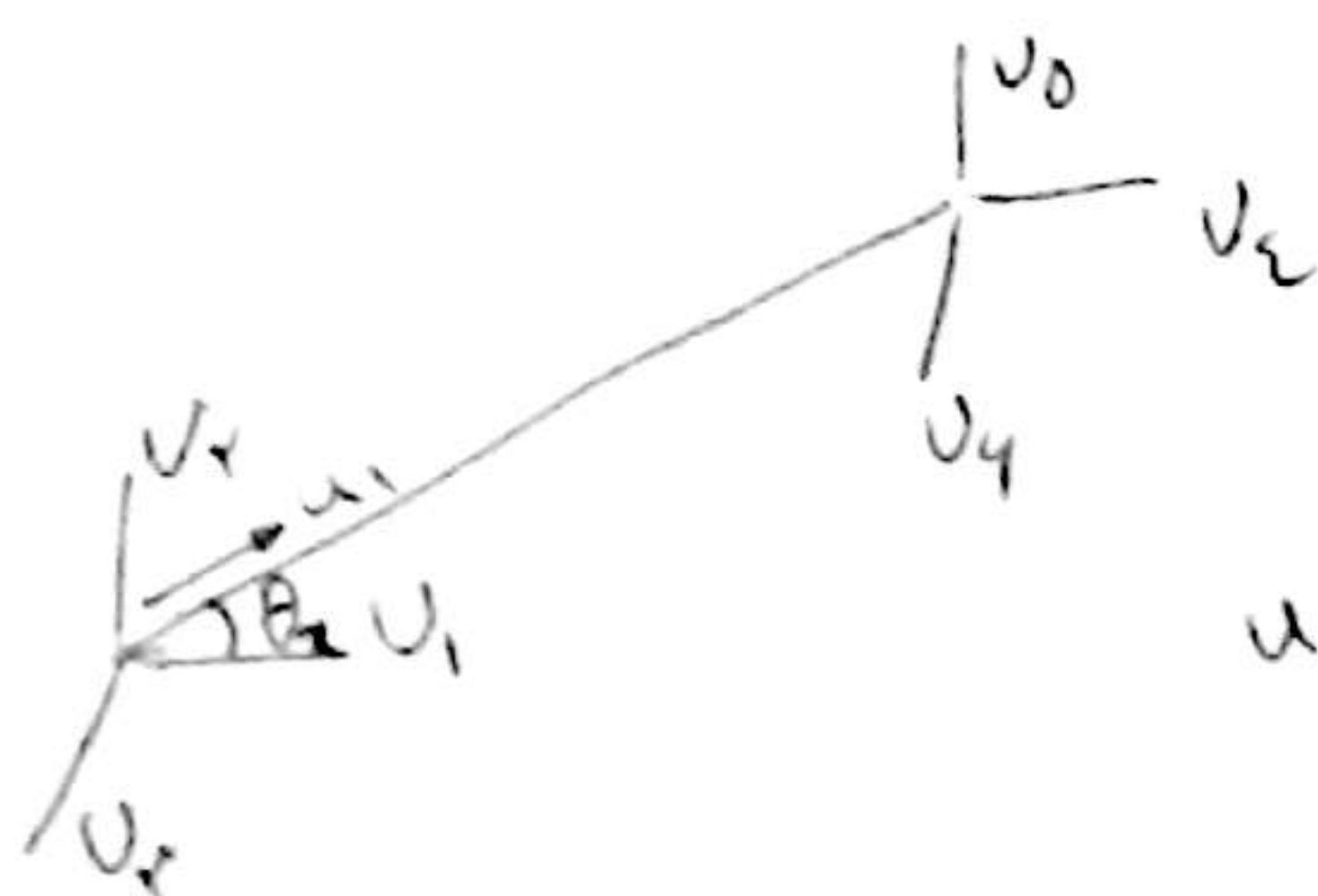
$$A_{23} = (-1)^{2+3} \begin{vmatrix} 0.1021 & 0.1022 \\ 0.1023 & 0.1024 \end{vmatrix} = 2.9 \times 10^{-2}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 0.1021 & 0.1022 \\ 0.1023 & -2.11 \times 10^{-2} \end{vmatrix} = +1.11 \times 10^{-2}$$

$$A^{-1} = \frac{1}{-8.1234 \times 10^{-4}} \begin{bmatrix} 4.142 & -4.94 & -1.9 \\ -1.9 & 2.9 & 1.9 \\ -1.9 & 1.11 & 1.9 \end{bmatrix}$$



$$\begin{bmatrix} -5,10 \times 10^{-4} & 1,10 \times 10^{-4} & 3,10 \times 10^{-4} \\ 3,10 \times 10^{-4} & -9,10 \times 10^{-4} & -1,10 \times 10^{-4} \\ 1,10 \times 10^{-4} & -1,10 \times 10^{-4} & -1,10 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 0 \\ -100 \\ 0 \end{bmatrix} = \begin{bmatrix} -0,1012 \text{ V} \\ 1,1019 \times 10^{-4} \\ 0,1015 \text{ V} \end{bmatrix} = \begin{bmatrix} U_F \\ U_g \\ U_y \end{bmatrix}$$



$$U_1 = U_x \cos \theta_x + U_y \cos \theta_y + U_z \cos \theta_z$$

$$\sigma_i = E_i \left[ \frac{1}{L_1} \quad \frac{1}{L_2} \right] \begin{bmatrix} \cos \theta_x & \cos \theta_y & \cos \theta_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \theta_x & \cos \theta_y & \cos \theta_z \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = [ \quad ]_{1 \times 1}$$

$$\sigma_i = 10^4 \left[ \frac{1}{2 \times 10^{-4}} \quad \frac{1}{2 \times 10^{-4}} \right] \begin{bmatrix} 0,10 & -0,10 & 0,10 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0,10 & -0,10 & 0,10 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix}$$

$$\sigma_i = 10^4 \left[ \frac{1}{2 \times 10^{-4}} \quad \frac{1}{2 \times 10^{-4}} \right] \begin{bmatrix} 0 \\ -1,10 \times 10^{-4} \end{bmatrix} = \begin{bmatrix} -1,10 \times 10^4 \end{bmatrix}$$

$$\sigma_r = 10^4 \left[ \frac{1}{2 \times 10^{-4}} \quad \frac{1}{2 \times 10^{-4}} \right] \begin{bmatrix} 0,10 & 0,10 & -0,10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,10 & 0,10 & -0,10 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix}$$

$$\sigma_r = 10^4 \left[ \frac{1}{2 \times 10^{-4}} \quad \frac{1}{2 \times 10^{-4}} \right] \begin{bmatrix} -0,10 \\ 0 \end{bmatrix} = -1,10 \times 10^4$$

$$\sigma_r = 10^4 \left[ \frac{1}{2 \times 10^{-4}} \quad \frac{1}{2 \times 10^{-4}} \right] \begin{bmatrix} 0,10 & 0,10 & -0,10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,10 & 0,10 & -0,10 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = 10^4 \left[ \frac{1}{2 \times 10^{-4}} \quad \frac{1}{2 \times 10^{-4}} \right] \begin{bmatrix} -0,10 \times 10^{-4} \\ 0 \end{bmatrix} = -1,10 \times 10^4$$